area correction: assume Gaussian beam and grazing incidence sample

beam

$$G(x) := \frac{1}{\sqrt{\pi}} \cdot \exp\left(-x^2\right)$$

$$\frac{1}{\sqrt{\pi}} = 0.564$$

FWHM of normed distribution

$$G\left(\sqrt{\ln(2)}\right) \cdot \sqrt{\pi} = 0.5 \qquad 2 \cdot \sqrt{\ln(2)} = 1.665$$

x -> x/b with b=slit/1.665

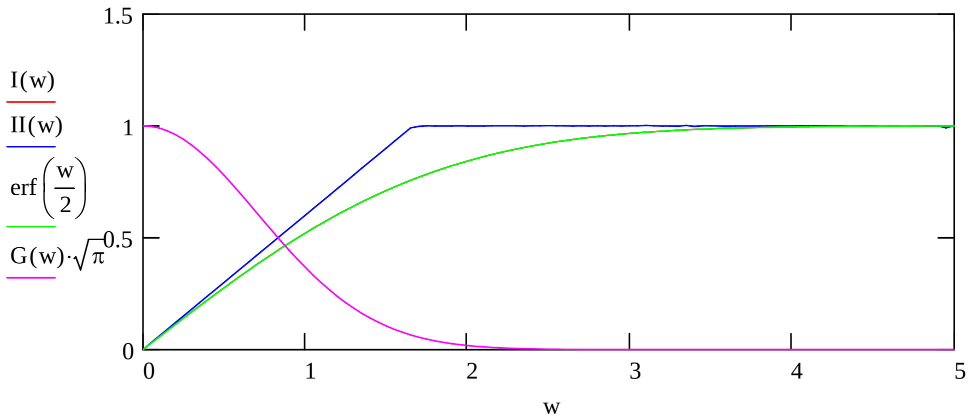
sample as effective slit with width: w=L*sin(th)

$$w := 0, .05 .. 5$$

$$I(w) := \int_{\frac{-w}{2}}^{\frac{w}{2}} G(x)\, dx$$

comparison: box shaped beam

$$II(w) := \frac{\int_{\frac{-w}{2}}^{\frac{w}{2}} \text{if}\left(|x| < \sqrt{\ln(2)}, 1, 0\right) dx}{2\sqrt{\ln(2)}}$$



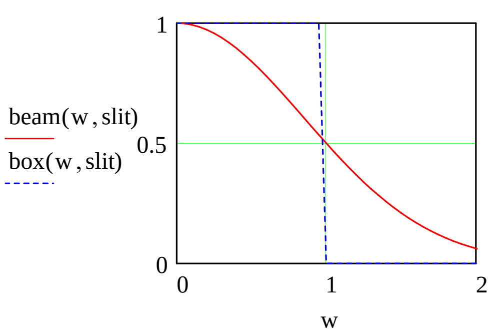
Gaussian beam profile:

$$\text{slit} := 2$$

$$\text{beam}(x, \text{slit}) := G\left(\frac{x}{\frac{\text{slit}}{1.665}}\right) \cdot \sqrt{\pi}$$

box-shaped beam profile

$$\text{box}(x, \text{slit}) := \text{if}\left(|x| < \frac{\text{slit}}{2}, 1, 0\right)$$



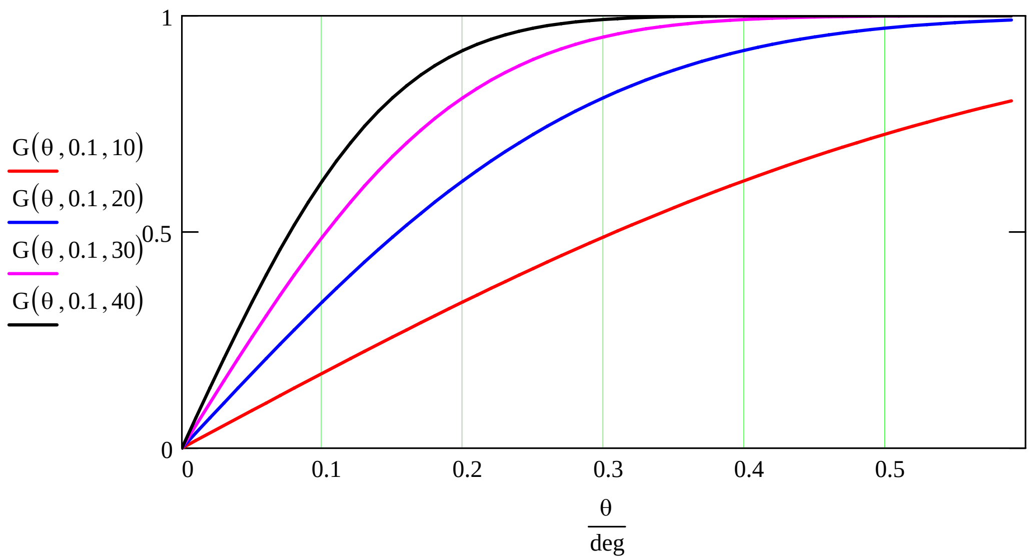
area correction properly normalized: effect of different sample lengths

$h := 0.1$

$\theta := 0, 0.01 \cdot deg .. 0.59 \cdot deg$

area correction for
Gaussian profile

$$G(\theta, h, L) := \mathrm{erf}\left(\frac{\frac{L \cdot \sin(\theta)}{2}}{\frac{h}{\sqrt{\pi}}}\right)$$



cut-off angles

$$\mathrm{asin}\left(\frac{0.1}{10}\right) = 0.573\,deg$$

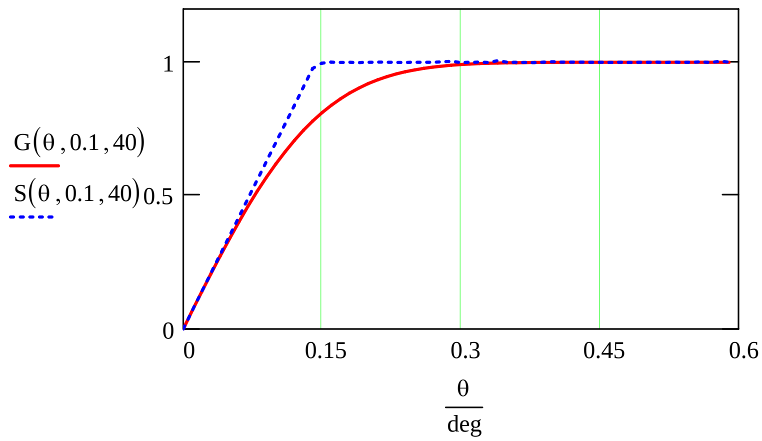
$$\mathrm{asin}\left(\frac{0.1}{20}\right) = 0.286\,deg$$

$$\mathrm{asin}\left(\frac{0.1}{30}\right) = 0.191\,deg$$

$$\mathrm{asin}\left(\frac{0.1}{40}\right) = 0.143\,deg$$

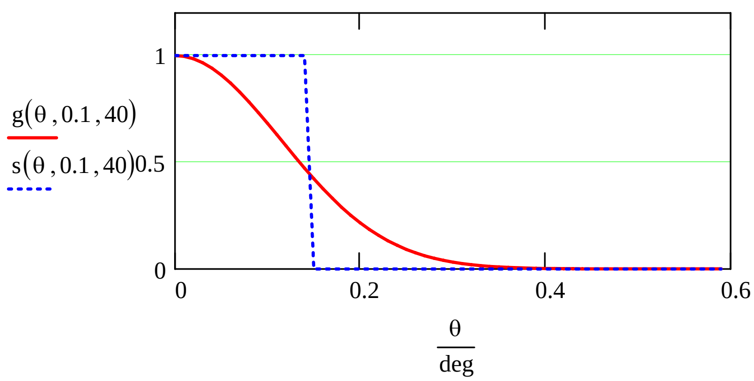
same for a box-shaped beam:

$$S(\theta,h,L) := \frac{\displaystyle\int_{\frac{-L}{2}\cdot\sin(\theta)}^{\frac{L}{2}\cdot\sin(\theta)} \mathrm{if}\left(|x| < \frac{h}{2},1,0\right)dx}{h}$$



associated beam profiles

$$s(\theta,h,L) := \mathrm{if}\left(\left|\frac{L}{2}\cdot\sin(\theta)\right| < \frac{h}{2},1,0\right) \qquad g(\theta,h,L) := \exp\left[-\left(\frac{\frac{L\cdot\sin(\theta)}{2}}{\frac{h}{\sqrt{\pi}}}\right)^2\right]$$



a more realistic beam profile model: convolution of slit and Gaussian

$$ss(z,\sigma) := \int_{\frac{-h}{2}}^{\frac{h}{2}} \exp\left[-\frac{(z-zz)^2}{2\cdot\sigma^2}\right] dzz$$

$$sss(z,\sigma) := \frac{-\mathrm{erf}\left(\sqrt{2}\cdot\frac{-h+2\cdot z}{4\sigma}\right)+\mathrm{erf}\left(\sqrt{2}\cdot\frac{h+2\cdot z}{4\sigma}\right)}{-\mathrm{erf}\left(\sqrt{2}\cdot\frac{-h}{4\sigma}\right)+\mathrm{erf}\left(\sqrt{2}\cdot\frac{h}{4\sigma}\right)}$$

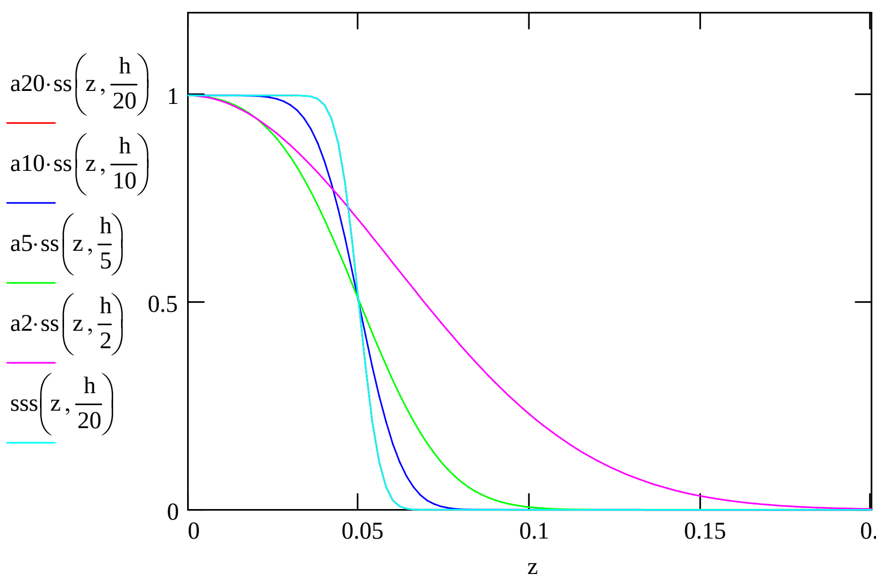
(my guess of the explicit form: correct)

normalization constants

$$a1 := \frac{1}{ss\left(0,\frac{h}{1}\right)} \qquad a2 := \frac{1}{ss\left(0,\frac{h}{2}\right)} \qquad a5 := \frac{1}{ss\left(0,\frac{h}{5}\right)} \qquad a10 := \frac{1}{ss\left(0,\frac{h}{10}\right)} \qquad a20 := \frac{1}{ss\left(0,\frac{h}{20}\right)}$$

-> the parameter σ is associated with the width of the "soft" edge of the beam

calculation: $h = 0.1$ $z := 0, 0.002 .. 0.2$



area effect with beam as convolution of slit and Gaussfuntion

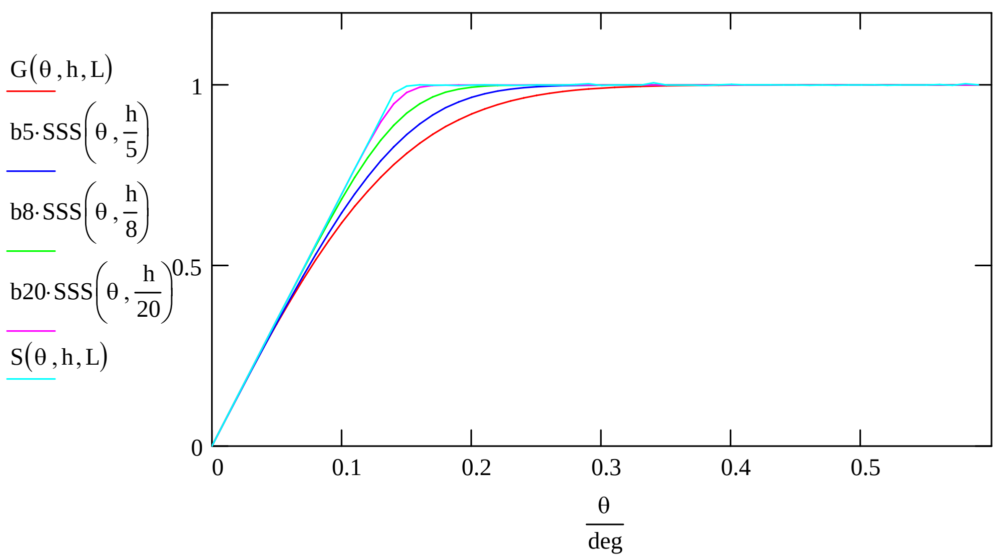
$h := 0.1 \qquad L := 40$

$$SSS(\theta,\sigma) := \int_{\frac{-L\cdot\sin(\theta)}{2}}^{\frac{L\cdot\sin(\theta)}{2}} sss(z,\sigma)\,dz$$

normalization factors

$$b20 := \frac{1}{SSS\left(1,\frac{h}{20}\right)} \qquad b10 := \frac{1}{SSS\left(1,\frac{h}{10}\right)} \qquad b5 := \frac{1}{SSS\left(1,\frac{h}{5}\right)} \qquad b8 := \frac{1}{SSS\left(1,\frac{h}{8}\right)}$$

most general area correction functions :



-> can describe anything between square and Gaussian beam profile
-> could be fitted to a reference sample with a large critical angles (Au, Pt)
-> the reflectivity curve gets corrected by dividing by the area correction function
-> a bit elaborate for my taste