Practical 7 :: Plotting the Characteristic for the First Order PDE

A general quasi-linear PDE is :

a (x,y,u) u_x + b(x,y,u) u_y - c(x,y,u) = 0

The characteristic equations of the quasi-linear equation :

dx/dt = a (x,y,u)

dy/dt = b (x,y,u)

du/dt = c (x,y,u)

Equivalently characteristic equations of the above equation in the non - parametric form are :

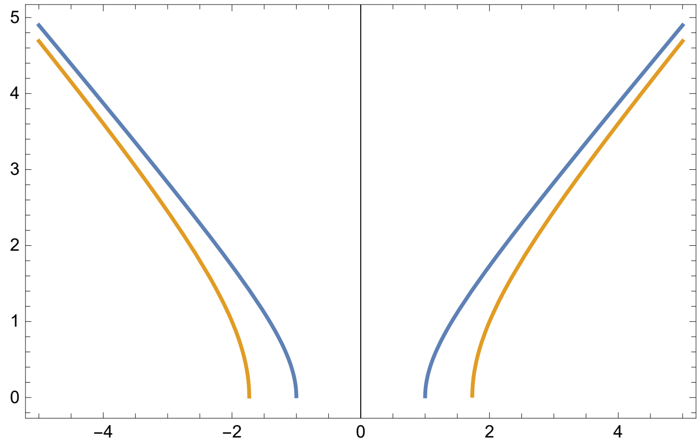
dx/a = dy/b = du/c

Integrate the above and plot the solutions

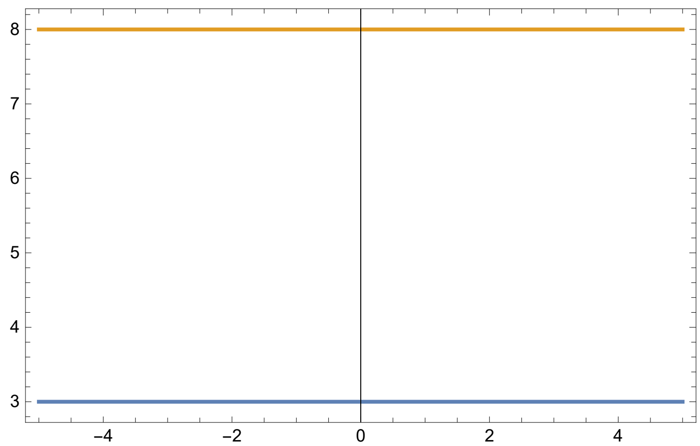
Ques 1. y u_x + x u_y = 0

The characteristic system is given by dx/y = dy/x = du=0 and the characteristic equations are given by $x^2 + y^2 = c_1$ and $u = c_2$. Taking c_1 = 1 and $u = c_2$. Taking c_1 = 1 and 3 and c_2 = 3 and 8.

In[∘]:=
```
Plot[{Sqrt[x^2 - 1], Sqrt[x^2 - 3]}, {x, -5, 5}, PlotStyle → Thick, Frame → True]
```

Out[∘]=


In[∘]:=
```
Plot[{3, 8}, {x, -5, 5}, PlotStyle → Thick, Frame → True]
```

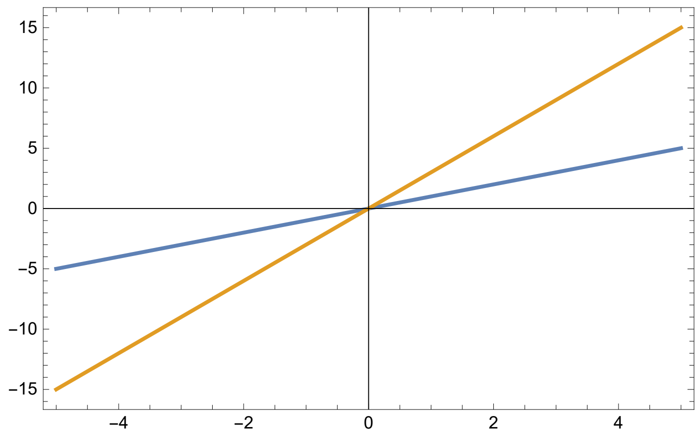
Out[∘]=


Ques 2. $x\, u_x + y\, u_y = u$

The characteristic system is given by dx/x = dy/y = du/u and the characteristic equations are given by y/x = c_1 **and** $u/x = c_2$**. Taking** $c_1 = 1$ **and 3 and** $c_2 = 3$ **and 8.**

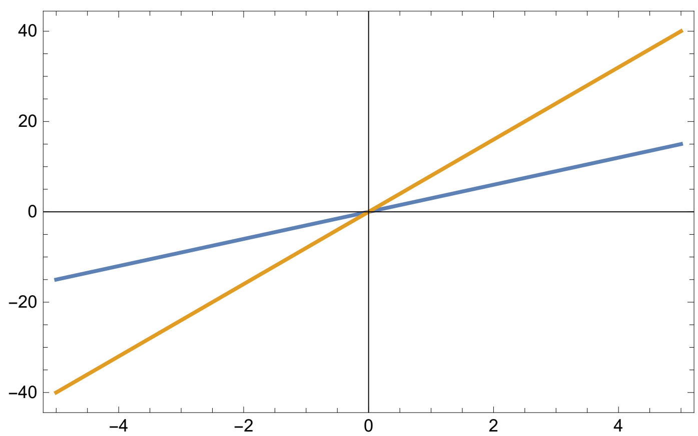
In[ ]:=
```
Plot[{x, 3 x}, {x, -5, 5}, PlotStyle → Thick, Frame → True]
```

Out[ ]=



In[ ]:=
```
Plot[{3 x, 8 x}, {x, -5, 5}, PlotStyle → Thick, Frame → True]
```

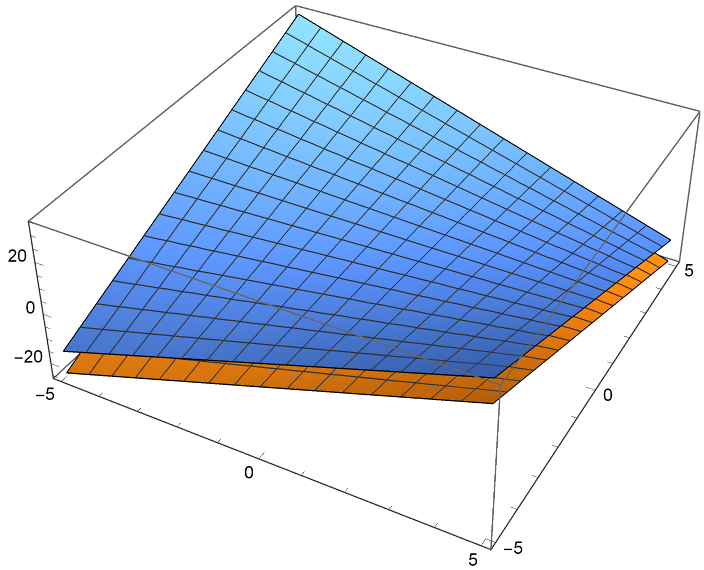
Out[ ]=



Ques 3. **(y+xu) u_x - (x+uy) u_y = x^2 - y^2**
The characteristic system is given by dx/y+ux = dy/-(x+uy) = du/x^2 $-y^2$ and the characteristic equations are given by xy+u = c_1 and $x^2 + y^2 - u^2 = c_2$. Taking $c_1 = 0$ and 9 and $c_2 = 0$ and 10.

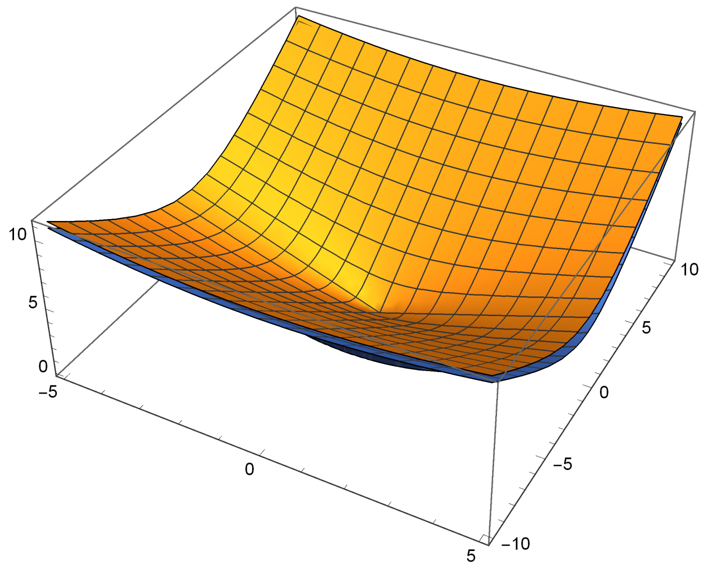
In[∘]:=
```
Plot3D[{-x*y, -x*y+9}, {x, -5, 5}, {y, -5, 5}, PlotStyle → Thick]
```

Out[∘]=



In[∘]:=
```
Plot3D[{Sqrt[x^2 + y^2], Sqrt[x^2 + y^2 - 10]}, {x, -5, 5}, {y, -10, 10}, PlotStyle → Thick]
```

Out[∘]=

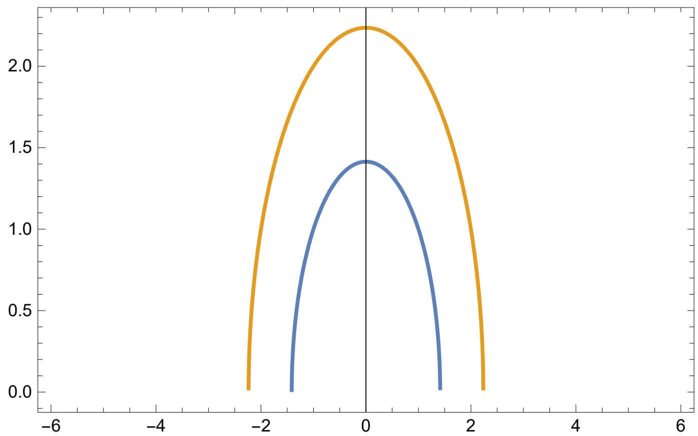


Ques 4. $x\, u_x + y\, u_y = u$

The characteristic system is given by dx/x = dy/y = du/u and the characteristic equations are given by y/x = c_1 and $u/x = c_2$. Taking $c_1 = 1$ and 3 and $c_2 = 3$ and 8.

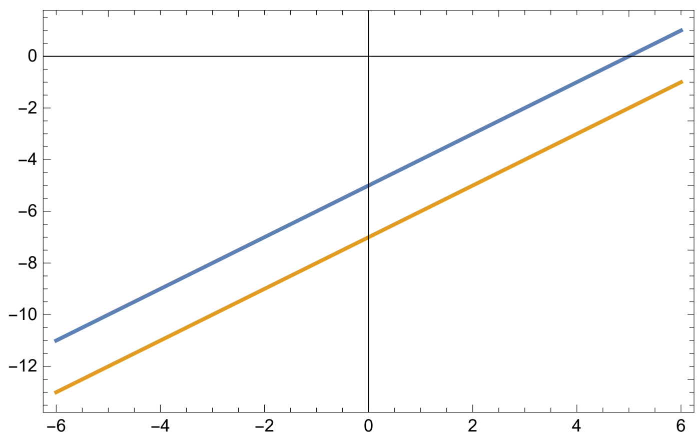
In[76]:=
```
Plot[{Sqrt[2 - x^2], Sqrt[5 - x^2]}, {x, -6, 6}, PlotStyle → Thick, Frame → True]
```

Out[76]=



In[77]:=
```
Plot[{y - 5, y - 7}, {y, -6, 6}, PlotStyle → Thick, Frame → True]
```

Out[77]=



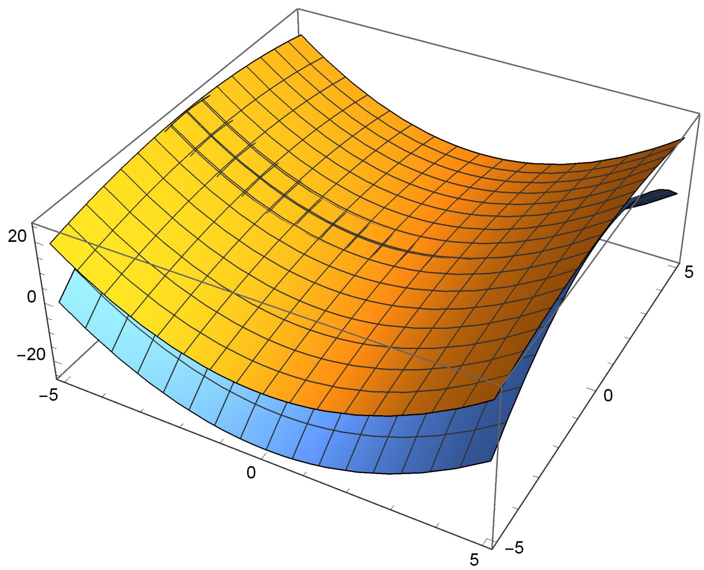
Ques 5 : $u(x+y)u_x + u(x-y)u_y = x^2+y^2$

The characteristic system is given by dx/u(x+y) = dy/u(x-y) = du/(x^2+y^2) and the characteristic equations are given by

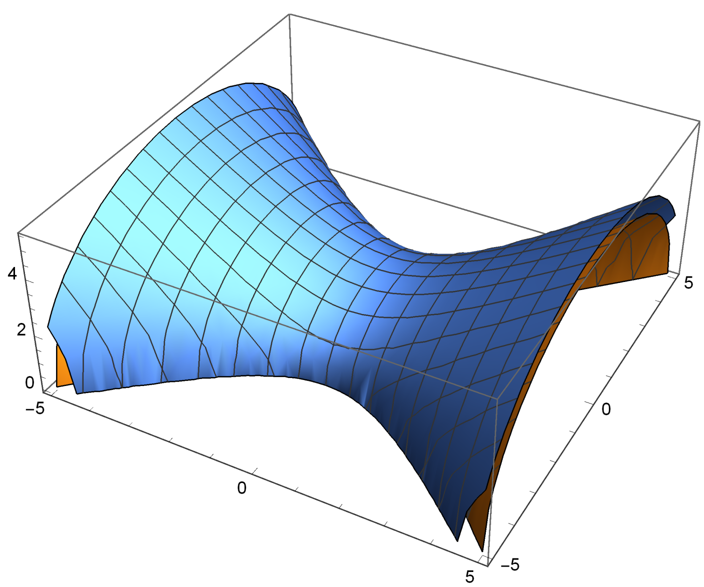
x^2/2 - y^2/2 - u = c1 and y^2 - u^2 - x^2 = c2.

Taking c1 = 2 and 5 and c2 = 5 and 7.

In[78]:=
```
Plot3D[{x^2 - y^2/4, x^2 - y^2 - 2/4}, {x, -5, 5}, {y, -5, 5}, PlotStyle → Thick]
```

Out[78]=


In[79]:=
```
Plot3D[{Sqrt[x^2 - y^2], Sqrt[x^2 - y^2 + 5]}, {x, -5, 5}, {y, -5, 5}, PlotStyle → Thick]
```

Out[79]=


Ques 6 : $u_x - u_y = 1$

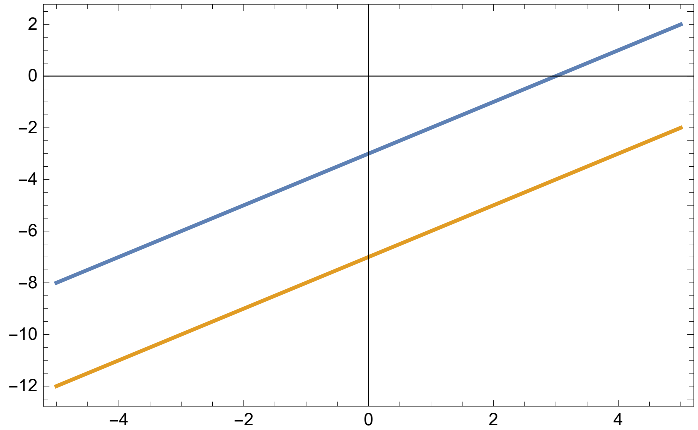
The characteristic system is given by dx/1 = dy/(-1) = du/1 and the characteristic equations are given by

x-y = c1 and -y-u = c2.

Taking c1 = 2 and 5 and c2 = 5 and 7.

In[80]:=
```
Plot[{x - 3, x - 7}, {x, -5, 5}, PlotStyle → Thick, Frame → True]
```

Out[80]=



In[81]:=
```
Plot[{1 - y, 5 - y}, {y, -5, 5}, PlotStyle → Thick, Frame → True]
```

Out[81]=