Practical 8 :: Plot the integral surfaces of First Order Partial Differential Equations with initial data

Ques 1. $x\,u_x + y\,u_y = 2xy$

```
In[ ]:= pde1 = x * D[u[x, y], x] + y * D[u[x, y], y] == 2 * x * y;
In[ ]:= sol1 = DSolve[pde1, u[x, y], {x, y}]
Out[ ]= {{u[x, y] → x y + C1[y/x]}}
```

```
In[ ]:= a.C[a] = Sin a
```

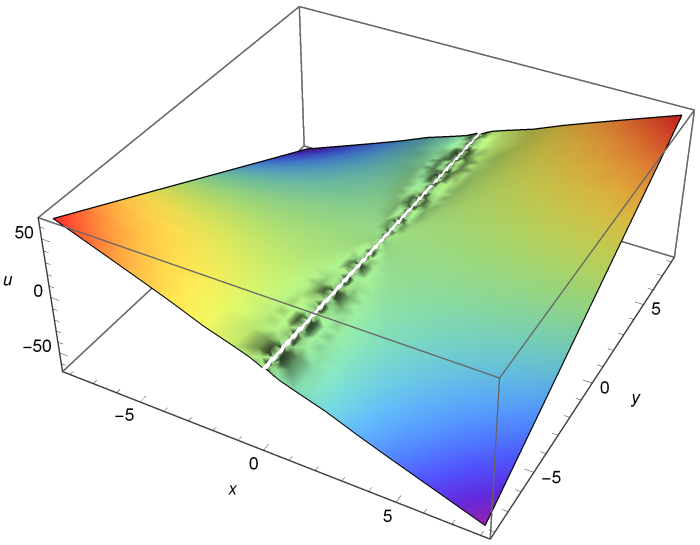
Set: Tag Dot in a.C_a is Protected.

```
Out[ ]= a Sin
```

```
In[ ]:= parsol = u[x, y] /. sol1[[1]] /. C[1][a_] → Sin[a]
Out[ ]= x y + Sin[y/x]
```

```
In[ ]:= Plot3D[parsol, {x, -8, 8}, {y, -8, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "Rainbow"]
```

Out[ ]=



In[ ]:=
```
b.C[a] = Cos a
```

Set: Tag Dot in b.C$_a$ is Protected.

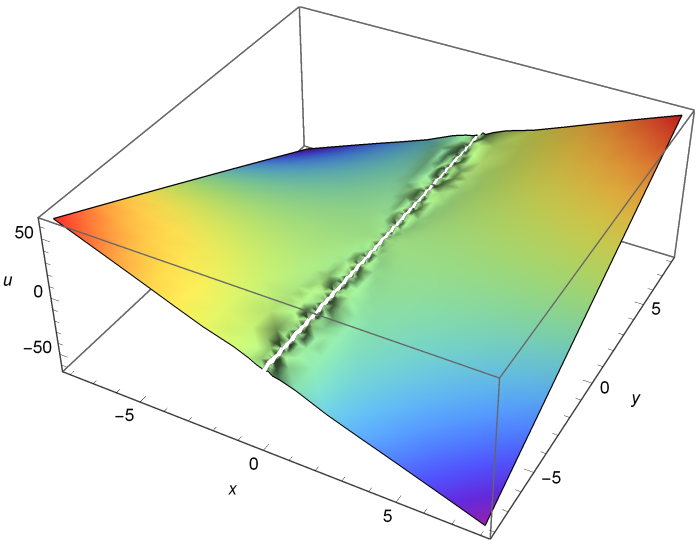
Out[ ]= a Cos

In[ ]:=
```
parsol = u[x, y] /. sol1[[1]] /. C[1][a_] → Cos[a]
```

Out[ ]= $x\,y + \text{Cos}\left[\frac{y}{x}\right]$

In[ ]:=
```
Plot3D[parsol, {x, -8, 8}, {y, -8, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "Rainbow"]
```

Out[ ]=



In[ ]:=
```
c.C[a] = a^3
```

Set: Tag Dot in c.C$_a$ is Protected.

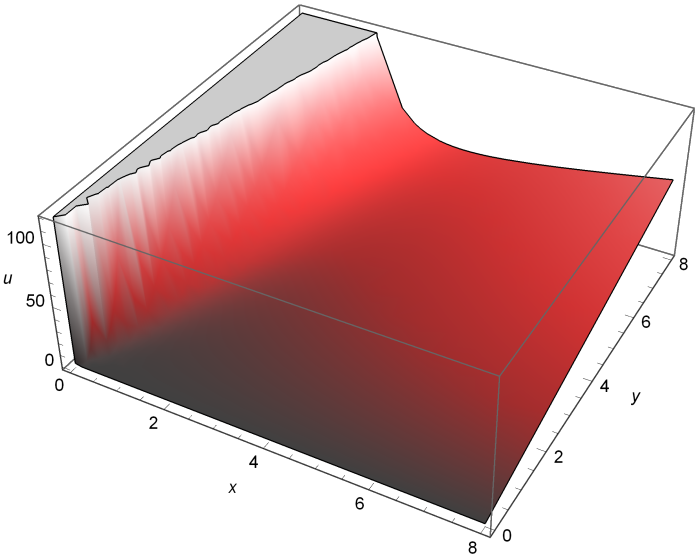
Out[ ]= a^3

In[ ]:=
```
parsol = u[x, y] /. sol1[[1]] /. C[1][a_] → a^3
```

Out[ ]= $x\,y + \frac{y^3}{x^3}$

In[ ]:=
```
Plot3D[parsol, {x, 0, 8}, {y, 0, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "CherryTones"]
```

Out[ ]=



Solve the following :

Ques 2 : $3\,u_x + 2\,u_y = 0$

In[ ]:=
```
pde2 = 3 * D[u[x, y], x] + 2 * D[u[x, y], y] == 0;
sol2 = DSolve[pde2, u[x, y], {x, y}]
```

Out[ ]=
$$\left\{\left\{u[x, y] \to c_1\left[\frac{1}{3}\left(-2\,x + 3\,y\right)\right]\right\}\right\}$$

In[ ]:=
```
a.C[a] = Sin a
```

Set: Tag Dot in a.c_a is Protected.

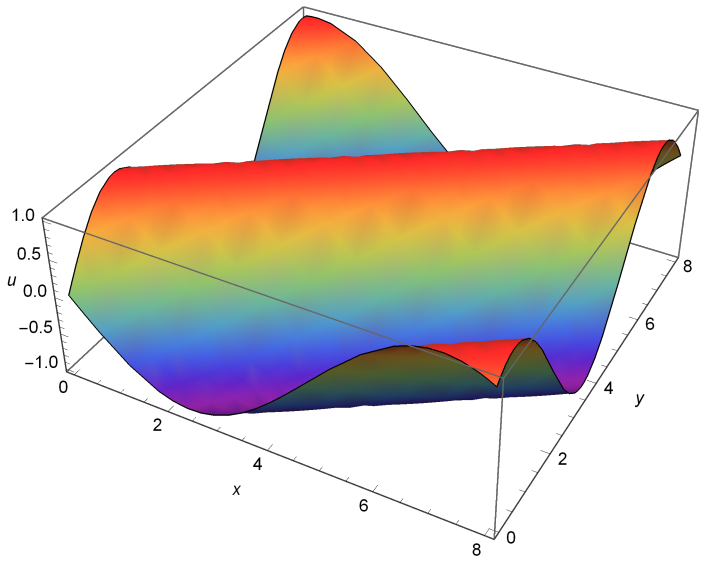
Out[ ]=
```
a Sin
```

In[ ]:=
```
parsol2 = u[x, y] /. sol2[[1]] /. C[1][a_] → Sin[a]
```

Out[ ]=
$$\text{Sin}\left[\frac{1}{3}\left(-2\,x + 3\,y\right)\right]$$

In[•]:=
```
Plot3D[parsol2, {x, 0, 8}, {y, 0, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "Rainbow"]
```

Out[•]=



In[•]:=
```
a.C[a] = Cos a
```

Set: Tag Dot in a.$\mathfrak{C}_a$ is Protected.

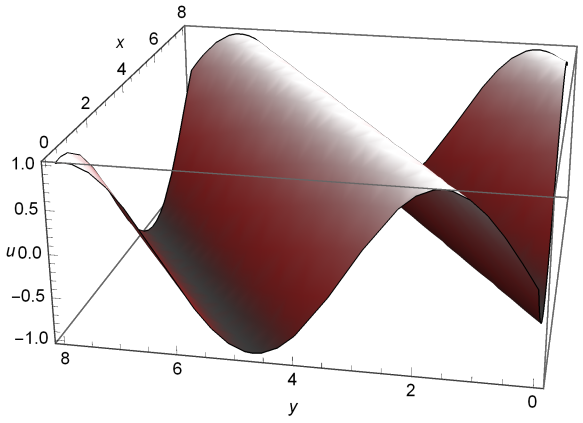
Out[•]= a Cos

In[•]:=
```
parsol2 = u[x, y] /. sol2[[1]] /. C[1][a_] → Sin[a]
```

Out[•]= $\text{Sin}\left[\frac{1}{3}\left(-2\,x+3\,y\right)\right]$

In[•]:=
```
Plot3D[parsol2, {x, 0, 8}, {y, 0, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "CherryTones"]
```

Out[•]=



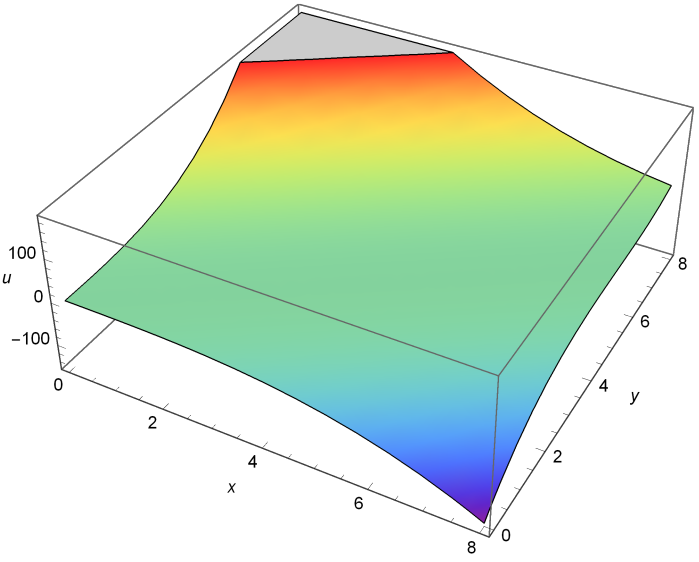
$a.C[a] = a^3$

In[ ]:=
```
parsol2 = u[x, y] /. sol2[[1]] /. C[1][a_] → a^3
```

Out[ ]=
$$\frac{1}{27}\left(-2x+3y\right)^3$$

In[ ]:=
```
Plot3D[parsol2, {x, 0, 8}, {y, 0, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "Rainbow"]
```

Out[ ]=

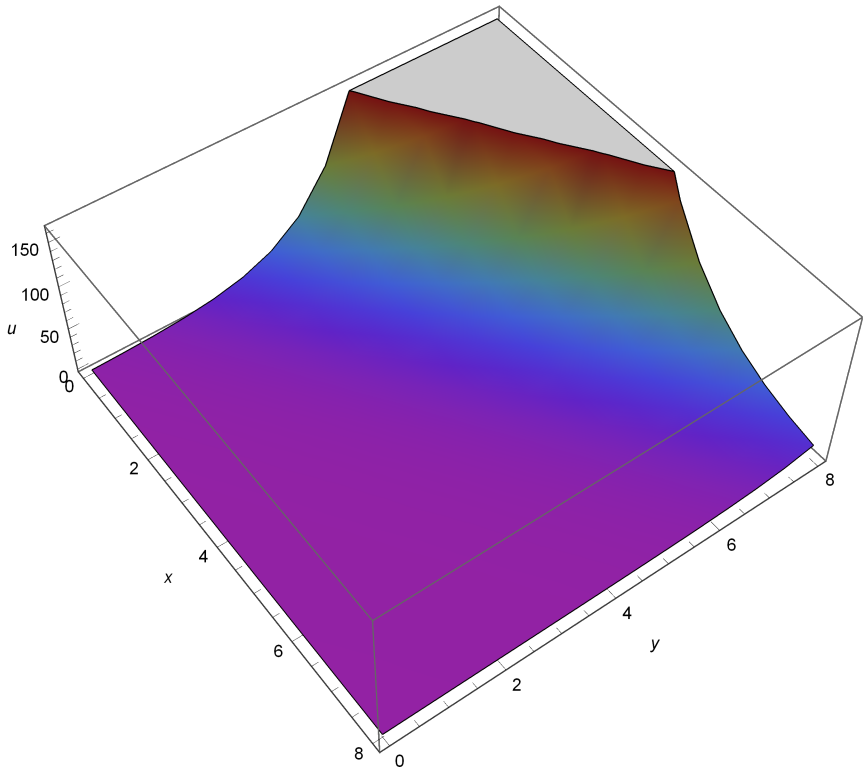


In[ ]:=
```
parsol2 = u[x, y] /. sol2[[1]] /. C[1][a_] → Exp[a]
```

Out[ ]=
$$e^{\frac{1}{3}(-2x+3y)}$$

In[ ]:=
```
Plot3D[parsol2, {x, 0, 8}, {y, 0, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "Rainbow"]
```

Out[ ]=



Ques 3 : $u_x - u_y = 1$

In[ ]:=
```
pde3 = D[u[x, y], x] - D[u[x, y], y] == 1;
sol3 = DSolve[pde3, u[x, y], {x, y}]
```

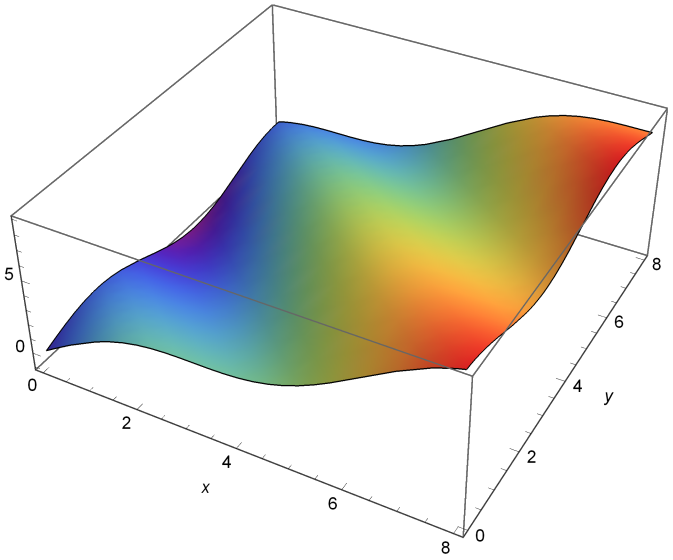
Out[ ]=
```
{{u[x, y] → x + C[1][x + y]}}
```

In[ ]:=
```
parsol3 = u[x, y] /. sol3[[1]] /. C[1][a_] → Sin[a]
```

Out[ ]=
```
x + Sin[x + y]
```

In[•]:=
```
Plot3D[parsol3, {x, 0, 8}, {y, 0, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "Rainbow"]
```

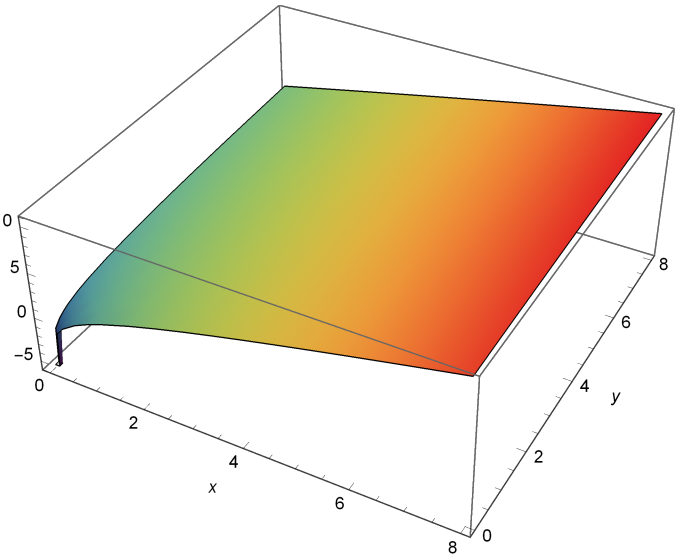
Out[•]=



In[•]:=
```
parsol3 = u[x, y] /. sol3[[1]] /. C[1][a_] → Log[a]
Plot3D[parsol3, {x, 0, 8}, {y, 0, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "Rainbow"]
```

Out[•]=
$$x + \text{Log}[x + y]$$

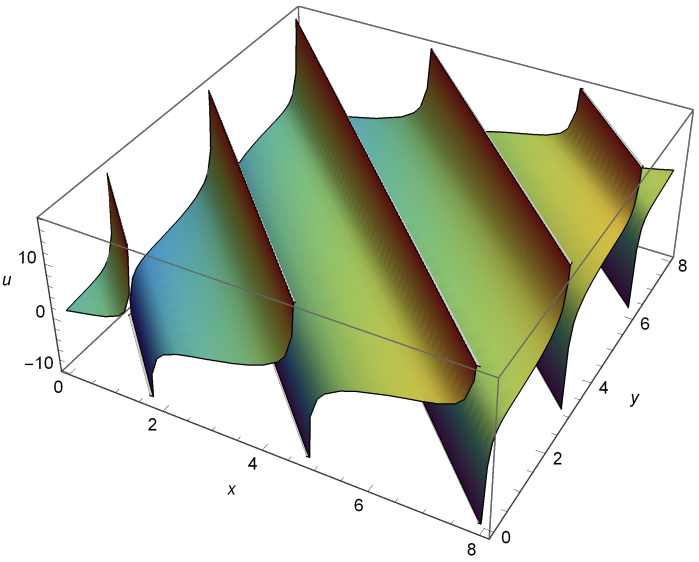
Out[•]=



In[•]:=
```
arsol3 = u[x, y] /. sol3[[1]] /. C[1][a_] → Tan[a]
Plot3D[parsol3, {x, 0, 8}, {y, 0, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "Rainbow"]
```

Out[•]= $x + \text{Tan}[x + y]$

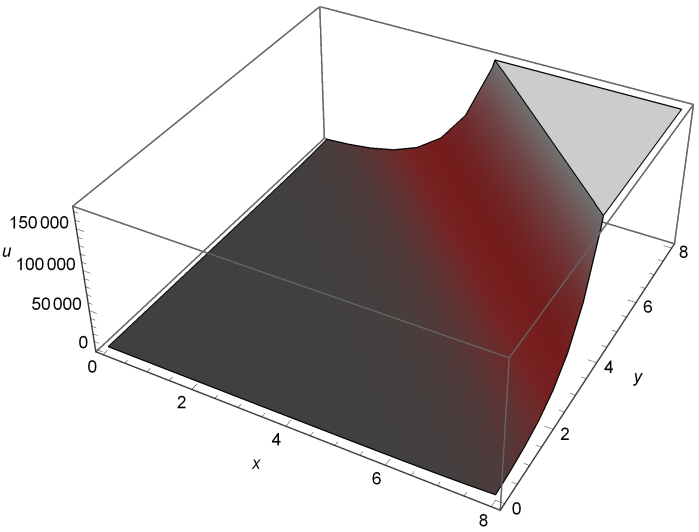
Out[•]=



In[•]:=
```
parsol3 = u[x, y] /. sol3[[1]] /. C[1][a_] → Exp[a]
Plot3D[parsol3, {x, 0, 8}, {y, 0, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "CherryTones"]
```

Out[•]= $e^{x+y} + x$

Out[•]=



In[ ]:=
```
parsol3 = u[x, y] /. sol3[[1]] /. C[1][a_] → a^2
Plot3D[parsol3, {x, 0, 8}, {y, 0, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "Rainbow"]
```

Out[ ]= $x + (x + y)^2$

Out[ ]=