Q1:

Proof that $\log(n!) = O(nlogn)$. (Hint: Use the definition of Big-O notation at p.28 of CH3 slide)

A1:

1. $\log(n!) = \log 1 + \log 2 + \cdots + \log(n) \leq \log n + \log n + \cdots + \log n$

$= n \cdot \log n$.

By definition, let $C = 1$, $n \geq 1$,

we have $0 \leq \log(n!) \leq n \cdot \log n$.

$\Rightarrow \log(n!) = O(n \log n)$.

Q2:

Proof that $\log(n!) = \Omega(n\log n)$. (Hint: Use the definition of Big-Omega notation at p.31 of CH3 slide)

A2:

2. $\log(n!) = (\log 1 + \log n) + (\log 2 + \log(n-1)) + \cdots + (\log \frac{n}{2} + \log(\frac{n}{2}+1))$

$\geq \frac{n}{2} \cdot \log n$.

By definition, let $c = \frac{1}{2}$, $n \geq 1$,

we have $0 \leq \frac{1}{2} n \log n \leq \log(n!)$.

$\Rightarrow \log(n!) = \Omega(n \log n)$