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Question One:

The dynamics of a hard disk drive servo system are given by

$$y = \frac{K_p}{s^2} \sum_{i=1}^{6} \frac{b_{1i} s + b_{0i}}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} u$$

where ω_i, $i = 1, \dots, 6$, are the resonant frequencies which are large, i.e., $\omega_1 = 11.2\pi \times 10^3$ rad/s, $\omega_2 = 15.5\pi \times 10^3$ rad/s, $\omega_3 = 16.6\pi \times 10^3$ rad/s, $\omega_4 = 18\pi \times 10^3$ rad/s, $\omega_5 = 20\pi \times 10^3$ rad/s, $\omega_6 = 23.8\pi \times 10^3$ rad/s. The unknown constants b_{1i} are of the order 10^4; b_{0i} are of the order of 10^8, and K_p is of the order 10^7. The damping coefficients ζ_i are of the order of 10^{-2}.

a. Derive a low order model for the servo system.

$$\frac{a}{\omega_i} \cong 0 \rightarrow \frac{a^2}{\omega_i^2} \cong 0 \text{ for } a < 10^3$$

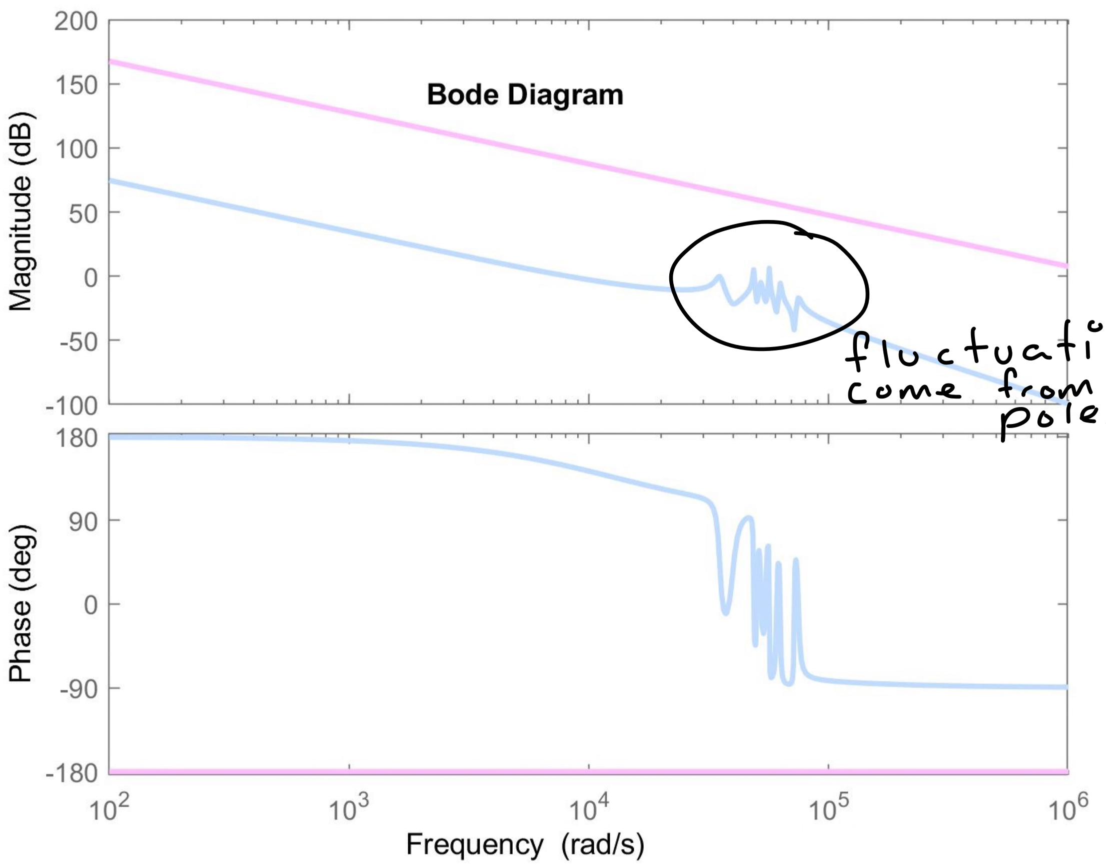
The system has 2 poles from the integrator, $1/s^2$, and 2 from the damped component.
To reduce the number of poles and, in turn, the complexity of the system, each term is divided by the highest order frequencies, ω_i^2, and compared with the others. The negligible terms can then be ignored.

$$y = \frac{K_p}{s^2}\left(\sum_{i=1}^{6} \frac{\frac{b_{1i}}{\omega_i^2} s + \frac{b_{0i}}{\omega_i^2}}{\underbrace{\frac{s^2}{\omega_i^2}}_{\text{small}} + \underbrace{\frac{\zeta_i \cancel{\omega_i}}{\cancel{\omega_i}^2}}_{\text{small}} s + 1} \right) u$$

the b_{1i} terms are on the order of 10^4, as are the ω_i terms. The b_{0i} terms have an order of 10^7.
Thus, the low order model is:

$$y_{red} = \frac{K_p}{s^2}\left(\frac{b_{01}}{\omega_1^2} + \frac{b_{02}}{\omega_2^2} + \dots + \frac{b_{06}}{\omega_6^2} \right) u$$

b. Obtain a Bode plot for the full-order and reduced-order models.



fluctuations come from damped poles

c. Use the reduced-order model in (a) to obtain a parametric model for the unknown parameters. Design a robust adaptive law to estimate the unknown parameters online.

reduced order model:

$$y_{red} = \frac{K_p}{s^2}\left(\frac{b_{01}}{\omega_1^2} + \frac{b_{02}}{\omega_2^2} + \dots + \frac{b_{06}}{\omega_6^2}\right)u$$

original model:

$$y = \frac{K_p}{s^2}\sum_{i=1}^{6}\frac{b_{1i}s + b_{0i}}{s^2 + 2\zeta_i\omega_i s + \omega_i^2}\,u$$

$$\rightarrow y = y_{red}\,(1 + \Delta(s))\,u$$

$$\Delta(s) = \left(\sum_{i=1}^{6}\frac{b_{1i}s}{s^2 + 2\zeta_i\omega_i s}\right)^{-1}$$

$$\rightarrow y = \frac{K_p}{s^2}\left(\frac{b_{01}}{\omega_1^2} + \frac{b_{02}}{\omega_2^2} + \dots + \frac{b_{06}}{\omega_6^2}\right)(1 + \Delta_m(s))\,u$$

$$y\,(s^2) = K_p\left(\frac{b_{01}}{\omega_1^2} + \frac{b_{02}}{\omega_2^2} + \dots + \frac{b_{06}}{\omega_6^2}\right)(1 + \Delta_m(s))\,u$$

filtering with $\underset{M}{\frac{1}{(s+\lambda)^2}}$ $\rightarrow$

$$\frac{s^2}{(s+\lambda)^2}y = \underbrace{K_p\left(\frac{b_{01}}{\omega_1^2} + \frac{b_{02}}{\omega_2^2} + \dots + \frac{b_{06}}{\omega_6^2}\right)}_{\text{constant}}\left(\frac{1}{(s+\lambda)^2}\right)\underbrace{(1 + \Delta_m(s))\,u}_{\text{filtered}}$$

$$\frac{s^2}{(s+\lambda)^2}y = K_p\left(\frac{b_{01}}{\omega_1^2} + \frac{b_{02}}{\omega_2^2} + \dots + \frac{b_{06}}{\omega_6^2}\right)\left(\frac{1}{(s+\lambda)^2}\right)u$$

$$+ K_p\left(\frac{b_{01}}{\omega_1^2} + \frac{b_{02}}{\omega_2^2} + \dots + \frac{b_{06}}{\omega_6^2}\right)\left(\frac{1}{(s+\lambda)^2}\right)(u)(\Delta m(s))$$

z, θ^*, ϕ, η

$$z = \frac{s^2}{(s+\lambda)^2}y\,(\omega_1 + \omega_2 + \dots + \omega_6)$$

$$z = \theta^* \phi + \eta$$

$$\theta^* = [K_p b_{01} \quad K_p b_{02} \quad \dots \quad K_p b_{06}]$$

$$\phi = \left[\frac{1}{(s+\lambda)^2}u\right]$$

$$\eta = 1 + \Delta m(s) = \sum_{i=1}^{6} \frac{b_{1i}\, s}{s^2 + 2\zeta_i \omega_i s}$$

Since the reduced order spm, $z = \theta^* \phi$, doesn't include all unknown parameters, the adaptive law will need to be robustified. I plan to implement a gradient adaptive law with instaneous cost, a σ-modification, and a dynamic normalization term.

$$\dot{\hat{\theta}} = \Gamma \varepsilon \phi - \sigma_s \Gamma \hat{\theta}$$

where $\varepsilon = \dfrac{z - \hat{\theta}^T \phi}{m^2}$ and $m^2(t) = 1 + \underbrace{n_s^2(t)}_{\text{static normalization}} + \underbrace{n_d(t)}_{\text{dynamic normalization}}$

static normalization $\phi^T \phi$

dynamic normalization $\dot{n}_d(t) = -\delta_0 n_d + u^2(t)$

$n_d(0) = 0$

$\delta_0 > 0$ is chosen such that $\Delta(s)$ has no poles to the right of $-\dfrac{\delta_0}{2}$ ("extra stable").

Additionally, using a switching modification σ_s will vary according to:

$$\sigma_s(t) = \begin{cases} 0 & \text{if } \sum b_{0i}^2 \le M_0^2 \\ \left(\dfrac{\sqrt{\sum_{i=1}^{6}(b_{0i}^2)}}{M_0} - 1\right)\sigma_0 & \text{if } M_0^2 < \sum b_{0i}^2 \le 2M_0^2 \\ \sigma_0 & \text{if } \sum b_{0i}^2 > 2M_0^2 \end{cases}$$

where $M_0 > 0$ is a constant such that $\sum b_{0i}^2 \le M_0^2$ and $\sigma_0 > 0$ is a design constant.

$$\rightarrow \dot{\hat{\theta}} = \Gamma \varepsilon \phi - \sigma_s \Gamma \hat{\theta}(t)$$

Question Two:

The linearized dynamics of a throttle angle θ to vehicle speed V subsystem are given by the third order system:

$$V = \frac{b p_1 p_2}{(s+a)(s+p_1)(s+p_2)}\theta + d$$

where $p_1, p_2 > 20$, $1 \geq a > 0$, and d is a load disturbance.

a. Obtain a parametric model for the parameters of the dominant part of the system.

the poles at p_1 and p_2 decay much faster than at a. Thus, the part of the system containing p_1 and p_2 represents the neglected dynamics $\rightarrow$

$$\Delta(s) = \frac{p_1 p_2}{(s+p_1)(s+p_2)} - 1$$

This means that the dominant, or nominal, part of the system is given by:

$$y = \frac{b}{s+a}u$$

Thus, a spm can be found for this reduced system:

$$y(s+a) = bu$$

$$\dot{y} + ay = bu$$

$$\rightarrow \dot{y} = bu - ay$$

Filtering with $\frac{1}{s+2}$ gives:

$$\underbrace{\frac{s}{s+2}y}_{z} = \underbrace{b}_{}\underbrace{\frac{1}{s+2}u}_{\phi} - \underbrace{a}_{\theta^*}\underbrace{\frac{1}{s+2}y}_{}$$

$$z = \frac{s}{s+2}y$$

$$\theta^{*T} = [a \;\; b]$$

$$\phi = \left[-\frac{1}{s+2}y \quad \frac{1}{s+2}u\right]$$

$$z = \theta^*\phi \rightarrow \text{spm}$$

b. Design a robust adaptive law for estimating these parameters online.

Including the unknown dynamics $\rightarrow$

$$y = \frac{b}{s+a}[1 + \Delta(s)]u + d$$

$$\rightarrow \dot{y} = -ay + bu + b\Delta(s)u$$

filtering with $\frac{1}{s+2}$:

$$\underbrace{\frac{s}{s+2}y(t)}_{z} = -\underbrace{a}_{\theta^*}\underbrace{\frac{1}{s+2}y(t)}_{} + \underbrace{b}_{}\underbrace{\frac{1}{s+2}u(t)}_{\phi} + \underbrace{b\frac{1}{s+2}\left(\frac{p_1 p_2}{(s+p_1)(s+p_2)} - 1\right)u(t) + d}_{\eta}$$

$$z = \frac{s}{s+2}y(t)$$

$$\theta^* = [a \;\; b]$$

$$\phi = \left[-\frac{1}{s+2}\dot{y}(t) \quad \frac{1}{s+2}\right]$$

$$\eta = \underbrace{b\frac{1}{s+2}\left(\frac{p_1 p_2}{(s+p_1)(s+p_2)} - 1\right)}_{\Delta_1}u(t) + d$$

$$z = \theta^*\phi + \eta$$

Since the system contains unknown dynamics, it will need to be robustified to prevent destabilization from those dynamics I plan to use an ε-modification with a dynamic normalization term.

$$\dot{\hat{\theta}} = \Gamma \varepsilon \phi - \sigma \Gamma \hat{\theta}$$

where $\varepsilon = \dfrac{z - \hat{\theta}^T \phi}{m^2}$

$\hookrightarrow m^2 = 1 + n_s^2(t) + n_d(t) = 1 + \phi^T \phi + n_d(t)$

n_d is a dynamic normalization term such that

$$\dot{n}_d = -\delta_0 n_d + u^2(t),\quad n_d(0) = 0$$

where $\delta_0 > 0$ is chosen so that $\Delta_1(s)$ has no poles to the right of $-\frac{\delta_0}{2}$.

$$\rightarrow \dot{\hat{a}} = \gamma_1 \varepsilon \phi_1 - \sigma \gamma_1 \hat{a}(t)$$

$$\dot{\hat{b}} = \gamma_2 \varepsilon \phi_2 - \sigma \gamma_2 \hat{b}(t)$$

where $\varepsilon = \dfrac{z(t) - \hat{a}(t)\phi_1(t) - \hat{b}(t)\phi_2(t)}{1 + \phi_1^2 + \phi_2^2 + n_d(t)}$ and

$$\sigma = |\varepsilon m| \nu_0$$

$\hookrightarrow \nu_0 > 0$ is a design constant.