

Probability Distribution Based Evolutionary Algorithms

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空间信息重点实验室

Outline

1. Background

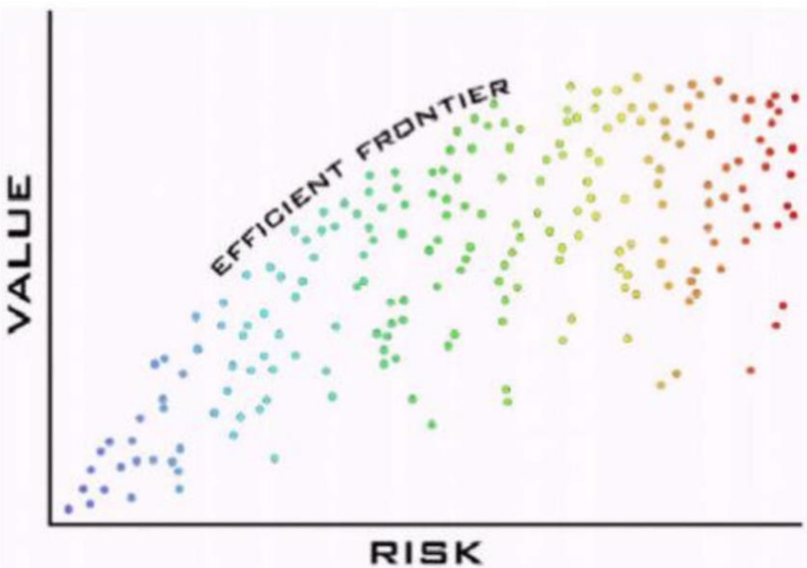
2. Probability Distribution Based EAs for Seeking Multiple Solutions

3. Probability Distribution Based EAs for Discrete Optimization

4. Applications & Future Work

Data-Driven Optimization

- ❑ **In big-data environment, many problems are driven by data**

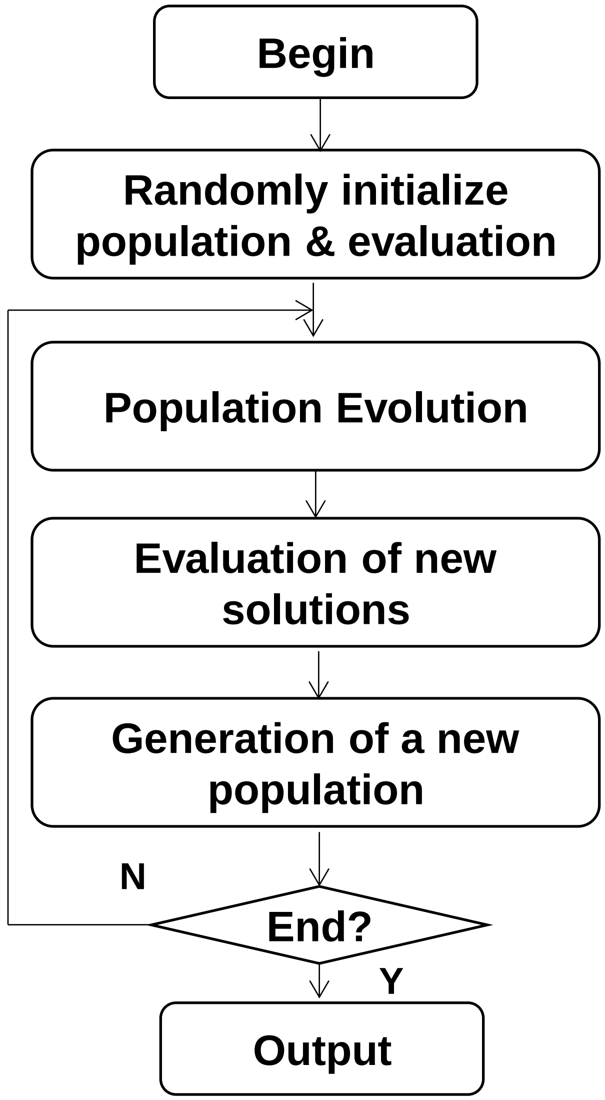


Portfolio Optimization

Order Dispatching

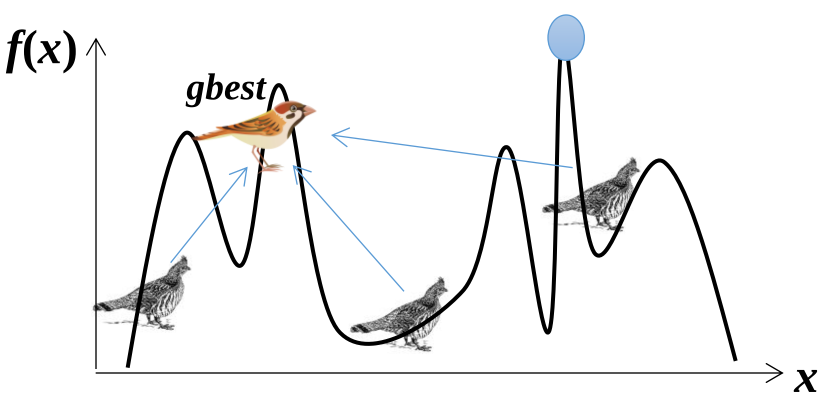
Evolutionary Computation

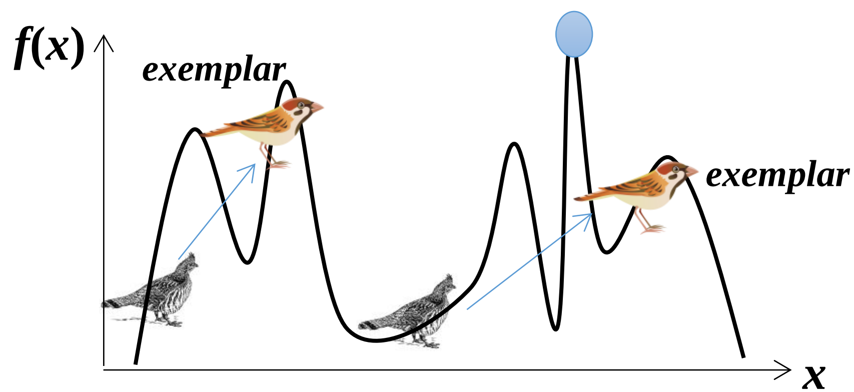
- Population-based stochastic algorithms which simulate intelligent behaviors
- Popular Algorithms
  - Genetic algorithms (GA)
  - Differential evolution (DE)
- Meta-heuristics
  - Particle swarm optimization (PSO)
  - Ant colony optimization (ACO)
  - …
- Advantages
  - Do not make any assumption about the underlying fitness landscape
  - Find approximated solutions within acceptable time



Challenge 1: Premature Convergence

- **How to escape from local optima in multimodal problems has long been a key issue in EC research**
  - **Parameter adaption** **(APSO, Zhan et al., 2009)**
  - **Modified mutation \ update rules** **(Local Topology PSO, Kennedy & Mendes, 2002, CLPSO, Liang et al. 2006, etc.)**
  - **Hybrid approaches**

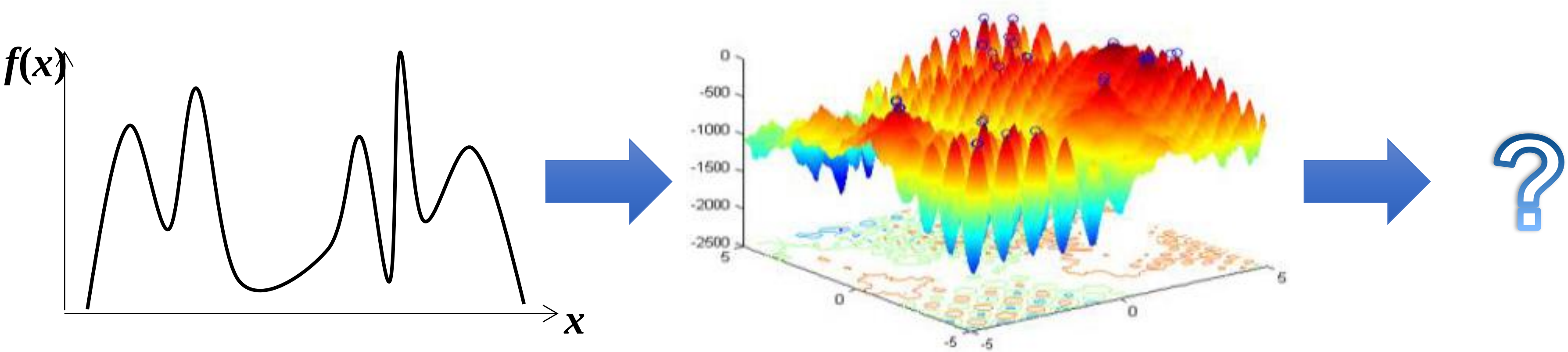




Improved PSO variants:

local neighborhood topology
multi-swarm PSOs
try to introduce more exemplars to improve search diversity

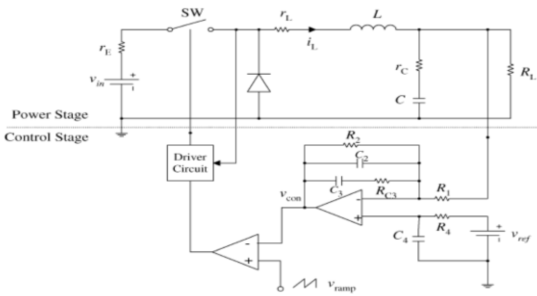
Challenge 2: Curse of Dimensionality

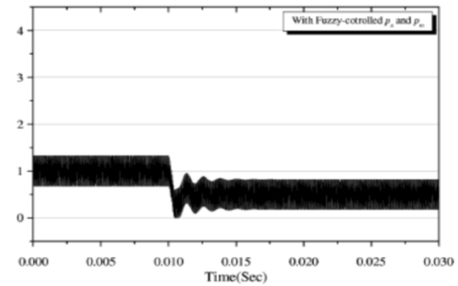


- **The search space is enlarged exponentially**
- **The number of local optima increases rapidly**
- **The execution time becomes too long**

Other Challenges

❑ Mixed Variable Optimization

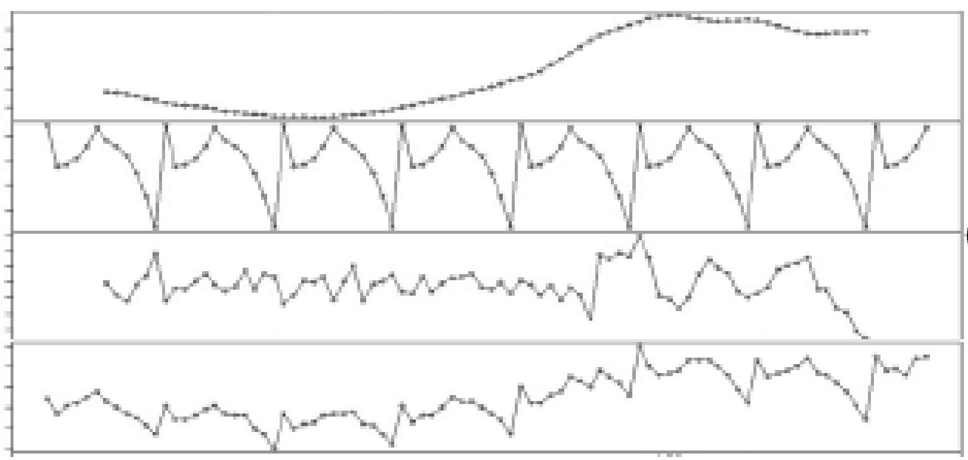




Optimal Design of Power Electronic Circuits

continuous variables
categorical variables

❑ Optimization under Uncertain / Dynamic Environment

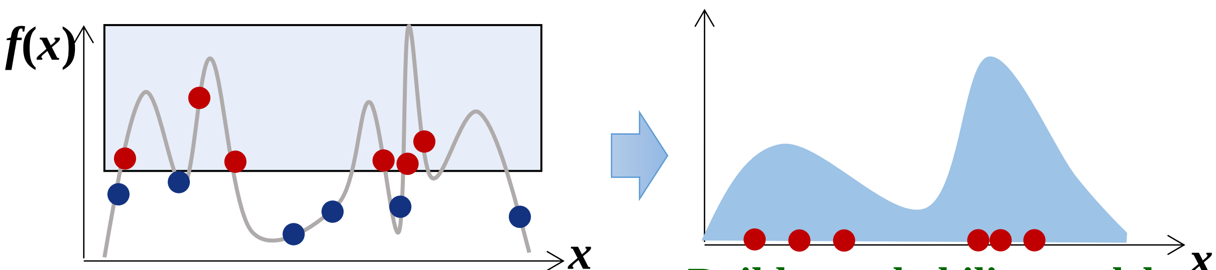


Maximize Profit
Minimize Risk

Robust Optimization

Probability-Based EAs (1)

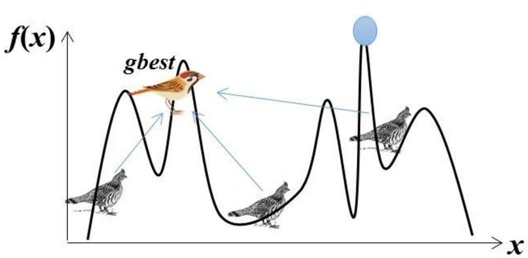
- **Estimation of Distribution Algorithm (EDA, Lozano et al., 2002)**: building and sampling explicit probabilistic models of promising candidate solutions



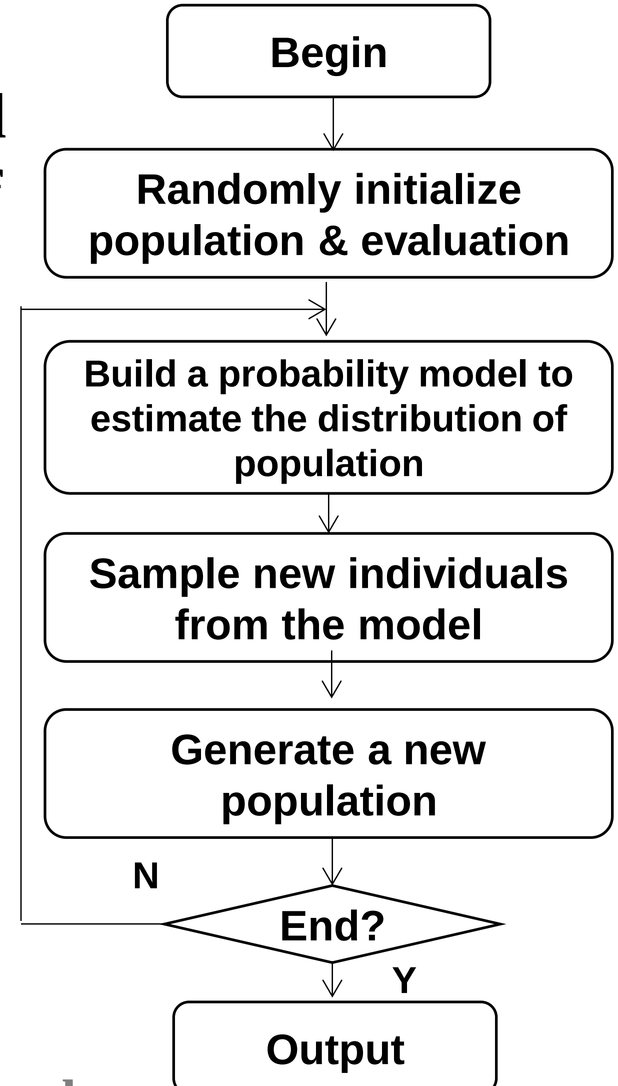
Select promising candidate solutions

Build a probability model to estimate the distribution of candidate solution

EDA: focus on the global fitness landscape

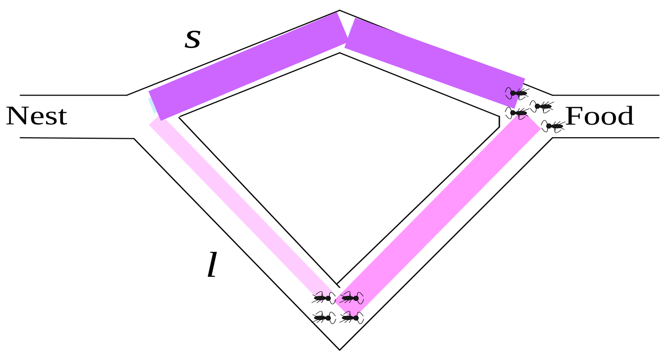


PSO: focus on the local neighborhood around the exemplar



Probability-Based EAs (2)

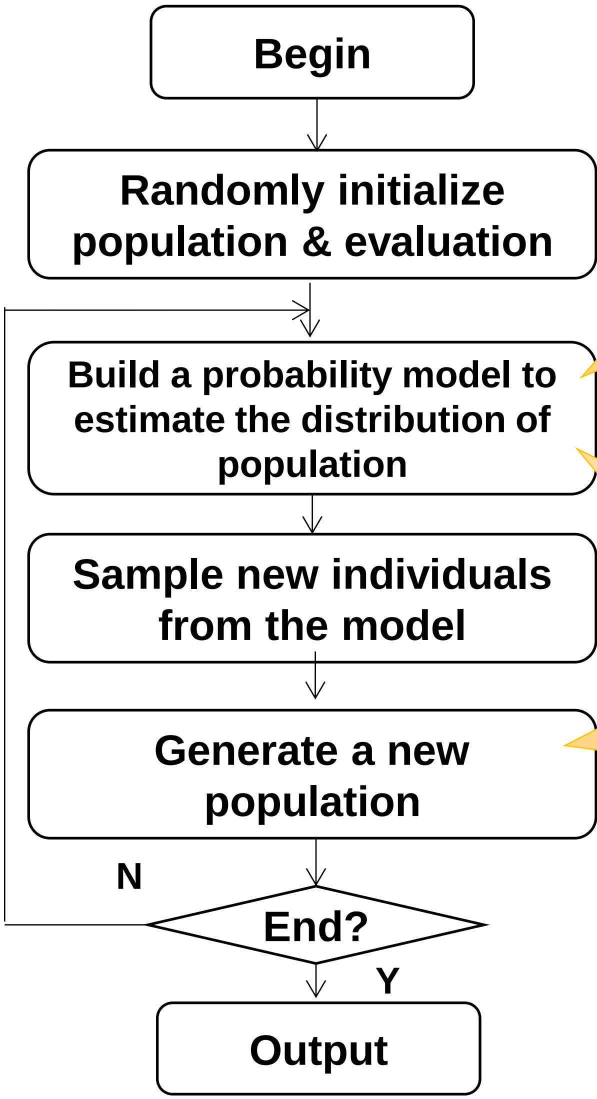
- In a broad sense, there are also some other EC algorithms that implicitly use probability distribution
  - Ant Colony Optimization (ACO, Dorigo et al., 1996)



The pheromone depositing behavior can be viewed as building a probability distribution in the search space.

- Strengths
  - Good at global optimization and diversity preservation
- Weakness
  - Speed, Precision

An Outline of Our Study



1 Probability Distribution Based EAs for Seeking Multiple Solutions

Build a framework with local (niched) probability distribution to improve search diversity of EC

2 Probability Distribution Based EAs for Discrete Optimization

Build a framework for probability distribution based EC algorithms for solving mixed-variable optimization problems

Outline

1. Background

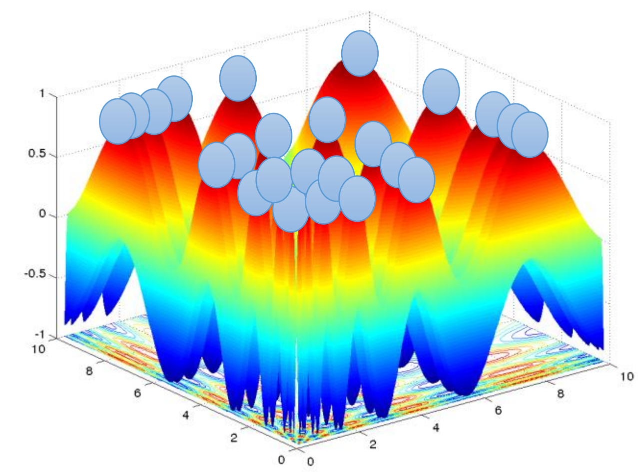
2. Probability Distribution Based EAs for Seeking Multiple Solutions

3. Probability Distribution Based EAs for Discrete Optimization

4. Applications & Future Work

Motivation

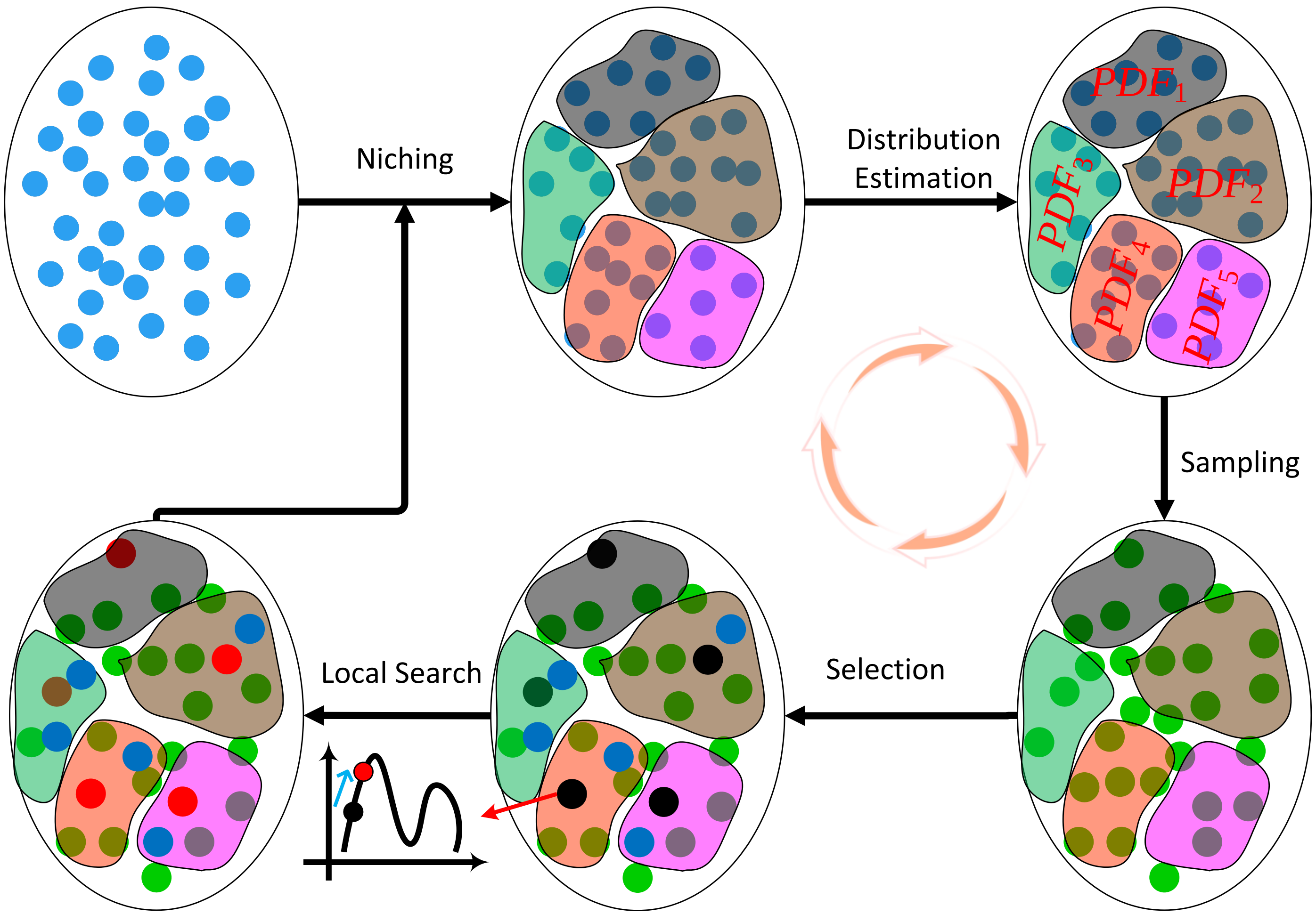
- Seeking multiple optimal solutions requires **extremely high search diversity**
  - By now, almost all approaches to seek multiple solutions are based on PSO or DE
  - On some high dimension problems with many local optima, the detection rates of many approaches are below 15%, or even 0%.



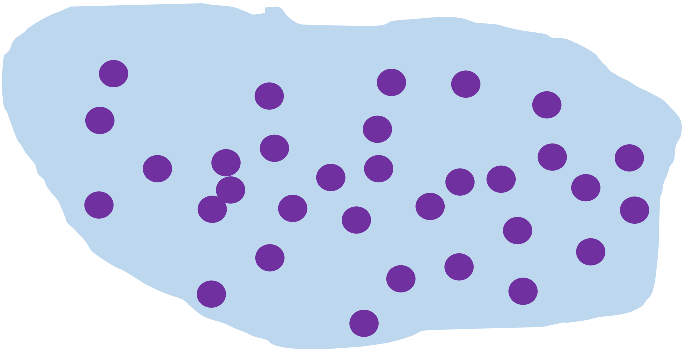
- EDA has shown good search diversity, but the potential of probability distribution based evolution for seeking multiple solutions have not been explored

Framework

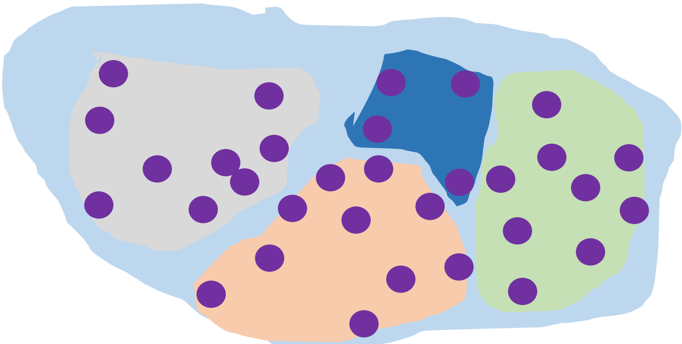
- ❑ **Combine Probability-Distribution Based EC with Niching**



Niching



Population



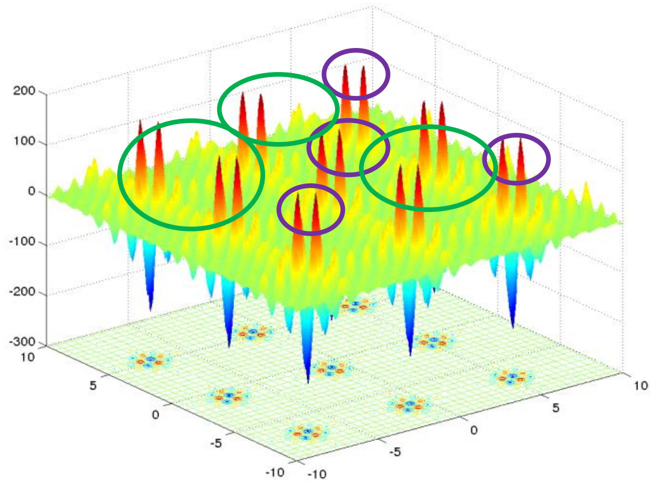
Niching

- Fitness Sharing (Goldberg and Richardson, 1987)
- Crowding (Thomsen, 2004)
- Speciation (Li, 2005)
- Clustering (Gao and Yen, et al., 2014; Qu and Suganthan, et al. 2012)

Multimodal EDA

1) Niching with Dynamic Niche Size

- $\boldsymbol{C}=\{c_1,c_2,\ldots,c_t\}$(different integers)
- Random selection before each generation
- Small size beneficial for exploitation
- Large size profitable for exploration
- Better balance between exploration and exploitation (than with a fixed size)

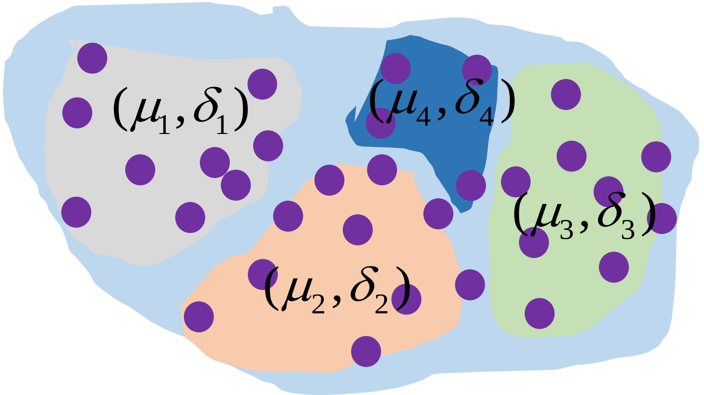


2) Distribution Estimation

- Estimating each niche separately
- All individuals in one niche participating in the estimation to keep high diversity

$$\mu_i^d = \frac{1}{M}\sum_{j=1}^{M} x_j^d$$

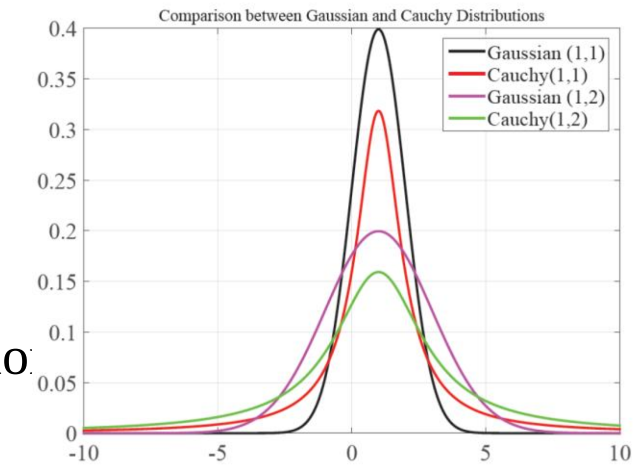
$$\delta_i^d = \sqrt{\frac{1}{M-1}\sum_{j=1}^{M}(x_i^d - \mu_i^d)^2}$$



Multimodal EDA

- **3) Sampling with Cauchy / Gaussian Distribution**

$$C_i = \begin{cases} Cauchy(\mu_i, \delta_i) & if\ rand() \le 0.5 \\ Gaussian(\mu_i, \delta_i) & \text{otherwise} \end{cases}$$



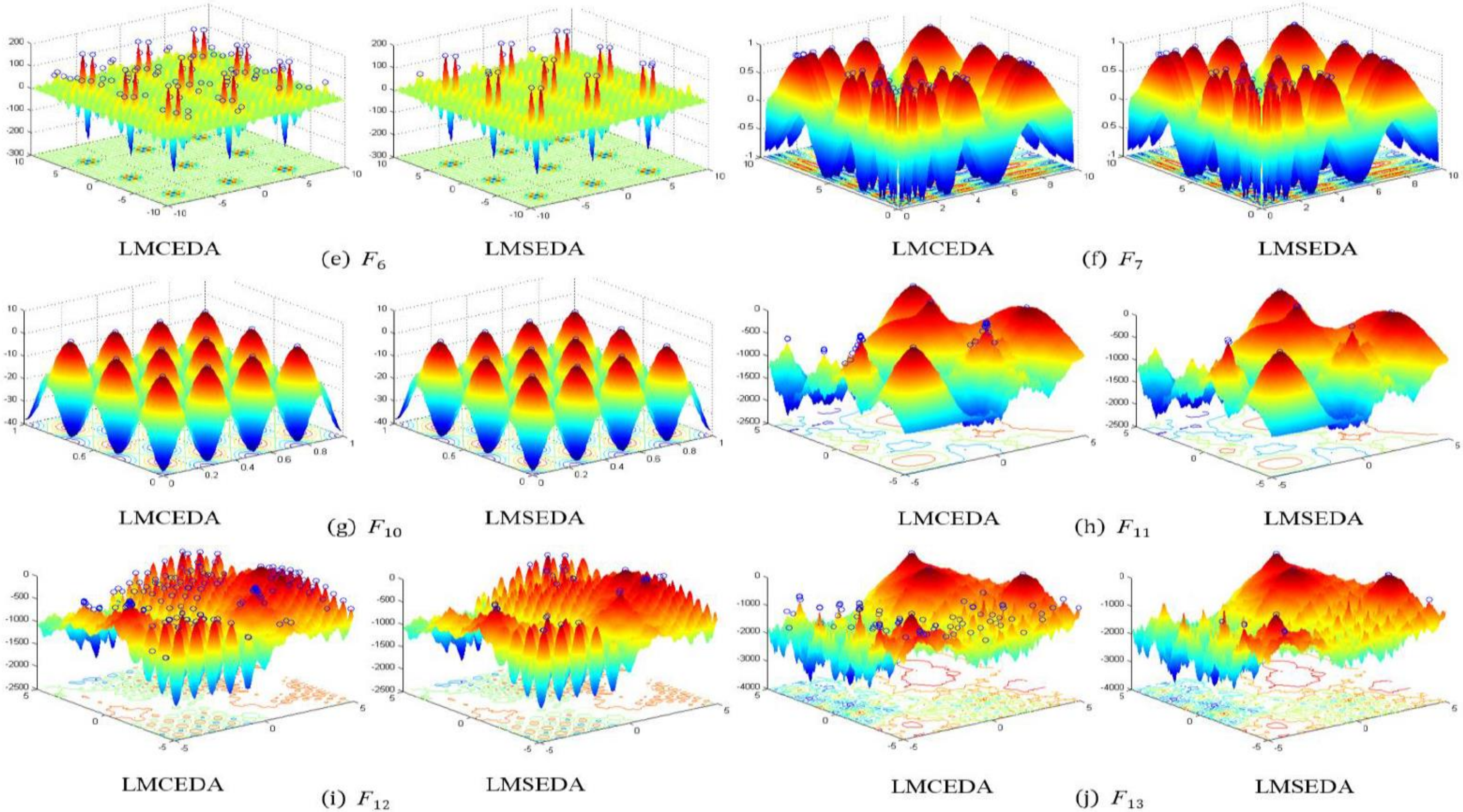
- Cauchy Distribution (long tail, beneficial for explo
- Gaussian Distribution (narrow tail, profitable for exploitation)
- Better balance between exploration and exploration

- **4) Selection with the Neighborhood Replacement Rule**
- **5) Local Search to Accelerate Search**
  - Adaptively conduct around seeds of niches
  - Local search by sampling *N* points using Gaussian Distribution around seeds

$$N(\mu, \delta) \quad \mu = seed_i, \delta = 1.0E-4$$

Results of Multimodal EDA

Clustering for Crowding (MCEDA) **Clustering for Speciation (MSEDA)**



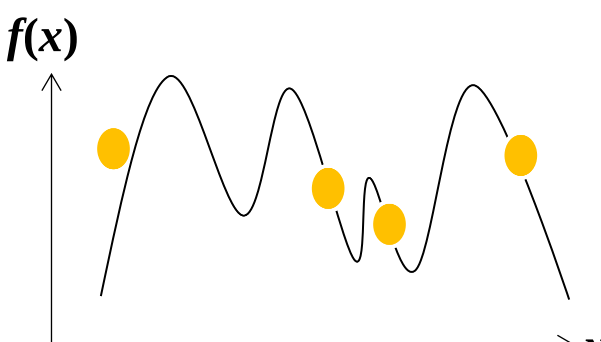
Results of Multimodal EDA

F	Self_CCDE			Self_CSDE			LoICDE			LoISDE			PNPCDE			LMCEDA			LMSEDA		
	PR	SR	CS	PR	SR	CS	PR	SR	CS	PR	SR	CS	PR	SR	CS	PR	SR	CS	PR	SR	CS
F_1	**1.000**	1.000	3.55E+2	**1.000**	1.000	7.15E+2	**1.000**	1.000	1.73E+2	**1.000**	1.000	1.68E+2	**1.000**	1.000	1.62E+2	**1.000**	1.000	3.23E+2	**1.000**	1.000	3.41E+2
F_2	**1.000**	1.000	7.61E+2	**1.000**	1.000	1.03E+3	**1.000**	1.000	1.24E+3	0.486	0.353	3.25E+4	**1.000**	1.000	1.08E+3	**1.000**	1.000	1.07E+3	**1.000**	1.000	8.97E+2
F_3	**1.000**	1.000	5.55E+2	**1.000**	1.000	7.23E+2	**1.000**	1.000	5.84E+2	**1.000**	1.000	7.48E+2	**1.000**	1.000	8.33E+2	**1.000**	1.000	5.06E+2	**1.000**	1.000	4.84E+2
F_4	**1.000**	1.000	7.07E+3	0.907	0.706	2.97E+4	**1.000**	1.000	1.48E+4	0.265	0.020	4.91E+4	**1.000**	1.000	2.28E+4	**1.000**	1.000	1.18E+4	**1.000**	1.000	6.37E+3
F_5	**1.000**	1.000	1.97E+3	**1.000**	1.000	3.38E+3	**1.000**	1.000	1.88E+3	0.814	0.627	1.91E+4	**1.000**	1.000	3.69E+3	**1.000**	1.000	3.78E+3	**1.000**	1.000	2.51E+3
F_6	0.972	0.647	1.37E+5	0.760	0.020	1.97E+5	**1.000**	1.000	9.58E+4	0.056	0.000	2.00E+5	0.806	0.157	1.97E+5	0.990	0.843	1.64E+5	0.973	0.588	1.43E+5
F_7	**0.884**	0.020	1.97E+5	0.696	0.000	2.00E+5	0.858	0.020	2.00E+5	0.029	0.000	2.00E+5	0.875	0.000	2.00E+5	0.782	0.000	2.00E+5	0.712	0.000	2.00E+5
F_8	**0.997**	0.902	2.35E+5	0.695	0.000	4.00E+5	0.000	0.000	4.00E+5	0.012	0.000	4.00E+5	0.000	0.000	4.00E+5	0.352	0.000	4.00E+5	0.622	0.000	4.00E+5
F_9	0.459	0.000	4.00E+5	0.265	0.000	4.00E+5	0.421	0.000	4.00E+5	0.005	0.000	4.00E+5	0.473	0.000	4.00E+5	0.333	0.000	4.00E+5	0.281	0.000	4.00E+5
F_{10}	**1.000**	1.000	8.15E+3	**1.000**	1.000	1.51E+4	**1.000**	1.000	3.04E+4	0.083	0.000	2.00E+5	**1.000**	1.000	2.16E+4	**1.000**	1.000	1.02E+4	0.998	0.980	1.23E+4
F_{11}	0.824	0.255	1.92E+5	0.565	0.000	2.00E+5	0.667	0.000	2.00E+5	0.167	0.000	2.00E+5	0.667	0.000	2.00E+5	0.667	0.000	2.00E+5	0.905	0.451	1.80E+5
F_{12}	0.591	0.000	2.00E+5	0.409	0.000	2.00E+5	0.615	0.000	2.00E+5	0.125	0.000	2.00E+5	0.015	0.000	2.00E+5	0.750	0.000	2.00E+5	**0.990**	0.922	1.09E+5
F_{13}	0.667	0.000	2.00E+5	0.493	0.000	2.00E+5	0.634	0.000	2.00E+5	0.167	0.000	2.00E+5	0.637	0.000	2.00E+5	0.667	0.000	2.00E+5	0.667	0.000	2.00E+5
F_{14}	**0.667**	0.000	4.00E+5	0.500	0.000	4.00E+5	0.663	0.000	4.00E+5	0.167	0.000	4.00E+5	0.592	0.000	4.00E+5	**0.667**	0.000	4.00E+5	**0.667**	0.000	4.00E+5
F_{15}	0.370	0.000	4.00E+5	0.287	0.000	4.00E+5	0.358	0.000	4.00E+5	0.125	0.000	4.00E+5	0.152	0.000	4.00E+5	0.699	0.000	4.00E+5	**0.738**	0.000	4.00E+5
F_{16}	0.663	0.000	4.00E+5	0.232	0.000	4.00E+5	0.621	0.000	4.00E+5	0.167	0.000	4.00E+5	0.010	0.000	4.00E+5	**0.667**	0.000	4.00E+5	**0.667**	0.000	4.00E+5
F_{17}	0.260	0.000	4.00E+5	0.103	0.000	4.00E+5	0.238	0.000	4.00E+5	0.076	0.000	4.00E+5	0.000	0.000	4.00E+5	0.456	0.000	4.00E+5	**0.620**	0.000	4.00E+5
F_{18}	0.353	0.000	4.00E+5	0.016	0.000	4.00E+5	0.222	0.000	4.00E+5	0.157	0.000	4.00E+5	0.160	0.000	4.00E+5	0.657	0.000	4.00E+5	**0.660**	0.000	4.00E+5
F_{19}	0.150	0.000	4.00E+5	0.000	0.000	4.00E+5	0.054	0.000	4.00E+5	0.027	0.000	4.00E+5	0.000	0.000	4.00E+5	0.451	0.000	4.00E+5	**0.458**	0.000	4.00E+5
F_{20}	0.069	0.000	4.00E+5	0.000	0.000	4.00E+5	0.125	0.000	4.00E+5	0.088	0.000	4.00E+5	0.000	0.000	4.00E+5	**0.250**	0.000	4.00E+5	**0.250**	0.000	4.00E+5
bprs		9			5			7			2			6			9			13	

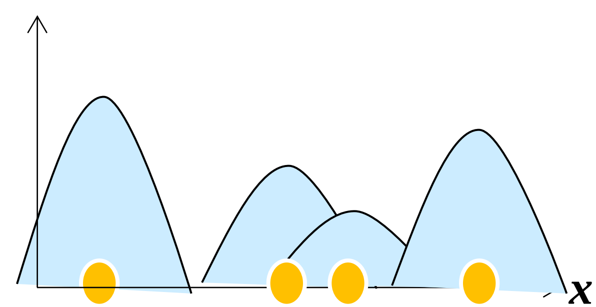
MEDA performs well on high-dimensional multimodal problems with many local optima, as they can locate more globally optimal solutions.

Multimodal ACO

- ACO is developed for discrete optimization (Dorigo et al., 1996)
- By modeling pheromone as probability distribution, ACO can also solve continuous problems (Socha and Dorigo, 2008)
- The capability of EDA is usually limited by the distribution model (usually Cauchy or Gaussian)
- The pheromone depositing behavior of ACO may provide a more flexible probability distribution



Distribution of Individuals



Pheromone Deposition

Multimodal ACO

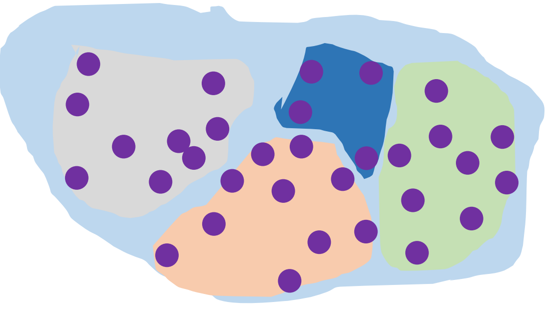
❑ 1) Niching with Dynamic Niche Size

❑ 2) Pheromone Deposition

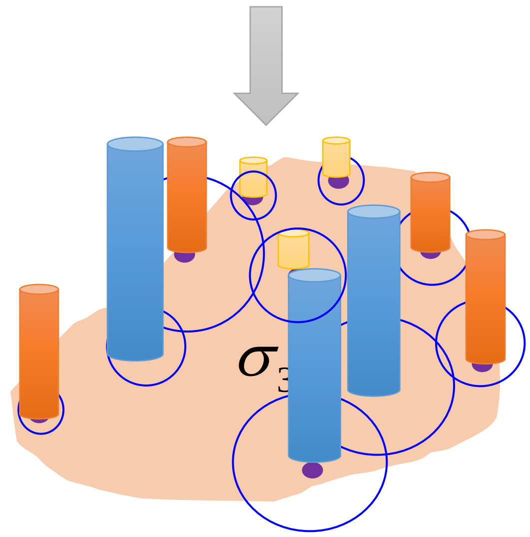
- Each individual deposits pheromone according to its fitness
- The pheromone of an individual determines the probability of sampling new children around this individual
- Adaptive configuration for the parameter σ

$$w_i = \frac{1}{\sigma NP\sqrt{2\pi}} e^{-\frac{(rank(i)-1)^2}{2\sigma^2 NP^2}}$$

$$p_j = \frac{w_j}{\sum_{i=1}^{NP} w_i}$$



Niching



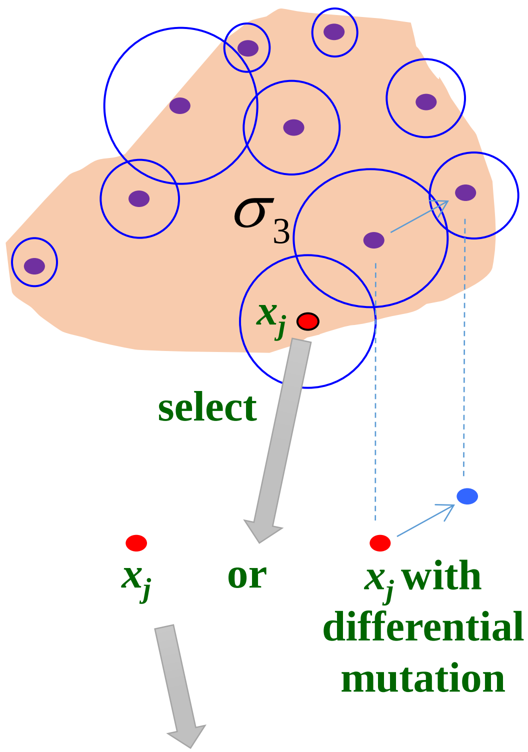
Pheromone Deposition to form a probability distribution for each niche

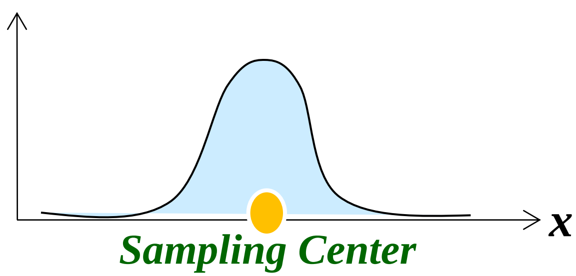
Multimodal ACO

- **3) Sampling new solutions**
  - Step 1. Select an individual according to their pheromone distribution
  - Step 2. Determine the center of sampling

$$\mu = \begin{cases} x_j \text{ (selected solution)} & rand() \le 0.5 \\ u = x_j + F(\mathbf{x}_{seed} - x_j) & \text{otherwise} \end{cases}$$

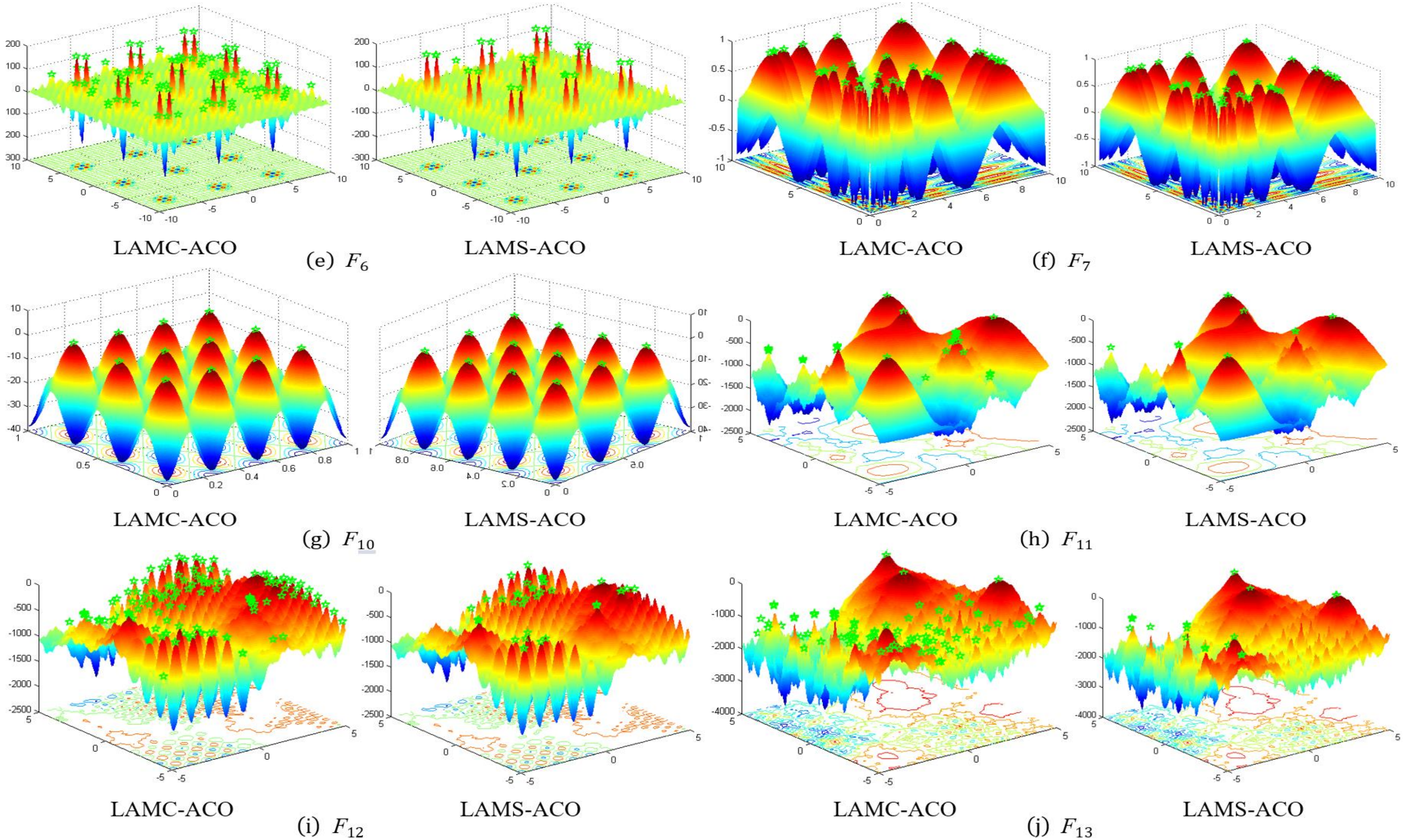
  - Step 3. Sampling with Gaussian Distribution
  - Step 4. Repeat 1-3 until a new generation has been built
- **4) Selection with the Neighborhood Replacement Rule**
- **5) Local Search**





Sampling using Gaussian Distribution

Results of AM-ACO



Results of AM-ACO

F	Self_CSDE			LoICDE			LoISDE			PNPCDE			MOMMOP			LAMC-ACO			LAMS-ACO		
	PR	SR	CS	PR	SR	CS	PR	SR	CS	PR	SR	CS	PR	SR	CS	PR	SR	CS	PR	SR	CS
F_1	**1.000**	1.000	7.15E+2	**1.000**	1.000	1.73E+2	**1.000**	1.000	1.68E+2	**1.000**	1.000	1.62E+2	**1.000**	1.000	1.66E+2	**1.000**	1.000	2.59E+2	**1.000**	1.000	2.14E+2
F_2	**1.000**	1.000	1.03E+3	**1.000**	1.000	1.24E+3	0.486	0.353	3.25E+4	**1.000**	1.000	1.08E+3	**1.000**	1.000	1.09E+3	**1.000**	1.000	5.35E+2	**1.000**	1.000	4.90E+2
F_3	**1.000**	1.000	7.23E+2	**1.000**	1.000	5.84E+2	**1.000**	1.000	7.48E+2	**1.000**	1.000	8.33E+2	**1.000**	1.000	9.58E+2	**1.000**	1.000	3.50E+2	**1.000**	1.000	3.53E+2
F_4	0.907	0.706	2.97E+4	**1.000**	1.000	1.48E+4	0.265	0.020	4.91E+4	**1.000**	1.000	2.28E+4	**1.000**	1.000	3.50E+4	**1.000**	1.000	5.04E+3	**1.000**	1.000	3.50E+3
F_5	**1.000**	1.000	3.38E+3	**1.000**	1.000	1.88E+3	0.814	0.627	1.91E+4	**1.000**	1.000	3.69E+3	**1.000**	1.000	1.48E+4	**1.000**	1.000	1.42E+3	**1.000**	1.000	1.10E+3
F_6	0.760	0.020	1.97E+5	**1.000**	1.000	9.58E+4	0.056	0.000	2.00E+5	0.806	0.157	1.97E+5	**1.000**	1.000	5.61E+4	0.999	0.980	1.10E+5	0.990	0.824	1.05E+5
F_7	0.696	0.000	2.00E+5	0.858	0.020	2.00E+5	0.029	0.000	2.00E+5	0.875	0.000	2.00E+5	**1.000**	1.000	6.32E+4	0.789	0.000	2.00E+5	0.716	0.000	2.00E+5
F_8	0.695	0.000	4.00E+5	0.000	0.000	4.00E+5	0.012	0.000	4.00E+5	0.000	0.000	4.00E+5	**1.000**	1.000	2.85E+5	0.680	0.000	4.00E+5	0.782	0.000	4.00E+5
F_9	0.265	0.000	4.00E+5	0.421	0.000	4.00E+5	0.005	0.000	4.00E+5	0.473	0.000	4.00E+5	**1.000**	1.000	2.95E+5	0.348	0.000	4.00E+5	0.295	0.000	4.00E+5
F_{10}	**1.000**	1.000	1.51E+4	**1.000**	1.000	3.04E+4	0.083	0.000	2.00E+5	**1.000**	1.000	2.16E+4	**1.000**	1.000	4.24E+4	**1.000**	1.000	9.47E+3	**1.000**	1.000	9.02E+3
F_{11}	0.565	0.000	2.00E+5	0.667	0.000	2.00E+5	0.167	0.000	2.00E+5	0.667	0.000	2.00E+5	0.938	0.647	1.73E+5	0.683	0.000	2.00E+5	**0.974**	0.843	1.40E+5
F_{12}	0.409	0.000	2.00E+5	0.615	0.000	2.00E+5	0.125	0.000	2.00E+5	0.015	0.000	2.00E+5	0.949	0.627	1.73E+5	0.824	0.098	1.99E+5	**0.983**	0.863	1.03E+5
F_{13}	0.493	0.000	2.00E+5	0.634	0.000	2.00E+5	0.167	0.000	2.00E+5	0.637	0.000	2.00E+5	0.667	0.000	2.00E+5	0.667	0.000	2.00E+5	0.676	0.000	2.00E+5
F_{14}	0.500	0.000	4.00E+5	0.663	0.000	4.00E+5	0.167	0.000	4.00E+5	0.592	0.000	4.00E+5	**0.667**	0.000	4.00E+5	**0.667**	0.000	4.00E+5	**0.667**	0.000	4.00E+5
F_{15}	0.287	0.000	4.00E+5	0.358	0.000	4.00E+5	0.125	0.000	4.00E+5	0.152	0.000	4.00E+5	0.627	0.000	4.00E+5	0.740	0.000	4.00E+5	**0.748**	0.000	4.00E+5
F_{16}	0.232	0.000	4.00E+5	0.621	0.000	4.00E+5	0.167	0.000	4.00E+5	0.010	0.000	4.00E+5	0.650	0.000	4.00E+5	**0.667**	0.000	4.00E+5	**0.667**	0.000	4.00E+5
F_{17}	0.103	0.000	4.00E+5	0.238	0.000	4.00E+5	0.076	0.000	4.00E+5	0.000	0.000	4.00E+5	0.512	0.000	4.00E+5	0.608	0.000	4.00E+5	**0.708**	0.000	4.00E+5
F_{18}	0.016	0.000	4.00E+5	0.222	0.000	4.00E+5	0.157	0.000	4.00E+5	0.160	0.000	4.00E+5	0.497	0.000	4.00E+5	**0.667**	0.000	4.00E+5	**0.667**	0.000	4.00E+5
F_{19}	0.000	0.000	4.00E+5	0.054	0.000	4.00E+5	0.027	0.000	4.00E+5	0.000	0.000	4.00E+5	0.223	0.000	4.00E+5	0.500	0.000	4.00E+5	**0.502**	0.000	4.00E+5
F_{20}	0.000	0.000	4.00E+5	0.125	0.000	4.00E+5	0.088	0.000	4.00E+5	0.000	0.000	4.00E+5	0.125	0.000	4.00E+5	0.272	0.000	4.00E+5	**0.348**	0.000	4.00E+5
bprs		5			7			2			6			11			9			15	

AM-ACO achieved even better results than MEDA. E.g., for the most difficult problem *F*20, AM-ACO yields a detection rate 34.8%, MEDA is 25%, while most other existing approaches are below 12.5%

Conclusion

- Probability Distribution Based EC algorithms are good at maintaining sufficient search diversity
- Combing Probability Distribution Based EC & Niching is promising for problems with high diversity requirement, e.g., seeking multiple solutions in multimodal optimization
- Local search and parameter adaption are helpful to improve search efficiency
- Implicit Probability Distribution Approaches, e.g., ACO, may sometimes provide a more flexible way to build distribution

"Multimodal Estimation of Distribution Algorithms", ***IEEE Transactions on Cybernetics***, vol. 47, no. 3, pp. 636-650, 2017.

"Adaptive Multimodal Continuous Ant Colony Optimization", ***IEEE Transactions on Evolutionary Computation***, vol. 21, no. 2, pp. 191-205, 2017.

Outline

1. Background
   - Multimodal Optimization
   - Probability Distribution Based EAs

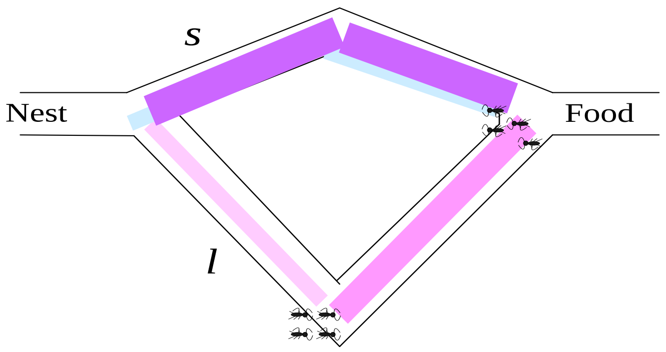
2. Probability Distribution Based EAs for Seeking Multiple Solutions

3. **Probability Distribution Based EAs for Discrete Optimization**

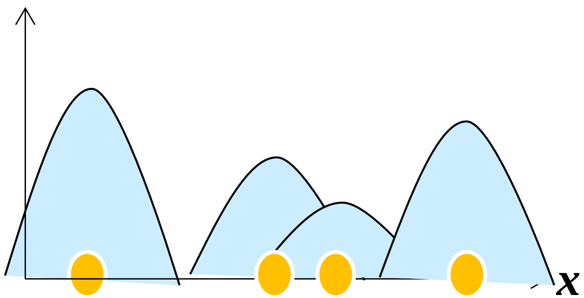
4. Applications & Future Work

Motivation

- ❑ **Probability Distribution is suitable on both continuous and discrete problems**



Probability distribution



Pheromone Deposition

ACO in discrete space **ACO in continuous space**

- ❑ **Some popular EC algorithms, e.g., PSO, are originally defined on continuous real vector space**

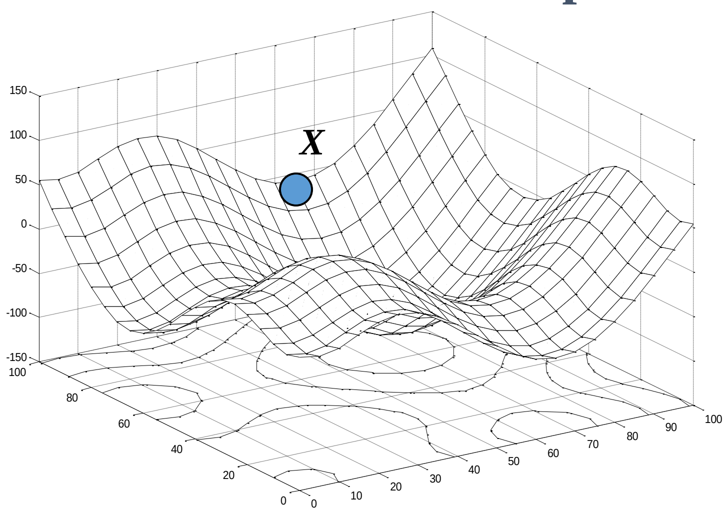
Velocity Update $v_i^j \leftarrow wv_i^j + c_1 r_1^j (pbest_i^j - x_i^j) + c_2 r_2^j (gbest^j - x_i^j), \quad j = 1,2,\ldots,n$

- ❑ **Is it possible to use Probability Distribution to build a more general discrete PSO (DE, etc.) framework?**

Set-Based Representation

COPs can be formulated in the abstract as “find from a set E a subset X that satisfies some constraints and optimizes the objective function f” (Lin and Kernighan 1973)

Continuous Search Space

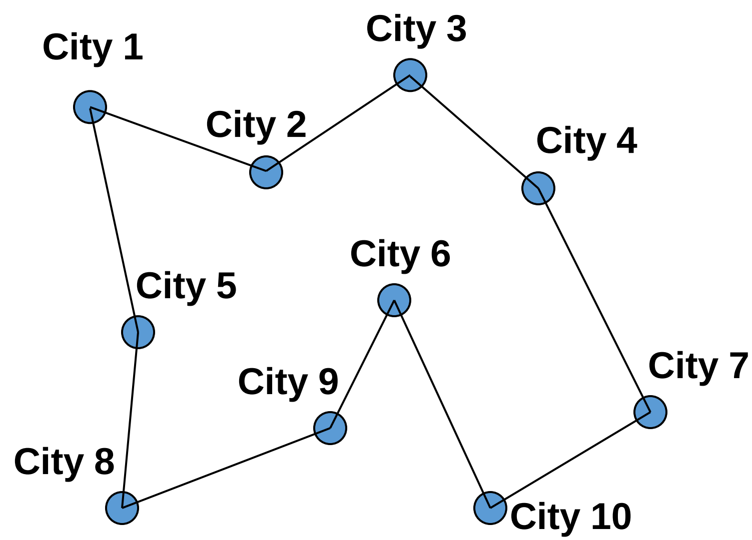


Representation: Real Vector

$$\boldsymbol{x_i}\,(x_i^1, x_i^2, \cdots, x_i^n)$$

?

Discrete Search Space

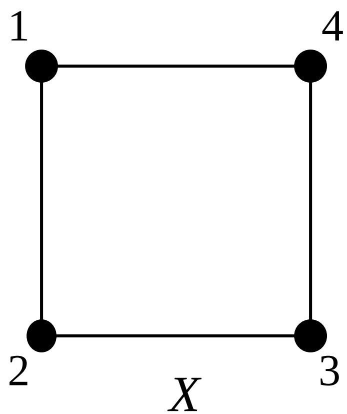


Representation: Set

{(1,2), (2,3), (3,4), …, (1,5)}

Set-Based Representation

- **Position: a crisp set**



$$E = \{(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)\}$$

$$E^1 = \{(1,2),(1,3),(1,4)\} \qquad E^2 = \{(1,2),(2,3),(2,4)\}$$

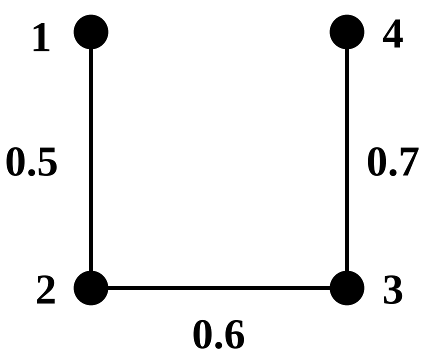
$$E^3 = \{(1,3),(2,3),(3,4)\} \qquad E^4 = \{(1,4),(2,4),(3,4)\}$$

$$X = \{(1,2),(2,3),(3,4),(1,4)\}$$

$$X^1 = \{(1,2),(1,4)\} \qquad X^2 = \{(1,2),(2,3)\}$$

$$X^3 = \{(2,3),(3,4)\} \qquad X^4 = \{(1,4),(3,4)\}$$

- **Velocity: a set with possibilities**



$$V = \{(1,2)/0.5,(2,3)/0.6,(3,4)/0.7\}$$

$$V^1 = \{(1,2)/0.5\}$$

$$V^2 = \{(1,2)/0.5,(2,3)/0.6\}$$

$$V^3 = \{(2,3)/0.6\}$$

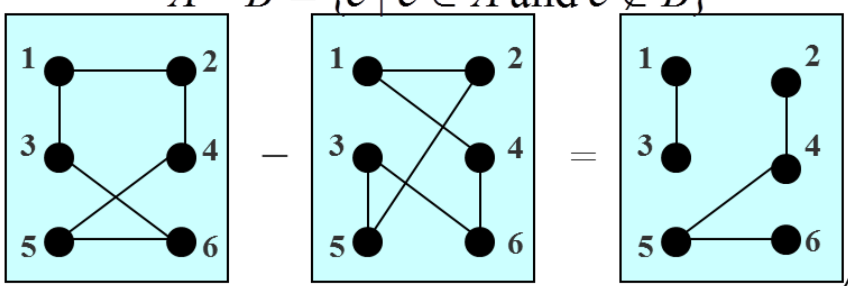
Velocity Update

- Essence of velocity update: build a **probability distribution of the "promising elements"** for particles to learn from

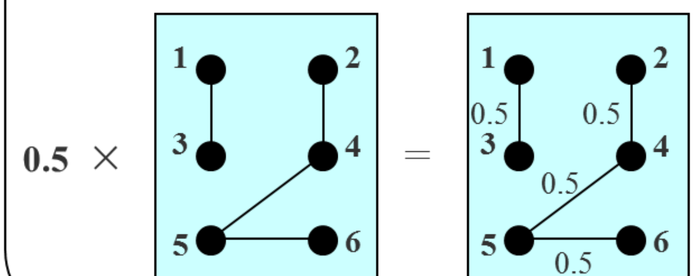
$$V_i^j \leftarrow \omega V_i^j + c_1 r_1^j \boxed{(PBest_i^j - X_i^j)} + \boxed{c_2 r_2^j (GBest^j - X_i^j)}, \qquad j = 1,2,\ldots,n$$

Position – Position
（crisp set – crisp et）

$$A - B = \{e \mid e \in A \text{ and } e \notin B\}$$



Coefficient * Crisp set



Union of two sets with possibilities

$V_1^1=\{(1, 2)/0.6, (1, 4)/0.9\}$ $\qquad$ $V_1^2=\{(1, 2)/0.9, (1, 3)/0.3\}$

$V_1^1 + V_1^2 = \{(1, 2)/0.9, (1, 3)/0.3, (1, 4)/0.9\}$

Position Update

- Particles uses the elements sampling from the probability distribution defined by “velocity” to update positions

Convert the velocity (probability distribution) into a crisp set

$$cut_\alpha(V_i^j) = \{e \mid e / p(e) \in V_i^j \text{ and } p(e) \geq \alpha\}$$

↓

Learn from the elements in $cut_\alpha(V_i^j)$

↓

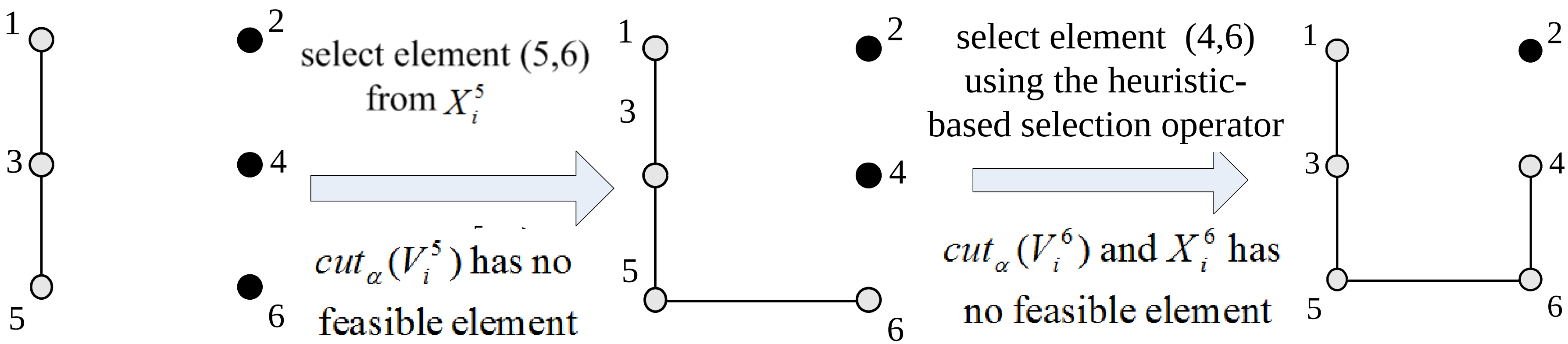
Reuse the elements from the previous position X_i^j

↓

Use other unselected elements to build a complete feasible solution

Constraint Handling

❑ Step-by-Step



❑ Build and Repair

X_i

Item 1	selected
Item 2	unselected
Item 3	selected
Item 4	unselected
Item 5	selected
Item 6	unselected

$cut_\alpha(V_i^j)$

{item 2, item 3, imtem4}

Build

New Position (infeasible)

Item 1	selected
Item 2	selected
Item 3	selected
Item 4	selected
Item 5	selected
Item 6	unselected

repair

Item 1	selected
Item 2	selected
Item 3	unselected
Item 4	selected
Item 5	selected
Item 6	unselected

Search Behavior

Velocity update in GPSO

Redefine **Discrete PSO**

$$v_i^j \leftarrow wv_i^j + c_1 r_1^j (pbest_i^j - x_i^j) + c_2 r_2^j (gbest^j - x_i^j), \quad j = 1,2,\ldots,n$$

→ **S-GPSO**

Velocity update in LPSO

$$v_i^j \leftarrow wv_i^j + c_1 r_1^j (pbest_i^j - x_i^j) + c_2 r_2^j (lbest_i^j - x_i^j), \quad j = 1,2,\ldots,n$$

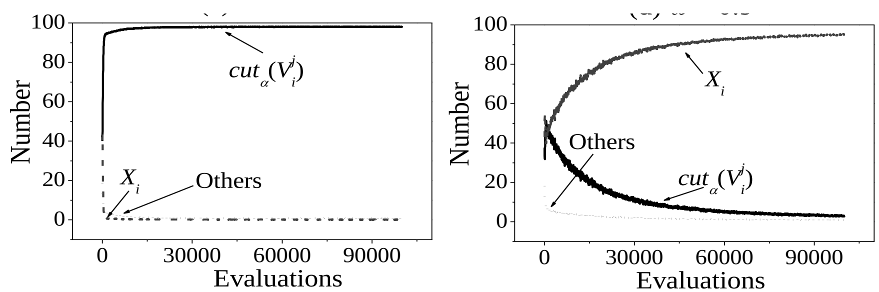
→ **S-LPSO**

Velocity update in CLPSO

$$v_i^j \leftarrow \omega \cdot v_i^j + cr^j (pbest_{f_i(j)}^j - x_i^j), \quad j = 1,2,\ldots,n$$

→ **S-CLPSO**

Different PSO variants can be directly extended to their discrete versions based on the S-PSO.



(e) $\omega = 1$ (f) decreasing ω

Parameter configurations in original PSO can be still used in S-PSO

Conclusion

- Probability Distribution can also be used to extend EC algorithms from continuous space to discrete space
- Different kinds of PSO variants (including multiobjective ones) can be extended to their discrete versions
- First verified on TSP and MKP,
- Solve other discrete optimization problems:
  - Vehicle routing (Gong et al., 2012)
  - Coverage array generation (Wu et al., 2015)

“A novel set-based particle swarm optimization method for discrete optimization problem”, ***IEEE Transactions on Evolutionary Computation***, 2010

"Set-Based Discrete Particle Swarm Optimization Based on Decomposition for Permutation-Based Multiobjective Combinatorial Optimization Problems", ***IEEE Trans. on Cybernetics***, in press

“Set-based discrete particle warm optimization and its applications: a survey”, *Frontiers of Computer Science*, 2018

Outline

1. Background

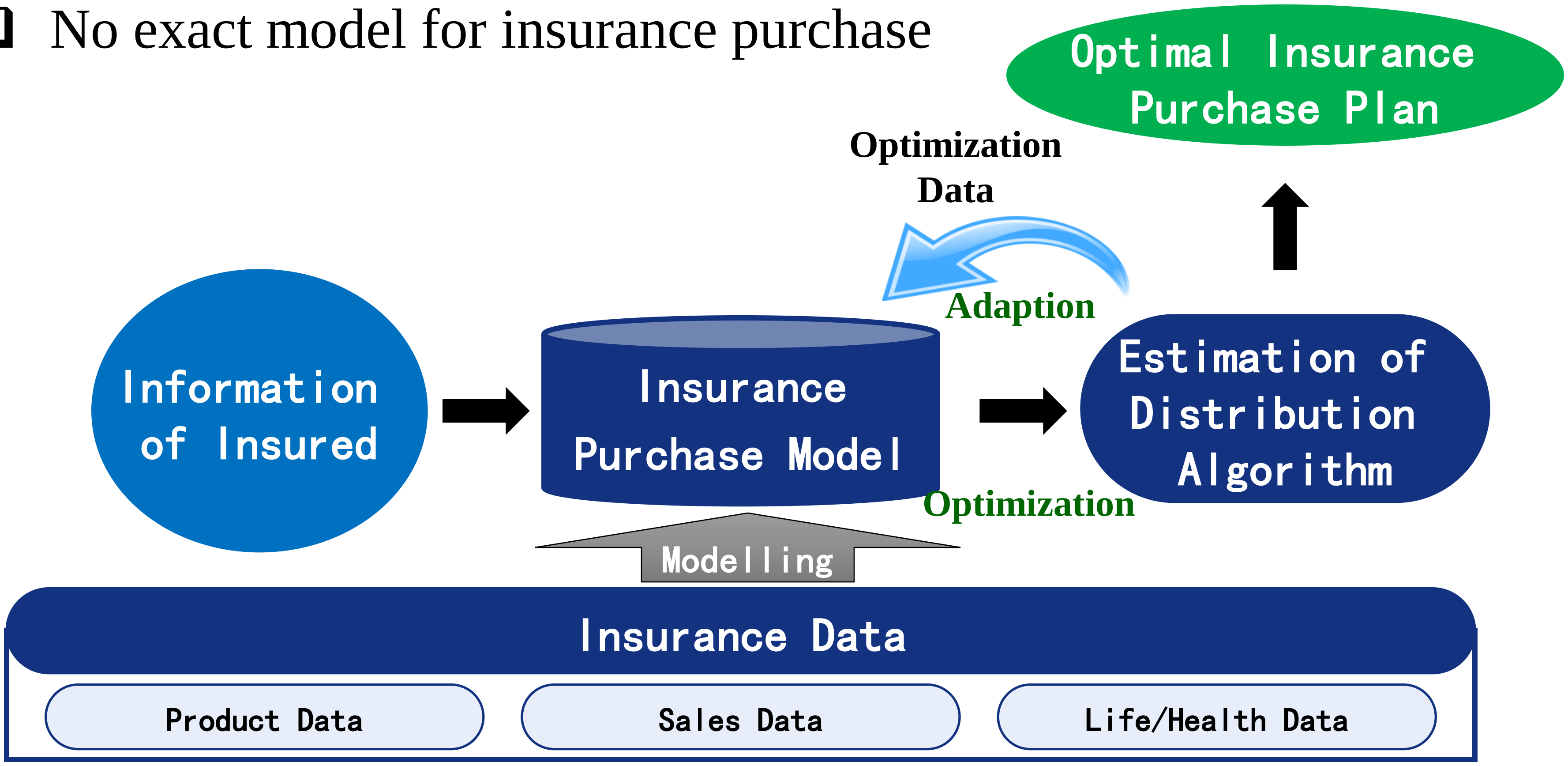
2. Probability Distribution Based EAs for Seeking Multiple Solutions

3. Probability Distribution Based EAs for Discrete Optimization

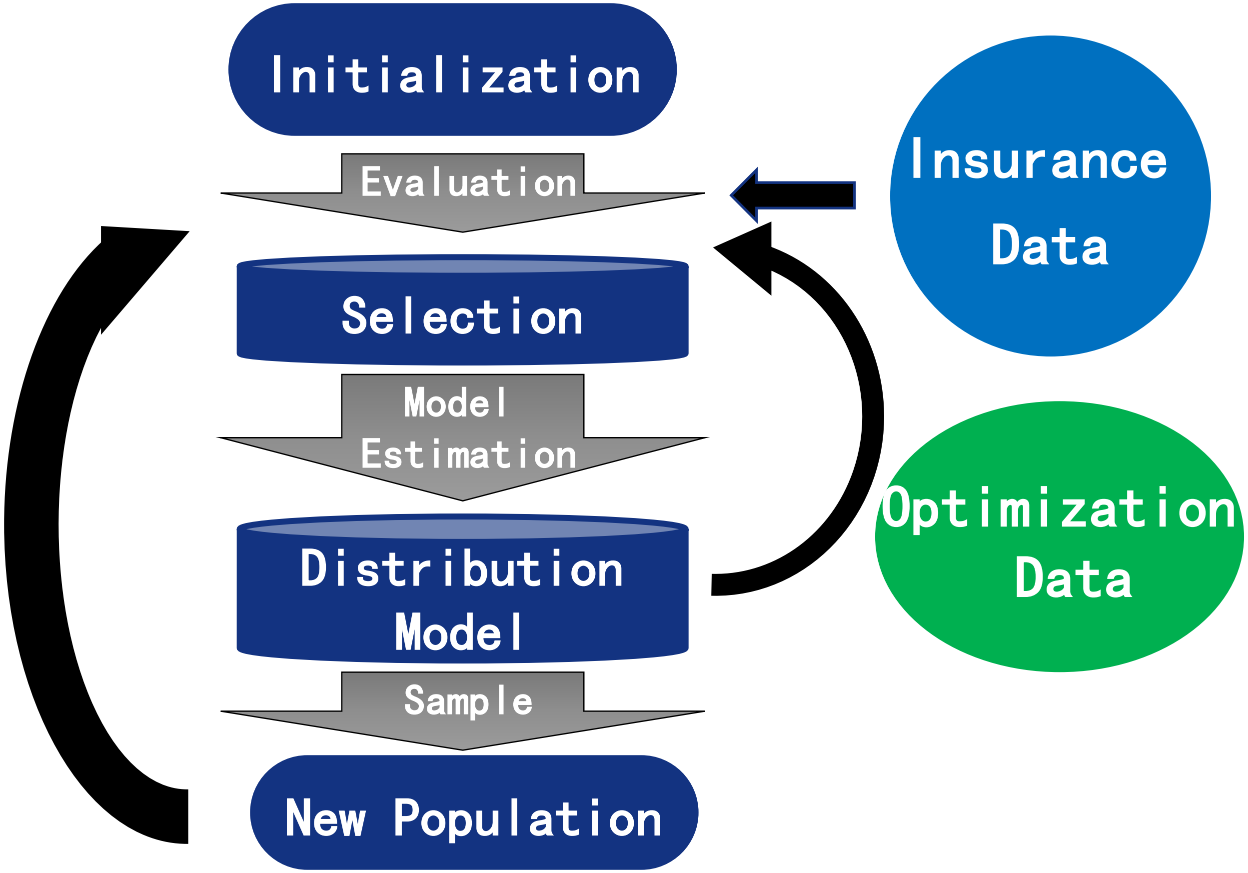
4. Applications & Future Work

Application 1: Insurance Purchase Optimization

- ❑ Insurance is an important way for investment and risk aversion
- ❑ Many insurance products exist
- ❑ No exact model for insurance purchase



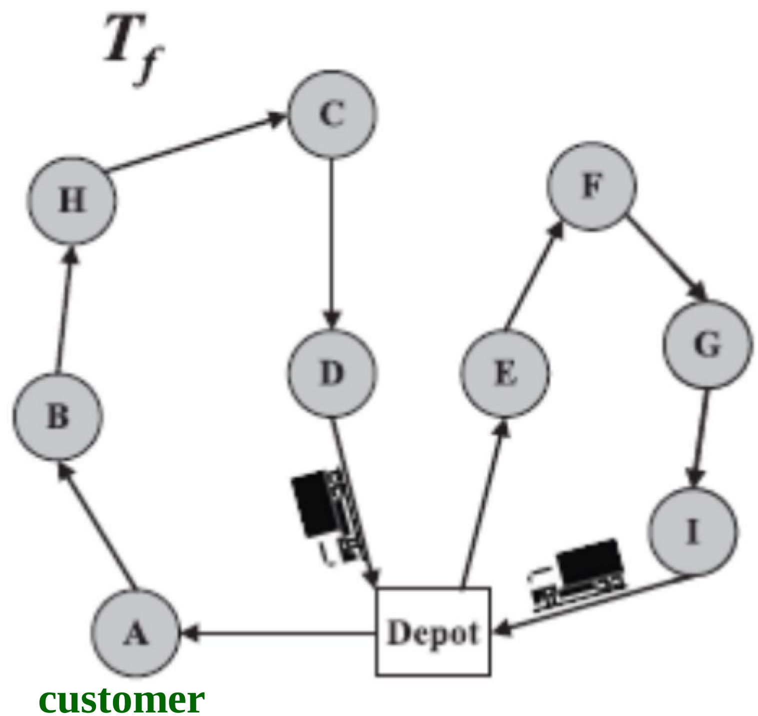
Estimation of Distribution Approach



"An Adaptive Estimation of Distribution Algorithm for Multi-Policy Insurance Investment Planning", ***IEEE Transactions on Evolutionary Computation***, Accepted in Nov. 2017

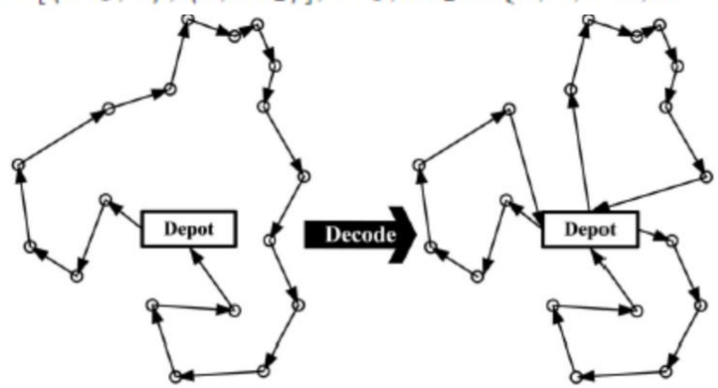
Application 2: Dynamic Vehicle Routing

- **Definition**
  - Designing routes for a fleet of vehicles to serve a set of customers
- **Objective:**
  - Minimize total travel distances subject to various constraints
- **A crucial issue in transportation and logistics systems**
- **NP-hard**



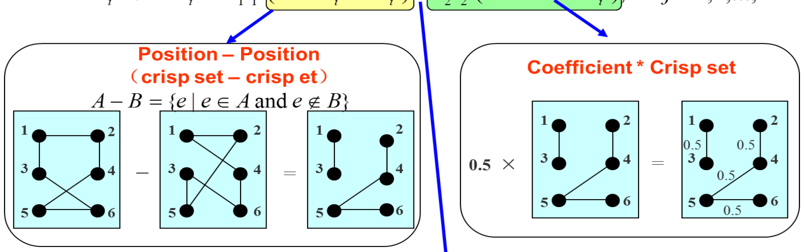
Probability-based EC for Vehicle Routing

$$X_i^d = [\langle nb_1, d\rangle, \langle d, nb_2\rangle], nb_1, nb_2 \in \{0, 1, \ldots, d-1, d+1, n\}$$



Set-based Representation
with a decoding scheme

$$V_i^j \leftarrow \omega V_i^j + c_1 r_1^j (PBest_i^j - X_i^j) + c_2 r_2^j (GBest^j - X_i^j), \quad j = 1,2,\ldots,n$$

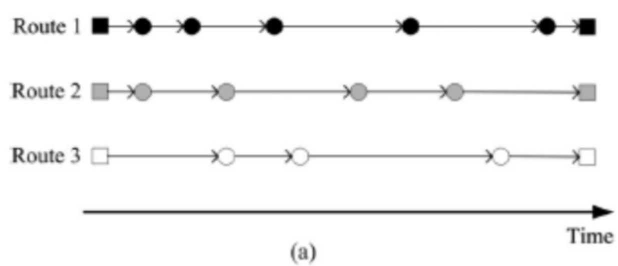


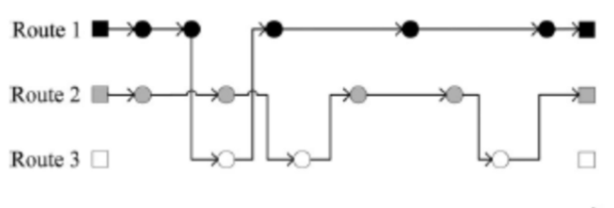
Union of two sets with possibilities

V_1^1={(1, 2)/0.6, (1, 4)/0.9} V_1^2={(1, 2)/0.9, (1, 3)/0.3}

$V_1^1 + V_1^2$= {(1, 2)/0.9, (1, 3)/0.3, (1, 4)/0.9}

Set-based comprehensive learning PSO



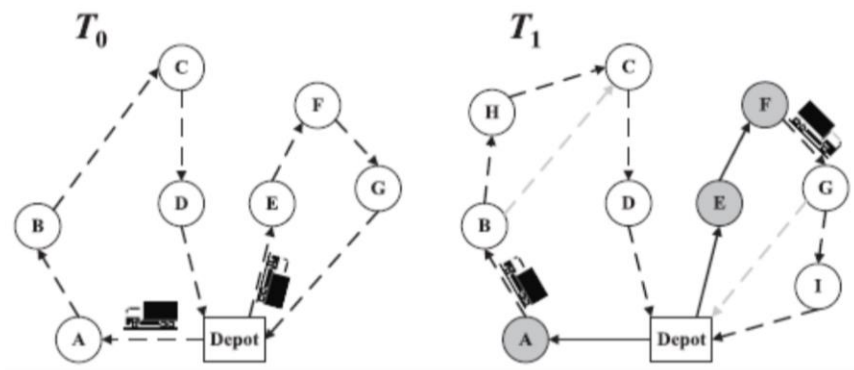


Local search

- ❑ Provide new best-known results on 60+ benchmark instances

Dynamic Logistics Dispatching System

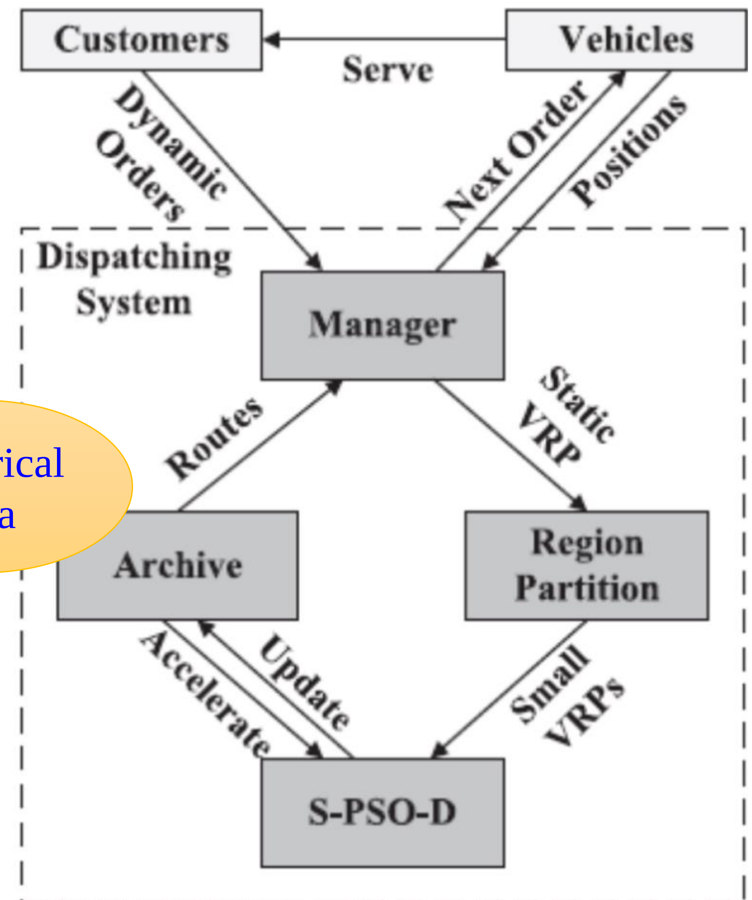
❑ Dynamic orders



❑ Dynamic optimization

- Archive strategy to store historical results and accelerate convergence
- Region partition to cut a large-scale problem into small pieces

Historical data

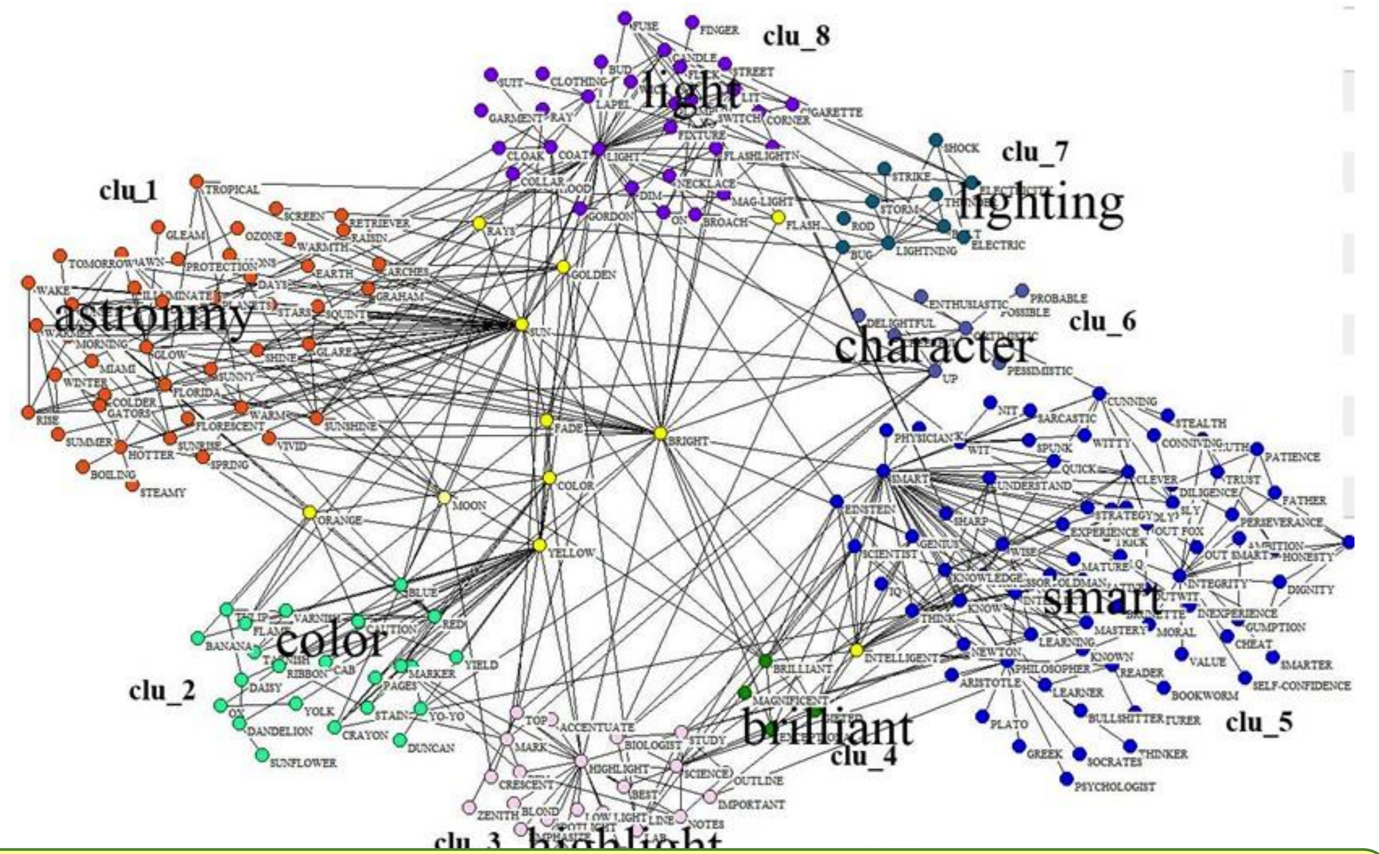


Architecture of the dispatching system

"A Dynamic Logistic Dispatching System With Set-Based Particle Swarm Optimization", ***IEEE TSMC-Systems***, in press, 2017

Application 3: Community Detection

❑ Detection of Overlapping Social Network Communities

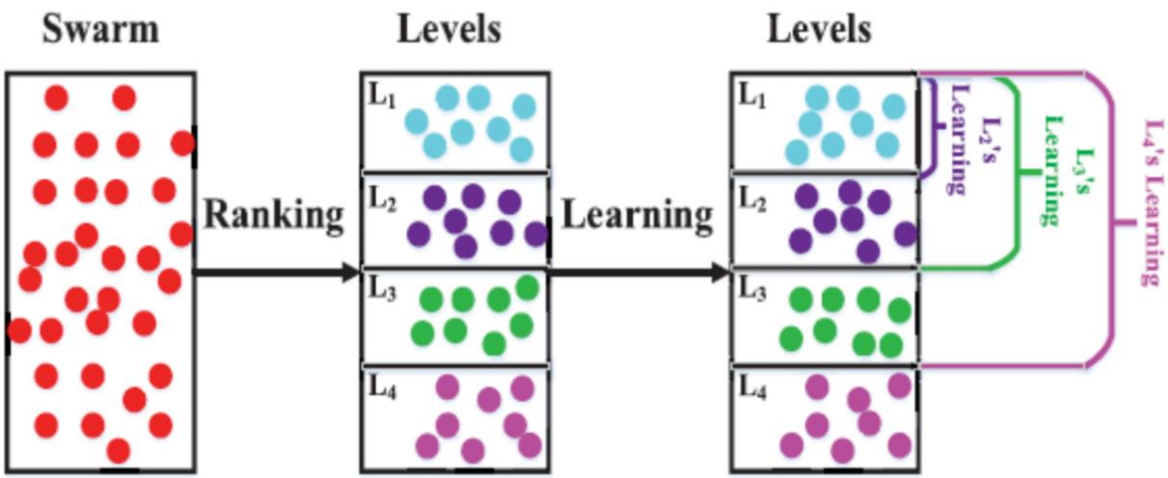


"A Maximal Clique Based Multiobjective Evolutionary Algorithm for Overlapping Community Detection ", ***IEEE Transactions on Evolutionary Computation***, 2017

Potential Future Work

- **Large-scale and high-dimensional problems**
  - **Cooperatively Coevolution** (Li and Yao, 2013 etc., …)
  - **Competitive Swarm Optimizer** (Cheng & Jin, 2015)

Level-based Learning / Segment-Based Predominant Learning Optimizer



Exploration:
- A lot of exemplars

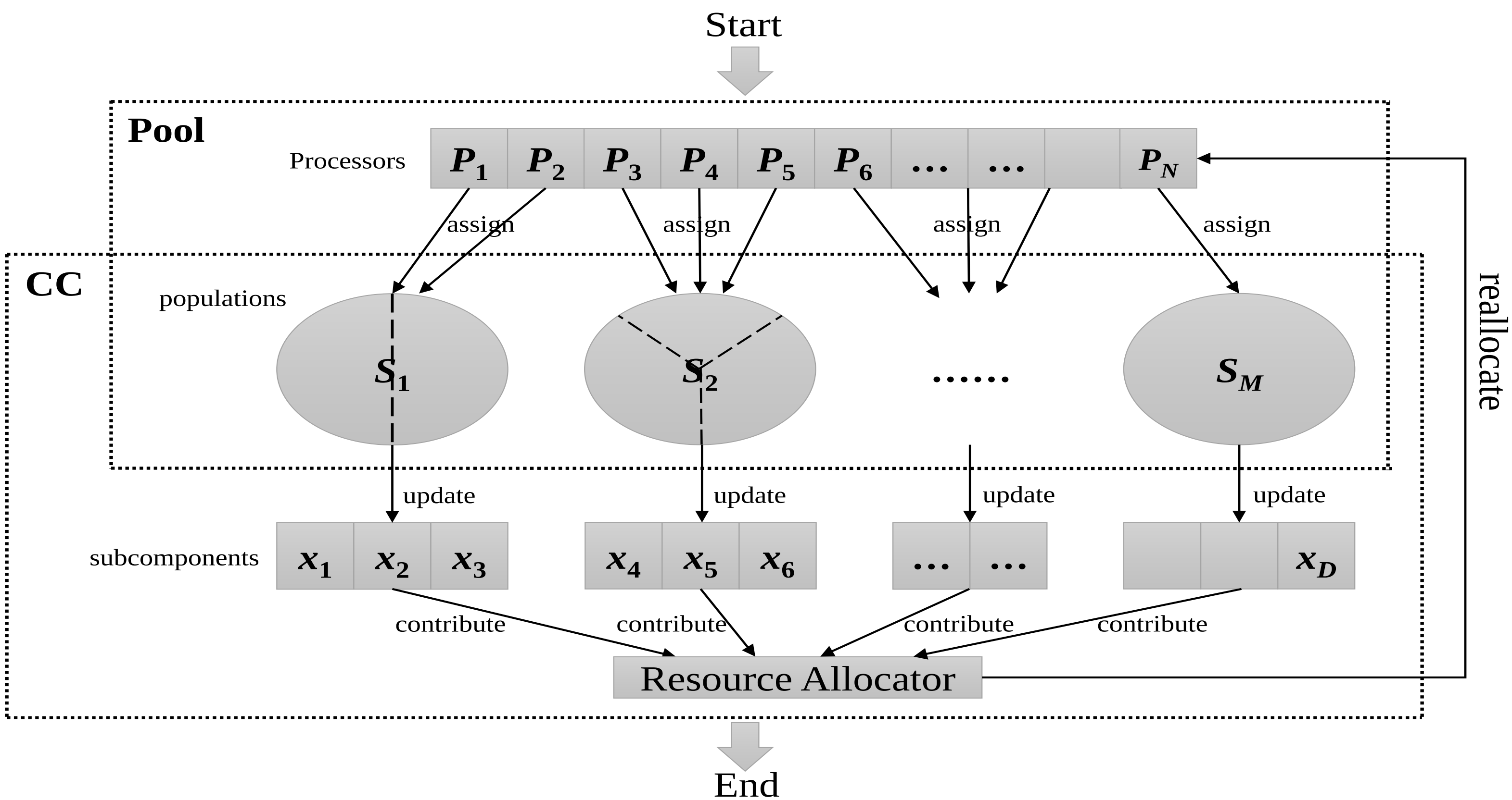
Exploitation:
- Predominant exemplars
- Short distances between learners and exemplars

"A Level-based Learning Swarm Optimizer for Large Scale Optimization", ***IEEE Transactions on Evolutionary Computation***, in press

"Segment-Based Predominant Learning Swarm Optimizer for Large-Scale Optimization," ***IEEE Transactions on Cybernetics***, vol. 47, no. 9, pp. 2896-2910, 2017

Potential Future Work

❑ Distributed Design & Implementation



"Distributed Cooperative Co-evolution with Adaptive Computing Resource Allocation for Large Scale Optimization", ***IEEE Transactions on Evolutionary Computation***, in press

Conclusions

- Probability distribution based EC algorithms are good at preserver search diversity
- Probability distribution is also helpful to extend applicable domain of EC algorithms
- Potential future studies
  - Large-scale optimization
    - Cooperatively coevolution
    - Parallel & distributed design & implementation
  - Optimization under uncertainty
  - Real world applications

Thanks for your attention!