CK-12

Geometry Teacher's Edition
(Being Reviewed)

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CHAPTER 1

Geometry TE - Teaching Tips

Chapter Outline

1.1 Basics of Geometry

Points, Lines, and Planes

Pacing: This lesson should take approximately three class periods.

Goal: This lesson introduces students to the basic principles of geometry. Students will become familiar with three primary undefined geometric terms and how these terms are used to define other geometric vocabulary. Finally, students are introduced to the concept of dimensions.

Study Skills Tips! Start your students off on the correct foot – vocabulary is a necessity in geometry success! Devote five minutes of each class period to creating flash cards of the major terminology of this text. Use personal whiteboards to perform quick vocabulary checks. Or, better yet, visit Discovery School's puzzle maker and make your own word searches and crosswords (http://puzzlemaker.discoveryeducation.com/)!

Language Arts Connection! To give an example of why some words are undefined, use the concept of circularity. Students use a dictionary, either electronic or paper (yes, they are still printed!) to complete this activity. Ask students to look up the word *point* in their reference. Find a key word in that definition. Students should continue this process until the word *point* is found. Repeat this process for *line* and *plane.* The rationale behind this activity is for students to see there is no one way to define these geometric terms, thus allowing them to be undefined but recognizable.

Real World Connection! Have students identify real-life examples of points, lines, planes in the classroom, as well as sets of collinear and coplanar. For example, points could be chairs, lines could be the intersection of the ceiling and wall, and the floor is a great model of a plane. If your chairs are four-legged, this is a fantastic example of why 3 points determine a plane, not four. Four legged chairs tend to wobble, while 3−legged stools remain stable.

To help students understand dimension, use the following table:

TABLE 1.1:

Zero-dimensional	1-dimensional (length)	2-dimensional (length and width)	3-dimensional (length, width, and height)

Have students write abstract examples of each dimension (point, line, plane, prism, etc) in the first row. Then have students brainstorm real-life examples of each dimension. Complete the table by gathering the responses of various students.

Segments and Distances

Pacing: This lesson should take one class period

Goal: Students should be familiar with using rulers to measure distances. This lesson incorporates geometric postulates and properties to measurement, such as the Segment Addition Property.

Real World Connection! To review the concept of measurement, use a map of your community. Label several things on your map important to students – high school, grocery store, movie theatre, etc. Have students practice finding

the distances between landmarks "as the crow flies."

Extension! Discuss with your students the rationale of using different units of distance – inch, foot, centimeter, mile, etc. Why are things measured in inches as opposed to fractional feet? This is also a great time to introduce the difference between the metric system and the U.S. measurement system. Have students perform research regarding why the United States continues to use its system while the majority of other countries use the metric system. Provide pros and cons to using each type of system.

Fun tip! Have students devise their own measurement device. Students can use their invention to measure a school hallway, parking lot, or football field. Engage in a whole-class discussion regarding the results.

Refresher! Students may need a refresher regarding multiplying units. Have the students write out the complete unit, as on page 18, and show students how units can be cross-canceled.

Look out! While the Segment Addition Property seems simple, students begin to struggle once proofs come into play. Remind students that the Segment Addition Property allows an individual to combine smaller measurements of a line segment into its whole.

Rays and Angles

Pacing: This lesson should take one class period

Goal: This lesson introduces students to rays and angles and how to use a protractor to measure angles. Several real world models are used to illustrate the concepts of angles.

Real World Connection! Have students Think-Pair-Share their answers to the opening question, "Can you think of other real-life examples of rays?" Choose several groups to share with the class.

Notation Tip! Beginning geometry students may get confused regarding the ray notation. Draw rays in different directions so students become comfortable with the concept that ray notation always points to the right, regardless of the drawn ray's orientation.

Teaching Strategy! Using a classroom sized protractor will allow students to check to make sure their calculations are the same as yours. Better yet, use an overhead projector or digital imager to demonstrate the proper way to use a protractor.

Teaching Strategy! A good habit for students is to name an angle using of all three letters. This becomes important when labeling vertices of triangles and labeling similar and congruent figures using the similarity statement. Furthermore, stress to students the use of double and triple arcs to denote angles of different measurements. Students can get caught up in the mass amounts of notation and forget this important concept, especially during triangle congruency.

Stress the parallelism between the Segment Addition Property and Angle Addition Property. Students will discover that many geometrical theorems and properties are quite similar, with perhaps one words changed. Yet, the meaning remains the same.

Arts and Crafts Time! Have students take a piece of paper and fold it at any angle of their choosing from the corner of the paper. Open the fold and refold the paper at a different angle, forming two "rays" and three angles. Show how the angle addition property can be used by asking students to measure their created angles and finding the sum – they should equal 90 degrees!

Physical Models! The angle formed at a person's elbow is a useful physical model of angles. Have the students put their arm straight out, illustrating a straight angle. Then have the student gradually turn their arm up (or down) gradually to demonstrate how the degree changes. Use several students as examples to show that the length of the forearm and bicep do not change the angle measurement.

Segments and Angles

Pacing: This lesson should take one to one and one-half class periods

Goal: The lesson introduces students to the concept of congruency and bisectors. Students will use algebra to write equivalence statements and solve for unknown variables.

Have Fun! Have students do a call-back, similar to what cheerleaders do. You call out "AB" and students would retort, "The distance between!" Continue this for several examples so students begin to see the difference between the distance notation and segment notation.

This is a great lesson for students to create a "dictionary" of all the notation and definitions learned thus far. In addition to the flashcards students are making, the dictionary provides an invaluable reference before assessments.

When teaching the Midpoint Postulate, reiterate to students that this really is the arithmetic average of the endpoints, incorporating algebra and statistics into the lesson.

Visualization! Students have not learned about a perpendicular bisector. Have students complete Example 3 without using their texts as guides. Have students show their bisectors. Hopefully your class will construct multiple bisectors, not simply those that are perpendicular. This helps students visualize that there are an infinite amount of bisectors, but only one that is perpendicular.

Fun Tip! To visualize the angle congruence theorem and provide a means of assessing the ability to use a protractor, give students entering your class an angle measure on a slip of paper (the measurements should repeat). Have the students construct the angle as a warm up. Then have the students find their "matching" partner and check their partner's angle using a protractor.

Physical Models! Once students have reviewed Example 5, have them copy the angle onto a sheet of notebook paper or patty paper and measure the degree of the bisector. Students will construct a fold at that particular angle measurement to see the angle bisector ray.

Real Life Application! Another method of illustrating angle bisectors is to show a compass rose, as shown below.

http://commons.wikimedia.org/wiki/File:Compass_rose_browns_00.svg

Students can see how directions such as SWS, NNW, etc bisect the traditional four-corner directions.

Angle Pairs

Pacing: This lesson should take one to one and one-half class periods

1.1. Basics of Geometry

Goal: Angle pairs are imperative to geometry. This lesson introduces students to common angle pairs.

Inquiry Learning! Students should be encouraged to learn through self-discovery whenever possible. To illustrate the concept of the Linear Pair Postulate, offer several examples of linear pairs. Have students measure each angle and find the sum of the linear pair. Students should discover any linear pair of angles is supplementary.

To further illustrate the idea of vertical angles, extend the adjacent ray of the previous linear pairs to a line. Have students repeat the process of measuring the angles, noting the linear pairs. Students will come to the conclusion that the angles opposite in the `[U+0080][U+009C]`X`[U+0080][U+009D]` are equal.

Students tend to get confused with the term *vertical,* as in vertical angles. Vertical angles are named because the angles share a vertex, not necessarily because they are in a vertical manner.

Interdisciplinary Connection! NASA has developed many lesson plans that infuse science, technology, and mathematics. The following link will take you to a lesson plan incorporating the seasons and vertical angles. http://sunea rthday.nasa.gov/2005/educators/AOTK_lessons.pdf

Classifying Triangles

Pacing: This lesson should take one class period

Goal: Students have previously experienced triangle terminology: scalene, equilateral, isosceles. This lesson incorporates these terms with other defining characteristics.

In Class Activity: Give pairs of students three raw pieces of spaghetti (you can also use non-bendy straws). Instruct one partner to recreate the below table while the second makes two breaks in the spaghetti. It is okay if some breaks away!

The students are to measure the three pieces formed by the two breaks and attempt to construct a triangle using these segments. Students will reach the conclusion that the sum of two segments must always be larger than the third if a triangle is to be formed. *The Triangle Inequality Theorem can be found in the lesson entitled Inequalities in Triangles*

TABLE 1.2:

Segment 1 Length (in cm)	Segment 2 Length (in cm)	Segment 3 Length (in cm)	Can a triangle be formed (Yes/No)

Showing students the difference between line segments and curves, introduce cooked spaghetti. The flexibility of the spaghetti demonstrates to students that segments must be straight in order to provide rigidity and follow the definitions of polygons.

Students can express the concepts presented in this lesson using a Venn diagram or a hierarchy. If students are not familiar with a hierarchy, remind students a hierarchy is an ordering of related objects from the most general to the most specific. An example is shown below.
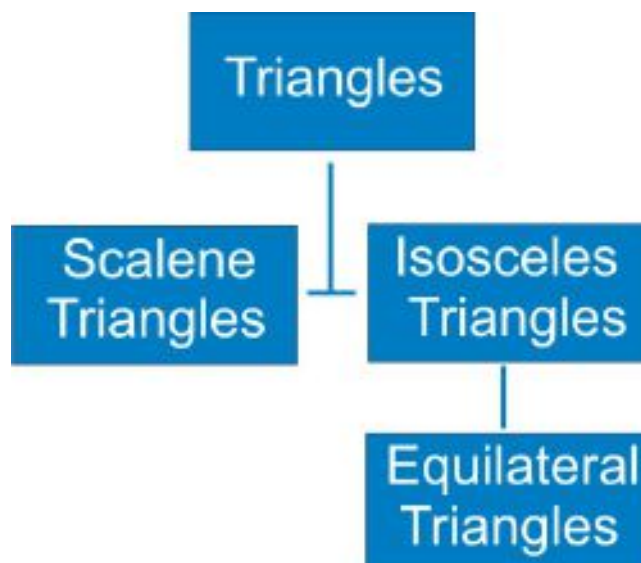
Classifying Polygons

Pacing: This lesson should take one class period

Goal: This lesson explains the characteristics of a polygon. Students should be able to classify polygons according to its number of sides and whether it's convex or concave.

Language Arts Connection! Have students find several high school textbooks and internet sites that provide a definition of polygon. Compare each definition for similarities and differences. Devise a workable classroom definition of a polygon, using the ones found as guides.

Study Skills Tip! Flashcards are imperative to geometry success! Students should construct flashcards of the important polygons. One side should be the drawing of the polygon with the reverse naming the polygon, listing the number of sides, and if applicable the sum of the interior angles in a polygon or how to separate the polygon into triangles (useful when determining polygonal area).

We love Pythagoras! When teaching the distance formula, relate this to Pythagorean's Theorem. The vertical distance (change in y) represents one leg of a right triangle and the horizontal distance (change in x) represents the other leg. Using $a^2 + b^2 = c^2$, students can derive the distance formula. Many students will use Pythagorean's Theorem as opposed to the distance formula when determining the length between two points.

What Did You Learn? Use this activity as a culminating activity or perhaps an alternative assessment. Devise a geometry scavenger hunt, listing the major concepts learned thus far. Equip students with a digital camera and their imagination. Ask students to find as many objects as possible, capture them with a photograph and incorporate the photos into a movie or slideshow presentation. Offer bonus points for such things as originality, nature made objects, etc.

Problem Solving in Geometry

Pacing: While this concept should permeate throughout this course, this particular lesson should take one class period.

Goal: Problem solving is necessary is daily life. Drawing diagrams, working backwards, checking multiple options, and answering reasoning questions are imperative for students to learn. This lesson introduces the key questions one

should ask when problem solving and some strategies students can use.

Problem solving is essential in daily life. Encouraging students to reflect upon the strategies offered in this lesson and incorporating word problems into your daily routine will help students become successful problem solvers. Allow students to struggle through these problems, facilitating their knowledge rather than providing direct instruction.

Provide a chart for students to use as reference that ask the five essential questions:

- What is the problem asking for?
- What do I have that could be used to answer the question?
- What do I need to know to find the answer?
- Did I provide the information the problem requested?
- Does my answer make sense?

By having students begin their problem solving using a chart outlining these questions, students will begin to see when they have answered the problem completely. This helps students later in the textbook by providing a method to answer the age-old question, "When is my proof complete?"

Gather multiple story problems that require various forms of problem solving techniques. Use personal whiteboards to do an immediate check regarding students' progress.

1.2 Reasoning and Proof

Inductive Reasoning

Pacing: This lesson should take one class period

Goal: This lesson introduces students to inductive reasoning. Inductive reasoning applies easily to algebraic patterns, integrating algebra with geometry.

A great way to start this lesson is to further expand upon inductive reasoning. Inductive reasoning uses patterns to make generalizations. Simply put, inductive reasoning takes repeated specific examples and extends it to a general conjecture.

Begin by writing an arithmetic or a geometric sequence such as $1, 4, 7, 10\ldots$ or $40, 20, 10, 5, \ldots$ Ask students to recognize the pattern and write the generalization in words (this also lends itself to exposure to sequences and series, a topic usually found in Advanced Algebra).

Challenge: Offer students this type of pattern: $14, 10, 15, 11, 16, 12, 17\ldots$ The pattern here is to subtract four then add five.

Take the opportunity to further discuss the triangular numbers, as seen on page 76. The triangular numbers are formed such that the dots form a triangle and also a numerical pattern of $s(n) = \frac{1}{2}n^2 - n$.

In examples 1 and 2 on page 76, relate the even and odd numbers to a symbolic pattern. For example, even numbers can be represented by the expression $2n$, while all odd numbers can be represented by $2n + 1$.

Real Life Connection! Apply the idea of counterexample to real life situations. Begin by devising a statement, such as, "If the sun is shining, then you can wear shorts." While this is true for warm weather states such as Florida and California, for those living in the Midwest or Northern states, it is quite common to be sunny and 12 degrees! Have students create their own statements and encourage other students to find counterexamples.

Conditional Statements

Pacing: This lesson should take two class periods

Goal: This lesson introduces the all-important conditional statements. Students will gain an understanding of how converses, inverses, and contrapositives are formed from a conditional and further explore truth values of each of these statements.

The first portion of this lesson may be best taught using direct instruction and several visual aids. Design phrases you can laminate, such as "you are sixteen" and "you can drive." Adhere magnets to the back of the phrases (to stick to the white board), or you can use a SMART board. Begin by writing the words "IF" and "THEN," giving ample space to place your phrases. When discussing each type of conditional, show students how each is constructed by rearranging your phrases, yet leaving the words "IF" and "THEN" intact.

Have students create a chart listing the type of conditional, its symbolic form and an example. This allows students an easy reference sheet when trying to decipher between converse, conditional, contrapositive, and inverse.

TABLE 1.3:

	Symbolic Form	Example
Conditional	$p \Rightarrow q$	
Inverse	$: p \Rightarrow : q$	
Converse	$q \Rightarrow p$	
Contrapositive	$: q \Rightarrow : p$	

Spend time reviewing example one on page 85 as a class. Stress the importance of counterexamples.

Interactive Lesson! Use the same setup as the opening activity when discussing biconditionals. Begin with a definition, such as example one on page 86. Set up your magnetic phrases in if and only if form, then illustrate to students how the biconditional can be separated into its conditional and converse.

Deductive Reasoning

Pacing: This lesson should take one class period

Goal: This lesson introduces deductive reasoning. Different than inductive reasoning, deductive reasoning begins with a generalized statement, and assuming the hypothesis is true, specific examples are deduced.

Differentiate between deductive and inductive reasoning to students by linking to the previous lessons. Deductive reasoning begins with a conjecture (hypothesis) and infers specific examples.

Stress example 5 with your students. Students can get confused with the inverse and contrapositive from the previous lesson that they make the mistake of using faulty reasoning.

When determining the truth value of $p^{\wedge}q$, students may be confused as to why the value is false if the hypothesis is false. Offer students a real life example. "If it is snowing, then it is cold." If the hypothesis is already false, stress that it doesn't matter the conclusion; the statement is not applicable.

Be sure the students understand the difference between $\wedge$ (exclusive) $\vee$ and (inclusive) before filling out the truth tables.

Real World Application! Show a portion of an episode of a courtroom drama scene. Ask students to apply the ideas of deductive and inductive reasoning to the lawyers. Determine which reasoning the prosecuting attorney is using. Is it different reasoning than what the defending attorney uses?

Algebraic Properties

Pacing: This lesson should take one class period

Goal: Students should have some familiarity with these properties. Here we can extend algebraic properties to geometric logic.

Fun Tip! Construct "I have, who has" cards for your class. Using the properties from this lesson (and other lessons if you have a large class), create as many cards as students in your class. The first card should read, "I have Reflexive Property of Equality. Who has the property that states if $a = b$, then $b = a$?" The next card should state, "I have the Symmetric Property of Equality. Who has...? Continue this process until the last card. The "Who has" of this card should state, "Who has the property that a equals a?" Shuffle the cards and give one to each student. Because the cards are all connected, it doesn't matter who starts. Time the class and then challenge the students to beat their previous time. Not only does this increase listening in the classroom, but it also reinforces the properties and encourages active participation.

Teaching Strategy: Use personal whiteboards or interactive "clickers" to do a spot check. Create two or three property questions each day and begin your class with these mini-quizzes. Encourage students to create flashcards for the properties and use them for two-column proofs.

Stress to students that the properties of congruence can only be used when given congruence ($\cong$), not equality ($=$). This also hold true for the properties of equality; these properties are reserved for objects that are equivalent.

Have students list properties not mentioned in this lesson. Students may come up with the distributive property of the multiplying fractions property. Students may offer notions that are incorrect – take the time to have students learn from incorrect thoughts!

Diagrams

The best way to describe what you can and cannot assume is "Looks are deceiving." Reiterate to students that nothing can be assumed. The picture must literally say one thousand words using notation such as tic marks, angle arcs, arrows, etc.

Additional Example! Use the following diagram and ask your students to list everything they can assume from the drawing and those things that cannot be assumed. For the latter list, ask students to list additional information needed to clarify the drawing.
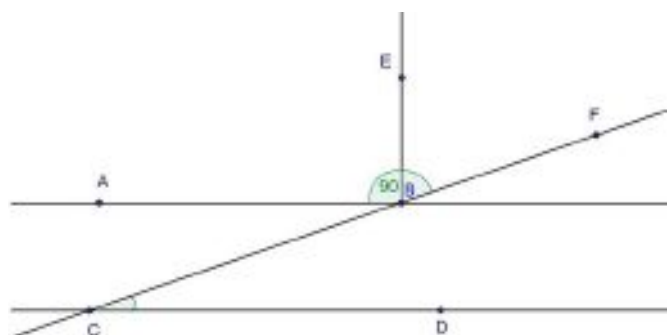
Two Column Proof

Pacing: While two-column proofs will be used for the remainder of the text, this lesson should take one to two class periods

Goal: Students are introduced to the format of a two-column proof in this lesson. The purpose of two-column proofs is not only to prove geometric theorems. Organizing one's thoughts in a logical manner allows students to become better writers and debaters.

Fun Tip! Use cut outs so students can begin to visualize two column proofs. Photocopy the following proof and cut it into sections. Shuffle the sections and place into an envelope. Give pairs of students the envelope and a sheet of paper with the given statement, the "to prove" statement, and the column separator. Have students sort through the rectangles and recreate the proof.

Given: $\overline{AB}$ bisects $\overline{DE}$; $\overline{DE}$ bisects $\overline{AB}$

Prove: $\triangle ABM \cong \triangle DCM$

1.2. Reasoning and Proof
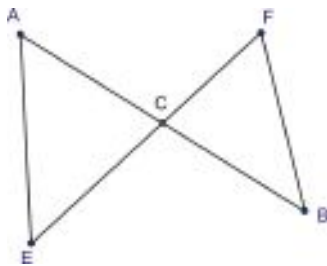
TABLE 1.4:

Reason	Justification
Segment *AD* bisects segment *BC*	Given
Segment *BC* bisects segment *AD*	
Segment *AM* ≅ segment *DM*	Midpoint Postulate
Segment *BM* ≅ segment *CM*	Midpoint Postulate
∠*AMB* ≅ ∠*DMC*	Vertical Angles Theorem
△*ABM* ≅ △*DCM*	Side-Angle-Side Congruence Theorem

Neat Idea! Whiteboard makers also make a 2-column personal whiteboard. You could purchase these to perform a spontaneous check or you could make your own. Purchase plain white paneling from a home improvement store. Cut out the desired lengths then use electrical tape to construct your proof T-chart.

1.3 Parallel and Perpendicular Lines

Lines and Angles

Pacing: This lesson should take one to one and one-half class periods

Goal: Students will be introduced to parallel, perpendicular, and skew lines in this lesson. Transversals and the angles formed by such are also introduced.

While example 1 shows students that it is possible for streets to be perpendicular or parallel, challenge students to find roads that begin as parallel then intersect (or begin perpendicular and then become parallel).

Visualization! Show a map that has zoned roads in the fashion of example 1, perhaps in rural Ohio or Kansas. Encourage students to compare this map with one of Atlanta, New York City, or Chicago.

Physical Model! Give each student a cube; it could be a die, box, etc. When discussing the definition of skew lines, have students point to the lines you are referencing. This provides students a physical model in addition to allowing you to do a quick assessment.

In Class Activity! Have students trace the top and bottom of a ruler to create pair of parallel lines. Then construct an oblique line crossing through both parallel lines. Instruct students to number all 8 angles and color code each of the terms found on page 131. Ask students to summarize each definition and how it relates to another angle in the diagram.

Vocabulary! There are five ways to determine parallel lines: showing congruent corresponding angles or congruent alternate interior angles, proving same side interior angles are supplementary, showing both lines are parallel to a third line or by showing both lines are perpendicular to the same line.

Have students prove the following lines are parallel using one of the above methods.
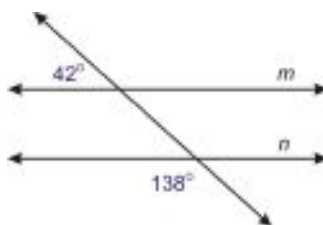
Challenge! There are 13 red and white alternating stripes on the United States Flag. Explain why the top red stripe must be parallel to the third white stripe. *Answer: Using the syllogism property and the idea of parallel lines, since each lines is parallel to the one before it, then the first red stripe must be parallel to the third white stripe. Also, the flag may look weird if the stripes were not parallel!*

http://www.steve4u.com/usflag/50star.gif

Parallel Lines and Transversals

Pacing: This lesson should take one class period

Goal: The textbook further extends the notion of transversals and parallel lines to illustrate the corresponding angles postulate and the alternate interior angles postulate. Additional theorems and postulates are proven in this lesson.

Use the in-class activity from the previous lesson as a refresher and guide to the lesson opener. This lesson provides several key theorems: corresponding angles postulate and alternate interior angles postulate. Students have experienced the definitions of these angles in previously lessons and have also been given brief introductory proofs. The goal of this lesson is to use these notions to prove alternate exterior angles are congruent and consecutive interior angles of parallel lines are supplementary.

In-Class Activity! Divide your class into six sections of pairs. Provide enough copies of the Corresponding Angles Postulate, its converse, the Alternate Interior Angles Postulate, its converse, and the Alternate Exterior Angles Postulate and its converse. Instruct each pair to prove their theorem, and then group homogenous sections in order to discuss the results. Taking a pair of each theorem and its respective converse to form a team of four, have the students discuss the proofs. As an assignment, have the groups create a visual poster of the proof of the theorem, the converse and its respective proof.

In-Class Activity! Demonstrate to students that these theorems do not apply to non-parallel lines. Each student should create two non-parallel lines and a transversal. Label the 8 angles formed, having students measure all angles. Students will see the alternate interior angles, corresponding angles, and vertical angles are not congruent, nor are the consecutive interior angles supplementary.

Proving Lines Parallel

Pacing: This lesson should take one class period

Goal: The converse of the previous lesson's theorems and postulates are provided in this lesson. Students are encouraged to read through this lesson and follow along with the proofs.

Vocabulary! The Parallel Lines Property can be stated, "If line *l* is parallel to line *m*, and line *m* is parallel to line *n*, then lines *l* and *n* are also parallel." Ask students to write the converse of this property and determine if it is true. If students determine the converse false, have them provide a counterexample.

Slopes of Lines

Pacing: This lesson should take one to two class periods

Goal: Students should feel comfortable with slopes and lines. Use this lesson as a review of key concepts needed to determine parallel and perpendicular lines in the coordinate plane.

Fun Fact! The word slope comes from the Middle English word *sloop*, meaning at an angle.

Most students have experienced slope in Algebra. However, students rarely have seen the delta symbol when determining slope. Use the following to stress the connection to pre-calculus:

$$\text{Slope} = \text{rate of change} \; \frac{\text{Rise}}{\text{run}} = \frac{\triangle y}{\triangle x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Vocabulary Connection! Ask students to brainstorm the many different interpretations of the word slope. Apply these to real world situations such as the slope of a mountain, or the part of a continent draining into a particular ocean (Alaska's North Slope), the slope of a wheelchair ramp, etc.

Include the synonym for slope – grade. Students should come up with more examples using this word.

Real Life Connection! Eldred Street in Los Angeles, California has a grade of 33%, Baldwin Street In Dunedin, New Zealand boasts a 35% incline, and Banton Avenue in Pittsburg, Pennsylvania officially measurers 37%! Have students reconstruct the incline of these streets using the rise over run notion of slope.

When discussing the rise over run triangles, begin making the right triangle connection to students, demonstrating that every rise/run triangle will form a 90 degree angle. When students are asked to find the distance between two oblique points, the distance formula is a derivation of Pythagorean's Theorem.

Fun Tip! To illustrate why vertical lines have an undefined slope, ask for volunteers for the following demonstration.

To illustrate a horizontal line, run a length of masking tape on your floor. Ask a student to walk over the line. Onlookers should see the student is walking at a zero incline (or slope).

To illustrate an oblique line, lay a 2" by 4" piece of wood on top of a chair, or something sturdy, creating a $2\% - 3\%$ incline. Ask a student to walk up the hill. Relate the percentage to a fraction, relating rise over run.

To illustrate a vertical, ask students to place their feet on a wall, lying parallel to the floor. Instruct the students to walk up the wall in this position, similar to what Spiderman can do. Students will tell you this is impossible! Dividing by zero is also impossible, thus illustrating why vertical lines have undefined (impossible) slopes.

To further demonstrate perpendicular slopes, use the formula to your advantage $\text{slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$ so the slope of the line perpendicular must be $-\frac{(x_2 - x_1)}{(y_2 - y_1)}$

Students may find that making an *xy* T-chart is an easy way to construct a line. Whichever your preference, make sure students can see a variety of ways to begin to solve a problem.

Equations of Lines

Pacing: This lesson should take one to two class periods

Goal: This lesson reinforces key concepts learned during Algebra to prepare students for geometric connections. Students will review slope intercept form, standard form for a linear equation, and introduce equations for parallel and perpendicular lines.

1.3. Parallel and Perpendicular Lines

Alternative Ways to Think! An alternative way to express slope-intercept form is $y = b + mx$. In some situations, this form will make much more sense to students that the "original" way. You could also try substituting m with a. Linear regressions found on graphing calculators often use this formula: $y = ax + b$. Students tend to feel frustrated with the constant replacement of variables. Determine which variable appear in later textbooks and feel free to use that variable from the beginning.

Inquiry based learning! Have students trace the top and bottom edges of a ruler onto a coordinate plane. Ask students to determine the equations for each line and compare the results. *Students should notice that, if done correctly, the slopes will be equal.* Follow this activity with the equations for parallel lines section.

Use the graph provided in example 3 for this activity. Once students have found the slope of the graphed equation, incorporate the previous lesson's concept to find the slope of the line perpendicular. Ask students to place a dot anywhere on the $y-$axis and use the newly found slope to construct a line. Using a projection device, ask several students to graph their equations. *Students should come to the conclusion there are infinitely many lines perpendicular in a coordinate plane.*

Algebra Review! Before discussing standard form for a linear equation, make sure students can clear fractions, something that is widely forgotten. During the warm up or opening set, ask students to clear the following fractions:

$$\frac{5}{6}x = 30 \qquad\qquad \frac{2}{3}x + 3 = 9 \qquad\qquad \frac{7}{6}x + \frac{1}{4} = \frac{1}{2}$$

This will allow you to determine the level of which you may have to re-teach before moving on to standard form.

Why do I need this? Many students ask why they need to know standard form. One reason is because many real life problems take form in a linear combination (standard form) approach. For example, one cheeseburger is \$1.69 and a small French fry is \$1.39. How many of each can you buy with \$15.25, excluding tax? The equation begins in standard form and many students will rewrite this into slope-intercept form.

Perpendicular Lines

Pacing: This lesson should take one class period

Goal: Students will extend their learning to include angle pairs formed with perpendicular lines. The properties presented in this lesson hold only for perpendicular lines pairs.

Extension: Connect the introduction of this lesson with circles. Draw a circle around the origin of the Cartesian plane found on page 180. Students should already know the sum of the degrees of a circle $(360°)$. Demonstrate to students what angles are formed when the axes split the circle into four congruent segments. This will aid students when discussing circles in Chapter 9.

Example 3 can also be solved using the notion of vertical angles. To find $m\angle WHO = 90°$, instruct students to visualize these two angles as being vertical angles.

Extension: Extend example 4 to review vertical angles. Turn ray L into a line and have students apply the Vertical Angle Theorem to the angles found in quadrant four. This will help keep Vertical Angles fresh in students' minds.

Take time to review how perpendicular angles are formed – *the product of slopes of perpendicular lines must equal* -1. Continuous review will help students prepare for the test.

Why Is This So? Students may question why the lesson is entitled "Perpendicular Lines" when most of the material presented is regarding angles. Explain to students that these properties only hold for lines intersecting at 90 degree angles. To further illustrate, have students construct non-perpendicular lines and attempt to draw in complementary adjacent angles.

Perpendicular Transversals

Pacing: This lesson should take one class period

Goal: The goal of this lesson is to introduce students to the concept that parallel lines are equidistant from each other and to prove lines parallel using the converse of the perpendicular to parallels theorem.

As students work through example 4, ask them to look at the slopes of the lines. Students should realize the slopes are the same, thus they will never intersect. Have students create their own property describing this concept.

Additional Example: Have students place a ruler in any direction on a coordinate plane. Then, by tracing the top and bottom of the ruler, the students will create parallel lines. Ask each student to find their equations for their personal lines. Check with a partner to see if the equations are correct.

Non-Euclidean Geometry

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to extend students' understanding of geometry beyond parallel and perpendicular lines, angle pairs, and abstract drawings. Most students will enjoy this lesson due to the real life application. However, even if students are unfamiliar with taxicabs, extend this lesson to rural areas with roads that intersect.

History Connection! Take time to discuss Euclid during the lesson. Show the following picture of his book, *Elements.* Go through the first five postulates. Use the following website to gather additional information. Or, have students write mini-reports of the impact Euclid had on present day Geometry. Offer "Euclid Day," a day of celebration on behalf of Euclid. The possibilities are endless!

Create a class discussion regarding Euclid's 5th Postulate. "If two lines are cut by a transversal, and consecutive interior angles have a total measure of less than 180 degrees, then the lines will intersect on that side of the transversal." Mathematicians tried to prove this true, thus making it a theorem as opposed to a postulate for 2000 years. Since many mathematicians did not regard this as truth, non-Euclidean geometries were founded.

Other types of non-Euclidean geometry are: spherical geometry, hyperbolic geometry and elliptic geometry. In spherical geometry, straight lines are great spheres, so any two lines meet in two points. There are also no parallel lines (think longitude lines meeting at the poles). Hyperbolic geometry satisfied all Euclid's postulates except the parallel postulate, replacing it with "For any infinite straight line L and any point P not on it, there are many other infinitely extending straight lines that pass through P and which do not intersect L." Elliptic geometry replaces Euclid's parallel postulate with "through any point in the plane, there exist no lines parallel to a give line."

1.4 Congruent Triangles

Triangle Sums

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to familiarize students with the polygonal sum theorem and its specific application, the triangle sum theorem. Students will incorporate algebra to find unknown polygons given an interior angle sum and find an interior angle sum given a specific polygon.

Physical Models! Using the triangle from the introduction, have students measure the interior angles of the triangle. Then, by extending segments $\overline{AB}, \overline{BC}, \overline{AC}$, students see three new exterior angles and should measure these too. *Students should make the connection that the interior and exterior angles form a linear pair, and by the Linear Pair Theorem, are supplementary.*

Extension! The Triangle Sum Theorem is a special case of the Polygonal Sum Theorem, in which the sum of interior angles of an $n-$gon is found by the following formula:

$T = 180(n-2)$, where $n \geq 3$

Ask students to brainstorm the reasoning behind $n \geq 3$. *Students should remember that a polygon cannot be formed with less than three segments.*

Physical Model! To demonstrate the explanation of the Triangle Sum Theorem found on page 209, students should draw a triangle and measure all three interior angles. Students can then rip or cut off any two angles and, like a puzzle, fit them with the third. The result is a straight line with a measurement of 180 degrees.

Technology Activity! Using a geometric software program, have students follow these steps:

    a. Place 3 noncollinear points on the plane, labeled A, B, C. Connect these three points to form $\triangle ABC$.
    b. Compute the measures of $\angle A, \angle B, \angle C$. How can we classify this triangle? Is it scalene, equilateral, or isosceles? Is it acute, obtuse, or right?
    c. Find the sum of all three angles. *It should equal* 180 degrees.
    d. Highlight points A, B (thus $\overline{AB}$), and point C. Using the appropriate menu, click on "construct a parallel line." There should a line parallel to $\overline{AB}$.
    e. Locate points on the line parallel to $\overline{AB}$, calling them F and E.
    f. Measure $\angle ABF$ and $\angle CBE$. Calculate the sum of these two angles and $\angle A$. *The sum should equal* 180 degrees.

Congruent Figures

Pacing: This lesson should take one class period

Goal: The goal of this lesson is to prepare students for the five triangle congruency theorems: Angle-side-angle, side-angle-side, side-side-side, angle-angle-side, and the special case of side-side-angle, the hypotenuse-leg theorem. This lesson provides a needed introduction by looking at congruent triangles in a non-formal manner.

Notation, Notation, Notation! Revisit congruence notation from earlier lessons: $\cong$ Stress the importance of labeling each congruency statement such that the congruent vertices match. For example, $ABCD \cong LMPQ$ shows $\angle A \cong$

$\angle L$, $\angle C \cong \angle P$, and so forth.

Stress the tic mark notation in relation to the congruency statement. Simply because the letters used are in alphabetical order does not necessarily mean they will line up this way in a congruency statement. Students must follow the tic marks around the figure when writing congruency statements.

Look Out! Students begin to become confused with notation at this point. Be consistent with notation. Have groups of students create classroom posters regarding symbols.

Use the following mantra, "Distances are equal and side lengths are congruent." While each lends to the other, students need to understand which value applies.

Ask students to determine if the below triangles are congruent and explain any reasoning. Use the following information: $\overline{DE} \cong \overline{AB}$ and $\overline{EF} \cong \overline{BC}$. *These are not congruent because the double tick marks do not match.*
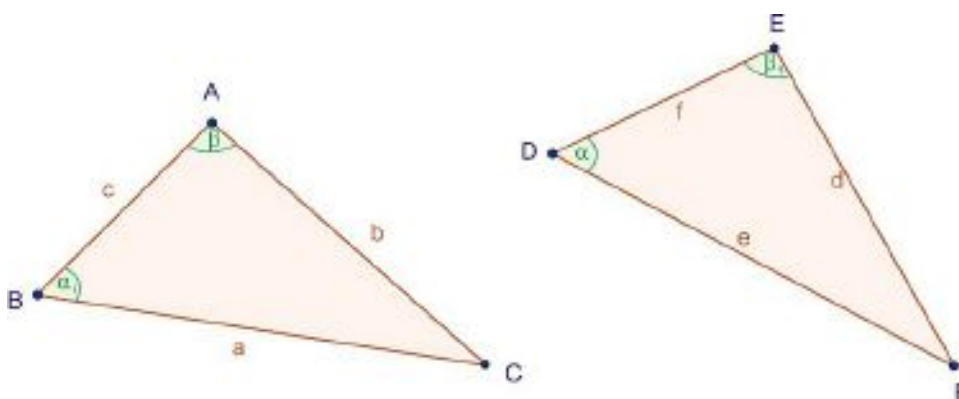
Proof Using SSS

Pacing: This lesson should take one class period

Goal: This lesson introduces students to the formal concept of triangle congruency. The easiest for students to visualize is the side-side-side (SSS) Congruence Postulate.

Differentiation! For students struggling with the distance formula, encourage them to create a right triangle using the $\frac{\text{rise}}{\text{run}}$ of the line. Then students can use Pythagorean's Theorem $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$ to find the length of the segment.

Arts and Crafts Time! Students can visualize the SSS Congruence Postulate in the following way. Using three $8.5[U+0080][U+009D] \times 11[U+0080][U+009D]$ sheets of paper, have students create three dowels by rolling tightly from corner to opposite corner. Cut the dowels to the following lengths: $4[U+0080][U+009D], 6[U+0080][U+009D]$, and $7.[U+0080][U+009D]$ Using tape, glue, or staples, the students should create a triangle and compare their figure with the figures of several classmates. *Students should see that all triangles are congruent, helping to demonstrate the rationale behind the SSS Congruence Postulate.*

Background Information! The SSS Congruence Postulate can be proved using the idea of congruence. In theory, as mentioned in the lesson, these two triangles represent a slide of 7 units right and 8 units down. A slide, or *translation,* is an isometry, preserving distance and angle measure. Thus, since the distances are equal, the lengths are congruent.

1.4. Congruent Triangles
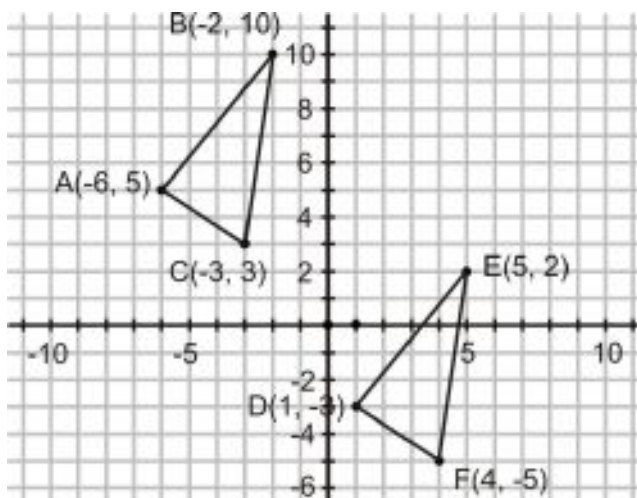
Proof Using ASA and AAS

Pacing: This lesson should take one class period

Goal: Students will learn how triangles can be determined congruent using Angle-Side-Angle and Angle-Angle-Side Theorems.

Look out! These two congruency postulates look identical to students. Use this method when explaining which to use. When **moving left to right of a triangle,** recite the information you have. If you have an angle-angle-side (or side-angle-angle), use the AAS Congruence Theorem. If you have an angle-side-angle, use the ASA Congruence Postulate. *In-class activity:* Have the students complete the following two activities:

Activity #1: Have students draw a 42 degree angle. Measure one ray 5 cm. Using the unmeasured ray, students will now draw a 70 degree angle. Connect both rays to form a triangle. Have students share their drawings. *All drawings should be congruent, according to AAS.*
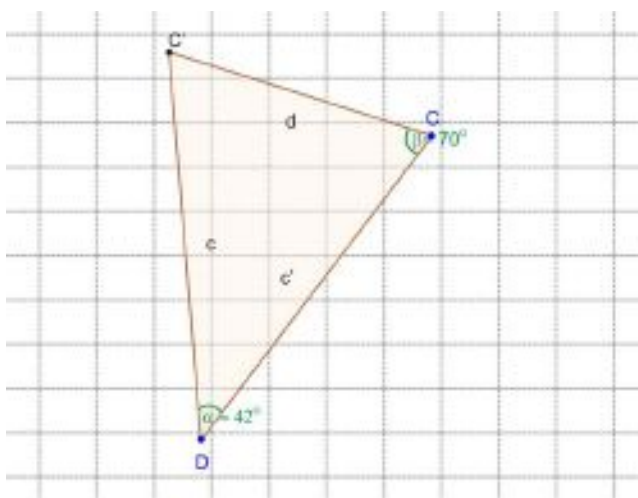
Activity #2: Have students draw a 55 degree angle, measuring one ray 5 cm. Using this same ray, construct a second angle of measure of 85 degrees. Connect both rays to form a triangle. Have students share their drawings *(all drawings should be congruent, according to ASA).*
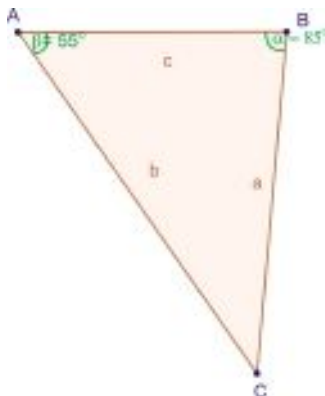
English Connection! There are three main types of proofs: two-column, flow charts, and paragraph form. Use all three methods when presenting this lesson (and subsequent lessons). Some students will be more comfortable with organizing information in a 2-column format and others will use a flow chart. Students with a strong language arts background will find that writing proofs using complete sentences in a paragraph will make the process of proving similar to drafting a persuasive essay.

Homework Check! Using the bonus question following #10 is a great review for students regarding the appropriate way to name angles. Be sure you review this question in class!

Proofs Using SAS and HL

Pacing: This lesson should take one class period

Goal: This lesson is to complete the triangle congruency postulates and theorems. There is a brief description of the anatomy of a right triangle and Pythagorean's Theorem. It also introduces the notion that AAA and SSA relationships will not produce congruent triangles.

Note Taking Time! Students need help organizing these congruency proofs. Use a table similar to the one shown below, projected on a digital imager or projector and take time to organize the material. Extend the organizer one more column and two more rows to accommodate the information gathered in the **Using Congruent Triangles** lesson.

TABLE 1.5:

Congruence Type	Definition	Diagram
SSS		
SAS		
ASA		
AAS		
HL		

Extension! Use the following diagram from example 1 to further assess your students' understanding of Triangle Congruencies. Have students list the information needed for each of the four congruencies learned thus far: ASA, SSS, SAS, and AAS.

1.4. Congruent Triangles
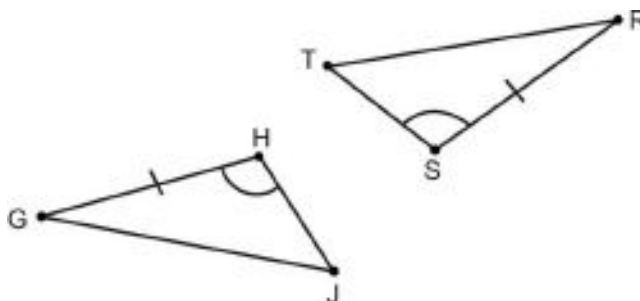
Triangle Anatomy! Understanding right triangle anatomy is crucial, especially once students move into trigonometry. Before discussing the Hypotenuse-leg Congruence Theorem, draw a blank right triangle. Have movable words of LEG, LEG, and HYPOTENUSE. Encourage students to correctly label the right triangle by moving the terms to the correct positions.

Pythagorean's Theorem! This is one of the most useful theorems in mathematics; it is used for distance, finding missing side lengths in a right triangle, and is the basis of the Law of Cosines. Use this silly memory device. You will use Pythagorean's Theorem enough times to cause *Post-traumatic Stress Disorder (PTSD).* Each time you say, "PTSD?" students should respond, `[U+0080][U+009C]`$a^2 + b^2 = c^2$! `[U+0080][U+009D]`

In-class Activity! Similar to the activities showing ASA and AAS congruencies, students will use the following two activities to show there are no such congruencies for AAA and SSA relationships.

Using Congruent Triangles

Pacing: This lesson should take one class period

Goal: The goal of this lesson to illustrate how congruent triangles can be used to determine congruent corresponding segments or vertices and find distance.

Extension! Using the graphic organizer from the **SAS and HL** lesson, include the information presented in the introduction.

Look out! This is approximately the lesson in which students will ask the question, "When will we ever need to know why triangles are congruent? How will this apply to my life?" Encourage students to research aviation, construction, manufacturing, and so forth to explore real world uses of triangles. Have students draft an essay with their findings and present it to the class. *Here are some real life examples: Architecture such as bridge construction and roof rafters; proving properties of other figures, such as parallelograms, squares, rhombuses; determining congruent sails on sailboats; ensuring stairs are the same (the risers have congruent triangles cut from them).*

Arts and Crafts Time! Students confuse "drawings" with "constructions." Stress to students that a true construction can only be made with the following tools: compass and a straightedge. Constructions cannot be measured using degrees from a protractor or units from a ruler. Once these two tools come into play, the construction is now considered a drawing.

For a beginner warm up, encourage students to play with the compass. Many will make faces, animals, or flowers. Have students decorate their drawings and post them on a bulletin board.

Take time and go through the perpendicular bisector constructions as a class.

Extension! Have students continue to practice constructions by constructing an angle bisector using the following directions.

a. Have students draw an angle of their choice, labeling it $\angle B$.
b. Place the compass on B and draw an arc through both sides of the angle. Call the points of intersection A and C.

c. Place the compass on point *B* and draw an arc across the interior of the angle.
d. Without changing the radius of the compass, place the compass on point *C* and draw an arc across the interior of the angle.
e. Label the intersection of the two arc as *D*. Draw $\overrightarrow{BD}$.
f. $\overrightarrow{BD}$ is the bisector of $\angle ABC$.

An example is show below.
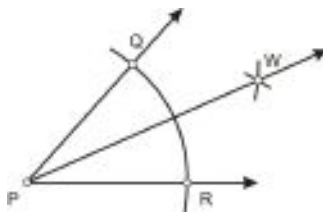
Isosceles and Equilateral Triangles

Pacing: This lesson should take one class period

Goal: There is a natural progression from triangle congruencies to isosceles and equilateral triangles. This lesson illustrates the special properties that arise from these two types of polygons.

It is helpful for students to reproduce the isosceles triangle drawing. They will benefit from using this diagram when completing the exercises.

Another useful theorem of isosceles triangles has an especially long name. The *Isosceles Triangle Coincidence Theorem* states, "If a triangle is isosceles, then the bisector of the vertex angle, the perpendicular bisector to the base, and the median to the base are the same line." Therefore, the perpendicular bisector to an isosceles triangle's base is the same line generated by the angle bisector of the vertex.

Because of this theorem, step 5 of the proof in example 1 can be alternatively justified using the HL Congruence Theorem. $\overline{AD}$ *creates two right angles,* $\angle ADC$ *and* $\angle ADB$.

An equilateral triangle is a special type of isosceles triangle. Some definitions of isosceles state, "An isosceles triangle has **at least two** sides of equal length." While this book does not use this exact definition, it is implied by using the Isosceles Triangle Base Angle Theorem with equilateral triangles.

Extension! Since the Isosceles Triangle Base Angle Theorem and its converse are true, have your students create a biconditional of this theorem. *The base angles of a triangle are congruent if and only if the triangle is isosceles.*

Extension! Students may fall into the trap of assuming figures must be both equiangular and equilateral. To show a counterexample to the belief, have students create equiangular polygons that are not equilateral. For example, students can draw a pentagon having five interior angle measurements of 108 degrees with varying side lengths.

Congruence Transformations

Pacing: This lesson should take one class period

Goal: This lesson introduces students to isometries, which can also be found in Chapter 12. The main focus of this lesson is on congruent triangle transformations.

Name That Transformation! Create slides or cards with images of transformations (preimage and image). Flash one at a time to the class. Have the students answer on a personal whiteboard or other monitoring system. Offer one

1.4. Congruent Triangles

point for each transformation the student correctly answers – offer double points if the student can explain why s/he chose that particular transformation.

1.5 Relationships Within Triangles

Midsegments of a Triangle

Pacing: This lesson should take one class period

Goal: This lesson introduces students to the concept of midsegments and the properties they hold.

Extension: Have students find the other midsegments to the triangle of the introduction
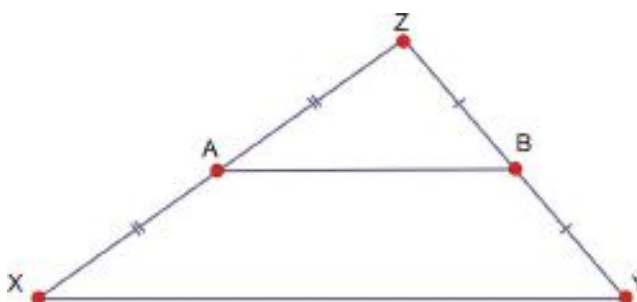
Students will need to measure XY to find its midpoint, call it C, and create two new line segments: $\overline{AC}$and $\overline{BC}$. Encourage students to analyze the three midsegments. What do the segments form? What relationships can be seen? *The midsegments form a second triangle. $\overline{AB}$is parallel to $\overline{XY}$, $\overline{AC}$is parallel to $\overline{ZY}$, and $\overline{BC}$is parallel to $\overline{XZ}$.*

Discuss the proof of the first section of the Midsegment Theorem with your class. It may be helpful to begin with the figure above, and with each new justification, add it to the drawing.

Extension! Have your students take the paragraph proof of the Midsegment Theorem and rewrite it into 2-column form.

Additional Example: Find *x*, and the lengths of *DE* and *BC*.
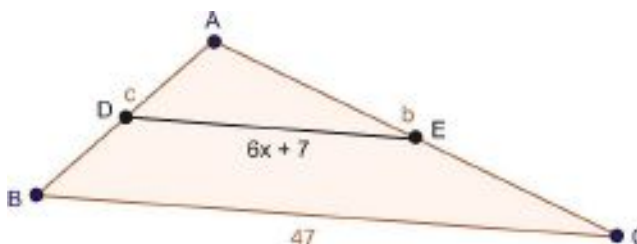
Perpendicular Bisectors in Triangles

Pacing: This lesson should take one class period

Goal: Students are introduced to the concept of the *circumcenter of a triangle.* The process of finding the circumcenter uses perpendicular bisectors, which students can either draw using a protractor or construct using the method found in the *Using Congruent Triangles* lesson.

Additional Example: Construct a circle passing through these three points:
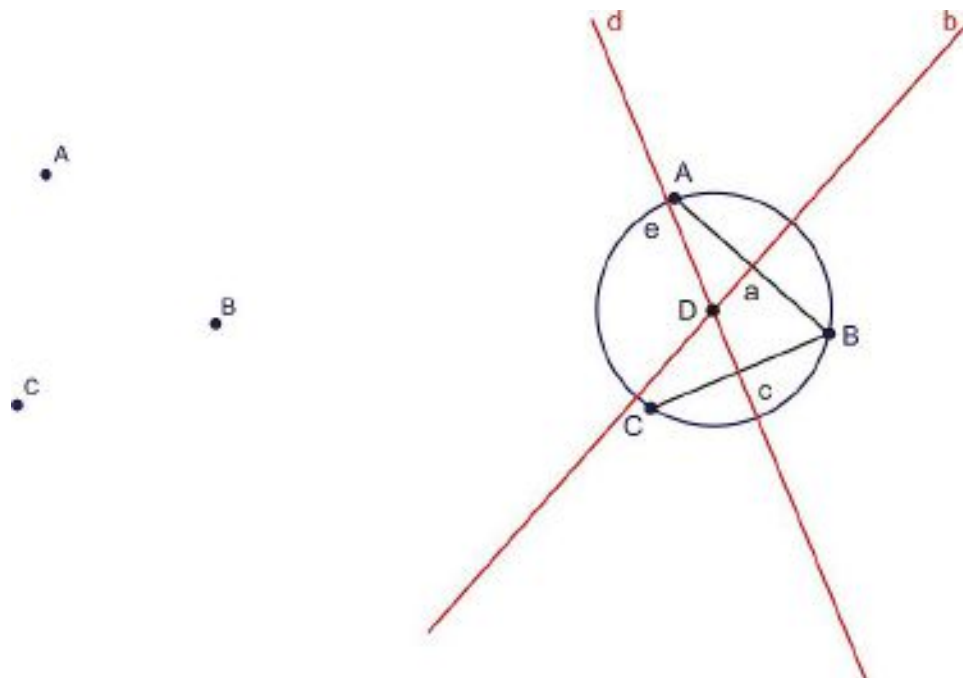
Angle Bisectors in Triangles

Pacing: This lesson should take one class period

Goal: Students will learn how to inscribe circles within triangles using the concept of angle bisectors. Students should be familiar with angle bisectors, as there were presented in chapter one.

Summary: To inscribe circles within triangles, the center of the circle is the intersection of angle bisectors. To circumscribe circles about triangles, the center of the circle is the intersection of perpendicular bisectors.

To construct the angle bisectors, repeat the *In-class Activity* found in *Using Congruent Triangles* lesson.

Additional Example: Locate the circle inscribed within the following triangle.
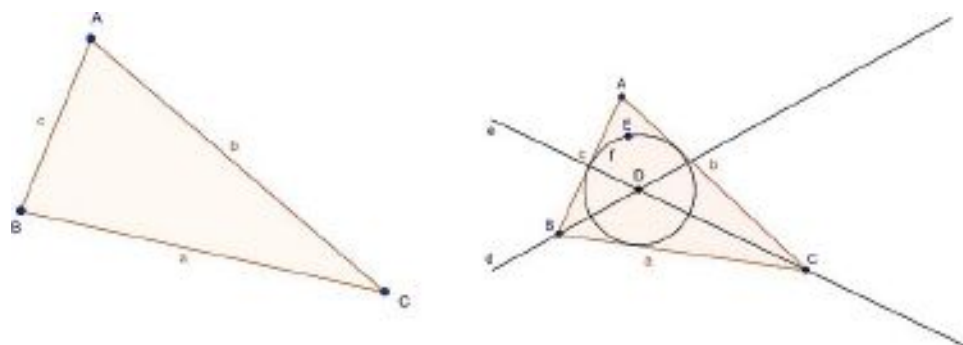
Medians in Triangles

Pacing: This lesson should take one class period

Goal: This lesson introduces the centroid of a triangle. By now, students should be familiar with the three main intersection points regarding triangles: the circumcenter, the incenter, and the centroid.

History Connection! In addition to an infamous dictator, it appears Napoleon Bonaparte was an excellent mathematician. He was the top mathematics student in his school, taking algebra, trigonometry, and conics. His favorite class, however, was geometry. After graduation, Bonaparte interviewed for a position in the Paris Military School and was accepted due to his mathematical ability. Bonaparte completed the curriculum in one year (it took average students two or three years to complete) and was appointed to the mathematics section of the French National Institute.

During his reign, Bonaparte appointed such men as Gaspard Monge, Joseph Fourier, and Pierre Laplace to recruit teachers and reform the curriculum to emphasize mathematics. Napoleon's Theorem is named as such because, while Napoleon was not the first person to discover it, he supposedly found it independently.

Altitudes in Triangles

Pacing: This lesson should take one class period

Goal: Students will learn how to construct an altitude and how this auxiliary line differs from the median. Altitudes are important in such geometrical concepts as area and volume.

Guided Discovery Questions! What is the difference between a median and an altitude? Is a median always an angle bisector? Can the perpendicular bisector be a median?

Review Question: Using the diagram below, ask students to label the following auxiliary items: median, circumcenter, incenter, orthocenter, altitude, perpendicular bisector
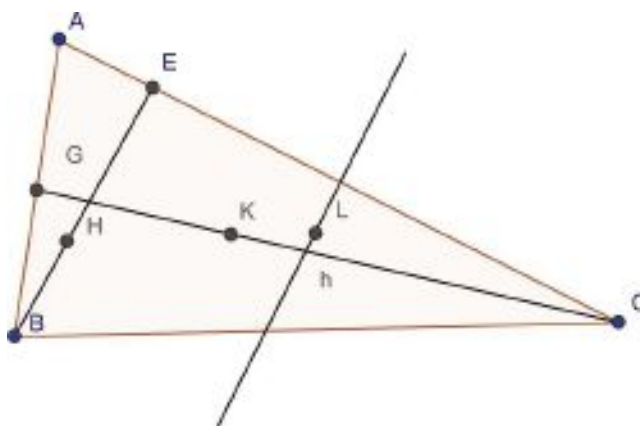
Inequalities in Triangles

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to familiarize students with the angle inequality theorems and the Triangle Inequality Theorem. The lesson further extends the concepts of perpendicular lines and triangles to deduce the shortest path between a point and a line is its perpendicular, thus leading to parallel lines.

If you have not used the in-class activity found in chapter 1, *Classifying Triangles* lesson, include it here. Otherwise, reintroduce the concept in this lesson.

Additional Example: Jerry is across the street in the following diagram. Draw the path she should travel to minimize the distance across the street.
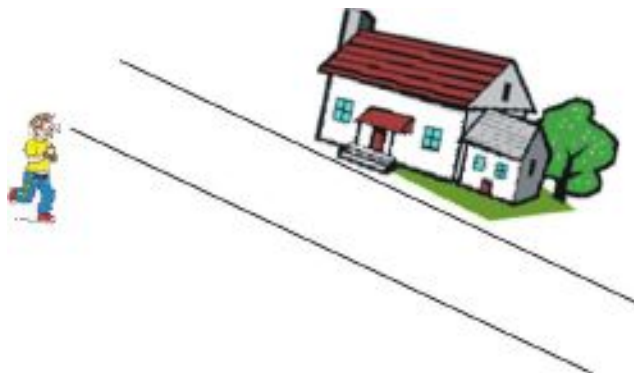
Inequalities in Two Triangles

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to utilize the concept of inequality to determine corresponding angle measures and side lengths of a triangle.

Review! Be sure your students can solve inequalities. Try these as a warm-up or brief review.

    a. $8x - 4 + x > -76 \quad x > -8$
    b. $-3(4x - 1) \geq 15 \quad x \leq -1$
    c. $8y - 33 > -1 \quad y > 4$

Additional Examples:

List the sides of each triangle in order from shortest to longest.

1. $\triangle ABC$ with $m\angle A = 90, m\angle B = 40, m\angle C = 50$. AC, AB, BC

2. $\triangle XYZ$ with $m\angle X = 51, m\angle Y = 59, m\angle Z = 70$. YZ, XZ, XY

List the angles of the triangle in order from largest to smallest.

3. $\triangle ABC$ where $AB = 10, BC = 3$, and $CA = 9$. $\angle C, \angle B, \angle A,$

Indirect Proof

Pacing: This lesson should take one class period

Goal: Students have seen several theorems proven using the indirect proof. Indirect proof is an invaluable resource to students attempting to prove theorems or postulates.

Indirect proofs typically have four sentences that can be summarized by the following acronym: ATBT.

A – Assume (the opposite of what you're trying to prove. Essentially, this is the negation of the conclusion of the conditional)

T – Then (by doing some mathematics or using reasoning, a conclusion can be made)

B – But (here lies the contradiction. The conclusion you made in the previous sentences defies a definition or previously proved theorem)

T – Therefore (your original conditional must be true)

Indirect proofs are usually used when the word *cannot* appears in the proof.

Example: Prove a triangle cannot have two right angles.

Assume a triangle can have two right angles. Then, using the Triangle Sum Theorem, $90 + 90 + x = 180$ degrees. Using the angle addition property and the addition property of equality, $x = 0$ (here lies your contradiction). But a triangle must have three angles greater than zero degrees. Therefore, a triangle cannot have two right angles.

Additional Example: Prove $\sqrt{15} \neq 4$.

Assume $\sqrt{15} = 4$. *Then, by squaring both sides,* $15 = 16$. But $15 \neq 16$. *Therefore,* $\sqrt{15} \neq 4$

1.6 Quadrilaterals

Interior Angles

Pacing: This lesson should take one class period

Goal: Students will use the Triangle Sum Theorem to derive the Polygonal Sum Theorem by dividing a convex polygon into triangles.

Inquiry Based Learning! Analyzing a pattern is another method to looking at the polygonal sum theorem. Begin by setting up the below chart. Ask students to fill in the second column, asking if the number of side lengths can form a polygon. Ask students to then complete the obvious interior angle sums such as triangle and quadrilateral. Encourage students to see a pattern and create its function. *Students should see that the "starting" sum is 180 and each subsequent polygonal sum is 180 degrees greater.*

Vocabulary! Reiterate to students that a diagonal is drawn from any vertex to a non-adjacent vertex.

Extension! Does this theorem work for non-convex polygons? Pose this question to students as you draw several non-convex polygons on the board. *Since the polygon is non-convex, the "indented" angle will always be obtuse, showing this theorem will only work for convex polygons.*

Additional Example: The sum of the interior angles of an $n-$gon is $3,960$ degrees. What is n? $n = 24$

Exterior Angles

Pacing: This lesson should take one class period

Goal: This lesson introduces students to exterior angles of polygons. The Linear Pair Theorem is used to determine the measures of exterior angles.

Stress to students that there are two possibilities for exterior angles and it will be extremely important to label the angle correctly.

Additional Examples: Using what you have learned thus far, determine the measure of $\angle FGH$. *Students will use the Polygonal Sum Theorem to determine the sum of the interior angles in the heptagon is* $900°$. *Dividing by 7, each interior angle has a measure of* $128.57°$. $\angle FGH$ *forms a linear pair with* $\angle AGF$ *and are supplementary. Therefore, the measure of angle* $FGH = 51.43$ *degrees.*
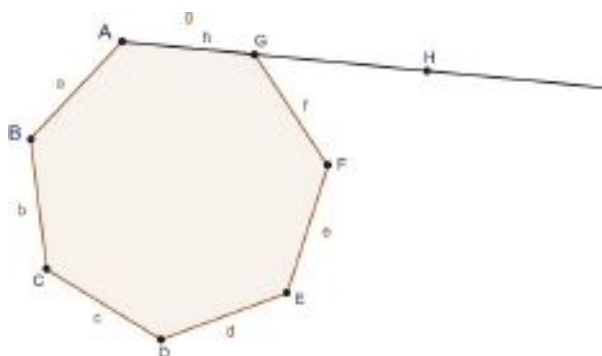
Classifying Quadrilaterals

Pacing: This lesson should take one to two class periods

Goal: Students are introduced to the most common quadrilaterals and relationships they share. A Venn diagram is provided as a visual to allow students to visualize how a quadrilateral such as a square fits with a rectangle, parallelogram, and trapezoid.

Have students take the Venn diagram and transfer it to a hierarchy, showing the most general quadrilateral to the most specific quadrilateral.

Discussion! Begin a discussion with students regarding trapezoids and parallelograms. Some textbooks describe a trapezoid as, "a quadrilateral with at least one pair of parallel sides." Discuss this possible definition with your students. How would the Venn diagram change if this definition were accepted as true? Should it be accepted as true? Why is the definition of a trapezoid provided in this text stating "exactly one pair of parallel sides?"

Additional Examples: Ask students to answer the following questions, either on a personal whiteboard, journal entry, or in a Think-Pair-Share group.

    a. *Always, sometimes, never.* All rhombi are squares. *Sometimes. A square is a special type of rhombus.*
    b. *Always, sometimes, never.* All rhombi are parallelograms. *Always.*
    c. *Always, sometimes, never.* Parallelograms are trapezoids. *This answer depends upon the discussion of your class.*

Pythagorean's Theorem AGAIN! Reiterate to your students the connection between Pythagorean's Theorem and the distance formula. This is especially helpful when determining the lengths of segments of a quadrilateral to determine its appropriate classification.

Trapezoids and Parallel Lines! Encourage your students to make the connection between consecutive interior angles of parallel lines and a trapezoid. A trapezoid is really a pair of parallel lines cut by two transversals. Therefore, consecutive interior angles are supplementary (according to Euclid's 5th Postulate).

Using Parallelograms

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to familiarize students with properties special to parallelograms. These properties are useful when proving a figure is a parallelogram. These properties also hold for rectangles, rhombi, and squares – those quadrilaterals that are "included" within parallelograms in the Venn diagram.

In-class Activity! Instead of using string, your students can also use raw spaghetti noodles. Be sure the segments are equal in length; otherwise, the models may not illustrate a parallelogram appropriately.

Making Connections! Make as many connections as possible. This will help your students see how geometrical concepts fit together. For example, students have learned parallel lines are equidistant from each other. Connect this to a parallelogram

Flash Fast Game! Have your students create flashcards with quadrilateral names on one side and important information or properties on the reverse. Have various types of quadrilaterals, both abstract and real world, ready to show students. Once students believe they have classified the quadrilateral, they are to hold up the appropriate name. You can keep score or use this as a summative assessment.

Proving Quadrilaterals are Parallelograms

Pacing: This lesson should take one class period

Goal: Students will use triangle congruence postulates and theorems to prove quadrilaterals are parallelograms. This lesson serves as an application of the concepts learned in the Triangle Congruence lesson.

Be sure to review each proof in the lesson with your class. Have the students perform a Think-Pair-Share by writing the conditional on the board and having students attempt to prove the statements on their own.

Another Way of Thinking! The proof of, "If a quadrilateral has two pairs of congruent sides, then it is a parallelogram," can be proven using the SSS Congruence Postulate. Instead of using same side interior angles, use the Reflexive Property to state $CE = CE$.

Rhombuses, Rectangles, and Squares

Pacing: This lesson should take one class period

Goal: This lesson demonstrates another application of triangle congruence. Students are shown important properties of rhombuses such as bisecting diagonals and opposite angles.

Refer students back to the Venn Diagram or the hierarchy of quadrilaterals. Make sure students understand that everything that falls within a rhombus possess the same characteristics and properties of a rhombus. Identifying this key relationship will help students understand this lesson.

Remind students that a diagonal is a segment drawn from one vertex to any non-adjacent vertex. *Question to think about:* Will there always be two diagonals for any quadrilateral? What is it is non-convex?

Arts and Crafts Time! Using patty paper and a pencil, have students trace a rectangle. Instruct students to fold the rectangle so the lower left angle fits on top of the upper right angle, thus forming a diagonal. Open the fold and repeat the process on the other diagonal. Overlay the patty paper onto a coordinate grid and have students work through the distance formula to determine the lengths of the diagonals.

Discuss the proof of this as a class, using the patty paper rectangle for further illustration, if necessary.

Be sure students can "take apart" a biconditional into its two separate statements. This may require more practice on behalf of your students before they can determine if the biconditional is true.

Additional examples: Separate these biconditionals into a conditional and its converse

   a. The rain will fall if and only if it is cloudy. *If the rain will fall, then it is cloudy. If it is cloudy, then the rain will fall.*
   b. An animal is a mammal if and only if it has whiskers. *If an animal is a mammal, then it has whiskers. If an animal has whiskers, then it is a mammal.*
   c. An object is a circle if an only if it is the set of points equidistant from a single point. *If an object is a circle, then it is the set of points equidistant from a single point. If an object is the set of points equidistant from a single point, then it is a circle.*

Trapezoids

Pacing: This lesson should take one class period

Goal: This lesson introduces students to the special properties of trapezoids, especially those of the isosceles trapezoid.

Real Life Application! Show a photograph of the John Hancock Center, located in Chicago, Illinois. This structure was the first skyscraper to be built using exterior tube technology, instead of using internal beams as support. Each face of the John Hancock Center is comprised of six isosceles trapezoids.

Try This! Have each student draw three different isosceles trapezoids. Exchange papers with one student. Ask students to check one trapezoid to be sure it is isosceles by measuring non-base sides for congruence and equivalent diagonals. Have correcting students "sign off" on their opinions. Switch papers a second a third time, repeating the process.

Kites

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to present special properties of kites.

Visualization! Have students draw two isosceles triangles sharing the same base. By erasing the shared base, they have just drawn a kite! *The shared base represents one diagonal of the kite.*

Vocabulary! Using the phrase "ends of the kite" can be misleading for students. The ends do not always mean the endpoints of the longer diagonal. Be sure students can identify the ends of non-convex kites, and non-traditional kites (as shown below).
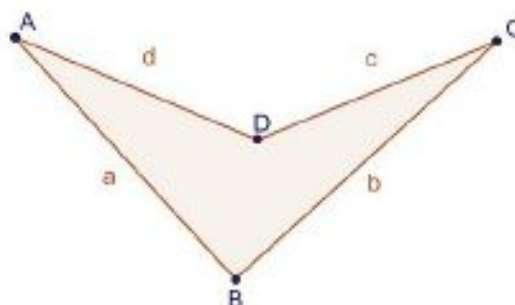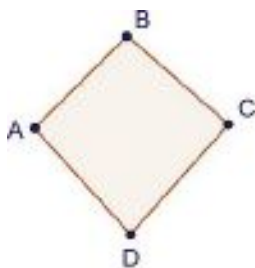
Beat the clock! Have students draw and cut out two copies of a scalene triangle. In one minute, have students form as many polygons as possible, drawing a sketch of each they form. *You can also use tangrams for this activity.*

1.7 Similarity

Ratios and Proportions

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to reinforce the algebraic concept of ratios and proportions. Proportions are necessary when discussing similarity of geometric objects.

What's the difference? Ratios and rates are both fractions. However, ratios compare same units, while rates compare different units. Ask students to brainstorm types of ratios and rates. *Rates are typically much easier for students to identify – miles per hour, cost per pound, etc.*

Look Out! Students easily get confused when we throw proportions into the mix. For some reason, students do not realize that the equal sign $(=)$ in a proportion is different than a multiplication sign $(*)$ when asking to find the product of two fractions. For example, students will attempt to solve these two statements the same way:

$$\frac{3}{4} * \frac{x}{7} \qquad\qquad\qquad \frac{3}{4} = \frac{x}{7}$$

Encourage your students to understand the difference between finding the product of two fractions and using the means-extremes method of cross-multiplication

Look Out! Another pitfall is cross-multiplication versus cross-reducing. You may have to take some time to discuss the difference and allow your students to practice doing both.

Additional Example: A model train is built $\frac{1}{64}$ scale. The stack of the model is 1.5". How tall is the real smokestack?

Food For Thought! "Why are these the means?" The best answer I have heard was, "The extremes are called such because they are on the far ends of the equation, meaning $ad = bc$." Before the fraction bar became commonplace, people would write fractions using the colon. $3 : 6 = 1 : 2$. Therefore, 6 and 1 represent the means (middle values), while 3 and 2 represent the extremes.

Properties of Proportions

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to demonstrate to students that the order in which you write the proportion is irrelevant, the answer comes out identical.

Try This! Prior to reading through the lesson, and using the following example, ask students to create their proportion. Look for several different proportions. Spread these students around the room. Have the remainder of the class match their proportion to the "totem pole." *This allows students to see that there is no one correct way to write a proportion.*

Question: A yardstick makes a shadow 6.5[U+0080][U+0099] long. Raul is $6'$ 3". How long is his shadow? *Be sure students convert a yardstick to* 3 *[U+0080] [U+0099] before continuing with the proportion.*

While there are many correct ways to write a proportion, encourage students to visualize what the proportion is stating. For example, the following are both correct (as are their reciprocals):

$$\frac{\text{length (stick)}}{\text{shadow (stick)}} = \frac{\text{length (person)}}{\text{shadow (person)}} \qquad \frac{\text{length (stick)}}{\text{length (person)}} = \frac{\text{shadow (stick)}}{\text{shadow (person)}}$$

Nonetheless, the units are still the same in each ratio. The first proportion uses length and shadow as units while the second uses stick and person.

Similar Polygons

Pacing: This lesson should take one class period

Goal: This lesson connects the properties of proportions to similar polygons. An introduction to scale factors is also presented within this lesson.

An alternative way of determining a scale factor is by using the fraction $\frac{\text{image}}{\text{preimage.}}$ This relates to the notion that similar figures are formed by an applying an isometry, mapping an image onto its preimage. The first figure written in the similarity statement represents the preimage and the second figure represents its corresponding image. Therefore, the scale factor k, is a ratio of the length of the image to the preimage.

Extension! A golden rectangle is a rectangle in which the ratio of the length to the width is the Golden Ratio, approximately 1.6180339888. The Golden Ratio is denoted by the Greek letter Phi and can be found in nature, biology, art, and architecture. For example, golden rectangles can be found all over the Parthenon in Athens, Greece.

Using the diagram below, have your students measure distances and find how many golden rectangles can be found.

http://www.geom.uiuc.edu/ demo5337/s97b/art.htm
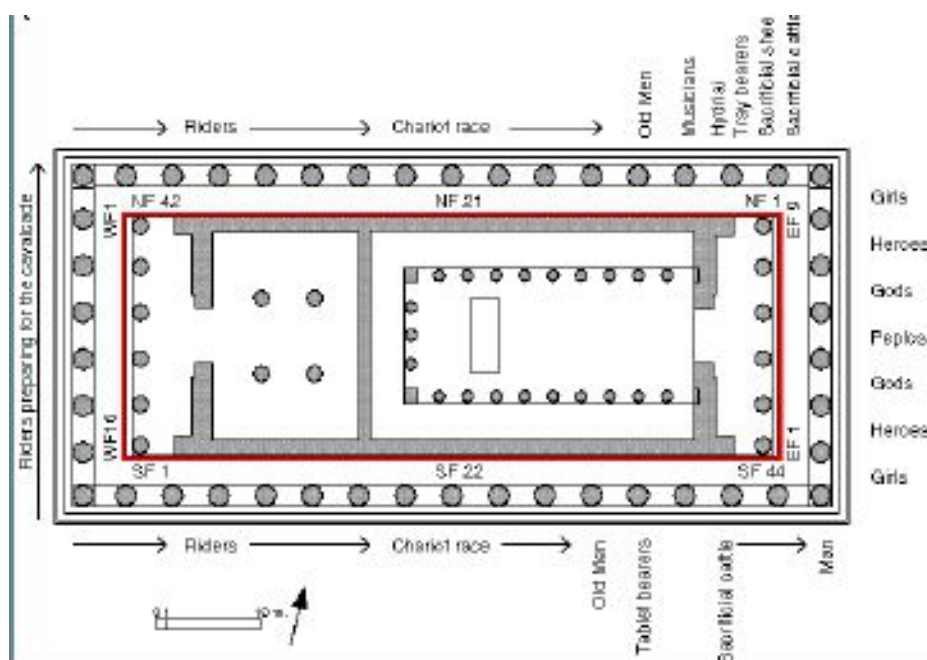
Similarity by AA

Pacing: This lesson should take one class period

1.7. Similarity

Goal: The purpose of this lesson is to enable students to see the relationship between triangle similarity and proportions. While the angle-angle relationship does not necessarily lead to congruence, its properties are still imperative to similarity.

How Does it Work? Indirect measurement utilizes the Law of Reflection, stating that the angle at which a ray of light (ray of incidence) approaches a mirror will be the same angle in which the light bounces off (ray of reflection). This method is the basis of reflecting points in real world applications such as billiards and miniature golf.

Additional Example: Pere Noel is shopping for a Christmas tree. The tree can be no more than 4 meters tall. Mary finds a tree that casts a shadow of 2 m, whereas Mary (120 cm tall) casts a shadow of 0.8 m. Will the tree fit in Pere Noel's room? *Yes, the tree is* 3 meters *tall, therefore, it will fit in the room.*

Similarity by SSS and SAS

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to extend the SSS and SAS Congruence Theorems to include similarity.

Visualization! Now may be a good time to discuss similarity and congruency by drawing a Venn diagram. Students may ask the question, "How can triangles be congruent and similar simultaneously?" The diagram below will help clear questions.

Similar figures are usually thought to be produced under dilations (size changes). However, congruent figures are a specific type of similarity transformation. Therefore, rotations, reflections, glide reflections, translations, and the identify transformation all yield similar figures.
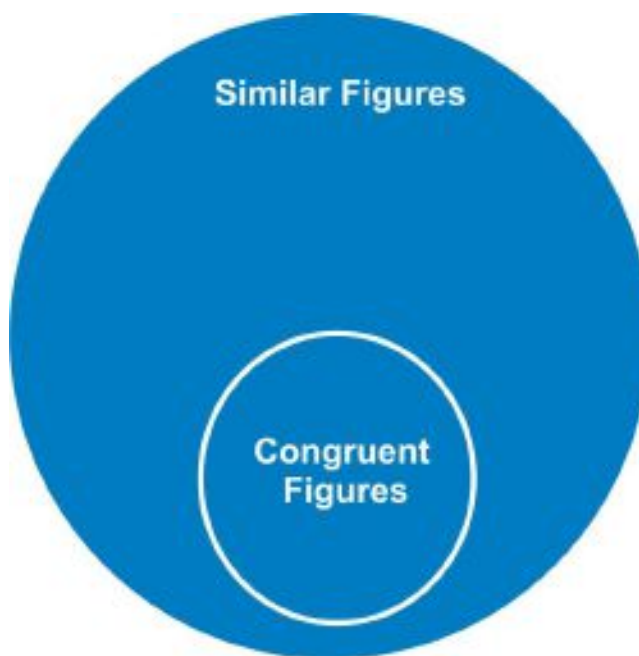
Similarity Transformations

Pacing: This lesson should take one class period

Goal: Dilations produce similar figures. This lesson introduces the algorithm to produce similar figures using measurements and a scale factor, k

An easy way to remember expansions versus contractions is a rhyme. Have your students repeat the rhyme until it sticks! "A contraction is a proper fraction!" Improper fractions are mixed numbers, thus creating expansions.

Dilations can also be clarified using a photograph. School pictures are great examples of dilations. Suppose a typical photograph is $4[\text{U+0080}][\text{U+009D}] \times 6[\text{U+0080}][\text{U+009D}]$. An $8[\text{U+0080}][\text{U+009D}] \times 10[\text{U+0080}][\text{U+009D}]$ enlargement (expansion) does not alter the appearance. This is also true for shrinking photos for wallets. Using a base picture, bring in several enlargements and contractions to further illustrate this concept.

The above visualization can be used when discussing notation. The first dilation is denoted using the apostrophe (') symbol. Sub sequent transformations add an additional apostrophe (", "', and so on). Labeling each picture you've made with this notation will allow your students to visualize how to use image notation.

In-Class Activity! Reproduce the smiley face drawn below for each student. Using the algorithm presented in this lesson, instruct students to enlarge the smiley face by a scale factor of 3.

www.clker.com/clipart-4263.html

Self-Similarity (Fractals)

Pacing: This lesson should take one class period

Goal: Fractals, a term coined only approximately twenty years ago, are a newly discovered genre of mathematics. This lesson introduces students to popular fractals. Fractals possess self-similarity, thus maintaining properties of similarity.

History Connection! Mathematician Benoit Mandelbrot derived the term "fractal" from the Latin word *frangere*, meaning to fragment. A fractal is a geometric figure in which its branches are smaller versions of the "parent" figure. Most fractals are explained using higher level mathematics, however, students can create their own fractal patterns easily.

Additional Example: Make your own fractals! Follow these instructions to create a cauliflower fractal.

  a. Hold your paper in landscape format.
  b. Draw a horizontal segment $\overline{AB}$ in the center of the paper.
  c. Find C, the midpoint of $\overline{AB}$.
  d. Measure AC and take half of that distance. This is essentially $\frac{1}{4}AB$.
  e. Draw $\overline{DC}$ (the length found in step 4) perpendicular to $\overline{AB}$.
  f. Draw lines $\overline{AC}$ and $\overline{BC}$, forming $\triangle ADC$.

1.7. Similarity

g. Repeat steps $3-6$ for $\overline{AC}$.
h. Repeat steps $3-6$ for $\overline{BC}$.

1.8 Right Triangle Trigonometry

The Pythagorean Theorem

Pacing: This lesson should take one class period

Goal: This lesson introduces the Pythagorean Theorem. It is arguably one of the most important theorems in mathematics, allowing for a multitude of uses, including the Law of Cosines.

Visualization! Here is a second proof of Pythagorean's Theorem that students can do in class.

a. Reproduce the following diagram for each student.
b. Students cut the red square and the blue square from the triangle.
c. Cut along the lines within the blue square. Students should have four pieces.
d. Students will fit these four puzzle pieces onto the yellow triangle, proving that the combined area of the two smaller squares equal the area of the largest square. Hence, $a^2 + b^2 = c^2$
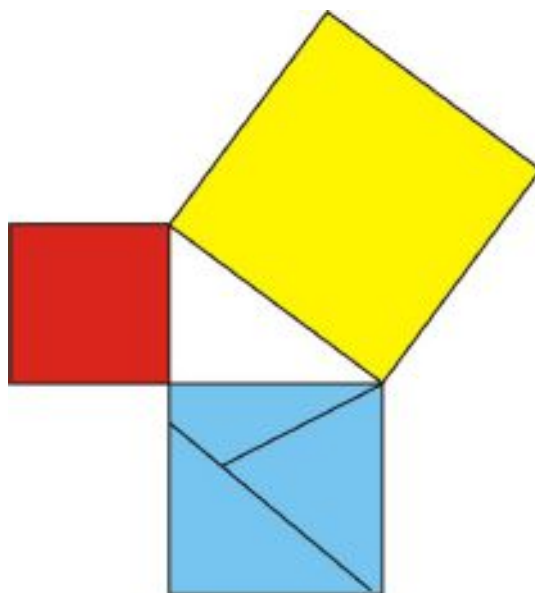
Illustration_to_Euclid's_proof_of_the_Pythagorean_theorem.svg

Additional Examples:

a. A rectangular park measures 500 m by 650 m. How much shorter is the path diagonally than walking around the outside edge?
b. Television sets are described according to its diagonal length. A 42[U+0080][U+009D] TV means the diagonal of the screen is 42[U+0080][U+009D] long. Suppose the TV below is 36[U+0080][U+009D] tall with a 47[U+0080][U+009D] diagonal. How wide is the TV?

Converse of the Pythagorean Theorem

Pacing: This lesson should take one class period

Goal: This lesson applies the converse of Pythagorean's Theorem to determine whether triangles are right, acute, or obtuse.

Additional Examples:

a. Can the following lengths form a right triangle? Explain your answer. $10, 15, 225$. *No,* $10^2 + 15^2 = 325$. *However, this will be much less than* 225^2.
b. Find an integer such that the three lengths represent an acute triangle: $9, 12,$ ____. *Sample:* $16, 17, 18, 22 \ldots$
c. Find an integer such that the three lengths represent an obtuse triangle: $8, 19,$ ____. *Sample:* $20, 10, 14, \ldots$

Using Similar Right Triangles

Pacing: This lesson may take two class periods, due to the difficulty of the material

Goal: The concept of geometric mean is used in Advanced Algebra to determine the mean of a widespread data set. In geometry, the geometric mean is illustrated using a right triangle and its altitude.

Look Out! The concept of geometric mean is easy to comprehend, but difficult for student to apply. Spend time in class reviewing this lesson and using additional examples.

An alternative to the abstract formula for geometric mean is, "The altitude of the hypotenuse equals the geometric mean between the segments of the hypotenuse."

Additional Examples:

a. Consider the diagram below. Suppose $h = 12$ and $x = 8$. Find y. $y = 18$
b. Consider the diagram below. Suppose $a = ?, x = 9$ and $y = 11$. Find the value of a. $a = 6\sqrt{5}$
c. Consider the diagram below. Suppose $x = 5$ and $y = 15$. Find the value of the altitude, h. $h = 5\sqrt{3}$
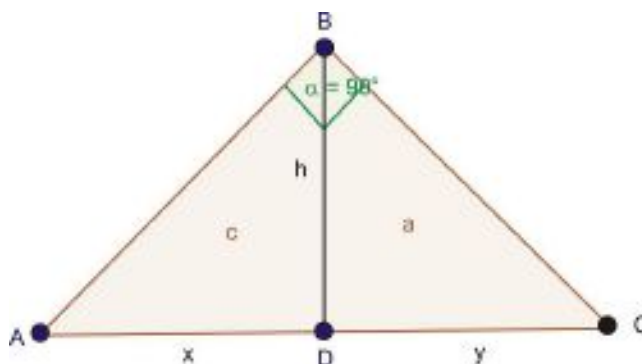
Special Right Triangles

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to encourage the use of shortcuts to find values of special right triangles. These triangles are extremely useful when relating the trigonometric functions to the exact values found within the unit

circle.

If students have trouble remembering these special shortcuts, encourage them to use Pythagorean's Theorem and simplify the answer. The resulting answer will equal the shortcut.

In-class Activity! Separate your class into six to ten groups. Write six to ten numbers on the board, one for each group. Instruct the groups to draw an isosceles right triangle with legs of the given length. Have the groups solve for the hypotenuse and share with the remaining groups.

What is the Connection? In any right isosceles triangle, if a leg has value of x, then by Pythagorean's Theorem, $x^2 + x^2 = h^2$. Adding like terms, you get $2x^2 = h^2$. To solve for the value of the hypotenuse, you must square root both sides, leaving the equation $x\sqrt{2} = h$. Therefore, the length of the hypotenuse in ANY right isosceles triangle is equal to leg $\sqrt{2}$

In any $30 - 60 - 90$ triangle, the relationship between the segments is as follows: Let the smallest leg have the value of d. The hypotenuse will always have length $2d$ and the other leg will always have length $d\sqrt{3}$. This again can be proved using Pythagorean's Theorem.

Tangent Ratios

Pacing: This lesson should take one class period

Goal: This lesson introduces the first trigonometric function, the tangent ratio. The tangent seems to be the most natural for students to understand, as opposed to the sine or cosine functions. Tangent ratios occur in many careers, from construction to machine operators.

Check Your Tech! If you are using calculators for the tangent (TAN), cosine (COS), and sine (SIN) functions, be sure to do a "Mode Check." Have each student check to ensure their calculator is set to degrees (DEG) instead of radians (RAD). Having a calculator in radians will provide incorrect answers and students at this level do not know what radians are to correct their answers.

Vocabulary Connection! Students must understand *adjacent* and *opposite* to be successful with trigonometric ratios. They have already had experience with adjacent in previous chapters. Begin by reviewing such vocabulary as adjacent angles and adjacent sides in regards to parallel lines and transversals.

Beginning Activity! Once the class has reviewed the term *adjacent,* offer students these triangles. Ask students to write the terms *adjacent* and *opposite* above the appropriate legs and label the hypotenuse. Always stress that the information stems from the given angle (not the 90 degree)!
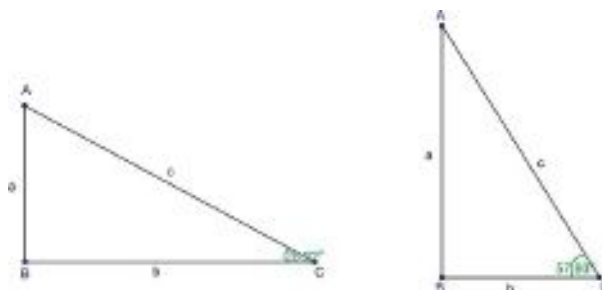
Extension! When discussing tangent values of common angles, such as $30°, 45°$, and $60°$, review how to rationalize the denominator with your students. This concept should have been presented in Algebra 1. While not as common with the increased use of technology, most standardized test questions will present answers in completely simplified form. For example, the tangent $(30°) = \frac{1}{\sqrt{3}}$. The objective of rationalizing a denominator is to clear it of decimals, radicals, and complex values. To do so, multiply the fraction by a value of 1, in this case, $\frac{\sqrt{3}}{\sqrt{3}}$. The new expression

becomes $\frac{\sqrt{3}}{\sqrt{3}}$, a simplified version of the tangent of 30 degrees. *Additional Examples:*

a. Pradnya's kite is 50″ away from her in the sky, forming a 27° angle with the ground. Pradnya is 4′ 6" tall. How high is the kite from the ground? *Approximately* 30″
b. The roof pitch is always described in terms of rise/run. Suppose the roof makes a 65° with the horizontal truss and forms the triangle below. How tall is the peak of the roof? 37.53″
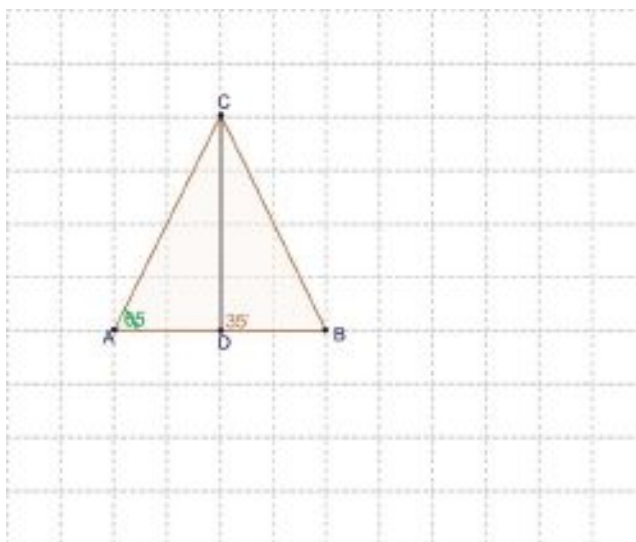
Arts and Crafts Time! Using paper, have each student create an astrolabe and use it to determine the height of a tall object, such as a skyscraper or tree.

a. Begin by folding an 8.5″ × 11″ sheet of notebook paper into a square and remove the excess.
b. Bisect on angle of the square – the segment represents a 45 – degree angle.
c. Bisect each 45 – degree angle. There should be three creases – two 22.5 degree angles and one 45 – degree angle.
d. Punch a hole in the opposite corner. Tie string through this hole and attach a pencil at the other end.
e. Go outside and line your astrolabe to the top of something, say a tree. Pretend you are hunting and plan to attack something with your astrolabe.
f. Gravity will show you the degree of your sight.
g. Use trigonometric functions to determine its height.

For more information, go to Berkley's website: http://cse.ssl.berkeley.edu/AtHomeAstronomy/activity_07.html

Sine and Cosine Ratios

Pacing: This lesson should take one to two class periods

Goal: The objective of this lesson is to complete the introduction of trigonometric functions by presenting the sine and cosine ratios.

Welcome to Camp SOH − CAH − TOA! To make trigonometry fun, invite students to Camp *SOH − CAH − TOA*. Wear a camp counselor outfit, arrange your students in a circle around a makeshift campfire, and begin with an old-fashioned tent (one you can use to point out right triangles).

Have students sketch your tent, splitting it into two right triangles at the altitude (good use of vocabulary!). State that the angle the tent makes to the ground is 55° (something you cannot use special triangles for). Ask students to label each triangle with the appropriate terms: adjacent leg, opposite leg, and hypotenuse. The question is, "How long is the outside edge of the tent?" Question why the tangent ratio cannot be used *(the question you want to answer is not the opposite nor adjacent leg)*. Ask for additional ways to solve the problem. Present the sine and cosine ratios.

Additional Examples/Extensions:

   a. Given the triangle below, find sine(A) and cos(B). What is special about these two answers?
   b. Evaluate $(\cos(65))^2 + (\text{sine}(65))^2$. List as much as you can about this expression and the answer you received. Generalize this question. $(\cos(65))^2 + (\text{sine}(65))^2 = 1$, *the degrees of the sine and cosines are the same value, so the cosine of an angle square plus the sine of an angle squared should equal* 1.
   c. Suppose a fireman's ladder is 39 `[U+0080]` `[U+0099]` long is placed against the side of a building at a 62 degree angle. How high will the ladder reach? 18.31 `[U+0080]` `[U+0099]`

Inverse Trigonometric Functions

Pacing: This lesson should take one to two class periods

Goal: In the previous two lessons, students used the special trigonometric values to determine approximate angle measurements. This lesson enables students to "cancel" a trigonometric function by applying its inverse to accurately find an angle measurement.

Using Previous Knowledge! Begin by listing several mathematical operations on the board in one column. In a second column, head it with "Inverse." Be sure students understand what an *inverse* means *(an inverse cancels an operation, leaving the original value undisturbed)*.

For example,

TABLE 1.6:

OPERATION	INVERSE
Addition	
Squaring	
Division	
Subtraction	
Tangent	
Sine	

The first four are typically easy for students *(Subtraction, square root, multiplication, and addition)*. You may have to lead students a little more on the last two *(inverse tangent and inverse sine)*. Students may say, "Un-tangent it." Use the correct terminology here, but also use their wording, if at all possible. Students will be able to cancel the trigonometric function using the inverse of that function, even though they may use incorrect terminology.

Outside at Camp SOH − CAH − TOA! Find the angle of inclination of the sun! Students love this activity, as it gets them outside and applying mathematics in real life. Explain how the Earth progresses around the sun, giving seasons. Also explain how the Earth's tilt lends to the number of hours of daylight. The combination of these principles describes the angle of elevation (inclination) of the sun in the sky. Create groups of three or four. One student is the statue, one student is the surveyor, and the third is the secretary. Once outside, the statue stands on a flat plane while the surveyor measures the statue's height and shadow length in the same units and relays this information to the secretary. The secretary draws a right-triangle sketch of the statue and shadow. The goal is to find the angle of elevation (the angle made between the horizon and sun).

Acute and Obtuse Triangles

Pacing: This lesson could take two to four class periods

Goal: The purpose of this lesson is to extend trigonometric ratios to non-right triangles. This is done using to new laws: the Law of Sines and the Law of Cosines. The Law of Sines is much easier to present to students than the Law of Cosines. The Law of Sines uses proportions, while the Law of Cosines uses a general form of Pythagorean's Theorem.

Shortcut! Students confuse themselves regarding which law to use. A shortcut to use is as follows: If the triangle has more side information than angle information, use the Law of Cosines. If the triangle has more angle information than side information, use the Law of Sines. And of course, if you cannot solve it with the law you have chosen, "Choose the Other One!"

Relate to Triangle Similarity! Have students recall the five basic types of triangle similarity: SSS, SAS, AAS, AAA, ASA. Reading the given information from left to right, if the information ends in an "angle," use the Law of Sines. If the information ends in a "side," use the Law of Cosines.

Additional Examples:

1. A fire is spotted in Yellowstone National Park by two forest ranger stations. Fire Station A is 15 km from Fire Station B. The angle at which the fire is spotted by Fire Station B is 75 and the angle at which the fire is spotted by Fire Station A is 70 degrees. Which fire station should report to the fire? $AB = 25.26$ km *and* $BC = 24.57$. *Therefore, Fire Station A should report to the fire.*
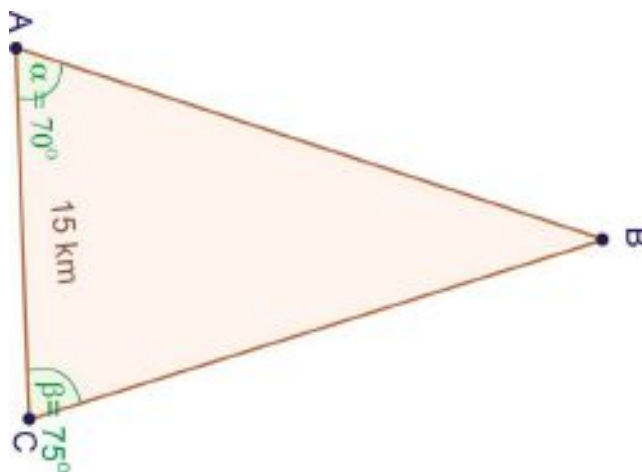
2. Suppose $\triangle ABC$ has the following values: $m\angle C = 40, a = 8, b = 9$. Find c. $c = 5.89$

1.9 Circles

About Circles

Pacing: This lesson could take two to four class periods

Goal: This lesson discusses numerous characteristics of circles. Inscribed polygons, equations of a circle, diameters, secants, tangents, and chords are all presented in this lesson.

Round Robin! Once students have read through this lesson, try this fun activity. Round robin tournaments are scheduled so each team players another exactly once. Circles and chords are used to schedule such a tournament.

Using a compass, have the students construct a circle that takes up most of an 8.5 [U+0080] [U+009D] $\times 11$ [U+0080] [U+009D] sheet of copy paper. Decide upon a collegiate conference, such as the Big 10 and instruct students to evenly space these dots around the circle. *To do this, divide* $360°$ *by* 11. *This will give students the number of degrees before each new dot is placed.* Choosing 11 colored pencils, begin at one dot and draw a chord to a second dot (see diagram below). Draw parallel chords until all but one team has an opponent; the leftover team has a bye. This color represents week 1. Using a second color, start at another dot and connect it to a different team. Continue this process until all 11 weeks have been "scheduled."

If there is an even number of teams to be scheduled, place one dot in the center of the circle and set each remaining dot on the boundary. Start by drawing a radius from the center to any point, then draw chords to the remaining teams. There will be no byes with even numbered teams.
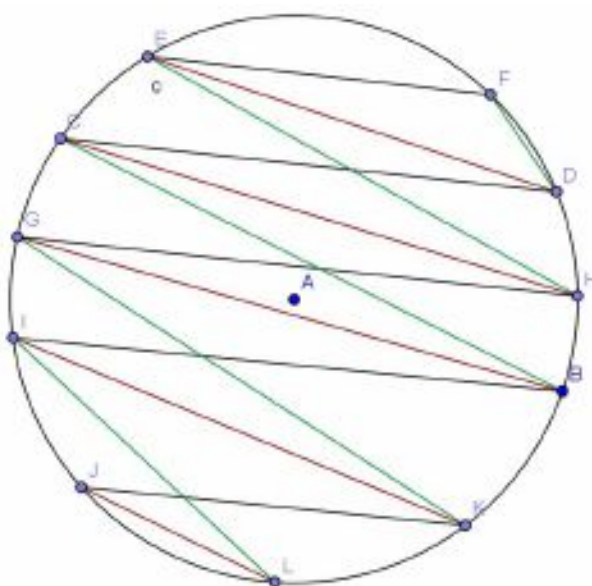
Visualization! Fill a clear bowl or container with water. Show how concentric circles are formed by dropping a rock into water, forming ripples. Concentric circles can also be formed when raindrops hit a body of water, such as a lake, puddle, etc.

1.9. Circles

Tangent Lines

Pacing: This lesson should take one to two class periods

Goal: The purpose of this lesson is to connect the radius of a circle to its tangent. The real life applications of tangents are found in the additional examples below.

Connection! Tangents are used in Calculus; the slope of the tangent line represents the derivative. Students should make this connection visually once Calculus begins.

Additional Examples:

a. Find the perimeter of *ABCD*. 50 cm
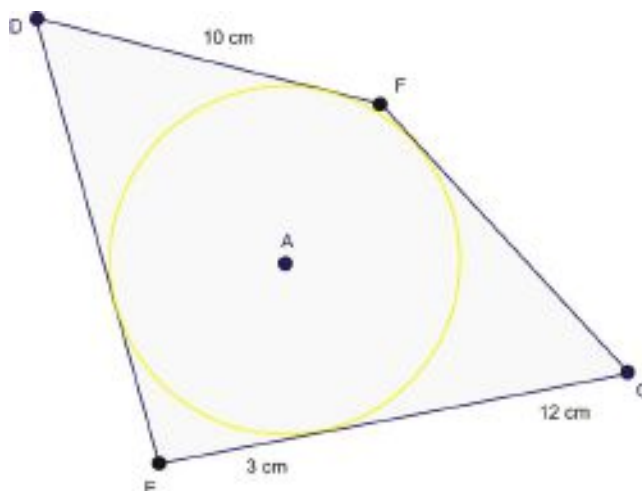
Common Tangent and Tangent Circles

Pacing: This lesson should take one class period

Goal: This lesson focuses on the properties shared with circles sharing common tangents.

Additional Example:

a. A dirt bike chain fits snugly around two gears, forming a diagram like the one below. Find the distance between the centers of the gears. Assume the distance from *B* to the top of the second circle is 30.25 [U+0080] [U+009D]. 30.254 [U+
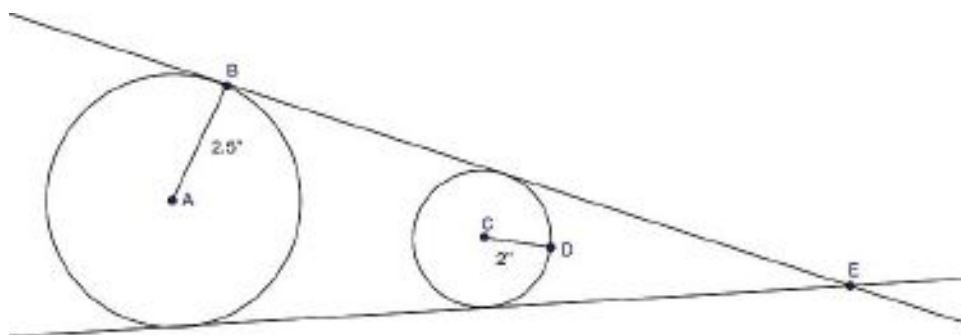
Arc Measures

Pacing: This lesson should take one class period

Goal: This lesson introduces arc measurements. Central angle knowledge is important when creating and interpreting pie charts and when determining arc length and the area of a sector.

Students may need extra practice when it comes to finding the value of the arc. Set up several questions and use personal whiteboards to check students' understanding.

The Arc Addition Property should be familiar to students; it is quite similar to the Angle Addition Property.

Additional Example:

Identify the following in $\odot A$

A. Minor arcs

B. Major arcs

C. Semicircles

D. Arcs with equal measurement

E. Identify the four major arcs that contain point F.
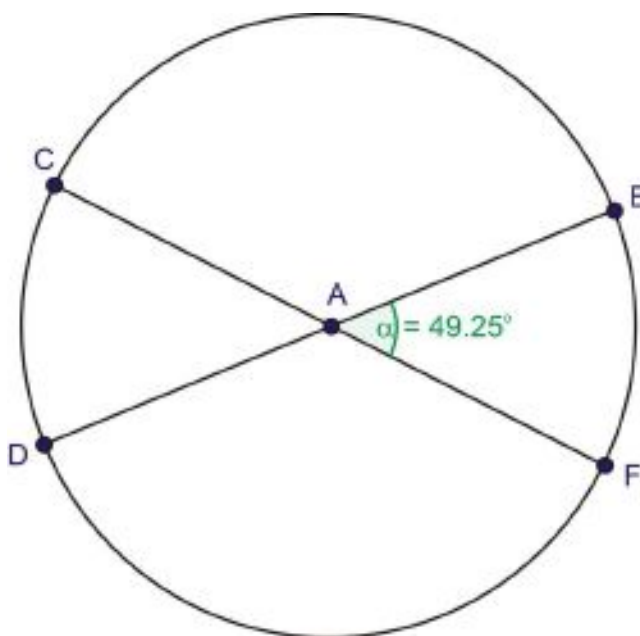
Inscribed Angles

Pacing: This lesson should take one to one and one-half class periods
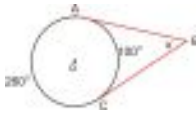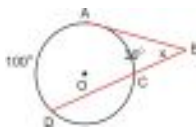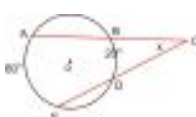
Goal: This lesson will demonstrate how to find measures of inscribed angles and how to find the measure of an angle formed by a tangent and a chord.

In-class activity! Have students copy the drawings below.

    a. In $\odot A$, use a protractor to find the measures of $\angle CDE, \angle CFE$, and $\angle CBE$. Determine the measure of arc CE.

a. Write a hypothesis regarding measure arc *CE* and ∠*CDE*.
b. Write a hypothesis regarding the measures of ∠*CDE*, ∠*CFE*, and ∠*CBE*.

b. Use a protractor to measure ∠*IKJ*, ∠*ILJ*, and *IHJ*.

a. Write a hypothesis about an angle whose vertex is on the boundary of a circle and whose sides intersect the endpoints of the diameter.

Angles of Chords, Secants, and Tangents

Pacing: This lesson should take one class period

Goal: This goal of this lesson is to explain the formulas for determining angle measures formed by secants and tangents.

Vocabulary: According to www.encylcopedia.com, "A secant is a line that intersects a curved surface." A chord is a line segment connecting two points on the boundary of a circle. Thus, a chord is a segment of the secant.

Set up a chart similar to the one below to help students organize their formulas for this lesson and the next.

TABLE 1.7:

	Example	Angle Formula	Segment Length
Tangent Chord		$\frac{1}{2}$ (intercepted arc)	
Angles formed by two intersecting chords		Sum of intercepted arcs	

TABLE 1.7: (continued)

	Example	Angle Formula	Segment Length
Angle formed by two intersecting tangents		Difference of intercepted arcs	
Angle formed by two intersecting secants		Difference of intercepted arcs	
Angle formed by a tangent and secant intersection		Difference of intercepted arcs	

Segments of Chords, Secants, and Tangents

Pacing: This lesson should take one class period

Goal: This goal of this lesson is to explain the formulas for determining segment lengths formed by intersecting secants and tangents.

Organize! Finish the chart began in the previous lesson.

TABLE 1.8:
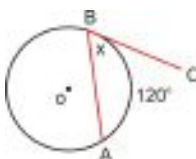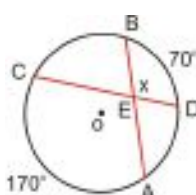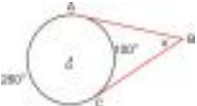
	Example	Angle Formula	Segment Length
Tangent Chord		$\frac{1}{2}$ (intercepted arc)	N/A
Angles formed by two intersecting chords		Sum of intercepted arcs	$a*b = c*d$ whole secant$_1$ * external part$_1$ = whole secant$_2$ * external part$_2$

1.9. Circles

TABLE 1.8: (continued)

	Example	Angle Formula	Segment Length
Angle formed by two intersecting tangents		Difference of intercepted arcs	N/A
Angle formed by two intersecting secants		Difference of intercepted arcs	$a * b = c * d$ whole secant$_1$ * external part$_1$ = whole secant$_2$ * external part$_2$
Angle formed by a tangent and secant intersection		Difference of intercepted arcs	$b * c = a^2$ whole secant * external part = tangent^2

1.10 Perimeter and Area

Triangles and Parallelograms

Pacing: This lesson should take one class period

Goal: This lesson introduces students to the area formulas for parallelograms and rectangles. It also illustrates the relationships between these formulas.

Flashcards! Creating another set of flashcards will be second-nature to our students by now. These flashcards should also be double-sided. The blank side should be a sketch of the figure and its special name. The flip side should repeat its definition, the sum of its interior angles, the expression for its perimeter, the formula for its area, and the formula for its perimeter. Have students create flashcards as the chapter presents the figures; this lesson only covers triangles, rectangles, and parallelograms.

Visualization! Encourage students to see how a parallelogram can be transformed into a rectangle by performing the activity presented in the lesson.

Extra Credit? You may want to offer extra credit for students who can correctly determine the total area of the eight circles found in the introduction of this lesson. $16 - 8 * (\pi * (\frac{1}{2})^2) = 16 - 4\pi \text{ft}^2$.

Extension! Using the hexagon below, find its area. *Students use the concept of triangles and interior angles.* Note** This may be more appropriate for students to attempt once the trapezoid area formula has been presented.
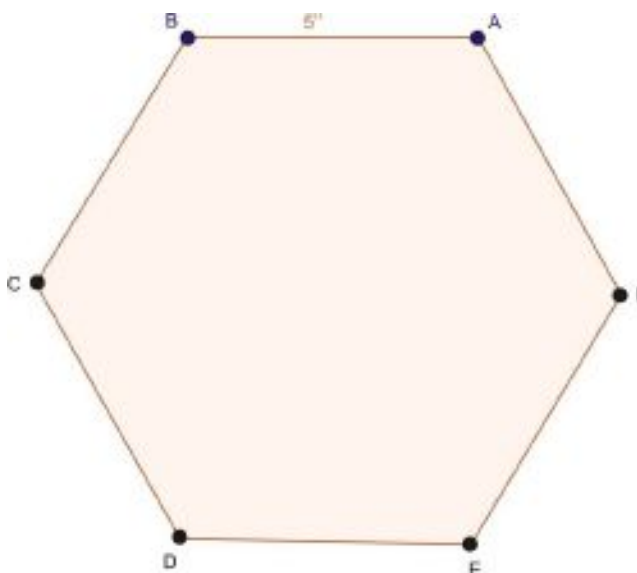
Trapezoids, Rhombi, and Kites

Pacing: This lesson should take one class period

Goal: This lesson further expands upon area formulas to include trapezoids, rhombi, and kites.

Additional Examples:

1. Trina has a rectangular flower garden, as shown below. The area of the garden is $1,602$ ft^2. How long is the bottom edge? *Students must realize the bottom edge represents the altitude of the trapezoid.*
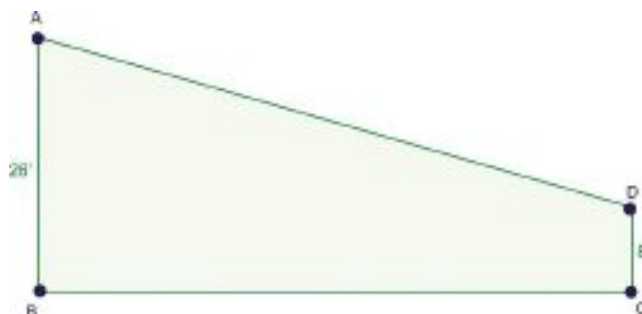
2. What is the area of the kite pictured at the right?
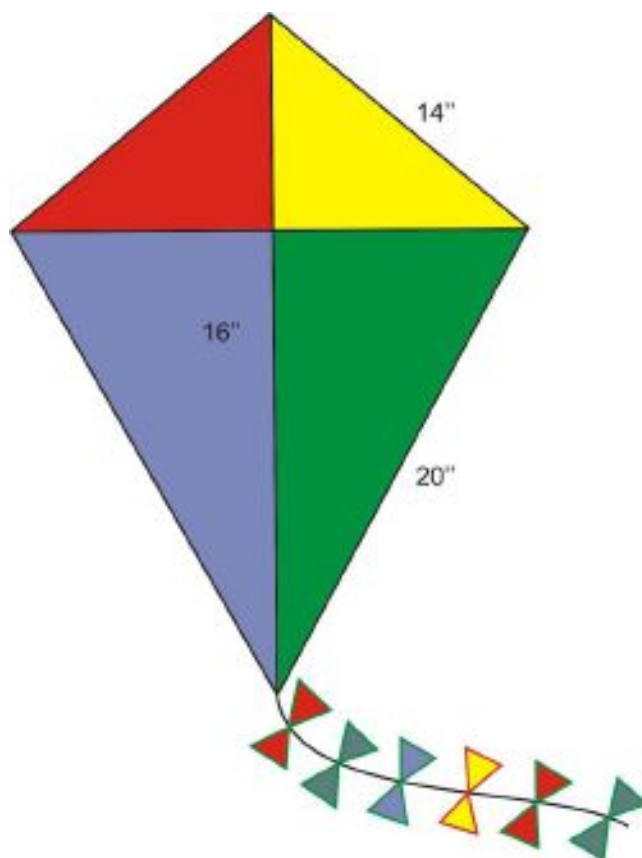
www.clker.com/clipart-kite.html

3. Assume the rhombus below has an area of 45 in^2. One diagonal measures 5 inches. What is the length of the second diagonal? *Extension:* How long is each segment of the rhombus?
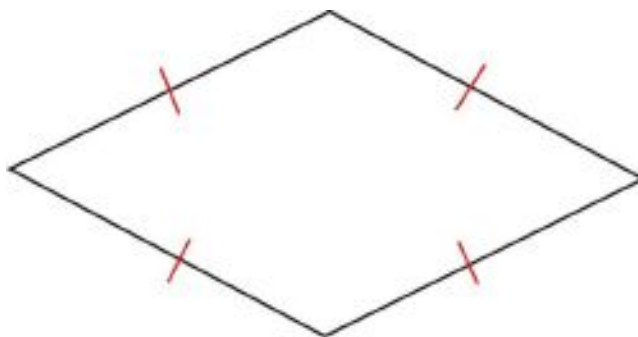
Areas of Similar Figures

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to connect similarity, areas, perimeters, and the scale factor k.

Look Out! Students may get confused when attempting to set up a proportion regarding similar area. Areas have a ratio of k^2, whereas perimeters have a ratio of k. This may be a good time to review the fraction $\frac{\text{image area}}{\text{preimage area}} = \frac{k^2}{1}$. Encourage students to use this proportion when referring to area.

Could There Be Giants? Before reading the section "Why There Are No 12' Giants," discuss Robert Wadlow, the tallest man on record. The following website is the "official" Robert Wadlow information guide. Students are captivated at Wadlow's shoe size, his growth chart, and the photographs that can be found on this site. Encourage students to further research people of great height and do quick summaries of their findings.

Gulliver's Travels! Written by Jonathan Swift, Gulliver's Travels tells of a man who visits two worlds, one where people are 12 times his size (Brobdignagians), and another where people are $\frac{1}{12}$ his size (Lilliputians). Use these values for k to describe such things as the area of footprints.
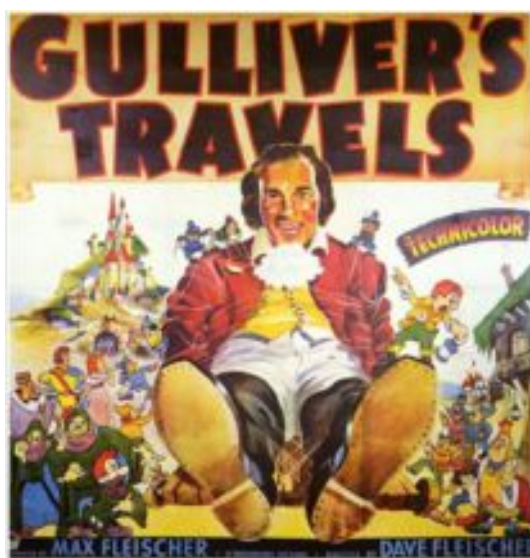
i445.photobucket.com/.../mathewcmills/gull2.jpg

1.10. Perimeter and Area

Circumference and Arc Length

Pacing: This lesson should take one to two class periods

Goal: The purpose of this lesson is to introduce the circumference formula and derive a formula for arc length (portions of the circumference)

Who Wants Pizza? Use pizza, pies, or cookies as a visual for this lesson's formulas. Give each student one of the aforementioned round objects and a piece of string. Ask students to measure how much string it takes to circle around the object. Explain to students that this is the circumference.

When discussing arc length, split the object into six, eight, ten, or twelve even sections. Revisit central angles and the fraction of the whole. This fraction $(\frac{1}{8}, \frac{1}{6}, \frac{1}{10}, \frac{1}{12})$ will be your multiplier to the entire circumference.

Look Out! Students can become confused regarding the Pi symbol (π). Students tend to view this as a variable instead of an approximate value.

Exact versus approximate. Students wonder why it is necessary to leave answer in exact value (π), instead of approximate (multiplying by 3.14). This is usually a teacher preference. By using the approximate value for Pi, the answer automatically has a rounding error. Rounding the decimal too short will cause a much larger error than using the decimal to the hundred-thousandths place. Whatever your preference, be sure to explain both methods to your students.

Circles and Sectors

Pacing: This lesson should take one to two class periods

Goal: The purpose of this lesson is to introduce the area of a circle formula and derive a formula for its fractional area, the sector.

Arts and Crafts Time! Use a compass to draw a large circle. Fold the circle horizontally and vertically along its diameters and cut into four $90°$ wedges. Fold each wedge into quarters and cut along lines. Students should have 16 wedges. Fit all 16 pieces together to form a parallelogram, where the width of the parallelogram is the radius of the circle and the length is some value *b*. *Students will see that the area of a sector must be a portion of the whole.*

Who Wants Pizza? Use pizza, pies, or cookies as a visual for this lesson's formulas. Give each student one of the aforementioned round objects. Illustrate area by discussing the amount of material needed to make the cookie, dough, etc. is an example of area. Have students discuss why the previously learned area formulas will not provide an accurate answer. Present the area of a circle formula and have students calculate the area of their individual object.

When discussing the area of a sector, divide the objects into $6, 8, 10,$ or 12 even sections. Each section represents a fraction of the whole, thus can be modeled by determining the fraction and multiplying it by the entire area.

Additional Examples:

a. How much more pizza is in a 16[U+0080][U+009D] diameter pizza than a 12[U+0080][U+009D] diameter pizza? 87.96 in^2
b. Suppose a 14[U+0080][U+009D] pizza is cut into 10 slices. What is the area of two slices? 30.79 in^2

Regular Polygons

Pacing: This lesson should take one to two class periods

Goal: The purpose of this lesson is to introduce the formulas to determine the areas of regular polgyons by defining the *apothem.*

Additional Examples:

   a. Find the area of a regular pentagon with 11.8 cm sides and a 9.2 cm apothem. 271.4 cm^2.
   b. Find the area of a regular hexagon is its side length is 20 [U+0080][U+009D]. 1039.23 in^2.
   c. Find the area of the regular quadrilateral below. 32 mi^2
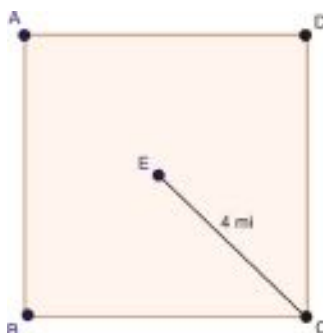
Geometric Probability

Pacing: This lesson should take one class period

Goal: Students will apply the formula for general probability to geometric objects.

Extension! The probability formula can also be applied to areas. $P = \frac{\text{area of favorable outcome}}{\text{total area}}$. Use this formula for the following additional examples.

Additional Examples:

1. What is the probability of landing in the bulls-eye of the dartboard below? Probability $= 1.56\%$

1.10. Perimeter and Area
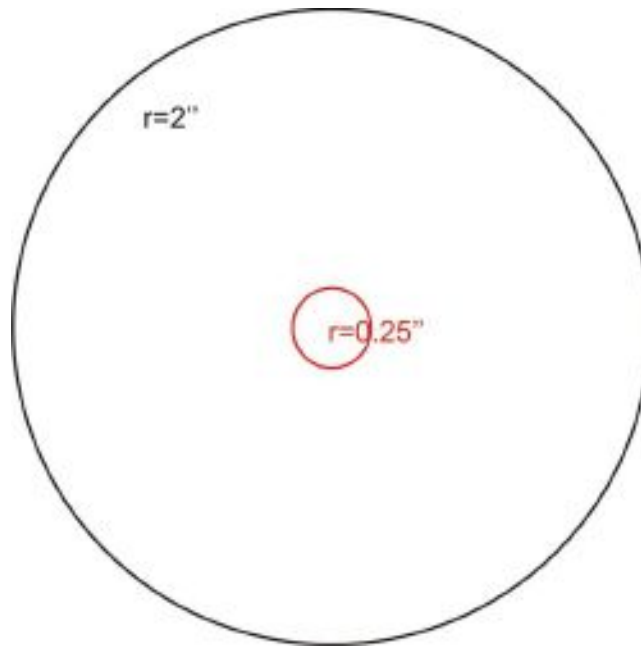
2. What is the probability that if you jump off the roof, you will land on the deck instead of the pool? Probability = 73.63%
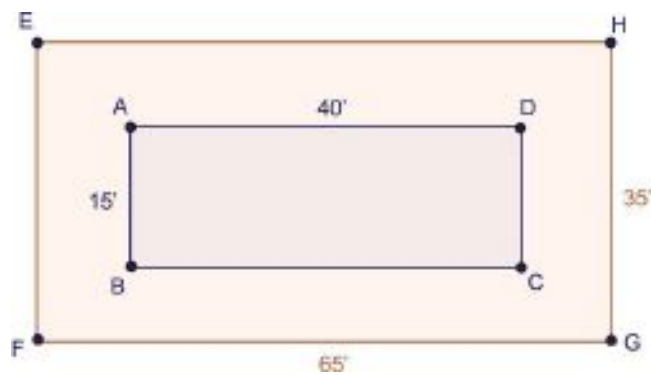
1.11 Surface Area and Volume

The Polyhedron

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to introduce students to three-dimensional figures. Polyhedral figures are presented in this lesson and common terms such as edge, vertex, and face are explained.

Grocery Shopping! Begin gathering objects from home that you can use in subsequent lessons. Collect empty cereal, rice, or pasta boxes, empty or full cans of soup, Stackers containers (triangular prisms), etc. These objects will help students visualize nets, presented in the next lesson.

Arts and Crafts Time! Download the nets found on Mathforum's website. Have students color, cut, and adhere the edges together to form Platonic polyhedra. http://mathforum.org/alejandre/workshops/net.html

For More Information - Click on the following link to gather more information regarding polyhedral figures. http://mathforum.org/alejandre/workshops/unit14.html

Upcoming Vocabulary! Lateral face and lateral edge are two common vocabulary words students should learn. *Lateral face* is a non-base face (usually the sides). *Lateral edge* is the segment where two lateral faces meet.

Representing Solids

Pacing: This lesson should take one to two class periods

Goal: The purpose of this lesson is to introduce students to the various types of representations of solid figures. Most will come naturally to students and should be presented as a fun lesson.

Become an Architect! After discussing orthographic views, collect students into groups of 2 or 3. Offer each group a collection of wooden blocks. Their only rule is to build something – a building, house, the Parthenon, etc. Explain to students that architects will often visualize the completed 3-D building from two-dimensional drawings.

Once all the creations are complete, students will rotate to a different structure and sketch its top, front, back, and side views. *Students are drawing the orthographic views of a 3-D structure.* Students may complete additional drawings as an assignment, in-class activity, or extra credit.

Cross Section View, Using a Breadknife! When discussing cross-sections, bring in a loaf of bread and a breadknife. Illustrate the perpendicular cross by cutting through the bread vertically. You could also show non-perpendicular cross section by cutting through the bread at different angles.

Nets. Using the collected cereal boxes, have students turn these into nets of prisms and draw sketches. *Students will use real life objects to visualize nets.*

Prisms

Pacing: This lesson should take one to two class periods

Goal: This lesson introduces students to the surface area and volume formulas of prisms.

Flashcards! The focus of these flashcards is to organize 3−dimensional formulas for area and volume. You could use the following chart to help students begin to organize their flashcards.

TABLE 1.9:

	Surface Area Formula	Volume Formula
Cube		
Rectangular Prism		
Triangular Prism		
Hexagonal Prism		

Additional Examples:

1. How much honey can 65 honeycomb cells hold if each hexagonal cell is $\frac{1}{8}$[U+0080][U+009D] long by $\frac{1}{4}$[U+0080][U+009D] deep? 2.64 in^3

http://www.flickr.com/photos/justusthane/1252907196/

Cylinders

Pacing: This lesson should take one to two class periods

Goal: This lesson introduces students to the surface area and volume formulas of cylinders.

Organization! Add the following rows to your table began in the previous lesson.

TABLE 1.10:

	Surface Area Formula	Volume Formula
Cylinder		

Visualization! Using soup cans or other cylindrical objects, show students the lateral face of a cylinder by peeling the label from the can. *Students will see the lateral face is a rectangle, not a circle.*

Additional Example:

a. A drinking straw has is 11[U+0080][U+009D] long with a 0.5[U+0080][U+009D] diameter. How much plastic is needed to form the straw? 17.28 in^2.

b. Using the same straw, how much liquid can it hold? 2.16 in^3

Pyramids

Pacing: This lesson should take one to two class periods

Goal: This lesson introduces students to the surface area and volume formulas of cylinders.

Lab Investigation! Before you read through the volume of a pyramid section, have your class complete this lab! This is a great way to demonstrate the relationship between the volume of a prism and the volume of a pyramid.

Fill a gallon jug with water colored with food coloring. Separate students into groups of three or four. Each group should receive a prism and its matching pyramid. **The bases and heights must be identical for this to work!** Instruct one student to measure the necessary values of the 3−dimensional figures (altitude and lengths of base) while another student records the information. A third student will fill the **pyramidal figure** with colored water and pour it into the prism. The goal is to determine how many times it will take to fill the prism. *The answer should be approximately* 3.

Encourage students to write a hypothesis regarding the relationship between these two volumes. *Students should state that* $3 * \text{pyramid} = \text{prism}$.

Additional Examples:

a. Draw a net for a right pentagonal pyramid.

Cones

Pacing: This lesson should take one to two class periods

Goal: This lesson introduces students to the surface area and volume formulas of cones.

Additional Examples:

1. Draw the net for a right cone with diameter 3 cm and height 5 cm.

2. How much material is needed to make the waffle cone shown below with dimensions 12[U+0080][U+009D] tall with 5[U+0080][U+009D] diameter? 96.29 in^2

1.11. Surface Area and Volume

3. The conical water cup has dimensions 6″ with a 3.5″ diameter. How much can it hold? 57.73 in^3

Cones

Pacing: This lesson should take one to two class periods

Goal: This lesson introduces students to the surface area and volume formulas of spheres.

Geography Connection! Using a globe, show students how a sphere is formed using rotation. The teardrop shaped pieces are called *gores.* Once placed together and folded so they meet at the top and bottom (poles), a sphere is formed.

The *great circle* is a cross section of a sphere cutting through the widest part of the sphere, the equator. Any other cross section is called a *small circle.* Using the globe, show students examples of each (i.e. The Artic Circle and the Equator).

Extra research! Have students research the different types of maps and list the pros and cons of each, relating to the true topography of Earth.

Additional Examples:

   a. How much leather is needed to make a baseball with a 6.5″ diameter? 132.72 in^2
   b. A plant container is a hemisphere with a radius of 17.″ How much dirt can it hold? *Don't forget to divide your answer in half* – 1286.22 in^3

Similar Solids

Pacing: This lesson should take one to two class periods

Goal: This lesson introduces students to the surface area and volume formulas of cones.

Gulliver's Travels Revisited! In a previous lesson, we connected Gulliver's Travels to areas of similar figures, such as the footprints of Gullivers versus the Lilliputians. Extend this topic to surface area and volume.

The surface area (amount of material needed to make clothing, etc.) has a ratio $\left(\frac{\text{image area}}{\text{preimage area}} \right)$ of k^2. Therefore, if a Lilliputian was $\frac{1}{12}$ the size of Gulliver and Gulliver was $\frac{1}{12}$ the size of a Brobdignagian, the amount of material needed to cloth a Brobdignagian would be 144^2 times the amount needed for a Lilliputian!

This also related to volume (weight). The ratio here would be k^3. Have fun and try lots of different ratios!

1.12 Transformations

Translations

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to introduce the concept of translations (slides) in the coordinate plane.

Vocabulary! The word *isometry* refers to any type of transformation...

Did You Know? Translations are formed by a composite of reflections over parallel lines. To illustrate this in further detail, see the **Reflections** lesson.

Vectors Outside the Coordinate Plane! Vectors can be used to translate an object not on a coordinate plane. A vector in this case tells the length and direction to translate a figure. Use the diagram below.
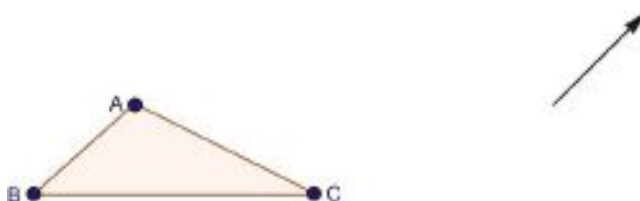
The black ray represents the vector. Therefore, $\triangle ABC$ should be translated *NNE* the length of the vector. The resulting translation is pictured below.
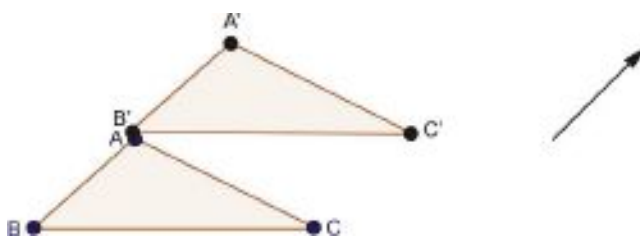
Additional Example:

Suppose A[U+0080][U+0099]B[U+0080][U+0099]C[U+0080][U+0099]D[U+0080][U+0099] is the image of $ABCD$ under a translation by vector $c = (-8, 3)$. What are the vertices of the preimage is the image has vertices at the following locations: A[U+0080][U+0099] $= (0,0), B$[U+0080][U+0099] $= (1,5), C$[U+0080][U+0099] $= (-4,4), D$[U+0080][U+00 $(-7,-10)$?$A = -8, -3), B = (9,2), C = (4,1), D = (-1,-13)$

Matrix Addition

Pacing: This lesson should take one class period

Goal: Matrices are useful in geometry, as well as algebra and business. This lesson introduces students to the basics of matrices, namely, matrix addition.

Basics of Matrices! Matrices are referred to according to its dimensions, rows by columns. To get students thinking about which is which, use this phrase. "You row ACROSS a lake and columns hold UP houses." By relating rowing across and columns up, students should correctly organize the information.

Matrices can only be added if the dimensions are equivalent. Because adding matrices requires adding the same cell, there must be equal numbers to combine.

Excel spreadsheets are excellent examples of matrices. If you have the ability to set up such spreadsheet matrices, students can see how businesses use these to organize and manipulative inventory.

Vocabulary! A 2×1 matrix organizing a point is called a *point matrix*. The $x-$values should go into row 1 and the $y-$values should go into row 2. The columns represent the points of the figure in the coordinate plane.

Additional Example: Target is processing its baby items inventory. Arrange the following into a matrix. Shirts: $24 - 2T, 0 - 3T, 9 - 4T$; shorts: $5 - 4T, 17 - 2T, 11 - 3T$; pants: $8 - 3T, 0 - 4T, 3 - 2T$.

Suppose another Target is shipping its excess inventory to this store. Write the sum of the two shipments into a single matrix.

TABLE 1.11:

	$2T$	$3T$	$4T$
Shirts	25	6	7
Shorts	8	19	12
Pants	1	4	30

Reflections

Pacing: This lesson should take one class period

Goal: Reflections are an important concept in geometry. Many objects can be explained with reflections. This manipulation is also related to similarity and triangle congruence.

Matrix Multiplication! Matrix Multiplication can be quite difficult for students to compute by hand. Here is a way to use a graphing calculator to achieve the multiplication.

a. Find the Matrix menu. If your students are using a Texas Instrument product, it is located by typing the 2nd and x^{-1} keys.
b. Edit matrix A by moving to the right to the edit menu and choosing $[A]$. Input the dimensions and data.
c. Choose 2nd & MODE to quit the menu
d. Repeat steps $1 - 3$ for the second matrix $[B]$.
e. Choose matrix $[A]$ by repeating step 1 and touching enter under the Name Menu
f. Choose the multiplication symbol
g. Repeat step 7 but choose $[B]$ instead.
h. Your working menu should look like this: $[A] * [B]$
i. Touch ENTER. The answer resulting is the product of the two matrices.

Extension In-Class Activity! Reflections can be performed without a coordinate plane, just as translations.

a. Using patty paper (or tracing paper), have students draw a small scalene triangle $(\triangle DEF)$ on the right side of the paper.
b. Fold the paper so that $\triangle DEF$ is covered.
c. Trace $\triangle DEF$.

1.12. Transformations

d. Unfold the patty paper and label the vertices as $D\prime\prime, E\prime\prime$, and $F\prime\prime$, the image points of the D, E, and F.
e. Darken in the fold – this is the reflecting line. Label a point on this line Q.
f. Use a ruler to draw $\overline{FF'}$. Mark the intersection of the reflecting line and $\overline{FF'}$ point M.
g. Measure the FM and $F\prime\prime M$. What do you notice about these distances?
h. Measure $\angle FMQ$ and $\angle F\prime\prime MQ$. What do you notice about these measurements?

Extension - Reflections and Translations! Use the diagram below. Reflect $\triangle CAT$ over line m, obtaining $\triangle C\prime\prime A$ Now reflect $\triangle C\prime\prime A\prime\prime T\prime\prime$ over line n, obtaining $\triangle C\prime\prime\prime A$ The resulting image is a translation of the preimage, double the distance between the parallel lines.
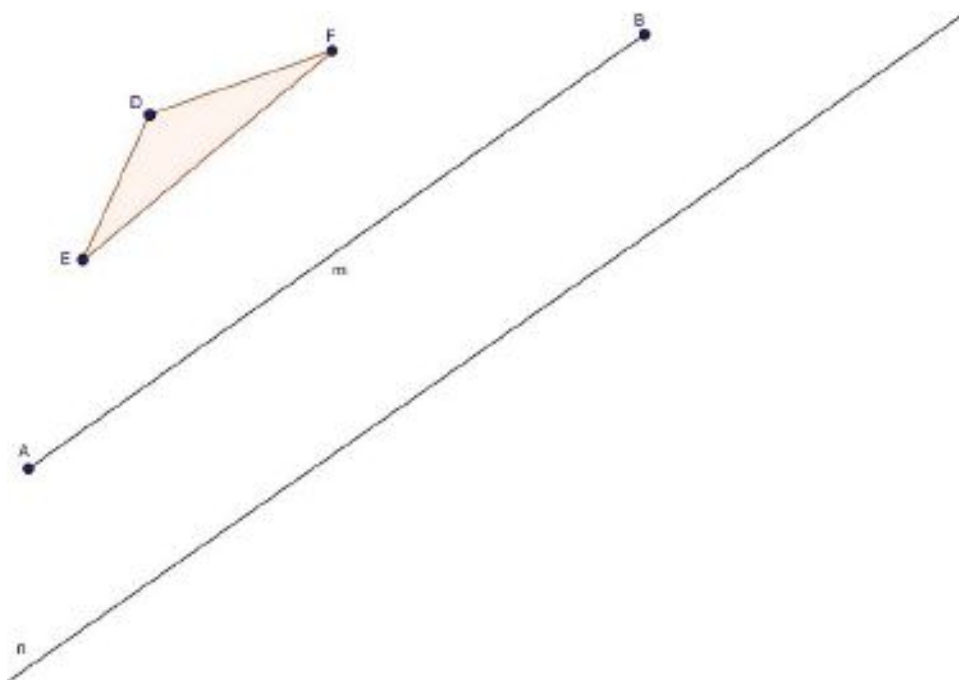
Rotations

Pacing: This lesson should take one class period

Goal: Rotations are also an important concept in geometry. Tires rotate in 360 degree increments, as do the hands on a clock. This lesson presents the concept of rotations and how matrix multiplication is used to compute the image points.

Extension – Reflections and Rotations! Just as translations are a composite of reflections over parallel lines, rotations are a composite of reflections. The only difference is that rotations occur when the reflecting lines intersect. Have your students complete the following:

Using the diagram below, reflect $ABCD$ over line f, resulting in $A\prime\prime B\prime\prime C\prime\prime$ Reflect $A\prime\prime B\prime\prime C\prime\prime D\prime\prime$ over line e, resulting in $A\prime\prime\prime B\prime\prime\prime C\prime\prime\prime D\prime\prime\prime$. The final image represents a rotation of the preimage double the acute angle formed by the intersecting reflecting lines.

Composition

Pacing: This lesson should take one class period

Goal: This lesson introduces students to the concept of composition. Composition is the process of applying two (or more) operations to an object. This concept is also in Advanced Algebra.

Look Out! Students can get easily confused when applying compositions. They may attempt to perform the composition from left to right, as in reading a sentence. Point out to the students they must begin with the object and, according to the order of operations, should perform the operation occurring within the parentheses first.

Notation! Composition notation can take two forms. To write the reflection of *ABCD* over line *m***following** a reflection over line *n*, you could:

A. Write $r_n \circ r_m(ABCD)$

B. Write $r_n(r_m(ABCD))$

The lowercase [U+0080][U+009C]r[U+0080][U+009D] stands for reflection and the subscript refers to the reflecting line.

Watch Your Feet! Your feet are a prime example of glide reflections. When you walk, one foot is translated above the other and are reflected about your body's center line.

Tessellations

Pacing: This lesson should take one class period

1.12. Transformations

Goal: This lesson introduces students to how tessellations are formed and which type of polygons will tessellate the plane.

Research! Have your students research M.C. Escher, an artist who has designed numerous pieces of artwork using tessellations. Have each student choose a piece of artwork, outline its preimage figures and give a short presentation.

Create Your Own Escher Print! An activity many students love to do is designed an unique piece of art. Complete the following steps:

a. Cut out a $2''$ square from a sheet of copy paper.
b. Draw a curve between two consecutive vertices. Be careful to not cut off a vertex!
c. Cut out the curve and slide the cutout to the opposite side of the square and tape it in place.
d. Repeat this process with the remaining two sides of the square.
e. This is your template, or preimage. Begin with an $11'' \times 14''$ piece of copy paper. Trace your preiamge and continue the pattern by rotating and translating until you cover the entire sheet.
f. Color and post for a bulletin board.

*Did You Know?* The first person to discover how to tessellate with a pentagon was....

Symmetry

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to introduce the concept of symmetry. Students have experienced symmetry in previous lessons: isosceles triangles and various quadrilaterals. This lesson incorporates two-dimensional and three-dimensional figures and their lines of symmetry.

Maximum Points! Have your students write the alphabet in uppercase letters. Using one colored pencil, show which letters possess horizontal symmetry by drawing in the symmetry line. For example, *B* has a line of symmetry, as does *E* and *K*. Using a second color, draw in the vertical lines of symmetry. Have a contest to determine who can write the longest word possessing one type of symmetry. For example, MAXIMUM is a word where all the letters have vertical symmetry. KICKBOXED is another.

Project! Using a digital camera, have students (or groups thereof) take photographs of objects possessing symmetry, either rotational or reflective. Give points for the most original, the most nature made, etc. Create a slide show presentation in PowerPoint or Microsoft Movie Maker.

Vocabulary! When discussing rotational symmetry, some textbooks may refer to it as $n-fold$ *rotational symmetry.* This simply means that the *n* is the number of times the figure can rotate onto itself. For example, a regular pentagon has $5-$fold rotation symmetry, because it can be rotated 5 times of 108 degrees before returning to its original position.

The Return of the Breadknife! The notion of cutting through a $3-$dimensional object with a breadknife was used in an earlier lesson to demonstrate cross sections. This concept can be used to illustrate planes of symmetry. The plane of symmetry essentially "cuts" through the $3-$dimensional solid so that each piece is identical.

Dilations

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to illustrate how scalar multiplication yield dilations. Figures under dilation are similar figures; all properties of the similarity chapter apply to these objects.

Vocabulary! Scale factors of the same value, such as S_2, are also called size changes. All properties of similar figures hold for size changes.

Extension! Scalar multiplication can be extended to multiplying the $x-$values and $y-$values by different values, yielding a non-similar figure. For example, you could multiply the points $(0,2), (1,7), (4,5), (6,2)$ by $S_{2,\ 3}$. The first value in the subscript is the multiplier for the $x-$values and the second value in the subscript is the multiplier for the $y-$values. The resulting ordered pairs are $(0,6), (2,21), (8,15)$, and $(12,6)$.

Technology! To find the image points of a size change, input a 2×2 matrix in matrix $[A]$, such as $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$. For a scale change, a matrix could look like this: $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$. Multiply the two matrices using the process found in **Reflections** lesson.

Additional Example:

    a. Suppose the image JAR has the following matrix: $\begin{bmatrix} 2 & -4 & 8 \\ 6 & -3 & 5 \end{bmatrix}$, occurring under a size change $S_{\frac{1}{2}}$. What are the coordinates of the preimage TIP?

CHAPTER 2

Geometry TE - Common Errors

Chapter Outline

2.1 Basics of Geometry

Points, Lines, and Planes

Naming Lines – Students often want to use all the labeled points on a line in its name, especially if there are exactly three points labeled. Tell them they get to pick two, any two, to use in the name. This means there are often many possible correct names for a single line.

Key Exercise: How many different names can be written for a line that has four labeled points?

Answer: 12

Student can get to this answer by listing all the combination of two letters. Recommend that they make the list in an orderly way so they do not leave out any possibilities. This exercise is good practice for counting techniques learned in probability.

Naming Rays – There is so much freedom in naming lines, that students often struggle with the precise way in which rays must be named. They often think that the direction the ray is pointing needs to be taken into consideration. The arrow "hat" always points to the right. The "hat" only indicates that the geometric object is a ray, not the ray's orientation in space. The first letter in the name of the ray is the endpoint; it does not matter if that point comes first or second when reading from left to right on the figure. It is helpful to think of the name of a ray as a starting point and direction. There is only one possible starting point, but often several points that can indicate direction. Any point on the ray other than the endpoint can be the second point in the name.

There is only one point B – English is an ambiguous language. Two people can have the same name; one word can have two separate meanings. Math is also a language, but is different from other languages in that there can be no ambiguity. In a particular figure there can be only one point labeled B.

Key Exercise: Draw a figure in which $\overleftrightarrow{AB}$ intersects $\overline{AC}$.

Answer: There are many different ways this can be drawn. There must be a line with the points A and B, and a segment with one endpoint at A and the other endpoint C could be at any location.

Segments and Distance

Number or Object – The measure of a segment is a number that can be added, subtracted and combine arithmetically with other numbers. The segment itself is an object to which postulates and theorems can be applied. Using the correct notation may not seem important to the students, but is a good habit that will work to their benefit as they progress in their study of mathematics. For example, in calculus whether a variable represents a scalar or a vector is critical. When clear notation is used, the mind is free to think about the mathematics.

Using a Ruler – Many Geometry students need to be taught how to use a ruler. The problems stems from students not truly understanding fractions and decimals. This is a good practical application and an important life skill.

Measuring in centimeters will be learned quickly. Give a brief explanation of how centimeters and millimeters are marked on the ruler. Since a millimeter is a tenth of a centimeter, both fractions and decimals of centimeters are easily written.

Using inches is frequently challenging for students because so many still struggle with fractions. Some may need to

be shown how an inch is divided using marks for $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$, and $\frac{1}{16}$. These fractions often need to be added and reduced to get a measurement in inches.

Review the Coordinate Plane – Some students will have forgotten how to graph an ordered pair on the coordinate plane, or will get the words vertical and horizontal confused. A reminder that the $x-$coordinate is first, and measures horizontal distance from the origin, and that the $y-$coordinate is second and measures vertical distance from the origin will be helpful. The coordinates are listed in alphabetical order.

Additional Exercises:

1. Points $A, B,$ and C are collinear, with B located between A and C.

$AB = 12$ cm and $AC = 20$ cm. What is BC?

(Hint: Draw and label a picture.)

Answer: 20 cm $- 12$ cm $= 8$ cm

Drawing a picture is extremely helpful when solving Geometry problems. It is good to get the students in this habit early. The process of going form a description to a picture also helps them review their vocabulary.

Rays and Angles

Naming Angles with Three Points – Naming, and identifying angles named with three points is often challenging for students when they first learn it. The middle letter of the angle name, the vertex of the angle, is the most important point. Instruct the students to start by identifying this point and working from there. With practice students will become adept at seeing and naming different angles is a complex picture. Review of this concept is also important. Every few months give the students a problem that requires using this important skill.

Using a Protractor – The two sets of numbers on a protractor are convenient for measuring angles oriented in many different directions, but often lead to errors on the part of the students. There is a simple way for students to check their work when measuring an angle with a protractor. Visual inspection of an angle usually can be used to tell if an angle is acute or obtuse. After the measurement is taken, students should notice if their answer matches with the classification.

Additional Exercises:

1. *True or False:* A ray can have a measure

Answer: False. A ray extends infinitely on one direction, so it does not have a length.

2. $< ABC$ has a measure of 100 degrees. Point D is located in the interior of $< ABC$ and $< ABD$ has a measure of 30 degrees. What is the measure of $< DBC$?

(Hint: Draw and label a picture.)

Answer: 100 degrees $- 30$ degrees $= 70$ degrees

3. $< XYZ$ has a measure of 45 degrees and $< ZYW$ has a measure of 75 degrees. What is the measure of $< XYW$?

(Hint: Draw and label a picture.)

Answer: 45 degrees $+ 75$ degrees $= 120$ degrees

Segments and Angles

Congruent or Equal – Frequently students interchange the words congruent and equal. Stress that equal is a word

that describes two numbers, and congruent is a word that describes two geometric objects. Equality of measure is often one of the conditions for congruence. If the students have been correctly using the naming conventions for a segment and its measure and an angle and its measure in previous lessons they will be less likely to confuse the words congruent and equal now.

The Number of Tick Marks or Arcs Does Not Give Relative Length – A common misconception is that a pair of segments marked with one tick, are longer than a pair of segments marked with two ticks in the same figure. Clarify that the number of ticks just groups the segments; it does not give any relationship in measure between the groups. An analogous problem occurs for angles.

Midpoint or Bisector – Midpoint is a location, a noun, and bisect is an action, a verb. One geometric object can bisect another by passing through its midpoint. This link to English grammar often helps students differentiate between these similar terms.

Intersects vs. Bisects – Many students replace the word intersects with bisects. Remind the students that if a segment or angle is bisected it is intersected, and it is know that the intersection takes place at the exact middle.

Orientation Does Not Affect Congruence – The only stipulation for segments or angles to be congruent is that they have the same measure. How they are twisted or turned on the page does not matter. This becomes more important when considering congruent polygons later, so it is worth making a point of now.

Labeling a Bisector or Midpoint – Creating a well-labeled picture is an important step in solving many Geometry problems. How to label a midpoint or a bisector is not obvious to many students. It is often best to explicitly explain that in these situations, one marks the congruent segments or angles created by the bisector.

Additional Exercises:

1. Does it make sense for a line to have a midpoint?

Answer: No, a line is infinite in one dimension, so there is not a distinct middle.

Angle Pairs

Complementary or Supplementary – The quantity of vocabulary in Geometry is frequently challenging for students. It is common for students to interchange the words complementary and supplementary. A good mnemonic device for these words is that they, like many math words, go in alphabetical order; the smaller one, complementary, comes first.

Linear Pair and Supplementary – All linear pairs have supplementary angles, but not all supplementary angles form linear pairs. Understanding how Geometry terms are related helps students remember them.

Angles formed by Two Intersection Lines – Students frequently have to determine the measures of the four angles formed by intersecting lines. They can check their results quickly when they realize that there will always be two sets of congruent angles, and that angles that are not congruent must be supplementary. They can also check that all four angles measures have a sum of 360 degrees.

Write on the Picture – In a complex picture that contains many angle measures which need to be found, students should write angle measures on the figure as they find them. Once they know an angle they can use it to find other angles. This may require them to draw or trace the picture on their paper. It is worth taking the time to do this. The act of drawing the picture will help them gain a deeper understanding of the angle relationships.

Proofs – The word proof strikes fear into the heart of many Geometry students. It is important to define what a mathematical proof is, and let the students know what is expected of them regarding each proof.

Definition: A mathematical proof is a mathematical argument that begins with a truth and proceeds by logical steps to a conclusion which then must be true.

The students' responsibilities regarding each proof depend on the proof, the ability level of the students, and where in the course the proof occurs. Some options are (1) The student should understand the logical progression of the steps in the proof. (2) The student should be able to reproduce the proof. (3) The student should be able to create proofs using similar arguments.

Classifying Triangles

Vocabulary Overload – Students frequently interchange the words isosceles and scalene. This would be a good time to make flashcards. Each flashcard should have the definition in words and a marked and labeled figure. Just making the flashcards will help the students organize the material in their brains. The flashcards can also be arranged and grouped physically to help students remember the words and how they are related. For example, have the students separate out all the flashcards that describe angles. The cards could also be arranged in a tree diagram to show subsets, for instance equilateral would go under isosceles, and all the triangle words would go under the triangle card.

Angle or Triangle – Both angles and triangles can be named with three letters. The symbol in front of the letters determines which object is being referred to. Remind the students that the language of Geometry is extremely precise and little changes can make a big difference.

Acute Triangles need all Three – A student may see one acute angle in a triangle and immediately classify it as an acute triangle. Remind the students that unlike the classifications of right and obtuse, for a triangle to be acute all three angles must be acute.

Equilateral Subset of Isosceles – In many instances one term is a subset of another term. A Venn diagram is a good way to illustrate this relationship. Having the students practice with this simple instance of subsets will make it easier for the students to understand the more complex situation when classifying quadrilaterals.

Additional Exercises:

1. Draw and mark an isosceles right and an isosceles obtuse triangle.

Answer: The congruent sides of the triangles must be the sides of the right or obtuse angle.

This exercise lays the groundwork for studying the relationship between the sides and angles of a triangle in later chapters. It is important that students take the time to use a straightedge and mark the picture. Using and reading the tick marks correctly helps the students think more clearly about the concepts.

Classifying Polygons

Vocab, Vocab, Vocab – If the students do not know the vocabulary well, they will have no chance at leaning the concepts and doing the exercises. Remind them that the first step is to memorize the vocabulary. This will take considerable effort and time. The student edition gives a good mnemonic device for remembering the word concave. Ask the students to create tricks to memorize other words and have them share their ideas.

Side or Diagonal – A side of a polygon is formed by a segment connecting consecutive vertices, and a diagonal connects nonconsecutive vertices. This distinction is important when student are working out the pattern between the number of sides and the number of vertices of a polygon.

Squaring in the Distance Formula – After subtracting in the distance formula, students will often need to square a negative number. Remind them that the square of a negative number is a positive number. After the squaring step there should be no negatives or subtraction. If they have a negative in the square root, they have made a mistake.

Additional Exercises:

1. Find the length of each side of the triangle with the following vertices with the distance formula. Then classify each triangle by its sides.

a) $(3, 1), (3, 5),$ and $(10, 3)$

Answer: The triangle is isosceles with side lengths: 4, $\sqrt{51}$, and $\sqrt{51}$.

b) $(-3, 2), (-8, 3),$ and $(-3, -5)$

Answer: This triangle is scalene with side lengths: 7, $\sqrt{26}$, and $\sqrt{89}$.

Sometimes students have trouble seeing that they need to take the points two at a time to find the side lengths. By graphing the triangle on graph paper before using the distance formula they can see how to find the side lengths. If they graph the triangle they can also classify it by its angles.

Problem Solving in Geometry

Don't Panic – Problem solving and applications are particularly challenging for many students. Sometimes they just give up. Let the students know that this is difficult. They are probably going to struggle, have to reread the information several time, and will be confused for a while. It is all part of the process. This section will give them strategies to work through the difficulties.

Highlight Important Information – It is nice when students can actually mark up the text of the exercise, but frequently this is not the case. As they read the paragraph have the students take notes or organize the information into a chart. Otherwise the students can just get lost in all the words. Translating from English to math is often the hardest part.

The Last Sentence – When the students are faced with a sizable paragraph of information the most important sentence, the one that asks the question, is usually at the end. Advise the students to read the last sentence first, then as they read the rest of the paragraph they will see how the information they are being given is important.

Does This Make Sense? – It is so hard to get the students to ask themselves this question at the end of a word problem or application. I think they are so happy to have an answer they do not want to know if it is wrong. Keep reminding them. Sometimes it is possible to not accept work with an obviously wrong answer. The paper can be returned to the student so they can look for their mistake. This is a good argument for the importance of showing clear, organized work.

Naming Quadrilaterals – When naming a quadrilateral the letter representing the vertices will be listed in a clockwise or counterclockwise rotation starting from any vertex. Students are accustomed to reading from left to right and will sometimes continue this pattern when naming a quadrilateral.

The Pythagorean Theorem – Most students have learned to use the Pythagorean Theorem before Geometry class and will want to use it instead of the distance formula. They are closely related; the distance formula is derived from the Pythagorean Theorem as will be explained in another chapter. If they are allowed to use the Pythagorean Theorem remind them that it can only be used for right triangles, and that the length of the longest side of the right triangle, the hypotenuse, must be substituted into the c variable if it is know. If the hypotenuse is the side of the triangle being found, the c stays a variable, and the other two side are substituted for a and b.

2.2 Reasoning and Proof

Inductive Reasoning

The nth Term – Students enjoy using inductive reasoning to find missing terms in a pattern. They are good at finding the next term, or the tenth term, but have trouble finding a generic term or rule for the number sequence. If the sequence is linear (the difference between terms is constant), they can use methods they learned in Algebra for writing the equation of a line.

Key Exercise: Find a rule for the nth term in the following sequence.

$$13, 9, 5, 1, \ldots.$$

Answer: The sequence is linear, each term decreases by 4. The first term is 13, so the point $(1, 13)$ can be used. The second term is 9, so the point $(2, 9)$ can be used. Applying what they know from Algebra I, the slope of the line is -4, and the $y-$intercept is 17, so the rule is $-4n + 17$.

True Means <u>Always</u> True – In mathematics a statement is said to be true if it is always true, no exceptions. Sometimes students will think that a statement only has to hold once, or a few times to be considered true. Explain to them that just one counterexample makes a statement false, even if there are a thousand cases where the statement holds. Truth is a hard criterion to meet.

Sequences – A list of numbers is called a sequence. If the students are doing well with the number of vocabulary words in the class, the term sequence can be introduced.

Additional Exercises:

1. What is the next number in the following number pattern? $1, 1, 2, 3, 5, 8, 13, \ldots$

Answer: This is the famous Fibonacci sequence. The next term in the sequence is the sum of the previous two terms.

$$8 + 13 = 21$$

2. What is the missing number in the following number pattern? $25, 18, ?, 10, 9, \ldots$

Answer: Descending consecutive odd integers are being subtracted from each term, so the missing number is 13.

Conditional Statements

The Advantages and Disadvantages of Non-Math Examples – When first working with conditional statements, using examples outside of mathematics can be very helpful for the students. Statements about the students' daily lives can be easily broken down into parts and evaluated for veracity. This gives the students a chance to work with the logic, without having to use any mathematical knowledge. The problem is that there is almost always some crazy exception or grey area that students will love to point out. This is a good time to remind students of how much more precise math is compared to our daily language. Ask the students to look for the idea of what you are saying in the non-math examples, and use their powerful minds to critically evaluate the math examples that will follow.

Converse and Contrapositive – The most important variations of a conditional statement are the converse and the contrapositive. Unfortunately, these two sound similar, and students often confuse them. Emphasize the converse and contrapositive in this lesson. Ask the students to compare and contrast them.

Converse and Biconditional – The converse of a true statement is not necessarily true! The important concept of implication is prevalent in Geometry and all of mathematics. It takes some time for students to completely understand the direction of the implication. Daily life examples where the converse is obviously not true is a good place to start. The students will spend considerable time deciding what theorems have true converses (are bicondtional) in subsequent lessons.

Key Exercise: What is the converse of the following statement? Is the converse of this true statement also true?

If it is raining, there are clouds in the sky.

Answer: The converse is: If there are clouds in the sky, it is raining. This statement is obviously false.

Practice, Practice, Practice – Students are going to need a lot of practice working with conditional statements. It is fun to have the students write and share conditional statements that meet certain conditions. For example, have them write a statement that is true, but that has an inverse that is false. There will be some creative, funny answers that will help all the members of the class remember the material.

Deductive Reasoning

Inductive or Deductive Reasoning – Students frequently struggle with the uses of inductive and deductive reasoning. With a little work and practice they can memorize the definition and see which form of reasoning is being used in a particular example. It is harder for them to see the strengths and weaknesses of each type of thinking, and understand how inductive and deductive reasoning work together to form conclusions.

Recognizing Reasoning in Action – Use situations that the students are familiar with where either inductive or deductive reasoning is being used to familiarize them with the different types of logic. The side by side comparison of the two types of thinking will cement the students' understanding of the concepts. It would also be beneficial to have the students write their own examples.

Key Exercise: Is inductive or deductive reasoning being used in the following paragraph? Why did you come to this conclusion?

1. The rules of Checkers state that a piece will be crowned when it reaches the last row of the opponent's side of the board. Susan jumped Tony's piece and landed in the last row, so Tony put a crown on her piece.

Answer: This is an example of detachment, a form of deductive reasoning. The conclusion follows from an agreed upon rule.

2. For the last three days a boy has walked by Ana's house at 5 pm with a cute puppy. Today Ana decides to take her little sister outside at 5 pm to show her the dog.

Answer: Ana used inductive reasoning. She is assuming that the pattern she observed will continue.

Which is Better? – Students quickly conclude that inductive reasoning is much easier, but often miss that deductive reasoning is more sure and frequently provides some insight into the answer of that important question, "Why?".

Additional Exercises:

1. What went wrong in this example of inductive reasoning?

Teresa learned in class that John Glenn (the first American to orbit Earth) had to eat out of squeeze tubes, and her mom says the food served in airplanes is not very good. She just had a yummy pizza for lunch. She sees a pattern. Food gets better as one approaches the center of the earth. Therefore the food in a submarine must be delicious!

Answer: She carried the pattern too far.

2.2. Reasoning and Proof

Algebraic Properties

Commutative or Associate – Students sometimes have trouble distinguishing between the commutative and associate properties. It may help to put these properties into words. The associate property is about the order in which multiple operations are done. The commutative is about the first and second operand having different roles in the operation. In subtraction the first operand is the starting amount and the second is the amount of change. Often student will just look for parenthesis; if the statement has parenthesis they will choose associate, and they will usually be correct. Expose them to an exercise like the one below to help break them of this habit.

Key Exercise: What property of addition is demonstrated in the following statement?

$$(x+y)+z = z+(x+y)$$

Answer: It is the commutative property that ensures these two quantities are equal. On the left-hand side of the equation the first operand is the sum of x and y, and on the right-hand side of the equation the sum if x and y is the second operand.

Transitive or Substitution – The transitive property is actually a special case of the substitution property. The transitive property has the additional requirement that the first statement ends with the same number or object with which the second statement begins. Acknowledging this to the students helps avoid confusion, and will help them see how the properties fit together.

Key Exercise: The following statement is true due to the substitution property of equality. How can the statement be changed so that the transitive property of equality would also ensure the statement's validity?
　　If $ab = cd$, and $ab = f$, then $cd = f$.

Answer: The equality $ab = cd$ can be changed to $cd = ab$ due to the symmetric property of equality. Then the statement would read:
　　If $cd = ab$, and $ab = f$, then $cd = f$.

This is justified by the transitive property of equality.

Diagrams

Keeping It All Straight – At this point in the class the students have been introduced to an incredible amount of material that they will need to use in proofs. Laying out a logic argument in proof form is, at first, a hard task. Searching their memories for terms at the same time makes it near impossible for many students. A notebook that serves as a "tool cabinet" full of the definitions, properties, postulates, and later theorems that they will need, will free the students' minds to concentrate on the logic of the proof. After the students have gained some experience, they will no longer need to refer to their notebook. The act of making the book itself will help the students collect and organize the material in their heads. It is their collection; every time they learn something new, they can add to it.

All Those Symbols – In the back of many math books there is a page that lists all of the symbols and their meanings. The use of symbols is not always consistent between texts and instructors. Students should know this in case they refer to other materials. It is a good idea for students to keep a page in their notebooks where they list symbols, and their agreed upon meanings, as they learn them in class. Some of the symbols they should know at this point in are the ones for equal, congruent, angle, triangle, perpendicular, and parallel.

Don't Assume Congruence! – When looking at a figure students have a hard time adjusting to the idea that even if two segments or angles look congruent they cannot be assumed to be congruent unless they are marked. A triangle

is not isosceles unless at least two of the sides are marked congruent, no matter how much it looks like an isosceles triangle. Maybe one side is a millimeter longer, but the picture is too small to show the difference. Congruent means exactly the same. It is helpful to remind the students that they are learning a new, extremely precise language. In geometry congruence must be communicated with the proper marks if it is known to exist.

Communicate with Figures – A good way to have the students practice communicating by drawing and marking figures is with a small group activity. One person in a group of two or three draws and marks a figure, and then the other members of the group tell the artist what if anything is congruent, perpendicular, parallel, intersecting, and so on. They take turns drawing and interpreting. Have them use as much vocabulary as possible in their descriptions of the figures.

Two-Column Proofs

Diagram and Plan – Students frequently want to skip over the diagramming and planning stage of writing a proof. They think it is a waste of time because it is not part of the end result. Diagramming and marking the given information enables the writer of the proof to think and plan. It is analogous to making an outline before writing an essay. It is possible that the student will be able to muddle through without a diagram, but in the end it will probably have taken longer, and the proof will not be written as clearly or beautifully as it could have been if a diagram and some thinking time had been used. Inform students that as proofs get more complicated, mathematicians pride themselves in writing simple, clear, and elegant proofs. They want to make an argument that undeniably true.

Teacher Encouragement – When talking about proofs and demonstrating the writing of proofs in class, take time to make a well-drawn, well-marked diagram. After the diagram is complete, pause, pretend like you are considering the situation, and ask students for ideas of how they would go about writing this proof.

Assign exercises where students only have to draw and mark a diagram. Use a proof that is beyond their ability at this point in the class and just make the diagram the assignment.

When grading proofs, use a rubric that assigns a certain number of points to the diagram. The diagram should be almost as important as the proof itself.

Start with "Given", but Don't End With "Prove" – After a student divides the statement to be proved into a given and prove statements he or she will enjoy writing the givens into the proof. It is like a free start. Sometimes they get a little carried away with this and when they get to the end of the proof write "prove" for the last reason. Remind them that the last step has to have a definition, postulate, property, or theorem to show why it follows from the previous steps.

Scaffolding – Proofs are challenging for many students. Many students have a hard time reading proofs. They are just not used to this kind of writing; it is very specialized, like a poem. One strategy for making students accustom to the form of the proof is to give them incomplete proofs and have them fill in the missing statements and reason. There should be a progression where each proof has less already written in, and before they know it, they will be writing proofs by themselves.

Segment and Angle Congruence Theorems

Number or Geometric Object – The difference between equality of numbers and congruence of geometric objects was addressed earlier in the class. Before starting this lesson, a short review of this distinction to remind students is worthwhile. If the difference between equality and congruence is not clear in students' heads, the proofs in this section will seem pointless to them.

Follow the Pattern – Congruence proofs are a good place for the new proof writer to begin because they are fairly

formulaic. Students who are struggling with proofs can get some practice with this style of writing while already knowing the structure of the proof.

1

st *State the if side in congruence form.*

2

nd *Change the congruence of segments into equality of numbers.*

3

rd *Apply the analogous property of equality.*

4

th *Change the equality of numbers back to congruence of segments.*

Theorems – The concept of a theorem and how it differs from a postulate has been briefly addressed several times in the course, but this is the first time theorems have been the focus of the section. Now would be a good time for students to start a theorem section in their notebook. As they prove, or read a proof of each theorem it can be added to the notebook to be used in other proofs.

Additional Exercises:

1. Prove the following statement.

If $AB = AC$, triangle ABC is isosceles.

Answer:

TABLE 2.1:

Statement	Reason
$AB = AC$	Given
$\overline{AB} \cong \overline{AC}$	Definition of congruent segments.
Triangle ABC is isosceles.	Definition of isosceles triangle.

Proofs About Angle Pairs

Mark-Up That Picture – Angles are sometimes hard to see in a complex picture because they are not really written on the page; they are the amount of rotation between two rays that are directly written on the page. It is helpful for students to copy diagram onto their papers and mark all the angles of interest. They can use highlighters and different colored pens and pencils. Each pair of vertical angles or linear pairs can be marked in a different color. Using colors is fun, and gives the students the opportunity to really analyze the angle relationships.

Add New Information to the Diagram – It is common in geometry to have multiple questions about the same diagram. The questions build on each other leading the student though a difficult exercise. As new information is found it should be added to the diagram so that it is readily available to use in answering the next question.

Try a Numerical Example – Sometimes students have trouble understanding a theorem because they get lost in all the symbols and abstraction. When this happens, advise the students to assign a plausible number to the measures of the angles in question and work form there to understand the relationships. Make sure the student understands that this does not prove anything. When numbers are assigned, they are looking at an example, using inductive reasoning to get a better understanding of the situation. The abstract reasoning of deductive reasoning must be used to write a proof.

Inductive vs. Deductive Again – The last six sections have given the students a good amount of practice drawing

diagrams, using deductive reasoning, and writing proofs, skills which are closely related. Before moving on to Chapter Three, take some time to review the first two sections of this chapter. It is quite possible that students have forgotten all about inductive reasoning. Now that they have had practice with deductive reasoning they can compare it to inductive reasoning and gain a deeper understanding of both. They should understand that inductive reasoning often helps a mathematician decide what should be attempted to be proved, and deductive reasoning proves it.

Review – The second section of chapter two contains information about conditional statements that will be used in the more complex proofs in later chapters. Since the students did not get to use most of it with these first simple proofs, it would be a good idea to draw their attention to it again and talk briefly about the more complex proof that will be coming.

2.3 Parallel and Perpendicular Lines

Lines and Angles

Marking the Diagram – Sometimes students confuse the marks for parallel and congruent. When introduction them to the arrows that represent parallel lines, review the ticks that represent congruent segments. Seeing the two at the same time helps avoid confusion.

When given the information that two lines or segments are perpendicular, students don't always immediately see how to mark the diagram accordingly. They need to use the definition of perpendicular and mark one of the right angles created by the lines with a box.

Symbol Update – Students should be keeping a list of symbols and how they will be used in this class in their notebooks. Remind them to update this page with the symbols for parallel and perpendicular.

Construction – The parallel and perpendicular line postulates are used in construction. Constructing parallel and perpendicular lines with a compass and straightedge is a good way to give students kinesthetic experience with these concepts. Construction can also be done with computer software. To construct a parallel or perpendicular line the student will select the line they want the new line to be parallel or perpendicular to, and the point they want the new line to pass through, and chose construct. The way the programs have the students select the line and then the point reinforces the postulates.

Additional Exercises:

1. Write a two-column proof of the following conditional statement.

If $\overline{AB}$ is perpendicular to $\overline{BC}$, triangle ABC is a right triangle.

Answer:

TABLE 2.2:

Statement	Reason
$\overline{AB}$ is perpendicular to $\overline{BC}$	Given.
$< B$ is right	Definition of perpendicular.
triangle ABC is a right triangle	Definition of right triangle.

Parallel Lines and Transversals

The Parallel Hypothesis – So far seven different pairs of angles that may be supplementary or congruent have been introduced. All seven of these pairs are used in the situation where two lines are being crossed by a transversal forming eight angles. Some of these pairs require the two lines to be parallel and some do not. Students sometimes get these confuse on when they need parallel lines to apply a postulate or theorem, and if a specific pair is congruent or supplementary. A chart like the one below will help them sort it out.

TABLE 2.3:

	Type of Angle Pair	Relationship
Do Not Require Parallel Lines	Linear Pairs	Supplementary
	Vertical Angles	Congruent
Parallel Lines Required	Corresponding Angles	Congruent
	Alternate Interior Angles	Congruent
	Alternate Exterior Angles	Congruent
	Consecutive Interior Angles	Supplementary
	Consecutive Exterior Angles	Supplementary

Patty Paper Activity – When two lines are intersected by a transversal eight angles are formed in two sets of four. When the lines are parallel, the two sets of four angles are exactly the same. To help students see this relationship, have them darken a set of parallel lines on their binder paper a few inches apart and draw a transversal through the parallel lines. Now they should trace one set of four angles on some thin paper (tracing paper or patty paper). When they slide the set of four angles along the transversal they will coincide with the other set of four angles. Have them try the same thing with a set of lines that are not parallel. This will help students find missing angle measures quickly and remember when they can transfer numbers down the transversal. It does not help them learn the names of the different pairs of angles which in important for communicating with others about mathematical concepts and for writing proofs.

Additional Exercises:

1. One angle of a linear pair has a measure twice as large as the other angle. What are the two angle measures?

Answer:

$$x + 2x = 180 \qquad \text{The angles measure 60 degrees and 120 degrees}$$
$$x = 60$$

Proving Lines Parallel

When to Use the Converse – It takes some experience before most students truly understand the difference between a statement and its converse. They will be able to write and recognize the converse of a statement, but then will have a hard time deciding which one applies in a specific situation. Tell them when you know the lines are parallel and are looking for angles, you are using the original statements; when you are trying to decide if the lines are parallel or not, you are using the converse.

Additional Exercise:

1. Prove the Converse of the Alternate Exterior Angle Theorem.

Answer: Refer to the image used to prove the Converse of the Alternate Interior Angle Theorem in the text.

TABLE 2.4:

Statement	Reason
$\angle ABC \cong \angle HFE$	Given

2.3. Parallel and Perpendicular Lines

TABLE 2.4: (continued)

Statement	Reason
$\angle HFE \cong \angle GFB$	Vertical Angles Theorem
$\angle ABC \cong \angle GFB$	Transitive Property of Angle Congruence
$\overleftrightarrow{AD}$ is parallel to $\overleftrightarrow{GE}$	Converse of the Corresponding Angles Postulate.

The Converse of the Alternate Exterior Angle Theorem could also be proved using the Converse of the Alternate Interior Angle Theorem. This would demonstrate to the students that once a theorem has been proved it, can be used in the proof of other theorems. It demonstrates the building block nature of math.

TABLE 2.5:

Statement	Reason
$\angle ABC \cong \angle HFE$	Given
$\angle HFE \cong \angle GFB$	Vertical Angles Theorem
$\angle DBF \cong \angle GFB$	
$\angle DBF \cong \angle GFB$	Transitive Property of Angle Congruence
$\overleftrightarrow{AD}$ is parallel to $\overleftrightarrow{GE}$	Converse of the Alternate Interior Angles Theorem.

Proving the theorem in several ways gives students a chance to practice with the concepts and their proof writing skills. Similar proofs can be assigned for the other theorems in this section.

Slopes

Order of Subtraction – When calculating the slope of a line using two points it is important to keep straight which point was made point one and which one was point two. It does not matter how these labels are assigned, but the order of subtraction has to stay the say in the numerator and the denominator of the slope ratio. If students switch the order they will get the opposite of the correct answer. If they have a graph of the line, ask them to compare the sign of the slope to the direction of the line. Is the line increasing or decreasing? Does that match the slope?

Graphing Lines with Integer Slopes – The slope of a line is the ratio of two numbers. When students are asked to graph a line with an integer slope they often fail to realize what and where the second number is. Frequently they will make the "run" of the line zero and graph a vertical line. It is helpful to have them write the integer that is the slope, as a ratio over one, before then do any graphing. Really, they only need to do this a few times on paper before they are able to graph the lines correctly. They will begin to see the ratio correctly in their heads.

Zero or Undefined – Students need to make these associations:

Zero in numerator – slope is zero – line is horizontal

Zero in denominator – slope is undefended – line is vertical

They frequently switch these around. After the relationships are explained in class, remind them frequently, maybe have a poster up in the room or write the relationship on a corner of the board that does not get erased.

Use Graph Paper – Making a connection between the numbers that describe a line and the line itself is an important skill. Requiring that the students use graph paper encourages them to make nice, thoughtful graphs, and helps them make this connection.

Additional Exercises:

1. Find the slope of the line that is perpendicular to the line passing through the points $(5, -7)$, and $(-2, -3)$.

Answer: The slope of the line passing through the given points is $\frac{-4}{7}$, so the line perpendicular to this line has the slope $\frac{7}{4}$.

Equations of Lines

The $y-$axis is Vertical – When using the slope-intercept form to graph a line or write an equation, it is common for students to use the $x-$intercept instead of the $y-$intercept. Remind them that they want to use the vertical axis, $y-$intercept, to begin the graph. Requiring that the $y-$intercept be written as a point, say $(0,3)$ instead of just 3, helps to alleviate this problem.

Where's the Slope – Students are quickly able to identify the slope as the coefficient of the $x-$variable when a line is in slope-intercept form, unfortunately they sometimes extend this to standard form. Remind the students that if the equation of a line is in standard form, or any other form, they must first algebraically convert it to slope-intercept form before they can easily read off the slope.

Key Exercises:

1. Write the equation $3x + 5y = 10$ in slope-intercept form.

Answer: $y = -\frac{3}{5}x + 2$

2. What is the slope of the line $2x - 3y = 7$?

Answer: $\frac{3}{2}$

3. Are the lines below parallel, perpendicular, or neither?

$$6x + 4y = 7$$
$$6x - 4y = 7$$

Answer: These lines are neither parallel nor perpendicular.

Why Use Standard Form – The slope-intercept form of the line holds so much valuable information about the graph of a line, that students probably won't understand why any other form would ever be used. Mention to them that standard form is convenient when putting equations into matrices, something they will be doing in their second year of algebra, to motivate them to learn and remember the standard form.

Perpendicular Lines

Complementary, Supplementary, or Congruent – When finding angle measures students generally need to decide between three possible relationships: complementary, supplementary, and congruent. A good way for them to practice with these and review their equation solving skills, is to assign variable expressions to angle measures, state the relationship of the angles, and have the students use this information to write an equation that when solved will lead to a numerical measurement for the angle.

Key Exercise:

1. Two vertical angles have measures $2x - 30°$ and $x + 60°$.

Set-up and solve an equation to find x. Then find the measures of the angles.

Answer:

2.3. Parallel and Perpendicular Lines

$$2x - 30° = x + 60°$$
$$x = 90°$$

Both angles have a measure of 150 degrees.

2. The outer rays of two adjacent angle with measures $4x + 10°$ and $5x - 10°$ are perpendicular. Find the measures of each angle.

Answer:

$$5x - 10 + 4x + 10 = 90$$
$$x = 10$$

The angles have measures of 50 degrees and 40 degrees.

3. The angles of a linear pair have measures $3x + 45°$ and $2x + 35°$. Find the measure of each angle.

Answer:

$$2x + 35 + 3x + 45 = 180$$
$$x = 20$$

The angles have measures of 105 degrees and 75 degrees.

Encourage students to take the time to write out and solve the equation neatly. This process helps them avoid errors. Many times students will find the value of x, and then stop without plugging in the value to the expression for the angle measures. Have the students verify that their final answers are angle measures that have the desired relationship.

Additional Exercise:

1. Perpendicular lines form an angle with measure $8x + 10°$. What is the value of x?

Answer:

$$8x + 10° = 90°$$
$$x = 10°$$

Perpendicular Transversals

The Perpendicular Distance – In theory, measuring along a perpendicular line makes sense to the students, but in practice, when lining up the ruler or deciding which points to put in the distance formula, there are many distractions. Students can evaluate their decision by taking a second look to see if the path they chose was the shortest one possible.

Multi-Step Procedures – When working on an exercise that requires many different steps, like the last problem in this section, students sometimes become lost in the process or overwhelmed before they begin. A good way to ground students, and help them move through the problem, is to create, or have them create, a To-do list. Writing out the steps that need to be completed will help them understand the process, give them a sense of satisfaction as the check off parts they have completed, and help them organize their work. Creating the list could be a good group activity.

Where to Measure? – Now that the students know to measure along a line that is perpendicular to both parallel lines, they might wonder where along the lines to measure. When working on a coordinate plane it is best to start with a

point that has integer coordinates, just to keep the problem simple and accurate. They will get the same distance no matter where they measure though. An alternate definition of parallel lines is two lines that are a constant distance apart.

Addition Exercises:

1. Prove the Converse of the Perpendicular Transversal Theorem.

Answer: Refer to the figure at the top of page 178, at the beginning of this lesson.

TABLE 2.6:

Statement	Reason
$\overleftrightarrow{KN}$ is perpendicular to $\overleftrightarrow{QT}$	Given
$\overleftrightarrow{OR}$ is perpendicular to $\overleftrightarrow{QT}$	Given
$\angle QPO$ is right	Definition of Perpendicular Lines
$\angle PST$ is right	Definition of Perpendicular Lines
$\angle QPO \cong \angle PST$	Right Angle Theorem
$\overleftrightarrow{OR}$ is parallel to $\overleftrightarrow{KN}$	Converse of Corresponding Angle Postulate

Non-Euclidean Geometry

Separate Worlds – The geometry presented in this section is completely separate from the geometry in the rest of the text. The study of non-Euclidean geometry is excellent for developing critical thinking skills. It also demonstrates to the students what an influential role postulates play and how important it is to carefully evaluate them before accepting them as true. This section is best used for enrichment and should be treated differently from the other sections. If the students attempt to memorize the postulates in this section it may compromise their ability to recall analogous postulates of Euclidean Geometry. Exploring taxicab geometry is a wonderful way to spend a day in class, but it is not something that has to be included on tests. This is a decision that the instruction can make based on the ability of the students in a particular class.

Projects – This section opens the door to many possible projects that students can complete as part of the class or for extra credit. More advanced students in particular will have the ability and interest to explore the topic of Non-Euclidean geometry independently. Topics can include further exploration of taxicab geometry, other types of Non-Euclidean geometry, like spherical geometry, or research into the mathematician who developed these fields. This may make a good group project, where each group presents its findings to the class.

Encourage Creativity - Have students write their own problems involving taxicab geometry. This type of geometry lends itself to application and story problems. Students can be creative and funny. They will enjoy sharing problems with their classmates and solving each other's challenges. Writing word application helps students solve similar exercises. When formulating their question and deciding what information to give and how to give it, they become more aware of the structure of a word problem. If the students are enjoying this line of study and there is time, they may create their own type of geometry by setting up a system of postulates.

Abstraction and Modeling– This section briefly addresses the fact that mathematics is an abstraction and that it usually needs to be modified before it can be helpful in applications to the world in which we live. This is an important concept applicable to all areas of mathematics that is easily seen while studying geometry. This knowledge will help students understand why math is useful and how they will benefit from what they are learning in this class.

2.4 Congruent Triangles

Triangle Sums

Interior vs. Exterior Angles – Students frequently have trouble keeping interior and exterior angles straight. They may fail to identify to which category a specific angle belongs and include an exterior angle in a sum with two interior. They also sometimes use the wrong total, 360 degrees verses 180 degrees. Encourage the students to draw the figure on their papers and color code it. They can highlight or use a specific color of pencil to label all the exterior angle measures and another color for the interior angle measures. Then it is easy to do some checks on their work. Each interior/exterior pair should have a sum of 180 degrees, all of the interior angles should add to 180 degrees, and the measures of the exterior angles total 360 degrees.

Find All the Angles You Can – When a student is asked to find a specific angle in a complex figure and they do not immediately see how they can do it, they can become stuck, and don't know how to proceed. A good strategy is to find any angle they can, even if it is not the one they are after. Finding other angles keeps their brains active and working, they practice using angle relationships, and the new information will often help they find the target angle. Many exercises are not designed to do in one step. It is important that the students know this.

Congruent Angles in a Triangle – In later sections students will study different ways of determining if two or more angles in a triangle are congruent, and will then have to use this information to find missing angles in a triangle. To start them on this process it is good to have them work with triangles in which two angles are stated to be congruent.

Key Exercises:

1. An acute triangle has two congruent angles each measuring 70 degrees. What is the measure of the third angle?

Answer: $180 - 2 * 70 = 40$ degrees

2. An obtuse triangle has two congruent angles. One angle of the triangle measures 130 degrees. What are the measures of the other two angles?

Answer: The two remaining angles must be congruent since a triangle can not have more than one obtuse angle.

$(180 - 130) \div 2 = 25$ degrees

Congruent Figures

Rotation Difficulties – When congruent triangles are shown with different orientations, many students find it difficult to rotate the figures in their head to align corresponding sides and angles. One recommendation is to redraw the figures on paper so that they have the same orientation. It may be necessary for students to physically rotate the paper at first. After students have had some time to practice this skill, most will be able to skip this step.

Stress the Definition – The definition of congruent triangles requires six congruencies, three pairs of angles and three pairs of sides. If students understand what a large requirement this is, they will be more motivated to develop the congruence shortcuts in subsequent lessons.

The Language of Math – Many students fail to see that math is a language, a form of communication, which is extremely dense. Just a few symbols hold great amounts of information. The congruence statements for example, not only tell the reader which triangles are congruent, but which parts of the triangle correspond. When put in terms

of communication students have an easier time understanding why they must put the corresponding vertices in the same order when writing the congruence statement.

Third Angle Theorem by Proof – In the text an example is given to demonstrate the Third Angle Theorem, this is inductive reasoning. A deeper understanding of the theorem, and different types of reasoning, can be gained by using deductive reasoning to write a proof. It will also reinforce the idea that theorems must be proved, and shows how inductive and deductive reasoning work together.

Key Exercise: Prove the Third Angle Theorem.

Answer: Refer to the figures on the top of page 213, where the example of the Third Angle Theorem is given.

TABLE 2.7:

Statement	Reason
$\angle W \cong \angle C$	Given
$\angle V \cong \angle A$	Given
$m\angle V + m\angle W + m\angle X = 180$	Triangle Sum Theorem
$m\angle C + m\angle A + m\angle T = 180$	Triangle Sum Theorem
$m\angle V + m\angle W + m\angle X = m\angle C + m\angle A + m\angle T$	Substitution Property of Equality
$m\angle C + m\angle A + m\angle X = m\angle C + m\angle A + m\angle T$	Subtraction Property of Equality
$m\angle X = m\angle T$	Substitution Property of Equality

Triangle Congruence Using SSS

One Triangle or Two – In previous chapters, students learned to classify a single triangle by its sides. Now students are comparing two triangles by looking for corresponding pairs of congruent sides. Evaluating the same triangle in both of these ways helps the students remember the difference, and is a good way to review previous material. For instance, students could be asked to draw a pair of isosceles triangles that are not congruent, and a pair of scalene triangles that can be shown to be congruent with the SSS postulate.

Correct Congruence Statements – Determining which vertices of congruent triangles correspond is more difficult when no congruent angles are marked. Once the students have determined that the triangles are in fact congruent using the SSS Congruence postulate, it is advisable for them to mark congruent angles before writing the congruence statement. Corresponding congruent angles are found by matching up side markings. The angle made by the sides marked with one and two tick marks corresponds to the angle made by the corresponding sides in the other triangle, and so on.

Translation Rotation – Translating a triangle on a coordinate plane in order to see if it fits exactly over another triangle is a good way to demonstrate that two triangles are congruent. The notation used to describe these translations can sometimes be confusing. The text writes out the movement in words "D is 7 units to the right and 8 units below A". If students use other materials for reference, they may see this same translation as $(7, -8)$. This could be confused with the point located at $(7, -8)$. It may be helpful to alert students to this difference.

Additional Exercises:

1. Use the congruence statement and given information to find the indicated measurements.

$$\triangle ABC \cong \triangle ZYX$$
$$m\angle A = 52°$$
$$m\angle Y = 85°$$
$$AC = 12 \text{ cm}$$

Find XY and $m\angle X$.

Answer: $XY = 12$ cm and $m\angle X = 43$ degrees.

Triangle Congruence Using ASA and AAS

An Important Distinction – At first students may not see why it is important to identify whether ASA or AAS is the correct tool to use for a specific set of triangles. They both lead to congruent triangles, right? Yes, but this will not always be the case, as they will see in the next lesson. Sometimes the configuration of the corresponding congruent sides and angles in the triangles determines if the triangles can be proved to be congruent or not. Knowing this will motivate students to study the difference between ASA and AAS.

Flowchart Proofs – Flowchart proofs do a much better job of showing implication than two-column proofs. In a two-column proof one statement following another does not necessarily mean that the previous statement implies the next. Sometimes all the given information is listed at the beginning or another parallel argument needs to be developed before the implication is made. This can be confusing for students without much experience with proofs, or who have trouble understanding the argument. In a flowchart proof the implications are clearly indicated with arrows, and when parallel arguments are being developed, they are arranged vertically. The flowchart holds much more information.

Different Folks – People think and learn in different ways. When teaching, it is best to provide a few different explanations and have a variety of ways to present content. Some students, the linear thinkers, will understand two-column proof perfectly, and others, the special thinkers, will find flowchart proofs clearer. It is best to use both so that all students understand and develop their reasoning skills. One option to introduce the flowchart format is to have the students go back to key two-column proofs provided in the text and convert them to flowchart proofs.

Patterns and Structure – All of the shortcuts to triangle congruence require three pieces of information, therefore the box of the flowchart proof that states that two triangles are congruent will have three boxes leading into it. These kinds of structural relationships help students write and understand flowchart proofs and should be noted. It is also helpful to give students incomplete flowchart proofs and have them fill in the missing information. Subsequent proofs can be given with less information provided each time, until the boxes are all empty, and then with no help at all. The only problem is that sometimes there is more than one way to write a proof and a different chart may be required for the proof that the student wants to write. In that case, students can start from scratch if they like.

Proof Using SAS and HL

AAA – Students sometimes have to think for a bit to realize that AAA does not prove triangle congruence. Ask them to think back to the definition of triangle. Congruent triangles have the same size and shape. Most students intuitively see that AAA guarantees that the triangles will have the same shape. To see that triangles can have AAA and be different sizes ask them to consider a triangle they are familiar with, the equiangular triangle. They can draw an equiangular triangle on their paper, and you can draw an equiangular triangle on the board. The triangles have AAA, but are definitely different in size. This is a counterexample to AAA congruence. Have the students note that the triangles are the same shape; this relationship is called similar and will be studied in later chapters.

SSA – Student will have a hard time seeing the two possible triangles with SSA. The best way to describe it when the congruent parts are set up, is to tell them to take that the last congruent side can bend in, so that the third side is short to make one triangle, and bent out, so that the third side is long, to make the other triangle. Some students will see it right away and others will really have to play around with their triangles for awhile in order to understand.

Why Not LL? – Some students may wonder why there is not a LL shortcut for the congruence of right triangles. It

also leads to SSS when the Pythagorean theorem is applied. Have the students explore the situation with a drawing. They can draw out two congruent right triangles and mark sides so that the triangles have LL. There is already a congruence guarantee for this, SAS. What would the non-right triangle congruence be for HL? Is this a guarantee? (It would be SSA, and no, this does not work in triangles that are right.)

Importance of Right Triangles – When using math to model situations that occur in the world around us the right triangle is used frequently. Have the students think of right angles that they see every day: walls with the ceilings and the floors, widows, desks, and many more constructed objects. Right triangles are also important in trigonometry which they will be studying soon. Stressing the usefulness of right triangles will motivate them to think about why HL guarantees triangle congruence but SSA, in general, does not.

Using Congruent Triangles

The Process – When students first start examining pairs of triangles to determine congruence it is difficult for them to sort out all the sides and angles.

The first step is for them to copy the figure onto their paper. It is helpful to color code the sides and angles, congruent sides marked in one color and the congruent angles in another. Some congruent parts will not be marked in the original figure that is given to the students in the text. For example, there could be an overlapping side that is congruent to itself, due to the reflexive property; mark it as well. Then they should do a final check to ensure that the congruent parts do correspond.

The next step is for them to count how many pairs of congruent corresponding sides and how many pairs of congruent corresponding angles there are. With this information they can eliminate some possibilities from the list of way to prove triangles congruent. If there is no right angle they can eliminate HL, or if they only have one set of corresponding congruent angles, they can eliminate both ASA and AAS.

If at this point there is still more than one possibility, they are going to need to decide if an angle is between two sides or if a side is between two angles. Remind them that both ASA and AAS can be used to guarantee triangle congruence, and that SAS works, but that SSA can not be used to prove two triangles are congruent.

If all postulates and theorems have been eliminated, then it is not possible to determine if the triangles are congruent.

AAS or SAA – Sometimes students try to list the congruent sides and angles in a circle as they move around the triangle. This could result in AAS or SAA when there are two pairs of congruent angles and one pair of congruent sides that is not between the angles. They know AAS proves congruence and want to know if SAA does as well. When this occurs it is best to redirect their thinking process. With two sets of angles and one set of sides there are only two possibilities, the side is between the angles or it is another side. When it is between the angles we have ASA, if it is either of the other two sides we use SAA. This same situation occurs with SSA, but is even more important since SSA is not a test for congruence. A good way for the students to remember this is that when the order of SSA is reversed it makes an inappropriate word. This word should not be used in class or in proofs, even if it is spelled backwards.

Isosceles and Equilateral Triangles

The Useful Definition of Congruent Triangles – The arguments used in the proof of the Base Angle Theorem apply what the students have learned about triangles and congruent figures in this chapter, and what they learned about reasoning and implication in the second chapter. It is a lot of information to bring together and students may need to review before they can fully understand the proof.

They have been practicing with proofs throughout the chapter, so they should be adept with the logic at this point.

2.4. Congruent Triangles

If they are having trouble, a review of the Deductive Reasoning section in Chapter Two: Reasoning and Proof will help. It could be assigned as reading the night before the current lesson will be done in class.

This is a good point to summarize what the students have learned in this chapter about congruent triangles and demonstrate how it can be put to use. To understand this proof, students need to remember that the definition of congruent triangles requires three pairs of congruent sides and three pairs of congruent angles, but realize that not all six pieces of information need to be verified before it is certain that the triangles are congruent. There are shortcuts. The proof of the Base Angle Theorem uses one of these shortcuts and jumps to congruence which implies that the base angles, a pair of corresponding angles of congruent triangles, are congruent.

To a student new to geometry this argument is not as straightforward as it may seem to an instructor experienced in mathematical proofs. Plan to take some time explaining this important proof.

A Proved Theorem Can Be Used – Now that the students have the proof of the Base Angle Theorem they can use it as opportunities present themselves. They should be on the lookout for isosceles triangles in the proofs of other theorems, in complex figures, and in all other situations. When they spot them, they need to immediately apply the Base Angle Theorem and mark those base angles congruent. This is true for the converse as well. When they spot a triangle with congruent angles, they should mark the appropriate sides congruent. Students sometimes do not realize what a powerful tool this theorem is and that they will be using it extensively throughout this class, and in math classes they will take in the future.

Additional Exercise:

1. What are the measures of the angles of an equilateral triangle? What postulates or theorems did you use to obtain your answer?

Answer: 60 degrees, Base Angle Theorem, Triangle Sum Theorem

Congruence Transformations

Reflection or Rotation – When looking a two triangle, where one is a transformation of the other, students sometimes have trouble distinguishing between a reflection and a rotation. This is particularly true when the triangles are almost equilateral. When demonstrating these transformations, it is best to use an obviously scalene triangle. Good use of labels is also helpful. The prime notation clearly indicates the new location of each vertex under the transformation. A rotation preserves the order of the vertices, and a reflection reverses the order of the vertices. If the students are unsure of what transformation has been applied to the figure, have them choose one vertex and then move counterclockwise around the polygon listing off the vertices as they occur. If they start with the image of that first vertex in the new figure, and again move counterclockwise, they will get the images of the vertices in the same order for a rotation, and in reverse order for a reflection.

Don't Just Memorize, Reason – This section contains many ordered pair rules for different transformation. Students will try to memorize them without really thinking about them or looking for patterns. This is challenging, if not impossible for most students. Have the students discuss similarities and differences between the rules. Ask them if the rule surprises them, or seems logical. Why? If they really get stuck, they can do a test. Graph a scalene triangle and apply different rules to it until the desired transformation occurs. Students will be motivated to use reason to shorten the guess and check process. In geometry problems are written so that students will have to think about them for awhile, and figure out an answer. Once students realize that they are not supposed to know the answer immediately, they are much more willing to spend time thinking about an exercise.

Additional Exercise:

1. Graph a scalene triangle in the first quadrant of the coordinate axis. Reflect the triangle over the $x-$axis. Take this new triangle and reflect it over the $y-$axis. What single transformation would take the first triangle to the final triangle? How can this be predicted by the ordered pair rules?

Answer: A single rotation of 180 degrees about the origin would result in the final triangle. The reflection over the $x-$axis takes the opposite of the $y-$coordinate and the reflection over the $y-$axis takes the opposite of the $x-$coordinate. If opposite of both coordinates are taken, the result is a rotation of 180 degrees about the origin.

2.5 Relationships Within Triangles

Midsegments of a Triangle

Don't Forget the $\frac{1}{2}$ - In this section there are two types of relationships that the students need to keep in mind when writing equations with variable expressions. The first involves the midpoint. When the expressions represent the two parts of a segment separated by the midpoint they just have to set the expressions equal to each other. The second is when comparing the length of a side of the triangle with the midsegment parallel to it. In this case they need to multiply the expression representing the side of the triangle by $\frac{1}{2}$, and then set it equal to the expression representing the midsegment. They may forget the $\frac{1}{2}$ or forget to use parenthesis and distribute. Remind them that they need to multiply the entire expression by $\frac{1}{2}$, not just the first term.

Additional Exercises:

1. The proof given in the text of the Midsegment Theorem is a paragraph proof. Write the second part of the proof as a two-column proof.

Answer: Refer to the triangle used in the text proof on the top of page 267.

TABLE 2.8:

Statement	Reason
$\overline{AB}, \overline{CB}$, and $\overline{AC}$ are midsegments of $\triangle XYZ$	Given
$\overline{AB}$ is parallel to $\overline{XY}$	Midsegment Theorem (1)
$\angle BAC \cong \angle XCA$	Alternate Interior Angle Theorem
$\overline{CB}$ is parallel to $\overline{XZ}$	Midsegment Theorem (1)
$\angle XAC \cong \angle BCA$	Alternate Interior Angle Theorem
$\overline{AC} \cong \overline{AC}$	Reflexive Property of Congruence
$\triangle AXC \cong \triangle CBA$	ASA Postulate
$\overline{AB} \cong \overline{XC}$	Definition of Congruent Triangles
C is the midpoint of $\overline{XY}$	Definition of Midsegment
$XC = CY$	Definition of Midpoint
$XY = XC + CY$	Segment Addition Postulate
$XY = XC + XC$	Substitution Property of Equality
$AB = XC$	Definition of Congruent Segments
$XY = AB + AB = 2AB$	Substitution Property of Equality
$AB = \frac{1}{2}XY$	Division Property of Equality

Perpendicular Bisectors in Triangles

Construction Frustrations – Using a compass and straightedge to make clean, accurate constructions takes a bit of practice. Some students will pick up the skill quickly and others will struggle. What is nice about doing construction in the classroom is that it is often the students that typically struggle with mathematics, the more artistically minded students that excel and learn from constructing figures.

Recommend that students invest in a decent compass. They can get a quality, metal compass with some weight behind it for less that $20. The $2 variety often does not hold the pencil steady.They slip, and are extremely frustrating.

Here are some other tips for good construction: (1) Hold the compass at an angle. (2) Try rotating the paper while holding the compass steady. (3) Work on a stack of a few papers so that the needle of the compass can really dig into the paper and will not slip.

Perpendicular Bisector Quirks – There are two key ways in which the perpendicular bisector of a triangle is different form the other segments in the triangle that the students will learn about in subsequent sections. Since they are learning about the perpendicular bisector first these differences do not become apparent until the end of the chapter.

The perpendicular bisector of the side of a triangle does not have to pass through a vertex. Have the students explore in what situations the perpendicular bisector does pass through the vertex. They should discover that this is true for equilateral triangles and for the vertex angle of isosceles triangles.

The point of concurrency of the three perpendiculars of a triangle, the circumcenter, can be located outside the triangle. This is true for obtuse triangles. The circumcenter will be on the hypotenuse of a right triangle. This is also true for the orthocenter, the point of concurrency of the altitudes.

Same Construction for Midpoint and Perpendicular Bisector – The Perpendicular Bisector Theorem is used to construct the perpendicular bisector of a segment and to find the midpoint of a segment. When finding the midpoint, the students should make the arcs, one from each endpoint with the same compass setting, to find two equidistant points, but instead of drawing in the perpendicular bisector, they can just line up their ruler and mark the midpoint. This will keep the drawing from getting overcrowded and confusing.

Angle Bisectors in Triangles

Check with a Third – When constructing the point of concurrency of the perpendicular bisectors or angle bisectors of a triangle, it is strictly necessary to construct only two of the three segments. The theorems proved in the texts ensure that all three segments meet in one point. It is advisable to construct the third segment as a check of accuracy. Sometimes the compass will slip a bit while the student is doing the construction. If the three segments form a little triangle, instead of meeting at a single point, the student will know that their drawing is not accurate and can go back and check their marks.

Inscribed Circles – For a circle to be inscribed in a triangle, all three sides of the triangle must be tangent to the circle. A tangent to a circle intersects the circle in exactly one point. After accurately finding the incenter, students may have a difficult time finding the correct compass setting that will construct the inscribed circle. The best method is to place the center of the compass at the incenter, choose one side, and adjust the compass setting until the compass brushes by that side of the triangle, without passing through it. The word tangent does not have to be introduced at this point if the students already have enough vocabulary to learn. When the incenter is correctly placed, the compass should also hit the other two sides of the triangle once, creating the inscribed circle.

Additional Exercises:

1. Construct an equilateral triangle. Now construct the perpendicular bisector of one of the sides. Construct the angle bisector from the angle opposite of the side with the perpendicular bisector. What do you notice about these two segments? Will this be true of a scalene triangle?

Answer: The segments should coincide on the equilateral triangle, but not on the scalene triangle.

2. Construct an equilateral triangle. Now construct one of the angle bisectors. This will create two right triangles. Label the measures of the angles of the right triangles. With your compass compare the lengths of the shorter leg to the hypotenuse of either right triangle. What do you notice?

2.5. Relationships Within Triangles

Answer: The hypotenuse should be twice the length of the shorter leg.

Medians in Triangles

Vocabulary Overload – So far this chapter has introduced to a large number of vocabulary words, and there will be more to come. This is a good time to stop and review the new words before the students become overwhelmed. Have them make flashcards, or play a vocabulary game in class.

Label the Picture – When using the Concurrency of Medians Theorem to find the measure of segments, it is helpful for the students to copy the figure onto their paper and write the given measures by the appropriate segments. When they see the number in place, it allows them to concentrate on the relationships between the lengths since they no longer have to work on remembering the specific numbers.

Median or Perpendicular Bisector – Students sometimes confuse the median and the perpendicular bisector since they both involve the midpoint of a side of the triangle. The difference is that the perpendicular bisector must be perpendicular to the side of the triangle, and the median must end at the opposite vertex.

Key Exercises:

1. In what type(s) of triangles are the medians also perpendicular bisectors? Is this true of all three medians?

Answer: This is true of all three medians of an equilateral triangle, and the median that intersects and vertex angle of an isosceles triangle.

Applications – Students are much more willing to spend time and effort learning about topics when they know of their applications. Questions like the ones below improve student motivation.

Key Exercises:

In the following situations would it be best to find the circumcenter, incenter, or centroid?

1. The drama club is building a triangular stage. They have supports on all three corners and want to put one in the middle of the triangle.

Answer: Centroid, because it is the center of mass or the balancing point of the triangle

2. A designer wants to fit the largest circular sink possible into a triangular countertop.

Answer: Incenter, because it is equidistant from the sides of the triangle.

Altitudes in Triangles

Extending the Side – Many students have trouble knowing when and how to extend the sides of a triangle when drawing in an altitude. First, this only needs to be done with obtuse triangles when drawing the altitude that intersects the vertex of one of the acute angles. It is the sides of the triangle that form the obtuse angle that need to be extended. The students should rotate their paper so that the vertex of the acute angle they want to start an altitude from is above the other two, and the segment opposite of this vertex is horizontal. Now they just need to extend the horizontal side until it passes underneath the raised vertex.

The Altitude and Distance – The distance between a point and a line is defined to be the shortest segment with one endpoint on the point and the other on the line. It has been shown that the shortest segment is the one that is perpendicular to the line. So, the altitude is the segment along which the distance between a vertex and the opposite side is measured. Seeing this connection will help students remember and understand why the length of the altitude is the height of a triangle when calculating the triangle's area using the formula $A = \frac{1}{2}bh$.

Explorations – When students discover a property or relationship themselves it will be much more meaningful. They will have an easier time remembering the fact because they remember the process that resulted in it. They will also have a better understanding of why it is true now that they have experience with the situation. Unfortunately, students sometimes become frustrated with explorations. They may not understand the instruction, or they may not be carefully enough and the results are unclear. Some of the difficulties can be alleviated by have the students work in groups. They can work together to understand the directions and interpret the results. Students strong in one area, like construction, can take on that part of the task and help the others with their technique.

Some guidelines for successful group work.

- Groups of three work best.
- The instructor should choose the groups before class.
- Students should work with new groups as often as possible.
- Desks or tables should be arranged so that the members of the group are physically facing each other.
- The first task of the group is to assign jobs: person one reads the directions, person two performs the construction, person three records the results. Students should regularly trade tasks.

Inequalities in Triangles

The Opposite Side/Angle – At first it may be difficult for students to recognize what side is opposite a given angle or what angle is opposite a given side. If it is not obvious to them from the picture, obtuse, scalene triangles can be confusing, they should use the names. For $\triangle ABC$, the letters are divided up by the opposite relationship, the angle with vertex A is opposite the side with endpoints B and C. Being able to determine these relationships without a figure is important when studying trigonometry.

Small, Medium, and Large - When working with the relationship between the sides and angles of a triangle, students will summarize the theorem to "largest side is opposite largest angle". They sometimes forget that this comparison only works within one triangle. There can be a small obtuse triangle in the same figure as a large acute triangle. Just because the obtuse angle is the largest in the figure, does not mean the side opposite of it is the longest among all the segments in the figure, just that it is the longest in that obtuse triangle. If the triangles are connected or information is given about the sides of both triangles, a comparison between triangles could be made. See exercise #9 is the text.

Add the Two Smallest – The triangle inequality says that the sum of the lengths of any two sides of a triangle is greater than the length of the third side. In practice it is enough to check that the sum of the lengths of the smaller two sides is larger than the length of the longest side. When given the three sides lengths for a triangle, students who do not fully understand the theorem will add the first two numbers instead of the smallest two. When writing exercises it is easy to always put the numbers in ascending order without thinking much about it. Have the students try to draw a picture of the triangle. After making a few sketches they will understand what they are doing, instead of just blindly following a pattern.

Indirect Proof – Students will not really understand the method of indirect proof the first time they see it. Let them know that this is just the first introduction, and that in subsequent lessons they will be given more examples and opportunities to learn this new method of proof. If students think they are supposed to understand something perfectly the first time they see it, and they don't, they will become frustrated with themselves and mathematics. Let them know that the brain needs time, and multiple exposures to master these challenging concepts.

Inequalities in Two Triangles

Use Color – The figures is this section now have two triangles instead of just one and are therefore more complex.

2.5. Relationships Within Triangles

The students may need some help sorting out the shapes. A good way for them to begin this process is to draw the figure on their paper and use highlighters to color code the information.

Both of the theorems presented in this section require two pairs of congruent sides. The first step is for student to highlight these four sides in a common color, let's say yellow. Once they have identified the two pairs of congruent sides, they know the hypothesis of the theorem has been filled and they can apply the conclusion.

The conclusions of these theorems involve the third side and the angle between the two congruent sides. These parts of the triangles can be highlighted in a different color, let's say pink.

Now the students need to determine if they need to use the SAS Inequality theorem or the SSS inequality theorem. If they know one of the pink angles is bigger than the other, than they will use the SAS Inequality theorem and write an inequality involving the pink sides. If they know that one of the pink sides is bigger than the other, they will apply the SSS Inequality theorem, and write an inequality involving the two pink angles.

Having a step-by-step process is good scaffolding for students as they begin working with new types of problems. After the students have gained some experience, they will no longer need to go through all the steps.

Solving Inequalities – Students learned to solve inequalities in algebra, but a short review may be in order. Solving inequalities involves the same process as solving equations except the equal sign is replaced with an inequality, and there is the added rule that if both sides of the inequality are multiplied or divided by a negative number the direction of the inequality changes. Students frequently want to change the direction of the inequality when it is not required. They might mistakenly change the inequality if they subtract from both sides, or if result of multiplication or division is a negative even if the number used to change the inequality was not negative. In geometry it is most common to be working with all positive numbers, but depending on how the students apply the Properties of Inequalities, they may create some negative values.

Indirect Proof

Why Learn Indirect Proof – For a statement to be mathematically true it must always be true, no exceptions. This frequently makes it easier to prove that a statement is false than to prove it is true. Indirect proof gives mathematician the choice between proving a statement true or proving a statement false and can therefore greatly simplify some proofs. Letting the students know that indirect proof can be a potential shortcut will motivate them to learn to use this type of logic.

Review the Contrapositive - Proving a statement using indirect proof is equivalent to proving the contrapositive of the statement. If students are having trouble setting up indirect proofs, or even if they are not, it is a good idea to have them review conditional statements and the contrapositive. The second section of Chapter Two: Reasoning and Proof is about conditional statements. Have the students reread this section before working on indirect proof in class. The first step to writing an indirect proof, can be to have them write out the contrapositive of the statement they want to prove. This will reduce confusion about what statement to start with, and what statement concludes the proof.

Does This Really Prove Anything? – Even after students have become adept with the mechanics of indirect proof, they may not be convinced that what they are doing really proves the original statement. This is the same as asking if the contrapositive is equivalent to the original statement. Using examples outside the field of mathematics can help students concentrate on the logic.

Start with the equivalence of the contrapositive. Does statement (1) have the same meaning as statement (2)?

   a. If you attend St. Peter Academy, you must wear a blue uniform.
   b. If you don't wear a blue uniform, you don't attend St. Peter Academy.

Let the students discuss the logic, and have them create and share their own examples.

If a good class discussion ensues, and the students provide many statements on a single topic, it may be possible to write some indirect proofs of statements not concerned with mathematics. This could be a good bonus assignment or project that when presented to the class will make the logic of indirect proof clearer for other students.

2.6 Quadrilaterals

Interior Angles

All Those Polygons – Although they have probably been taught it before, not all students will remember the names of the different polygons. There are not very many opportunities in life to use the word heptagon. Add these words to their vocabulary list.

This is most likely the first time they have been introduced to a polygon with a variable number of sides, an $n-$gon. This notation can be used when referring to a polygon that does not have a special name in common use, like a $19-$gon. It can also be used when the number of sides of the polygon is unknown.

Key Exercise:

1. What is the measure of each interior angle of a regular $n-$gon if the sum of the interior angles is 1080 degrees?

Answer: 135 degrees

First the number of sides needs to be found:

$$1080 = 180(n-2)$$
$$n = 8$$

Now the total of 1080 degrees needs to be divided into 8 congruent angles.

$$1080 \div 8 = 135$$

Sketchpad Alternatives – Many students become particularly engaged in a topic when they are able to investigate it while playing around with the computer. Here are a couple of ways to use Geometers' Sketchpad in the classroom as an alternative or supplement to direct instruction.

Angle Sum Conjecture – Have student draw different convex polygons and measure the sum of their interior angles.

    a. The students should observe that for each type of polygon, no matter how many were drawn, they all have the same interior angle sum.
    b. The students should drag a vertex of each polygon toward the center to create a concave polygon, and notice if the sum stays the same. (It won't.)
    c. Put the sums in order on the board: $180, 360, 540, \ldots$ Ask the students to find the pattern in this sequence of numbers. Lead them to discovering the Angle Sum formula from the pattern.

Exterior Angles

Clockwise or Counterclockwise But Not Both – At this point in the class, student are usually good at recognizing vertical angles. They will understand that the exterior angles made by extending the sides of the polygon in a clockwise rotation are congruent, at each vertex, to the exterior angle formed by extending the sides counterclockwise.

What they will sometimes do is include both of these angles when using the Exterior Angle Sum theorem. Reinforce that the number of exterior angles is the same as the number of interior angles and sides, one at each vertex.

Interior or Exterior – Interior and exterior angles come in linear pairs. If one of these angles is known at a particular vertex, it is simple to find the other. When finding missing angles in a polygon, students need to decide from the beginning if they are going to use the interior or exterior sum. Most likely, if the majority of the known angle measures are from interior angles they will use the interior sum. They need to convert the exterior angle measures to interior angle measures before including them in the sum. If there are more exterior angle measures given, they can convert the interior angle measures and use the sum of 360 degrees. It is important that they make a clear choice. They may mix the two types of angles in one summation if they are not careful.

Do a Double Check – Students often do not take the time to think about their answers. Going over the arithmetic and logic is one way to check work, but it is common to not recognize the error the second time either. A better strategy is to use other relationships to do the checking. In this lesson if the exterior sum was used, the work can be checked with the interior sum.

What's the Interior Sum of a Nonagon Again? – If students do not remember the interior sum for a specific polygon, and do not remember the formula, they can always convert to the exterior angle measures using the linear pair relationship. The sum of the exterior angles is always 360 degrees. This strategy will work just as well as using the interior sum. Remind the students to be creative. When taking a test, they may not know an answer directly, but many times they can figure out the answer in an alternative way.

Sketchpad Alternative – The activities designed for students to explore interior angles in the pervious section can be easily adapted for exterior angles, and be used with this section. Using Sketchpad to extend the sides of the polygon helps students gain an understanding of where the exterior angle is in relation to the polygon.

Classifying Quadrilaterals

Tree Diagram – Most students will need practice working with the classification of quadrilaterals before they completely understand and remember all of the relationships. The Venn diagram is an important mathematical tool and should definitely be used to display the relationships among the different types of quadrilaterals. A tree diagram will also make an informative visual. Using both methods will reinforce the students' understanding of quadrilaterals, and their ability to make good diagrams.

Parallel Line Properties – In the second section of Chapter Three: Parallel and Perpendicular Lines, the students learn about the relationships between the measures of the angles formed by parallel lines and a transversal. Many of the quadrilaterals studied in this section have parallel sides. The students can apply what they learned in chapter three to the quadrilaterals in this chapter. They may have trouble seeing the relationships because instead of lines the quadrilaterals are made of segments. Recommend that the students draw the figures on their papers and extend the sides of the quadrilaterals so they can see all four angles made by the intersection of the lines. These angles will be useful when looking for specific information about the quadrilateral.

Show Clear, Organized Work – When using the distance or slope formula to verify information about a quadrilateral on the coordinate plane, students will often do messy scratch work as if they are the only ones that will need to read it. In this situation, the work is a major part of the answer. They need to communicate their thoughts on the situation. They should write as if they are trying to convince the reader that they are correct. As students progress in their study of mathematics, this is more often the case than the need for a single numerical answer. They should start developing good habits now.

Symmetry – Most students have already studied symmetry at some point in their education. A review here may be in order. When studying quadrilaterals, symmetry is a good property to consider. Symmetry is also important when discussing the graphs of key functions that the students will be studying in the next few years. It will serve the students well to be adept in recognizing different types of symmetry.

Using Parallelograms

Proofs Using Congruent Triangles – The majority of the proofs in this section use congruent triangles. The quadrilateral of interest is somehow divided into triangles that can be proved congruent with the theorems and postulates of the previous chapters. Once the triangles are known to be congruent, the definition of congruent triangles ensures that certain parts of the quadrilateral are also congruent. Students should be made aware of this pattern if they are having difficulty writing or understanding the proofs of the properties of various quadrilaterals. If they are still struggling they should spend some time reviewing sections two through six of Chapter Four: Congruent Triangles.

The Diagonals of Parallelograms – The properties concerning the sides and angles of parallelograms are fairly intuitive, and students pick them up quickly. More emphasis should be placed on what is known, and not known about the diagonals. Students frequently try to use the incorrect fact that the diagonally of a parallelogram are congruent. Rectangles are the focus of an upcoming lesson, but demonstrating to students that the diagonals of a quadrilateral are only congruent in the special case where all the angles of the parallelogram are congruent. For a general parallelogram, the measures of the two pieces of the same diagonal separated by the other diagonal, can be set equal to each other, but no comparison can be made between diagonals.

Additional Exercises:

1. Quadrilateral $ABCD$ is a parallelogram.

$AB = 2x + 5, BC = x - 3$, and $DC = 3x - 10$

Find the measures of all four sides of the quadrilateral.

(Hint: Draw and label a picture. Remember the name of a polygon lists the vertices in a circular order.)

Answer:

$$2x + 5 = 3x - 10 \qquad\qquad AB = CD = 35 \text{ and } BC = AD = 12$$
$$x = 15$$

2. JACK is a parallelogram.

$m\angle A - 10x - 60°$ and $m\angle C = 2x + 45°$

Find the measures of all four angles.

Answer:

$$10x - 60 + 2x + 45 = 180 \qquad\qquad m\angle C = m\angle J = 77.5°$$
$$x = 16\,\frac{1}{4} \qquad\qquad m\angle A = m\angle K = 102.5°$$

Proving Quadrilaterals are Parallelograms

Proof Practice – The proofs in this section may seem a bit repetitive, but students will benefit from practicing these proofs since they review important concepts learned earlier in the course. To avoid loosing the students' attention, find different ways of presenting the proofs. One idea is to divide the students into groups, and have each group demonstrate a different proof to the class.

Parallel or Congruent – When looking at a marked figure students will sometimes see the arrows that designate parallel segments and take that the segments to be congruent. This could be due to the misreading of the marks, or mistakenly thinking parallel always implies congruence. Warn students not to make this error. The last method of proof in this section which utilizes that one pair of sides are both congruent and parallel, along with an example of a trapezoid where the parallel sides are not congruent, will help students remember the difference.

Additional Exercises:

1. KATE is a parallelogram with a perimeter of 40 cm.

$KA = 3x + 8$, and $AT = x + 4$

Find the length of each side.

(Hint: Draw and label a picture. Remember the name of a polygon lists the vertices in a circular order.)

Answer:

$$2(3x + 8) + 2(x + 4) = 40 \qquad\qquad KA = ET = 14 \text{ cm}$$
$$x = 2 \qquad\qquad AT = KE = 6 \text{ cm}$$

2. SAMY is a parallelogram with diagonals intersecting at point X.

$SX = x + 5, XM = 2x - 7, AX = 12x$

Find the length of each diagonal.

Answer:

$$x + 5 = 2x - 7 \qquad\qquad SM = 34 \text{ cm}$$
$$x = 12 \qquad\qquad AY = 288 \text{ cm}$$

3. JEDI is a parallelogram.

$m\angle J = 2x + 60$, and $m\angle D = 3x + 45$

Find the measures of the four angles of the parallelogram.

Does this parallelogram have a more specific categorization?

Answer:

$$2x + 60 = 3x + 45 \qquad\qquad \text{All four angles measure 90 degrees.}$$
$$x = 15 \qquad\qquad \text{JEDI is a rectangle.}$$

Rhombi, Rectangles, and Squares

The Power of the Square – Students should know by the classification of quadrilaterals that all the theorems for parallelograms, rectangles, and rhombi, also apply to squares. It is a good idea to talk about this in class though in case they have not put it together on their own. A combination of these theorems and the definition of a square can be combined to from some interesting exercises.

Key Exercises:

2.6. Quadrilaterals

1. SQUR is a square.

$m\angle XUR = 3x - 9$, and $SU = x$

Find the length of both diagonals.

(Hint: Draw and label a picture. Remember the name of a polygon lists the vertices in a circular order.)

Answer:

$$3x - 9 = 45 \qquad\qquad\qquad\qquad SU = QR = 18$$
$$x = 18$$

Information Overload – Quite a few theorems are presented in this chapter. Remembering them all and which quadrilaterals they apply to can be a challenge for students. If they are unsure, and cannot check reference material, a test case can be drawn. For example: Do the diagonals of a parallelogram bisect the interior angles of that parallelogram? First they need to draw a parallelogram that clearly does not fit into any subcategory. It should be long and skinny, so no rhombi properties are mistakenly attributed to it. It should also be well slanted over, so as not to be mistaken for a rectangle. Now they can draw in the diagonals. It will be obvious that the diagonals are not bisecting the interior angles. They could also try to recreate the proof, but that will probably be more time consuming and it requires a bit of skill.

Additional Exercises:

1. DAVE is a rhombus with diagonals that intersect at point X.

$DX = 3$ cm, and $AX = 4$ cm

How long is each side of the rhombus?

Answer:

$$3^2 + 4^2 = DA^2 \qquad\qquad\qquad \text{since } \triangle DXA \text{ is right}$$
$$DA = 5$$
$$DA = AV = VE = ED = 5 \text{ cm}$$

Trapezoids

Average for the Median – Students who have trouble memorizing formulas may be intimidated by the formula for the length of the median of a trapezoid. Inform them that they already know this formula; it is just the average. The application of the formula makes since, the location of the median is directly between the two bases, and the length of the median is exactly between the lengths of the bases. They will have no problem finding values involving the median.

Where Are We? – Most students have five other classes and a demanding social and family life. It is easy for them to forget how what they are learning today relates to the chapter and to the class. Use the Venn diagram of the classification of quadrilaterals to orient them in the chapter. They are no longer learning about parallelograms, but have moved over to the separate trapezoid area. When student are able to organize their new knowledge, they are better able to retain and apply it.

Does it Have to be Isosceles? – Students may have trouble remembering which theorems in this section apply only to isosceles trapezoids. Note that base angle, and diagonal congruence apply only to isosceles trapezoids, but the relationship of the length of the median to the bases is the same for all trapezoids.

Additional Exercises:

1. TRAP is a trapezoid. The median has length 4 cm, and one of the bases has length 7 cm. What is the length of the other base?

Answer: 1 cm

Seven is three more than four, so the other base must be three less than four.

OR solve the equation $4 = (7 + x) \div 2$

2. $WXYZ$ is a trapezoid. The length of one base is twice the length of the other base, and the median is 9 cm. How long is each base?

Answer:

$$(x + 2x) \div 2 = 9 \qquad\qquad \text{The bases are 6 cm and 12 cm.}$$
$$x = 6$$

Kites

Only One Congruent Set – It is important to note that in a kite, only one set of interior angles are congruent, and only one of the diagonals is bisected. Sometimes students struggle with identifying where these properties hold. It is the nonvertex angles that are congruent, and the diagonal connecting the nonvertex angles that is bisected. The single line of symmetry of a kite shows both these relationships.

Break it Up – When working with a kite, it is sometimes easier to think of it as two isosceles triangles, or four right triangles, instead of one quadrilateral.

At this point in the class, students have had extensive experience working with isosceles triangles, and can easily apply the Base Angle theorem to see that the nonvertex angles of the kite are congruent. They have also seen that the segment from the vertex angle creates many symmetries in the triangle, and it will make since to them that the diagonal connecting the nonvertex angles is bisected.

They can also think of a kite as four right triangles. This will help them remember that the diagonals are perpendicular, and remind them that the Pythagorean theorem can be used to find missing segment measures. Noticing that the right triangles are in two congruent sets will help them identify congruent segments and angles.

Additional Exercises:

1. Refer to the kite used to prove the diagonal properties on page 396.

Prove that $\triangle AYR$ is congruent to $\triangle TYR$.

Answer:

TABLE 2.9:

Statement	Reason
$\overline{AR} \cong \overline{TR}$	Given
$\overline{AT} \perp \overline{PR}$	Kite Diagonal Theorem
$\angle AYR$ is right	Definition of Perpendicular
$\angle TYR$ is right	Definition of Perpendicular
$\angle TYR \cong \angle AYR$	Right Angle Theorem
$\triangle AYR \cong \triangle TYR$	HL

2.7 Similarity

Ratio and Proportion

Keep it in Order – When writing a ratio, the order of the numbers is important. When the ratio is written in fraction form the amount mentioned first goes in the numerator, and the second number goes in the denominator. Remind the students it is important to keep the values straight, especially if they are looking at the male to female student ratio at their top three college choices.

To Reduce or Not to Reduce – When a ratio is written in fraction form it can be reduced like any other fraction. This will often make the arithmetic simpler and is frequently required by instructors for fractions in general. But when reducing a ratio, useful information can be lost. If the ratio of girls to boys in a classroom is 16 to 14, it may be best to use the fraction $\frac{16}{14}$ because it gives the total number of students in the class where the reduced ratio $\frac{8}{7}$ does not.

Consistent Proportions – A proportion can be correctly written in many ways. As long as the student sets up the ratios in a consistent, orderly fashion, they will most likely have written a correct proportion. There should be a common tie between the two numerators, the two denominators, the numbers in the first ratio, and the numbers in the second ratio. They should think about what the numbers represent, and not just use them in the order given in the exercise, although the numbers are usually given in the correct order.

Key Example:

1. Junior got a new hybrid. He went 525 miles on the first five gallons that came with the car. He just put 12 gallons in the tank. How far can he expect to go on that amount of gas?

Answer:

$$\frac{525}{5} = \frac{x}{12}$$
$$x = 1260$$

He can expect to go $1,260$ miles.

Note: Students will be tempted to put the 12 in the numerator of the second ratio because it was the third number given in the exercise, but it should go in the denominator with the other amount of gas.

The Fraction Bar is a Grouping Symbol – Students know that parenthesis are a grouping symbol and that they need to distribute when multiplying a number with a sum or difference. A fraction bar is a more subtle grouping symbol that students frequently overlook, causing them to forget to distribute. To help them remember have them put parenthesis around sums and differences in proportions before they cross-multiply.

Example: $\frac{x+3}{5} = \frac{x-8}{7}$ becomes $\frac{(x+3)}{5} = \frac{(x-8)}{7}$

Properties of Proportions

Everybody Loves to Cross-Multiply – There is something satisfying about cross-multiplying and students are prone to overusing this method. Remind them that cross-multiplication can only be used in proportions, when two rations are <u>equal</u> to each other. It is not appropriate to cross-multiply when two fractions are being added or subtracted.

Examples:

TABLE 2.10:

Cross Multiply	Don't Cross Multiply
$\frac{3}{4} = \frac{10}{x}$	$\frac{3}{4} + \frac{10}{x}$
$\frac{x+3}{4} = \frac{10}{x}$	$\frac{x+3}{4} - \frac{10}{x}$

Only Cancel Common Factors – When reducing a fraction or putting a ratio in simplest terms, students often try to cancel over an addition or subtraction sign. This problem occurs most frequently when students work with fractions that contain variable expressions. To combat this error, go back to numerical examples. Students will see that what they are doing does not make sense when the variables are removed. Then go back to example with variables. Hopefully the students will be able to carry over the concept.

Examples:

TABLE 2.11:

Can be Reduced	Can't be Reduced
$\frac{3 \cdot 2}{5 \cdot 2}$	$\frac{3+2}{5+2}$
$\frac{3(x-4)}{3 \cdot 2}$	$\frac{x-4}{4}$

Color-Code the Proofs – The proof in this section requires many substitutions of similar looking expressions. It is difficult to see where everything is coming from and moving to. When presenting the proof in class use colors so the variables will be easier to follow. Another option is to have the students do the color-coding. Once they understand the mechanics of the proof in the lesson, they will be able to do the similar proof in the exercises.

Similar Polygons

A Common Vocabulary Error – Students frequently interchange the words proportional and similar. Remind them that proportional describes a relationship between numbers, and similar describes a relationship between figures, like equal and congruent.

Compare and Contrast Similar with Congruent – If your students have already learned about congruent figures, now would be a good time to review. The definitions of congruent and similar are very close. Ask the students if they can identity the difference; it's only one word. You can also point out that congruent is a subset of similar like square is a subset of rectangle, or mother is a subset of women. Understanding the differences between congruent and similar will be important in upcoming lessons when proving triangles similar.

Use that Similarity Statement – In some figures, which sides of similar polygons correspond is obvious, but when the polygons are almost congruent, or oriented differently, the figure can be misleading. Students usually begin by using the figure and then forget to use the similarity statement when necessary. Remind them about this information as they start working on more complicated problems. The similarity statement is particularly useful for students that have a hard time with visual-spatial processing.

Who's in the Numerator – When writing a proportion students sometimes carelessly switch which polygon's measurements are in the numerator. To combat this I tell the students to choose right from the beginning and BE CONNSISTENT throughout the problem. When it comes to writing proportions if the students focus on being orderly and consistent, they will usually come up with a correct setup.

Bigger or Smaller – After completing a problem it is always a good idea to take a minute to decide if the answer makes sense. This is hard to get students to do. When using a scale factor, a good way to check that the correct ratio

was used is to notice if the number got bigger or smaller. Is that what we expected to happen?

Update the List of Symbols – In previous lessons it has been recommended that students create a reference page in their note books that contains a list of all the symbols and how they are being used in this class. Students should add the symbol for similar to the list, and take few minutes to compare it to the symbols they already know. Sometimes students will read the similarity symbol as "approximately equal". It is standard to use two wavy lines for approximately equal and one wavy line for similar, but this is not always the case.

Similarity by AA

Definition of Similar Triangles vs. AA Shortcut – Let the students know what a deal they are getting with the AA Triangle Similarity Postulate. The definition of similar polygons requires that all three corresponding pairs of angles be congruent, and that all three pairs of corresponding sides are proportional. This is a significant amount of information to verify, especially when writing a proof. The AA postulate is a significant shortcut; only two piece of information need to be verified and all the rest comes for free. When students see how much this reduces the work, they will be motivated to understand the proof and will enjoy using the postulate. Everybody likes to use a tricky shortcut.

Get Some Sun – It is always a good idea to create some variety in the class. It will keep students' minds active. Although it is time consuming, get some yard sticks and take the students outside to measure a tree or a flagpole using their shadows and similar triangles. Have them evaluate their accuracy. They will have to measure carefully if they are to get a reasonable numbers. This will give them some practice using a rule and converting units. The experience will also help them put what they are learning about similar triangles into their long term memory.

Trigonometry – Let the students know that the next chapter is a about trigonometry, and that the AA Triangle Similarity Postulate is what make trigonometry possible. If the students know what an important postulate this is, they will be motivated to understand and learn how to apply it. Mentioning what is to come will start to prepare their minds and make learning the material in the next chapter that much easier. Here are some problems that involve similar right triangles to accustom the students to this new branch of mathematics.

Key Exercises:

1. $\triangle ABC$ is a right triangle with right angle C, and $\triangle ABC \sim \triangle XYZ$.

Which angle in $\triangle XYZ$ is the right angle?

Answer: $\angle Z$

2. $\triangle CAT \sim \triangle DOG$, $\angle A$ is a right angle

$CA = 5$ cm, $CT = 13$ cm

What is DG?

Answer: $DG = 13$ cm,

Students must use the Pythagorean theorem and the definition of similar polygons.

Similarity by SSS and SAS

The [U+0080][U+009C]S[U+0080][U+009D] of a Triangle Similarity Postulate – At this point in the class, students have shown that a significant number of triangles are congruent. They have learned the process well. When teaching them to show that triangles are similar, it is helpful to build on what they have learned. The similarity postulates have $S's$ and $A's$ just like the congruence postulates and theorems. The $A's$ are treated exactly the same

in similarity postulates as they were in congruence theorems. Each "A" in a similarity shortcut stands for one pair of congruent corresponding angles in the triangles.

The S's represent a different requirement in similarity postulates then they did in congruence postulates and theorems. Congruent triangles have congruent sides, but similar triangles have proportional sides. Each "S" in a similarity postulate represents a ratio of corresponding sides. Once the ratios (two for SAS and three for SSS) are written, equality of the ratios must be verified. If the ratios are equal, the sides in question are proportional, and the postulate can be applied.

It is sometimes hard for student to adjust to this new side requirement. They have done so much work with congruent triangles that it is easy for them to slip back into congruent mode. Warn them not to fall into the old way of thinking.

TABLE 2.12:

Triangle <u>Congruence</u> Postulates and Theorems "S" $\leftrightarrow$ congruent sides	Triangle <u>Similarity</u> Postulates "S" $\leftrightarrow$ proportional sides
SSS	AA
SAS	SSS
ASA	SAS
AAS	
HL	

Only Three Similarity Postulates – Students will sometimes try to use ASA, or other congruence theorems to show that two triangles are similar. Bring it to their attention that there are only three postulates for similarity, and that they do not all have the same side and angle combinations as congruence postulates or theorems.

Proportionality Relationships

Similar Triangles Formed by an Interior Parallel Segment – Students frequently are presented with a triangle that contains a segment that is parallel to one side of the triangle and intersects the other two sides. This segment creates a smaller triangle in the tip of the original triangle. There are two ways to consider this situation. The two triangles can be considered separately, or the Triangle Proportionality Theorem can be applied.

(1) Consider the two triangles separately.

The original triangle and the smaller triangle created by the parallel segment are similar as seen in the proof of the Triangle Proportionality theorem. One way students can tackle this situation is to draw the triangles separately and use proportions to solve for missing sides. The strength of this method is that it can be used for all three sides of the triangles. Students need to be careful when labeling the sides of the larger triangle; often the lengths will be labeled as two separate segments and the students will have to add to get the total length.

(2) Use the Triangle Proportionality theorem.

When using this theorem it is much easier to setup the proportions, but there is the limitation that the theorem can not be used to find the lengths of the parallel segments.

Ideally student will be able to identify the situations where each method is the most efficient, and apply it. This may not happen until the students have had some experience with these types of problem. It is best to have students use method (1) at first, then after they have worked a few exercises on their own, they can use (2) as a shortcut in the appropriate situations.

Additional Exercises:

1. $\triangle ABC$ has point E on $\overline{AB}$, and F on $\overline{BC}$ such that $\overline{EF}$ is parallel to $\overline{AC}$.

$AE = 5$ cm, $EB = 3$ cm, $BF = 4$ cm, $AC = 10$ cm

Find EF and FC.

(Hint: Draw and label a picture, then draw another figure where the two triangles are shown separately.)

Answer:

$$\frac{3}{8} = \frac{EF}{10}$$
$$EF = 3\frac{3}{4} \text{ cm}$$

$$\frac{3}{5} = \frac{4}{FC} \quad \text{or} \quad \frac{3}{8} = \frac{4}{FC+4}$$
$$FC = 6\frac{2}{3} \text{ cm}$$

Similarity Transformations

Scale Factor Compared to Segment and Area Ratios – When a polygon is dilated using scale factor k, the ratio of the image of the segment to the original segment is k. This is true for the sides of the polygon, all the special segments of triangles studied in chapter five, and the perimeter of the polygon. The relationship holds for any linear measurement. Area is not a linear measurement and has a different scale factor. The ratio of the area of the image to the area of the original polygons is k^2. Student frequently forget to square the scale factor when working with the ratios of a figure and its image. This is an important concept that is frequently used on the SAT and on other standardized tests.

Key Exercises:

$\triangle ABC$ has coordinates at the following vertices. $A(1,13), B(6,1)$, and $C(1,1)$.

1. Graph $\triangle ABC$.

2. Use the distance formula to find the length of each side of $\triangle ABC$.

3. Calculate the perimeter of $\triangle ABC$.

4. Calculate the area of $\triangle ABC$.

$\triangle A[U+0080][U+0099]B[U+0080][U+0099]C[U+0080][U+0099]$ is the image of $\triangle ABC$ under a dilation centered at the origin with scale factor 3.

5. Graph $\triangle A[U+0080][U+0099]B[U+0080][U+0099]C[U+0080][U+0099]$.

6. Use the distance formula to find the length of each side of $\triangle A[U+0080][U+0099]B[U+0080][U+0099]C[U+0080][U+0099]$.

7. Calculate the perimeter of $\triangle A[U+0080][U+0099]B[U+0080][U+0099]C[U+0080][U+0099]$.

8. Calculate the area of $\triangle A[U+0080][U+0099]B[U+0080][U+0099]C[U+0080][U+0099]$.

Compare $\triangle A[U+0080][U+0099]B[U+0080][U+0099]C[U+0080][U+0099]$ to $\triangle ABC$.

9. What is the ratio of each set of corresponding side lengths, the perimeters, and the areas? What do you notice when these ratios are compared to the scale factor.

Answers:

1.

2. $AC = 12, BC = 5, AB = 13$

3. 30

4. 30

5.

6. $A''C'' = 36, B''C'' = 15, A''B''$
39

7. 90

8. 270

9. The ratios of the side lengths and the perimeter are $3 : 1$ the same as the scale factor. The ratio of the areas is $9 : 1$, the square of the scale factor.

Self-Similarity (Fractals)

More Complex Fractals – Students need to begin learning about fractals with the simple examples given in the text. Once they have taken some time to work with, and understand the self-similar relationship, it is amazing to see how complex and beautiful fractal can become. Numerous examples of exquisite fractals can be found on-line. If you are lucky enough to have access to computers and a projector, have the students search for fractals and choose their favorite to share with the class. Student will begin to realize the importance of what there are learning when they see what a huge ocean they are dipping their toe into.

Applications – Many students need to know how a subject is useful before they are motivated to spend time and energy learning about it. Throughout the text there have been references to modeling and how mathematical concepts often need to be adjusted to fit the world around us. Fractals are used to model many aspects of nature including tree branches, shells, and the coast line. Knowing of the applications of fractals motivates students. If time permits give a more in-depth explanation, or use this topic to assign research projects.

Video Time – Self-similarity and fractals make up an extremely complex visual topic. There are many videos in common use that can give a much more exciting and attention grabbing explanation than most teachers can deliver while standing in front of the classroom. These videos are not hard to come by, and they give an excellent explanation of the material. It is a nice change of pace for the students, and it gives the instructor some precious time to catch-up on paperwork. It is the best approach for all.

Create Your Own Fractal – Having the students create their own fractal outside of class is a fun, creative project. This gives the more artistically minded students an opportunity to shine in the class, and the products make beautiful wall decorations. Here are some guidelines for the assignment.

    a. The fractal should fill the top half of a piece of $8\frac{1}{2} \times 11$ inch plain white paper turned vertically. To give them more space, provide them with legal size paper. Be aware that each student will probably require more than one piece before they create their final product.
    b. The fractal should be boldly colored to accentuate the self-similarity.
    c. The students should be encouraged to be creative and original in their design.
    d. The bottom half of the paper will have a paragraph explaining the self-similarity in the fractal. They should explain why their design is a fractal.
    e. Create a rubric to give to the students at the time the project is assigned so that they will feel like they are being graded fairly. It is hard to evaluate artwork in a way that everyone feels is objective.

2.8 Right Triangle Trigonometry

The Pythagorean Theorem

Presenting the Proof – The proof of the Pythagorean theorem given in this lesson provides a wonderful review of, and use for, what the students just learned about similar triangles. Sometimes it is difficult for students to see the three right triangles contained in the figure and how the sides correspond. It is helpful to make an additional drawing of the three triangles so that they are separate, and oriented in the same direction. Using both figures for reference students can more easily verify the proportions used in this proof.

Skipping Around – Not all texts present material in the same order, and many instructors have a preferred way to develop concepts that is not always the same as the one used in the text. The Pythagorean theorem is frequently moved from place to place. If the students have not done similar figures yet, or if area has already been covered, the proof of the Pythagorean theorem given in the exercises may be the better place to focus the students' attention. Proofs are hard for most students to understand. It is important to choose one that the students can feel good about. Don't limit the possibilities to these two, research other methods, and pick the one that is most appropriate for your class. Or better yet, pick the best two or three. Different proofs will appeal to different students.

The Height Must be Measured Along a Segment That is Perpendicular to the Base – When given an isosceles triangle where the altitude is not explicitly shown, student will frequently try to use the length of one of the sides of the triangle for the height. The will do this repeatedly, even after you tell them that they must find the length of the altitude that is perpendicular to the segment that's length is being used for the base in the formula $A = \frac{1}{2}bh$. Sometimes they do not know what to do, and are just trying something, which is, in a way, admirable. The more common explanation though is that they forget. The students have been using this formula for years, they think they know this material, so they just plug and chug, not realizing that the given information has changed. Remind the students that now that they are in Geometry class, there is an extra step. The new challenge is to find the height, and then they can do the easy part and plug it into the formula.

Derive the Distance Formula – After doing an example with numbers to show how the distance formula is basically just the Pythagorean theorem, use variables to derive the distance formula. Most students will understand the proof if they have seen a number example first. Point out to the students that the number example was inductive reasoning, and the proof was deductive reasoning. Taking the time to do this is a good review of logic and algebra as well as great proof practice.

Converse of The Pythagorean Theorem

Mnemonic Devise for Acute and Obtuse Triangles – Many students have trouble remembering that the inequality with the greater tan is true when the triangle is acute, and that the equation with the less than is true for obtuse triangles. It seams backwards to them. One way to present this relationship is to compare the longest side and the angle opposite of it. In a right triangle, the equation has an equal sing; the hypotenuse is the perfect size. When the longest side of the triangle is shorter than what it would be in a right triangle, the angle opposite that side is also smaller, and the triangle is acute. When the longest side of the triangle is longer than what is would be in a right triangle, the angle opposite that side is also larger, and the triangle is obtuse.

Review Operations with Square Roots – Some of the exercises in this section require students to do operations

with square roots. This is an essential skill for working with special right triangle which is an important topic that is also covered in this chapter. Many students struggle with using roots in algebra, and they have probably not thought about this topic for a year. Depending on the level of the class, it may be wise to take a day, or half a day, to review operations with square roots. Here are some sample problems of the basic operations with square roots that student will have to know how to do in order to be successful in this chapter.

Simplify:

1. $\sqrt{9} =$

2. $\sqrt{50} =$

3. $5\sqrt{96} =$

Multiply:

4. $\sqrt{2} * \sqrt{5} =$

5. $9\sqrt{6} * 4\sqrt{7} =$

6. $\sqrt{10} * \sqrt{14} =$

Square:

7. $(\sqrt{7})^2 =$

8. $(3\sqrt{2})^2 =$

Add or Subtract:

9. $\sqrt{3} + 7\sqrt{3} =$

10. $3\sqrt{5} - \sqrt{20} =$

Answers:

1. 3

2. $5\sqrt{2}$

3. $20\sqrt{6}$

4. $\sqrt{10}$

5. $36\sqrt{42}$

6. $\sqrt{140} = 2\sqrt{35}$

7. 7

8. $9 * 2 = 18$

9. $8\sqrt{3}$

10. $3\sqrt{5} - 2\sqrt{5} = \sqrt{5}$

Using Similar Right Triangles

Separate the Three Triangles – The altitude from the right angle of a triangle divides the triangle into two smaller right triangles that are similar to each other, and to the original triangle. All the relationships among the segments in this figure are based on the similarity of the three triangles. Many students have trouble rotating shapes in their minds, or seeing individual polygons when they are overlapping. It is helpful for these students to draw the triangles separately and oriented in the same direction. After going through the process of turning and redrawing the triangles a few times, they will remember how the triangles fit together, and this step will no longer be necessary.

2.8. Right Triangle Trigonometry

Color-Coded Flashcards – It is difficult to describe in words which segments to use in the geometric mean to find the desired segment. Labeling the figure with variables and using a formula is the standard method. The relationship is easier to remember if the labeling of the triangles is kept the same every time the figure is drawn. What the students need to remember, is the location of the segments relative to each other. Making color-coded pictures or flashcards will be helpful. For each relationship the figure should be drawn on both sides of the card. The segment whose measure is to be found should be highlighted in one color on the front, and on the back, the two segments that need to be used in the geometric mean should be highlighted with two different colors. Using two colors on the back is important because the segments often overlap. Making these cards will be helpful even if the students never use them. Those that have trouble remembering the relationship will use these cards frequently as a reference.

Add a Step and Find the Areas – The exercises in this section have the students find the base or height of triangles. They have all the information that they need to also calculate the areas of these triangles. Students need practice with multi-step problems. Having them find the area will help them think through a more complex problem, and give them practice laying out organized work for calculations that are more complex. Chose to extend the assignment or not based on how well the students are doing with the material, and how much time there is to work on this section.

Additional Exercise:

1. Refer to the figure used to give the relationship of the altitude as the geometric mean of the lengths of the two segments of the hypotenuse on page 478 of the text.

Let $f = 3$ cm and $c = 10$ cm. What are the values of d and e?

Answer:

$$3 = \sqrt{e} * (10 - e)$$
$$9 = 10e - e^2$$
$$0 = e^2 - 10e + 9$$

$e = 1$ or 9 cm, so $d = 9$ or1 cm such that the sum is 10

Special Right Triangles

Memorize These Ratios – There are some prevalent relationships and formulas in mathematics that need to be committed to long term memory, and the ratios made by the sides of these two special right triangles are definitely among them. Students will use these relationships not only in the rest of this class, but also in trigonometry, and in other future math classes. Students are expected to know these relationships, so the sooner learn to use them and commit them to memory, the better off they will be.

Two is Greater Than the Square Root of Three – One way that students can remember the ratios of the sides of these special right triangles, is to use the fact that in a triangle, the longest side is opposite the largest angle, and the shortest side is opposite the smallest angle. At this point in the class, students know that the hypotenuse is the longest side in a right triangle. What sometimes confuses them is that in the $30 - 60 - 90$ triangle, the ratio of the sides is $1 : 2 : \sqrt{3}$, and if they do not really think about it, they sometimes put the $\sqrt{3}$ as the hypotenuse because it might seem bigger than 2. Using the opposite relationship is a good method to use when working with these triangles. Just bring to the students' attention that $2 > \sqrt{3}$.

TABLE 2.13:

$45 - 45 - 90$ Triangle	$30 - 60 - 90$ Triangle
$x^2 + x^2 = c^2$	$x^2 + b^2 = (2x)^2$
$2x^2 = c^2$	$x^2 + b^2 = 4x^2$
$x\sqrt{2} = c$	$b^2 = 3x^2$

TABLE 2.13: (continued)

45 − 45 − 90 Triangle	30 − 60 − 90 Triangle
	$b = x\sqrt{3}$

Derive with Variables – The beginning of the last chapter offers students a good amount of experience with ratios. If they did well on those sections, it would benefit them to see the derivation of the ratios done with variable expressions. It would give them practice with a rigorous derivation, review and apply the algebra they have learned, and help them see how the triangles can change in size.

Exact vs. Decimal Approximation – Many students do not realize that when they enter $\sqrt{2}$ into a calculator and get 1.414213562, that this decimal is only an approximation of $\sqrt{2}$. They also do not realize that when arithmetic is done with an approximation, that the error usually grown. If 3.2 is rounded to 3, the error is only 0.2, but if the three is now multiplied by five, the result is 15, instead of the 16 it would have been if original the original number had not been rounded. The error has grown to 1.0. Most students find it more difficult to do operations with radical expressions than to put the numbers into their calculator. Making them aware of error magnification will motivate them to learn how to do operations with radicals. In the last step, it may be nice to have a decimal approximation so that the number can be easily compared with other numbers. It is always good to have an exact form for the answer so that the person using your work can round the number to the desired degree of accuracy. Less accuracy is needed for building a deck than sending a robot to Mars?

Tangent Ratio

Trig Thinking – Students sometimes have a difficult time understanding trigonometry when they are first introduced to this new branch of mathematics. It is quite a different way of thinking when compared to algebra or even geometry. Let them know that as they begin their study of trigonometry in the next few sections the calculations won't be difficult, the challenge will be to understand what is being asked. Sometimes students have trouble because they think it must be more difficult than it appears to be. Most students find they like trigonometry once they get the feel of it.

Ratios for a Right Angle – Students will sometimes try to take the sine, cosine or tangent of the right angle in a right triangle. They should soon see that something is amiss since the opposite leg is the hypotenuse. Let them know that there are other methods of finding the tangent of angles 90 degrees or more. The triangle based definitions of the trigonometric functions that the students are learning in this chapter only apply to angles in the interval 0 degrees $< m <$ 90 degrees.

The Ratios of an <u>Angle</u> – The sine, cosine, and tangent are ratios that are associated with a specific angle. Emphasize that there is a pairing between an acute angle measure, and a ratio of side lengths. Sine, cosine, and tangent is best described as functions. If the students' grasp of functions is such that introducing the concept will only confuse matters, the one-to-one correspondence between acute angle and ratio can be taught without getting into the full function definition. When students understand this, they will have an easier time using the notation and understanding that the sine, cosine, and tangent for a specific angle are the same, no matter what right triangle it is being used because all right triangles with that angle will be similar.

Use Similar Triangles – Many students have trouble understanding that the sine, cosine, and tangent of a specific angle measure do not depend on the size of the right triangle used to take the ratio. Take some time to go back and explain why this is true using what the students know about similar triangle. It will be a great review and application.

Sketchpad Activity:

a. Students can construct similar right triangles using dilation from the transformation menu.

b. After choosing a specific angle they should measure the corresponding angle in all the triangles. Each of these measurements should be equal.
c. The legs of all the right triangles can be measured.
d. Then the tangents can be calculated.
e. Student should observe that all of ratios are the same.

Remind the students that if the right triangles have one set of congruent acute angles, then they are similar by the AA Triangle Similarly Postulate. Once the triangles are known to be similar it follows that their sides are proportional. The ratios are written using two sides of one triangle and compared to the ratios of the corresponding sides in the other triangle. This is different but equivalent to the ratios students probably used to find missing sides of similar triangles in previous sections.

Sine and Cosine Ratio

Trig Errors are Hard to Catch – The math of trigonometry is, at the point, not difficult. Not much computation is necessary to chose two number and put them in a ratios. What students need to be aware of is how easy it is to make a little mistake and not realize that there is an error. When solving an equation the answer can be substituted back into the original equation to be checked. The sine and cosine for acute angles do not have a wide range. It is extremely easy to mistakenly use the sine instead of the cosine in an application and. The difference often is small enough to seem reasonable, but still definitely wrong. Ask the student to focus on accuracy as they work with these new concepts. Remind them to be slow and careful.

Something to Consider – Ask the students to combine their knowledge of side-angle relationships in a triangle with the definition of sine. How does the length of the hypotenuse compare to the lengths of the legs of a right triangle? What does that mean about the types of numbers that can be sine ratios? With leading questions like these students should be able to see that the sine ratio for an acute angle will always be less than one. This type of analysis will prepare them for future math classes and increase their analytical thinking skills. It will also be a good review of previous material and help them check there work when they first start writing sine and cosine ratios.

Rationalizing the Denominator – Sometimes student will not recognize that $\frac{1}{\sqrt{2}}$ and $\frac{\sqrt{2}}{2}$ are equivalent. Most likely, they learned how to rationalize denominators in algebra, but it is nice to do a short review before using these types of ratios in trigonometry. Student will have to be able to easily switch between the two forms of the number when working with the unit circle in later classes.

Two-Step Problems – Having the students write sine, cosine, and tangent ratios as part of two-step problems will help them connect the new material that they have learned to other geometry they know. They will remember it longer, and be better able to see where it can be applied.

Key Exercise:

1. $\triangle ABC$ is a right triangle with the right angle at vertex C.

$AC = 3$ cm and $BC = 4$ cm

What is the sign of $\angle A$?

Answer: $AB = 5$ cm by the Pythagorean theorem, therefore $\sin A = \frac{4}{5}$.

Note: The sine of an angle does not have units. The units will cancel out in the ratio.

Inverse Trigonometric Ratios

Regular or Arc – Students will sometimes be confused about when to use the regular trigonometric function and when to use the inverse. They understand to concepts, but do not want to go through the entire thought process each time they must make the decision. I give them this short rule of thumb to help them remember: When looking for a ratio or side length, use regular and when looking for an angle use arc. They can associate "angle" and "arc" in their minds. Use the alliteration.

Which Trig Ratio – A common mistake students make when using the inverse trigonometric functions to find angles in right triangles is to use the wrong function. They may use arcsine instead of arccosine for example. There is a process that students can use to reduce the number of these kinds of errors.

a. First, the students should mark the angle whose measure is to be found. With the angle in question highlighted, it is easier for the students to see the relationship the sides have to that angle. It is fun for the students to use colored pencils, pens, or highlighters.
b. Next, the students should look at the sides with known side measures and determine their relationship to the angle. They can make notes on the triangle, labeling the hypotenuse, the adjacent leg and the opposite let. If they are having trouble with this I have them look for the hypotenuse first and always highlight it green, then they and decide between opposite and adjacent for the remaining to sides.
c. Now, they need to look at the two sides they have chosen, and decide if they need to use sine, cosine, or tangent. It might help to have a mnemonic device to help them remember the definitions of the trigonometric functions. A common one is *soh-cah-toa*. The student can write this abbreviation on the top of every paper and refer to it when necessary. For example, in an exercise, if they decide it is the adjacent leg to the angle, and the hypotenuse that they have measures for, that is the *"ah"* portion of *cah*. They will know to use cosine, and be reminded that the length of the adjacent leg will e in the numerator of the ratio.

Make a Graph – Sometimes student will have a hard time seeing a pattern in a list of numbers. One way to help them remember the general trends in the trigonometric ratios is two have them make a graph. They can put the angle measure on the horizontal axis and the ratio, in decimal form, on the vertical axis. They will have to use different scales, of course. Now they can use their calculators to find the trig values of different angles at every five or ten degrees between zero and ninety and plot points on their graph. The comparison would be most meaningful if they put all three on the same set of axes with different colors. The process of making the graph and the visual representation of the pattern will form an impression in the students' minds that will be useful and lasting.

Acute and Obtuse Triangles

Law of Sines or Law of Cosines – At first, it may be difficult for student to determine if they need to use the Law of Sines or the Law of Cosines to find a measure in a particular situation. Here is a good thought process for them to use.

a. First have them look for the two, fairly easy to recognize, Law of Cosines situations. They have all three sides and are looking for an angle, or have two angles and the included side and are looking for the third side.
b. If it is not one of these, then they need to try to set-up a Law of Sines proportion.

The Third Angle – Remind students that the three angles of a triangle have a sum of 180 degrees, and that this fact is often helpful when applying the Law of Sines or Cosines. Sometimes they may not be able to fine the angle they want directly, but if they find the third angle, they can use the Triangle Sum Theorem to get the measure they need.

Two Exercises in One - Sometimes the students will have to use both the Law of Sines and the Law of Cosines to find a measure in a specific triangle. For instance, let's say they have two sides and the included angle of a triangle,

but they do not want to find the third side, they want to find the other angles. It will be necessary to use the Law of Cosines to get the third side, and then use that third side to get the ratio for the triangle so the Law of Sines could be used. Remind students to be creative when solving exercises. They should use all of their mathematical knowledge to figure out the solution.

Triangle Labeling – Stress the labeling convention of using a capital letter for a vertex and the same letter in lower case for the opposite side. It is especially important when using the Law of Cosines to fine an angle. Students need to verify that they start the Law of Cosines with the side opposite of the angle they are finding.

Multiplication Before Addition – At this level, students usually faithfully follow this application of the order of operations except, when using the Law of Cosines to fine an angle. The last term of the Law of Cosines is $-2ab\cos C$. This is four values being multiplied together. The cosine function is new to students. They will not see it as the representation of a number and will separate it from the other terms. Frequently, they will subtract the $-2ab$ from the $a^2 + b^2$ or add it to the c^2 on the other side of the equation. Use a big multiplication symbol when writing and using the formula. It can be written, $-2ab * \cos C$, to remind students to use the proper order of operations.

2.9 Circles

About Circles

Circle Vocabulary – This section has quite a few vocabulary words. Some the students will already know, like radius, and some, like secant, will be new. Encourage the students to make flashcards or a vocabulary list. They should know the word definition and have pictures drawn and labeled. It is also important for students to know the relationships between the words. The radius is half the length of the diameter and the diameter is the longest chard in a circle. Make knowing the vocabulary a specific assignment, otherwise many students will forget to take the time to learn the vocabulary well.

Circle or Disk – The phrase "a point on the circle" is commonly used. This will confuses the students that do not realize that the circle is the set of points *exactly* some set distance from the center, and not the points less than or equal to that distance from the center or the circle. What is happening is that they are confusing the definition of a disk and a circle. Emphasize to the students that a circle is one dimensional; it only contains the points on the edge. Another option is to give them the definition of a disc along with that of a circle, so that they can compare and contrast the two definitions.

Inscribed or Circumscribed – An inscribed circle can also be described as a circumscribed polygon. The different ways that these vocabulary words can be used can make learning the relationships complicated. As a guide, tell the students that the object inscribed is on the inside. Starting with that, they can work out the rest. For practice, ask the students to draw different figures that are described in words, like a circumscribed hexagon, or a circle inscribed in an octagon.

Square the Radius – When working with the equation of a circle, students frequently forget that the radius is squared in the equation, especially when the radius is an irrational number. Explaining the equation of the circle in terms of the Pythagorean theorem will help the students remember and understand how to graph this conic section.

Completing the Square – Completing the square to put the equation of a conic section in standard form is a nice little math trick. It exemplifies the kinds of moves mathematicians use to manipulate expressions and equations. Students find it difficult to do especially when fractions are involved and they have trouble retaining the process for more than a few days. Give them many opportunities to practice.

Tangent Lines

Bringing It All Together – This section makes use of many concepts students have previously learned in the class. It will help students to start to prepare for the final, or for an end of the quarter cumulative test. Students will need time to go back and review the topics used in this section as well as the normal time allotted to learn the new material. Below is a list of subjects the students must be competent at to be successful with this section. A day spent reviewing these will help avoid frustration.

Review Topics:

a. The converse of a conditional statement and proof by contradiction
b. Proofs that employ congruent triangles and there corresponding parts
c. The Pythagorean theorem and its converse

   d. Equations of lines and circles, including slopes of perpendicular lines
   e. The proportionality of the sides of similar triangles
   f. Polygons: the sum of interior angles and regular polygons

All the Radii of a Circle Are Congruent – It may seem obvious, but frequently students forget to use the fact that all the radii of a circle are congruent. This follows directly from the definition of a circle. Remind students to use this fact when setting up equations and assigning variables to different radii in the same circle.

Congruent <u>Tangents</u> – In this section the Tangent Segment Theorems is proved and applied. Remind student that this is only true for tangents and does not extend to secants. Sometimes student will see a secant enter a circle and think the distance from the exterior point to where the secant intersects the circle is the same as a tangent or another secant from that same point.

Hidden Tangent Segments – Sometimes it is difficult for students to recognize tangent segments because they are imbedded in a more complex figure, or the tangent segment is extended in some way. A common situation where this occurs is when there is an inscribed circle. Tell the students to be on the lookout for tangent segments. They should look at segments individually and as part of the whole. Sometimes it is helpful to use a small sticky note to cover parts of the figure so they do not distract from the area of focus.

Common Tangents and Tangent Circles

Using Trigonometry to Find Side Measurers in Right Triangles – Using the definitions of sine, cosine, or tangent to find the measures of sides in a right triangle is a common application of trigonometry that is put to use in this section. Students will need a bit of practice and perhaps a step-by-step process when learning this skill. With some experience though, this will become an easy, enjoyable task.

Step-by-Step Process:

   a. Highlight the side of the right triangle that's measure is to be found. Place a variable, say *x*, by that side.
   b. Chose one of the acute angles of the right triangle whose measure is known to work from. Highlight this angle in another color.
   c. Chose another sides of the triangle whose measure is known. Highlight that side in the same color as the other side.
   d. Decide what relationships (opposite leg, adjacent leg, or hypotenuse) the highlighted sides have to the highlighted angle.
   e. Decide which of the three trigonometric ratios utilize those side relationships.
   f. Write out the definition of that trigonometric ratio.
   g. Substitute in the highlighted values.
   h. Solve the equation by either multiplying or dividing. It is best to not round the decimal approximation of the trigonometric ratio taken from the calculator. Round after the multiplication or division has taken place.

Key Exercises:

1. $\triangle ABC$ is a right triangle with the right angle at vertex C.

$m\angle A = 52°$ and $AC = 10$ cm. Find BC.

Answer:

$$\tan 52° = \frac{BC}{10} \qquad\qquad\qquad BC \approx 12.8 \text{ cm}$$

2. $\triangle DEF$ is a right triangle with the right angle at vertex F.

$m\angle D = 22°$ and $DF = 1143$ ft. Find DE.

Answer:

$$\cos 22° = \frac{1143}{DE} \qquad\qquad DE \approx 1200 \text{ ft}$$

Arc Measure

Naming Major Arcs and Semicircles – When naming and reading the names of major arcs and semicircles, the three letter system is sometimes confusing for students. When naming an angle with three letters, the first place to look is to the middle letter, the vertex. It is just the opposite for a three letter arc name. First, the students should locate the endpoints of the arc at the ends of the name. For a major arc they have two arcs to choose from. The major arc uses three letters and is the long way around. Any of the other points on the major arc can be used to designate that the long path is being taken. A semicircle divides the circle into two congruent arcs. A third letter is needed to designate which half of the circle is being named.

Look For Diameters – When working exercises that call for students to find the measures of arcs by adding and subtracting arc and angle measures in a circle, students often forget that a diameter divides the circle in half, or into two 180 degree arcs. Remind the students to be on the lookout for diameters when finding arc measures.

Using Trigonometry to Find Angle Measures in Right Triangles – A similar process is needed for finding angles in right triangles as for finding sides in right triangles in the previous lesson. Students need some scaffolding when they first learn to use this method.

   a. Highlight the angle whose measure is to be found.
   b. Two sides of the right triangle must be known. Highlight these two sides.
   c. Decide what relationship the highlighted sides have to the angle in question.
   d. Decide which trigonometric ratio used those side relationships.
   e. Write and solve an equation. Remember to use the inverse of the trigonometric ratio on the calculator since it is the angle that needs to be found.

Key Example:

1. $\triangle ABC$ is a right triangle with the right angle at vertex C.

$AC = 12$ cm, and $AB = 17$ cm. Find $m\angle B$.

Answer:

$$\sin B = \frac{12}{17} \qquad\qquad m\angle B \approx 45°$$

Chords

Update the Theorem List – Students should be keeping a notebook full of all the theorems they have learned in geometry class. These theorems are like tools that can be used to work exercises and write proofs. This section has quite a few different theorems about the relationships or chords and angles that need to be included in their notebook. Each entry should have the name of the theorem, the written statement of the theorem, and a picture to illustrate the

2.9. Circles

relationship. Not only will this be good reference material, making the notebook will help the students to remember the material.

Algebra Review – Students may need a bit of a review before correctly squaring algebraic expressions and solving quadratic equations in geometric applications.

a. In example three the equation of a line is substituted into the equation of a circle so points of intersection can be found. When the binomial is substituted for the $y-$variable in the circle equation, it must be squared. Students frequently try to "distribute" the square instead of using the FOIL method. Make a point of writing out the binomial twice, and multiplying. Students should know and be able to use the pattern for a perfect square binomial, but they will understand why they have to use the pattern when they see the long way written out once and awhile, and will be more likely to remember.

b. In the same example the quadratic formula is used to solve for the two possible values of the $x-$variable. Students will benefit from a brief explanation of how quadratic equations are solved. First, when the student realized that it is a second degree equation they need to solve for zero. Then the equation can be factored or the quadratic formula can be applied. The students should remember the process quickly when they see it. This is an important topic of algebra, and it is always good to review to eliminate misconceptions.

Tips and Suggestions – There are a few strategies that students should keep in mind when working on the exercises in this section.

a. Draw in segments to create right triangles, central angles, and any other useful geometric objects.
b. Remember to split the length of the chord in half if only half of it is used in a right triangle. Don't just use the numbers that are given. The theorems must be applied to get the correct number, and multiple steps will usually be necessary.
c. Use trigonometry of right triangles to find the angles and segment lengths needed to complete the exercise.
d. Don't forget that all radii are congruent. If you have the length of one radius, you have them all, including the ones you add to the figure.
e. Employ the Pythagorean theorem and any other tool you have from previous lessons that might be useful.

Inscribed Angles

Inscribed Angle or Central Angle – When students spot an arc/angle pair to use in solving a complex circle exercise, the first step is to identify the angle as a central angle, an inscribed angle, or possibly neither. If necessary, they can trace the sides of the angle back from the arc to see where the vertex is located. If the vertex is at the center of the circle, it is a central angle, and the measure of the arc and the angle are equal. If the vertex is on the circle, it is an inscribed angle, and the students must remember to double the angle measure. A good mnemonic device is to think of the arc of an inscribed angle being farther away from the vertex than the arc of a central angle. Therefore the measure of the arc will be larger. If the vertex is at neither the exact center or on the circle, no arc/angle relationship can be determined with only one arc.

What to Look For – Students can be overwhelmed by the number of different relationships that need to be used to solve these circle exercises. Sometimes they can just get paralyzed and not know where to start. In small groups, or as a class, have them create a list of possible tools that are commonly used in these types of situations.

Does the figure contain?

a. A triangle with a sum of 180 degrees
b. A convex quadrilateral with a sum of 360 degrees
c. A right triangle formed with a tangent
d. An isosceles triangle formed with two radii

e. A diameter creating a 180 degree semicircle
f. Arcs covering the entire 360 degrees of the circle
g. Central or Inscribed angles
h. Tangents that form right angles
i. Similar triangle with proportional sides
j. Congruent triangles with congruent corresponding parts

Any New Information is Good – If students can not immediately see how to find the measure they are after, advise them to find any measure they can. This keeps their mind active and working. Frequently, they will be able to use the new information to find other measures, and will eventually work their way around to the desired answer. This might not br the most efficient method, but the students' technique will improve with practice.

Angles of Chords, Secants, and Tangents

Where's the Vertex? – When determining the relationships between angles and arcs in a circle the location of the vertex of the angle is the determining factor. There are four possibilities.

a. The vertex of the angle is at the center of the circle, it is a central angle, and the arc and angle have the same measure.
b. The vertex of the angle is on the circle. The angle could be made by two cords, an inscribed angle, or by a chord and a tangent. In either situation, the measure of the arc is twice that of the angle.
c. The vertex of the angle is inside the circle, but not at the center. In this case two arcs are necessary, and the angle measure is the average of the measures of the arcs cut off by the chords that form the vertical angles.
d. The vertex of the angle is outside the circle. Then the two intersected arcs have to be subtracted and the difference divided by two. Note the similarity to an average.

Students often need help organizing information in this way. It is best to do this with them, as a class activity so that in the future they will be able to do it for themselves.

Use the Arcs – It is typical to have more than one angle intercepting a specific arc. In this case a measure can be moved to an arc and then back out to another angle. Another situation students should look for is when a circle is divided into two arcs. One arc can be represented as $360-$ (an expression for the other arc). Students sometimes miss these kinds of moves. It may be beneficial to have students share with the class the different strategies and patterns they see when working on these exercises.

Additional Exercises:

1. Two tangent segments with a common endpoint intercept a circle dividing it into two arcs, one of which is twice as big as the other. What is the measure of the angle formed by the by the two tangents?

Answer:

$$x + 2x = 360 \qquad\qquad \text{angle measure} = (240 - 120) \div 2 = 60 \text{ degrees}$$
$$x = 120$$

2. Two intersecting chords intercept congruent arcs. What kind of angles do the chords form?

Answer: central angles

2.9. Circles

Segments of Chords, Secants, and Tangents

Chapter Study Sheet – This chapter contains many relationships for students to remember. It would be helpful for them to summarize all of these relationships on a single sheet of paper to use when studying. Some instructors allow students to use these sheets on the exam in order to encourage students to make the sheets. The value of a study sheet is in its making. Students should know this and make them regardless of whether they can be used on the exams. Sometimes if students know that they will be able to use the study sheet, they will not work to remember all of the relationships, and their ability to learn the material is compromised. It is a hard issue to work around and each instructor needs to deal with it as he or she feels best with their particular classes.

When to Add – When writing proportions involving secants, students will have a difficult time remembering to add the two segments together to form the second factor. A careful study of the proof will help them remember this detail. When they see secants, have them picture the similar triangles that could be drawn. Remind them, and give them ample opportunity to practice.

Have Them Subtract – One way to give students more practice with the lengths of secants in circles is to give them exercises where the entire length of the secant is given, and they have to setup an expression using subtraction to use in the proportion.

Key Examples:

1. A secant and a tangent segment have a common exterior endpoint. The secant has a total length of 12 cm and the tangent has length 7 cm. What is the measure of the both segments of the secant?

Answer:

Let one segment of the secant be x, so the other can be represented by $20 - x$.

$$7^2 = (12 - x) * 12$$

$$x \approx 7.9$$

The secant is composed of two segments with approximate lengths of 4.1 cm and 7.9 cm

2. Two secant segments have a common endpoint outside of a circle. One has interior and exterior segments of lengths 10 ft and 12 ft respectively and the other has a total measure of 18 ft. What is the measure of the two segments composing the other secant?

Answer:

$$12(10 + 12) = (18 - x) * 18$$

$$x = 3\frac{1}{3}$$

The secant is composed of two segments with lengths $3\frac{1}{3}$ ft and $14\frac{2}{3}$ ft

2.10 Perimeter and Area

Triangles and Parallelograms

The Importance of Units – Students will give answers that do not include the proper units, unless it is required by the instructor. When stating an area, square units should be included, and when referring to a length, linear units should be used. Using proper units helps reinforce the basic concepts. With these first simple area problems including the units seems like a small detail, but as the students move to more complex situations combining length, area, and volume, units can be a helpful guide. In physics and chemistry dimensional analysis is an important tool.

The Power of Labeling – When doing an exercise where a figure needs to be broken into polynomials with known area formulas, it is important for the student to draw on and label the figure well. Each polygon, so far only parallelograms and triangles, should have their base and height labeled and the individual area should be in the center of each. By solving these exercises in a neat, orderly way student will avoid errors like using the wrong values in the formulas, overlapping polygons, or leaving out some of the total area.

Subtracting Areas – Another way of finding the area of a figure that is not a standard polygon is to calculate a larger known area and then subtracting off the areas of polygons that are not included in the target area. This can often result in fewer calculations than adding areas. Different minds work in different ways, and this method might appeal to some students. It is nice to give them as many options as possible so they feel they have the freedom to be creative.

The Height Must Be Perpendicular to the Base – Students will frequently take the numbers from a polygon and plug them into the area formula without really thinking about what the numbers represent. In geometry there will frequently be more steps. The students will have to use what they have learned to find the correct base and height and then use those numbers in an area formula. Remind students that they already know how to use a formula; many exercises in this class will require more conceptual work.

Write Out the Formula – When using an area formula, it is a good idea to have the students first write out the formula they are using, substitute numbers in the next step, and then solve the resulting equation. Writing the formula helps them memorize it and also reduces error when substituting and solving. It is especially important when the area is given and the student is solving for a length measurement in the polygon. Students will be able to do these calculations in their heads for parallelograms, and maybe triangles as well, but it is important to start good habits for the more complex polygons to come.

Trapezoids, Rhombi, and Kites

It's Arts and Crafts Time – Student have trouble remembering how to derive the area formulas. At this level it is required that they understand the nature of the formulas and why the formulas work so they can modify and apply them in less straightforward situations. An activity where student follow the explanation by illustrating it with shapes that they cut out and manipulate is much more powerful then just listening and taking notes. It will engage the students, keep their attention, and make them remember the lesson longer.

Trapezoid

    a. Have student use the parallel lines on binder paper to draw a trapezoid. They should draw in the height and

label it h. They should also label the two bases b_1 and b_2.
b. Now they can trace and cut out a second congruent trapezoid and label it as they did the first.
c. The two trapezoids can be arranged into a parallelogram and glued down to another piece of paper.
d. Identify the base and height of the parallelogram in terms of the trapezoid variables. Then substitute these expressions into the area formula of a parallelogram to derive the area formula for a trapezoid.
e. Remember that two congruent trapezoids were used in the parallelogram, and the formula should only find the area of one trapezoid.

Kite

a. Have the students draw a kite. They should start by making perpendicular diagonals, one of which is bisecting the other. Then they can connect the vertices to form a kite.
b. Now they can draw in the rectangle around the kite.
c. Identify the base and height of the parallelogram in terms of d_1 and d_2, and then substitute into the parallelogram area formula to derive the kite area formula.
d. Now have the students cut off the four triangles that are not part of the kite and arrange them over the congruent triangle in the kite to demonstrate that the area of the kite is half the area of the rectangle.

Rhombus

The area of a rhombus can be found using either the kite or parallelogram area formulas. Use this as an opportunity to review subsets and what they mean in terms of applying formulas and theorems.

Areas of Similar Polygons

Adjust the Scale Factor - It is difficult for students to remember to square and cube the scale factor when writing proportions involving area and volume. Writing and solving a proportion is a skill they know well and have used frequently. Once the process is started, it is hard to remember to add that extra step of checking and adjusting the scale factor in the middle of the process. Here are some ways to reinforce this step in the students' minds.

a. Inform students that this material is frequently used on the SAT and other standardized tests in some of the more difficult problems.
b. Play with graph paper. Have students draw similar shape on graph paper. They can estimate the area by counting squares, and then compare the ratio of the areas to the ratio of the side lengths. Creating the shapes on graph paper will give the students a good visual impression of the areas.
c. Write out steps, or have the students write out the process they will use to tackle these problems. (1) Write a ratio comparing the two polygons. (2) Identify the type of ratio: linear, area, or volume. (3) Adjust the ratio using powers or roots to get the desired ratio. (4) Write and solve a proportion.
d. Mix-up the exercises so that students will have to square the ratio in one problem and not in the next. Keep them on the lookout. Make them analyze the situation instead of falling into a habit.

Additional Exercises:

1. The ratio of the lengths of the sides of two squares is $1 : 2$. What is the ratio of their areas?

Answer: $1 : 4$

2. The area of a small triangle is 15 cm^2, and has a height of 5 cm. A larger similar triangle has an area of 60 cm^2. What is the corresponding height of the larger triangle?

Answer:

area ratio $15 : 60$ or $1 : 4$ height linear ratio $1 : 2 = 10$ cm

Height of larger triangle is twice the of the smaller triangle. $5 * 2$

3. The ratio of the lengths of two similar rectangles is $2 : 3$. The larger rectangle has a width of 18 cm. What is the width of the smaller rectangle?

Answer:

$$\frac{2}{3} = \frac{x}{18}$$
$$x = 12$$

The width of the smaller rectangle is 12 cm.

Circumference and Arc Length

Pi is an Irrational Number – Many students can give the definition of an irrational number. They know that an irrational number has an infinite decimal that has no pattern, but they have not really internalized what this means. Infinity is a difficult concept. A fun way to help the students develop this concept is to have a pi contest. The students can chose to compete by memorizing digits of pi. They can be given points, possible extra credit, for ever ten digits or so, and the winner gets a pie of their choice. The students can also research records for memorizing digits of pi. The competition can be done on March 14th, pi day. When the contest is introduced, there is always a student who asks "How many points do I get if I memorize it all?" It is a fun way to reinforce the concept of irrational numbers and generate a little excitement in math class.

How is Pi Calculated? - Students frequently ask how mathematicians calculate pi and how far they have gotten. One method is by approximating the circumference of a circle with inscribed or circumscribed polygons. Inspired students can try writing the code themselves, and possibly sharing it with the class. There are many other more commonly used methods, but they involve calculus or other mathematics that is beyond geometry students.

There Are <u>Two</u> Values That Describe an Arc – The measure of an arc describes how curved the arc is, and the length describes the size of the arc. Whenever possible, have the students give both values with units so that they will remember that there are two different numerical descriptions of an arc. Often student will give the measure of an arc when asked to calculate its length.

Arc Length Fractions – Fractions are a difficult concept for many students even when they have come as far as geometry. For many of them putting the arc measure over 360 does not obviously give the part of the circumference included in the arc. It is best to start with easy fractions. Use a semi-circle and show how 180/360 reduces to $\frac{1}{2}$, then a ninety degree arc, and then a 120 degree arc. After some practice with fractions they can easily visualize, the students will be able to work with that eighty degree arc.

Exact or Approximate – When dealing with the circumference of a circle there are often two ways to express the answer. The students can give exact answers, such as 2π cm or the decimal approximation 6.28 cm. Explain the strengths and weaknesses of both types of answers. It is hard to visualize 13π feet, but that is the only way to accurately express the circumference of a circle with diameter 13 ft. The decimal approximations $41, 40.8, 40.84$, can be calculated to any degree of accuracy, are easy to understand in terms of length, but are always slightly wrong. Let the students know if they should give one, the other, or both forms of the answer.

Circles and Sectors

Reinforce – This section on area of a circle and the area of a sector is analogous to the previous section about circumference of a circle and arc length. This gives students another chance to go back over the arguments and logic to better understand, remember, and apply them. Focus on the same key points and methods in this section,

and compare it to the previous section. Mix-up exercises so students will see the similarities and learn each more thoroughly.

Don't Forget the Units – Remind students that when they calculate an area the units are squared. When an answer contains the pi symbol, students are more likely to leave off the units. In the answer 7π cm^2, the π is part of the number and the cm^2 are units of area.

Draw a Picture – When applying geometry to the world around us, it is helpful to draw, label, and work with a picture. Visually organize information is a powerful tool. Remind students to take the time for this step when calculating the areas of the irregular shapes that surround us.

Additional Exercises:

1. What is the area between two concentric circles with radii 5 cm and 12 cm?

(Hint: Don't subtract the radii.)

Answer: $144\pi - 25\pi = 119\pi$ cm^2

2. The area of a sector of a circle with radius 6 m, is 12π cm^2. What is the measure of the central angle that defines the sector?

Answer:

$$12\pi = \frac{x}{360} * \pi * 6^2 \qquad\qquad \text{The central angle measures 120 degrees.}$$
$$x = 120$$

3. A square with side length $5\sqrt{2}$ cm is inscribed in a circle. What is the area of the region between the square and the circle?

Answer: $\pi * 5^2 - (5\sqrt{2})^2 = 25\pi - 50 \approx 28.5$ cm^2

Regular Polygons

The Regular Hypothesis – Make it clear to students that these formulas only work for regular polygons, that is, polygons will all congruent sides and angles. The regular restriction is part of the hypothesis. Many times the hypotheses of important theorems in mathematics are quite restrictive, but that does not necessarily limit the value of the theorem. If a polygon is approximately regular, then the formula can be used to get an approximate area. Also, the method of breaking the polygon into triangles can be applied to non-regular polygons, but each triangle may be different and therefore each area computed separately. It is important to understand how the theorem or formula was derived so it can be adapted to other situations. Knowing this will motivate students to work to understand the formulas.

Numerous Variables and Relationships – Polygons come with an entire new set of variable. Students need to learn what these knew variable represent, how they are related to the triangles that makeup the polygon, and how they are related to each other. This will take some time and practice. If students do not have time to memorize what the variables represent, they will not understand how they are being put together in the various formulas. Find convenient breaking points and give students time, examples, or activities to help them become familiar with the material. If it all comes too fast, student will get lost and frustrated.

What is n? - Students will frequently be given the value of n, but will not realize it because it is not given in the form they are expecting. An exercise may state, "Each side of a regular hexagon is nine inches long." The students will see the nine and assign it to the variable s, but not notice that they are also being given the value of n; a hexagon has six sides.

Let the Radius be One – The simplification of only considering polygons inscribed in a unit circle by letting the radius be equal to one, may seem a bit odd to students. Let them know that this is being done in preparation for more advanced trigonometry. In trigonometry ratios of side lengths of similar triangles are considered, and the size of the triangles in not important. Letting them know that this simplification becomes useful in the future will reassure them that the course of their mathematics education is well designed.

Additional Exercises:

1. The area of a regular hexagon is $24\sqrt{3}$ cm^2. What is the length of each side?

Answer:

$$24\sqrt{3} = \frac{1}{2} * s * \frac{1}{2}s\sqrt{3} * 6 \qquad\qquad \text{Each side has a length of 4 cm.}$$
$$s = 4$$

Geometric Probability

Count Carefully – Counting is the most challenging aspect of probability for students. It is easy to make an error when thinking of all the possible outcomes and determining how many of them are favorable. The best way to guard against errors is to make logical, orderly lists. The goal is for the students to see a pattern so that eventually they will be able to get the count without listing all of the possibilities.

TABLE 2.14:

Outcome	Favorable?	Outcome	Favorable?
(N_1, N_2)	No	(D, N_1)	No
(N_1, D)	No	(D, N_2)	No
(N_1, Q)	Yes	(D, Q)	Yes
(N_2, N_1)	No	(Q, N_1)	Yes
(N_2, D)	No	(Q, N_2)	Yes
(N_2, Q)	Yes	(Q, D)	Yes

Does Order Matter? – One of the hardest decisions for a new probability student to make when analyzing a situation is to determine if different permutations should be counted separately or not. Take Example Two from this section. Is there a first coin and a second coin? Does it matter if Charmane sees the quarter of the dime first? In this case, it does not because the two coins are taken at the same time. To compare have the students consider the situation where first one coin is drawn and then a second. In this case there are more possible outcomes. The probability remains the same, 50%, but now it is calculated as $\frac{6}{12}$ instead of $\frac{3}{6}$. It is not always the case that considering order results in the same probability as when order is not considered. Note that this example can be reduced to the simpler question of whether a quarter is one of the two coins drawn.

Additional Exercises:

1. A man throws a dart at a circular target with radius 6 inches. He is equally likely to hit anywhere in the target. What is the probability that he is within 2 inches of the center of the target?

Answer:

$$\frac{\pi 2^2}{\pi 6^2} = \frac{1}{9} \approx .11 \qquad \text{There is an approximately 11\% chance that he will hit within 2 inches of the target.}$$

2.10. Perimeter and Area

2. There is a 110 mile stretch of road between the centers of two cities. A hospital is located 30 miles from the center of one city. If an accident is equally likely to occur anywhere between the two cities, what is the probability it is within ten miles of the hospital?

Answer:

$\dfrac{20}{110} \approx .18$ There is an approximately 18% chance that the accident will be within 10 miles of the hospital.

2.11 Surface Area and Volume

The Polyhedron

Polygon or Polyhedron – A polyhedron is defined using polygons, so in the beginning students will understand the difference. After some time has passes though, students tend to get these similar sounding words confused. Remind them that polygons are two-dimensional and polyhedrons are three-dimensional. The extra letters in polyhedron represents it spreading out into three-dimensions.

The Limitations of Two Dimensions – It is difficult for students to see the two-dimensional representations of three-dimensional figures provided in books and on computer screens. A set of geometric solids is easily obtained through teacher supply companies, and are extremely helpful for students as they familiarize themselves with three-dimensional figures. When first counting faces, edges, and vertices most students need to hold the solid in their hands, turn it around, and see how it is put together. After they have some experience with these objects, student will be better able to read the figures drawn in the text to represent three-dimensional objects.

Assemble Solids – A valuable exercise for students as they learn about polyhedrons is to make their own. Students can cut out polygons from light cardboard and assemble them into polyhedrons. Patterns are readily available. This hands-on experience with how three-dimensional shapes are put together will help them develop the visualization skills required to count faces, edges, and vertices of polyhedrons described to them.

Computer Representations – When shopping on-line it is possible to "grab" and turn merchandise so that they can be seen from different perspectives. The same can be done with polyhedrons. With a little poking around students can find sights that will let them virtually manipulate a three-dimensional shape. This is another possible option to develop the students' ability to visualize the solids they will be working with for the remainder of this chapter.

Using the Contrapositive – If students have already learned about conditional statements, point out to them that Example Six in this section makes use of the contrapositive. Euler's formula states that if a solid is a polyhedron, then $v + f = e + 2$. The contrapositive is that if $v + f \neq e + 2$, then the solid is not a polyhedron. Students need periodic review of important concepts in order to transfer them to their long-term memory. For more review of conditional statements, see the second section of the second chapter of this text.

Representing Solids

Each Representation Has Its Use – Each of the methods for making two-dimensional representations of three-dimensional figures was developed for a specific reason and different representation are most appropriate depending on what aspect of the geometric solid is of interest.

Perspective – used in art, and when one wants to make the representation look realistic

Isometric View – used when finding volume

Orthographic View – used when finding surface area

Cross Section – used when finding volume and the study of conic sections (circles, ellipses, parabolas, and hyperbolas) is based on the cross sections of a cone

Nets – used when finding surface area or assembling solids

Ask the students to think of other uses for these representations.

When students know and fully understand the options, they will be able to choose the best tool for each task they undertake.

Isometric Dot Paper – If students are having trouble making isometric drawings, they might benefit from the use of isometric dot paper. The spacing of the dots allows students to make consistent lengths and angles on their polyhedrons. After some practice with the dot paper, they should be able to make decent drawings on any paper. A good drawing will be helpful when calculating volumes and surface areas.

Practice – Most students will need to make quite a few drawings before the result is good enough to be helpful when making calculations. The process of making these representations provides the student with an opportunity to contemplate three-dimensional polyhedrons. The better their concept of these solids, the easier it will be for them to calculate surface areas and volumes in the sections to come.

Additional Exercises:

1. In the next week look around you for polyhedrons. Some example may be a cereal box or a door stop. Make a two-dimensional representation of the object. Choose four objects and use a different method of representation for each.

These can make nice decorations for classroom walls and the assignment makes students look for way to apply what they will learn in this chapter about surface area and volume.

Prisms

The Proper Units – Students will frequently use volume units when reporting a surface area. Because the number describes a three-dimensional figure, the use of cubic units seems appropriate. This shows a lack of understanding of what exactly it is that they are calculating. Provide students with some familiar applications of surface area like wrapping a present or painting a room, to improve their understanding of the concept. Insist on the use of correct units so the student will have to consider what exactly is being calculated in each exercise.

Review Area Formulas – Calculating the surface area and volume of polyhedrons requires the students to find the areas of different polygons. Before starting in on the new material, take some time to review the area formulas for the polygons that will be used in the lesson. When students are comfortable with the basic area calculations, they can focus their attention on the new skill of working with three-dimensional solids.

A Prism Does Will Not Always Be Sitting On Its Base – When identifying prisms, calculating volumes, or using the perimeter method for calculating surface area, it is necessary to locate the bases. Students sometimes have trouble with this when the polyhedron in question is not sitting on its base. Remind students that the mathematical definition of the bases of a prism is two parallel congruent polygons, not the common language definition of a base, which is something an object sits on. Once students think they have identified the bases, they can check that any cross-section taken parallel to the bases is congruent to the bases. Thinking about the cross-sections will also help them understand why the volume formula works.

Additional Exercises:

1. A prism has a base with area 15 cm^2 and a height of 10 cm. What is the volume of the prism?

Answer: $V = 15 * 10 = 150$ cm^3

2. A triangular prism has a height of 7 cm. Its base is an equilateral triangle with side length 4 cm. What is the volume of the prism?

Answer: $V = Bh = \frac{1}{2} * 4 * 2\sqrt{3} * 7 = 28\sqrt{3}$ cm$^3 \approx 48.5$ cm^3

3. The volume of a cube is 27 cm^3. What is the cube's surface area?

the figure is probably a prism. If there are more triangles, the figure is most likely a pyramid. Once the student has located the lateral faces, then they can make a more detailed inspection of the base or bases.

Height, Slant Height, or Edge – A pyramid contains a number of segments with endpoints at the vertex of the pyramid. There is the altitude which is located inside right pyramids, the slant height of the pyramid is the height of the triangular lateral faces, and there are lateral edges, where two lateral faces intersect. Students frequently get these segments confused. To improve their understanding, give them the opportunity to explore with three-dimensional pyramids. Have the students build pyramids out of paper or light cardboard. The slant height of the pyramid should be highlighted along each lateral face in one color, and the edges where the lateral faces come together in another color. A string can be hung form the vertex to represent the altitude of the pyramid. The lengths of all of these segments should be carefully measured and compared. They should make detailed observations before and after the pyramid is assembled. Once the students have gained some familiarity with pyramids and these different segments, it will make intuitive sense to them to use the height when calculating volume, and the slant height for surface area.

Additional Exercises:

1. A square pyramid is placed on top of a cube. The cube has side length 3 cm. The slant height of the triangular lateral faces of the pyramid is 2 cm. What is the surface area of this composite solid?

Answer:

$$A = 5 * \text{area of the square} + 4 * \text{area of the triangle}$$
$$= 5 * (3 * 3) + 4 * \left(\frac{1}{2} * 3 * 2 \right)$$
$$= 57 \text{ cm}^2$$

Note: The top face of the cube is covered by the base of the pyramid so neither square is included in the surface area of the composite figure.

Cones

Mix'em Up – Students have just learned to calculate the surface area and volume of prisms, cylinders, and cones. Most students do quite well when focused on one type of solid. They remember the formulas and how to apply them. It is a bit more difficult when students have to choose between the formulas for all four solids. Take a review day here. Have the students work in small groups during class on a worksheet or group quiz that has a mixture of volume and surface area exercises for these four solids. The extra day will greatly help to solidify the material learned in the last few lessons.

Additional Exercises:

1. A cone of height 9 cm sits on top of a cylinder of height 12 cm. Both cone and cylinder have radius with length 4 cm. Find the volume and surface area of the composite solid.

Answer:

$$V = \pi r^2 h_1 + \frac{1}{3} \pi r^2 h_2 = \pi * 4^2 * 12 + \frac{1}{3} * \pi * 4^2 * 9 = 240\pi \text{ cm}^3 \approx 750 \text{ cm}^3$$

$$A = \pi r^2 + 2\pi rh + \pi rl \qquad\qquad \text{and} \qquad\qquad r^2 + h_2 = l^2$$
$$4^2 + 9^2 = l^2$$
$$l \approx 9.8$$

$$A \approx \pi 4^2 + 2\pi * 4 * 12 + \pi * 4 * 9.8$$
$$A \approx 475 \text{ cm}^2$$

2. A cone of radius 7 cm is carved out of a square prism of the same height. The square base of the prism has area 225 cm^2 and height 30 cm. What is the volume of this composite solid?

Answer:

$$V = \text{Volume of Prism}[U+0080][U+0093]\text{Volume of Cone}$$
$$V = 225 * 30 [U+0080][U+0093] \frac{1}{3} * \pi 7^2 * 30 = 6750 [U+0080][U+0093] 490\pi \text{ cm}^3 \approx 5210 \text{ cm}^3$$

Spheres

Expand on Circles – Students learned about circles earlier in the course. Review and expand on this knowledge as they learn about spheres. Ask the students what they know about circles. Being able to demonstrate their knowledge will build their confidence and activate their minds. Now modify the definitions that the students have provided to fit the three-dimensional sphere. Students will learn the new material quickly and will remember it because it is now neatly filed away with their knowledge of circles.

Explore Cross-Sections – One of the goals of this chapter is to develop the students' ability to think about three-dimensional objects. Most students will need a significant amount of practice before becoming competent at this skill. Take some time and ask the students to think about what the cross-sections of a sphere and a plane will look like. Explore trends. What happens to the cross-section as the plane moves farther away from the center of the circle? A cross-section that passes through the center of the sphere makes the largest possible circle, or the great circle of the sphere.

Cylinder to Sphere – It would be a good exercise for students to take the formula for the surface area of a cylinder and derive the formula for the surface area of a sphere. It is just a matter of switching a few variables, but it would be a good exercise for them. During the lesson, ask them to do it in their notes, wait a few minutes and then do it on the board or ask one of them to put their work on the board. It should look something like this:

$$A_{\text{cylinder}} = \text{bases} + \text{lateral area}$$
$$A_{\text{cylinder}} = 2 * \pi r^2 + 2\pi rh$$
$$A_{\text{sphere}} = 2 * \pi r^2 + 2\pi r * r \qquad\qquad \text{substitute in } h = r$$
$$A_{\text{sphere}} = 2 * \pi r^2 + 2\pi \ r^2$$
$$A_{\text{sphere}} = 4\pi r^2$$

Note to students that in the last line the like terms where combine. This can only be done because both terms had the factor πr^2. The coefficients could have been different, but to combine terms using the distributive property they

must have the exact same variable combination. Here the π is being treated as a variable even though it represents a number. This is a more complex application of like terms than students are used to seeing.

Limits – The formula for the volume of a sphere is developed using the idea of a limit. Explain this to the students or the logic might seem fuzzy to them. The limit is a fundamental concept to all of calculus. It is worthwhile to give it some attention here.

Similar Solids

Surface Area is Squared – Surface area is a two-dimensional measurement taken of a three-dimensional object. Students are often distracted by the solid and use cubed units when calculating surface area or mistakenly cube the ratio of linear measurements of similar solids when trying to find the ratio of the surface areas. Remind them, and give them many opportunities to practice with exercises where surface area and volume are both used.

Don't Forget to Adjust the Ratio – There are three distinct ratios that describe the relationship between similar solids. When the different ratios and their uses are the subject of the lesson, students usually remember to use the correct ratio for the given situation. In a few weeks when it comes to the chapter test or on the final at the end of the year, students will frequently forget that the area ratio is different from the volume ratio and the linear ratio. They enjoy writing proportions and when they recognize that a proportion will be used, they get right to it without analyzing the ratios. One way to remind them is to have them use units when writing proportions. The units on both sides of the equal sign have to match before they can cross-multiply. Give them opportunities to consider the relationship between the different ratios with questions like the one below.

Key Exercise:

1. If a fully reduced ratio is raised to a power, will the resulting ratio be fully reduced? Explain your reasoning.

Answer: Yes, two numbers make a fully reduced ratio if they have no common factors. Raising a number to a power increases the exponent of each factor already present, but does not introduce new factors. Therefore, the resulting two numbers will still not have any common factors.

These concepts frequently appear on the SAT. It will serve the students well to practice them from time to time to keep the knowledge fresh.

Additional Exercises:

1. The ratio of the surface area of two cubes is 25 : 49. What is the ratio of their volumes?

Answer:

$$(\sqrt{25})^3 : (\sqrt{49})^3$$
$$(5)^3 : (7)^3$$
$$125 : 343$$

2.12 Transformations

Translations

Translation or Transformation – The words translation and transformation look and sound quite similar to students at first. Emphasis their relationship. A translation is just one of the many transformations the students will be learning about in this chapter.

Point or Vector – There are two mathematical objects being use in this lesson that have extremely similar looking notation. An ordered pair is use to represent a location on the coordinate plane, and it also is used to represent movement in the form of a vector. Some texts represent the translation vector as a mapping. The vector $v = (-3, 7)$ would be written $(x, y) \rightarrow (x - 3, y + 7)$. This makes distinguishing between the two easier, but does not introduce the student to the important concept of a vector. It can be used though if the students are having a really hard time with notation.

The Power of Good Notation – There is a lot going on in these exercises. There are the points that make the preimage, the corresponding points of the image, and the vector that describes the translation. Good notation is the key to keeping all of this straight. The points of the image should be labeled with capital letters, and the prime marks should be used on the points of the image. In this way it is easy to see where each point has gone. This will be even more important when working with more complex transformations in later sections. Start good habits now. A vector should be named with a bold, lower case letter, usually from the end of the alphabet. Just writing $(9, 6)$ is a bit ambiguous, but labeling the vector $u = (9, 6)$, will make the meaning clear. In time students will be able to understand the meaning from the context, but when they are first learning good notation can avoid confusion and frustration.

Use Graph Paper and a Ruler – When making graphs of these translations by had, insist that the students use graph paper and a ruler. If students try to graph on binder paper, the result is frequently messy and inaccurate. It is beneficial for students to see that the preimage and image are congruent to reinforce the knowledge that a translation is an isometry. It is also important that students take pride in producing quality work. They will learn so much more when they take the time to do an assignment well, instead of just rushing through the work.

Translations of Sketchpad – Geometers' Sketchpad uses vectors to translate figures. The program will display the preimage, vector, and image at the same time. Students can type in the vector and can also drag points on the screen to see how the image moves when the vector is changed. It is a quick and engaging way to explore the relationships. If the students have access to Sketchpad and there is a little class time available, it is a worthwhile activity.

Matrices

Rows Then Columns – The dimensions of a matrix are stated by first stating the number of rows and then the number of columns. It may take some time for the students to remember this convention. Give them many opportunities to practice. In this lesson it is important to state the dimensions correctly because only matrices with the same dimensions can be added. The next lesson requires students to decide if two matrices can be multiplied. The order of the dimensions is critical in making that determination.

Additional Exercises:

2.12. Transformations

1. What are the dimensions of any matrix that translates the vertices of a heptagon on the coordinate plane? Explain the significance of the numbers you used.

Answer: 7×2 The number of rows is seven because each of the seven vertices of the heptagon must be moved. The second number is two because each vertex has an $x-$coordinate and a $y-$coordinate that must be changed.

2. The additive identity for real numbers is the number zero because:

$a + 0 = a$, for all real numbers a.

What is the additive identity for 3×2 matrices?

Answer: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

3. If the matrix $\begin{bmatrix} 2 & -3 \\ 2 & -4 \\ 2 & -3 \end{bmatrix}$ were added to matrix a 3×2 matrix containing the vertices of a triangle would the resulting transformation be an isometry? Why or why not?

Answer: No, in an isometry each point must be moved the same distance in order to preserve size and shape. This matrix moves the second vertex farther down than it moves the other two vertices.

Reflections

Matrix Multiplication – It will take some time and practice for students to become proficient with matrix multiplication. On first inspection of the formula it is sometimes hard to see where all the numbers are coming from and where they are going. For many students a spatial representation is more useful. Here are some guidelines that will help students master matrix multiplication.

  a. Use the rows of the first matrix and the columns of the second matrix.
  b. Move across and down using each number only once.
  c. The resulting sum of the products goes in the slot determined by the row of the first matrix and the column of the second matrix.

Think Don't Memorize – Many students will try to learn the reflection matrices using rote memory. This is difficult to begin with since the matrices are fairly similar, but it is also not a good method of learning the material because the knowledge will not last. As soon as the students stop regularly using the matrices, they will be forgotten. Instead have the students think about why the matrices produce their intended effect. When the students really understand what is happening, they will not need to memorize patterns of ones, negative ones, and zeros. The knowledge will be long lasting, and they will be able to develop new matrices that represent other types of transformation and other operations.

Additional Exercises:

1. Let matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and matrix $B = \begin{bmatrix} 4 & 3 \\ 3 & 4 \end{bmatrix}$. Calculate AB and BA. What do you notice about the products? Will this happen with all 2×2 matrices? What is special about matrix A and matrix B that allowed this special result?

Answer: $AB = BA = \begin{bmatrix} 10 & 11 \\ 11 & 10 \end{bmatrix}$. Matrix A and B commute. This property does not hold for most matrices. These matrices commute because they are symmetric matrices. A symmetric matrix is one in which the rows and columns of the matrix are the same.

Rotation

The Trigonometry – The general rotation matrix uses the trigonometric functions, sine and cosine of the angle of rotation. Students are generally introduced to right triangle trigonometry in the third quarter of geometry. This means they understand the meaning of the trigonometric ratios sine, cosine, and tangent for acute angles in right triangles. In later courses students are introduced to the unit circle which enables them to expand the domain of the trigonometric functions to all real numbers. This is a good point to preview the upcoming material. Tell students that they will shortly learn a method for finding $\sin 90°$ and $\cos 120°$; let them know that it is a brilliant method used to expand these extremely useful definitions. For now though, they will have to trust the number given to them by their calculator when using the general rotation matrix. It is not practical to give them the full explanation now, but letting them know that there is an explanation, and that they will learn it soon, will avoid confusion.

Degrees or Radians – If students have followed standard high school mathematical curriculum, they have no idea what a radian is, but somehow calculators frequently end up in radian mode. Students are not familiar enough with trigonometric functions to realize that they are not getting the correct ratio, and continue with their calculations resulting in incorrect answers. Show the students how to check if their calculators are in radian or degree mode and how to change the mode. This is also a good opportunity to start students thinking about different ways to measure angles. Let them know that radians are another unit used to measure rotation. Similar to how both inches and centimeters both measure length. Making students aware of radians now, will help them avoid errors and ready their minds to learn about radians in the future.

Additional Exercises:

1. Graph $\triangle ABC$ with $A(2,1), B(5,3)$, and $C(4,4)$.

2. Put the vertices of the triangle in a 3×2 matrix and use matrix multiplication to rotate the triangle 45 degrees. Graph the image of $\triangle ABC$ on the same set of axis using prime notation.

3. Use the matrix multiplication to rotate $\triangle A'B'C'$ 60 degrees. To do this, take the matrix produced in #2 and multiply it by the rotation matrix for 60 degrees. Graph the resulting triangle on the same set of axis as $\triangle A''B''C''$

4. What single matrix could have rotated $\triangle ABC$ to $\triangle A''B''C''$ in one step? How does this matrix compare to the 45 degree and 60 degree rotation matrix?

Answers:

1.

2. $\begin{bmatrix} 2 & 1 \\ 5 & 3 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} .707 & .707 \\ -.707 & .707 \end{bmatrix} = \begin{bmatrix} .707 & 2.12 \\ 1.41 & 5.66 \\ 0 & 5.66 \end{bmatrix}$

3. $\begin{bmatrix} .707 & 2.12 \\ 1.41 & 5.66 \\ 0 & 5.66 \end{bmatrix} \begin{bmatrix} .5 & .866 \\ -.866 & .5 \end{bmatrix} = \begin{bmatrix} -1.48 & 1.67 \\ -4.20 & 4.05 \\ -4.90 & 2.83 \end{bmatrix}$

4. $\begin{bmatrix} -0.26 & 1.00 \\ -1.00 & -0.26 \end{bmatrix} = \begin{bmatrix} .707 & .707 \\ -.707 & .707 \end{bmatrix} \begin{bmatrix} .5 & .866 \\ -.866 & .5 \end{bmatrix}$

Composition

Use the Image – When first working with compositions student often try to apply both operations to the original figure. Emphasis that a composition is a two-step process 83s. The second step of which is performed on the result

of the first step. Remind them of the composition of two functions if they have already learned about this topic.

Associative Property of Matrices – Students have heard about the commutative and associate properties many times during their education in mathematics. These are important properties in the study of matrices and become more meaningful for the students when applied to this new set. The fact that the commutative property does not hold for matrix multiplication is surprising at first, and is a concept that needs to be revisited. Although it has been discussed in recent lessons, it would be beneficial to go over it again here before discussing the property that is really of interest in this section, the associate property of matrix multiplication. So far the students have seen that the image of points can be found under a rotation or reflection by multiplying a matrix made up of the coordinates of the points by a matrix specific to the chosen transformation. In this section they should discover that because matrix multiplication is associative, they can multiply two or more transformation matrices together to get a matrix for the composition.

Key Exercises:

Consider the triangle with matrix representation $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \\ 4 & 4 \end{bmatrix}$.

Matrix Multiplication is NOT Commutative

1. Use matrix multiplication to rotate the triangle 90 degrees, then take the image and reflect it in the $x-$axis.

2. Use matrix multiplication to reflect the original triangle in the $x-$axis, then take the image and rotate it 90 degrees.

3. Are the results of #1 and #2 the same?

Matrix Multiplication is Associate

4. Multiply the matrix used to rotate the triangle 90 degrees by the matrix used to reflect the triangle in the line $y = x$.

5. Use the matrix found in #4 to transform the triangle. Is the result the same as that in #1?

Answers:

1. $\begin{bmatrix} 2 & 1 \\ 5 & 3 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -3 & 5 \\ -4 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -3 & -5 \\ -4 & -4 \end{bmatrix}$

2. $\begin{bmatrix} 2 & 1 \\ 5 & 3 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 5 & -3 \\ 4 & -4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 5 \\ 4 & 4 \end{bmatrix}$

3. no

4. $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

5. $\begin{bmatrix} 2 & 1 \\ 5 & 3 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -3 & -5 \\ -4 & -4 \end{bmatrix}$, yes

Tessellations

Review Interior Angles Measures for Polygons – Earlier in the course students learned how to calculate the sum of the measures of interior angles of a convex polygon, and how to divide by the number of angles to find the measures of the interior angle of regular polygons. Now would be a good time to review this lesson. The students will need this knowledge to see which regular polygons will tessellate and the final is fast approaching.

Move Them Around – When learning about regular and semi-regular tessellation it is helpful for students to have

a set of regular triangles, squares, pentagons, hexagons, and octagons that they can slide around and fit together. These shapes can be bought from a mathematics education supply company or made with paper. Exploring the relationships in this way gives the students a fuller understanding of the concepts.

Use On-Line Resources – A quick search on tessellations will produce many beautiful, artistic examples like the work of M. C. Escher and cultural examples like Moorish tiling. This bit of research will inspire students and show them how applicable this knowledge is to many areas.

Tessellation Project – A good long-term project is to have the students create their own tessellations. This is an artistic endeavor that will appeal to students that typically struggle with mathematics, and the tessellations make nice decorations for the classroom. Here are some guidelines for the assignment.

a. Fill an $8\frac{1}{2}$ by 11 inch piece of solid colored paper with a tessellation of your own creation.
b. Make a stencil from cardboard and trace it to make the figures congruent.
c. Be creative. Make your tessellation look like something.
d. Color your design to enhance the tessellation.
e. Your tessellation will be graded on complexity, creativity, and presentation.
f. Write a paragraph explaining how you made your tessellation, and why your design is a tessellation. Use vocabulary from this section.

This project could also be done on a piece of legal sized paper. The tessellation can fill the top portion and the paragraph written on the lower part.

Symmetry

360 Degrees**Doesn't Count –** When looking for rotational symmetries students will often list 360 degree rotational symmetry. When a figure is rotated 360 degrees the result is not congruent to the original figure, it is the original figure itself. This does not fit the definition of rotational symmetry. This misconception can cause error when counting the numbers of symmetries a figure has or deciding if a figure has symmetry or not.

Review Quadrilateral Classifications – Earlier in the course students learned to classify quadrilaterals. Now would be a good time to break out that Venn diagram. Students will have trouble understanding that some parallelograms have line symmetry if they do not remember that squares and rectangles are types of parallelograms. As the course draws to an end, reviewing helps students retain what they have learned past the final. It is possible to redefine the classes of quadrilaterals based on symmetry. This pursuit will make the student use and combine knowledge in different ways making what they have learned more flexible and useful.

Applications – Symmetry has numerous applications both in and outside of mathematics. Knowing some of the uses for symmetry will motivate student, especially those who are not inspired by pure mathematics, to spend their time and energy learning this material.

Biology – Most higher level animals have bilateral symmetry, starfish and flowers often have 72 degree rotational symmetry. Naturally formed nonliving structures like honeycomb and crystals have 60 degree rotational symmetry. These patterns are fascinating and can be used for classification and study.

Trigonometry – Many identities of trigonometry are based on the symmetry of a circle. In the next few years of mathematics the students will see how to simplify extremely complex expressions using these identities.

Advertising – Many company logos make use of symmetry. Ask the students to bring in examples of logos with particular types of symmetry and create a class collection. Analyze the trends. Are certain products more appropriately represented by logos that contain a specific type of symmetry? Does the symmetry make the logo more pleasing to the eye or more easily remembered?

Functions – A function can be classified as even or odd based on the symmetry of its graph. Even functions have

symmetry around the $y-$axis, and odd functions have 180 degree rotational symmetry about the origin. Once a function is classified as even or odd, properties and theorems can be applied to it.

Draw – Have students be creative and create their own logos or designs with specific types of symmetry. Using these concepts in many ways will build a deeper understanding and the ability to apply the new knowledge in different situations.

Dilations

Naming Conventions – Mathematics is a language, an extremely precise method of communication. While matrices are named with uppercase letters, scalars are represented by lower case letters. Many times students do not know to look for these types of patterns. Point out these conventions when an appropriate example arises and tell the students to look for the subtle differences that have major significance when communicating with mathematics.

The Scale Factor and Area – Students frequently forget to square the scale factor when comparing the area of the original figure to the image under a dilation. This relationship has been covered several times before when studying area, similar figures, and when dilation was first introduced. Make a point of it again. This omission is quite common and the concept is often used on the SAT and other standardized tests.

Additional Exercises:

1. Does the multiplication of a scalar and a 2×2 matrix commute? If so, write a proof. If not give a counterexample.

Answer: Yes, multiplication of a scalar with a 2×2 matrix does commute.

Let k be a scalar and $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Then $kA = k\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix} = \begin{bmatrix} ak & bk \\ ck & dk \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} k = Ak$

Here a 2×2 matrix was used, but this same proof can be done with a matrix of any dimensions.

2. What scalar could be multiplied by a matrix containing the vertices of a polygon to produce an image with half the area as the original figure?

Answer: $\frac{1}{\sqrt{2}}$ since $\left(\frac{1}{\sqrt{2}} \right)^2 = \frac{1}{2}$

3. Will a dilation followed by a reflection produce the same image if the order of the transformations is reversed? Why or why not?

Answer: The image will be the same regardless of the order in which the transformations are applied. This can be justified with the commutative property of scalar multiplication.

Chapter 2. Geometry TE - Common Errors

CHAPTER **3**   **Geometry TE - Enrichment**

Chapter Outline

Geometry- Teacher's Edition- Enrichment   **Jen Kershaw**

The goal of an enrichment section is just what is implied in the title, "to enrich." By enrichment, we mean something that breathes a new or different life into something else- to make it better to enliven it. This is the goal of this branch of the teacher's edition. This is an opportunity for you and your students to locate and explore the wonderful world of geometry in other subjects such as architecture or music or art. It is a chance for students to see how the world of mathematics can connect to other subjects that they are passionate about.

Our goal is that using this Enrichment Flexbook will help you to expand your own personal creativity as well as the creativity of your students. The projects/topics in this flexbook can be used in several different ways. They can be used as a discussion point, an example to highlight during a lesson, a project to expand on whether students complete the project in class or at home or as a way to broaden student thinking by using a web search once per week as an example. It is not the intention that every single lesson be used in this flexbook. Take what inspires you and use it to inspire your students. Isn't that what the world of mathematics is all about!

3.1 Basics of Geometry

Points, Lines and Planes

I. <u>Section Objectives</u>

- Understand the undefined terms *point, line and plane.*
- Understand defined terms, including *space, segment and ray.*
- Identify and apply basic postulates of points, lines and planes.
- Draw and label terms in a diagram

II. <u>Cross- curricular Study-Art</u>

- Use the painting on this website.
- www.nationalgalleries.org/media_collection/6/GMA 1279.jpg
- You can print it out or have students look at it on the computer.
- One way to use this painting is to have students work in small groups to identify the points, lines, planes, segments and rays in the painting.
- You could also use it as a spring board for the students to create their own black, white and gray piece of art that contains points, lines, planes, segments and rays.
- You could even pair this up with the art teacher to really incorporate other disciplines into the mathematics classroom.

III. <u>Technology Integration</u>

- One way to integrate technology is to use a drawing/painting program and have the students work to design their own artwork on the computer.
- This could take several days, so be sure that you have access to the computers.
- In the end, the students could present their material to the other students in the class and explain the geometric properties of their work.

IV. <u>Notes on Assessment</u>

- When looking at student work, you want to include creativity in your assessment, but also look at the mathematics incorporated.
- Did the student include all of the geometric elements?
- Is the student able to describe the different elements to his/her peers?
- Has the work been completed with care?
- Offer feedback/correction as needed.

Segments and Distance

I. <u>Section Objectives</u>

- Measure distances using different tools
- Understand and apply the ruler postulate to measurement
- Understand and apply the segment addition postulate to measurement
- Use endpoints to identify distances on a coordinate grid

II. Cross- curricular Study- Surveying

- Land surveyors measure distance all of the time.
- They determine land boundaries, and property lines, as well as determine roads etc.
- One possible activity is to have a land surveyor visit the class.
- Have the surveyor bring in his/her tools also have them demonstrate the actual measuring to the students.
- Maybe work on measuring a part of the school grounds.
- Ask the presenter to be prepared to discuss the ways that geometry and measurement are featured in his/her job.
- Have the students prepare questions and write a short report demonstrating what they have learned following the visit.

III. Technology Integration

- Have the students research land surveying.
- This can be done in class or as an at home assignment.
- Completing this assignment will help the students to be prepared when the land surveyor comes to visit the class.
- Ask students to keep track of the sites that they search and to jot down at least ten facts about land surveying as a career.
- An extension into future careers could have the students research schooling and salary options for land surveyors.
- Ask students to report on their findings.

IV. Notes on Assessment

- One of the biggest points to assess in this activity is the student questions.
- Ask the students to write down their questions and the answers to them.
- This will help you to confirm that the students gave thought to their questions and were alert to the answers.

Rays and Angles

I. Section Objectives

- Understand and identify rays.
- Understand and classify angles.
- Understand and apply the protractor postulate.
- Understand and apply the angle addition postulate.

II. Cross- curricular- Astronomy

- Begin this lesson by asking students to observe the night sky.
- Tell them to make note of the different constellations that they observe.
- Also ask the students to find these constellations on line or in a book and to bring the hard

3.1. Basics of Geometry

copy of a picture of the constellation to class.

- When students do this, you can begin a whole discussion about the angles in the constellations and the distance between stars.
- This can lead right into the technology integration.

III. Technology Integration

- www.geocities.com/angolano/Astronomy/PIinSky.html
- Use the website for an in depth student of measurement in the sky.
- This website looks at many different facets of geometry, measurement and astronomy.
- Although some topics have not been discussed yet, have the student read it all anyway.
- Then focus on the last section where students can see how to measure angles in the sky using their hands.
- Then send them back out at night to rediscover the initial constellations using their hands to measure distances.

IV. Notes on Assessment

- Assess student work in three sections.
- First, discuss the initial constellations with the students.
- What did the students discover?
- Were they able to make connections between angles and the constellations?
- Then move on to the astronomy site.
- Finally, assess student observations once they had an understanding of how to measure using their hands.
- What did this do to their initial observations?
- Were the students able to expand on the initial sightings?

Segments and Angles

I. Section Objectives

- Understand and identify congruent line segments
- Identify the midpoint of line segments
- Identify the bisector of a line segment
- Understand and identify congruent angles
- Understand and apply the Angle Bisector Postulate

II. Cross- curricular-Architecture

- Use the image from Wikipedia on A Frame Homes in this lesson. This is Figure01.04.01
- www.en.wikipedia.org/wiki/File:Aframe2.jpg
- You want the students to use the design of the A frame home to prove the theorems in this lesson.
- Students are going to use the image to demonstrate the following:
- 1. Congruent line segments
- 2. Bisecting line segments
- 3. Midpoints of line segments
- 4. Congruent angles
- 5. Bisecting angles
- 6. Angle bisector postulate
- Have students work in small groups, and then present their findings.

- Another option is to have the students design their own A frame home.
- When designing, the students will have to use the concepts in the chapter and apply them to the design of the home.

III. Technology Integration- Websearch

- A great way to study the concept of bisection is through a websearch.
- Have the students google "bisecting"
- When they do this, many pages of images will pop up. For example, one image is of a fence bisecting two mountains. Another is an aerial view of a highway.
- Ask students to select three different real- world images to work with.
- You want the students to draw connections between the concepts in the text and the images that they have selected.
- Ask the students to investigate how the terms bisect, congruent and midpoint applies to each image.
- For example, the student might see that in the highway picture the roads are bisecting by other roads. In the mountain picture, the fence crosses the midpoint between the two mountains bisecting the distance between the two.
- This activity will cause students to use higher level thinking skills. The connections may not be obvious.

IV. Notes on Assessment

- Look at student work.
- Are the students able to apply how each concept applies to the A frame home design?
- Look at student designs- is the A frame home congruent?
- Are the angles congruent?
- You could choose to do some or all of the suggestions in this lesson, you are looking to see that the students understand the concepts and can apply them in real life situations. They could be doing this in a diagram, a presentation or a written explanation.
- Assess student work accordingly.

Angle Pairs

I. Section Objectives

- Understand and identify complementary angles
- Understand and identify supplementary angles
- Understand and utilize the Linear Pair Postulate
- Understand and identify vertical angles

Cross- curricular- Map of NYC

- Use the image of a street map of Manhattan. This is Figure01.05.01.
- www.aaccessmaps.com/show/map/us/ny/manhattan
- Print a copy of this image for students to work with during the activity.
- This map has many different examples of complementary and supplementary angles. As well as vertical angles.
- Have the students work in pairs with a highlighter, colored pencils or markers.
- The students are going to identify examples of each of the types of angles in the map.
- Remind students to look at Broadway and at the way Broadway intersects the other streets.

3.1. Basics of Geometry

- Ask the students to make a list of the intersections on paper and how each angle fits the description.
- You could also have the students enlarge the map (or do this ahead of time), and then use a protractor to measure the angles.
- Ask students to share their work in small groups.

III. Technology Integration

- Search maps from different cities.
- You could use the map of the city that you live in.
- You could ask the students to identify a city of their choosing.
- Then complete the same exploration with a protractor on this map.

IV. Notes on Assessment

- Check student maps.
- Collect the student maps and their notes.
- Have the students identified the angle pairs correctly.
- If measurement was completed, is it accurate?
- Provide students with feedback/comments on their work.

Classifying Triangles

I. Section Objectives

- Define triangles
- Classify triangles as acute, right, obtuse, equiangular
- Classify triangles as scalene, isosceles, or equilateral

II. Cross- curricular-Triangle Creations

- This is a fun way to explore triangles.
- To prepare, you will need an assortment of one or more of the following items: gumdrops, marshmallows, toothpicks, tinkertoys, kynex
- Be sure that the students understand the different types of triangles and have an example of each type.
- Then have them create an example of each of the following triangles using the materials provided.
- The students will have a GREAT time with this. It is very hands- on and since so much work has been done on the different triangle types, this will make the lesson fun.
- Students need to label each type and example and be able to explain how and why it is that type of triangle.
- Allow time for the students to present their creations.

III. Technology Integration- Geometric paintings

- Use the following website for an investigation in geometric paintings using triangles. This is Figure 01.06.01
- www.4.bp.blogspot.com/_wLt09kFTsi4/RdsN0RvBmgI/AAAAAAAAAAk/Oo6kiVl78AE/s400/BlackBeetle.jpg
- Ask the students to work in small groups and identify the different types of triangles in this painting.
- Then have the students create their own painting using the different types of triangles.
- They should create a key and color code to show each type of triangle.
- An alternative to this would be to have the students search and find triangles in other geometric paintings.
- They can then use this one as a springboard for their own design.

IV. <u>Notes on Assessment</u>

- Create a specific set of directions for the student art piece.
- How many triangles do the students need to have in their design?
- How many of each type?
- Consider creativity.
- Provide students with comments/feedback on their work.

Classifying Polygons

I. <u>Section Objectives</u>

- Define polygons
- Understand the difference between convex and concave polygons
- Classify polygons by the number of sides
- Use the distance formula to find side lengths on a coordinate grid

II. <u>Cross- curricular-Architecture</u>

- You will need a computer or a way to show this video during class.
- You can go to www.futureschannel.com and have the students watch the short video on polygons in architecture.
- Use this video as a discussion prompt.
- Ask the students to identify how architecture is shaped by the use of polygons and how it depends on polygons.
- Ask the students to brainstorm the many different types of polygons in the classroom.
- Extension- ask the students to go home- take one room and write down all of the different types of polygons that they can find in that room.
- Allow time for students to share their work.

III. <u>Technology Integration</u>

- Have the students complete a websearch on different types of lens.
- They can use Wikipedia for this.
- Students are going to explore the concepts of concave and convex as it applies to lenses.
- Ask the students to make a list of the different characteristics of concave polygons.
- Then ask students to make a list of the characteristics of convex polygons.
- Then have the student select five different lenses.
- They need to create a description/explanation of how each lens is either concave, convex or neither.
- Allow time for the students to share their work when finished.

IV. <u>Notes on Assessment</u>

- Student work is assessed through discussion.
- Be sure that all students have an opportunity to share.
- You want to encourage this class lesson to include a lively engaging discussion.

Problem Solving in Geometry

I. <u>Section Objectives</u>

3.1. Basics of Geometry

- Read and understand given problem situations
- Use multiple representations to restate problem situations
- Identify problem- solving plans
- Solve real- world problems using planning strategies

II. Cross-curricular- Putting It Together

- In this chapter, students have been working with all of the basics of geometry.
- Now they are going to be combining these ideas to create their own word problem.
- Students can use magazine pictures, clip art or other pictures in their problem.
- Have the students work in pairs or small groups to design a word problem that uses some or all of the following concepts: lines, angles, triangles, polygons.
- Students should demonstrate an answer key where students use the problem solving techniques from the chapter to solve the problem.
- This may seem to be too broad an activity.
- If so, you can give the students a topic to write their problems about such as sports.
- Then all of the problems that the students write will have to do with the topic of sports.
- Collect all student problems and answers when finished.

III. Technology Integration

- Use the following website and have the students watch the video and geometry and dance.
- www.thefutureschannel.com/dockets/realworld/dancing/
- Use this as a discussion starting point.
- Then you could show students a short clip from The Nutcracker (you will need to prep this ahead of time).
- Ask the students to use the concepts from the first movie clip and apply it to the dancing in the Nutcracker.
- This can be a fun way of showing how math is in places where you least expect it.

IV. Notes on Assessment

- Collect student problems and answers.
- Check all work for accuracy.
- Provide students with feedback/correction.
- You can use these problems as quiz questions or extra credit work.

3.2 Reasoning and Proof

Inductive Reasoning

I. Section Objectives

- Recognize visual patterns and number patterns
- Extend and generalize patterns
- Write a counterexample to a pattern rule

II. Cross- curricular- Music

- Prepare several different examples of repetitive music.
- For example, rap, classical, folk song, childrens song.
- You want a few with a refrain or a clear consistent pattern.
- You will need to prepare this ahead of time.
- You want the students to develop a rule for each selection.
- Brainstorm a list of possible pattern rules and decide on one.
- Then write this one on the board.
- Ask the students to come up with a counterexample to the pattern rule.
- You can do this first one as a whole class so that the students understand the idea.
- Then break students off into groups.
- Have them listen to each of the other selections and write a rule and a counter example for each.
- When finished, allow time for students to share their work.

III. Technology Integration

- Have the students complete a google search on patterns in nature.
- Students are going to select one or more examples of patterns in nature.
- Ask them to write a rule for each pattern.
- Then write a counter example for each pattern.
- Finally allow time for students to share their work.

IV. Notes on Assessment

- Assessment is done through class sharing.
- Do the students understand how to write a pattern rule?
- Are the rules accurate?
- Do the students understand how to write a counterexample?
- Provide feedback as needed.

Conditional Statements

I. Section Objectives

3.2. Reasoning and Proof

- Recognize if- then statements
- Identify the hypothesis and conclusion of an if-then statement
- Write the converse, inverse and contrapositive of an if-then statement
- Understand a biconditional statement

II. Cross- curricular-Literature

- Provide students with a copy of the poem "The Road Not Taken" by Robert Frost.
- Read the poem with the class.
- Discuss the meaning of the poem and the thoughts behind it.
- Then tell the students that they are going to change the poem to be written in all conditional statements.
- They can reword it if they wish.
- Allow time for the students to work on this in small groups.
- When they have finished, ask them if the meaning of the poem has changed with their conditional statements.
- Ask them how conditional statements can impact different statements.
- Allow time for the students to read their poems.

III. Technology Integration

- Have the students use the following website to investigate conditional statements further.
- www2.edc.org/makingmath/mathtools/conditional/conditional.asp
- Ask the students to use the diagrams to write three different conditional statements.
- Allow time for the students to share their work in small groups.

IV. Notes on Assessment

- You will hear how well the students understand conditional statements by listening to their poems.
- Provide feedback/correction as needed.

Deductive Reasoning

I. Section Objectives

- Recognize and apply some basic rules of logic
- Understand the different parts that inductive reasoning and deductive reasoning play in logical reasoning
- Use truth tables to analyze patterns of reasoning

II. Cross- curricular-Mount Everest

- Begin this activity with a discussion about deductive and inductive reasoning.
- Review these concepts so that the students are not confused when working on this activity.
- Have students research through books or technology facts about people who have climbed Mt. Everest.
- Ask the students to make a list of at least ten facts about people who have climbed Everest.
- Then write this on the board, "If you have climbed Mt. Everest, then you...."
- Tell the students that they are to write at least five different statements using deductive reasoning to complete this statement.
- When finished, have students share their work in small groups.
- Ask each group to assess whether or not the students have successfully written statements using deductive reasoning.

III. Technology Integration

- Complete a websearch on Aristotle.
- Ask students to do some research about Aristotle and how he developed the concept of logic.
- Students can write a short essay about this or apply it to a real life example.
- Collect student work.

IV. Notes on Assessment

- Collect student statements.
- Assess them for accuracy.
- Provide students with feedback/correction as needed.
- When working on the technology integration, ask the students to share what they have discovered about Aristotle and logic.
- This can become a lively discussion about how the actions of someone in the past impacts the way we work today- draw a connection to the judicial system.

Algebraic Properties

I. Section Objectives

- Identify and apply properties of equality
- Recognize properties of congruence "inherited" from the properties of equality
- Solve equations and cite properties that justify the steps in the solution
- Solve problems using properties of equality and congruence

II. Cross- curricular- Scale Design

- This activity involves students exploring the concept of equality.
- Bring in several different scales for students to work with.
- Then prepare an assortment of items for students to work with. For example, apples, bananas, bags of flour, bags of rice, oranges, etc. You can use non food items too.
- Students need to come up with collections of items that demonstrate equality.
- For example, apples and oranges- can you put so many apples to equal so many oranges?
- Have students make a list of the items that equal other items.
- Then ask students to use the properties from the chapter and write a reflexive statement, a symmetric statement and a transitive statement about two of their equal statements.
- Finally, allow time for the students to share their work.

III. Technology Integration

- Have students explore how properties apply to circuits.
- Use the following website for this exploration.
- www.allaboutcircuits.com/vol_4/chpt_7/4.html
- Begin a discussion about the information. Create a list of important facts on the board.
- You can also have the students do this in small groups.
- Students can then create their own diagrams to demonstrate how the circuit works and how algebraic properties impact circuits.
- Allow time for the students to share their diagrams.

3.2. Reasoning and Proof

IV. <u>Notes on Assessment</u>

- Observe students as they work.
- Then collect all student statements and diagrams.
- Check student work for accuracy.
- Provide feedback/correction as needed.

Diagrams

I. <u>Section Objectives</u>

- Provide the diagram that goes with a problem or proof.
- Interpret a given diagram.
- Recognize what can be assumed from a diagram and what can not be
- Use standard marks for segments and angles in diagrams.

II. <u>Cross- curricular-Airports</u>

- Begin this activity by reviewing each of the eleven postulates in the chapter.
- Make a list of them and their characteristics on the board.
- Then move on to the activity.
- Use a copy of the map of the runway at O'Hare International Airport. This is Figure02.05.01.
- www.en.wikipedia.org/wiki/O'Hare_International_Airport
- Ask the students to use colored pencils to find an example of each of the eleven postulates.
- They need to use a color to highlight each example.
- Then they can use this color as an indicator and write a description of HOW the example illustrates the postulate.
- Do this for all of the eleven postulates.
- When students are finished, allow time for them to share their work.

III. <u>Technology Integration</u>

- Have the students do a search for different housing floor plans.
- They can use the following website for the search or another of their own choosing.
- www.thehousedesigners.com/
- Then ask the students to make a list of how the different postulates apply to housing floor plans.
- Would it be possible for houses to be designed without these postulates?
- Conduct a class discussion on this topic.

IV. <u>Notes on Assessment</u>

- Collect the airport maps and student notes.
- Check them for accuracy.
- Did the students follow the directions?
- Is each of the eleven postulates represented?
- Were the students able to write a written explanation of how the postulate is shown in the map?
- Provide feedback/correction as needed.

Two- Column Proof

I. <u>Section Objectives</u>

- Draw a diagram to help set up a two- column proof.
- Identify the given information and statement to be proved in a two- column proof.
- Write a two- column proof.

II. <u>Cross-curricular- Cooking</u>

- In this activity, the students are going to need to prove the following statement.
- "You must have eggs to make a chocolate cake."
- Assign half of the class the job of proving that this is a true statement.
- Assign the other half of the class the job of disproving the statement.
- This can branch off into technology as well.
- If students have access to computers, they can search recipes and cake information on line.
- Some students will break right off and talk about dairy- free or vegan cakes.
- This is great because students can talk about that, but they will need to prove it.
- Tell students that they need at least four different statements.
- Tell students that they will need to use resources to back up their statements.
- Allow students time to work.
- When finished, allow them time to share their arguments.
- The class can assess whether they successfully proved it or not.
- You may want to do this first in small groups.
- Have each group select the best proof.
- Then have a whole class debate using the best proofs.
- Ask the students to share what worked or was challenging about this assignment.
- Students may figure out that they can be very specific in their proof.

III. <u>Technology Integration</u>

- Incorporate technology into the above activity by allowing students computer access to do their recipe/cooking searches.

IV. <u>Notes on Assessment</u>

- Assess student work through the debates and discussions.
- Collect students work and read through their proofs.
- This is a GREAT class for demonstrating how challenging it can be to prove or disprove something.
- For fun, you could serve chocolate cake when finished.

Segment and Angle Congruence Theorems

I. <u>Section Objectives</u>

- Understand basic congruence properties.
- Prove theorems about congruence.

3.2. Reasoning and Proof

II. <u>Cross- curricular-Roller Coasters</u>

- Use the following image from Wikipedia for the first part of this lesson.
- This is Figure02.07.01
- www.en.wikipedia.org/wiki/File:Wooden_roller_coaster_txgi.jpg
- Review the segment and angle congruence theorems from the lesson in the text.
- Make a list of them on the board.
- Then distribute this image to the students.
- Students are going to work in pairs or small groups.
- They need to use the image to explain why segment and angle congruence theorems are important to roller coaster design.
- Allow time for the students to work on this.
- This is a written explanation and should include the definitions from the text applied in a real life context.
- Allow time for the students to share their work when finished.

III. <u>Technology Integration</u>

- Have students complete a websearch of roller coasters.
- Ask each student to select one that best uses the segment and angle congruence theorems.
- Then conduct a large class discussion on this.
- Be sure that the students see how the theorems apply in real life.
- If segments and angles weren't congruent, how would this impact the operations of the roller coaster?

IV. <u>Notes on Assessment</u>

- Assessment is completed through class discussion.
- Observe students as they work and listen to their ideas in the discussion.
- Are the students connecting the theorems to the design?
- Help them to make the connections.

Proofs about Angle Pairs

I. <u>Section Objectives</u>

- State theorems about special pairs of angles.
- Understand proofs of the theorems about special pairs of angles.
- Apply the theorems in problem solving.

II. <u>Cross- curricular-Theorems in Art</u>

- Students are going to use art to prove the different theorems.
- Use the following image from this website for this activity. This is Figure 02.08.01.
- www.prestonsteed.com/Sale_pages/Right_Angles/Right.Angles.html
- Then ask the students to come up with an example of each of the following theorems in this painting.
- 1. Right Angle Theorem
- 2. Supplements of the Same Angle Theorem
- 3. Complements of the Same Angle Theorem
- 4. Vertical Angles Theorem
- Have students discuss their findings in small groups.

- Allow time for sharing in the large group as well.

III. Technology Integration

- Students can use the power point presentation in this website to explore different angles and their relationships.
- www.learninginhand.com/lessonplans/angles.html
- The activities themselves are an excellent integration of technology.

IV. Notes on Assessment

- Assess student work through observation.
- Walk around and listen to students as they discuss the painting.
- Sit in and interject thoughts are ideas.
- Are the students connecting the theorems to the painting?
- Are they discovering the angle relationships?
- Offer feedback/suggestions as needed.

3.2. Reasoning and Proof

3.3 Parallel and Perpendicular Lines

Lines and Angles

I. Section Objectives

- Identify parallel lines, skew lines, and perpendicular lines
- Know the statement of and use the Parallel Line Postulate.
- Know the statement of and use the Perpendicular Line Postulate.
- Identify angles made by transversals.

II. Cross- curricular-Architecture

- Use this image from Wikipedia for a discussion on lines and angles.
- This is Figure 03.01.02
- www.en.wikipedia.org/wiki/English_Gothic_architecture
- Discuss the lines and angles in the picture.
- Show students how perpendicular angles are a major feature in the structure.
- This can be a fun lively discussion on how architecture and geometry come together.
- Write student points on the board.

III. Technology Integration-Artist Todd Hoover

- Use the following painting by Todd Hoover titled "Coming Together."
- It can be found at this website. This is Figure 03.01.01.
- www.fineartamerica.com/featured/coming-together-todd-hoover.html
- Have the students discuss the different lines and angles in this painting.
- Then use this image as a springboard to have the students create their own painting.
- Simplicity is key.
- Allow time for the students to create their work.
- Display art in the classroom.

IV. Notes on Assessment

- Create a rubric for grading student art.
- Establish how many types of lines need to be in the painting.
- Be sure to include creativity when grading.
- How well did the students take Todd Hoover's simplicity and make it their own?
- Provide feedback to students on their work.

Parallel Lines and Transversals

I. Section Objectives

- Identify angles formed by two parallel lines and a non- perpendicular transversal.
- Identify and use the Corresponding Angles Postulate.
- Identify and use the Alternate Interior Angles Theorem.
- Identify and use the Alternate Exterior Angles Theorem.
- Identify and use the Consecutive Interior Angles Theorem.

II. Cross- curricular-Tube Map in London

- Use the following image from Wikipedia. This is Figure 03.02.01.
- www.en.wikipedia.org/wiki/File:Tube_map_thumbnail.png
- Be sure that each student has a copy of the map.
- Students are going to use this map to find an example of each of the postulates/theorems in this lesson.
- 1. Corresponding Angles Postulate
- 2. Alternate Interior Angles Theorem
- 3. Alternative Exterior Angles Theorem
- 4. Consecutive Interior Angles Theorem
- Students will need to prove that each example in the map is accurate.
- Have them use a protractor to measure and provide a list of statements and proof for each postulate/theorem.
- Then allow time for students to share their work.

III. Technology Integration- Transportation Search

- Have students use this map of the Tube and compare it with the map of the subway in NYC and the map of the "L" in Chicago.
- Compare and contrast each map and the use of angles, parallel lines and transversals.
- Have students write a few concluding statements to describe each in mathematical terms.
- Then allow time for students to share their work.

IV. Notes on Assessment

- Collect student tube maps and statements.
- Did the students justify each theorem/postulate correctly and accurately?
- Did they use angle measures in their justifications?
- Provide students with feedback/correction when needed.

Proving Lines Parallel

I. Section Objectives

- Identify and use the Converse of the Corresponding Angles Postulate.
- Identify and use the Converse of Alternate Interior Angles Theorem.
- Identify and use the Converse of Alternate Exterior Angles Theorem.
- Identify and use the Converse of Consecutive Interior Angles Theorem.
- Identify and use the Parallel Lines Property.

II. Cross- curricular-Washington DC

- Use the following map of the mall in Washington DC.
- This is Figure 03.03.01

3.3. Parallel and Perpendicular Lines

- www.visitingdc.com/images/national-mall-map.jpg
- There are several different examples of parallel lines and transversals in this map.
- Students are going to write a series of directions to take someone on a tour of the mall.
- Ask them to start their directions at the American History Museum and write a list of directions in a mathematical way.
- Students can work in small groups on this assignment.
- When finished, have the students swap directions with a neighboring group and check to be sure that the directions work.
- Have the groups provide each other with feedback on their directions.
- Make corrections as needed.

III. Technology Integration

- Have the students explore proving lines parallel by watching the video.
- Use the following website.
- www.yourteacher.com/geometry/provinglinesparallel.php
- When finished, use this as a discussion starter.

IV. Notes on Assessment

- Collect student maps and directions.
- Check work for accuracy.
- Provide students with feedback/correction as needed.

Slopes of Lines

I. Section Objectives

- Identify and compute slope in the coordinate plane.
- Use the relationship between slopes of parallel lines.
- Use the relationship between slopes of perpendicular lines.
- Plot a line on a coordinate plane using different methods.

II. Cross- curricular-Construction/Architecture

- Ask a roof designer, architect or contractor to visit the class and present information on designing a roof.
- Prepare the presenter that he/she needs to be able to talk about slope or pitch and how mathematics plays an important role in construction.
- Ask the students to write questions for the presenter to answer.
- Conduct a follow- up discussion with the students on career connections between architecture and slope.

III. Technology Integration- Roof Design

- Have the students use the following website to explore slopes and roof design.
- Given that the pitch of the roof is connected to the slope of the roof, students can see and explore the real life application of how slope is used in construction.
- www.roofgenius.com/roofpitch.htm
- There are several different websites that do a great job at this.
- Ask the students to begin with this one, and then explore further with other websites.

- A possible extension is for students to design their own roof plan.

IV. Notes on Assessment

- Observe students during the presentation.
- Listen to student questions and answers.
- Be sure that the students understand how roofing and slopes are connected.

Equations of Lines

I. Section Objectives

- Identify and write equations in slope- intercept form.
- Identify equations of parallel lines.
- Identify equations of perpendicular lines.
- Identify and write equations in standard form.

II. Cross- curricular-Ramp Design

- Have a presenter from the local skate shop come in to explain ramps and how they are constructed.
- Be sure that the person that you are having as a speaker is knowledgeable about skateboard ramps and how the ramps are designed.
- You could also have someone come in who is an expert in snowboarding and ramps too.
- Ask the person to bring in some designs or ramps and compare the slope to the equation of the line.
- You can expand this after the presentation by asking the students to draw a diagram representing a skateboard ramp and demonstrate the slope and equation of the line in the diagram.

III. Technology Integration

- Have students complete a websearch on parallel and perpendicular lines.
- Students will find several different websites to explore about parallel lines and the equations of a line.
- Also, they can search for perpendicular lines and equations of a line.
- Use these websites to expand student understanding and prompt discussion.

IV. Notes on Assessment

- Assessment is done through observation in this lesson.
- You want to be sure that the students are engaging in exploring the concepts of the lesson.
- There is not a specific measureable content piece for this lesson.

Perpendicular Lines

I. Section Objectives

- Identify congruent linear pairs of angles.
- Identify the angles formed by perpendicular intersecting lines.
- Identify complementary adjacent angles.

3.3. Parallel and Perpendicular Lines

II. Cross- curricular-Gymnastics

- Use the following image from Wikipedia.
- This is Figure 03.06.01
- www.en.wikipedia.org/wiki/Parallel_bars
- Use this image as a discussion point about perpendicular lines of the gymnast and the high bar.
- One of the ways that gymnasts are scored is on their ability to reach a perfectly perpendicular point.
- This is the basis for the discussion.
- Ask the students to identify a linear pair of angles.
- Also ask the students to find the angles formed by the perpendicular lines.
- Begin this conversation as a springboard to extend into the technology integration.

III. Technology Integration

- Have students continue to search gymnastics through Wikipedia.
- There are several different examples of angles and geometric components of gymnastics.
- Ask the students to make a list of the ways that geometry is integrated into gymnastics.
- Allow time for a class discussion.

IV. Notes on Assessment

- Assess student understanding through discussion.
- Ask the students to point out different examples of geometric terms as they are illustrated in gymnastics.
- Then participate with the students during discussion.

Perpendicular Transversals

I. Section Objectives

- Identify the implications of perpendicular transversals on parallel lines.
- Identify the converse theorems involving perpendicular transversals and parallel lines.
- Understand and use the distance between parallel lines.

II. Cross- curricular-Airport Map

- For this activity, download the map of the main terminal from the following website for the Atlanta International Airport.
- Use this website, and consider this Figure 03.07.01
- www.atlanta-airport.com/forms/passenger/frmPassengerInformation_terminallayout.aspx
- Then use this map to show the main terminal and each of the concourses A- E at the bottom of the map to show a perpendicular transversal and the angles formed by the perpendicular transversal.
- Have the students explore the rest of the map and discover ways that the perpendicular transversals are represented in the other places on the map.
- Allow students to discuss this in small groups first.
- Then bring the students back together and have them share in a large group.

III. Technology Integration

- Have the students google perpendicular transversals.

- Then after doing this, allow students the time to work through some of the websites and explore the information.
- You can ask them to search through particular sites or allow this to be a general investigation time.
- Ask students to make a list of the sites that they explore and at least three things that are presented on the site.

IV. Notes on Assessment

- Look at the student maps and at the notes that the students make about the map.
- Is their work accurate?
- Are there patterns that they can find in the map having to do with perpendicular transversals?
- How can they be sure that the lines are perpendicular?
- What does this do to the angles formed?

Non- Euclidean Geometry

I. Section Objectives

- Understand non- Euclidean geometry concepts.
- Find taxicab distances.
- Identify and understand taxicab circles.
- Identify and understand taxicab midpoints.

II. Cross- curricular-Game Time

- Review the basics of taxicab geometry, distances, circles and midpoints with the students.
- Have them work in small groups.
- Their task is to create a board game that uses the concepts of taxicab geometry.
- You can provide students with a piece of cardboard for a game board, index cards, dice or number cubes, and small colored circle pieces.
- Then set them to work.
- The students will need to create a grid for the "taxis" to move on.
- When finished, let the students play each other's games.
- This can be very in depth and take several days for the students to work on.

III. Technology Integration

- This is a very fun website that has the students go on a treasure hunt while using taxicab geometry.
- www.learner.org/teacherslab/math/geometry/shape/taxicab/
- Allow time for students to explore this website and hunt for the treasure.
- Then allow them time to play and then discuss what they have learned about taxicab geometry while hunting for treasure.

IV. Notes on Assessment

- Create a rubric that gives the students guidelines on how their game will be graded.
- Then walk around and observe students as they work.
- When assessing the game, be sure to play it yourself or observe students playing it so that you can assess whether the game works or not.
- Provide feedback/correction as needed.

3.3. Parallel and Perpendicular Lines

3.4 Congruent Triangles

Triangle Sums

I. <u>Section Objectives</u>

- Identify interior and exterior angles in a triangle.
- Understand and apply the Triangle Sum Theorem.
- Utilize the complementary relationship of acute angles in a right triangle.
- Identify the relationship of the exterior angles in a triangle.

II. <u>Cross- curricular-Hang Gliders</u>

- Have students examine the triangles in the image from Wikipedia.
- This is Figure 04.01.01.
- www.en.wikipedia.org/wiki/Hang_gliding
- Ask students what they notice about the number of triangles that are in the hang glider.
- Then have the students identify all of the different angles of the triangles, also include the interior and exterior angles.
- There are angles created by the strings too.
- Complete this as a whole class discussion.

III. <u>Technology Integration</u>

- Have students complete a google search on triangles in nature.
- There they will find hundreds of different images of how triangles are found in nature.
- Ask the students to look for triangles and assign them the task of finding a real example of triangles in nature.
- Have students bring in these examples the next day and show them to the class.

IV. <u>Notes on Assessment</u>

- This class is a discussion class.
- You want the students to see the connection between the different angles of the triangles both interior and exterior.
- Although it is not directly mentioned, you can draw students back to the Triangle Sum Theorem and explain how the measurement of the angles still equal 180°.
- Also make note of any congruent triangles and why they are important to the hang glider being able to fly.
- If the top sails weren't congruent, what would happen then?

Congruent Figures

I. <u>Section Objectives</u>

- Define congruence in triangles.
- Create accurate congruence statements.
- Understand that if two angles of a triangle are congruent to two angles of another triangle, the remaining angles will also be congruent.
- Explore properties of triangle congruence.

II. Cross- curricular-Bridge Construction

- Begin by showing students some truss bridge designs.
- For this activity, the students are going to use popsicle sticks or toothpicks to build a truss bridge.
- In younger grades, there are several workbooks on how to do this.
- Given that this is a high school course, have the students design and then build the bridge themselves.
- They need to draw a design first and get it approved.
- Then they can move on to the construction piece of the project.
- When finished, have students explain the importance of congruent triangles in building a solid bridge.

III. Technology Integration

- Students are going to work on a bridge exploration in this activity.
- Have the students google "triangles in bridges"
- Then the students need to look at the different types of bridges.
- Have the students explore two different types bridge designs.
- Ask the students to write congruence statements explaining the congruence of the triangles in the different bridge designs.
- When finished, allow students time to share their work.

IV. Notes on Assessment

- Grade student work in two parts.
- First, grade the design. Is it accurate? Is it neat? Is it labeled? Are the triangles congruent?
- Then grade the construction. Is it complete? Did the students demonstrate with congruent triangles? Is it accurate with the design?
- Provide students with feedback on their work.

Triangle Congruence Using SSS

I. Section Objectives

- Use the distance formula to analyze triangles on a coordinate grid.
- Understand and apply the SSS postulate of triangle congruence.

II. Cross- curricular-Quiltmaking

- Students are going to be creating their own quilt squares.
- This will extend into the next two lessons.
- In this first lesson, the students are going to design a square that uses triangles that can be proven congruent using the SSS postulate.
- Students should use certain colors in this square and design a key card to explain the color code and that the triangles can be proven congruent using the SSS postulate.

3.4. Congruent Triangles

- For example, this red and blue quilt square is made up of triangles that can be proven congruent using the SSS Postulate.
- Then the student would include measurements.
- Students can create this as a poster on poster board or using cloth and sewing by hand.
- If sewing, students could have a small quilt by the time they have finished this chapter.
- If working on poster board, they will have a poster when finished.

III. Technology Integration

- Have students search on quilt making.
- There they can find directions on making quilt squares as well as different stitches to use.
- Students can also see examples of different quilts and quilt squares on different websites.

IV. Notes on Assessment

- Assess the student's quilt square.
- Does it represent the SSS postulate?
- Is it clearly explained on the note card?
- Provide students with feedback as needed.

Triangle Congruence Using ASA and AAS

I. Section Objectives

- Understand and apply the ASA Congruence Postulate.
- Understand and apply the AAS Congruence Postulate.
- Understand and practice two- column proofs.
- Understand and practice flow proofs.

II. Cross- curricular-Quilt making

- In this lesson, students are going to add on to their quilts.
- They need to design two different squares.
- One is going to use triangles that can be proven congruent using the ASA Congruence Postulate.
- One is going to use triangles that can be proven congruent using the AAS Congruence Postulate.
- Students need to create a color card with these two squares as well.
- They need to include measurements and explain how the triangles are proven congruent using the different postulates.
- This can be done as an addition to the poster that was started in the last lesson.
- It can also be done as an addition to a sewn quilt.

III. Technology Integration

- Have students complete some research on the history of quilt making.
- Request that each student write a short report on its history and relevance in American society.
- Students can research their material at home or school and write the report as part of a final grade on the quilt.

IV. Notes on Assessment

- Check each quilt square.
- Is the postulate represented?
- Is the note card clearly written?
- Is the report on quilt making clearly written?
- Provide feedback as needed.

Proof Using SAS and HL

I. Section Objectives

- Understand and apply the SAS Congruence Postulate.
- Identify the distinct characteristics and properties of right triangles.
- Understand and apply the HL Congruence Theorem.
- Understand that SSA does not necessarily prove triangles are congruent.

II. Cross- curricular- Quilt making

- Students are working on adding on to their quilts.
- In this lesson, they are going to be creating quilt squares using right triangles.
- After the student has created his/her square, ask them to create a color card to demonstrate congruence using the SAS Congruence Postulate.
- These quilt squares should contain right triangles.
- Add these quilt squares to the poster.
- Students can also sew these to the quilt.

III. Technology Integration

- Ask students to look at patterns using right triangles.
- Then have them find quilt patterns using right triangles.
- This investigation can impact their design work on the quilt squares.
- Have students look at some examples of Amish Quilts.
- How do they differ from other quilt designs?
- Ask the students to identify some of their favorite patterns and explain why they were selected.

IV. Notes on Assessment

- Examine student work.
- Is the student caught up on the work on the quilt?
- Is each square in today's lesson using right triangles?
- Does the student understand the SAS Congruence Postulate?
- Is this clearly demonstrated on the color card?
- Provide students with feedback/coaching as needed.

Using Congruent Triangles

I. Section Objectives

3.4. Congruent Triangles

- Apply various triangles congruence postulates and theorems.
- Know the ways in which you can prove parts of a triangle congruent.
- Find distances using congruent triangles.
- Use construction techniques to create congruent triangles.

II. Cross- curricular-Quilt making

- Today have the students use what they have already been working on with regard to their quilts to explain the different congruence postulates.
- This can be a discussion piece that takes place in small groups.
- As the students discuss each of the triangles and how to prove congruence, the students will expand their understanding of the information.
- Next, allow time for students to "catch up" on unfinished work with regard to the quilts.
- If students are sewing, they will probably need an extra day to sew their quilt squares.

III. Technology Integration

- Have students go to the following website to explore the concepts behind proving triangles are congruent.
- www.onlinemathlearning.com/congruent-triangles.html
- This website not only has information for students to learn with, but also has short videos for students to watch.
- This is created as a support for students to expand what they have already learned.

IV. Notes on Assessment

- Listen to student explanations during the presentations.
- Listen for accuracy in student explanations.
- If the students are missing important information stop them and provide correction/feedback.
- If the students are not clear in their explanations, help them to clarify their explanation on how to determine congruence.
- You can also use this class as a way for students to complete their quilt squares.
- Help the students to make a backing for the quilt if it is made of cloth.
- If it is in poster form, then display the student quilt posters in the class.

Isosceles and Equilateral Triangles

I. Section Objectives

- Prove and use the Base Angles Theorem.
- Prove that an equilateral triangle must also be equiangular.
- Use the converse of the Base Angles Theorem.
- Prove that an equiangular triangle must also be equilateral.

II. Cross- curricular- Geodesic Domes

- For this activity, students are going to examine the equilateral triangles in a geodesic dome.
- Use this website to see this image. This is Figure 04.07.01.
- www.en.wikipedia.org/wiki/File:Epcot07.jpg
- Ask students to use to image to justify the Base Angles Theorem.

- Ask students to use the image to prove that an equilateral triangle is also equiangular.
- Show how an equiangular triangle is also equilateral.
- Allow time for the students to share their work in small groups.
- Have students work on designing their own geodesic dome.
- They can draw it out on graph paper.
- Once they decide on the size of the equilateral triangle, the rest comes together quite easily by repeating the pattern.
- The technology integration can help with this.

III. Technology Integration

- In designing their geodesic domes, the students may want some support from technology.
- Students can research geodesic domes and look at some designs for them.
- Here is another option. Use this website.
- www.fetchaphrase.com/dome/index.html
- This website shows you how to build a geodesic dome out of cardboard.
- Students can use this to construct small geodesic domes.

IV. Notes on Assessment

- Collect student explanations of the different concepts and theorems from the text.
- Be sure that the students have an understanding of how to prove each one of them using the image of the geodesic dome.
- If anything is unclear, provide students with correction and feedback.

Congruence Transformations

I. Section Objectives

- Identify and verify congruence transformations.
- Identify coordinate notation for translations.
- Identify coordinate notation for reflections over the axes.
- Identify coordinate notation for rotations about the origin.

II. Cross- curricular-Art Images

- Have the students select a singular image.
- They can choose any image that they would like to choose as long as it is singular and simple.
- Then have the students make several different copies of the image.
- If it is in book, they can use a copy machine.
- The students are going to create a piece of art using the image and what they have learned about transformations.
- Each of the transformations needs to be represented.
- Students are going to include a reflection, a rotation, a slide and a dilation of their image.
- Students should do this in a creative way.
- Students are welcome to use more copies of the image as long as at least one of the above listed transformations is in the art piece.

III. Technology Integration

3.4. Congruent Triangles

- Have students use the following website to explore all of the different types of transformations.
- www.mathsnet.net/transform/index.html
- On this website, students can explore, understand and work with transformations in an interactive way.
- This is a great way to integrate technology into the lesson.
- You can have students work on this individually or in pairs.

IV. <u>Notes on Assessment</u>

- Examine each student's piece of art.
- Does it contain each of the required transformations?
- Is there more that the student could have done creatively?
- Provide students with feedback/criticism.
- Display work in the classroom.

3.5 Relationships Within Triangles

Midsegments of a Triangle

I. Section Objectives

- Identify the midsegment of a triangle.
- Apply the Midsegment Theorem to solve problems involving side lengths and midsegments of triangles.
- Use the Midsegment Theorem to solve problems involving variable side lengths and midsegments of triangles.

II. Cross- curricular-Mapping

- Use the following image of the Bermuda Triangle in this activity.
- This is Figure 05.01.01
- www.en.wikipedia.org/wiki/File:Bermuda_Triangle.png
- Each student will need a copy of the image to work with.
- Use a scale and a ruler to determine the distance between each of the vertices of the triangle.
- Then determine the midsegment of the triangle.
- Draw the midsegment into the image of the triangle.
- After drawing in the midsegment, write a proof that proves that this is the correct midsegment of the triangle.
- Students can work on this in pairs so that they have peer support when writing the proof.
- Students may want to name each of the vertices to help with writing the proof.
- When finished, allow time for the students to share their work.

III. Technology Integration

- Use Wikipedia or another website to research facts about the Bermuda Triangle.
- What are some of the mysteries surrounding this area?
- When was it discovered to be a "triangle" in shape?
- Be prepared to share your findings with the others in the class.

IV. Notes on Assessment

- Assess student understanding by examining the proof.
- Is it clear?
- Is the given information clear?
- Is it clear which information needs to be proven and which does not?
- Provide students with feedback on their work.
- Share strong example with the others in the class so that all can improve their proof writing.

Perpendicular Bisectors in Triangles

I. Section Objectives

- Construct the perpendicular bisector of a line segment.
- Apply the Perpendicular Bisector Theorem to identify the point of concurrency of the perpendicular bisectors of the sides (the circumcenter).
- Use the Perpendicular Bisector Theorem to solve problems involving the circumcenter of triangles.

II. Cross- curricular-Origami

- There are several different origami designs that you can do that require the use of an equilateral triangle.
- First, use this website to help the students move from a circle to an equilateral triangle.
- www.cyffredin.co.uk/The equilateral triangle.htm
- This will help the students to have an equilateral triangle in design.
- Then you can move on to folding in perpendicular bisectors of the triangle.
- This will help you to identify and mark the circumcenter.
- After the exploration is complete, you can ask the students what they have learned about the perpendicular bisectors of a triangle and the circumcenter of the triangle.
- Brainstorm a list of conclusions on the board.

III. Technology Integration

- Complete a websearch on origami.
- There are several different sites and patterns that students can explore.
- Ask them to select patterns that begin with an equilateral triangle.
- Use this pattern and the equilateral triangle to fold a dolphin or another animal of choice.
- Allow students time to share their work.

IV. Notes on Assessment

- Assessment is completed through observation.
- You can walk around and see students working with the equilateral triangles and the perpendicular bisectors as they fold their designs.

Angle Bisectors in Triangles

I. Section Objectives

- Construct the bisector of an angle.
- Apply the Angle Bisector Theorem to identify the point of concurrency of the perpendicular bisectors of the sides (the incenter).
- Use the Angle Bisector Theorem to solve problems involving the incenter of triangles.

II. Cross- curricular-Art

- This activity builds on the origami that the students completed in the last lesson.
- This time, students aren't going to be working with equilateral triangles but with three different sized triangles.
- Ask the students to cut out triangles that are three different sizes.
- Then with each triangle, students are to fold the paper to show the three bisecting lines of each of the angles of the triangle.
- In the end, the students will have the point of concurrency.
- From there, they can inscribe the circle into the triangle.

- Students need to complete this with all three triangles.
- Allow time for the students to share their work in small groups when finished.

III. Technology Integration

- Students can use the following website to explore bisecting angles.
- www.geom.uiuc.edu/ demo5337/Group2/incenter.html
- When finished, ask the students to share what they discovered about angle bisectors and inscribing circles.
- Write the conclusions on the board.

IV. Notes on Assessment

- Walk around and observe the students as they work on the paper folding.
- Assist students who are having difficulty.
- Students should see this as a hands- on way to work through the point of concurrency and inscribing circles.

Medians in Triangles

I. Section Objectives

- Construct the medians of a triangle.
- Apply the Concurrency of Medians Theorem to identify the point of concurrency of the medians of the triangle (the centroid).
- Use the Concurrency of Medians Theorem to solve problems involving the centroid of triangles.

II. Cross- curricular-Art

- Use the concept of Napolean's Theorem to create a new design/stained glass window effect.
- Review Napolean's Theorem and how it works.
- Then have students begin to work on a design.
- Students can explore trying different sizes of triangles.
- They can also see if it makes sense to integrate different shapes.
- Since the goal is a stained glass window of sorts, students are going to create a frame and then place different colored tissue paper inside the frame.
- Students can complete this and hang them in the window and the sunlight will come through the design.

III. Technology Integration

- One of the ways to integrate technology into this lesson is to have the students look at some of the other designs of Napolean's Theorem.
- They can begin by googling Napolean's Theorem.
- There will be several different websites that will come up where students can read about Napoleans Theorem and see different patterns and designs.
- When finished, students will have expanded their thinking on this theorem.

IV. Notes on Assessment

- Walk around and assist students as they work.
- When finished, assess student designs.

3.5. Relationships Within Triangles

- Does it work according to Napolean's Theorem?
- If not, what would make a difference?
- Is the shape incorrect or does there need to be more shapes?
- How detailed did the student get?
- Is the stained glass window colorful and creative?
- Provide students with feedback on their work.

Altitudes in Triangles

I. <u>Section Objectives</u>

- Construct the altitude of a triangle.
- Apply the Concurrency of Altitudes Theorem to identify the point of concurrency of the altitudes of the triangle (the orthocenter).
- Use the Concurrency of Altitudes Theorem to solve problems involving the orthocenter of triangles.

II. <u>Cross- curricular-Sculpture</u>

- Investigate the concept of altitude by using the following image. This is Figure 05.05.01
- This is an image of the Mihashira Torii sculpture.
- www.en.wikipedia.org/wiki/File:Yamato_mihasira006.jpg
- Use this image and ask the students to share how they think the concept of altitude impacts this sculpture.
- Brainstorm ideas and write them on the board.
- Then set students to work on designing and building their own sculpture.
- You will need dowels, small hand saws, sand paper and wood glue or fast drying glue.
- You can use dowels with a small diameter so that they will be easy to cut.
- The students need to work with a triangle as the basic shape of the sculpture and demonstrate the altitude of the triangle in their sculpture.
- Allow time for the students to work and then present their sculptures when finished.

III. <u>Technology Integration</u>

- Investigate the concept of altitude using the computer.
- Ask the students to search all of the different ways that altitude impacts our way of life.
- Have them keep a list of the websites that they visit.
- They also need to make notes on at least ten different ways that altitude impacts how we live.
- Allow time for students to share their research when finished.

IV. <u>Notes on Assessment</u>

- Assessment is completed through observation.
- Walk around and see how the students are doing on their sculptures.
- Help out when needed.

Inequalities in Triangles

I. <u>Section Objectives</u>

- Determine relationships among the angles and sides of a triangle.
- Apply the Triangle Inequality Theorem to solve problems.

II. Cross- curricular-Sculpture

- In this activity, you are going to design a sculpture using triangles.
- You want to show that your triangles represent an inequality.
- To do this, you will need to design your sculpture before building it.
- This design should have measurements and demonstrate an inequality.
- When finished with the design, students can use clay to build their triangles.
- Have tools available to work with.
- When finished, have students write a short explanation of their sculpture, what they designed, how it was created and how it demonstrates the concept of an inequality.

III. Technology Integration

- Have the students research triangular sculptures.
- There are so many different sites to select and sculptures to see.
- Students need to select one triangle sculpture that they appreciate.
- Then there is a writing piece to this assignment.
- Students need to write about why they selected the piece and to use principles already learned to describe it.
- What kind of triangle(s) are in the sculpture?
- Is it in three- dimensions or two?
- Are there congruent triangles involved?
- How can you determine congruence?

IV. Notes on Assessment

- Assess student designs and sculptures.
- Does the design and the sculpture match up?
- Does the sculpture represent an inequality?
- Is student writing clear?
- Does the student have an understanding of the concept of triangle inequalities?
- Provide students with feedback/notes.

Inequalities in Two Triangles

I. Section Objectives

- Determine relationships among the angles and sides of two triangles.
- Apply the SAS and SSS Triangle Inequality Theorems to solve problems.

II. Cross- curricular- Architecture

- Use the concept of gable windows during this lesson.
- Students can use the two Theorems to determine the relationship between triangles in gable windows.
- Use the following image. This is Figure 05.07.01.
- www.loghomebuilders.org/files/images/log-home-bham-gable-windows.preview.jpg
- Have students work in small groups.

3.5. Relationships Within Triangles

- In each group, the students need to come up with a way to prove the relationship between the two triangles in the image of gable windows.
- They are going to be using the SAS and the SSS Triangle Inequality Theorem to do this.
- When finished, have the students present their work to the class.

III. Technology Integration

- Expand this websearch into many different types of triangular windows.
- Some windows will show congruent triangles, but others won't.
- Have the students select a window pair that is congruent and then prove congruency.
- Have the students select a window pair that demonstrates an inequality and then prove this using the theorems.
- Have the students repeat the exercise that they did in small groups with a new window design.

IV. Notes on Assessment

- Assessment is completed through observation.
- Walk around as students work.
- Ask questions and probe into student thinking.

Indirect Proofs

I. Section Objectives

- Reason indirectly to develop proofs of statement.

II. Cross- curricular-Sports

- You have the job of being a sports announcer at a basketball game.
- To do this, you will be reporting on the actions of the game.
- However, you can only report your findings using if-then statements.
- You are going to prepare a short broadcast and then present it with a peer to the class.
- This is meant to be a fun short assignment to help students to see how to use if then statements in real life.
- Students will have fun with this.
- Give them time to work and props are fine to use as well.
- When finished, allow time for each pair to present their skit.

III. Technology Integration

- What is proof?
- Who uses proof?
- What kinds of careers or projects require people to prove something?
- Complete a web investigation on the topic of proof.
- Make a list of the websites that you visit.
- Keep a record of the data you discover.
- Write a one page paper to share/explain your findings.

IV. Notes on Assessment

- Assessment is complete through observation.

- Did the students use if- then statements?
- Were they prepared?
- Were they focused?
- Offer feedback to students as needed.

3.6 Quadrilaterals

Interior Angles

I. <u>Section Objectives</u>

- Identify the interior angles of convex polygons.
- Find the sums of interior angles in convex polygons.
- Identify the special properties of interior angles in convex quadrilaterals.

II. <u>Cross- curricular-Mandalas</u>

- Use the following image to discuss interior angles of quadrilaterals.
- This is Figure 06.01.01
- www.isibrno.cz/ gott/mandala/sriclr2.gif
- This is an image of a mandala that is composed of triangles that can also be interpreted to be quadrilaterals.
- You can use this image to discuss the measure of the interior angles of the quadrilateral with students.
- Show them how two triangles can be combined together to become a quadrilateral.
- Then remind students that the interior angles of a triangle add up to be $180°$ according to the Triangle Sum Theorem.
- Then ask the students to look at how many degrees are in a quadrilateral based on the fact that it is made up of two triangles.
- The students will conclude that it is equal to $360°$.

III. <u>Technology Integration</u>

- Have students complete some research on mandalas.
- Where do they come from?
- When were they first used?
- What is the purpose of a mandala?
- Have students keep a record of the websites that they visit.
- Allow time for students to share their findings.

IV. <u>Notes on Assessment</u>

- Assessment is completed through student discussion.
- Listen to the students as they share their thoughts and ideas.
- Be sure that they understand how the interior angles of a quadrilateral are equal to $360°$.

Exterior Angles

I. <u>Section Objectives</u>

- Identify the exterior angles of convex polygons.
- Find the sums of exterior angles in convex polygons.

II. Cross- curricular- Mandalas

- Use the information that you developed in the last lesson to work on this project.
- Use the image of the mandala on this website.
- This is Figure 06.02.01
- www.isibrno.cz/ gott/mandala/sriclr2.gif
- Now tell students that today they are going to be working to design their own mandalas.
- The students need to include triangles and quadrilaterals in their design.
- Then they also need to identify the interior and exterior angles of the quadrilaterals.
- Be sure to tell students to use color and creativity in their designs.
- Then have the students write a paragraph describing the mandala and explaining the connections between the interior angles and the exterior angles and the measurements of 180° and 360°.
- Allow time for students to share their work when finished.
- Display the mandalas in the classroom.

III. Technology Integration

- Students can go to the following website and investigate the interior and exterior angles of quadrilaterals.
- This site is fun, interactive and colorful.
- www.slideshare.net/guest4210b1/quadrilaterals
- Students can participate in this as an independent study activity.

IV. Notes on Assessment

- Assess student mandalas.
- Did the students incorporate the triangles and quadrilaterals?
- Are the interior angles labeled or identified in some way?
- Are the exterior angles labeled or identified in some way?
- Is the writing piece clearly written?
- Does it explain the angle measures?
- Provide students with feedback on their work.

Classifying Quadrilaterals

I. Section Objectives

- Identify and classify a parallelogram.
- Identify and classify a rhombus.
- Identify and classify a rectangle.
- Identify and classify a square.
- Identify and classify a kite.
- Identify and classify a trapezoid.
- Identify and classify an isosceles trapezoid.
- Collect the classifications in a Venn diagram.
- Identify how to classify shapes on a coordinate grid.

II. Cross- curricular-Art

3.6. Quadrilaterals

- Use the following website and image of a geometric pattern for the following activity.
- This is Figure06.03.01
- www.tfaoi.com/cm/2cm/2cm511.jpg
- Ask the students to work in teams and identify all of the different quadrilaterals in the drawing.
- Students need to identify the quadrilateral and then write the characteristics of that quadrilateral.
- Next, have the students work to complete their own quadrilateral art piece.
- All of the different types of quadrilaterals should be included in the piece.
- Also, the art design should only consist of quadrilaterals.
- Design and color need to be included in this work.
- Once they have done the design, have the students write a description of their work identifying each quadrilateral in the design and its characteristics.

III. Technology Integration

- Students can go to the following website to see a video on how to identify and classify quadrilaterals.
- www.onlinemathlearning.com/quadrilaterals.html
- Then the students can use this information in the first part of this lesson, or this can be used to help students to solidify the information that they have already learned.

IV. Notes on Assessment

- Assess student designs.
- Have the students created a design composed only of quadrilaterals?
- Is it creative?
- Does the writing piece identify and describe each quadrilateral according to its characteristics?
- Provide students with feedback on their work.

Using Parallelograms

I. Section Objectives

- Describe the relationships between opposite sides in a parallelogram.
- Describe the relationship between opposite angles in a parallelogram.
- Describe the relationship between consecutive angles in a parallelogram.
- Describe the relationship between the two diagonals in a parallelogram.

II. Cross- curricular-Architecture

- Select several of the images on the following website.
- Print these images and distribute them to the students or have the students use computers to look at the images.
- www.trendir.com/house-design/
- All of the homes here are constructed using many different parallelograms.
- There are large parallelograms, small ones, all kinds of different ones.
- Ask the students to select one of the houses and work with it to identify the elements of the parallelograms in the designs.
- Students need to be looking for the relationship between the opposite sides of a parallelogram.
- The consecutive angles and the diagonals- how can the relationship be determined?
- Ask students to take notes on their home and then to share their findings in small groups.

III. Technology Integration

- One possible integration is to have the students explore this website and the house designs further.
- www.trendir.com/house-design/
- Then they can actually work to design their own home using parallelograms.
- Students can draw this out on grid paper or on plain paper.
- The key element or focus of the home must be the parallelogram.
- Have students share their work with their peers.

IV. Notes on Assessment

- Assessment is completed through observation.
- Walk around and participate in student groups as they look at the different elements of the house that they have been given.
- Examine each house design.
- Is the key element a parallelogram?
- Provide students with feedback on their work.

Proving Quadrilaterals are Parallelograms

I. Section Objectives

- Prove a quadrilateral is a parallelogram given congruent opposite sides.
- Prove a quadrilateral is a parallelogram given congruent opposite angles.
- Prove a quadrilateral is a parallelogram given that the diagonals bisect each other.
- Prove a quadrilateral is a parallelogram if one pair of sides is both congruent and parallel.

II. Cross- curricular- Drama

- Assign students the task of creating a skit to prove that a quadrilateral is a parallelogram.
- Begin this lesson by reviewing the different characteristics that make a quadrilateral a parallelogram.
- The synopsis of the skit is "Mr./Ms. Quadrilateral needs to prove that he/she is a parallelogram. He/she has selected Geo Geometry to prove the case."
- Then let the students go to work.
- They can use props, scenery (simple) and costumes.
- Allow time for them to write a skit and rehearse it.
- Then the students need to be given time to perform their skit.

III. Technology Integration

- Students can explore the properties of a parallelograms with the following website.
- www.mathwarehouse.com/geometry/quadrilaterals/parallelograms/interactive-parallelogram.php
- On this website, there is a place where it lists the criteria for proving that a quadrilateral is a parallelogram.
- Then it also has an interactive part where you can click and drag the vertices of the parallelogram to alter the side lengths, angle measures, etc.
- The numbers change instantly on the screen as you move the vertices around.
- A great interactive site to work with.

IV. Notes on Assessment

- Assess each skit.

3.6. Quadrilaterals

- Did the students get the job done?
- Did they prove that the quadrilateral is a parallelogram?
- If so, what did they do well?
- If not, what was missing?
- Provide students with feedback on their work.

Rhombi, Rectangles, and Squares

I. <u>Section Objectives</u>

- Identify the relationship between the diagonals in a rectangle.
- Identify the relationship between the diagonals in a rhombus.
- Identify the relationship between the diagonals and opposite angles in a rhombus.
- Identify and explain biconditional statements.

II. <u>Cross- curricular-Design Collage</u>

- In this activity, have the students work in groups of three.
- Each student is going to select one of the three quadrilaterals.
- Then they are going to create a collage about the quadrilateral that they have chosen.
- Included in the collage should be pictures of their shape out in the world.
- Have each group pick a theme for their collage.
- For example, sports or nature or furniture.
- Students can hunt through magazines for these pictures.
- On one of the pictures, they need to draw in the angles and diagonals of the figure.
- This is a way to demonstrate the characteristics of that figure.
- When finished, each group should have a complete description of all three types of quadrilaterals.

III. <u>Technology Integration</u>

- Students can explore the properties of rectangles, squares and rhombi on the following website.
- www.mathsisfun.com/quadrilaterals.html
- This is a basic site, but since these are basic figures, the information should be review.
- There is also a place where they have interactive quadrilaterals and students can manipulate the size and configuration of the quadrilateral.

IV. <u>Notes on Assessment</u>

- Assess the continuity of each design trio.
- Is there a consistent theme?
- Does each design show the characteristics of each quadrilateral?
- Provide students with feedback on their work.

Trapezoids

I. <u>Section Objectives</u>

- Understand and prove that the base angles of isosceles trapezoids are congruent.
- Understand and prove that if base angles in a trapezoid are congruent, it is an isosceles trapezoid.
- Understand and prove that the diagonals in an isosceles trapezoid are congruent.
- Understand and prove that if the diagonals in a trapezoid are congruent, the trapezoid is isosceles.
- Identify the median of a trapezoid and use its properties.

II. Cross- curricular-Buildings/Construction

- Use the following image of the Flatiron Building in NYC from Wikipedia.
- This is Figure 06.07.01
- www.en.wikipedia.org/wiki/File:Flatiron_crop_20040522_114306_1.jpg
- Print the image so that the students can look at the image in their seats.
- Use this as a feature of discussion.
- Show the students how the sides of the building are the trapezoids.
- This building is formed by three trapezoids and then the top is a triangle.
- Are the students able to identify the type of triangle?
- What would the shape of the building change to if the sides were composed of four trapezoids instead of three?
- You can either have this part as a discussion or as an exploration.
- If you want students to explore this, be sure that they have cardboard, scissors, rulers, tape and a copy of the image to work with.
- Have the student begin by building a rough model of the Flatiron Building as it is now.
- Then they can look at altering it by adding another trapezoid.
- This gives students a discussion point about the building.
- Allow time to discuss and share findings and conclusions following the exploration.

III. Technology Integration

- Ask students to complete a websearch.
- Students are going to google "trapezoid images" for this websearch.
- There will be tons of different images that use trapezoids.
- Ask the students to select five different ones to share.
- If possible, have them print these images, if not they should make notes about the images and where the trapezoid is in the image.
- Also be sure that students write down any websites where the images are located.
- Allow time for students to share their work.

IV. Notes on Assessment

- Assess student understanding of trapezoids through class discussions and sharing.
- Provide students with feedback on their work.

Kites

I. Section Objectives

- Identify the relationship between diagonals in kites.
- Identify the relationship between opposite angles in kites.

II. Cross- curricular-Kites

3.6. Quadrilaterals

- Have students work to design their own kite.
- They can look at the different kinds of kites that are possible by looking at the Wikipedia website.
- www.en.wikipedia.org/wiki/Kite
- In the kite, the students should show the diagonals and the opposite angles of the kite.
- Students should be very creative with their kite and have it say something about them.
- Use this in connection with the technology integration section.

III. Technology Integration

- Have students do some research on kites.
- They can do a web search on kites and see many different designs.
- Students should be able to report on the origin of kites.
- Three cultural elements of kites
- Three countries known for kites
- How and why kites fly
- How kites are used in science and technology
- Ask students to write a report on this topic.

IV. Notes on Assessment

- Assess student reports and kite designs.
- Is the design creative?
- Does it say something about the student?
- Is the report complete?
- Are all of the components in it?
- Provide students with feedback/correction as needed.

3.7 Similarity

Ratios and Proportions

I. Section Objectives

- Write and simplify ratios.
- Formulate proportions.
- Use ratios and proportions in problem solving.

II. Cross- curricular-Greek Architecture

- Provide students with an image of the Parthenon from Wikipedia.
- This is Figure 08.01.01
- www.en.wikipedia.org/wiki/File:Parthenon-2008.jpg
- Then provide students with an image of the Acropolis from Wikipedia.
- www.en.wikipedia.org/wiki/File:AthensAcropolisDawnAdj06028.jpg
- Now use the images as a discussion about the golden ratio of approx. 1.6 and how this is shown in the dimensions of each building.

III. Technology Integration

- Students can look at this website using The Golden Ratio and talking about how beauty has to do with ratios. Check it out first.
- www.intmath.com/Numbers/mathOfBeauty.php
- Students can also use this website which looks at ratios in nature.
- www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibInArt.html
- This website has a ton of different links for students to explore when looking at how ratios play into different topics.

IV. Notes on Assessment

- The content of this lesson is assessed through the discussion.
- You want students to understand that when you compare different facets, the ratios impact the design.
- With the golden ratio, the ratio is the same.
- Then you want the students to begin to make the connections on their own.
- Students can see the real life examples of ratios, especially the golden ratio.

Properties of Proportions

I. Section Objectives

- Prove theorems about proportions.

3.7. Similarity

- Recognize true proportions.
- Use proportions theorem in problem solving.

II. <u>Cross- curricular- Astronomy</u>

- Use the following map of the constellations in this activity.
- This is Figure 08.02.01
- www.nightskyinfo.com/sky_highlights/july_nights/july_sky_map.png
- Use the image of Ursa Major and Ursa Minor to explore the concepts of proportions.
- Are the two images in proportion?
- How can we tell?
- Complete an in class discussion on what makes two images or two ratios a proportion.
- What kinds of measurements would we need to prove that the two constellations were proportional?
- Encourage students to work with the concepts of proportions and apply it to the constellation map.

III. <u>Technology Integration</u>

- Students can use this youtube video to study the planets in proportion.
- www.youtube.com/watch?v=PZNrQGCEXzs
- Students can follow this up by researching and comparing two planets.
- Have them choose two to compare and write ratios and proportions to compare them both.
- Allow time for students to share their work when finished.

IV. <u>Notes on Assessment</u>

- Assess student work through the discussion and through student notes.
- Were the students able to decide how to write proportions and ratios on the planets and constellations?
- Then provide students with feedback on their work.

Similar Polygons

I. <u>Section Objectives</u>

- Recognize similar polygons.
- Identify corresponding angles and sides of similar polygons from a statement of similarity.
- Calculate and apply scale factors.

II. <u>Cross- curricular-Model Design</u>

- This is a great opportunity to include scale and design into the mathematics classroom.
- You can work with this lesson in two different ways.
- The first way is to have the students choose a polygon and to build a model of two polygons that are similar using a scale model.
- This way, the students can actually have a hands- on experience of figuring out the dimensions of a scale model and then put these measurements to work building the model.
- The second way is to choose a mountain or a building for the students to use to create a scale design or model of.
- For example, if you chose the Empire State Building, the students would figure out the actual measurements, and then build a model or draw a design using a scale.

- You could do 1[U+0080][U+009D] per foot, etc.
- Allow time for students to share their work when finished.

III. Technology Integration

- Students can go to the following website to explore similar polygons.
- www.saskschools.ca/curr_content/byersjmath/geometry/students/polygon/intmovie.html
- When the students go to this website, they need to go to the section on similar polygons.
- From there, they can watch the animation which explains all how to determine similar polygons and how to create similar polygons.

IV. Notes on Assessment

- Check student work for accuracy.
- Is the scale accurate?
- Does the model or design match the scale?
- Do the students have a good understanding of similar polygons?
- Provide students with correction/feedback on their work.

Similarity by AA

I. Section Objectives

- Determine whether triangles are similar.
- Understand AAA and AA rules for similar triangles.
- Solve problems about similar triangles.

II. Cross- curricular-Pyramids

- This lesson will work best with the technology integration.
- Have students complete the study of Thales first and then move to a hands- on activity.
- Once students have selected a pyramid, they are going to work on this activity.
- Students are going to use the researched dimensions of the pyramid to build a model to scale.
- Students can build this model out of sugar cubes and glue.
- Sugar cubes tend to work well.
- After completing the model, use a darkened room and a high powered flashlight to demonstrate the shadow of the pyramid.
- Is it accurate according to Thales?
- See if the students can develop a way to test out this theory.
- Allow time for students to share their work when finished.

III. Technology Integration

- Have students complete some research on Thales and on indirect measurement.
- Students can read about Thales at the following website.
- www.phoenicia.org/thales.html
- Conduct a discussion on Thales and on how he discovered and figured out the height of the pyramids using indirect measurement.
- Once students have a good understanding of this, move on to the next part of this lesson.

3.7. Similarity

- Then have the students do a search and choose a pyramid.
- Students are going to use the dimensions of this pyramid to build a model.

IV. Notes on Assessment

- Assess student work through discussion and observation.
- Do the students understand who Thales was and the significance of his discovery?
- Is the student model to scale?
- Were the students able to come up with a way to test Thales' findings?
- What are students sharing about this assignment?
- Is higher level thinking involved?
- Provide students with feedback as needed.

Similarity by SSS and SAS

I. Section Objectives

- Use SSS and SAS to determine whether triangles are similar.
- Apply SSS and SAS to solve problems about similar triangles.

II. Cross- curricular-Literature/Poetry

- In this activity, students need to create a poem, song or story that explains the three ways to figure out if two triangles are similar.
- The first is AA- angle angle
- The second is side- side- side.
- The third is side- angle- side.
- You can begin this lesson by reviewing the definitions of each and how to use them to figure out if two triangles are similar.
- Then divide students into groups of three.
- Have the groups work on their expression of figuring out if two triangles are similar.
- When finished, allow time for the students to share their work.

III. Technology Integration

- Students can go to the following class zone website and see the animation on similar triangles.
- www.classzone.com/cz/books/geometry_2007_na/get_chapter_group.htm?cin=2&rg=animated_math&at=animations&va
- This is a fun interactive way to see the work done.
- Because class zone is affiliated with another textbook, the students can have a difficult time navigating the site.
- Use the link above for it.
- This will bring the students to the animation.
- If you don't wish to use class zone, students can also go to futureschannel.com and see a short movie on triangles and architecture.

IV. Notes on Assessment

- Assess each group's poem or story.
- Does it explain how to figure out if triangles are similar?
- Is each theorem well explained?
- Provide students with feedback as needed.

Proportionality Relationships

I. <u>Section Objectives</u>

- Identify proportional segments when two sides of a triangle are cut by a segment parallel to the third side.
- Divide a segment into any given number of congruent parts.

II. <u>Cross- curricular-Proportional Divisions</u>

- Have students participate in a hands- on activity to explore the section objectives.
- Students are going to work with several different triangles.
- The triangles should all be the same size.
- You can either prepare the triangles ahead of time or have the students cut them out themselves.
- Then have students work in small groups.
- In each group, the students are going to explore the proportional segments that are created when two sides of a triangle are cut by a segment parallel to the third side.
- They should try this will three different line segments each parallel to a different side.
- This means that the activity will get repeated with three different triangles.
- The students need to measure each side and write proportions to represent the different sections of the triangle.
- For example, when the triangle is cut, there are two polygons- how do the side lengths compare? Are they in proportion?
- Students need to make notes on these comparisons and share them with the other students.

III. <u>Technology Integration</u>

- Use Wikipedia to explore the concept of proportionality.
- www.en.wikipedia.org/wiki/Proportionality
- Students can look at proportionality in mathematics, but also in human design and architecture.
- There are several different links to explore.

IV. <u>Notes on Assessment</u>

- Assess student understanding by observing their work in small groups.
- Were the students able to successfully cut the triangles into proportions?
- Were they able to write proportions that demonstrate that the two polygons are similar?
- Provide feedback as needed.

Similarity Transformations

I. <u>Section Objectives</u>

- Draw a dilation of a given figure.
- Plot the image of a point when given the center of dilation and scale factor.
- Recognize the significance of the scale factor of a dilation.

II. <u>Cross- curricular- Art</u>

- The name of this activity is "Honey I Shrunk the Polygon!"

3.7. Similarity

- Students are going to take any polygon that they would like to and create an art piece that shows the dilations of the polygon.
- The polygon that is the beginning polygon should be in red.
- That way you can tell which polygon is being transformed.
- Students should create dilations which are smaller and larger.
- The scale factor can be decided by the student.
- The scale should be the same whether the polygon is being dilated smaller or larger.
- Allow students time to work.
- Display student work when finished.

III. Technology Integration

- To look at different dilations, students can do some research on Christmas Tree Farms.
- Because farms often use the same kind of tree, there will be small versions of the tree and large versions of the tree.
- This is a real life look at dilations.
- Students can do some work drawing different trees.
- Have them choose one to begin with and then dilated two or three times.
- This will show a "growth progression" of the tree.

IV. Notes on Assessment

- Ask the students to share their dilated polygons.
- What works about the polygon and what doesn't work?
- Is there an accurate scale factor?
- Are both images correctly dilated?
- Provide students with feedback.

Self- Similarity (Fractals)

I. Section Objectives

- Appreciate the concept of self- similarity.
- Extend the pattern in a self- similar figure.

II. Cross- curricular- T-shirt Design

- Review the concept of fractals and what makes a fractal image.
- Then show students the image on this website.
- This is Figure 07.08.01
- www.redbubble.com/people/archimedesart/art/3390955-4-bright-lights
- Then show students this second fractal.
- This is Figure 07.08.02
- www.zazzle.com/right_angles_tshirt-235230222951842274
- Discuss these fractals with the students.
- Notice the quadrilaterals in the image.
- This is a T- shirt design.
- Have students design their own fractal t- shirt.
- This can be as complicated or simple as you wish.

- Students can use fabric paint and fabric markers to actually draw their fractal on their shirt.
- They could also create a pattern with a piece of cardboard and then use fabric paint to paint over the image and have it displayed on the shirt.

III. Technology Integration

- Have students research vegetable fractals.
- There are so many interesting images of fractals.
- Ask the students to select a few and write about why they chose the one that they did.
- Also, ask the students to explain, to the best of their ability, how the image is a fractal.
- What characteristics/qualities make it a fractal?
- Allow time for students to share their thinking when finished.

IV. Notes on Assessment

- When looking at student t-shirt designs, you are looking for a representation of a fractal.
- This can be assessed by looking at each t- shirt.
- Provide students with feedback when finished.

3.8 Right Triangle Trigonometry

The Pythagorean Theorem

I. <u>Section Objectives</u>

- Identify and employ the Pythagorean Theorem when working with right triangles.
- Identify common Pythagorean triples.
- Use the Pythagorean Theorem to find the area of isosceles triangles.
- Use the Pythagorean Theorem to derive the distance formula on a coordinate grid.

II. <u>Cross- curricular-Toy Construction</u>

- <u>If possible, complete this after watching the movie.</u>
- <u>Divide the students into groups of three or four.</u>
- <u>You will need Kynex for this activity.</u>
- <u>Students may be able to bring in some from home.</u>
- <u>If Kynex are not available, just have this be a design project.</u>
- <u>Tell the students that they are going to be designing a toy that has a right triangle as its core component.</u>
- <u>Students can use other shapes as well, but the triangle is central.</u>
- <u>Students are to draw a design of their toy.</u>
- <u>Then, students are to build a model using the Kynex.</u>
- <u>Allow time for students to share their work when finished.</u>

<u>III.</u> Technology Integration

- Use the following website so that students can watch a short movie on creating triangular toys.
- www.thefutureschannel.com/dockets/realworld/inventing_toys/
- This video shows how two designers working for Kynex design toys.
- Tell the students to notice all of the uses of polygons and triangles in the designs.
- When finished, discuss the video.
- What did the students observe?
- What did they notice about the shapes used in the toy designs?
- How did patterns impact the work of the designers?
- How does geometry impact their work?

IV. <u>Notes on Assessment</u>

- Assess each toy design and construction.
- You may want to create a rubric for grading the toys.
- Observe students as they work.
- Provide students with feedback when necessary.

Converse of the Pythagorean Theorem

I. <u>Section Objectives</u>

- Understand the converse of the Pythagorean Theorem.
- Identify acute triangles from side measures.
- Identify obtuse triangles from side measures.
- Classify triangles in a number of different ways.

II. <u>Cross- curricular-Architecture/Design</u>

- Use the following image from Wikipedia to show students an image of St. Basil's Cathedral.
- This is Figure 08.02.01
- www.en.wikipedia.org/wiki/File:RedSquare_SaintBasile_(pixinn.net).jpg
- You can either use this image as a discussion point or have students work with it in small groups.
- In small groups, have the students identify the equilateral and acute triangles in the cathedral.
- There are many of them to choose from.
- Then ask the students to identify how they know that these are equilateral and acute.
- The students should be able to discuss the different characteristics of what makes an acute triangle acute and what makes an equilateral triangle equilateral.
- Have students discuss this in small groups.

III. <u>Technology Integration</u>

- Ask students to research triangles and bridge designs.
- What is the most common type of triangle used in bridge designs?
- Why is it the most common?
- Have the students do some research on this and then report on their findings.
- Students should keep track of any websites they visit to refer back to when reporting on their findings.

IV. <u>Notes on Assessment</u>

- Observe students as they work.
- Listen to the discussions and you will hear whether the students have an understanding of acute, obtuse and equilateral triangles.
- Ask questions to expand student thinking.

Using Similar Right Triangles

I. <u>Section Objectives</u>

- Identify similar triangles inscribed in a larger triangle.
- Evaluate the geometric mean of various objects.
- Identify the length of an altitude using the geometric mean of a separated hypotenuse.
- Identify the length of a leg using the geometric mean of a separated hypotenuse.

II. <u>Cross- curricular-Triangular Lodge</u>

3.8. Right Triangle Trigonometry

- Have students use this website, or show them the image and give them the measurements that they will need to work with.
- www.daviddarling.info/encyclopedia/T/Triangular_Lodge.html
- This is a building that is composed on a triangle.
- We know that each side of the triangle is 33 feet long.
- If this is the case, what is the altitude of the building?
- Have student work in small groups or pairs to solve this problem.
- Students will need to work through the formula for geometric mean in the text.
- If they are having trouble, refer them back to the text for this information.
- Solution:
- $33 \times 33 = 1089$ feet
- $\sqrt{1089} = 33$ feet
- Be sure that the students understand how the measurements are all the same.
- Allow time for questions and feedback.

III. Technology Integration

- Have students complete some research on circus tents.
- Circus tents use poles and canvas to hold up the tent.
- The use of the poles impacts the height or altitude of the tent.
- Ask the students to report on the most common design of a circus tent.
- Have them make a list of the websites that they visit and to select one type of tent or image to discuss.
- You can conduct a discussion about how geometric mean, altitude and triangles connect with circus tents.
- How are they interconnected?
- This will require the students to use higher level thinking skills since the connections may not be obvious.

IV. Notes on Assessment

- Assess student understanding through discussion.
- Try to have time for each group to share.
- You will see how much the students understand through their sharing and conversation.

Special Right Triangles

I. Section Objectives

- Identify and use the ratios involved with right isosceles triangles.
- Identify and use the ratios involved with $30 - 60 - 90$ triangles.
- Identify and use ratios involved with equilateral triangles.
- Employ right triangle ratios when solving real- world problems.

II. Cross- curricular-Sports

- Use the following image of a baseball diamond from Wikipedia.
- This is Figure 08.04.01
- www.en.wikipedia.org/wiki/File:Baseball_diamond_marines.jpg
- This is a problem to solve.
- Here is the problem.
- If the distance between the bases is 90 feet, how far will the first baseman throw the ball to reach the third baseman?

- Solution:
- To solve this problem, you can use the Pythagorean Theorem since each of the bases is at a 90° angle.
- Therefore, you can split up the baseball diamond into $45 - 45 - 90$ triangles.
- $90^2 + 90^2 = c^2$
- $8100 + 8100 = c^2$
- $16200 = c^2$
- 127.2 feet is the distance from first to third base.

III. Technology Integration

- Have the students complete a websearch on baseball fields across the United States.
- Students can select their favorite one and report on its dimensions.
- Does the Pythagorean Theorem work for all baseball diamonds?
- Conduct a discussion exploring the angles and dimensions of baseball diamonds.

IV. Notes on Assessment

- Were the students able to solve the problem?
- Were there struggles?
- Did the students see the right angles in the diamond?
- Did they notice that they could divide the diamond into two $45 - 45 - 90$ triangles?
- Where is the hypotenuse of the triangles?
- Assess student work and provide feedback as needed.

Tangent Ratios

I. Section Objectives

- Identify the different parts of right triangles.
- Identify and use the tangent ratio in a right triangle.
- Identify complementary angles in right triangles.
- Understand tangent ratios in special right triangles.

II. Cross- curricular-Art/Furniture Making

- Have the students look at the website or show them the images of the triangle table.
- You can use this as a discussion piece.
- Ask the students to identify the parts of the right triangle.
- Then ask them to identify the tangent ratio of the right triangle.
- Finally, students can be given the task of constructing their own right triangle table.
- Students will need tools and saws to do this.
- You may want to see if you can combine this activity with woodshop, if offered in your school.
- Have the students share their work when finished.

III. Technology Integration

- Have the students explore the concept of dragon tiles that have right angles in them.
- The students can go to the following website to explore this.
- www.ecademy.agnesscott.edu/ lriddle/ifs/levy/tiling.htm

3.8. Right Triangle Trigonometry

- This will provide students with step by step directions on how to complete the dragon tiles.
- Have students work in small groups.
- When the students have finished studying the information on the website, have them go ahead and create their own pattern of dragon tiles.
- Students can work in pairs on this.
- They can either draw in each tile, or create a pattern to trace.
- Either way, the dragon tiling will be completely made of right triangles.

IV. Notes on Assessment

- Assessment can be completed by looking at each student's work product.
- If you built triangle tables, are the measurements of the table accurate?
- Is the table a right triangle?
- If you completed a dragon tiling, is it accurate?
- Does it show right triangles?
- Offer students feedback as needed.

Sine and Cosine Ratios

I. Section Objectives

- Review the different parts of right triangles.
- Identify and use the sine ratio in a right triangle.
- Identify and use the cosine ratio in a right triangle.
- Understand sine and cosine ratios in special right triangles.

II. Cross- curricular-Land Surveying

- Find a local land surveyor and ask him/her to visit the classroom.
- This is an opportunity to have a speaker come and teach the students about how geometry can be applied in real life situations.
- Ask the speaker to be prepared to show the connection between land surveying and geometry.
- Also ask him/her to please bring tools for students to see.
- Students will need to complete some research on land surveying prior to the speakers presentation.
- Have the students prepare five questions each to ask the speaker, and be sure that the students ask questions of the speaker when he/she is there.
- Students can prepare a written report sharing how land surveying is connected to geometry following the presentation.

III. Technology Integration

- Use the internet to research land surveying.
- Be sure that the students understand what land surveyors do, some of the tools used, and how right triangles play a part in land surveying.
- There are several websites that students can visit to do this.
- They will find a lot of information simply by using google or Wikipedia.

IV. Notes on Assessment

- Read each students report on the speaker.

- Assess student knowledge and ability to connect this career with geometry.
- Did the students simply repeat the presentation, or did he/she bring their own thoughts and opinions into the paper?
- Provide students with feedback on their work.

Inverse Trigonometric Ratios

I. <u>Section Objectives</u>

- Identify and use the arctangent ratio in a right triangle.
- Identify and use the arcsine ratio in a right triangle.
- Identify and use the arccosine ratio in a right triangle.
- Understand the general trends of trigonometric ratios.

II. <u>Cross- curricular-Environmental Studies</u>

- Use the following page of information to show students how tangents and arctangents are used in real world applications.
- www.e-education.psu.edu/natureofgeoinfo/c7_p10.html
- This website shows the students that when people are studying the environment and changes in elevation, that they use the measurements of slope to do this.
- Review slope with the students.
- Then you can move on to connecting slope with the tangent ratios.
- These ratios can show how a slope or how elevation is changing over time.
- For example, take beach erosion.
- When the beach or coast is eroding away due to a storm or hurricane, the slope of the land before the storm and after the storm can be compared.
- In the same example, the change in the degrees of the triangle (the arctangent) can be used to compare or demonstrate change as well.
- Have the students think of other types of elevation changes.
- Brainstorm examples and write them on the board.

III. <u>Technology Integration</u>

- Here is a great video showing a word problem and how to figure it out.
- www.video.yahoo.com/watch/3008744/8600194
- Students can watch this video for some extra practice on solving trigonometric word problems.
- Then they can practice writing their own.
- Have the students write an answer key too.
- When finished, collect the word problems for further use.

IV. <u>Notes on Assessment</u>

- Collect student word problems.
- Read them and assess whether or not the students have a good grasp of the material.
- Use the word problems for a quiz or homework assignment.
- Provide students with feedback as needed.

3.8. Right Triangle Trigonometry

Acute and Obtuse Triangles

I. Section Objectives

- Identify and use the Law of Sines.
- Identify and use the Law of Cosines.

II. Cross- curricular-Comic Strip

- Students are going to write a comic strip that tells what happens when someone breaks the Law of Sines.
- Students can make this comical and draw characters to go with it.
- It is a creative assignment, but one that also incorporates mathematical information in it.
- It should be considered a fun assignment, but one that also needs to be accurate.
- Students can work on their comic strip in pairs.
- Allow time for students to work.
- Each strip should have writing and animation with it.
- When finished, allow time for the students to share their work.

III. Technology Integration

- Here is a movie that students can watch about landscape architecture and triangulation.
- www.thefutureschannel.com/hands-on_math/survey_team.php
- After watching the film, conduct a discussion on the film and what students learned about geometry and being a landscape architect.

IV. Notes on Assessment

- Collect each comic strip.
- Read them and assess them on two levels.
- 1. Is the mathematical content accurate?
- 2. Is it presented in a creative way?
- Provide students with feedback/correction as needed.

3.9 Circles

About Circles

I. <u>Section Objectives</u>

- Distinguish between radius, diameter, chord, tangent, and secant of a circle.
- Find relationships between congruent and similar circles.
- Examine inscribed and circumscribed polygons.
- Write the equation of a circle.

II. <u>Cross- curricular-Nature</u>

- Look at the following examples of circles in nature.
- These images are Figure 09.01.01
- www.naturesmightypictures.blogspot.com/2006/06/circles-in-nature.html
- While these don't specifically name all of the parts of a circle, use these images to discover the different parts of a circle.
- Where is the radius or the diameter?
- Is there a polygon inscribed in any of the circles?
- For example, look at the sunflower or the rose.
- Are any of the circles similar?
- For example, look at the patterns in the different images. Do you see any similar circles?
- Have a discussion with the students that broadens their thinking about circles and the parts of a circle.
- Then ask the students to find an example of circles in nature.
- Bring it into class the next day.

III. <u>Technology Integration</u>

- Students are going to be working to make connections between circles and real world activities.
- How are circles used in different careers?
- This first example is a designer who makes wheels.
- This designer makes wheels that are used in performance racing.
- As students watch this video, have them make notes on the different geometric elements that are mentioned in the video.
- Then following the video, conduct a discussion on how geometry and wheel design are related.
- www.thefutureschannel.com/hands-on_math/spoke_math.php

IV. <u>Notes on Assessment</u>

- Assessment is completed through class discussion.
- Work to have all students participate in the discussion.
- Ask questions of the students and provide feedback as needed.

Tangent Lines

I. Section Objectives

- Find the relationship between a radius and a tangent to a circle.
- Find the relationship between two tangents draw from the same point.
- Circumscribe a circle.
- Find equations of concentric circles.

II. Cross- curricular-Design

- Have students look at the following image from Wikipedia.
- This is Figure 09.02.01
- www.en.wikipedia.org/wiki/File:Concentric_(PSF).png
- This is a picture of concentric circles.
- Have students discuss the characteristics of concentric circles.
- Are they similar?
- How can we design a concentric circle?
- Ask the students to create a black and white art design using concentric circles.
- Students will need white paper, black pencil or marker, a compass.
- Have the students work to create their own design.
- Also have them insert one tangent line somewhere in the design.
- Allow time for students to share their work when finished.

III. Technology Integration

- This is a very fascinating website for students to explore.
- www.cut-the-knot.org/Curriculum/Geometry/TangentTwoCirclesI.shtml
- In working with this website, students will be manipulating the center of one of the circles.
- They can click on the center and drag the center anywhere that they wish to.
- When they do this, they will alter the diagram of the two circles and their tangents.
- It is a great visual and very interactive.

IV. Notes on Assessment

- Assess student work with the art design.
- Is the design of the concentric circles accurate?
- Are the circles organized around a common center?
- Is there a tangent in the design?
- Does the student understand what a tangent is based on what he/she has drawn?
- Provide students with feedback as needed.

Common Tangents and Tangent Circles

I. Section Objectives

- Solve problems involving common internal tangents of circles.
- Solve problems involving common external tangents of circles.

- Solve problems involving externally tangent circles.
- Solve problems involving internally tangent circles.
- Common tangent

II. Cross- curricular-Mad Tea Party

- Use the following images from Wikipedia on the Mad Tea Party.
- This is Figure 09.03.01
- www.en.wikipedia.org/wiki/Mad_Tea_Party
- Students can even complete the technology integration first to see some real pictures of the tea cups in action.
- Tell the students that their task is to draw the design of the Mad Tea Party using circles that are connected.
- The design of the Mad Tea Party consists of three small turntables, which rotate counter clock-wise, each holding six teacups, within one large turntable, rotating clockwise.
- Students are to draw this design and how they hypothesize that the circles are or are not connected.
- Do the students think that tangents play a role in this?
- Why or why not?
- Ask the students to write a short paragraph explaining their thinking about the ride.

III. Technology Integration

- Have students complete a websearch for the Mad Tea Party at Disney World.
- Students will see images and can even see a film clip of the ride on youtube.
- Students can use this information to assist them in drawing the design of the ride.

IV. Notes on Assessment

- Assess student work.
- How did the students draw the design of the ride?
- What was the student's hypothesis about tangents?
- Does the reasoning make sense?
- Provide students with comments on their work.

Arc Measures

I. Section Objectives

- Measure central angles and arcs of circles.
- Find relationships between adjacent arcs.
- Find relationships between arcs and chords.

II. Cross-curricular-Plate Design

- Use the image of the dinner plate with the stripes by Cynthia Rowley.
- This is Figure 09.04.01
- www.prontohome.com/product/whim-by-cynthia-rowley-melamine-p_1213285046
- Use this to show the students where there are arcs and chords.
- Then show them major and minor arcs as well.
- The assignment is for the students to design their own plate design using lines, chords and circles.

3.9. Circles

- Tell the students that they are free to design the plate however they would like as long as they label the major arcs and the minor arcs.
- They also need to figure out the measure of one of the arcs and explain how they completed this task.
- Allow time for the students to share their work when finished.

III. Technology Integration

- Have students go to www.brittanica.com the encyclopedia Brittanica's website and search for Eratosthenes of Cyrene.
- Have them research how he used arcs to figure out the circumference of the earth.
- This may be challenging for the students to understand, so you may want to either allow them to work in small groups or to discuss this as a whole class.
- Begin by having them take notes on their own, then begin the discussion.

IV. Notes on Assessment

- Assess each plate design.
- Is it creative?
- Does it use the concepts of chords and arcs?
- Are the major and minor arcs labeled?
- Did the students figure out the measure of one of the arcs?
- Is the work written out and explained?
- Provide students with comments/feedback on their work.

Chords

I. Section Objectives

- Find the lengths of chords in a circle.
- Find the measure of arcs in a circle.

II. Cross- curricular-Archimedes

- Use the image and information at the following website.
- Display this image for the students to see.
- This is Figure 09.05.01
- Now have the students copy this image on a piece of paper.
- In small groups, the students need to use this image to prove that the sum of the intercepted opposite arcs is equal.
- Students can use the text to refer back to the information that they have learned.
- They need to write five statements that demonstrate that this is a true statement.
- Students should be prepared to present their findings.

III. Technology Integration

- Begin with this statement, "It could be said that a spoke is the chord of a wheel."
- Use different wheel designs to demonstrate how this is true or untrue.
- You may use a drawing program to draw and design support for your answer.
- You may also use a collection of images to support your answer.

- Be prepared to share your work when finished.

IV. <u>Notes on Assessment</u>

- When the students present their findings, listen to their reasoning.
- Challenge the others in the class to do the same thing.
- How does it support what we know about perpendicular lines and angles?
- How does it support what we have learned about arcs?
- Does the reasoning of the group make sense or is something missing?
- Is there a diagram to support their thinking?
- Did the students complete any measurements?
- Provide students with feedback on their work.

Inscribed Angles

I. <u>Section Objectives</u>

- Find the measure of inscribed angles and the arcs they intercept.

II. <u>Cross- curricular-Theater</u>

- This is a problem that needs to be solved. It will require the students to use angle measures.
- This is picture of a seating chart for the Fichander Theater.
- This is Figure 09.05.01
- www.gotickets.com/venues/dc/fichandler_theatre.php
- Be sure that each student has a copy of the image.
- Show students how this is a theater in the round.
- The seating is arranged in a circle.
- The students need to use what they have learned about angles and arcs to determine which seats have the best angle to see the stage.
- Note: Students may determine right away that all of the seats are equal due to their angles. Why is this? Have them prove their thinking.

III. <u>Technology Integration</u>

- Students can go to the following website for a worksheet where they can practice finding the measure of inscribed angles.
- This is a great site for simple practice and drill of skills already learned.
- www.regentsprep.org/Regents/math/geometry/GP15/PcirclesN.htm
- Students can also go to any of several websites to find further explanation of inscribed angles and of the measure of those angles.
- Any of these sites will support students in expanding their understanding.

IV. <u>Notes on Assessment</u>

- Walk around and observe students as they work.
- Then have the students share their thinking about the theater problem.
- Be sure students are able to articulate their reasoning by using content from geometry.
- Diagrams are an excellent way for students to share their thinking.

3.9. Circles

Angles of Chords, Secants and Tangents

I. <u>Section Objectives</u>

- Find the measures of angles formed by chords, secants and tangents.

II. <u>Cross- curricular-Poetry</u>

- Students are assigned the task of writing a poem or rap about the theorems in the text.
- Students can choose to write their poem about one of the theorems or all of the theorems.
- Students could also write a poem that defines and explains the relationship between chords, secants and tangents.
- It isn't necessary to give too many directions for this assignment.
- Let the students work in small groups, and they will illustrate their level of understanding of the material through the poem.
- When finished, allow students time to present their work.

III. <u>Technology Integration</u>

- Have students complete this chapter by completing a websearch on circles in architecture.
- They can google this topic.
- Have the students keep track of the sites that they visit.
- They need to select three different images that best illustrates the content of the chapter.
- The students need to write a paragraph explaining how each one illustrates the concepts of the chapter, and which concepts it illustrates.
- Have students share their work when finished.

IV. <u>Notes on Assessment</u>

- Assess student work through their presentations.
- How well does the poem explain the theorem or theorems?
- How well does the poem explain the definitions from the text?
- Are the images that the student selected in line with the content from the chapter?
- Did the student explain which concepts are illustrated in the image?
- Is the information accurate?
- Provide students with feedback on their work.

Segments of Chords, Secants and Tangents

I. <u>Section Objectives</u>

- Find the lengths of segments associated with circles.

II. <u>Cross- curricular- Circus math</u>

- This is a problem having to do with the circus.
- Here is the problem.
- A circus ring has a diameter of 42 feet.

- A high wire is stretched across the diameter of the circle
- A second wire is stretched across the diameter of the circle.
- The two wires intersect at one point.
- On the first wire, the lengths of the wire are ten feet and eight feet.
- On the second wire, only one section of the wire is known and that is five feet.
- What is the length of the second section of the wire?
- Have students work in small groups on this problem.
- It is a great idea to have students draw a diagram of the solution of the problem.
- Solution:
- $10 \times 8 = 5x$
- $80 = 5x$
- $x = 16$ feet
- The diameter of the circle has no impact on the answer of this problem.

III. Technology Integration

- Have students go to the following website to do some research about high wire acts in the circus.
- www.reachoutmichigan.org/funexperiments/agesubject/lessons/newton/hwire.html
- What kind of math is involved in this art?
- Does the diameter of the wire impact the act?
- Have students write a short report on what they have learned about math and the high wire.
- Students can even complete the activity at the end of the web page and experience walking a "high wire" of sorts themselves.

IV. Notes on Assessment

- Check the solution to the problem.
- Did the students use a diagram?
- Is the diagram accurate?
- Were they able to solve the problem accurately?
- Provide students with feedback and comments.

3.10 Perimeter and Area

Triangles and Parallelograms

I. Section Objectives

- Understand the basic concepts of the meaning of area.
- Use formulas to find the area of specific types of polygons.

II. Cross- curricular-Reflecting Pool dimensions

- Use the following image from Wikipedia of the Reflecting Pool in Washington, DC.
- This is Figure 10.01.01
- www.en.wikipedia.org/wiki/File:Reflecting_pool.jpg
- Here is the problem.
- According to Wikipedia, the dimensions of the Reflecting Pool are 2029 ft long and 167 feet wide.
- Given this information, what shape is the Reflecting Pool?
- What is the perimeter of the pool?
- What is the area of the pool?
- Draw a diagram to explain your answer.
- Solution:
- The shape is a rectangle.
- The perimeter is $2029 + 2029 + 167 + 167 = 4392$ ft.
- The area is $2029 \times 167 = 338,843$ sq. ft.

III. Technology Integration

- There are two great short videos on this website for area and length.
- One is an architect and one is on an apartment design.
- www.thefutureschannel.com/hands-on_math/apartment.php
- Have students watch the videos.
- Then you can expand on this by having the students draw a design of their room at home.
- Students will need to go home and do some measurements and then come back with the area and perimeter of their room.
- Rooms with unconventional shapes will be the most fun and challenge.
- Allow time for students to share their work.

IV. Notes on Assessment

- Assess student work on the Reflecting Pool problem.
- Is the diagram accurate?
- Did the students calculate the area correctly?
- Did the students calculate the perimeter correctly?
- Provide students with feedback on their work.

Trapezoids, Rhombi and Kites

I. Section Objectives

- Understand the relationships between the areas of two categories of quadrilaterals: basic quadrilaterals and special quadrilaterals.
- Derive area formulas for trapezoids, rhombi and kites.
- Apply the area formula for these special quadrilaterals.

II. Cross- curricular-Room Design

- Tell students that they are going to design a room that has a trapezoidal shape.
- Students can complete this in connection with the Technology Integration if you choose.
- If not, have the students use the dimensions of their own bedroom (they did this in the last lesson), or the classroom or a standard size bedroom (11×10) for example.
- Students are going to redesign this area as a trapezoid.
- They want to come as close to the original area as possible.
- So if the room was 11×10, the area is 110 sq feet.
- How can you come close to the same area if the shape of the room is a trapezoid?
- Students should draw their design on grid paper and explain their thinking.
- Allow time for students to share their work when finished.

III. Technology Integration

- This is a website that shows a house designed as a trapezoid.
- www.momoy.com/2009/04/02/l-house-beautiful-trapezoid-house-design-by-philippe-steubi-architekten-gmbh/
- Students can look at the trapezoid shape of the house and the floor plan is also included.
- There are views of the inside of the house and the outside of the house as well as some of the rooms.
- Conduct a discussion about the house. What would be the challenges of designing and building such a house?

IV. Notes on Assessment

- Assessment will come with student presentations and work product.
- What did students learn about the relationship between rectangles and trapezoids?
- Were they able to come up with a room with an area close to the original?
- Who got the closest?
- Provide students with feedback on their work.

Area of Similar Polygons

I. Section Objectives

- Understand the relationship between the scale factor of similar polygons and their areas.
- Apply scale factors to solve problems about areas of similar polygons.
- Use scale models or scale drawings.

II. Cross- curricular-National Mall Mapping

3.10. Perimeter and Area

- Ask students to use the Wikipedia image of the National Mall to create a map of it.
- This is Figure 10.03.01
- www.en.wikipedia.org/wiki/National_Mall
- Then tell the students that the mall is 1.9 miles $\times$ 1.2 miles.
- They are going to use what they have learned about scale and measurement to create their own map of the mall.
- They need to choose a scale to work with.
- Then they use grid paper to design the mall.
- When students have the area of the mall correct, they can draw in as many different museums and monuments as they can.
- Extra details add extra credit to their work.
- When finished, allow time for students to share their work.

III. Technology Integration

- Use the following website on the National Mall in Washington DC.
- www.en.wikipedia.org/wiki/National_Mall
- Have students complete some research about the mall.
- Possible questions include:
- Who designed it?
- When was it built?
- What is at the North end?
- What is at the South end?
- How many different museums can you visit there?
- Have you been to the mall?
- Which museum would you most like to visit or did you enjoy and why?

IV. Notes on Assessment

- Assess each student map.
- Is the use of scale done correctly?
- Are the measurements correct?
- Is the map accurate?
- Has the student take the time to add in details?
- Provide students with feedback on their work.

Circumference and Arc Length

I. Section Objectives

- Understand the basic idea of a limit.
- Calculate the circumference of a circle.
- Calculate the length of an arc of a circle.

II. Cross- curricular-The Pantheon

- Have students use the image of the floor plan of the rotunda of the Pantheon to calculate the circumference of it.
- This is Figure 10.04.01

- www.en.wikipedia.org/wiki/Pantheon,_Rome
- The diameter of the dome is 142 ft.
- Given this measurement, what is the circumference?
- Have the students draw a diagram to explain their work.
- Allow time for students to share their diagrams in small groups.

III. Technology Integration

- Have students use the following Wikipedia site to research information on the Pantheon.
- www.en.wikipedia.org/wiki/Pantheon,_Rome
- Students can use this information to write a short essay.
- Students should hunt for mathematical information about the Pantheon for their essay.
- For example, height of the columns.
- What is a portico?
- What is a rotunda?
- Have the students complete this work and then collect it for your review.
- Extension on initial exercise- have students research the dimensions of the rectangle that connect the portico and the rotunda.
- What is the area of the rectangle?
- What is the perimeter?

IV. Notes on Assessment

- Look at student work.
- Is it accurate?
- Does the diagram represent student work?
- Provide students with feedback on their work.

Circles and Sectors

I. Section Objectives

- Calculate the area of a circle.
- Calculate the area of a sector.
- Expand understanding of the limit concept.

II. Cross- curricular-History

- Use the following image from the round table used by King Arthur.
- This is Figure 10.05.01
- www.crystalinks.com/roundtable.gif
- The diameter of the round table was 18 feet.
- Given this measurement, calculate the area of the round table.
- If the table was divided between each of the knights evenly, what is the area of one of the sectors?
- Draw a diagram to explain your work.
- Allow students time to share their diagrams when finished.

III. Technology Integration

- Have students use the following website as a tutorial on area and circumference of circles.

3.10. Perimeter and Area

- www.mathgoodies.com/lessons/vol2/circle_area.html
- Students can review already learned material.
- There is also a worksheet section for them to work with and practice solving problems.

IV. Notes on Assessment

- Examine student diagrams.
- Were they able to find the correct area of the table?
- How about the sectors?
- Does the diagram accurately show their work?
- Is there anything missing?
- Provide students with feedback/correction on their work.

Regular Polygons

I. Section Objectives

- Recognize and use the terms involved in developing formulas for regular polygons.
- Calculate the area and perimeter of a regular polygon.
- Relate area and perimeter formulas for regular polygons to the limit process in prior lessons.

II. Cross- curricular-Architecture

- Use the following image of a roof in the shape of a hexagon.
- This is Figure 10.06.01
- www.space-frames.com/commercial buildings/xha28.htm
- Have the students use the dimensions of this design to figure out the area of the roof of this hexagon.
- Then have the students draw a diagram and explain how they figured out the area of the hexagon.
- Allow time for students to share their work when finished.

III. Technology Integration

- Have students complete some research on where to find hexagons and pentagons.
- Students can search architecture, nature or their own subject.
- Ask the students to keep track of the websites that they visit.
- Students should prepare a presentation of at least five examples of pentagons or hexagons in their given subject area.
- Students should include diagrams or images with their work.

IV. Notes on Assessment

- Assess student diagrams.
- How did the students figure out the area of the roof?
- Does their method make sense?
- Did they divide it into triangles?
- Did they divide it into trapezoids?
- Provide students with feedback on their work.

Geometric Probability

I. <u>Section Objectives</u>

- Identify favorable outcomes and total outcomes.
- Express geometric situations in probability terms.
- Interpret probabilities in terms of lengths and areas.

II. <u>Cross- curricular-Target Practice</u>

- Use the following image of a dartboard.
- This is Figure 10.07.01.
- www.home.wlu.edu/ mcraea/GeometricProbabilityFolder/Introduction/Problem0/images/images/dartboard.gif
- Here is the problem.
- What is the geometric probability of hitting the center of the internal square of the dartboard?
- Use probability to figure this out.
- Show your work with a diagram and be prepared to explain your answer.
- Allow time for students to share their work when finished.

III. <u>Technology Integration</u>

- Visit the same website that the image came from and explore the solution to the problem.
- www.home.wlu.edu/ mcraea/GeometricProbabilityFolder/Introduction/Problem0/images/images/dartboard.gif
- The answer to the problem that the student solved above is there.
- Have students use this to correct their own work.
- Show any changes/corrections that they completed.
- Then explore the other problems on the site.

IV. <u>Notes on Assessment</u>

- Because students are going to correct their own work during the technology integration, use this as a time to assess student work through observation.
- Are the students able to apply the concepts of probability to geometry?
- Refer students back to the text if they are having difficulty.

3.11 Surface Area and Volume

The Polyhedron

I. Section Objectives

- Identify polyhedral.
- Understand the properties of polyhedral.
- Use Euler's formula to solve problems.
- Identify regular (Platonic) polyhedral.

II. Cross- curricular-Cubic Houses

- Use the following image from the Wikipedia website.
- This is Figure 11.01.01
- www.en.wikipedia.org/wiki/File:Rotterdam_Cube_House.jpg
- Use the image of the cubic houses to conduct a discussion with the students about the different parts of the polyhedron.
- Students should be able to identify some of the edges, the vertices and the faces of the cubes.
- Have the students identify what is unique about these houses.
- Brainstorm a list and write them on the board.
- When finished, have the students move on to drawing their own design of a cubic house.
- Have them label the faces, edges and vertices of their house design.

III. Technology Integration

- Have students select one specific polyhedron to research.
- Then have them research this polyhedron as it is connected to a theme such as photography, architecture or nature.
- Ask the students to keep track of the websites that they visit.
- Have the students take notes on where and how they discover their specific solid.
- Then have them share their findings in small groups.

IV. Notes on Assessment

- Assess student work through their house design.
- Did the students identify the faces, edges and vertices?
- Were the students creative in their design?
- Also look at the technology integration section.
- Did the students find examples of their solid according to theme?
- Were any of the results surprising?
- Assess student understanding through their sharing.

Representing Solids

I. <u>Section Objectives</u>

- Identify isometric, orthographic, cross- sectional views of solids.
- Draw isometric, orthographic, cross- sectional views of solids.
- Identify, draw and construct nets for solids.

II. <u>Cross- curricular-Designs of Polyhedrons</u>

- This is also the <u>Technology Integration</u> section because this activity depends on the technology.
- Use the following website for nets of polyhedrons. This website also contains different patterns for many different polyhedrons.
- www.korthalsaltes.com/
- The patterns can be downloaded in pdf form and printed.
- Students will require access to a computer and printer.
- Have each student select two different polyhedrons to work with.
- After printing out the pattern, have the student create/build a model of each solid.
- Then each student is to draw orthographic, cross- sectional views of each solid.
- Students need to be sure that their work is complete and that the solid is correctly represented.
- Then have the students create their own model of a third solid.
- For this one, they can't use the already created patterns.
- They must create their own model using what they have already learned.
- When finished, allow time for students to share their work.

IV. <u>Notes on Assessment</u>

- Begin by observing students as they work.
- Do the students understand the difference between the different views of the solid?
- Did the students successfully create each model?
- Is the orthographic, cross- sectional view of each solid accurate?
- Provide students with feedback on their work.

Prisms

I. <u>Section Objectives</u>

- Use nets to represent prisms.
- Find the surface area of a prism.
- Find the volume of a prism.

II. <u>Cross- curricular-Prism Collage</u>

- Have students use magazines to find pictures of different prisms.
- For example, a triangular prism could be piece of pie, a rectangular prism could be a box, etc.
- Students will need scissors, magazines of all kinds, glue and large posterboard.
- Have students identify the prisms in their collage.
- When finished, allow time for the students to share their work.

3.11. Surface Area and Volume

III. Technology Integration

- Use the following image of a deck prism
- This is Figure 10.03.01
- www.defender.com/expanded.jsp?path=-1|74081|316411&id=86235
- Have the students go to Wikipedia and research what a deck prism was and what it was used for.
- Then have the students write a short paragraph explaining the purpose of a deck prism.
- After the students have finished this, have them draw a diagram of a ship with a deck prism to illustrate where the deck prism would have been placed and the function that it would serve.
- When finished, allow students time to share their work.

IV. Notes on Assessment

- Assess the student collages.
- Are the pictures in the collage all prisms?
- Did the students label the different prisms?
- Assess student work on the deck prism and the ship.
- Do the students understand the purpose of the prism?
- Is it drawn correctly on the ship?
- Provide students with comments/feedback on their work.

Cylinders

I. Section Objectives

- Find the surface area of cylinders.
- Find the volume of cylinders.
- Find the volume of composite three- dimensional figures.

II. Cross- curricular-Cheese Press

- Use an image from the technology section to have a picture to work with for the following problem.
- Here is the problem.
- Given the following dimensions, figure out the surface area and the volume of the cylinder of the cheese press.
- 8.5 in high
- Diameter of 6 inches
- Base Area of 72 sq. inches
- Draw a diagram to explain your work on both parts of the problem.
- Be sure to show your work.
- Allow time for students to share their work when finished.

III. Technology Integration

- Use the following website for information on a cheese press that makes cheese.
- www.thegrape.net/browse.cfm/4,10188.html
- This will give you an image to work with for the first activity.
- Then research the cheese press.
- How does it work?
- When was it first used?

- What are the necessary ingredients for making cheese?
- How long does it take?
- Write a short essay on the cheese press to accompany your mathematical work.
- Students can also go to this website and watch a video about how volume impacts space flight.
- www.thefutureschannel.com/dockets/hands-on_math/orion_space_capsule/

IV. Notes on Assessment

- Assess student work and diagrams.
- Is it accurate?
- Were the students able to figure out the surface area of the cylinder?
- Were the students able to figure out the volume of the cylinder?
- Provide students with feedback/comments on their work.

Pyramids

I. Section Objectives

- Identify pyramids.
- Find the surface area of a pyramid using a net or formula.
- Find the volume of a pyramid.

II. Cross- curricular-Pyramids

- Begin by using this image to show students an aerial view of three pyramids.
- This is Figure 11.05.01.
- www.alienworld.files.wordpress.com/2008/09/pyramids.jpg
- Looking at this image will also give students a great idea of what a net of a pyramid can look like.
- Students are going to be working on drawing nets of different sized pyramids.
- Assign them the task of drawing three nets for three different sized pyramids.
- They can choose which type of pyramids they wish to draw too.
- Have the students label each net with the type of pyramid and be sure that the pyramids are proportional.
- Allow time for students to share their work when finished.

III. Technology Integration

- This is a great website to explore the volume of a pyramid.
- www.mathsisfun.com/geometry/pyramids.html
- It is very simple and basic in its approach.
- It would be excellent for a student who is having difficulty with the concepts or who just needs more practice.

IV. Notes on Assessment

- Assess the nets of the pyramids.
- Are the pyramids labeled to show the type of pyramid that they are?
- Are the pyramids accurately drawn?
- Are they proportional?
- Provide students with comments/feedback on their work.

3.11. Surface Area and Volume

Cones

I. <u>Section Objectives</u>

- Find the surface area of a cone using a net or formula.
- Find the volume of a cone.

II. <u>Cross- curricular-Sculpture</u>

- Students are going to design their own sculpture using different cones.
- These can be cones that they create out of paper, or cones from nature such as a pine cone.
- Begin by conducting a discussion about cones.
- Be sure to review the parts of a cone.
- Tell students that they need to present work to show the surface area and volume of one of the cones that they create.
- Then let them work on their sculpture.
- Students will need paper, markers, colored paper, a surface to build on and glue.
- When finished, allow time for students to share their work.

III. <u>Technology Integration</u>

- Students can go to the following website and look at many different images of cones in architecture.
- www.fiveprime.org/hivemind/Tags/architecture,cone
- They need to select one cone to work with.
- Cones of specific buildings are named. Students can use this information to research about the specific cone.
- Students should write a short essay on their cone and draw a design to represent the cone that they have chosen.
- Allow time for students to share their work when finished.

IV. <u>Notes on Assessment</u>

- Assess student work.
- Is the work creative?
- Are the cones accurate?
- Is there anything that the student needs to improve upon?
- Provide students with feedback/comments.

Spheres

I. <u>Section Objectives</u>

- Find the surface area of a sphere.
- Find the volume of a sphere.

II. <u>Cross- curricular-Spheres for Space</u>

- Begin with the article on MIT and space spheres.
- Print the article and either have the students read it silently or read it as a whole class.
- Tell the students that they are going to be designing spheres for this space project.

- The spheres need to be the same size as a volleyball.
- You will need some volleyballs and tape measures for this class.
- Have students measure each model and then build a model of their space sphere.
- Students should create a 3D model and draw a design of their space sphere as well.
- Have students work in groups of three.
- Allow time for students to share their work when finished.

III. Technology Integration

- Begin by having students look at this website which looks at MIT students who are designing spheres to go into space.
- www.spacedaily.com/news/microsat-00e.html
- This is a great video which show architecture and space together.
- The engineers in the video work with spheres and also explain how area and volume impact the design of anything that is sent into space.
- www.thefutureschannel.com/dockets/hands-on_math/space_architecture/
- This is a great place to begin a discussion with students about careers that use mathematics.
- Here is another fun website that looks at spheres.
- www.cotf.edu/ete/modules/msese/earthsysflr/spheres.html

IV. Notes on Assessment

- Assess each work product.
- Were the directions followed?
- Did students accomplish the objective?
- Provide students with feedback on their work.

Similar Solids

I. Section Objectives

- Find the volumes of solids with bases of equal areas.

II. Cross- curricular-Kaleidocycles

- For this activity, you will need to use the information at the following website.
- www.mathematische-basteleien.de/kaleidocycles.htm
- This website provides pictures and directions of how to make different kaleidocycles.
- It is a great way for students to see how similar solids can be combines together.
- Some of the solids are congruent and some are similar.
- Become familiar with some of the patterns and designs before assigning this to the students.
- Then give them the instructions, by printing or providing technology and let them go to work.
- Students need to select at least two different kaleidocycles to create.
- They can also design their different ones.
- Allow time for students to present their work when finished.

III. Technology Integration

- The activity above integrates technology into the making of kaleidocycles.

3.11. Surface Area and Volume

- www.maa.org/mathland/mathtrek_11_13_06.html
- Have students explore the website on the math trek.
- Discuss the different aspects of the trek.
- You could even take your students on a math trek around the school or town.
- Have the students make notes of all of the different places where mathematics/geometry can be found.

IV. Notes on Assessment

- For this activity, assessment can be completed through observation.
- Be sure to interact with students as they work.
- Offer assistance where needed.

3.12 Transformations

Translations

I. <u>Section Objectives</u>

- Graph a translation in a coordinate plane.
- Recognize that a translation is an isometry.
- Use vectors to represent a translation.

II. <u>Cross- curricular-Sculpture</u>

- Use this image of Rinus Roelof's tetrahedron sculpture.
- This is Figure 12.01.01
- www.mathpaint.blogspot.com/2007/04/structures-by-rinus-roelofs.html
- Use this to show the students the vectors that can be drawn from one tetrahedron to the next tetrahedron.
- This shows length and direction.
- Discuss the various components of the sculpture.
- Ask students to identify all of the different elements of the sculpture.
- Students can then draw a design of their own using different three- dimensional solids or one solid as Roelof did.
- Have students identify any and all solids as well as the vectors in the design.
- Allow time for the students to share their designs.

III. <u>Technology Integration</u>

- This is a great website to explore translations.
- www.cut-the-knot.org/Curriculum/Geometry/Translation.shtml
- Students can read all about vectors and isometry.
- Then there is an interactive section where the students can manipulate the figure in the box.
- Students can use this to demonstrate their understanding.
- Have students work in pairs on this task.

IV. <u>Notes on Assessment</u>

- Assess student understanding through discussion.
- Ask questions to be sure that the students understand the key elements of this lesson.
- They should have an understanding of isometry, vectors and translations.

Matrices

I. <u>Section Objectives</u>

- Use the language of matrices.
- Add matrices.
- Apply matrices to translations.

II. Cross- curricular-Progressive Matrices

- This website has an image of a progressive matrix.
- www.en.wikipedia.org/wiki/File:RavenMatrix.gif
- Have the students discuss the elements of how this image is representative of a matrix.
- Then have them use this as a model to create their own matrix pattern.
- Students should complete at least three steps of the progressive matrix.
- Students can use black and white or color.
- Students could expand this idea into a design with dimensions.
- Students can also use paint chips from a hardware store, or small mosaic tiles.
- Students could use elements of nature such as rocks or small leaves.
- This could be a very creative assignment.
- Allow time for the students to share their work when finished.

III. Technology Integration

- Complete a research assignment on how the banking industry uses matrices.
- Students will need to visit several different websites to do this.
- Have them write a short essay and include examples on how the banking industry relies on matrices to support their work.

IV. Notes on Assessment

- Assess each design.
- Is it modeled off of the example on Wikipedia?
- Does it show a progression?
- What would be the next step in the progression?
- Provide students with feedback on their work.

Reflections

I. Section Objectives

- Find the reflection of a point in a line on a coordinate plane.
- Multiple matrices.
- Apply matrix multiplication to reflections.
- Verify that a reflection is an isometry.

II. Cross- curricular-Art

- Students are going to create a reflection.
- They can choose a picture, a symbol, a shape or an image.
- The key thing is that they can reproduce it as a reflection.
- The students are going to show how this image is reflected in a horizontal or vertical plane.
- They can work in small groups on this.

- The task will involve spatial thinking and organization to be sure that the students can "see" the correct positioning of the image.
- Then they need to reproduce this.
- Students can choose to use as simple or as complicated an image as they choose.
- The key is that they need to be able to explain their work and have it be accurate.
- Allow time for students to share their work when finished.

III. Technology Integration

- Have students use the following website to work on reflections.
- www.mathwarehouse.com/transformations/compositions/reflections-in-math.php
- The website provides a tutorial on how to create a reflection.
- It also provides students with an interactive way to work on reflections.
- Students can practice designing reflections.
- Provide an opportunity for students to ask questions as they work.

IV. Notes on Assessment

- Check student work on reflections.
- Is the reflection accurate?
- Is there anything missing in its representation?
- Is the image too complicated?
- Provide students with feedback on their work.

Rotations

I. Section Objectives

- Find the image of a point in a rotation in a coordinate plane.
- Recognize that a rotation is an isometry.
- Apply matrix multiplication to rotations.

II. Cross- curricular- Sports

- Provide students with three or four copies of this image of a skateboarder.
- www.en.wikipedia.org/wiki/File:Skateboarder1.jpg
- This is Figure 12.04.01.
- Tell the students that they are to use these images to create a scene showing the rotations of a skateboarder.
- Students can create this any way that they choose.
- Ask for students who are knowledgeable about skateboarding.
- Pair these students up with students who don't consider themselves knowledgeable.
- Then have the students work together to create the scenes.
- Students can show as many different rotations as they would like.
- Be sure to give students an opportunity to share their work.
- Some students may want to extend this scene to include other skateboarding images- that is fine as long as the concept of rotations is included.

III. Technology Integration

- A great website to explore rotations.

3.12. Transformations

- www.cut-the-knot.org/Curriculum/Geometry/Rotation.shtml
- Students can review information on rotations here.
- Then they can work to manipulate and create different rotations.
- There are directions on the screen which help them in accomplishing this task.
- You can use this as extra practice or for a student who needs remedial work in this area.

IV. Notes on Assessment

- Assess the student rotation scenes.
- Did the students accomplish the task of showing the skateboarder in different rotations?
- If not, what would have worked better?
- Did the students expand on the assignment?
- Were the students able to explain the use of rotations in their scene?
- Provide students with feedback on their work.

Composition

I. Section Objectives

- Understand the meaning of composition.
- Plot the image of a point in a composite transformation.
- Describe the effect of a composition on a point or polygon.
- Supply a single transformation that is equivalent to a composite of two transformations.

II. Cross- curricular-Movie Posters

- Have students watch the video first.
- Discuss the elements of a great movie poster.
- What works and what doesn't work.
- Tell the students that their job is to create a movie poster for a new movie.
- You can use one that is popular with the students at this time or use an old favorite like "Star Wars" that probably all of the students have seen.
- Tell the students that they are going to create a poster for this movie using the elements of transformations.
- There needs to be a use of rotation, translation and reflection in their posters.
- Students can work in pairs on this task.
- Have students share their work when finished.

III. Technology Integration

- If possible, have the students watch this short video first.
- www.thefutureschannel.com/dockets/hands-on_math/movie_posters/
- You want the students to be looking for elements or transformations in the posters.
- Students are going to use this information in the activity.

IV. Notes on Assessment

- Assess student work based on the student's use of transformations.
- Are there rotations in the poster?
- Are there translations in the poster?

- Are there reflections in the poster?
- Did the student focus on one or all of the elements?
- How successful were they?
- Provide students with feedback on their work.

Tessellations

I. <u>Section Objectives</u>

- Understand the meaning of tessellation.
- Determine whether or not a given shape will tessellate.
- Identify the regular polygons that will tessellate.
- Draw your own tessellation.

II. <u>Cross- curricular-Honeycombs</u>

- Show students the following images of honey combs.
- This is Figure 12.06.01.
- www.en.wikipedia.org/wiki/File:Cubic_honeycomb.png
- This is Figure 12.06.02
- www.en.wikipedia.org/wiki/File:Honey_comb.jpg
- This is Figure 12.07.03
- www.en.wikipedia.org/wiki/File:Apis_florea_nest_closeup2.jpg
- Tell students that their task is to create a honeycomb piece of art.
- They can use any shape that will tessellate as they saw with the cubic honeycomb.
- The key is that the honeycomb, according to a Wikipedia definition, is a space *filling* or *close packing* of polyhedral or higher-dimensional *cells,* so that there are no gaps.
- Students can use any size that they choose and can incorporate color too.
- They will need rulers, pencils, colored pencils or markers, paper and scissors.

III. <u>Technology Integration</u>

- Have students study the work of M.C. Escher who was famous for his tessellations.
- www.en.wikipedia.org/wiki/M._C._Escher
- They can begin with the Wikipedia site, but there are so many other sites to work with as well.
- Have the students select one piece of his work as a favorite piece and share in small groups the elements that tessellate and how they tessellate.
- Conduct a small group discussion on the power of tessellations.

IV. <u>Notes on Assessment</u>

- Assess student honeycombs.
- Were they successful in their tessellations?
- Provide students with feedback on their work.

Symmetry

I. <u>Section Objectives</u>

3.12. Transformations

- Understand the meaning of symmetry.
- Determine all the symmetries for a given plane figure.
- Draw or complete a figure with a given symmetry.
- Identify planes of symmetry for three- dimensional figures.

II. Cross- curricular-Puzzle Creation

- Review the basics of symmetry with the students.
- Have them define symmetry and describe what makes something symmetrical.
- Review symmetry in nature or in other objects or buildings.
- Then assign students the task of taking a symmetrical image and making it into a puzzle.
- Students can use an image from a magazine, a computer image, or a hand drawn image.
- They are going to use cardboard to create a puzzle.
- They can make it as simple or complex as they wish.
- Have students create their puzzle and then exchange puzzles with a peer and work to assemble the other person's puzzle.
- Allow time for students to share their work when finished.

III. Technology Integration

- Explore the sculptures of Quark Park on the following website.
- www.symmetrymagazine.org/cms/?pid=1000396
- Have the students work to discuss each different sculpture in small groups.
- Students should make notes on the symmetrical elements of each sculpture and be prepared to share them with the class.

IV. Notes on Assessment

- Assess student work through observation.
- Observe students as they create their puzzles.
- Inquire into how symmetry can assist someone in creating or putting together a puzzle.
- Then listen in as students discuss the symmetrical elements of the sculpture in Quark Park.

Dilations

I. Section Objectives

- Use the language of dilations.
- Calculate and apply scalar products.
- Use scalar products to represent dilations.

II. Cross- curricular-Dilations in context

- Ask the students to think about the concept of dilations and to come up with one career where people would use dilation in their work.
- Students need to write a hypothesis on how they think that this profession would use dilations.
- Have them write down their hypothesis.
- Ask students to make a list of questions that they are going to explore.
- If you have use of technology, complete this with the use of the computer.

- If not, visit the school library so that students can research there.

III. Technology Integration

- Have students research their chosen profession.
- They need to prove that their hypothesis is true or not.
- Each student should have reasons and explanations on how dilations are used in the chosen profession.
- Students need to write a short essay and provide one diagram or image to support their findings.
- Ask students to keep track of websites that they visit for documentation purposes.
- Allow time for students to present their work when finished.

IV. Notes on Assessment

- Assess student work.
- Was the student able to prove their hypothesis?
- What corrections did he/she make?
- Is the essay well written?
- Does it explain how this profession uses dilations?
- Does the diagram support student research?
- Provide students with feedback on their work.

3.12. Transformations

CHAPTER 4 Geometry TE - Differentiated Instruction

Chapter Outline

Geometry- Teacher's Edition- Differentiated Instruction

Introduction

Differentiated Instruction is a term that teachers often hear regarding instructing and successfully educating students. In the context of this teacher's edition, differentiated instruction refers to instruction that does not simply present material, but works to engage every student through multiple methods. As educators, we know that students all learn in different ways whether it is visually or kinesthetically or interpersonally. Today, there are also many students who come into our classrooms with learning challenges that are not addressed by traditional teaching methods. Differentiated instruction is about reaching each one of these different learners. In our classrooms, students can have multiple options for learning new ideas and concepts, while teachers can utilize many methods of presenting these ideas to the students. Differentiated instruction can assist all students in using their gifts and talents to learn, and as educators, we can assist students in developing and discovering these gifts and talents. By providing students with things like flexible grouping, projects and interactive activities, our students will become the active explorers of the information presented. This flexbook is designed to be a guide for you, the educator to assist you in differentiating the material in this geometry text. There are many suggestions and ideas in this flexbook, not all of them are meant to be incorporated into every lesson. Select those things that work best for you and your students so that the world of mathematics can really come to life.

4.1 Basics of Geometry

Points, Lines and Planes

I. <u>Section Objectives</u>

- Understand the undefined terms *point, line and plane*.
- Understand defined terms, including *space, segment and ray*.
- Identify and apply basic postulates of points, lines and planes.
- Draw and label terms in a diagram

II. <u>Multiple Intelligences</u>

This section is designed to assist educators in differentiating instruction with the multiple intelligences in mind.

- Visual Learners- one way to assist visual learners with this lesson is to use the actual objects mentioned in the lesson. Where there is a map or a globe mentioned, use an actual map and a globe. This will also assist students with special needs in making a connection with the material.
- Kinesthetic Learners- allow move time so that students can walk around the classroom identifying points, lines and planes in their surroundings. Request that students make a list of the things that they find.
- Interpersonal Learners- have students work in pairs or small groups to discuss their findings from the "walk around" activity. This engages students who need to talk about their work to gain a better understanding of a concept.

III. <u>Special Needs/Modifications</u>

This section is designed to assist with any modifications or to assist students who face learning challenges.

- Be sure that all of the vocabulary words are written on a board or overhead as they are presented and discussed. Request that students copy this information into a notebook. Reading the terms, hearing them discussed, seeing them written again and writing the words themselves assists students in retaining information.
- Write each postulate on the board as it is discussed.
- Example 7- Expand this for all learners.

The goal here is to assist students in grasping and learning each term/postulate and its definition. The more students interact with each term and concept, the more they will remember what has been taught.

- Draw an example of each vocabulary word. Example, draw three collinear points.
- Draw an example that illustrates each postulate.
- Allow students to have an interpersonal connection by discussing their drawings with a peer.

IV. <u>Alternative Assessment</u>

There are many ways to assess student understanding during a lesson. This section provides a few ideas for that.

- Walk around and observe students as they work. Are students on task? Are they working diligently? Is the conversation appropriate to what is being taught?

- Use peers to assess each other. With the activity in Example 7, have the students assess the accuracy of each other's work and correct any inconsistencies. If time allows, you could even have a presentation part where students share their findings.

Segments and Distance

I. Section Objectives

- Measure distances using different tools
- Understand and apply the ruler postulate to measurement
- Understand and apply the segment addition postulate to measurement
- Use endpoints to identify distances on a coordinate grid

II. Multiple Intelligences

- Activity with Example 3- This activity will address the following intelligences- visual, interpersonal, kinesthetic, logical- mathematical.

The students work in pairs. One member in the pair draws a line segment that he or she has measured to find the distance. The other member also draws a line segment that he/she has measured. Then they switch drawings. Student A must figure out the length of student B's line segment, and student B must figure out the length of student A's line segment.

- Extension with postulates. In this lesson, there are two key postulates. One is the Ruler Postulate and the other is the Segment Addition Postulate. The students then take their work and determine which line segments are examples of the postulates. They can even exchange with another group to accomplish this.

This extension includes visual, interpersonal, kinesthetic, logical- mathematical intelligences.

- Discussion of activity- by having the students share their examples, answers and reasoning with the entire class or in small groups, the intrapersonal intelligence is included as students share their personal insights into their work.
- Activity with Example 5 and 6- hand out grid paper. Ask students to draw a coordinate plane and provide given distances on the board/overhead. Then allow the student time to draw a line segment with this distance. Then provide a time for sharing/feedback from the exercise.

III. Special Needs/Modifications

- Begin class with a brief review of previously learned material. This can be done with words on the board/overhead or as a class discussion.
- Review what distance means and what it means to estimate.
- Write all vocabulary on the board as it is brought up in the lesson. Request that students take the time to copy this information in their notebooks.
- Ruler Postulate defined on board.
- Segment Addition Postulate defined on board.
- Be sure that students are given plenty of time to think through their work and be sure that all students have finished examples before going over the answers. Sometimes, special needs students require more time to complete tasks and will stop working if the answers to a particular question are given before they have finished.

IV. Alternative Assessment

- Observe students as they work in groups. Notice which students need assistance or seem lost. Make a note of who each student is and set aside a time to check in with each of these students.
- Create an observation checklist of things to watch for when students are completing exercises in a group.
- Pay close attention to student thinking during discussions before and after an activity.

Rays and Angles

I. Section Objectives

- Understand and identify rays.
- Understand and classify angles.
- Understand and apply the protractor postulate.
- Understand and apply the angle addition postulate.

II. Multiple Intelligences

- Activity with identifying rays and angles.

Have students work in small groups. Assign one group rays and the other group angles. Using rulers, the students need to design a series of either rays or angles. You can use index cards for this activity. Then have the groups switch cards. The angle group needs to name all of the rays that the other group has drawn. The ray group needs to name all of the angles that the angle group has drawn. Then the groups exchange answers and check each other's work. This involves discussion and peer tutoring as well.

Addresses the following intelligences

- – Linguistic- students discuss their answers and thinking
  – Logical- mathematical- students draw their angles and rays
  – Spatial- visual- students draw angles and rays
  – Interpersonal- students share their thinking in a group
  – Intrapersonal- students explain their answers in a group
- Activity with Protractors

Provide students with drawings of several different angles. You can use the angles that were drawn in the previous activity. Have students measure their angles using protractors. Then have the students all share in a class discussion.

III. Special Needs/Modifications

- Begin each lesson with a review of previously learned vocabulary words and information. This helps students to recall what they have learned in a previous lesson. It also decreases the number of confused students once an assignment has been given.
- Write all vocabulary on the board or overhead. Request that students write these terms in their notebooks.
- Vocabulary for this lesson
  - – Ray- include symbol notation and an example
  - – Angle- include symbol notation and a diagram with sides and vertex labeled.
  - – Right angle- include drawing
  - – Perpendicular- include symbol
  - – Acute angle- include drawing
  - – Obtuse angle- include drawing
  - – Straight angle- include drawing

4.1. Basics of Geometry

– Protractor Postulate
– Angle Addition Postulate

IV. Alternative Assessment

- Use an observation checklist to observe students as they work.
- Pay attention to the questions asked during the lesson.
- Make a note of students who are having difficulty. Consider flexible grouping to assist these students in their work.

Segments and Angles

I. Section Objectives

- Understand and identify congruent line segments
- Identify the midpoint of line segments
- Identify the bisector of a line segment
- Understand and identify congruent angles
- Understand and apply the Angle Bisector Postulate

II. Multiple Intelligences

- We can differentiate this lesson by organizing the content into a table. This is done as part of a class discussion. It is not done ahead of time and then presented. Creating the chart is meant to be interactive. Since this lesson works with line segments and angles, we can use these as the two columns of our table. Here is a sample of a table and how to organize it for the students.

TABLE 4.1:

Line segment	Angles
Congruent (show example)	Congruent (show example)
Segment midpoint	Show vertex and sides
Show symbols	Show symbols
Segment midpoint postulate	Angle bisector postulate

- Be sure to explain each concept and how they are different and similar depending on whether you are working with line segments or angles.
- This helps the students to see the connections between the concepts.

II. Multiple Intelligences : Linguistic, logical- mathematical, spatial- visual, interpersonal, intrapersonal

III. Special Needs/Modifications

- Review previously learned information. One way to do this is with students working in pairs to quiz each other.
- Write all vocabulary on the board/overhead. Request that students copy this information in their notebooks.
- Vocabulary
  – Congruent with symbol
  – Segment

- Midpoint
- Segment Midpoint Postulate
- Segment bisector
- Angle bisector postulate

IV. Alternative Assessment

- When creating the table, be sure to include all students in the discussion.
- Refer students back to the information in the lesson to assist with adding in the information.
- Make a note of which students have a strong grasp of the material. Be sure to pair those students up with students that seem to be having difficulty when working on in class assignments.

Angle Pairs

I. Section Objectives

- Understand and identify complementary angles
- Understand and identify supplementary angles
- Understand and utilize the Linear Pair Postulate
- Understand and identify vertical angles

II. Multiple Intelligences

- In this lesson, students are going to work on understanding the relationship between pairs of angles. One way to assist students in doing this is to create a chart that compares and contrasts the different relationships.
  - Go through all of the material in the lesson first. Be sure that the students have a basic understanding of the terms and concepts in this lesson. You want to use the activity to expand student knowledge and understanding.
  - To do this, students are going to work in small groups. Review what that compare means to look at the similarities between things, and that contrast means to look at the differences between things.
  - Hand out chart paper and markers to each group.
  - Request that students compare and contrast supplementary angles, complementary angles, linear pairs and vertical angles. Ask them to include drawings to justify what they are comparing and contrasting.
- Then allow time for the students to share their chart work with the rest of the class.
- II. Multiple Intelligences - linguistic, logical- mathematical, bodily- kinesthetic, spatial- visual, interpersonal, intrapersonal

III. Special Needs/Modifications

- Sometimes, special needs students will have difficulty remembering how to do previously learned skills. Here are some prerequisite skills to review prior to beginning this lesson.
  - Solving one- step equations
  - Solving multi- step equations
  - Combining like terms to solve an equation
- Write all vocabulary words on the board/overhead. Request that students copy this information in their notebooks. Students will need this information to complete the activity.
- Vocabulary
  - Adjacent

4.1. Basics of Geometry

- – Congruent
- – Complementary angles
- – Supplementary angles
- – Linear pairs
- – Line pair postulate
- – Vertical angles
- – Vertical angles theorem

IV. Alternative Assessment

- This is a great lesson to use an observation checklist. Make the checklist prior to teaching the lesson. Then use it while groups complete and present their charts. It will provide you with clear things to look and listen for when teaching this lesson.

Classifying Triangles

I. Section Objectives

- Define triangles
- Classify triangles as acute, right, obtuse, equiangular
- Classify triangles as scalene, isosceles, or equilateral

II. Multiple Intelligences

- This activity is a drawing activity that involves students creating a design and then classifying the triangles within the design.
  - – The students are given plain paper and a ruler. They are told to create a page of triangles created by intersecting lines.
  - – Once they have finished, ask them to create a key and to color (use crayons or colored pencils) to color in the different triangles found in the design.
  - – Students can be asked to finish this design for homework.
- Multiple Intelligences- logical- mathematical and spatial- visual

III. Special Needs/Modifications

- Provide students with a diagram of a triangle with the vertices labeled, the sides labeled and the angles labeled. Be sure that students understand where to find the interior angle. They will need this to classify the triangles.
- Write all vocabulary words on the board/overhead. Request that students copy these notes into their notebooks.
- Vocabulary for classifying by angles
  - – Right
  - – Obtuse
  - – Acute
  - – Equiangular
- Vocabulary for classifying by side lengths
  - – Scalene
  - – Isosceles
  - – Equilateral

IV. Alternative Assessment

- Use observation to assess students as they work. Most students will need assistance creating a key to show how their design has been colored. You may want to provide an example of this and then see how the students do following directions.
- You can pair students up to work together too. This may help students who are having a difficult time with the activity.

Classifying Polygons

I. Section Objectives

- Define polygons
- Understand the difference between convex and concave polygons
- Classify polygons by the number of sides
- Use the distance formula to find side lengths on a coordinate grid

II. Multiple Intelligences

- This activity has students begin working alone and then they work in a small group. The purpose is to assist students with developing a deeper understanding of concave and convex figures.
    - Students begin alone. They can choose to draw either four concave polygons or four convex polygons. They don't tell anyone else what they have chosen.
    - When finished, the students join a small group. Then they exchange papers and they must show, by drawing lines, whether the figures they have been given or concave or convex.
    - Students need to justify their answers.
    - Peers correct each other's work.
- Multiple Intelligences- linguistic, logical- mathematical, spatial- visual, interpersonal, intrapersonal

III. Special Needs/Modifications

- Polygon drawing with sides and vertices labeled on the board.
- Write all vocabulary words on the board. Request that students copy these words in their notebooks.
- Complete the distance formula examples slowly on the board/overhead. Be sure that the students are following along.
- Add another example using the distance formula. Use exercise 5 and find the length of $\overline{BA}, \overline{AD}$ and $\overline{DC}$.

IV. Alternative Assessment

- Collect all student work when the groups have finished. Review their work and see how the students have justified whether their figure was concave or convex. This will show you a lot about how students were thinking as they worked on the assignment.

Problem Solving in Geometry

I. Section Objectives

4.1. Basics of Geometry

- Read and understand given problem situations
- Use multiple representations to restate problem situations
- Identify problem- solving plans
- Solve real- world problems using planning strategies

II. Multiple Intelligences

- The great thing about this lesson is that each way of solving a problem can be identified with a specific intelligence. This lesson can assist each student in understanding how he/she works best.
- Begin by presenting all of the information in the lesson. Request that students take notes too.
- Then go through a brief discussion on multiple intelligences. Ask the students to try to identify how they learn best. Have them write this down on a piece of paper. You can refer back to this discussion throughout your teaching and help students to further define the ways that each of them learns best.
- Once students have identified how they learn best, reorganize the class according to learning styles.
- Then ask each group (you may need to subdivide if groups are large) to solve the exercises at the end of the section according to how the group learns.
- Students will easily leap into this, but if not help them with an example or two.

III. Special Needs/Modifications

- Review concept- the Pythagorean Theorem- it is mentioned in the lesson, but not reviewed.
- Write the steps to simplifying a problem on the board. Review what it means to "simplify" something.

  – What is this problem asking for?
  – What do I need to know to find the answer?

IV. Alternative Assessment

- Make notes about the groups of students when organized according to how they learn best.
- This can be very valuable when assisting students in learning.
- For example, a visual learner could better understand a concept presented verbally by drawing a picture. When you are aware of which category each student falls in, you can better address his/her needs when teaching.

4.2 Reasoning and Proof

Inductive Reasoning

I. Section Objectives

- Recognize visual patterns and number patterns
- Extend and generalize patterns
- Write a counterexample to a pattern rule

II. Multiple Intelligences

- Include group work in this lesson. Rather than explaining all of the information in the lesson and then assigning group work, intersperse the group work with the lesson.
- Begin by going over visual patterns.
- Then have students work in pairs. Each student draws a visual pattern. Then they exchange papers with their partner. Our next step is to write a rule for the pattern they have been given, and to extend the pattern two steps.
- Next, go over number patterns.
- Then have the students work in pairs. Each student writes a number pattern. Then they exchange papers with their partner. Our next step is to write a rule (an equation) for the number pattern and to extend the pattern two steps.
- Finally teach about conjectures and counterexamples.
- Have students work with the patterns that they have previously worked with and write a conjecture and a counterexample for each pattern.
- Multiple Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, spatial- visual, interpersonal, intrapersonal

III. Special Needs/Modifications

- Write all vocabulary on the board. Request that students copy this information down in their notebooks.
- Vocabulary

  – Conjecture
  – Counterexample

IV. Alternative Assessment

- Collect student papers.
- Review each student's work to assess understanding.
- Use this to review at the beginning of the next class. You can use different student patterns in the beginning of the next class to review the previously learned material.
- This will be especially helpful to special needs students who require a lot of review to recall previously learned concepts.

Conditional Statements

I. Section Objectives

- Recognize if- then statements
- Identify the hypothesis and conclusion of an if-then statement
- Write the converse, inverse and contrapositive of an if-then statement
- Understand a biconditional statement

II. Multiple Intelligences

- Students need to develop a good understanding of conditional statements and related statements in this section. Because of this, teach all of the material in the lesson and then you can use the following activity to expand student understanding.
- Divide students into four groups. Assign each group one of the following: converse, inverse, contrapositive and biconditional.
- Then write a conditional statement on the board/overhead.
- Each group needs to work together to write a related statement for the given conditional statement.
- When finished, go over the student answer as a class. Use other peers in the class to do any correcting that is needed.
- Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, visual- spatial, interpersonal, intrapersonal

III. Special Needs/Modifications

- This is a difficult lesson for special needs students to understand because the language and symbols are so verbal. Many students with language based learning disabilities will find this challenging. Here is one option on how to scaffold the information in this text.
- Alter these definitions and provide an example (in words not symbols) for each.
- Converse- switch the hypothesis and the conclusion of the conditional statement.
- Inverse- add not to the conditional statement to negate it.
- Contrapositive- add not to the converse to negate it.
- Biconditional statement- combine the conditional statement and its converse together.
- Allow time for the students to work with these definitions and to copy them into their notebooks.

IV. Alternative Assessment

- Use the answers from each group to assess student learning.
- If the students are having difficulty with the activity, then after the first example add another example.
- Repeat as necessary until the students have a good grasp of the information.

Deductive Reasoning

I. Section Objectives

- Recognize and apply some basic rules of logic
- Understand the different parts that inductive reasoning and deductive reasoning play in logical reasoning
- Use truth tables to analyze patterns of reasoning

II. Multiple Intelligences

- Teach the concepts in this lesson prior to completing the activity. This activity involves music and will assist students in a concrete example of how to write a conclusion using inductive reasoning and then use deductive reasoning to prove their conclusion.
- Prepare a five or six music samples for students to listen to. Be sure that all of the samples are from the same genre of music, for example, all jazz. You can even make a few of them by the same artist. The students will have a couple of different options to draw a conclusion from.
- Then allow students time to work in a pair and to write a conclusion about the samples that they have heard.
- Have each pair exchange their conclusion with another pair. Then the students must use deductive reasoning to prove that the statement is correct.
- Intelligences: linguistic, logical- mathematical, musical, interpersonal, intrapersonal

III. Special Needs/Modifications

- Begin the class with a review of the following terms. Knowledge and understanding of these terms is implied in this lesson.
    - Linear pair
    - Adjacent angles
    - Vertical angles
    - Supplementary angles
    - Complementary angles
- Write the following new definitions on the board/overhead. Request that students copy this information down in their notebooks.
    - Law of Detachment
    - Law of Syllogism
    - Inductive Reasoning
    - Deductive Reasoning
- Truth Tables- go over this information very slowly. Be sure that the students understand all of the symbols prior to going over the section in the textbook. You could even create a chart of symbols and explanations on the board for easier understanding.

IV. Alternative Assessment

- Notice how students react to the music cues. Then notice the conclusions that they write about the music. Be sure that they prove their statements clearly. If they have difficulty, then provide examples or use another peer group to coach these students.

Algebraic Properties

I. Section Objectives

- Identify and apply properties of equality
- Recognize properties of congruence "inherited" from the properties of equality
- Solve equations and cite properties that justify the steps in the solution
- Solve problems using properties of equality and congruence

II. Multiple Intelligences

4.2. Reasoning and Proof

- There is a lot of information given in this lesson. Rather than using an activity, here are some suggestions on how to differentiate all of this information so that all learners are engaged.
- Begin lesson by writing the intention of the lesson on the board/overhead. Intention- "to combine geometric building blocks with reasoning."
- Throughout the lesson, refer back to this statement. Identify the geometric building blocks for the students. For example, when there is an example on angles, refer back to student notes on angles. Ask students to brainstorm a few things that they have learned about angles. Then continue with the lesson.
- Review equality definition.
- Write all Properties on one half of the board.
- Write the statements of congruence on the other half of the board.
- Show students how to combine these two together visually. Use different colored chalk or pens (on a whiteboard) to illustrate how combining these statements together can help to prove the given statement.
- Practice this with a few examples. Encourage class participation.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

- If you have many special needs students in the class, you may want to break up this lesson over two days.
- Day one- review properties and statements of congruence. Review basics of geometry.
- Day two- show students how to combine the two together.
- Here are some steps for combining statements and properties.

a. Look at whether you are working with line segments or angles. This will help you choose a statement of congruence.
b. Choose a property that explains the given statement.
c. Combine the statement of congruence and the property together for a final answer.

IV. Alternative Assessment

- Verbally and visually check- in with students to be sure that they are following the lesson. If necessary, go back over previously learned information so that you are sure to have everyone following along.

Diagrams

I. Section Objectives

- Provide the diagram that goes with a problem or proof.
- Interpret a given diagram.
- Recognize what can be assumed from a diagram and what can not be
- Use standard marks for segments and angles in diagrams.

II. Multiple Intelligences

- Complete this activity half-way through the lesson. Once you have gone over the definitions of the eleven postulates, divide the students into eleven groups.
- Assign each group a postulate.
- Request that each group design a diagram that best proves their given postulate.
- When finished, have each group share and justify their diagram.
- Request that they explain how the diagram illustrates the proof.

- Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, visual- spatial, interpersonal, intrapersonal.
- When finished, go back to the text and work on the section where we combine the postulates, properties and statements of congruence together.

III. Special Needs/Modifications

- Review all properties from the previous lesson.
- Review the statements of congruence.
- Review the steps for combining the two together.
- Write all of the postulates on the board/overhead. Request that the students make a three column chart for each.

TABLE 4.2:

Postulate	Definition	example

IV. Alternative Assessment

- Work with the student groups to understand proving each postulate.
- Assessment is done through observation and verbal questions.
- Provide students with plenty of "think time" so that you receive the most accurate response.

Two- Column Proof

I. Section Objectives

- Draw a diagram to help set up a two- column proof.
- Identify the given information and statement to be proved in a two- column proof.
- Write a two- column proof.

II. Multiple Intelligences

- To work through this lesson, allow students to work in pairs and discuss their work. There are logical-mathematical, visual, linguistic, interpersonal and intrapersonal aspects to working with a peer.
- Since many students have a difficult time with proofs, one suggestion is to provide students with a fill- in the blank proof before working on the exercises at the end of the section.
- You can use the proof on page 101- 102 to do this or write one of your own.
- Provide the students with a series of statements on the board. The students fill- in the reasons.
- Provide students with a two- column proof where some of the statements are blank and some of the reasons are blank. Where there are blank statements, the students will need to use the reason to write a statement. Where the reason is blank, the students will need to use the statement to write the reason.
- This will assist the students in interacting with the information in the proof and discussing it will help with retention.
- Now students should be able to complete number fifteen in the exercises which requires that they write a two-column proof without any assistance.

III. Special Needs/Modifications

4.2. Reasoning and Proof

- Be sure that the students have a current list of postulates, properties and vocabulary where they can access it easily.
- Here are some helpful hints for students in working on two- column proofs.

a. Draw a diagram to better understand the vocabulary in the given. For example, if you are working on proving that points are collinear, then draw a diagram of the collinear points. Then look at what statements and reasons you can write about it.
b. Look at the vocabulary in the given. Example 2 has the word "bisects" in it. Therefore, you will need a statement and reason that explains bisects.

Example 2 also has a congruent symbol in it. Therefore, you will need a statement and a reason that addresses congruency.

IV. Alternative Assessment

- Give a homework assignment where students write their own two- column proof based on a common given.
- The next day review the assignment and answers with the students. In small groups, have them write one "best" proof for the given.

Segment and Angle Congruence Theorems

I. Section Objectives

- Understand basic congruence properties.
- Prove theorems about congruence.

II. Multiple Intelligences

- The best way to address different learning styles in this lesson is to use diagrams and to teach this lesson as a class discussion.
- The students will need to break down the concepts provided to gain an excellent understanding of the material.
- Prior to teaching the lesson, write the intention on the board or overhead. This will assist all visual learners and help special needs students too.
- "To prove congruence properties, we turn congruence statements into number statements, and use properties of equality.
- Here are some steps to write on the board:

a. Take the given and notice whether you are working with segments or angles.
b. Think of converting to measurement. For example $\overline{AB} \cong \overline{AB}$, as a statement, we can say that $AB = AB$. We are working with the measurement or length of the segment here. We have changed this to numbers. With angles, change to show the measurement of the angles is equal.

III. Special Needs/Modifications

- Be sure that students have a page of notes out that explain the properties of equality.
- Review each of the properties and what each means.
- Show students how to draw a diagram to illustrate a given statement. A picture often helps special needs students.
- Explain that postulates don't need to be proven.

- Explain that theorems need to be proven.

IV. <u>Alternative Assessment</u>

- Use an observation checklist to observe students as they work.
- Notice who is having difficulty and who isn't. Be sure to make a note of those students so that they can be offered assistance or a peer tutor.
- When possible, use peers to help explain concepts. The student teaching and the student learning both benefit greatly.

Proofs about Angle Pairs

I. <u>Section Objectives</u>

- State theorems about special pairs of angles.
- Understand proofs of the theorems about special pairs of angles.
- Apply the theorems in problem solving.

II. <u>Multiple Intelligences</u>

- Use the following activity to assist students in understanding theorems about special pairs of angles.
- Teach the content in the lesson. Be sure that the students have a good understanding of the concepts in the lesson. Then divide students into groups.
- Each group must make a diagram that illustrates each of the following theorems. They need to write a two column proof to show how the diagram illustrates the theorem.
- When all have finished, allow students time to share their work with the class.
- If time does not allow for students to work on all four of the theorems, assign each group a different theorem to work with.
- Students can complete their work using colored pencils, rulers and large chart paper.
- Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, visual- spatial, interpersonal, intrapersonal.

III. <u>Special Needs/Modifications</u>

- Review the basic definitions of right angles, supplementary angles, complementary angles, and vertical angles.
- Draw an example of each on the board/overhead.
- Explain to students how to move from the basic definition of each angle relationship to the theorem.
- Write each of the theorems on the board/overhead and request that students copy this information in their notebooks.

IV. <u>Alternative Assessment</u>

- Examine the work that the students completed on their chart paper. Notice which groups were more accurate than others.
- Pay attention to which groups expressed their reasoning well verbally.
- Notice which groups expresses their reasoning well in the diagram.
- If a letter grade is needed, assign the same assignment for homework and then grade student work the following day. This will give you an excellent understanding of which students have a good grasp of the concepts and who still needs more practice.

4.2. Reasoning and Proof

4.3 Parallel and Perpendicular Lines

Lines and Angles

I. Section Objectives

- Identify parallel lines, skew lines, and perpendicular lines
- Know the statement of and use the Parallel Line Postulate.
- Know the statement of and use the Perpendicular Line Postulate.
- Identify angles made by transversals.

II. Multiple Intelligences

- Teach half of the lesson, complete the walk around activity, and then finish the rest of the lesson.
- Write the vocabulary on the board for the visual- spatial learners. Include the following.

    - parallel lines with symbol
    - perpendicular lines with symbol
    - parallel planes
    - skew lines

- Have students walk around the room and make a list of all of the places where they can locate each type of lines.
- When finished, allow time for students to share their findings.
- Go on to the postulates. Break each postulate down into simple steps. This will assist visual- spatial learners and special needs students. Here are some suggestions.
- Parallel Line Postulate

    - Given a line and a point not on that line
    - One line parallel to the given line goes through that point

- Perpendicular Line Postulate

    - Given a line and a point not on that line
    - One line perpendicular to the line passes through that point

- When working with transversals, use color to indicate the different angles in a diagram. Use color for definitions too. This will help students to keep things clear.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Before working on the section of the lesson on transversals, review the meaning of the following words: adjacent, vertical, interior, exterior, corresponding and consecutive.
- Break down the transversal section. Use a diagram too.
- Teach adjacent and vertical angles first.
- Then teach interior, exterior and corresponding
- Finally, teach alternate interior, alternate exterior and consecutive angles.

Parallel Lines and Transversals

I. <u>Section Objectives</u>

- Identify angles formed by two parallel lines and a non- perpendicular transversal.
- Identify and use the Corresponding Angles Postulate.
- Identify and use the Alternate Interior Angles Theorem.
- Identify and use the Alternate Exterior Angles Theorem.
- Identify and use the Consecutive Interior Angles Theorem.

II. <u>Multiple Intelligences</u>

- This activity engages several of the intelligences, but also will demonstrate student understanding of the postulates and theorems in this lesson.
- Divide the students into a group of four.
- Assign each of the four students a different topic- Corresponding Angles Postulate, Alternate Interior Angles Theorem, Alternate Exterior Angles Theorem, Consecutive Interior Angles Theorem.
- Let the students know that their assignment is to prepare a lesson and teach the other students in the group about their topic. They can use their notes, a diagram, real life examples, a poem, a song, whatever they would like to make the topic clear. The other students in the groups will let the "teacher" know what he/she did well and also offer suggestions to improve the presentation.
- Intelligences- linguistic, logical- mathematical, musical, bodily- kinesthetic, visual- spatial, interpersonal, intrapersonal

III. <u>Special Needs/Modifications</u>

- Review the angles formed by a transversal
- Write all new postulates and theorems on the board/overhead. Request that the students copy this information down in their notebooks.
- Be sure to place students in a group where they will be well supported by their peers. Some groups are more encouraging than others.

IV. <u>Alternative Assessment</u>

- Create a rubric that covers the points that you want each student presentation to have.
- Share this rubric with the students.
- Observe students as they present their material.
- Include group feedback as part of the rubric.
- This could be used to calculate a quiz/classwork grade if necessary.

Proving Lines Parallel

I. <u>Section Objectives</u>

- Identify and use the Converse of the Corresponding Angles Postulate.
- Identify and use the Converse of Alternate Interior Angles Theorem.
- Identify and use the Converse of Alternate Exterior Angles Theorem.
- Identify and use the Converse of Consecutive Interior Angles Theorem.

4.3. Parallel and Perpendicular Lines

- Identify and use the Parallel Lines Property.

II. Multiple Intelligences

- The best way to differentiate this lesson is to do so as part of a discussion. You want the students to make connections between the parallel lines, the transversal, and proving that the lines are parallel.
- Demonstrate that it is possible to draw two lines and a transversal and have the lines not be parallel. This is where things being true or not true comes into the lesson.
- The students are going to work with you as you work on the board/overhead. Begin by drawing two parallel lines and a transversal on the board.
- Request that the students mirror this work at their seats. They will need paper, rules, pencils and protractors.
- Then go through measuring each pair of angles.
- Once this is finished, go though each postulate and theorem and demonstrate proving that the lines are parallel using the postulates and theorems.
- Remind students that they are "proving" the accuracy of the statement.
- Completing this lesson this way engages the following intelligences- linguistic, logical- mathematical, visual-spatial, interpersonal, intrapersonal

III. Special Needs/Modifications

- Review that what a conditional statement is and how to write the converse of a conditional statement.
- Practice writing converse statements from conditional statements by using real life examples.
- Be sure that the students understand this concept before moving on to the material in the lesson.
- Review the Transitive Property.
- Write all new terms on the board. Request that the students copy these notes in their notebooks.

IV. Alternative Assessment

- The best way to assess student understanding is through observation.
- Ask probing questions, allow plenty of think time, and listen carefully to student responses during the work of the lesson.

Slopes of Lines

I. Section Objectives

- Identify and compute slope in the coordinate plane.
- Use the relationship between slopes of parallel lines.
- Use the relationship between slopes of perpendicular lines.
- Plot a line on a coordinate plane using different methods.

II. Multiple Intelligences

- For this lesson, be sure that students have grid paper, rulers and colored pencils.
- Complete each of the exercises in the text as a whole class.
- This will assist the students in practicing the constructions of lines and finding the slope of a line as you work with them.
- It might even make sense to have a list of the ordered pairs in each example prepared ahead of time and not use the text at first. This way, you can go through each example with the students constructing lines and figuring slopes on their own without the answers presented in the text.

- When you get to the parallel line section, you can introduce the theorem, and then have them construct the line and a line parallel to that given line.
- When you get to the perpendicular line section, you can introduce the theorem and work with the students to draw in a line perpendicular to the line drawn.
- Finally, work with students on using ordered pairs to graph different lines and to find the slopes of the lines. This is from the section on graphing strategies.
- Intelligences: linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal, bodily- kinesthetic

III. Special Needs/Modifications

- Review the following prior to beginning the lesson.
- Drawing a coordinate grid
- Labeling the coordinate grid
- How to locate the x and y axis'
- Review the origin as $(0,0)$
- Review ordered pairs (x,y)
- Review finding the reciprocal of a number/fraction
- Write the two new theorems on the board and request that the students copy this information in their notebooks.

IV. Alternative Assessment

- Alternative Assessment in this lesson can be done through observation. As the students work on the exercises, walk around the room and observe them as they work.
- This is also a good time to notice students who need assistance.
- If several students are needing assistance, consider allowing students to work in pairs.

Equations of Lines

I. Section Objectives

- Identify and write equations in slope- intercept form.
- Identify equations of parallel lines.
- Identify equations of perpendicular lines.
- Identify and write equations in standard form.

II. Multiple Intelligences

- In this lesson, one way to differentiate this lesson is to break the material down into sections.
- There is only one example per section, so you may want to include more than one so that the students have a chance to practice the concept before moving on to something new.
- Include constructions whenever possible.
- When working with slope- intercept form, show how in the equation $y = mx + b$, that m is the slope of the line and that the b is the y intercept.
- Explain that the y intercept is where the line intersects with the y axis.
- When figuring out which equation represents a line parallel to a line already graphed, provide students with these steps.

a. Find the slope of the graphed line.
b. Look at the equation choices

4.3. Parallel and Perpendicular Lines

c. Any equation with the same slope will be parallel to the graphed line.

- To figure out the equation for a line perpendicular to a graphed line, follow these steps.

a. Find the slope of the graphed line.
b. Find the reciprocal of the slope.
c. Any equation with the reciprocal as the slope is perpendicular to the line.

- Intelligences- linguistic, logical- mathematical, visual- spatial

III. Special Needs/Modifications

- Use the information above to scaffold this lesson for the students.
- Ex 4- Standard Form- begin with a simpler example first.
- Possible example $2y = 6x + 12$
- Write all steps on the board and request that the students copy those notes into their notebooks.

IV. Alternative Assessment

- Observe students as they work. If there is a lot of confusion, review concepts more than once to be certain that students are following the material.

Perpendicular Lines

I. Section Objectives

- Identify congruent linear pairs of angles.
- Identify the angles formed by perpendicular intersecting lines.
- Identify complementary adjacent angles.

II. Multiple Intelligences

- Begin this lesson by reviewing the definition of perpendicular lines and the symbol for perpendicular lines.
- List the description for each type of linear pair on the board.
- Then ask the students to use the description to draw an example of the pair being described.
- Example- Draw a linear pair of angles.
- Next, show students how a triangle can be drawn into their diagram.
- Request that they mark the right angle in the triangle.
- Then assign one of the other angles in the triangle a measurement.
- Request that students work to figure out the measurement of the missing angle.
- Explain to students that because they know that the interior angles of a triangle add up to be $180°$ that they can use this to figure out the missing angle of a triangle.
- To expand upon this, ask the students to draw a diagram for a peer to solve. They need to include angle measurements too.
- Then have them switch papers and solve each other's problems.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal

III. Special Needs/Modifications

- Review the following vocabulary prior to beginning the lesson.
- Congruent
- Perpendicular
- Complementary
- Supplementary
- Vertical angles
- Adjacent angles
- Linear pair

IV. Alternative Assessment

- Walk around the room as students work.
- Collect the problems/diagrams that they created for a peer to solve.
- Use these as a classwork grade or to assess student understanding.

Perpendicular Transversals

I. Section Objectives

- Identify the implications of perpendicular transversals on parallel lines.
- Identify the converse theorems involving perpendicular transversals and parallel lines.
- Understand and use the distance between parallel lines.

II. Multiple Intelligences

- Prior to teaching this lesson, provide students with another example. Request that they draw it out and then discover the 90° angles for themselves. Then move on to the text.
- Having the students engage right away and draw conclusions about the material will help to reaffirm the new information in their minds.
- Assist students in completing as many constructions as they can throughout this lesson. Drawing out the examples will expand student understanding.
- Have students practice one or two more examples of finding the distance between straight parallel lines before moving on to slanted parallel lines.
- When you begin working on slanted parallel lines, list these steps on the board. Request that the students copy these steps into their notebooks.

a. Choose two points on one of the lines.
b. Use those points to find the slope of the line.
c. Take the slope and find the slope of the segment perpendicular to the line- the opposite of the reciprocal.
d. Select a point on the line and use the slope to draw a segment perpendicular to the line.
e. Take the coordinates of where this segment intersects both lines.
f. Use the distance formula and these coordinates to find the distance between the parallel lines.

Writing out the steps in this fashion assists students with verbally seeing something, writing it down, and talking about it. Students will tend to remember the information better.

III. Special Needs/Modifications

- Review the following vocabulary.

4.3. Parallel and Perpendicular Lines

- Perpendicular lines
- Parallel lines
- Transversals
- Corresponding angles
- Alternate interior angles
- Alternate exterior angles
- Consecutive interior angles
- Review converse and remind students that this is working backwards in a way.

Non- Euclidean Geometry

I. Section Objectives

- Understand non- Euclidean geometry concepts.
- Find taxicab distances.
- Identify and understand taxicab circles.
- Identify and understand taxicab midpoints.

II. Multiple Intelligences

- This is a great lesson to differentiate because taxicab geometry lends itself to creative problems.
- Go through the material in this lesson, and then divide students into groups. Assign each group either a taxicab distance or a taxicab circle.
- The students then work to write a problem to determine a taxicab distance or a taxicab circle.
- After writing the problem, the students need to draw out a diagram that shows their problem and the solution.
- Finally, the students act out the problem and the solution to the problem. They can use desks or different markers in the room to represent the area or "blocks" that they are working with.
- Be sure that students understand the directions and whether or not they will be graded on their work.
- If you are using a rubric for grading, share it with the students prior to their work.
- Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Review circles including radius
- Write a definition for taxicab geometry, taxicab circles, taxicab distance and taxicab midpoint on the board.
- Request that students copy the information down in their notebooks.

IV. Alternative Assessment

- This is a great opportunity to design a rubric for the activity.
- The rubric can be divided into three sections: the problem itself, the diagram and the skit.
- Students have many opportunities to excel in this assignment because of all of the different ways to participate.
- This could be used as a quiz grade or a classwork grade.

4.4 Congruent Triangles

Triangle Sums

I. Section Objectives

- Identify interior and exterior angles in a triangle.
- Understand and apply the Triangle Sum Theorem.
- Utilize the complementary relationship of acute angles in a right triangle.
- Identify the relationship of the exterior angles in a triangle.

II. Multiple Intelligences

- Begin this lesson by talking about interior and exterior angles of a triangle. Request that students draw an example in their notebooks of a triangle with interior and exterior angles.
- Request that they label the interior angles red and the exterior angles in blue.
- Move on to the Triangle Sum Theorem.
- Remind students that this is information that they already know, but that it has now been written as a theorem.
- One way to make this part of the lesson interactive is to draw a diagram of a triangle on the board and to play "fill in the blank" with the students. You can even have them work on teams and see which team can complete the addition the quickest.
- Another fun way to do this is to use only mental math. This is great for students to practice their thinking skills. The big thing to watch for is students calling out- remind them to raise a hand when done or to stand up.
- If you wish to keep this more traditional, then have students draw the triangles at their seats but use a protractor to measure angles and solve for missing angles.

III. Special Needs/Modifications

- Review the following terms.
- Interior angles
- Triangle
- Exterior angles
- Write new theorems on the board.
- Request that students copy this information down in their notebooks.
- Allow time for questions to check on student understanding.

IV. Alternative Assessment

- The best way to assess this lesson is through observation.
- If you are playing the game, observe which students are actively participating and which ones aren't.
- If completing seat work, walk around and check in with students as they work. Answer questions and offer assistance as needed.

Congruent Figures

I. <u>Section Objectives</u>

- Define congruence in triangles.
- Create accurate congruence statements.
- Understand that if two angles of a triangle are congruent to two angles of another triangle, the remaining angles will also be congruent.
- Explore properties of triangle congruence.

II. <u>Multiple Intelligences</u>

- A way to differentiate this lesson is to work with the determining the congruence of triangles in a hands- on way.
- Begin by teaching the material in the lesson, then move on to the activity.
- Students begin by drawing two congruent triangles. They should use letters to label the vertices of each triangle.
- Then they need to use a protractor to be certain that the two triangles are congruent.
- Once they have determined congruency, the student adds in the tic marks to show congruent sides.
- Finally, they exchange papers with a peer. Once they have exchanged papers, the students need to write three statements that demonstrate and explain that the two triangles are congruent.
- Check student work as part of a class discussion.
- Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, visual- spatial, interpersonal, intrapersonal.

III. <u>Special Needs/Modifications</u>

- Review what it the word congruent means.
- Review the properties of a triangle- meaning corresponding sides and corresponding angles being congruent.
- Explain the use of tic marks to show congruency. Be sure that students understand that they may see tic marks on other figures as well.
- Write the congruence properties on the board. Draw the similarities between these properties and the properties of equality.
- Allow time for any questions.

IV. <u>Alternative Assessment</u>

- Complete an assessment of student understanding by reviewing student diagrams and congruence statements during the class discussion.

Triangle Congruence Using SSS

I. <u>Section Objectives</u>

- Use the distance formula to analyze triangles on a coordinate grid.
- Understand and apply the SSS postulate of triangle congruence.

II. <u>Multiple Intelligences</u>

- After reviewing the first example, ask students to participate in this activity.
- Student one draws a triangle on the coordinate grid and the passes their paper to the right. All students are doing this simultaneously.
- The next student takes the triangle passed to them and uses the distance formula to figure out the lengths of each side of the triangle. Then he/she passes the paper to the right.
- The next student takes the measurements and draws a triangle congruent to the first triangle somewhere on the coordinate grid. Then he/she passes the paper to the right.
- The final student checks the work of all of the others.
- Discuss work when all have finished.
- This is great practice for the students and keeps them engaged because of the paper passing.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal

III. Special Needs/Modifications

- Review the definition for a postulate.
- Review distance formula and how to use it. Provide students with two examples.
- Practice drawing triangles on the coordinate grid.
- Review ordered pairs and how to use the ordered pairs with the distance formula.
- Write the SSS Triangle Congruence Postulate on the board/overhead. Request that students copy these notes down in their notebooks.

IV. Alternative Assessment

- Students can be assessed during the class discussion.
- Also, walk around during the paper passing exercise. You will be able to observe students as they work.
- Allow time for questions.
- Make a note of any students who are having difficulty during the lesson.

Triangle Congruence Using ASA and AAS

I. Section Objectives

- Understand and apply the ASA Congruence Postulate.
- Understand and apply the AAS Congruence Postulate.
- Understand and practice two- column proofs.
- Understand and practice flow proofs.

II. Multiple Intelligences

- When you introduce the ASA Congruence Postulate, review that a postulate is assumed true.
- Then go through the directions in the text, but have the students follow along with you and do the steps themselves in their seats.
- Once students have a good grasp on the ASA Congruence Postulate, then move on to the AAS Congruence Theorem. Be sure that the students understand that a theorem can be proved.
- Demonstrate the example.
- Ask the students what they can notice about the ASA Congruence Postulate and the AAS Congruence Theorem. Write their ideas on the board.
- You want the students to realize that they can be used equally. If the students aren't making this connection on their own, use an example from the text to guide them in discovering it. Having them discover it on their own is much more valuable than telling them the information.

4.4. Congruent Triangles

- Move on to the two- column proofs. Go through the material. Request that the students participate in completing the proof.
- Flow proofs- go through the material.
- If time allows, have students write their own flow proofs and share them in small groups. You could also assign this as a homework assignment.

III. Special Needs/Modifications

- Write the new terms and vocabulary on the board/overhead.
- ASA Congruence Postulate
- AAS Congruence Theorem
- Review the difference between a postulate and a theorem.
- Review the Third Angle Theorem
- Review two- column proofs
- Allow time for questions.

IV. Alternative Assessment

- The best way to assess student learning in this lesson is through question and answer sessions.
- Be sure that you allow time for the students to participate in the lesson. Do not assume that they understand the material. Verify that they do through their responses.

Proof Using SAS and HL

I. Section Objectives

- Understand and apply the SAS Congruence Postulate.
- Identify the distinct characteristics and properties of right triangles.
- Understand and apply the HL Congruence Theorem.
- Understand that SSA does not necessarily prove triangles are congruent.

II. Multiple Intelligences

- Write these two points on the board/overhead. Write that students are going to know all of the theorems and postulates that can prove congruence, and that they are going to understand all of the combinations of sides and angles that do not prove congruence.
- Use the uncooked spaghetti throughout this lesson with the protractors.
- As each exercise is described in the text, walk the students through using the uncooked spaghetti and the protractors to test out each theorem.
- Then use the uncooked spaghetti to show how the AAA does not prove congruence but similarity.
- Demonstrate this by creating two different size triangles that have the same angle measurements. Then the students will see that although the angle measurements are the same, the triangles are not congruent.
- Teach the Pythagorean Theorem. Connect this theorem with the HL Congruence Theorem.
- Make this lesson as interactive as possible by using the protractors and the uncooked spaghetti to model each part of the lesson.
- Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, visual- spatial, interpersonal.

III. Special Needs/Modifications

- Write all of the theorems on the board/overhead as they are taught. Request that the students copy these notes down in their notebooks.
- Review the different types of triangles: acute, obtuse, equilateral, right.
- Review the parts of a triangle- sides and hypotenuse
- Review Pythagorean Theorem
- Show students how to connect the Pythagorean Theorem to the HL Congruence Theorem.

IV. Alternate Assessment

- Observe students as they work through the lesson with the uncooked spaghetti.
- Allow time for student thinking and feedback.

sing Congruent Triangles

I. Section Objectives

- Apply various triangles congruence postulates and theorems.
- Know the ways in which you can prove parts of a triangle congruent.
- Find distances using congruent triangles.
- Use construction techniques to create congruent triangles.

II. Multiple Intelligences

- Divide the students into seven groups. Each group is assigned one of the theorems from the review.
- Then each group must create a diagram to show how each illustrates or does not illustrate congruence.
- Allow time for the students to work.
- When they have finished, allow time for each group to present their work to the class.
- When completing constructions, be sure that students have both a compass and a straightedge.
- Review the steps in the text on drawing a perpendicular bisector of the segment.
- Expand Example 4- Have the students practice drawing segments and practice drawing perpendicular bisectors of each segment.
- Students could work in pairs on this lesson.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

- Complete the Congruence Theorem Review. Create a chart with all of this information and be sure that students copy it into their notebooks.
- Write out the notes for proving parts congruent.
- Be sure students understand that they need to use the distance formula, and the reflexive property of congruence, but that using a protractor does not necessarily mean that the triangles are congruent but similar.

IV. Alternative Assessment

- Create a rubric to grade students on their group presentations.
- Share the rubric with them prior to assigning the group work.
- Then use the rubric to give students a quiz or class work grade.

4.4. Congruent Triangles

Isosceles and Equilateral Triangles

I. Section Objectives

- Prove and use the Base Angles Theorem.
- Prove that an equilateral triangle must also be equiangular.
- Use the converse of the Base Angles Theorem.
- Prove that an equiangular triangle must also be equilateral.

II. Multiple Intelligences

- In this lesson, the students are going to work with the Base Angles Theorem.
- They are going to need to prove the Base Angles Theorem with both isosceles and equilateral triangles.
- Prior to teaching the lesson, ask students to recall information about isosceles and equilateral triangles. Ask them to make a list of the characteristics of each in their notebooks.
- When finished, use a class discussion to generate a list of characteristics for both isosceles and equilateral triangles on the board.
- Then present the information in the text.
- As you teach about the Base Angles Theorem, point out which characteristics apply when working with this theorem.
- Do this for both the isosceles triangle and the equilateral triangle.
- Be sure that the students take notes on both triangle examples.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal.

III. Special Needs/Modifications

- Review isosceles triangles and their parts on the board.
- Present the material in words and in a diagram.
- Review converse statements. Be sure that students understand converse statements.
- Review equilateral triangles and their parts on the board.
- Present the material in words and in a diagram.
- Write out any points or conclusions that you make while discussing this lesson with the students.
- Write this information out on the board and request that the students copy these notes down in their notebooks.

IV. Alternative Assessment

- Alternative Assessment can be completed through observation and listening during the brainstorming session and during the discussion.

Congruence Transformations

I. Section Objectives

- Identify and verify congruence transformations.
- Identify coordinate notation for translations.
- Identify coordinate notation for reflections over the axes.
- Identify coordinate notation for rotations about the origin.

II. <u>Multiple Intelligences</u>

- To differentiate this lesson, begin by teaching all of the content in the lesson. The content will be necessary to do this activity.
- Begin the activity by giving students the coordinates of one triangle on the coordinate grid.
- Have students use colored pencils and a ruler during this lesson.
- Request that the students graph this triangle on the coordinate grid in one color.
- Then have students take this triangle and draw each of the following by using this triangle as a starting point.
- Students are going to have multiple pages of diagrams when they are finished.
- One page consists of a translation or slide.
- One page consists of a reflection or flip.
- One page consists of a rotation or turn.
- One page consists of a dilation.
- Stress the point that a dilation is the only image where the two images are not congruent. The two images are similar.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. <u>Special Needs/Modifications</u>

- Write all new vocabulary words on the board.
- Request that students copy these words in their notebooks.
- Review the meaning of clockwise and counterclockwise.
- Review the difference between things that are congruent and things that are similar.

IV. <u>Alternative Assessment</u>

- Collect the packet of diagrams/drawings that the students have created during this lesson.
- This packet will allow you to assess student understanding of the information presented in this lesson.
- If letter grades are used, than a classwork or quiz grade can be given for this packet.

4.5 Relationships Within Triangles

Midsegments of a Triangle

I. Section Objectives

- Identify the midsegment of a triangle.
- Apply the Midsegment Theorem to solve problems involving side lengths and midsegments of triangles.
- Use the Midsegment Theorem to solve problems involving variable side lengths and midsegments of triangles.

II. Multiple Intelligences

- The best way to differentiate this lesson is to be sure to draw out each of the diagrams in this text on the board/overhead.
- Begin by reviewing the Parallel Postulate.
- Review with a few examples.
- Introduce students to the Midsegment Theorem through a diagram first.
- Draw the diagram on the board. Ask students to brainstorm some of the conclusions that they can draw about the diagram.
- List these conclusions on the board.
- Then take the conclusions that have been generated and use them to write the Midsegment Theorem.
- Include the two statements that are proven in the lesson.
- 1. Parallel to the third side
- 2. Half as long as the third side
- Show students how to prove these two statements.
- Use the examples in the text to do this, but be sure that the students are following through on the examples.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

- Review symbols for parallel and congruent.
- The notes on how numbers 1 and 2 are proven are written out in paragraph form. Rewrite these notes in step form so that the students can follow it easier. This will assist students who have any problem with visually tracking information.
- Review solving multi- step equations.
- Show students how to take the diagram and write an equation from the information.
- Then review solving the equation.
- Show how to apply solving to equation to Example 2 on page 268.

IV. Alternative Assessment

- In this lesson, alternative assessment is done through observation.

Perpendicular Bisectors in Triangles

I. Section Objectives

- Construct the perpendicular bisector of a line segment.
- Apply the Perpendicular Bisector Theorem to identify the point of concurrency of the perpendicular bisectors of the sides (the circumcenter).
- Use the Perpendicular Bisector Theorem to solve problems involving the circumcenter of triangles.

II. Multiple Intelligences

- Begin by reviewing notes on the perpendicular bisector of a line segment.
- Show how the bisector divides the line segment into two congruent segments.
- Show how it intersects the line at a right angle.
- Ask students to draw two different line segments, measure them and draw in the perpendicular bisector.
- Either walk around and check student work or do a peer check. It is important to establish understanding before moving on to the next section.
- Go over the Perpendicular Bisector Theorem and its converse.
- Have students use a compass and colored pencils in the next activity.
- 1. Students draw a triangle of their own design.
- 2. Students draw in the perpendicular bisectors of each line segment.
- 3. Students use a compass to draw in a circle that encompasses the triangle.
- 4. Students label the circumcenter of the diagram.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic

III. Special Needs/Modifications

- Review the definitions of perpendicular and bisector.
- Write the Theorems on the board.
- Be sure that students copy those notes into their notebooks.
- Define circumcenter.
- Demonstrate how to draw in the segment bisectors and label the circumcenter.
- Review using a compass.

IV. Alternative Assessment

- Collect and examine student drawings/diagrams to assess student understanding.
- Be sure to allow time for student questions.
- If students are having a difficult time with the in class assignments, allow them the option of working with a peer.
- Be sure that peer work is on task through observation and walking around.

Angle Bisectors in Triangles

I. Section Objectives

- Construct the bisector of an angle.

4.5. Relationships Within Triangles

- Apply the Angle Bisector Theorem to identify the point of concurrency of the perpendicular bisectors of the sides (the incenter).
- Use the Angle Bisector Theorem to solve problems involving the incenter of triangles.

II. Multiple Intelligences

- Write the intention of this lesson on the board/overhead.
- The intention is to inscribe circles in triangles.
- Go through the material in the lesson.
- After teaching the material in the lesson, give the students an opportunity to work with large triangles and inscribe circles in these triangles.
- This can be a lot of fun.
- Encourage students to use colored pencils and to make their diagram as colorful as they wish.
- Also use large chart paper.
- Allow students the option of working in a small group or by themselves.
- Then ask them to draw a diagram that shows the Concurrency of Angle Bisector Theorem.
- Be sure that they label each part of the diagram.

III. Special Needs/Modifications

- Review how to bisect and angle.
- Review that the bisector of an angle is the ray that divides the angle into two congruent angles.
- Remind students that with the Concurrency of Angle Bisectors Theorem, that we are going to show the point of intersection.
- Write these steps on the board.
- 1. Draw in angle bisectors.
- 2. Draw in perpendicular bisectors of each line segment.
- 3. Show the point of intersection
- 4. Use a compass to inscribe the circle inside the triangle.

IV. Alternative Assessment

- Allow time for the students to present their work to the class or in small groups.
- Walk around and listen to the students discuss and explain their work.
- Use this as a way to assess student understanding.

Medians in Triangles

I. Section Objectives

- Construct the medians of a triangle.
- Apply the Concurrency of Medians Theorem to identify the point of concurrency of the medians of the triangle (the centroid).
- Use the Concurrency of Medians Theorem to solve problems involving the centroid of triangles.

II. Multiple Intelligences

- Go through the initial material in this lesson first.
- Review the median of a triangle and how to find it.

- Show a diagram on the board that introduces the students to the Concurrency of Medians Theorem.
- Use the diagram to show each median being drawn in then show the point of intersection, the centroid.
- Introduce the vocabulary word as the material is covered.
- You may want to allow time for the students to try this with a triangle of their own creation.
- This will give them a good understanding of the concepts.
- Then complete the second part of the lesson.
- Pg. 300 -Complete the activity in Example 2 together as a class.
- Ask the students to write down the answers to the two questions.
- Then open up the discussion to a brainstorming session.
- Write student responses on the board.

III. Special Needs/Modifications

- Write all definitions on the board/overhead.
- Define median of a triangle.
- Define concurrent
- Define centroid and show the connection between concurrent and centroid.
- Review the midpoint formula.
- Review the distance formula.
- Review using the Geometer's Sketchpad.
- Define Napolean's Theorem.

IV. Alternative Assessment

- Assessment is completed through observation and discussion.

Altitudes in Triangles

I. Section Objectives

- Construct the altitude of a triangle.
- Apply the Concurrency of Altitudes Theorem to identify the point of concurrency of the altitudes of the triangle (the orthocenter).
- Use the Concurrency of Altitudes Theorem to solve problems involving the orthocenter of triangles.

II. Multiple Intelligences

- Begin this lesson with a real life example about altitude. You could use a plane and review the meaning of the word altitude with the students. This will help them to draw associations as they work with the concept.
- Define altitude.
- Use these steps to find the altitude of a triangle.
- 1. Identify the vertex you are using.
- 2. Find the side of the triangle opposite the vertex or where this side should be located.
- 3. Draw a straight line from the vertex to that opposite side, draw the side in if it does not exist in the original triangle.
- Show the two examples with the acute triangle and the obtuse triangle. Have students draw these two examples in their notebooks.
- Define the Concurrency of Triangles Altitude Theorem
- Define orthocenter

4.5. Relationships Within Triangles

- Pg. 307 Example 1- do each step of the example with the students.
- "What do you observe?" Request that the students write their observations in their notebooks. Then allow some time for sharing and write these notes on the board.
- 8- What do you observe about the four points? Repeat the brainstorming activity to expand student thinking.
- Have students draw conjectures about whether or not the points would be collinear for all other kinds of triangles.
- Expand this as an extra credit assignment or homework assignment for students to draw different triangles and demonstrate whether or not the four points are collinear. Request that they include a writing piece in their assignment and describe their findings in words.
- Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, visual- spatial, intrapersonal, interpersonal

III. Special Needs/Modifications

- Write all vocabulary words on the board and request that students copy these notes into their notebooks.
- Allow a lot of time for questions.
- Be sure that the students have copied down the steps for locating the altitude of a triangle in their notes.
- Students are most likely to be concerned when the altitude is drawn outside of the triangle. Use some extra time to show students why this is the case.

IV. Alternative Assessment

- Assess student learning through observation and through feedback during brainstorming sessions.
- Be alert to students who are not writing during the independent writing times.
- Be sure that all students are following the lesson. Review concepts or use peer tutoring if students are having challenges.

Inequalities in Triangles

I. Section Objectives

- Determine relationships among the angles and sides of a triangle.
- Apply the Triangle Inequality Theorem to solve problems.

II. Multiple Intelligences

- Teach the information in this lesson, and then expand it using a coordinate grid.
- The activity has the students draw triangles with different side lengths to prove the Triangle Inequality Theorem.
- Students use graph paper, colored pencils and rulers for this activity.
- Give students the dimensions for five different triangles on the board/overhead. For example, can we have a triangle with the lengths $6, 7, 12$.
- Students need to draw out the triangle on the coordinate grid to demonstrate whether it can be a triangle or not.
- Then they also need to write an inequality demonstrating whether or not these side lengths work for a triangle.
- When finished, allow time for the students to prove their findings.
- Include the different theorems into their explanations when they occur. Point these out to the students.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- (Sides) Theorem- walk through the proof step by step. Provide students with a brief explanation of each step.
- Review briefly each previously learned term as it is introduced. Example- ruler postulate, angle addition, substitution
- (Angle) Theorem- on board
- Review what is meant by indirect reasoning- by assumption or conjecture
- Define corollary

IV. Alternative Assessment

- Collect student work on with the triangles on the coordinate grid.
- Use these diagrams and the inequality statements to assess student understanding.
- You can use this as a classwork grade.
- It is also useful to assess the students that are in need of assistance.

Inequalities in Two Triangles

I. Section Objectives

- Determine relationships among the angles and sides of two triangles.
- Apply the SAS and SSS Triangle Inequality Theorems to solve problems.
- Multiple Intelligences
- Expand this lesson by creating a model for students to use throughout the lesson.
- Before beginning, have students use three long strips of paper. Put fasteners to connect the strips of paper into a long strip.
- The fasteners will be moveable so that the students can manipulate the pieces into different shaped triangles. This is how they can demonstrate proving each of the theorems in the lesson.
- Have each student work with a partner since we will be working with two triangles.
- After going through the first example on SAS Inequality Theorem, have students work in pairs to test it out themselves.
- Offer time for feedback.
- Then go through the other examples in the lesson, after each example, have students work to test out the theorems with their model.
- Offer time for feedback.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Review inequalities in one triangle first.
- Review congruent triangles.
- Write all of the theorems on the board/overhead.
- Request that students copy these notes into their notebooks.
- Allow time for questions.

IV. Alternative Assessment

- Assess student understanding through observation of the partner work.
- Interact with students as they work through proving each theorem.
- Listen to student feedback and correct any unclear information.

4.5. Relationships Within Triangles

- Expand this lesson into a writing assignment by having students write their observations and conclusions about the theorems in narrative form.
- If time allows, have students share their conclusions with the whole class or in small groups.

Indirect Proofs

I. Section Objectives

- Reason indirectly to develop proofs of statement.

II. Multiple Intelligences

- This is a short lesson but scaffold it into three sections. This will work for both multiple intelligences and for special needs students.
- Begin by defining an indirect proof.
- Define conjecture and what is meant by a conjecture.
- 1. Begin by writing if- then statements using real life examples.
- For example- "If Mary plays soccer then she is an athlete."
- Request that the students write three if- then statements in their notebooks.
- Allow time for the students to share their work.
- 2. Algebraic Examples- use the one in the text to begin with.
- Then have students write three more algebraic examples.
- Exchange papers with a partner.
- Each partner must prove the if- then statement as true or false.
- Allow time for students to share their work.
- This helps students to make the connection between if- then statements and whether the statement is true or false.
- 3. Geometric Examples- use the example in the text.
- Then divide students into small groups.
- Request that they prove the following using the same diagram from the text.
- $\angle 2 = \angle 3$
- After students are finished writing the proof, allow time for sharing.
- Take the best parts of each written proof to compose a proof on the board.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Write all theorems on the board.
- Use the above activity to scaffold this lesson for the students.

IV. Alternative Assessment

- Prior to teaching the lesson, compose a list of essential elements for the proof that the students are going to write.
- When composing the group proof on the board, be sure that the final example has each of these elements in it.
- Request that students copy this proof into their notebooks.

4.6 Quadrilateral

Interior Angles

I. <u>Section Objectives</u>

- Identify the interior angles of convex polygons.
- Find the sums of interior angles in convex polygons.
- Identify the special properties of interior angles in convex quadrilaterals.

II. <u>Multiple Intelligences</u>

- Divide this lesson into two parts. The first part is going to focus on finding the sum of interior angles of polygons. The second part is going to focus on finding the sum of the interior angles of a quadrilateral.
- Define convex.
- Define polygon.
- Define interior angles.
- Note that the number of interior angles matches the number of sides of the polygon.
- Show how to use the Triangle Sum Theorem to divide the polygon into triangles.
- Demonstrate using an equation to solve for the sum of the measure of the interior angles.
- Have students do this with two new polygons in their notebooks- a pentagon and a decagon.
- Allow time for students to share their work when finished.
- Be sure student answers were found using the equation.
- Part 2 – move onto quadrilaterals
- Test out the equation with a rectangle and a trapezoid.
- Intelligences- logical- mathematical, visual- spatial, interpersonal, intrapersonal

III. <u>Special Needs/Modifications</u>

- Define all terms on the board. Request that students copy these notes into their notebooks.
- Here are the steps to solving these problems.
- 1. Divide the figure into triangles.
- 2. Use an equation to find the sum of the angles.
- 3. With a quadrilateral- check does the sum equal $360°$?

IV. <u>Alternative Assessment</u>

- Check student work. Be sure that the students are writing an equation when solving for the sum of the interior angles of both polygons and quadrilaterals.
- Provide correction when necessary.

Exterior Angles

I. <u>Section Objectives</u>

- Identify the exterior angles of convex polygons.
- Find the sums of exterior angles in convex polygons.

II. Multiple Intelligences

- Each of the examples in this lesson can be used as an extension to include multiple intelligences as you teach this lesson.
- I recommend not using the text with the students but teaching this lesson as an exploration.
- Have students work with rulers, protractors, paper and pencils at their desks in small groups.
- Have students work in pairs and/or groups of three.
- Then, present each section of the lesson.
- Have the students explore each example in their seats.
- For example, you draw the diagram on the board or overhead. Present a leading question, and then ask the students to work with the figure to solve the dilemma.
- When finished, allow time for class discussion.
- After going through each example in the text. Provide students with the notes for the lesson. They will have an experiential understanding of the information through working with each example.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Write all vocabulary and terms on the board/overhead.
- Define exterior angles.
- 1. Formed by extending the side of a polygon.
- 2. Two possible exterior angles for any given vertex.
- Define supplementary angles.
- Define vertical angles.
- Exterior Angle Sum

IV. Alternative Assessment

- Alternative Assessment is done through observation of student group work.
- Use an observation checklist to be sure that the groups are working to discover the big ideas of this lesson.

Classifying Quadrilaterals

I. Section Objectives

- Identify and classify a parallelogram.
- Identify and classify a rhombus.
- Identify and classify a rectangle.
- Identify and classify a square.
- Identify and classify a kite.
- Identify and classify a trapezoid.
- Identify and classify an isosceles trapezoid.
- Collect the classifications in a Venn diagram.
- Identify how to classify shapes on a coordinate grid.

II. Multiple Intelligences

- To differentiate this lesson, I recommend beginning by having students design a Venn diagram to classify the quadrilaterals. Do this BEFORE teaching the lesson.
- Begin by going through a brief explanation of a Venn diagram. Most students will be familiar with them.
- Then tell students that they are going to work with the text in small groups and design a Venn diagram to classify the following quadrilaterals.
- List the quadrilaterals on the board and provide students with chart paper and colored pencils.
- Students have been working with these figures for a long time. This exercise allows you an opportunity to walk around and assess student understanding about these figures.
- Allow students to devise their own classifications system to engage higher level thinking.
- When finished, ask students to explain how and why they chose to classify the figures the way that they did.
- Then move to the text.
- Go through the material in the text with the students having a deeper understanding of different quadrilaterals.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Define each type of quadrilateral on the board.
- Help students to create a chart of each with a drawing and a brief description.
- Define Opposite Sides of Parallelogram Theorem
- Define Opposite Angles of Parallelogram Theorem
- Review distance formula
- Review slope

IV. Alternative Assessment

- Use the Venn diagram exercise to assess student learning and understanding.
- Look at how the students have chosen to classify the different figures- does this make sense? What adjustments are needed?
- Did the students classify according to sides and angles?
- Provide feedback to expand student understanding.

Using Parallelograms

I. Section Objectives

- Describe the relationships between opposite sides in a parallelogram.
- Describe the relationship between opposite angles in a parallelogram.
- Describe the relationship between consecutive angles in a parallelogram.
- Describe the relationship between the two diagonals in a parallelogram.

II. Multiple Intelligences

- Begin by handing out pieces of string to each student. Some of the strings can be the same length and some can be different lengths.
- Have the students use these strings to work through the examples at their seats.
- This is a hands- on way to demonstrate the power of congruent side lengths.
- Then have the students design a quadrilateral on the coordinate grid. They can each decide whether it is a parallelogram or not.

4.6. Quadrilateral

- Exchange papers with a peer. Each peer needs to use the distance formula to test out whether the figure that they have been given is a parallelogram or not.
- Allow time for sharing.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Review the meaning of congruent.
- List out the following description of a parallelogram. Request that the students copy this information down in their notebooks.
- Parallelogram
- 1. Quadrilateral with 2 pairs of parallel sides.
- 2. Opposite sides are congruent.
- 3. Opposite angles are congruent.
- 4. Consecutive angles are supplementary.
- 5. Diagonals bisect each other.
- Walk through each step of filling in the proofs.
- Provide a review of each "Reason" as it is presented.

IV. Alternative Assessment

- Listen to the student sharing and assess whether students understand what makes a parallelogram a parallelogram.
- Allow time for questions.

Proving Quadrilaterals are Parallelograms

I. Section Objectives

- Prove a quadrilateral is a parallelogram given congruent opposite sides.
- Prove a quadrilateral is a parallelogram given congruent opposite angles.
- Prove a quadrilateral is a parallelogram given that the diagonals bisect each other.
- Prove a quadrilateral is a parallelogram if one pair of sides is both congruent and parallel.

II. Multiple Intelligences

- To differentiate this lesson, begin by going through the material in the lesson and stop when you get to the diagram in Example 4.
- Divide students into five groups.
- Each group is going to use this diagram to PROVE that the figure is a parallelogram.
- Each group is assigned a characteristic of a parallelogram to prove. Explain that they will also need to prove the converse of each statement.
- Group 1- quadrilateral with two pairs of parallel sides.
- Group 2- opposite sides are congruent
- Group 3- opposite angles are congruent
- Group 4- consecutive angles are supplementary
- Group 5- Diagonals bisect each other.
- When finished, have students present their work.

- As a class decide if the group was successful in proving their statement.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

- List characteristics of a parallelogram on the board.
- 1. Quadrilateral with two pairs of parallel sides.
- 2. Opposite sides are congruent.
- 3. Opposite angles are congruent.
- 4. Consecutive angles are supplementary.
- 5. Diagonals bisect each other.
- Be sure that students understand how to find each characteristic in a diagram.
- Walk through each proof.
- Explain each "Reason" as it is covered. This will require students to review previously learned information.

IV. Alternative Assessment

- Students will provide the assessment in this lesson when they decide whether each group has successfully proven their statement.

Rhombi, Rectangles, and Squares

I. Section Objectives

- Identify the relationship between the diagonals in a rectangle.
- Identify the relationship between the diagonals in a rhombus.
- Identify the relationship between the diagonals and opposite angles in a rhombus.
- Identify and explain biconditional statements.

II. Multiple Intelligences

- Break down the information in this lesson to provide students with the following notes on rectangles and rhombi.
- Rectangle
- 1. Demonstrate diagonals are congruent using the distance formula
- Provide students with an example on the overhead that they can they figure out on grid paper using the distance formula. You can even divide the class in half. Ask one half of the class to work on one diagram and the other half of the class to work on another diagram.
- Rhombi
- 1. Diagonals are perpendicular bisectors of each other.
- 2. Diagonals bisect the interior angles.
- Define a biconditional statement as a conditional statement that also has a true converse. "if and only if"
- In pairs, have students write a biconditional statement for a rectangle and a biconditional statement for a rhombus.
- Allow time for students to share their statements.
- The class decides whether it is true biconditional statement or not.
- If not, provide counterexamples.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

4.6. Quadrilateral

- Walk through each proof in the lesson.
- Provide a brief explanation of each "Reason" as it is presented. Do not assume students remember the definitions of each.
- Review the following:
- Distance formula
- Definition of Perpendicular bisector
- Review finding the slope of a line.
- Review conditional statements.

IV. Alternative Assessment

- Create a checklist of what would be acceptable correct biconditional statements. Use this checklist to assist students in evaluating the biconditional statements written by each pair.

Trapezoids

I. Section Objectives

- Understand and prove that the base angles of isosceles trapezoids are congruent.
- Understand and prove that if base angles in a trapezoid are congruent, it is an isosceles trapezoid.
- Understand and prove that the diagonals in an isosceles trapezoid are congruent.
- Understand and prove that if the diagonals in a trapezoid are congruent, the trapezoid is isosceles.
- Identify the median of a trapezoid and use its properties.

II. Multiple Intelligences

- Break down the information in this lesson into sections to assist student understanding.
- Define a trapezoid.
- 1. One pair of parallel sides
- 2. NOT parallelograms
- Define an Isosceles trapezoid.
- 1. One pair of non- parallel sides that are the same length.
- 2. Base angles are congruent.
- 3. Diagonals are congruent.
- Activity- have students draw two trapezoids and two isosceles trapezoids.
- With the isosceles trapezoids request that they do the following.
- 1. Label angles, sides and diagonals to show that it is an isosceles trapezoid.
- 2. Write a statement and its converse for each label to explain it.
- Notes of Trapezoid Medians
- 1. Connects the medians of the non- parallel sides in a trapezoid.
- 2. Located half-way between the bases in a trapezoid.
- Theorem$-\frac{\text{sum of base lengths}}{2}$
- Use this equation with the example in the text. Request that students practice this as well.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Write all notes on the board/overhead. Request that students copy this information in their notebooks.
- Define symmetry.

- Use an example so that students understand symmetry in connection with trapezoids.

IV. Alternative Assessment

- Collect student diagrams.
- Assess understanding based on labels and statements.

Kites

I. Section Objectives

- Identify the relationship between diagonals in kites.
- Identify the relationship between opposite angles in kites.

II. Multiple Intelligences

- Write out the following notes on kites.
- 1. No parallel sides.
- 2. Two pairs of congruent sides adjacent to each other.
- 3. Two vertex angles
- 4. Two non- vertex angles that are congruent
- 5. Diagonals are perpendicular
- Have students design a kite on a coordinate grid.
- They must prove it is a kite by providing proof that the diagonals are perpendicular, and that the non- vertex angles are congruent.
- They can do this through statements or by writing a proof with reasons.
- Finally, once their diagram has been approved, they can use it to design and decorate their own kite. Students can use chart paper and colored pencils to do this.
- Hang all work up in the class.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Define adjacent.
- Define vertex angles.
- Define non- vertex angles.
- Walk through each proof.
- Explain each "Reason" in the proof.

IV. Alternative Assessment

- Approve all students kite diagrams before students design their final kite.
- Be sure that the points have been successfully proven.

4.7 Similarity

Ratios and Proportions

I. Section Objectives

- Write and simplify ratios.
- Formulate proportions.
- Use ratios and proportions in problem solving.

II. Multiple Intelligences

- Differentiate this lesson by providing opportunities to expand each example for the students. This will provide students with a way to practice the concepts as the information is presented.
- Ratio is a fraction that compares two things.
- Three ways to write a ratio. As a fraction, with a colon or using the word to.
- Expand Example 1: What is the ratio of everything bagels to sesame bagels?
- Answer: $\frac{50}{25} = \frac{2}{1}$
- Expand the equation example with the dancers and the singers.
- "What if the ratio of dancers to singers was 5 to 4 and there were forty- five dancers? How many singers are there? How many dancers are there?
- Answer: $5n =$ dancers
- $4n =$ singers
- $5n + 4n = 45$
- $9n = 45$
- $n = 5$
- $5(5) = 25$ dancers
- $4(5) = 20$ singers
- Proportion- an equation that compares two equal ratios.
- Expand Barn Dimensions example.
- "What if the water line was actually 20 ft instead of ten? What would the length be on a scale drawing?"
- Answer $x = 5$ inches
- Intelligences- linguistic, logical- mathematical, visual- spatial

III. Special Needs/Modifications

- Write each definition and its examples on the board. Request students write down the information in their notebook.
- Explain the Cross Multiplication Theorem as something that the students already know from previous classes about how to solve a proportion. This is a formal way of writing it.

IV. Alternative Assessment

- Throughout the expansion of each exercise, allow students to contribute their answers to class discussion.

Properties of Proportions

I. Section Objectives

- Prove theorems about proportions.
- Recognize true proportions.
- Use proportions theorem in problem solving.

II. Multiple Intelligences

- This is a brief lesson used to explain the corollary theorems associated with the Cross Multiplication Theorem.
- To make this lesson more interesting and interactive, use group work.
- Divide the students into five groups.
- After teaching the lesson and going through the material, assign each group one of the corollary theorems.
- Then put an example of two similar triangles (with measurements) on the board/overhead.
- Ask each group to use the example on the board to create an example that illustrates their corollary theorem.
- Allow time for the students to work and then have each group present their example to the class.
- When finished, encourage the other students to ask questions to see how well the students in the group can answer them.
- Answering questions is a great way to assess student understanding.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Write these notes on the board/overhead. Request that students copy them down.
- Cross Multiplication Theorem- defining property of proportions.
- Subtheorems are called corollary theorems.
- Review which terms in a proportion are the means and the extremes.
- Corollary 1- swap means.
- Corollary 2- swap extremes.
- Corollary 3- flip it upside down.
- Corollary $4 - \frac{a+b}{b} = \frac{c+d}{d}$
- Corollary $5 - \frac{a-b}{b} = \frac{c-d}{d}$

IV. Alternative Assessment

- Assessment can be done through student questions and answers following the exercise.

Similar Polygons

I. Section Objectives

- Recognize similar polygons.
- Identify corresponding angles and sides of similar polygons from a statement of similarity.
- Calculate and apply scale factors.

II. Multiple Intelligences

4.7. Similarity

- Activity to differentiate this lesson.
- Ask students to draw a polygon and label the lengths of the sides of their polygon.
- Then ask the students to exchange polygons with someone else. Students may exchange more than once just be sure that everyone has a different polygon than the one that they started with.
- Students need to complete the following with this new polygon.
- 1. Draw a similar polygon to the one that you have been given.
- 2. Write proportions to demonstrate that the side lengths are similar.
- 3. Determine the scale factor.
- 4. Determine the ratio of the perimeters.
- When finished, divide into small groups to share their findings.
- Use peers to correct any errors in the work of each individual.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal

III. Special Needs/Modifications

- Define similar.
- Similar in the context of polygons.
- 1. Same number of sides
- 2. For each angle there is a corresponding angle in the other polygon that is congruent.
- 3. Lengths of all corresponding sides are proportional.
- Write all assignment directions on the board so that students can refer back to what is needed for each step.
- Use flexible grouping to assist students in understanding the activity.
- Ratios of similar perimeters- same as scale factor- be sure that students understand these two concepts.

IV. Alternative Assessment

- Walk around and listen in on group discussions.
- Interject important information, offer feedback or constructive criticism when needed.

Similarity by AA

I. Section Objectives

- Determine whether triangles are similar.
- Understand AAA and AA rules for similar triangles.
- Solve problems about similar triangles.

II. Multiple Intelligences

- The technology integration is a nice way to differentiate this lesson by adding the interactive element of technology.
- In addition, after covering the material in the lesson, the shadow problems with the similar triangles are a fun way to help the students to gain a deeper understanding of the concepts in the lesson.
- You could have the students write their own problems and solve each other's using diagrams and drawings.
- You could also take the students outside, use a tree or a flagpole, the height of a student, and their shadow to measure and actually create a real life problem.
- This can be a fun way to bring the outdoors, nature and real life into the math classroom.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal

III. Special Needs/Modifications

- Walk students through the directions on how to use the technology.
- AAA Rule- if angles of a triangle are congruent to the corresponding angles of another triangle, then the triangles are similar.
- SAME AS: The AA Triangle Similarity Postulate except that it uses two angles and not three.
- Both are true and both work.
- Indirect measurement- provide students with a visual example of the two similar triangles and the proportions.

IV. Alternative Assessment

- Alternative Assessment in this lesson is done through observation and through interacting with the students during the activity.

Similarity by SSS and SAS

I. Section Objectives

- Use SSS and SAS to determine whether triangles are similar.
- Apply SSS and SAS to solve problems about similar triangles.

II. Multiple Intelligences

- There are two great ways to differentiate this lesson included in the text. One is the technology integration and one is the hands- on activity.
- Here is how we can take the hands- on activity and expand it a bit to include an interactive part.
- Complete the hands- on activity as it has been written in the text.
- When finished, ask students to write a few observations that they have made into their notebooks.
- Then ask students to contribute their observations to a class discussion.
- Write all of the student observations on the board. Point out the SSS for Similar Triangles.
- SSS for Similar Triangles- if the lengths of the sides of the two triangles are proportional, the triangles are similar.
- SAS for Similar Triangles- use Cheryl's examples to create your own example to illustrate the SAS for Similar Triangles.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

- Write the intention of the lesson on the board.
- Intention is to explore relationships between proportional side lengths and congruent angles of similar triangles.
- Review the directions for using the technology.
- Write the directions to the hands- on activity on the board.

IV. Alternative Assessment

- Observe student work through the technology integration activity.
- Observe student work through the hands- on activity.

4.7. Similarity

Proportionality Relationships

I. <u>Section Objectives</u>

- Identify proportional segments when two sides of a triangle are cut by a segment parallel to the third side.
- Divide a segment into any given number of congruent parts.

II. <u>Multiple Intelligences</u>

- Complete the technology integration as one way to differentiate this lesson.
- After working through the technology integration ask students to write their observations down in their notebooks.
- Conduct a sharing session and write student observations on the board.
- Midsegment of a Triangle- midsegment that divides the sides of a triangle proportionately.
- Activity- have the student use construction paper to design a triangle.
- Then using a ruler and a pencil, draw in the midsegment of the triangle.
- Ask students to make notes about the measurements of the triangle and how they have been altered with the midsegment.
- Then, use scissors to actually divide the triangle into two proportionate sections.
- Request that students write their observations down in their notebooks.
- The Lined Notebook Paper Corollary- demonstrate this as a whole class. Ask students to use notebook paper and a ruler.
- Write student observations on the board.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal.

III. <u>Special Needs/Modifications</u>

- Review ways to figure out if two triangles are similar using side lengths and angles.
- Define midsegment of a triangle on the board.
- Write out the Triangle Proportionality Theorem.
- Go through each step of each proof and explain each "Reason" as it is presented.
- Do not assume that the students remember previously learned information.

IV. <u>Alternative Assessment</u>

- Create an observation checklist to use during both activities.
- For the technology piece, make a list of things that you would like students to gain from the activity.
- For the hands- on piece, make a list of things that you would like students to gain.
- Notice during the discussion sessions whether or not these goals have been met.
- If not, make these points to the students and explain how they were discovered.

Similarity Transformations

I. <u>Section Objectives</u>

- Draw a dilation of a given figure.
- Plot the image of a point when given the center of dilation and scale factor.
- Recognize the significance of the scale factor of a dilation.

II. <u>Multiple Intelligences</u>

- Teach the information in this lesson.
- Then use the information on dilations to complete the following activity.
- Divide the students into groups and have them measure the classroom.
- Each group is working on the same assignment. Having each group work on the same assignment will produce different diagrams to explain the same information.
- Students are going to complete a drawing that shows the measurement of the classroom. They can create a scale to represent the measurement of the room.
- Next, students create a dilation of the classroom where the scale factor is $\frac{1}{2}$.
- Students need to draw a diagram of this and label it with the correct measurements.
- Then, students create a dilation of the classroom with a scale factor of $\frac{1}{4}$.
- Next, students create a dilation of the classroom with a scale factor of 3.
- Then have students complete the area and the perimeter of the room.
- Find a perimeter with a scale factor of $\frac{1}{3}$.
- Find an area with a scale factor of $\frac{1}{3}$.
- Allow time for the students to share their work and any observations.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. <u>Special Needs/Modifications</u>

- Review congruence transformations that preserve length- translations, rotations, and reflections.
- Define a dilation.
- Define scale factor.
- Review finding the perimeter of a figure.
- Review finding the area of a figure.

IV. <u>Alternative Assessment</u>

- Walk around and observe students as they work.
- Collect diagrams and use for a classwork or a quiz grade.

Self- Similarity (Fractals)

I. <u>Section Objectives</u>

- Appreciate the concept of self- similarity.
- Extend the pattern in a self- similar figure.

II. <u>Multiple Intelligences</u>

- Differentiate this lesson by completing a construction of each of the fractals. Ue rulers and measure to ensure accuracy.
- Here are the steps.
- 1. Begin with a segment.
- 2. Divide the segment into three congruent parts.
- 3. Remove the middle part leaving two parts.
- 4. Divide each segment into three congruent parts.

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- 5. Remove the middle part of each segment.
- Allow students to work in groups.
- You can extend this lesson by having students work with both horizontal lines and vertical lines.
- With the pattern for the Sierpinski Triangle, have students see which other polygons can be used to create a similar pattern.
- Allow students to work in pairs or groups on this assignment.
- Display student work.

III. Special Needs/Modifications

- Write the steps for each fractal on the board/overhead.
- Pattern for Sierpinski Triangle.
- 1. Draw a triangle.
- 2. Connect the midpoints of the sides of the triangle. Shade in the center triangle.
- 3. Repeat this process with each triangle.

4.8 Right Triangle Trigonometry

The Pythagorean Theorem

I. Section Objectives

- Identify and employ the Pythagorean Theorem when working with right triangles.
- Identify common Pythagorean triples.
- Use the Pythagorean Theorem to find the area of isosceles triangles.
- Use the Pythagorean Theorem to derive the distance formula on a coordinate grid.

II. Multiple Intelligences

- This lesson uses the Pythagorean Theorem in several different ways.
- You can differentiate this lesson by expanding each of the examples in the lesson.
- Begin by drawing and labeling the parts of right triangle. Be sure that students understand which are the legs and the hypotenuse.
- Prove the Pythagorean Theorem
- 1. Start with a right triangle.
- 2. Construct the altitude
- 3. Use it in an example.
- Start with an example where the hypotenuse is missing.
- Ask students to use the Pythagorean Theorem to find the length of the hypotenuse.
- Example: leg $1 = 4$, leg $2 = 6$
- Answer $-c = 7.2$
- Be sure that students understand that they will probably need to round to the nearest tenth.
- Move to finding a missing side length.
- Example, leg $1 = a$, leg $2 = 4$, hypotenuse $= 5$
- Have students solve this for leg a.
- Answer is 3.
- Introduce the concept of a Pythagorean Theorem. Show the difference between the first example where we did not have a perfect square and needed to round, and the second example where our answer was a perfect square.
- Return to the text and demonstrate the other Pythagorean Triples.
- Move on to finding the area of an isosceles triangle.
- Walk through this example in the text.
- Complete the exercise on the board step by step.
- Then allow time for student questions.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

- Review constructing an altitude.
- Review symbol for similar.
- Review finding square roots/radicals.

- Review the concept of a perfect square.

IV. Alternative Assessment

- Observe students as they work.
- Check in periodically throughout the lesson to be sure that students understand the material.
- Review any information that is not clear.

Converse of the Pythagorean Theorem

I. Section Objectives

- Understand the converse of the Pythagorean Theorem.
- Identify acute triangles from side measures.
- Identify obtuse triangles from side measures.
- Classify triangles in a number of different ways.

II. Multiple Intelligences

- Begin by teaching all of the information in this lesson.
- You will need to prepare this activity by creating triangles of different sizes to put around the room. Be sure that there are some acute, obtuse and right triangles.
- Then, let students know that they are going to go on a triangle hunt. They are going to search around the room and locate different triangles.
- Each student needs to find a triangle and test it out to figure out if the triangle is an acute, obtuse or right triangle.
- They need to be prepared to justify their answer.
- The students should repeat this process with three different triangles.
- When finished, have the students gather in small groups to share their findings.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Define the Converse of the Pythagorean Theorem.
- Review that a converse statement switches the if and the then part of a conditional statement.
- Write out the formula for finding out whether a triangle is right triangle, an acute triangle or an obtuse triangle.

IV. Alternative Assessment

- Listen in on the group discussions.
- Be sure to ask questions and probe into student thinking.
- Also check each student's work on the triangles.
- This will help you to assess the accuracy of the student work.
- You may want to collect the work for a classwork grade.

Using Similar Right Triangles

I. Section Objectives

- Identify similar triangles inscribed in a larger triangle.
- Evaluate the geometric mean of various objects.
- Identify the length of an altitude using the geometric mean of a separated hypotenuse.
- Identify the length of a leg using the geometric mean of a separated hypotenuse.

II. Multiple Intelligences

- One way to differentiate this lesson is to have the students teach the concepts in the lesson.
- You can do this by dividing up the content as follows.
- 1. Group 1- teaches a review of arithmetic mean.
- 2. Group 2- teaches geometric mean
- 3. Group 3- teaches finding the length of an altitude
- 4. Group 4- teaches finding the length of a leg
- If you have a large class, you can assign one group the same topic for a different perspective.
- If you choose to do this activity, DO NOT teach the content first.
- Assign students the text and let them decipher it.
- This will also give you an opportunity to observe student understanding.
- Allow time for group work and request that students use diagrams in their presentations.
- When finished, each group "teaches" their concept to the others.
- Allow time for feedback, questions and clarification.
- Intelligences- logical- mathematical, linguistic, visual- spatial, interpersonal, intrapersonal, bodily- kinesthetic

III. Special Needs/Modifications

- Inscribed- remind the students of the circles
- Define altitude.
- Review definition for similar objects.
- Review finding the arithmetic mean.
- Provide students with these notes to help clarify the material.
- 1. To find the length of the altitude- take the length of the segments of the divided hypotenuse and find the geometric mean.
- 2. To find the length of the leg- multiple line segment of divided hypotenuse times the length of the hypotenuse and take the square root of the product.

IV. Alternative Assessment

- Provide feedback during presentations.
- Assess student learning during group work and presentations.

Special Right Triangles

I. Section Objectives

- Identify and use the ratios involved with right isosceles triangles.
- Identify and use the ratios involved with 30-60-90 triangles.
- Identify and use ratios involved with equilateral triangles.
- Employ right triangle ratios when solving real- world problems.

II. Multiple Intelligences

4.8. Right Triangle Trigonometry

- Begin this lesson with an exploration about what happens when you divide up different shapes. Do this before teaching the content of the lesson.
- Start by having the students draw an equilateral triangle.
- Pose the question "What happens when you divide an equilateral triangle in half?"
- Have students actually cut their triangles in half using scissors.
- Then brainstorm answers to the questions.
- Then begin a new exploration.
- Have students draw a square.
- Pose the question, "What happens when you cut a square in half along the diagonal?"
- Have students cut their squares along the diagonal using scissors.
- Then brainstorm answers to the question.
- You should be able to create two columns of this information on the board.
- Label one side $45 - 45 - 90$ and the other side $30 - 60 - 90$
- Tell students that these are the concepts that you are going to be working with in the lesson.
- As you teach the lesson, keep referring back to the information that the students have already discovered during the exploration at the beginning of the class.
- Then move on to the content in the text.
- Intelligences- linguistic, logical-mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

- List this description of the right isosceles triangle on the board/overhead.
- Two sides the same length
- Congruent base angles of $45°$.
- One right angle
- Review the Pythagorean Theorem.

IV. Alternative Assessment

- Have students work through the problems in their notebooks as they are covered in the text.
- Then allow time for questions and answers.

Tangent Ratios

I. Section Objectives

- Identify the different parts of right triangles.
- Identify and use the tangent ratio in a right triangle.
- Identify complementary angles in right triangles.
- Understand tangent ratios in special right triangles.

II. Multiple Intelligences

- To differentiate this lesson, keep it active by including students in designing triangles to determine the tangent ratio.
- Begin by covering the material in the lesson.
- When finished, ask students to work with a partner and design three different right triangles.
- Ask students to measure and label the side lengths of each triangle, and label each angle.
- Then have the students exchange papers.

- The students each find the tangent ratios for each angle in each of the three triangles.
- Once this is completed, ask them to compare their answers with the chart which gives the angle measure for different special triangles.
- Have the students note if any of the triangles drawn fall into the category of these special triangles.
- When finished, ask the students to check each other's work.
- Allow time for whole class feedback.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

- Write on board "Trigonometric ratios show the relationship between the sides of a triangle and the angles inside it."
- Define Tangent Ratio- of an angle is

$$\text{Tan}x = \frac{\text{Length of opposite side}}{\text{Length of adjacent side}}$$

- The x refers to the angle we are focused on.

IV. Alternative Assessment

- Collect student work and use the triangles to assess student understanding.
- Listen to student comments following the activity.
- Allow time for student questions.

Sine and Cosine Ratios

I. Section Objectives

- Review the different parts of right triangles.
- Identify and use the sine ratio in a right triangle.
- Identify and use the cosine ratio in a right triangle.
- Understand sine and cosine ratios in special right triangles.

II. Multiple Intelligences

- To differentiate this lesson, teach the material in the lesson and then complete a "working backwards" activity with the students.
- In the last lesson, we worked with a right triangle and wrote out the tangent ratios for the angles in the triangle.
- In this lesson, we are going to start with the cosine and sine for the different angles of a right triangle. Then the students need to take these ratios and design a triangle that matches the ratios.
- In this way, the activity is called "working backwards."
- The angles of the triangle are A, B and C.
- $\sin A = \frac{6}{7}$
- $\sin B = \frac{5}{7}$
- $\cos A = \frac{5}{7}$
- $\cos B = \frac{6}{7}$
- Allow time for the students to work with these ratios and draw a right triangle that matches the ratios.
- When finished, allow time for student sharing.
- Intelligences- logical- mathematical, linguistic, visual- spatial, interpersonal, intrapersonal.

4.8. Right Triangle Trigonometry

III. Special Needs/Modifications

- Review the parts of a triangle.
- Remind students that the *x* in the cosine and sine ratios refers to the angle that we are focusing on.
- Break down the two formulas and write them on the board. Request that the students copy them down in their notebooks.
- Allow extra time for questions.

IV. Alternative Assessment

- Collect student work after the activity.
- Use this to assess student understanding and provide extra support for students who are having difficulty.

Inverse Trigonometric Ratios

I. Section Objectives

- Identify and use the arctangent ratio in a right triangle.
- Identify and use the arcsine ratio in a right triangle.
- Identify and use the arccosine ratio in a right triangle.
- Understand the general trends of trigonometric ratios.

II. Multiple Intelligences

- The best way to differentiate this lesson is to break down the information in the lesson. This will help all students.
- Here are some notes to give the students as you teach the information in the lesson.
- Inverse of a trigonometric function has the word arc in front of it.
- Inverse Tangents
- Convert measurement to degrees in two ways.
- 1. Use a table of trigonometric ratios.
- 2. Use a calculator with "arctan", "atan" or "tan_1"
- This will give you the measure of the angle in degrees.
- Notice that we use the approximately symbol for measurements that are not exact.
- Point this out for students in the lesson examples.
- Inverse Sine
- You can find the arcsine by the same two methods as the arctangent.
- This converts the measurement to degrees.
- Inverse Cosine
- You can find the arccosine the same two ways.
- This will convert the measurement to degrees.

III. Special Needs/Modifications

- Begin with some work on inverses.
- Be sure that students understand an inverse of an operation undoes the operation.
- Use a one- step equation to show students this.
- Then use the notes in the Multiple Intelligences section to break down the content for the students.

IV. Alternative Assessment

- Observe students through this lesson.
- Allow plenty of time for the students to ask questions.
- Repeat examples or information that seems unclear.

Acute and Obtuse Triangles

I. Section Objectives

- Identify and use the Law of Sines.
- Identify and use the Law of Cosines.

II. Multiple Intelligences

- There is a lot of information in this chapter. I recommend breaking it down and going through the examples slowly so that students are given a visual aid, an auditory aid and a chance to verbally ask questions.
- Intention of lesson- to apply the sine and cosine ratios to angles in acute and obtuse triangles.
- Law of Sines- is constant. It can be used to find the missing lengths in triangles.
- Review using a calculator to find the value of sines.
- Law of Cosines- works on acute, obtuse and right triangles.
- If the students seem lost during this lesson, break them up into small groups. Assign each group either the Law of Sines or the Law of Cosines and have them create a poster explaining the steps to using these laws in an example.
- The students can even use one of the examples in the text.
- By having the students create a poster to explain the information, the students will learn to assimilate the information themselves.
- Allow time for each group to explain their poster when finished.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal

III. Special Needs/Modifications

- Begin with a review of sines and cosines.
- Allow time for student questions.
- Review acute triangles have all angles that are less than 90.
- Review that obtuse triangles have one angle that is greater than 90.
- Review using a calculator to find the value of sines.

IV. Alternative Assessment

- Walk around while the students are working on their posters.
- Listen to the conversation in the groups.
- Are the students on task? Are they having difficulty?
- Often if the conversation has strayed from the content of the assignment, the students are lost and not sure what to do next.
- Assess student understanding through posters and presentations. Clarify points that have been missed or are incorrect.

4.8. Right Triangle Trigonometry

4.9 Circles

About Circles

I. <u>Section Objectives</u>

- Distinguish between radius, diameter, chord, tangent, and secant of a circle.
- Find relationships between congruent and similar circles.
- Examine inscribed and circumscribed polygons.
- Write the equation of a circle.

II. <u>Multiple Intelligences</u>

- To differentiate this lesson, break the lesson down into two sections. In the first section, cover all of the basic information about circles.
- Have the students work on creating a diagram of a circle. Their diagram must have the following things labeled: radius, chord, diameter, secant, tangent line.
- Encourage students to make their diagrams colorful.
- Allow time for sharing when students are finished.
- The second part of the lesson involves more of the operations associated with circles.
- For this lesson, be sure that students have graph paper to work with.
- Complete the examples on the board and walk through each of the examples and all of the steps needed to complete each one.
- Point out where to find the radius and the ordered pair in the equation.
- Make this section interactive so that you work through the example on the board/overhead while the students work through it in their notebooks.
- Working through this as a whole class will help the students to follow the steps of each problem.
- Intelligences- linguistic, visual- spatial, logical- mathematical, interpersonal, intrapersonal.

III. <u>Special Needs/Modifications</u>

- Include the following notes for students.
- Two circles are congruent if they have the same radius. Two circles are similar if they have different radii. Their similarity is shown through a ratio.
- When writing similarity ratios be sure to simplify.
- Remember that you can write the ratios in three ways. The text uses a colon, but you can use a fraction or the word "to".
- Define chord.
- Define diameter.
- Define secant.
- Tangent line- touches a circle at one point. This is called the point of tangency.
- Inscribed polygon- convex polygon inside circle.
- Circumscribed polygon- convex polygon around circle.
- Review convex polygons.
- Equations with graphing- work through slowly.

- Concentric circles- practice drawing them.

IV. Alternative Assessment

- Assess this lesson through student drawings.
- If the students can draw and label the parts of the circle, then they have an understanding of it.
- If the students can graph the circles, then they have an understanding of the process.

Tangent Lines

I. Section Objectives

- Find the relationship between a radius and a tangent to a circle.
- Find the relationship between two tangents draw from the same point.
- Circumscribe a circle.
- Find equations of concentric circles.

II. Multiple Intelligences

- Review tangents of a circle and the point of tangency.
- Tangent to a Circle Theorem- work with this theorem by first having the students complete an exploration.
- Have them begin by drawing a circle, the radius and a tangent line.
- Then ask what they can observe about the relationship between the radius and the tangent line. You want them to discover the theorem on their own and then you can put a name to it for them.
- When working with the Pythagorean Theorem and finding the hypotenuse of the triangle, ask students "How can we find the length of the hypotenuse?"
- Once again, you want the students to come up with the Pythagorean Theorem on their own.
- Show Pythagorean Theorem in Example 1.
- When working with the Tangent Segments from a Common External Point Theorem- begin with an exploration.
- Have the students draw it out and then record student observations on the board.
- You want them to discover the theorem on their own.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal

III. Special Needs/Modifications

- There is a lot of skills/vocabulary to review in this lesson. Do not assume that the students remember.
- Review radius.
- Define circumscribe.
- Define concentric circles.
- Review what is meant by a contradiction.
- Example 1- rather than beginning by proving this theorem through a contradiction, use the exploration first. Then go through and show how to use a contradiction. Beginning with the contradiction can confuse many students.
- Review the Pythagorean Theorem.
- Review converse statements.
- Example 5- show students where the hypotenuse is located.
- Review how to rationalize a denominator.
- Review HL congruence.

4.9. Circles

IV. Alternative Assessment

- Review student thinking by observing their work during the observations.
- Are the students able to come to conclusions about each theorem before it is actually taught?
- Is there higher level thinking involved?
- Can the students verbalize the steps to working through an example?

Common Tangents and Tangent Circles

I. Section Objectives

- Solve problems involving common internal tangents of circles.
- Solve problems involving common external tangents of circles.
- Solve problems involving externally tangent circles.
- Solve problems involving internally tangent circles.
- Common tangent

II. Multiple Intelligences

III. Special Needs/Modifications

- This lesson focuses on problem solving.
- These notes will work for multiple intelligences and special needs students.
- The work that we are going to do to differentiate this lesson is to be sure that the steps to working through the problems are clear and understood.
- List out these steps as you teach the lesson. Request that the students copy these notes into their notebooks.
- Define common internal tangent.
- Define common external tangent.
- Steps to common External Tangents
- 1. Label the diagram.
- 2. Draw a line segment that joins the centers of the two circles.
- 3. Draw in the perpendicular segments.
- 4. Look for any polygons – Example = rectangle.
- Steps to Common Internal Tangents
- 1. Look for similarity. Here in Example 2, there are similar triangles.
- "Who can we find the length of x? Think back to our work on similar triangles."
- Lead students to discover working with ratios.
- 2. Then we can find the length of the hypotenuse of the two triangles to identify the distance between the two circles.
- Use the Pythagorean Theorem to do this.
- Intersecting circles- define internally and externally tangent
- Walk through each example with the students. Rely on previously learned material from this lesson.

IV. Alternative Assessment

- Be sure that the students are making the connections between the circles, perpendicular segments and where polygons and right triangles can help them in solving each problem.
- The students need to combine previously learned material to be successful with this lesson.
- Review information and skills when necessary.

Arc Measures

I. Section Objectives

- Measure central angles and arcs of circles.
- Find relationships between adjacent arcs.
- Find relationships between arcs and chords.

II. Multiple Intelligences

- Define Central angle
- Define arc
- In each part of this lesson, work with each diagram and then have the students brainstorm different observations about each diagram. Then give them the information from each diagram.
- For example, begin with the diagram that demonstrates the Arc Addition Postulate. It is common sense that the two arcs would add up to be the total. Give students the name of the postulate and ask them to come up with the meaning of the postulate given the diagram. Then walk them through it.
- Do the same with the diagram on the congruent chords.
- In Example 1- show that all angles must add to be 360°.
- Use the straight line to show the 180°, then talk to the students about working through the puzzle of figuring out the measure of each angle inside the circle.
- Keep this lesson interactive and engage students in participating in the discussion.
- Refrain from simply presenting the material to them.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

- Define semicircle with a diagram.
- Define major arc with a diagram- named with three letters.
- Define minor arc with a diagram- named with two letters (endpoints).
- Review solving multi- step equations.
- Present diagrams first and then engage the students in a discussion about the diagram.

IV. Alternative Assessment

- Create a checklist of important points that you want students to discover in the lesson.
- Then listen for these points while you discuss the information and present the material to the students.
- During the practice exercises, pair students up to work together. This will help students to clarify the given information.

Chords

I. Section Objectives

- Find the lengths of chords in a circle.
- Find the measure of arcs in a circle.

II. Multiple Intelligences

4.9. Circles

- To differentiate this lesson, have the students work on an activity in small groups.
- The students are going to create a diagram to teach a theorem to the other students in the class.
- Divide the students into groups of four. Each group is assigned a different theorem.
- Group 1- The perpendicular bisector of a chord is the diameter.
- Group 2- the perpendicular bisector of a chord bisects the arc intercepted by the chord.
- Group 3- Congruent chords in the same circle are equidistant from the center of a circle.
- Group 4- Two chords equidistant from the center of a circle are congruent.
- Allow time for the students to work and then have each group teach the class about their theorem.
- Allow time for students to ask questions.
- From this activity, move to the longer examples in the text. The students should have an easier time working through these examples now that the theorems are very clear.
- Intelligences- logical- mathematical, linguistic, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal

III. Special Needs/Modifications

- Begin by defining a chord as a line segment whose endpoints are both on a circle.
- Show students a diagram to define a chord.
- Write out each theorem on the board. Request that students write the notes in their notebooks.
- Review the terms diameter, bisector, perpendicular, congruent.
- Point of the significance of the word "equidistant" in two of the theorems.

IV. Alternative Assessment

- Use flexible grouping to engage all learners.
- Walk around and observe students as they work on preparing their lesson.
- Be sure that each presentation accurately teaches the content of the lesson.
- Provide correction and feedback when necessary.

Inscribed Angles

I. Section Objectives

- Find the measure of inscribed angles and the arcs they intercept.

II. Multiple Intelligences

- Begin by teaching the material in the first part of this lesson. Stop before you get to the practical examples where students are actually figuring out angle measures.
- To expand student understanding, make the corollary section interactive.
- Have students work in pairs to draw out an example of each corollary.
- Tell students that you will be collecting the examples at the end of the class.
- Then move on to the actual examples in the lesson.

III. Special Needs/Modifications

- Provide students with the following notes.
- Inscribed angles- vertex on circles, sides are chords, intercepts an arc of the circle.
- Review parts of an angle.
- Review definition of a chord.

- Measure of inscribed angle is $\frac{1}{2}$ of the measure of the arc it intercepts.
- Measure of center angle is 2 (measure of inscribed angle)
- List out the inscribed angle corollaries.
- When working on the multi-step examples, use color to help students to differentiate between which angles are being worked with and which ones aren't being worked with. The color will help students to focus on the appropriate part of the diagram.

IV. Alternative Assessment

- Walk around and help students as they work.
- Collect student work from the corollaries activity.
- Examine each example and see if it clearly demonstrates or shows the corollary.
- Provide students with feedback/correction in the next class.

Angles of Chords, Secants and Tangents

I. Section Objectives

- Find the measures of angles formed by chords, secants and tangents.

II. Multiple Intelligences

- In this lesson, you are going to be working with three main theorems. The students need to learn these theorems and then prove each of the theorems.
- Here are some notes to help students to break down each theorem.
- Theorem- the measure of an angle formed by a chord and a tangent that intersects on the circle equals half of the measure of the intercepted arc.
- mangle $= \frac{1}{2}$ marc
- Look at the first diagram and label the chord, tangent, intercepted arc and possible angles to measure.
- Theorem- angles inside a circle
- mangle $= \frac{1}{2}($marc$1 +$marc$2)$
- Theorem- angles outside circle
- mangle $= \frac{1}{2}($arc$°1 +$arc$°2)$
- Once the students understand the three theorems, go through the example and proofs in the lesson. Ask the students to point out where the theorems are illustrated and explained in each.
- Discuss each example and proof.
- Allow time for questions.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

- Review chords.
- Review secants.
- Review tangents.
- Review interior angles.
- Review exterior angles.
- Review what an intercepted arc is and how to locate it.

IV. Alternative Assessment

4.9. Circles

- As you work through each example, have the students work through the example in their seats.
- When all have finished, ask the students to explain how they solved the problem.
- Then provide feedback.
- You can verbally check in with the students by having them raise their hands if they had the same answer. This will give you a visual cue of how many students were successful and how many were not.

Segments of Chords, Secants and Tangents

I. Section Objectives

- Find the lengths of segments associated with circles.

II. Multiple Intelligences

- Begin this lesson by having the students draw a circle. Then they need to use previously learned information and their text to draw in the following.
- Tangent
- Chord
- Secant
- Tangent segment
- Chord segment
- Secant segment
- Have them use color to draw in each item.
- When students are finished, explain that we are going to be using these diagrams to illustrate three different theorems.
- When working through each theorem, give students the measurements for each section of the circle, and then have them work to figure things out.
- Theorem- If two chords intersect, the product of segments of chord1 = product of segments of chord2.
- Then we create two similar triangles.
- Similar triangles- ratios
- Add in measures and solve for the missing segment length.
- Theorem- If two secants are drawn to a common point, $a(a+b) = c(c+d)$
- The a[U+0080][U+0099]$s = 1^{\text{st}}$ secant
- The c[U+0080][U+0099]$s = 2^{\text{nd}}$ secant
- Draw in two triangles inside the circle.
- Similar triangles- ratios
- Use formula to find the length of the segment of the secant.
- Theorem- tangent and secant$-a(a+b) = c^2$
- The a[U+0080][U+0099]$s =$ secant
- The c = the tangent
- Use it to find the value of the missing tangent length.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, bodily- kinesthetic, intrapersonal.

III. Special Needs/Modifications

- Review chords.
- Review tangents.
- Review secants.

- Review squaring a number.
- Review the distributive property.

IV. <u>Alternative Assessment</u>

- Assess student learning through observation and class discussion.

4.10 Perimeter and Area

Triangles and Parallelograms

I. <u>Section Objectives</u>

- Understand the basic concepts of the meaning of area.
- Use formulas to find the area of specific types of polygons.

II. <u>Multiple Intelligences</u>

- Teach the material in this lesson, and then move on to the activity.
- To complete this activity, you will need to prepare enough drawings for one- half of the students in the class. One- half of the class receives a drawing of a complex figure.
- You will need to prepare the area measurements for this complex figure for the other half of the class.
- Then hand out one to each student. Some students will receive drawings and some will receive measurements.
- The students will need to figure out the measure of their figure and find the person in the room who has the correct area measurement for their figure.
- The activity is complete when both persons are sure that they have been matched up correctly.
- This is a noisy activity, but the students will have a lot of fun doing it. It also has a lot of movement in it which is excellent for kinesthetic learners.
- Then repeat this activity. Be sure that those who received drawings get area measurements and those who had measurements receive drawings.
- Intelligence- linguistic, logical- mathematical, bodily- kinesthetic, visual- spatial, interpersonal, intrapersonal.

III. <u>Special Needs/Modifications</u>

- Provide students with notes to refer to throughout the lesson and activity.
- Area of rectangle $= l \times w$
- Area of parallelogram$= bh$
- Area of triangle $= \frac{1}{2}bh$
- Write out the Congruent Area Postulate
- Write out the Area of Whole is Sum of Parts Postulate.
- Be sure that students copy these notes down in their notebooks.

IV. <u>Alternative Assessment</u>

- Student assessment is done through the activity. Were the students able to find the correct "match- up?"
- Assist students who have difficulty with the assignment.

Trapezoids, Rhombi and Kites

I. <u>Section Objectives</u>

- Understand the relationships between the areas of two categories of quadrilaterals: basic quadrilaterals and special quadrilaterals.
- Derive area formulas for trapezoids, rhombi and kites.
- Apply the area formula for these special quadrilaterals.

II. Multiple Intelligences

- Be sure to teach the material in this lesson in an interactive way. I would recommend going through the material without the text first. That way each student can explore the different characteristics of the three figures without using the information in the text as a guide.
- Then go back to the text and go over the information in it. The students will see this in a new way because they will have already "discovered" it.
- Here are some notes on each figure in the section. This will help the students to "break down" the content.
- Trapezoid- quadrilateral with one pair of parallel sides.
- Formula- $\frac{1}{2}(b_1 + b_2)h$
- Finding the Area of a Rhombus
- 1. Frame a rhombus in a rectangle.
- 2. Notice all of the triangles.
- 3. 4 triangles to fill in the rhombus.
- 4. 8 triangles fill in the rectangle.
- 5. 4 is half of 8
- 6. Area of rhombus $= \frac{1}{2}$ area of a rectangle.
- Formula $= A = \frac{1}{2}d_1 d_2$
- Finding the Area of a Kite
- 1. Frame in a rectangle.
- 2. Notice the similarities with the rhombus.
- 3. Use the same formula as a rhombus.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal.

III. Special Needs/Modifications

- Review finding the area of a rectangle.
- Review finding the area of a parallelogram.
- Review finding the area of a triangle.
- Show how more than one figure can be combined together to create a new figure.
- Example- a rectangle and a triangle together.
- Review finding the area of such a figure.

IV. Alternative Assessment

- Observe students as they work through the explorations.
- Allow a lot of time for students to speculate and share their findings.

Area of Similar Polygons

I. Section Objectives

- Understand the relationship between the scale factor of similar polygons and their areas.
- Apply scale factors to solve problems about areas of similar polygons.

4.10. Perimeter and Area

- Use scale models or scale drawings.

II. Multiple Intelligences

- There are several different components to this lesson.
- First, we can start with the basic information to write the formula for finding the area of similar polygons. In working through this section, use an example on the board and take the students through each step in the text as you do the work out on the board. This will help them to "see" where the formula really comes from.
- The next section is on scale drawings and scale models.
- One of the best ways for the students to understand scale drawings is to complete one.
- You could break the students off into pairs and have them create a scale drawing of the classroom. Allow students to use chart paper, rulers, tape measures, colored pencils and to create their own scale for the diagram.
- Have students work in pairs to complete the table. You could also expand this activity and add Mt. Everest to the table.
- The section on the giant can be fun. Ask the students to create a drawing to show how there aren't any twelve foot giants. They can use the information in the text as a guide.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Review perimeter.
- Review area of different figures.
- Review Pythagorean Theorem.
- Review finding the area of a rhombus.
- Define squaring a number.

IV. Alternative Assessment

- Use observation and student work product to assess student understanding.
- You can collect work for a class work grade when students are finished.

Circumference and Arc Length

I. Section Objectives

- Understand the basic idea of a limit.
- Calculate the circumference of a circle.
- Calculate the length of an arc of a circle.

II. Multiple Intelligences

- When working through this lesson, it is a good idea to begin by reviewing previously learned information about a circle.
- Have students brainstorm a list and then write them on the board.
- These include the labels for radius, diameter, center angle, arc, interior, etc.
- Also review that there are $360°$ in a circle.
- Then move on to the measurement for pi and having students understand the measurement for pi.
- Use the exploration in the text for this.

- The activity to differentiate this lesson comes in the example where the circle is inscribed inside the square on the graph paper.
- The students can count the units to figure out that the length of the side of the square is also the diameter of the square.
- Here is the activity.
- 1. Have students draw their own circle inscribed in a square.
- 2. Exchange papers with a partner.
- 3. Each student must label the length of the diameter.
- 4. Find the circumference of the circle.
- Allow time for sharing when students have finished.
- Walk through the section on how to find the arc measures. Be sure that the students understand the $\frac{60}{360}$ ratio and how it makes sense to multiply the diameter with the measure of the arc to find the arc measure.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Write formulas for finding the diameter and the radius of a circle on the board.
- You can even do a few examples to have students practice finding these measures.
- Write the formula for circumference on the board.
- Allow time for student questions.

IV. Alternative Assessment

- Observe students as they work on the circle dilemma.
- Offer assistance when needed.
- Listen for how the students solved the dilemmas when the students are sharing their work after the activity.

Circles and Sectors

I. Section Objectives

- Calculate the area of a circle.
- Calculate the area of a sector.
- Expand understanding of the limit concept.

II. Multiple Intelligences

- Teach the material in this lesson and then differentiate it with the following activity.
- Activity- have students work in pairs.
- Students begin by drawing a square on graph paper and then inscribing a circle within the square. The students can decide how much of the square is taken up by the circle.
- Have students shade in the area of the square around the circle.
- Exchange papers.
- Students work with each other's papers.
- They need to find the area of each circle.
- Then they need to find the area of the shaded region of each circle.
- Request that students write out the steps that they did to complete this assignment.
- Allow time for students to share their work at the end of the activity.
- When working with the sectors, be sure that the students understand what is meant by a sector.

4.10. Perimeter and Area

- Use a diagram to show students a sector in a circle.
- Then show how it has an arc measure and how it also has a measure of the area of the circle.
- This will help students to make sense of the formula to find the area of the sector.
- Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Review finding the circumference of the circle.
- Review pi.
- Review the concept of the limit and how it leads to pi.
- Write out the formulas on the board. Request students write these notes in their notebooks.

IV. Alternative Assessment

- Collect the student worksheet from the activity.
- Read all of the steps that the students wrote and check their process.
- Is there anything missing? Do the students understand where each part of the formula comes from?
- Is there higher level thinking here or are students just "using" the formula?

Regular Polygons

I. Section Objectives

- Recognize and use the terms involved in developing formulas for regular polygons.
- Calculate the area and perimeter of a regular polygon.
- Relate area and perimeter formulas for regular polygons to the limit process in prior lessons.

II. Multiple Intelligences

- When working through this lesson, be sure to explain each formula and how it was arrived at slowly and with detail.
- I recommend beginning the lesson without the text.
- Use the text as a teacher guide and break down the information in it for the students.
- Use the board/overhead to show each step.
- Begin by labeling the regular polygon and its parts in different colors.
- You can use these colors to track through to the formulas.
- For example, if you used red for the *n* in the diagram, then whenever the n is presented in a formula, you can put it in red.
- Color will help the students to track the information from the diagram to the examples and back again.
- Allow plenty of time for student questions and repeat material as necessary.
- Intelligences- linguistic, logical- mathematical, visual- spatial

III. Special Needs/Modifications

- Go to the simplest version of each of these formulas for the students to make sense of this unit.
- Because there is so much processing in this lesson, special needs students will have difficulty following all of the different possible options.
- Simplify it as much as possible.

- You want the students to understand the core concepts involved.
- Using color, as suggested above, will help special needs students.

IV. Alternative Assessment

- Ask questions and answer a lot of questions in this lesson.
- Since most of this lesson is about student process, be sure that the students are following the lesson.

Geometric Probability

I. Section Objectives

- Identify favorable outcomes and total outcomes.
- Express geometric situations in probability terms.
- Interpret probabilities in terms of lengths and areas.

II. Multiple Intelligences

- Review the basics of probability.
- Review the ratio for probability.
- Activity 1- Basic Probability- have the students use two number cubes and figure out what the probability would be to roll an even number.
- Students can work in pairs during this activity.
- Allow time for them to share their work when finished.
- Are there any surprising results?
- How did the students arrive at their answers?
- Activity 2- Geometric Probability
- Students work in pairs again.
- The students work together to design their own problem for determining geometric probability.
- Have the students write their problems out, use diagrams and not solve the problems.
- You can use these problems at a later date.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Provide the students with some notes on probability.
- Review how to convert fractions, decimals and percentages.

IV. Alternative Assessment

- Collect student problems.
- Check them for accuracy.
- Reassign them to students for a homework or classwork assignment.
- They could also be used for an extra credit problem.
- Have students explain their answers and write them out in words not just show a solution.

4.10. Perimeter and Area

4.11 Surface Area and Volume

The Polyhedron

I. Section Objectives

- Identify polyhedral.
- Understand the properties of polyhedral.
- Use Euler's formula to solve problems.
- Identify regular (Platonic) polyhedral.

II. Multiple Intelligences

- Teach the material in this lesson, and then differentiate it by having the students work in groups to test out Euler's formula.
- Provide the students with actual solids or with diagrams of different polyhedral.
- The students need to use the solids or diagrams to label each polyhedra and to come up with a way to demonstrate how Euler's formula works.
- Allow students time to choose one solid that they are going to use for their "teaching" session.
- When students are finished preparing, allow time for them to teach the other students in the class how Euler's formula works.
- They need to be clear on the number of faces, edges and vertices of their solid.
- Allow time for the students to answer questions.
- Provide feedback to the students.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal

III. Special Needs/Modifications

- Provide students with notes on polyhedrons.
- Polyhedrons
- 3D
- Made of polygons and only polygons- faces
- Polygons join at the edges.
- Edges meet in points called vertices.
- No gaps between them.
- Review the definition of a polygon.
- Write out Euler's Formula.

IV. Alternative Assessment

- Pay close attention to the teaching session.
- Do the students prove Euler's formula or simply state reasons.
- Ask questions and stretch the students to really demonstrate how Euler's formula works and makes sense.

Representing Solids

I. <u>Section Objectives</u>

- Identify isometric, orthographic, cross- sectional views of solids.
- Draw isometric, orthographic, cross- sectional views of solids.
- Identify, draw and construct nets for solids.

II. <u>Multiple Intelligences</u>

- Make this lesson very interactive by giving students the following hands- on tasks.
- Students may work in small groups for this activity.
- In the activity, be sure that students have graph paper, dot paper, plain paper, rulers, tape and colored pencils.
- Students are going to choose a solid to work with. You can provide students with a model of a solid if you have them.
- Then students are going to create four different things.
- 1. Create an orthographic projection of their solid.
- 2. Create a cross- section of the solid.
- 3. Create a net for the solid.
- 4. Use the net to create an actual model of the solid.
- Intelligences- linguistic, visual- spatial, bodily- kinesthetic, logical- mathematical, interpersonal, intrapersonal.

III. <u>Special Needs/Modifications</u>

- Review polyhedral.
- Review faces, edges and bases.
- Define isometric.
- Define perspective.
- Define orthographic projection.
- Define cross- section.
- When students draw an orthographic projection, they will need the following views:
- 1. Top
- 2. Left side
- 3. Back
- 4. Right side
- 5. Front
- 6. Bottom

IV. <u>Alternative Assessment</u>

- An assessment of student understanding can easily be completed by looking at the student work.
- Were the students able to create a net that built a solid?
- Did they use the same solid for all of the pieces of the activity?
- Where did students have challenges?

Prisms

I. <u>Section Objectives</u>

4.11. Surface Area and Volume

- Use nets to represent prisms.
- Find the surface area of a prism.
- Find the volume of a prism.

II. Multiple Intelligences

- When differentiating this lesson, you want to divide it into two sections. The first section is going to be on surface area and the second section is going to be on volume.
- The first thing to do is to present the information on surface area.
- When you teach surface area, have the students create a net of a prism (on graph paper) to work with.
- Once the students have created their nets, then use the formulas for surface area to show students how to calculate the surface area of the prism that they created.
- When finished, allow time for the students to complete their work.
- Then move on to volume.
- Use the same net to calculate the volume of the solid.
- Have the students see where the formula for finding the volume of a solid comes from.
- Building the lesson in this way connects the last few lessons together. We have connected surface area and volume with polyhedra.
- Allow time for students to share their work in the large class or in small groups.
- Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, interpersonal, intrapersonal

III. Special Needs/Modifications

- Write out all new terms and information on the board. Be sure that students copy this information in their notebooks.
- Define prism. What makes a prism a prism?
- Show students the difference between a right prism and an oblique prism.
- Define Area Congruence Postulate
- Define Area Addition Postulate
- Define Surface Area.
- Review formulas for area of a triangle, parallelogram, rectangle and square.
- Show the difference between Lateral area and surface area.
- Define Volume.
- Define Volume Congruence Postulate.
- Define Volume Addition Postulate.
- Show how these two postulates are similar to the ones on surface area.
- Be sure that students know that the capital B in the volume formula means the area of the base not the length of the base.

IV. Alternative Assessment

- Listen to student responses during class discussions.
- More students will have a chance to share their work in small groups.
- If you use small groups, walk around and listen in on each group.
- Make notes of any students who are having difficulties.

Cylinders

I. Section Objectives

- Find the surface area of cylinders.
- Find the volume of cylinders.
- Find the volume of composite three- dimensional figures.

II. Multiple Intelligences

- To differentiate this lesson, you can make it very hands- on by using some different cylinders. Example- Quaker Oats containers, soda cans, etc.
- Teach the material in the lesson.
- Then have the students work to find the surface area and volume of each cylinder.
- The students will need string to determine circumference, rulers, colored pencils and paper.
- Have students draw a net for their cylinder and label all of the measurements involved.
- Then they complete their work.
- Provide time for students to share their work in small groups.

III. Special Needs/Modifications

- Review finding the area and circumference of a circle.
- Define cylinders.
- Show students the difference between right cylinders and oblique cylinders.
- Write out the formulas for surface area and volume of cylinders.
- Steps to Working with Composite Solids
- 1. Break each composite solid into its smaller solid parts.
- 2. Select the correct formula for either surface area or volume based on the problem.
- 3. Find the surface area or volume of each smaller solid.
- 4. Add/subtract the results of the surface area or volume based on the question.

IV. Alternative Assessment

- Observe students as they work in small groups.
- Have the students share their work from the cylinder activity when finished.
- Check student work for accuracy.
- Provide assistance and feedback when necessary.

Pyramids

I. Section Objectives

- Identify pyramids.
- Find the surface area of a pyramid using a net or formula.
- Find the volume of a pyramid.

II. Multiple Intelligences

- Differentiate this lesson by having students complete the following activities.
- Students are going to be working with a pyramid of their choosing.
- Begin by dividing students into groups. While the students will be working with different pyramids, they will have the support of the other students in the group as they work.
- Each student is to choose a type of pyramid: triangular, square, pentagonal, hexagonal, etc.

4.11. Surface Area and Volume

- It is fine if two students in the group choose the same pyramid.
- Next, the students need to draw a net for their pyramid. Have them include measurements of each part of the pyramid.
- Then, students need to find the surface area of the pyramid.
- The lateral area of the pyramid
- The volume of the pyramid
- Have students show all of their work.
- When finished, they need to check the work of one other student in their group.
- Finally, collect student work to assess levels of understanding.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modification

- Define regular pyramid.
- Identify types of pyramids according to each base.
- Define slant height.
- Write the formulas for surface area, lateral area and volume on the board.
- Be sure students understand how to find each measurement.

IV. Alternative Assessment

- Collect and review student work.
- Is the net correctly drawn and labeled?
- Is the surface area of each net correct?
- Is the lateral area of each correct?
- Is the volume correct?
- Have there been any corrections to the work of the students by a peer?

Cones

I. Section Objectives

- Find the surface area of a cone using a net or formula.
- Find the volume of a cone.

II. Multiple Intelligences

- Begin by having the students work to build cones.
- Students begin by drawing a half circle and measure the top of it.
- This top measurement becomes the circumference of the cone.
- Then they cut out the half circle.
- Have the students turn it into a cone.
- Then use the descriptions of the parts of the cone in the text to help the students to understand the parts of the cone.
- After this teach the material in the lesson and then move on to the next activity.
- Next, divide the students into groups.
- In each group, ask the students to choose either the surface area formula or the volume formula.
- With each formula, the students need to find a way to teach how the formula is put together for the other students in the class.

- The students will demonstrate the meaning of each part of the formula and present an example of how to use the formula.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Define cone as a single curved base that tapers to a single point. This point is called the **apex**.
- Base can be a circle or an oval.
- Right cone- apex in the center
- Oblique cone- apex not in the center
- Write the formulas for surface area and volume on the board.

IV. Alternative Assessment

- Create a checklist of the things that the students should be teaching in their lessons.
- Use these checklists during the presentations.
- Make notes of the things that the students cover.
- Be sure to provide feedback for the things that the students miss in their presentations.

Spheres

I. Section Objectives

- Find the surface area of a sphere.
- Find the volume of a sphere.

II. Multiple Intelligences

- Begin by showing students some spheres of different sizes. You can use a baseball, a globe and a basketball for example.
- Often the measurement of a ball is given according to the diameter. Have the diameter of any object that you show the students close by. Use one- for example, a fourteen inch basketball to demonstrate the following.
- Parts of a circle
- 1. $O =$ center point
- 2. $r =$ radius ($\frac{1}{2}$ of diameter)
- 3. $d =$ diameter
- 4. Chord- intersects the center of the circle or sphere.
- 5. Secant- line, ray or line segment that intersects in two places and extends OUTSIDE the sphere
- 6. Tangent- intersects the sphere at only one point.
- SA of a Sphere- use the formula and then use the examples to have students work to find the SA of one or more of the given objects.
- For example, find the SA of the 14" basketball.
- Volume of a sphere- use the formula and then use the examples to have students work to find the volume of each given object.
- Allow time for students to share their work.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

4.11. Surface Area and Volume

- Review circles.
- Compare circles to spheres.
- Show how the parts of a circle relate to the parts of a sphere.
- Review the meaning of surface area.
- Review the definition of volume.

IV. Alternative Assessment

- Assess student understanding of the material through the discussion and through student answers when working with the given objects.

Similar Solids

I. Section Objectives

- Find the volumes of solids with bases of equal areas.

II. Multiple Intelligences

- To differentiate this lesson, begin by teaching the content in the lesson.
- Ask the student's to draw an example of Cavalieri's Principle (Volume of a solid postulate)
- Have students share their example with a peer and then allow time for student sharing.
- The class participation will give you time to see if the students understand the principle.
- Then move on to working with similar solids. The students are going to draw a pair of similar solids and then work to problem solve with the similar solids.
- Tell students to draw a solid that is similar to a rectangular prism with a depth of 4, a width of 6, and a height of 9.
- Students should begin by drawing this given rectangular prism and then draw one similar to it.
- Once this is similar, ask them to write ratios to demonstrate that the prisms are similar.
- Next, have the students find the surface area of each prism and demonstrate that they are similar through the Similar Solids Postulate.
- Finally, ask students to find the volume of each prism and demonstrate that they are similar through the Similar Solids Postulate.
- Allow time for the students to share their work.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Review surface area.
- Review volume.
- Review scale factor.
- Write Cavalieri's Principle on the board. Rename it as the Volume of a Solid Postulate.
- Review similar solids and writing equal ratios.

IV. Alternative Assessment

- Walk around as students work and assess their understanding through observation.
- You can collect student work to use as a classwork grade.
- Offer assistance to students who are in need of help.
- Use flexible grouping to assist these students.

4.12 Transformations

Translations

I. Section Objectives

- Graph a translation in a coordinate plane.
- Recognize that a translation is an isometry.
- Use vectors to represent a translation.

II. Multiple Intelligences

- To differentiate this lesson, make it very interactive.
- Be sure that students have graph paper, colored pencils and rulers at their seats. Work through this lesson on the overhead projector with graph paper yourself so that the students can model the examples and work them out themselves at their seats.
- Begin by reviewing some information about translation.
- Have students draw a translation. Use the example from the text or create one of your own.
- Use the distance formula as was done in the text to review finding the coordinates to graph. Go the extra step and graph an example with the students.
- Define isometry- explain how the distance between the two points of an image is the same as the distance between the two images.
- When you look at the example that the students have just drawn, illustrate this.
- Then name it with the Translation Isometry Theorem.
- Move on to vectors. Begin by having students graph two line segments on a coordinate grid. You can use the same line segments as in Example 2, or you can create your own.
- The key is that you want the students to understand that the vector is the horizontal and vertical direction connected with each graphed line segment.
- Drawing in vectors is a great opportunity for students to use color to differentiate the vectors.
- Then ask students to name the horizontal component and the vertical component of the graphed line segments.
- Expand Example 3. Read through it with the students and then give them time to graph the two triangles and to explore what happens to the triangles.
- Allow time for student sharing.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal

III. Special Needs/Modifications

- Review translations and what makes a translation.
- Review using the distance formula with a translation.
- Define isometry.
- Define the Translation Isometry Theorem.
- Define Vector.

IV. Alternative Assessment

- Assess student understanding through discussion.
- Allow time for student feedback.
- Assess understanding again with the work done to expand Example 3. How did the students do with this? Do they understand vectors?
- What conclusions did they draw from the example?

Matrices

I. Section Objectives

- Use the language of matrices.
- Add matrices.
- Apply matrices to translations.

II. Multiple Intelligences

- Begin by teaching the material in the lesson. Then move on to the activity.
- Students will need graph paper, rulers and colored pencils.
- Students can work with a partner for this activity.
- Ask students to draw a triangle, a square or a rectangle on the coordinate grid.
- Then have them create a matrix of the coordinates of their polygon.
- Next, students are going to create a translation of the polygon that is two units down and three units to the right.
- *Note if this doesn't work with the student's image change it to two units up and three units to the left.*
- Then have the students design a matrix to represent this translation.
- Finally students will add the two matrices together.
- Ask them to exchange papers with a peer for a check of their work.
- After their peer review, make any necessary changes.
- Allow time for student's to share their work.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal.

III. Special Needs/Modifications

- Notes on Matrices
- 1. A multidimensional way to show data.
- 2. They have their own arithmetic.
- 3. In brackets, a matrix is an array of numbers.
- 4. Numbers are arranged in rows and columns.
- You add the elements of a matrix by adding the value in each place in one matrix with the matching value in the same place in the other matrix.
- Matrices can represent real- life data.
- Matrices can represent the vertices of a polygon.
- Operation with matrices and translations = ADDITION

IV. Alternative Assessment

- Collect student work and use it as a way to check student understanding.

Reflections

I. <u>Section Objectives</u>

- Find the reflection of a point in a line on a coordinate plane.
- Multiple matrices.
- Apply matrix multiplication to reflections.
- Verify that a reflection is an isometry.

II. <u>Multiple Intelligences</u>

- To differentiate this lesson, teach the material in the lesson first, and then use this activity to give the students a hands- on way of practicing multiplying matrix reflections.
- Have the students work in pairs.
- They will need graph paper, rulers and colored pencils.
- Students may choose a polygon and draw it on the coordinate grid.
- Then have them show that it is reflected by the line $y = x$.
- Students take the vertices of their polygon to create a matrix for it.
- Then multiply the matrix of the polygon by the matrix represented by $y = x$.
- Finally, students show the product in a new matrix.
- Allow time for student sharing in a whole class discussion or in small groups.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal.

III. <u>Special Needs/Modifications</u>

- Operation with reflections = MULTIPLICATION
- Matrix multiplication-
- Multiply firsts by firsts and seconds by seconds- then add the products
- You can't multiply a smaller matrix by a larger one.
- You can multiply a larger matrix by a smaller one.

IV. <u>Alternative Assessment</u>

- Observe students as they work through the assignment.
- Offer assistance when necessary.
- Check for student understanding when discussing the activity in the whole class discussion.
- If students are discussing in small groups, walk around and check in with them.

Rotations

I. <u>Section Objectives</u>

- Find the image of a point in a rotation in a coordinate plane.
- Recognize that a rotation is an isometry.
- Apply matrix multiplication to rotations.

II. <u>Multiple Intelligences</u>

4.12. Transformations

- To differentiate this lesson, teach the material in the lesson first, and then let students practice working with rotations.
- The students are going to play a game called "Pass the Image"
- To prepare this activity, you will need to prepare some small square images. You can design a square where it is divided on the diagonal and one- half of it is blue and the other half is red, etc.
- Students are going to be given an image. They need to draw the image according to the rotation specified in the instruction.
- For example, a student is given an image card, they start by drawing the image as it is.
- Then they are told to draw it at 180° rotation.
- Then they draw it at a 90 degree rotation.
- A 45 degree rotation.
- A 270 degree rotation.
- Then when finished, they pass the image and are given a new one.
- You can do this several times and the students can then compare their work with other students who had the same images.
- Break up students in pairs to have them compare and discuss their work.
- Next, you can move to working with an image on the coordinate plane.
- You can have students draw their own or work with the exercises in the text.
- If you do this activity first, the students will have an excellent understanding of a rotation before moving to the coordinate grid.
- Intelligences- linguistic, logical- mathematical, interpersonal, intrapersonal, visual- spatial, bodily- kinesthetic.

III. Special Needs/Modifications

- Review the basics of matrices.
- Review how to multiply a matrix.
- Review how to draw a matrix using the vertices of a polygon.
- Define rotation
- 1. Center at the origin with an angle of rotation of $n°$.
- 2. Point moves counterclockwise along an arc of a circle.

IV. Alternative Assessment

- Create a "key" of what each image looks like after it is rotated.
- Then use this key to check student work.

Composition

I. Section Objectives

- Understand the meaning of composition.
- Plot the image of a point in a composite transformation.
- Describe the effect of a composition on a point or polygon.
- Supply a single transformation that is equivalent to a composite of two transformations.

II. Multiple Intelligences

- Begin by introducing the concept of a composition.

- A composition is when transformations are "put together". In this lesson, we will be putting together translations, reflections and rotations.
- Glide Reflection- a composition of a reflection and a translation. The translation is in a direction parallel to the line of reflection.
- Expand Example 1- Before moving to the matrix, have the students draw out this glide reflection. This will give them a hands- on way to see the two images without first moving to the matrices. This will keep it in a visual way, before moving to an arithmetic way.
- Once students have practices drawing in the glide reflection, move to using the matrix to figure out the same information. At this point, you can refer back to the text.
- The technology integration in this chapter is also a great way to provide students with a visual and hands- on way of working with the material.
- Provide time for feedback, discussion and questions after completing the work with technology.
- Intelligences- linguistic, logical- mathematical, bodily- kinesthetic, visual spatial, interpersonal.

III. Special Needs/Modifications

- Review translations, reflections and rotations.
- Review matrices.
- Review the matrix for a 180° rotation $-\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
- Review the matrix for a 90° rotation $-\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

IV. Alternative Assessment

- Allow plenty of time for the students to ask questions during this lesson.
- Spend time on reviewing previously learned skills (ie. How to multiply a matrix) if necessary.

Tessellations

I. Section Objectives

- Understand the meaning of tessellation.
- Determine whether or not a given shape will tessellate.
- Identify the regular polygons that will tessellate.
- Draw your own tessellation.

II. Multiple Intelligences

- This is a fun lesson and students usually love working with tessellations.
- Provide students with some information on tessellations and then have them work on two activities.
- The first activity, you will need to prepare.
- Students will work in groups, so you will need a few polygons/shapes for each group. If you can provide different polygons/shapes for each group- great.
- Give each group their polygons and tell them that they will need to prove whether each one tessellates or not.
- Students need to demonstrate that it has no gaps, no overlapping shapes, that the entire plane is covered in all directions.
- Also have students demonstrate that it surrounds a point.
- Allow students time for this exploration and then have students share their work.

4.12. Transformations

- The second activity is to have students create their own tessellation.
- Encourage students to be creative and design a colorful tessellation of their own creation.
- Intelligences- linguistic, logical- mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Write the notes of how to tell if a shape tessellates on the board.
- Provide students with a few visual examples of some tessellations.
- Have them use actual pattern blocks to explore.

IV. Alternative Assessment

- Grade student tessellations.
- You will be able to tell if the students executed the concept well by looking at their tessellations.
- Be sure to include design and color in your evaluation.
- Also check the edges of the plane- did the students successfully fill- in partial images?

Symmetry

I. Section Objectives

- Understand the meaning of symmetry.
- Determine all the symmetries for a given plane figure.
- Draw or complete a figure with a given symmetry.
- Identify planes of symmetry for three- dimensional figures.

II. Multiple Intelligences

- To differentiate this lesson, divide the students into groups.
- Ask each group to come up with an example to explain line symmetry, rotational symmetry, point symmetry and planes of symmetry.
- When finished, have each group share their images.
- Then move on to the next part of the activity.
- Ask each group to draw half of an image that has line symmetry, rotational symmetry, point symmetry. Students can use objects in the room to help them brainstorm which image to draw for each. Students may use examples from biology as well.
- Then they are going to pass their papers to a group near them.
- The next group must finish the drawings according to each description.
- When finished, allow time for sharing.
- Intelligences- linguistic, logical-mathematical, visual- spatial, bodily- kinesthetic, interpersonal, intrapersonal.

III. Special Needs/Modifications

- Two- dimensional
- 1. Line symmetry- left- right symmetry. Divides the figure into two congruent halves. When flipped over the line of symmetry, it is exactly the same.
- 2. Rotational symmetry- rotated image looks exactly like it did before the rotation.

- 3. Point symmetry- looks the same right side up and upside down. It looks the same from the left and from the right.
- Three dimensional
- Planes of symmetry- divide a 3*D* figure into two parts that are reflections of each other. Think of a cylinder or cube.

IV. Alternative Assessment

- There are several ways to assess student understanding in this lesson.
- The first way is with the images to represent each type of symmetry.
- The second is with the partial images and completions.
- Collect student work and check to see that student work is complete and accurate.
- Provide students with feedback or corrections.

Dilations

I. Section Objectives

- Use the language of dilations.
- Calculate and apply scalar products.
- Use scalar products to represent dilations.

II. Multiple Intelligences

- To differentiate this lesson, begin by teaching the concepts in the lesson to the students.
- Then, students are going to create their own dilations using scalar multiplication.
- Students will need graph paper, rulers and colored pencils.
- Ask the students to show all of their work.
- Here are the steps to the activity.
- 1. Draw a polygon of choice on the coordinate grid.
- 2. Use the vertices of the polygon to create a matrix.
- 3. Select or use a given scale factor.
- 4. Multiply the scale factor with the matrix.
- 5. The product is a new matrix- the new matrix is the vertices of the dilated matrix.
- 6. Draw in the figure on the coordinate grid.
- Allow time for the students to share their work when finished.
- Intelligences- linguistic, logical- mathematical, visual- spatial, interpersonal, intrapersonal

III. Special Needs/Modifications

- Review that a dilation is an image "blown up" or decreased in size.
- Transformations are also dilations.
- Dilations can be written as a matrix.
- Review scale factor.
- Scalar Multiplication- Take the real number and multiply it with each element in a matrix. The product is a new matrix.
- To create a dilation on the coordinate grid
- 1. Design a matrix based on the vertices of a polygon drawn on the coordinate grid.
- 2. Decide on a scale factor for the dilation.

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- 3. Multiply the scale factor with the matrix.
- 4. The product is a new matrix that is the vertices of the dilated figure.
- 5. Draw in the new figure on the coordinate grid.

IV. Alternative Assessment

- Collect student work.
- Check to be sure that the scalar multiplication is accurate.
- Be sure that the images match the elements of each matrix.
- Assign students a classwork grade based on their work.

CHAPTER **5**

Geometry TE - Problem Solving

Chapter Outline

5.1 Basics of Geometry

Points, Lines and Planes

I. Section Objectives

- Understand the undefined terms point, line and plane.
- Understand the defined terms, including space, segment, and ray.
- Identify and apply basic postulates of points, lines and planes.
- Draw and label terms in a diagram.

II. Problem Solving Activity- Global Architecture

- Objective: The objective of this activity is to have students recognize the postulates connected with points, lines and planes in real life architecture.
- Here are the postulates.

  - Line Postulate - There is exactly one line through any two points.
  - Plane Postulate - There is exactly one plane that contains any three non- collinear points.
  - Postulate - A line connecting points in a plane also lines with the plane.
  - Postulate - the intersection of two distinct lines will be a single point.
  - Postulate - the intersection of two planes is a line.

- You can either print the pictures, use computer displays or slides.
- The students need to use the following postulates and identify examples of each postulate as displayed in the picture.
- Students should be encouraged to use mathematical language as they describe and write about each example of a postulate.
- Figure01.01.01- The Guggenheim Museum in Spain http://en.wikipedia.org/wiki/Guggenheim_Museum_Bilbao
- Figure 01.01.02- Eiffel Tower http://en.wikipedia.org/wiki/File:Tour_Eiffel_Wikimedia_Commons.jpg
- Figure01.01.03- St. Basil's Cathedral http://en.wikipedia.org/wiki/Saint_Basil%27s_Cathedral

III. Meeting Objectives

- Students found examples of points, lines and planes in each architectural figure.
- Students identified and applied the basic postulates associated with points, lines and planes.

IV. Notes on Assessment

- There are several different illustrations of each postulate in the architecture provided. This is an activity where students are not only expected to be able to locate an example of each, but also where they need to use mathematical language to write about how each postulate is shown in the architecture.
- An example of this is in the picture of the Guggenheim. You can see that two of the planes intersect in exactly one line. The sun is even shining directly on the line.

Segments and Distance

I. <u>Section Objectives</u>

- Measure distances using different tools.
- Understand and apply the ruler postulate to measurement.
- Understand and apply the segment addition postulate to measurement.
- Use endpoints to identify distances on a coordinate grid.

II. <u>Problem Solving Activity- Town Design</u>

- Students are assigned the task of using measurement to design their own town.
- Students will need rulers, pencils, colored pencils and chart paper.
- Each town needs to have the following buildings in it: a post office, a police station, a bank, a park, a school and some houses. The students can expand this list if they choose.
- Each town map needs a scale to determine the distances from one building to another building. This scale could be expanded to include standard and metric measurement.
- Each students needs to develop a key that shows the measurements from one building to another.
- Distances must be actual distances measured with rulers and matched to scale. This could be expanded to include both standard and metric measurement.
- After the design has been completed, the students need to write a series of directions and distances for someone to travel around their town. The distances should be clear enough for any other student to follow.
- Students pair up with a peer to check and make sure that student directions/measurements are accurate.
- Finally, allow students time to share their designs in small groups or with the entire class.

III. <u>Meeting Objectives</u>

- Students were required to measure distance using different tools.
- Street measurement is a real life example of the ruler postulate.

IV. <u>Notes on Assessment</u>

- Observe students as they work on this assignment.
- You can create a rubric where each piece of the assignment is worth points.
- For example, the scale is worth 4 points.
- Then student grading is calculated out of the total possible points.
- Check final work for accuracy.
- Are the directions/measurements clear and accurate?
- Does the town contain all of the essential buildings?
- Provide student feedback and grading according to a rubric.

Rays and Angles

I. <u>Section Objectives</u>

- Understand and identify rays.
- Understand and classify angles.
- Understand and apply the protractor postulate.

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- Understand and apply the angle addition postulate.

II. Problem Solving Activity- Angle Hunt

- Use Figure01.03.01 in this activity.
- Students are going to use the figure to apply each of the section objectives.
- Students will need rulers, colored pencils or markers and protractors.
- First, students take the drawing and find ten different angles.
- They need to use letters and label each of the following angles.
- Next, they make a list of each of the ten angles and classify each.
- Then, they apply the protractor postulate to measure each of the ten angles.
- Finally, they apply the angle addition postulate and create four different combinations of angles to calculate total measures.
- When finished, pair up students and have them check each other's work.
- Each student needs to provide their peer partner with verbal and written feedback.
- Then allow students time to share feedback in small groups.

III. Meeting Objectives

- Students demonstrated understanding angles.
- Students demonstrated classifying angles.
- Students applied the protractor postulate.
- Students applied the angle addition postulate.

IV. Notes on Assessment

- Create a rubric to grade each students work.
- Were ten angles labeled and identified?
- Are the measurements of each angle accurate?
- Did students successfully use the angle addition postulate?
- Provide students with feedback and grading on their work.

Segments and Angles

I. Section Objectives

- Understand and identify congruent line segments.
- Identify the midpoint of line segments.
- Identify the bisector of a line segment.
- Understand and identify congruent angles.
- Understand and apply the Angle Bisector Postulate.

II. Problem Solving Activity-Revolving Door Design

- Use the two diagrams from this Wikipedia site. These are Figure01.04.01 and Figure01.04.02
- http://en.wikipedia.org/wiki/Revolving_door
- Students are going to be assigned the task of designing their own revolving door.
- Point out that the diagram of the revolving door has four wings to it.
- The students are going to be assigned the task of designing a revolving door with at least six wings in it.

- Students will need rulers, protractors, pencils, and paper.
- They can choose to add more wings, but the revolving door needs to have at least six in it.
- Here are the specifics of the assignment:
- Design a revolving door with at least six wings.
- Each angle must be congruent.
- Label each angle measure using a protractor.
- Identify line segments that are bisected.
- Identify the midpoint of each line segment.
- Label each part of the revolving door and demonstrate congruency.

III. Meeting Objectives

- Students will demonstrate an understanding of line segments, angles, congruency and bisecting angles in this lesson.
- Students will also demonstrate measuring angles and identifying angles.

IV. Notes on Assessment

- Assess student work by thinking about each of the following points.
- Were the students successful in executing a design that matches the specifics of the assignment?
- Are the angles of the wings congruent?
- Are the angle measures labeled?
- Is it clear that students understand the concepts discussed in the lesson?

Angle Pairs

I. Section Objectives

- Understand and identify complementary angles.
- Understand and identify supplementary angles.
- Understand and utilize the Linear Pair Postulate.
- Understand and identify vertical angles.

II. Problem Solving Activity- Visualize It

- Students are going to go on a search for different types of angle pairs. This can be done in the classroom, but it would be best to expand it to the entire school or outside.
- If possible, allow the use of digital cameras.
- If this is not possible, students can draw sketches of the places where they locate each type of angle pairs.
- Students can photograph or draw each.
- They will need rulers, pencils, chart paper, clip boards.
- Students need to locate three examples of each.
- They first find three examples of complementary angles.
- Three examples of supplementary angles.
- Three examples of vertical angles.
- Students must write a description of each example and explain why it is a complementary angle pair, supplementary angle pair or vertical angle pair.
- Print student pictures and create a display of student work.

III. Meeting Objectives

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- Students will identify and write about complementary angles. This demonstrates understanding.
- Students will identify and write about supplementary angles. This demonstrates understanding.
- Students will identify and write about vertical angles. This demonstrates understanding.

IV. Notes on Assessment

- Have students work in groups to assess each other's work.
- Request that students read each description of the angle pair to be sure that it describes each angle pair in mathematical terms.
- You want to see that students are using measurements such as 90° for complementary angles, and that they are demonstrating that vertical angles are congruent.

Classifying Triangles

I. Section Objectives

- Define triangles.
- Classify triangles as acute, right, obtuse or equiangular.
- Classify triangles as scalene, isosceles or equilateral.

II. Problem Solving Activity- Bicycle Design

- Begin by showing students the following short movie clip from this website.
- http://www.thefutureschannel.com/dockets/hands-on_math/bicycle_design/
- Allow time for a student discussion about the short video.
- Ask students what they observed about bicycle design from the video.
- Write these notes on the board.
- Allow time for questions and then give students the assignment.
- Students are going to create their own bicycle design.
- They need to use at least two different types of triangles in the design.
- Students will need paper, rulers, pencils and colored pencils.
- Students can design their own bicycle.
- When finished, they need to label each type of triangle used and label it according to side length of and angles.
- Then students may decorate their design.
- Allow time for students to share their work.

III. Meeting Objectives

- Students will define triangles by using them in their bicycle design.
- Students will classify the triangles used according to side length.
- Students will classify the triangles used according to angle measure.

IV. Notes on Assessment

- Study each student design.
- Does the design have at least two different types of triangles in it?
- Are the triangles labeled according to side length?
- Are the triangles labeled according to angle measure?
- Provide students with feedback/corrections.

Classifying Polygons

I. Section Objectives

- Define polygons.
- Understand the difference between convex and concave polygons.
- Classify polygons by number of sides.
- Use the distance formula to find side lengths on a coordinate grid.

II. Problem Solving Activity- Polygon Sort

- This activity requires students to sort polygons in three different ways.
- 1. According to whether or not it is a polygon
- 2. Convex or concave
- 3. According to the number of sides
- To prepare this activity, you will need to create or copy different polygons. You want an assortment of polygons and non- polygons, convex polygons, concave polygons and regular polygons (i.e. quadrilaterals, hexagons, etc.). Then you can place these all around the room.
- When students begin the activity, they need to hunt for a specific number of figures. You could have each student find three different ones to work with. Then they can choose one for each exercise.
- Then you can do a sorting exercise.
- For example, "All of the polygons sit down. All of the non- polygons stand up."
- Then you can ask for a few examples from each group to explain why they are or are not a polygon.
- Next, you can do another sort. Concave figures to the front of the room. Convex to the back of the room.
- Same thing- ask for students to demonstrate why the figure is concave or convex.

III. Meeting Objectives

- Students will be required to define polygons.
- Students will demonstrate an understanding between concave and convex polygons.
- Students will classify polygons according to the number of sides.

IV. Notes on Assessment

- Assess student understanding by checking each "sorting exercise"
- Also ask different student for feedback about why they "sorted" their polygons the way that they did.
- Allow time for feedback and student questions.

Problem Solving in Geometry

I. Section Objectives

- Read and understand given problem situations.
- Use multiple representations to restate problem situations.
- Identify problem- solving plans.
- Solve real- world problems using planning strategies.

II. Problem Solving Activity- Camping Fire Expansion

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- Use the diagram on page 65 of the text. This will be Figure01.08.01.
- Here is an expansion on the earlier problem.
- Students can work in groups on this problem.
- The fire has begun to spread. It had spread to a tent that is fifty feet north of her tent. It has also spread to additional tent that is twenty- five miles south of the river and fifty feet south of the original tent. Two other campers have begun helping with the fire problem. How can all three minimize their distances? What is the shortest distance any one of them can run to put out the fire?

III. Meeting Objectives

- Be sure that the students have completed the work in problem 7 of the text before tackling this problem.
- If they have, then this problem should be a natural extension of the original one.
- Encourage students to follow the problem solving steps. There are two parts to this problem. Be sure that the students identify each part.
- Then have students draw a diagram to show the original tent and the two new tents as well. Students can label them A, B and C.
- Finally, using a scale, have students measure the distances.
- Who has the shortest distance to run?
- Ask students to show their work and to justify their thinking.

IV. Notes on Assessment

- Walk around as students work on this problem.
- Offer assistance when necessary and remind the students of the problem solving steps.
- When finished, allow students an opportunity to share their work.
- If there are different answers, ask groups to justify their answer.

5.2 Reasoning and Proof

Inductive Reasoning

I. Section Objectives

- Recognize visual patterns and number patterns.
- Extend and generalize patterns.
- Write a counterexample to a pattern rule.

II. Problem Solving Activity-Pascal's Triangle

- Students are going to work with a diagram of Pascal's Triangle for this activity.
- Pascal's Triangle is Figure02.01.01
- http://en.wikipedia.org/wiki/Pascal%27s_triangle
- Student are going to problem solve to find a rule to Pascal's Triangle.
- Have students work in small groups.
- They can use color on the triangle to point out different patterns.
- Allow a lot of time for the students to explore the patterns of the triangle.
- Ask them to use the Wikipedia pattern to write the rule for the triangle.
- Once they have the rule, they need to write the next two rows of the triangle.
- Then demonstrate two ways that you know that your rule is accurate.
- Finally, write a conjecture and a counterexample for the rule.
- Allow time for student sharing.

III. Meeting Objectives

- Students will recognize visual patterns and number patterns.
- Students will be required to extend and generalize patterns in Pascal's Triangle.
- Students will write conjectures and counterexamples of their rule.

IV. Notes on Assessment

- Do some independent study on Pascal's Triangle prior to completing this activity.
- Ask leading questions if students are stuck, but refrain from offering solutions.
- Encourage students to help each other with the patterns if they are having difficulties.

Conditional Statements

I. Section Objectives

- Recognize if- then statements.
- Identify the hypothesis and conclusion of an if- then statement.

- Write the converse, inverse and contrapositive of an if- then statement.
- Understand a biconditional statement.

II. Problem Solving Activity-Advertisements

- Students are going to use newspapers and magazines for this problem solving activity.
- Begin the activity by talking about how advertisers use conditional statements to lure people into purchasing their products. For example, a phone company will often offer a free phone for a cell phone plan.
- *Note: If you can find one such add it would be great to bring it in for a demonstration.*
- Tell students that their assignment is to use newspapers and magazines to find one such conditional advertisement.
- Then they are to take that advertisement and create a display using it to show the converse, inverse, contrapositive and biconditional statement of the advertisement.
- Students can decorate and design their display.
- Allow time for students to share their work when finished.

III. Meeting Objectives

- Students will recognize if- then statements in advertisements.
- Students will write converse, inverse and contrapositive statements.
- Students will understand biconditional statements by writing them.
- Students will present their work to their peers.

IV. Notes on Assessment

- Be sure that the students have selected a conditional statement in an advertisement.
- Check their work for accuracy when writing each of the different statements.
- Allow time for students to share their work.
- Include creativity in student evaluations.
- Students could receive a classwork or homework grade for this assignment.

Deductive Reasoning

I. Section Objectives

- Recognize and apply some basic rules of logic
- Understand the different parts that inductive reasoning and deductive reasoning play in logical reasoning
- Use truth tables to analyze patterns of reasoning

II. Problem Solving Activity-Write It Out

- Students are going to write statements based on Figure02.03.01 of vertical angles.
- Draw the figure on the board/overhead.
- Underneath it write the words "Vertical angles are congruent."
- Put the students in groups.
- There are four "starters" on index cards. A starter card gives the students a beginning geometric statement. They then need to take this statement and complete it using the Law of Detachment or the Law of Syllogism.
- For example, if the starter is $\angle 1 \cong \angle 2$, then the students would need to write two other statements using the figure as a guide.

- Possible answers could be: ∠1 and ∠2 are vertical angles. Vertical ∠'s are congruent.
- Here are the three other starters: ∠1 and ∠4 are supplementary angles.
- ∠3 and ∠4 each equal 55°.
- ∠2 and ∠3 are adjacent angles.
- ∠3 and ∠4 are congruent.
- Finally, when finished, ask students to identify whether the Law of Detachment or the Law of Syllogism was at work in each set of statements.

III. Meeting Objectives

- Students will use the basic rules of logic.
- Students will understand the role of inductive and deductive reasoning.
- Students will apply this reasoning to geometric content.

IV. Notes on Assessment

- Observe students as they work.
- If groups are struggling, refer them back to their notes on previously learned material.
- Refrain from offering suggestions.
- Give feedback based on content and accuracy.

Algebraic Properties

I. Section Objectives

- Identify and apply properties of equality
- Recognize properties of congruence "inherited" from the properties of equality
- Solve equations and cite properties that justify the steps in the solution
- Solve problems using properties of equality and congruence

II. Problem Solving Activity-Match It Up

- Students are going to create a matching game that they can then play in small groups.
- Each small group needs to create a pair for each of the properties. One card will have the name of the property on it, and the match will be a numerical example, and or a geometric example.
- Be sure that the students write out an actual example of the property and not just variables as they did in class.
- This will help them to take the lesson in the text to a new level.
- Be sure that the students have index or small cards, pens, rulers, etc.
- After the cards have all been created, have one group exchange with another group and play that team's game.
- When finished, ask the teams to give each other feedback on the examples used.
- Here are the properties to use:
  - Reflexive Property
  - Symmetric Property
  - Transitive Property
  - Substitution Property
  - Addition Property of Equality
  - Multiplication Property of Equality
  - Reflexive Property of Congruence with segments and angles
  - Symmetric Property of Congruence with segments and angles

5.2. Reasoning and Proof

– Transitive Property of Congruence with segments and angles

III. <u>Meeting Objectives</u>

- Students will identify and apply properties of equality.
- Students will solve equations and cite properties in their examples.
- Students solve problems using the properties.

IV. <u>Notes on Assessment</u>

- This activity has two parts. The first part is to observe the students as they work on creating the game.
- The second part is to watch them play it.
- Because they are switching game cards with another team, any errors will quickly come to light.
- Be sure to allow time for feedback/correction.

Diagrams

I. <u>Section Objectives</u>

- Provide the diagram that goes with a problem or proof.
- Interpret a given diagram.
- Recognize what can be assumed from a diagram and what can not be
- Use standard marks for segments and angles in diagrams.

II. <u>Problem Solving Activity-Name That Postulate!</u>

- This is a game. The students will create the game cards and then a "Jeopardy" kind of game can be played in the large class or in small groups.
- Students are assigned the task of creating an index card with a diagram that represents each postulate.
- Students should use diagrams and also standard marks for segments and angles in their examples.
- There are eleven postulates, so if there are twenty- two students in the class, each postulate would be represented by two different diagrams. You need to assign the students the postulates to avoid too many repeats.
- Allow time for the students to create their diagrams and then use peers to check each other's work for accuracy.
- When finished, collect the cards and play the game with the students.

III. <u>Meeting Objectives</u>

- Students create diagrams to better understand postulates.
- Students interpret given diagrams when playing the game.
- Students use standard marks for segments and angles when creating their game cards.

IV. <u>Notes on Assessment</u>

- Assessment is easier with this lesson because the students will be playing the game. You will be able to see who understands the postulates and who doesn't.
- Also, having students check each other's work before playing the game will definitely help to catch any errors.
- You can help add any corrections when playing the game and looking at each game card.

Two- Column Proof

I. <u>Section Objectives</u>

- Draw a diagram to help set up a two- column proof.
- Identify the given information and statement to be proved in a two- column proof.
- Write a two- column proof.

II. <u>Problem Solving Activity-Wind Generators</u>

- Use a figure like this one of a wind generator. This is Figure 02.06.01
- www.blaineschools.org/Schools/WRMS/Tech/Zsupportin_documents/Images/wind_generator.jpg
- Here is the problem.
- "Mike Eisele did an experiment for his science project to figure out which angle of degree on a propeller of a wind generator would be the most efficient. He figured out that 75° was the most efficient. Your task is to take this given information and write a proof to using geometric principles. We'll call one angle of the propeller angle 1 and the other angle 2."
- Show students the diagram of the wind generator. Point out the two angles that you are working with and then write this information on the board.
- *On Board:*

Given:

$$m\angle 1 = 75°$$
$$\angle 1 \cong \angle 2$$

Prove: $m\angle 2 = 75°$

III. <u>Meeting Objectives</u>

- Students will use a diagram to help set up a two- column proof.
- Students can draw a diagram of a wind generator and label the given angles.
- Students will write a two- column proof.

IV. <u>Notes on Assessment</u>

- Here is a possible answer for the given proof.

TABLE 5.1:

Statements	Reasons
$m\angle 1 = 75\angle 1 \cong \angle 2$	Given
$m\angle 1 = m\angle 2$	Definition of Congruent Angles
$75 = m\angle 2$	Substitution
$m\angle 2 = 75$	Symmetric Property

- Observe students while they work. Offer assistance when necessary.
- If you want to learn more about Mike Eisele and his experiment, see the Enrichment section of this Teacher's Edition. The website about Mike and his experiment is www.share3.esd105.wednet.edu/mcmillend/02SciProj/ReeseC/rees...

5.2. Reasoning and Proof

Segment and Angle Congruence Theorems

I. Section Objectives

- Understand basic congruence properties.
- Prove theorems about congruence.

II. Problem Solving Activity-Angle or Segment?

- For this problem solving activity, you will need to prepare a set of cards with angle statements and a set of cards with segment statements.
- Use numbers or letters to mark each card. Then you will know which statements the students were working with.
- Write each angle statement as reflexive, symmetric or transitive.
- Write each segment statement as reflexive, symmetric or transitive.
- Students are going to each be given a card with either an angle statement or a segment statement on it.
- Remind students to label their work with a letter or number that matches the card that they have been given.
- Then the students need to write out the property for the card and draw a diagram that illustrates the statement on the card.
- Students will need rulers, pencils and protractors for this assignment.
- All work should be accurate and measured.
- Allow a certain amount of time for this first card, when finished, ask the students to pass the card to a neighbor and repeat this assignment. You want the students to each work on two different angle cards and two different segment cards.
- This will help to secure student understanding.
- Have students share their work in small groups when finished.

III. Meeting Objectives

- Students will understand basic congruence properties.
- Students will prove basic congruence properties through diagrams and group discussions.

IV. Notes on Assessment

- Collect student work.
- Check each student's work for accuracy and offer written feedback.

Proofs about Angle Pairs

I. Section Objectives

- State theorems about special pairs of angles.
- Understand proofs of the theorems about special pairs of angles.
- Apply the theorems in problem solving.

II. Problem Solving Activity-Judges Table

- Before explaining the activity, select four students to serve as judges.

- Explain that the students are going to need to use theorems and proofs to "PROVE" each statement.
- The judges will be deciding if the students have successfully proved their statement.
- Students should work in groups of three for this assignment.
- The judges are also going to need to complete the work for all of the statements that way they know whether or not students have successfully proven their statement.
- Use Figure02.08.01- provide each group with a copy of the diagram.
- Here are some possible statements:
- $\overline{AD} \cong \overline{BC}$
- Given that $\angle 1 \cong \angle 2$, which other angles are congruent?
- $m\angle 5$ and $m\angle 6 = 90°$
- You can create as many different statements as you would like.
- Allow time for the students to work and then they present their case to the judges.
- The judges accept it or decline it. If accepted, students can work on another statement. If declined, the students need to go back and try again.

III. Meeting Objectives

- Students will state theorems about special pairs of angles.
- Students will understand how to prove theorems about special pairs of angles.
- Students will apply the theorems in problem solving.

IV. Notes on Assessment

- Sit on the panel with the judges.
- Listen to the statements and offer feedback.
- The students can be given extra credit for the number of statements that they are able to prove.
- Students could also be given a classwork grade for this assignment.

5.3 Parallel and Perpendicular Lines

Lines and Angles

I. <u>Section Objectives</u>

- Identify parallel lines, skew lines, and perpendicular lines
- Know the statement of and use the Parallel Line Postulate.
- Know the statement of and use the Perpendicular Line Postulate.
- Identify angles made by transversals.

II. <u>Problem Solving Activity-School Map</u>

- Students are going to use parallel lines, perpendicular lines and skew lines to create a map of the school.
- You can begin this lesson by using a fire exit map of the school to assist students in getting started.
- The students are going to work in groups of three to create a map of the school.
- Students will need chart paper, rulers, yard sticks, colored pencils.
- Take students on a walk around the school to begin taking notes for their design.
- Note: If your school is very large, you can either challenge students with the whole school or select a floor to design. This could create a complete school map in the end with groups putting their "floors" together.
- Have students draw and design their map.
- When finished with the map, students must identify two sets of parallel lines and two sets of perpendicular lines.
- Extension- you can extend this activity even further by asking the students to identify an example of the Parallel Line Postulate and an example of the Perpendicular Line Postulate on their map.
- Allow time at the end for the students to share their work.
- Create a class display of student maps.

III. <u>Meeting Objectives</u>

- Students will identify parallel lines.
- Students will identify skew lines.
- Students will identify perpendicular lines.

IV. <u>Notes on Assessment</u>

- Use each map to assess student work.
- Do the students have a good understanding of parallel lines?
- Of perpendicular lines?
- Of skew lines?
- Are students able to identify an example of each postulate?

Parallel Lines and Transversals

I. <u>Section Objectives</u>

- Identify angles formed by two parallel lines and a non- perpendicular transversal.
- Identify and use the Corresponding Angles Postulate.
- Identify and use the Alternate Interior Angles Theorem.
- Identify and use the Alternate Exterior Angles Theorem.
- Identify and use the Consecutive Interior Angles Theorem.

II. Problem Solving Activity-Airport Map

- For this problem solving activity, students are going to use an aerial map of Logan International Airport in Boston, Massachusetts.
- Figure 03.02.01 can be found at www.en.wikipedia.org/wiki/File:KBOS_Aerial_NGS.jpg
- Students need to use the picture to draw their own version of the map.
- Then using color, they need to identify the following:
- In red, two parallel lines and a non- perpendicular transversal.
- In blue- two corresponding angles
- In green- two alternate interior angles
- In orange- two alternate exterior angles
- In purple- two consecutive interior angles
- When finished, you can extend this by having the students use a protractor to determine angle measures.
- Allow time for students to share their work.

III. Meeting Objectives

- Students will identify the different angles created by parallel lines and transversals.
- Students will also identify the types of angles indentified in the different theorems.

IV. Notes on Assessment

- Assess student understanding through student sharing.
- Look at each student's diagram.
- Is it labeled correctly?
- Did the students follow directions?
- Is there anything missing?
- Does the diagram demonstrate that students understand the different angle pairs?

Proving Lines Parallel

I. Section Objectives

- Identify and use the Converse of the Corresponding Angles Postulate.
- Identify and use the Converse of Alternate Interior Angles Theorem.
- Identify and use the Converse of Alternate Exterior Angles Theorem.
- Identify and use the Converse of Consecutive Interior Angles Theorem.
- Identify and use the Parallel Lines Property.

II. Problem Solving Activity-Forest Tower

- Students are going to design a tower to be used in National Park by a Forest Ranger.

5.3. Parallel and Perpendicular Lines

- The students are going to need to design the tower so that the posts of the tower are parallel and are connected or braced by a transversal.
- They will need to demonstrate how the angles of the transversal prove that the tower poles are parallel.
- Begin the lesson by explaining, "Today you are going to design a tower to be used in a National Park by a Forest Ranger."
- To design the tower, you must create four parallel poles to place your platform on top of. The poles must be connected by a supporting transversal.
- You must use what you have learned about angles to design the tower and prove that the poles are parallel.
- Students will need protractors, rulers, chart paper and pencils.
- All angles must be labeled and measured.
- Look at Figure 03.03.01 to get an idea of a possible example.
- When finished, allow students time to share their work.

III. Meeting Objectives

- Students will show interior and exterior angles in their diagram.
- Students will show corresponding angles in their diagram.
- Students will apply what they have learned in a real- life example.
- Students will demonstrate the Parallel Lines Property.

IV. Notes on Assessment

- Look at each student diagram and assess student work.
- Is the diagram labeled?
- Are the angle measures correct?
- Were the students able to label the diagram to demonstrate that the poles are parallel?
- Offer students corrections and feedback.

Slopes of Lines

I. Section Objectives

- Identify and compute slope in the coordinate plane.
- Use the relationship between slopes of parallel lines.
- Use the relationship between slopes of perpendicular lines.
- Plot a line on a coordinate plane using different methods.

II. Problem Solving Activity-Wheelchair Ramps

- Students are going to use what they have learned about slope to design a wheelchair ramp.
- A wheelchair ramp has a slope of $\frac{1}{12}$ ft. - www.newdisability.com/wheelchairramp.htm
- Here is the problem.
- A new home has a front door that is $3\frac{1}{2}$ feet off of the ground. A wheelchair ramp needs to have a slope of $\frac{1}{12}$ ft. Based on this fact and on the height of the door, design a wheelchair ramp that will work for this new home. Show all of your work in your diagram.
- Allow time for the students to work on this dilemma.
- Then tell students that every unit on a coordinate grid represents one foot. Ask them to draw their wheelchair ramp on coordinate grid.
- When finished, allow time for students to share their work.

III. <u>Meeting Objectives</u>

- Students will use what they have learned about slope to determine the rise and run of the ramp.
- Students will demonstrate an understanding of how the slope of a line impacts the rise and run of the line.

IV. <u>Notes on Assessment</u>

- Examine student work.
- Is the rise and run of the ramp accurate?
- According to the slope of $\frac{1}{12}$, the rise is 3.5 ft and the run is 42 inches.
- Is this drawn accurately?
- Is it graphed correctly on the coordinate grid?
- Allow time for students to share their work and offer feedback and correction when necessary.

Equations of Lines

I. <u>Section Objectives</u>

- Identify and write equations in slope- intercept form.
- Identify equations of parallel lines.
- Identify equations of perpendicular lines.
- Identify and write equations in standard form.

II. <u>Problem Solving Activity-The Park Path</u>

- To work on this problem, students are going to be designing a path for a park using Parallel and Perpendicular Lines.
- Here is the problem:
- "Maria is on a team that is designing paths through a local park. The team has cleared some of the brush and has created one path through the park. Here is the path graphed on the coordinate grid."
- Figure03.05.01
- "The equation for this path is $y = 3x + 2$."
- "The team needs to draw in two more paths. The first one will be parallel to this one, and the next one will be perpendicular to this one."
- "Use what you have learned to draw these three paths on a coordinate grid. Use your problem solving skills to name the equation of each line. Be sure to write your equations in slope- intercept form."

III. <u>Meeting Objectives</u>

- Students will write equations in slope- intercept form.
- Students will identify the equations of parallel lines.
- Students will identify the equations of perpendicular lines.
- Students will graph these lines on the coordinate grid.

IV. <u>Notes on Assessment</u>

- The line parallel to $y = 3x + 2$ can be several different things. The key is that it must have the same slope. As long as the line has the same slope, then it is parallel to this first line.
- The perpendicular line must have a slope that is the reciprocal of the $\frac{3}{1}$. Therefore, any line with a slope of $\frac{1}{3}$ will be perpendicular to this first line.
- Allow time for student questions.
- Check student work for accuracy.

5.3. Parallel and Perpendicular Lines

Perpendicular Lines

I. <u>Section Objectives</u>

- Identify congruent linear pairs of angles.
- Identify the angles formed by perpendicular intersecting lines.
- Identify complementary adjacent angles.

II. <u>Problem Solving Activity-Rug Angles</u>

- In this problem solving activity, students are going to identify the angles formed by perpendicular lines.
- Use this rug design as Figure03.06.01 www.homedecorators.com/detail.php?parentid=26996&aid=bzrt
- Ask students to use this figure to draw their own rug design.
- They can use color and different sizes of polygons on the rug. The big point is to be sure that they are using perpendicular lines on the rug.
- In the design, the students need to include two linear pairs.
- They need to include two sets of perpendicular intersecting angles.
- They need to show one set of complementary adjacent angles.
- These rugs can be designed on chart paper or graph paper- you can leave it up to the students on the size of the rug.
- Allow time for the students to share their work when finished.

III. <u>Meeting Objectives</u>

- Students will identify and draw congruent linear pairs of angles.
- Students will identify and draw angles formed by perpendicular intersecting lines.
- Students will identify and draw complementary adjacent angles.

IV. <u>Notes on Assessment</u>

- Did the students accomplish this task?
- Are all of the angles present?
- Are all of the perpendicular lines accurate?
- Is there anything missing in the design?
- Offer feedback to students during group discussions.

Perpendicular Transversals

I. <u>Section Objectives</u>

- Identify the implications of perpendicular transversals on parallel lines.
- Identify the converse theorems involving perpendicular transversals and parallel lines.
- Understand and use the distance between parallel lines.

II. <u>Problem Solving Activity-Magazine Hunt</u>

- For this problem solving activity, you will need an assortment of magazines.
- Students are going to go on a magazine hunt for parallel lines and their transversals.

- Students are to find a picture that illustrates an example of parallel lines and perpendicular transversals.
- Then, students need to cut out the picture.
- Label all of the angles.
- Label all of the measures of all of the angles.
- Finally, ask students to explain how the converse theorems impact the perpendicular lines in each picture.
- If time allows, you can request that students work with more than one picture.
- You can also extend this activity to include a presentation piece, so that students are required to verbalize what they have learned through the activity.

III. Meeting Objectives

- Students will identify perpendicular transversals and parallel lines.
- Students will identify the angles associated with perpendicular transversals and parallel lines.
- Students will identify the converse theorems associated with parallel lines and perpendicular transversals.

IV. Notes on Assessment

- Have the students selected an appropriate picture?
- Does this picture show parallel lines and perpendicular transversals?
- Are all of the angles accurately labeled?
- Have the students made notes of the converse theorems involved?
- Assess student understanding through written work and presentations.

Non- Euclidean Geometry

I. Section Objectives

- Understand non- Euclidean geometry concepts.
- Find taxicab distances.
- Identify and understand taxicab circles.
- Identify and understand taxicab midpoints.

II. Problem Solving Activity- Taxicab Geometry

- In this activity, students are going to work on the following problem. This can be done individually or in pairs.
- Here is the problem.
- "Juan rides his bike 2800 feet from him home to the park to play baseball. Using a scale of 1 unit: 200 feet, draw a possible path for Juan on a coordinate grid."
- Students will need to work backwards to solve this problem.
- There are several different solutions to this problem.
- The important thing to note is that Juan travels 14 units according to the scale.
- Any combination of Juan traveling a combination of 14 units is correct.
- You will get many different pictures of this problem.
- Allow time for the students to share their work and explain how they got their answers.
- Extension of this is to say that Juan travels $5,820$ feet to school. This is the equivalent of one mile. Give the students the same scale and see what they can do with it.
- Have students draw a diagram.
- The solution is that Juan walks 29.1 units to school. There are many different diagrams that could come out of this problem. Watch for the .1 in the diagram too.

5.3. Parallel and Perpendicular Lines

III. Meeting Objectives

- Students will work to understand non- Euclidean geometry.
- Students will find taxicab distances.
- Students will draw diagrams to show taxicab distances.
- Students will explain their work to their peers.

IV. Notes on Assessment

- Do not provide the students with much direction in this lesson.
- Provide them with the scale and the problem and let them go to work.
- Using what they have learned in the text, the students should be able to work backwards and figure out that Juan walks 14 units to the park.
- Then you can assess their diagrams.
- Are the diagrams accurate?
- Are the students able to explain their process of finding the solution to the problem?
- Provide correction/feedback when necessary.

5.4 Congruent Triangles

Triangle Sums

I. Section Objectives

- Identify interior and exterior angles in a triangle.
- Understand and apply the Triangle Sum Theorem.
- Utilize the complementary relationship of acute angles in a right triangle.
- Identify the relationship of the exterior angles in a triangle.

II. Problem Solving Activity-Triangle Sums

- Provide students with a copy of Figure04.01.01
- Students are going to work in pairs with this figure.
- Part one- students need to figure out the angle measures for each of the missing angles.
- Part two- students are going to write down the theorem that helped them to figure out the measure of each angle.
- Allow time for the students to explain their work in small groups.
- Solution: $a = 140°$ Exterior angles in a Triangle Theorem

$b = 100°$ Triangle Sum Theorem

$c = 60°$ Acute Angles Theorem

$d = 30°$ Triangle Sum Theorem or Exterior angles in a Triangle Theorem

Extension- have students draw their own puzzle for others to solve.

III. Meeting Objectives

- Students will identify interior and exterior angles in a triangle.
- Students will apply the Triangle Sum Theorem.
- Students will apply the Exterior Angles Theorem.
- Students will apply the Acute Angles Theorem.
- Students will demonstrate understanding through diagrams and verbal explanations.

IV. Notes on Assessment

- Is the student's work accurate?
- Can the student talk about why each theorem is appropriate for each angle measure?
- Ask questions and provide feedback when necessary.

Congruent Figures

I. Section Objectives

5.4. Congruent Triangles

- Define congruence in triangles.
- Create accurate congruence statements.
- Understand that if two angles of a triangle are congruent to two angles of another triangle, the remaining angles will also be congruent.
- Explore properties of triangle congruence.

II. <u>Problem Solving Activity-Congruent Triangles Game</u>

- This game involves a bit of prep by the teacher.
- First, you are going to divide the class numbers in thirds. Then, you design three cards.
- Card 1- a congruent triangle.
- Card 2- it's matching triangle
- Card 3- a congruence statement about the two triangles.
- Create enough cards so that each student receives one. Be sure that your triangles have tic marks on them.
- Add some challenge by using repeated letters in the diagrams and congruence statements. Students will need to pay close attention to the order of the angles in the congruence statement to find the correct matches.
- Then shuffle and hand out the cards.
- Students need to move around the room and find their matches. Each group should have three students in each group when finished.
- This is a fun activity that has a lot of movement in it.

III. <u>Meeting Objectives</u>

- Students will define congruence in triangles.
- Students will understand congruent angles in a triangle.
- Students will use triangle congruence statements.

IV. <u>Notes on Assessment</u>

- Are the groups correct?
- Have students been able to find their team mates?
- Listen to student conversation as they work, are the students talking about properties of congruence?
- Allow time for the students to share the strategies that they used to find each other.

Triangle Congruence Using SSS

I. <u>Section Objectives</u>

- Use the distance formula to analyze triangles on a coordinate grid.
- Understand and apply the SSS postulate of triangle congruence.

II. <u>Problem Solving Activity- Triangle Measurement</u>

- Students are going to use the distance formula to analyze triangles on a coordinate grid.
- Here is the problem:
- "Maria is designing a triangular garden in her yard. Her neighbor wants an identical triangle garden in her yard too. Maria is going to use the same dimensions to build her neighbor a garden. To plan her work, Maria plots her garden out using a coordinate grid. Here are the coordinates for Maria's triangular garden.
- $(-3,7)(-1,5)(-4,1)$

- "If Maria uses these dimensions, and moves the plot five units to the left and three units down, what will the coordinates of her neighbor's garden be?"
- Students can work on this problem in pairs or individually.
- Have the students draw out the coordinate grid with the two triangles on them.
- Allow time for the students to share their work when finished.
- Extension- students can transfer these triangles to measurement. For example, they could use 1 foot for every one unit on the coordinate grid.

III. Meeting Objectives

- Students will use the distance formula to plot the triangle on the coordinate grid.
- Students will draw Congruent Triangles on the coordinate grid.
- Students will use the distance formula when converting units to feet in the extension.

IV. Notes on Assessment

- Were the students able to follow the directions accurately?
- Is the triangle in the correct location?
- Here is the solution graph. It is Figure04.03.01
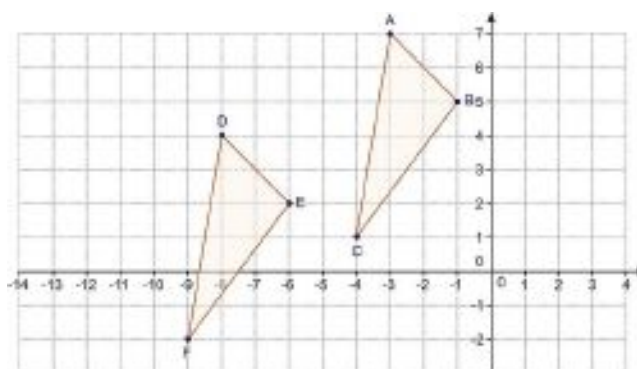
Triangle Congruence Using ASA and AAS

I. Section Objectives

- Understand and apply the ASA Congruence Postulate.
- Understand and apply the AAS Congruence Postulate.
- Understand and practice two- column proofs.
- Understand and practice flow proofs.

II. Problem Solving Activity- Double Triangles

- For this activity, there is the preparation of each student drawing a small triangle to work with.
- Then have the students work in groups. The students need to choose two of the triangles that have been drawn to work with.
- Then, the students need to draw two triangles that are congruent to the selected triangles.
- This means that each group will have two pairs of Congruent Triangles.
- Each team will have two pairs of students on it. Each pair selects one set of Congruent Triangles to work with.

5.4. Congruent Triangles

- Now, the teams quiz each other.
- Each team needs to prove that their triangles are congruent.
- The other team can question and challenge any claims that the teammates use to justify congruency.
- When the team has successfully proven that their triangles are congruent, the team can draw either a flow proof or a two- column proof with their statements and reasons in it.

III. Meeting Objectives

- Students will apply the ASA Congruence Postulate.
- Students will apply the AAS Congruence Postulate.
- Students will use these postulates to prove that their triangles are congruent.

IV. Notes on Assessment

- Question each team, were they successful in proving that their triangles are congruent?
- Is the proof written to show SSS, ASA, or AAS?
- Allow time for the students to share their work.
- Offer corrections when necessary.

Proof Using SAS and HL

I. Section Objectives

- Understand and apply the SAS Congruence Postulate.
- Identify the distinct characteristics and properties of right triangles.
- Understand and apply the HL Congruence Theorem.
- Understand that SSA does not necessarily prove triangles are congruent.

II. Problem Solving Activity- The Wall Design

- Here is the problem.
- "Harry has a square wall in his bedroom. He knows it is square because he has measured it. He decides to paint a bold red diagonal down the wall. Harry starts at the upper right corner and measuring carefully, paints a red diagonal from the upper right corner to the bottom left corner of the wall. He stands back to admire his work. His brother comes in, looks at the wall and says, "It's great, but your triangles aren't even." Harry says that they are."
- Your task is to draw a diagram and write a two- column proof showing that Harry's triangles are congruent. Because Harry has not given us the dimensions of his wall, use the knowledge that it is square and decide your own dimensions.
- Allow time for the students to work on this task.
- If stuck, remind students that the triangles can be proved congruent using SAS. Refer them back to the text for clarification and for right triangle properties.
- Students need to show the measure of two sides, the measure of two angles to compare- these measurements should be labeled in their diagrams.

III. Meeting Objectives

- Students will apply the SAS Congruence Postulate.
- Students will identify the distinct characteristics and properties of right triangles.

- Students will apply the HL Congruence Theorem.

IV. Notes on Assessment

- Check student diagrams.
- Is the work accurately labeled with measurements?
- Did the students make note of the two triangles?
- Which postulate or theorem did the student use to prove congruency?
- Is the student aware of the properties of right triangles?
- Are the right triangles noted in the student's work?

Using Congruent Triangles

I. Section Objectives

- Apply various triangles congruence postulates and theorems.
- Know the ways in which you can prove parts of a triangle congruent.
- Find distances using Congruent Triangles.
- Use construction techniques to create Congruent Triangles.

II. Problem Solving Activity-The Kite Design

- Students will need chart paper, compasses, rulers and colored pencils.
- Here is the problem.
- "Jonas is working on designing a kite. He knows that for his kite to fly well, that it needs to be created with four Congruent Triangles. Jonas starts by drawing a line segment that will serve as the center line of his kite. Now he needs to draw in the four triangles. He can't remember how to do it.
- Your task is to use a compass and a straightedge to help Jonas design his kite.
- Remember to measure your work so that it is accurate.
- Here is the line segment that Jonas started with.
- Figure04.06.01
- Allow time for the students to work.
- When finished, the students can decorate their kites.

III. Meeting Objectives

- Students will apply various triangle congruence postulates and theorems.
- Students will use construction techniques to create Congruent Triangles.
- Students will show their work in a design.

IV. Notes on Assessment

- Leave students alone to work on this problem.
- If students are having trouble, refer them back to the text to follow the steps for the construction.
- Assess student work for accuracy and creativity.
- Provide feedback/correction when students are finished.
- Create a class display with the designs.

5.4. Congruent Triangles

Isosceles and Equilateral Triangles

I. Section Objectives

- Prove and use the Base Angles Theorem.
- Prove that an equilateral triangle must also be equiangular.
- Use the converse of the Base Angles Theorem.
- Prove that an equiangular triangle must also be equilateral.

II. Problem Solving Activity-Triangle Proofs

- Provide students with the definition for the Base Angles Theorem.
- Base Angles Theorem- If two sides of a triangle are congruent, then their opposite sides are congruent, and the angles opposite the congruent sides are congruent.
- Next, assign students the task of proving that this is correct.
- The students need to draw a diagram involving isosceles triangles to demonstrate this theorem.
- Then, they need to write a two- column proof or a flow proof.
- When finished, the students are going to present their work in small groups.
- Allow time for the other students in the group to provide feedback.
- Extension- repeat the exercise with the students working with the converse of the Base Angles Theorem.
- Possibility- you can also assign half the class the Base Angles Theorem and the other half the converse of the Base Angles Theorem.

III. Meeting Objectives

- Students will draw a design using isosceles triangles.
- The design is to prove the Base Angles Theorem.
- Students will demonstrate understanding through a design and through a verbal explanation.

IV. Notes on Assessment

- Did the students complete the design?
- Does it use isosceles triangles?
- Did the students show the bisection of the triangle?
- Does the diagram show the angles and sides that are congruent?
- Does the proof have the correct statements and reasons?
- Is the student able to explain his/her thinking in words?
- Provide comments/feedback when necessary.

Congruence Transformations

I. Section Objectives

- Identify and verify congruence transformations.
- Identify coordinate notation for translations.
- Identify coordinate notation for reflections over the axes.
- Identify coordinate notation for rotations about the origin.

II. Problem Solving Activity-Transformation Game

- In this game, students are going to manipulate one figure to represent the clues that the team has been given. You can play this as a whole class and make it about the speed with which a team can complete the transformation.
- Each team will need a triangle (made of cardstock) and a large coordinate grid. You can do this on chart paper.
- Each team starts with their triangle in the first quadrant. The team can decide where to put it. They draw it in or trace it in as the starting point.
- Now the team has a moveable triangle and a drawn one.
- Then call out the first command, "Rotate 90°[U+0080][U+009D]
- The team rotates the triangle using the moveable triangle and places the triangle in a new place to show the rotation. When the team has completed the task they raise their hands.
- Teams that are correct and finish first get a point.
- Then begin again. The team moves the triangle back to the starting point and starts again. Call out a new command.
- Example "Translate 5 right and 3 down."
- Then the game repeats.

III. Meeting Objectives

- Students will identify and use congruence transformations.
- Students will identify coordinate notation for translations.
- Students will identify coordinate notation for reflections.
- Students will identify coordinate notation for rotations.
- Students will practice all of these in a creative game.

IV. Notes on Assessment

- Assessment is completed through observation as each team is playing the game.
- Make note of teams that are having difficulty and offer assistance after class.
- If necessary, stop the game and offer corrections or review.

5.5 Relationships within Triangles

Midsegments of a Triangle

I. <u>Section Objectives</u>

- Identify the midsegment of a triangle.
- Apply the Midsegment Theorem to solve problems involving side lengths and midsegments of triangles.
- Use the Midsegment Theorem to solve problems involving variable side lengths and midsegments of triangles.

II. <u>Problem Solving Activity- Midsegment Mystery</u>

- Here is the problem.
- "Sari is working on a design for the roof of a dog house. Here is her design."
- Figure05.01.01
- "Sari has forgotten her notes, but she knows that *x* is equal to 7. What is her measurement for *y* if *x* is equal to 7?"
- Use what you have learned about midsegments and triangles to help her work this out.
- Allow students time to work. Ask them to show all of their work when finished.

III. <u>Meeting Objectives</u>

- Students will identify the midsegment of a triangle.
- Students will apply the Midsegment Theorem to solve problems involving side lengths and midsegments.
- Students will demonstrate understanding by showing all of their work.

IV. <u>Notes on Assessment</u>

- Here is the solution to the problem.
-

$$x = 7$$
$$3x + 5 = \frac{1}{2}y$$
$$3(7) + 5 = \frac{1}{2}y$$
$$26 = \frac{1}{2}y$$
$$y = 52$$

- Be sure that student work is labeled and that students show all of their work.
- Provide feedback when necessary.

Perpendicular Bisectors in Triangles

I. Section Objectives

- Construct the perpendicular bisector of a line segment.
- Apply the Perpendicular Bisector Theorem to identify the point of concurrency of the perpendicular bisectors of the sides (the circumcenter).
- Use the Perpendicular Bisector Theorem to solve problems involving the circumcenter of triangles.

II. Problem Solving Activity-The Camping Dilemma

- Here is the problem.
- "Boy Scout Troop 462 is going on a camping trip. When they arrive at the campground, they are given a huge triangular field to set up in. The boys decide to place their tents along the perimeter of the triangle and to put the campfire in the center of the triangle. To do this, they will need to use perpendicular bisectors to identify the circumcenter."
- Your task is to use the following diagram to help the boy scouts. Draw in the perpendicular bisectors and label the place where the campfire should go.
- Figure05.02.01
- Students can work individually or in pairs on this problem.
- Allow students time to share their work when they are finished.

III. Meeting Objectives

- Students will construct perpendicular bisectors of line segments.
- Students will use the Perpendicular Bisector Theorem to identify the circumcenter of the triangles.
- Students will solve problems by helping the boy scouts with their campfire location.

IV. Notes on Assessment

- Did the students draw in all of the perpendicular bisectors accurately?
- Is the circumcenter in the correct location?
- Did the students identify where the campfire should be?
- Are students able to verbalize how they went about solving the problem?
- Offer feedback and support as students are working.

Angle Bisectors in Triangles

I. Section Objectives

- Construct the bisector of an angle.
- Apply the Angle Bisector Theorem to identify the point of concurrency of the perpendicular bisectors of the sides (the incenter).
- Use the Angle Bisector Theorem to solve problems involving the incenter of triangles.

II. Problem Solving Activity- Inscribing Circles

- This activity will focus on the students inscribing circles into already designed triangles.

5.5. Relationships within Triangles

- To prepare the activity, have several triangles drawn out for the students.
- Divide the students into groups.
- Each group needs to receive three different triangles.
- The students are completing these tasks at the same time. Each students completes his/her part of inscribing the triangle.
- The first student draws in one angle bisector and passes the triangle to the right.
- The next student draws in the next angle bisector and passes the triangle to the right.
- The next student draws in the perpendicular bisectors.
- The final student uses a compass to inscribe the circle.
- When finished, the students will have completed this task for three different triangles.

III. Meeting Objectives

- Students will construct the bisectors of angles in a triangle.
- Students will draw in perpendicular bisectors of the angles in a triangle.
- Students will use a compass to inscribe a circle into a triangle.
- Students will share their work with peers.

IV. Notes on Assessment

- Check student work for accuracy.
- Are the angles bisected corrected?
- Are the perpendicular bisectors correct?
- Have the students located the circumcenter?
- Is the circle correctly inscribed into the triangle?
- Can the students explain how and why they completed each piece the way that they did?
- Offer feedback/correction when necessary.

Medians in Triangles

I. Section Objectives

- Construct the medians of a triangle.
- Apply the Concurrency of Medians Theorem to identify the point of concurrency of the medians of the triangle (the centroid).
- Use the Concurrency of Medians Theorem to solve problems involving the centroid of triangles.

II. Problem Solving Activity-Napolean's Theorem

- Begin by sharing Napolean's Theorem with the students from the Wikipedia site. Use the diagram as well this is Figure05.04.01
- http://en.wikipedia.org/wiki/Napoleon%27s_theorem
- Students are going to prove Napoleans Theorem.
- Tell students that they are going to create a design three levels in complexity to prove Napoleans Theorem.
- Show them that the Wikipedia diagram is three levels of complexity.
- Students can use chart paper, colored pencils, and rulers.
- They need to be prepared to show, through their diagram, how Napolean's Theorem is accurate and true.
- Each student works on his or her own design, but you may want to allow them to work in groups to help each other.

- Extension- does this continue to prove true if more triangles are added? Where can they be added?

III. Meeting Objectives

- Students will construct the medians of a triangle.
- Students will show how Napoleans Theorem works.
- Students will explain their thinking through a presentation.
- Students will demonstrate understanding through each person's design.

IV. Notes on Assessment

- Is student work accurate?
- Does it show accurate equilateral triangles?
- Are the students able to explain their work to prove Napolean's Theorem?
- What happens when the pattern is extended beyond three levels? Are there more equilateral triangles to be found?
- Offer feedback and support as students work.

Altitudes in Triangles

I. Section Objectives

- Construct the altitude of a triangle.
- Apply the Concurrency of Altitudes Theorem to identify the point of concurrency of the altitudes of the triangle (the orthocenter).
- Use the Concurrency of Altitudes Theorem to solve problems involving the orthocenter of triangles.

II. Problem Solving Activity-Drawing Triangles

- In this activity, students are going to demonstrate that they understand the concepts associated with altitude by constructing different triangles.
- 1. Students need to construct an acute triangle.
- Label the altitude of the triangle.
- Label the orthocenter.
- 2. Students need to construct an obtuse triangle.
- Label the altitude of the triangle.
- Label the orthocenter of the triangle.
- 3. Students need to construct a right triangle.
- Label the altitude of the triangle.
- Label the orthocenter of the triangle.
- Allow students time to share their work in small groups.

III. Meeting Objectives

- Students will construct the altitude of a triangle.
- Students will locate and label the orthocenter of a triangle.
- Students will demonstrate understanding through their constructions.

IV. Notes on Assessment

5.5. Relationships within Triangles

- Look at each student's triangles.
- Is the orthocenter of the acute triangle inside the triangle?
- Is the orthocenter of the obtuse triangle outside the triangle?
- Is the orthocenter of the right triangle at the vertex of the right triangle?
- Assess student understanding by assessing each individual triangle.
- Offer feedback/correction when necessary.

Inequalities in Triangles

I. Section Objectives

- Determine relationships among the angles and sides of a triangle.
- Apply the Triangle Inequality Theorem to solve problems.

II. Problem Solving Activity-Prove the Theorem

- In this problem, students need to use the information given to draw a diagram and prove the theorem.
- Students are going to prove the theorem that states that the angle opposite the longest side of a triangle with unequal sides will have the greatest measure.
- Here is the problem.
- "In triangle $ABC, \overline{AB} < \overline{BC}$, the measure of angle A is $80°$. The measure of angle C is $40°$, and the measure of angle B is $60°$. Given the theorem on side lengths and angle measures, can this be a true statement? Why or why not?"
- Provide students time to work on this problem.
- Students need to provide a diagram, a written explanation and a verbal explanation to explain their thinking.
- When finished, allow students time to present their work to the class.

III. Meeting Objectives

- Students will determine relationships among the angles and sides of a triangle.
- Students will apply the Triangle Inequality Theorem to solve problems.
- Students will demonstrate understanding through diagrams, written explanations and verbal explanations.

IV. Notes on Assessment

- Here is the solution to the problem.
- This is a true statement because the length of BC is longer than AB. The angle opposite BC is the greatest of the three angles in the triangle. It measures $80°$. Therefore, the theorem is accurate and proven through this statement.
- Be sure that the students have diagram that resembles Figure05.06.01
- Offer feedback and correction when necessary.

Inequalities in Two Triangles

I. Section Objectives

- Determine relationships among the angles and sides of two triangles.

- Apply the SAS and SSS Triangle Inequality Theorems to solve problems.

II. Problem Solving- Name that Inequality

- This problem solving activity is a game.
- Preparation is to prepare two Congruent Triangles for each group to work with.
- Students play it in groups of four. In the groups of four, they split up into pairs.
- Each pair is a team that plays against each other.
- When the students play, they are trying to "stump" the other party.
- The play begins like this, one team comes up with a problem for the other team to solve.
- For example, "If I lengthen side *AB* what inequality compares side *AB* to *CD*?"
- Then the other team has to answer it.
- If they answer it correctly, the team receives a point.
- If not, the other team gets a point.
- Then they repeat the process by switching team positions.
- Both teams play until time is up.
- Students need to be encouraged to use the SAS Triangle Inequality Theorem and the SSS Triangle Inequality Theorem as well as the converse Theorems.
- Students can create as many different types of questions as they would like.
- Students can be very creative in their approach to writing questions.

III. Meeting Objectives

- Students will determine relationships among the angles and sides of two triangles.
- Students will apply the SAS and SSS Triangle Inequality Theorems to solve problems.
- Students will demonstrate their knowledge and understanding through the quiz game.

IV. Notes on Assessment

- Walk around as students play and assist students when necessary.
- Offer suggestions and challenge students to create difficult questions.
- Notice which students are having difficulty with the assignment and offer assistance and coaching.

Indirect Proofs

I. Section Objectives

- Reason indirectly to develop proofs of statement.

II. Problem Solving Activity-Draw it Out!

- Assign students the task of drawing an example of a geometric proof and an algebraic proof.
- Tell the students that they are going to use indirect reasoning to develop these proofs of statements.
- Here is one possible example for an algebraic problem.
- "Marcy is selling candy bars for the school band. She starts out selling five bars. But in the end, she sells three times as many as her friend John does. The band teacher congratulates her on selling over forty candy bars. If John sold less than 12 bars, Marcy did not sell more than forty bars."
- Students need to write a proof to show this statement is true.
- Here is the answer:

5.5. Relationships within Triangles

*

$$3x + 5 = 41$$
$$3(11) + 5 = 38$$
$$38 \neq 41$$

- Since the band teacher congratulated Marcy on selling more than forty bars, we can have our equation equal 41.
- John sold less than 12 bars, so we can use 11 as a possible answer for *x*.
- This shows that our statement that Marcy did not sell more than forty bars is a true statement.
- Then have students write their own statement and proof of statement for a geometric proof.
- You could provide them with a diagram or have them draw in their own.

III. Meeting Objectives

- Students will reason indirectly to write proofs of statements.
- Students will work with both algebraic and geometric examples.

IV. Notes on Assessment

- Look at student examples.
- Is the work accurate?
- Does the reasoning make sense?
- Are the students applying the correct theorems to prove their statements?
- Offer feedback and correction when necessary.

5.6 Quadrilaterals

Interior Angles

I. <u>Section Objectives</u>

- Identify the interior angles of convex polygons.
- Find the sums of interior angles in convex polygons.
- Identify the special properties of interior angles in convex quadrilaterals.

II. <u>Problem Solving Activity-Pentagon Cleaning</u>

- Figure 06.01.01 Pentagon
- Here is the problem.
- "Washers Cleaning Company is in charge of cleaning the entire pentagon. This cleaning company is unique because it is made up of people who love math. Because of this, they clean the Pentagon in triangular sections, figuring out how much has been completed and how much is left based on degrees. If they have cleaned $\frac{2}{3}$ of the Pentagon, how many degrees are left to clean?"
- Steps to solving this problem:
- To solve this problem, the students will need to figure out some things.
- 1. How many triangles are there in a pentagon?
- 2. How many degrees are there in a pentagon?
- 3. If $\frac{2}{3}$ is clean, than $\frac{1}{3}$ is left to be cleaned.
- 4. How many degrees are in the $\frac{1}{3}$?
- 5. Finally, use all of this information to figure out the solution to the problem.
- Students need to use diagrams to demonstrate their solution as well as writing.

III. <u>Meeting Objectives</u>

- Students will identify the interior angles of convex polygons.
- Students will find the sums of interior angles in convex polygons.
- Students will demonstrate understanding through diagrams and writing.

IV. <u>Notes on Assessment</u>

- Does student work show all of the information in steps $1-5$?
- Did the students figure out that the missing number of degrees is $180°$?
- Listen to students explain their solutions.
- Offer suggestions and feedback when necessary.

Exterior Angles

I. <u>Section Objectives</u>

- Identify the exterior angles of convex polygons.
- Find the sums of exterior angles in convex polygons.

II. <u>Problem Solving Activity- The Garden Dilemma</u>

- Here is the problem.
- "Johanna has designed a garden in the shape of a hexagon for the local botanical garden. She is very proud of her work. Each vertex of the hexagon has one garden path that extends from it making the garden a central feature of the botanical garden. The carpenter who works at the botanical garden wants to design an edging for each angle of the pentagon. To do this, he needs your help. He knows the measure for three of the exterior angles and he needs to find the measure of the other two. He knows that these two angles are congruent. Use the figure to find the measure of the two missing angles."
- Figure 06.02.01
- Students need to draw a picture to illustrate this problem.
- Students need to write about how they solved the problem.
- Students need to explain their work to the class.

III. <u>Meeting Objectives</u>

- Students will identify the exterior angles of convex polygons.
- Students will find the sums of exterior angles in convex polygons.
- Students will show their work in a diagram or drawing.
- Students will write a written explanation of their work.

IV. <u>Notes on Assessment</u>

- Here is the simple solution to the problem.
- $50 + 110 + 80 = 240$
- $360 - 240 = 120$
- Because the two remaining angles are congruent, we can divide this number by two.
- This leaves us with each remaining angle being equal to $60°$.
- $a = 60°$
- $b = 60°$
- Check student work to be sure that the students have included all of the important components of solving this problem.
- Listen as students explain their work and offer suggestions/feedback when needed.

Classifying Quadrilaterals

I. <u>Section Objectives</u>

- Identify and classify a parallelogram.
- Identify and classify a rhombus.
- Identify and classify a rectangle.
- Identify and classify a square.
- Identify and classify a kite.
- Identify and classify a trapezoid.
- Identify and classify an isosceles trapezoid.
- Collect the classifications in a Venn diagram.

- Identify how to classify shapes on a coordinate grid.

II. Problem Solving Activity-Quad Design

- This is a creative design activity.
- The students are going to need colored pencils, crayons, markers, rulers, large blank sheets of paper.
- The task is to create a design that has each of the seven figures in it.
- Students need to create a color key to identify the seven figures in the design.
- They can include any other shapes/color that they would like, as long as the seven figures are in the design.
- Here are the figures that must be in the design: parallelogram, rhombus, rectangle, square, kite, trapezoid, isosceles trapezoid
- Allow students the entire class to work.
- Tell students that creativity does count in the activity.

III. Meeting Objectives

- Students will identify and classify a parallelogram.
- Students will identify and classify a rhombus.
- Students will identify and classify a rectangle.
- Students will identify and classify a square.
- Students will identify and classify a kite.
- Students will identify and classify a trapezoid.
- Students will identify and classify an isosceles trapezoid.

IV. Notes on Assessment

- Check each design to be sure that all of the seven figures are in it.
- Be sure that there is a key that is clear and easy to read.
- Include color and creativity in the assessment of each student's work.
- Create a bulletin board to display student work.

Using Parallelograms

I. Section Objectives

- Describe the relationships between opposite sides in a parallelogram.
- Describe the relationship between opposite angles in a parallelogram.
- Describe the relationship between consecutive angles in a parallelogram.
- Describe the relationship between the two diagonals in a parallelogram.

II. Problem Solving Activity-Parallelogram Exploration

- Walk the students through this activity.
- Allow time for students to share their responses after each step of the activity.
- Students will need paper, pencils and rulers to complete this task.
- First, ask the students to draw two sets of intersecting parallel lines on a piece of paper.
- When finished, ask them to label the vertices A, B, C, D
- Ask "What do you notice about this figure?"
- Some students may recognize the parallelogram right away.

- If so, help the students to expand their thinking to see the other properties of the figure as well.
- Ask "What do you notice about the opposite sides of the figure?"
- Answer- They are congruent and parallel.
- Ask "What do you notice about the opposite angles?"
- Answer- They are also congruent.
- Ask- "Which angles are supplementary?"
- For this answer, have the students demonstrate the answer by using a protractor.
- Ask- "How can you use the diagonals of the shape to figure out the number of degrees in this figure?"
- Students should be using triangles for this.

III. Meeting Objectives

- Students will describe the relationships between opposite sides in a parallelogram.
- Students will describe the relationship between opposite angles in a parallelogram.
- Students will describe the relationship between consecutive angles in a parallelogram.
- Students will describe the relationship between the two diagonals in a parallelogram.

IV. Notes on Assessment

- Assessment for this lesson is completed as the students work through each step of the activity.

Proving Quadrilaterals are Parallelograms

I. Section Objectives

- Prove a quadrilateral is a parallelogram given congruent opposite sides.
- Prove a quadrilateral is a parallelogram given congruent opposite angles.
- Prove a quadrilateral is a parallelogram given that the diagonals bisect each other.
- Prove a quadrilateral is a parallelogram if one pair of sides is both congruent and parallel.

II. Problem Solving Activity-Is it really a Parallelogram?

- For this activity, students will need to cut two strips of paper that are the same length and two strips that aren't.
- Then have the students attach the four strips of paper together at the ends with fasteners.
- This will form a quadrilateral.
- Explain to the students that some quadrilaterals are parallelograms and some aren't.
- Then divide the students into groups.
- Students need to come up with ways to demonstrate the following points using their moveable figures.
- These points will help students to see how to prove that a quadrilateral is a parallelogram or that the shape that they have created is NOT a parallelogram.
- 1. Prove a point about opposite sides.
- 2. Prove a point about opposite angles.
- 3. Demonstrate the Supplement Theorem.
- 4. Show how the number of degrees in a quadrilateral is the same as a parallelogram by using the Triangle Sum Theorem.
- When students are finished working in their groups, give them time for each group to demonstrate one point to the rest of the class.
- Ask students to write what they have learned in their notebooks.

III. Meeting Objectives

- Students will prove a quadrilateral is a parallelogram given congruent opposite sides.
- Students will prove a quadrilateral is a parallelogram given congruent opposite angles.
- Students will prove a quadrilateral is a parallelogram given that the diagonals bisect each other.
- Students will prove a quadrilateral is a parallelogram if one pair of sides is both congruent and parallel.

IV. Notes on Assessment

- Check student work for accuracy. Offer feedback during presentations.
- Notice how the students demonstrate each point in their presentations.

Rhombi, Rectangles, and Squares

I. Section Objectives

- Identify the relationship between the diagonals in a rectangle.
- Identify the relationship between the diagonals in a rhombus.
- Identify the relationship between the diagonals and opposite angles in a rhombus.
- Identify and explain biconditional statements.

II. Problem Solving Activity-Can you prove it?

- To prepare this activity, you will need to draw either a rectangle or a rhombus on a coordinate grid. You can have some be accurate and some close.
- The students are going to need to figure out if the figure is a rectangle or a rhombus or does it just look like one.
- Students will be using the principles that they learned in the text to determine whether the figure is really a rectangle or a rhombus.
- Students can work in pairs or small groups on this activity.
- In a rectangle, the students should be pointing out that or proving that the diagonals are congruent.
- In a rhombus, the students should be proving or pointing out that the diagonals intersect at a right angle.
- Students can also use the angles of both figures and the relationship between the angles.
- Allow time for the students to investigate and prepare to prove what their figure is or is not.
- Then allow time for each group to present their discovery.

III. Meeting Objectives

- Students will use the relationship of the diagonals in a rectangle to prove whether a figure is a rectangle or not.
- Students will use the relationship of the diagonals in a rhombus to prove whether a figure is a rhombus or not.
- Students will explain their thinking to their peers.

IV. Notes on Assessment

- Listen to each group prove their figure.
- Challenge their thinking by asking questions.
- Be sure that student answers are clear and precise.
- Offer correction/feedback when needed.

5.6. Quadrilaterals

Trapezoids

I. Section Objectives

- Understand and prove that the base angles of isosceles trapezoids are congruent.
- Understand and prove that if base angles in a trapezoid are congruent, it is an isosceles trapezoid.
- Understand and prove that the diagonals in an isosceles trapezoid are congruent.
- Understand and prove that if the diagonals in a trapezoid are congruent, the trapezoid is isosceles.
- Identify the median of a trapezoid and use its properties.

II. Problem Solving Activity- Trapezoidal Towers

- In this problem, the students are going to have to use what they have learned about trapezoids to justify why or why not a tower characteristic is an isosceles trapezoid or a non- isosceles trapezoid.
- Here is the problem.
- "Jonas is studying architecture. He is really interested in unique geometric architecture. Jonas decides to share two of his favorite buildings with his friend Sam. One is of the Shanghai World Financial Center and the other is of Sutton Place in NYC. Jonas looks at the two towers and says, "It's a shame that the hole in the top of the Shanghai World Financial Center isn't an isosceles trapezoid like Sutton Place that would be really cool." Sam looks puzzled."
- Figure 06.07.01 http://en.wikipedia.org/wiki/Shanghai_World_Financial_Center
- Figure 06.07.02 http://www.thecityreview.com/sutton/rivtow.html
- Look at each building. Why does Jonas think that the hole in the Shanghai World Financial Center is not an isosceles triangle? Use what you have learned to help Sam understand.
- Students need to use a written explanation and diagrams to complete this problem.

III. Meeting Objectives

- In completing this problem, the students will use the properties of isosceles and non- isosceles trapezoids.
- Students will use the Base Angles Theorem.
- Students will write out and justify their answers.

IV. Notes on Assessment

- Read student answers and look at student diagrams.
- Were they able to see that the base angles of Sutton Place are congruent?
- Were they able to see that the opposite sides of the Shanghai building are not congruent?
- Is student writing clearly written?
- Are there diagrams to illustrate student understanding?

Kites

I. Section Objectives

- Identify the relationship between diagonals in kites.
- Identify the relationship between opposite angles in kites.

II. Problem Solving Activity-Let's Go Fly a Kite!

- For this activity, use the kite dimensions found in the text under the exercises on page 398-399. This is Figure06.08.01.
- Give students material for making a kite.
- A great material is Tyvek which is used in housing for wall coverage. You can complete about 100 kites with one roll.
- If cost is an issue, then use paper or plastic, but the Tyvek works the best.
- Also, use wooden dowels for the supports of the kite.
- Ask the students to use the angle measurements in the exercises to design a kite.
- They can make it as large or small as they would like as long as the angle measures are the same.
- *Note: This will be used again in the next chapter on Similarity. Because the angle measures are the same, the small and large kites will be similar.*
- Then let the students work.
- You can do a whole project on this too complete with flying the kites on a windy day.
- Students will LOVE it!!

III. Meeting Objectives

- Students will identify the relationship between diagonals in kites.
- Students will identify the relationship between opposite angles in kites.
- Students will use their knowledge to construct a kite.

IV. Notes on Assessment

- Assess student work on three different levels.
- 1. Did the students correctly measure the angles to construct an accurate kite?
- 2. Did the students construct their kite?
- 3. Was student work accurate and completed on time?
- Create a rubric for grading students on each element of the kite.
- Include creativity in your grading scale.

5.6. Quadrilaterals

5.7 Similarity

Ratios and Proportions

I. <u>Section Objectives</u>

- Write and simplify ratios.
- Formulate proportions.
- Use ratios and proportions in problem solving.

II. <u>Problem Solving Activity- Ratio/Proportion Relay</u>

- Since this lesson is mostly review, use this fun game to review the concepts in the lesson.
- To prepare, bring in a bunch of assorted items from home. Make these random household items, but have more than one of each type of items. For example, three hairbrushes or five apples.
- Then put the students are four teams.
- This activity is timed.
- Students come up one at a time.
- Students have $15 - 20$ seconds to look at the table and write as many ratios as they can.
- Then buzz it and the next person comes up.
- The last person has to take the ratios of the first three and create as many proportions as he/she can in two minutes.
- Students are not allowed to coach each other in their work.
- The team with the most ratios earns points.
- The team with the most correct proportions wins.
- You can repeat this more than once. Students will love it.

III. <u>Meeting Objectives</u>

- Students will write and simplify ratios.
- Students will write proportions.
- Students will use ratios and proportions in problem solving.

IV. <u>Notes on Assessment</u>

- How many ratios did each team write?
- Are the proportions accurate?
- Assess the time constraints, did the students need more or less time- adjust as necessary.

Properties of Proportions

I. <u>Section Objectives</u>

- Prove theorems about proportions.
- Recognize true proportions.
- Use proportions theorem in problem solving.

II. Problem Solving Activity-Proportional Teamwork

- For this activity, students are going to draw two proportional triangles.
- Be sure to include measurements for each side of each triangle.
- Then, when instructed, students are going to pass the triangles to the person to their right. The person on the end of the room passes across the room to the "first"person.
- Then each person must write one proportion that represents the two triangles.
- Then they pass them again.
- Once again this repeats and the next student writes one proportion about the two triangles.
- Now there are two proportions on the paper under the triangles.
- Next it is passes again, and the last student writes the one remaining proportion.
- Finally, it is given back to the starting person (who drew the triangles).
- That student needs to correct the work of the other three.
- This student needs to write at least one theorem that is represented by these proportions and explain why he/she selected that theorem.

III. Meeting Objectives

- Students will prove theorems about proportions.
- Students will recognize true proportions.
- Students will use proportions theorem in problem solving.

IV. Notes on Assessment

- Collect all student work at the end of the class.
- First, check the triangles. Are they proportional?
- Next, check all of the proportions that were written about the triangles.
- Finally, check the notes/corrections and the theorem used.
- Is it explained well?
- Is it correct?
- Provide students with feedback and comments.

Similar Polygons

I. Section Objectives

- Recognize similar polygons.
- Identify corresponding angles and sides of similar polygons from a statement of similarity.
- Calculate and apply scale factors.

II. Problem Solving Activity- Let's Go Fly a Kite- Part Two

- For this activity, students are going to revisit the work they did on kites.
- Particularly, students are going to use their work on creating kite designs from pg. 398-399.
- Ask students to identify what makes these kites similar. Brainstorm a list and write them on the board.

5.7. Similarity

- Students should be commenting that because the angle measures are all the same, that the side lengths will be proportional.
- Then have students pair up.
- These two students need to compare their kite designs.
- They need to write four different proportions comparing the side lengths of the kites.
- Then have students design a statement of similarity that best compares their kites.
- If they can figure out the scale factor between the two kites have them include that in their work.
- Allow time for the student to share their work when finished.

III. Meeting Objectives

- Students will recognize similar polygons in their kite designs.
- Students will identify and explain similar side lengths of the kite design.
- Students will write proportions to show similarity.
- Students will calculate scale factors.

IV. Notes on Assessment

- Assess student work by walking around and observing students as they work.
- Then assess student understanding during student presentations.
- Help students to make connections about similar polygons by asking questions and providing feedback.

Similarity by AA

I. Section Objectives

- Determine whether triangles are similar.
- Understand AAA and AA rules for similar triangles.
- Solve problems about similar triangles.

II. Problem Solving Activity- Thales and the Pyramids

- This is a great lesson to use Thales and the simple way that Thales measured the pyramids.
- You can use this website to find out more information about Thales.
- http://educ.queensu.ca/ fmc/april2002/Pyramids.htm
- Essentially, Thales figured out the height of the pyramids by using his own height. He waited until his shadow equaled his height. Then he measured the height of the pyramid and he knew that the height of the pyramid was equal to the pyramid's height.
- Here is the problem.
- "Stan is 6 feet tall. When he goes outside, his shadow is only nine feet long. The shadow of the tree in his yard is eighteen feet long. Based on these numbers, what is the height of the tree?"
- Use a drawing to figure out the answer to this problem.
- Have students write a paragraph to explain their process and how they arrived at their answer.
- When finished, have the students share their work in small groups.

III. Meeting Objectives

- Students will determine similar triangles.
- Students will understand AAA and AA for similar triangles.

- Students will use AAA and AA with indirect measurement to solve problems.
- Students will share their understanding in their written work.
- Students will explain their thinking in small groups.

IV. Notes on Assessment

- Assess each diagram to assess student understanding.
- Are the triangles similar?
- Are they labeled correctly?
- Was a proportion used to solve for the height of the tree?
- Then assess student writing.
- Is student thinking clear?
- Offer notes/feedback.

Similarity by SSS and SAS

I. Section Objectives

- Use SSS and SAS to determine whether triangles are similar.
- Apply SSS and SAS to solve problems about similar triangles.

II. Problem Solving Activity-Triangle Jeopardy

- To play this game, divide students into small groups.
- To prepare this game, use a set of index cards and write one of the ways to prove similarity among triangles on each card.
- You should have cards that say SSS, SAS, AA and AAA
- Be sure that you have several of each card and mix them up.
- Each student in the group takes a turn.
- The student selects a card.
- Then he/she must come up with an example that illustrates the way to prove similar triangles.
- Each team can have 1 helpful hint- that is from you, and 1 lifeline from their group.
- If the student completes the challenge correctly, the team receives a point.
- You can play this game for quite a while.
- Some variations can include scale factor or diagrams on the board and then the group needs to show how the triangles are similar.

III. Meeting Objectives

- Students will use and apply the SSS and SAS when determining whether triangles are similar.
- Students will use and apply the SSS and SAS to solve problems about similar triangles.
- Students will explain their work verbally and through diagrams.

IV. Notes on Assessment

- Assessment comes through the process of the game.
- Because students work individually, you will have a really good idea of who understands about similar triangles and who needs more assistance.
- Provide coaching/feedback when necessary through "helpful hints."

5.7. Similarity

Proportionality Relationships

I. <u>Section Objectives</u>

- Identify proportional segments when two sides of a triangle are cut by a segment parallel to the third side.
- Divide a segment into any given number of congruent parts.

II. <u>Problem Solving Activity-Midsegment Match- up</u>

- To prepare this activity, you will need to draw a bunch of triangles and cut them along the midsegment line.
- Then pass out one part of a triangle to each student.
- Students need to measure their part of the triangle.
- Then, they need to find the student who has their match.
- Students walk around the room and find a match for their triangle.
- When finished, all of the triangles should be complete.
- Once students have found a match, or think that they have, they need to write proportions to justify their thinking.
- Allow time for the students to share their work when finished.

III. <u>Meeting Objectives</u>

- Students will identify proportional segments when two sides of a triangle are cut by a segment parallel to the third side.

IV. <u>Notes on Assessment</u>

- Observe students as they walk around finding the match for their triangle part.
- Notice if students are using measurements or not.
- If not, remind students to look for the proportionality in the measurements of each section of the triangle.
- Be sure to listen as students share their work.
- Are they able to articulate why the triangles are similar?
- Is the match that they selected an accurate match?
- Offer feedback/correction as needed.

Similarity Transformations

I. <u>Section Objectives</u>

- Draw a dilation of a given figure.
- Plot the image of a point when given the center of dilation and scale factor.
- Recognize the significance of the scale factor of a dilation.

II. <u>Problem Solving Activity- T- Shirt Dilation</u>

- Use the picture of Mickey Mouse for this problem- Figure 07.07.01
- http://en.wikipedia.org/wiki/Mickey_mouse
- Provide students with a copy of the picture.
- Then have students measure the height of Mickey Mouse according to the picture.

- Have them also measure the width of Mickey's arms.
- Tell them to keep this measurement as they will need it to solve the problem.
- Here is the problem.
- "Sasha is creating a t-shirt for her sister with Mickey Mouse on it. She takes a picture of Mickey Mouse from Wikipedia and measures it. Now she knows that she needs to triple the size of the picture for it to be perfect on the t-shirt. Given this information, help Sasha by creating proportions that show how the original picture compares to the one on the t- shirt."
- Students need to use a diagram to show their work.
- Students need to show all of their problem solving.
- Remind them to show all work.
- Have students share their work when finished.

III. <u>Meeting Objectives</u>

- Students will draw a dilation of a given figure.
- Students will use a scale factor to show a dilation.
- Students will explain their thinking through writing and through diagrams.
- Students will share their work to demonstrate understanding.

IV. <u>Notes on Assessment</u>

- Is the diagram accurate?
- Is the dilation correct?
- Have the students shown all of the measurements?
- Are the proportions correct?
- Have the students shown all of their work?
- Is the student able to verbally explain their work?

Self- Similarity (Fractals)

I. <u>Section Objectives</u>

- Appreciate the concept of self- similarity.
- Extend the pattern in a self- similar figure.

II. <u>Problem Solving Activity-Fractal Fun</u>

- If you have access to technology, then use this site to have students watch the different fractals being formed.
- http://en.wikipedia.org/wiki/Fractal
- Then tell students that they are going to create their own fractal.
- They can begin with a triangle, a pentagon or a hexagon.
- Students need to show four levels of the fractal.
- Remind them of the steps on how the fractal is created.
- You can use the steps in the text to assist the students.
- Then provide them with rulers, colored pencils and paper to create their fractals.
- Hint: There are bound to be errors at first. Remind the students to use the text to support their work. They can also revisit the website to help them to brainstorm ideas and create an exciting fractal.
- Leave students alone as much as possible.
- Allow students time to think and struggle a bit. It will help them to come to a clearer understanding of the concepts in the lesson.

5.7. Similarity

III. Meeting Objectives

- Students will appreciate the concept of self- similarity.
- Students will extend the pattern in a self- similar figure.
- Students will create their own fractal.

IV. Notes on Assessment

- Walk around and help students who are really struggling.
- Be sure to allow students some time to work before you jump in as it is always better for students to try to solve problems on their own.
- Create a rubric to help you in grading each fractal.
- First, does the fractal work?
- Is their work complete?
- Does it show four levels?
- Make notes and share corrections/feedback with students.

5.8 Right Triangle Trigonometry

The Pythagorean Theorem

I. <u>Section Objectives</u>

- Identify and employ the Pythagorean Theorem when working with right triangles.
- Identify common Pythagorean triples.
- Use the Pythagorean Theorem to find the area of isosceles triangles.
- Use the Pythagorean Theorem to derive the distance formula on a coordinate grid.

II. <u>Problem Solving Activity-The Ramp Dilemma</u>

- For this problem, students will be using the Pythagorean Theorem to figure out whether or not the following dimensions work for a bike ramp. They will be using the concept of Pythagorean triples in their work.
- Here is the problem.
- "Jonas is building a bike ramp. He has a pattern for a small model of a bike ramp and he wants to build a larger version of the bike ramp for his yard. His pattern uses measurements in inches. The pattern says that the dimensions of the bike ramp model will be $9 - 12 - 15$. Jonas wants to enlarge this pattern. Use what you have learned about the Pythagorean Theorem and Pythagorean Triples to design a bike ramp that is not larger than $21\text{ feet} - 28\text{ feet} - 34\text{ feet}$."
- Solution Notes:
- This problem requires several steps. The first thing that the students are going to need to do is to convert feet into inches. Then they will know how many inches the ramp design needs to be.
- Next, the students need to begin to build proportions that work for triples.
- By multiplying by three, the students will find that they can build several different ramps.
- Here are some options:
- $27 - 36 - 45, 81 - 108 - 135$
- The largest one without going over is $243 - 324 - 405$.
- Students need to draw a design with a scale to show their ramp.
- Then they need to write the final dimensions in feet.
- In feet- $20\frac{1}{4} - 27 - 33\frac{3}{4}$ feet.

III. <u>Meeting Objectives</u>

- Students will use the Pythagorean Theorem in designing a bike ramp.
- Students will use their knowledge of Pythagorean triples in designing their ramp.
- Students will draw a design to show their work.
- The design will be drawn to scale.

IV. <u>Notes on Assessment</u>

- See the problem solving activity section for notes and solutions.

Converse of the Pythagorean Theorem

I. <u>Section Objectives</u>

- Understand the converse of the Pythagorean Theorem.
- Identify acute triangles from side measures.
- Identify obtuse triangles from side measures.
- Classify triangles in a number of different ways.

II. <u>Problem Solving Activity-Figure Those Triangles</u>

- This is a game that has the students use number cards to figure out whether triangles are right triangles, acute triangles or obtuse triangles.
- To prepare for this game, use index cards and write one number on each card. Number the cards $2-25$. Each group will need a set of cards to play the game.
- Divide the students into groups.
- Students are going to use the number cards to create as many different types of triangles as they can.
- Begin with right triangles.
- Students are given a short period of time to use the number cards to create as many different combinations of numbers that will equal right triangles. For example, using $3-4-5$, the students will have a right triangle because $3^2 + 4^2 = 5^2$.
- Students work together to write out as many as possible using the numbers $2-25$.
- Then move on to acute triangles. Remind students that $a^2 + b^2 > c^2$.
- Then move on to obtuse triangles. Remind students that $a^2 + b^2 < c^2$.
- Finally have students share their combinations.
- Winning teams can be determined based on the number of combinations created and on accuracy.

III. <u>Meeting Objectives</u>

- Students will use number combinations to determine side lengths of right triangles.
- Students will use number combinations to determine side lengths of acute triangles.
- Students will use number combinations to determine side lengths of obtuse triangles.

IV. <u>Notes on Assessment</u>

- Walk around as students play the game.
- Offer assistance when needed.
- Have students share their combinations to determine accuracy.

Using Similar Right Triangles

I. <u>Section Objectives</u>

- Identify similar triangles inscribed in a larger triangle.
- Evaluate the geometric mean of various objects.
- Identify the length of an altitude using the geometric mean of a separated hypotenuse.
- Identify the length of a leg using the geometric mean of a separated hypotenuse.

II. Problem Solving Activity-The Camping Question

- Students are going to use the geometric mean to figure out the altitude of a triangle.
- Here is the problem.
- "Mariah is going on a teen wilderness trip. She will be using a tent that is triangular in shape and she has been told that the height of the tent can't exceed nine feet. For this to be possible, the tent pole can not be greater than nine feet. She has a diagram of her tent. Use what you have learned about the geometric mean and the given the dimensions, to figure out the height of the tent. Will Mariah's tent work for her trip or does she need to get a different tent?"
- Figure08.03.01
- Students first need to convert inches to feet. This will show them that the tent is divided into two 8 foot sections.
- Solution: $8 \times 8 = 64$
- $\sqrt{64} = 8$
- The height of the tent is 8 feet, so Mariah's tent will fit the specifications of the trip.

III. Meeting Objectives

- Students will evaluate the geometric mean of various objects.
- Students will identify the length of an altitude using the geometric mean of a separated hypotenuse.
- Students will justify their answers.

IV. Notes on Assessment

- Check student work for accuracy.
- Did the students convert inches to feet correctly?
- Did they remember to multiply the dimensions of the divided hypotenuse?
- Did they come up with a solution of 8 feet for the altitude of the tent?
- Offer feedback/correction when needed.

Special Right Triangles

I. Section Objectives

- Identify and use the ratios involved with right isosceles triangles.
- Identify and use the ratios involved with $30 - 60 - 90$ triangles.
- Identify and use ratios involved with equilateral triangles.
- Employ right triangle ratios when solving real- world problems.

II. Problem Solving Activity-Triangle Tiling

- For this activity, you will need to cut out a bunch of squares of different sizes. Each square needs to have a diagonal dividing it into two right triangles.
- Figure 08.04.01
- Each student is going to receive a square to work with.
- Tell students that they need to measure their square and figure out the length of the hypotenuse of each square.
- The measurements must be labeled on the back of the design.
- Then they can decorate the square however they would like.
- Finally, the students are going to create a square tiling on a piece of paper.

5.8. Right Triangle Trigonometry

- They must be sure that the dimensions of each square is accurate (many students will trace the initial square).
- Creativity counts for this assignment.
- Students could repeat this assignment using $30 - 60 - 90$ triangles too.

III. Meeting Objectives

- Students will use ratios involved in right isosceles triangles.
- Students will see how a square and a right triangle are related.
- Students will understand the properties of $45 - 45 - 90$ triangles.
- Students will demonstrate understanding by creating their tiling.
- Students will employ right triangle ratios with real- world problems.

IV. Notes on Assessment

- Create a rubric to grade student work.
- Share the rubric grading scale with the students before beginning.
- Be sure that student measurements are accurate.
- Encourage students to use color and creativity in their designs.
- Create a wall display of student work.

Tangent Ratios

I. Section Objectives

- Identify the different parts of right triangles.
- Identify and use the tangent ratio in a right triangle.
- Identify complementary angles in right triangles.
- Understand tangent ratios in special right triangles.

II. Problem Solving Activity-Ladder Tangents

- Students will use what they have learned about tangents to solve the following problem.
- Here is the problem.
- "A building is 20 feet tall. Mark and his roofing company are going to replace the roof on the building. The ladder that they are using 10 feet from the building creating an angle of $55°$. Use this information and the diagram to solve the following questions."
- Figure08.05.01
- Write these questions on the board.
- 1. What is the tangent of angle x?
- 2. What is the length of the ladder?
- Solution:
- Tan $x = \frac{3}{5}$
- Hypotenuse: $a^2 + b^2 = c^2$
- $12^2 + 10^2 = c^2$
- $144 + 100 = c^2$
- $244 = c^2$
- $\sqrt{244} = c^2$
- $15.6 \text{ feet} = c$

III. Meeting Objectives

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- Students will understand the different parts of right triangles.
- Students will use the tangent ratio in a right triangle.
- Students will identify complementary angles in right triangles.
- Students will understand tangent ratios in special right triangles.

IV. Notes on Assessment

- Walk around as the students are working.
- Notice which students are having difficulty and offer support.
- Allow students time to share their work when finished.

Sine and Cosine Ratios

I. Section Objectives

- Review the different parts of right triangles.
- Identify and use the sine ratio in a right triangle.
- Identify and use the cosine ratio in a right triangle.
- Understand sine and cosine ratios in special right triangles.

II. Problem Solving Activity-Dion's Yard Design

- Here is the problem.
- "Dion has a square yard. He wants to divide the yard on the diagonal by planting a beautiful row of fruit trees. The yard measures 16×16. If this is the case, what is the measurement of the diagonal? Draw a diagram of the yard and the diagonal to also answer the following questions about the ratios in the yard."
- Students begin by drawing a diagram of the yard.
- Using the Pythagorean Theorem, the diagonal is 22.6 feet long.
- Now use these measurements to answer the following questions.
- 1. What is $\sin a$?
- 2. What is $\sin b$?
- 3. What is $\cos a$?
- 4. What is $\cos b$?
- Solutions:
- 1. 1.1425
- 2. .7
- 3. .7
- 4. .7
- Allow time for the students to share their work at the end.
- Extension: Would the ratios be the same if the diagonal extended the opposite way as well?

III. Meeting Objectives

- Students will review the different parts of right triangles.
- Students will identify and use the sine ratio in a right triangle.
- Students will identify and use the cosine ratio in a right triangle.

IV. Notes on Assessment

- Check the student diagram first.

5.8. Right Triangle Trigonometry

- Is it labeled the correct way?
- Do the students understand the difference between the adjacent, the opposite and the hypotenuse?
- Are the sin ratios accurate?
- Are the cosine ratios accurate?
- Listen for understanding and clarity when students share their work.

Inverse Trigonometric Ratios

I. Section Objectives

- Identify and use the arctangent ratio in a right triangle.
- Identify and use the arcsine ratio in a right triangle.
- Identify and use the arccosine ratio in a right triangle.
- Understand the general trends of trigonometric ratios.

II. Problem Solving Activity-Teaching Time

- The students are going to be teaching the class about how to find trigonometric ratios and inverse trigonometric ratios.
- Divide the students into six groups.
- Assign each of the groups one of the following topics.
- 1. Tangents
- 2. Sines
- 3. Cosines
- 4. Arctangents
- 5. Arcsines
- 6. Arccosines
- Then tell students that they need to prepare a five minute presentation on their topic to teach the others in the class about it.
- Students with the arc tangents can use calculators.
- Each presentation must have a diagram, an example and a problem for the class to solve.
- Allow time for the students to work.
- Then have each group present their topic.

III. Meeting Objectives

- Students will identify and use the arctangent ratio in a right triangle.
- Students will identify and use the arcsine ratio in a right triangle.
- Students will identify and use the arccosine ratio in a right triangle.

IV. Notes on Assessment

- Create a checklist for each point that needs to be in the student presentation.
- Evaluate each group as they present their topic.
- Is the presentation complete?
- Are there any missing pieces?
- Do the students have a good understanding of the material?
- Are the students able to speak clearly about it?

Acute and Obtuse Triangles

I. <u>Section Objectives</u>

- Identify and use the Law of Sines.
- Identify and use the Law of Cosines.

II. <u>Problem Solving Activity-Teaching Time</u>

- For this activity, the students will be divided into groups of $4 - 6$. Then in this larger group, divide the group into smaller sections of $2 - 3$ people.
- Each team will be assigned one of the Laws.
- Each team has the assignment of teaching the other team about their Law.
- First, the team must design a problem that they think best illustrates the Law of Sines or the Law of Cosines.
- Then they must prepare their presentation.
- Finally, the teams present their work to the other teams.
- Each team needs to evaluate the other.
- You can use a rubric for evaluations.
- Some possible points might be:
- 1. Clarity of explanation
- 2. Is the work accurate?
- 3. Did you learn something new about the law?
- Allow time for students to share about their experience when finished.

III. <u>Meeting Objectives</u>

- Students will identify and use the Law of Sines.
- Students will identify and use the Law of Cosines.
- Students will demonstrate understanding through their presentation.

IV. <u>Notes on Assessment</u>

- Observe each group as it presents.
- You won't get to see all of it, but do your best to get an idea of how each group is doing. Make notes on the presentations.
- Collect all notes from each group.
- Then compare student evaluations with your evaluations.
- Assign each group a final grade.

5.9 Circles

About Circles

I. Section Objectives

- Distinguish between radius, diameter, chord, tangent, and secant of a circle.
- Find relationships between congruent and similar circles.
- Examine inscribed and circumscribed polygons.
- Write the equation of a circle.

II. Problem Solving Activity- Circle Rhymes

- Create a rhyme about circles.
- Students can make it a rap, a song or a poem.
- Here are the components of the rhyme.
- Each must have:
- Radius
- Diameter
- Chord
- Tangent
- Secant
- Defined in it.
- Students can choose to include other elements, but these key ones must be in there.
- In addition, if students want to mention pi, they can, just be sure that they have the other parts of a circle involved.
- Allow students time to work on this poem.
- Students can perform/present it in small groups.
- If you have a really creative group that likes to perform, you might have them do it for the whole class. If this is the case, then presentations may take a whole class period.

III. Meeting Objectives

- Students will distinguish between radius, diameter, chord, tangent, and secant of a circle.
- Students will present their material in a creative fun way.

IV. Notes on Assessment

- Be sure that all of the important components are present.
- Students really can't do this wrong.
- Sit back and enjoy watching students use their creativity to bring circles to life.

Tangent Lines

I. Section Objectives

- Find the relationship between a radius and a tangent to a circle.
- Find the relationship between two tangents draw from the same point.
- Circumscribe a circle.
- Find equations of concentric circles.

II. Problem Solving Activity-Lawn Sprinklers

- Here is the problem.
- "Tomas is putting in a sprinkler system in his back yard. He has divided the yard into six square sections. Inside the center of each square he has planted a sprinkler. The sprinkler spray extends to a distance of 56 feet. If this is the case, how much area will Tomas cover with his six sprinklers? Here is a diagram of one of the square plots to help you out."
- Figure 09.02.01
- Tell students to show all of their work in their answer.
- Solution:
- 56 is the radius, so 112 is the diameter of the circle of the spray.
- This is also the side length of one of the square.
- Since we are looking for area, the formula for area of a square is s^2
- $A = 112^2 = 12,544$ feet.
- For six squares we multiply this number $x\ 6 = 75,264$ feet.

III. Meeting Objectives

- Students will use circles in problem solving.
- Students will use what they have learned about inscribed circles in problem solving.

IV. Notes on Assessment

- Look at all student work.
- Is the work accurate?
- Did the students solve for area and not for perimeter?
- Did the students figure the distance for the diameter?
- Did the students arrive at the correct answer?
- Provide students with feedback/correction.

Common Tangents and Tangent Circles

I. Section Objectives

- Solve problems involving common internal tangents of circles.
- Solve problems involving common external tangents of circles.
- Solve problems involving externally tangent circles.
- Solve problems involving internally tangent circles.
- Common tangent

II. Problem Solving Activity-The Gear Problem

- Here is the problem.

5.9. Circles

- "Ron is constructing a board with moveable gears for his little brother to play with. He constructs the first circle gear and puts the second gear right next to the first one. He wants to figure out the distance between the centers of the two gears so that he can use the correct size band to tie the two together. Use the diagram to help Ron figure out the distance of $\overline{AB}$."
- Figure 09.03.01
- Students need to include a diagram and a written explanation of their solution.
- Here is the solution:
- Figure 09.03.02
- The distance is approximately 14.4.

III. Meeting Objectives

- Students will solve problems involving common external tangents of circles.
- Students will solve problems involving externally tangent circles.

IV. Notes on Assessment

- Collect both diagrams and written work from students.
- Does the written work match the diagram?
- Is the diagram clear?
- Is the written explanation clear?
- Did the students arrive at the correct answer?
- Provide students with correction/feedback.

Arc Measures

I. Section Objectives

- Measure central angles and arcs of circles.
- Find relationships between adjacent arcs.
- Find relationships between arcs and chords.

II. Problem Solving Activity-The Pie Dilemma

- Here is the problem.
- "Jessica has made a delicious blueberry pie. Unfortunately, she cut the pie into uneven pieces."
- Figure 09.04.01
- "Given the diagram, help Jessica figure out the dimensions of each section of the pie based on her uneven cuts."
- Find the measure of arc CD, arc AC and arc $BDCA$.
- Students need to use a diagram, a written explanation and a worksheet showing all work.
- Solution:
- The angle of arc AB is $80°$.
- Since AB and CD are congruent, then CD is also $80°$.
- Arc $AC = 180 - 80 = 100°$
- Arc $BDCA = 100 + 180 = 280°$

III. Meeting Objectives

- Students will find the measure central angles and arcs of circles.

- Students will find relationships between adjacent arcs.
- Students will find relationships between arcs and chords.

IV. Notes on Assessment

- Collect student work.
- Check student work for accuracy.
- Offer feedback/notes as needed.

Chords

I. Section Objectives

- Find the lengths of chords in a circle.
- Find the measure of arcs in a circle.

II. Problem Solving Activity-Circle Dilemma

- Here is the problem.
- "Mrs. Watson decided to teach her class about chords in a new way. She began by taking them out to the elementary school playground where there is a huge circle painted on the ground. Then she drew a straight line across and marked the center of the circle. Mrs. Watson gave her students a tape measure and asked them to figure out the hypotenuse of the right triangle."
- "Jeff was puzzled, what right triangle? But Maria wasn't. She took the tape measure and went to work. Here is what she found."
- Figure 09.05.01
- "Help Maria finish the problem. What is the measure of the hypotenuse?"
- Have students show their work through a diagram and written explanation.
- Solution:
- Use the Pythagorean Theorem.
- $5^2 + 10^2 = c^2$
- $25 + 100 = c^2$
- $125 = c^2$
- $11.2 \approx c$

III. Meeting Objectives

- Students will find the lengths of chords in a circle.
- Students will find the lengths of the sides of a right triangle.
- Students will use this information in problem solving.

IV. Notes on Assessment

- Collect student work.
- Check student work for accuracy.
- Does the diagram accurately show the given information?
- Is the arithmetic correct?
- Does the written explanation make sense?
- Offer correction/feedback as needed.

5.9. Circles

Inscribed Angles

I. <u>Section Objectives</u>

- Find the measure of inscribed angles and the arcs they intercept.

II. <u>Problem Solving Activity- Find the Inscribed Angles</u>

- For this activity, students are going to work in small groups to write a problem.
- Each problem must have a circle with a triangle or a quadrilateral inscribed in it.
- Students are going to use a protractor to measure the angles of the inscribed angles.
- Students are assigned the task of writing a word problem for another student to solve.
- Students must include a diagram and have an answer key to check student work.
- Allow time for students to write their problems.
- If students are having difficulty, have them brainstorm things with circles and other things in them- like a circus ring with a stand in it, an amphitheater, a raft in a lake, etc.
- When students are finished, have them switch papers with another group and solve each other's problem.
- Allow time for students to share when finished.
- There is a learning curve for this and don't be surprised if some of the initial attempts at writing a problem are missing essential components. If this is the case, have the students try again and rewrite the problem.
- Be sure that diagrams are accurate- this will make a huge difference.

III. <u>Meeting Objectives</u>

- Students will find the measure of inscribed angles and the arcs they intercept.
- Students will use what they have learned to write a word problem.
- Students will use what they have learned in problem solving.

IV. <u>Notes on Assessment</u>

- Walk around and observe students as they work.
- If students are having difficulty, offer suggestions.
- Help students to understand how to solve the problems.
- Help students to be sure that the problems that they are writing are complete.
- Offer suggestions/feedback as needed.

Angles of Chords, Secants and Tangents

I. <u>Section Objectives</u>

- Find the measures of angles formed by chords, secants and tangents.

II. <u>Problem Solving Activity-Folding It Out</u>

- Students are going to be examining different chords, secants and tangents as found with a circle model.
- Begin by having each students cut out a circle from a sheet of colored paper.
- Then ask students to fold the circle in half.
- Then fold it in thirds.

- Then fold it in half again.
- This way you have a circle that has been divided.
- Finally, trace each line of the folds of the circle so that they can be seen.
- Then glue the circle with its lines to a piece of white paper.
- Have the students choose one pair of lines to extend outside the circumference of the circle.
- Then have students work in small groups to figure out the measures of each of the angles created by the different folds of paper.
- Some students may have folded their circles differently.
- This is alright as long as the students are going to work together.
- Ask them to work together on figuring out the angle measures.
- Allow students time to work and then to share their work when finished.

III. Meeting Objectives

- Students will find the measures of angles formed by chords, secants and tangents.
- Students will work in groups to figure out the different angle measures.
- Students will share their work when finished.

IV. Notes on Assessment

- Walk around as students work and offer assistance as needed.
- If students are having difficulty, remind them to refer back to the text.
- Collect student work when finished, and assess student understanding based on the accuracy of student work.
- Offer feedback and correction as needed.

Segments of Chords, Secants and Tangents

I. Section Objectives

- Find the lengths of segments associated with circles.

II. Problem Solving Activity-Criss- Cross Applesauce

- Here is the problem.
- "Jack and Andy decided to play a game called Criss Cross Applesauce in the school yard. They each took a jump rope and ran across the circular playground. Their ropes crossed at one point. They wondered who had a longer rope. Using this diagram, help them to figure this out."
- Figure 09.08.01
- Allow time for students to share their work when finished.
- Solution:
- The produce of one line segment is equal to the product of the other line segment.
- Therefore, we can solve for the missing segment of the line by writing an equation and multiplying.
- $12x = (6)(10)$
- $12x = 60$
- $x = 5$
- One rope is 16 feet long.
- One rope is 17 feet long.
- The boy with the 17 foot long rope has the longer rope.
- Extension- What happens when an 18 foot rope is added to the mix- where does this rope cross the circle? Does it intersect with the other two ropes?

5.9. Circles

III. <u>Meeting Objectives</u>

- Students will find the lengths of segments associated with circles.
- Students will use this information in problem solving.
- Students will share their work with a peer.

IV. <u>Notes on Assessment</u>

- Were the students able to arrive at the correct answer?
- Can they explain how they got their answer?
- Is their work accurate?
- Listen in as students work and offer feedback and correction as needed.

5.10 Perimeter and Area

Triangles and Parallelograms

I. Section Objectives

- Understand the basic concepts of the meaning of area.
- Use formulas to find the area of specific types of polygons.

II. Problem Solving Activity-Flooring Dilemma

- Here is the problem.
- "Ron is working for a flooring company for the summer. He is assigned the task of figuring out the area of a strangely shaped room at the library. Here is a diagram of the room that Ron needs to measure."
- Figure 10.01.01
- "Ron thinks that the room looks like two rectangles and one triangle. He has some of the measurements from his diagram, but needs to figure out some of the other measurements. Is Ron correct? Can the room be divided into two rectangles and a triangle? What are the missing measurements? What is the area of the room?"
- Students will work in pairs to figure out these answers.
- Here are the solutions.
- The length of x is 8 feet because $12 - 4 = 8$.
- The area of the first rectangle is $6 \times 12 = 72$ feet.
- The area of the second rectangle is $8 \times 8 = 64$ feet.
- The area of the triangle is $= \frac{1}{2}(6)(6) = 18$ feet
- Total area $= 72 + 64 + 18 = 154$ feet

III. Meeting Objectives

- Students will understand the basic concepts of the meaning of area.
- Students will use formulas to find the area of specific types of polygons.
- Students will use this understanding in problem solving.

IV. Notes on Assessment

- Be sure that the students answered all of the questions in the problem.
- Are the measurements accurate?
- Did they divide the figure into two rectangles and a triangle?
- Did they find a different variation that works?
- Check all work and offer correction/feedback when necessary.

Trapezoids, Rhombi and Kites

I. Section Objectives

- Understand the relationships between the areas of two categories of quadrilaterals: basic quadrilaterals and special quadrilaterals.
- Derive area formulas for trapezoids, rhombi and kites.
- Apply the area formula for these special quadrilaterals.

II. Problem Solving Activity-Jeffrey's Kite

- Students will use what they have learned in the lesson to find the area of Jeffrey's kite.
- Here is the problem.
- "Jeffrey has designed a kite to be flown in the city parade. He has made a large version of a small kite. His kite fits perfectly in a rectangular box that is $2[U+0080][U+0099] \times 3[U+0080][U+0099]$. In inches, what is the area of Jeffrey's kite?"
- Students need to use what they have learned to solve this problem. They can draw an interpretation of the kite design if they wish to.
- Students will need to convert feet to inches to begin with.
- Solution:
- 2 feet = 24 inches
- 3 feet = 36 inches
- The rectangle is 24×36
- Use the formula for finding the area of a kite.
- $\frac{1}{2}d_1 d_2 = \frac{1}{2}(24)(36)$
- $\frac{1}{2}(864)$
- Area = 432 inches = 36 feet
- Conclusion: Jeffrey has created a HUGE kite.

III. Meeting Objectives

- Students will use the formula for area to find the area of a kite.
- Students will learn to compare rectangles and the area of a kite.
- Students will demonstrate understanding through problem solving.

IV. Notes on Assessment

- Be sure that the students have solved the problem accurately.
- Did the students convert the measurement units?
- Is the problem labeled correctly?
- Did the students draw a diagram to explain their answer?
- Offer correction/feedback when necessary.

Area of Similar Polygons

I. Section Objectives

- Understand the relationship between the scale factor of similar polygons and their areas.
- Apply scale factors to solve problems about areas of similar polygons.
- Use scale models or scale drawings.

II. Problem Solving Activity-Mt. Rushmore

- You can use the following website for reference.

- http://library.thinkquest.org/04oct/00398/ThinkquestHome_files/Thinkquest/Lia%20Web%20Page.htm
- "Jessica is designing a model of Mt. Rushmore. Mt. Rushmore is 500 feet high and 400 feet wide. Based on a scale of 1[U+0080][U+009D] $= 25$ feet, what are the dimensions of Jessica's model?"
- "When you are sure that you have the correct measurement, use grid paper and draw a diagram of Jessica's model. Be sure to draw your diagram to scale."
- Solution:
- 1[U+0080][U+009D] $= 25$ feet
- 500 feet high $= 20$[U+0080][U+009D] high
- 400 feet wide $= 16$[U+0080][U+009D] wide
- The students can draw this free hand using a ruler and large sheets of paper, or they can use grid paper to create another scale using the units on grid paper and then draw a diagram there.
- Allow time for the student to share their work when finished.

III. Meeting Objectives

- Students will use scale models or scale drawings.
- Students will apply what they have learned about scale models and drawing when problem solving.

IV. Notes on Assessment

- Is student work accurate?
- Does the diagram accurately represent the measurement of Mt. Rushmore?
- Have the students used the scale and proportion to come up with the correct measurements?
- Provide students with coaching and feedback.

Circumference and Arc Length

I. Section Objectives

- Understand the basic idea of a limit.
- Calculate the circumference of a circle.
- Calculate the length of an arc of a circle.

II. Problem Solving Activity- The Cul-de- sac Question

- Here is the problem.
- Figure 10.04.01
- "Janine lives on a cul-de-sac. The diameter of the cul- de- sac is twenty- four feet. There is one road into the cul- de- sac and the measure of that arc is $60°$. Janine would like to plant flowers along the edge of the cul-de-sac, but she does not need to plant them where the road is. How many feet can Janine plant?"
- Solution:
- Students will need to do several steps to figure this one out.
- First, find the circumference.
- The circumference is 75.36 feet.
- Next figure out the arc length.
- The measure of the arc is $60°$.
- The arc length is 6.28 feet.
- Now subtract the arc length from the circumference.
- $75.36 - 6.28 = 69.08$

5.10. Perimeter and Area

- Janine will plant approximately 69 feet.

III. <u>Meeting Objectives</u>

- Students will calculate the circumference of a circle.
- Students will calculate the length of an arc of a circle.
- Students will use what they have learned in problem solving.

IV. <u>Notes on Assessment</u>

- Check student work for accuracy.
- Have the students correctly figured the circumference?
- Have the students correctly figured the arc length?
- Is the answer clearly labeled?
- Offer support/correction when needed.

Circles and Sectors

I. <u>Section Objectives</u>

- Calculate the area of a circle.
- Calculate the area of a sector.
- Expand understanding of the limit concept.

II. <u>Problem Solving Activity-Circle Shading</u>

- Students will use what they have learned about area and circles to figure out the area of the shaded region of the figure.
- Figure 10.04.01
- Here is the problem:
- "Use what you have learned about circles and area to find the area of the shaded region of the figure. Show all of your work in your answer."
- Solution:
- The solution to this problem has two parts.
- First, students need to figure out the area of the circle. Since the length of the side of the square is ten inches, that is also the diameter of the circle since the circle fills almost the entire square. Therefore, the radius is five inches.
- $A = \pi r^2$
- $A = (3.14)(25)$
- $A = 78.5 \text{ in}^2$
- Next, the students need to find the area of the square.
- $A = s^2$
- $A = 10^2$
- $A = 100 \text{ in}^2$
- Now since the shaded region is what we are looking to find, we can subtract the area of the circle from the area of the square, and the remaining inches will indicate the shaded region.
- $100 - 78.5 = 21.5$ inches

III. <u>Meeting Objectives</u>

- Students will calculate the area of a circle.
- Students will use area in problem solving.

IV. Notes on Assessment

- Be sure that each of the pieces of the solution is in the student's answer.
- Offer correction/feedback when necessary.

Regular Polygons

I. Section Objectives

- Recognize and use the terms involved in developing formulas for regular polygons.
- Calculate the area and perimeter of a regular polygon.
- Relate area and perimeter formulas for regular polygons to the limit process in prior lessons.

II. Problem Solving Activity-Teaching Time

- In this activity, students are going to teach the other students in the class.
- Divide students into small groups.
- Assign each group one of the following.
- 1. Explain what makes a polygon a regular polygon
- 2. Parts and terms of a regular polygon
- 3. Figuring Perimeter- one group gets $P = ns$
- 4. Figuring Perimeter- one group gets $P = 2nx$
- 5. Calculating Area using $A = \frac{1}{2} Pa$
- Once each group is given their assignment, their work is to demonstrate/teach their topic to the other students.
- Each group needs to use a diagram.
- Each group needs to have an example that demonstrates.
- Each group needs to have an example for the rest of the class to work with.
- Allow time for students to work and then present their topics.

III. Meeting Objectives

- Students will recognize and use the terms involved in developing formulas for regular polygons.
- Students will calculate the area and perimeter of a regular polygon.
- Students will demonstrate understanding by teaching this material to their peers.

IV. Notes on Assessment

- Take notes on each group's presentation.
- Is the material clearly presented?
- Have the students successfully used a diagram?
- Are the students able to answer questions from others in the class?
- Offer correction/feedback as needed.

Geometric Probability

I. Section Objectives

5.10. Perimeter and Area

- Identify favorable outcomes and total outcomes.
- Express geometric situations in probability terms.
- Interpret probabilities in terms of lengths and areas.

II. Problem Solving Activity-What are the Odds?

- Write the following list of things on the board. Students are going to play a game with the different topics.
- 6 bananas
- 5 apples
- 4 oranges
- 10 pineapples
- 12 lemons
- 8 bunches of grapes
- 7 melons
- To prepare the game, you need to create cards with statements that compare these items- for example, oranges to apples. Prepare a bunch of cards.
- The students play in pairs or teams of four.
- They play against each other.
- The students flip the top card over.
- Then whoever can say the probability first wins the point.
- It is a noisy game, but fun.
- Students play this similar to the card game "War"

III. Meeting Objectives

- Students will identify total outcomes and favorable outcomes.
- Students will practice these skills while playing a fun game.

IV. Notes on Assessment

- Walk around and observe students as they play the game.
- Offer feedback when needed.
- Extension of this activity- play the same game again but this time use a list of geometric measurements.

5.11 Surface Area and Volume

The Polyhedron

I. Section Objectives

- Identify polyhedral.
- Understand the properties of polyhedral.
- Use Euler's formula to solve problems.
- Identify regular (Platonic) polyhedral.

II. Problem Solving Activity-Using Euler

- During this activity, students are going to use Euler's formula to determine the number of vertices, faces and edges for the pyramid.
- These pictures can be printed or if technology is available students can use a website to find the picture.
- Here is a picture of a pyramid with a rectangular base. This is Figure 11.01.01
- http://en.wikipedia.org/wiki/File:01_khafre_north.jpg
- Students need to create a three dimensional diagram of the pyramid.
- Then they can label the faces, vertices and edges of the pyramid.
- Finally, the students need to use Euler's formula and demonstrate the accuracy of their answers.
- Extension is for students to draw the pyramid design to scale. This would involve more research as students would need to search for the dimensions of the pyramid.
- Then allow time for the students to share their work.

III. Meeting Objectives

- Students will identify polyhedrons.
- Students will understand the properties of polyhedra.
- Students will use Euler's formula to solve problems.
- Students will demonstrate understanding through a diagram and presentation.

IV. Notes on Assessment

- Walk around as students are working on this assignment.
- Students may have difficulty picturing the rest of the pyramid given they can only see it from one perspective.
- Offer support by referring students back to the text.
- Another option is to provide students with a solid of a pyramid for them to work with.
- During presentations, listen for whether or not students have a grasp of Euler's formula.
- Provide coaching/feedback as needed.

Representing Solids

I. Section Objectives

- Identify isometric, orthographic, cross- sectional views of solids.
- Draw isometric, orthographic, cross- sectional views of solids.
- Identify, draw and construct nets for solids.

II. Problem Solving Activity-Solids

- If you used the activity in the Differentiated Instruction lesson of the Teacher's Edition, you can move right on to the second part of this activity.
- If not, then begin with this part.
- Part One- Give students a geometric solid to work with.
- Then students are going to create four different things.
- 1. Create an orthographic projection of their solid.
- 2. Create a cross- section of the solid.
- 3. Create a net for the solid.
- 4. Use the net to create an actual model of the solid.
- Part Two- students are going to use the net that they created to design a mobile of solids.
- For this, students can create the same solid in different sizes, or they can create nets for different solids.
- Encourage students to use color and creativity in their mobile.
- Students can connect all of the created solids together with string and wooden dowels.
- Have materials on hand for students to complete their mobiles when finished.
- You can create a great display of mobiles and hang them all around the classroom.

III. Meeting Objectives

- Students will identify isometric, orthographic, cross- sectional views of solids.
- Students will draw isometric, orthographic, cross- sectional views of solids.
- Students will identify, draw and construct nets for solids.
- Students will show their work in a mobile.

IV. Notes on Assessment

- Look at each mobile and assess it.
- Are the nets created correctly?
- Did the student use different sized versions of the same solid or different solids?
- Is the mobile finished?
- Did the student use color and creativity in his/her design?

Prisms

I. Section Objectives

- Use nets to represent prisms.
- Find the surface area of a prism.
- Find the volume of a prism.

II. Problem Solving Activity-Prism Pair- Up

- This activity requires some preparation.
- First, you will need a bunch of boxes.

- Then you will need to figure out the surface area of each of the boxes.
- Write each surface area on an index card.
- You will need as many boxes as you have students in one class.
- Then you bring all of the boxes to the classroom.
- To complete the activity, each of the students is given a ruler/tape measure and an index card with a surface area on it.
- Students will need to take notes as they work- remind them to do this.
- Also encourage students to think of strategies before beginning the task.
- Does a large box have a large surface area or a small surface area?
- Then the students need to figure out which box has their correct surface area.
- This will take some time.
- Don't rush the students, but encourage them to work together.
- Once they have found their box, the students need to create a diagram of it.
- Label the surface area and find the volume of the prism.
- Allow time for the students to share their work when finished.

III. Meeting Objectives

- Students will find the surface area of a prism.
- Students will find the volume of a prism.
- Students will show their work in a diagram.

IV. Notes on Assessment

- Did the students match up the correct prism with their measurements?
- Is the diagram labeled accurately?
- Is the volume of the prism correct?
- Offer correction/feedback when necessary.

Cylinders

I. Section Objectives

- Find the surface area of cylinders.
- Find the volume of cylinders.
- Find the volume of composite three- dimensional figures.

II. Problem Solving Activity-Cylinder Sizing

- For this activity, the students are going to need a variety of different size cylinders to work with.
- This activity is going to combine our work with cylinders and scale factor together.
- Students can work in groups.
- Each group selects a cylinder to work with.
- First, the students need to figure out the surface area and volume of that cylinder.
- Next, the students are going to create a cylinder that is three times as large as the one that they have in their hands.
- Students will need large pieces of paper, scissors, rulers, string, tape or glue.
- The students will have to figure out the dimensions of the new cylinder.
- They will need to figure the surface area and the volume of the new cylinder as well.

5.11. Surface Area and Volume

- Then they will draw a net of the cylinder.
- Finally, they can cut the net out and build the actual cylinder.
- When finished, each group should have two cylinders constructed.
- Allow time for the students to share their work.

III. Meeting Objectives

- Students will find the surface area of cylinders.
- Students will find the volume of cylinders.
- Students will use their work in problem solving.

IV. Notes on Assessment

- Check student work for accuracy.
- Are the cylinders in proportion?
- Did the students calculate the surface area correctly?
- Did the students calculate the volume correctly?
- Did the students construct the second cylinder?
- Listen for student understanding of surface area and volume during student presentations.

Pyramids

I. Section Objectives

- Identify pyramids.
- Find the surface area of a pyramid using a net or formula.
- Find the volume of a pyramid.

II. Problem Solving Activity-Create Your Own Pyramid

- For this activity, the students are going to design their own pyramids.
- First, they need to decide which polygon to use as a base.
- The project has two parts to be completed- the first is a diagram of the pyramid done with a scale, the second is an actual model of the pyramid.
- Students need to decide how large they want their pyramid.
- They can use these measurements to design the actual pyramid.
- Once they know how large they want to actually build a pyramid, they can draw a design to show their work.
- After the design has been completed, the students must figure out the surface area and the volume of the pyramid.
- The final stage is to draw a net (in actual size) and cut it out to construct the pyramid.
- Allow students time to share their work.
- Some students will build large pyramids, others small- either is fine as long as the work is accurate.

III. Meeting Objectives

- Students will identify pyramids.
- Students will find the surface area of a pyramid using a net or formula.
- Students will find the volume of a pyramid.
- Students will demonstrate their understanding through a design and a model.

IV. Notes on Assessment

- Check student work for accuracy.
- The design should be drawn to scale- so the design should not be the same size as the model unless the students are making a small model. Then that should be indicated on the diagram.
- Listen during presentations for student understanding.
- Can the student explain what they did and why they did it?
- Is the surface area measurement accurate?
- Is the volume accurate?
- Offer correction/feedback as needed.

Cones

I. Section Objectives

- Find the surface area of a cone using a net or formula.
- Find the volume of a cone.

II. Problem Solving Activity-Ice Cream Cones

- Here is the problem.
- "George works at an ice cream parlor. He wants to design a new cone for them to put ice cream in. The standard cone that they use now has a radius of 2[U+0080][U+009D] and a height of 4[U+0080][U+009D]. This cone easily holds 2 scoops of ice cream. George wants to design a cone that holds 6 scoops of ice cream. Using these dimensions, figure out what George's cone will look like."
- To complete your work:
- 1. Draw a design of the new cone. Be sure to include measurements.
- 2. Figure out the surface area of the cone.
- 3. Figure out the volume of the cone.
- 4. Decide whether this design is even possible for an ice cream cone- justify your answer.
- Solution:
- Based on the ratios- the new cone would have a radius of 6[U+0080][U+009D] and a height of 12[U+0080][U+009D].
- The surface area $= C = 37.68$
- $A = 113.04$
- SA $= 2,556$[U+0080][U+009D] ≈ 213 feet
- Volume $= 452$ units
- This is a ridiculous size for a cone!!!

III. Meeting Objectives

- Students will figure out the surface area of a cone.
- Students will figure out the volume of a cone.
- Students will use what they have learned while problem solving.

IV. Notes on Assessment

- Check student work.
- Is the surface area calculated correctly?
- Is the volume calculated correctly?
- What conclusions did the students come to regarding the size of the cone?
- Did they justify their answer?

5.11. Surface Area and Volume

Spheres

I. <u>Section Objectives</u>

- Find the surface area of a sphere.
- Find the volume of a sphere.

II. <u>Problem Solving Activity- Measuring Spheres</u>

- Bring in a bunch of different size balls.
- The students will need string, measuring tape, rulers, paper and pencils to complete this assignment.
- Students may complete this assignment in small groups.
- First, divide up the groups and have the students select one ball to work with.
- Using their tools, and any other given information on the ball, students need to figure out the surface area and the volume of this first ball.
- Then they select a ball that appears to be proportional to this first ball.
- Again, students are going to figure out the surface area and the volume for the second ball.
- Then students need to draw a diagram comparing the two and write about any conclusions that they can determine about the two balls.
- Allow time for students to share their work when finished.

III. <u>Meeting Objectives</u>

- Students will use tools to calculate the dimensions of a sphere.
- Students will find the surface area of a sphere.
- Students will find the volume of the sphere.
- Students will compare two different spheres.
- Students will draw a diagram explaining their work.
- Students will write about their conclusions.
- Students will explain and share their work with the class.

IV. <u>Notes on Assessment</u>

- Be sure that each step of the objectives is accurate.
- Listen for student understanding during the presentations.
- What conclusions did the students draw about the two balls?

Similar Solids

I. <u>Section Objectives</u>

- Find the volumes of solids with bases of equal areas.

II. <u>Problem Solving Activity-Building Dimensions</u>

- Here is the problem.

- "Kara has a sister who is an architect. Her sister is designing a building which is in the shape of a rectangular prism. When Kara visits, she sees her sister is building a model of the building. Her sister says that the model will be 2 feet high, 1.5 inches wide and 4.2 inches deep. She tells Kara that it has a scaling factor of 200/1[U+0080][U+009D].
- Based on this information, help Kara to figure out the dimensions of the real building."
- Solution:
- First- the students will need to have all of the measurements in inches since the scaling factor is in inches.
- That means that the model with be 36 inches high, 1.5 inches wide and 4.2 inches deep.
- They can multiply these by 200.
- Then convert back to feet by dividing by 12– since most buildings aren't measured in inches.
- The final dimensions are: 300 feet high, 25 feet wide and 70 feet deep.

III. Meeting Objectives

- Students will use the scaling factor with similar solids.
- Students will problem solve using this information.
- Students will share their work with their peers.

IV. Notes on Assessment

- Is student work accurate?
- Did the students convert the original dimensions to inches?
- Is their final answer in feet?
- What conclusions did the students draw about the building?

5.12 Transformations

Translations

I. <u>Section Objectives</u>

- Graph a translation in a coordinate plane.
- Recognize that a translation is an isometry.
- Use vectors to represent a translation.

II. <u>Problem Solving Activity-Translation Teamwork</u>

- For this activity, the students will be creating a game.
- Divide students onto teams of two.
- The first step is for students to design translations on a coordinate grid. They will need to create four different translations.
- Then have them write the coordinates for each translation on a separate index card.
- Finally, they need to write directions indicating the spacing of the translation. For example, three up and four to the right.
- Now they can put the whole thing together.
- Each team joins another team.
- They put all of the game cards together and they have to match up the correct diagram with the card coordinates and the directions.
- This can have a time limit for students as well.

III. <u>Meeting Objectives</u>

- Students will graph translations on a coordinate plane.
- Students will recognize that a translation is an isometry.

IV. <u>Notes on Assessment</u>

- When students are finished, collect each teams "playing cards"
- Check student work for accuracy.
- Offer correction/feedback when necessary.

Matrices

I. <u>Section Objectives</u>

- Use the language of matrices.
- Add matrices.
- Apply matrices to translations.

II. Problem Solving Activity-Matrix Translations

- Students will use what they have learned about matrices and translations and apply the two together.
- First, the students need to be given a sheet of graph paper, a blank paper, a ruler and colored pencils.
- Tell students to begin by drawing a quadrilateral on the coordinate grid.
- They don't need to do anything else.
- Then have them pass their papers to the right.
- Next write 2 up and four down on the board.
- The students need to use this information and the diagram in front of them to write a matrix and use that matrix to figure out the coordinates of a new triangle.
- Tell them to draw this new triangle in a new color.
- Have students sign their name in the color of the triangle they drew.
- Then pass papers again.
- Students should go back to the original triangle.
- Next write 3 down and 2 up on the board.
- Students use a matrix again to design a new triangle.
- Ask the students to draw this one in with a different color.
- You can repeat this as many times as you wish- the students will a translation collection when finished.

III. Meeting Objectives

- Students will use the language of matrices.
- Students will add matrices.
- Students will apply matrices to translations.

IV. Notes on Assessment

- Collect all student work when finished.
- Check student work for accuracy.
- Offer correction/feedback when necessary.

Reflections

I. Section Objectives

- Find the reflection of a point in a line on a coordinate plane.
- Multiple matrices.
- Apply matrix multiplication to reflections.
- Verify that a reflection is an isometry.

II. Problem Solving Activity-Reflection Matrices

- First, the students will need graph paper, rulers and colored pencils for this activity.
- Tell students to begin by drawing a quadrilateral on a coordinate grid.
- They can use color if they wish to.
- Then tell them that they will be drawing a reflection of this quadrilateral where $y = x$.
- If students need more help- tell them that they will be using $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ to multiply and find the coordinates of the new polygon.

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- Students write a matrix and show all matrix multiplication.
- Then the students draw in the new quadrilateral on the coordinate grid.
- Finally, have students exchange papers and complete a peer check.
- Each person needs to check the other person's work.
- Allow time for the students to share their reflections in small groups when finished.

III. Meeting Objectives

- Students will find the reflection of a point in a line on a coordinate plane.
- Students will multiple matrices.
- Students will apply matrix multiplication to reflections.
- Students will verify that a reflection is an isometry.
- Students will use what they have learned to draw a reflection of a quadrilateral.
- Students will demonstrate understanding by explaining their work in small groups.

IV. Notes on Assessment

- Walk around and observe students as they work.
- Which strategies are the students using?
- Listen in on small group discussions.
- Is the student understanding lines of reflection?
- Offer assistance/feedback as needed.

Rotations

I. Section Objectives

- Find the image of a point in a rotation in a coordinate plane.
- Recognize that a rotation is an isometry.
- Apply matrix multiplication to rotations.

II. Problem Solving Activity-Rotating Triangles

- For this problem, the students need to practice drawing rotations using matrices.
- Students will need graph paper, rulers and colored pencils.
- Tell students to use color to show the different rotations that they are going to be drawing on the coordinate grid.
- First, have the students draw a triangle on the coordinate grid.
- Then ask the students to write a matrix to describe this triangle.
- Next, they use matrix multiplication to rotate the triangle 180°.
- Finally, they draw in the triangle.
- Then using the same original triangle, students are going to use matrix multiplication to rotate a triangle 90°.
- After multiplying, they draw in the triangle.
- When finished, have students exchange papers so that a peer can check their work.
- Staple the work sheet with the matrix multiplication to the back of the sheet with the coordinate grid drawings.

III. Meeting Objectives

- Students will find the image of a point in a rotation in a coordinate plane.

- Students will recognize that a rotation is an isometry.
- Students will apply matrix multiplication to rotations.
- Students will draw in new rotations on a coordinate grid.

IV. Notes on Assessment

- Collect student work.
- Compare the matrix multiplication to the triangles on the coordinate grid.
- Does the work match up?
- If not, what is missing?
- Offer correction/feedback as needed.
- This could be graded as a classwork or homework assignment.

Composition

I. Section Objectives

- Understand the meaning of composition.
- Plot the image of a point in a composite transformation.
- Describe the effect of a composition on a point or polygon.
- Supply a single transformation that is equivalent to a composite of two transformations.

II. Problem Solving Activity-Glide Rotations

- Students are going to draw a glide rotation of their own choosing.
- Tell students that the figure is going to be reflected in the x axis.
- Then tell students to draw in their figure.
- Next, they need to use matrix multiplication to design a reflection of that figure.
- Finally, they can move it 4 units to the right and one up.
- Students draw in the final figure.
- Allow time for them to share their work when finished.
- Solution:
- Because the figure is reflected in the x axis, students will multiply the coordinates of the figure by $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
- Then they use $(4, 1)$ to add to the product of the first two steps.
- Finally, the students can draw in the figure.

III. Meeting Objectives

- Students will understand the meaning of composition.
- Students will plot the image of a point in a composite transformation.
- Students will describe the effect of a composition on a point or polygon.
- Students will supply a single transformation that is equivalent to a composite of two transformations.
- Students will share their work with their peers.

IV. Notes on Assessment

- Collect student work.
- Be sure to collect the diagram and the written work.
- Check it for accuracy.
- Provide feedback/correction as needed.

5.12. Transformations

Tessellations

I. <u>Section Objectives</u>

- Understand the meaning of tessellation.
- Determine whether or not a given shape will tessellate.
- Identify the regular polygons that will tessellate.
- Draw your own tessellation.

II. <u>Problem Solving Activity-Designing Tessellations</u>

- For this lesson, the students are going to design a tessellation that uses two different polygons.
- Students first need to identify which polygons will tessellate and which won't.
- Then the students can work on their design.
- Each student will need colored pencils, rulers, paper, cardstock, and large paper.
- Explain to the students that they will be graded on their work.
- Each student needs to create a pattern to demonstrate that the tessellation that they have suggested does work.
- Students need to have a peer check their work.
- Students need to have an instructor check their work.
- When the pattern has been approved, students may begin work on their final design.
- This will help students problem solve and eliminate redoing work.
- Allow time for the students to present their work when finished.

III. <u>Meeting Objectives</u>

- Students will understand the meaning of tessellation.
- Students will determine whether or not a given shape will tessellate.
- Students will identify the regular polygons that will tessellate.
- Students will draw their own tessellations.
- Students will share their work when finished.

IV. <u>Notes on Assessment</u>

- Design a rubric to grade student work.
- Share the rubric with the students ahead of time.
- Possible sections include focus, creativity, design, accuracy, etc.
- Provide written feedback to students on their work.

Symmetry

I. <u>Section Objectives</u>

- Understand the meaning of symmetry.
- Determine all the symmetries for a given plane figure.
- Draw or complete a figure with a given symmetry.
- Identify planes of symmetry for three- dimensional figures.

II. <u>Problem Solving Activity-Magazine Symmetry Hunt</u>

- For this activity, students are going to hunt through magazines of example of symmetry.
- You will need to prepare and bring a stack of magazines to class. Architecture and nature magazines will be very helpful.
- They are going to create a collage of the pictures that they find.
- Students will need paper, glue, scissors, a ruler and a pencil.
- Students can use pictures that show symmetry of a plane figure and symmetry of a three- dimensional figure.
- When the students select a given picture, they need to cut it out and attach it to the collage.
- Then they draw in the lines of symmetry.
- Students do this until their collage is complete.
- Then have the students write a paragraph explaining their collage- any themes and the types of symmetry found in the collage.
- Allow time for students to share their work when finished.

III. Meeting Objectives

- Students will understand the meaning of symmetry.
- Students will determine all the symmetries for a given plane figure.
- Students will draw or complete a figure with a given symmetry.
- Students will identify planes of symmetry for three- dimensional figures.
- Students will find real- life examples of symmetry.

IV. Notes on Assessment

- Look at student collages.
- Read student descriptions of their collages.
- Develop a grading rubric for the assignment.
- Use observation as a piece of the rubric.
- Were the students organized?
- Were students focused?
- Offer feedback on student work.

Dilations

I. Section Objectives

- Use the language of dilations.
- Calculate and apply scalar products.
- Use scalar products to represent dilations.

II. Problem Solving Activity-Dilation Design

- This activity is an extension of the work that was done in the Differentiation Lesson of the Teachers Edition.
- Students will create two dilations in this activity.
- Students will create a large dilation and a small dilation.
- Tell students that they will be working with the scale factors of $\frac{1}{3}$ and 3.
- Next, students will need a coordinate grid to work with.
- Students begin by selecting a polygon that they would like to work with.
- The students can draw this polygon anywhere that they would like to on the coordinate grid.
- Once this is done, have the students write the coordinates out as a matrix.

- Then tell students to use scalar multiplication and the given scale factors to design two new polygons.
- Have students complete the work with the matrices on a separate sheet of paper.
- When finished, draw the two new polygons on the coordinate grid.
- Allow time for students to share their work when finished.

III. Meeting Objectives

- Students will use the language of dilations.
- Students will calculate and apply scalar products.
- Students will use scalar products to represent dilations.

IV. Notes on Assessment

- Collect both the coordinate grid with the dilated polygons and the matrix worksheet.
- Then check student work for accuracy.
- Offer correction/feedback as needed.