Centre of Mass

Questions Asked in Examination (2008-2020)

1. (a) Show that **J** and **ω** are not necessarily in the same direction for a rigid body.
   (b) What are the principal moments of inertia and principal axes of a rigid body? Explain these in terms of inertia tensor.
2. (a) Show that the angular momentum **J** of a system of particles can be expressed in the form $\vec{J} = \vec{J}_{cm} + \vec{R} \times \vec{P}$, Where $\vec{J}_{cm}$ is angular momentum about the centre of mass, ***R*** is the position vector of the centre of mass, and ***P*** is the total momentum.
   (b) Find the centre of mass of a homogeneous semi-circular plate of radius r.
3. (a) For a body of arbitrary shape and mass distribution show that angular momentum is given by $\vec{J} = \sum_N M_n \, [\vec{\omega} r_n^2 - \vec{r}_n (\vec{r}_n . \vec{\omega})]$ for the x-component of J compute the inertial coefficients Ixx, Ixy and Ixz. Hence prove that Ixx+Iyy+Izz=$2 \int \rho(r) r^2 \, dv$.
   (b) What is the centre of mass of a system of particles? Find the centre of mass of a system of three particles of masses 1 kg, 2 kg, and 3 kg placed at a corner of an equilateral triangle of 1 m side.
4. Three particles of masses 1, 2, and 3 kg are located at (0,0,0), (0, a, 3a) and (a, 2a, 3a) respectively. Find the inertial coefficients of the system.
5. Write short notes on the following:
   (a) Moment of Inertia Tensor
   (b) Inertial Coefficients