homework

TWO

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1. $y = G(s)u(s)$

$$G(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}$$

a. Obtain parametric models for the plant in the form of SPM and DP when $\theta^* = [b_2, b_1, b_0, a_2, a_1, a_0]^T$

A static parametric model (spm) has the form, $z = \theta^* \phi$

$$y = G(s)u(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0} u(s)$$

$$\longrightarrow y(s^3 + a_2 s^2 + a_1 s + a_0) = (b_2 s^2 + b_1 s + b_0) u(s)$$

Using a filter of $\frac{1}{(s+\lambda)^3}$, where $\lambda > 0$ gives:

$$\frac{s^3}{(s+\lambda)^3} y + a_2 \frac{s^2}{(s+\lambda)^3} y + a_1 \frac{s}{(s+\lambda)^3} y + a_0 \frac{1}{(s+\lambda)^3} y$$
$$= b_2 \frac{s^2}{(s+\lambda)^3} u + b_1 \frac{s}{(s+\lambda)^3} u + b_0 \frac{1}{(s+\lambda)^3} u$$

Now, the SPM can be represented by:

$$z = \frac{s^3}{(s+\lambda)^3} y$$

$$\phi = \left[-\frac{s^2}{(s+\lambda)^3} y, \; -\frac{s}{(s+\lambda)^3} y, \; -\frac{1}{(s+\lambda)^3} y, \; \frac{s^2}{(s+\lambda)^3} u, \; \frac{s}{(s+\lambda)^3} u, \; \frac{1}{(s+\lambda)^3} u \right]$$

$$\theta^* = \begin{bmatrix} a_2 \\ a_1 \\ a_0 \\ b_2 \\ b_1 \\ b_0 \end{bmatrix}$$

A dynamic parametric model (dpm) has the form,
$z = W(s)\theta^* \phi$

Using the same procedure as above,

$$z = \frac{s^3}{(s+\lambda)^3} y$$

$$W = \frac{1}{(s+\lambda)^3}$$

Since W is the filter, that leaves,

$$\phi = [-s^2 y, -sy, -y, s^2 u, su, u]$$

θ^* remains the same as it contains the unknowns:

$$\theta^* = [a_2, a_1, a_0, b_2, b_1, b_0]^T$$

b. If a_0, a_1, and a_2 are Known, i.e., $a_0 = 2$, $a_1 = 1$, and $a_2 = 3$, obtain a parametric model for the plant in terms of $\theta^* = [b_2, b_1, b_0]^T$.

$$y = \frac{b_2 s^2 + b_1 s + b_0}{s^3 + 3s^2 + s + 2} u$$

$$\rightarrow y(s^3 + 3s^2 + s + 2) = (b_2 s^2 + b_1 s + b_0) u$$

Filtering with $\frac{1}{(s+\lambda)^3}$:

$$\frac{s^3}{(s+\lambda)^3} y + 3\frac{s^2}{(s+\lambda)^3} y + \frac{s}{(s+\lambda)^3} y + 2\frac{1}{(s+\lambda)^3}$$

$$= b_2 \frac{s^2}{(s+\lambda)^3} u + b_1 \frac{s}{(s+\lambda)^3} u + b_0 \frac{1}{(s+\lambda)^3} u$$

$$z = y\left(\frac{s^3}{(s+\lambda)^3} + \frac{3s^2}{(s+\lambda)^3} + \frac{s}{(s+\lambda)^3} + \frac{2}{(s+\lambda)^3}\right)$$

$$\theta^* = \begin{bmatrix} b_2 \\ b_1 \\ b_0 \end{bmatrix}, \quad \phi = \left[\frac{s^2}{(s+\lambda)^3} u, \frac{s}{(s+\lambda)^3} u, \frac{1}{(s+\lambda)^3} u\right]$$

$$z = \theta^* \phi$$

c. If b_0, b_1, and b_2 are Known, i.e., $b_0 = 2$, $b_1 = b_2 = 0$, obtain a parametric model in terms of $\theta^* = [a_2, a_1, a_0]^T$.

$$y = \frac{b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a s + a_0} u$$

$$y = \frac{2s^2}{s^3 + a_2 s^2 + a s + a_0} u$$

$$y(s^3 + a_2 s^2 + a s + a_0) = 2s^2 u$$

$$\frac{s^3}{(s+\lambda)^3} y + a_2 \frac{s^2}{(s+\lambda)^3} y + a_1 \frac{s}{(s+\lambda)^3} y + a_0 \frac{1}{(s+\lambda)^3} y = 2\frac{s^2}{(s+\lambda)^3} u$$

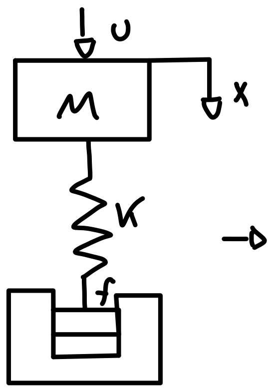
$$\rightarrow \frac{s^3}{(s+\lambda)^3} y - 2\frac{s^2}{(s+\lambda)^3} u = -a_2 \frac{s^2}{(s+\lambda)^3} y - a_1 \frac{s}{(s+\lambda)^3} y - a_0 \frac{1}{(s+\lambda)^3} y$$

$$z = \frac{s^3}{(s+\lambda)^3} y - 2\frac{s^2}{(s+\lambda)^3} u$$

$$\theta^* = \begin{bmatrix} a_2 \\ a_1 \\ a_0 \end{bmatrix}, \quad \phi = \left[\frac{-s^2}{(s+\lambda)^3} y, \frac{-s}{(s+\lambda)^3} y, \frac{-1}{(s+\lambda)^3} y\right]$$

Consider the mass-spring-dashpot system of Figure 2.1 described by 2.5 with x, u as the only signals available for measurement. Let us assume that $M = 100$ Kg and f, K are unknown parameters that we want to estimate online. Develop a parametric model for estimating f & K.

2.1:



$$M\ddot{x} = u - Kx - f\dot{x}$$

$$\theta^* = [f, K]^T$$
$$z = u$$
$$\phi = [\ddot{x}, \dot{x}, x]$$

→ solving for u:

$$u = \overset{100}{M}\ddot{x} + f\dot{x} + Kx$$

Filtering with $\frac{1}{\Lambda(s)} = \frac{1}{(s+\lambda)^2}$, $\lambda > 0$

$$\rightarrow \frac{Ms^2 + fs + K}{\Lambda(s)}(x) = \frac{1}{\Lambda(s)} u$$

$$= \frac{100s^2 + fs + K}{\Lambda(s)} x$$

$$z = \theta^{*T}\phi$$

$$z = \frac{1}{\Lambda(s)} u = \frac{1}{(s+\lambda)^2} u$$

$$\phi = \left[\frac{s^2}{\Lambda(s)}x, \frac{s}{\Lambda(s)}x, \frac{1}{\Lambda(s)}x\right] = \left[\frac{s^2}{(s+\lambda)^2}x, \frac{s}{(s+\lambda)^2}x, \frac{1}{(s+\lambda)^2}x\right]$$

$$\theta^* = [f, K]^T = \begin{bmatrix} f \\ K \end{bmatrix}$$

Consider the second-order ARMA model,

$$y(k) = -1.3y(k-1) - a_2 y(k-2) + b_1 u(k-1) + u(k-2)$$

where the parameters a_2 and b_1 are known constants. Express the unknown parameters in the form of a linear parametric model.

$$y(k) - u(k-2) = -1.3y(k-1) - a_2 y(k-2) + b_1 u(k-1)$$

$$\rightarrow \underbrace{y(k) - u(k-2)}_{z(t)} = \underbrace{[-1.3, -a_2, b_1]}_{\theta^*} \underbrace{\begin{bmatrix} y(k-1) \\ y(k-2) \\ u(k-1) \end{bmatrix}}_{\phi(t)}$$

Consider the nonlinear system

$$\dot{x} = f(x) + g(x)u$$

where the state x and the input u are available for measurement and $f(x)$, $g(x)$ are smooth but unknown functions of x. In addition, it is known that $g(x) > 0$ $\forall x$. We want to estimate the unknown functions f, g online using neural network approximation techniques. It is known that there exist constant parameters W^*_{fi}, W^*_{gi}, refered to as weights, such that

$$f(x) \approx \sum_{i=1}^{n} W^*_{fi} \varphi_{fi}(x)$$

$$g(x) \approx \sum_{i=1}^{m} W^*_{gi} \varphi_{gi}(x)$$

where $\varphi_{fi}(\cdot)$, $\varphi_{gi}(\cdot)$ are some basis functions that are known, and n, m are known integers representing the number of nodes of the neural network. Obtain a parameterization of the system in the form of SPM that can be used to identify the weights online.

$$\dot{x} = f(x) + g(x)u$$

$$= W_f \varphi_f(x) + W_g \varphi_g(x) u$$

Filtering with $\frac{1}{s+\lambda}$

$$\frac{s}{s+\lambda} x = W_f \varphi_f(x) \frac{1}{s+\lambda} + W_g \varphi_g(x) \frac{1}{s+\lambda} u$$

$$z = \frac{s}{s+\lambda} x$$

$$\theta^* = \begin{bmatrix} \sum_{i=1}^{m} W_{fi} \varphi_{fi}(x) \\ \sum_{i=1}^{n} W_{gi} \varphi_{gi}(x) \end{bmatrix}$$

$$\phi = \left[\frac{1}{s+\lambda}, \frac{1}{s+\lambda} u\right]$$