PRACTICAL 2 : Solution of Second Order Differential Equation

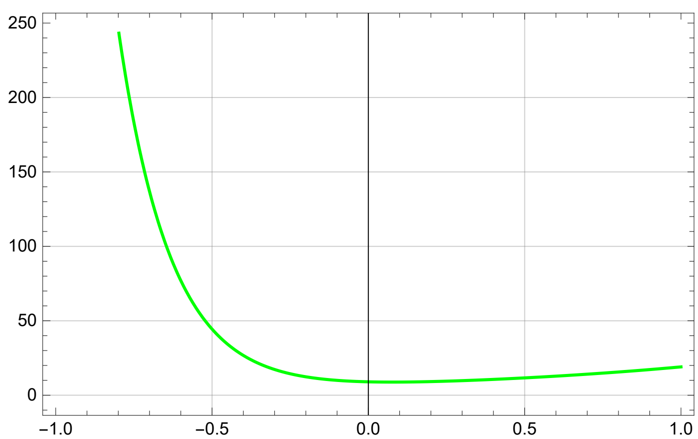
Homogenous Linear ODEs of Second Order

Real and Distinct Roots

In[ ]:=
```
sol = DSolve[y''[x] + 5 * y'[x] - 6 * y[x] == 0 , y[x], x]
```
Out[ ]= $\{\{y[x] \to e^{-6x} c_1 + e^{x} c_2\}\}$

In[ ]:=
```
sol1 = y[x] /. sol[[1]] /. {C[1] → 2, C[2] → 7}
```
Out[ ]= $2 e^{-6x} + 7 e^{x}$

In[ ]:=
```
Plot[{sol1} , {x, -1, 1}, PlotStyle → {Green},
 Frame → True, AxesOrigin → {0, 0}, GridLines → Automatic]
```
Out[ ]=



PLOTTING FAMILY OF SOLUTIONS

Solve and plot four solutions of the following Differential Equation

y’’+y=0

In[ ]:=
```
Sol = DSolve[y''[x] + y[x] == 0, y[x], x]
```
Out[ ]=
```
{{y[x] → c1 Cos[x] + c2 Sin[x]}}
```

Taking C[1] as a constant

In[ ]:=
```
Sol1 = y[x] /. Sol /. {C[1] → 1, C[2] → 2}
```
Out[ ]=
```
{Cos[x] + 2 Sin[x]}
```

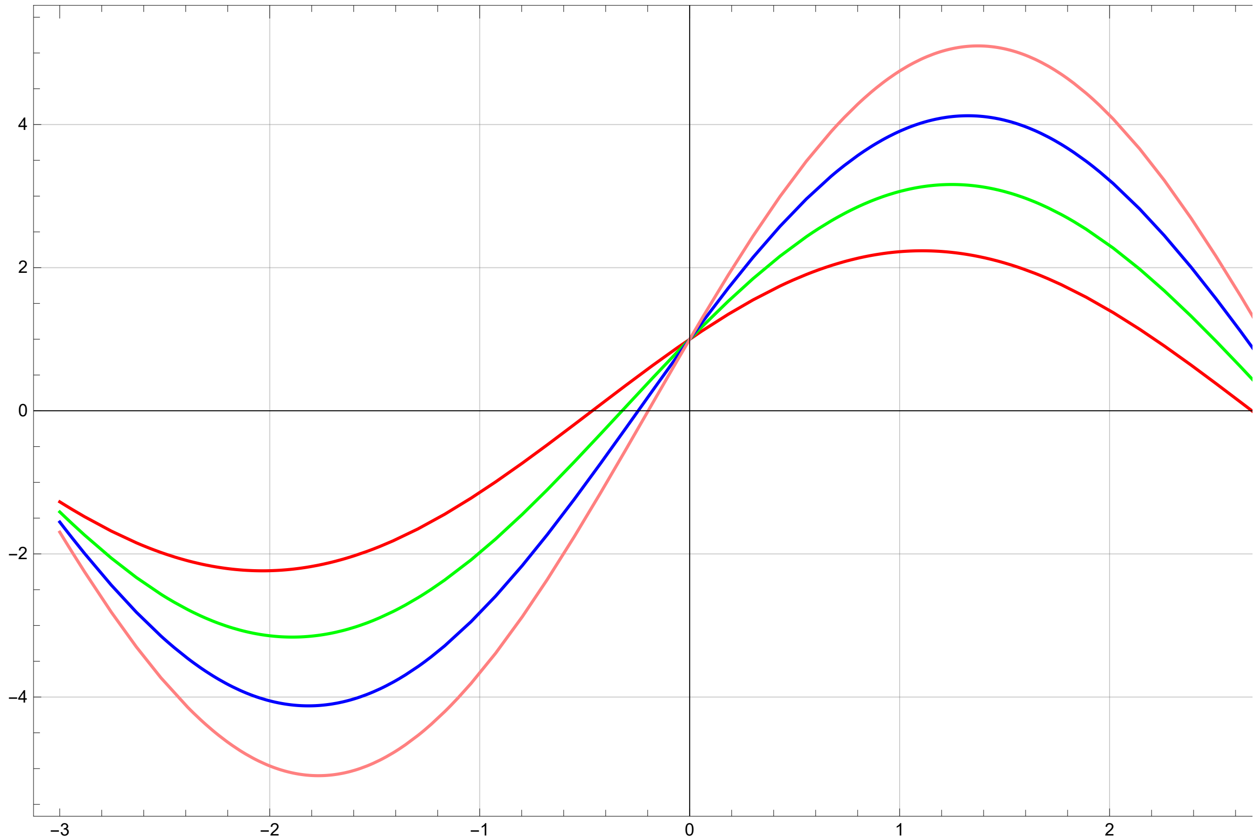
In[ ]:=
```
Sol2 = y[x] /. Sol /. {C[1] → 1, C[2] → 3 }
```
Out[ ]=
```
{Cos[x] + 3 Sin[x]}
```

In[ ]:=
```
Sol3 = y[x] /. Sol /. {C[1] → 1, C[2] → 4}
```
Out[ ]=
```
{Cos[x] + 4 Sin[x]}
```

In[ ]:=
```
Sol4 = y[x] /. Sol /. {C[1] → 1, C[2] → 5}
```
Out[ ]=
```
{Cos[x] + 5 Sin[x]}
```

In[ ]:=
```
Plot[{Sol1, Sol2, Sol3, Sol4}, {x, -3, 3}, PlotStyle → {Red, Green, Blue, Pink},
  Frame → True, AxesOrigin → {0, 0}, GridLines → Automatic, ImageSize → 700,
  PlotLegends → LineLegend[{"Sol1", "Sol2", "Sol3", "Sol4"}, LegendFunction → "Frame"]]
```

Out[ ]=



In[ ]:=

Real and Equal Roots :

In[ ]:=
```
sol2 = DSolve[y''[x] - 6 * y'[x] + 9 y[x] == 0, y[x], x]
```

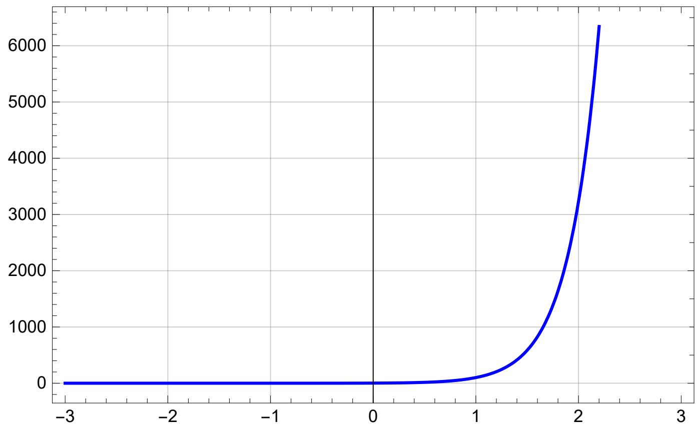
Out[ ]= $\{\{y[x] \to e^{3x} c_1 + e^{3x} x\, c_2\}\}$

In[ ]:=
```
sol3 = y[x] /. sol2[[1]] /. {C[1] → 2, C[2] → 3}
```

Out[ ]= $2 e^{3x} + 3 e^{3x} x$

In[∘]:=
```
Plot[{sol3}, {x, -3, 3}, PlotStyle → {Blue},
 Frame → True, AxesOrigin → {0, 0}, GridLines → Automatic]
```

Out[∘]=



4y''+12y'+9y=0

In[∘]:=
```
B = DSolve[y''[x] - 6 * y'[x] + 9 y[x] == 0, y[x], x]
```

Out[∘]= $\{\{y[x] \to e^{3x} c_1 + e^{3x} x\, c_2\}\}$

Taking C[1] as constant

In[∘]:=
```
B1 = Table[y[x] /. B /. {C[1] → 1, C[2] → k}, {k, 2, 5}]        // TableForm
```

Out[∘]//TableForm=

$e^{3x} + 2\, e^{3x}\, x$

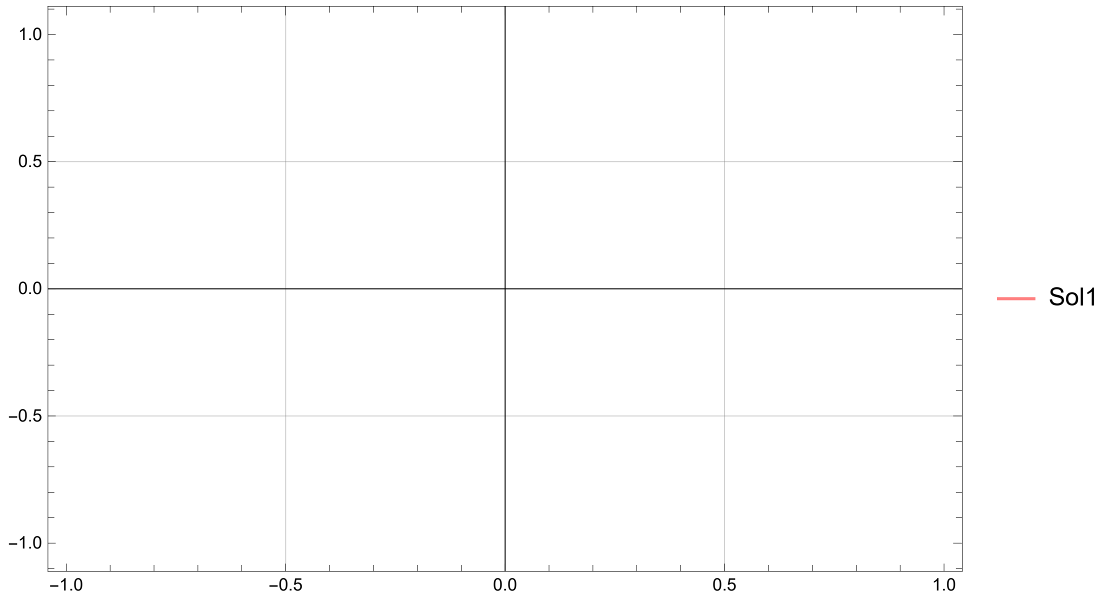
$e^{3x} + 3\, e^{3x}\, x$

$e^{3x} + 4\, e^{3x}\, x$

$e^{3x} + 5\, e^{3x}\, x$

In[3]:=
```
Plot[B1, {x, -1, 1}, PlotStyle → {Red, Green, Blue , Pink},
 GridLines → Automatic, Frame → True, AxesOrigin → {0, 0}, ImageSize → 500,
 PlotLegends → LineLegend[{"Sol1", "Sol2", "Sol3", "Sol4"}, LegendFunction → "Frame"]]
```

Out[3]=



Imaginary Roots

In[ ]:=
```
sol4 = DSolve[y''[x] - y'[x] + y[x] == 0, y[x], x]
```

Out[ ]= $\left\{\left\{y[x] \to e^{x/2}\, c_1 \operatorname{Cos}\left[\frac{\sqrt{3}\, x}{2}\right] + e^{x/2}\, c_2 \operatorname{Sin}\left[\frac{\sqrt{3}\, x}{2}\right]\right\}\right\}$

In[ ]:= $\left\{\left\{\mathbf{y[x] \to e^{x/2}\, c_1 \operatorname{Cos}\left[\frac{\sqrt{3}\, x}{2}\right] + e^{x/2}\, c_2 \operatorname{Sin}\left[\frac{\sqrt{3}\, x}{2}\right]}\right\}\right\}$

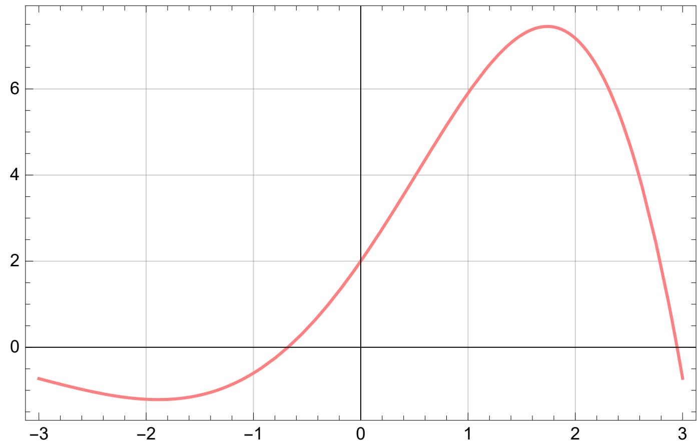
Out[ ]= $\left\{\left\{y[x] \to e^{x/2}\, c_1 \operatorname{Cos}\left[\frac{\sqrt{3}\, x}{2}\right] + e^{x/2}\, c_2 \operatorname{Sin}\left[\frac{\sqrt{3}\, x}{2}\right]\right\}\right\}$

In[ ]:=
```
sol5 = y[x] /. sol4[[1]] /. {C[1] → 2, C[2] → 3}
```

Out[ ]= $2\, e^{x/2} \operatorname{Cos}\left[\frac{\sqrt{3}\, x}{2}\right] + 3\, e^{x/2} \operatorname{Sin}\left[\frac{\sqrt{3}\, x}{2}\right]$

In[∘]:=
```
Plot[{sol5}, {x, -3, 3}, PlotStyle → {Red, Green, Blue, Pink},
  Frame → True, AxesOrigin → {0, 0}, GridLines → Automatic]
```

Out[∘]=



y''-4y'+13y=0

In[∘]:=
```
c = DSolve[y''[x] - 4 * y'[x] + 13 * y[x] == 0, y[x], x]
```

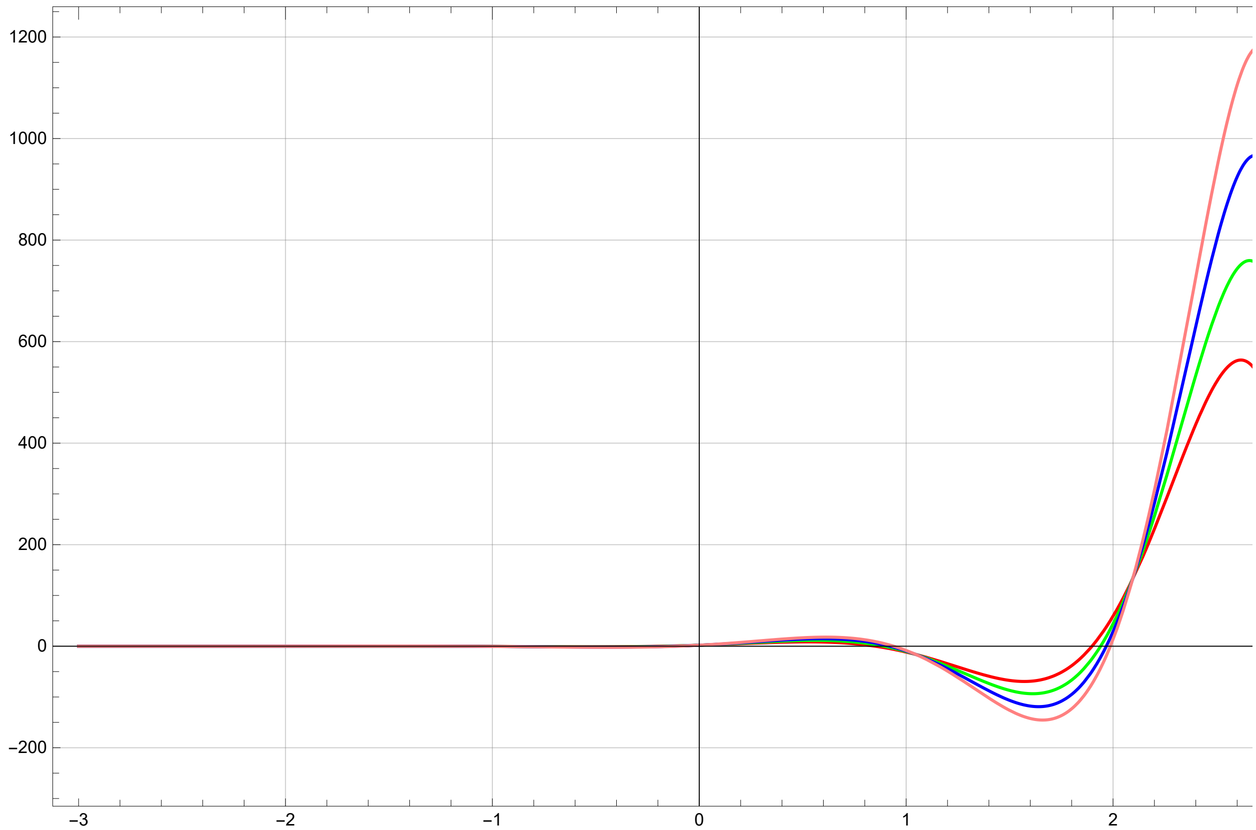
Out[∘]= $\{\{y[x] \to e^{2x} c_2 \text{Cos}[3x] + e^{2x} c_1 \text{Sin}[3x]\}\}$

In[∘]:=
```
c1 = Table[y[x] /. c /. {C[1] → k, C[2] → 2}, {k, 3, 6}]
```

Out[∘]= $\{\{2 e^{2x} \text{Cos}[3x] + 3 e^{2x} \text{Sin}[3x]\}, \{2 e^{2x} \text{Cos}[3x] + 4 e^{2x} \text{Sin}[3x]\}, \{2 e^{2x} \text{Cos}[3x] + 5 e^{2x} \text{Sin}[3x]\}, \{2 e^{2x} \text{Cos}[3x] + 6 e^{2x} \text{Sin}[3x]\}\}$

In[•]:=
```
Plot[{c1}, {x, -3, 3}, PlotStyle → {Red, Green, Blue, Pink}, GridLines → Automatic,
 Frame → True, AxesOrigin → {0, 0}, PlotRange → All, ImageSize → 700]
```

Out[•]=



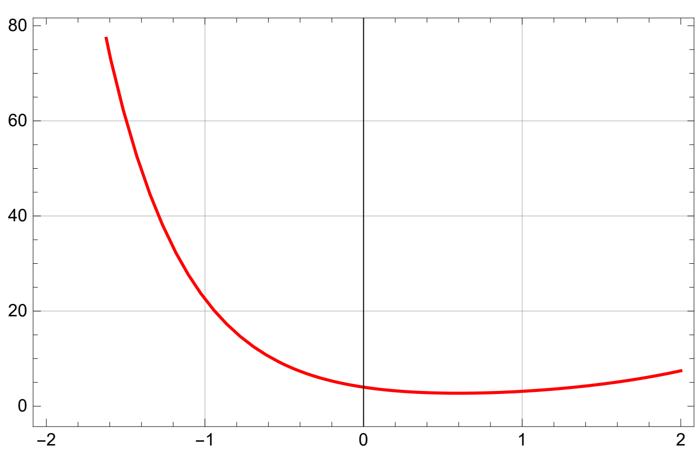
Initial value problem --- no

In[1]:=
```
pp = DSolve[{y''[x] + y'[x] - 2 * y[x] == 0 , y[0] == 4, y'[0] == -5}, y[x], x]
```

Out[1]= $\{\{y[x] \to e^{-2x} (3 + e^{3x})\}\}$

In[2]:=
```
Plot[y[x] /. pp, {x, -2, 2}, PlotStyle → {Red}, GridLines → Automatic, Frame → True]
```

Out[2]=



Non - Homogenous Equations

In[8]:=
```
r = DSolve[y''[x] - 2 * y'[x] - 3 * y[x] == 30 * Exp[2 * x], y[x], x]
```

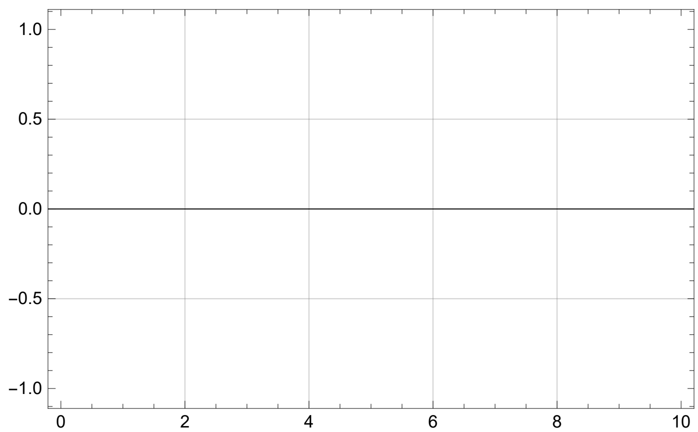
Out[8]= $\{\{y[x] \to -10\, e^{2x} + e^{-x}\, c_1 + e^{3x}\, c_2\}\}$

In[6]:=
```
a = Table[y[x] /. r /. {C[1] → k, C[2] → 2}, {k, 3, 6}]
```

Out[6]= $\{\{3\, e^{-x} - 10\, e^{2x} + 2\, e^{3x}\}, \{4\, e^{-x} - 10\, e^{2x} + 2\, e^{3x}\}, \{5\, e^{-x} - 10\, e^{2x} + 2\, e^{3x}\}, \{6\, e^{-x} - 10\, e^{2x} + 2\, e^{3x}\}\}$

In[7]:=
```
Plot[{sol4}, {x, 0, 10}, PlotStyle → {Red, Green, Blue , Pink},
  GridLines → Automatic, Frame → True, AxesOrigin → {0, 0}]
```

Out[7]=



y''-2y'-3y=2sinx

In[ ]:=
```
p = DSolve[y''[x] - 2 * y'[x] - 3 * y[x] == 2 * Sin[x], y[x], x]
```

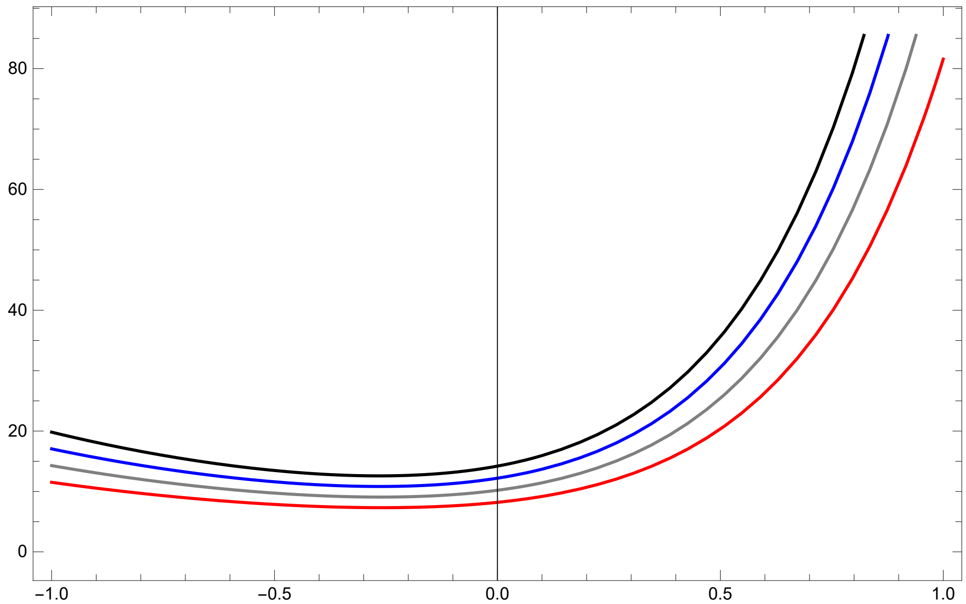
Out[ ]= $\{\{y[x] \to e^{-x}\, c_1 + e^{3x}\, c_2 + \frac{1}{5}(\mathrm{Cos}[x] - 2\,\mathrm{Sin}[x])\}\}$

taking C[1] and C[2] both same and varying

In[ ]:=
```
p1 = Table[y[x] /. p /. {C[1] → m, C[2] → m}, {m, 4, 7}]
```

Out[ ]= $\{\{4\, e^{-x} + 4\, e^{3x} + \frac{1}{5}(\mathrm{Cos}[x] - 2\,\mathrm{Sin}[x])\}, \{5\, e^{-x} + 5\, e^{3x} + \frac{1}{5}(\mathrm{Cos}[x] - 2\,\mathrm{Sin}[x])\},$
$\{6\, e^{-x} + 6\, e^{3x} + \frac{1}{5}(\mathrm{Cos}[x] - 2\,\mathrm{Sin}[x])\}, \{7\, e^{-x} + 7\, e^{3x} + \frac{1}{5}(\mathrm{Cos}[x] - 2\,\mathrm{Sin}[x])\}\}$

In[∘]:=
```
Plot[{p1}, {x, -1, 1}, PlotStyle → {Red, Gray, Blue, Black},
  Frame → True, ImageSize → 500, AxesOrigin → {0, 0}]
```

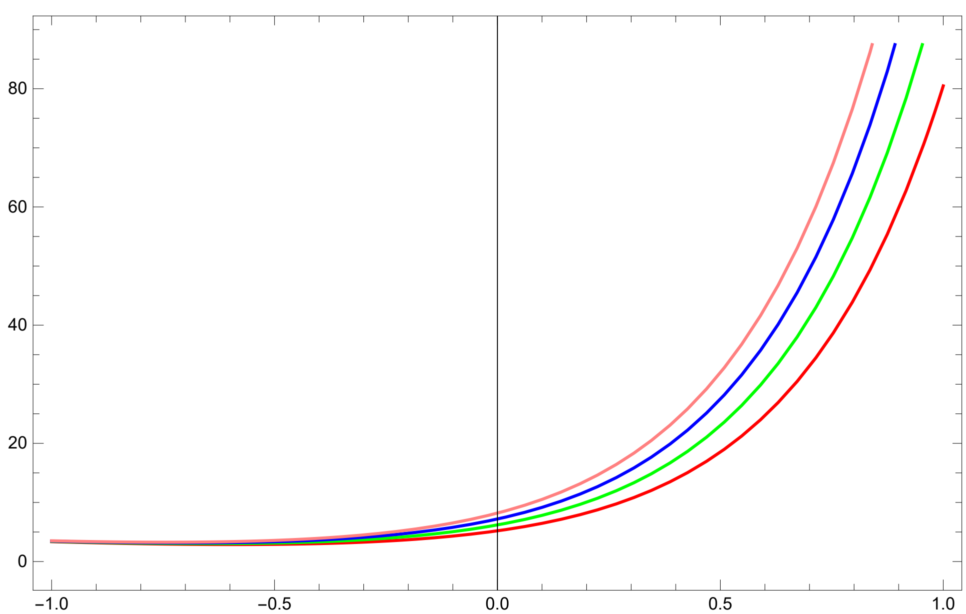
Out[∘]=


taking C[1] constant

In[∘]:=
```
p2 = Table[y[x] /. p /. {C[1] → 1, C[2] → m}, {m, 4, 7}]
```

Out[∘]=
$$\left\{\left\{e^{-x}+4\,e^{3x}+\frac{1}{5}\left(\mathrm{Cos}[x]-2\,\mathrm{Sin}[x]\right)\right\},\left\{e^{-x}+5\,e^{3x}+\frac{1}{5}\left(\mathrm{Cos}[x]-2\,\mathrm{Sin}[x]\right)\right\},\left\{e^{-x}+6\,e^{3x}+\frac{1}{5}\left(\mathrm{Cos}[x]-2\,\mathrm{Sin}[x]\right)\right\},\left\{e^{-x}+7\,e^{3x}+\frac{1}{5}\left(\mathrm{Cos}[x]-2\,\mathrm{Sin}[x]\right)\right\}\right\}$$

In[∘]:=
```
Plot[{p2} , {x, -1, 1}, PlotStyle → {Red, Green, Blue , Pink},
  ImageSize → 500, Frame → True, AxesOrigin → {0, 0}]
```

Out[∘]=


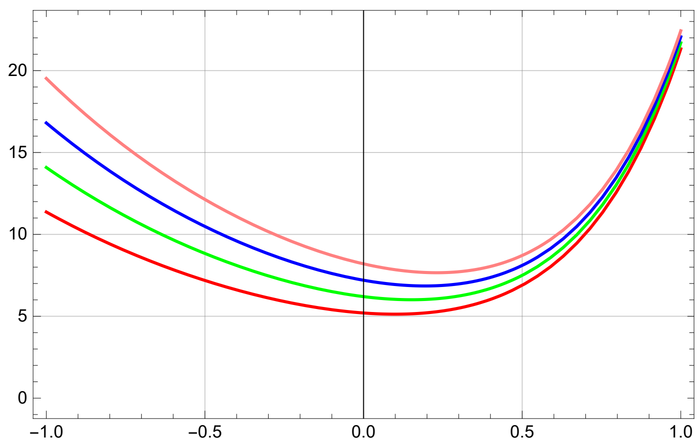
taking C[2] constant.

In[∘]:=
```
p3 = Table[y[x] /. p /. {C[1] → m , C[2] → 1}, {m, 4, 7}]
```

Out[∘]= $\left\{\left\{4 e^{-x} + e^{3x} + \frac{1}{5}(\text{Cos}[x] - 2\,\text{Sin}[x])\right\}, \left\{5 e^{-x} + e^{3x} + \frac{1}{5}(\text{Cos}[x] - 2\,\text{Sin}[x])\right\}, \left\{6 e^{-x} + e^{3x} + \frac{1}{5}(\text{Cos}[x] - 2\,\text{Sin}[x])\right\}, \left\{7 e^{-x} + e^{3x} + \frac{1}{5}(\text{Cos}[x] - 2\,\text{Sin}[x])\right\}\right\}$

In[∘]:=
```
Plot[{p3}, {x, -1, 1}, PlotStyle → {Red, Green, Blue, Pink},
 GridLines → Automatic, Frame → True, AxesOrigin → {0, 0}]
```

Out[∘]=



Initial value problems for non - homogenous

y’’+y=0.001x^2 , y(0)=0 , y’[0]=1.5 no

In[9]:=
```
q2 = DSolve[{y''[x] + y[x] == 0.001 * x^2 , y[0] == 0, y'[0] == 1.5}, y[x], x]
```

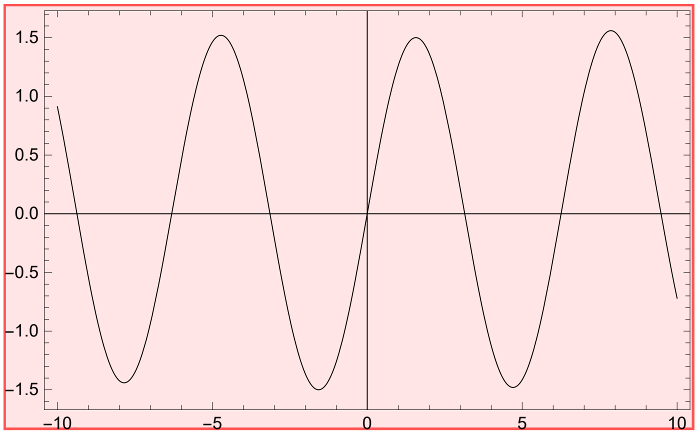
Out[9]= $\{\{y[x] \to -0.002 + 0.001\,x^2 + 0.002\,\text{Cos}[1.\,x] + 1.5\,\text{Sin}[1.\,x]\}\}$

In[10]:=
```
q3 = Table[y[x] /. q2]
```

Out[10]= $\{-0.002 + 0.001\,x^2 + 0.002\,\text{Cos}[1.\,x] + 1.5\,\text{Sin}[1.\,x]\}$

In[11]:=
```
Plot[q3, {x, -10, 10}, PlotStyle → {Red}. GridLines → Automatic,
 Frame → True, AxesOrigin → {0, 0}, PlotLegends → Automatic]
```

Out[11]=



Euler an Cauchy Equations

In[∘]:=
```
b = DSolve[x^2 * y''[x] - 2 * x * y'[x] - 4 * y[x] == 0, y[x], x]
```

Out[∘]= $\left\{\left\{y[x] \to \frac{\mathbb{C}_1}{x} + x^4\,\mathbb{C}_2\right\}\right\}$

In[∘]:=
```
c = Table[y[x] /. b /. {C[1] → k, C[2] → 2}, {k, 3, 6}]
```

Out[∘]= $\left\{\left\{\frac{3}{x} + 2\,x^4\right\}, \left\{\frac{4}{x} + 2\,x^4\right\}, \left\{\frac{5}{x} + 2\,x^4\right\}, \left\{\frac{6}{x} + 2\,x^4\right\}\right\}$

In[∘]:=
```
Plot[{c}, {x, 0, 2}, PlotStyle → {Red, Blue, Black, Green},
 GridLines → Automatic, Frame → True, AxesOrigin → {0, 0}, PlotLegends → Automatic]
```

Out[∘]=