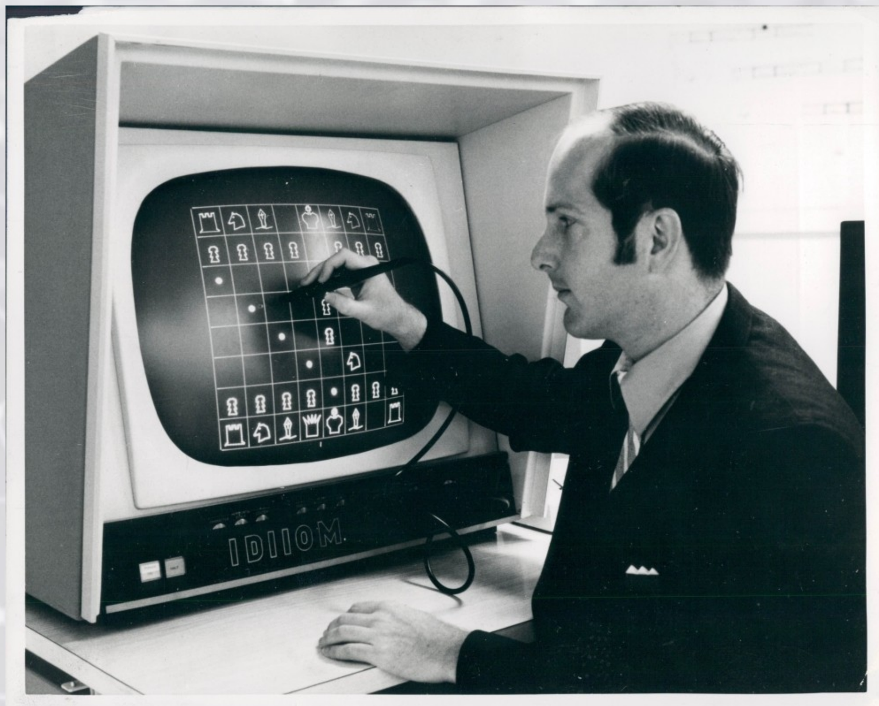
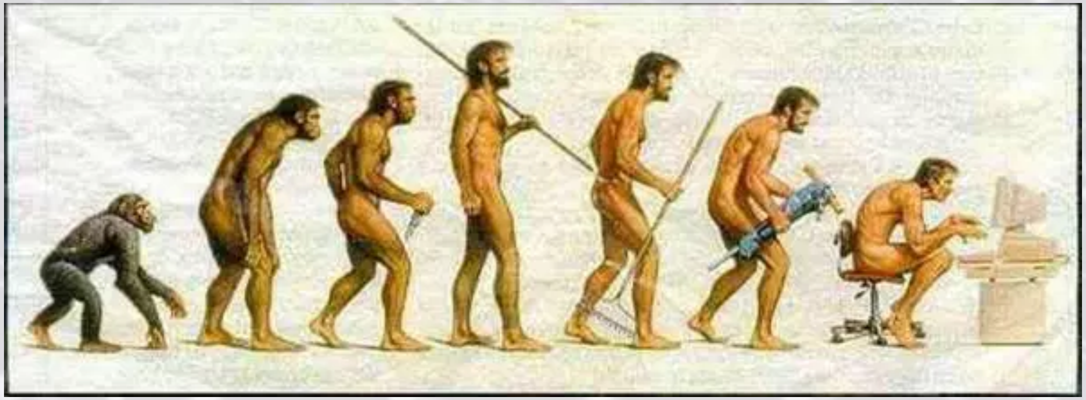


Chapter III: Part Deux

Outline

- Best-first search
- Greedy best-first search
- A* search
- Heuristics



Overview

- **Informed Search**: uses problem-specific knowledge.
- General approach: **best-first search**; an instance of TREE-SEARCH (or GRAPH-SEARCH) – where a search strategy is defined by picking the order of node expansion.
- With best-first, node is selected for expansion based on **evaluation function** *f(n)*.
- Evaluation function is a *cost estimate*; expand lowest cost node first (same as uniform-cost search but we replace *g* with *f*).

Overview (cont'd)

- The choice of *f* determines the search strategy (one can show that best-first tree search includes DFS as a special case).
- Often, for best-first algorithms, f is defined in terms of a **heuristic function**, h(n).

  h(n) = *estimated* cost of the cheapest path from the state at node *n* to a *goal state*. (for goal state: *h(n)*=0)
- Heuristic functions are the most common form in which **additional knowledge** of the problem is passed to the search algorithm.

Overview (cont'd)

- Best-First Search algorithms constitute a large family of algorithms, with different evaluation functions.
  - Each has a heuristic function h(n)

- Example: in route planning the estimate of the cost of the cheapest path might be the straight line distance between two cities.

Recall:

- $g(n)$ = cost from the initial state to the current state n.
- $h(n)$ = estimated cost of the cheapest path from node n to a goal node.
- $f(n)$ = evaluation function to select a node for expansion (usually the lowest cost node).

Best-First Search

- Idea: use an evaluation function *f(n)* for each node
  - f(n) provides an estimate for the total cost.

  →Expand the node n with smallest f(n).

- Implementation:

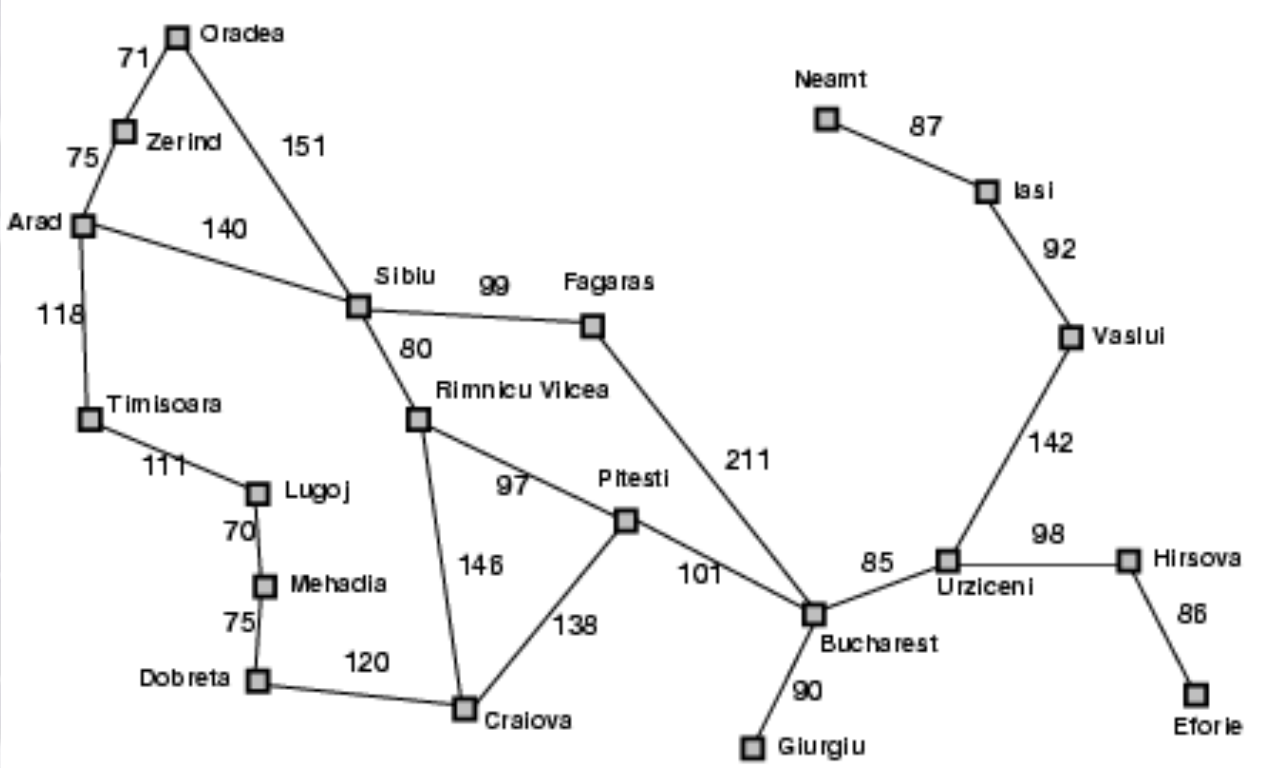
  Order the nodes in the frontier increasing order of cost.

- Special cases:
  - Greedy best-first search
  - A* search

Greedy best-first search

- Evaluation function $f(n) = h(n)$ (*heuristic*), the estimate of cost from *n* to *goal*.
- We use the straight-line distance heuristic: $h_{SLD}(n)$ = straight-line distance from *n* to Bucharest.
- Note that the heuristic values cannot be computed from the problem description itself!
- In addition, we require **extrinsic knowledge** to understand that h_{SLD} is correlated with the actual road distances, making it a useful heuristic.
- Greedy best-first search expands the node that appears to be closest to goal.

Romania with step costs in km



Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
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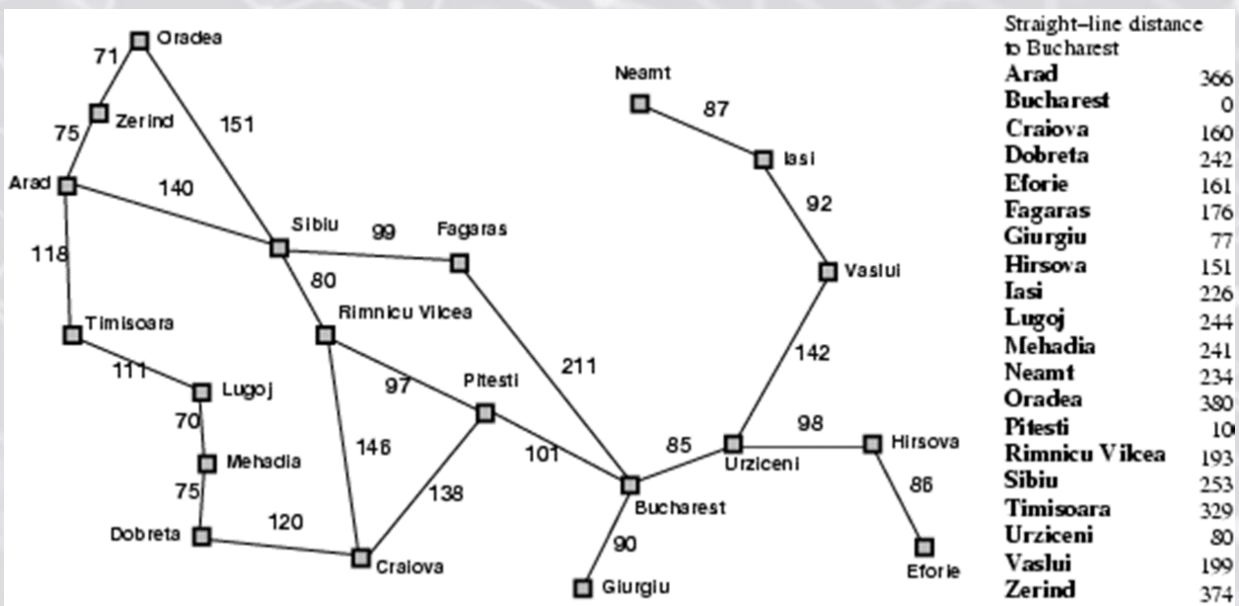
Greedy best-first search example



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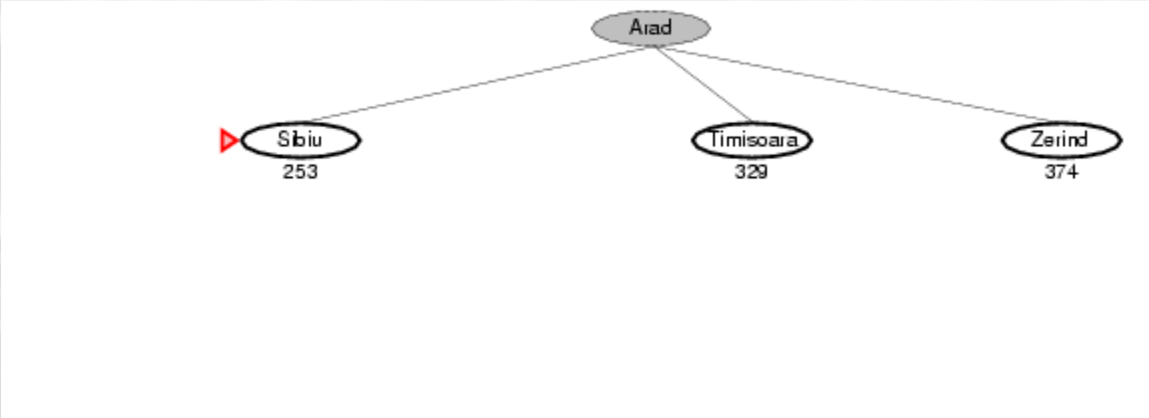
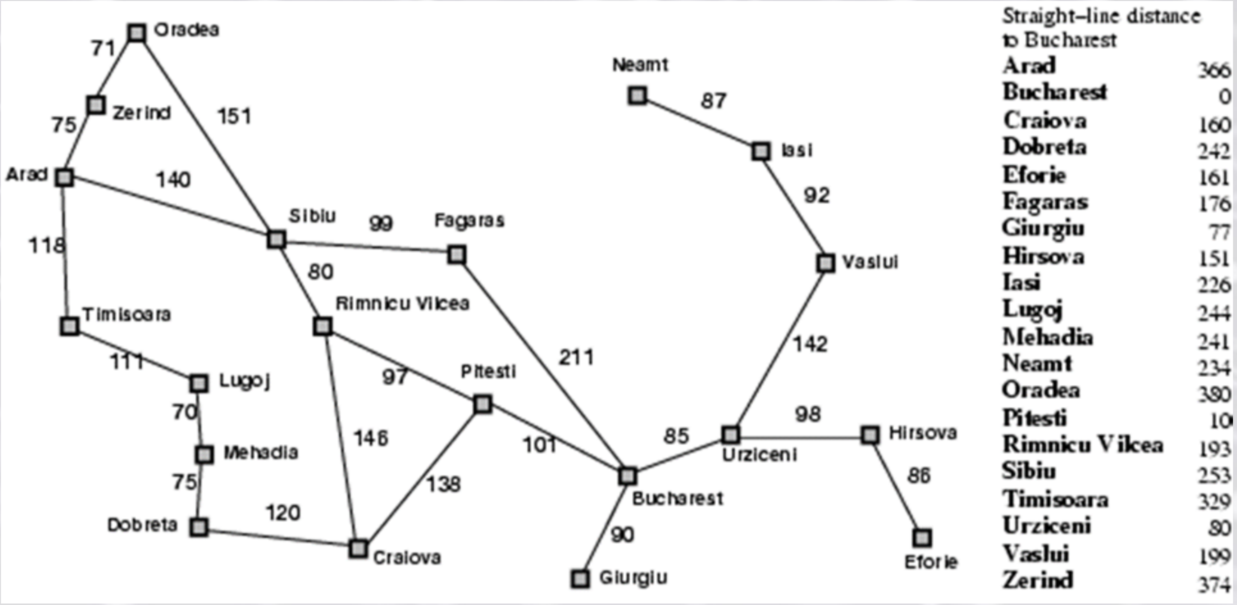
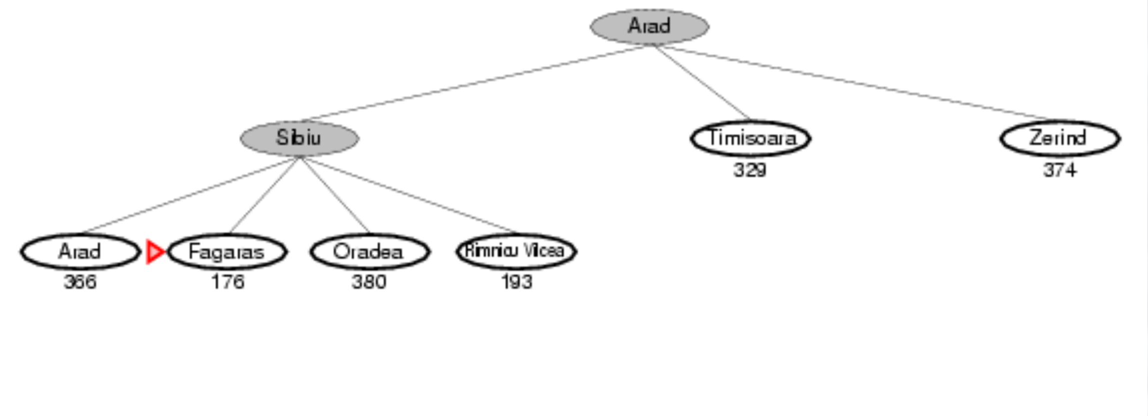
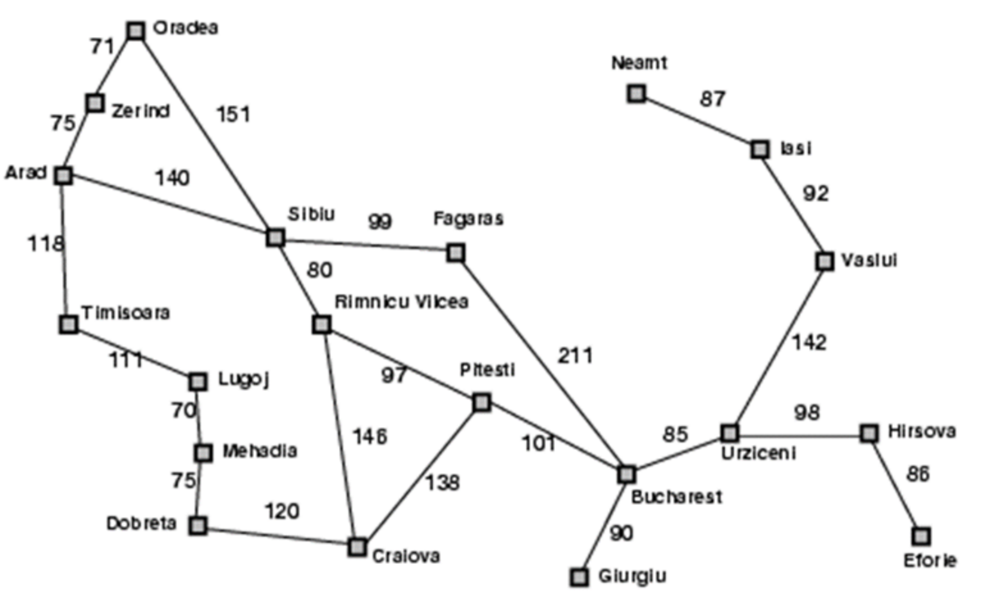




Greedy best-first search example





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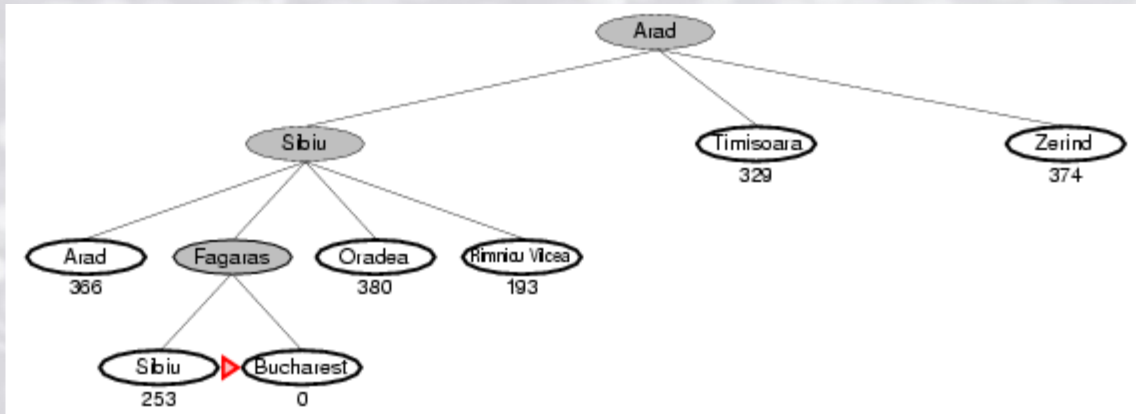
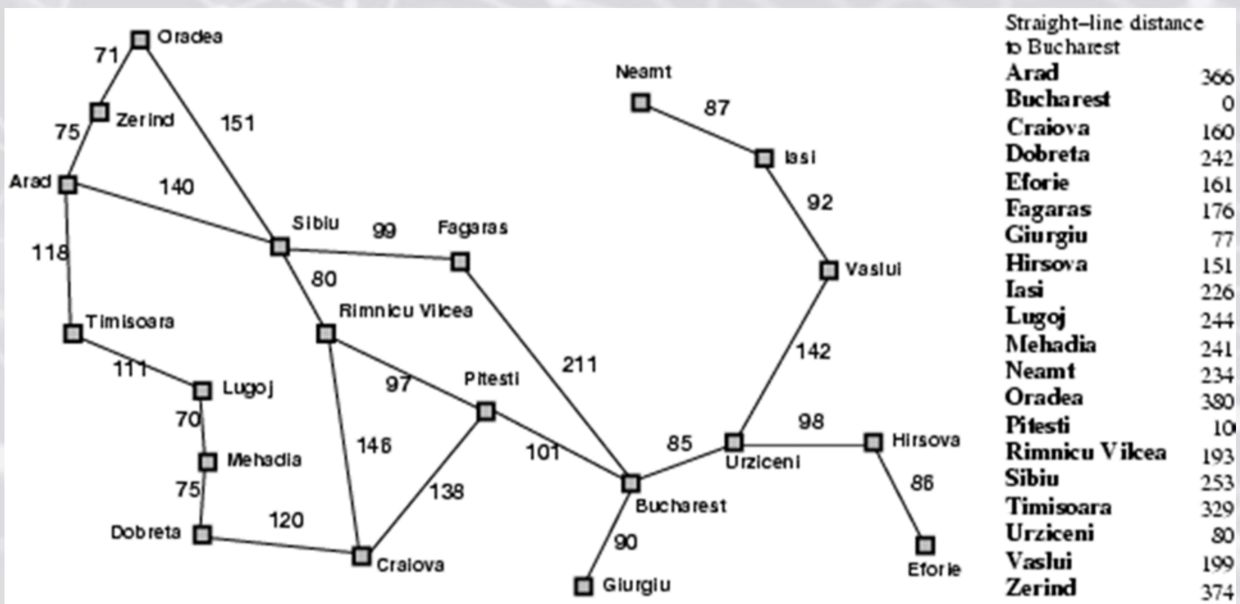
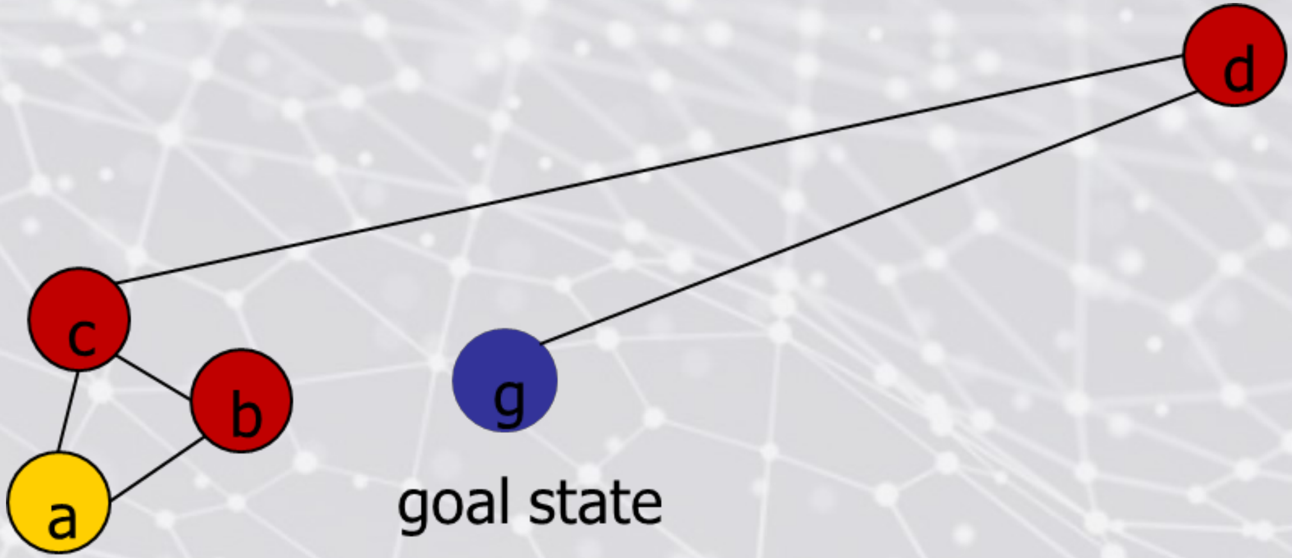
Greedy best-first search example





Greedy best-first search

- GBFS is *incomplete*!
- Why?



- Graph-Search version is, however, complete in *finite* spaces.

Properties of greedy best-first search

- Complete? No – can get stuck in loops, e.g., Iasi → Neamt → Iasi → Neamt →
- Time? $O(b^m)$, (in worst case) but a good heuristic can give dramatic improvement (m is max depth of search space).
- Space? $O(b^m)$ -- keeps all nodes in memory.
- Optimal? No (not guaranteed to render lowest cost solution).

A* Search

- Most widely-known form of best-first search.

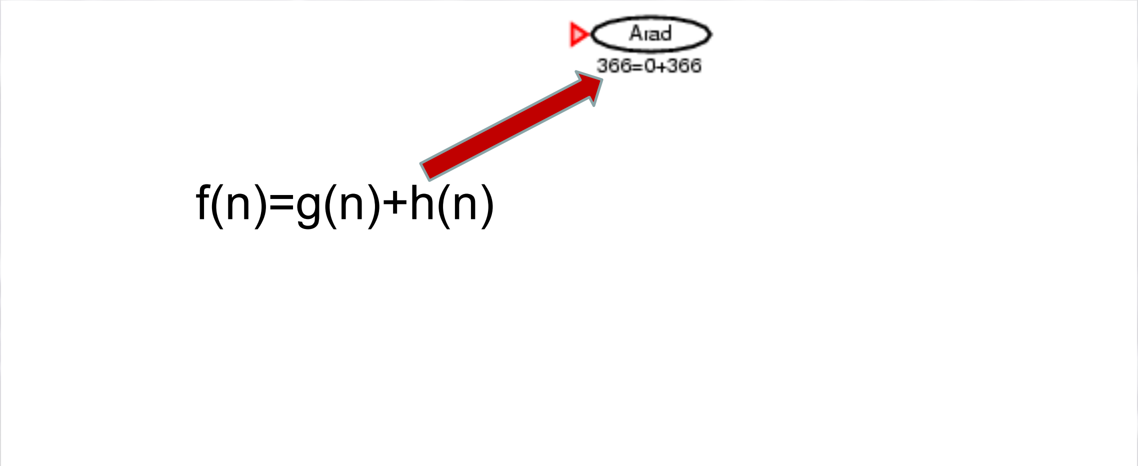
- It evaluates nodes by combining $g(n)$, the cost to reach the node, and $h(n)$, the cost to get from the node to the goal:

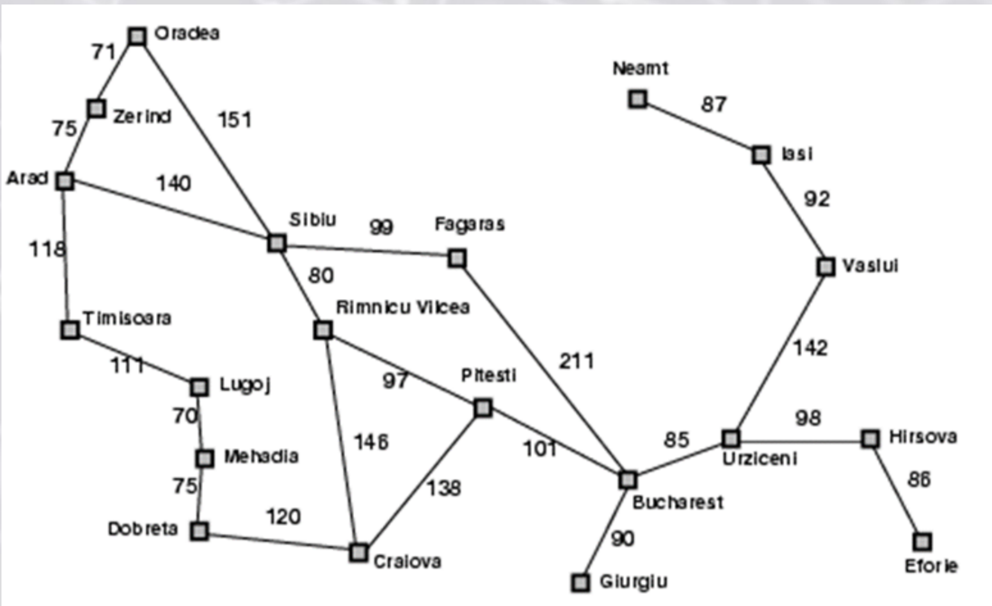
  $f(n) = g(n) + h(n)$ (estimated cost of cheapest solution through n).

- A reasonable strategy: try node with the lowest $g(n) + h(n)$ value!

- Provided heuristic meets some basic conditions, A* is both **complet**e and **optimal**.

A* search example





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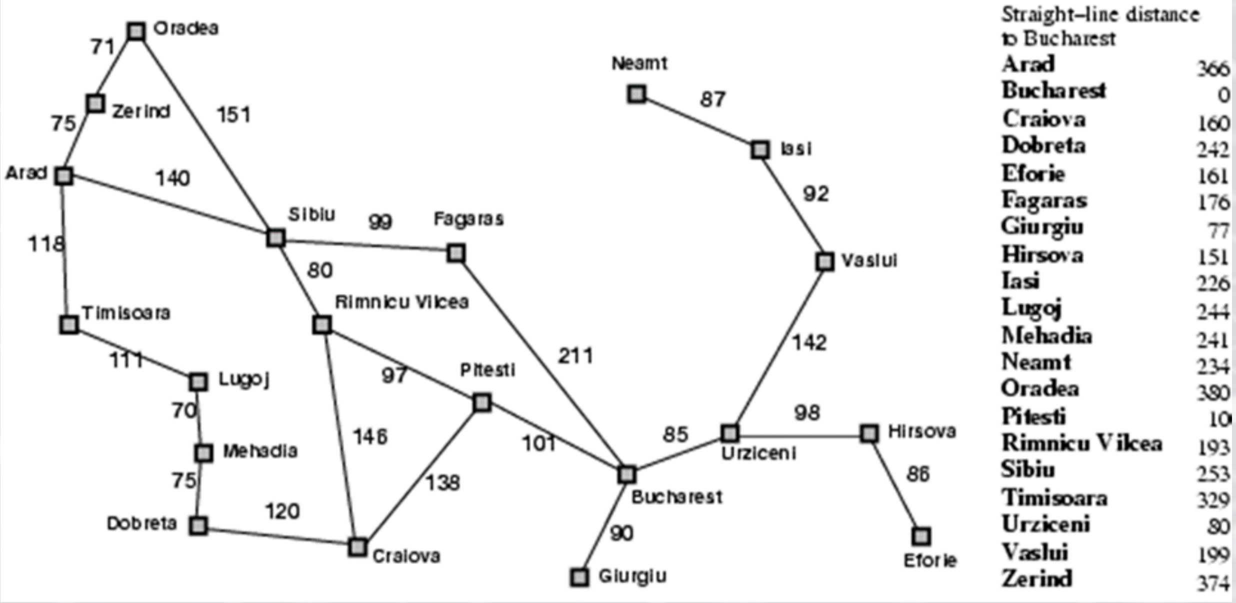
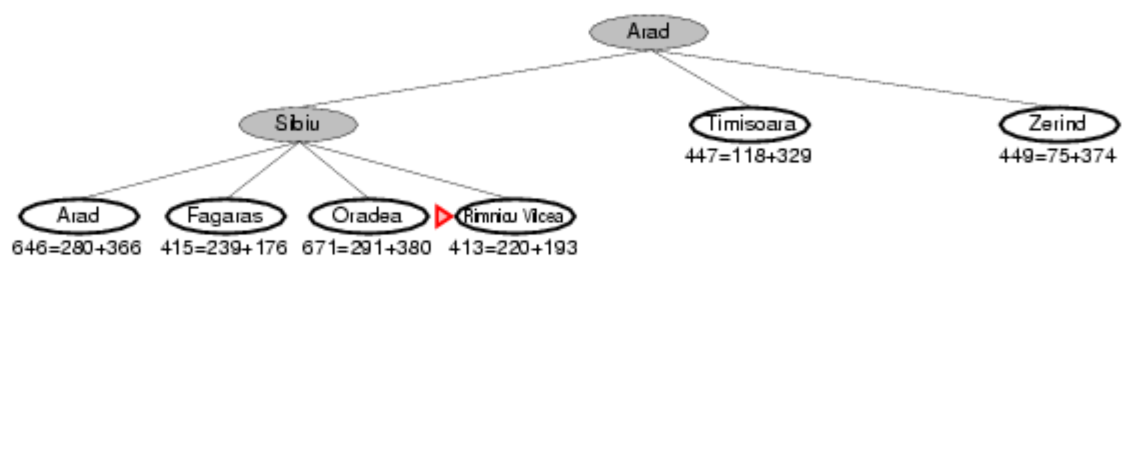
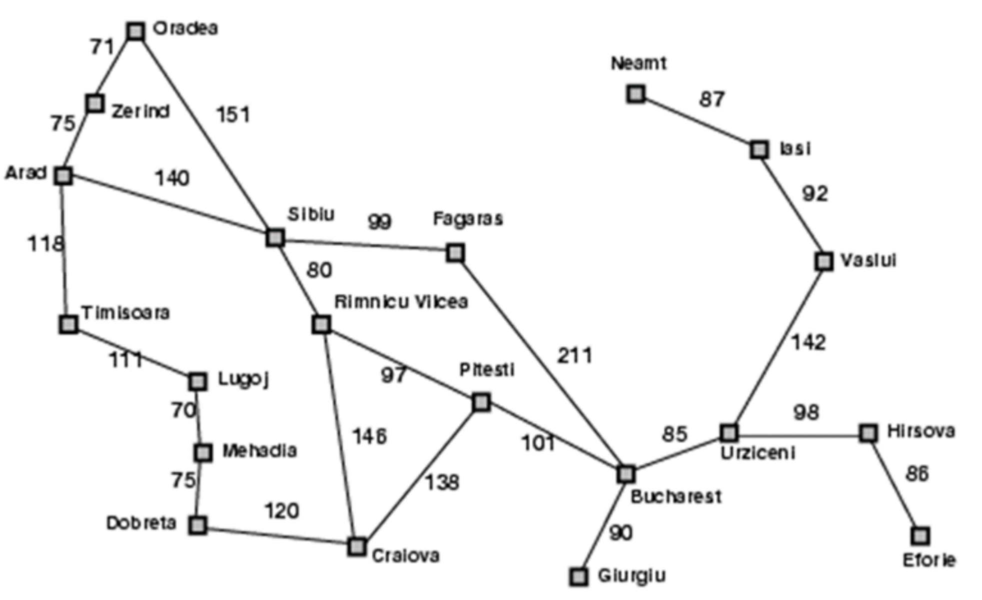
A* search example



A* search example





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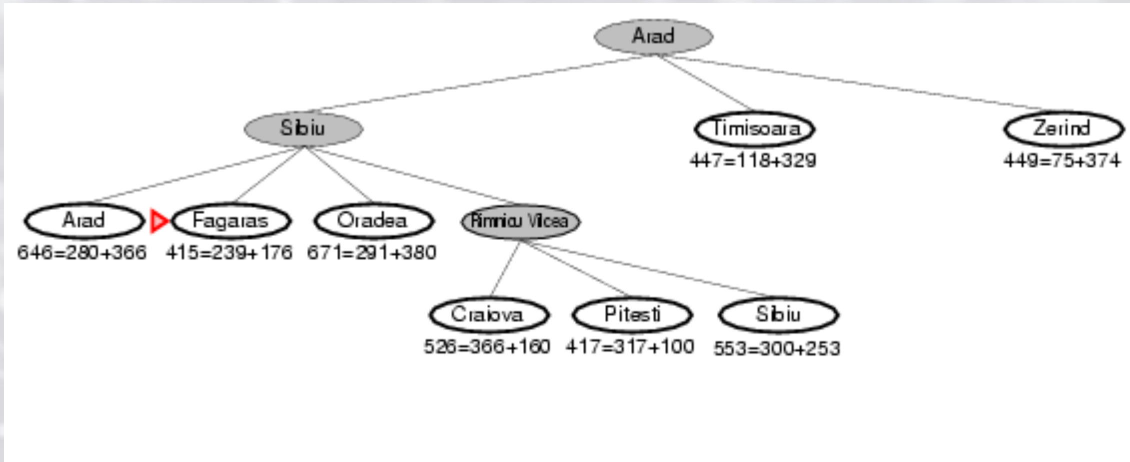
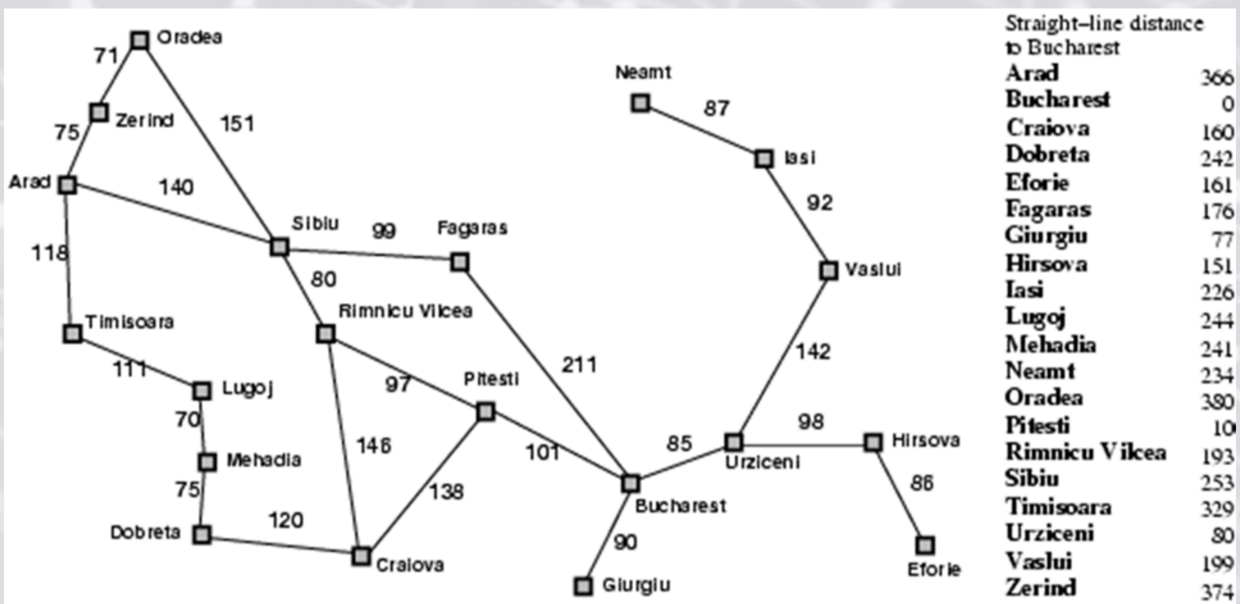
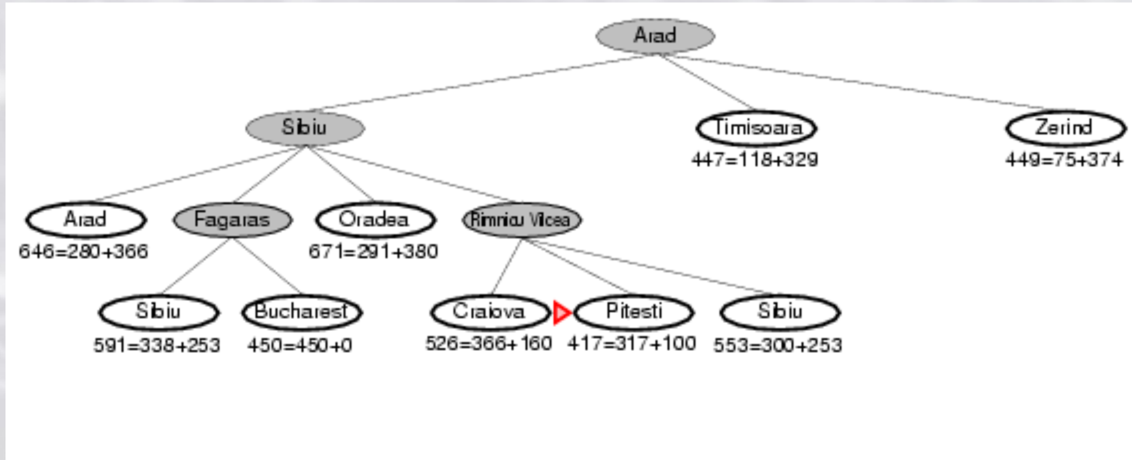
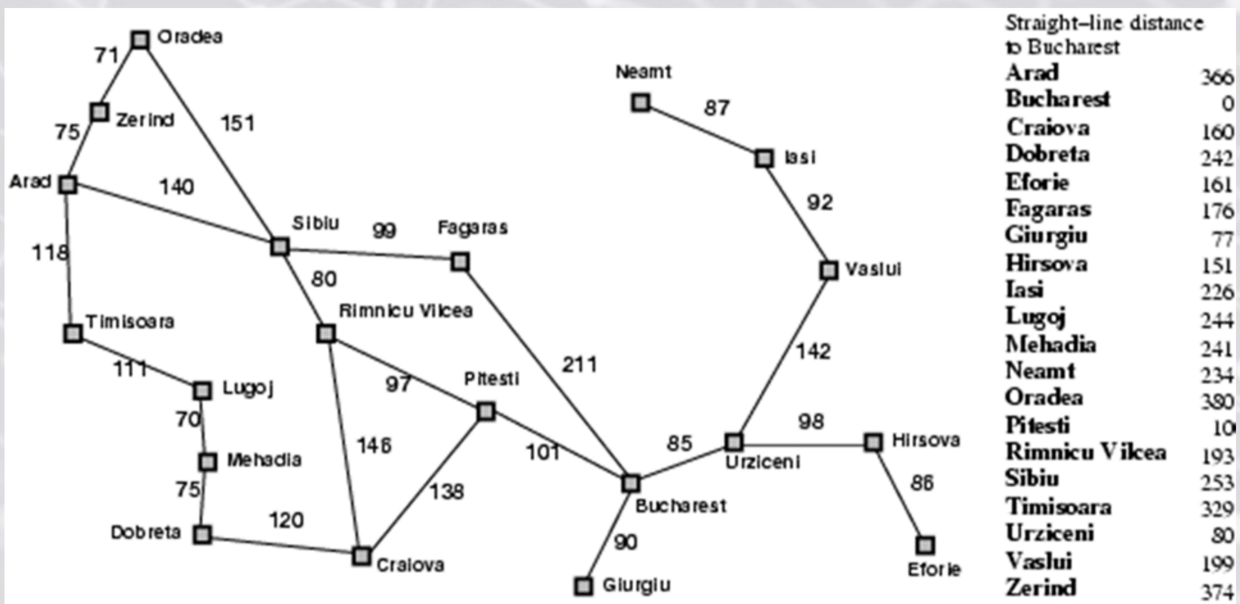
A* search example



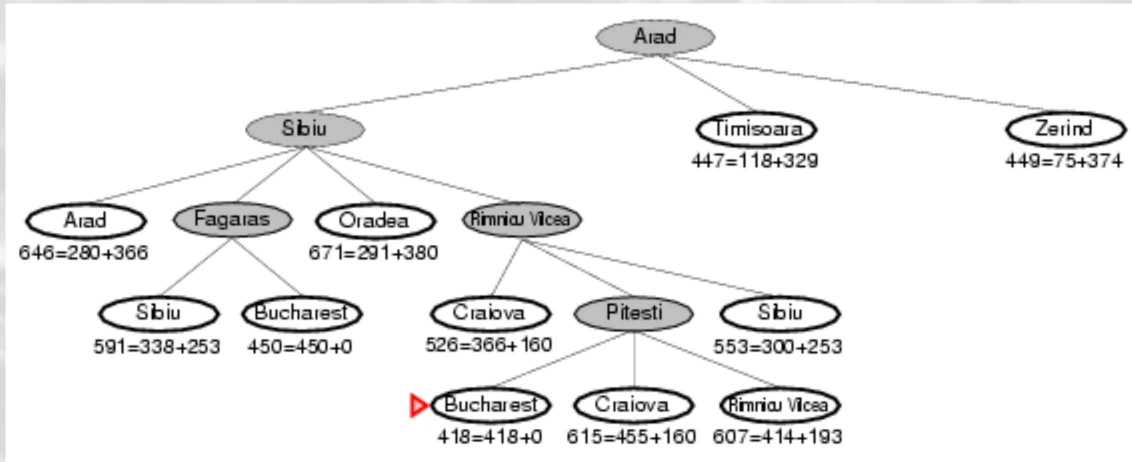


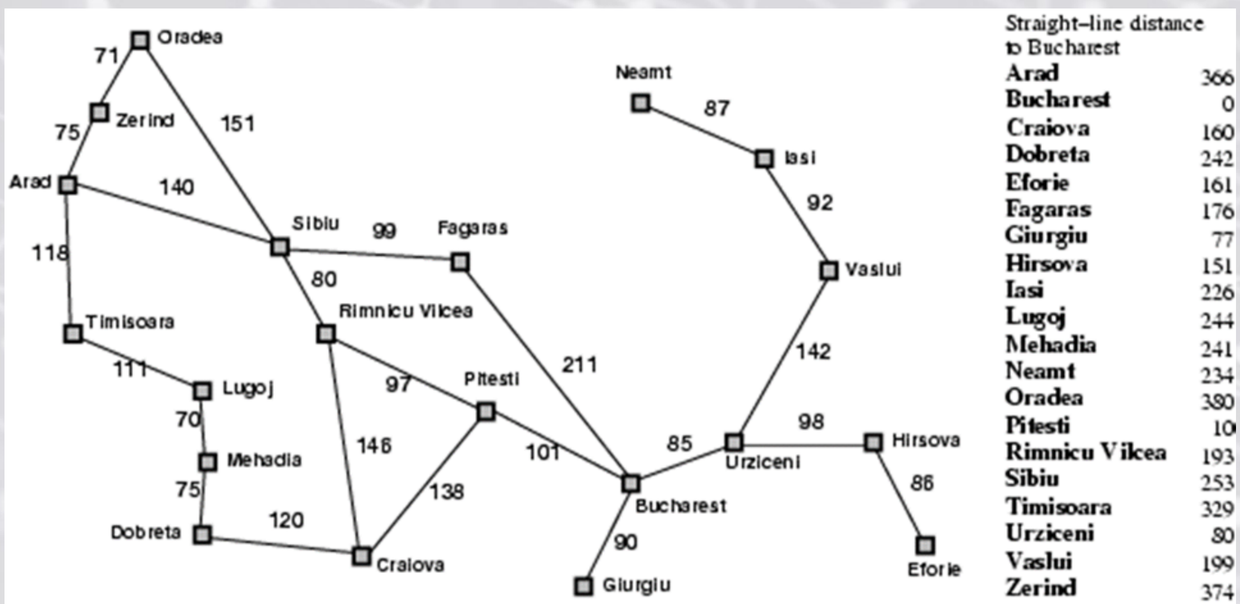
A* search example





A* search example





Admissible heuristics

- A heuristic $h(n)$ is admissible if for every node n,
  $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n.

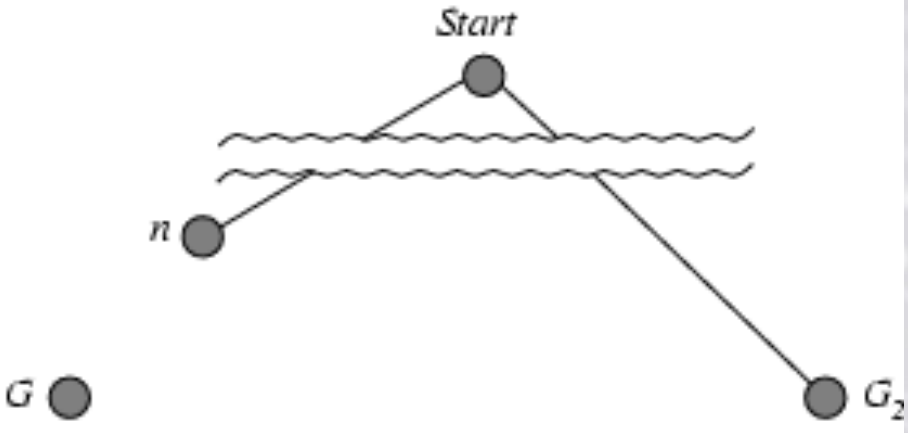
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic.

- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)

» Theorem: If $h(n)$ is admissible, A^* using TREE-SEARCH is optimal.

Optimality of A* (proof)

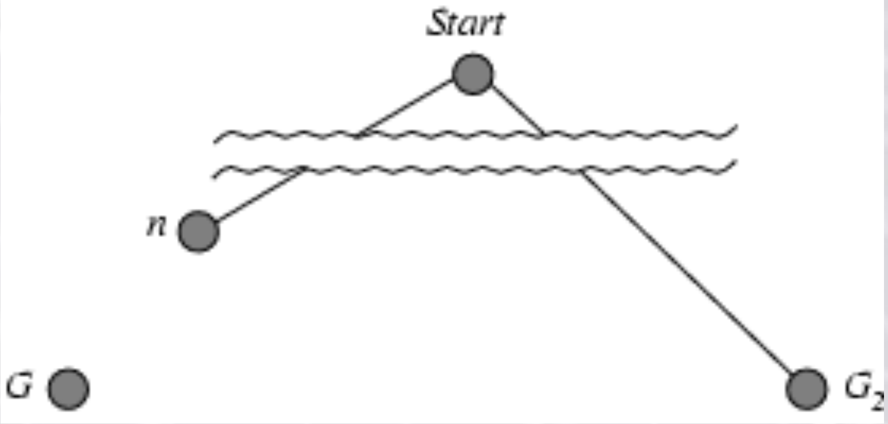
- Suppose some **suboptimal goal** G_2 has been generated and is in the frontier. Let n be an unexpanded node in the frontier such that n is on a shortest path to an optimal goal G.



- $f(G_2) = g(G_2)$ since $h(G_2) = 0$
- $g(G_2) > g(G)$ since G_2 is suboptimal
- $f(G) = g(G)$ since $h(G) = 0$
- $f(G_2) > f(G)$ from above

Optimality of A* (proof)

- Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let *n* be an unexpanded node in the fringe such that *n* is on a shortest path to an optimal goal *G*.



- $f(G_2) = g(G_2)$
- $g(G_2) > g(G)$
- $f(G) = g(G)$
- $f(G_2) > f(G)$

- $f(G_2) > f(G)$ (from above)
- $h(n) \le h^*(n)$ (since h is **admissible**)

-> $g(n) + h(n) \le g(n) + h^*(n)$

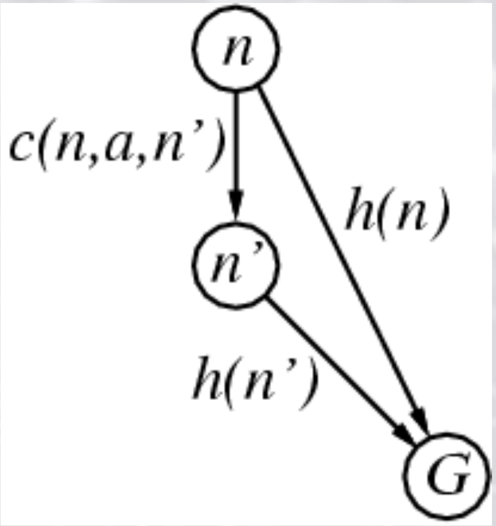
- $f(n) \le g(n) + h^*(n) < f(G) < f(G_2)$

Hence $f(G_2) > f(n)$, and A* will never select G_2 for expansion.

Consistent Heuristics

- A heuristic is consistent (or **monotonic**) if for every node *n*, every successor *n'* of *n* generated by any action *a:*

$$h(n) \leq c(n,a,n') + h(n')$$



- If *h* is **consistent**, we have:

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n,a,n') + h(n') \\ &\geq g(n) + h(n) \end{aligned}$$

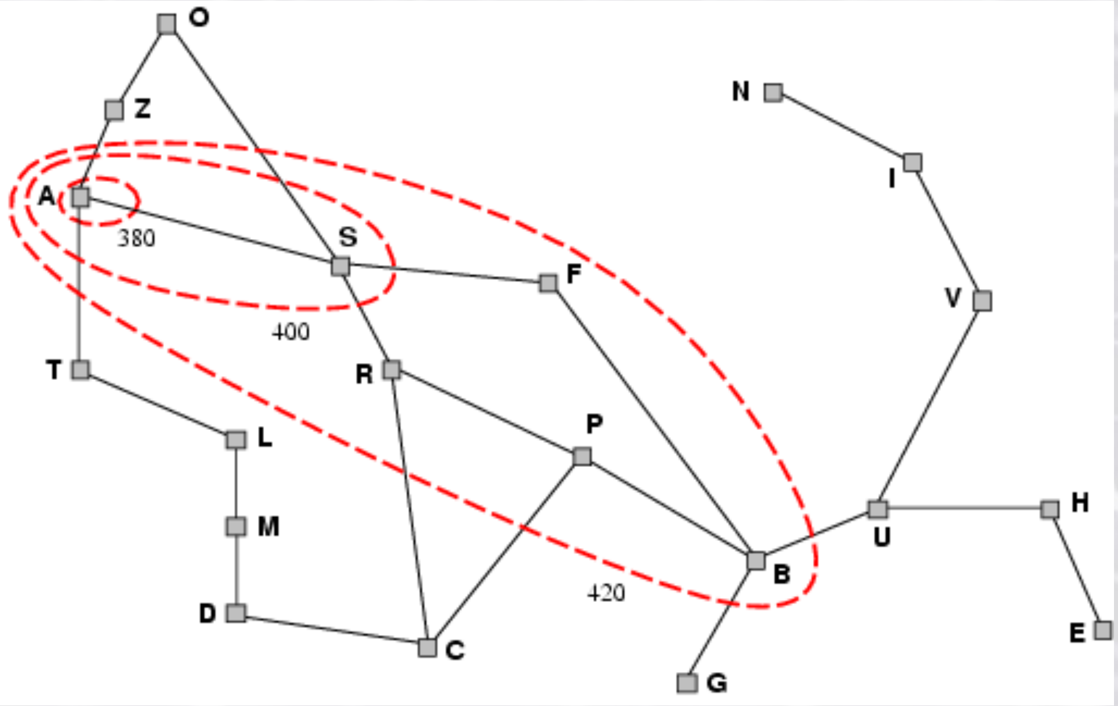
» $= f(n)$

» i.e., *f(n)* is **non-decreasing along any path**.

» Theorem: If *h(n)* is consistent, A* using GRAPH-SEARCH is optimal.

Optimality of A*

- A* expands nodes in order of increasing *f* value.
- Gradually adds "*f*-contours" of nodes.
- Contour *i* has all nodes with $f=f_i$, where $f_i < f_{i+1}$.
- That is to say, nodes inside a given contour have f-costs less than or equal to contour value.



Properties of A*

- Complete: Yes (unless there are infinitely many nodes with f $\leq f(G)$).
- Time: Exponential.
- Space: Keeps all nodes in memory, so also exponential.
- Optimal: Yes (provided *h* admissible or consistent).
- *Optimally Efficient*: Yes (no algorithm with the

  same heuristic is guaranteed to expand fewer nodes).

- *NB*: Every consistent heuristic is also admissible (Pearl).

Q: What about the converse?

Admissible Heuristics

E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles

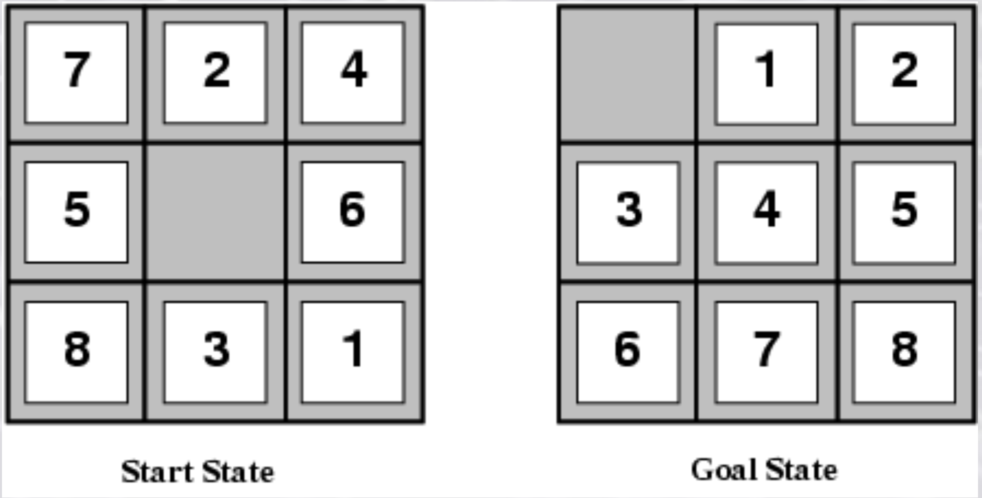
$h_2(n)$ = total Manhattan distance (i.e. 1-norm)

(i.e., no. of squares from desired location of each tile)

Q: Why are these admissible heuristics?

» $h_1(S) = ?$

» $h_2(S) = ?$



Admissible Heuristics

E.g., for the 8-puzzle:

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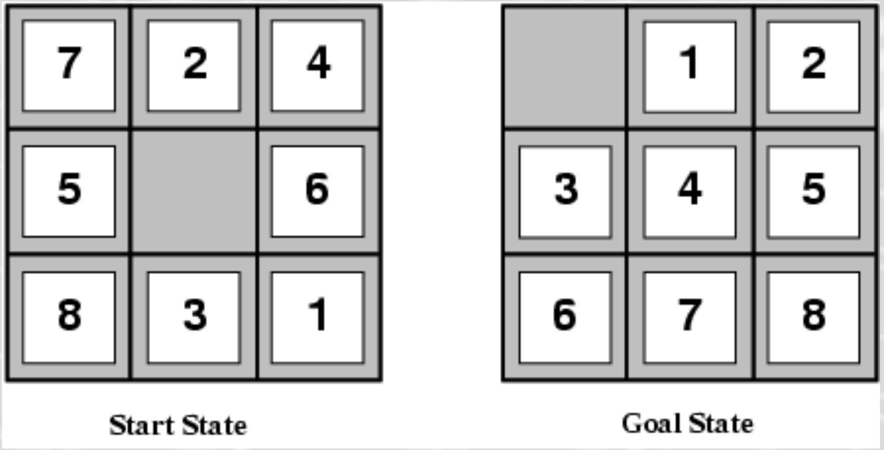


» $h_1(S)$ = ? 8

» $h_2(S)$ = ? 3+1+2+2+2+3+3+2 = 18

Dominance

- If $h_2(n) \geq h_1(n)$ <u>for all n</u> (both admissible), then h_2 dominates h_1 .

- *Essentially, domination translates directly into efficiency: "h_2* is better for search.
- A* using h_2 will never expand more nodes than A* using h_1.

- Typical search costs (average number of nodes expanded):

d=12 IDS = 3,644,035 nodes
$A^*(h_1)$ = 227 nodes
$A^*(h_2)$ = 73 nodes
d=24 IDS = too many nodes
$A^*(h_1)$ = 39,135 nodes
$A^*(h_2)$ = 1,641 nodes

» (IDS=iterative deepening search)

Memory Bounded Heuristic Search: Recursive BFS (best-first)

- How can we solve the memory problem for A* search?

- Idea: Try something like depth-first search, but let's <u>not forget everything about the branches we have partially explored</u>.

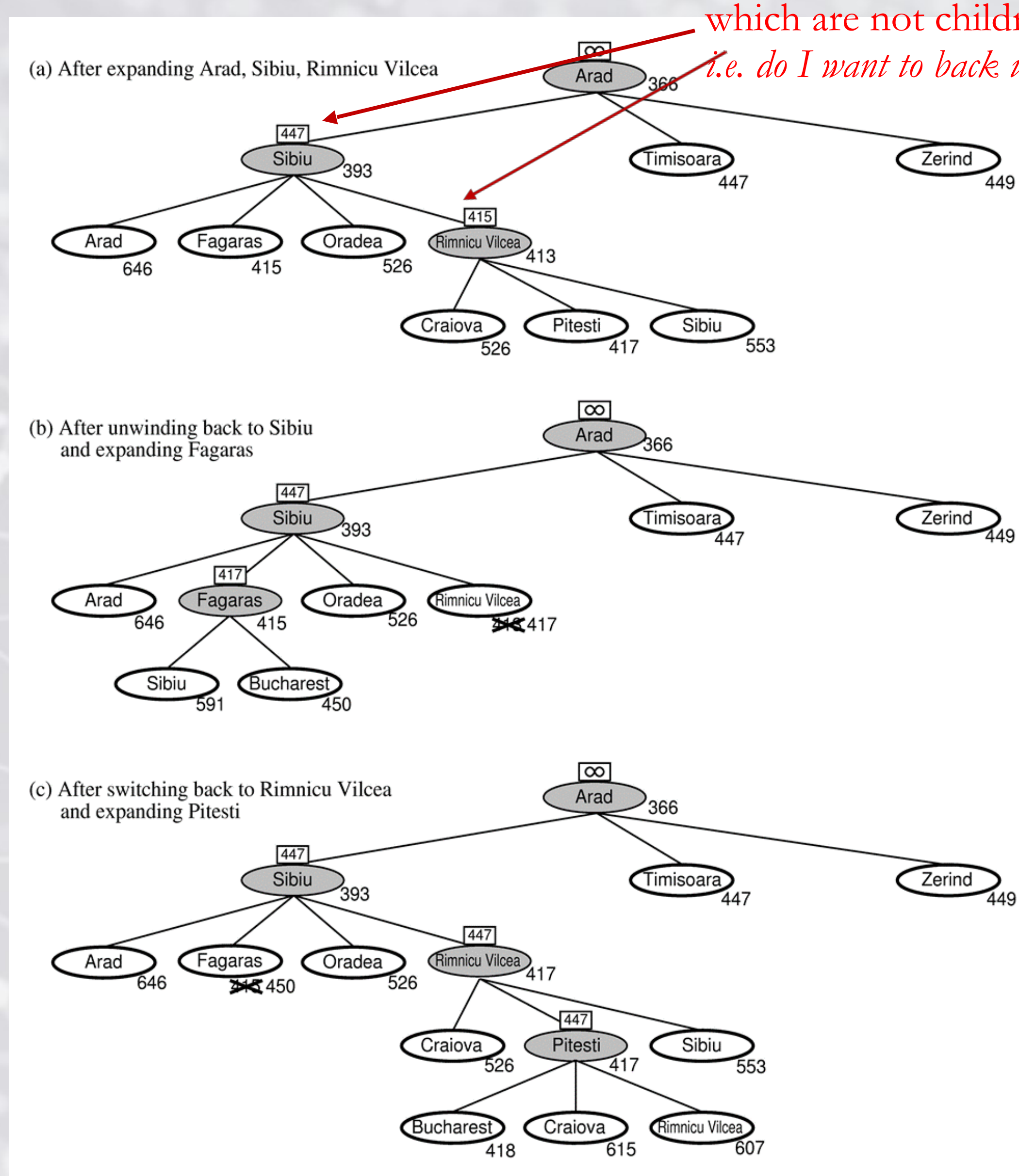
- *We remember the best f-value we have found so far in the branch we are deleting.*

Memory Bounded Heuristic Search: Recursive BFS

Best alternative over frontier nodes, which are not children: *i.e. do I want to back up?*

- RBFS changes its mind very often in practice. This is because f=g+h become more accurate (less optimistic) as we approach the goal. Hence, higher level nodes have smaller f-values and will be explored first.

- Problem: We should keep
- in memory whatever we can.



Simple Memory-Bounded A*

- This is like A*, but <u>when memory is full</u> we delete the worst node (largest f-value).
- Like RBFS, we remember the best descendent in the branch we delete.
- If there is a tie (equal f-values) we delete the oldest nodes first.
- Simple-MBA* finds the optimal *reachable* solution given the memory constraint (reachable means path from root to goal fits in memory).
- Can also use **iterative deepening** with A* (IDA*).
- Time can still be exponential.

Relaxed Problems

- A problem with fewer restrictions on the actions is called a relaxed problem.

- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem. (why?)

- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution.

- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution.

Summary

- **Informed search** methods may have access to a **heuristic function** *h(n)* that estimates the cost of a solution from *n*.
- The generic **best-first search** algorithm selects a node for expansion according to an **evaluation function**.
- **Greedy best-first search** expands nodes with minimal h(n). It is not optimal, but is often efficient.
- **A*** search expands nodes with minimal f(n)=g(n)+h(n).
- A* s **complete** and **optimal**, provided that h(n) is admissible (for TREE-SEARCH) or consistent (for GRAPH-SEARCH).
- The space complexity of A* is still prohibitive.
- The performance of heuristic search algorithms depends on the quality of the *h(n)* function.
- One can sometimes construct good heuristics by **relaxing** the problem definition.