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Math189R SU20.1
Homework 3
Thursday, June 18, 2019

1 (**Murphy 2.16**) Suppose $\theta \sim \text{Beta}(a, b)$ such that

$$\mathbb{P}(\theta; a, b) = \frac{1}{B(a,b)}\theta^{a-1}(1-\theta)^{b-1} = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\theta^{a-1}(1-\theta)^{b-1}$$

where $B(a,b) = \Gamma(a)\Gamma(b)/\Gamma(a+b)$ is the Beta function and $\Gamma(x)$ is the Gamma function. Derive the mean, mode, and variance of θ.

Mean

$$\begin{aligned}
E(x) &= \int \theta \mathbb{P}(\theta; a, b) d\theta \\
&= \frac{1}{B(a,b)} \int \theta\theta^{a-1}(1-\theta)^{b-1} d\theta \\
&= \frac{1}{B(a,b)} \int \theta^{a}(1-\theta)^{b-1} d\theta \\
&= \frac{1}{B(a,b)} \int (1-\theta)^{b-1}\theta^{a} d\theta \\
\text{(by parts)} &= \frac{1}{B(a,b)}[(1-\theta)^{b-1}\frac{\theta^{a+1}}{a+1}) + \frac{b-1}{a+1}\int \theta^{a+1}(1-\theta)^{b-2} d\theta]
\end{aligned} \tag{1}$$

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2 (**Murphy 9**) Show that the multinoulli distribution

$$\mathrm{Cat}(\mathbf{x}|\boldsymbol{\mu}) = \prod_{i=1}^{K} \mu_i^{x_i}$$

is in the exponential family and show that the generalized linear model corresponding to this distribution is the same as multinoulli logistic regression (softmax regression).

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