

PRACTICAL 4 : SOLUTION OF DIFFERENTIAL EQUATION BY VARIATION OF PARAMETER

Second Order DE

$$d^2y/dx^2 + p dy/dx + qy = f(x)$$

where p and q are constants, f(x) is a non zero function of x.

General solution of homogenous equation $d^2y/dx^2 + p dy/dx + qy = 0$

Particular solutions of the non- homogenous equation $d^2y/dx^2 + p dy/dx + qy = f(x)$

Ques 1 : $y''[x] + y[x] == 2\sin[x]$

```
In[ ]:= ClearAll
```

```
Out[ ]:= ClearAll
```

```
In[ ]:= gsh = DSolve[y''[x] + y[x] == 0, y[x], x]
gsh1 = y[x] /. gsh
```

```
Out[ ]:= { {y[x] -> c1 Cos[x] + c2 Sin[x] } }
```

```
Out[ ]:= { c1 Cos[x] + c2 Sin[x] }
```

```

In[ ]:= y1 := Cos[x]
        y2 := Sin[x]
        f := 2 * Sin[x]
        w = y1 * D[y2, x] - y2 * D[y1, x]
        w = Simplify[w]

```

```
Out[ ]:= Cos[x]^2 + Sin[x]^2
```

```
Out[ ]:= 1
```

```

In[ ]:= psn = -y1 * Integrate[y2 * (f/w), x] + y2 * Integrate[y1 * (f/w), x]
        psn1 = Simplify[psn]

```

```
Out[ ]:= -1/2 Cos[2 x] Sin[x] - 2 Cos[x] (x/2 - 1/4 Sin[2 x])
```

```
Out[ ]:= 1/2 (-2 x Cos[x] + Sin[x])
```

```
In[ ]:= gsh1 + psn1
```

```
Out[ ]:= {c1 Cos[x] + c2 Sin[x] + 1/2 (-2 x Cos[x] + Sin[x])}
```

```
In[ ]:= ClearAll
```

```
Out[ ]:= ClearAll
```

Ques 2 : $y'' + 3y' + 2y = 30e^{2x}$

psn (particular solution), gsh (general solution)

```
In[ ]:=
```

```

In[ ]:= yc2 = DSolve[y''[x] + 3 * y'[x] + 2 * y[x] == 0, y[x], x]
y1 := Exp[-2 * x]
y2 := Exp[-1 * x]
f := 30 * Exp[2 * x]
w = y1 * D[y2, x] - y2 * D[y1, x]
w = Simplify[w]
yp2 = -y1 * Integrate[y2 * (f/w), x] + y2 * Integrate[y1 * (f/w), x]
yp2 = Simplify[yp2]
yc2 + yp2

```

Out[]:= $\left\{ \left\{ y[x] \rightarrow e^{-2x} c_1 + e^{-x} c_2 \right\} \right\}$

Out[]:= e^{-3x}

Out[]:= e^{-3x}

Out[]:= $\frac{5 e^{2x}}{2}$

Out[]:= $\frac{5 e^{2x}}{2}$

Out[]:= $\left\{ \left\{ \frac{5 e^{2x}}{2} + \left(y[x] \rightarrow e^{-2x} c_1 + e^{-x} c_2 \right) \right\} \right\}$

Ques 3 : $y'' + y = \cot x$

```
In[ ]:= gsh = DSolve[y''[x] + y[x] == 0, y[x], x]
gsh1 = y[x] /. gsh
y1 := Cos[x]
y2 := Sin[x]
f := Cot[x]
w = y1 * D[y2, x] - y2 * D[y1, x]
w = Simplify[w]
psn = -y1 * Integrate[y2 * (f/w), x] + y2 * Integrate[y1 * (f/w), x]
psn1 = Simplify[psn]
gsh1 + psn1
```

```
Out[ ]:= { {y[x] -> c1 Cos[x] + c2 Sin[x]} }
```

```
Out[ ]:= { c1 Cos[x] + c2 Sin[x] }
```

```
Out[ ]:= Cos[x]^2 + Sin[x]^2
```

```
Out[ ]:= 1
```

```
Out[ ]:= -Cos[x] Sin[x] + (Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]]) Sin[x]
```

```
Out[ ]:= (-Log[Cos[x/2]] + Log[Sin[x/2]]) Sin[x]
```

```
Out[ ]:= { c1 Cos[x] + c2 Sin[x] + (-Log[Cos[x/2]] + Log[Sin[x/2]]) Sin[x] }
```

```
In[ ]:= ClearAll
```

```
Out[ ]:= ClearAll
```

Ques 4 : $y'' + 4y' + 5y = e^{-2x} \cdot \sec x$

```

In[ ]:= gsh = DSolve[y''[x] + 4 * y'[x] + 5 * y[x] == 0, y[x], x]
gsh1 = y[x] /. gsh
y1 := Exp[-2 * x] * Cos[x]
y2 := Exp[-2 * x] * Sin[x]
f := Exp[-2 * x] * Sec[x]
w = y1 * D[y2, x] - y2 * D[y1, x]
w = Simplify[w]
psn = -y1 * Integrate[y2 * (f/w), x] + y2 * Integrate[y1 * (f/w), x]
psn1 = Simplify[psn]
gsh1 + psn1

Out[ ]:= {{y[x] -> e^{-2 x} c2 Cos[x] + e^{-2 x} c1 Sin[x]}}

Out[ ]:= {e^{-2 x} c2 Cos[x] + e^{-2 x} c1 Sin[x]}

Out[ ]:= e^{-2 x} Cos[x] (e^{-2 x} Cos[x] - 2 e^{-2 x} Sin[x]) - e^{-2 x} Sin[x] (-2 e^{-2 x} Cos[x] - e^{-2 x} Sin[x])

Out[ ]:= e^{-4 x}

Out[ ]:= e^{-2 x} Cos[x] Log[Cos[x]] + e^{-2 x} x Sin[x]

Out[ ]:= e^{-2 x} (Cos[x] Log[Cos[x]] + x Sin[x])

Out[ ]:= {e^{-2 x} c2 Cos[x] + e^{-2 x} c1 Sin[x] + e^{-2 x} (Cos[x] Log[Cos[x]] + x Sin[x])}

In[ ]:= ClearAll
Out[ ]:= ClearAll

```

Ques 5 : $y'' + 6y' + 9y = e^{-3x} / x^3$

```

In[ ]:= gsh = DSolve[y''[x] + 6 * y'[x] + 9 * y[x] == 0, y[x], x]
gsh1 = y[x] /. gsh

Out[ ]:= {{y[x] -> e^{-3 x} c1 + e^{-3 x} x c2}}

Out[ ]:= {e^{-3 x} c1 + e^{-3 x} x c2}

```

```

In[ ]:= y1 := Exp[-3 * x]
        y2 := Exp[-3 * x] * x
        f := Exp[-3 * x] / x^3
        w = y1 * D[y2, x] - y2 * D[y1, x]
        w = Simplify[w]
        psn = -y1 * Integrate[y2 * (f/w), x] + y2 * Integrate[y1 * (f/w), x]
        psn1 = Simplify[psn]
        gsh1 + psn1
Out[ ]:= 3 e^{-6 x} x + e^{-3 x} (e^{-3 x} - 3 e^{-3 x} x)

Out[ ]:= e^{-6 x}

Out[ ]:= \frac{e^{-3 x}}{2 x}

Out[ ]:= \frac{e^{-3 x}}{2 x}

Out[ ]:= \left\{ \frac{e^{-3 x}}{2 x} + e^{-3 x} c_1 + e^{-3 x} x c_2 \right\}

In[ ]:= ClearAll
Out[ ]:= ClearAll

```

Ques 6 : $y'' - 2y' + y = x e^x \ln x$

```

In[ ]:= gsh = DSolve[y''[x] - 2 * y'[x] + y[x] == 0, y[x], x]
        gsh1 = y[x] /. gsh
Out[ ]:= {{y[x] -> e^x c1 + e^x x c2}}
Out[ ]:= {e^x c1 + e^x x c2}

```

```

In[ ]:= y1 := Exp[x]
        y2 := Exp[x] * x
        f := x * Exp[x] * Log[x]
        w = y1 * D[y2, x] - y2 * D[y1, x]
        w = Simplify[w]
        psn = -y1 * Integrate[y2 * (f/w), x] + y2 * Integrate[y1 * (f/w), x]
        psn1 = Simplify[psn]
        gsh1 + psn1

```

```
Out[ ]:= -e2 x x + ex (ex + ex x)
```

```
Out[ ]:= e2 x
```

```
Out[ ]:= ex x ( - x2/4 + 1/2 x2 Log[x] ) - ex ( - x3/9 + 1/3 x3 Log[x] )
```

```
Out[ ]:= 1/36 ex x3 (-5 + 6 Log[x])
```

```
Out[ ]:= { ex c1 + ex x c2 + 1/36 ex x3 (-5 + 6 Log[x]) }
```

```
In[ ]:= ClearAll
```

```
Out[ ]:= ClearAll
```

Ques 7 : y''+y=1/1+Sin[x]

```

In[ ]:= gsh = DSolve[y''[x] + y[x] == 0, y[x], x]
        gsh1 = y[x] /. gsh

```

```
Out[ ]:= { {y[x] → c1 Cos[x] + c2 Sin[x] } }
```

```
Out[ ]:= { c1 Cos[x] + c2 Sin[x] }
```

```

In[ ]:= y1 := Cos[x]
        y2 := Sin[x]
        f := 1 / (1 + Sin[x])
        w = y1 * D[y2, x] - y2 * D[y1, x]
        w = Simplify[w]
        psn = -y1 * Integrate[y2 * (f/w), x] + y2 * Integrate[y1 * (f/w), x]
        psn1 = Simplify[psn]
        gsh1 + psn1

```

Out[]:= $\cos^2[x] + \sin^2[x]$

Out[]:= 1

Out[]:= $-\cos[x] \left(x - \frac{2 \sin\left[\frac{x}{2}\right]}{\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]} \right) + \log[1 + \sin[x]] \sin[x]$

Out[]:= $-1 + \cos[x] - x \cos[x] + \sin[x] + \log[1 + \sin[x]] \sin[x]$

Out[]:= $\{-1 + \cos[x] - x \cos[x] + c_1 \cos[x] + \sin[x] + c_2 \sin[x] + \log[1 + \sin[x]] \sin[x]\}$

```

In[ ]:= ClearAll

```

Out[]:= ClearAll