PRACTICAL 4 : SOLUTION OF DE BY VARIATION OF PARAMETER

Second Order DE

d2y/dx2 + p dy/dx + qy = f(x)

where p and q are constants, f(x) is a non zero function of x.

General solution of homogenous equation d2y/dx2 + p dy/dx + qt =0

Particular solutions of the non- homogenous equation d2y/dx2 + pdy/dx+qy = f(x)

Example 1 : y''[x]+y[x]==2Sin[x]

```
In[•]:= ClearAll
Out[•]= ClearAll

In[•]:= gsh = DSolve[y''[x] + y[x] == 0, y[x], x]
        gsh1 = y[x] /. gsh
Out[•]= {{y[x] → 𝕔1 Cos[x] + 𝕔2 Sin[x]}}
Out[•]= {𝕔1 Cos[x] + 𝕔2 Sin[x]}

In[•]:= y1 := Cos[x]
        y2 := Sin[x]
        f := 2 * Sin[x]
        w = y1 * D[y2, x] - y2 * D[y1, x]
        w = Simplify[w]
Out[•]= Cos[x]^2 + Sin[x]^2
Out[•]= 1
```

In[∘]:=
```
psn = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x]
psn1 = Simplify[psn]
```

Out[∘]= $-\frac{1}{2}\text{Cos}[2x]\,\text{Sin}[x]-2\,\text{Cos}[x]\left(\frac{x}{2}-\frac{1}{4}\text{Sin}[2x]\right)$

Out[∘]= $\frac{1}{2}\left(-2x\,\text{Cos}[x]+\text{Sin}[x]\right)$

In[∘]:=
```
gsh1 + psn1
```

Out[∘]= $\left\{c_1\,\text{Cos}[x]+c_2\,\text{Sin}[x]+\frac{1}{2}\left(-2x\,\text{Cos}[x]+\text{Sin}[x]\right)\right\}$

In[∘]:=
```
ClearAll
```

Out[∘]= ClearAll

Example 2 : y’’+3*y’+2*y=30*e^2x
psn (particular solution), gsh (general solution)

In[∘]:=

In[∘]:=
```
yc2 = DSolve[y''[x] + 3 * y'[x] + 2 * y[x] == 0, y[x], x]
y1 := Exp[-2 * x]
y2 := Exp[-1 * x]
f := 30 * Exp[2 * x]
w = y1 * D[y2, x] - y2 * D[y1, x]
w = Simplify[w]
yp2 = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x]
yp2 = Simplify[yp2]
yc2 + yp2
```

Out[∘]= $\{\{y[x]\to e^{-2x}c_1+e^{-x}c_2\}\}$

Out[∘]= e^{-3x}

Out[∘]= e^{-3x}

Out[∘]= $\frac{5e^{2x}}{2}$

Out[∘]= $\frac{5e^{2x}}{2}$

Out[∘]= $\left\{\left\{\frac{5e^{2x}}{2}+\left(y[x]\to e^{-2x}c_1+e^{-x}c_2\right)\right\}\right\}$

Ques 1 : y’’+y=cotx

In[ ]:=
```
gsh = DSolve[y''[x] + y[x] == 0, y[x], x]
gsh1 = y[x] /. gsh
y1 := Cos[x]
y2 := Sin[x]
f := Cot[x]
w = y1 * D[y2, x] - y2 * D[y1, x]
w = Simplify[w]
psn = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x]
psn1 = Simplify[psn]
gsh1 + psn1
```

Out[ ]= $\{\{y[x] \to c_1 \text{Cos}[x] + c_2 \text{Sin}[x]\}\}$

Out[ ]= $\{c_1 \text{Cos}[x] + c_2 \text{Sin}[x]\}$

Out[ ]= $\text{Cos}[x]^2 + \text{Sin}[x]^2$

Out[ ]= 1

Out[ ]= $-\text{Cos}[x]\,\text{Sin}[x] + \left(\text{Cos}[x] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]\right) \text{Sin}[x]$

Out[ ]= $\left(-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]\right) \text{Sin}[x]$

Out[ ]= $\left\{c_1 \text{Cos}[x] + c_2 \text{Sin}[x] + \left(-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]\right) \text{Sin}[x]\right\}$

In[ ]:=
```
ClearAll
```

Out[ ]= ClearAll

Ques 2 : y''+4y' + 5y=e^-2x . Sec x

In[ ]:=
```
gsh = DSolve[y''[x] + 4 * y'[x] + 5 * y[x] == 0, y[x], x]
gsh1 = y[x] /. gsh
y1 := Exp[-2 * x] * Cos[x]
y2 := Exp[-2 * x] * Sin[x]
f := Exp[-2 * x] * Sec[x]
w = y1 * D[y2, x] - y2 * D[y1, x]
w = Simplify[w]
psn = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x]
psn1 = Simplify[psn]
gsh1 + psn1
```

Out[ ]= $\{\{y[x] \to e^{-2x} c_2 \operatorname{Cos}[x] + e^{-2x} c_1 \operatorname{Sin}[x]\}\}$

Out[ ]= $\{e^{-2x} c_2 \operatorname{Cos}[x] + e^{-2x} c_1 \operatorname{Sin}[x]\}$

Out[ ]= $e^{-2x} \operatorname{Cos}[x] \left(e^{-2x} \operatorname{Cos}[x] - 2 e^{-2x} \operatorname{Sin}[x]\right) - e^{-2x} \operatorname{Sin}[x] \left(-2 e^{-2x} \operatorname{Cos}[x] - e^{-2x} \operatorname{Sin}[x]\right)$

Out[ ]= e^{-4x}

Out[ ]= $e^{-2x} \operatorname{Cos}[x] \operatorname{Log}[\operatorname{Cos}[x]] + e^{-2x} x \operatorname{Sin}[x]$

Out[ ]= $e^{-2x} \left(\operatorname{Cos}[x] \operatorname{Log}[\operatorname{Cos}[x]] + x \operatorname{Sin}[x]\right)$

Out[ ]= $\{e^{-2x} c_2 \operatorname{Cos}[x] + e^{-2x} c_1 \operatorname{Sin}[x] + e^{-2x} \left(\operatorname{Cos}[x] \operatorname{Log}[\operatorname{Cos}[x]] + x \operatorname{Sin}[x]\right)\}$

In[ ]:=
```
ClearAll
```

Out[ ]= ClearAll

Ques 3 : y''+6y' + 9y=e^-3x / x^3

In[ ]:=
```
gsh = DSolve[y''[x] + 6 * y'[x] + 9 * y[x] == 0, y[x], x]
gsh1 = y[x] /. gsh
```

Out[ ]= $\{\{y[x] \to e^{-3x} c_1 + e^{-3x} x c_2\}\}$

Out[ ]= $\{e^{-3x} c_1 + e^{-3x} x c_2\}$

In[∘]:=
```
y1 := Exp[-3 * x]
y2 := Exp[-3 * x] * x
f := Exp[-3 * x] / x^3
w = y1 * D[y2, x] - y2 * D[y1, x]
w = Simplify[w]
psn = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x]
psn1 = Simplify[psn]
gsh1 + psn1
```

Out[∘]= $3\,e^{-6x}\,x + e^{-3x}\left(e^{-3x} - 3\,e^{-3x}\,x\right)$

Out[∘]= e^{-6x}

Out[∘]= $\frac{e^{-3x}}{2x}$

Out[∘]= $\frac{e^{-3x}}{2x}$

Out[∘]= $\left\{\frac{e^{-3x}}{2x} + e^{-3x}\,c_1 + e^{-3x}\,x\,c_2\right\}$

In[∘]:=
```
ClearAll
```

Out[∘]= ClearAll

Ques 4 : y''-2y' + y=x e^x ln x

In[∘]:=
```
gsh = DSolve[y''[x] - 2 * y'[x] + y[x] == 0, y[x], x]
gsh1 = y[x] /. gsh
```

Out[∘]= $\{\{y[x] \to e^{x}\,c_1 + e^{x}\,x\,c_2\}\}$

Out[∘]= $\{e^{x}\,c_1 + e^{x}\,x\,c_2\}$

In[ ]:=
```
y1 := Exp[x]
y2 := Exp[x] * x
f := x * Exp[x] * Log[x]
w = y1 * D[y2, x] - y2 * D[y1, x]
w = Simplify[w]
psn = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x]
psn1 = Simplify[psn]
gsh1 + psn1
```

Out[ ]= $-e^{2x} x + e^x \left(e^x + e^x x\right)$

Out[ ]= e^{2x}

Out[ ]= $e^x x \left(-\frac{x^2}{4} + \frac{1}{2} x^2 \text{Log}[x]\right) - e^x \left(-\frac{x^3}{9} + \frac{1}{3} x^3 \text{Log}[x]\right)$

Out[ ]= $\frac{1}{36} e^x x^3 (-5 + 6 \text{Log}[x])$

Out[ ]= $\left\{e^x c_1 + e^x x c_2 + \frac{1}{36} e^x x^3 (-5 + 6 \text{Log}[x])\right\}$

In[1]:=
```
ClearAll
```

Out[1]= ClearAll

Ques 5 : y''+y=1/1+Sin[x]

In[ ]:=
```
gsh = DSolve[y''[x] + y[x] == 0, y[x], x]
gsh1 = y[x] /. gsh
```

Out[ ]= $\{\{y[x] \to c_1 \text{Cos}[x] + c_2 \text{Sin}[x]\}\}$

Out[ ]= $\{c_1 \text{Cos}[x] + c_2 \text{Sin}[x]\}$

In[ ]:=
```
y1 := Cos[x]
y2 := Sin[x]
f := 1/(1 + Sin[x])
w = y1 * D[y2, x] - y2 * D[y1, x]
w = Simplify[w]
psn = -y1 * Integrate[y2 * (f/w), x] + y2 * Integrate[y1 * (f/w), x]
psn1 = Simplify[psn]
gsh1 + psn1
```

Out[ ]= $\text{Cos}[x]^2 + \text{Sin}[x]^2$

Out[ ]= 1

Out[ ]= $-\text{Cos}[x]\left(x - \frac{2\,\text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]}\right) + \text{Log}[1 + \text{Sin}[x]]\,\text{Sin}[x]$

Out[ ]= $-1 + \text{Cos}[x] - x\,\text{Cos}[x] + \text{Sin}[x] + \text{Log}[1 + \text{Sin}[x]]\,\text{Sin}[x]$

Out[ ]= $\{-1 + \text{Cos}[x] - x\,\text{Cos}[x] + \mathbb{c}_1\,\text{Cos}[x] + \text{Sin}[x] + \mathbb{c}_2\,\text{Sin}[x] + \text{Log}[1 + \text{Sin}[x]]\,\text{Sin}[x]\}$

In[2]:=
```
ClearAll
```

Out[2]= ClearAll