INFERENTIAL STATISTICS

PROJECT REPORT

SEPTEMBER 3, 2023

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Problem 1

A physiotherapist with a male football team is interested in studying the relationship between foot injuries and the positions at which the players play from the data collected.

	Striker	Forward	Attacking Midfielder	Winger	Total
Players Injured	**45**	**56**	**24**	**20**	**145**
Players Not Injured	**32**	**38**	**11**	**9**	**90**
Total	**77**	**94**	**35**	**29**	**235**

Table 1 Table for Foot Injuries and Position of Male Football Team

Based on the above data, answer the following questions.

1.1. What is the probability that a randomly chosen player would suffer an injury?

Answer: - Total number of player =235,   Total number of Player Injured = 145

Probability that a randomly chosen player would suffer an injury = Total Player Injured/ Total number of player

= 145/235

= 0.617

Probability that a randomly chosen player would suffer an injury is 0.617

1.2. What is the probability that a player is a forward or a winger?

Answer: - total number of forward player = 94, Total number of winger = 29

Total number of player forward or a winger = 94+29 = 123

Total number of player = 235

Probability that a player is a forward or a winger =

Total number of forward/winger/Total number of Player

= 123/235

= 0.523

Probability that a player is a forward or a winger is 0.523

	Striker	Forward	Attacking Midfielder	Winger	**Total**
Players Injured	45	56	24	20	**145**
Players Not Injured	32	38	11	9	**90**
Total	**77**	**94**	**35**	**29**	**235**

Table 1 Table for Foot Injuries and Position of Male Football Team

1.3. What is the probability that a randomly chosen player plays in a striker position and has a foot injury?

Answer: - Total number of player plays in striker position = 77

Total number of player plays in striker position and has a foot injury= 45

probability that a randomly chosen player plays in striker position and has a foot injury =

Total no. of player plays in striker position and has a foot injury/ Total no. of player plays in striker position

= 45/77

= 0.58

Probability that a randomly chosen player plays in a striker position and has a foot injury is 0.58

1.4. What is the probability that a randomly chosen injured player is a striker?

Answer: - Total no. of injured player is a striker =45, Total no. of injured player = 145

Probability that a randomly chosen injured player is a striker = Total no. of injured player is a striker/ Total no. of injured player

= 45/145

= 0.31

Probability that a randomly chosen injured player is a striker is 0.31

Problem 2

The breaking strength of gunny bags used for packaging cement is normally distributed with a mean of 5 kg per sq. centimeter and a standard deviation of 1.5 kg per sq. centimeter. The quality team of the cement company wants to know the following about the packaging material to better understand wastage or pilferage within the supply chain; Answer the questions below based on the given information; **(Provide an appropriate visual representation of your answers, without which marks will be deducted)**

Mu = 5, sigma = 1.5

2.1. What proportion of the gunny bags have a breaking strength of less than 3.17 kg per sq cm

Answer: -

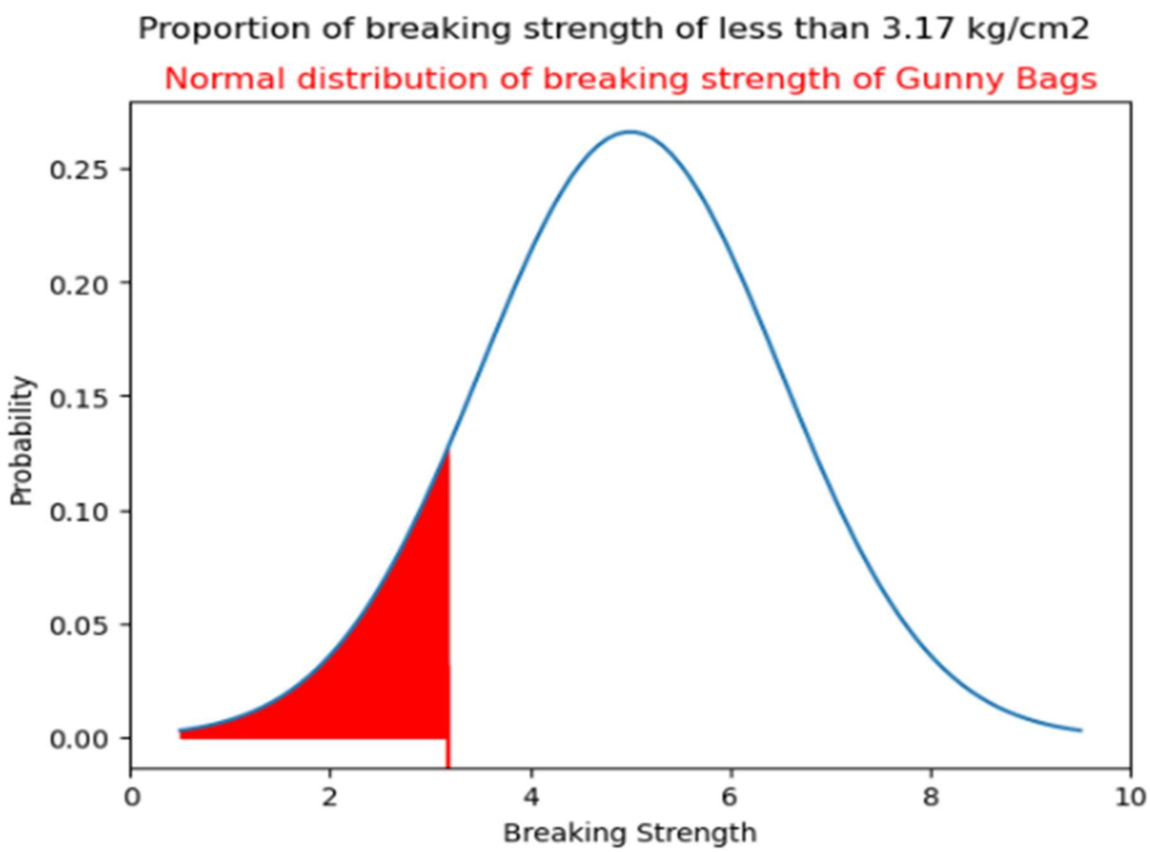

Fig 2.1 Proportion of breaking strength less than 3.17 kg/cm2

In the above figure the graph covered with red color is the proportion of the gunny bags have a breaking strength of less than 3.17 kg per sq cm.

Proportion of the gunny bags have a breaking strength of less than 3.17 kg per sq cm is 0.111

2.2. What proportion of the gunny bags have a breaking strength of at least 3.6 kg per sq. cm.?

Answer: -

Proportion of the gunny bags have a breaking strength of at least 3.6 kg per sq. cm., so here we need to look for the proportion equal to or more than 3.6 kg per sq. cm.

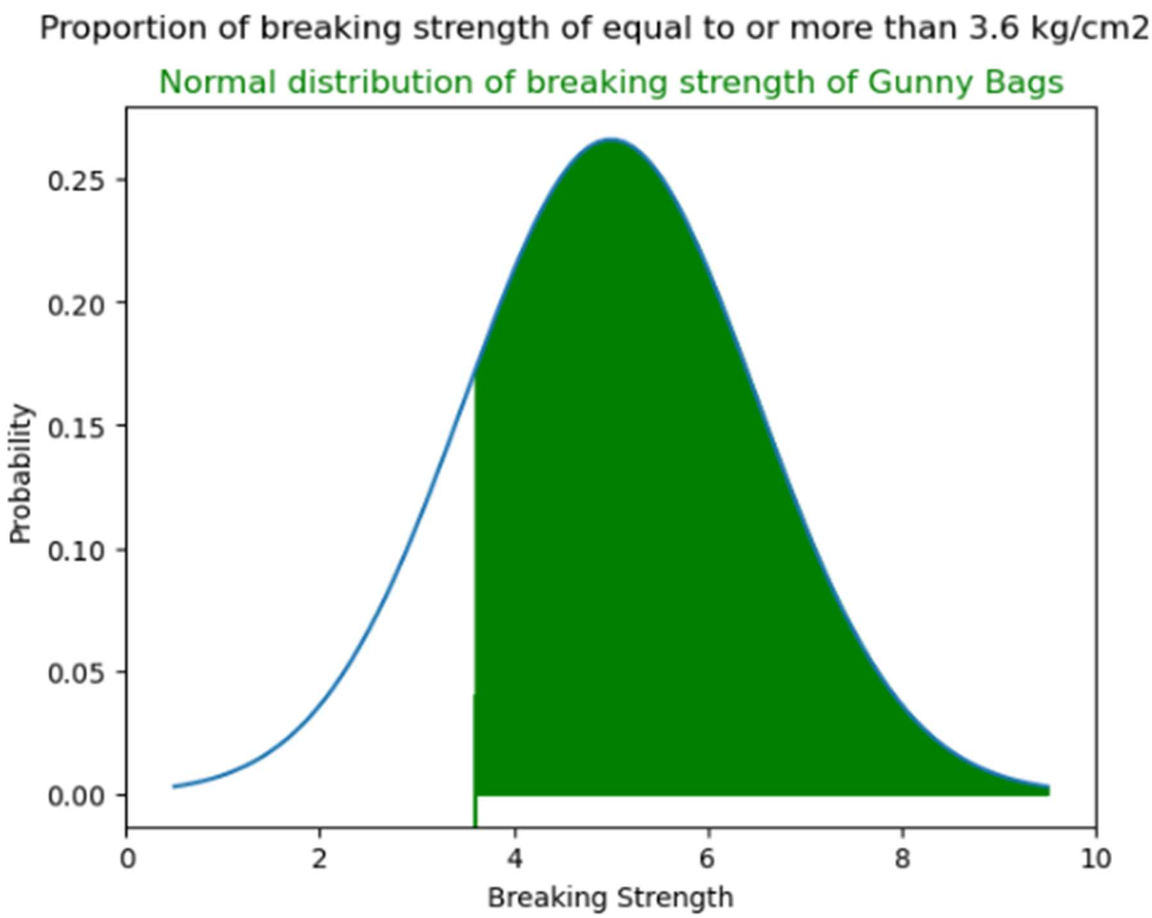

Fig 2.2 **Proportion of breaking strength equal to more than 3.60 kg/cm2**

In the above figure the proportion of graph covered with green color is the proportion of the gunny bags have a breaking strength of at least 3.6 kg per sq. cm.

Proportion of the gunny bags have a breaking strength of at least 3.6 kg per sq. cm. is 0.825

2.3. What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq. cm.?

Answer: -

Proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq. cm. So here we need to look for the proportion greater than 5 kg per sq. cm. and less than 5.5 kg per sq. cm.

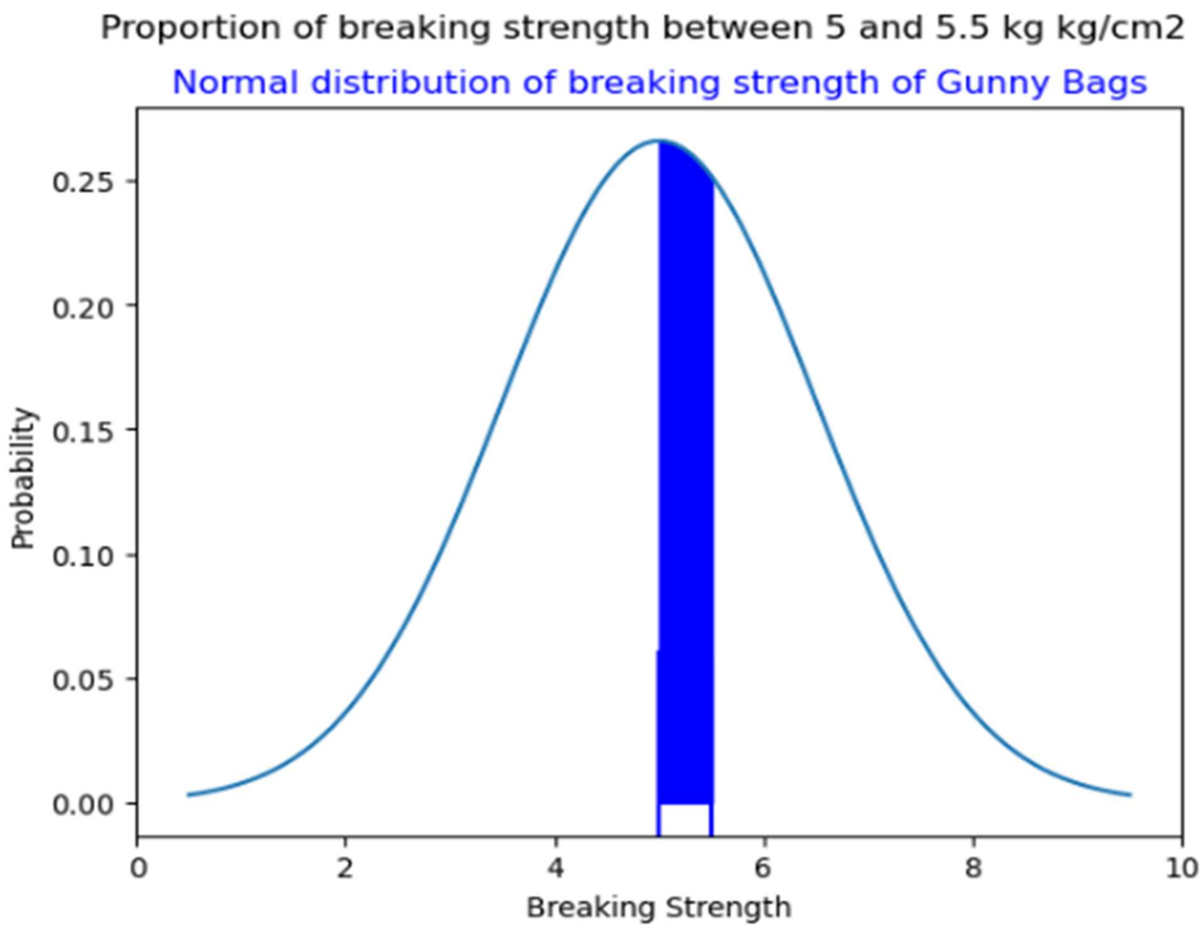

Fig 2.3 Proportion of breaking strength between 5 & 5.5 kg/cm2

In the above figure the proportion of graph covered with blue color is the proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq. cm.

Proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq. cm. is 0.131

2.4. What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq. cm.?

Answer: -

Proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq. cm. So here we need to look for the proportion less than 3 kg per sq. cm. and more than 7.5 kg per sq. cm.

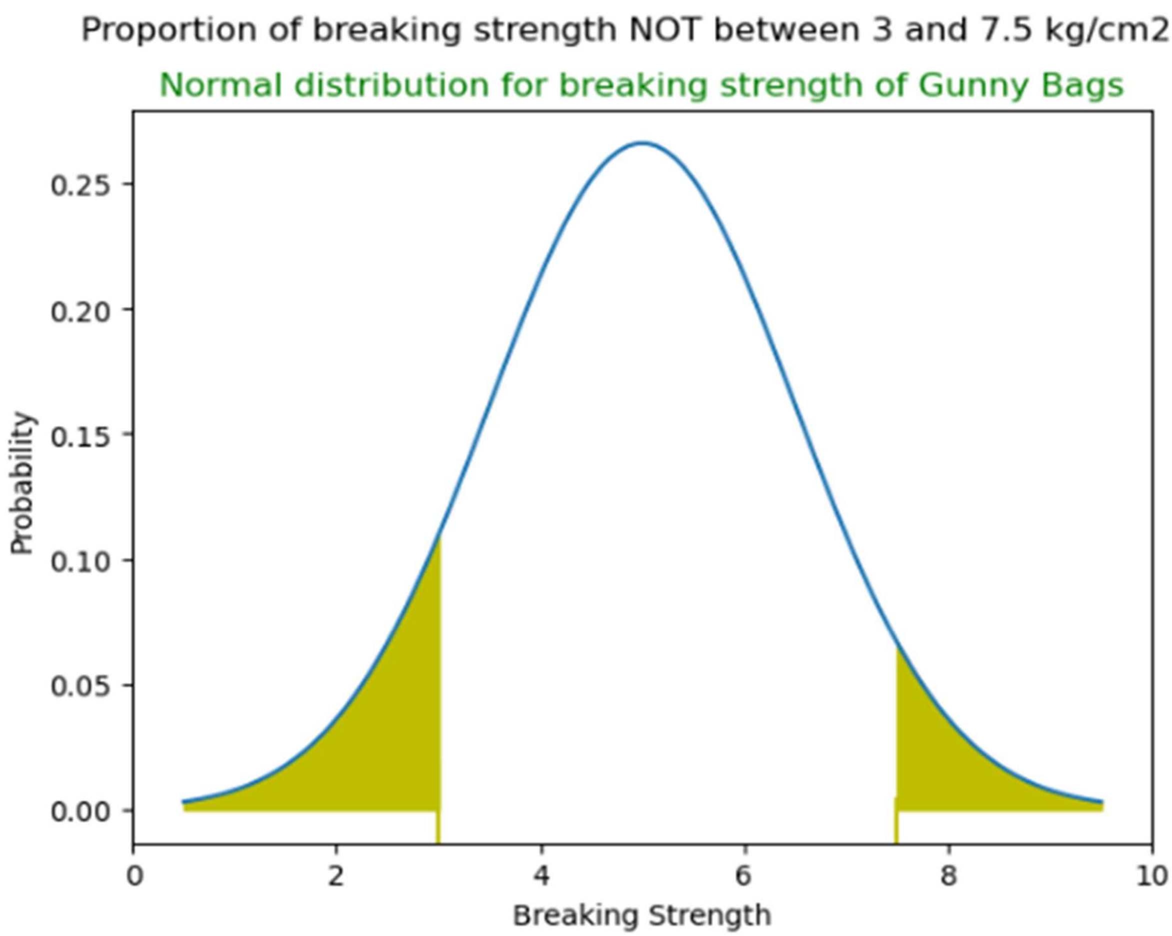

Fig. 2.4 Proportion of breaking strength NOT between 3 & 7.5 kg/cm2

In the above figure the proportion of graph covered with yellow color is the proportion of the gunny bags have a breaking strength less than 3 kg per sq. cm. and more than 7.5 kg per sq. cm.

Proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq. cm. is 0.139

Problem 3.

Zingaro stone printing is a company that specializes in printing images or patterns on polished or unpolished stones. However, for the optimum level of printing of the image, the stone surface has to have a Brinell's hardness index of at least 150. Recently, Zingaro has received a batch of polished and unpolished stones from its clients. Use the data provided to answer the following (assuming a 5% significance level);

3.1. Zingaro has reason to believe that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?

Answer: -

Let's check the given Data Summary (Descriptive summary): -

	count	mean	std	min	25%	50%	75%	max
Unpolished	75.0	134.110527	33.041804	48.406838	115.329753	135.597121	158.215098	200.161313
Treated and Polished	75.0	147.788117	15.587355	107.524167	138.268300	145.721322	157.373318	192.272856

Table 3.1A Descriptive Summary of Zingaro Dataset

From the above table 3.1.A we can see that the mean of the Unpolished and Polished stones are not same or different. Also the mean Brinell's Hardness of the surface for polished stone are much higher than the unpolished stone.

Let's check the distribution of our samples: -

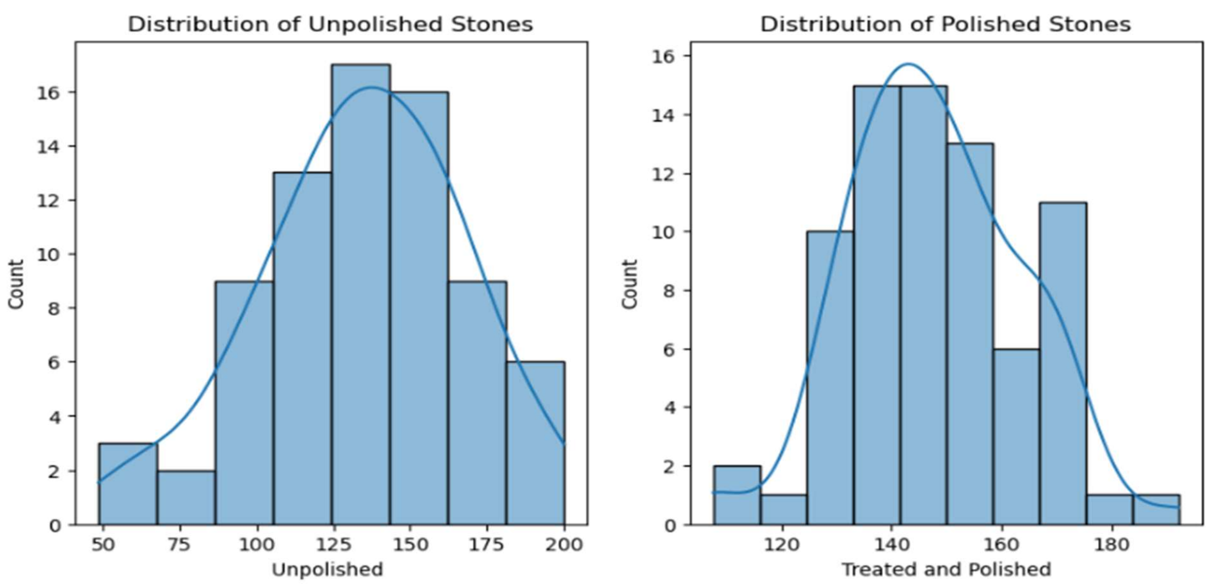

Fig.3.1A Descriptive Summary of Zingaro Dataset

From the above Fig.3.1A we can say that both the samples are normally distributed.

Let's calculate estimated mean of Unpolished stone and Polished stone.

We know that we can calculate the estimated mean of the population sample using norm.interval() function.

```
Estimated mean Brinell’s Hardness of Unpolished Stone is in range (126.63259963321897, 141.58845343424767)
```

```
Estimated mean Brinell’s Hardness of Polished Stone is in range (144.26043140952547, 151.31580295314123)
```

Fig 3.1B Estimated mean of Brinell’s Hardness of Polished and Unpolished

Estimated mean Brinell’s Hardness of Unpolished Stone from 126.63 to 141.59

Estimated mean Brinell’s Hardness of Polished Stone from 144.26 to 151.32

We can see that the estimated mean Brinell’s Hardness of Unpolished stone are quite less compared to Polished Stone surface.

Let’s check further with the hypothesis testing

Null Hypothesis: H0: Brinell's hardness index for Unpolished Stone is equal to or greater than 150

Alternate Hypothesis: HA: Brinell's hardness index for Unpolished stone is less than 150

Given: - Level of significance(alpha) = 0.05

Let’s calculate our t-statistic and p-value: -

We know that the two sample t-test can be done using ttest_1samp from scipy.stats

Results of t-test: -

```
t-statistic -4.164629601426757
p_value 4.171286997419652e-05
```

Table 3.1B Stone Result of ttest_1samp (t-stats, p-value)

Since p-value is less than the level of significance we reject the Null Hypothesis i.e. Brinell's hardness index for Unpolished stone is less than 150.

Answer 3.1.

Based on above Estimated means of Unpolished and Polished stone surface and t-test it is concluded that the Brinell’s Hardness Index of are not same. Hence Zingaro has enough reason to believe that the unpolished stones may not be suitable for printing.

3.2 Is the mean hardness of the polished and unpolished stones the same?

Answer: - Let's Perform hypothesis test. Define our Null and Alternate Hypothesis:

Null Hypothesis:

H0: Brinell's Hardness of Unpolished stone and Treated and Polished stone are same.

Alternate Hypothesis:

HA: Brinell's Hardness of Unpolished stone and Treated and Polished stone are not same.

Given: - Level of significance(alpha) = 0.05

Since Population standard deviation is not given we will perform two sample t-test.

Let's calculate our t-statistic and p-value: -

We know that the two sample t-test can be done using ttest_ind from scipy.stats

Results of t-test: -

```
t_stats -3.242232050141405З
p_value 0.0014655150194628353
```

Table 3.2 Result of ttest_ind (t-stats, p-value)

p-value = 0.00147

p-value is less than the level of significance (0.05). So we reject our Null Hypothesis

Hence we can say that the Brinell's Hardness of Unpolished stone and Treated and Polished stone are not same.

Answer 3.2.

t-test results -

p-value = 0.0015 alpha = 0.05

Since p-value is less than alpha (level of significance). Hence we can say that the mean hardness of the polished and unpolished stones is not same.

Problem 4

Dental implant data: The hardness of metal implants in dental cavities depends on multiple factors, such as the method of implant, the temperature at which the metal is treated, the alloy used as well as the dentists who may favor one method above another and may work better in his/her favorite method. The response is the variable of interest.

Answer: -

Let's check our Dataset: -

	Dentist	Method	Alloy	Temp	Response	Unnamed: 5	Unnamed: 6	Unnamed: 7	Unnamed: 8	Unnamed: 9	Unnamed: 10	Unnamed: 11	Unnamed: 12	Unnamed: 13
0	1.0	1.0	1.0	1500.0	813.0	NaN	NaN	Anova: Two-Factor Without Replication	NaN	NaN	NaN	NaN	NaN	NaN
1	1.0	1.0	1.0	1600.0	792.0	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2	1.0	1.0	1.0	1700.0	792.0	NaN	NaN	SUMMARY	Count	Sum	Average	Variance	NaN	NaN
3	1.0	1.0	2.0	1500.0	907.0	NaN	NaN	1	4	2315	578.75	523721.583333	NaN	NaN
4	1.0	1.0	2.0	1600.0	792.0	NaN	NaN	1	4	2394	598.5	584819	NaN	NaN

Table 4A Dental Implant Dataset (Sample)

Let's check the statistical summary of our dataset: -

	count	unique	top	freq	mean	std	min	25%	50%	75%	max
Dentist	90.0	NaN	NaN	NaN	3.0	1.422136	1.0	2.0	3.0	4.0	5.0
Method	90.0	NaN	NaN	NaN	2.0	0.821071	1.0	1.0	2.0	3.0	3.0
Alloy	90.0	NaN	NaN	NaN	1.5	0.502801	1.0	1.0	1.5	2.0	2.0
Temp	90.0	NaN	NaN	NaN	1600.0	82.107083	1500.0	1500.0	1600.0	1700.0	1700.0
Response	90.0	NaN	NaN	NaN	741.777778	145.767845	289.0	698.0	767.0	824.0	1115.0
Unnamed: 5	0.0	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Unnamed: 6	0.0	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Unnamed: 7	102.0	17.0	1.0	18.0	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Unnamed: 8	100.0	8.0	4.0	90.0	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Unnamed: 9	100.0	88.0	2220.0	2.0	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Unnamed: 10	99.0	87.0	582.0	2.0	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Unnamed: 11	98.0	91.0	524696.0	2.0	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Unnamed: 12	3	3	P-value	1	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Unnamed: 13	3	3	F crit	1	NaN	NaN	NaN	NaN	NaN	NaN	NaN

Table 4B Descriptive Statistic Summary of Dental Implant Dataset

From Table 4B we only require Dentist, Method, Alloy, Temperature, Response columns. Rest columns can be dropped or left as it is.

Let's check the datatypes of each variable/columns: -

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 107 entries, 0 to 106
Data columns (total 14 columns):
 #   Column       Non-Null Count  Dtype  
---  ------       --------------  -----  
 0   Dentist      90 non-null     float64
 1   Method       90 non-null     float64
 2   Alloy        90 non-null     float64
 3   Temp         90 non-null     float64
 4   Response     90 non-null     float64
 5   Unnamed: 5   0 non-null      float64
 6   Unnamed: 6   0 non-null      float64
 7   Unnamed: 7   102 non-null    object 
 8   Unnamed: 8   100 non-null    object 
 9   Unnamed: 9   100 non-null    object 
 10  Unnamed: 10  99 non-null     object 
 11  Unnamed: 11  98 non-null     object 
 12  Unnamed: 12  3 non-null      object 
 13  Unnamed: 13  3 non-null      object 
dtypes: float64(7), object(7)
memory usage: 11.8+ KB
```

Table 4C Datatype of variables of Dental Implant Dataset

From Table 4C we can see that Dentist, Method, Alloy and Temp columns to Categorical Datatypes in order to perform ANOVA tests

Let's check the distribution of required variables i.e. Dentist, Method, Temperature, Response: -

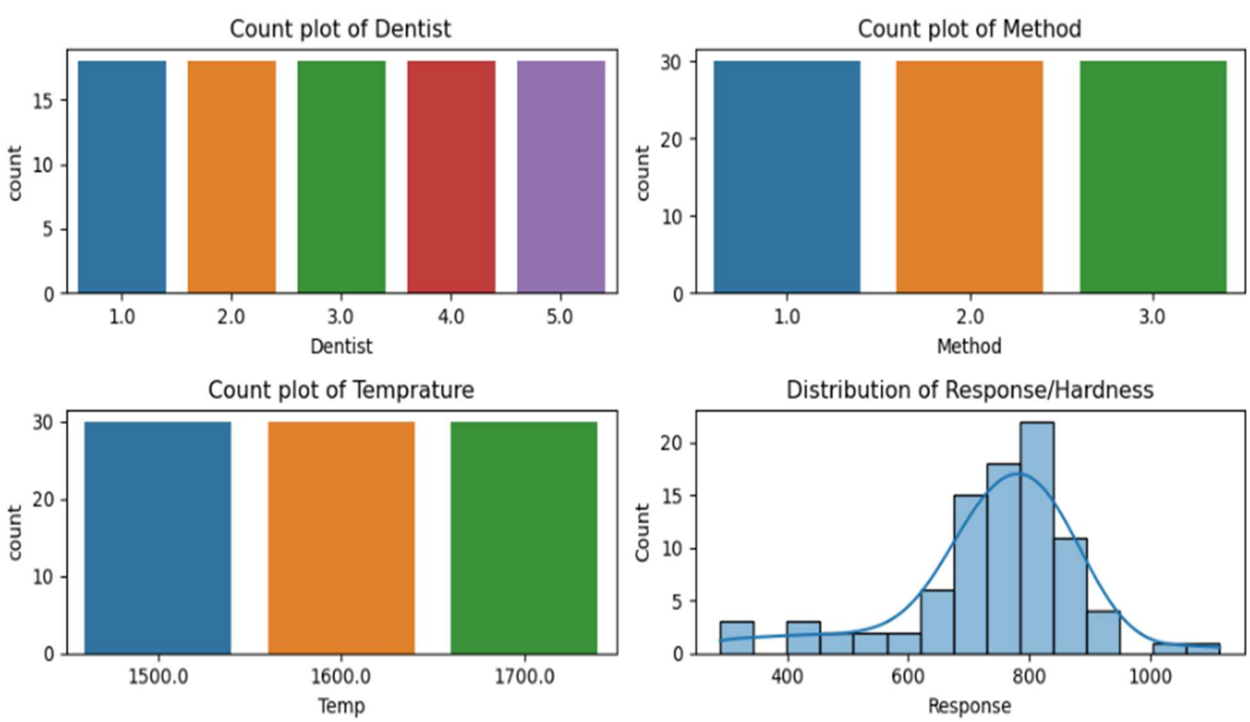

Fig. 4A Count plot of Dentist, Method, Temperature and Distribution of Response

From above fig 4A we can see that Dentist, Method and Temperature ideally normally distributed whereas Response is slightly left skewed distribution but we can consider it as normally distributed.

4.1. How does the hardness of implants vary depending on dentists?

<u>First let's consider Alloy Type 1-</u>

Let's check the assumptions of ANOVA test: -

```
For Alloy Type 01 Dentist 1 ShapiroResult(statistic=0.9113541841506958, pvalue=0.3254688084125519)
For Alloy Type 01 Dentist 2 ShapiroResult(statistic=0.9642462134361267, pvalue=0.8415456414222717)
For Alloy Type 01 Dentist 3 ShapiroResult(statistic=0.8721169233322144, pvalue=0.12953516840934753)
For Alloy Type 01 Dentist 4 ShapiroResult(statistic=0.8368974328041077, pvalue=0.05333680287003517)
For Alloy Type 01 Dentist 5 ShapiroResult(statistic=0.8534296751022339, pvalue=0.08127813786268234)

Dentist LeveneResult(statistic=1.3847146992797106, pvalue=0.2565537418543795)
```

Fig. 4.1A Result of Shapiro & Levene's Test for Dentist of Alloy Type 1

<u>Assumption 1.</u> i.e. Randomization: - In the above Fig.4.1A we can see that the Shapiro result has p-value all 5 dentists is more than 0.5 (level of significance) i.e. 100 %. So we can say that sample drawn from population are independent and random.

<u>Assumption 2.</u> Same or nearly equal variance: - In the above Fig.4.1A we can see that the Levene Test result p-value (0.256) is more than 0.5(level of significance).

This means we don't have sufficient evidence to say that the variance in hardness between the five doctors is significantly same. So we can say that the sample drawn from the populations have equal or nearly same variance.

Null Hypothesis $H0$: The mean hardness of alloy type 1 for all dentist is same at all 5 levels of treatment Response.

Alternate Hypothesis Ha: For at least one level of treatment Response, mean number of dentist treatment are different

	df	sum_sq	mean_sq	F	PR(>F)
Dentist	4.0	106683.688889	26670.922222	1.977112	0.116567
Residual	40.0	539593.555556	13489.838889	NaN	NaN

Table.4.1A Result of One-way ANOVA for Dentist of Alloy Type 1

Looking at above table we see that F-Test, p-value is 0.116. For level of significance of 0.05 we cannot reject our Null hypothesis i.e. The hardness mean of Dentist is same at all 5 levels of treatment.

For Alloy Type 1 the hardness of implants doesn't vary depending on dentists

Alloy Type 2

Let's check the assumptions of ANOVA test: -

```
For Alloy Type 02 Dentist 1 ShapiroResult(statistic=0.9039731621742249, pvalue=0.27593979239463806)
For Alloy Type 02 Dentist 2 ShapiroResult(statistic=0.9392004013061523, pvalue=0.5735077857971191)
For Alloy Type 02 Dentist 3 ShapiroResult(statistic=0.934097112251282, pvalue=0.5213080644607544)
For Alloy Type 02 Dentist 4 ShapiroResult(statistic=0.7613219022750854, pvalue=0.007332688197493553)
For Alloy Type 02 Dentist 5 ShapiroResult(statistic=0.9131584167480469, pvalue=0.33861100673675537)

For Alloy Type 02 Dentist LeveneResult(statistic=1.4456166464566966, pvalue=0.23686777576324952)
```

Fig. 4.1B Result of Shapiro & Levene's Test for Dentist of Alloy Type 2

<u>Assumption 1.</u> i.e. Randomization: - In the above Fig.4.1B we can see that the Shapiro result has p-value of 4 out of 5 is more than 0.5 (level of significance) i.e. 80 %. So we can say that sample drawn from population are independent and random.

<u>Assumption 2.</u> Same or nearly equal variance: - In the above Fig.4.1B we can see that the Levene Test result p-value (0.239) is more than 0.5(level of significance).

This means we don't have sufficient evidence to say that the variance in hardness between the five doctors is significantly same. So we can say that the sample drawn from the populations have equal or nearly same variance.

Null Hypothesis $H0$: The mean of dentist is same at all 5 levels of treatment Response.
Alternate Hypothesis Ha: For at least one level of treatment Response, mean number of dentist treatment are different

	df	sum_sq	mean_sq	F	PR(>F)
Dentist	4.0	5.679791e+04	14199.477778	0.524835	0.718031
Residual	40.0	1.082205e+06	27055.122222	NaN	NaN

Table.4.1B Result of One-way ANOVA for Dentist of Alloy Type 2

Looking at above Table.4.1B we see that , p-value is 0.718. For level of significance of 0.05 we cannot reject our Null hypothesis i.e. The hardness mean of Dentist is same at all 5 levels of treatment.

For Alloy Type 2 the hardness of implants doesn't vary depending on dentists

4.2. How does the hardness of implants vary depending on methods?

<u>First let's consider Alloy Type 1-</u>

Let's check the assumptions of ANOVA test: -

```
For Alloy Type 01 Method 1 ShapiroResult(statistic=0.918382287025451‍7, pvalue=0.18198540806770325)
For Alloy Type 01 Method 2 ShapiroResult(statistic=0.9732585549354553, pvalue=0.903033554553985‍6)
For Alloy Type 01 Method 3 ShapiroResult(statistic=0.9114548563957214, pvalue=0.14254699647426605)

For Alloy Type 01 Methods LeveneResult(statistic=6.52140454403598, pvalue=0.003416038146023397‍5)
```

Fig. 4.2A Result of Shapiro & Levene's Test for Method of Alloy Type 1

<u>Assumption 1: -</u> In the Above fig.4.2A we can see Shapiro Test result p-value for 2 all methods are less than 0.05 i.e. 100%. So we can say that sample drawn from population are independent and random

<u>Assumption 2: -</u> Same or nearly equal variance: - In the above Fig.4.2 we can see that the Levene-test result p-value (0.0034) is less than 0.5(level of significance).

This means we don't have sufficient evidence to say that the variance of each population sample is same or nearly equal.

Null Hypothesis $H0$: The hardness mean for Different Method is same at all 5 levels of treatment Response.
Alternate Hypothesis Ha: For at least one level of treatment Response, hardness mean of Method treatment are different

	df	sum_sq	mean_sq	F	PR(>F)
Method	2.0	148472.177778	74236.088889	6.263327	0.004163
Residual	42.0	497805.066667	11852.501587	NaN	NaN

Table.4.2A Result of One-way ANOVA for Method of Alloy Type 1

Looking at above Table.4.2A we see that F-Test, p-value is 0.0042. For level of significance of 0.05 p-value is quite less, so we reject our Null hypothesis i.e. For at least one level of treatment Response, hardness mean of different Method are different

For Alloy Type 1 the mean hardness of implants varies depending on Methods for at least one pair.

Let's find for which pair of Method the mean hardness of implants varies: -

```
  Multiple Comparison of Means - Tukey HSD, FWER=0.05
=====================================================
group1 group2  meandiff p-adj    lower    upper  reject
-----------------------------------------------------
   1.0    2.0   -6.1333  0.987 -102.714  90.4473  False
   1.0    3.0    -124.8 0.0085 -221.3807 -28.2193   True
   2.0    3.0 -118.6667 0.0128 -215.2473  -22.086   True
-----------------------------------------------------
```

Table 4.2.1A Result of Pairwise_tukey test for Method of Alloy Type 1

From above table 4.2.1A

- P-value for the difference in means between Method 1 and Method 2: 0.987
- P-value for the difference in means between Method 1 and Method 3: 0.0085
- P-value for the difference in means between Method 2 and Method 3: 0.0128

Thus, we would conclude that there is a statistically significant difference between the means of Method 1 and Method 2 , but not a statistically significant difference between the means of Method 1 and Method 3 , Method 2 and Method 3.

Alloy Type 2

Let's check the assumptions of ANOVA test: -

```
For Alloy Type 02 Method 1 ShapiroResult(statistic=0.963810384273529, pvalue=0.7582374811172485)
For Alloy Type 02 Method 2 ShapiroResult(statistic=0.755793035030365, pvalue=0.001051110913977027)
For Alloy Type 02 Method 3 ShapiroResult(statistic=0.9021322131156921, pvalue=0.1025901660323143)

For Alloy Type 02 Methods LeveneResult(statistic=3.349707184158617, pvalue=0.04469269939158668)
```

Fig.4.2B Result of Shapiro & Levene's Test for Method of Alloy Type 2

Assumption 1: - In the Above fig.4.2B we can see Shapiro Test result p-value for 2 out of 3 methods are less than 0.05 i.e. 66.67%. So we can say that sample drawn from population are independent and random

Assumption 2. Same or nearly equal variance: - In the above Fig.4.2 we can see that the Levene-test result p-value (0.045) is less than 0.5(level of significance).

This means we don't have sufficient evidence to say that the variance of each population sample is same or nearly equal.

Null Hypothesis $H0$: The hardness mean for Different Method is same at all 5 levels of treatment Response.
Alternate Hypothesis Ha: For at least one level of treatment Response, hardness mean of Method treatment are different

	df	sum_sq	mean_sq	F	PR(>F)
Method	2.0	499640.4	249820.200000	16.4108	0.000005
Residual	42.0	639362.4	15222.914286	NaN	NaN

Table 4.2B Result of One-way ANOVA for Method of Alloy Type 2

Looking at above Table.4.2B we see that F-Test, p-value is 0.000005. For level of significance of 0.05 the p-value is very less, so we reject our Null hypothesis i.e. For at least one level of treatment Response, hardness mean of different Method are different

For Alloy Type 2 the mean hardness of implants varies depending on Methods for at least one pair.

Let's find for which pair of Method mean hardness of implants varies

Multiple Comparison of Means - Tukey HSD, FWER=0.05

group1	group2	meandiff	p-adj	lower	upper	reject
1.0	2.0	27.0	0.8212	-82.4546	136.4546	False
1.0	3.0	-208.8	0.0001	-318.2546	-99.3454	True
2.0	3.0	-235.8	0.0	-345.2546	-126.3454	True

Table 4.2.1B Result of Pairwise_tukey test for Method of Alloy Type 2

From above table 4.2.1B

- P-value for the difference in means between Method 1 and Method 2: 0.8212
- P-value for the difference in means between Method 1 and Method 3: 0.0001
- P-value for the difference in means between Method 2 and Method 3: 0.0

Thus, we would conclude that there is a statistically significant difference between the means of Method 1 and Method 2 , but not a statistically significant difference between the means of Method 1 and Method 3 , Method 2 and Method 3.

4.3. What is the interaction effect between the dentist and method on the hardness of dental implants for each type of alloy?

Alloy Type 1

Let’s check the interaction effect between the dentist and method on the hardness of dental implants for each type of Alloy 1 from the Interaction Plot.

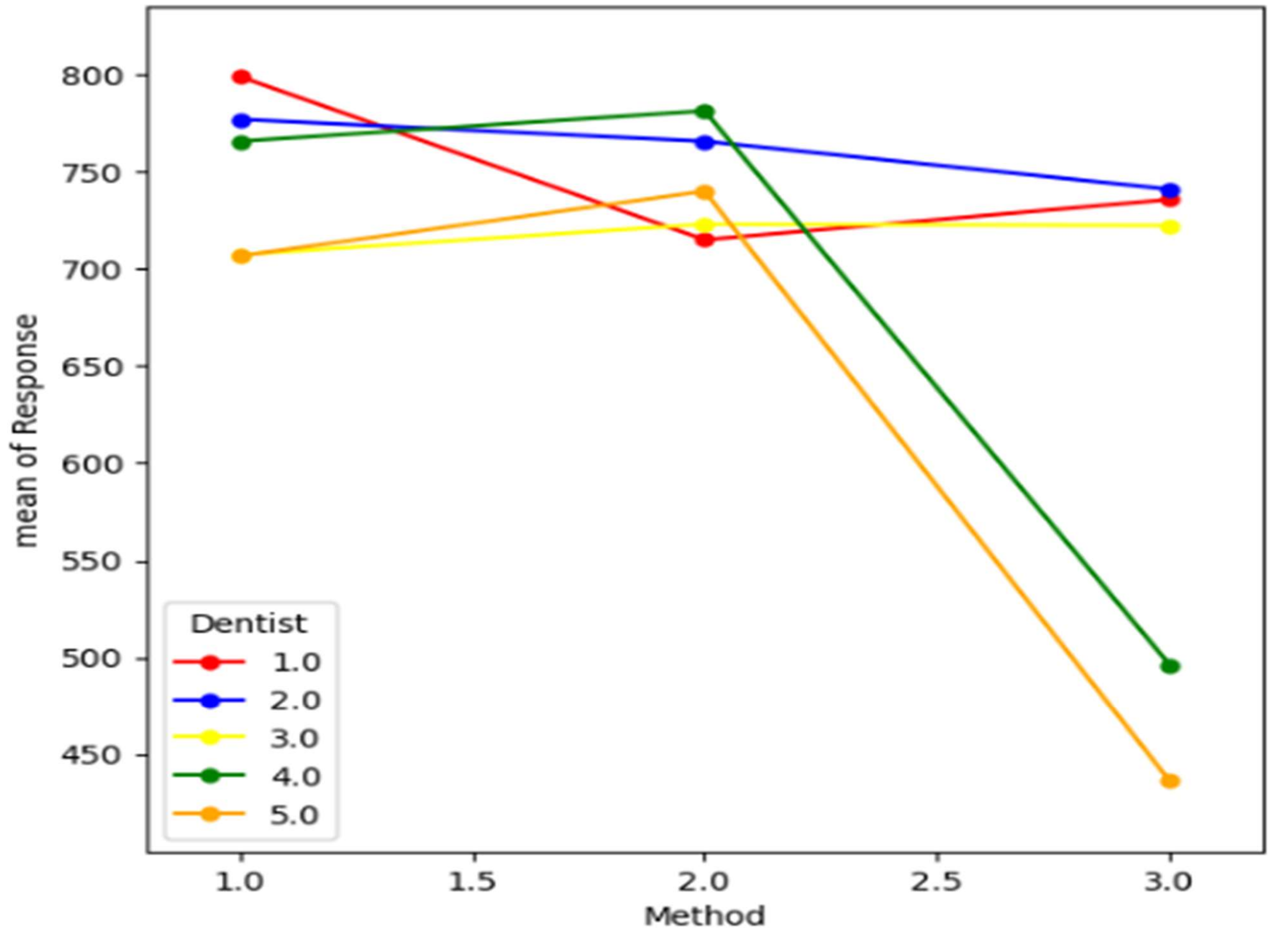

Fig 4.3A Interaction Plot Between Method: Dentist for Alloy Type 1

Inferences: -

In this interaction plot, the lines are not parallel. This interaction effect indicates that the relationship between Dentist and response depends on the Method used.

For Dentist 1 (red line) Hardness decreases from Method 1 to Method 2 and it increases from Method 2 to Method 3

For Dentist 2 (blue line) Hardness decreases from Method 1 to Method 2 and it also decreases Method 2 to Method 3

For Dentist 3 (Yellow line) Hardness decreases from Method 1 to Method 2 and is almost similar for Method 2 to Method 3

For Dentist 4 (Green line) Hardness increases from Method 1 to Method 2 and it decreases from Method 2 to Method 3

For Dentist 5 (Orange line) Hardness increases from Method 1 to Method 2 and it decreases from Method 2 to Method 3

	df	sum_sq	mean_sq	F	PR(>F)
Dentist:Method	1.0	260231.884058	260231.884058	28.986156	0.000003
Residual	43.0	386045.360386	8977.799079	NaN	NaN

Table 4.3A Result of two-way for Interaction between Method and Dentist of Alloy Type 1

The crossed lines on the graph suggest that there is an interaction effect, which the significant p-value (0.000003) for the Dentist*Method term confirms.

Alloy Type 2

Let's check the interaction effect between the dentist and method on the hardness of dental implants for each type of Alloy 1 from the Interaction Plot.

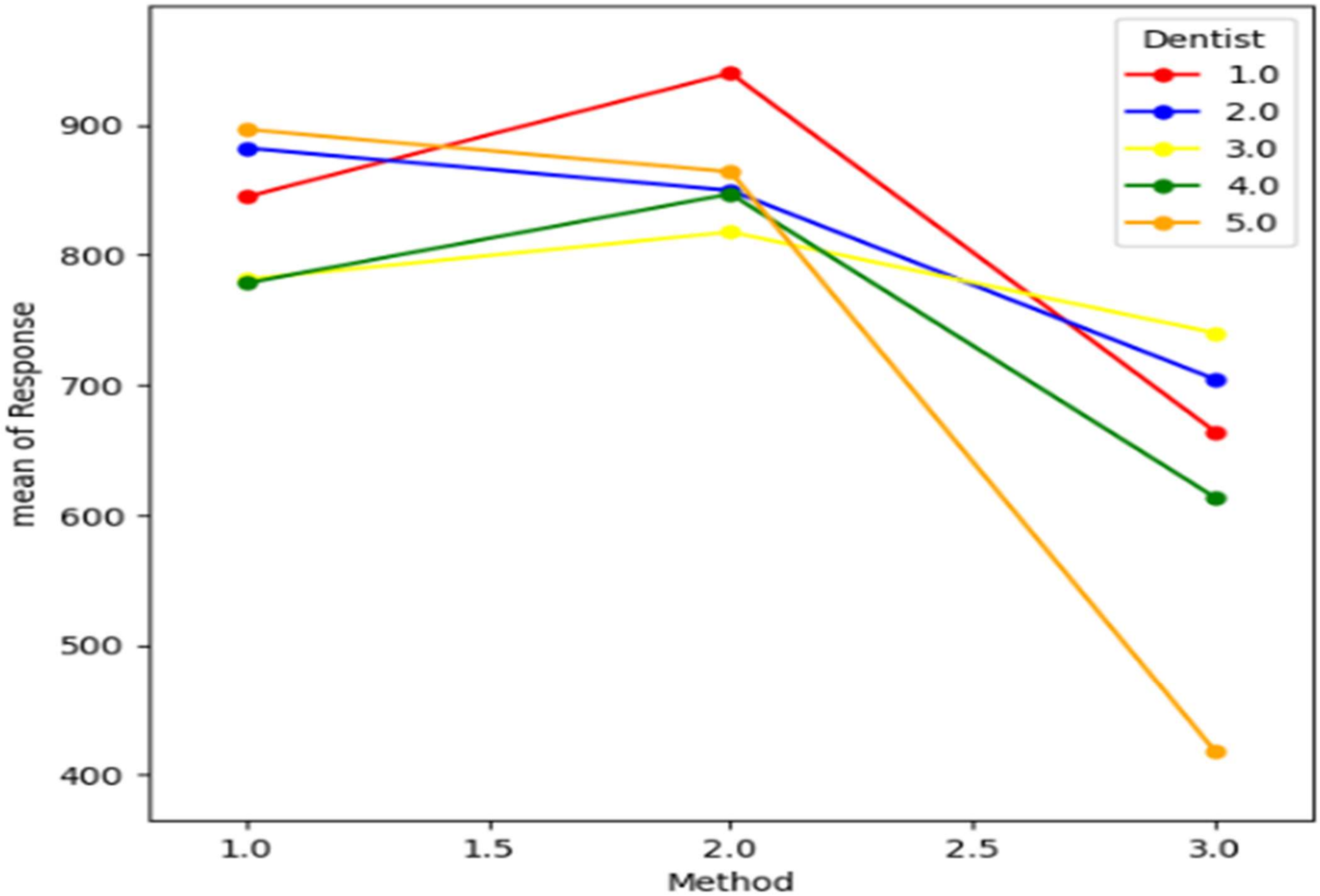

Fig 4.3B Interaction Plot Between Method: Dentist for Alloy Type 2

Inferences: -

In this interaction plot, the lines are not parallel. This interaction effect indicates that the relationship between Dentist and response depends on the Method used.

For Dentist 1 (red line) Hardness increases from Method 1 to Method 2 and it decreases from Method 2 to Method 3

For Dentist 2 (blue line) Hardness decreases from Method 1 to Method 2 and it also decreases Method 2 to Method 3

For Dentist 3 (Yellow line) Hardness increases from Method 1 to Method 2 and it decreases from Method 2 to Method 3

For Dentist 4 (Green line) Hardness increases from Method 1 to Method 2 and it decreases from Method 2 to Method 3

For Dentist 5 (Orange line) Hardness decreases from Method 1 to Method 2 and it decreases from Method 2 to Method 3

	df	sum_sq	mean_sq	F	PR(>F)
Dentist:Method	1.0	351656.418841	351656.418841	19.205303	0.000074
Residual	43.0	787346.381159	18310.380957	NaN	NaN

Table 4.3B Result of two-way for Interaction between Method and Dentist of Alloy Type 2

The crossed lines on the graph suggest that there is an interaction effect, which the significant p-value (0.000074) for the Dentist*Method term confirms.

4.4. How does the hardness of implants vary depending on dentists and methods together?

<u>First lets check for Alloy Type 1</u>

Null Hypothesis:
H0: All group means are equal at each level of the Dentists
H0: All group means are equal at each level of the Methods
H0: There is no interaction effect between the Dentists and Methods
Alternate Hypothesis:
HA: At least one group mean is not equal at each level of the Dentists
HA: At least one group mean is not equal at each level of the Methods
HA: There is an interaction effect between the Dentists and methods

	df	sum_sq	mean_sq	F	PR(>F)
Dentist	4.0	106683.688889	26670.922222	3.899638	0.011484
Method	2.0	148472.177778	74236.088889	10.854287	0.000284
Dentist:Method	8.0	185941.377778	23242.672222	3.398383	0.006793
Residual	30.0	205180.000000	6839.333333	NaN	NaN

Table 4.4A Result of two-way ANOVA between Dentist and Method with interaction for Alloy type 1

We can see the following p-values for each of the factors in the table4.4A: -
Dentist: p-value = 0.0115
Method: p-value = 0.000284
Dentist*Method: p-value = 0.0068

Since the p-values for Dentists and Methods are both less than 0.05, this means that both factors have a statistically significant effect on Hardness

The p-value of interaction effect is also less than 0.05, this tells us that there is significant interaction between Dentists and Methods.

Let's find for which pair of Method the mean hardness of implants varies: -

Multiple Comparison of Means - Tukey HSD, FWER=0.05

group1	group2	meandiff	p-adj	lower	upper	reject
1.0	2.0	-6.1333	0.987	-102.714	90.4473	False
1.0	3.0	-124.8	0.0085	-221.3807	-28.2193	True
2.0	3.0	-118.6667	0.0128	-215.2473	-22.086	True

Table 4.4B Result of Pairwise_tukey for Method of Alloy Type 1

From above table 4.4B

- P-value for the difference in means between Method 1 and Method 2: 0.987

- P-value for the difference in means between Method 1 and Method 3: 0.0085
- P-value for the difference in means between Method 2 and Method 3: 0.0128

Thus, we would conclude that there is a statistically significant difference between the means of Method 1 and Method 2 , but not a statistically significant difference between the means of Method 1 and Method 3 , Method 2 and Method 3.

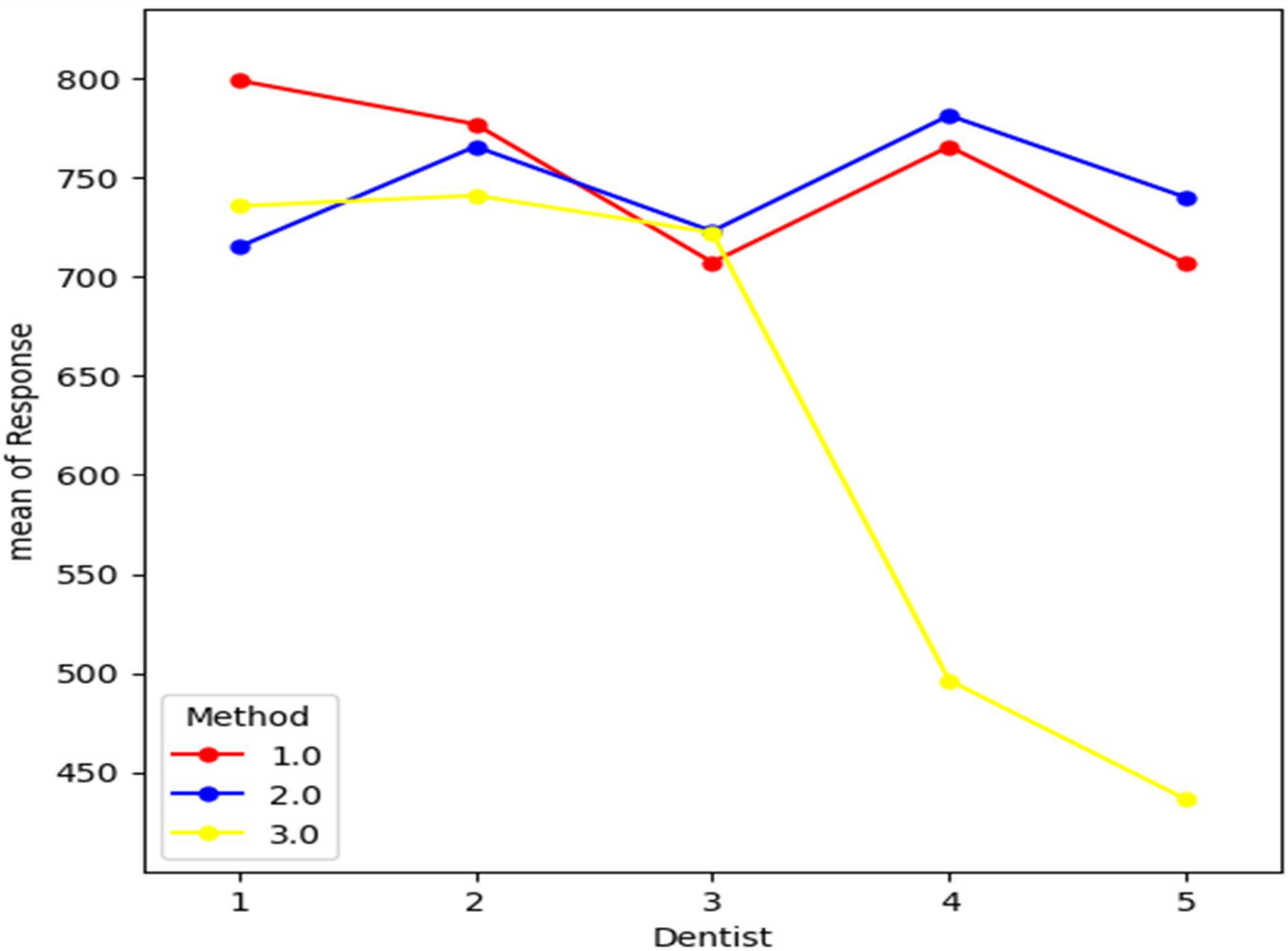

Fig 4.4C Interaction Plot Between Method: Dentist for Alloy Type 1

From the above interaction plot: -
For Dentist 1: - Hardness of the implant is most for Method 1 and least for Method 2, whereas harness of implant for method 3 is in between Method 1 & Method 2

For Dentist 2: - Hardness of the implant is most for Method 1 and least for Method 3, whereas harness of implant for method 2 is in between Method 1 & Method 3

For Dentist 3: - Hardness of the implant is most for Method 2 and Method 2 or we can say the implant hardness is same, whereas harness of implant for Method 1 is least.

For Dentist 4: - Hardness of the implant is most for Method 2 and least for Method 3, whereas harness of implant for method 1 is in between Method 2 & Method 3

For Dentist 5: - Hardness of the implant is most for Method 2 and least for Method 3, whereas harness of implant for method 1 is in between Method 2 & Method 33

Alloy Type 2

Null Hypothesis:

H0: All group means are equal at each level of the Dentists
H0: All group means are equal at each level of the Methods
H0: There is no interaction effect between the Dentists and Methods

Alternate Hypothesis:

HA: At least one group mean is not equal at each level of the Dentists
HA: At least one group mean is not equal at each level of the Methods
HA: There is an interaction effect between the Dentists and methods

	df	sum_sq	mean_sq	F	PR(>F)
Dentist	4.0	56797.911111	14199.477778	1.106152	0.371833
Method	2.0	499640.400000	249820.200000	19.461218	0.000004
Dentist:Method	8.0	197459.822222	24682.477778	1.922787	0.093234
Residual	30.0	385104.666667	12836.822222	NaN	NaN

Table 4.4C Result of two-way ANOVA between Dentist and Method with interaction for Alloy type 2

We can see the following p-values for each of the factors in the table 4.4C: -
Dentist: p-value = 0.372
Method: p-value = 0.000004
Dentist*Method: p-value = 0.0932

The p-values for Dentists (0.372) is more than 0.05, this means that Dentist doesn't have statistical significant effect on the hardness.
Whereas p-value of Methods (0.000004) is very less than 0.05 and has statistically significant effect on Hardness

The p-value (0.0932) of interaction effect is also slightly more than 0.05, this tells us that there is no significant interaction between Dentists and Methods.

Let's find for which pair of Method mean hardness of implants varies

Multiple Comparison of Means - Tukey HSD, FWER=0.05

group1	group2	meandiff	p-adj	lower	upper	reject
1.0	2.0	27.0	0.8212	-82.4546	136.4546	False
1.0	3.0	-208.8	0.0001	-318.2546	-99.3454	True
2.0	3.0	-235.8	0.0	-345.2546	-126.3454	True

Table 4.4D Result of Pairwise_tukey for Method of Alloy 2

From above table 4.4D

- P-value for the difference in means between Method 1 and Method 2: 0.8212
- P-value for the difference in means between Method 1 and Method 3: 0.0001
- P-value for the difference in means between Method 2 and Method 3: 0.0

Thus, we would conclude that there is a statistically significant difference between the means of Method 1 and Method 2 , but not a statistically significant difference between the means of Method 1 and Method 3 , Method 2 and Method 3.