Practical 6 :: Solution of Cauchy Problem for First Order Partial Differential Equation
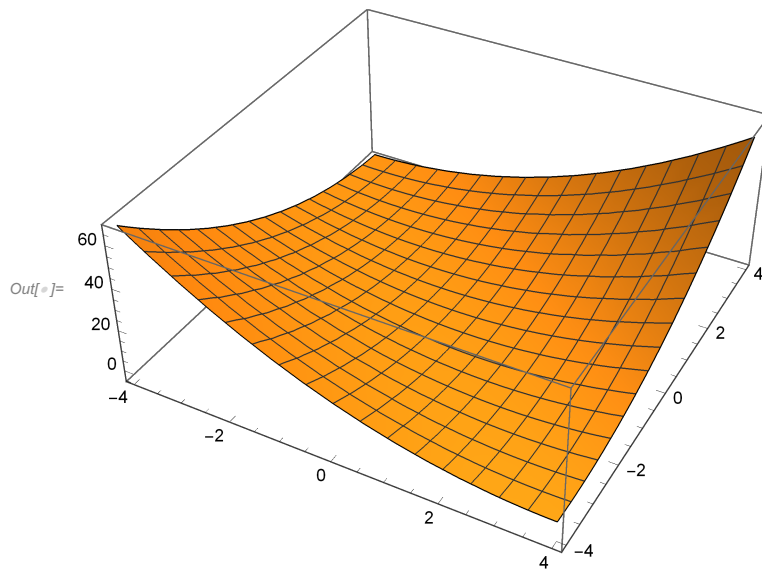
Ques 1 . u_x - u_y= 1, u(x,0)=x^2

In[ ]:= `sol = DSolve[{D[u[x, y], x] - D[u[x, y], y] == 1, u[x, 0] == x^2}, u[x, y], {x, y}]`

Out[ ]= $\{\{u[x, y] \rightarrow x^2 - y + 2 x y + y^2\}\}$

In[ ]:= `Plot3D[u[x, y] /. sol, {x, -4, 4}, {y, -4, 4}]`

Out[ ]=
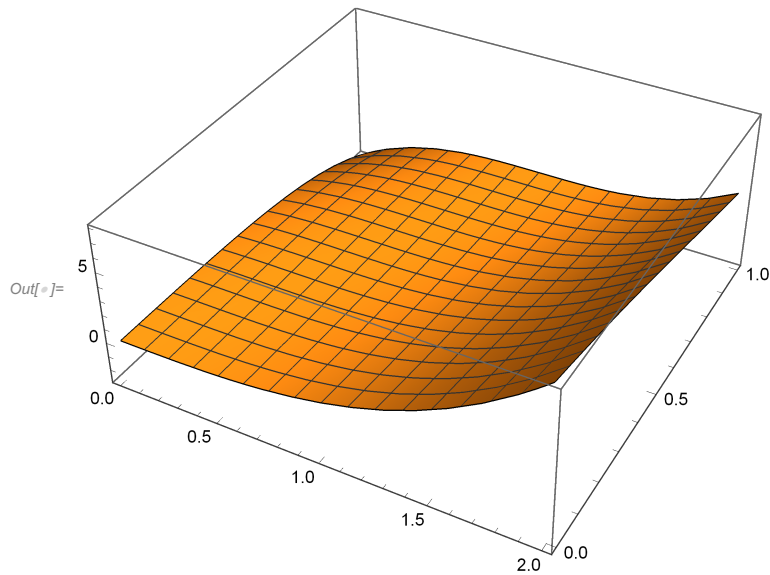

Ques 2 . u_x+u_y = u, u(x,0)=x^3

In[ ]:= `sol2 = DSolve[{D[u[x, y], x] + D[u[x, y], y] == u[x, y], u[x, 0] == x^3}, u[x, y], {x, y}]`

Out[ ]= $\{\{u[x, y] \rightarrow -e^y (-x + y)^3\}\}$

In[ ]:= `Plot3D[u[x, y] /. sol2, {x, 0, 2}, {y, 0, 1}]`

Out[ ]=

Ques 3. yu_x+ xu_y = u , u(0,y)=y^3

In[ ]:= `sol11 =`
`DSolve[{y D[u[x, y], x] + x D[u[x, y], y] == u [x, y], u[0, y] == y^3}, u[x, y], {x, y}]`

Out[ ]= $\left\{\left\{u[x, y] \to -\dfrac{\left(-x^2+y^2\right)^{3/2}\sqrt{1-\dfrac{x}{\sqrt{y^2}}}}{\sqrt{1+\dfrac{x}{\sqrt{y^2}}}}\right\}, \left\{u[x, y] \to -\dfrac{\left(-x^2+y^2\right)^{3/2}\sqrt{1+\dfrac{x}{\sqrt{y^2}}}}{\sqrt{1-\dfrac{x}{\sqrt{y^2}}}}\right\}\right\}$
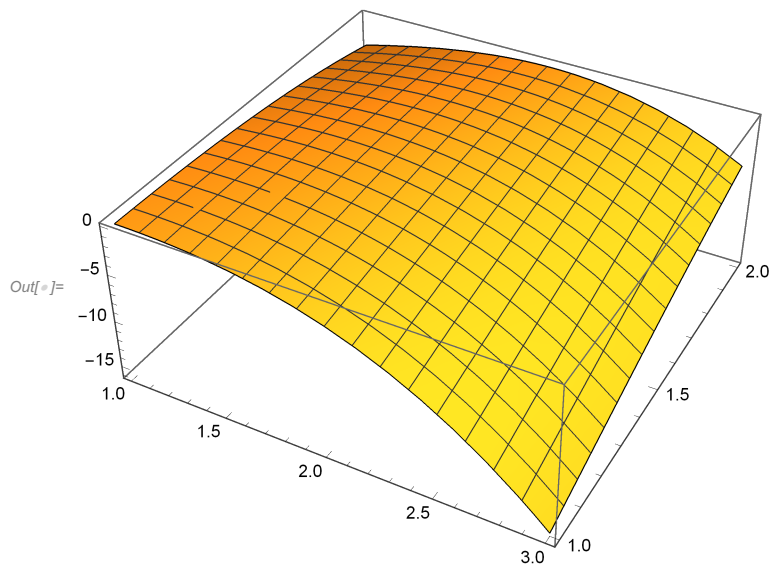
In[ ]:= `sol11[[1, 1]]`

Out[ ]= $u[x, y] \to -\dfrac{\left(-x^2+y^2\right)^{3/2}\sqrt{1-\dfrac{x}{\sqrt{y^2}}}}{\sqrt{1+\dfrac{x}{\sqrt{y^2}}}}$

In[ ]:= `sol11[[2, 1]]`

Out[ ]= $u[x, y] \to -\dfrac{\left(-x^2+y^2\right)^{3/2}\sqrt{1+\dfrac{x}{\sqrt{y^2}}}}{\sqrt{1-\dfrac{x}{\sqrt{y^2}}}}$

In[ ]:= **Plot3D[u[x, y] /. sol11[[1, 1]], {x, 1, 3}, {y, 1, 2}]**

Out[ ]=

In[ ]:= **Plot3D[u[x, y] /. sol11[[2, 1]], {x, 1, 3}, {y, 1, 2}]**
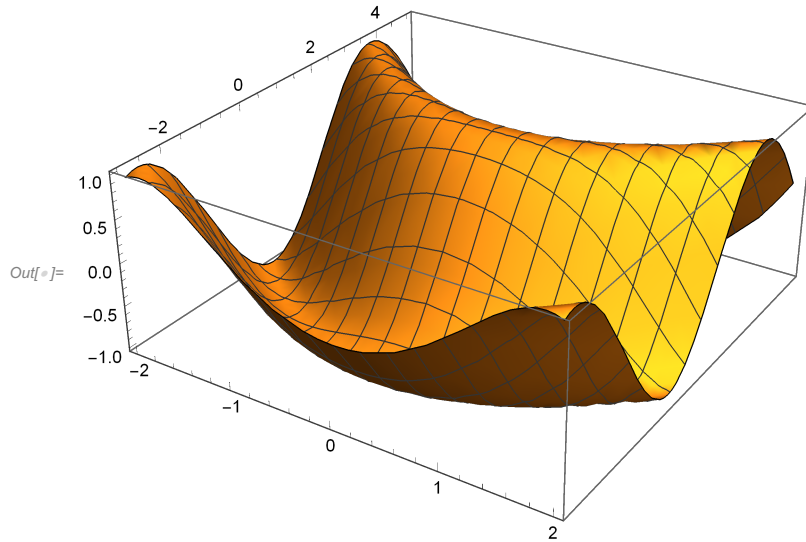
Out[ ]=
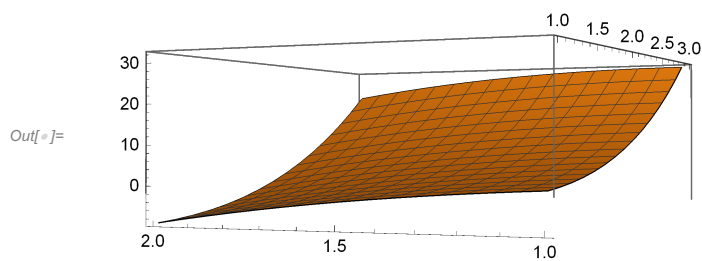
Questions :

Q 4 : $u_x + xu_y = 0$, $u(0,y) = \sin y$

In[ ]:= **sol4 = DSolve[{D[u[x, y], x] + x D[u[x, y], y] == 0 , u[0, y] == Sin[y]}, u[x, y], {x, y}]**

Out[ ]= $\left\{\left\{u[x, y] \rightarrow \mathrm{Sin}\left[\frac{1}{2}\left(-x^2 + 2\,y\right)\right]\right\}\right\}$

Here the initial values have been given for PDE, value of c1 is obtained and hence do not need to put any values (like given in questions above)

In[ ]:= **Plot3D[u[x, y] /. sol4, {x, -2, 2}, {y, -3, 5}]**

Out[ ]=

Q 5 : u (x+y) u_x + u (x-y) u_y = $x^2 + y^2$, u=0 , y=2x

In[ ]:= **sol5 = DSolve[{ u[x, y] * (x + y) D[u[x, y], x] + u[x, y] * (x - y) D[u[x, y], y] == x^2 + y^2,**
      u[x, 2 x] == 0}, u[x, y], {x, y}]

⬝⬝⬝ Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution
    information.

Out[ ]= $\left\{ \left\{ u[x, y] \rightarrow -\sqrt{\frac{2}{7}} \sqrt{2 x^2 + 3 x y - 2 y^2} \right\}, \left\{ u[x, y] \rightarrow \sqrt{\frac{2}{7}} \sqrt{2 x^2 + 3 x y - 2 y^2} \right\}, \right.$

$\left. \left\{ u[x, y] \rightarrow -\sqrt{\frac{2}{7}} \sqrt{2 x^2 + 3 x y - 2 y^2} \right\}, \left\{ u[x, y] \rightarrow \sqrt{\frac{2}{7}} \sqrt{2 x^2 + 3 x y - 2 y^2} \right\} \right\}$
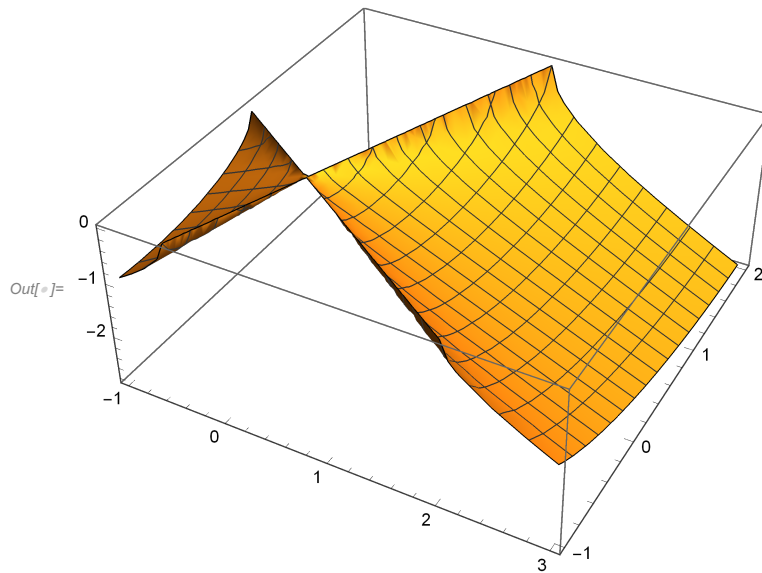
In[ ]:= **sol5[[1, 1]]**

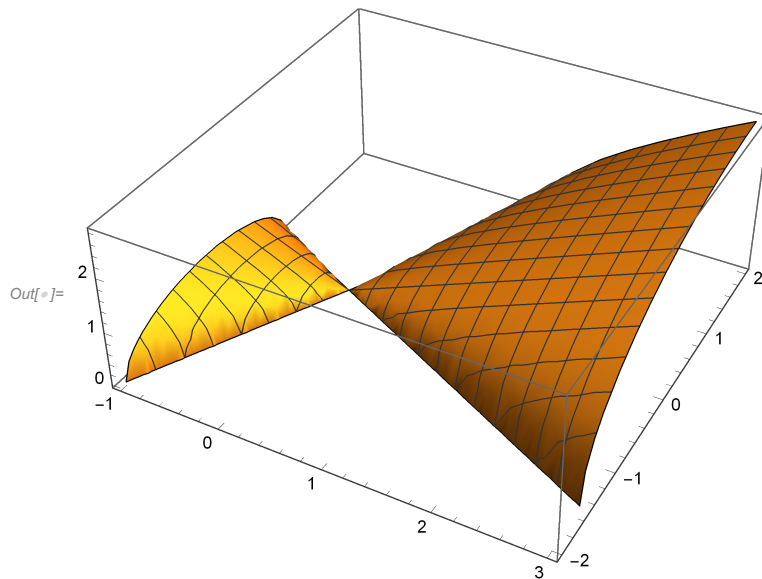*Out[ ]=* $u[x, y] \rightarrow -\sqrt{\frac{2}{7}} \sqrt{2 x^2 + 3 x y - 2 y^2}$

In[ ]:= **sol5[[2, 1]]**

Out[ ]= $u[x, y] \rightarrow \sqrt{\frac{2}{7}} \sqrt{2 x^2 + 3 x y - 2 y^2}$

In[ ]:= **Plot3D[u[x, y] /. sol5[[1, 1]], {x, -1, 3}, {y, -1, 2}]**

Out[ ]=

In[ ]:= **Plot3D[u[x, y] /. sol5[[2, 1]], {x, -1, 3}, {y, -2, 2}]**

Out[ ]=

Q 6 : $u_x + x\, u_y = (y - 1/2\, x^2)$, $u(0, y) = e^y$

In[ ]:= **sol6 = DSolve[**
  {D[u[x, y], x] + x D[u[x, y], y] == y - 1/2 * x^2, u[0, y] == Exp[y]}, u[x, y], {x, y}]

Out[ ]= $\left\{\left\{u[x, y] \rightarrow -\dfrac{1}{2}\, e^{-\frac{x^2}{2}}\left(-2\, e^{y} + e^{\frac{x^2}{2}}\, x^3 - 2\, e^{\frac{x^2}{2}}\, x\, y\right)\right\}\right\}$

In[ ]:= **Plot3D[u[x, y] /. sol6, {x, -2, 2}, {y, -5, 5}]**

Out[ ]=