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Department : Computer Science

Subject : Differential Equations (GE)

PRACTICAL 1 : Solution of First Order Differential Equation

ODEs in which there is a single independent variable and one or more dependent variable.

DSolve[eqn, y[x], x] : solving a differential equation for y[x]
DSolve[{eqn1, eqn2, ...}, {y1[x], y2[x],.....}, x] :
Solving a system of differential equation for yi[x]

Ques 1 : Solve $\frac{dy}{dx} = y$

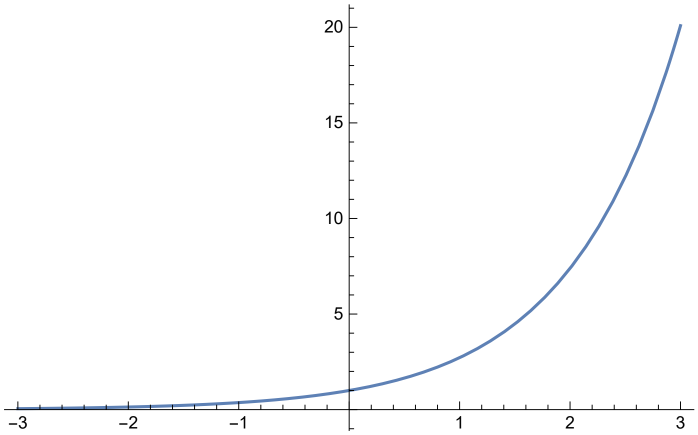
```
In[ ]:= x =.
In[ ]:= y =.
In[ ]:= z =.
In[ ]:= sol1 = DSolve[{ y'[x] == y[x] }, y[x], x]
```

Out[ ]= $\{\{y[x] \to e^{x} c_1\}\}$

```
In[ ]:= A = y[x] /. sol1 /. {C[1] → 1}
```

Out[ ]= $\{e^{x}\}$

In[∘]:=
```
y[x]
```
Out[∘]=
```
y[x]
```

In[∘]:=
```
Plot[{A} , {x, -3, 3}]
```
Out[∘]=


You can pick out a specific solution by using /. (Replace All)

Straight Integration

In[∘]:=
```
x =.
```

In[∘]:=
```
y =.
```

In[∘]:=
```
Clear All
```
Out[∘]=
```
All Clear
```

This equation is solved by simply integrating the right hand side with respect to x:

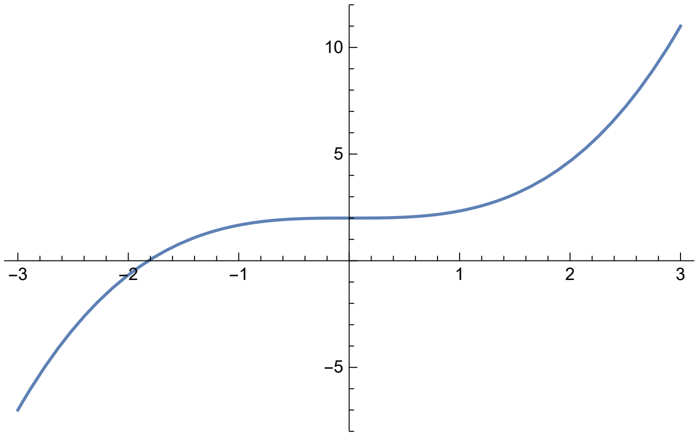
Ques 2 : Solve $y' = x^2$

In[∘]:=
```
sol22 = DSolve[y'[x] == x^2, y[x], x]
```
Out[∘]= $\left\{\left\{y[x] \to \frac{x^3}{3} + c_1\right\}\right\}$

In[∘]:=
```
B1 = y[x] /. sol22 /. {C[1] → 2}
```
Out[∘]= $\left\{2 + \frac{x^3}{3}\right\}$

In[∘]:= Plot[B1, {x, -3, 3}]

Out[∘]=



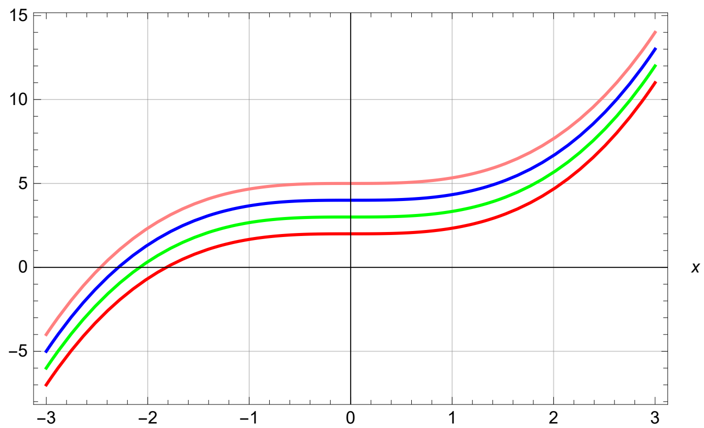
ANOTHER METHOD :

In[∘]:= B1 = Table[y[x] /. sol22 /. {C[1] → k}, {k, 2, 5}]

Out[∘]= $\left\{\left\{2+\frac{x^3}{3}\right\},\left\{3+\frac{x^3}{3}\right\},\left\{4+\frac{x^3}{3}\right\},\left\{5+\frac{x^3}{3}\right\}\right\}$

In[∘]:= Plot[B1, {x, -3, 3}, PlotStyle → {Red, Green, Blue , Pink}, GridLines → Automatic, Frame → True, AxesOrigin → {0, 0}, AxesLabel → Automatic, ImageSize → Medium]

Out[∘]=



Separable Equations

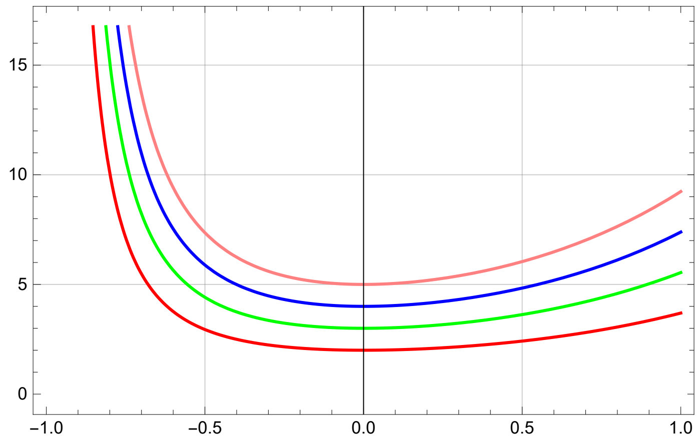
The general solution to this equation is found by separation of variable:

Ques 3 : Solve y’=2xy

In[∘]:= A = DSolve[y'[x] == 2 * x * y[x] / (x + 1), y[x], x]

Out[∘]= $\{\{y[x] \to e^{2\,(x-\mathrm{Log}[1+x])}\, c_1\}\}$

In[ ]:=
```
B1 = Table[y[x] /. A /. {C[1] → k}, {k, 2, 5}]
```
Out[ ]=
$$\{\{2\, e^{2\,(x-\text{Log}[1+x])}\},\ \{3\, e^{2\,(x-\text{Log}[1+x])}\},\ \{4\, e^{2\,(x-\text{Log}[1+x])}\},\ \{5\, e^{2\,(x-\text{Log}[1+x])}\}\}$$

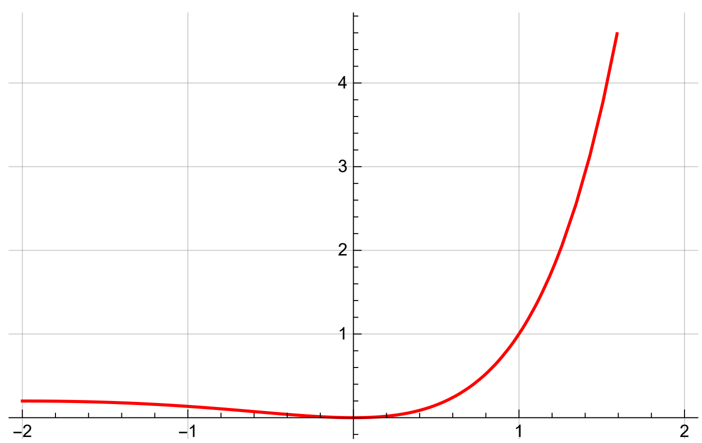
In[ ]:=
```
Plot[{B1}, {x, -1, 1}, PlotStyle → {Red, Green, Blue, Pink},
 GridLines → Automatic, Frame → True, AxesOrigin → {0, 0}]
```
Out[ ]=


Initial value problem

The solution of this type differential equations does not contain constant

Ques 4 : Solve y'=$\dfrac{y+2}{xy}$, y[1] =1

In[ ]:=
```
sol2 = DSolve[{y'[x] == y[x] + 2/x * y[x], y[1] == 1}, y[x], x]
```
Out[ ]=
$$\{\{y[x] \to e^{-1+x}\, x^2\}\}$$

In[ ]:=
```
Plot[y[x] /. sol2, {x, -2, 2}, PlotStyle → {Red}, GridLines → Automatic]
```
Out[ ]=


Homogenous Equations

here is a homogenous equation in which the total degree of both the numerator and the denominator of the righthand side is name. The two parts of the solution list give branches of the integral curves in the form :

```
In[]:= sol3 = DSolve[y'[x] == (x + y[x]) / (x), y[x], x]
```

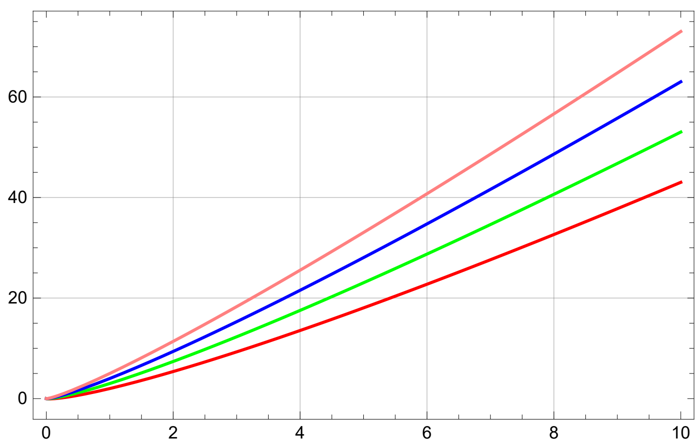
Out[]= $\{\{y[x] \to x\, c_1 + x\, \text{Log}[x]\}\}$

```
In[]:= sol4 = Table[y[x] /. sol3 /. {C[1] → k}, {k, 2, 5}]
```

Out[]= {{2 x + x Log[x]}, {3 x + x Log[x]}, {4 x + x Log[x]}, {5 x + x Log[x]}}

```
In[]:= Plot[{sol4}, {x, 0, 10}, PlotStyle → {Red, Green, Blue , Pink},
        GridLines → Automatic, Frame → True, AxesOrigin → {0, 0}]
```

Out[]=



Linear First-Order Equations

The following is a linear first order ODE because both y[x] and its derivative y’[x] occur in it with power 1 and is the highest derivative. Note that the solution contains the imaginary error function Erfi:

Ques 6 : Solve y’+ y/(x) = x^2

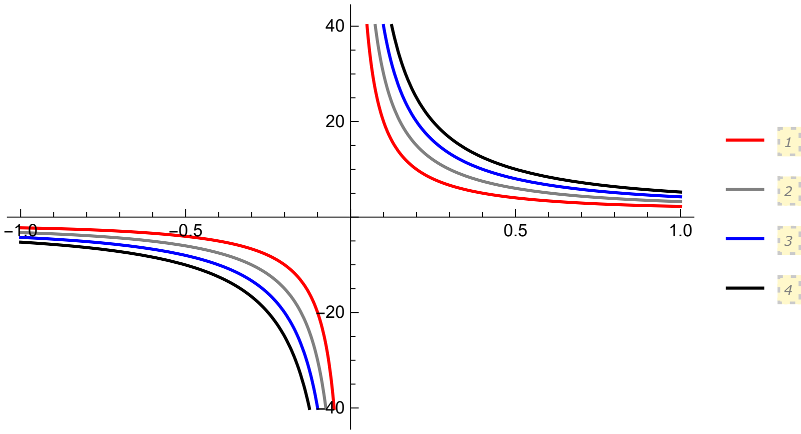
```
In[]:= sol5 = DSolve[y'[x] + y[x] / (x) == x^2, y[x], x]
```

Out[]= $\left\{\left\{y[x] \to \frac{x^3}{4} + \frac{c_1}{x}\right\}\right\}$

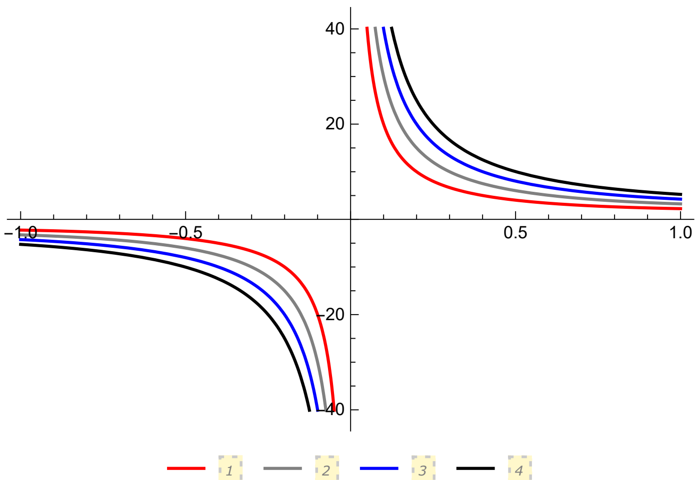
```
In[]:= sol6 = Table[y[x] /. sol5 /. {C[1] → k}, {k, 2, 5}]
```

Out[]= $\left\{\left\{\frac{2}{x} + \frac{x^3}{4}\right\}, \left\{\frac{3}{x} + \frac{x^3}{4}\right\}, \left\{\frac{4}{x} + \frac{x^3}{4}\right\}, \left\{\frac{5}{x} + \frac{x^3}{4}\right\}\right\}$

In[ ]:= Plot[{sol6}, {x, -1, 1}, PlotStyle → {Red, Gray, Blue, Black}, PlotLegends → Automatic]

Out[ ]=


In[ ]:= Plot[{sol6}, {x, -1, 1}, PlotStyle → {Red, Gray, Blue, Black},
PlotLegends → Placed[Automatic, Below]]

Out[ ]=


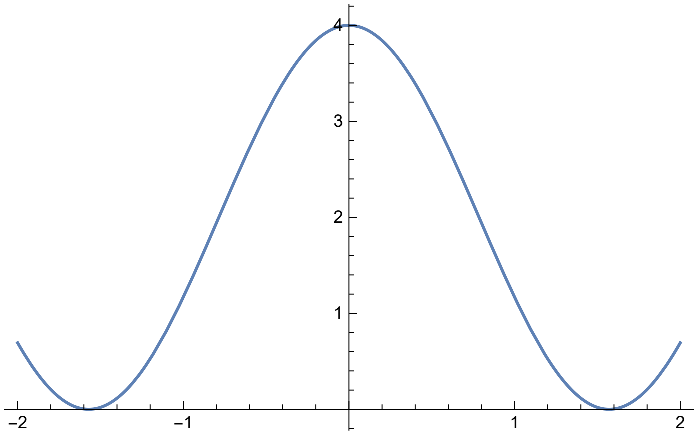
Ques 7 : $\frac{dy}{dx}$ * tan(x) = 2y - 8 , y[Pi/2] = 0

In[ ]:= sol7 = DSolve[{y'[x] * Tan[x] == 2 * y[x] - 8, y[Pi / 2] == 0}, y[x], x]

Out[ ]= {{y[x] → -4 (-1 + Sin[x]^2)}}

In[∘]:= `Plot[y[x] /. sol7, {x, -2, 2}]`

Out[∘]=



Bernoulli Equations

A Bernoulli Equation is a first - order equation of the form y’(x) + P(x) y(x) = Q(x) y (x)^n

Ques 8 : Solve $x\frac{dy}{dx} + y = y^2 x^2 + 1$

In[∘]:= `sol8 = DSolve[x * y'[x] + y[x] == y[x]^2 x^2 + 1, y[x], x]`

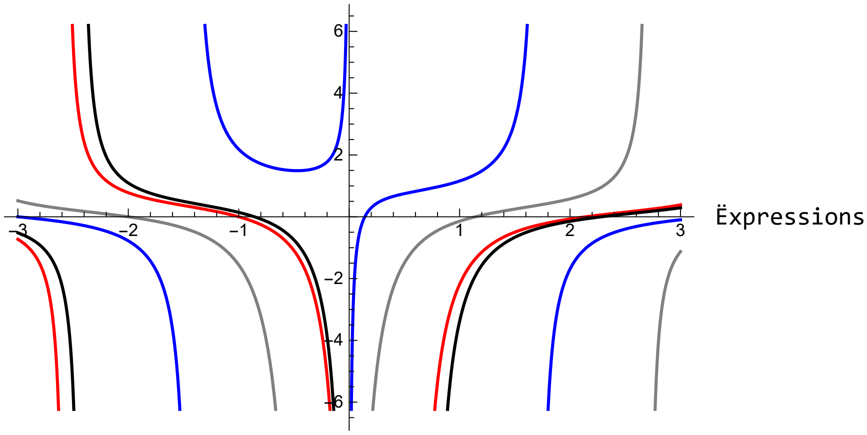
Out[∘]= $\left\{\left\{y[x] \to \frac{\text{Tan}[x + \mathbb{c}_1]}{x}\right\}\right\}$

In[∘]:= `sol9 = Table[y[x] /. sol8 /. {C[1] → k}, {k, 1, 4}]`

Out[∘]= $\left\{\left\{\frac{\text{Tan}[1 + x]}{x}\right\}, \left\{\frac{\text{Tan}[2 + x]}{x}\right\}, \left\{\frac{\text{Tan}[3 + x]}{x}\right\}, \left\{\frac{\text{Tan}[4 + x]}{x}\right\}\right\}$

In[∘]:= `Plot[{sol9}, {x, -3, 3}, PlotStyle → {Red, Gray, Blue, Black},`
`PlotLegends → Placed["Ëxpressions", Right]]`

Out[∘]=



Exact Equations

Let M(x,y) and N(x,y) be two smooth functions having continuous partial derivatives in some domain R^2 without holes. A differential equation, written in differentials :

M(x,y) dx + N(x,y)dy = 0

is called **Exact** if and only if

dM (x, y)/dy = dN(x,y)/dz

or there exists a smooth function v(x,y) called the potential function such that its total differential :

dv=M(x,y) dx + N(x,y) =0

Classify the differential equation as exact or not

```
D[f (x, y), differentiatewithrespectto]
D[f (x, y), x] diffwrtx
```

Ques 9 : (xy^2 + x) dx + (x^2 y)dy = 0

In[ ]:= `M1[x_, y_] := (x * y^2 + x)`

In[ ]:= `N1[x_, y_] := (y * x^2)`

In[ ]:= `Simplify[D[M1[x, y], y] - D[N1[x, y], x]]`

Out[ ]= 0

In[ ]:= `eqn = y'[x] == -M1[x, y[x]] / N1[x, y[x]]`

Out[ ]= $y'[x] == \frac{-x - x\,y[x]^2}{x^2\,y[x]}$

In[ ]:= `sol11 = DSolve[eqn, y[x], x]`

In[ ]:= $\left\{\left\{y[x] \to -\frac{\sqrt{e^{2 c_1} - x^2}}{x}\right\}, \left\{y[x] \to \frac{\sqrt{e^{2 c_1} - x^2}}{x}\right\}\right\}$

Out[ ]= $\left\{\left\{y[x] \to -\frac{\sqrt{e^{2 c_1} - x^2}}{x}\right\}, \left\{y[x] \to \frac{\sqrt{e^{2 c_1} - x^2}}{x}\right\}\right\}$

In[ ]:= `p[x_, y_] := -(5 x^2 - 2 y^2 + 11)`

In[ ]:= `q[x_, y_] := (Sin[y] + 4 xy + 3)`

In[ ]:= `Simplify[D[p[x, y], y] - D[q[x, y], x]]`

Out[ ]= 4 y

Ques 10 : (3x + 2y)dx + (2x+y) dy = 0

In[2]:=
```
p[x_, y_] := 3 x + 2 y
q[x_, y_] := 2 x + y
Simplify[D[p[x, y], y] - D[q[x, y], x]]
```

Out[4]= 0

Ques 11 : (y^2+3) **dx + (2xy - 4)dy = 0**

In[14]:=
```
p[x_, y_] := (y^2 + 3)
q[x_, y_] := (2 * x * y - 4)
Simplify[D[p[x, y], y] - D[q[x, y], x]]
```

Out[16]= 0

Ques 12 : $(4x + 3y^2)$dx + (2xy) dy = 0

In[17]:=
```
p[x_, y_] := (4 x + 3 y^2)
q[x_, y_] := (2 * x * y)
Simplify[D[p[x, y], y] - D[q[x, y], x]]
```

Out[19]= 4 y