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Subject : Differential Equations (GE)

# PRACTICAL 1 : Solution of First Order Differential Equation

## ODEs in which there is a single independent variable and one or more dependent variable.

DSolve[eqn, y[x], x] : solving a differential equation for y[x]
DSolve[{eqn1, eqn2, ......}, {y1[x], y2[x],.....}, x] :
Solving a system of differential equation for yi[x]

Ques 1 : Solve $\frac{dy}{dx} = y$

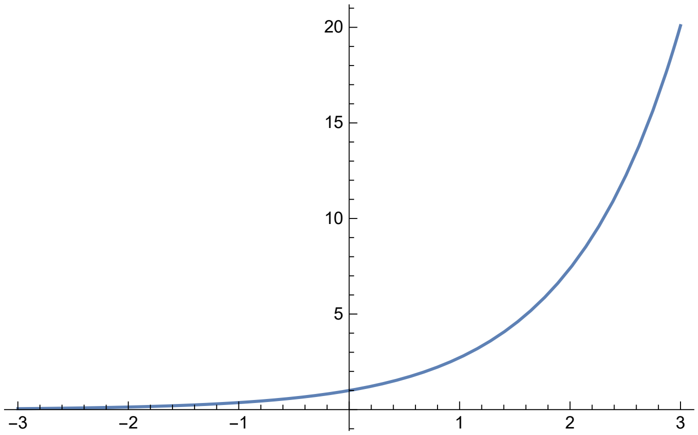
```
In[]:= x =.
In[]:= y =.
In[]:= z =.
In[]:= sol1 = DSolve[{ y'[x] == y[x] }, y[x], x]
Out[]= {{y[x] → e^x C1}}
In[]:= A = y[x] /. sol1 /. {C[1] → 1}
Out[]= {e^x}
```

In[ ]:= y[x]

Out[ ]= y[x]

In[ ]:= Plot[{A} , {x, -3, 3}]

Out[ ]=




You can pick out a specific solution by using /. (Replace All)

---

# Straight Integration

In[ ]:= x =.

In[ ]:= y =.

In[ ]:= Clear All

Out[ ]= All Clear

This equation is solved by simply integrating the right hand side with respect to x:

**Ques 2 : Solve $y' = x^2$**

In[ ]:= sol22 = DSolve[y'[x] == x^2, y[x], x]

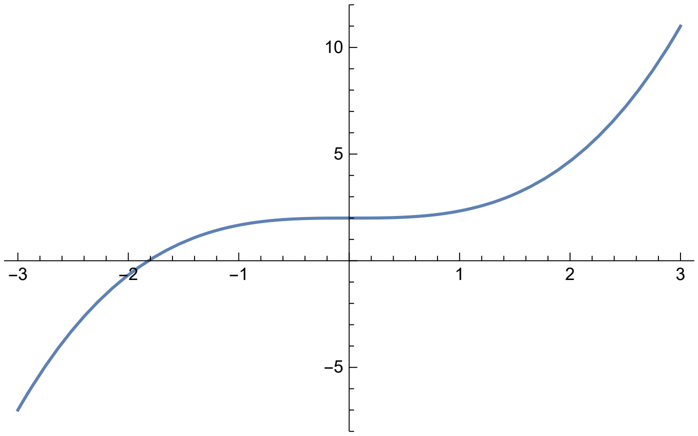
Out[ ]= $\left\{\left\{y[x] \to \frac{x^3}{3} + c_1\right\}\right\}$

In[ ]:= B1 = y[x] /. sol22 /. {C[1] → 2}

Out[ ]= $\left\{2 + \frac{x^3}{3}\right\}$

In[ ]:=
```
Plot[B1, {x, -3, 3}]
```

Out[ ]=



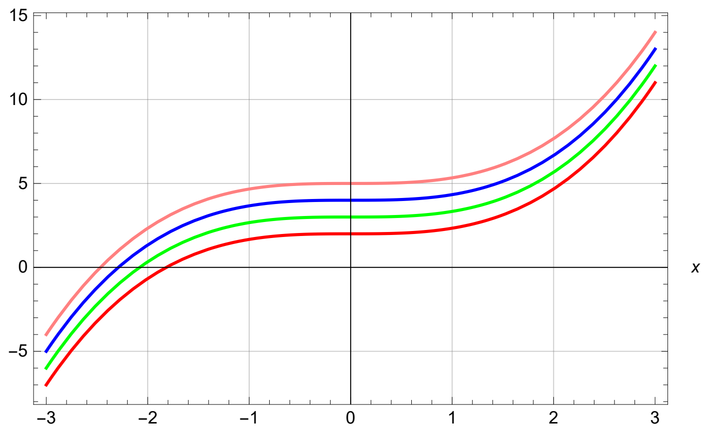

## ANOTHER METHOD :

In[ ]:=
```
B1 = Table[y[x] /. sol22 /. {C[1] → k}, {k, 2, 5}]
```

Out[ ]= $\left\{\left\{2+\frac{x^3}{3}\right\}, \left\{3+\frac{x^3}{3}\right\}, \left\{4+\frac{x^3}{3}\right\}, \left\{5+\frac{x^3}{3}\right\}\right\}$

In[ ]:=
```
Plot[B1, {x, -3, 3}, PlotStyle → {Red, Green, Blue , Pink}, GridLines → Automatic,
  Frame → True, AxesOrigin → {0, 0}, AxesLabel → Automatic, ImageSize → Medium]
```

Out[ ]=




## Separable Equations

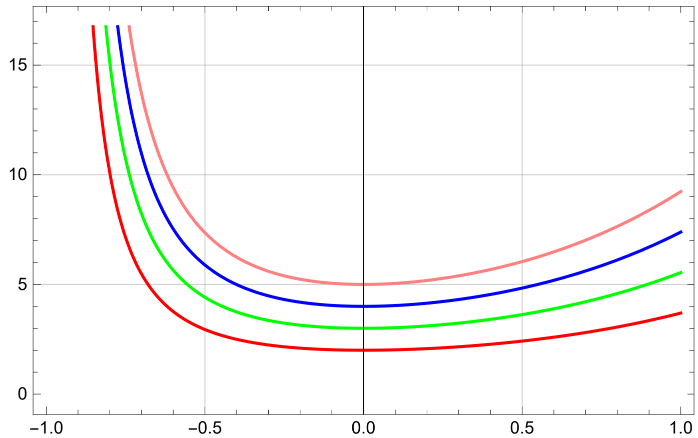
### The general solution to this equation is found by separation of variable:

**Ques 3 : Solve y’=2xy**

In[ ]:=
```
A = DSolve[y'[x] == 2 * x * y[x] / (x + 1), y[x], x]
```

Out[ ]= $\{\{y[x] \to e^{2\,(x-\mathrm{Log}[1+x])}\, c_1\}\}$

In[∘]:=
```
B1 = Table[y[x] /. A /. {C[1] → k}, {k, 2, 5}]
```
Out[∘]=
$$\left\{\left\{2\, e^{2\,(x-\mathrm{Log}[1+x])}\right\},\left\{3\, e^{2\,(x-\mathrm{Log}[1+x])}\right\},\left\{4\, e^{2\,(x-\mathrm{Log}[1+x])}\right\},\left\{5\, e^{2\,(x-\mathrm{Log}[1+x])}\right\}\right\}$$

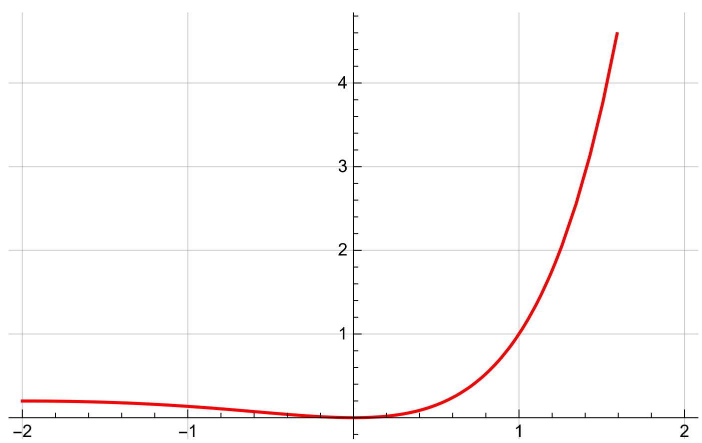
In[∘]:=
```
Plot[{B1}, {x, -1, 1}, PlotStyle → {Red, Green, Blue, Pink},
 GridLines → Automatic, Frame → True, AxesOrigin → {0, 0}]
```
Out[∘]=



## Initial value problem

The solution of this type differential equations does not contain constant

**Ques 4 : Solve $y' = \dfrac{y+2}{xy}$ , y[1] =1**

In[∘]:=
```
sol2 = DSolve[{y'[x] == y[x] + 2/x * y[x], y[1] == 1}, y[x], x]
```
Out[∘]=
$$\left\{\left\{y[x] \to e^{-1+x}\, x^2\right\}\right\}$$

In[∘]:=
```
Plot[y[x] /. sol2, {x, -2, 2}, PlotStyle → {Red}, GridLines → Automatic]
```
Out[∘]=



## Homogenous Equations

here is a homogenous equation in which the total degree of both the numerator and the denominator of the righthand side is name. The two parts of the solution list give branches of the integral curves in the form :

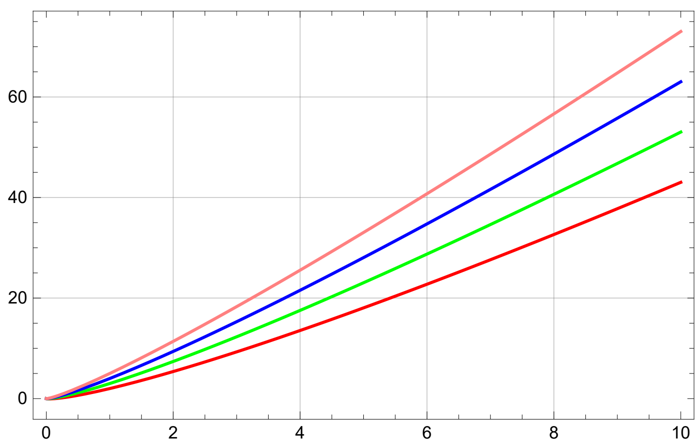
In[ ]:=
```
sol3 = DSolve[y'[x] == (x + y[x]) / (x), y[x], x]
```

Out[ ]= $\{\{y[x] \to x\, c_1 + x \,\text{Log}[x]\}\}$

In[ ]:=
```
sol4 = Table[y[x] /. sol3 /. {C[1] → k}, {k, 2, 5}]
```

Out[ ]= $\{\{2x + x\,\text{Log}[x]\}, \{3x + x\,\text{Log}[x]\}, \{4x + x\,\text{Log}[x]\}, \{5x + x\,\text{Log}[x]\}\}$

In[ ]:=
```
Plot[{sol4}, {x, 0, 10}, PlotStyle → {Red, Green, Blue , Pink},
 GridLines → Automatic, Frame → True, AxesOrigin → {0, 0}]
```

Out[ ]=



## Linear First-Order Equations

The following is a linear first order ODE because both y[x] and its derivative y’[x] occur in it with power 1 and is the highest derivative. Note that the solution contains the imaginary error function Erfi:

**Ques 6 : Solve y’+ y/(x) = $x^2$**

In[ ]:=
```
sol5 = DSolve[y'[x] + y[x] / (x) == x^2, y[x], x]
```

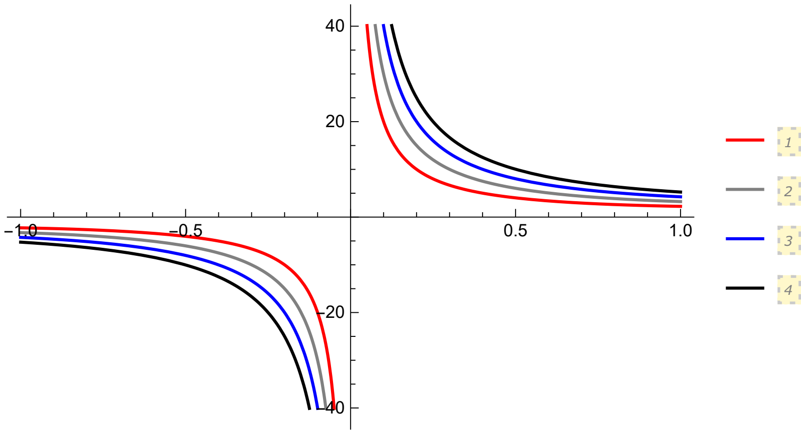
Out[ ]= $\left\{\left\{y[x] \to \frac{x^3}{4} + \frac{c_1}{x}\right\}\right\}$

In[ ]:=
```
sol6 = Table[y[x] /. sol5 /. {C[1] → k}, {k, 2, 5}]
```

Out[ ]= $\left\{\left\{\frac{2}{x} + \frac{x^3}{4}\right\}, \left\{\frac{3}{x} + \frac{x^3}{4}\right\}, \left\{\frac{4}{x} + \frac{x^3}{4}\right\}, \left\{\frac{5}{x} + \frac{x^3}{4}\right\}\right\}$

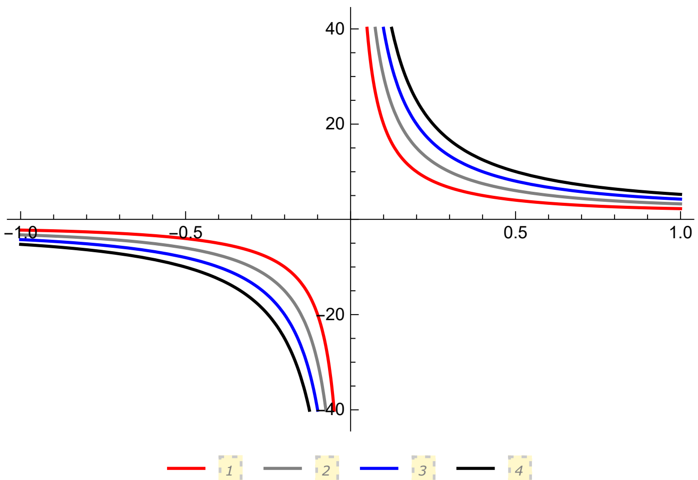
In[ ]:=
```
Plot[{sol6}, {x, -1, 1}, PlotStyle → {Red, Gray, Blue, Black}, PlotLegends → Automatic]
```

Out[ ]=




In[ ]:=
```
Plot[{sol6}, {x, -1, 1}, PlotStyle → {Red, Gray, Blue, Black},
 PlotLegends → Placed[Automatic, Below]]
```

Out[ ]=



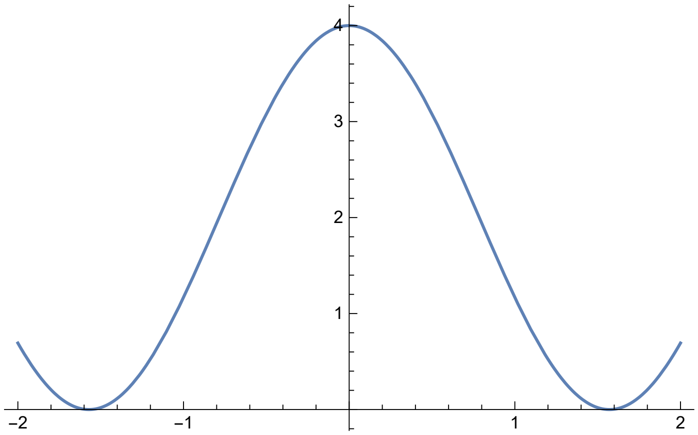

**Ques 7 : $\frac{dy}{dx}$ * tan(x) = 2y - 8 , y[Pi/2] = 0**

In[ ]:=
```
sol7 = DSolve[{y'[x] * Tan[x] == 2 * y[x] - 8, y[Pi / 2] == 0}, y[x], x]
```

Out[ ]=
```
{{y[x] → -4 (-1 + Sin[x]^2)}}
```

In[ ]:= 
```
Plot[y[x] /. sol7, {x, -2, 2}]
```

Out[ ]=



## Bernoulli Equations

A Bernoulli Equation is a first - order equation of the form y’(x) + P(x) y(x) = Q(x) y (x)^n

**Ques 8 : Solve** $x \frac{dy}{dx} + y = y^2 x^2 + 1$

In[ ]:= 
```
sol8 = DSolve[x * y'[x] + y[x] == y[x]^2 x^2 + 1, y[x], x]
```

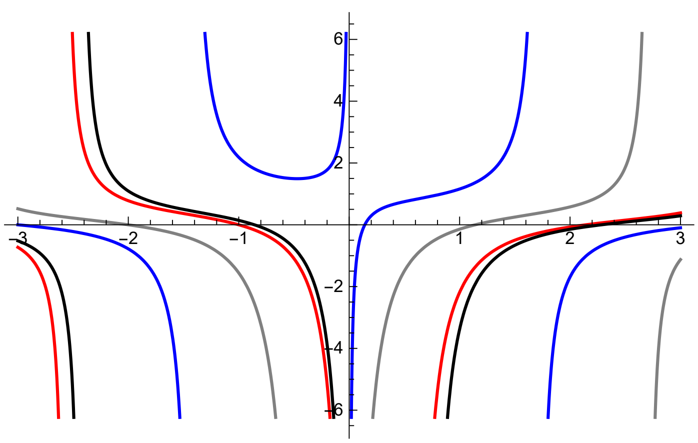
Out[ ]= $\left\{\left\{y[x] \to \frac{\mathrm{Tan}[x + c_1]}{x}\right\}\right\}$

In[ ]:= 
```
sol9 = Table[y[x] /. sol8 /. {C[1] → k}, {k, 1, 4}]
```

Out[ ]= $\left\{\left\{\frac{\mathrm{Tan}[1+x]}{x}\right\}, \left\{\frac{\mathrm{Tan}[2+x]}{x}\right\}, \left\{\frac{\mathrm{Tan}[3+x]}{x}\right\}, \left\{\frac{\mathrm{Tan}[4+x]}{x}\right\}\right\}$

In[ ]:= 
```
Plot[{sol9}, {x, -3, 3}, PlotStyle → {Red, Gray, Blue, Black},
 PlotLegends → Placed["Ëxpressions", Right]]
```

Out[ ]=



## Exact Equations

Let M(x,y) and N(x,y) be two smooth functions having continuous partial derivatives in some domain R^2 without holes. A differential equation, written in differentials :

M(x,y) dx + N(x,y)dy = 0

is called **Exact** if and only if

dM $(x, y)$/dy = dN(x,y)/dz

or there exists a smooth function v(x,y) called the potential function such that its total differential :

dv=M(x,y) dx + N(x,y) =0

### Classify the differential equation as exact or not

```
D[f (x, y), differentiatewithrespectto]
D[f (x, y), x] diffwrtx
```

**Ques 9 : $(xy^2 + x)$ dx + $(x^2 y)$dy = 0**

```
In[•]:= M1[x_, y_] := (x * y^2 + x)

In[•]:= N1[x_, y_] := (y * x^2)

In[•]:= Simplify[D[M1[x, y], y] - D[N1[x, y], x]]
Out[•]= 0

In[•]:= eqn = y'[x] == -M1[x, y[x]] / N1[x, y[x]]
```

Out[•]= $y'[x] == \frac{-x - x\,y[x]^2}{x^2\,y[x]}$

```
In[•]:= sol11 = DSolve[eqn, y[x], x]
```

In[•]:= $\left\{\left\{y[x] \to -\frac{\sqrt{e^{2 c_1} - x^2}}{x}\right\}, \left\{y[x] \to \frac{\sqrt{e^{2 c_1} - x^2}}{x}\right\}\right\}$

Out[•]= $\left\{\left\{y[x] \to -\frac{\sqrt{e^{2 c_1} - x^2}}{x}\right\}, \left\{y[x] \to \frac{\sqrt{e^{2 c_1} - x^2}}{x}\right\}\right\}$

```
In[•]:= p[x_, y_] := -(5 x^2 - 2 y^2 + 11)

In[•]:= q[x_, y_] := (Sin[y] + 4 xy + 3)

In[•]:= Simplify[D[p[x, y], y] - D[q[x, y], x]]
Out[•]= 4 y
```

**Ques 10 : (3x + 2y)dx + (2x+y) dy = 0**

In[2]:=
```
p[x_, y_] := 3 x + 2 y
q[x_, y_] := 2 x + y
Simplify[D[p[x, y], y] - D[q[x, y], x]]
```

Out[4]= 0

**Ques 11 :** $(y^2+3)$ **dx + (2xy - 4)dy = 0**

In[14]:=
```
p[x_, y_] := (y^2 + 3)
q[x_, y_] := (2 * x * y - 4)
Simplify[D[p[x, y], y] - D[q[x, y], x]]
```

Out[16]= 0

**Ques 12 : $(4x + 3y^2)$dx + (2xy) dy = 0**

In[17]:=
```
p[x_, y_] := (4 x + 3 y^2)
q[x_, y_] := (2 * x * y)
Simplify[D[p[x, y], y] - D[q[x, y], x]]
```

Out[19]= 4 y

# PRACTICAL 2 : Solution of Second Order Differential Equation

## Homogenous Linear ODEs of Second Order

**Real and Distinct Roots**

**Ques 1 : y’’ + 5y’- 6y = 0**

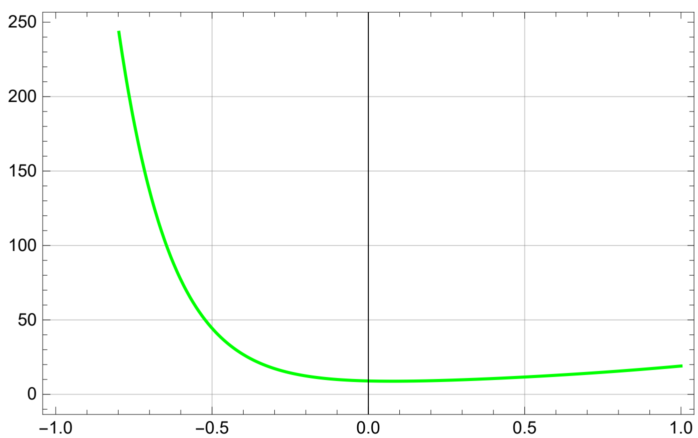
In[ ]:=
```
sol = DSolve[y''[x] + 5 * y'[x] - 6 * y[x] == 0 , y[x], x]
```

Out[ ]= $\{\{y[x] \to e^{-6x} c_1 + e^{x} c_2\}\}$

In[ ]:=
```
sol1 = y[x] /. sol[[1]] /. {C[1] → 2, C[2] → 7}
```

Out[ ]= $2 e^{-6x} + 7 e^{x}$

In[ ]:=
```
Plot[{sol1} , {x, -1, 1}, PlotStyle → {Green},
 Frame → True, AxesOrigin → {0, 0}, GridLines → Automatic]
```

Out[ ]=



## *PLOTTING FAMILY OF SOLUTIONS*

Solve and plot four solutions of the following Differential Equation

y’’+y=0

### Ques 2 : y’’ + y = 0

In[ ]:=
```
Sol = DSolve[y''[x] + y[x] == 0, y[x], x]
```
Out[ ]=
```
{{y[x] → C1 Cos[x] + C2 Sin[x]}}
```

Taking C[1] as a constant

In[ ]:=
```
Sol1 = y[x] /. Sol /. {C[1] → 1, C[2] → 2}
```
Out[ ]=
```
{Cos[x] + 2 Sin[x]}
```

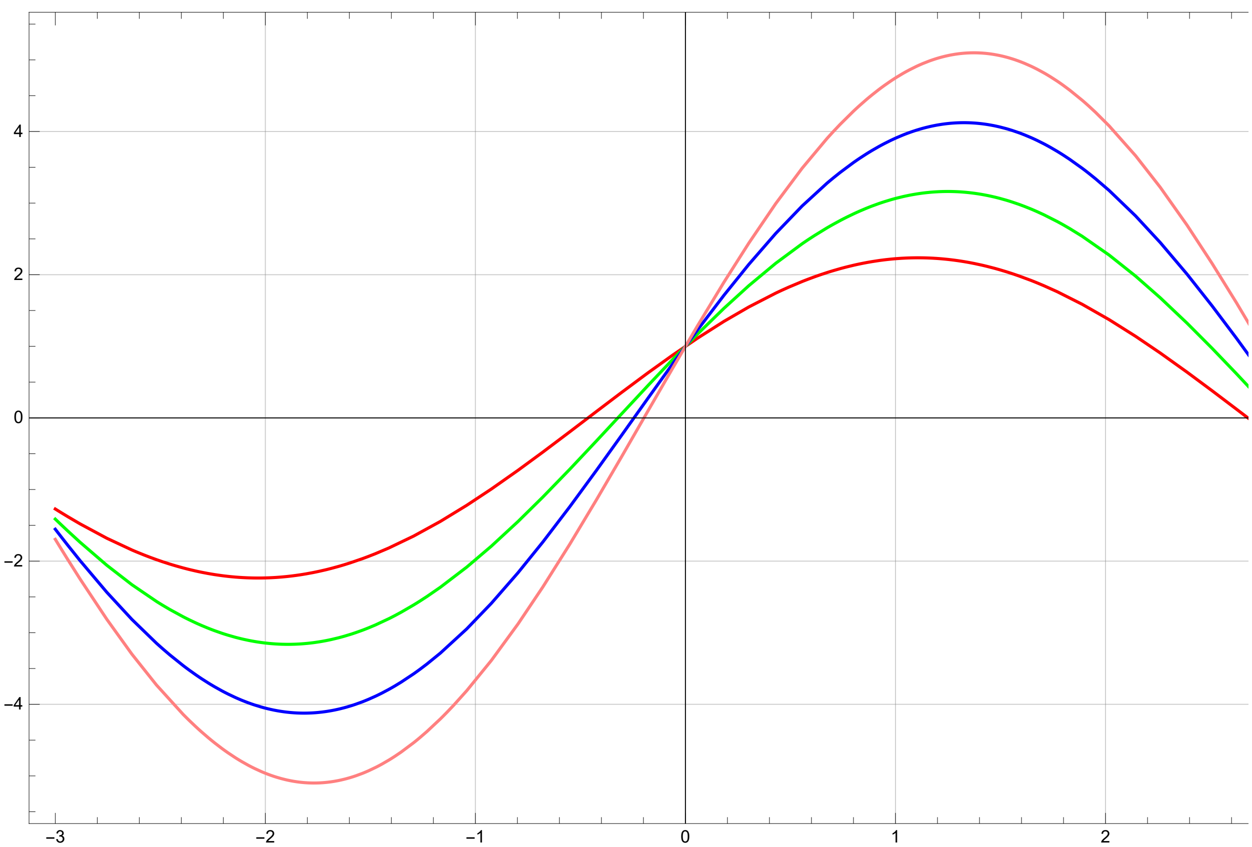
In[ ]:=
```
Sol2 = y[x] /. Sol /. {C[1] → 1, C[2] → 3 }
```
Out[ ]=
```
{Cos[x] + 3 Sin[x]}
```

In[ ]:=
```
Sol3 = y[x] /. Sol /. {C[1] → 1, C[2] → 4}
```
Out[ ]=
```
{Cos[x] + 4 Sin[x]}
```

In[ ]:=
```
Sol4 = y[x] /. Sol /. {C[1] → 1, C[2] → 5}
```
Out[ ]=
```
{Cos[x] + 5 Sin[x]}
```

In[ ]:=
```
Plot[{Sol1, Sol2, Sol3, Sol4}, {x, -3, 3}, PlotStyle → {Red, Green, Blue, Pink},
 Frame → True, AxesOrigin → {0, 0}, GridLines → Automatic, ImageSize → 700,
 PlotLegends → LineLegend[{"Sol1", "Sol2", "Sol3", "Sol4"}, LegendFunction →  "Frame" ]]
```

Out[ ]=



In[•]:=

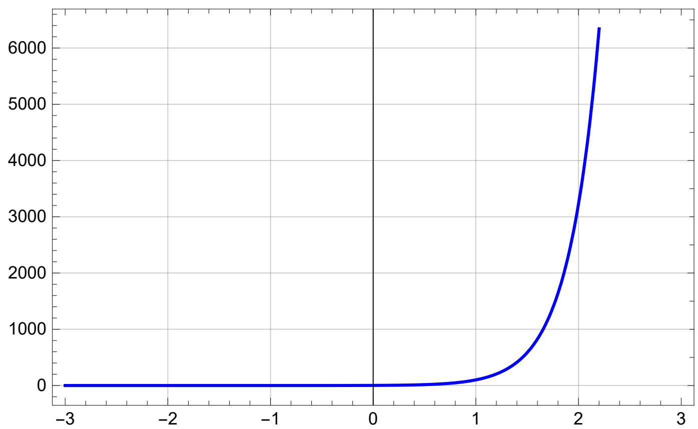
# Real and Equal Roots :

### Ques 3 : y’’ - 6y’+ 9y = 0

In[•]:=
```
sol2 = DSolve[y''[x] - 6 * y'[x] + 9 y[x] == 0, y[x], x]
```
Out[•]= $\{\{y[x] \to e^{3x} c_1 + e^{3x} x c_2\}\}$

In[•]:=
```
sol3 = y[x] /. sol2[[1]] /. {C[1] → 2, C[2] → 3}
```
Out[•]= $2 e^{3x} + 3 e^{3x} x$

In[•]:=
```
Plot[{sol3}, {x, -3, 3}, PlotStyle → {Blue},
 Frame → True, AxesOrigin → {0, 0}, GridLines → Automatic]
```
Out[•]=




### Ques 4 : 4y’’+12y’+9y=0

In[24]:=
```
B = DSolve[y''[x] - 6 * y'[x] + 9 y[x] == 0, y[x], x]
```
Out[24]= $\{\{y[x] \to e^{3x} c_1 + e^{3x} x c_2\}\}$

**Taking C[1] as constant**

In[25]:=
```
B1 = Table[y[x] /. B /. {C[1] → 1, C[2] → k}, {k, 2, 5}]        // TableForm
```
Out[25]//TableForm=

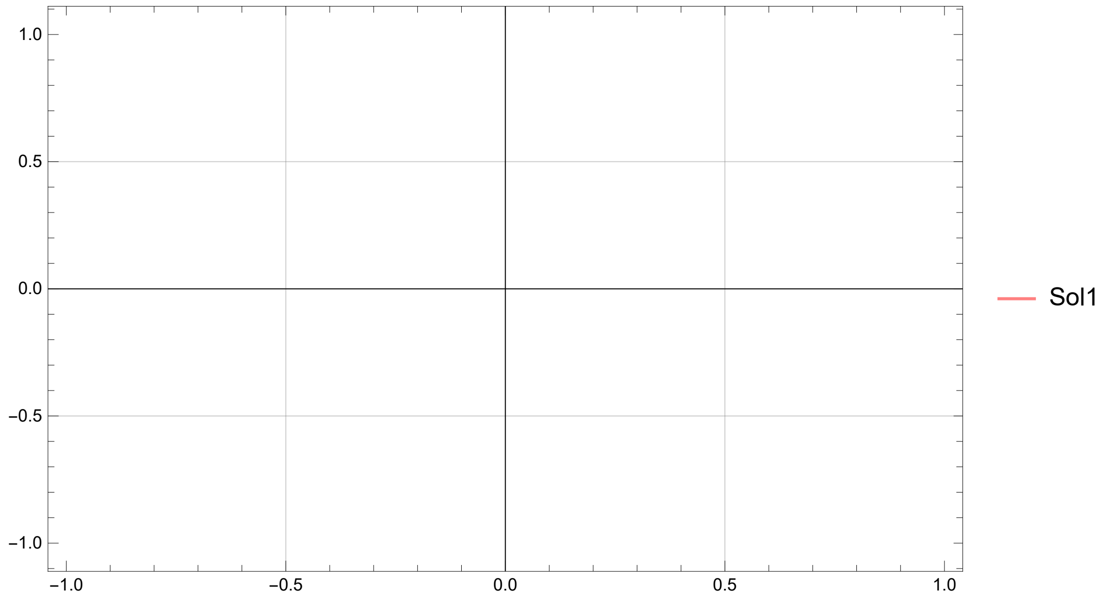
$e^{3x} + 2 e^{3x} x$

$e^{3x} + 3 e^{3x} x$

$e^{3x} + 4 e^{3x} x$

$e^{3x} + 5 e^{3x} x$

In[27]:=
```
Plot[B1, {x, -1, 1}, PlotStyle → {Red, Green, Blue , Pink},
 GridLines → Automatic, Frame → True, AxesOrigin → {0, 0}, ImageSize → 500,
 PlotLegends → LineLegend[{"Sol1", "Sol2", "Sol3", "Sol4"}, LegendFunction → "Frame"]]
```

Out[27]=



## Imaginary Roots

**Ques 5 : 4y''+ y'+ y = 0**

In[ ]:=
```
sol4 = DSolve[y''[x] - y'[x] + y[x] == 0, y[x], x]
```

Out[ ]= $\left\{\left\{y[x] \to e^{x/2} c_1 \mathrm{Cos}\left[\frac{\sqrt{3}\ x}{2}\right] + e^{x/2} c_2 \mathrm{Sin}\left[\frac{\sqrt{3}\ x}{2}\right]\right\}\right\}$

In[ ]:= $\left\{\left\{y[x] \to e^{x/2} c_1 \mathrm{Cos}\left[\frac{\sqrt{3}\ x}{2}\right] + e^{x/2} c_2 \mathrm{Sin}\left[\frac{\sqrt{3}\ x}{2}\right]\right\}\right\}$

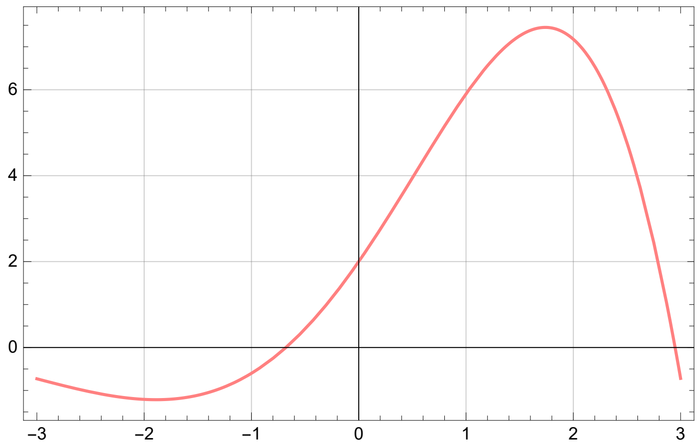
Out[ ]= $\left\{\left\{y[x] \to e^{x/2} c_1 \mathrm{Cos}\left[\frac{\sqrt{3}\ x}{2}\right] + e^{x/2} c_2 \mathrm{Sin}\left[\frac{\sqrt{3}\ x}{2}\right]\right\}\right\}$

In[ ]:=
```
sol5 = y[x] /. sol4[[1]] /. {C[1] → 2, C[2] → 3}
```

Out[ ]= $2 e^{x/2} \mathrm{Cos}\left[\frac{\sqrt{3}\ x}{2}\right] + 3 e^{x/2} \mathrm{Sin}\left[\frac{\sqrt{3}\ x}{2}\right]$

In[ ]:=
```
Plot[{sol5}, {x, -3, 3}, PlotStyle → {Red, Green, Blue, Pink},
 Frame → True, AxesOrigin → {0, 0}, GridLines → Automatic]
```

Out[ ]=




**Ques 6 : y’’-4y’+13y=0**

In[ ]:=
```
c = DSolve[y''[x] - 4 * y'[x] + 13 * y[x] == 0, y[x], x]
```

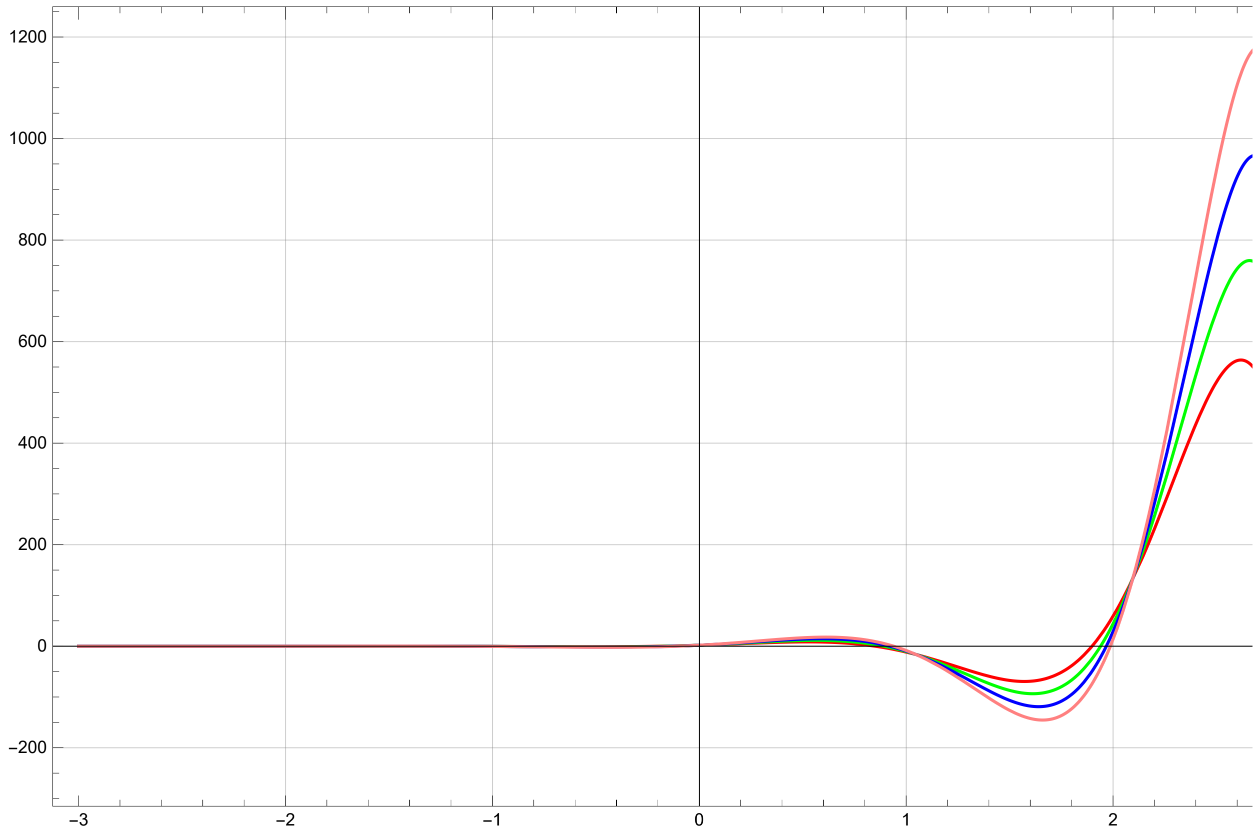
Out[ ]= $\{\{y[x] \to e^{2x} c_2 \text{Cos}[3x] + e^{2x} c_1 \text{Sin}[3x]\}\}$

In[ ]:=
```
c1 = Table[y[x] /. c /. {C[1] → k, C[2] → 2}, {k, 3, 6}]
```

Out[ ]= $\{\{2 e^{2x} \text{Cos}[3x] + 3 e^{2x} \text{Sin}[3x]\}, \{2 e^{2x} \text{Cos}[3x] + 4 e^{2x} \text{Sin}[3x]\}, \{2 e^{2x} \text{Cos}[3x] + 5 e^{2x} \text{Sin}[3x]\}, \{2 e^{2x} \text{Cos}[3x] + 6 e^{2x} \text{Sin}[3x]\}\}$

In[ ]:=
```
Plot[{c1}, {x, -3, 3}, PlotStyle → {Red, Green, Blue, Pink}, GridLines → Automatic,
  Frame → True, AxesOrigin → {0, 0}, PlotRange → All, ImageSize → 700]
```

Out[ ]=




## Initial value problem

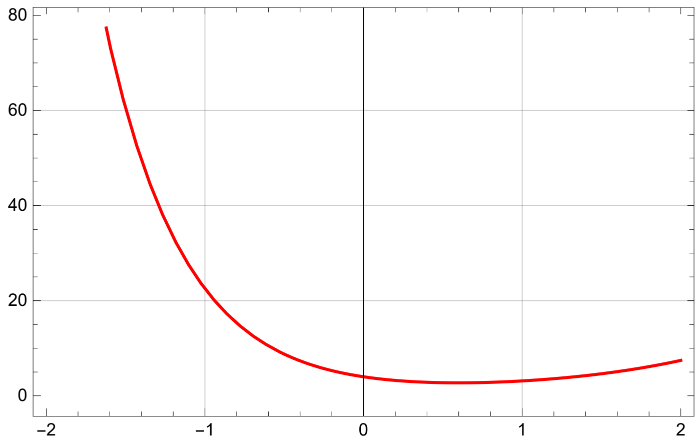
**Ques 7 : y’’ + y’ - 2y = 0**

In[ ]:=
```
pp = DSolve[{y''[x] + y'[x] - 2 * y[x] == 0 , y[0] == 4, y'[0] == -5}, y[x], x]
```

Out[ ]= $\{\{y[x] \to e^{-2x}(3 + e^{3x})\}\}$

In[∘]:=
```
Plot[y[x] /. pp, {x, -2, 2}, PlotStyle → {Red}, GridLines → Automatic, Frame → True]
```

Out[∘]=



## Non - Homogenous Equations

**Ques 8 : y'' - 2y'- 3y = 30$^{2x}$**

In[28]:=
```
r = DSolve[y''[x] - 2*y'[x] - 3*y[x] == 30*Exp[2*x], y[x], x]
```

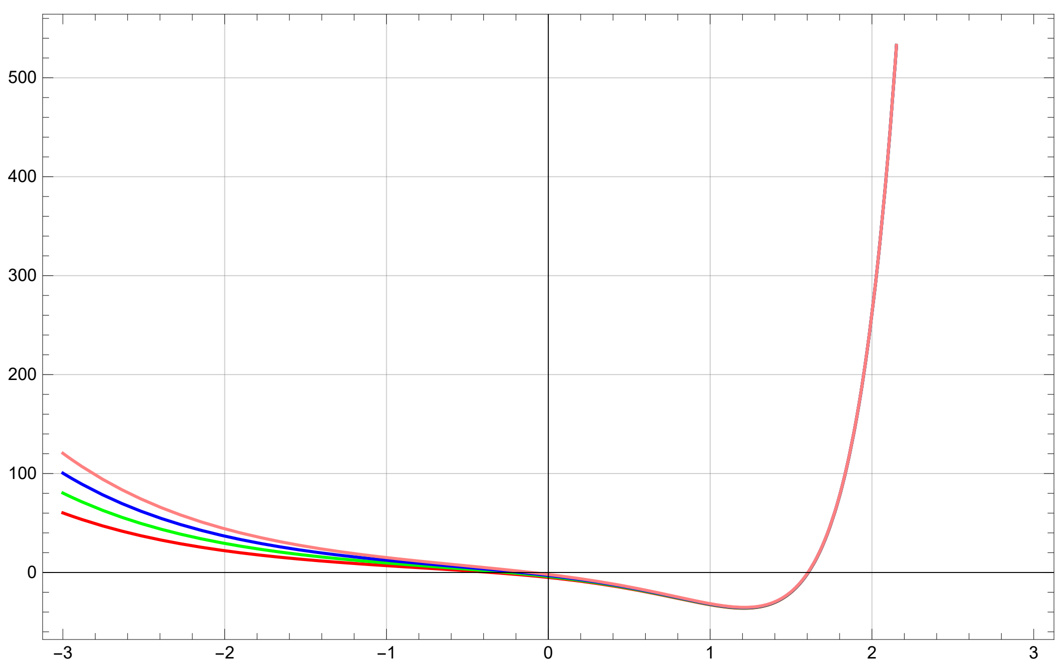
Out[28]= $\{\{y[x] \to -10\, e^{2x} + e^{-x} c_1 + e^{3x} c_2\}\}$

In[29]:=
```
a = Table[y[x] /. r /. {C[1] → k, C[2] → 2}, {k, 3, 6}]
```

Out[29]= $\{\{3\, e^{-x} - 10\, e^{2x} + 2\, e^{3x}\}, \{4\, e^{-x} - 10\, e^{2x} + 2\, e^{3x}\}, \{5\, e^{-x} - 10\, e^{2x} + 2\, e^{3x}\}, \{6\, e^{-x} - 10\, e^{2x} + 2\, e^{3x}\}\}$

In[32]:=
```
Plot[{a}, {x, -3, 3}, PlotStyle → {Red, Green, Blue , Pink},
 GridLines → Automatic, Frame → True, AxesOrigin → {0, 0}]
```

Out[32]=



## Ques 9 : y''-2y'-3y=2sinx

In[ ]:=
```
p = DSolve[y''[x] - 2 * y'[x] - 3 * y[x] == 2 * Sin[x], y[x], x]
```

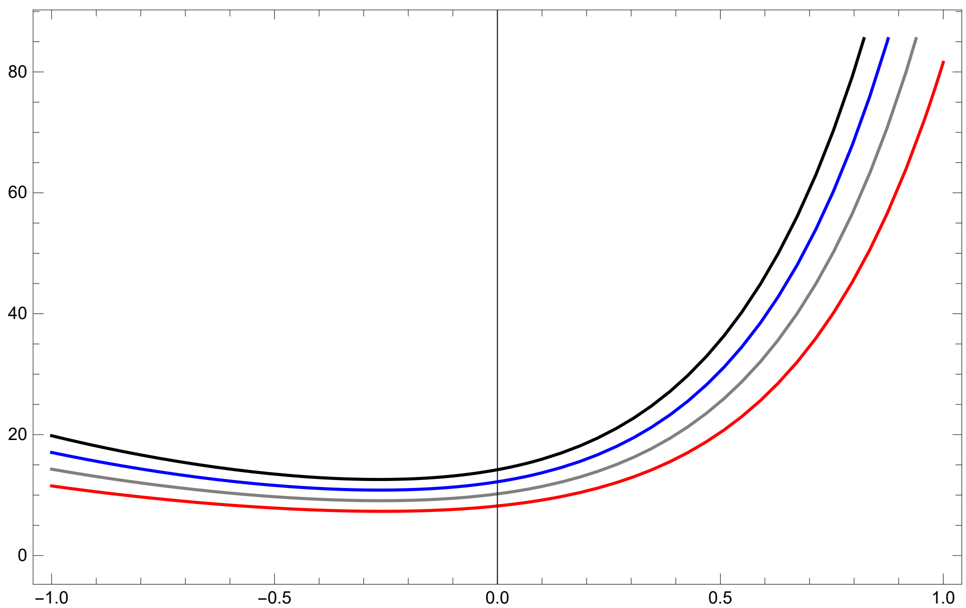
Out[ ]= $\left\{\left\{y[x] \to e^{-x} c_1 + e^{3x} c_2 + \frac{1}{5} (\text{Cos}[x] - 2\,\text{Sin}[x])\right\}\right\}$

**taking C[1] and C[2] both same and varying**

In[ ]:=
```
p1 = Table[y[x] /. p /. {C[1] → m, C[2] → m}, {m, 4, 7}]
```

Out[ ]= $\left\{\left\{4\,e^{-x} + 4\,e^{3x} + \frac{1}{5} (\text{Cos}[x] - 2\,\text{Sin}[x])\right\}, \left\{5\,e^{-x} + 5\,e^{3x} + \frac{1}{5} (\text{Cos}[x] - 2\,\text{Sin}[x])\right\}, \left\{6\,e^{-x} + 6\,e^{3x} + \frac{1}{5} (\text{Cos}[x] - 2\,\text{Sin}[x])\right\}, \left\{7\,e^{-x} + 7\,e^{3x} + \frac{1}{5} (\text{Cos}[x] - 2\,\text{Sin}[x])\right\}\right\}$

In[ ]:=
```
Plot[{p1}, {x, -1, 1}, PlotStyle → {Red, Gray, Blue, Black},
 Frame → True, ImageSize → 500, AxesOrigin → {0, 0}]
```

Out[ ]=


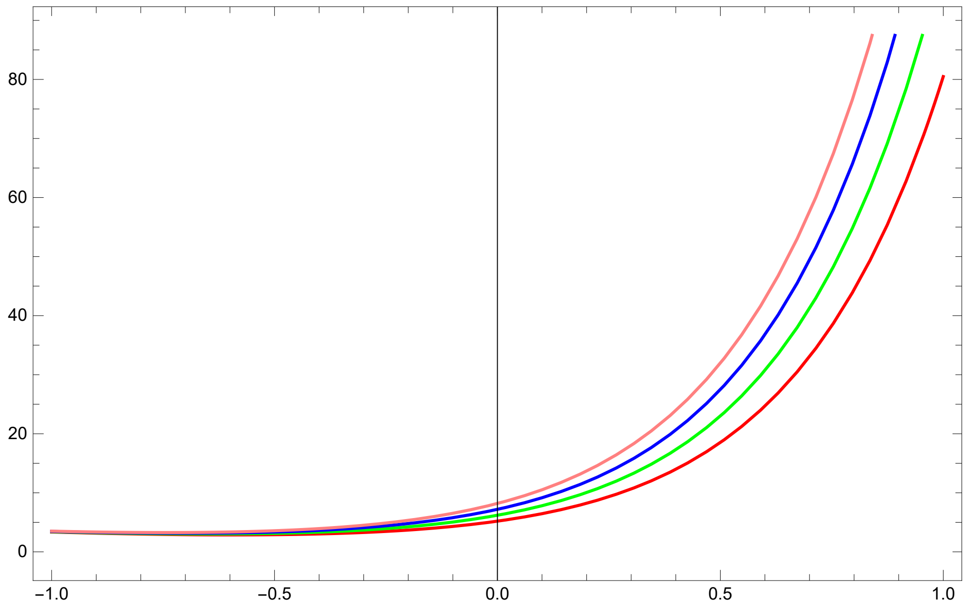

**taking C[1] constant**

In[ ]:=
```
p2 = Table[y[x] /. p /. {C[1] → 1, C[2] → m}, {m, 4, 7}]
```

Out[ ]= $\left\{\left\{e^{-x} + 4\,e^{3x} + \frac{1}{5} (\text{Cos}[x] - 2\,\text{Sin}[x])\right\}, \left\{e^{-x} + 5\,e^{3x} + \frac{1}{5} (\text{Cos}[x] - 2\,\text{Sin}[x])\right\}, \left\{e^{-x} + 6\,e^{3x} + \frac{1}{5} (\text{Cos}[x] - 2\,\text{Sin}[x])\right\}, \left\{e^{-x} + 7\,e^{3x} + \frac{1}{5} (\text{Cos}[x] - 2\,\text{Sin}[x])\right\}\right\}$

In[∘]:=
```
Plot[{p2} , {x, -1, 1}, PlotStyle → {Red, Green, Blue , Pink},
 ImageSize → 500, Frame → True, AxesOrigin → {0, 0}]
```

Out[∘]=




**taking C[2] constant.**

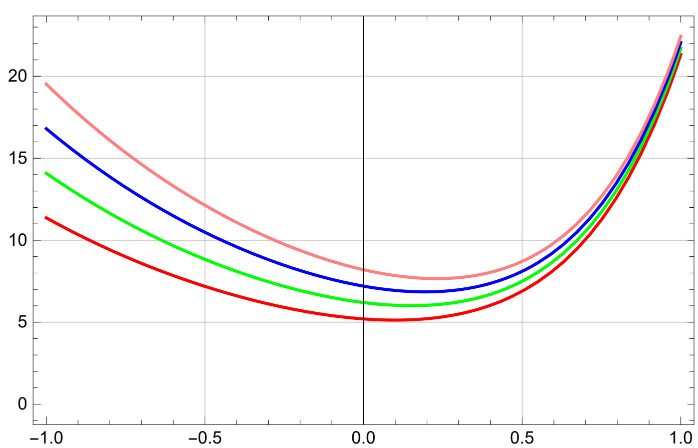
In[∘]:=
```
p3 = Table[y[x] /. p /. {C[1] → m , C[2] → 1}, {m, 4, 7}]
```

Out[∘]=

$$\left\{\left\{4\,e^{-x}+e^{3x}+\frac{1}{5}\left(\mathrm{Cos}[x]-2\,\mathrm{Sin}[x]\right)\right\},\left\{5\,e^{-x}+e^{3x}+\frac{1}{5}\left(\mathrm{Cos}[x]-2\,\mathrm{Sin}[x]\right)\right\},\left\{6\,e^{-x}+e^{3x}+\frac{1}{5}\left(\mathrm{Cos}[x]-2\,\mathrm{Sin}[x]\right)\right\},\left\{7\,e^{-x}+e^{3x}+\frac{1}{5}\left(\mathrm{Cos}[x]-2\,\mathrm{Sin}[x]\right)\right\}\right\}$$

In[∘]:=
```
Plot[{p3}, {x, -1, 1}, PlotStyle → {Red, Green, Blue, Pink},
 GridLines → Automatic, Frame → True, AxesOrigin → {0, 0}]
```

Out[∘]=




## Initial value problems for non - homogenous

**Ques 10 : y’’+y=0.001x^2 , y(0)=0 , y’[0]=1.5**

In[ ]:=
```
q2 = DSolve[{y''[x] + y[x] == 0.001 * x^2 , y[0] == 0, y'[0] == 1.5}, y[x], x]
```

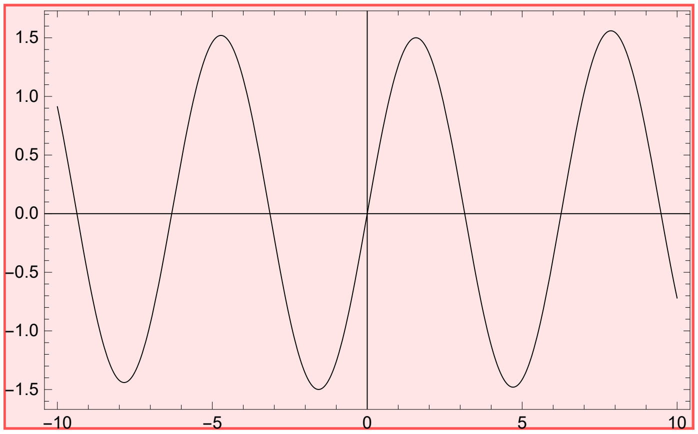
Out[ ]= $\{\{y[x] \to -0.002 + 0.001\, x^2 + 0.002\, \text{Cos}[1.\, x] + 1.5\, \text{Sin}[1.\, x]\}\}$

In[ ]:=
```
q3 = Table[y[x] /. q2]
```

Out[ ]= $\{-0.002 + 0.001\, x^2 + 0.002\, \text{Cos}[1.\, x] + 1.5\, \text{Sin}[1.\, x]\}$

In[ ]:=
```
Plot[q3, {x, -10, 10}, PlotStyle → {Red}. GridLines → Automatic,
 Frame → True, AxesOrigin → {0, 0}, PlotLegends → Automatic]
```

Out[ ]=




## Euler and Cauchy Equations

**Ques 11 :** $x^2 y'' - 2xy' - 4y = 0.001x^2$

In[ ]:=
```
b = DSolve[x^2 * y''[x] - 2 * x * y'[x] - 4 * y[x] == 0, y[x], x]
```

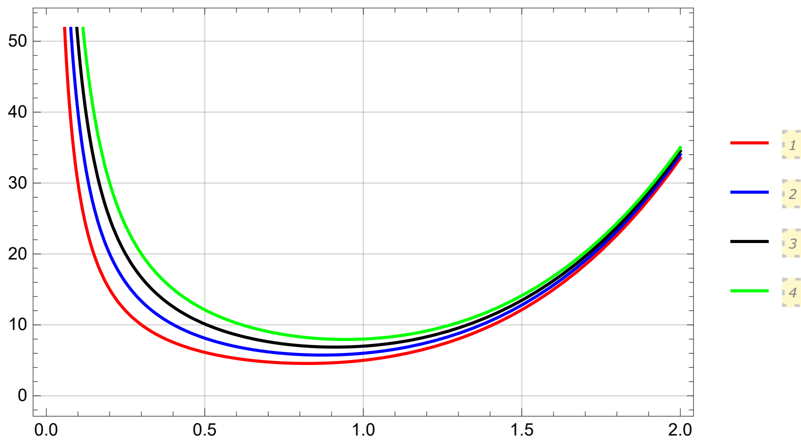
Out[ ]= $\left\{\left\{y[x] \to \frac{\mathbb{c}_1}{x} + x^4\, \mathbb{c}_2\right\}\right\}$

In[ ]:=
```
c = Table[y[x] /. b /. {C[1] → k, C[2] → 2}, {k, 3, 6}]
```

Out[ ]= $\left\{\left\{\frac{3}{x} + 2\,x^4\right\}, \left\{\frac{4}{x} + 2\,x^4\right\}, \left\{\frac{5}{x} + 2\,x^4\right\}, \left\{\frac{6}{x} + 2\,x^4\right\}\right\}$

In[ ]:=
```
Plot[{c}, {x, 0, 2}, PlotStyle → {Red, Blue, Black, Green},
  GridLines → Automatic, Frame → True, AxesOrigin → {0, 0}, PlotLegends → Automatic]
```

Out[ ]=



# PRACTICAL 3 : Plotting of third order solution family of differential equations

## Solve third order differential equations and plot its any three solutions

**Ques 1 : y'' + y = 0.001 x^2**

**Real and Distinct Roots**

In[36]:=
```
sol = DSolve[y'''[x] - 5 * y''[x] + 8 * y'[x] - 4 y[x] == 0 , y[x], x]
```

Out[36]= $\{\{y[x] \to e^{x} c_1 + e^{2x} c_2 + e^{2x} x c_3\}\}$

In[37]:=
```
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 2/3}]
```

Out[37]= $e^{x} + 2 e^{2x} + \frac{2}{3} e^{2x} x$

In[38]:=
```
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 0.5, C[2] → 2, C[3] → 3}]
```

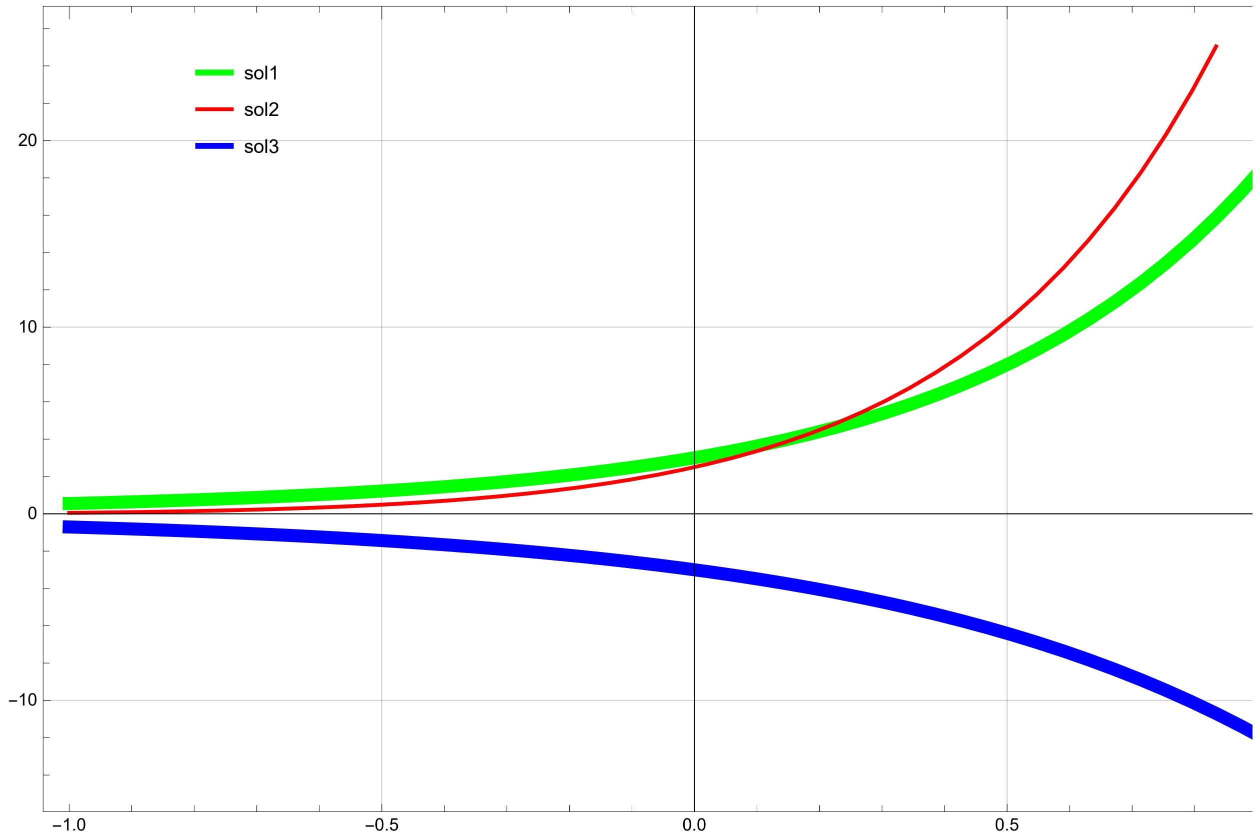
Out[38]= $0.5 e^{x} + 2 e^{2x} + 3 e^{2x} x$

In[39]:=
```
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -1, C[2] → -2, C[3] → 0.5}]
```

Out[39]= $-e^{x} - 2 e^{2x} + 0.5 e^{2x} x$

In[41]:=
```
Plot[{sol1, sol2, sol3} , {x, -1, 1}, ImageSize → 700,
 PlotStyle → {{Green, Thickness[0.01]}, {Red, Thick}, {Blue, Thickness[0.01]}},
 Frame → True, AxesOrigin → {0, 0}, GridLines → Automatic,
 PlotLegends → Placed[LineLegend["Expressions",
    LegendLayout → "Column" , LegendFunction → "Frame"], {0.15, 0.87}]]
```

Out[41]=



## Ques 2 : y’’’+3y’’-25y’+21y=0

```
In[ ]:= sol = DSolve[y'''[x] + 3 * y''[x] - 25 y'[x] + 21 * y[x] == 0, y[x], x]
        sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 3}]
        sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 2, C[2] → 4, C[3] → 8}]
        sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -3, C[2] → -2, C[3] → -4}]
        Plot[{sol1, sol2, sol3}, {x, -1, 1},
         PlotStyle → {{Red, Thickness[0.01]}, {Green, Thicker}, {Blue, thick}},
         Frame → True, ImageSize → 750, PlotLegends → Placed[{"sol1", "sol2", "sol3"}, Left]]
```

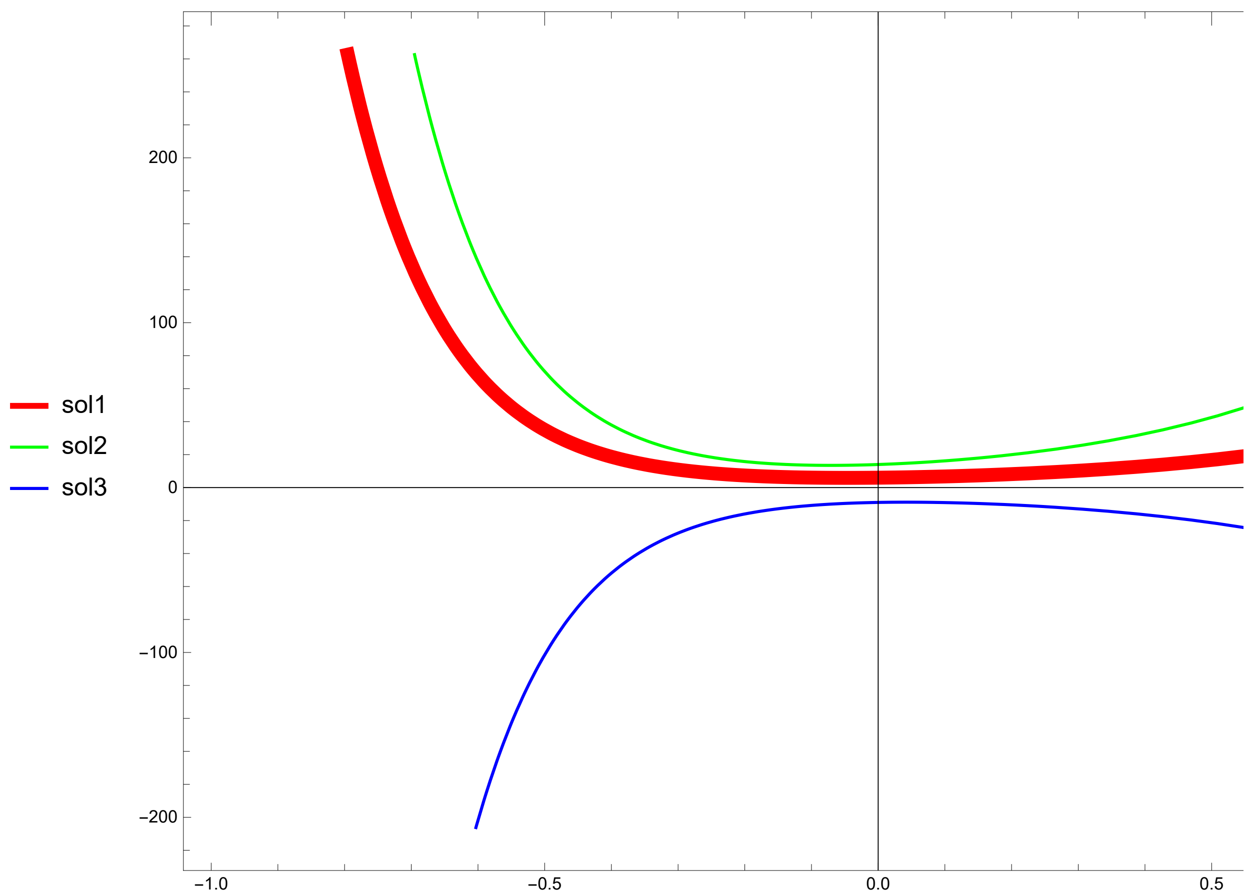
Out[ ]= $\{\{y[x] \to e^{-7x} c_1 + e^{x} c_2 + e^{3x} c_3\}\}$

Out[ ]= $e^{-7x} + 2 e^{x} + 3 e^{3x}$

Out[ ]= $2 e^{-7x} + 4 e^{x} + 8 e^{3x}$

Out[ ]= $-3 e^{-7x} - 2 e^{x} - 4 e^{3x}$

Out[ ]=



## Ques 3 : y’’’-4y’’-25y’+28y=0

In[42]:=
```
Sol = DSolve[y'''[x] - 13 y''[x] + 19 y'[x] + 33 y[x] == Cos[2 x], y[x], x]
sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 3}]
sol2 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → -1, C[2] → -2, C[3] → 1.3}]
sol3 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 3/2, C[2] → 0.5, C[3] → -2.5}]
Plot[{sol1, sol2, sol3}, {x, -1, 1},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick}, {Blue, Thickness[0.01]}},
 Frame → True, ImageSize → 750, PlotLegends → Placed[{"sol1", "sol2", "sol3"}, Below]]
```

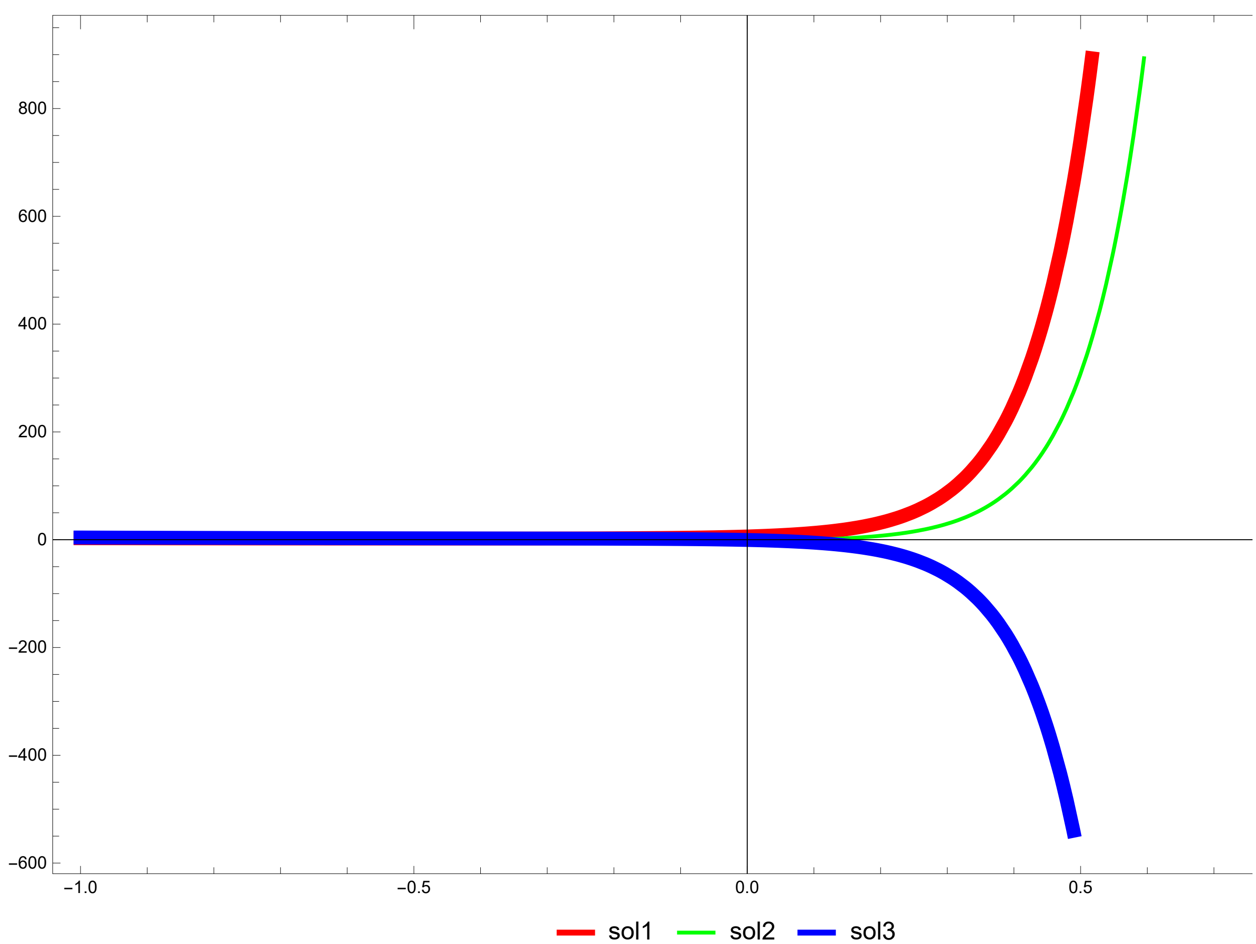
Out[42]= $\left\{\left\{y[x] \to e^{-x} c_1 + e^{3x} c_2 + e^{11x} c_3 + \frac{17 \text{Cos}[2x] + 6 \text{Sin}[2x]}{1625}\right\}\right\}$

Out[43]= $e^{-x} + 2 e^{3x} + 3 e^{11x} + \frac{17 \text{Cos}[2x] + 6 \text{Sin}[2x]}{1625}$

Out[44]= $-e^{-x} - 2 e^{3x} + 1.3 e^{11x} + \frac{17 \text{Cos}[2x] + 6 \text{Sin}[2x]}{1625}$

Out[45]= $\frac{3 e^{-x}}{2} + 0.5 e^{3x} - 2.5 e^{11x} + \frac{17 \text{Cos}[2x] + 6 \text{Sin}[2x]}{1625}$

Out[46]=



## Ques 4 : Solve third order differential Equation and plot its any three solutions

## y'''-13y''+19y'+33y=Cos2x

In[ ]:=
```
sol = DSolve[y'''[x] - 13 * y''[x] + 19 * y'[x] + 33 * y[x] == Cos[2 x] , y[x] , x]
```

Out[ ]=
$$\left\{\left\{y[x] \to e^{-x} c_1 + e^{3x} c_2 + e^{11x} c_3 + \frac{17 \operatorname{Cos}[2x] + 6 \operatorname{Sin}[2x]}{1625}\right\}\right\}$$

In[ ]:=
```
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 3}]
```

Out[ ]=
$$e^{-x} + 2 e^{3x} + 3 e^{11x} + \frac{17 \operatorname{Cos}[2x] + 6 \operatorname{Sin}[2x]}{1625}$$

In[ ]:=
```
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 2, C[2] → 4, C[3] → 8}]
```

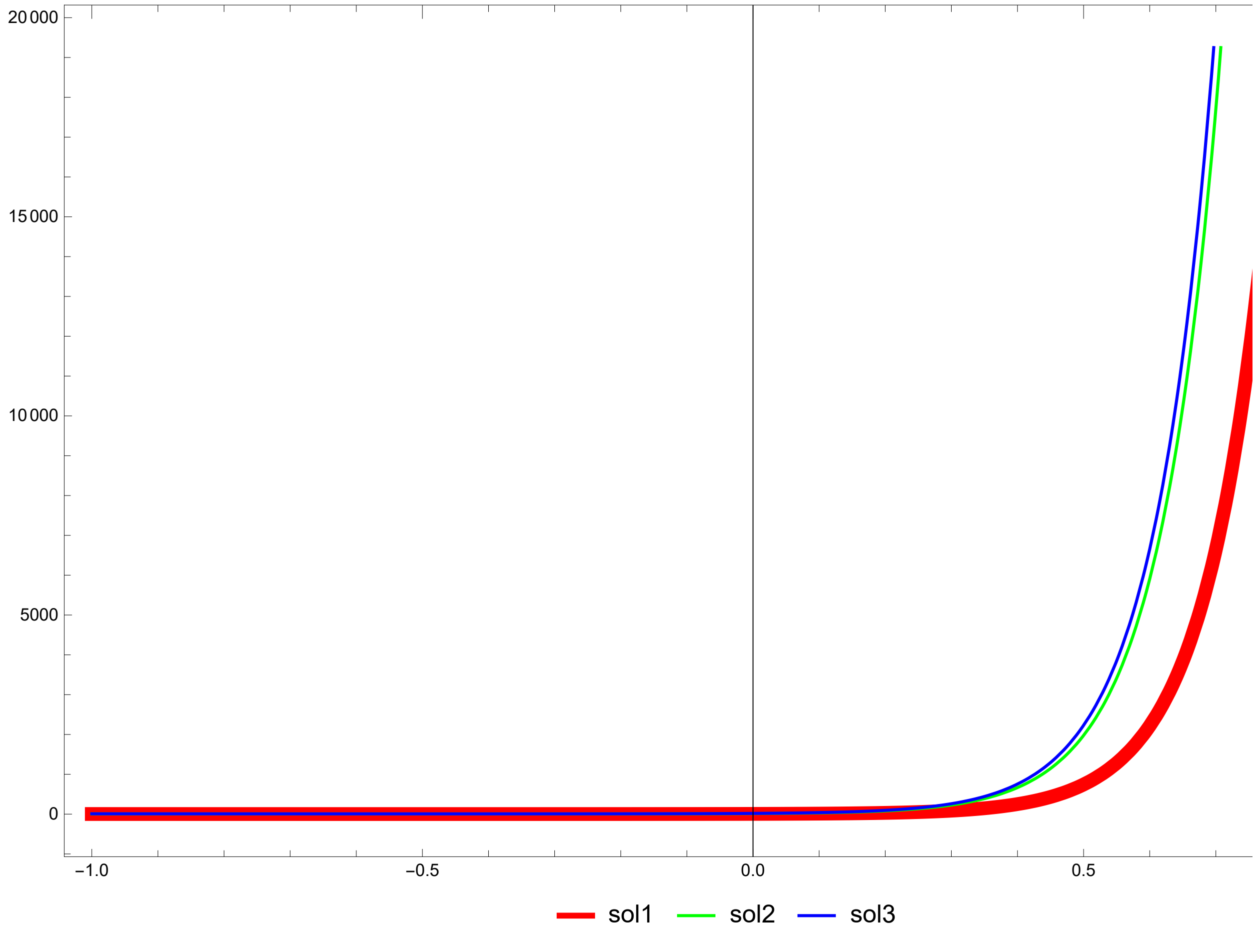
Out[ ]=
$$2 e^{-x} + 4 e^{3x} + 8 e^{11x} + \frac{17 \operatorname{Cos}[2x] + 6 \operatorname{Sin}[2x]}{1625}$$

In[ ]:=
```
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 3, C[2] → 6, C[3] → 9}]
```

Out[ ]=
$$3 e^{-x} + 6 e^{3x} + 9 e^{11x} + \frac{17 \operatorname{Cos}[2x] + 6 \operatorname{Sin}[2x]}{1625}$$

In[ ]:=
```
Plot[{sol1, sol2, sol3}, {x, -1, 1},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thicker}, {Blue, thick}},
 Frame → True, ImageSize → 750, PlotLegends → Placed[{"sol1", "sol2", "sol3"}, Below]]
```

Out[ ]=



# Solve the following differential equations

## Ques 5 : y''' -6y''+5y' + 12y = 0

In[•]:=
```
sol = DSolve[y'''[x] - 6 * y''[x] + 5 y'[x] + 12 * y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 3}]
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 2, C[2] → 4, C[3] → 8}]
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1/3, C[2] → 1/2, C[3] → 2/3}]
Plot[{sol1, sol2, sol3}, {x, -1, 1},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thicker}, {Blue, thick}},
 Frame → True, ImageSize → 750, PlotLegends → Placed[{"sol1", "sol2", "sol3"}, Top]]
```

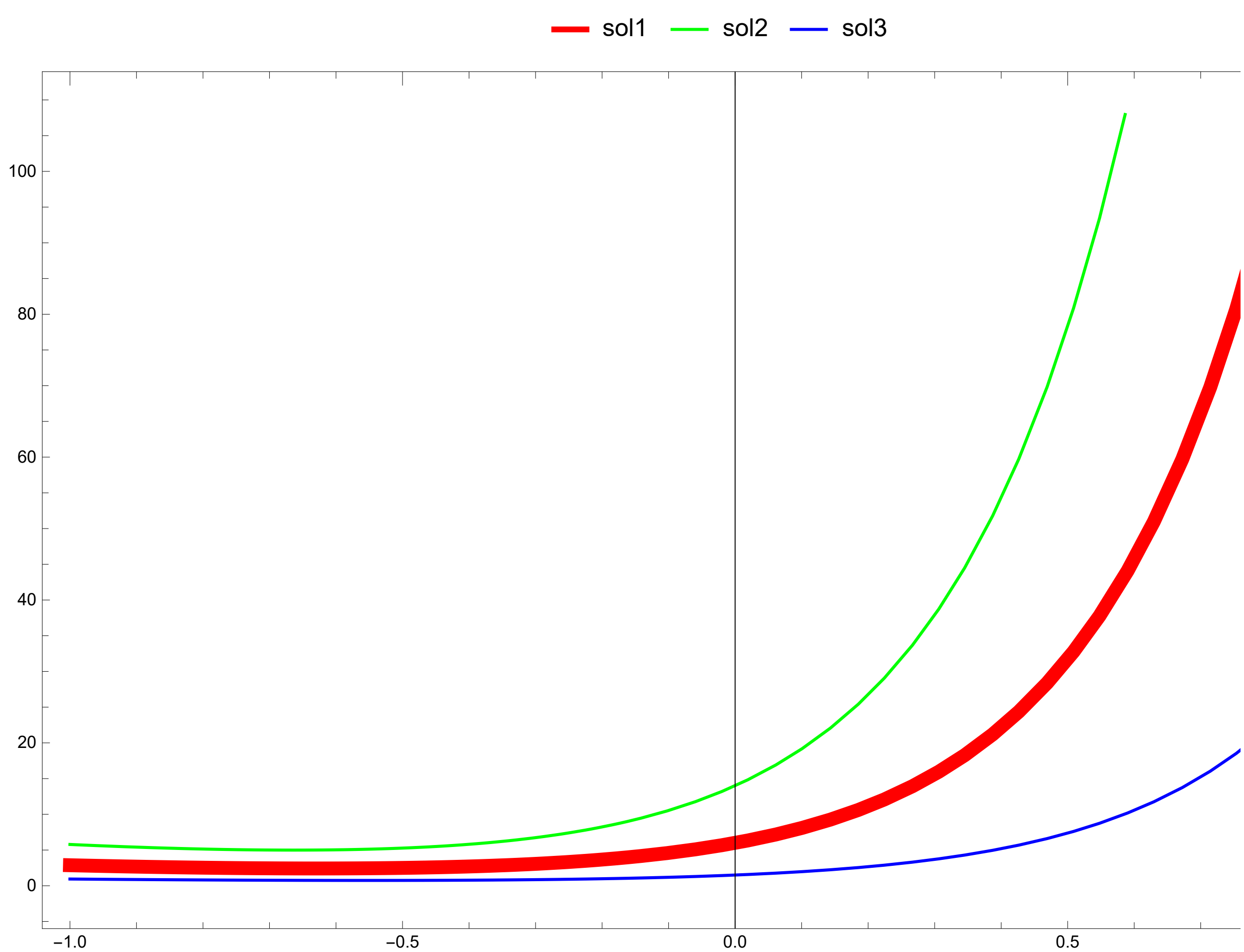
Out[•]= $\{\{y[x] \to e^{-x} c_1 + e^{3x} c_2 + e^{4x} c_3\}\}$

Out[•]= $e^{-x} + 2 e^{3x} + 3 e^{4x}$

Out[•]= $2 e^{-x} + 4 e^{3x} + 8 e^{4x}$

Out[•]= $\frac{e^{-x}}{3} + \frac{e^{3x}}{2} + \frac{2 e^{4x}}{3}$

Out[•]=



In[ ]:=

## Ques 6 : y'''-6y''+11y' -6y=0, y(0)=0, y'(0)=0, y''(0)=2

In[ ]:=
```
sol = DSolve[y'''[x] - 6 * y''[x] + 11 y'[x] - 6 * y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 3}]
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 2, C[2] → 4, C[3] → 8}]
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -3, C[2] → -2, C[3] → -4}]
Plot[{sol1, sol2, sol3}, {x, -1, 1},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thicker}, {Blue, thick}},
 Frame → True, ImageSize → 750, PlotLegends → Placed[{"sol1", "sol2", "sol3"}, Left]]
```

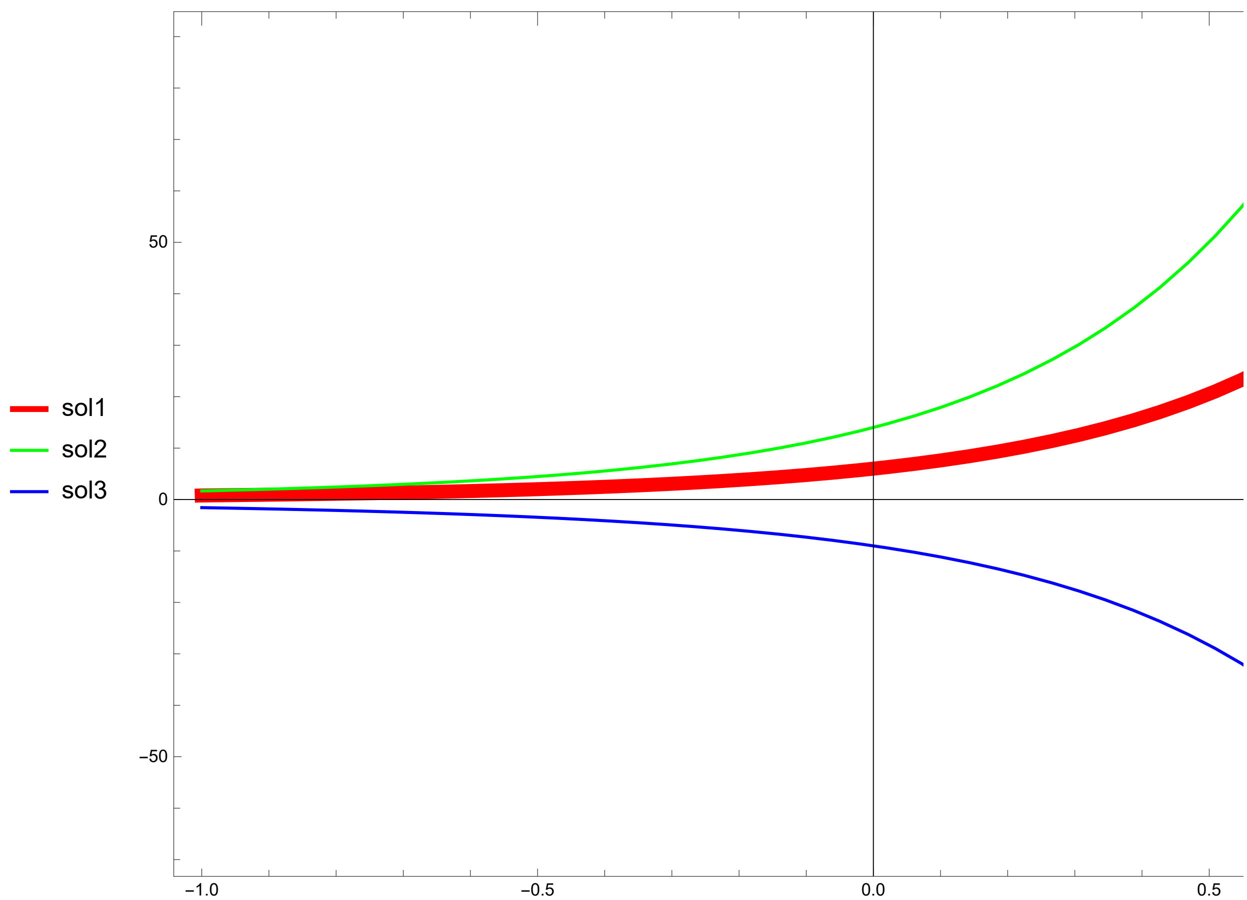
Out[ ]= $\{\{y[x] \to e^{x} c_1 + e^{2x} c_2 + e^{3x} c_3\}\}$

Out[ ]= $e^{x} + 2 e^{2x} + 3 e^{3x}$

Out[ ]= $2 e^{x} + 4 e^{2x} + 8 e^{3x}$

Out[ ]= $-3 e^{x} - 2 e^{2x} - 4 e^{3x}$

Out[ ]=



## Ques 7: y''' + y' = sec x

```
sol = DSolve[y'''[x] + 0 * y''[x] + y'[x] + 0 * y[x] == Sec[x], y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 18}]
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -2, C[2] → 1.0, C[3] → 0}]
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -1, C[2] → 0.5, C[3] → -2}]
Plot[{sol1, sol2, sol3}, {x, -3, 3},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thickness[0.1]}, {Blue, thick}},
 Frame → True, ImageSize → 750, PlotLegends → Placed[{"sol1", "sol2", "sol3"}, Below]]
```

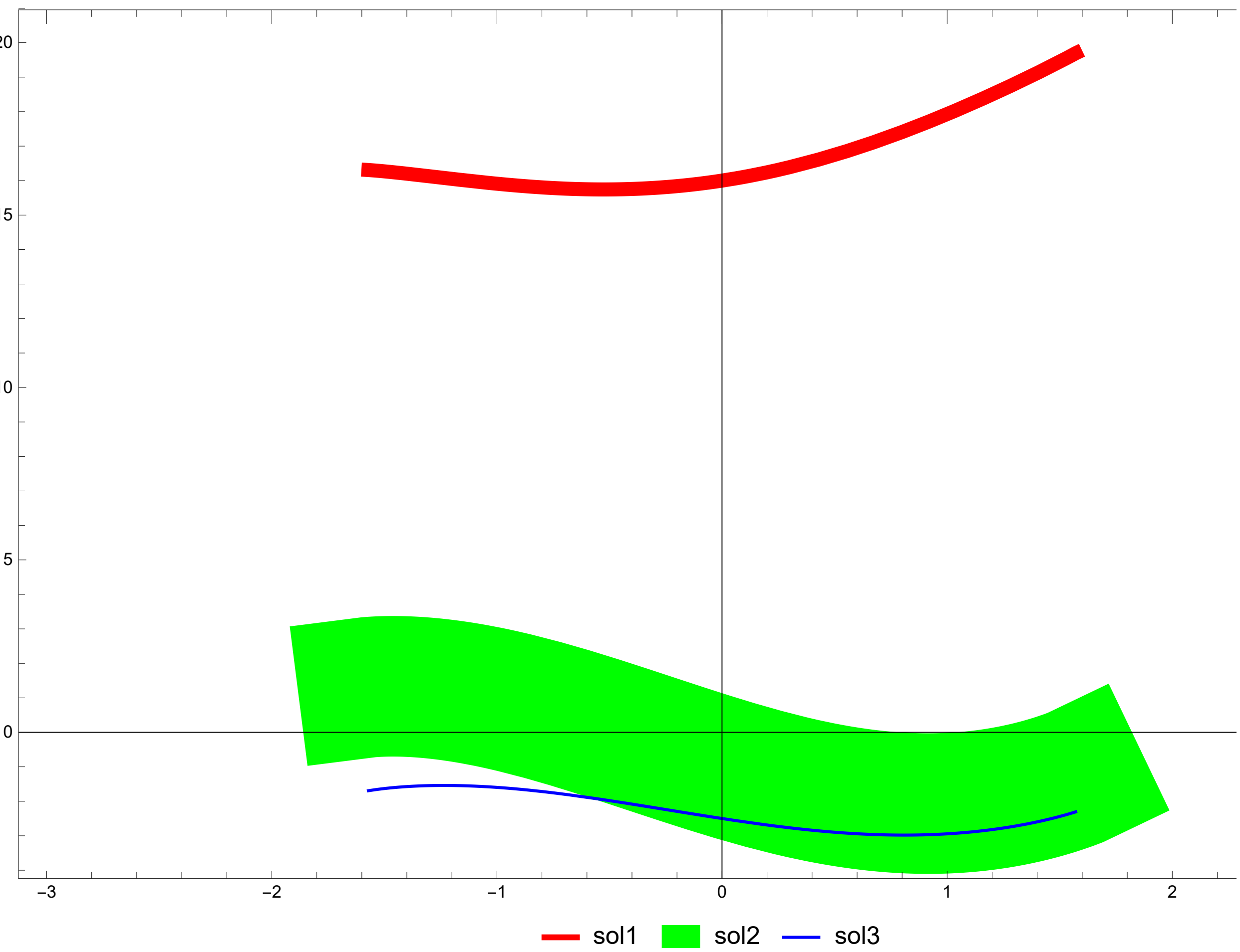
Out[57]= $\left\{\left\{y[x] \to c_3 - x\,\mathrm{Cos}[x] - c_2\,\mathrm{Cos}[x] - \mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right] - \mathrm{Sin}\left[\frac{x}{2}\right]\right] + \mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right] + \mathrm{Sin}\left[\frac{x}{2}\right]\right] + c_1\,\mathrm{Sin}[x] + \mathrm{Log}[\mathrm{Cos}[x]]\,\mathrm{Sin}[x]\right\}\right\}$

Out[58]= $18 - 2\,\mathrm{Cos}[x] - x\,\mathrm{Cos}[x] - \mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right] - \mathrm{Sin}\left[\frac{x}{2}\right]\right] + \mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right] + \mathrm{Sin}\left[\frac{x}{2}\right]\right] + \mathrm{Sin}[x] + \mathrm{Log}[\mathrm{Cos}[x]]\,\mathrm{Sin}[x]$

Out[59]= $-1.\,\mathrm{Cos}[x] - x\,\mathrm{Cos}[x] - \mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right] - \mathrm{Sin}\left[\frac{x}{2}\right]\right] + \mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right] + \mathrm{Sin}\left[\frac{x}{2}\right]\right] - 2\,\mathrm{Sin}[x] + \mathrm{Log}[\mathrm{Cos}[x]]\,\mathrm{Sin}[x]$

Out[60]= $-2 - 0.5\,\mathrm{Cos}[x] - x\,\mathrm{Cos}[x] - \mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right] - \mathrm{Sin}\left[\frac{x}{2}\right]\right] + \mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right] + \mathrm{Sin}\left[\frac{x}{2}\right]\right] - \mathrm{Sin}[x] + \mathrm{Log}[\mathrm{Cos}[x]]\,\mathrm{Sin}[x]$

Out[61]=



# PRACTICAL 4 : SOLUTION OF DIFFERENTIAL EQUATION BY VARIATION OF PARAMETER

Second Order DE

d2y/dx2 + p dy/dx + qy = f(x)

where p and q are constants, f(x) is a non zero function of x.

General solution of homogenous equation d2y/dx2 + p dy/dx + qt =0

Particular solutions of the non- homogenous equation d2y/dx2 + pdy/dx+qy = f(x)

## Ques 1 : y’’[x]+y[x]==2Sin[x]

In[ ]:=
```
ClearAll
```
Out[ ]=
```
ClearAll
```
In[ ]:=
```
gsh = DSolve[y''[x] + y[x] == 0, y[x], x]
gsh1 = y[x] /. gsh
```
Out[ ]= $\{\{y[x] \to c_1 \text{Cos}[x] + c_2 \text{Sin}[x]\}\}$

Out[ ]= $\{c_1 \text{Cos}[x] + c_2 \text{Sin}[x]\}$

In[ ]:=
```
y1 := Cos[x]
y2 := Sin[x]
f := 2 * Sin[x]
w = y1 * D[y2, x] - y2 * D[y1, x]
w = Simplify[w]
```

Out[ ]= $\text{Cos}[x]^2 + \text{Sin}[x]^2$

Out[ ]= 1

In[ ]:=
```
psn = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x]
psn1 = Simplify[psn]
```

Out[ ]= $-\frac{1}{2}\text{Cos}[2x]\,\text{Sin}[x] - 2\,\text{Cos}[x]\left(\frac{x}{2} - \frac{1}{4}\text{Sin}[2x]\right)$

Out[ ]= $\frac{1}{2}\left(-2x\,\text{Cos}[x] + \text{Sin}[x]\right)$

In[ ]:=
```
gsh1 + psn1
```

Out[ ]= $\left\{\mathbb{c}_1\,\text{Cos}[x] + \mathbb{c}_2\,\text{Sin}[x] + \frac{1}{2}\left(-2x\,\text{Cos}[x] + \text{Sin}[x]\right)\right\}$

In[ ]:=
```
ClearAll
```

Out[ ]= ClearAll

# Ques 2 : $y'' + 3*y' + 2*y = 30*e^{2x}$
# psn (particular solution), gsh (general solution)

In[ ]:=

```
In[]:= yc2 = DSolve[y''[x] + 3 * y'[x] + 2 * y[x] == 0, y[x], x]
       y1 := Exp[-2 * x]
       y2 := Exp[-1 * x]
       f := 30 * Exp[2 * x]
       w = y1 * D[y2, x] - y2 * D[y1, x]
       w = Simplify[w]
       yp2 = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x]
       yp2 = Simplify[yp2]
       yc2 + yp2
```

Out[]= $\{\{y[x] \to e^{-2x} c_1 + e^{-x} c_2\}\}$

Out[]= $e^{-3x}$

Out[]= $e^{-3x}$

Out[]= $\frac{5 e^{2x}}{2}$

Out[]= $\frac{5 e^{2x}}{2}$

Out[]= $\left\{\left\{\frac{5 e^{2x}}{2} + \left(y[x] \to e^{-2x} c_1 + e^{-x} c_2\right)\right\}\right\}$

# Ques 3 : y’’+y=cotx

In[∘]:=
```
gsh = DSolve[y''[x] + y[x] == 0, y[x], x]
gsh1 = y[x] /. gsh
y1 := Cos[x]
y2 := Sin[x]
f := Cot[x]
w = y1 * D[y2, x] - y2 * D[y1, x]
w = Simplify[w]
psn = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x]
psn1 = Simplify[psn]
gsh1 + psn1
```

Out[∘]= $\{\{y[x] \to c_1 \text{Cos}[x] + c_2 \text{Sin}[x]\}\}$

Out[∘]= $\{c_1 \text{Cos}[x] + c_2 \text{Sin}[x]\}$

Out[∘]= $\text{Cos}[x]^2 + \text{Sin}[x]^2$

Out[∘]= 1

Out[∘]= $-\text{Cos}[x]\,\text{Sin}[x] + \left(\text{Cos}[x] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]\right)\text{Sin}[x]$

Out[∘]= $\left(-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]\right)\text{Sin}[x]$

Out[∘]= $\left\{c_1 \text{Cos}[x] + c_2 \text{Sin}[x] + \left(-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]\right)\text{Sin}[x]\right\}$

In[∘]:=
```
ClearAll
```

Out[∘]= ClearAll

# Ques 4 : y’’+4y’ + 5y=e^-2x . Sec x

In[ ]:=
```
gsh = DSolve[y''[x] + 4 * y'[x] + 5 * y[x] == 0, y[x], x]
gsh1 = y[x] /. gsh
y1 := Exp[-2 * x] * Cos[x]
y2 := Exp[-2 * x] * Sin[x]
f := Exp[-2 * x] * Sec[x]
w = y1 * D[y2, x] - y2 * D[y1, x]
w = Simplify[w]
psn = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x]
psn1 = Simplify[psn]
gsh1 + psn1
```

Out[ ]= $\{\{y[x] \to e^{-2x} c_2 \text{Cos}[x] + e^{-2x} c_1 \text{Sin}[x]\}\}$

Out[ ]= $\{e^{-2x} c_2 \text{Cos}[x] + e^{-2x} c_1 \text{Sin}[x]\}$

Out[ ]= $e^{-2x} \text{Cos}[x] \left(e^{-2x} \text{Cos}[x] - 2 e^{-2x} \text{Sin}[x]\right) - e^{-2x} \text{Sin}[x] \left(-2 e^{-2x} \text{Cos}[x] - e^{-2x} \text{Sin}[x]\right)$

Out[ ]= $e^{-4x}$

Out[ ]= $e^{-2x} \text{Cos}[x] \text{Log}[\text{Cos}[x]] + e^{-2x} x \text{Sin}[x]$

Out[ ]= $e^{-2x} (\text{Cos}[x] \text{Log}[\text{Cos}[x]] + x \text{Sin}[x])$

Out[ ]= $\{e^{-2x} c_2 \text{Cos}[x] + e^{-2x} c_1 \text{Sin}[x] + e^{-2x} (\text{Cos}[x] \text{Log}[\text{Cos}[x]] + x \text{Sin}[x])\}$

In[ ]:=
```
ClearAll
```

Out[ ]= ClearAll

# Ques 5 : y’’+6y’ + 9y=e^-3x / x^3

In[ ]:=
```
gsh = DSolve[y''[x] + 6 * y'[x] + 9 * y[x] == 0, y[x], x]
gsh1 = y[x] /. gsh
```

Out[ ]= $\{\{y[x] \to e^{-3x} c_1 + e^{-3x} x c_2\}\}$

Out[ ]= $\{e^{-3x} c_1 + e^{-3x} x c_2\}$

In[ ]:=
```
y1 := Exp[-3 * x]
y2 := Exp[-3 * x] * x
f := Exp[-3 * x] / x^3
w = y1 * D[y2, x] - y2 * D[y1, x]
w = Simplify[w]
psn = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x]
psn1 = Simplify[psn]
gsh1 + psn1
```

Out[ ]= $3\,e^{-6x}\,x + e^{-3x}\left(e^{-3x} - 3\,e^{-3x}\,x\right)$

Out[ ]= $e^{-6x}$

Out[ ]= $\frac{e^{-3x}}{2x}$

Out[ ]= $\frac{e^{-3x}}{2x}$

Out[ ]= $\left\{\frac{e^{-3x}}{2x} + e^{-3x}\,c_1 + e^{-3x}\,x\,c_2\right\}$

In[ ]:= `ClearAll`

Out[ ]= `ClearAll`

# Ques 6 : y’’-2y’ + y=x e^x ln x

In[ ]:=
```
gsh = DSolve[y''[x] - 2 * y'[x] + y[x] == 0, y[x], x]
gsh1 = y[x] /. gsh
```

Out[ ]= $\{\{y[x] \to e^{x}\,c_1 + e^{x}\,x\,c_2\}\}$

Out[ ]= $\{e^{x}\,c_1 + e^{x}\,x\,c_2\}$

In[ ]:=
```
y1 := Exp[x]
y2 := Exp[x] * x
f := x * Exp[x] * Log[x]
w = y1 * D[y2, x] - y2 * D[y1, x]
w = Simplify[w]
psn = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x]
psn1 = Simplify[psn]
gsh1 + psn1
```

Out[ ]= $-e^{2x} x + e^x \left(e^x + e^x x\right)$

Out[ ]= $e^{2x}$

Out[ ]= $e^x x \left(-\frac{x^2}{4} + \frac{1}{2} x^2 \text{Log}[x]\right) - e^x \left(-\frac{x^3}{9} + \frac{1}{3} x^3 \text{Log}[x]\right)$

Out[ ]= $\frac{1}{36} e^x x^3 (-5 + 6 \text{Log}[x])$

Out[ ]= $\left\{e^x c_1 + e^x x c_2 + \frac{1}{36} e^x x^3 (-5 + 6 \text{Log}[x])\right\}$

In[ ]:=
```
ClearAll
```

Out[ ]= ClearAll

# Ques 7 : y''+y=1/1+Sin[x]

In[ ]:=
```
gsh = DSolve[y''[x] + y[x] == 0, y[x], x]
gsh1 = y[x] /. gsh
```

Out[ ]= $\{\{y[x] \to c_1 \text{Cos}[x] + c_2 \text{Sin}[x]\}\}$

Out[ ]= $\{c_1 \text{Cos}[x] + c_2 \text{Sin}[x]\}$

In[∘]:=
```
y1 := Cos[x]
y2 := Sin[x]
f := 1/(1 + Sin[x])
w = y1 * D[y2, x] - y2 * D[y1, x]
w = Simplify[w]
psn = -y1 * Integrate[y2 * (f/w), x] + y2 * Integrate[y1 * (f/w), x]
psn1 = Simplify[psn]
gsh1 + psn1
```

Out[∘]= $\text{Cos}[x]^2 + \text{Sin}[x]^2$

Out[∘]= 1

Out[∘]= $-\text{Cos}[x]\left(x - \frac{2\,\text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]}\right) + \text{Log}[1 + \text{Sin}[x]]\ \text{Sin}[x]$

Out[∘]= $-1 + \text{Cos}[x] - x\,\text{Cos}[x] + \text{Sin}[x] + \text{Log}[1 + \text{Sin}[x]]\ \text{Sin}[x]$

Out[∘]= $\{-1 + \text{Cos}[x] - x\,\text{Cos}[x] + \mathbb{c}_1\,\text{Cos}[x] + \text{Sin}[x] + \mathbb{c}_2\,\text{Sin}[x] + \text{Log}[1 + \text{Sin}[x]]\ \text{Sin}[x]\}$

In[∘]:=
```
ClearAll
```

Out[∘]= ClearAll

# PRACTICAL 5 : SIMULTANEOUS DIFFERENTIAL EQUATION

A two dimensional linear system is a system of the form :

dx/dt = a x + by

dy/dt= c x + dy

where a,b,c and d are parameters. This system can be written in matrix form as

X = AX, where

A = ( a  b      and    X = ( x

c  d )                      y )

The solutions of X = AX can be visualized as trajectories moving on the x- y plane called the phase plane.

## QUes 1 : Solve the following system of equations :

## dx/dt = -3x- y

## dy/dt = x-3y

In[62]:=
```
eq1 = {x'[t] == -3*x[t] - y[t], y'[t] == -3 y[t] - x[t]}
```

Out[62]= $\{x'[t] == -3\,x[t] - y[t],\ y'[t] == -x[t] - 3\,y[t]\}$

In[63]:=
```
sol = DSolve[eq1, {y[t], x[t]}, t]
```

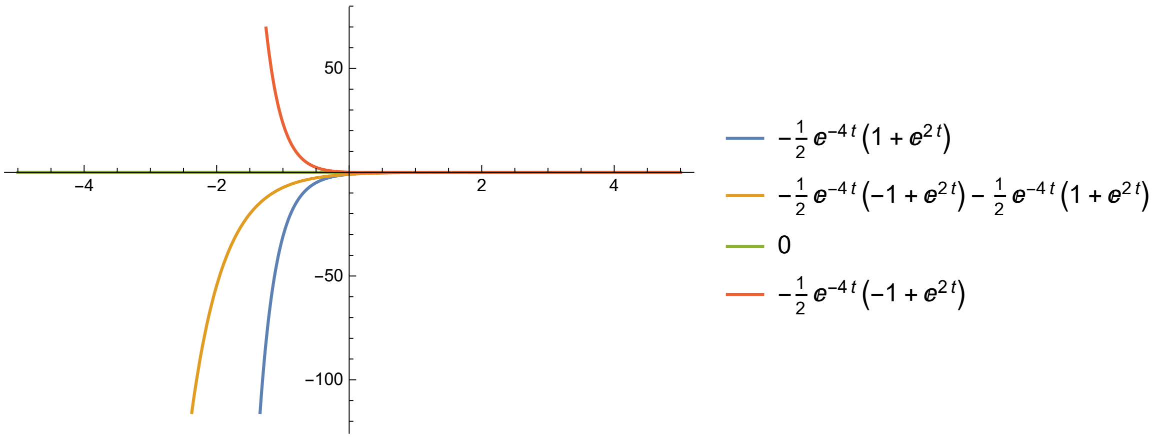
Out[63]= $\left\{\left\{x[t] \to \frac{1}{2} e^{-4t}\left(1 + e^{2t}\right) c_1 - \frac{1}{2} e^{-4t}\left(-1 + e^{2t}\right) c_2,\right.\right.$
$\left.\left. y[t] \to -\frac{1}{2} e^{-4t}\left(-1 + e^{2t}\right) c_1 + \frac{1}{2} e^{-4t}\left(1 + e^{2t}\right) c_2\right\}\right\}$

In[67]:=
```
tabx = Table[x[t] /. sol[[1, 1]] /. {C[1] → i, C[2] → j}, {i, -1, 0}, {j, 0, 1}] // Flatten
```

Out[67]= $\left\{-\frac{1}{2} e^{-4t}\left(1 + e^{2t}\right),\ -\frac{1}{2} e^{-4t}\left(-1 + e^{2t}\right) - \frac{1}{2} e^{-4t}\left(1 + e^{2t}\right),\ 0,\ -\frac{1}{2} e^{-4t}\left(-1 + e^{2t}\right)\right\}$

In[68]:= `Plot[Evaluate[tabx], {t, -5, 5}, PlotLegends → "Expressions"]`

Out[68]=

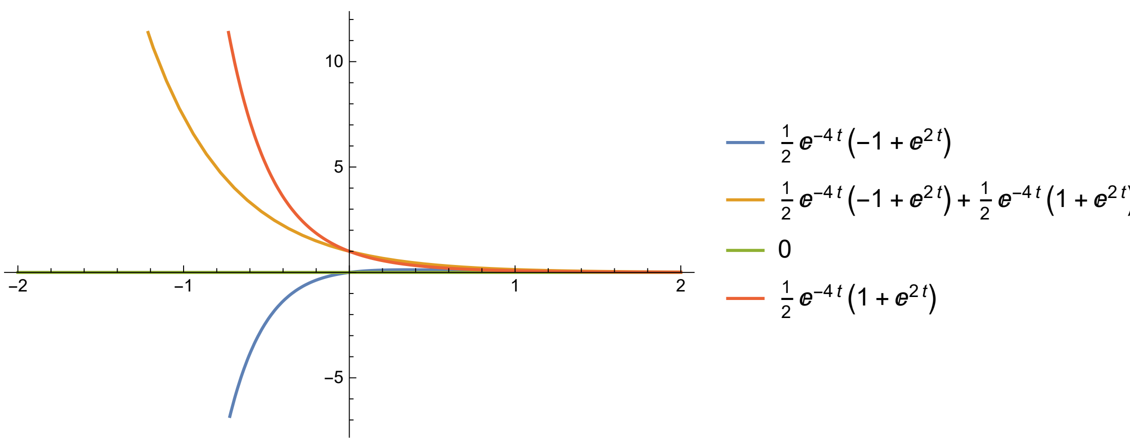



In[69]:= `taby = Table[y[t] /. sol[[1, 2]] /. {C[1] → i , C[2] → j}, {i, -1, 0}, {j, 0, 1}] // Flatten`

Out[69]= $\left\{\frac{1}{2}e^{-4t}\left(-1+e^{2t}\right), \frac{1}{2}e^{-4t}\left(-1+e^{2t}\right)+\frac{1}{2}e^{-4t}\left(1+e^{2t}\right), 0, \frac{1}{2}e^{-4t}\left(1+e^{2t}\right)\right\}$

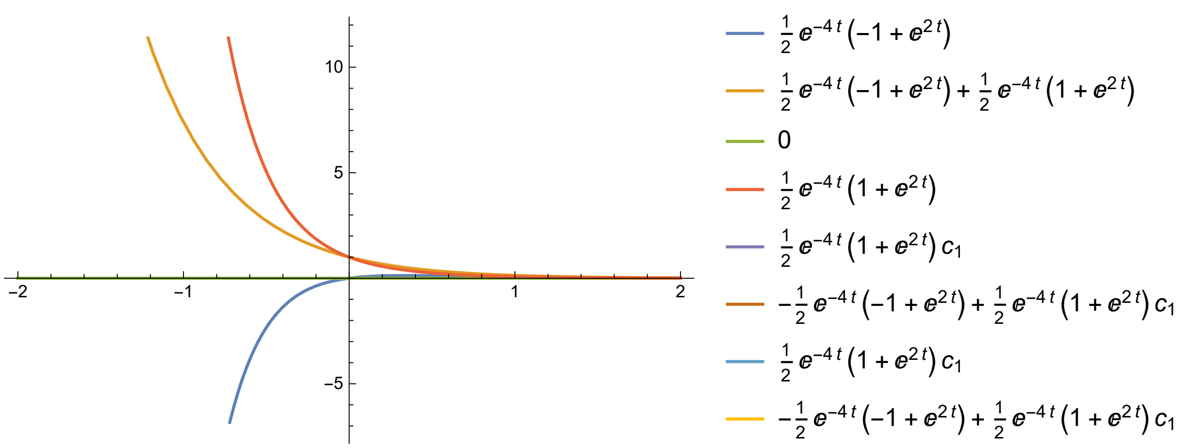
In[70]:= `Plot[Evaluate[taby], {t, -2, 2}, PlotLegends → "Expressions"]`

Out[70]=




In[ ]:= `Plot[Evaluate[{taby, tabx}], {t, -2, 2}, PlotLegends → "Expressions"]`

Out[ ]=




# Ques 2 : Solve the following system of equations :

# dx/dt = y
# dy/dt = 6x- y
# with initial condition x(0)= 1, y(0) = -2

In[ ]:=
```
eq2 = {{x'[t] == y[t], y'[t] == -y[t] + 6 x[t]}, x[0] == 1, y[0] == -2}
```

Out[ ]= $\{\{x'[t] == y[t], y'[t] == 6\,x[t] - y[t]\}, x[0] == 1, y[0] == -2\}$

In[ ]:=
```
DSolve[eq2, {x[t], y[t]}, t]
```

Out[ ]= $\left\{\left\{x[t] \to \frac{1}{5} e^{-3t}\left(4 + e^{5t}\right), y[t] \to \frac{2}{5} e^{-3t}\left(-6 + e^{5t}\right)\right\}\right\}$

In[ ]:=
```
{xsol[t_], ysol[t_]} = ExpandAll[{x[t], y[t]} /. Flatten[DSolve[eq2, {x[t], y[t]}, t]]]
```

Out[ ]= $\left\{\frac{4\,e^{-3t}}{5} + \frac{e^{2t}}{5}, -\frac{12}{5} e^{-3t} + \frac{2\,e^{2t}}{5}\right\}$

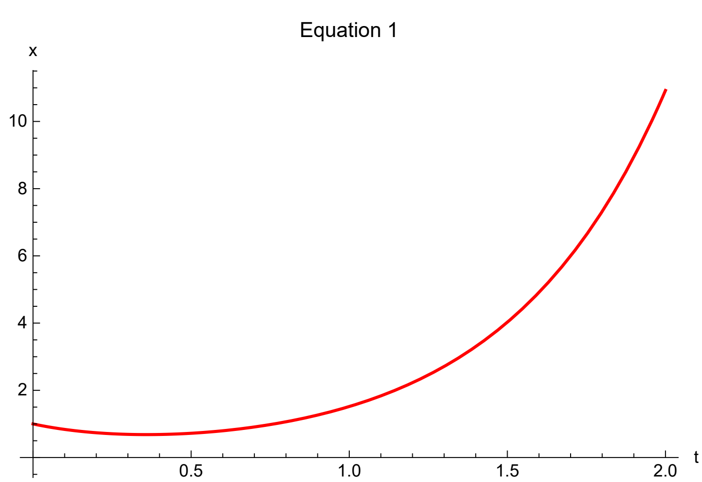
In[ ]:=
```
xsol[t]
```

Out[ ]= $\frac{4\,e^{-3t}}{5} + \frac{e^{2t}}{5}$

In[ ]:=
```
ysol[t]
```

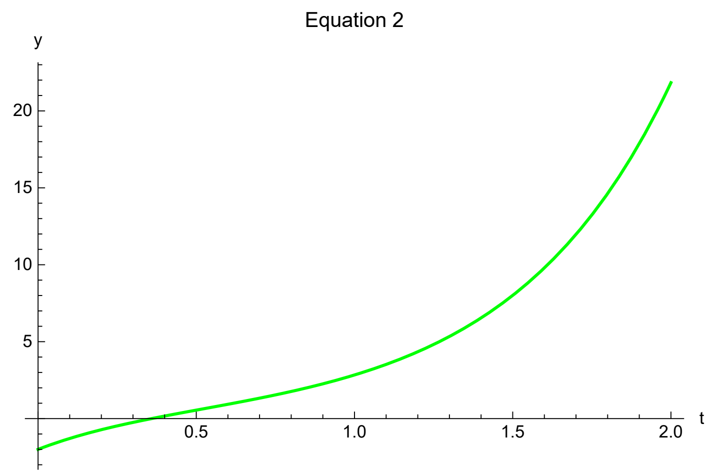
Out[ ]= $-\frac{12}{5} e^{-3t} + \frac{2\,e^{2t}}{5}$

In[ ]:=
```
plot1 = Plot[xsol[t], {t, 0, 2},
  AxesLabel → {"t", "x"}, PlotLabel → "Equation 1", PlotStyle → {Red}]
```

Out[ ]=


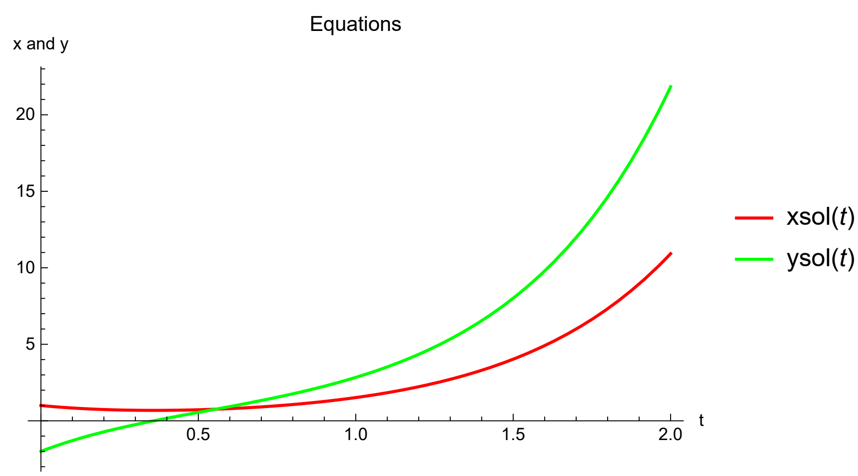
In[ ]:=
```
plo2 = Plot[ysol[t], {t, 0, 2},
  AxesLabel → {"t", "y"}, PlotLabel → "Equation 2", PlotStyle → {Green}]
```

Out[ ]=




In[ ]:=
```
Plot[{xsol[t], ysol[t]}, {t, 0, 2}, AxesLabel → {"t", "x and y" },
 PlotLabel → "Equations", PlotStyle → {Red, Green}, PlotLegends → "Expressions"]
```

Out[ ]=




## Solve the following Simultaneous DE and hence plot the solutions :

## **Ques 3 : dx/dt = 5x - 2y**

## **dy/dt= 4x-y**

In[ ]:=
```
eq1 = {x'[t] == 5 * x[t] - 2 * y[t], y'[t] == -y[t] + 4 * x[t]}
```

Out[ ]= $\{x'[t] == 5\,x[t] - 2\,y[t],\ y'[t] == 4\,x[t] - y[t]\}$

In[ ]:=
```
sol = DSolve[eq1, {y[t], x[t]}, t]
```

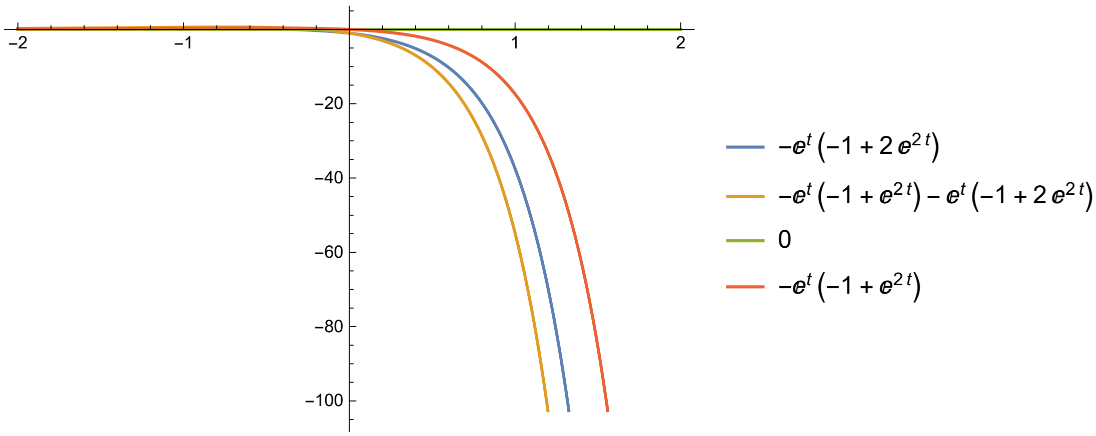
Out[ ]= $\{\{x[t] \to e^{t}\left(-1 + 2\,e^{2t}\right) c_1 - e^{t}\left(-1 + e^{2t}\right) c_2,\ y[t] \to 2\,e^{t}\left(-1 + e^{2t}\right) c_1 - e^{t}\left(-2 + e^{2t}\right) c_2\}\}$

In[∘]:=
```
tabx = Table[x[t] /. sol[[1, 1]] /. {C[1] → i, C[2] → j}, {i, -1, 0}, {j, 0, 1}] // Flatten
```

Out[∘]= $\left\{-e^{t}\left(-1+2\,e^{2t}\right),\ -e^{t}\left(-1+e^{2t}\right)-e^{t}\left(-1+2\,e^{2t}\right),\ 0,\ -e^{t}\left(-1+e^{2t}\right)\right\}$

In[∘]:=
```
Plot[Evaluate[tabx], {t, -2, 2}, PlotLegends → "Expressions"]
```

Out[∘]=

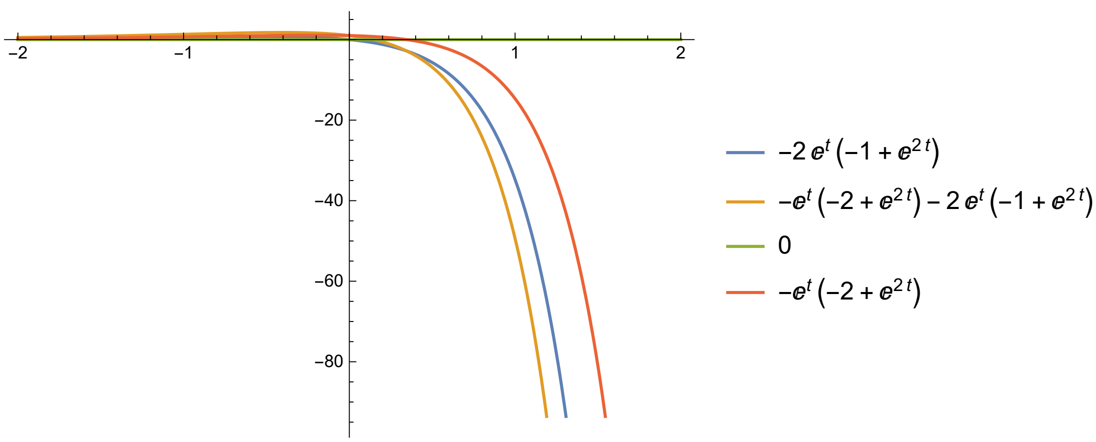



In[∘]:=
```
taby = Table[y[t] /. sol[[1, 2]] /. {C[1] → i, C[2] → j}, {i, -1, 0}, {j, 0, 1}] // Flatten
```

Out[∘]= $\left\{-2\,e^{t}\left(-1+e^{2t}\right),\ -e^{t}\left(-2+e^{2t}\right)-2\,e^{t}\left(-1+e^{2t}\right),\ 0,\ -e^{t}\left(-2+e^{2t}\right)\right\}$

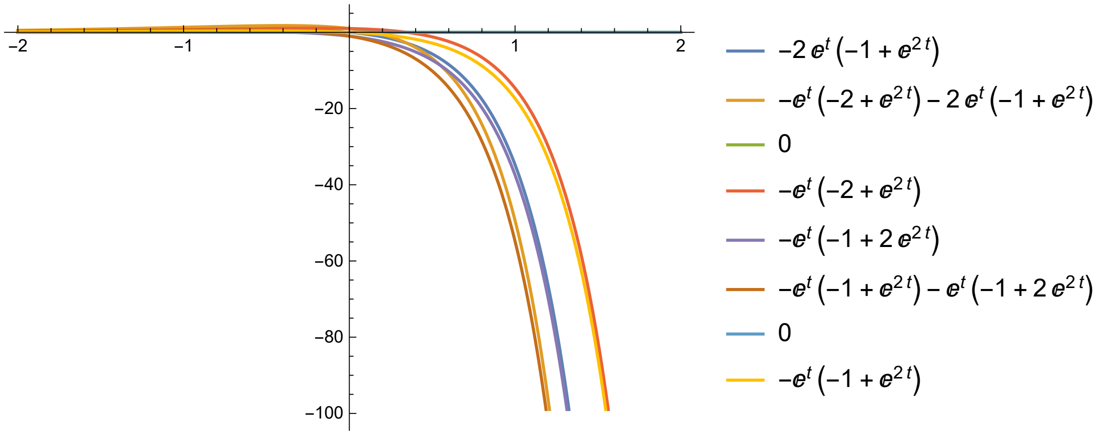
In[∘]:=
```
Plot[Evaluate[taby], {t, -2, 2}, PlotLegends → "Expressions"]
```

Out[∘]=



In[∘]:=
```
Plot[Evaluate[{taby, tabx}], {t, -2, 2}, PlotLegends → "Expressions"]
```

Out[∘]=




# Ques 4 : dx/dt = 3x - 4y
# dy/dt= 2x-y

In[∘]:=
```
eq1 = {x'[t] == 3*x[t] - 4*y[t], y'[t] == -y[t] + 2*x[t]}
```

Out[∘]= $\{x'[t] == 3x[t] - 4y[t],\ y'[t] == 2x[t] - y[t]\}$

In[∘]:=
```
sol = DSolve[eq1, {y[t], x[t]}, t]
```

Out[∘]= $\{\{x[t] \to -2e^t c_2 \mathrm{Sin}[2t] + e^t c_1 (\mathrm{Cos}[2t] + \mathrm{Sin}[2t]),\ y[t] \to e^t c_2 (\mathrm{Cos}[2t] - \mathrm{Sin}[2t]) + e^t c_1 \mathrm{Sin}[2t]\}\}$

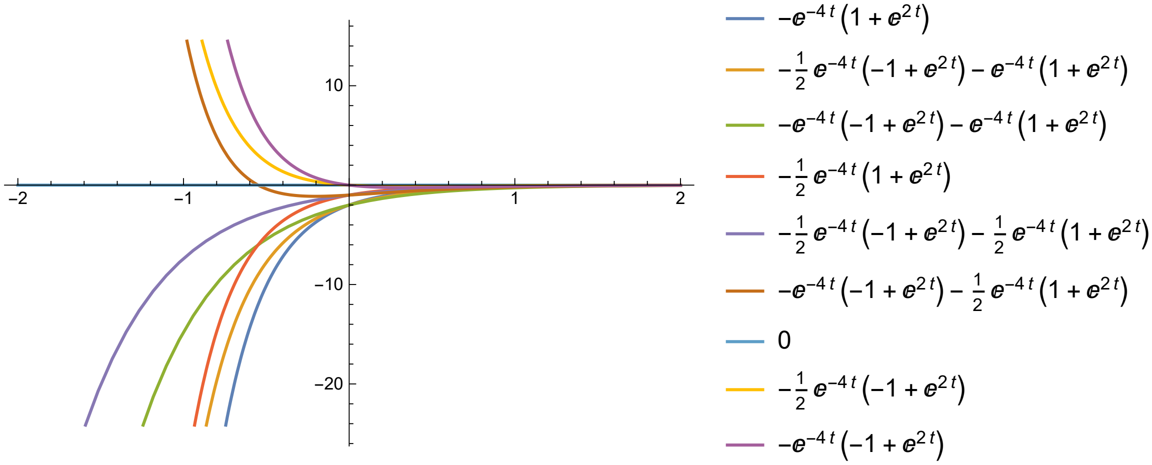
In[71]:=
```
tabx = Table[x[t] /. sol[[1, 1]] /. {C[1] → i, C[2] → j}, {i, -2, 0}, {j, 0, 2}] // Flatten
taby = Table[y[t] /. sol[[1, 2]] /. {C[1] → i, C[2] → j}, {i, -2, 0}, {j, 0, 2}] // Flatten
```

Out[71]= $\{-e^{-4t}(1+e^{2t}),\ -\frac{1}{2}e^{-4t}(-1+e^{2t}) - e^{-4t}(1+e^{2t}),\ -e^{-4t}(-1+e^{2t}) - e^{-4t}(1+e^{2t}),\ -\frac{1}{2}e^{-4t}(1+e^{2t}),\ -\frac{1}{2}e^{-4t}(-1+e^{2t}) - \frac{1}{2}e^{-4t}(1+e^{2t}),\ -e^{-4t}(-1+e^{2t}) - \frac{1}{2}e^{-4t}(1+e^{2t}),\ 0,\ -\frac{1}{2}e^{-4t}(-1+e^{2t}),\ -e^{-4t}(-1+e^{2t})\}$

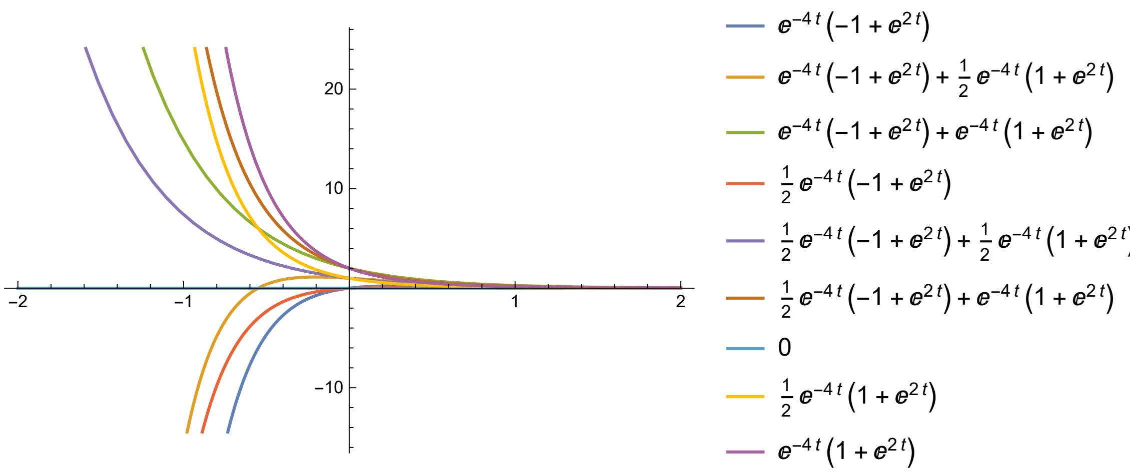
Out[72]= $\{e^{-4t}(-1+e^{2t}),\ e^{-4t}(-1+e^{2t}) + \frac{1}{2}e^{-4t}(1+e^{2t}),\ e^{-4t}(-1+e^{2t}) + e^{-4t}(1+e^{2t}),\ \frac{1}{2}e^{-4t}(-1+e^{2t}),\ \frac{1}{2}e^{-4t}(-1+e^{2t}) + \frac{1}{2}e^{-4t}(1+e^{2t}),\ \frac{1}{2}e^{-4t}(-1+e^{2t}) + e^{-4t}(1+e^{2t}),\ 0,\ \frac{1}{2}e^{-4t}(1+e^{2t}),\ e^{-4t}(1+e^{2t})\}$

In[73]:=
```
Plot[Evaluate[tabx], {t, -2, 2}, PlotLegends → "Expressions"]
Plot[Evaluate[taby], {t, -2, 2}, PlotLegends → "Expressions"]
Plot[Evaluate[{taby, tabx}], {t, -2, 2}, PlotLegends → "Expressions"]
```

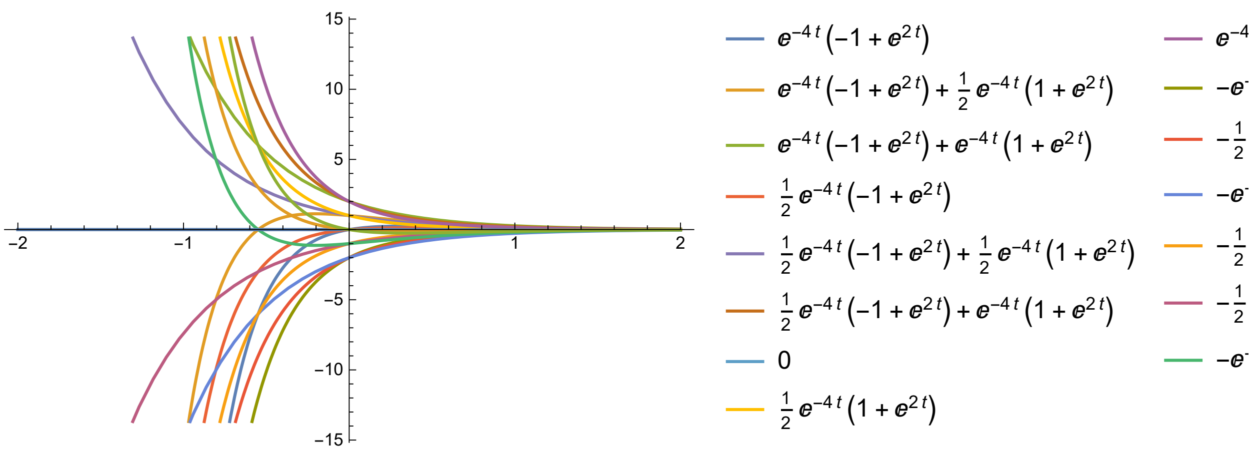
Out[73]=




Out[74]=




Out[75]=



# Ques 5 : dx/dt = 2x + 7y
# dy/dt= 3x+2y
# with x[0]=9, y[0]=-1

In[∘]:=
```
eq2 = {{x'[t] == 7 * y[t] + 2 * x[t], y'[t] == 2 * y[t] + 3 * x[t]}, x[0] == 9, y[0] == -1}
DSolve[eq2, {x[t], y[t]}, t]
{xsol[t_], ysol[t_]} =
  ExpandAll[{x[t], y[t]} /. Flatten[DSolve[eq2, {x[t], y[t]}, t]]]
xsol[t]
ysol[t]
plot1 = Plot[xsol[t], {t, 0, 2},
   AxesLabel → {"t", "x"}, PlotLabel → "Equation 1", PlotStyle → {Red}]
plo2 = Plot[ysol[t], {t, 0, 2}, AxesLabel → {"t", "y"},
   PlotLabel → "Equation 2", PlotStyle → {Green}]
Plot[{xsol[t], ysol[t]}, {t, 0, 3}, AxesLabel → {"t", "x and y" },
  PlotLabel → "Equations", PlotStyle → {Red, Green}, PlotLegends → "Expressions"]
```

Out[∘]= $\{\{x'[t] == 2\,x[t] + 7\,y[t],\ y'[t] == 3\,x[t] + 2\,y[t]\},\ x[0] == 9,\ y[0] == -1\}$

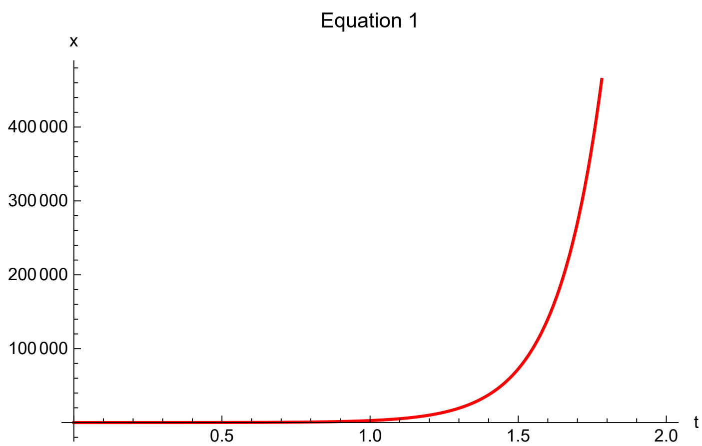
Out[∘]= $\left\{\left\{x[t] \to -\frac{1}{6} e^{2t-\sqrt{21}\,t}\left(-27-\sqrt{21}-27\,e^{2\sqrt{21}\,t}+\sqrt{21}\,e^{2\sqrt{21}\,t}\right),\right.\right.$
$\left.\left. y[t] \to \frac{1}{14} e^{2t-\sqrt{21}\,t}\left(-7-9\sqrt{21}-7\,e^{2\sqrt{21}\,t}+9\sqrt{21}\,e^{2\sqrt{21}\,t}\right)\right\}\right\}$

Out[∘]= $\left\{\frac{9}{2} e^{2t-\sqrt{21}\,t}+\frac{1}{2}\sqrt{\frac{7}{3}}\,e^{2t-\sqrt{21}\,t}+\frac{9}{2} e^{2t+\sqrt{21}\,t}-\frac{1}{2}\sqrt{\frac{7}{3}}\,e^{2t+\sqrt{21}\,t},\right.$
$\left.-\frac{1}{2} e^{2t-\sqrt{21}\,t}-\frac{9}{2}\sqrt{\frac{3}{7}}\,e^{2t-\sqrt{21}\,t}-\frac{1}{2} e^{2t+\sqrt{21}\,t}+\frac{9}{2}\sqrt{\frac{3}{7}}\,e^{2t+\sqrt{21}\,t}\right\}$

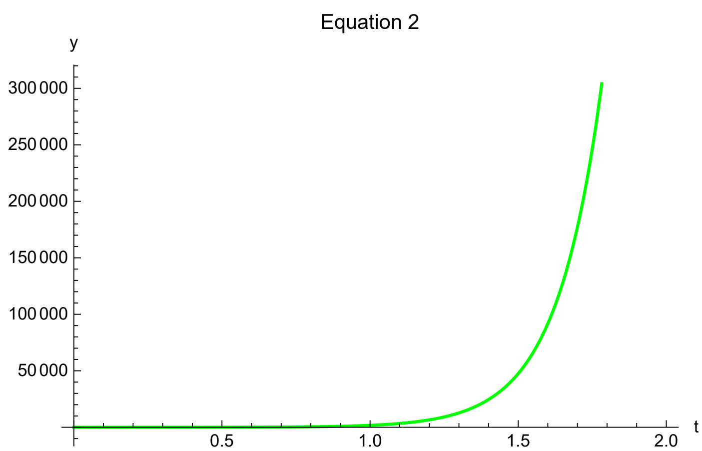
Out[∘]= $\frac{9}{2} e^{2t-\sqrt{21}\,t}+\frac{1}{2}\sqrt{\frac{7}{3}}\,e^{2t-\sqrt{21}\,t}+\frac{9}{2} e^{2t+\sqrt{21}\,t}-\frac{1}{2}\sqrt{\frac{7}{3}}\,e^{2t+\sqrt{21}\,t}$

Out[∘]= $-\frac{1}{2} e^{2t-\sqrt{21}\,t}-\frac{9}{2}\sqrt{\frac{3}{7}}\,e^{2t-\sqrt{21}\,t}-\frac{1}{2} e^{2t+\sqrt{21}\,t}+\frac{9}{2}\sqrt{\frac{3}{7}}\,e^{2t+\sqrt{21}\,t}$

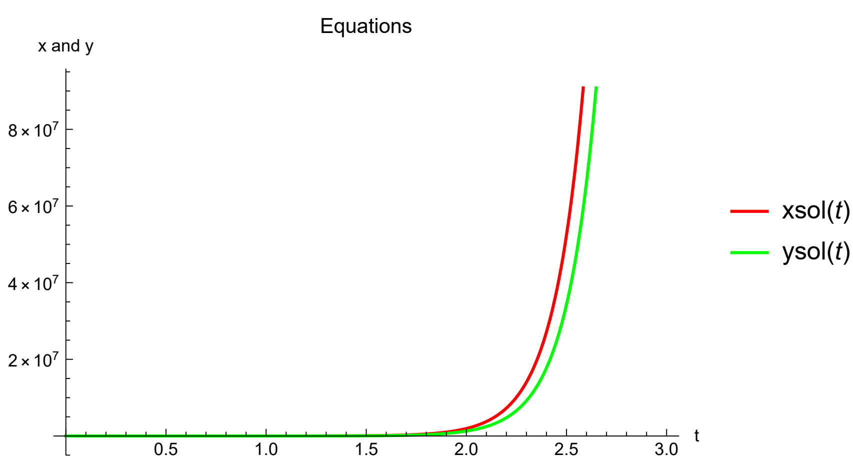
Out[ ]=


Equation 1
x
400 000
300 000
200 000
100 000
0.5
1.0
1.5
2.0
t

Out[ ]=


Equation 2
y
300 000
250 000
200 000
150 000
100 000
50 000
0.5
1.0
1.5
2.0
t

Out[ ]=


Equations
x and y
8 × 10^7
6 × 10^7
4 × 10^7
2 × 10^7
0.5
1.0
1.5
2.0
2.5
3.0
t
xsol(t)
ysol(t)

# Ques 6 : dx/dt = 7x - y
# dy/dt= 4x + 3y
# with initial conditions x[0]=1, y[0]=3

```
In[•]:= eq2 = {{x'[t] == -y[t] + 7 * x[t], y'[t] == 3 * y[t] + 4 * x[t]}, x[0] == 1, y[0] == 3}
DSolve[eq2, {x[t], y[t]}, t]
{xsol[t_], ysol[t_]} =
 ExpandAll[{x[t], y[t]} /. Flatten[DSolve[eq2, {x[t], y[t]}, t]]]
xsol[t]
ysol[t]
plot1 = Plot[xsol[t], {t, 0, 2},
  AxesLabel → {"t", "x"}, PlotLabel → "Equation 1", PlotStyle → {Red}]
plo2 = Plot[ysol[t], {t, 0, 2}, AxesLabel → {"t", "y"},
  PlotLabel → "Equation 2", PlotStyle → {Blue}]
Plot[{xsol[t], ysol[t]}, {t, 0, 2}, AxesLabel → {"t", "x and y" },
 PlotLabel → "Equations", PlotStyle → {Red, Blue}, PlotLegends → "Expressions"]
```

Out[•]= $\{\{x'[t] == 7\,x[t] - y[t],\ y'[t] == 4\,x[t] + 3\,y[t]\},\ x[0] == 1,\ y[0] == 3\}$

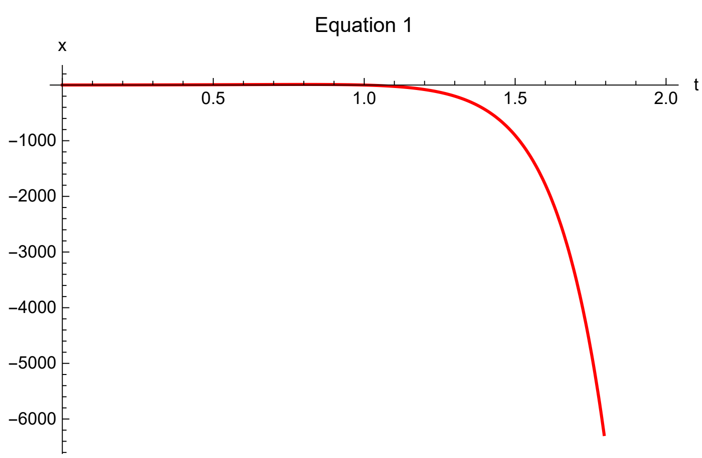
Out[•]= $\{\{x[t] \to -e^{5t}(-1+t),\ y[t] \to -e^{5t}(-3+2t)\}\}$

Out[•]= $\{e^{5t} - e^{5t}\,t,\ 3\,e^{5t} - 2\,e^{5t}\,t\}$

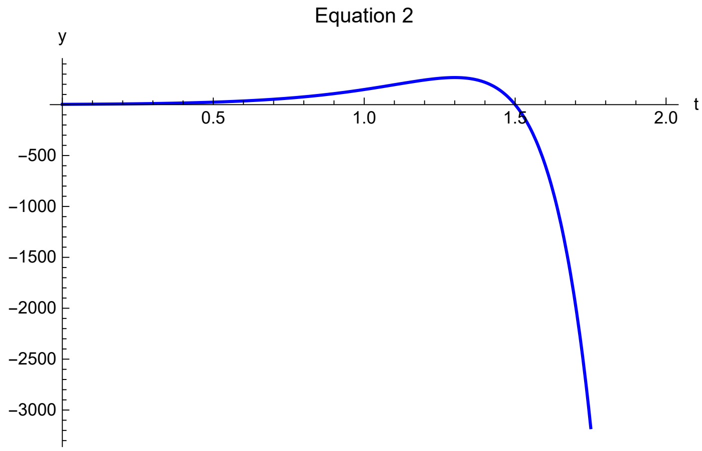
Out[•]= $e^{5t} - e^{5t}\,t$

Out[•]= $3\,e^{5t} - 2\,e^{5t}\,t$

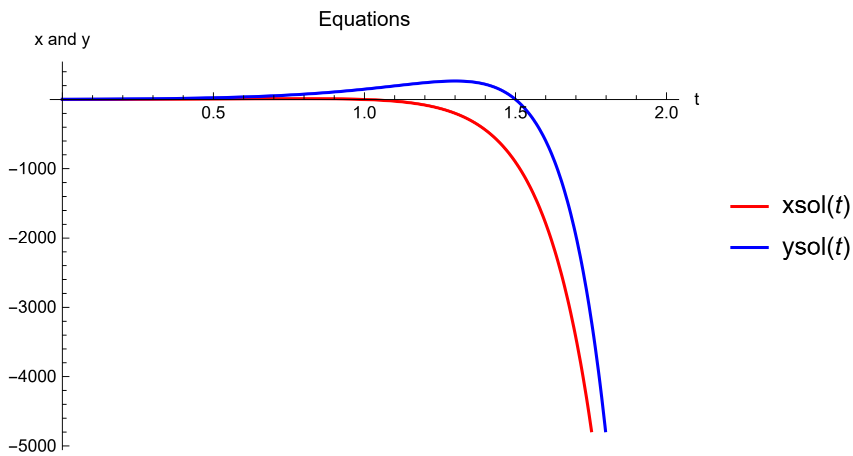
Out[•]=



Out[ ]=


Equation 2
y
t
0.5
1.0
1.5
2.0
−500
−1000
−1500
−2000
−2500
−3000

Out[ ]=


Equations
x and y
t
0.5
1.0
1.5
2.0
−1000
−2000
−3000
−4000
−5000
xsol(t)
ysol(t)

# Practical 6 :: Solution of Cauchy Problem for First Order Partial Differential Equation

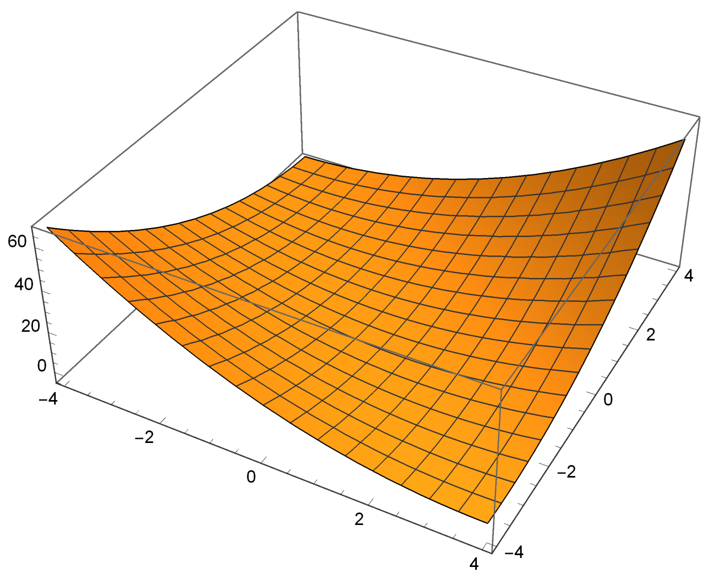
**Ques 1 . u_x - u_y= 1, u(x,0)=x^2**

```
In[ ]:= sol = DSolve[{D[u[x, y], x] - D[u[x, y], y] == 1, u[x, 0] == x^2}, u[x, y], {x, y}]
```

Out[ ]= $\{\{u[x, y] \to x^2 - y + 2\,x\,y + y^2\}\}$

```
In[ ]:= Plot3D[u[x, y] /. sol, {x, -4, 4}, {y, -4, 4}]
```

Out[ ]=



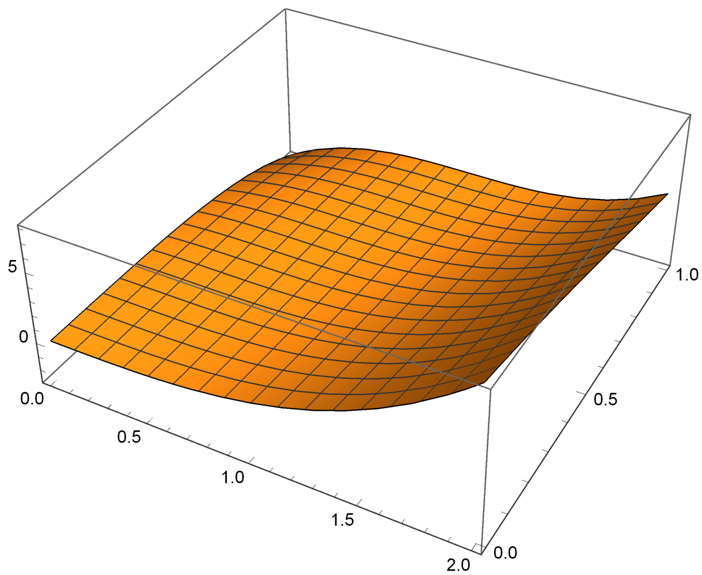

**Ques 2 . u_x+u_y = u, u(x,0)=x^3**

```
In[ ]:= sol2 = DSolve[{D[u[x, y], x] + D[u[x, y], y] == u[x, y], u[x, 0] == x^3}, u[x, y], {x, y}]
```

Out[ ]= $\{\{u[x, y] \to -e^{y}\,(-x + y)^3\}\}$

In[∘]:=
```
Plot3D[u[x, y] /. sol2, {x, 0, 2}, {y, 0, 1}]
```

Out[∘]=




**Ques 3. yu_x+ xu_y = u , u(0,y)=y^3**

In[∘]:=
```
sol11 =
  DSolve[{y D[u[x, y], x] + x D[u[x, y], y] == u [x, y], u[0, y] == y^3}, u[x, y], {x, y}]
```

Out[∘]= $\left\{\left\{u[x,y] \to -\frac{\left(-x^2+y^2\right)^{3/2}\sqrt{1-\frac{x}{\sqrt{y^2}}}}{\sqrt{1+\frac{x}{\sqrt{y^2}}}}\right\}, \left\{u[x,y] \to -\frac{\left(-x^2+y^2\right)^{3/2}\sqrt{1+\frac{x}{\sqrt{y^2}}}}{\sqrt{1-\frac{x}{\sqrt{y^2}}}}\right\}\right\}$

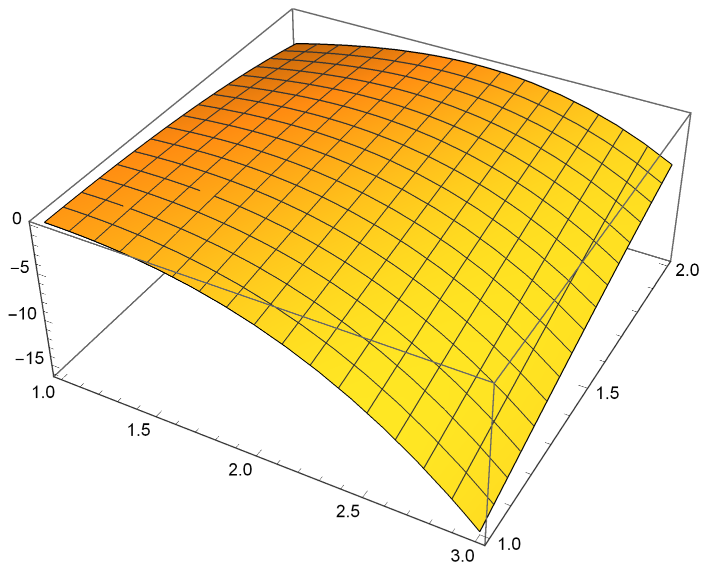
In[∘]:=
```
sol11[[1, 1]]
```

Out[∘]= $u[x,y] \to -\frac{\left(-x^2+y^2\right)^{3/2}\sqrt{1-\frac{x}{\sqrt{y^2}}}}{\sqrt{1+\frac{x}{\sqrt{y^2}}}}$

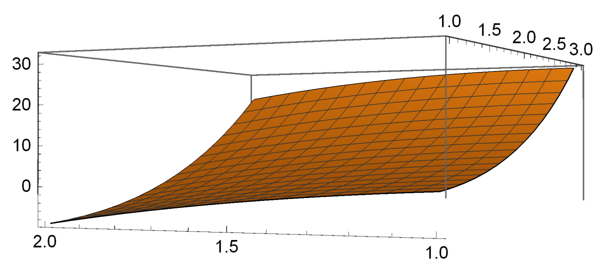
In[∘]:=
```
sol11[[2, 1]]
```

Out[∘]= $u[x,y] \to -\frac{\left(-x^2+y^2\right)^{3/2}\sqrt{1+\frac{x}{\sqrt{y^2}}}}{\sqrt{1-\frac{x}{\sqrt{y^2}}}}$

In[∘]:= `Plot3D[u[x, y] /. sol11[[1, 1]], {x, 1, 3}, {y, 1, 2}]`

Out[∘]=



In[∘]:= `Plot3D[u[x, y] /. sol11[[2, 1]], {x, 1, 3}, {y, 1, 2}]`

Out[∘]=



Questions :

**Q 4 : $u_x + xu_y = 0, \quad u(0,y) = \sin y$**

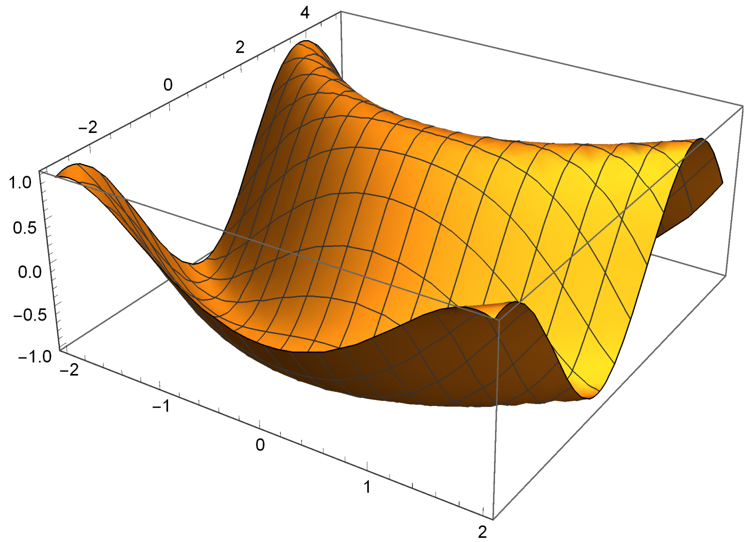
In[∘]:= `sol4 = DSolve[{D[u[x, y], x] + x D[u[x, y], y] == 0 , u[0, y] == Sin[y]}, u[x, y], {x, y}]`

Out[∘]= $\{\{u[x, y] \to \mathrm{Sin}[\frac{1}{2}(-x^2 + 2y)]\}\}$

Here the initial values have been given for PDE, value of c1 is obtained and hence do not need to put any values (like given in questions above)

In[ ]:=
```
Plot3D[u[x, y] /. sol4, {x, -2, 2}, {y, -3, 5}]
```

Out[ ]=




**Q 5 : u (x+y) $u_x$ + u (x-y) $u_y$ = $x^2 + y^2$, u=0 , y=2x**

In[ ]:=
```
sol5 = DSolve[{ u[x, y] * (x + y) D[u[x, y], x] + u[x, y] * (x - y) D[u[x, y], y] == x^2 + y^2 ,
    u[x, 2 x] == 0}, u[x, y], {x, y}]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[ ]= $\left\{\left\{u[x, y] \to -\sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\}, \left\{u[x, y] \to \sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\}, \left\{u[x, y] \to -\sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\}, \left\{u[x, y] \to \sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\}\right\}$

In[ ]:=
```
sol5[[1, 1]]
```

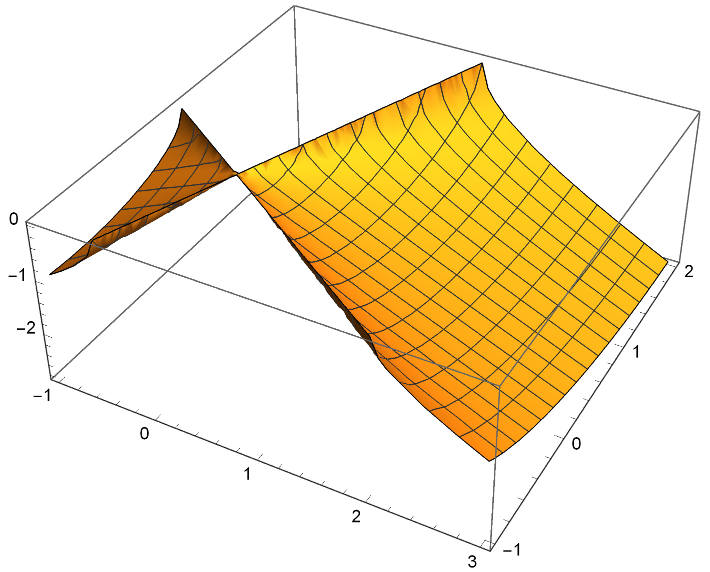
Out[ ]= $u[x, y] \to -\sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}$

In[ ]:=
```
sol5[[2, 1]]
```

Out[ ]= $u[x, y] \to \sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}$

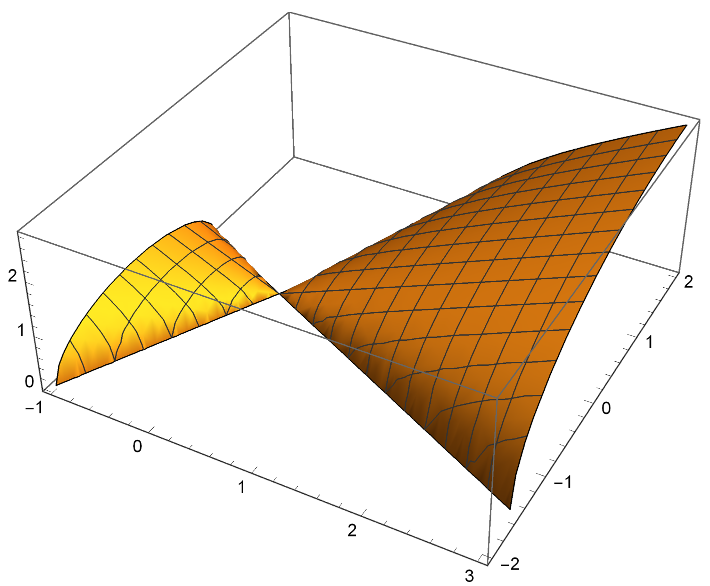
In[∘]:=
```
Plot3D[u[x, y] /. sol5[[1, 1]], {x, -1, 3}, {y, -1, 2}]
```

Out[∘]=




In[∘]:=
```
Plot3D[u[x, y] /. sol5[[2, 1]], {x, -1, 3}, {y, -2, 2}]
```

Out[∘]=

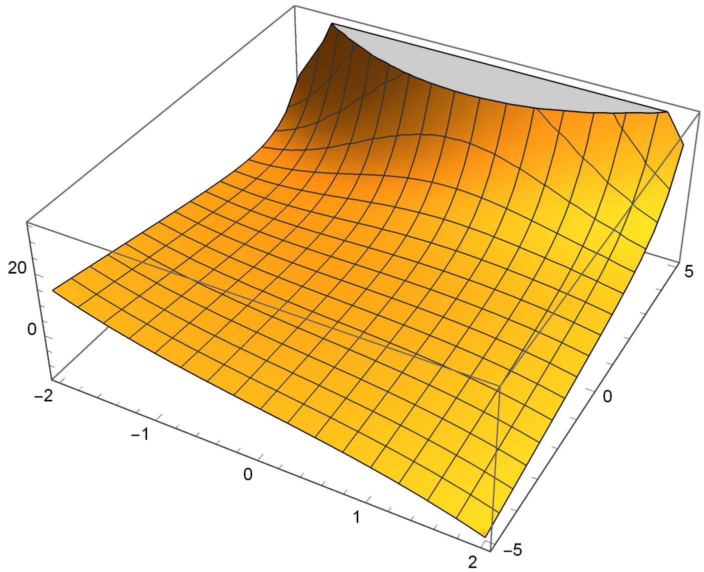



**Q 6 : $u_x + x\,u_y = (y - 1/2\,x^2)$ ,  $u(0, y) = e^y$**

In[∘]:=
```
sol6 = DSolve[
  {D[u[x, y], x] + x D[u[x, y], y] == y - (1/2) * x^2 , u[0, y] == Exp[y]}, u[x, y], {x, y}]
```

Out[∘]= $\left\{\left\{u[x, y] \to -\frac{1}{2} e^{-\frac{x^2}{2}} \left(-2 e^{y} + e^{\frac{x^2}{2}} x^3 - 2 e^{\frac{x^2}{2}} x\, y\right)\right\}\right\}$

In[ ]:=
```
Plot3D[u[x, y] /. sol6, {x, -2, 2}, {y, -5, 5}]
```

Out[ ]=


# Practical 7 :: Plotting the Characteristic for the First Order PDE

A general quasi-linear PDE is :

a (x,y,u) $u_x$ + b(x,y,u) $u_y$ - c(x,y,u) = 0

The characteristic equations of the quasi-linear equation :

dx/dt = a (x,y,u)

dy/dt = b (x,y,u)

du/dt = c (x,y,u)

Equivalently characteristic equations of the above equation in the non - parametric form are :

dx/a = dy/b = du/c

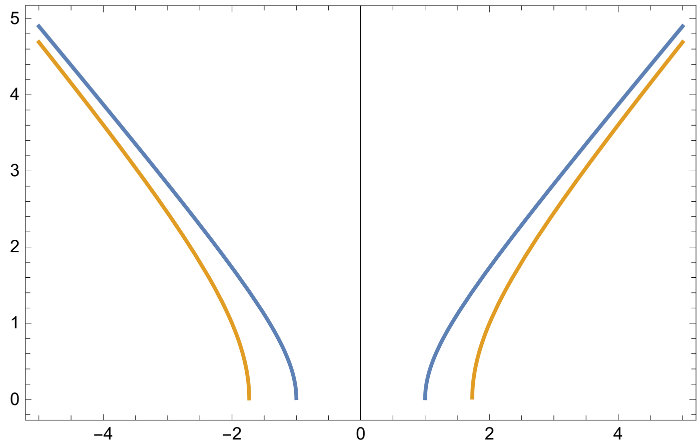
Integrate the above and plot the solutions

**Ques 1.** y $u_x$ + x $u_y$ = 0

**The characteristic system is given by dx/y = dy/x = du=0 and the characteristic equations are given by $x^2 + y^2 = c_1$ and $u = c_2$. Taking $c_1$ = 1 and $u = c_2$ . Taking $c_1$ = 1 and 3 and $c_2$ = 3 and 8.**

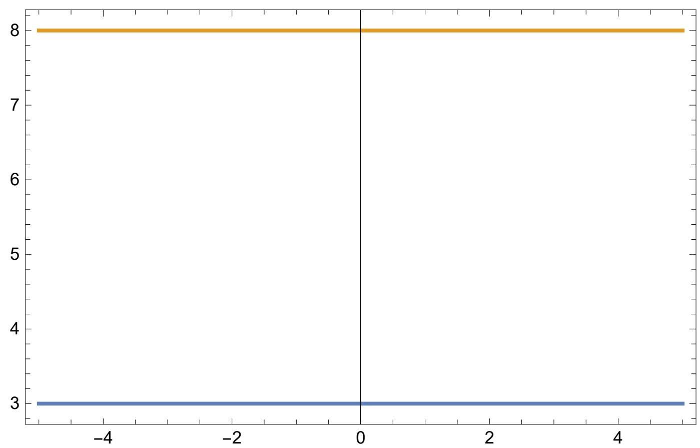
In[◦]:=
```
Plot[{Sqrt[x^2 - 1], Sqrt[x^2 - 3]}, {x, -5, 5}, PlotStyle → Thick, Frame → True]
```

Out[◦]=




In[◦]:=
```
Plot[{3, 8}, {x, -5, 5}, PlotStyle → Thick, Frame → True]
```

Out[◦]=

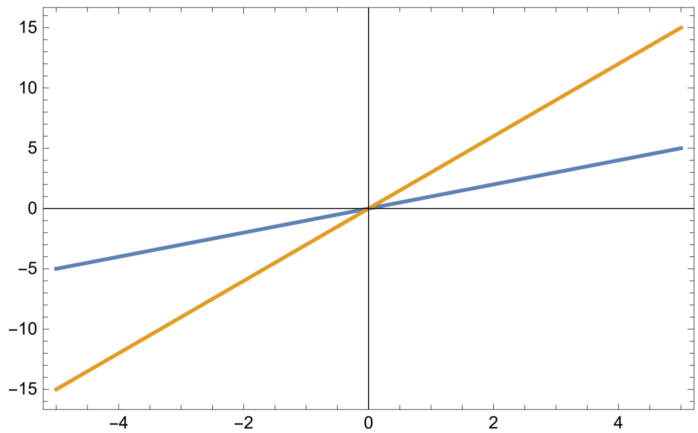



**Ques 2.** $x\, u_x + y\, u_y = u$

**The characteristic system is given by dx/x = dy/y = du/u and the characteristic equations are given by y/x = $c_1$ and $u/x = c_2$. Taking $c_1 = 1$ and 3 and $c_2 = 3$ and 8.**

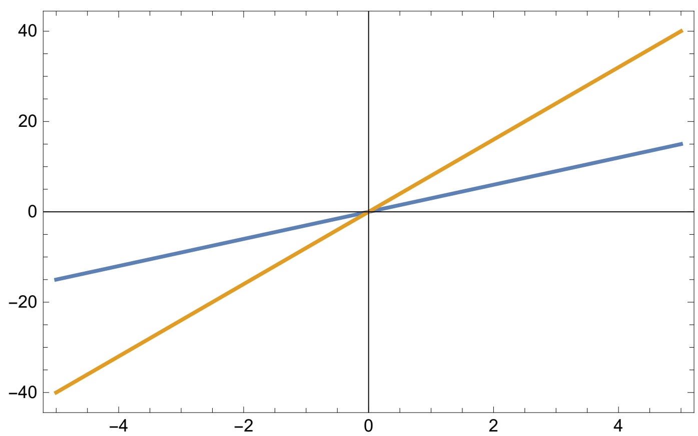
In[∘]:=
```
Plot[{x, 3 x}, {x, -5, 5}, PlotStyle → Thick, Frame → True]
```

Out[∘]=



In[∘]:=
```
Plot[{3 x, 8 x}, {x, -5, 5}, PlotStyle → Thick, Frame → True]
```

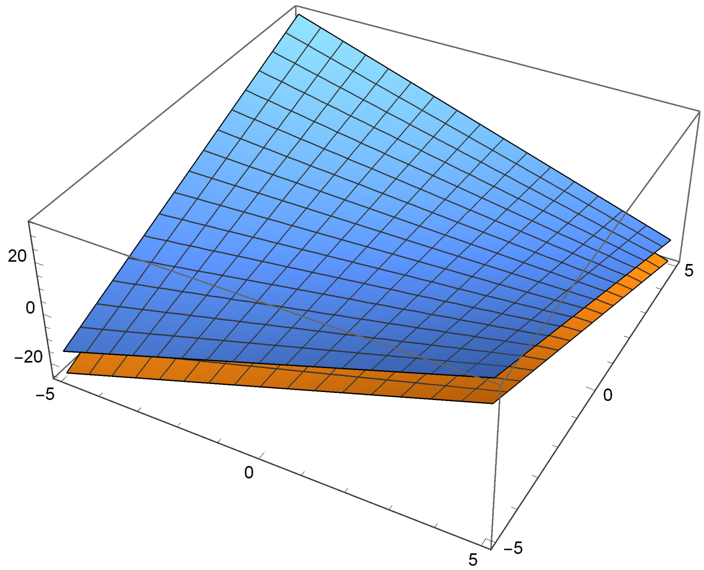
Out[∘]=



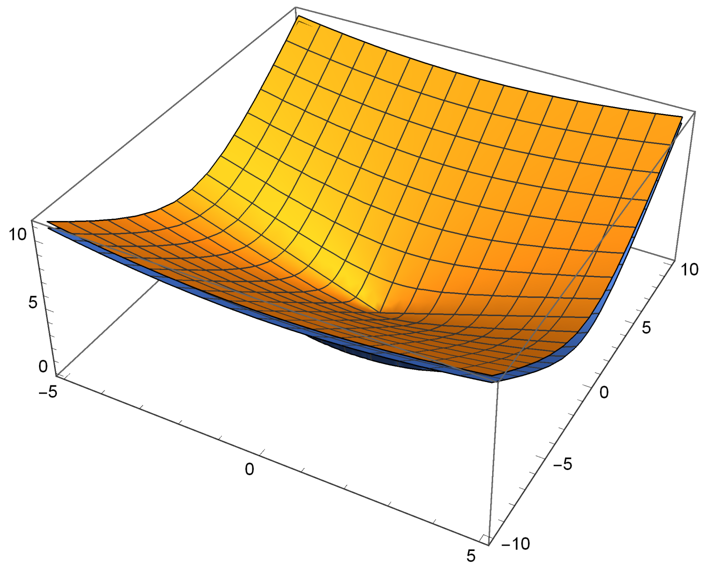
**Ques 3.** (y+xu) $u_x$ - (x+uy) $u_y$ = $x^2 - y^2$

**The characteristic system is given by dx/y+ux = dy/-(x+uy) = du/$x^2 - y^2$ and the characteristic equations are given by xy+u = $c_1$ and $x^2 + y^2 - u^2 = c_2$. Taking $c_1 = 0$ and 9 and $c_2 = 0$ and 10.**

In[ ]:=
```
Plot3D[{-x*y, -x*y+9}, {x, -5, 5}, {y, -5, 5}, PlotStyle → Thick]
```

Out[ ]=



In[ ]:=
```
Plot3D[{Sqrt[x^2 + y^2], Sqrt[x^2 + y^2 - 10]}, {x, -5, 5}, {y, -10, 10}, PlotStyle → Thick]
```

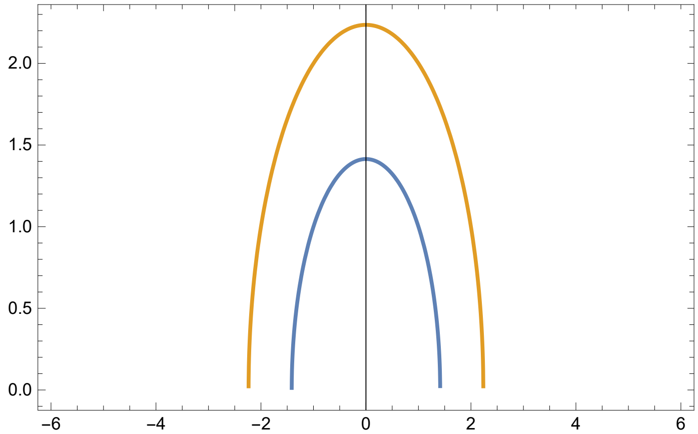
Out[ ]=



**Ques 4.** $x\, u_x + y\, u_y = u$

**The characteristic system is given by dx/x = dy/y = du/u and the characteristic equations are given by y/x = $c_1$ and $u/x = c_2$. Taking $c_1 = 1$ and 3 and $c_2 = 3$ and 8.**

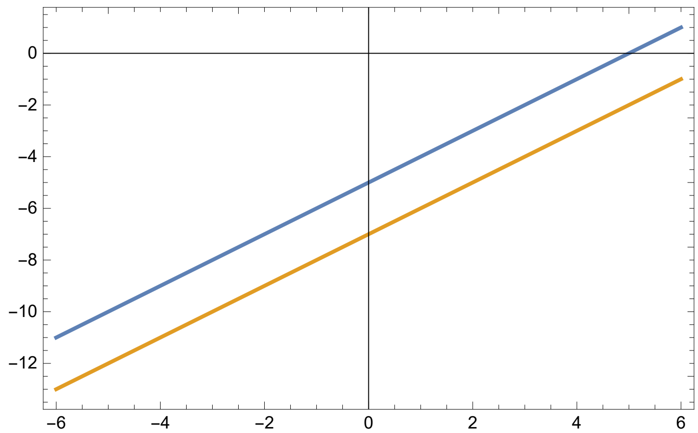
In[76]:=
```
Plot[{Sqrt[2 - x^2], Sqrt[5 - x^2]}, {x, -6, 6}, PlotStyle → Thick, Frame → True]
```

Out[76]=




In[77]:=
```
Plot[{y - 5, y - 7}, {y, -6, 6}, PlotStyle → Thick, Frame → True]
```

Out[77]=




## Ques 5 : $u(x+y)u_x + u(x-y)u_y = x^2 + y^2$

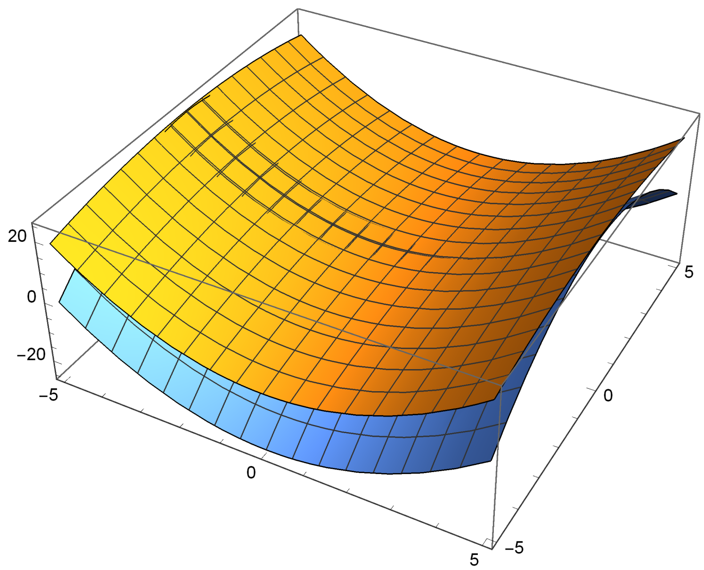
The characteristic system is given by dx/u(x+y) = dy/u(x-y) = du/(x^2+y^2) and the characteristic equations are given by

x^2/2 - y^2/2 - u = c1 and y^2 - u^2 - x^2 = c2.

Taking c1 = 2 and 5 and c2 = 5 and 7.

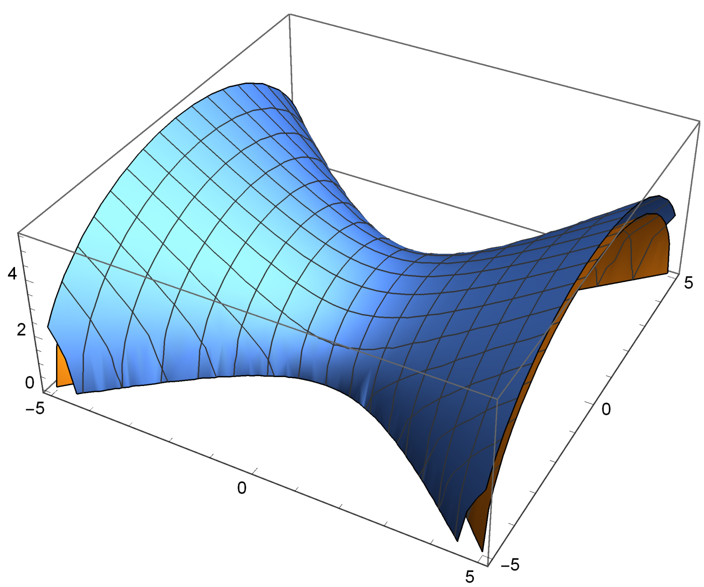
In[78]:= `Plot3D[{x² - y² / 4, x² - y² - 2 / 4}, {x, -5, 5}, {y, -5, 5}, PlotStyle → Thick]`

Out[78]=




In[79]:= `Plot3D[{Sqrt[x^2 - y^2], Sqrt[x^2 - y^2 + 5]}, {x, -5, 5}, {y, -5, 5}, PlotStyle → Thick]`

Out[79]=




## Ques 6 : $u_x - u_y = 1$

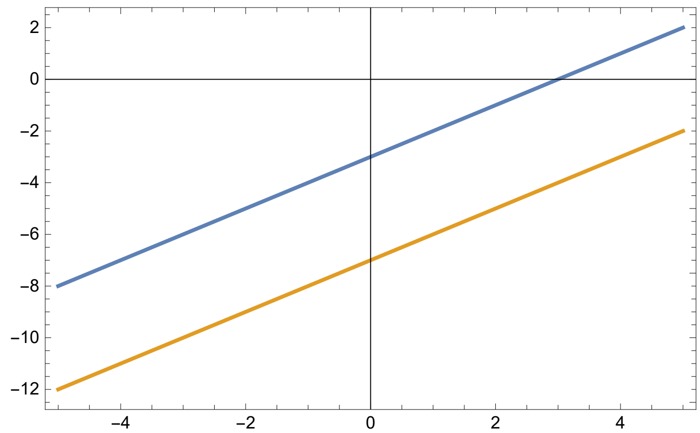
The characteristic system is given by dx/1 = dy/(-1) = du/1 and the characteristic equations are given by

x-y = c1 and -y-u = c2.

Taking c1 = 2 and 5 and c2 = 5 and 7.

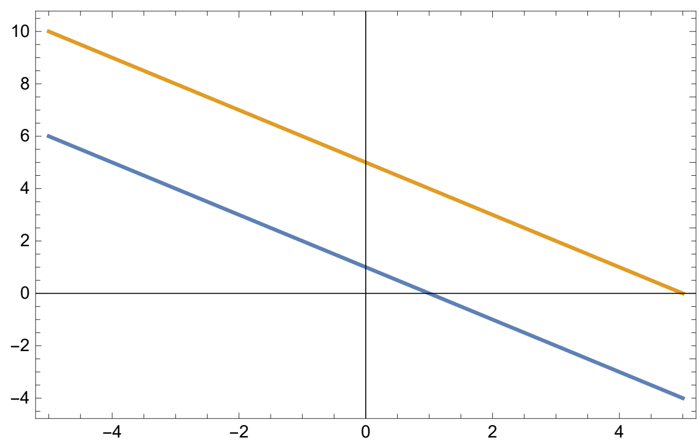
In[80]:= 
```
Plot[{x - 3, x - 7}, {x, -5, 5}, PlotStyle → Thick, Frame → True]
```

Out[80]=




In[81]:= 
```
Plot[{1 - y, 5 - y}, {y, -5, 5}, PlotStyle → Thick, Frame → True]
```

Out[81]=



# Practical 8 :: Plot the integral surfaces of First Order Partial Differential Equations with initial data

**Ques 1.** $x\,u_x + y\,u_y = 2xy$

```
In[∘]:= pde1 = x * D[u[x, y], x] + y * D[u[x, y], y] == 2 * x * y;

In[∘]:= sol1 = DSolve[pde1, u[x, y], {x, y}]
```

Out[∘]= $\left\{\left\{u[x, y] \to x\,y + c_1\left[\frac{y}{x}\right]\right\}\right\}$

```
In[∘]:= a.C[a] = Sin a
```

Set: Tag Dot in a.$c_a$ is Protected.

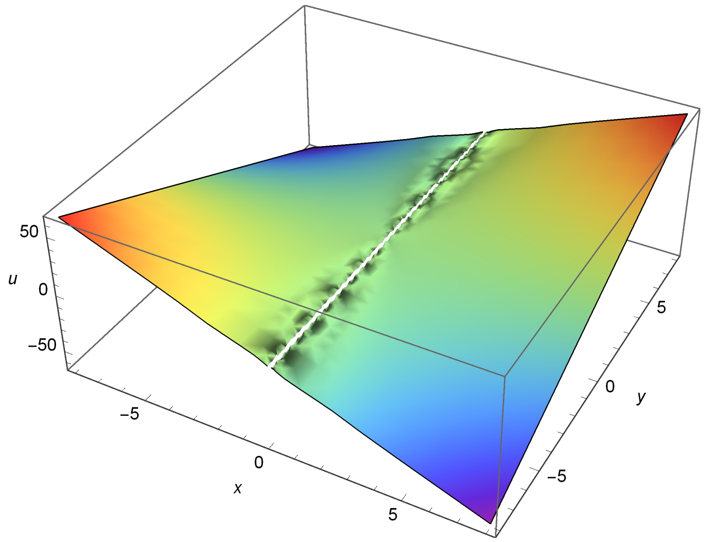
Out[∘]= a Sin

```
In[∘]:= parsol = u[x, y] /. sol1[[1]] /. C[1][a_] → Sin[a]
```

Out[∘]= $x\,y + \text{Sin}\left[\frac{y}{x}\right]$

In[ ]:=
```
Plot3D[parsol, {x, -8, 8}, {y, -8, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "Rainbow"]
```

Out[ ]=




In[ ]:=
```
b.C[a] = Cos a
```

Set: Tag Dot in b.$\mathbb{c}_a$ is Protected.

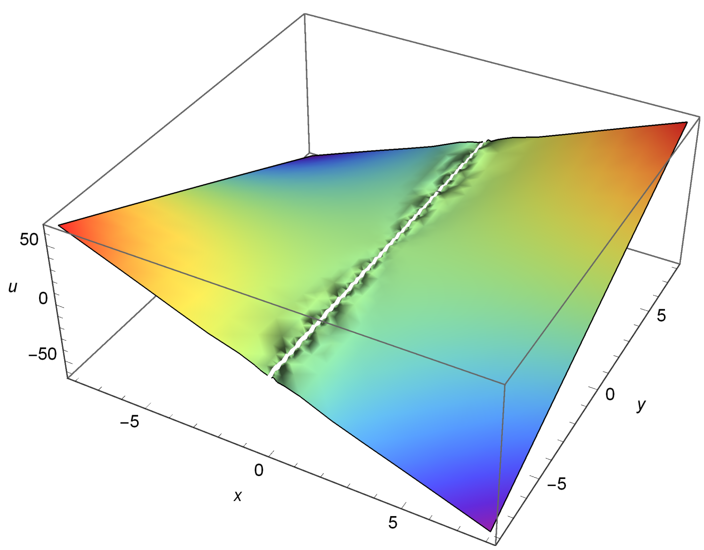
Out[ ]= a Cos

In[ ]:=
```
parsol = u[x, y] /. sol1[[1]] /. C[1][a_] → Cos[a]
```

Out[ ]= $x\,y + \mathrm{Cos}\left[\frac{y}{x}\right]$

In[ ]:=
```
Plot3D[parsol, {x, -8, 8}, {y, -8, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "Rainbow"]
```

Out[ ]=



In[ ]:=
```
c.C[a] = a^3
```

Set: Tag Dot in c.$\mathbb{c}_a$ is Protected.

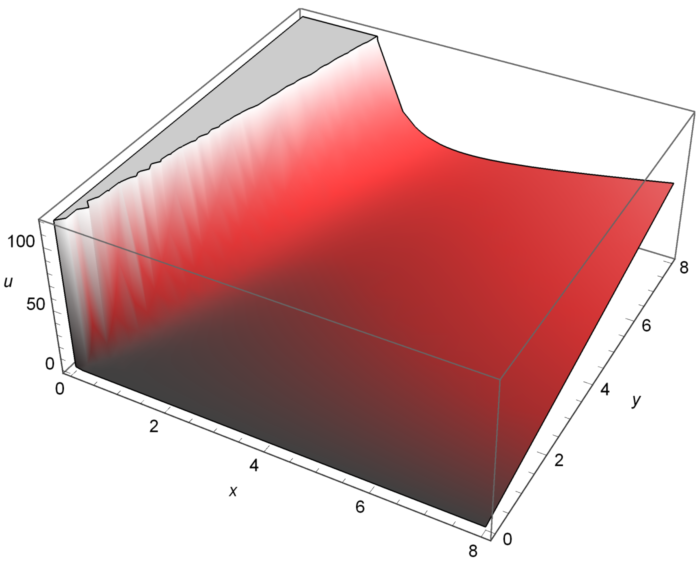
Out[ ]= $a^3$

In[ ]:=
```
parsol = u[x, y] /. sol1[[1]] /. C[1][a_] → a^3
```

Out[ ]= $x\,y + \frac{y^3}{x^3}$

In[ ]:=
```
Plot3D[parsol, {x, 0, 8}, {y, 0, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "CherryTones"]
```

Out[ ]=




Solve the following :

Ques 2 : $3\,u_x + 2\,u_y = 0$

In[ ]:=
```
pde2 = 3 * D[u[x, y], x] + 2 * D[u[x, y], y] == 0;
sol2 = DSolve[pde2, u[x, y], {x, y}]
```

Out[ ]= $\left\{\left\{u[x, y] \to \mathbb{c}_1\left[\frac{1}{3}\left(-2\,x + 3\,y\right)\right]\right\}\right\}$

In[ ]:=
```
a.C[a] = Sin a
```

Set: Tag Dot in a.$\mathbb{c}_a$ is Protected.

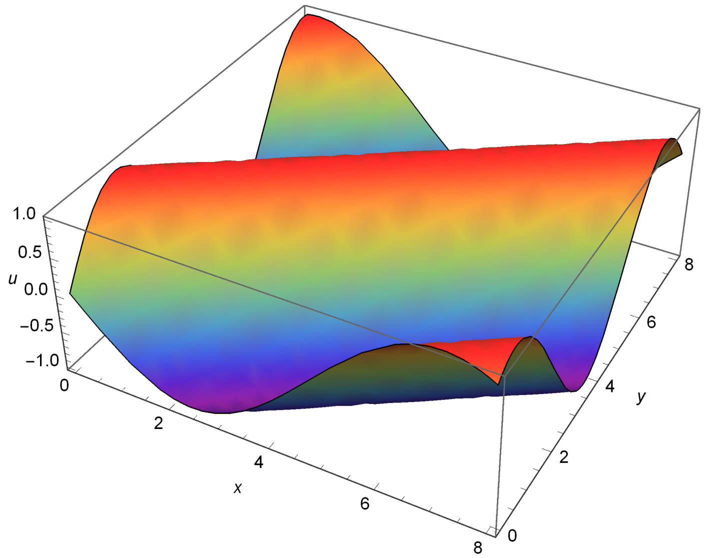
Out[ ]= a Sin

In[ ]:=
```
parsol2 = u[x, y] /. sol2[[1]] /. C[1][a_] → Sin[a]
```

Out[ ]= $\text{Sin}\left[\frac{1}{3}\left(-2\,x + 3\,y\right)\right]$

In[∘]:=
```
Plot3D[parsol2, {x, 0, 8}, {y, 0, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "Rainbow"]
```

Out[∘]=




In[∘]:=
```
a.C[a] = Cos a
```

Set: Tag Dot in a.ℭa is Protected.

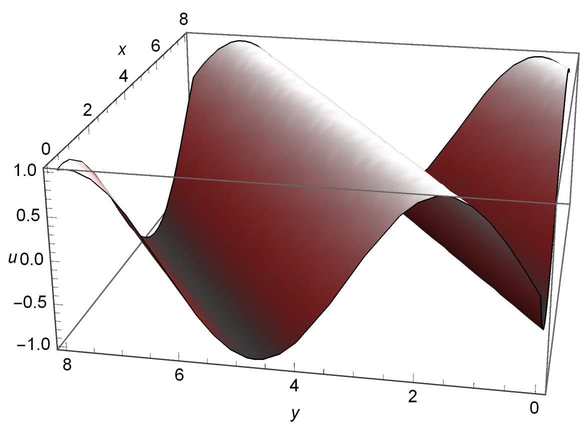
Out[∘]= a Cos

In[∘]:=
```
parsol2 = u[x, y] /. sol2[[1]] /. C[1][a_] → Sin[a]
```

Out[∘]= $\text{Sin}\left[\frac{1}{3}\left(-2\,x+3\,y\right)\right]$

In[∘]:=
```
Plot3D[parsol2, {x, 0, 8}, {y, 0, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "CherryTones"]
```

Out[∘]=

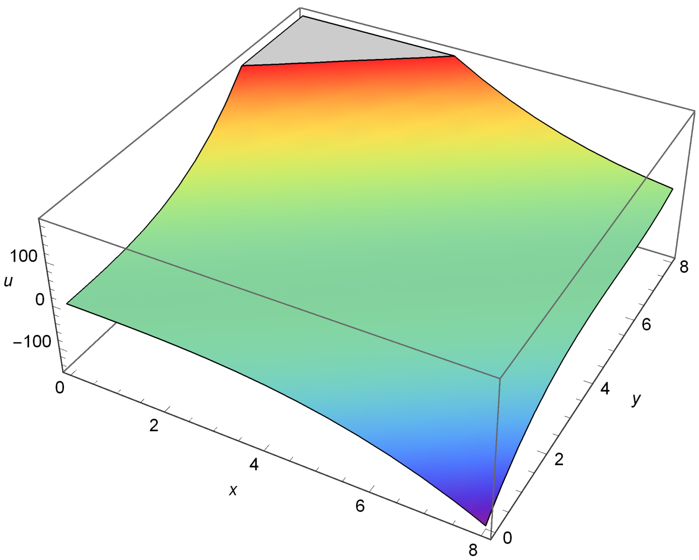



$a.C[a] = a^3$

In[ ]:=
```
parsol2 = u[x, y] /. sol2[[1]] /. C[1][a_] → a^3
```

Out[ ]= $\frac{1}{27}\left(-2x+3y\right)^3$

In[ ]:=
```
Plot3D[parsol2, {x, 0, 8}, {y, 0, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "Rainbow"]
```

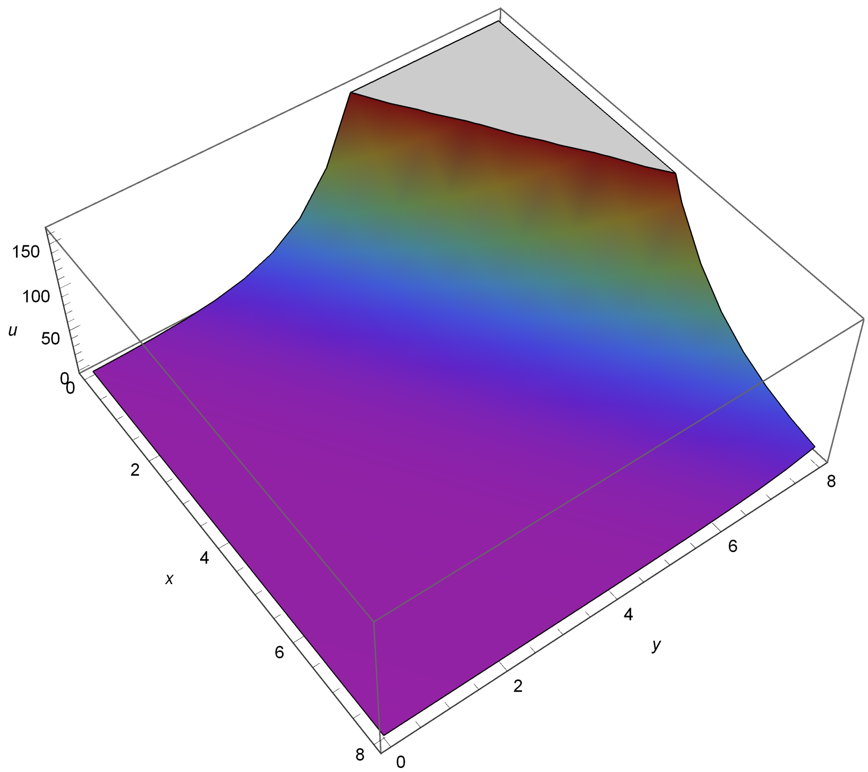
Out[ ]=



In[ ]:=
```
parsol2 = u[x, y] /. sol2[[1]] /. C[1][a_] → Exp[a]
```

Out[ ]= $e^{\frac{1}{3}(-2x+3y)}$

In[ ]:=
```
Plot3D[parsol2, {x, 0, 8}, {y, 0, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "Rainbow"]
```

Out[ ]=




**Ques 3 : $u_x - u_y = 1$**

In[ ]:=
```
pde3 = D[u[x, y], x] - D[u[x, y], y] == 1;
sol3 = DSolve[pde3, u[x, y], {x, y}]
```

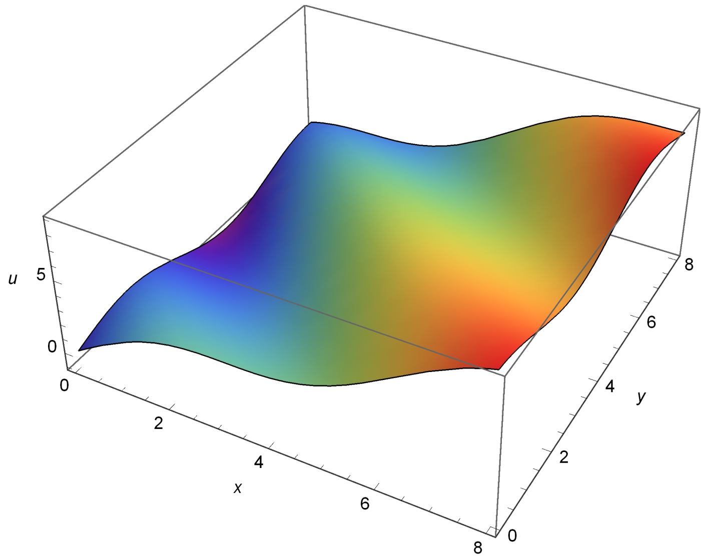
Out[ ]= {{u[x, y] → x + $\mathbb{c}_1$[x + y]}}

In[ ]:=
```
parsol3 = u[x, y] /. sol3[[1]] /. C[1][a_] → Sin[a]
```

Out[ ]= x + Sin[x + y]

In[∘]:=
```
Plot3D[parsol3, {x, 0, 8}, {y, 0, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "Rainbow"]
```

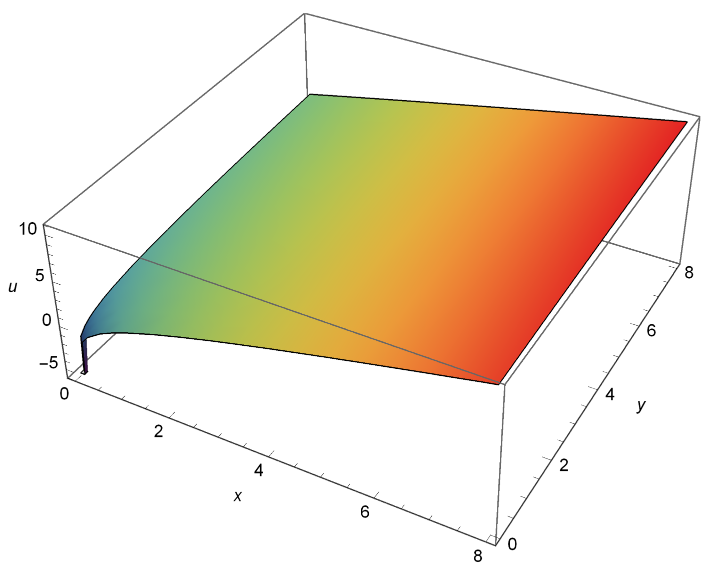
Out[∘]=




In[∘]:=
```
parsol3 = u[x, y] /. sol3[[1]] /. C[1][a_] → Log[a]
Plot3D[parsol3, {x, 0, 8}, {y, 0, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "Rainbow"]
```

Out[∘]= x + Log[x + y]

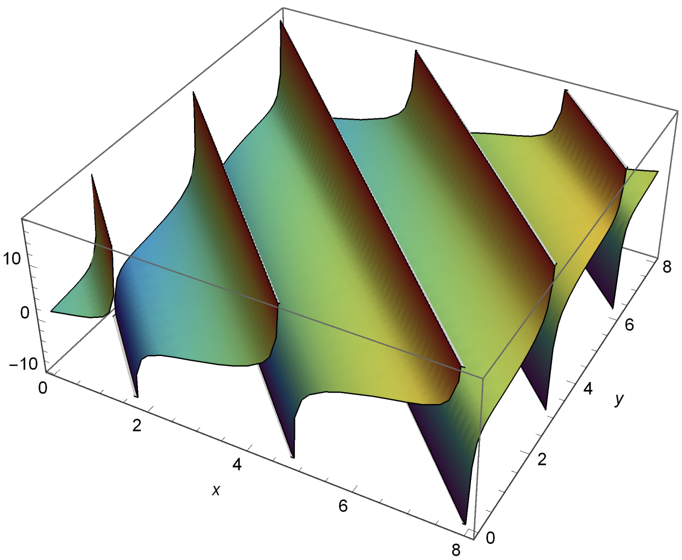
Out[∘]=



```
In[•]:= arsol3 = u[x, y] /. sol3[[1]] /. C[1][a_] → Tan[a]
        Plot3D[parsol3, {x, 0, 8}, {y, 0, 8},
         AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "Rainbow"]
```

Out[•]= $x + \mathrm{Tan}[x + y]$

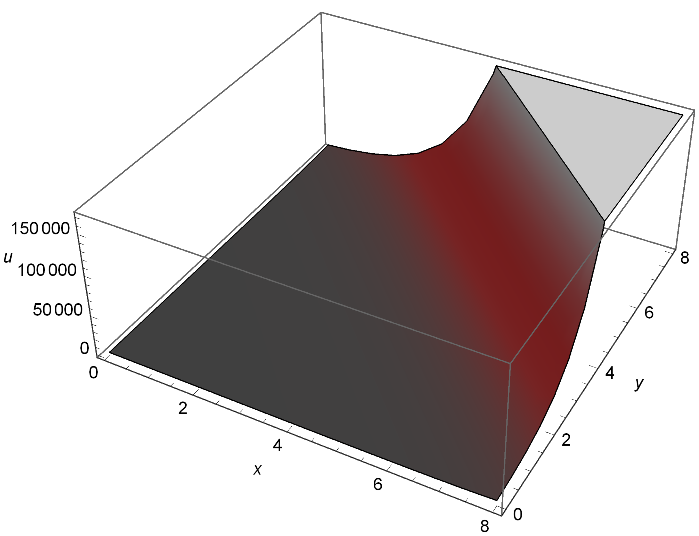
Out[•]=




```
In[•]:= parsol3 = u[x, y] /. sol3[[1]] /. C[1][a_] → Exp[a]
        Plot3D[parsol3, {x, 0, 8}, {y, 0, 8},
         AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "CherryTones"]
```

Out[•]= $e^{x+y} + x$

Out[•]=



In[ ]:=
```
parsol3 = u[x, y] /. sol3[[1]] /. C[1][a_] → a^2
Plot3D[parsol3, {x, 0, 8}, {y, 0, 8},
 AxesLabel → {x, y, u}, Mesh → None, ColorFunction → "Rainbow"]
```

Out[ ]= $x + (x + y)^2$

Out[ ]=