Algorithms Fifth Quiz

10:10 ~ 11:10

(2023/12/)

(**Note that if you design an algorithm, you must have pseudo code to show your algorithm and analyze the algorithm's time complexity in the worst case. It would help to put comments after your pseudo code to clarify your algorithm.**)

1. (10%) Give an algorithm determining whether an undirected graph $G = (V, E)$ contains a simple cycle. Your algorithm should run in O(V) time, independent of $|E|$.

2. (10%) Give an $O(V + E)$-time algorithm that, given a directed graph $G = (V, E)$, constructs another graph $G' = (V, E')$ such that G and G' have the same strongly connected components, G' has the same component graph as G, and $|E'|$ is as small as possible.

3. (10%) Let T be a minimum spanning tree of a graph G, and let L be the sorted list of the edge weights of T. Show that for any other minimum spanning tree T' of G, the list L is also the sorted list of edge weights of T'.

4. (10%) Give a simple implementation of Prim's algorithm that runs in $O(V^2)$ time when the graph $G = (V, E)$ is represented as an adjacency matrix.

5. (10%) Given a weighted directed graph $G = (V, E)$ with no negative-weight cycles, let m be the maximum over all vertices $v \in V$ of the minimum number of edges in a shortest path from the source s to v. (here, the shortest path is by weight, not the number of edges.) Modify the Bellman-Ford algorithm that allows it to terminate in m+1 passes, even if m is not known in advance.

6. (10%) Give a weighted directed graph $G = (V, E)$ with the positive edge weight. We can use Dijkstra's algorithm to find single-source shortest paths, i.e., find a shortest path from a given source vertex $s \in V$ to every vertex $v \in V$. Please explain why Dijkstra's algorithm cannot work with the negative edge weight even there is no negative-weight cycle.