PRACTICAL 3 : Plotting of third order solution family of differential equations

---

Ques 1 : Solve third order DE dE and plot its any three solutions

Real and Distinct Roots

In[66]:=
```
sol = DSolve[y'''[x] - 5 * y''[x] + 8 * y'[x] - 4 y[x] == 0, y[x], x]
```

Out[66]=
$$\left\{\left\{y[x] \to e^{x} C_1 + e^{2x} C_2 + e^{2x} x C_3\right\}\right\}$$

In[67]:=
```
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 2/3}]
```

Out[67]=
$$e^{x} + 2\, e^{2x} + \frac{2}{3} e^{2x} x$$

In[68]:=
```
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 0.5, C[2] → 2, C[3] → 3}]
```

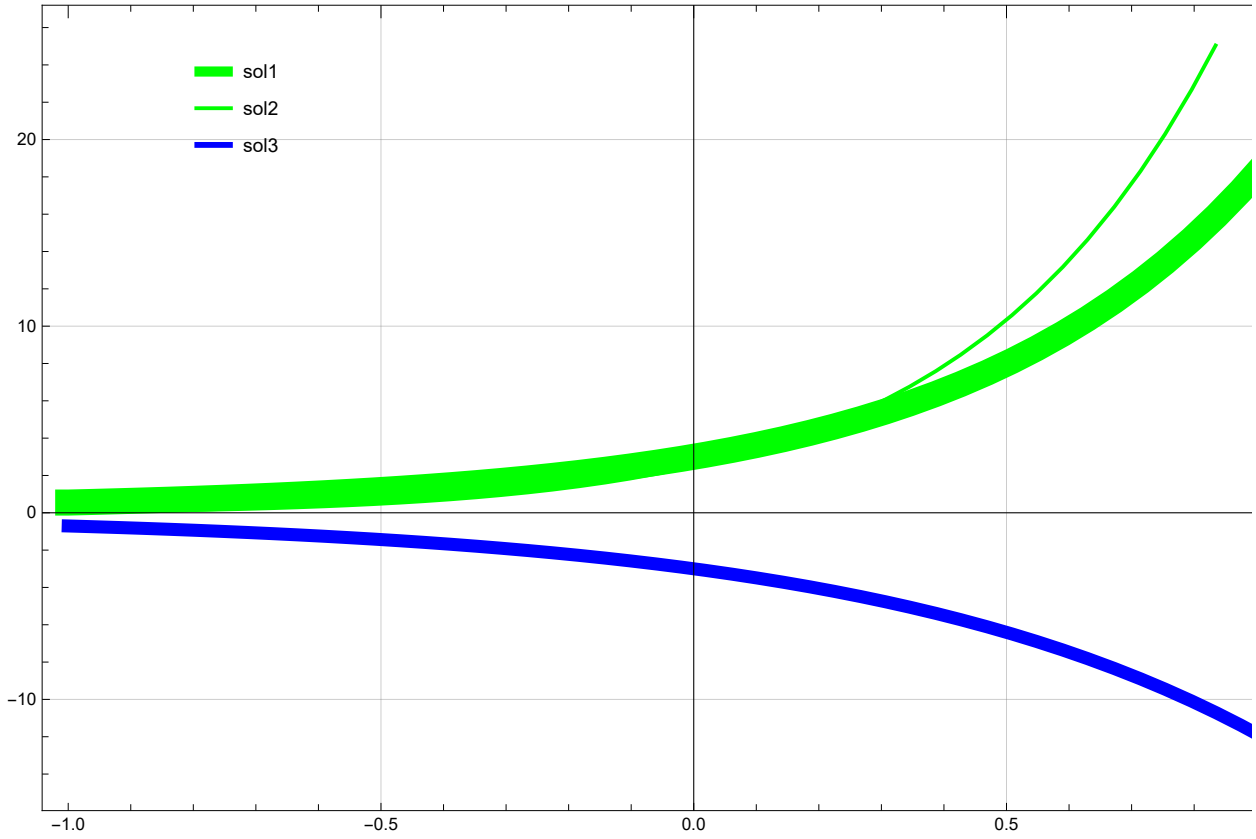
Out[68]=
$$0.5\, e^{x} + 2\, e^{2x} + 3\, e^{2x} x$$

In[69]:=
```
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -1, C[2] → -2, C[3] → 0.5}]
```

Out[69]=
$$-e^{x} - 2\, e^{2x} + 0.5\, e^{2x} x$$

In[70]:=
```
Plot[{sol1, sol2, sol3} , {x, -1, 1}, ImageSize → 700,
 PlotStyle → {{Green, Thickness[0.02]}, {Green, Thick}, {Blue, Thickness[0.01]}},
 Frame → True, AxesOrigin → {0, 0}, GridLines → Automatic,
 PlotLegends → Placed[LineLegend["Expressions",
    LegendLayout → "Column" , LegendFunction → "Frame"], {0.15, 0.87}]]
```

Out[70]=



Ques 2 :

In[71]:=
```
sol = DSolve[y'''[x] + 3 * y''[x] - 25 y'[x] + 21 * y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 3}]
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 2, C[2] → 4, C[3] → 8}]
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -3, C[2] → -2, C[3] → -4}]
Plot[{sol1, sol2, sol3}, {x, -1, 1},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thicker}, {Blue, thick}},
 Frame → True, ImageSize → 750, PlotLegends → Placed[{"sol1", "sol2", "sol3"}, Left]]
```

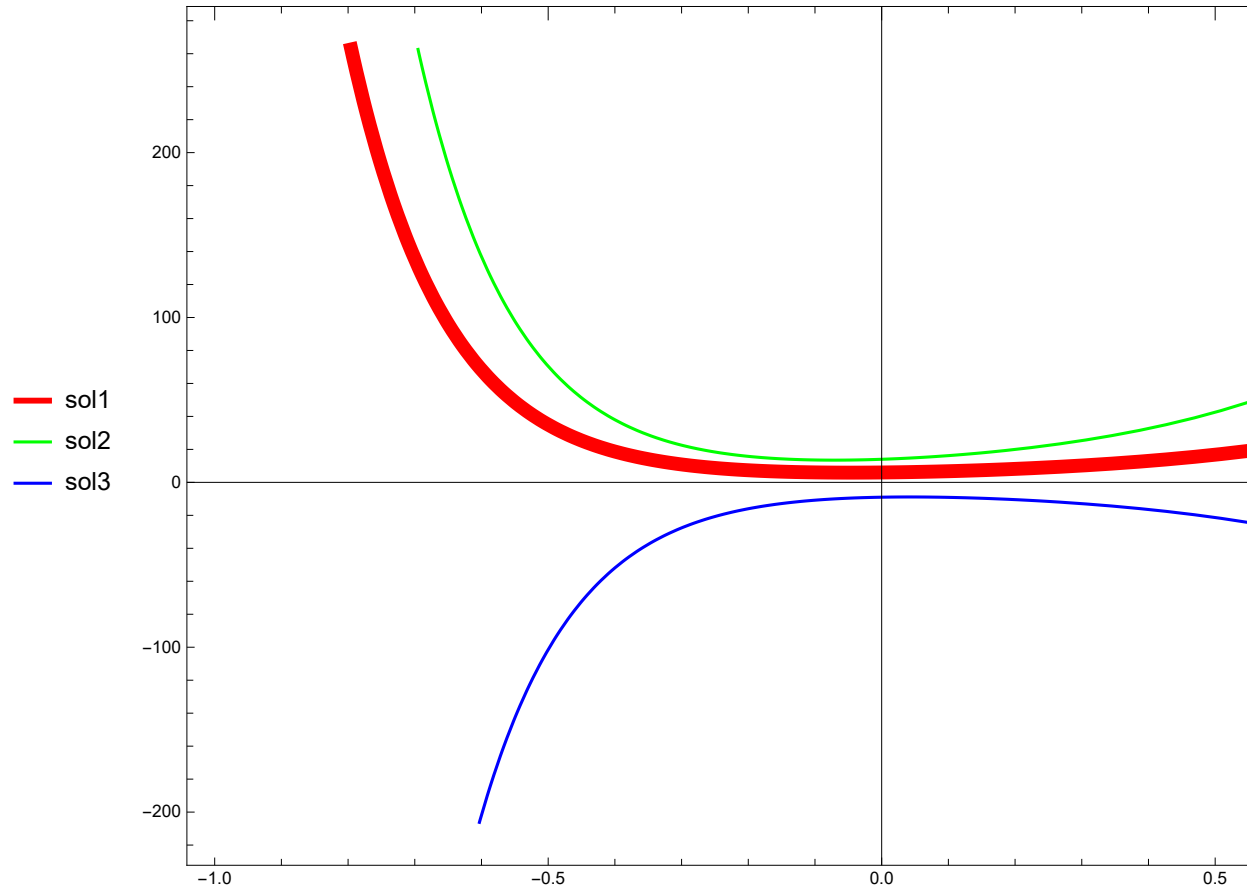
Out[71]= $\left\{\left\{y[x] \to e^{-7x} C_1 + e^{x} C_2 + e^{3x} C_3\right\}\right\}$

Out[72]= $e^{-7x} + 2\, e^{x} + 3\, e^{3x}$

Out[73]= $2\, e^{-7x} + 4\, e^{x} + 8\, e^{3x}$

Out[74]= $-3\, e^{-7x} - 2\, e^{x} - 4\, e^{3x}$

Out[75]=



Ques 3 :

Ques 4 : Solve third order differential Equation and plot its any three solutions

In[76]:=
```
sol = DSolve[y'''[x] - 13 * y''[x] + 19 * y'[x] + 33 * y[x] == Cos[2 x] , y[x] , x]
```

Out[76]=
$$\left\{\left\{y[x] \to e^{-x} c_1 + e^{3 x} c_2 + e^{11 x} c_3 + \frac{17 \cos[2 x] + 6 \sin[2 x]}{1625}\right\}\right\}$$

In[77]:=
```
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 3}]
```

Out[77]=
$$e^{-x} + 2 e^{3 x} + 3 e^{11 x} + \frac{17 \cos[2 x] + 6 \sin[2 x]}{1625}$$

In[78]:=
```
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 2, C[2] → 4, C[3] → 8}]
```

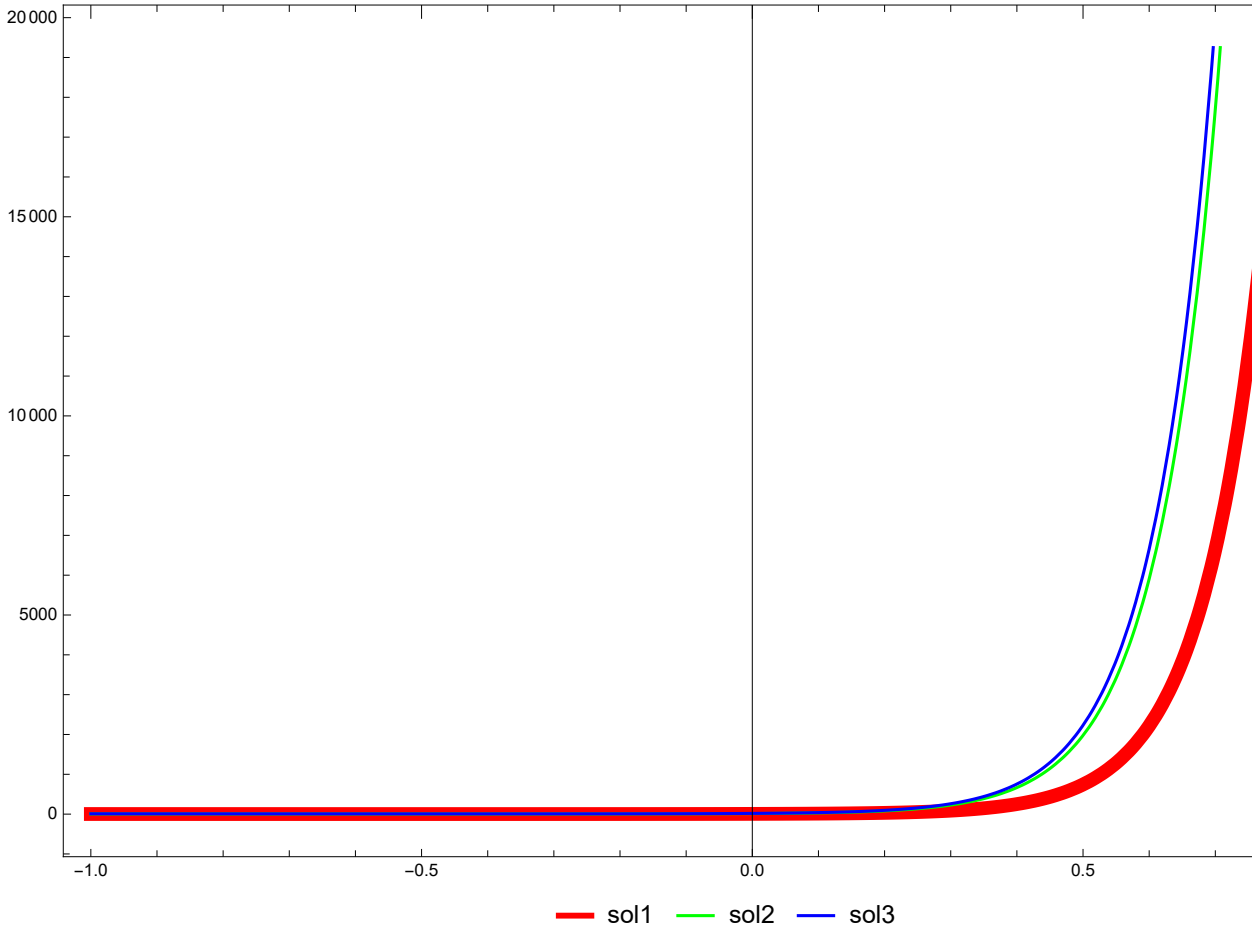
Out[78]=
$$2 e^{-x} + 4 e^{3 x} + 8 e^{11 x} + \frac{17 \cos[2 x] + 6 \sin[2 x]}{1625}$$

In[79]:=
```
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 3, C[2] → 6, C[3] → 9}]
```

Out[79]=
$$3 e^{-x} + 6 e^{3 x} + 9 e^{11 x} + \frac{17 \cos[2 x] + 6 \sin[2 x]}{1625}$$

In[80]:=
```
Plot[{sol1, sol2, sol3}, {x, -1, 1},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thicker}, {Blue, thick}},
 Frame → True, ImageSize → 750, PlotLegends → Placed[{"sol1", "sol2", "sol3"}, Below]]
```

Out[80]=



Solve the following differential equations

Q 1 . y''' -6y''+5y' + 12y = 0

In[81]:=
```
sol = DSolve[y'''[x] - 6 * y''[x] + 5 y'[x] + 12 * y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 3}]
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 2, C[2] → 4, C[3] → 8}]
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1/3, C[2] → 1/2, C[3] → 2/3}]
Plot[{sol1, sol2, sol3}, {x, -1, 1},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thicker}, {Blue, thick}},
 Frame → True, ImageSize → 750, PlotLegends → Placed[{"sol1", "sol2", "sol3"}, Top]]
```

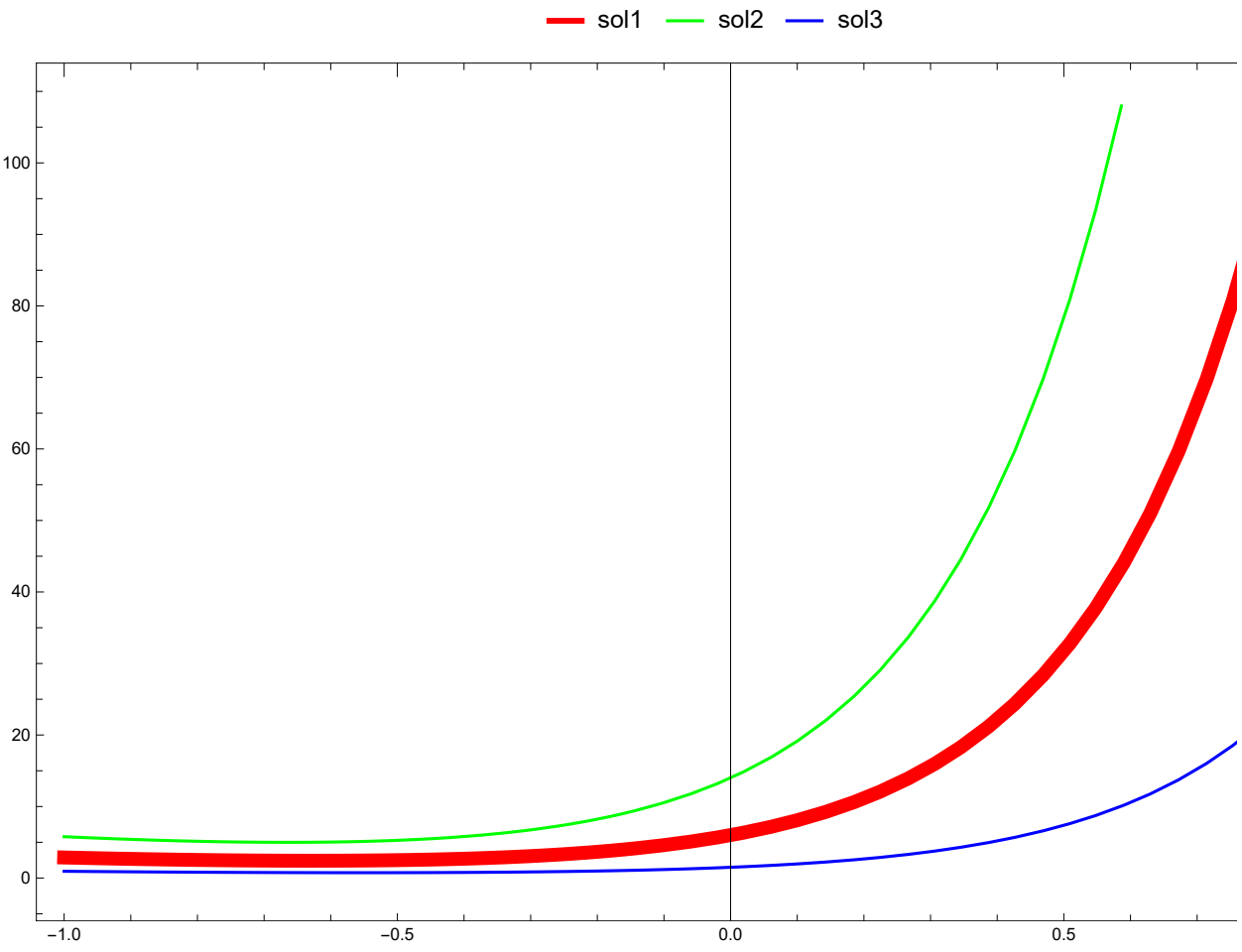
Out[81]= $\left\{\left\{y[x] \to e^{-x} c_1 + e^{3x} c_2 + e^{4x} c_3\right\}\right\}$

Out[82]= $e^{-x} + 2\, e^{3x} + 3\, e^{4x}$

Out[83]= $2\, e^{-x} + 4\, e^{3x} + 8\, e^{4x}$

Out[84]= $\frac{e^{-x}}{3} + \frac{e^{3x}}{2} + \frac{2\, e^{4x}}{3}$

Out[85]=



In[86]:=

Q2 . y'''-6y''+11y' -6y=0, y(0)=0, y'(0)=0, y''(0)=2

In[87]:=
```
sol = DSolve[y'''[x] - 6 * y''[x] + 11 y'[x] - 6 * y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 3}]
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 2, C[2] → 4, C[3] → 8}]
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -3, C[2] → -2, C[3] → -4}]
Plot[{sol1, sol2, sol3}, {x, -1, 1},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thicker}, {Blue, thick}},
 Frame → True, ImageSize → 750, PlotLegends → Placed[{"sol1", "sol2", "sol3"}, Left]]
```

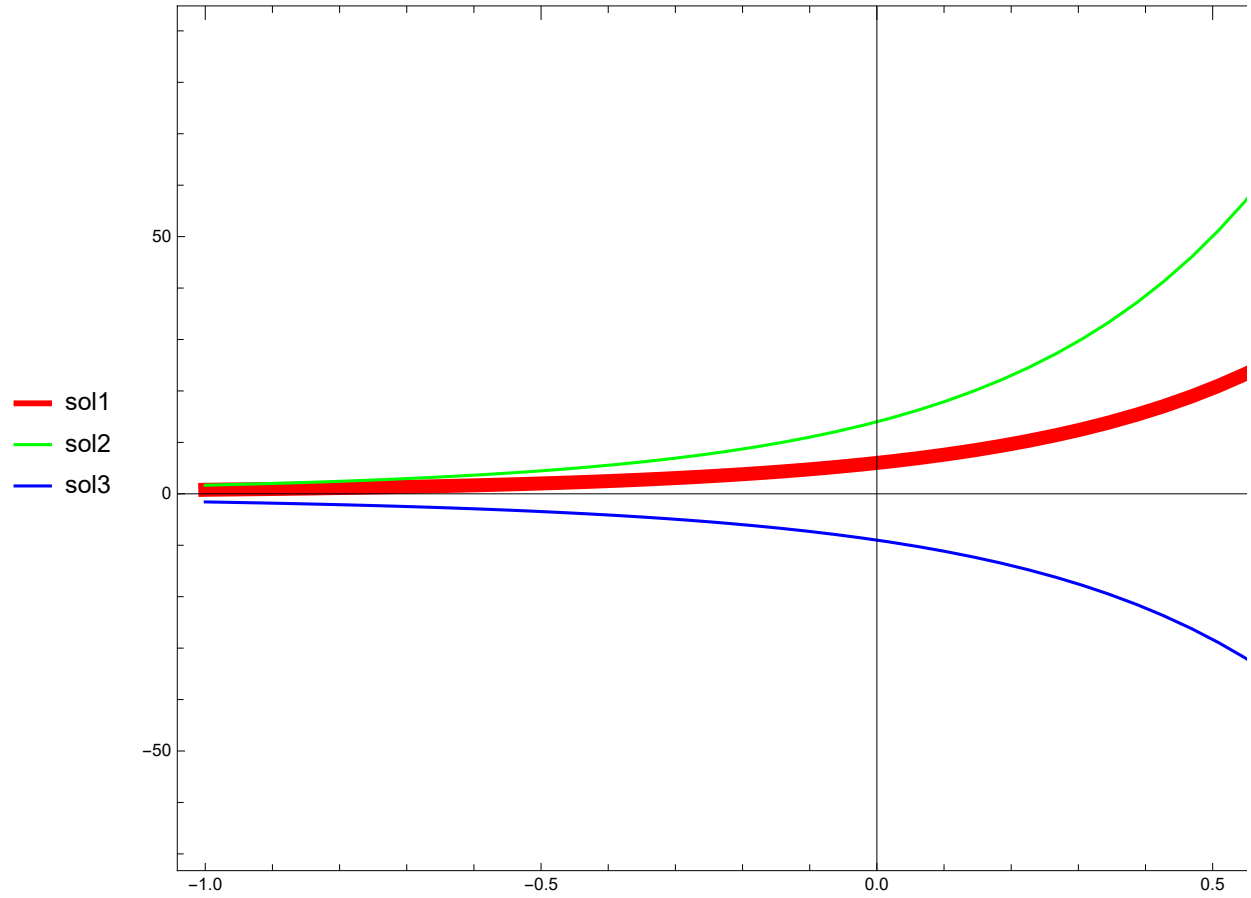
Out[87]= $\left\{\left\{y[x] \to e^{x} C_1 + e^{2x} C_2 + e^{3x} C_3\right\}\right\}$

Out[88]= $e^{x} + 2\, e^{2x} + 3\, e^{3x}$

Out[89]= $2\, e^{x} + 4\, e^{2x} + 8\, e^{3x}$

Out[90]= $-3\, e^{x} - 2\, e^{2x} - 4\, e^{3x}$

Out[91]=



Q3. y''' + y' = sec x

In[102]:=
```
sol = DSolve[y'''[x] + 0 * y''[x] + y'[x] + 0 * y[x] == Sec[x], y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 3, C[2] → 4.5, C[3] → 18}]
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 0.6, C[2] → 1.0, C[3] → 0}]
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -9, C[2] → -3, C[3] → -2}]
Plot[{sol1, sol2, sol3}, {x, -3, 3},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thickness[0.8]}, {Blue, thick}},
 Frame → True, ImageSize → 750, PlotLegends → Placed[{"sol1", "sol2", "sol3"}, Below]]
```

Out[102]=
$$\left\{\left\{y[x] \to \mathbb{C}_3 - x\,\text{Cos}[x] - \mathbb{C}_2\,\text{Cos}[x] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + \mathbb{C}_1\,\text{Sin}[x] + \text{Log}[\text{Cos}[x]]\,\text{Sin}[x]\right\}\right\}$$

Out[103]=
$$18 - 4.5\,\text{Cos}[x] - x\,\text{Cos}[x] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + 3\,\text{Sin}[x] + \text{Log}[\text{Cos}[x]]\,\text{Sin}[x]$$

Out[104]=
$$-1.\,\text{Cos}[x] - x\,\text{Cos}[x] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + 0.6\,\text{Sin}[x] + \text{Log}[\text{Cos}[x]]\,\text{Sin}[x]$$

Out[105]=
$$-2 + 3\,\text{Cos}[x] - x\,\text{Cos}[x] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] - 9\,\text{Sin}[x] + \text{Log}[\text{Cos}[x]]\,\text{Sin}[x]$$

Out[106]=