Practical 6 :: Solution of Cauchy Problem for First Order Partial Differential Equation

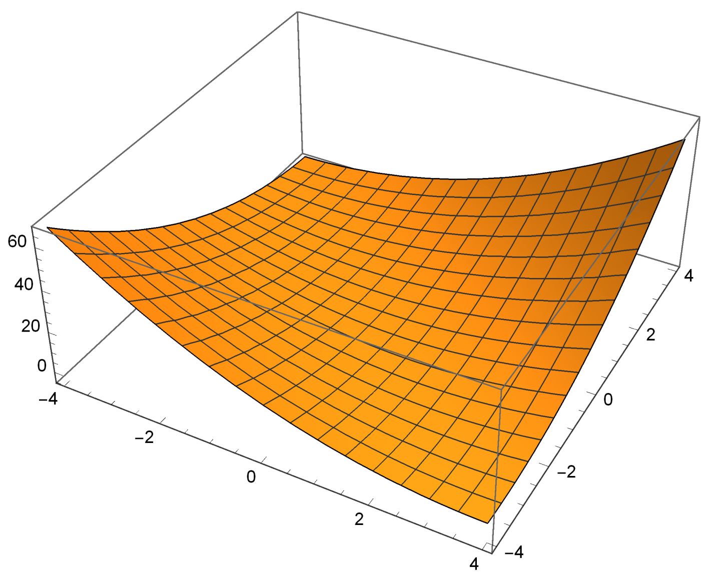
Ques 1 . u_x - u_y= 1, u(x,0)=x^2

In[∘]:=
```
sol = DSolve[{D[u[x, y], x] - D[u[x, y], y] == 1, u[x, 0] == x^2}, u[x, y], {x, y}]
```

Out[∘]= $\{\{u[x, y] \to x^2 - y + 2\,x\,y + y^2\}\}$

In[∘]:=
```
Plot3D[u[x, y] /. sol, {x, -4, 4}, {y, -4, 4}]
```

Out[∘]=



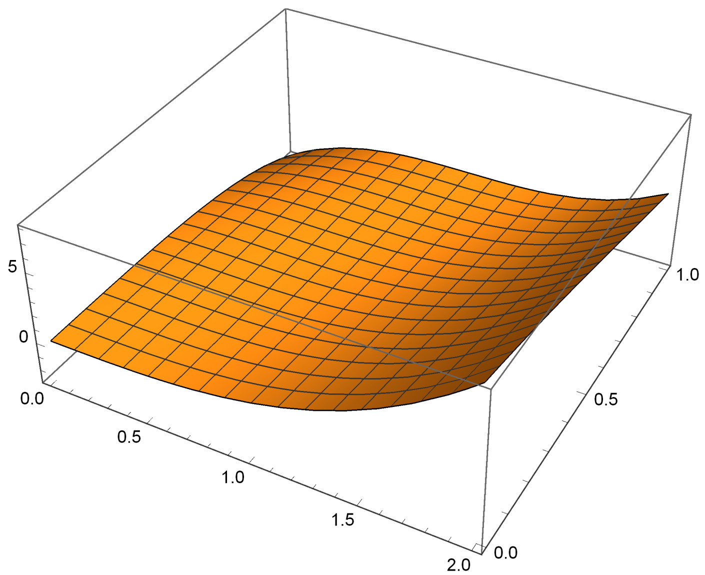
Ques 2 . u_x+u_y = u, u(x,0)=x^3

In[∘]:=
```
sol2 = DSolve[{D[u[x, y], x] + D[u[x, y], y] == u[x, y], u[x, 0] == x^3}, u[x, y], {x, y}]
```

Out[∘]= $\{\{u[x, y] \to -e^{y}\,(-x + y)^3\}\}$

In[ ]:=
```
Plot3D[u[x, y] /. sol2, {x, 0, 2}, {y, 0, 1}]
```

Out[ ]=



Ques 3. yu_x+ xu_y = u , u(0,y)=y^3

In[ ]:=
```
sol11 =
 DSolve[{y D[u[x, y], x] + x D[u[x, y], y] == u [x, y], u[0, y] == y^3}, u[x, y], {x, y}]
```

Out[ ]= $\left\{\left\{u[x, y] \to -\frac{\left(-x^2+y^2\right)^{3/2}\sqrt{1-\frac{x}{\sqrt{y^2}}}}{\sqrt{1+\frac{x}{\sqrt{y^2}}}}\right\}, \left\{u[x, y] \to -\frac{\left(-x^2+y^2\right)^{3/2}\sqrt{1+\frac{x}{\sqrt{y^2}}}}{\sqrt{1-\frac{x}{\sqrt{y^2}}}}\right\}\right\}$

In[ ]:=
```
sol11[[1, 1]]
```

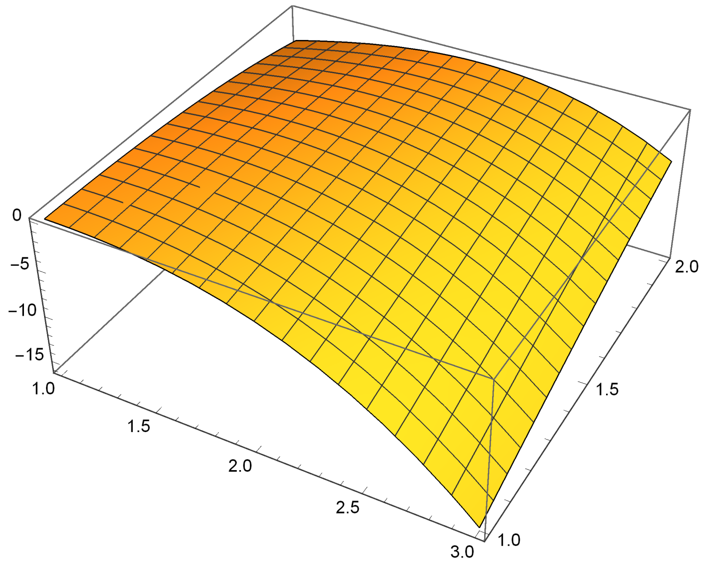
Out[ ]= $u[x, y] \to -\frac{\left(-x^2+y^2\right)^{3/2}\sqrt{1-\frac{x}{\sqrt{y^2}}}}{\sqrt{1+\frac{x}{\sqrt{y^2}}}}$

In[ ]:=
```
sol11[[2, 1]]
```

Out[ ]= $u[x, y] \to -\frac{\left(-x^2+y^2\right)^{3/2}\sqrt{1+\frac{x}{\sqrt{y^2}}}}{\sqrt{1-\frac{x}{\sqrt{y^2}}}}$

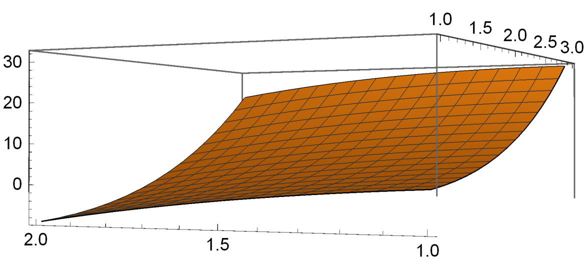
In[ ]:=
```
Plot3D[u[x, y] /. sol11[[1, 1]], {x, 1, 3}, {y, 1, 2}]
```

Out[ ]=



In[ ]:=
```
Plot3D[u[x, y] /. sol11[[2, 1]], {x, 1, 3}, {y, 1, 2}]
```

Out[ ]=



Questions :

Q 4 : $u_x + xu_y = 0, \quad u(0,y) = \sin y$

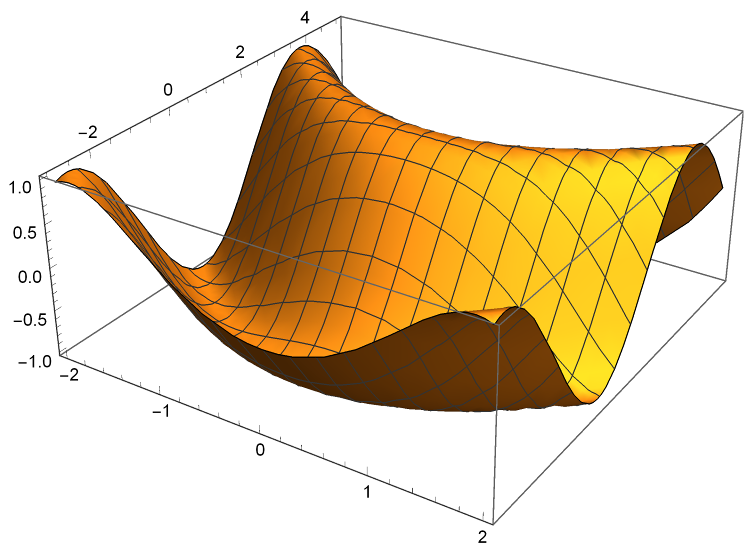
In[ ]:=
```
sol4 = DSolve[{D[u[x, y], x] + x D[u[x, y], y] == 0 , u[0, y] == Sin[y]}, u[x, y], {x, y}]
```

Out[ ]= $\{\{u[x, y] \to \text{Sin}[\frac{1}{2}(-x^2 + 2y)]\}\}$

Here the initial values have been given for PDE, value of c1 is obtained and hence do not need to put any values (like given in questions above)

In[∘]:=
```
Plot3D[u[x, y] /. sol4, {x, -2, 2}, {y, -3, 5}]
```

Out[∘]=



Q 5 : u (x+y) u_x + u (x-y) u_y = $x^2 + y^2$, u=0 , y=2x

In[∘]:=
```
sol5 = DSolve[{ u[x, y] * (x + y) D[u[x, y], x] + u[x, y] * (x - y) D[u[x, y], y] == x^2 + y^2 ,
    u[x, 2 x] == 0}, u[x, y], {x, y}]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[∘]= $\left\{\left\{u[x, y] \to -\sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\}, \left\{u[x, y] \to \sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\}, \left\{u[x, y] \to -\sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\}, \left\{u[x, y] \to \sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\}\right\}$

In[∘]:=
```
sol5[[1, 1]]
```

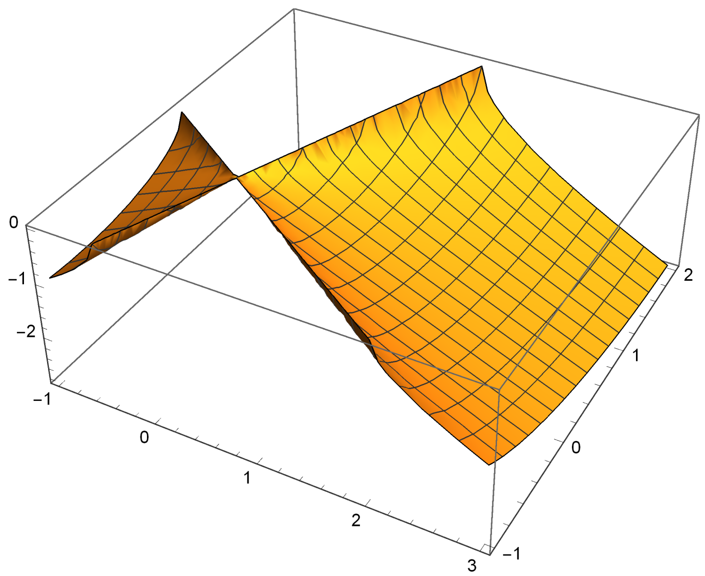
Out[∘]= $u[x, y] \to -\sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}$

In[∘]:=
```
sol5[[2, 1]]
```

Out[∘]= $u[x, y] \to \sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}$

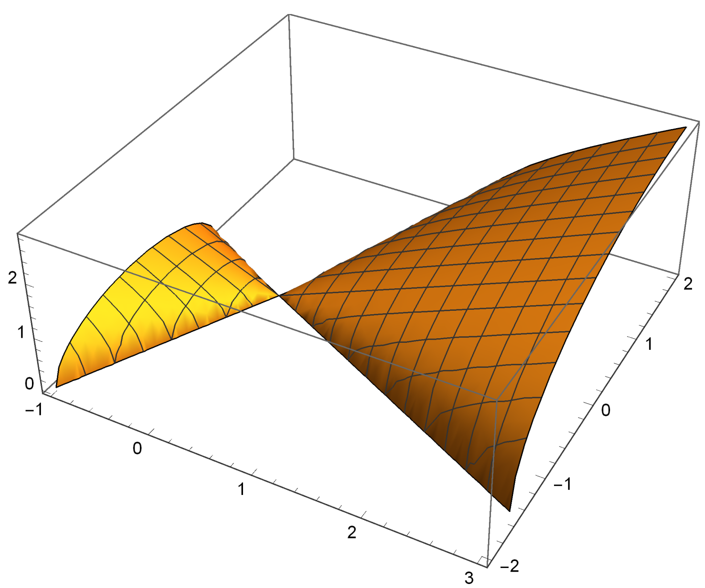
In[ ]:=
```
Plot3D[u[x, y] /. sol5[[1, 1]], {x, -1, 3}, {y, -1, 2}]
```

Out[ ]=



In[ ]:=
```
Plot3D[u[x, y] /. sol5[[2, 1]], {x, -1, 3}, {y, -2, 2}]
```

Out[ ]=



Q 6 : $u_x + x\, u_y = (y - 1/2\, x^{\wedge}2)$, $u(0, y) = e^y$

In[ ]:=
```
sol6 = DSolve[
  {D[u[x, y], x] + x D[u[x, y], y] == y - (1/2) * x^2 , u[0, y] == Exp[y]}, u[x, y], {x, y}]
```

Out[ ]= $\left\{\left\{u[x, y] \to -\frac{1}{2} e^{-\frac{x^2}{2}} \left(-2 e^y + e^{\frac{x^2}{2}} x^3 - 2 e^{\frac{x^2}{2}} x\, y\right)\right\}\right\}$

In[ ]:=
```
Plot3D[u[x, y] /. sol6, {x, -2, 2}, {y, -5, 5}]
```

Out[ ]=