PRACTICAL 5 : SIMULTANEOUS DIFFERENTIAL EQUATION

A two dimensional linear system is a system of the form :
dx/dt = a x + by
dy/dt= c x + dy
where a,b,c and d are parameters. This system can be written in matrix form as
X = AX, where
A = (a b and X = (x
c d) y)
The solutions of X = AX can be visualized as trajectories moving on the x- y plane called the phase plane.

Example : Solve the following system of equations :
dx/dt = -3x- y
dy/dt = x-3y

In[∘]:=
```
eq1 = {x'[t] == -3*x[t] - y[t], y'[t] == -3 y[t] - x[t]}
```

Out[∘]=
$$\{x'[t] == -3\,x[t] - y[t],\ y'[t] == -x[t] - 3\,y[t]\}$$

In[∘]:=
```
sol = DSolve[eq1, {y[t], x[t]}, t]
```

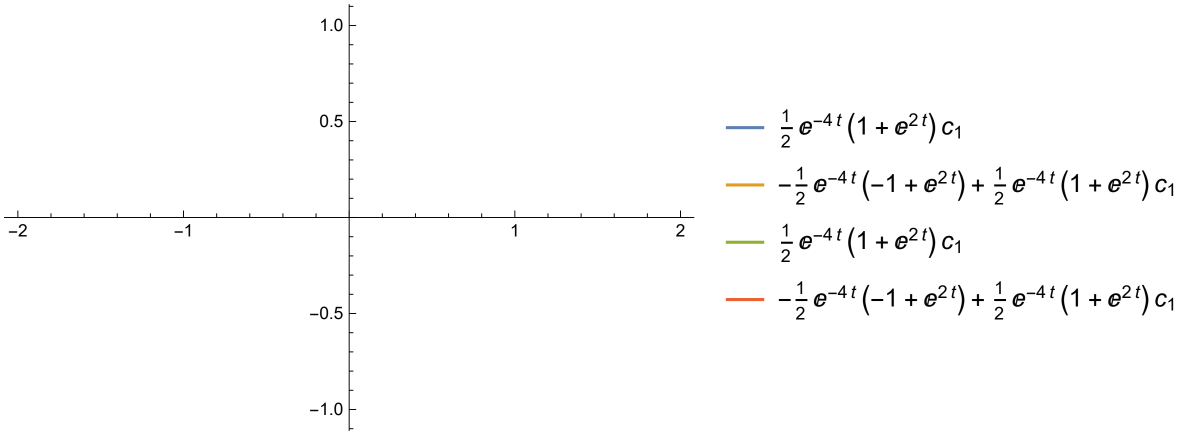
Out[∘]=
$$\left\{\left\{x[t] \to \frac{1}{2} e^{-4t}\left(1+e^{2t}\right) c_1 - \frac{1}{2} e^{-4t}\left(-1+e^{2t}\right) c_2,\ y[t] \to -\frac{1}{2} e^{-4t}\left(-1+e^{2t}\right) c_1 + \frac{1}{2} e^{-4t}\left(1+e^{2t}\right) c_2\right\}\right\}$$

In[∘]:=
```
tabx = Table[x[t] /. sol[[1, 1]] /. {c[1] → i, C[2] → j}, {i, -1, 0}, {j, 0, 1}] // Flatten
```

Out[∘]=
$$\left\{\frac{1}{2} e^{-4t}\left(1+e^{2t}\right) c_1,\ -\frac{1}{2} e^{-4t}\left(-1+e^{2t}\right) + \frac{1}{2} e^{-4t}\left(1+e^{2t}\right) c_1,\ \frac{1}{2} e^{-4t}\left(1+e^{2t}\right) c_1,\ -\frac{1}{2} e^{-4t}\left(-1+e^{2t}\right) + \frac{1}{2} e^{-4t}\left(1+e^{2t}\right) c_1\right\}$$

In[∘]:=
```
Plot[Evaluate[tabx], {t, -2, 2}, PlotLegends → "Expressions"]
```

Out[∘]=

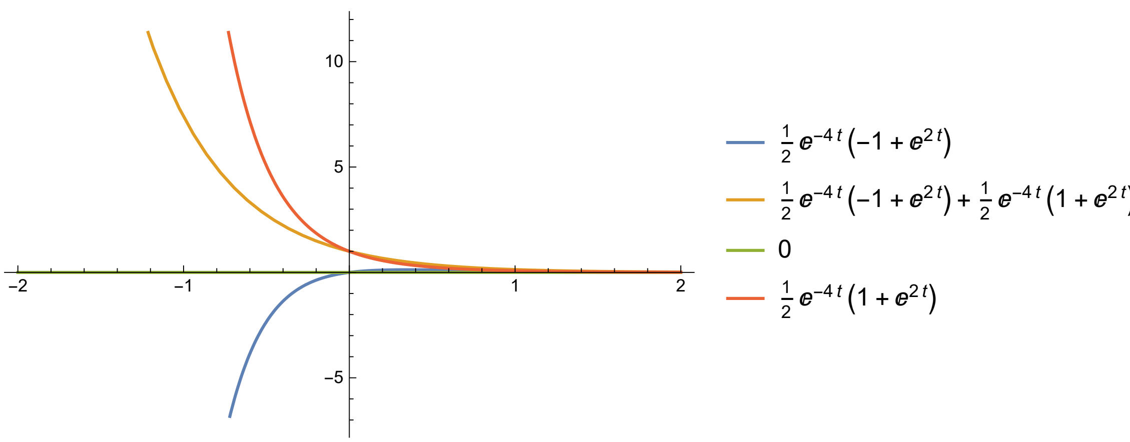


In[∘]:=
```
taby = Table[y[t] /. sol[[1, 2]] /. {C[1] → i , C[2] → j}, {i, -1, 0}, {j, 0, 1}] // Flatten
```

Out[∘]= $\left\{\frac{1}{2} e^{-4t}\left(-1+e^{2t}\right), \frac{1}{2} e^{-4t}\left(-1+e^{2t}\right)+\frac{1}{2} e^{-4t}\left(1+e^{2t}\right), 0, \frac{1}{2} e^{-4t}\left(1+e^{2t}\right)\right\}$

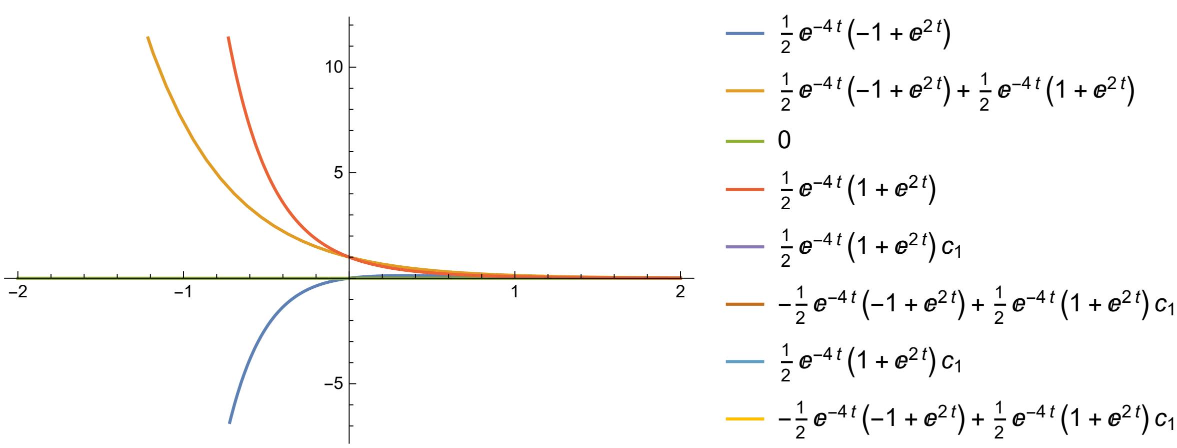
In[∘]:=
```
Plot[Evaluate[taby], {t, -2, 2}, PlotLegends → "Expressions"]
```

Out[∘]=



In[∘]:=
```
Plot[Evaluate[{taby, tabx}], {t, -2, 2}, PlotLegends → "Expressions"]
```

Out[∘]=



Example : Solve the following system of equations :

dx/dt = y
dy/dt = 6x- y
with initial condition x(0)= 1, y(0) = -2

In[ ]:=
```
eq2 = {{x'[t] == y[t], y'[t] == -y[t] + 6 x[t]}, x[0] == 1, y[0] == -2}
```

Out[ ]= $\{\{x'[t] == y[t], y'[t] == 6 x[t] - y[t]\}, x[0] == 1, y[0] == -2\}$

In[ ]:=
```
DSolve[eq2, {x[t], y[t]}, t]
```

Out[ ]= $\left\{\left\{x[t] \to \frac{1}{5} e^{-3t} \left(4 + e^{5t}\right), y[t] \to \frac{2}{5} e^{-3t} \left(-6 + e^{5t}\right)\right\}\right\}$

In[ ]:=
```
{xsol[t_], ysol[t_]} = ExpandAll[{x[t], y[t]} /. Flatten[DSolve[eq2, {x[t], y[t]}, t]]]
```

Out[ ]= $\left\{\frac{4 e^{-3t}}{5} + \frac{e^{2t}}{5}, -\frac{12}{5} e^{-3t} + \frac{2 e^{2t}}{5}\right\}$

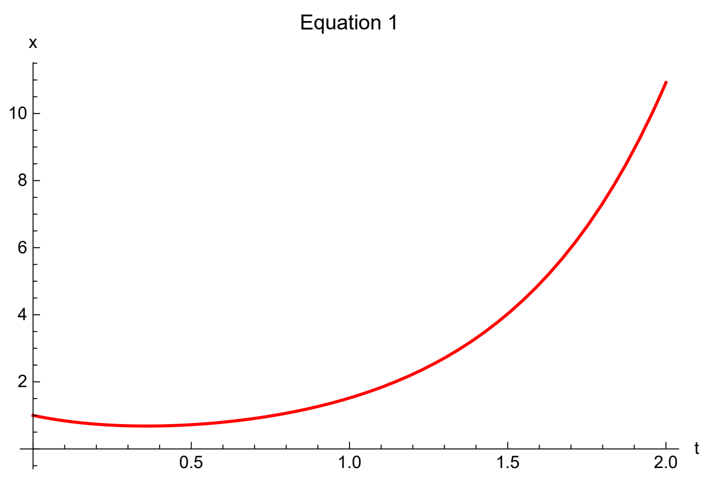
In[ ]:=
```
xsol[t]
```

Out[ ]= $\frac{4 e^{-3t}}{5} + \frac{e^{2t}}{5}$

In[ ]:=
```
ysol[t]
```

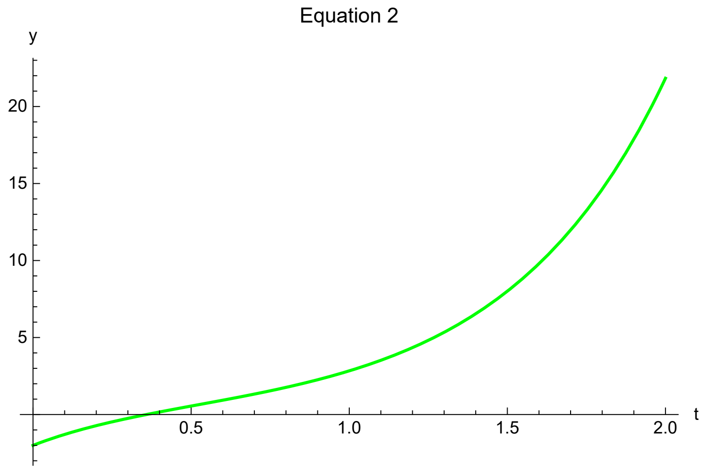
Out[ ]= $-\frac{12}{5} e^{-3t} + \frac{2 e^{2t}}{5}$

In[ ]:=
```
plot1 = Plot[xsol[t], {t, 0, 2},
  AxesLabel → {"t", "x"}, PlotLabel → "Equation 1", PlotStyle → {Red}]
```

Out[ ]=


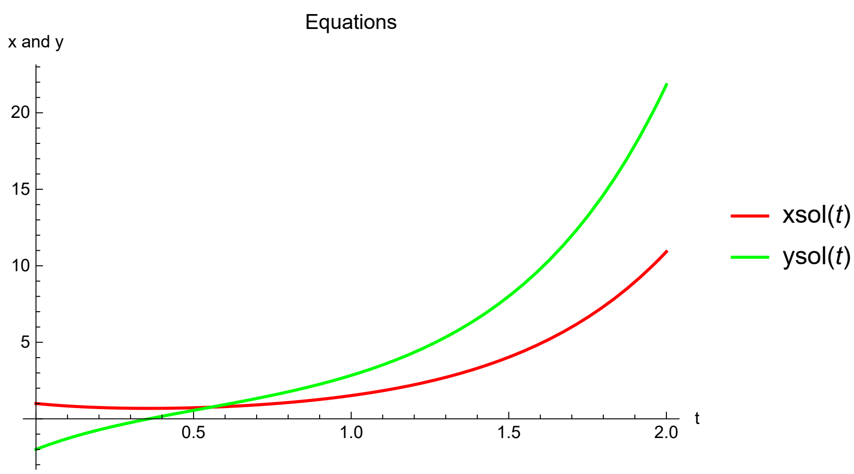
In[∘]:=
```
plo2 = Plot[ysol[t], {t, 0, 2},
  AxesLabel → {"t", "y"}, PlotLabel → "Equation 2", PlotStyle → {Green}]
```

Out[∘]=



In[∘]:=
```
Plot[{xsol[t], ysol[t]}, {t, 0, 2}, AxesLabel → {"t", "x and y" },
 PlotLabel → "Equations", PlotStyle → {Red, Green}, PlotLegends → "Expressions"]
```

Out[∘]=



Solve the following Simultaneous DE and hence plot the solutions :

1. dx/dt = 5x - 2y
dy/dt= 4x-y

In[∘]:=
```
eq1 = {x'[t] == 5 * x[t] - 2 * y[t], y'[t] == -y[t] + 4 * x[t]}
```

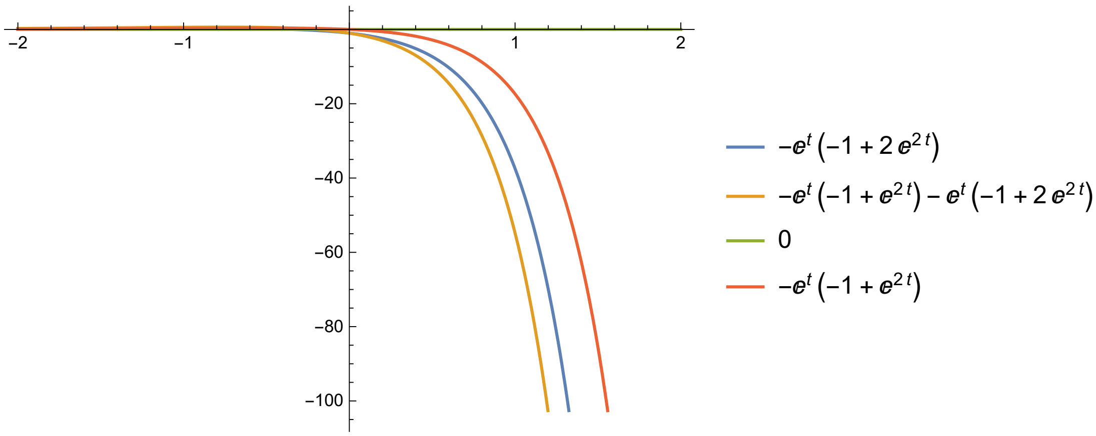
Out[∘]= $\{x'[t] == 5\,x[t] - 2\,y[t],\ y'[t] == 4\,x[t] - y[t]\}$

In[∘]:=
```
sol = DSolve[eq1, {y[t], x[t]}, t]
```

Out[∘]= $\{\{x[t] \to e^{t}\left(-1 + 2\,e^{2t}\right) c_1 - e^{t}\left(-1 + e^{2t}\right) c_2,\ y[t] \to 2\,e^{t}\left(-1 + e^{2t}\right) c_1 - e^{t}\left(-2 + e^{2t}\right) c_2\}\}$

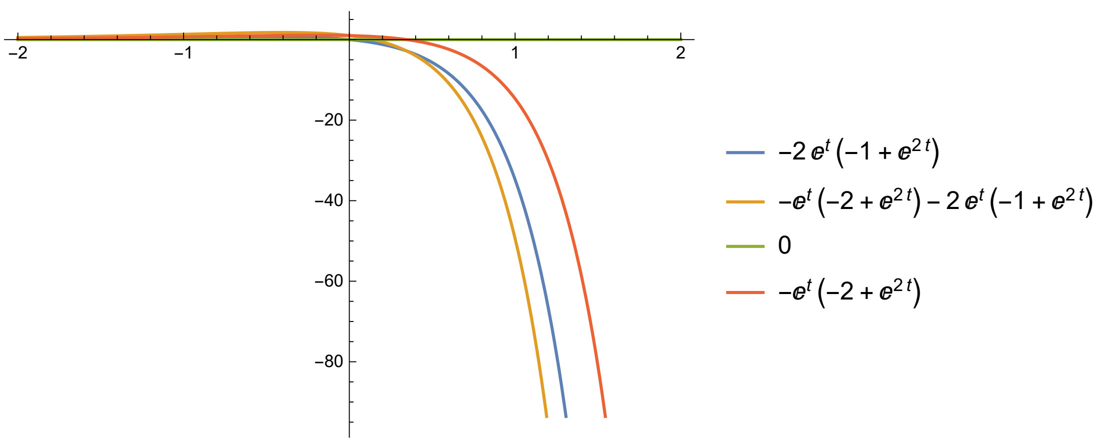
In[ ]:=
```
tabx = Table[x[t] /. sol[[1, 1]] /. {C[1] → i, C[2] → j}, {i, -1, 0}, {j, 0, 1}] // Flatten
```
Out[ ]= $\{-e^{t}(-1+2e^{2t}),\ -e^{t}(-1+e^{2t})-e^{t}(-1+2e^{2t}),\ 0,\ -e^{t}(-1+e^{2t})\}$

In[ ]:=
```
Plot[Evaluate[tabx], {t, -2, 2}, PlotLegends → "Expressions"]
```
Out[ ]=



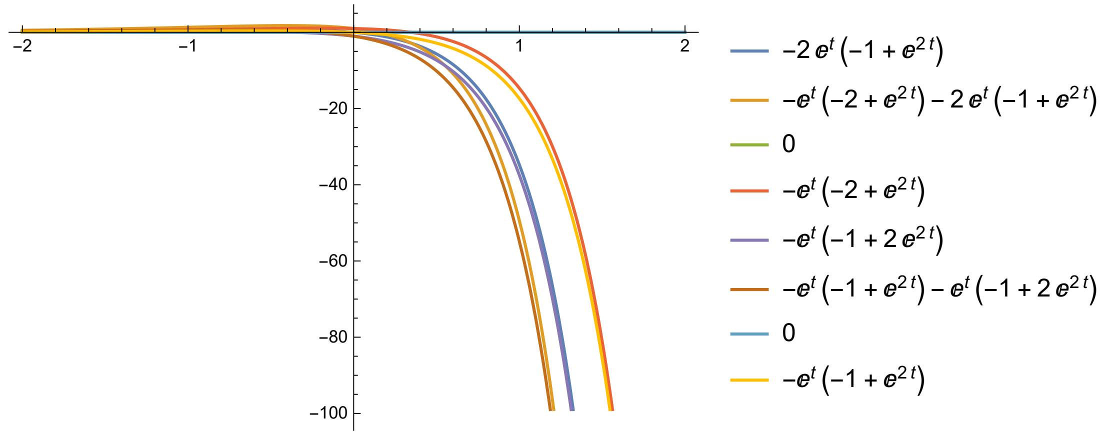
In[ ]:=
```
taby = Table[y[t] /. sol[[1, 2]] /. {C[1] → i, C[2] → j}, {i, -1, 0}, {j, 0, 1}] // Flatten
```
Out[ ]= $\{-2e^{t}(-1+e^{2t}),\ -e^{t}(-2+e^{2t})-2e^{t}(-1+e^{2t}),\ 0,\ -e^{t}(-2+e^{2t})\}$

In[ ]:=
```
Plot[Evaluate[taby], {t, -2, 2}, PlotLegends → "Expressions"]
```
Out[ ]=



In[∘]:=
```
Plot[Evaluate[{taby, tabx}], {t, -2, 2}, PlotLegends → "Expressions"]
```

Out[∘]=



2. dx/dt = 3x - 4y
dy/dt= 2x-y

In[∘]:=
```
eq1 = {x'[t] == 3 * x[t] - 4 * y[t], y'[t] == -y[t] + 2 * x[t]}
```

Out[∘]= $\{x'[t] == 3x[t] - 4y[t],\ y'[t] == 2x[t] - y[t]\}$

In[∘]:=
```
sol = DSolve[eq1, {y[t], x[t]}, t]
```

Out[∘]= $\{\{x[t] \to -2e^t c_2 \mathrm{Sin}[2t] + e^t c_1 (\mathrm{Cos}[2t] + \mathrm{Sin}[2t]),\ y[t] \to e^t c_2 (\mathrm{Cos}[2t] - \mathrm{Sin}[2t]) + e^t c_1 \mathrm{Sin}[2t]\}\}$

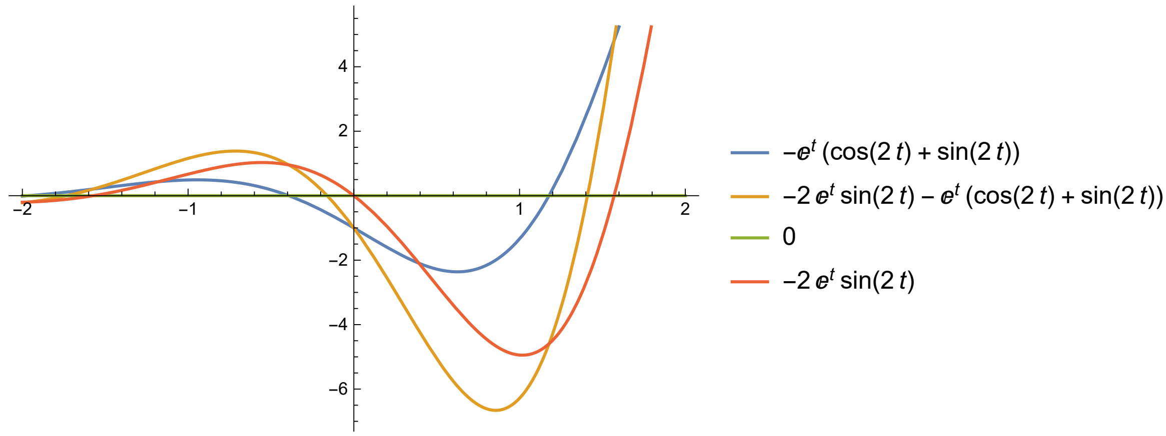
In[∘]:=
```
tabx = Table[x[t] /. sol[[1, 1]] /. {C[1] → i, C[2] → j}, {i, -1, 0}, {j, 0, 1}] // Flatten
taby = Table[y[t] /. sol[[1, 2]] /. {C[1] → i, C[2] → j}, {i, -1, 0}, {j, 0, 1}] // Flatten
```

Out[∘]= $\{-e^t(\mathrm{Cos}[2t] + \mathrm{Sin}[2t]),\ -2e^t \mathrm{Sin}[2t] - e^t(\mathrm{Cos}[2t] + \mathrm{Sin}[2t]),\ 0,\ -2e^t \mathrm{Sin}[2t]\}$

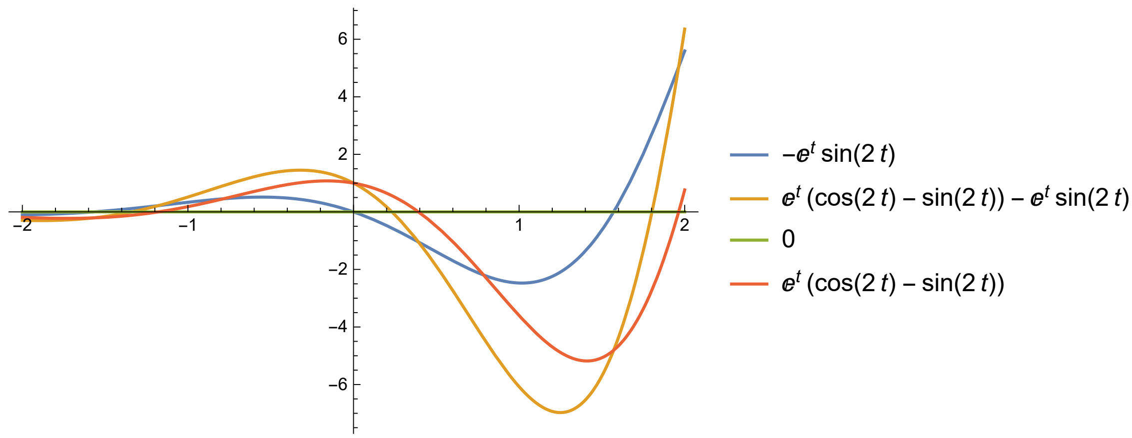
Out[∘]= $\{-e^t \mathrm{Sin}[2t],\ e^t(\mathrm{Cos}[2t] - \mathrm{Sin}[2t]) - e^t \mathrm{Sin}[2t],\ 0,\ e^t(\mathrm{Cos}[2t] - \mathrm{Sin}[2t])\}$

In[ ]:=
```
Plot[Evaluate[tabx], {t, -2, 2}, PlotLegends → "Expressions"]
Plot[Evaluate[taby], {t, -2, 2}, PlotLegends → "Expressions"]
Plot[Evaluate[{taby, tabx}], {t, -2, 2}, PlotLegends → "Expressions"]
```

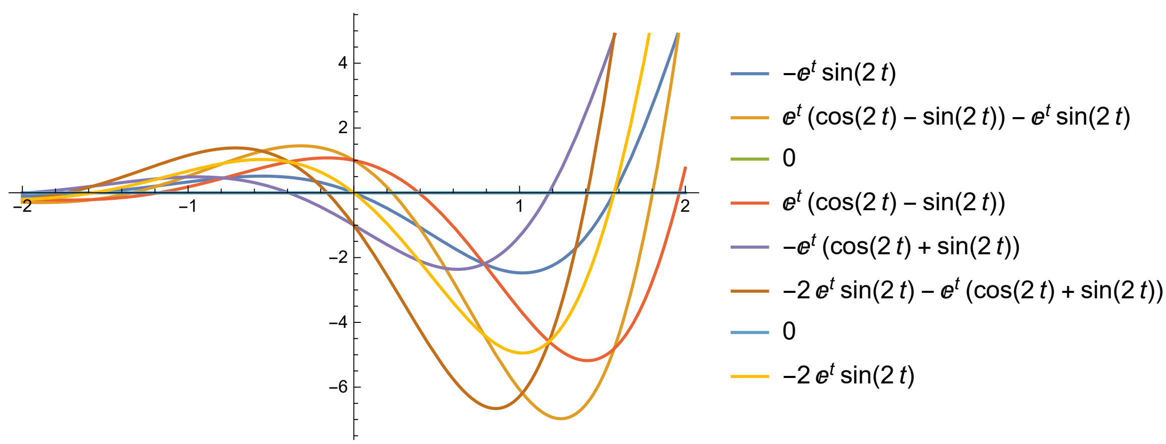
Out[ ]=



Out[ ]=



Out[ ]=



3. dx/dt = 2x + 7y
dy/dt= 3x+2y

with x[0]=9, y[0]=-1

In[35]:=
```
eq2 = {{x'[t] == 7 * y[t] + 2 * x[t], y'[t] == 2 * y[t] + 3 * x[t]}, x[0] == 9, y[0] == -1}
DSolve[eq2, {x[t], y[t]}, t]
{xsol[t_], ysol[t_]} =
 ExpandAll[{x[t], y[t]} /. Flatten[DSolve[eq2, {x[t], y[t]}, t]]]
xsol[t]
ysol[t]
plot1 = Plot[xsol[t], {t, 0, 2},
  AxesLabel → {"t", "x"}, PlotLabel → "Equation 1", PlotStyle → {Red}]
plo2 = Plot[ysol[t], {t, 0, 2}, AxesLabel → {"t", "y"},
  PlotLabel → "Equation 2", PlotStyle → {Green}]
Plot[{xsol[t], ysol[t]}, {t, 0, 3}, AxesLabel → {"t", "x and y" },
 PlotLabel → "Equations", PlotStyle → {Red, Green}, PlotLegends → "Expressions"]
```

Out[35]= $\{\{x'[t] == 2\,x[t] + 7\,y[t],\ y'[t] == 3\,x[t] + 2\,y[t]\},\ x[0] == 9,\ y[0] == -1\}$

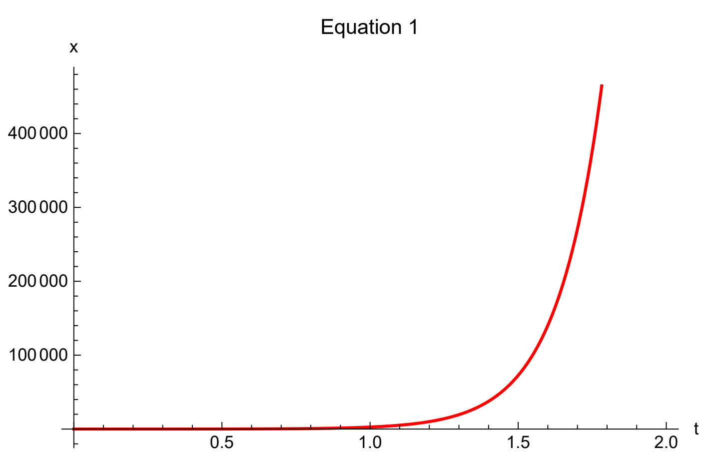
Out[36]= $$\left\{\left\{x[t] \to -\frac{1}{6} e^{2t-\sqrt{21}\,t}\left(-27-\sqrt{21}-27\,e^{2\sqrt{21}\,t}+\sqrt{21}\,e^{2\sqrt{21}\,t}\right),\ y[t] \to \frac{1}{14} e^{2t-\sqrt{21}\,t}\left(-7-9\sqrt{21}-7\,e^{2\sqrt{21}\,t}+9\sqrt{21}\,e^{2\sqrt{21}\,t}\right)\right\}\right\}$$

Out[37]= $$\left\{\frac{9}{2}e^{2t-\sqrt{21}\,t}+\frac{1}{2}\sqrt{\frac{7}{3}}\,e^{2t-\sqrt{21}\,t}+\frac{9}{2}e^{2t+\sqrt{21}\,t}-\frac{1}{2}\sqrt{\frac{7}{3}}\,e^{2t+\sqrt{21}\,t},\ -\frac{1}{2}e^{2t-\sqrt{21}\,t}-\frac{9}{2}\sqrt{\frac{3}{7}}\,e^{2t-\sqrt{21}\,t}-\frac{1}{2}e^{2t+\sqrt{21}\,t}+\frac{9}{2}\sqrt{\frac{3}{7}}\,e^{2t+\sqrt{21}\,t}\right\}$$

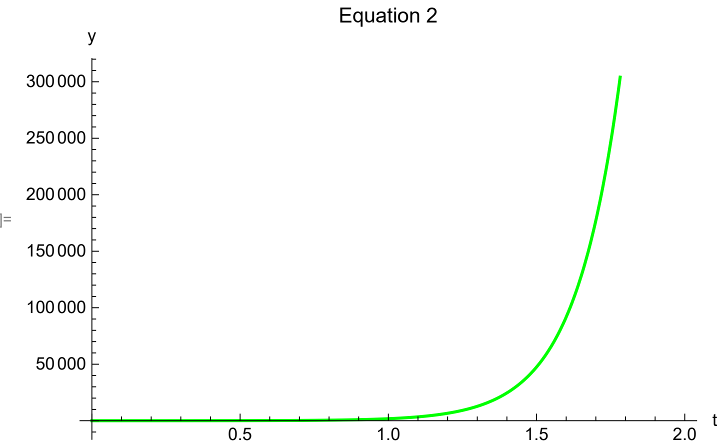
Out[38]= $$\frac{9}{2}e^{2t-\sqrt{21}\,t}+\frac{1}{2}\sqrt{\frac{7}{3}}\,e^{2t-\sqrt{21}\,t}+\frac{9}{2}e^{2t+\sqrt{21}\,t}-\frac{1}{2}\sqrt{\frac{7}{3}}\,e^{2t+\sqrt{21}\,t}$$

Out[39]= $$-\frac{1}{2}e^{2t-\sqrt{21}\,t}-\frac{9}{2}\sqrt{\frac{3}{7}}\,e^{2t-\sqrt{21}\,t}-\frac{1}{2}e^{2t+\sqrt{21}\,t}+\frac{9}{2}\sqrt{\frac{3}{7}}\,e^{2t+\sqrt{21}\,t}$$

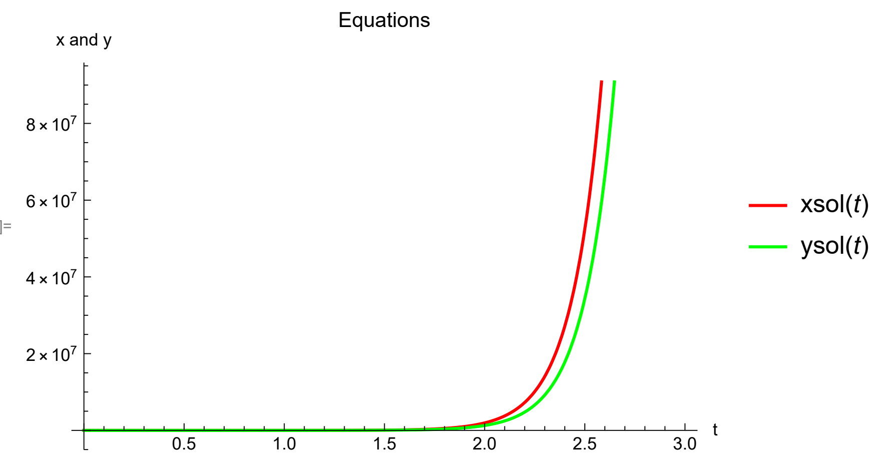
Out[40]=



Out[41]=



Out[42]=



4. dx/dt = 7x - y
dy/dt= 4x + 3y
with initial conditions x[0]=1, y[0]=3

In[43]:=

```
eq2 = {{x'[t] == -y[t] + 7 * x[t], y'[t] == 3 * y[t] + 4 * x[t]}, x[0] == 1, y[0] == 3}
DSolve[eq2, {x[t], y[t]}, t]
{xsol[t_], ysol[t_]} =
 ExpandAll[{x[t], y[t]} /. Flatten[DSolve[eq2, {x[t], y[t]}, t]]]
xsol[t]
ysol[t]
plot1 = Plot[xsol[t], {t, 0, 2},
  AxesLabel → {"t", "x"}, PlotLabel → "Equation 1", PlotStyle → {Red}]
plo2 = Plot[ysol[t], {t, 0, 2}, AxesLabel → {"t", "y"},
  PlotLabel → "Equation 2", PlotStyle → {Blue}]
Plot[{xsol[t], ysol[t]}, {t, 0, 2}, AxesLabel → {"t", "x and y" },
 PlotLabel → "Equations", PlotStyle → {Red, Blue}, PlotLegends → "Expressions"]
```

Out[43]= $\{\{x'[t] == 7\,x[t] - y[t],\ y'[t] == 4\,x[t] + 3\,y[t]\},\ x[0] == 1,\ y[0] == 3\}$

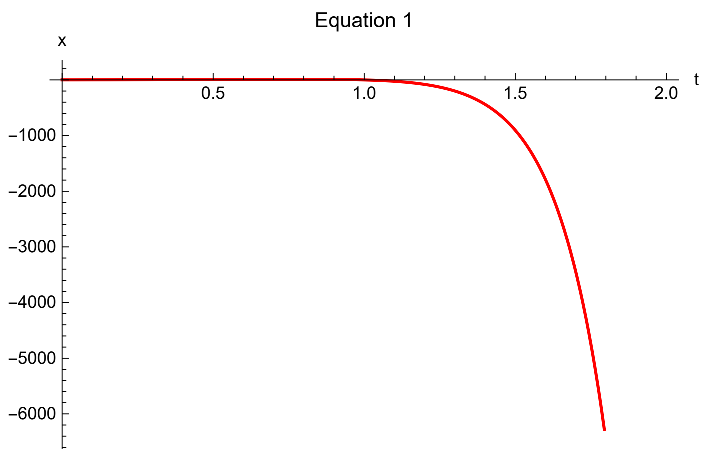
Out[44]= $\{\{x[t] \to -e^{5t}(-1+t),\ y[t] \to -e^{5t}(-3+2t)\}\}$

Out[45]= $\{e^{5t} - e^{5t}\,t,\ 3\,e^{5t} - 2\,e^{5t}\,t\}$

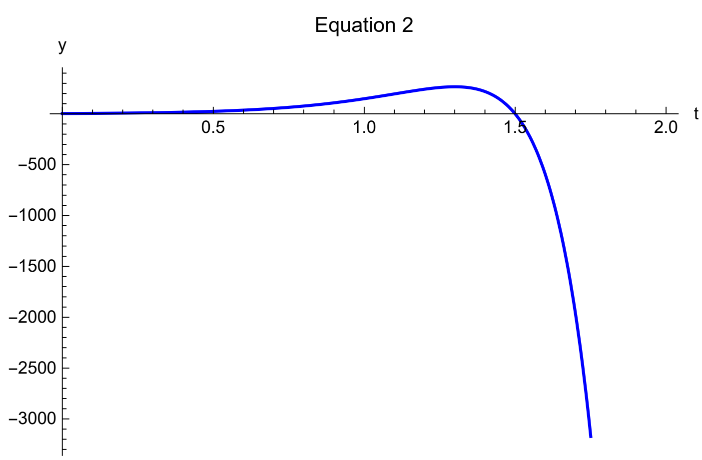
Out[46]= $e^{5t} - e^{5t}\,t$

Out[47]= $3\,e^{5t} - 2\,e^{5t}\,t$

Out[48]=



Out[49]=



Out[50]=