

## MATH 137 Quiz 3

Wednesday, September 27, 2023. Duration: 35 minutes.

### Notes:

1. Answer all questions in the space provided.
2. For multiple choice and true/false questions, answer by filling in the bubble on the last page of the quiz.
3. Your grade will be influenced by how clearly you express your ideas and how well you organize your solutions. Show all details to get full marks. Numerical answers should be in exact values (no approximations).
4. No calculators are allowed.
5. There are a total of 23 possible points.
6. DO NOT write on the Crowdmark QR code at the top of the pages or your quiz will not be scanned (and will receive a grade of zero).
7. Use a dark pen or pencil.

(MC) Answer the following multiple choice questions on the last page of the quiz. Bubble (a),  
[6] (b), (c), or (d). Note there is only one correct answer for each question.

1. Suppose that  $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} b_n = \infty$ . Then,  $\lim_{n \rightarrow \infty} a_n - b_n =$ 
  - (a) 0
  - (b)  $\infty$
  - (c) 1
  - (d) Unable to determine.
2. The least upper bound of the sequence  $\{a_n\}$  that is defined for  $n \geq 1$  by
$$a_n = \begin{cases} 2^{-n} & \text{if } n \text{ is odd} \\ -n & \text{if } n \text{ is even} \end{cases} \quad \text{is}$$
  - (a)  $\frac{1}{2}$
  - (b) 2
  - (c) 0
  - (d) None of the above
3. Which of the following conditions guarantees that the sequence  $\{a_n\}$  converges?
  - (a)  $\{a_n\}$  is bounded below
  - (b)  $\{a_n\}$  is monotonic and bounded below
  - (c)  $\{a_n\}$  is increasing and bounded above
  - (d) None of the above

(TF) Answer the following true or false questions on the last page of the quiz. Bubble (a) for  
[2] True, (b) for False.

4. TRUE or FALSE: If  $a_n \leq b_n \leq c_n$  for all  $n \in \mathbb{N}$  and both  $\{a_n\}$  and  $\{c_n\}$  converge, then so does  $\{b_n\}$ .
5. TRUE or FALSE: If a sequence converges, then it is bounded.

(SA) Short answer questions, marks only awarded for a correct final answer, you do not need to show any work.

[2] 1. Give an example of a divergent monotonic sequence.

[2] 2. Determine the limit of  $\lim_{n \rightarrow \infty} \frac{3n^2 - n + 1}{\sqrt{4n^4 + n} + 7n}$

(LA) The remaining questions are long answer questions, please show all of your work.

[4] 1. Compute  $\lim_{n \rightarrow \infty} \frac{n!}{n^n}$

2. Define a sequence  $\{a_n\}$  by  $a_1 = 1$  and  $a_{n+1} = \sqrt{1 + a_n}$ .

[4] (a) By induction, show that  $\{a_n\}$  is an increasing sequence that is bounded above by 3.

[3] (b) Does the sequence  $\{a_n\}$  converge? Justify your answer. If  $\{a_n\}$  converges, find  $\lim_{n \rightarrow \infty} a_n$ .

This page is meant for rough work. Clearly indicate in the original question if part of your solution is here.