PRACTICAL 3 : Plotting of third order solution family of differential equations

Solve third order differential equations and plot its any three solutions

Ques 1 : y'' + y = 0.001 x^2

Real and Distinct Roots

In[36]:=
```
sol = DSolve[y'''[x] - 5 * y''[x] + 8 * y'[x] - 4 y[x] == 0, y[x], x]
```

Out[36]=
$$\left\{\left\{y[x] \to e^{x} C_1 + e^{2x} C_2 + e^{2x} x C_3\right\}\right\}$$

In[37]:=
```
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 2/3}]
```

Out[37]=
$$e^{x} + 2\, e^{2x} + \frac{2}{3} e^{2x} x$$

In[38]:=
```
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 0.5, C[2] → 2, C[3] → 3}]
```

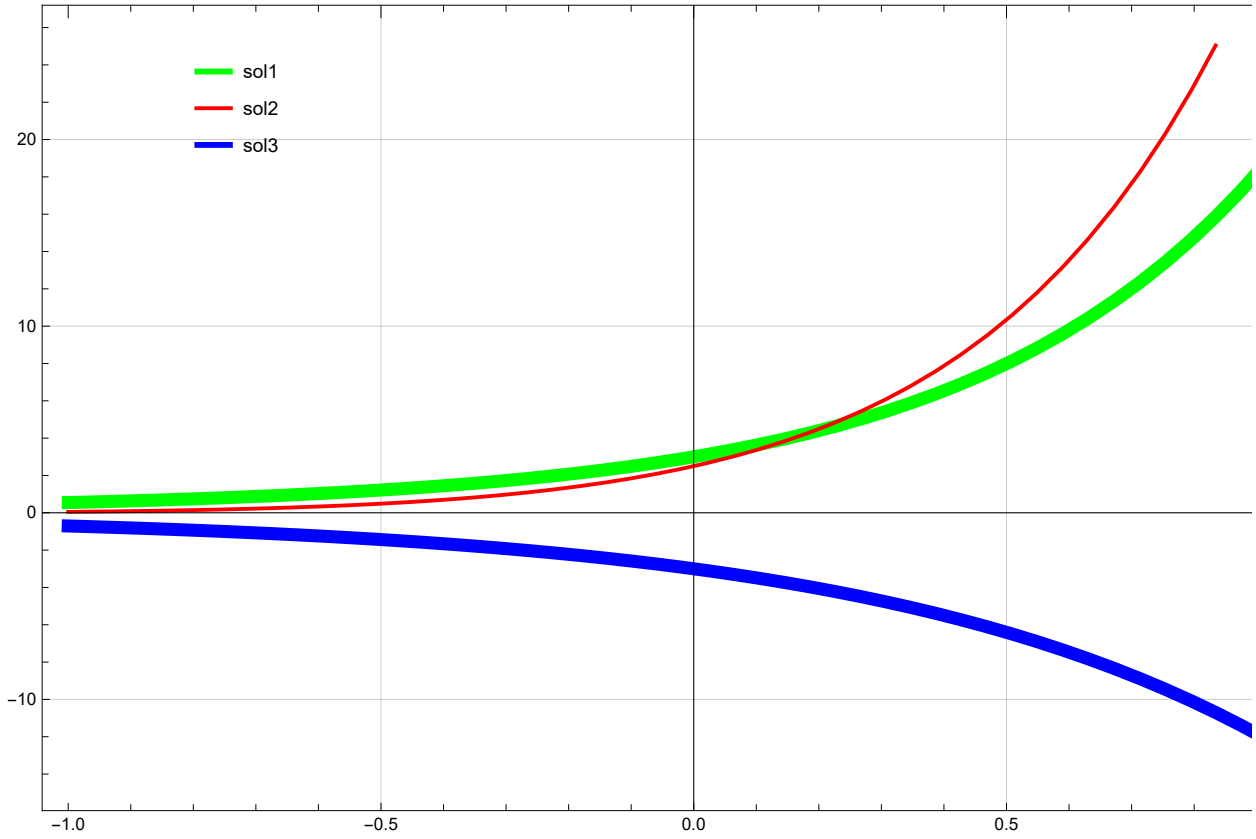
Out[38]=
$$0.5\, e^{x} + 2\, e^{2x} + 3\, e^{2x} x$$

In[39]:=
```
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -1, C[2] → -2, C[3] → 0.5}]
```

Out[39]=
$$-e^{x} - 2\, e^{2x} + 0.5\, e^{2x} x$$

In[41]:=
```
Plot[{sol1, sol2, sol3} , {x, -1, 1}, ImageSize → 700,
 PlotStyle → {{Green, Thickness[0.01]}, {Red, Thick}, {Blue, Thickness[0.01]}},
 Frame → True, AxesOrigin → {0, 0}, GridLines → Automatic,
 PlotLegends → Placed[LineLegend["Expressions",
    LegendLayout → "Column" , LegendFunction → "Frame"], {0.15, 0.87}]]
```

Out[41]=



Ques 2 : y’’’+3y’’-25y’+21y=0

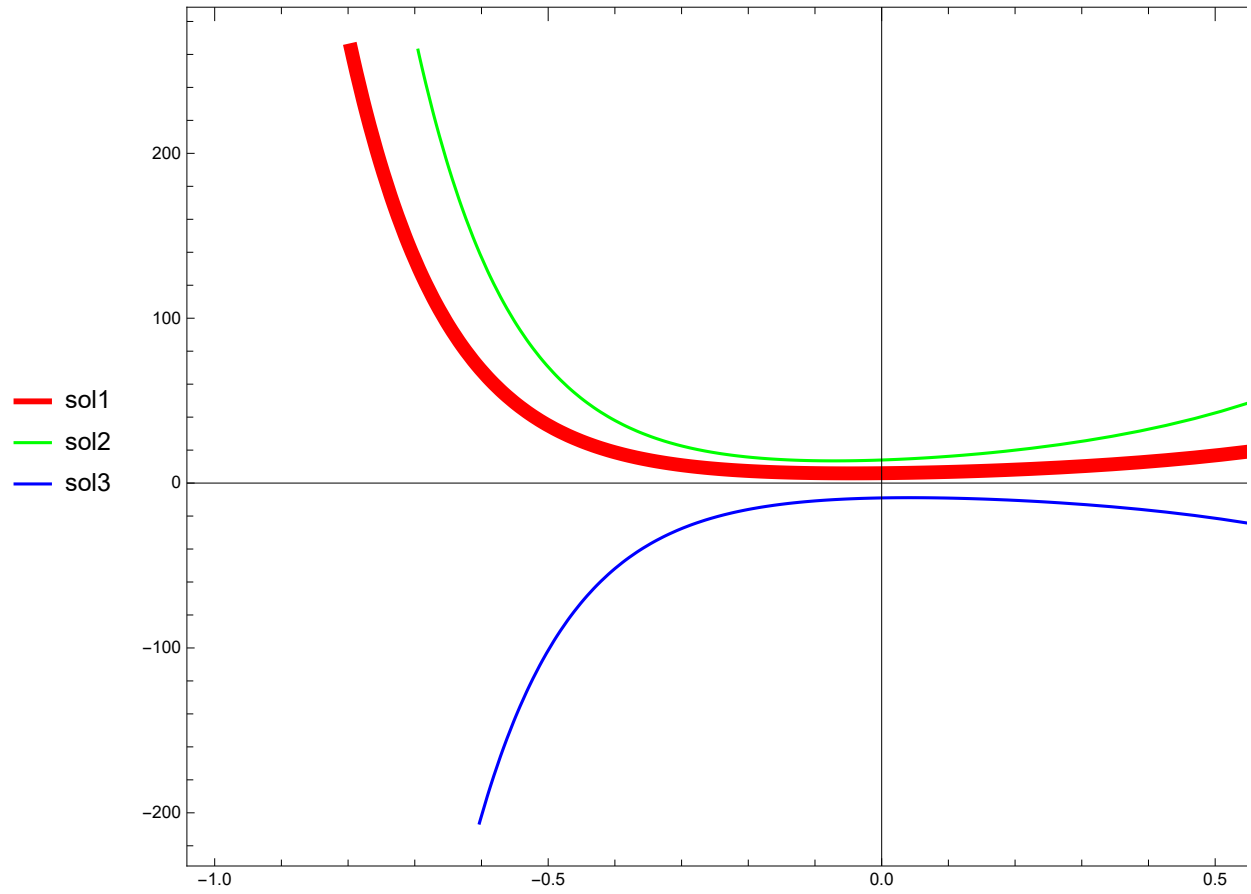
In[•]:=
```
sol = DSolve[y'''[x] + 3 * y''[x] - 25 y'[x] + 21 * y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 3}]
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 2, C[2] → 4, C[3] → 8}]
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -3, C[2] → -2, C[3] → -4}]
Plot[{sol1, sol2, sol3}, {x, -1, 1},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thicker}, {Blue, thick}},
 Frame → True, ImageSize → 750, PlotLegends → Placed[{"sol1", "sol2", "sol3"}, Left]]
```

Out[•]= $\left\{\left\{y[x] \to e^{-7x} c_1 + e^{x} c_2 + e^{3x} c_3\right\}\right\}$

Out[•]= $e^{-7x} + 2\, e^{x} + 3\, e^{3x}$

Out[•]= $2\, e^{-7x} + 4\, e^{x} + 8\, e^{3x}$

Out[•]= $-3\, e^{-7x} - 2\, e^{x} - 4\, e^{3x}$



Ques 3 : y'''-4y''-25y'+28y=0

In[42]:=
```
Sol = DSolve[y'''[x] - 13 y''[x] + 19 y'[x] + 33 y[x] == Cos[2 x], y[x], x]
sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 3}]
sol2 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → -1, C[2] → -2, C[3] → 1.3}]
sol3 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 3/2, C[2] → 0.5, C[3] → -2.5}]
Plot[{sol1, sol2, sol3}, {x, -1, 1},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick}, {Blue, Thickness[0.01]}},
 Frame → True, ImageSize → 750, PlotLegends → Placed[{"sol1", "sol2", "sol3"}, Below]]
```

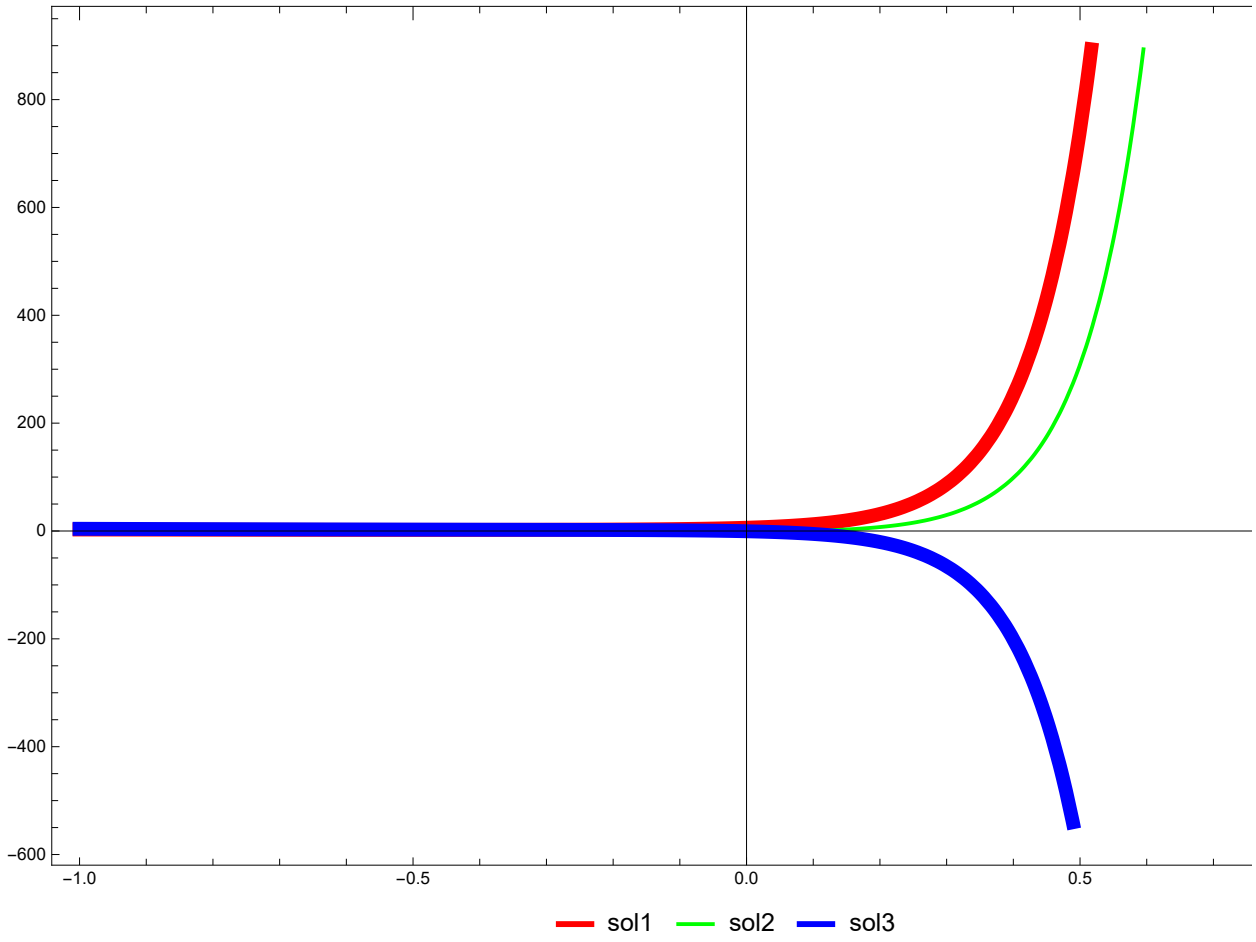
Out[42]= $\left\{\left\{y[x] \to e^{-x} c_1 + e^{3x} c_2 + e^{11x} c_3 + \frac{17 \cos[2x] + 6 \sin[2x]}{1625}\right\}\right\}$

Out[43]= $e^{-x} + 2 e^{3x} + 3 e^{11x} + \frac{17 \cos[2x] + 6 \sin[2x]}{1625}$

Out[44]= $-e^{-x} - 2 e^{3x} + 1.3 e^{11x} + \frac{17 \cos[2x] + 6 \sin[2x]}{1625}$

Out[45]= $\frac{3 e^{-x}}{2} + 0.5 e^{3x} - 2.5 e^{11x} + \frac{17 \cos[2x] + 6 \sin[2x]}{1625}$

Out[46]=



Ques 4 : Solve third order differential Equation and plot its any three solutions

y’’’-13y’’+19y’+33y=Cos2x

In[•]:=
```
sol = DSolve[y'''[x] - 13 * y''[x] + 19 * y'[x] + 33 * y[x] == Cos[2 x] , y[x] , x]
```

Out[•]=
$$\left\{\left\{y[x] \to e^{-x} C_1 + e^{3x} C_2 + e^{11x} C_3 + \frac{17 \operatorname{Cos}[2x] + 6 \operatorname{Sin}[2x]}{1625}\right\}\right\}$$

In[•]:=
```
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 3}]
```

Out[•]=
$$e^{-x} + 2 e^{3x} + 3 e^{11x} + \frac{17 \operatorname{Cos}[2x] + 6 \operatorname{Sin}[2x]}{1625}$$

In[•]:=
```
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 2, C[2] → 4, C[3] → 8}]
```

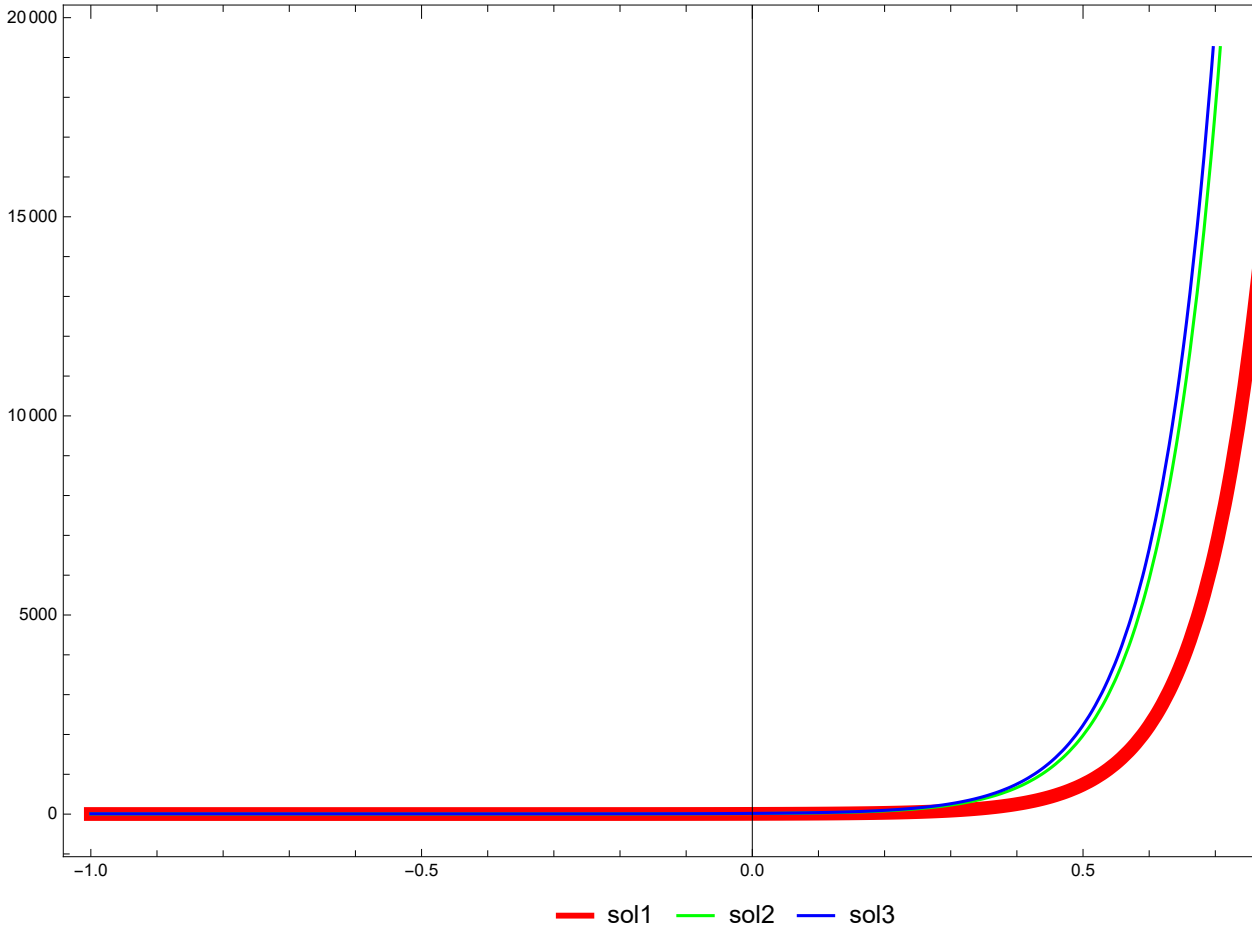
Out[•]=
$$2 e^{-x} + 4 e^{3x} + 8 e^{11x} + \frac{17 \operatorname{Cos}[2x] + 6 \operatorname{Sin}[2x]}{1625}$$

In[•]:=
```
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 3, C[2] → 6, C[3] → 9}]
```

Out[•]=
$$3 e^{-x} + 6 e^{3x} + 9 e^{11x} + \frac{17 \operatorname{Cos}[2x] + 6 \operatorname{Sin}[2x]}{1625}$$

In[ ]:=
```
Plot[{sol1, sol2, sol3}, {x, -1, 1},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thicker}, {Blue, thick}},
 Frame → True, ImageSize → 750, PlotLegends → Placed[{"sol1", "sol2", "sol3"}, Below]]
```

Out[ ]=



Solve the following differential equations

Ques 5 : y''' -6y''+5y' + 12y = 0

In[•]:=
```
sol = DSolve[y'''[x] - 6 * y''[x] + 5 y'[x] + 12 * y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 3}]
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 2, C[2] → 4, C[3] → 8}]
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1/3, C[2] → 1/2, C[3] → 2/3}]
Plot[{sol1, sol2, sol3}, {x, -1, 1},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thicker}, {Blue, thick}},
 Frame → True, ImageSize → 750, PlotLegends → Placed[{"sol1", "sol2", "sol3"}, Top]]
```

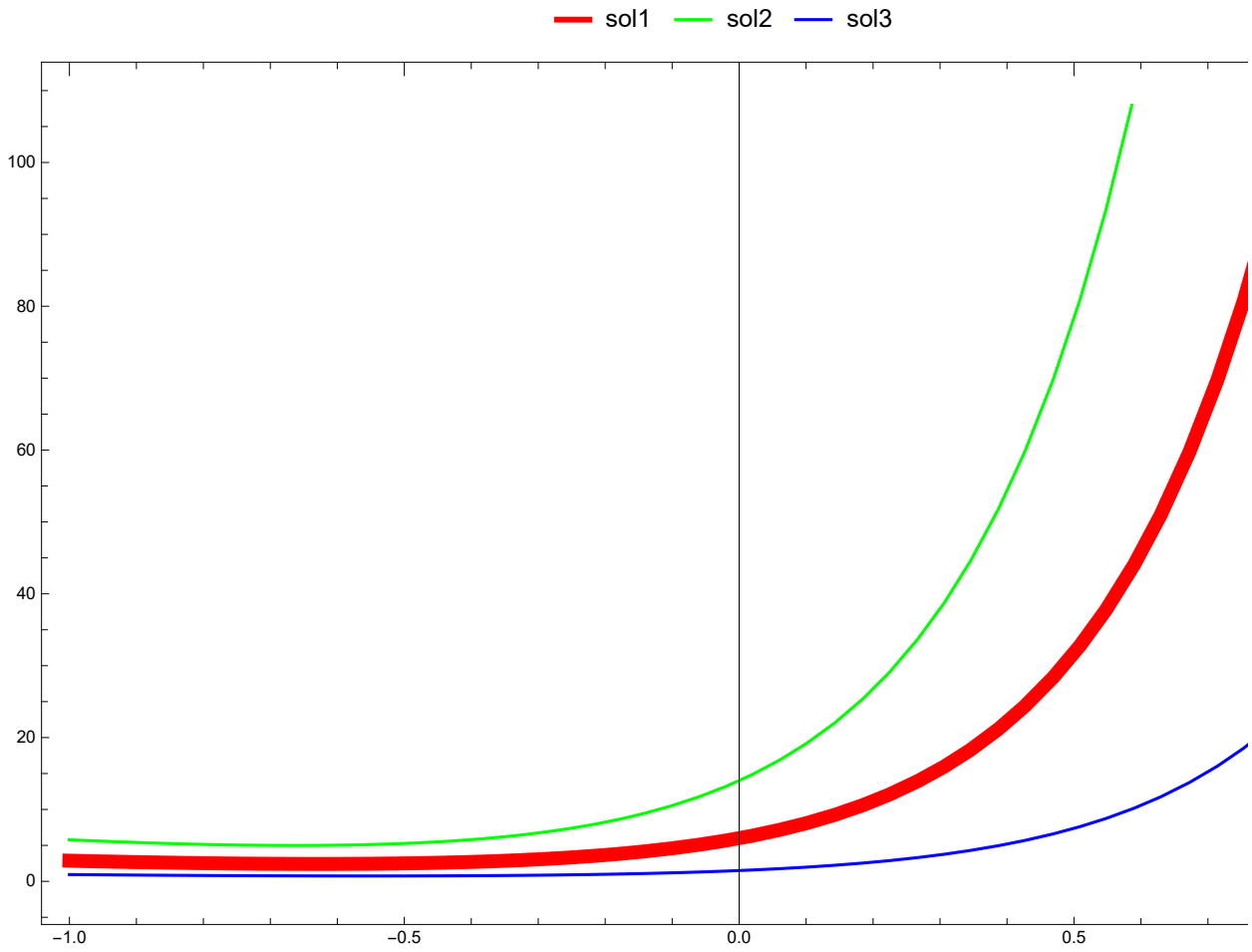
Out[•]= $\{\{y[x] \to e^{-x} C_1 + e^{3x} C_2 + e^{4x} C_3\}\}$

Out[•]= $e^{-x} + 2 e^{3x} + 3 e^{4x}$

Out[•]= $2 e^{-x} + 4 e^{3x} + 8 e^{4x}$

Out[•]= $\frac{e^{-x}}{3} + \frac{e^{3x}}{2} + \frac{2 e^{4x}}{3}$

Out[•]=



In[ ]:=

Ques 6 : y'''-6y''+11y' -6y=0, y(0)=0, y'(0)=0, y''(0)=2

In[ ]:=
```
sol = DSolve[y'''[x] - 6 * y''[x] + 11 y'[x] - 6 * y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 3}]
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 2, C[2] → 4, C[3] → 8}]
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -3, C[2] → -2, C[3] → -4}]
Plot[{sol1, sol2, sol3}, {x, -1, 1},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thicker}, {Blue, thick}},
 Frame → True, ImageSize → 750, PlotLegends → Placed[{"sol1", "sol2", "sol3"}, Left]]
```

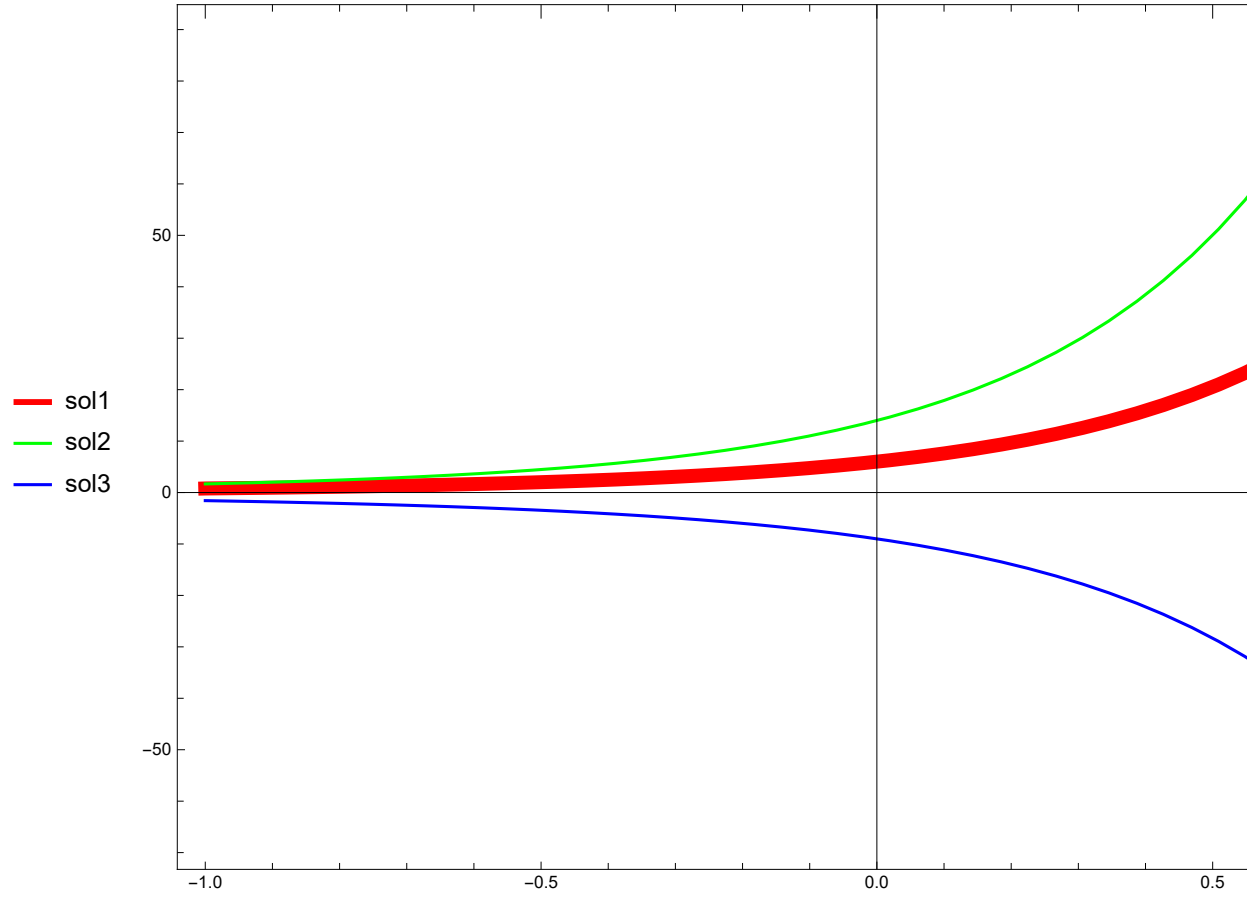
Out[ ]= $\{\{y[x] \to e^{x} C_1 + e^{2x} C_2 + e^{3x} C_3\}\}$

Out[ ]= $e^{x} + 2\, e^{2x} + 3\, e^{3x}$

Out[ ]= $2\, e^{x} + 4\, e^{2x} + 8\, e^{3x}$

Out[ ]= $-3\, e^{x} - 2\, e^{2x} - 4\, e^{3x}$

Out[ ]=



Ques 7: y''' + y' = sec x

```
sol = DSolve[y'''[x] + 0 * y''[x] + y'[x] + 0 * y[x] == Sec[x], y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 18}]
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -2, C[2] → 1.0, C[3] → 0}]
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -1, C[2] → 0.5, C[3] → -2}]
Plot[{sol1, sol2, sol3}, {x, -3, 3},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thickness[0.1]}, {Blue, thick}},
 Frame → True, ImageSize → 750, PlotLegends → Placed[{"sol1", "sol2", "sol3"}, Below]]
```

Out[57]= $\left\{\left\{y[x] \to c_3 - x \operatorname{Cos}[x] - c_2 \operatorname{Cos}[x] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] - \operatorname{Sin}\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right] + c_1 \operatorname{Sin}[x] + \operatorname{Log}[\operatorname{Cos}[x]] \operatorname{Sin}[x]\right\}\right\}$

Out[58]= $18 - 2 \operatorname{Cos}[x] - x \operatorname{Cos}[x] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] - \operatorname{Sin}\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right] + \operatorname{Sin}[x] + \operatorname{Log}[\operatorname{Cos}[x]] \operatorname{Sin}[x]$

Out[59]= $-1. \operatorname{Cos}[x] - x \operatorname{Cos}[x] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] - \operatorname{Sin}\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right] - 2 \operatorname{Sin}[x] + \operatorname{Log}[\operatorname{Cos}[x]] \operatorname{Sin}[x]$

Out[60]= $-2 - 0.5 \operatorname{Cos}[x] - x \operatorname{Cos}[x] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] - \operatorname{Sin}\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right] - \operatorname{Sin}[x] + \operatorname{Log}[\operatorname{Cos}[x]] \operatorname{Sin}[x]$

Out[61]=