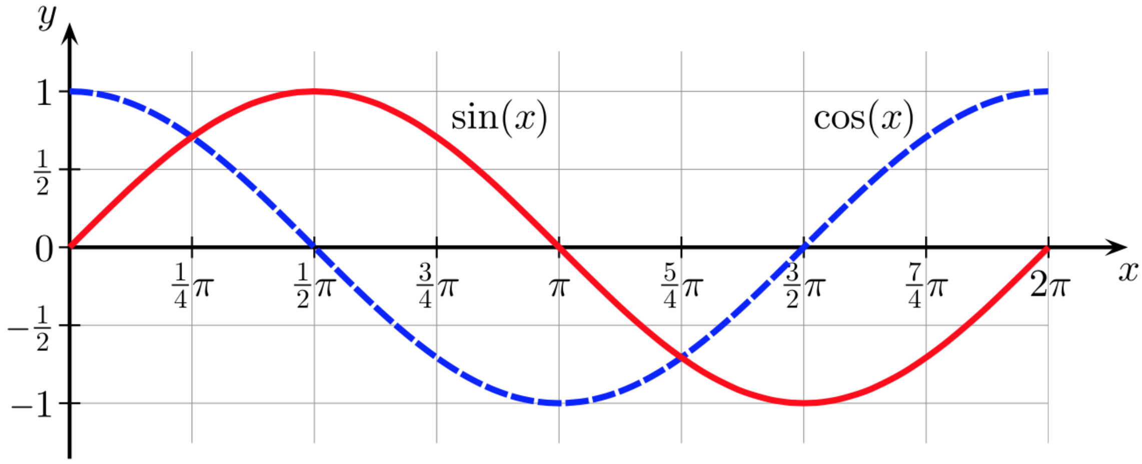
1 Allgemein

1.1 Trigonometrie



Bogenmaß	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Winkel	0°	30°	45°	60°	90°	120°	135°	150°	180°
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$
$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$
$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$
$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$
$$\sin^2(x) + \cos^2(x) = 1$$

1.2 Potenzgesetze

$$a^0 = 1$$
$$a^1 = a$$
$$a^m \cdot a^n = a^{m+n}$$
$$(a^n)^m = a^{nm}$$
$$\frac{a^n}{a^m} = a^{n-m}$$
$$a^{-n} = \frac{1}{a^n}$$
$$a^{\frac{b}{n}} = \sqrt[n]{a^b}$$

1.3 Logarithmus

$$\log(0) = \text{undef.}$$
$$\log(1) = 0$$
$$x\log_a(y) \Leftrightarrow a^x = y$$
$$-\log(x) = \log(\frac{1}{x})$$
$$\log(x) - \log(y) = \log(\frac{x}{y})$$
$$\frac{\log(x)}{\log(a)} = \log_a(x)$$

2 Integration

2.1 Elementare Integrale

$f'(x)$	$f(x)$	$F(x)$
$\frac{f'(x)g(x)-f(x)g'(x)}{g(x)^2}$	$\frac{f(x)}{g(x)}$	
0	c	cx
$r \cdot x^{r-1}$	x^r	$\frac{x^{r+1}}{r+1}$
$-\frac{1}{x^2} = -x^{-2}$	$\frac{1}{x} = x^{-1}$	$\ln\|x\|$
$\frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-\frac{1}{2}}$	$\sqrt{x} = x^{\frac{1}{2}}$	$\frac{2}{3}x^{\frac{3}{2}}$
$\cos(x)$	$\sin(x)$	$-\cos(x)$
$-\sin(x)$	$\cos(x)$	$\sin(x)$
$1+\tan^2(x) = \frac{1}{\cos^2(x)}$	$\tan(x)$	$-\ln\|\cos(x)\|$
e^x	e^x	e^x
$c \cdot e^{cx}$	e^{cx}	$\frac{1}{c} \cdot e^{cx}$
$\ln(c) \cdot c^x$	c^x	$\frac{c^x}{\ln(c)}$
$\frac{1}{x}$	$\ln\|x\|$	$x(\ln\|x\|-1)$
$\frac{1}{\ln(a)\cdot x}$	$\log_a\|x\|$	$\frac{x}{\ln(a)}(\ln\|x\|-1)$
$\frac{1}{\sqrt{1-x^2}}$	$\arcsin(x)$	$x \cdot \arcsin(x) + \sqrt{1-x^2}$
$-\frac{1}{\sqrt{1-x^2}}$	$\arccos(x)$	$x \cdot \arccos(x) - \sqrt{1-x^2}$
$\frac{1}{1+x^2}$	$\arctan(x)$	$x \cdot \arctan(x) - \frac{1}{2}\ln(1+x^2)$
$\cosh(x)$	$\sinh(x) = \frac{e^x-e^{-x}}{2}$	$\cosh(x)$
$\sinh(x)$	$\cosh(x) = \frac{e^x+e^{-x}}{2}$	$\sinh(x)$
$\frac{1}{\cosh^2(x)}$	$\tanh(x)$	$\log(\cosh(x))$

2.2 Regeln

Direkter Integral $\displaystyle\int f(g(x))g'(x)\ dx = F(g(x))$

Partielle Integration $\displaystyle\int f' \cdot g\ dx = f \cdot g - \int f \cdot g'\ dx$

mit Polynomen $\displaystyle\int \frac{p(x)}{q(x)}\ dx \Rightarrow$ Partialbruchzerlegung

Substitution $\displaystyle\int_a^b f(\varphi(t))\varphi'(t)\ dt = \int_{\varphi(a)}^{\varphi(b)} f(x)\ dx$ mit $x = \varphi(t)$

2.3 Tipps

$$\int \tan x\ dx = \int \frac{\sin x}{\cos x}\ dx = -\log|\cos(x)|$$

$$\int \frac{1}{x-\alpha}\ dx = \log(x-\alpha)$$

$$\int \frac{\frac{1}{\alpha}}{1+(\frac{x}{\alpha})^2}\ dx = \arctan(x)$$

$$\int \sin^2(x)\ dx = \frac{1}{2}(x - \sin(x)\cos(x)) + C$$

$$\int \cos^2(x)\ dx = \frac{1}{2}(x + \sin(x)\cos(x)) + C$$

$$\int \sqrt{x^2+1}\ dx = \sinh(x) + C$$

3 Vektorfelder

3.1 Differenzial (für f: $\mathbb{R}^n \mapsto \mathbb{R}^m$)

$$df = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & ... & \frac{\partial f_1}{\partial x_n} \\ ... & ... & ... \\ \frac{\partial f_m}{\partial x_1} & ... & \frac{\partial f_m}{\partial x_n} \end{pmatrix}$$

3.2 Gradient (für f: $\mathbb{R}^n \mapsto \mathbb{R}$)

$$\mathrm{grad}(f) = \nabla f = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ ... \\ \frac{\partial f}{\partial x_n} \end{pmatrix}$$

Der Gradient zeigt in die Richtung der maximalen Zuwachsrate von f und seine Länge ist gleich der maximalen Änderung von f.

3.3 Hessematrix (für f: $\mathbb{R}^n \mapsto \mathbb{R}$)

$$\mathrm{Hess}(f) = \begin{pmatrix} \frac{\partial^2 f}{\partial^2 x_1^2} & ... & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ ... & ... & ... \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & ... & \frac{\partial^2 f}{\partial x_n^2} \end{pmatrix}$$

3.4 Rotation (für f: $\mathbb{R}^3 \mapsto \mathbb{R}^3$ oder f: $\mathbb{R}^2 \mapsto \mathbb{R}^2$)

$$\text{In } \mathbb{R}^3: \ \mathrm{rot}(\vec{v}) = \nabla \times \vec{v} = \begin{pmatrix} \frac{\partial v_3}{\partial y} - \frac{\partial v_2}{\partial z} \\ \frac{\partial v_1}{\partial z} - \frac{\partial v_3}{\partial x} \\ \frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y} \end{pmatrix}, \ \text{in } \mathbb{R}^2: \ \mathrm{rot}(\vec{v}) = \frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y}$$

Bemerkung: Falls $\mathrm{rot}(\vec{v}) = 0$, dann ist $\vec{v}$ konservativ (Potenzialfeld).

3.5 Divergenz (für f: $\mathbb{R}^n \mapsto \mathbb{R}^n$)

$$\mathrm{div}(v) = \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + ...$$

3.6 Potenzialfeld

Ein Potenzialfeld ist konservativ. Das Potenzial Φ eines Potenzialfeldes ist gleich:

$$\nabla \Phi = \vec{v}$$

Für ein Potenzialfeld gilt $\mathrm{rot}(\vec{v}) = 0$ und es erfüllt die **Integrabilitätsbedinungen**:

$$\frac{\partial v_i}{\partial x_j} = \frac{\partial v_j}{\partial x_i}, \forall i \neq j$$

Berechnung eines Potenzials

$$\begin{aligned} \text{gegeben:} \quad & \vec{v} = \begin{pmatrix} e^{xy}(1+xy) \\ e^{xy}x^2 \end{pmatrix} \\ \textbf{Nach y integrieren:} \quad & \frac{\partial \Phi}{\partial y} = e^{xy}x^2 \Rightarrow \Phi = \int e^{xy}x^2 \ dy = xe^{xy} + C(x) \\ \textbf{Nach x ableiten:} \quad & \frac{\partial \Phi}{\partial x} = e^{xy} + xye^{xy} + C' \stackrel{!}{=} e^{xy} + xye^{xy} \Rightarrow C' = 0 \Rightarrow C = \text{konst.} \\ \textbf{Potenzial:} \quad & \Phi = xe^{xy} + \text{konst.} \end{aligned}$$

3.7 Koordinatentransformationen

3.7.1 Polarkoordinaten ($\mathbb{R}^2$)

Variablen: $\begin{aligned} x &= r\cos(\phi) \\ y &= r\sin(\phi) \end{aligned}$ **Volumenelement:** $\iint dxdy = \int_0^{2\pi} d\phi \int_0^R r\ dr$

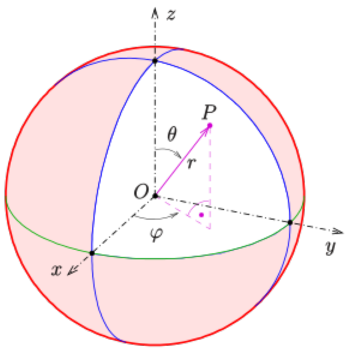
3.7.2 Elliptische Koordinaten ($\mathbb{R}^2$)

Variablen: $\begin{aligned} x &= ra\cos(\phi) \\ y &= rb\sin(\phi) \end{aligned}$ **Volumenelement:** $\iint dxdy = ab \int_0^{2\pi} d\phi \int_0^R r dr$

3.7.3 Zylinderkoordinaten ($\mathbb{R}^3$)

Variablen: $\begin{aligned} x &= r\cos(\phi) \\ y &= r\sin(\phi) \\ z &= z \end{aligned}$ **Volumenelement:** $\iiint dxdydz = \int_{-Z}^{Z} dz \int_0^{2\pi} d\phi \int_0^R r\ dr$

3.7.4 Kugelkoordinaten ($\mathbb{R}^3$)



Variablen: $\begin{aligned} x &= r\sin(\theta)\cos(\varphi) \\ y &= r\sin(\theta)\sin(\varphi) \\ z &= r\cos(\theta) \end{aligned}$

Volumenelement: $\iiint dxdydz = \int_0^{2\pi} d\varphi \int_0^{\pi} \sin(\theta)\ d\theta \int_0^R r^2\ dr$