

## Algorithms Fourth Quiz

10:10 ~ 11:10

(2023/11/29)

**(Note that if you design an algorithm, you must have pseudo code to show your algorithm and analyze the algorithm's time complexity in the worst case. It would help to put comments after your pseudo code to clarify your algorithm.)**

1. (10%) Prove that a non-full binary tree cannot correspond to an optimal prefix-free code.
2. (10%) Suppose that instead of always selecting the first activity to finish, you instead select the last activity to start that is compatible with all previously selected activities. Describe how this approach is a greedy algorithm and prove that it yields an optimal solution.
3. (10%) Show that if a DECREMENT operation is included in the  $k$ -bit counter,  $n$  operations can cost as much as  $\Theta(nk)$  time.
4. (10%) Suppose you have a potential function  $\Phi$  such that  $\Phi(D_i) \geq \Phi(D_0)$  for all  $i$ , but  $\Phi(D_0) \neq 0$ . Show that there exists a potential function  $\Phi'$  such that  $\Phi'(D_0) = 0$ ,  $\Phi'(D_i) \geq 0$  for all  $i$ , and the amortized costs using  $\Phi'$  are the same as the amortized costs using  $\Phi$ .
5. (10%) What is the running time and space of BFS if we present its input graph by an adjacency list?
6. (10%) Let  $G = (V, E)$  be a connected, undirected graph. Give an  $O(V + E)$ -time algorithm to compute a path in  $G$  that traverses each edge in  $E$  exactly once in each direction.