Homework One

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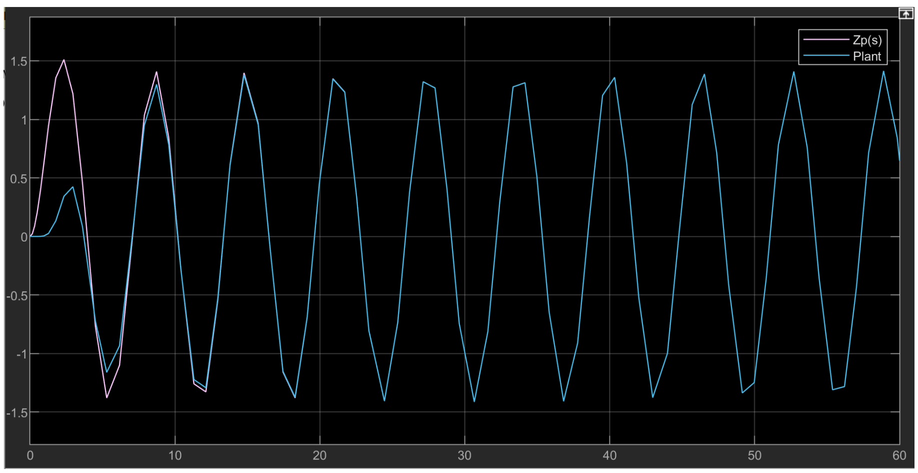
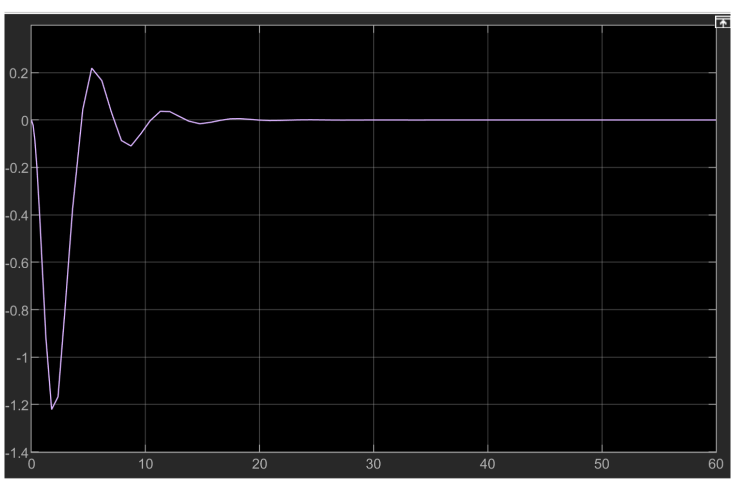
QUESTION ONE, PART A

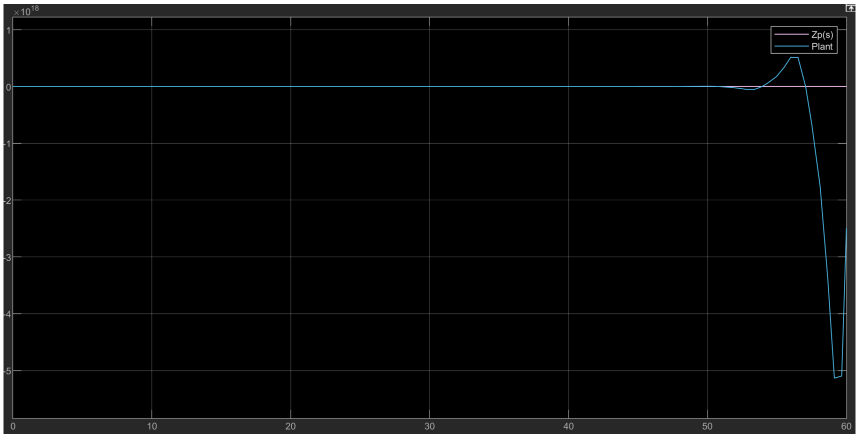
The tracking error of the system decreased and the reference and actual outputs converged. This means that the controller was effective in matching the two plants and giving the desired output for the system. The following graph shows the two outputs (the reference is shown in pink and the actual is shown in blue).



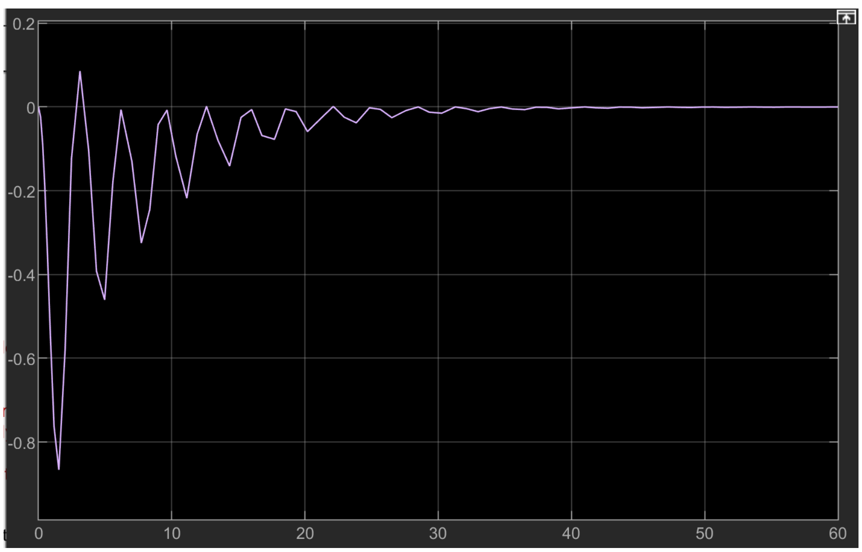
This can also be seen by examining the error itself, which is the difference between the actual output and the reference output. As the following plot shows, the error, e(t), stabilizes at zero, indicating that both models are exactly equal to each other.



Changing the gain also affects the stability of the system. A positive gain will cause it to be unstable, moving from a constant position to oscillatory behaviour. For example, the figure below shows the output with a gain of 5.



The actual output begins to oscillate drastically, on a scale of 10^{18}, after about 50 seconds. A gain of -5, on the other hand, still exhibits the same form of behaviour as depicted above:



The error reduces to 0 after about 45 seconds. The lower gain, therefore, took longer to stabilize than seen previously with a gain of -1.

QUESTION ONE, PART B

Problem Statement:

An essential feedback control system was given with the following structure,

$$\frac{dy}{dt} = -ay + bu$$

where u(t) is the controller and y(t) is the plant output. The constant parameters are unknown, but the sign of the high frequency gain is assumed to be known. The desired system behaviour is depicted through the following reference model:

$$\frac{\mathrm{dy}_m}{\mathrm{dt}} = -a_m y_m + b_m r,$$

in which a_m and b_m are chosen constants that guarantee stability. The task is to design a controller, u(t), that will adjust the plant so that it converges with the reference model. This means that $y(t)$ should equal $y_m(t)$. This controller should be of the following form:

$$\mathrm{u(t)} = \theta_1 r(t) - \theta_2 y(t)$$

Inputting this controller for u above,

$$\frac{\mathrm{dy}}{\mathrm{dt}} = -\mathrm{ay}(t) + b(\theta_1 r(t) - \theta_2 y(t))$$

Since the parameters, a and b, are unknown, we cannot set θ_1 and θ_2 directly. So, we define the ‘ideal’ control gains in which the output matches that of the reference model as:

$$\theta_1{}^o = \frac{b_m}{b} \text{ and } \theta_2{}^o = \frac{a_m - a}{b}.$$

Now, the MIT rule can be implemented to determine the online estimates of these parameters in real time, resulting in:

$$\frac{\mathrm{d}\theta_1}{\mathrm{dt}} = -\gamma_1 e \frac{\partial e}{\partial \theta_1} \text{ and } \frac{\mathrm{d}\theta_2}{\mathrm{dt}} = \gamma_2 e \frac{\partial e}{\partial \theta_2},$$

where the adaptation gains, γ_1 and γ_2, are greater than zero and the output error, e, is y - y_m.

Thus, we can define the output, y, as follows.

$$y(t) = \frac{\mathrm{b}\theta_1}{s + a + \mathrm{b}\theta_2} r(t)$$

This indicates that the input, r(t), is being filtered, which gives the output. Further, the sensitivities can be defined as:

$$\frac{\partial e}{\partial \theta_1} = \frac{b}{s + a + \mathrm{b}\theta_2} r(t) \text{ and } \frac{\partial e}{\partial \theta_2} = -\frac{b}{s + a + \mathrm{b}\theta_2} y(t).$$

Since a and b are still unknown, we must approximate the sensitivity derivates by assuming that the estimates, θ_1 and θ_2 eventually converge to their ideal values. This results in the sensitivity derivatives becoming

$$\frac{\partial e}{\partial \theta_1} \approx \frac{b}{s + a_m} r(t) \text{ and } \frac{\partial e}{\partial \theta_2} \approx \frac{b}{s + a_m} y(t)$$

The adaptive laws are thus:

$$\frac{\mathrm{d}\theta_1}{\mathrm{dt}} = -\gamma_1 e\left(\frac{b}{s + a_m} r(t)\right) \text{ and } \frac{\mathrm{d}\theta_1}{\mathrm{dt}} = \gamma_2 e\left(\frac{b}{s + a_m} y(t)\right)$$

Since we know that b is positive and constant, it can be absorbed into the adaptation gains as follows.

$$\frac{\mathrm{d}\theta_1}{\mathrm{dt}} = -\gamma_1' e\left(\frac{a_m}{s + a_m} r(t)\right) \text{ and } \frac{\mathrm{d}\theta_1}{\mathrm{dt}} = \gamma_2' e\left(\frac{a_m}{s + a_m} y(t)\right)$$

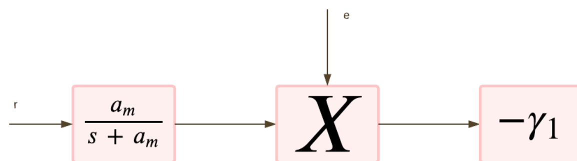
Now the task is to simulate this adaptive control system taking a = 1, b = 0.5, and $a_m = b_m = 2$ along with a square wave reference input that has an amplitude of 1 and a period of 20 seconds.

Procedure:

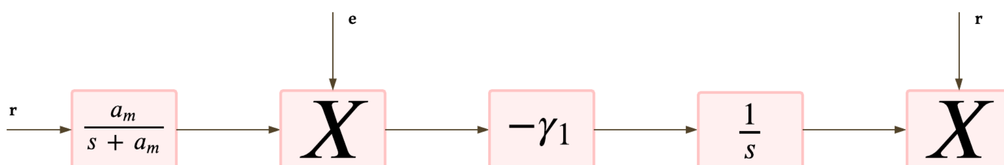
First, I looked at the resulting equation for $\frac{d\theta_1}{dt}$. This meant starting by filtering the input signal using:

$$\frac{a_m}{s + a_m} = \frac{2}{s + 2} \text{ (with the given values).}$$

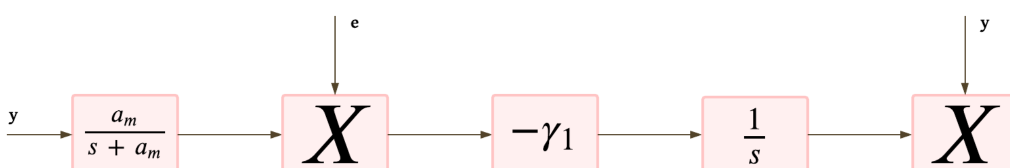
Next, that result was multiplied with the error of the system, simply depicted by taking the difference in the actual output and the reference output. Finally, the gain, γ, was incorporated into the system (it was set to -1 to start, as the problem stated). This resulted in the following component:



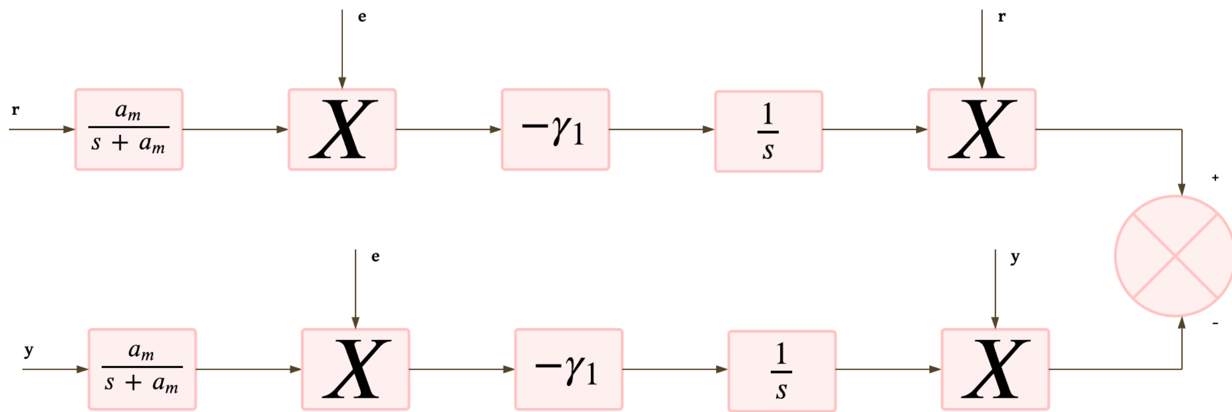
With this stage complete, the actual controller for r(t) is needed. To get θ_1 from $\frac{d\theta_1}{dt}$, an integrator is necessary. Following this, θ_1 must be implemented on the input signal, r(t). Thus, the above block becomes:



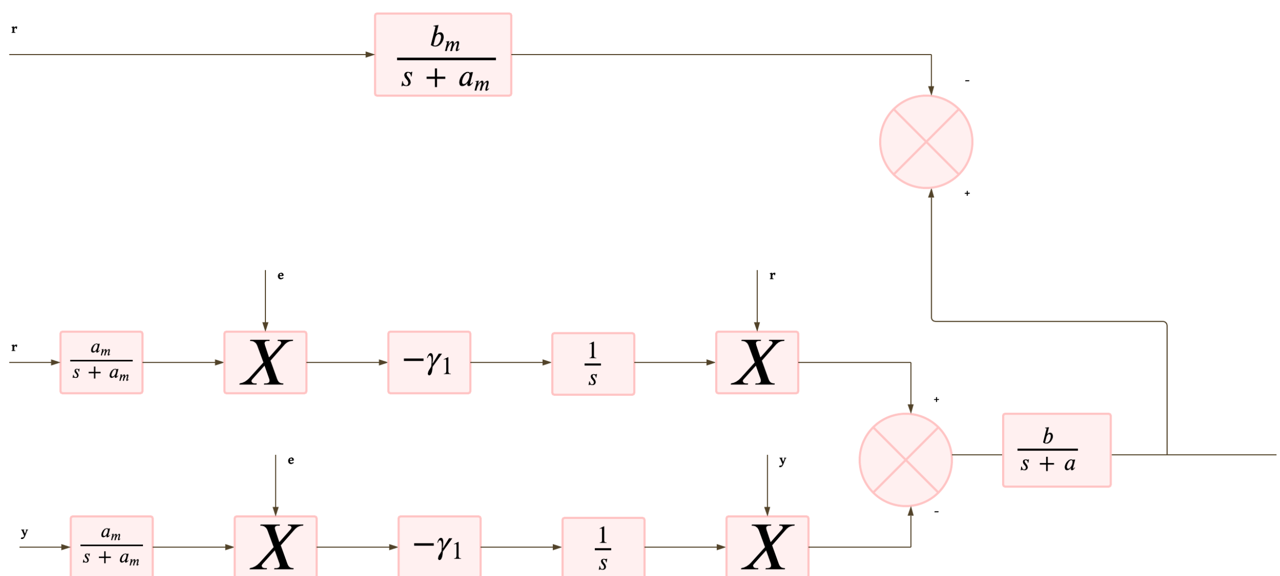
A similar process was followed to get θ_2, but, rather than acting on the input signal, it uses the output , y(t). Thus, the following shows the block for θ_2.



Now that both controllers had been designed, it was time to combine them and to implement the feedback structure. First, both controllers are summed based on the designated controller system, θ_1r(t) - θ_2y(t). This results in:



Now the only thing left to do was to apply this controller to the plant in order to get the output of the system. Additionally, the error section needed to be built by simply subtracting the reference output from the actual one, or $y - y_m$. Thus, the following image depicts the final design of the system.



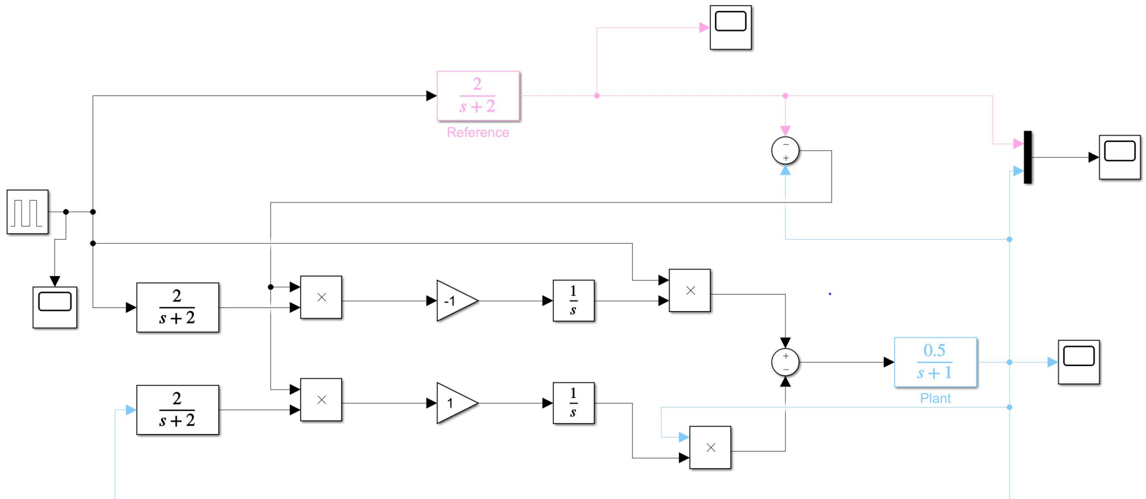
Simulink Design:

The above design was implemented in Simulink with the following parameter values:

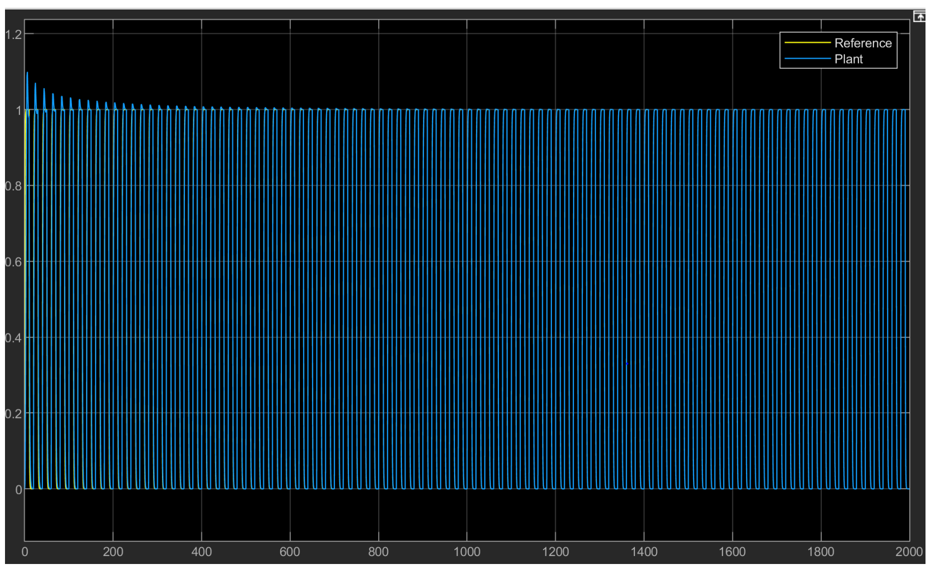
$$a_m = b_m = 2, \ a = 1, \ \text{and}\ b = 0.5.$$

Additionally, the result was simulated with with a square wave of amplitude 1 with a 20 second period.

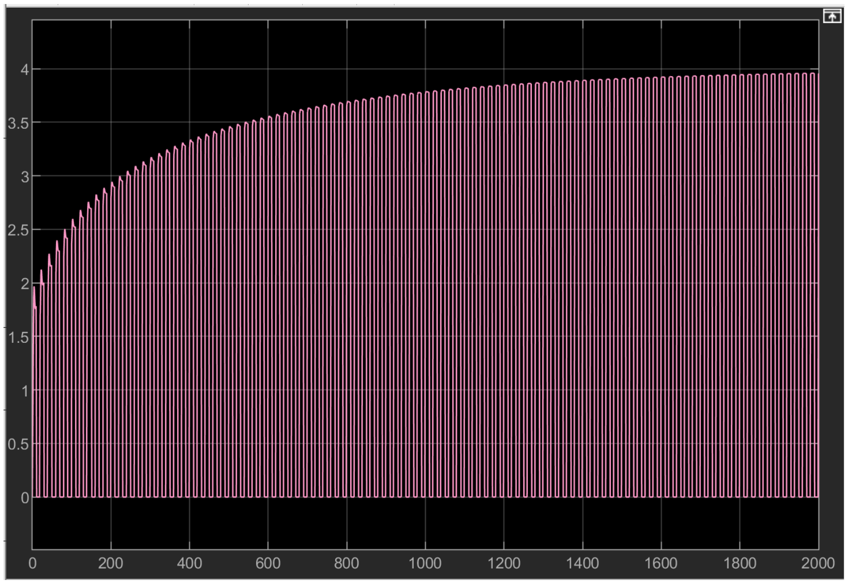
The following image depicts the final system design in Simulink:



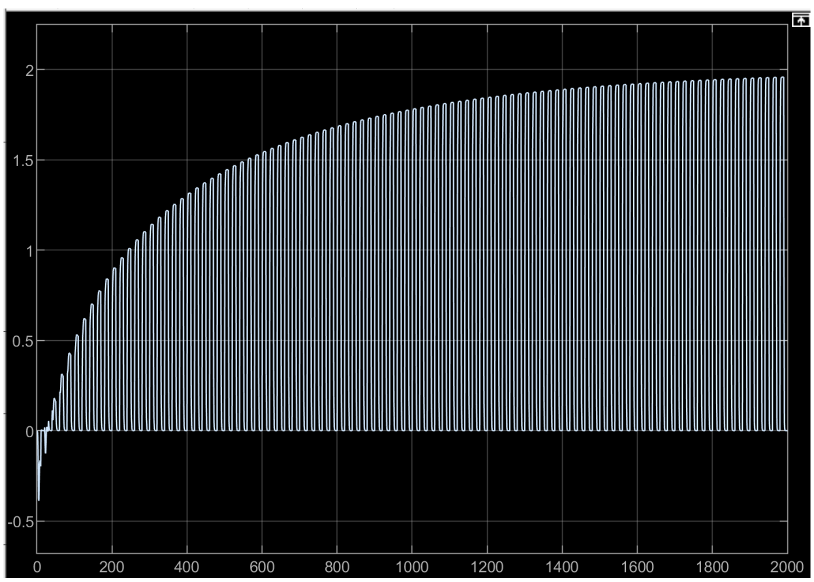
The following graph indicates the evolution of the plant's output over time. It shows both the reference output and the plant output. As the figure depicts, the actual output of the plant slowly converges with the reference model's output, indicating that the controller was effective.



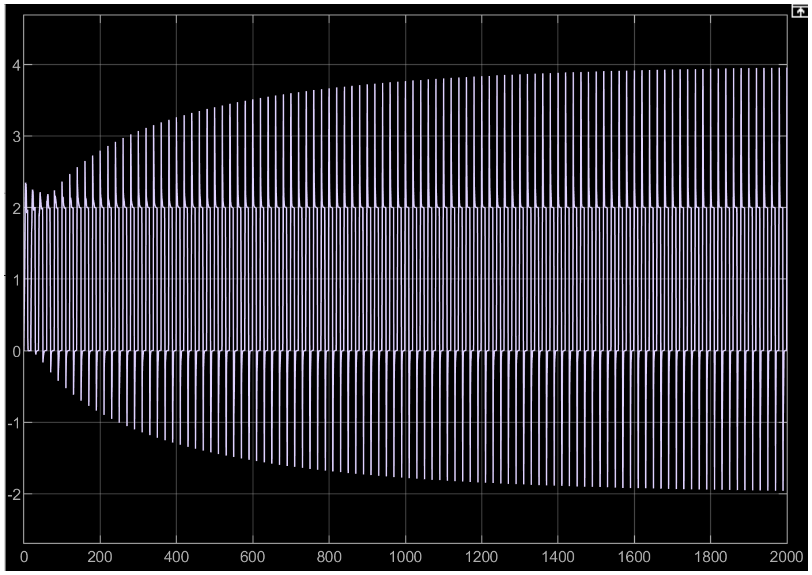
This next figure represents $\theta_1(t)$. As the graph depicts, θ_1 varies significantly over time. It seems to increase in value.



Additionally, the second part of the controller, $\theta_2(t)$ results in the following plot. Both increase over time and seem to converge to something similar to the square wave input signal.



These two signals are combined to make the complete control system, with a structure of $\theta_1(t) * r(t) - \theta_2(t) * y(t)$. Looking at the output of this system, we can see that the input signal is replicated in the center, or overlapping, areas of $\theta_1(t)$ and $\theta_2(t)$. The controller seems to counteract changes to the signal and ultimately acts to bring the input signal closer to our desired output.



In conclusion, the designed controller was efficient in modifying the plant and aligning with the desired output.