Artificial Intelligence
Chapter 5: Adversarial Search

Pop Quiz

- White to mate…

Pop Quiz

- White to mate…in 292 moves!

Games

- Multiagent environment
- Cooperative vs. competitive
  - Competitive environment is where the agents' goals are in conflict
  - Adversarial Search
- Game Theory
  - A branch of economics
  - Views the impact of agents on others as significant rather than competitive (or cooperative).

Properties of Games

- Game Theorists
  - Deterministic, turn-taking, two-player, zero-sum games of perfect information

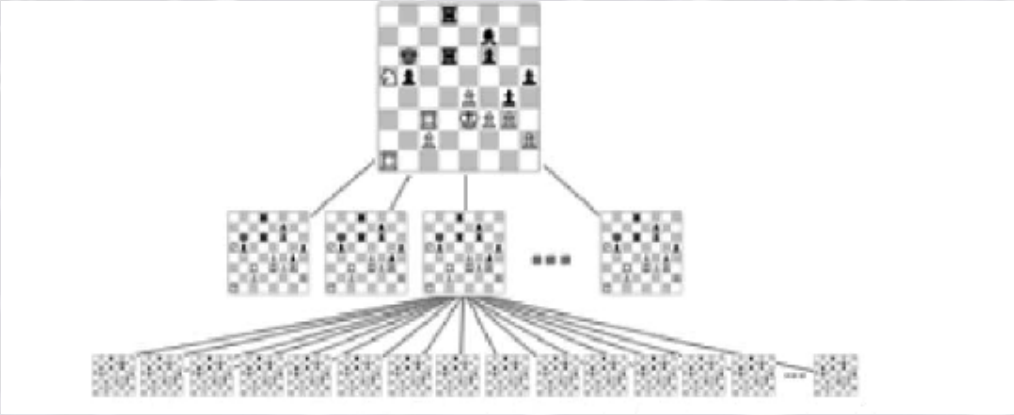
- AI
  - Deterministic
  - Fully-observable
  - Two agents whose actions must alternate
  - Utility values at the end of the game are equal and opposite
    - In chess, one player wins (+1), one player loses (-1)
    - It is this opposition between the agents' utility functions that makes the situation adversarial

Why Games?

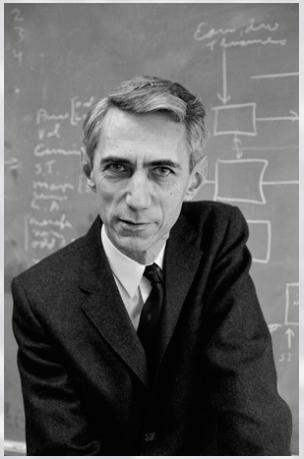
- Small defined set of rules
- Well defined knowledge set
- Easy to evaluate performance
- Large search spaces
  - Too large for exhaustive search
- Fame and Fortune
  - e.g. Chess and Deep Blue

Games as Search Problems

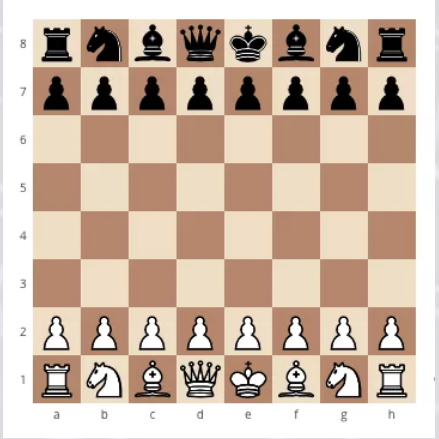
- Games have a state space search
  - Each potential board or game position is a state
  - Each possible move is an operation to another state
  - The state space can be HUGE!!!!!!!
    - Large branching factor (about 35 for chess)
    - Terminal state could be deep (about 50 for chess)

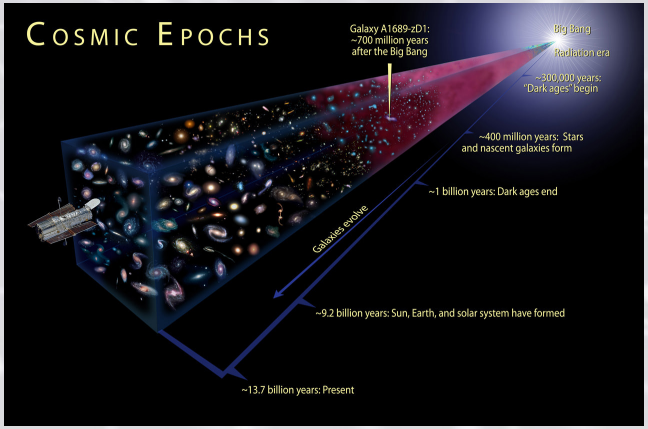


Aside: "Shannon number" for chess



- The "Shannon number" is a conservative lower-bound of the search complexity of chess: 10^{120} (cf. number of atoms in known universe is approximated as 10^{80}).





Aside: "Shannon number" for chess

- Shannon produced this calculation to demonstrate demonstrate the impracticality of solving chess by brute force in his 1950 paper "Programming a Computer for Playing Chess."
- Branching factor ~30; average game is 40 moves (80 plies); $30^{80} \sim 10^{120}$.
- He also estimated the number of possible positions, of the general order of: $64!/(32!8!^2 2!^6)$ – see paper for details.
- By comparison to the Shannon number, if chess is analyzed from the number of "sensible" games that can be played (not counting ridiculous or obvious game-losing moves such as moving a queen to be immediately captured by a pawn without compensation, then the result is closer to 10^{40} games.

Shannon paper (1950): https://vision.unipv.it/IA1/aa2009-2010/ProgrammingaComputerforPlayingChess.pdf

Games vs. Search Problems

- Unpredictable opponent
- Solution is a strategy
  – Specifying a move for every possible opponent reply
- Time limits
  – Unlikely to find the goal…agent must approximate
- In AI, the most common games are deterministic, turn-taking, two-player, zero-sum games of perfect information (i.e. fully observable environments).

Types of Games

	Deterministic	**Chance**
Perfect Information	Chess, checkers, go, othello	Backgammon, monopoly
Imperfect Information		Bridge, poker, scabble, nuclear war

Example Computer Games

- Chess – Deep Blue (World Champion 1997)
- Checkers – Chinook (World Champion 1994)
- Othello – Logistello
  - Beginning, middle, and ending strategy
  - Generally accepted that humans are no match for computers at Othello
- Backgammon – TD-Gammon (Top Three)
- Go – AlphaGo
- Bridge (Bridge Barron 1997, GIB 2000)
  - Imperfect information
  - multiplayer with two teams of two

Formal Elements of Games

- S_o: **Initial state**, specifies how game is set up to start.
- PLAYER(s): Defines which player has the move in a state.
- ACTION(s): Returns the set of legal moves in a state.
- RESULT(s,a): The transition mode, which defines the result of a move.
- TERMINAL-TEST(s):A terminal test, which is true when the game is over and false otherwise.

Formal Elements of Games

- UTILITY(s,p): A **utility function** (also called a payoff function) that defines the final numeric value for a game that ends in terminal state *s* for player *p*.
- Some games have a wider variety of possible outcomes; the payoffs in backgammon range from 0 to +192.
- A **zero-sum game** is defined as one where the total payoff to all players is the same for every instance of the game.
- Chess is zero-sum because every game has payoff 0+1, 1+0, or ½ + ½.

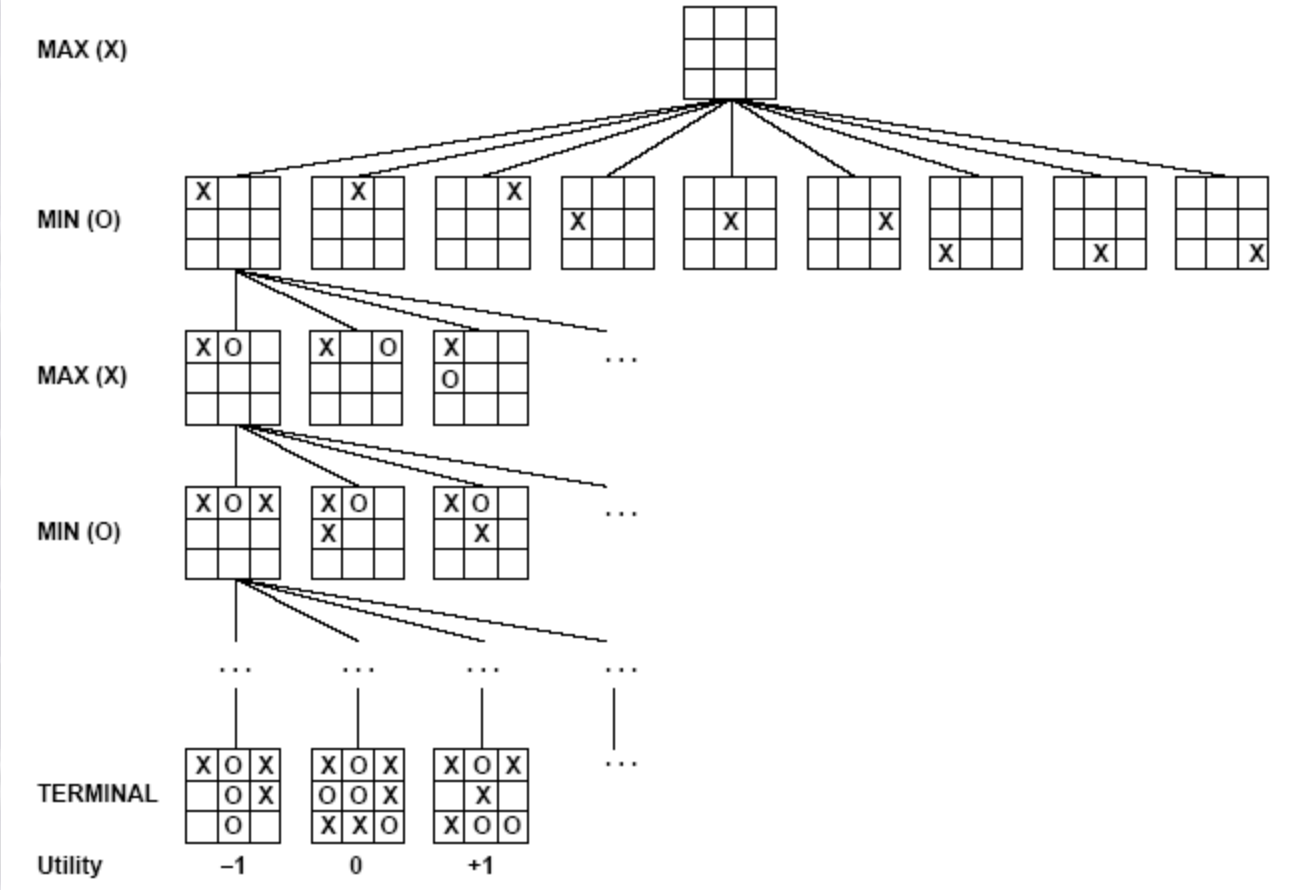
Optimal Decisions in Games

- Consider games with two players (MAX, MIN)
- Initial State
  - Board position and identifies the player to move
- Successor Function
  - Returns a list of (move, state) pairs; each a legal move and resulting state
- Terminal Test
  - Determines if the game is over (at terminal states)
- Utility Function
  - Objective function, payoff function, a numeric value for the terminal states (+1, -1) or (+192, -192)

Game Trees

- The root of the tree is the initial state
  - Next level is all of MAX's moves
  - Next level is all of MIN's moves
  - …
- Example: Tic-Tac-Toe
  - Root has 9 blank squares (MAX)
  - Level 1 has 8 blank squares (MIN)
  - Level 2 has 7 blank squares (MAX)
  - …
- Utility function:
  - win for X is +1
  - win for O is -1

Game Trees



Minimax Strategy

- Basic Idea:
  - Choose the move with the highest minimax value
    - **best achievable payoff against best play**
  - Choose moves that will lead to a win, even though min is trying to block

- Max's goal: get to 1
- Min's goal: get to -1

- Minimax value of a node (backed up value):
  - If N is terminal, use the utility value
  - If N is a Max move, take max of successors
  - If N is a Min move, take min of successors

Minimax Algorithm

- Minimax algorithm
  - Perfect play for deterministic, 2-player game
  - Max tries to maximize its score
  - Min tries to minimize Max's score (Min)
  - Goal: move to position of highest minimax value
    - → Identify best achievable payoff against best play

Minimax Algorithm

function MINIMAX-DECISION($state, game$) **returns** *an action*

action, *state* $\leftarrow$ the a, s **in** SUCCESSORS(s_{state})
such that MINIMAX-VALUE($s, game$) is maximized

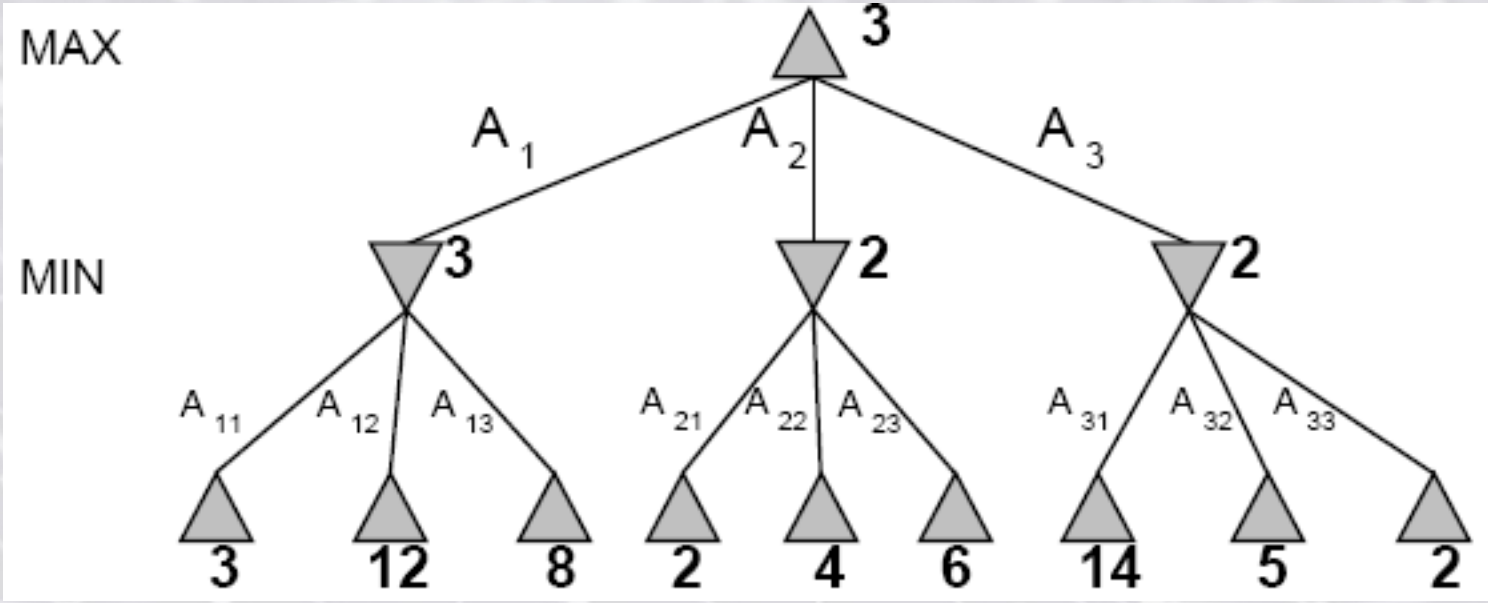
return *action*

---

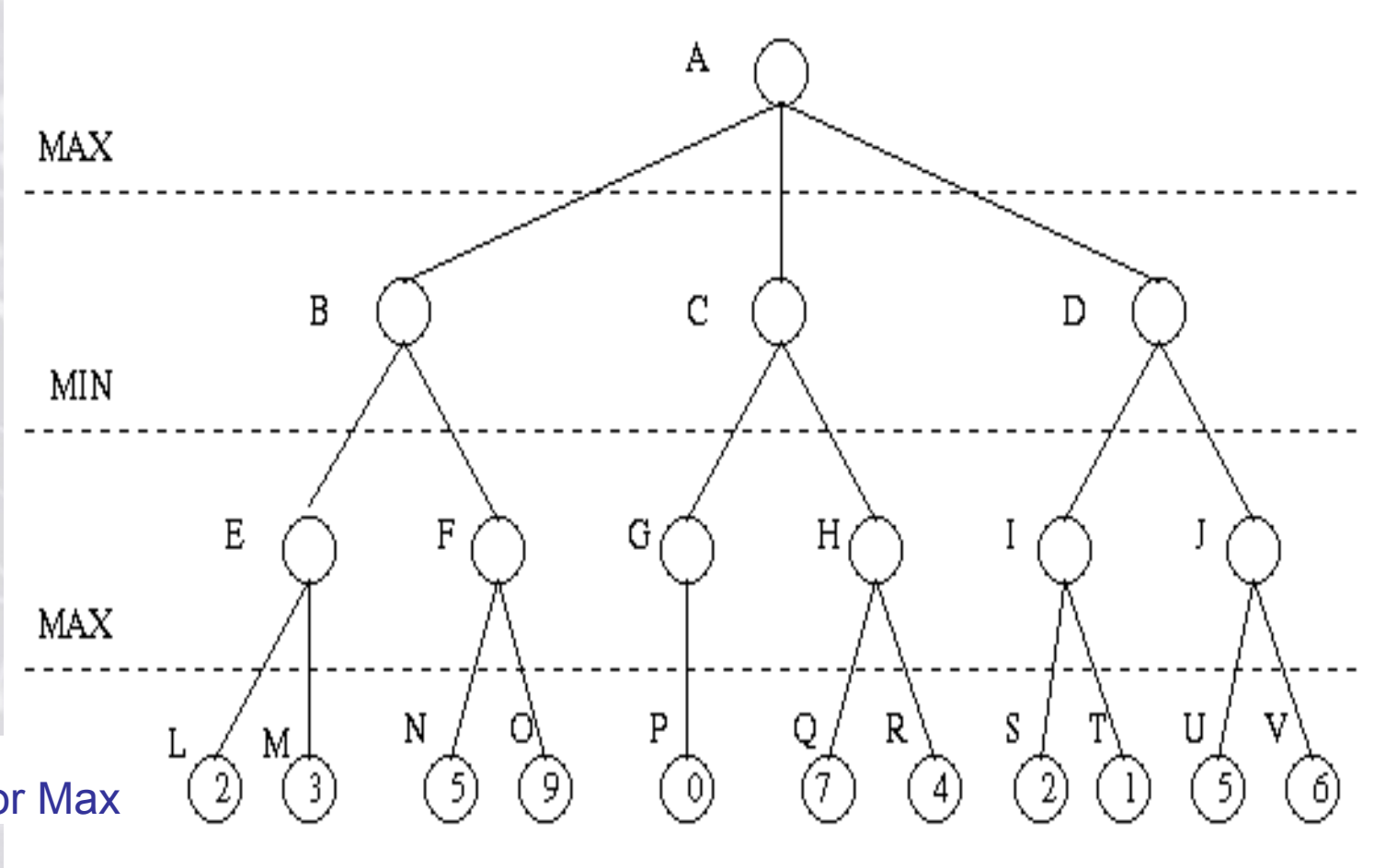
function MINIMAX-VALUE($s_{state}, game$) **returns** *a utility value*

if TERMINAL-TEST($state$) **then**
return UTILITY(s_{state})
else if MAX is to move in *state* **then**
return the highest MINIMAX-VALUE of SUCCESSORS($state$)
else
return the lowest MINIMAX-VALUE of SUCCESSORS(s_{state})

Minimax Strategy

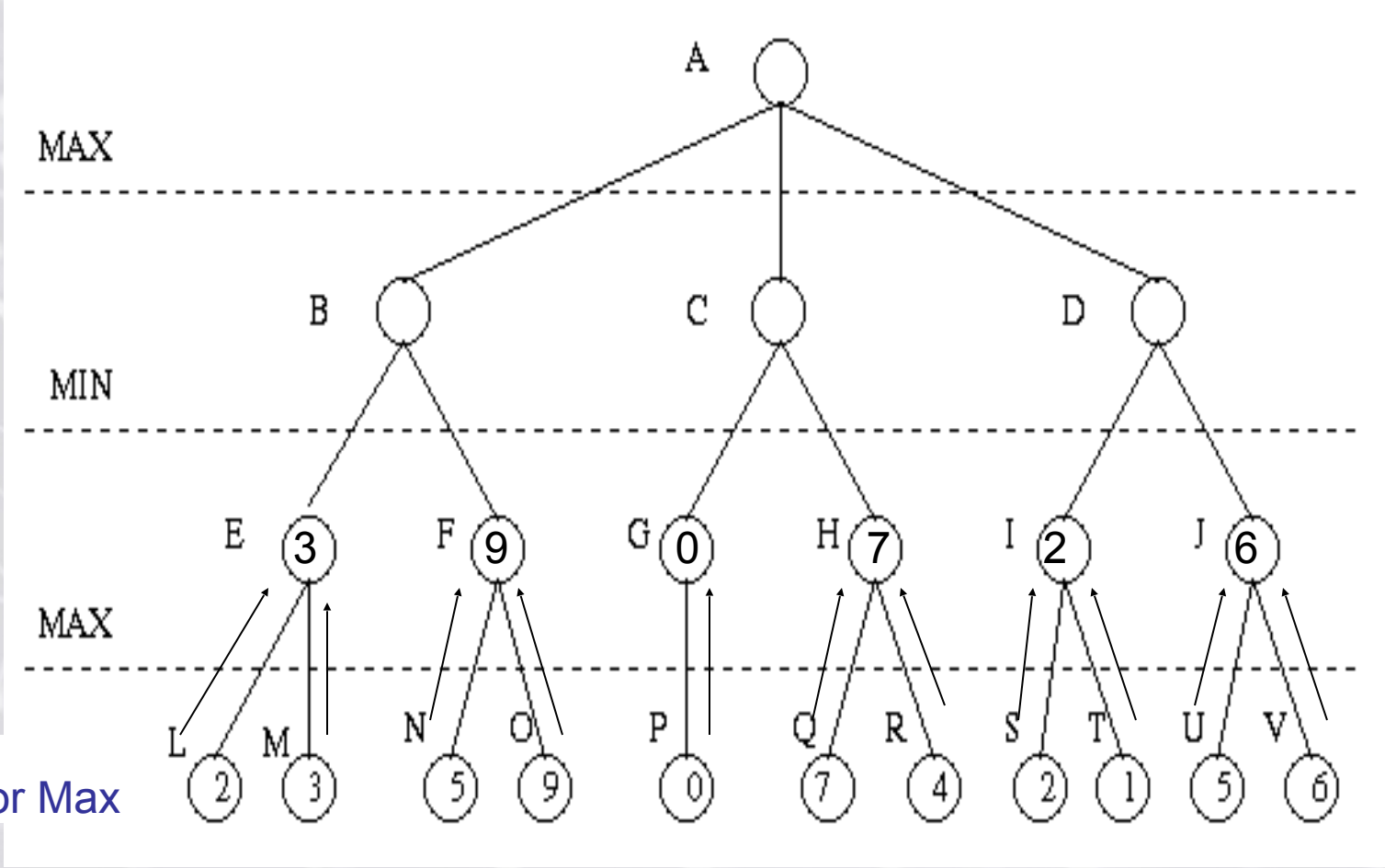


Minimax Algorithm



Payoff for Max

Minimax Algorithm (cont'd)



Payoff for Max

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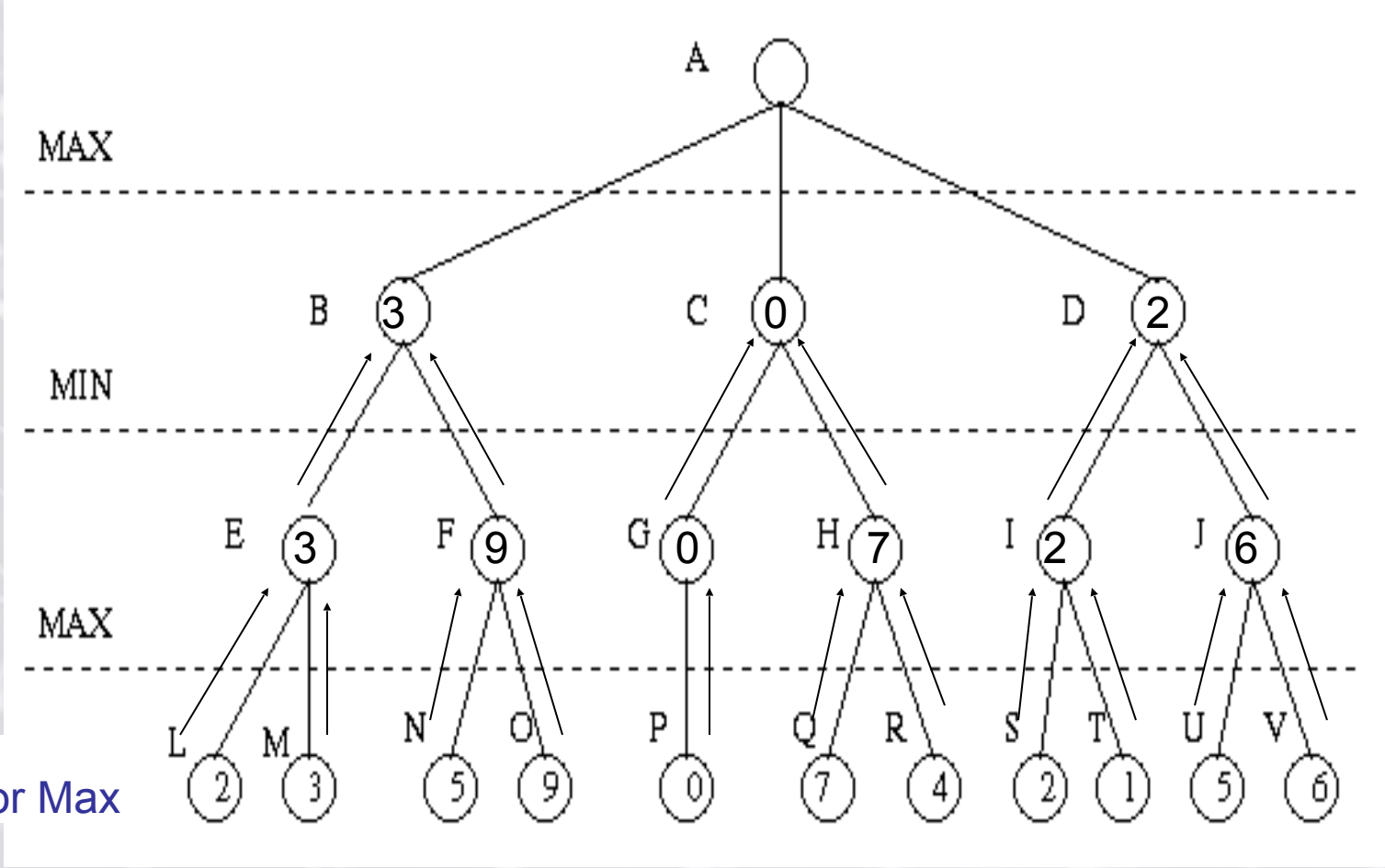

Payoff for Max

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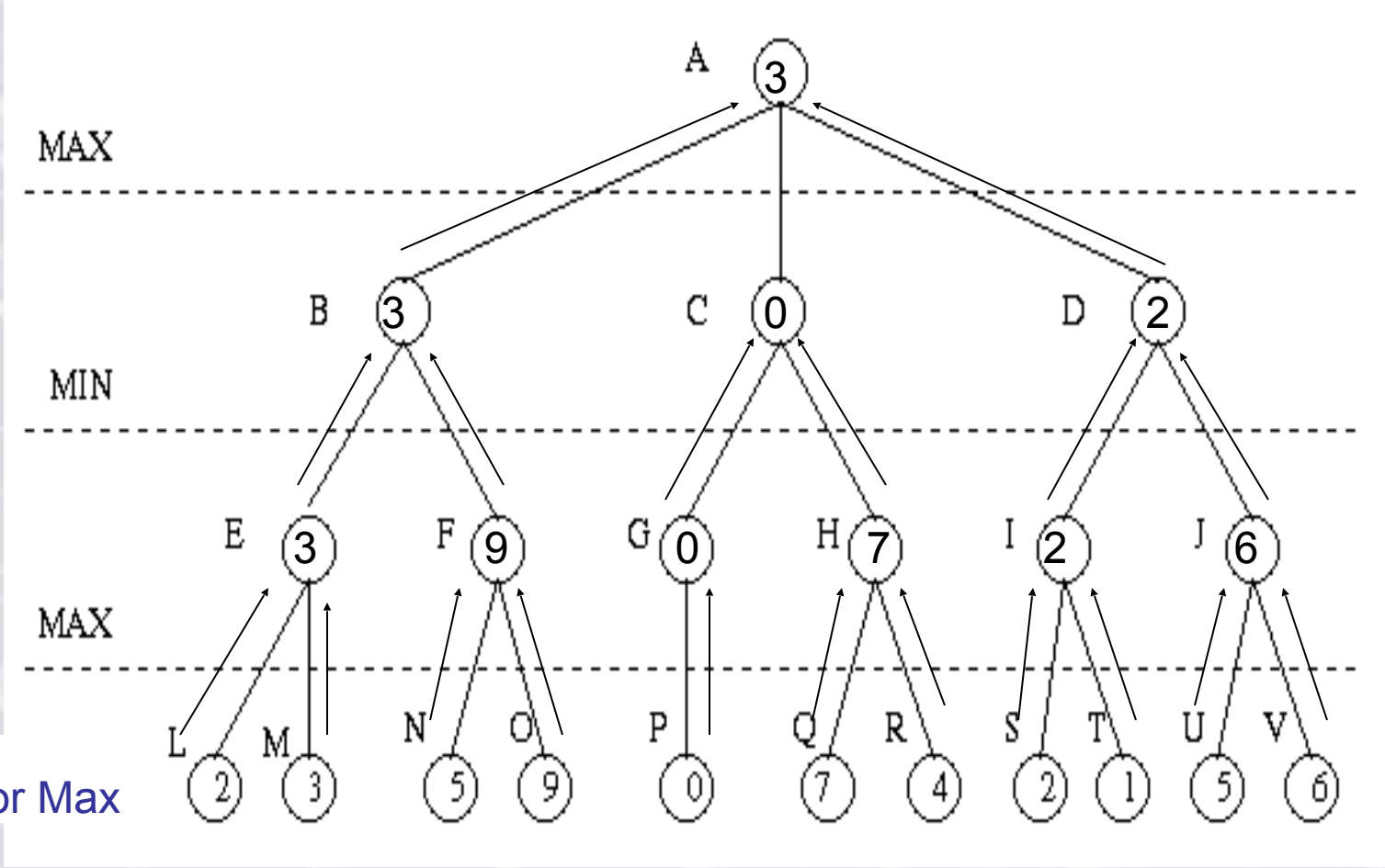


Payoff for Max

Minimax Algorithm

- Limitations
  - Not always feasible to traverse entire tree
  - Time limitations
- Key Improvement
  - Use evaluation function instead of utility
    - Evaluation function provides estimate of utility at given position

Properties of Minimax

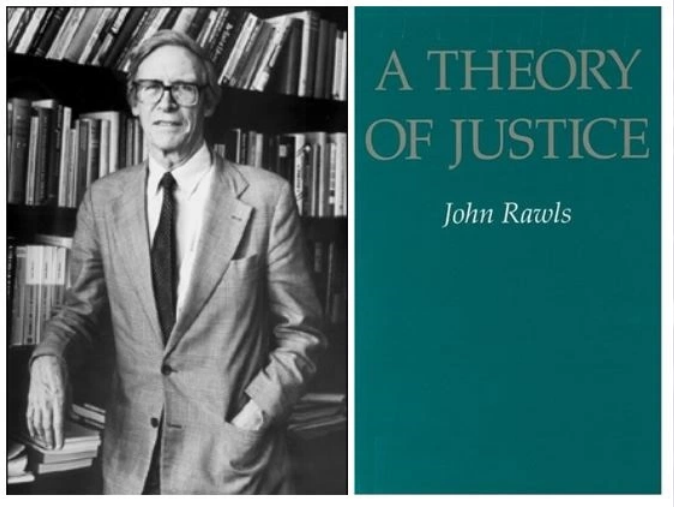
- Complete
  - Yes if the tree is finite (e.g. chess has specific rules for this)
- Optimal
  - Yes, against an optimal opponent, otherwise???
- Time
  - $O(b^m)$ (m is the maximum tree depth, b is the number of legal moves at each point)
- Space
  - O(bm) depth first exploration of the state space

Resource Limits

- Suppose there are 100 seconds, explore 10^4 nodes / second
- 10^6 nodes per move
- Standard approach
  - Cutoff test – depth limit
    - quiesence search – values that do not seem to change
  - Change the evaluation function

Minimax in philosophy

- In philosophy, the term "maximin" is often used in the context of moral/political philosopher John Rawl's ***A Theory of Justice*** (1971), where he refers to in the context of the **difference principle**. Rawls defined this principle as the rule which states that social and economic inequalities should be arranged so that "they are to be of the greatest benefit to the least-advantaged members of society".



Evaluation Functions

- In 1950, Shannon proposed that programs should cut off the search earlier to apply a heuristic **evaluation function** to states in the search (recall the infeasibility of exhaustive search for chess).
- This technique effectively turns non-terminal nodes into terminal leaves.
- The idea is to replace the utility function by a heuristic evaluation function (EVAL), which estimates the position's utility, and replace the terminal test by a **cutoff test** that decides when to apply EVAL.
- An evaluation function returns an estimate of the expected utility of the game from a given position, just as heuristic functions return an estimate of the distance to the goal.

Evaluation Functions

- The performance of a game-playing program depends strongly on the quality of its evaluation function – but how to design?

(1)The evaluation function should order the terminal states in the same way as the true utility function: states that are wins must evaluate better than draws, etc.

(2)The computation must not take too long!

(3)For nonterminal states, the evaluation function should be strongly correlated with the actual "chances of winning."

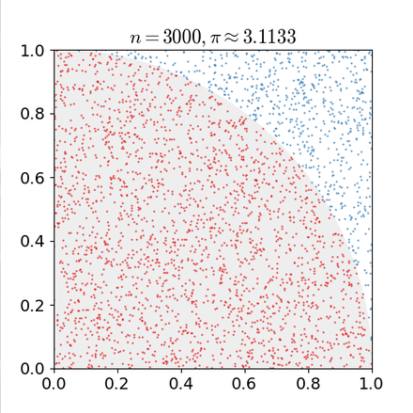
Evaluation Functions

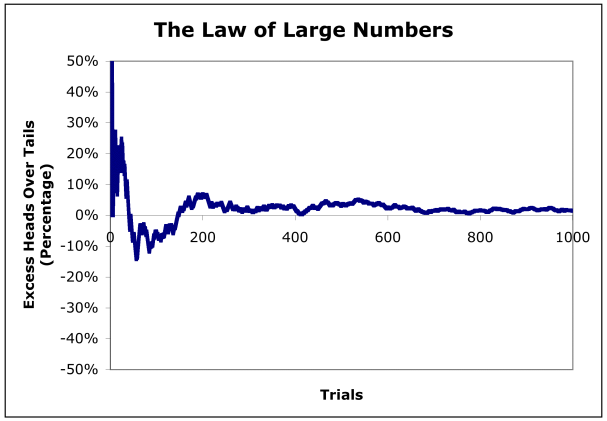
- Note that if the search must be cut off at nonterminal states, then the algorithm will necessarily be *uncertain* about the final outcomes of those states.
- Usually, an evaluation function calculates various features of a state (e.g. number of pawns, etc.).
- The features define various categories or equivalence classes of states: the states in each category have the same values for all features.
- For example: suppose experience suggests that 72% of states encountered in a two-pawn vs. one-pawn category lead to a win (utility: +1); 20% to a loss (0) and 8% to a draw (1/2).
- Expected value: (0.72 x 1)+(0.20 x 0) + (0.08 x ½) = 0.76.

Evaluation Functions

- Most evaluation functions compute separate numerical contributions from each feature and then combine them to find the total value.
  - Typical evaluation function is a linear sum of features
  - Eval(s) = $w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$
    - $w_1 = 9$
    - $f_1(s)$ = number of white queens) – number of black queens
    - etc.

Monte Carlo methods to estimate utility/evaluation functions

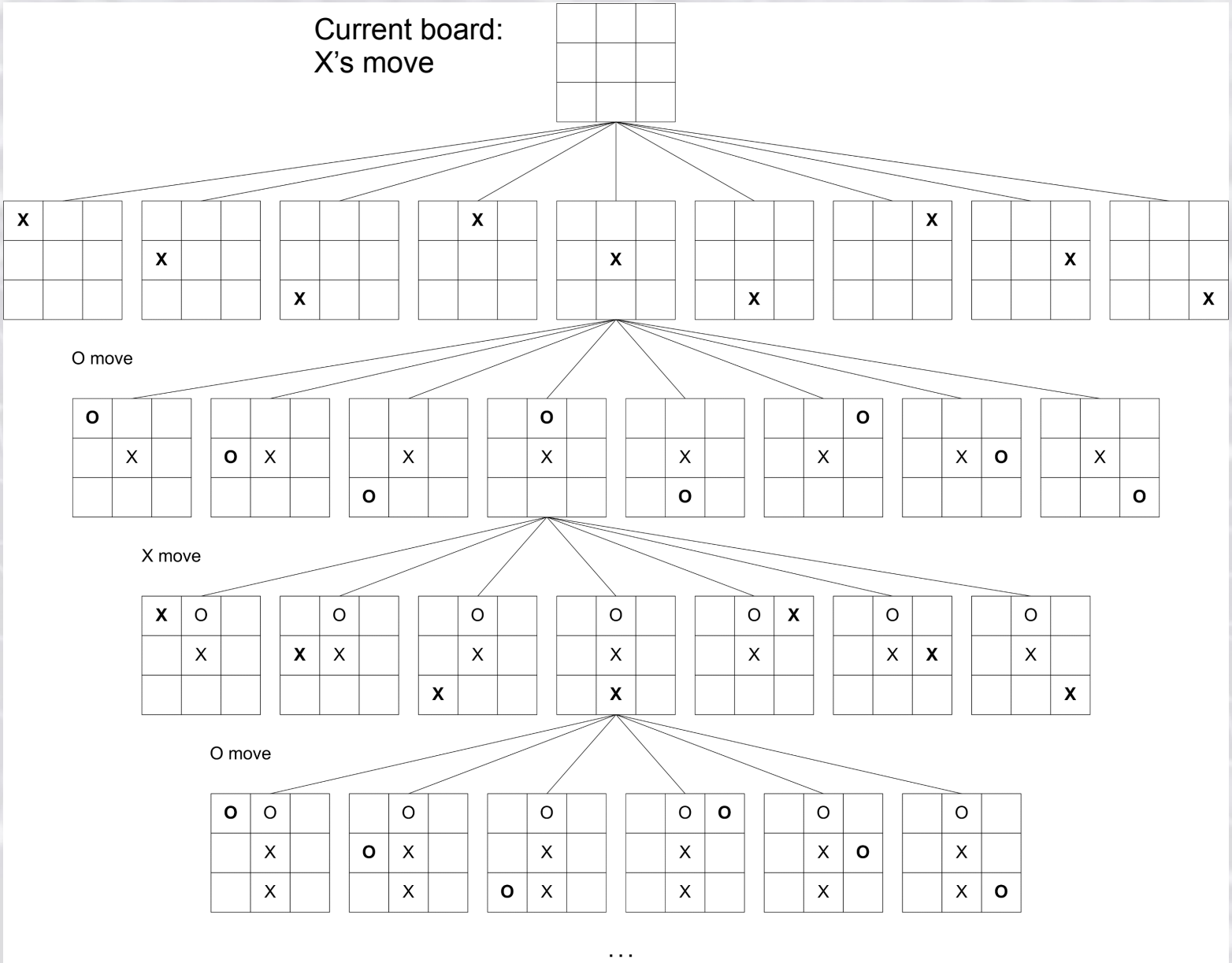




- Basic Idea: take many experiential samples (i.e. game plays); approximate utility based on experience.

Game tree

From M. T. Jones, *Artificial Intelligence: A Systems Approach*



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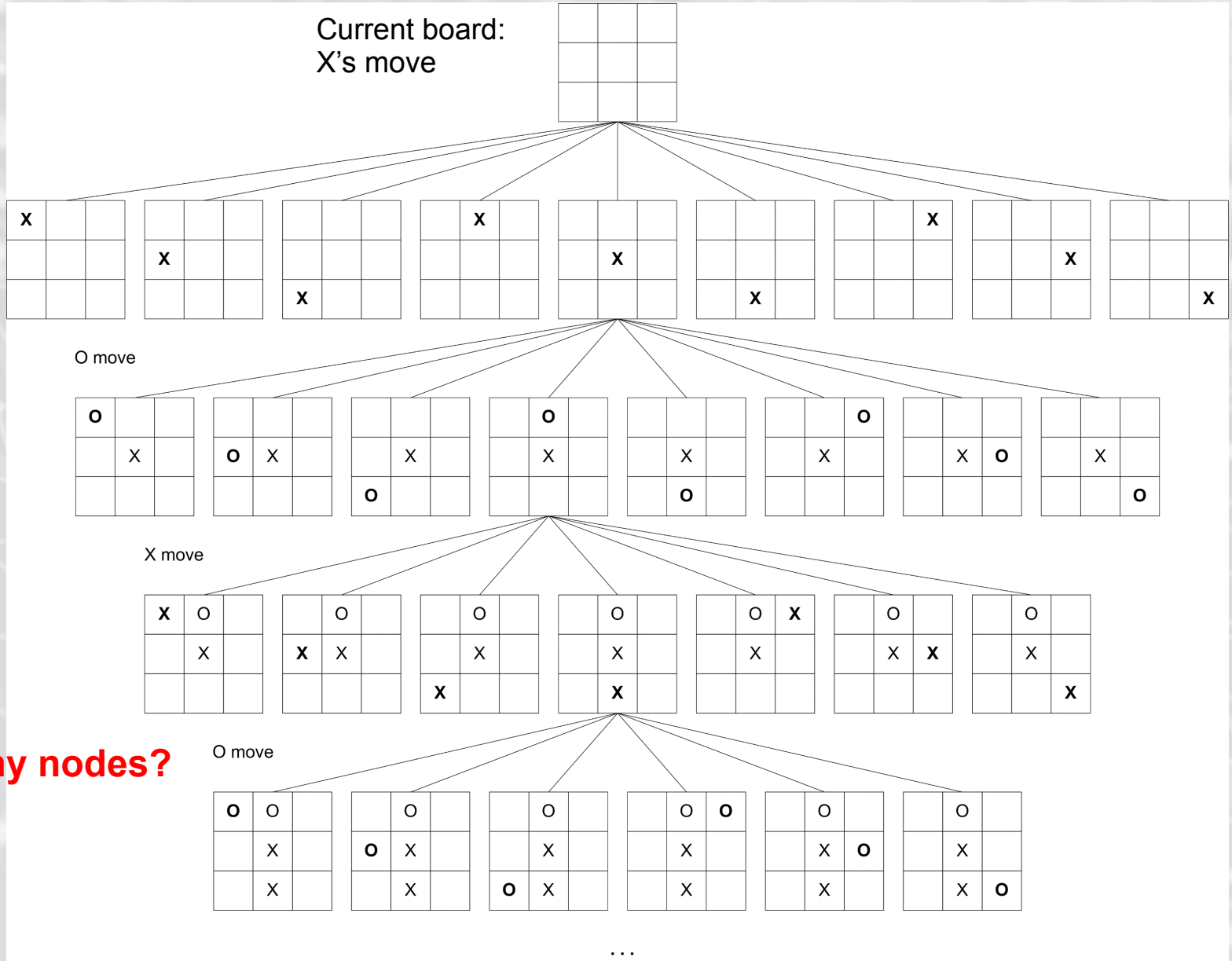
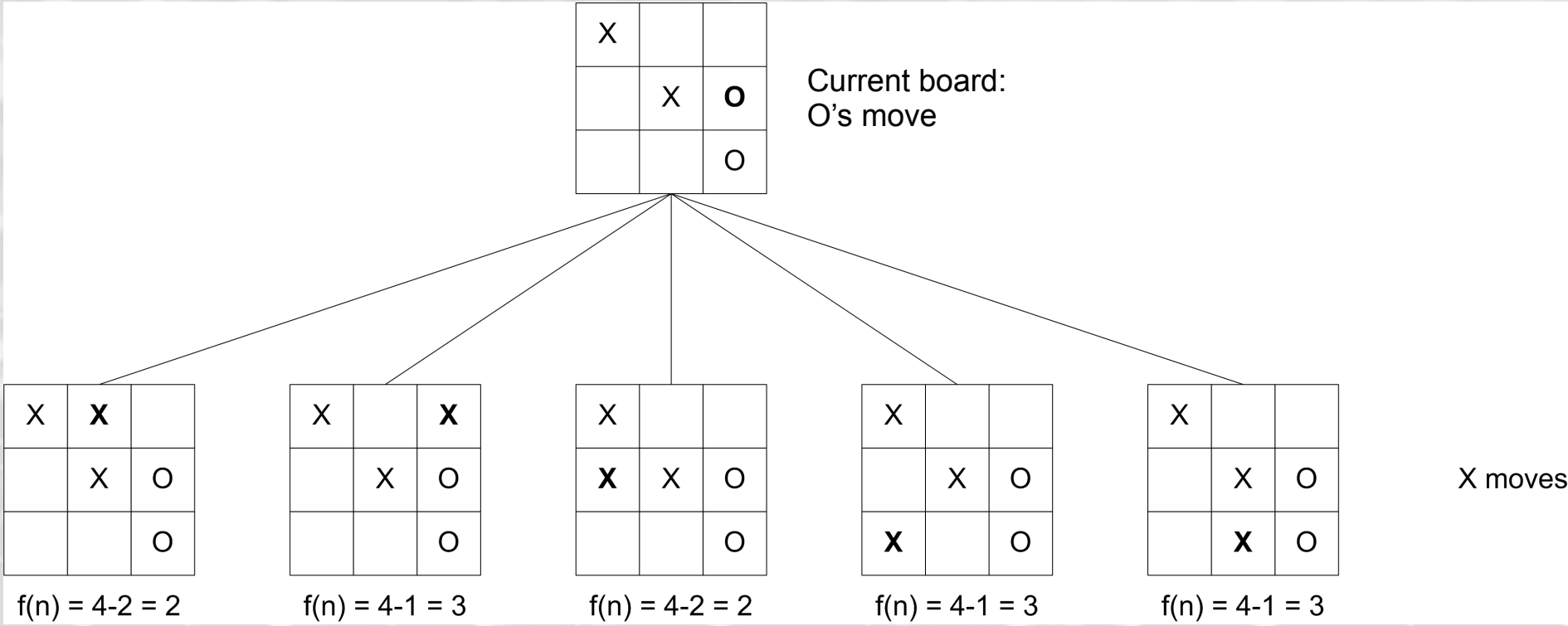


How many nodes?

Evaluation Function

From M. T. Jones, *Artificial Intelligence: A Systems Approach*

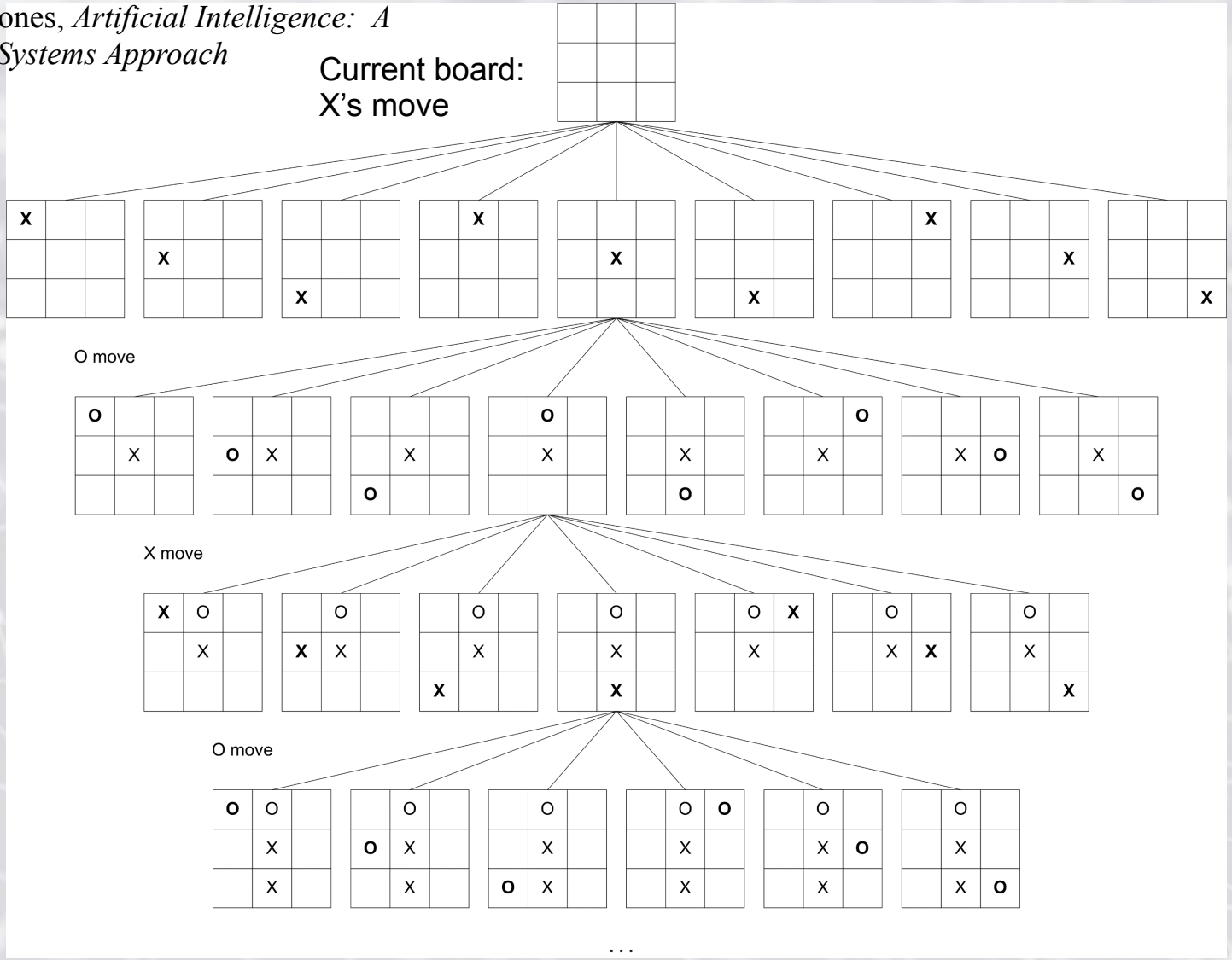


Evaluation function $f(n)$ measures "goodness" of board configuration n. Assumed to be better estimate as search is deepened (i.e., at lower levels of game tree).

Evaluation function here: "Number of possible wins (rows, columns, diagonals) not blocked by opponent, minus number of possible wins for opponent not blocked by current player."

Minimax search

From M. T. Jones, *Artificial Intelligence: A Systems Approach*



Minimax search: Expand the game tree by *m* ply (levels in game tree) in a limited depth-first search. Then apply evaluation function at lowest level, and propagate results back up the tree.

Minimax search

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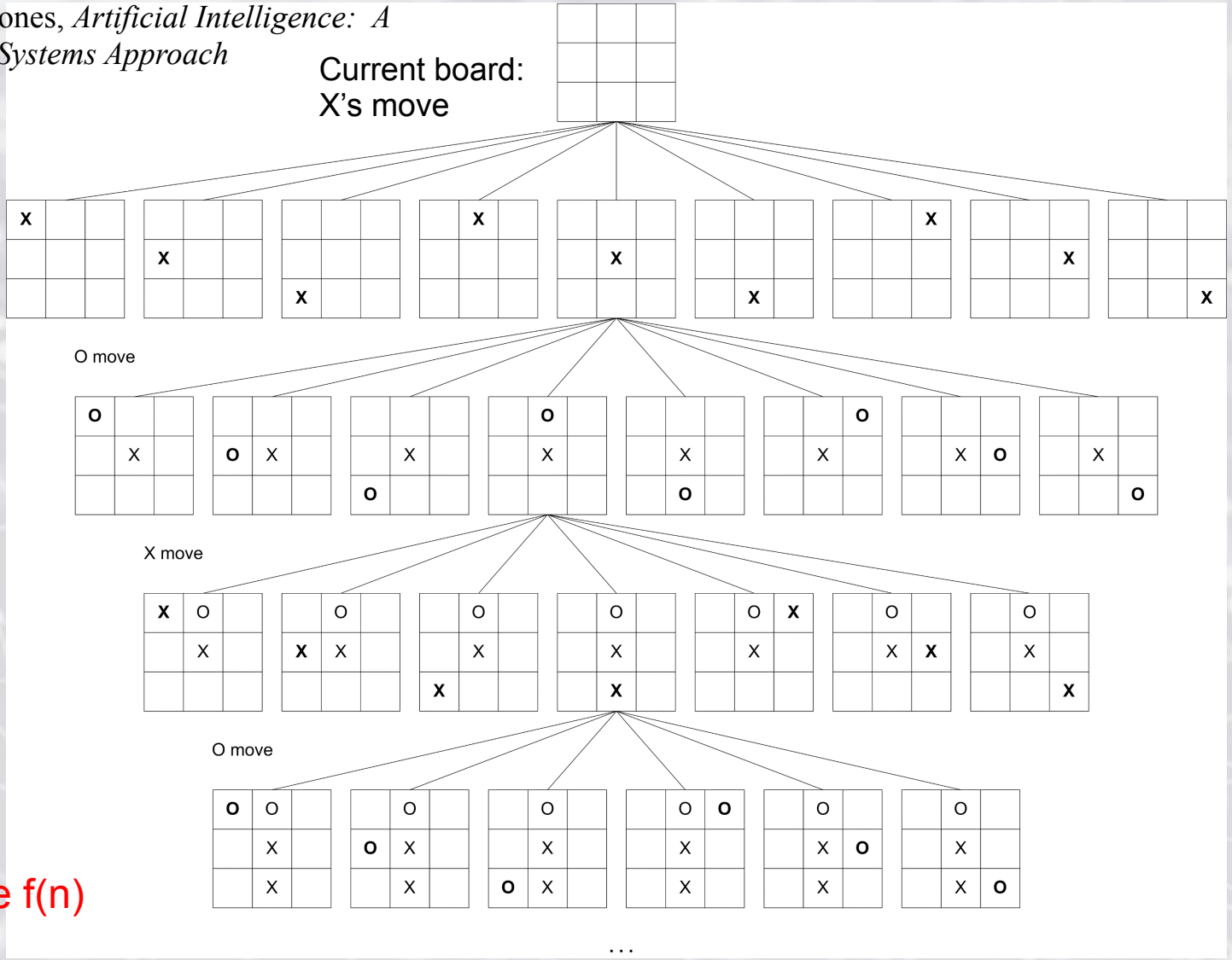

Calculate f(n)

Minimax search

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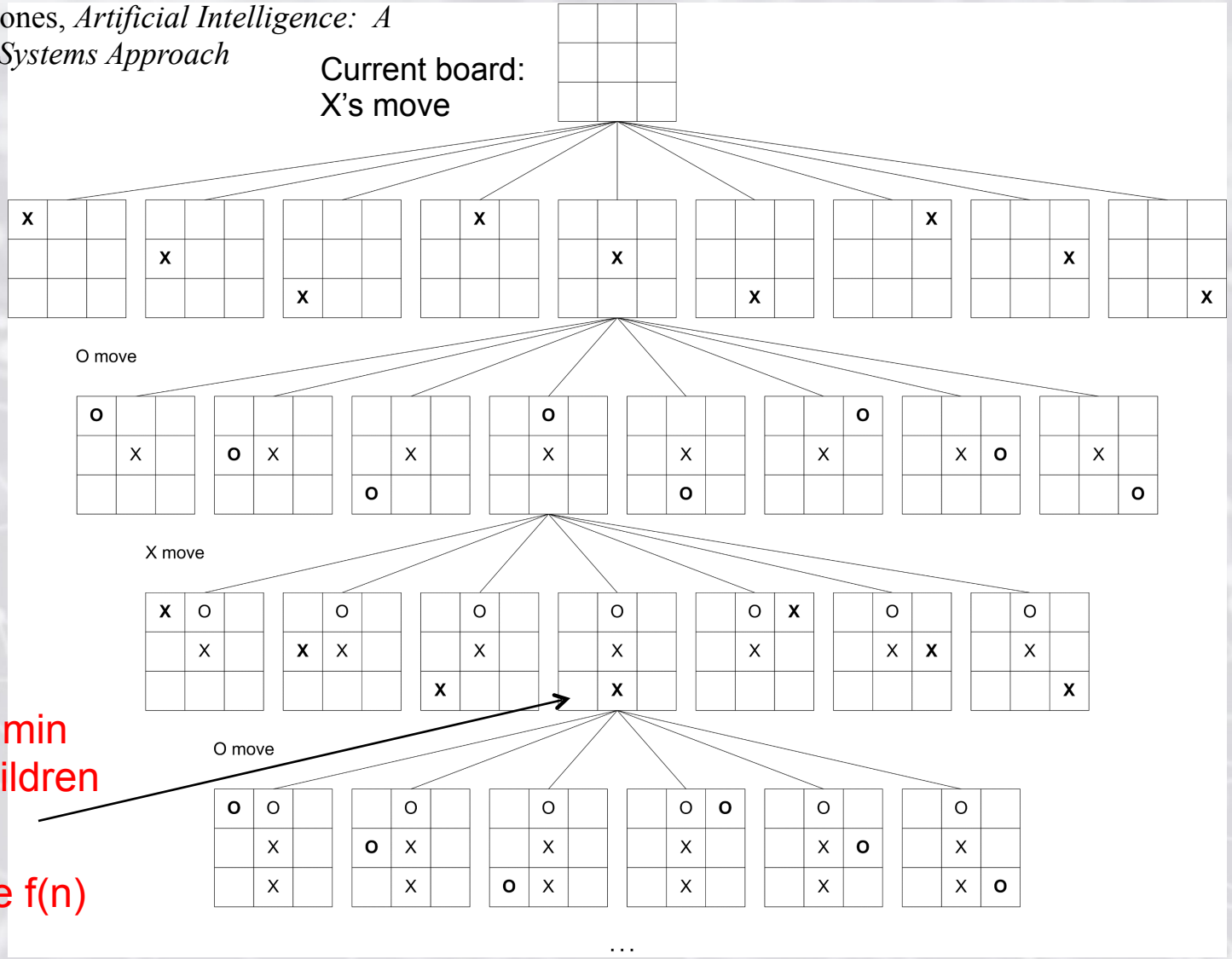

Propagate min value of children to parent

Calculate f(n)

Minimax search

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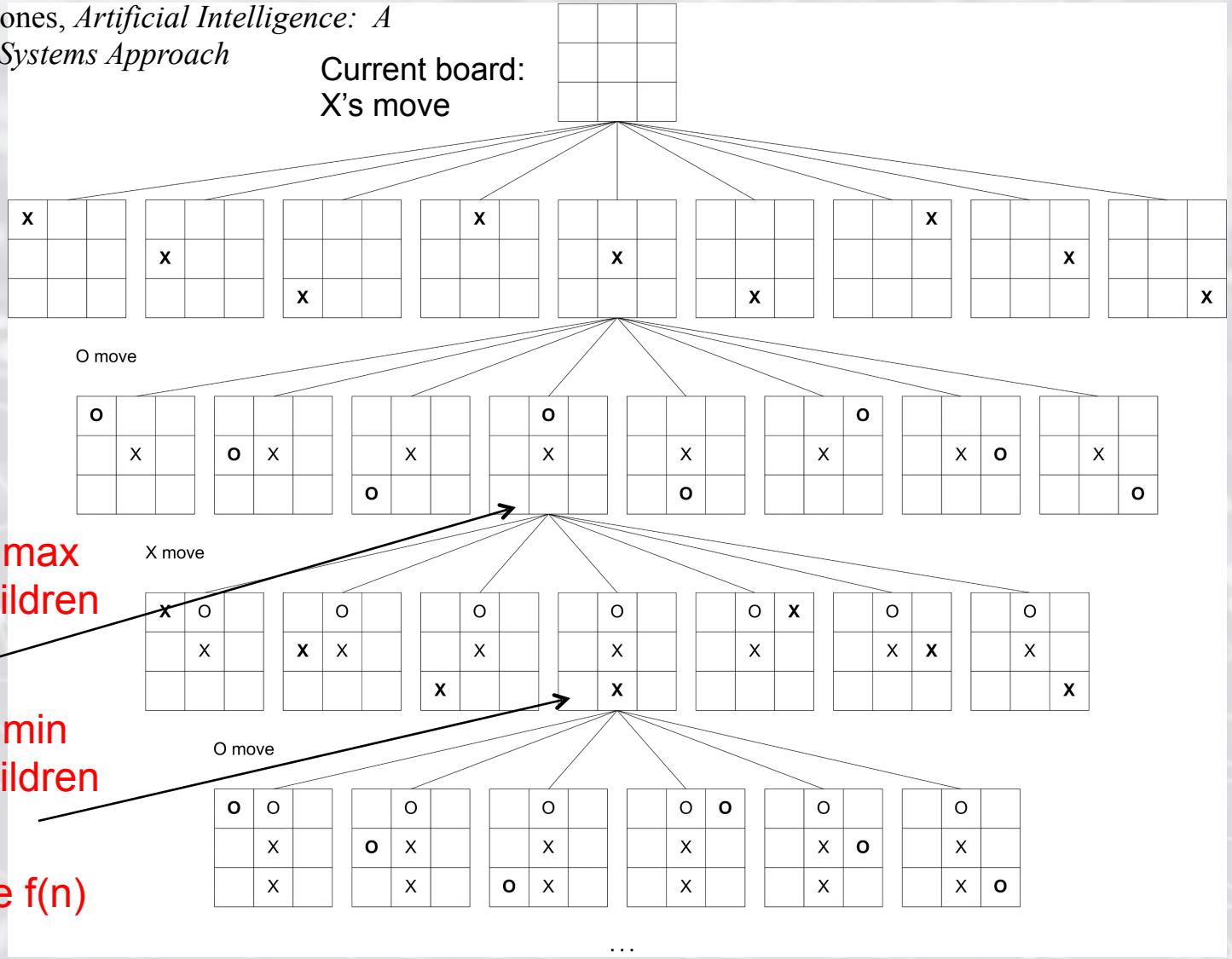

Propagate max value of children to parent

Propagate min value of children to parent

Calculate f(n)

Minimax search

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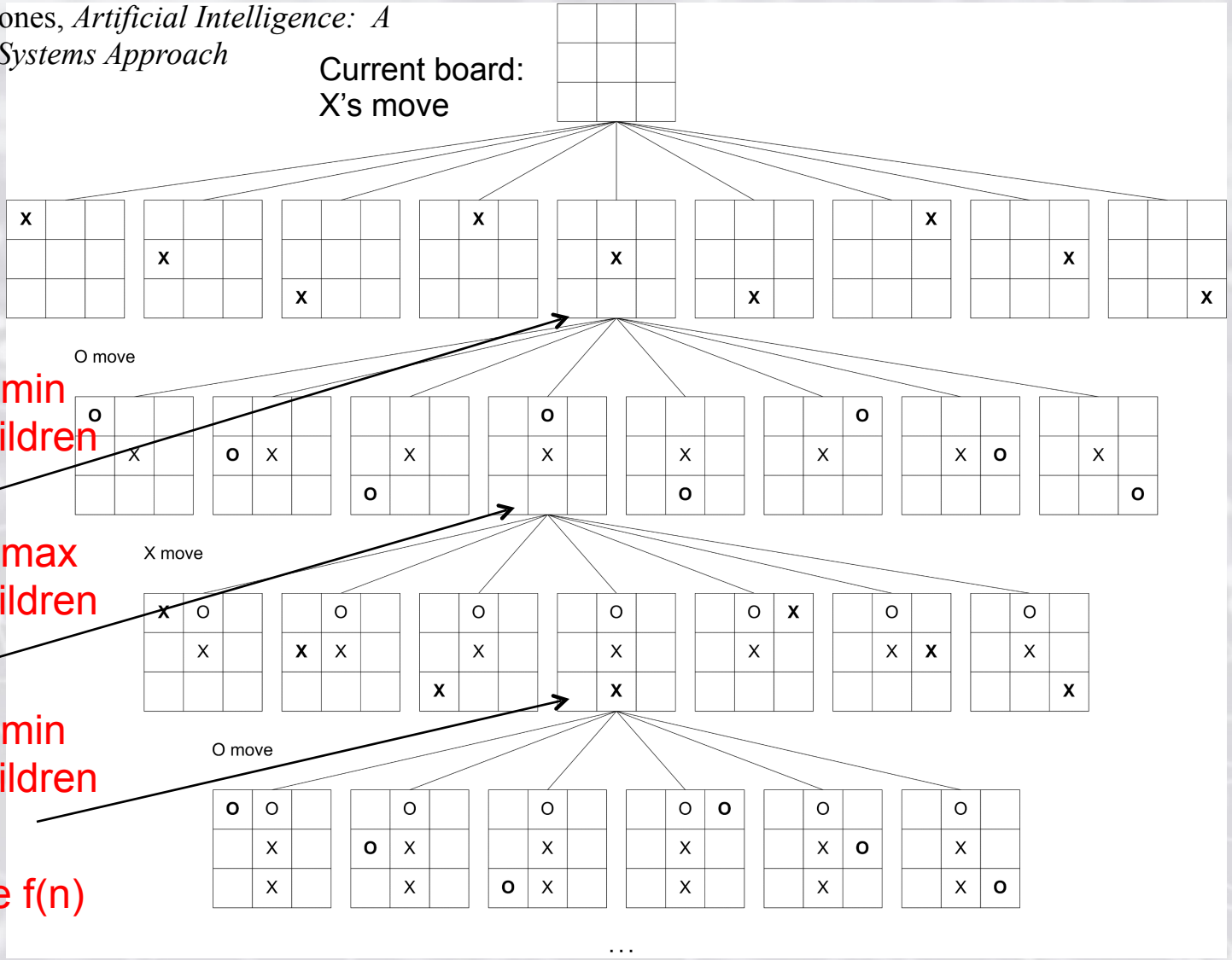

Propagate min value of children to parent

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Calculate f(n)

Minimax search

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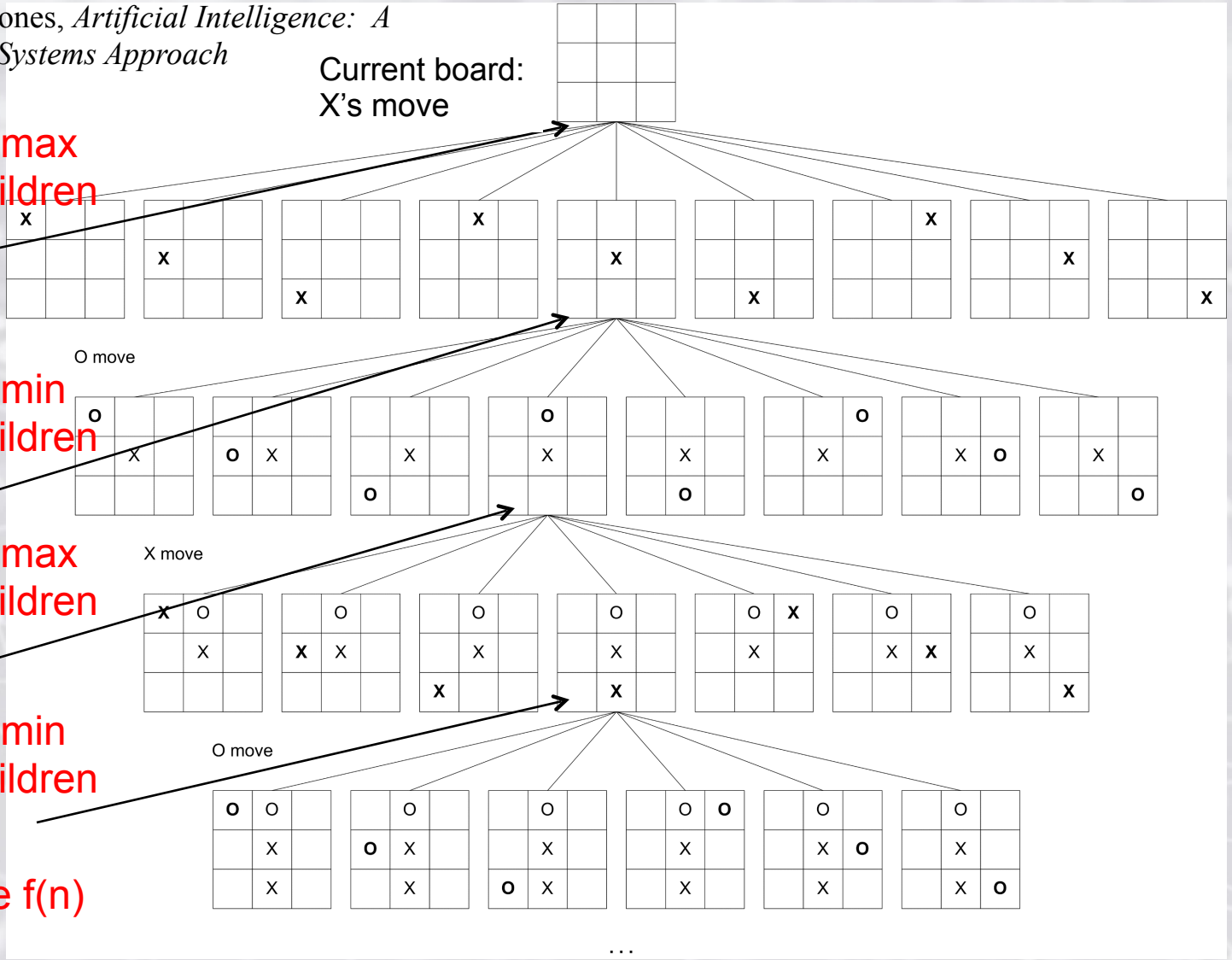

Minimax algorithm: Example

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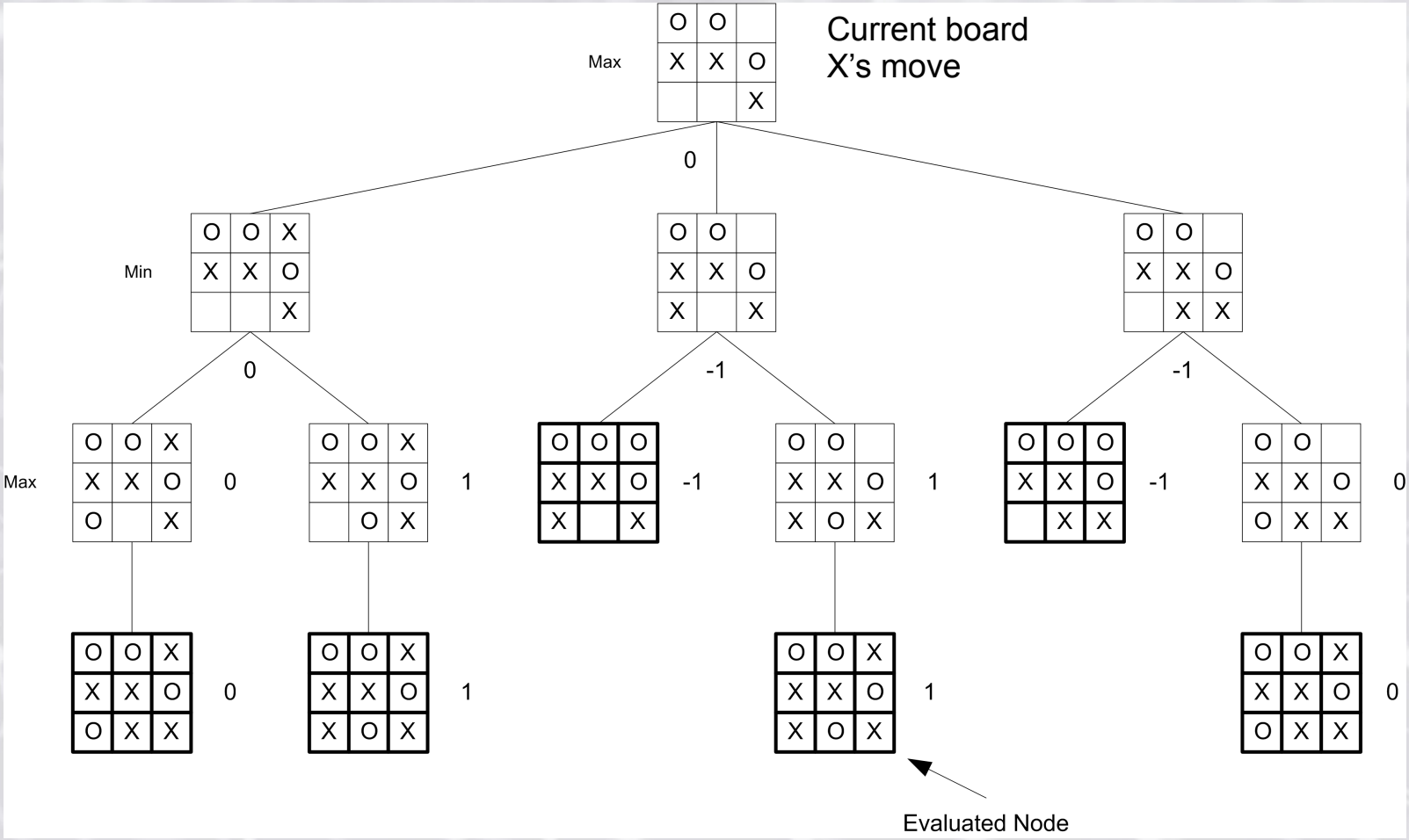
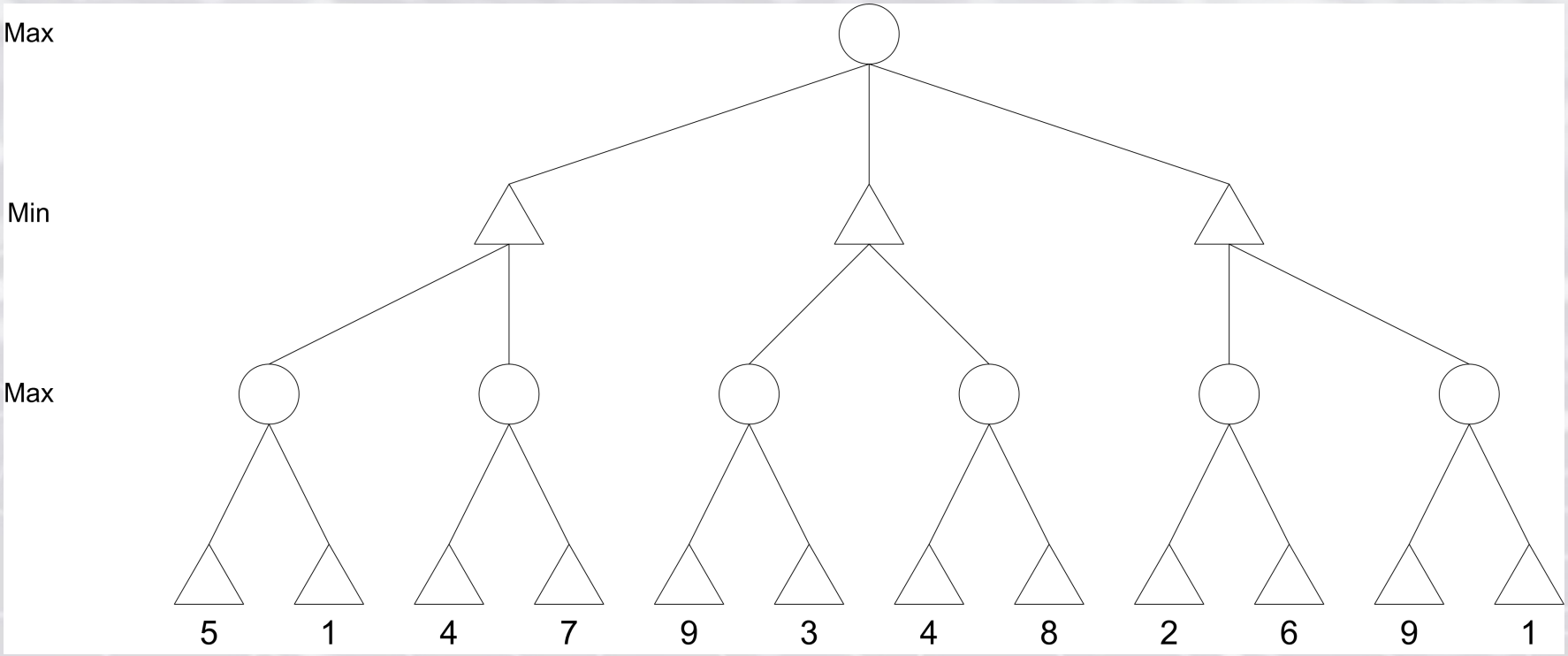


http://inst.eecs.berkeley.edu/~cs61b/fa14/ta-materials/apps/ab_tree_practice/

Exercise



What is value at the root?

Alpha-Beta Pruning

- The problem with minimax search is that the number of games states it has to examine is exponential in the depth of the tree.
- We can't eliminate the exponent – however, we can effectively cut it in half!
- The key idea is that it is possible to compute the correct minimax decision without looking at every node in the game true.
- We "prune" away branches in the game tree that cannot possibly influence the final decision.

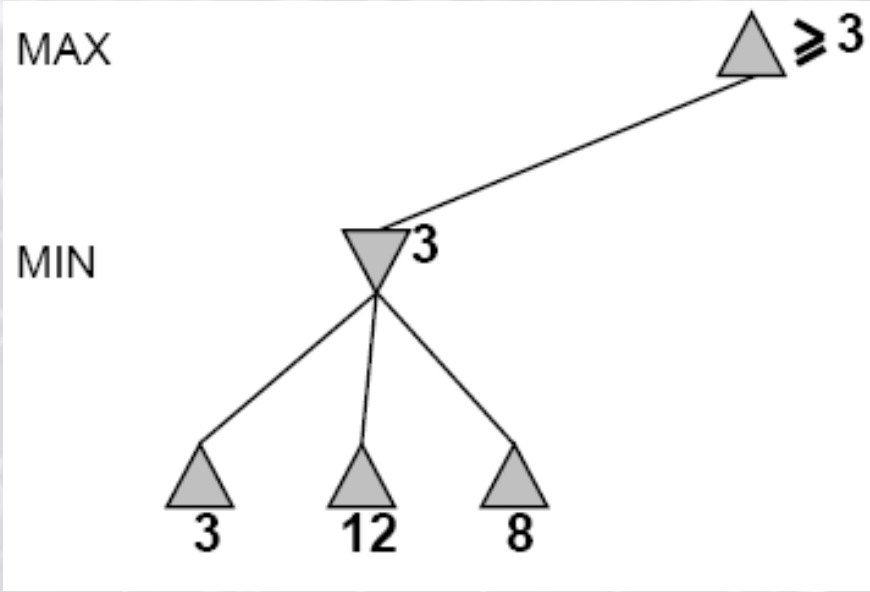
Alpha-Beta Pruning

- Recognize when a position can never be chosen in minimax *no matter what its children are*
  - Max (3, Min(2,x,y) …) is always ≥ 3
  - Min (2, Max(3,x,y) …) is always ≤ 2
  - We know this without knowing x and y!

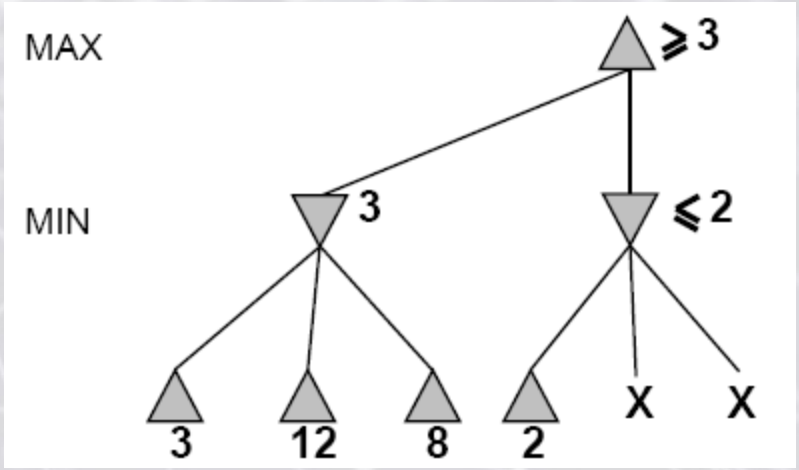
Alpha-Beta Pruning

- Alpha = the value of the best choice we've found so far for MAX (highest)
- Beta = the value of the best choice we've found so far for MIN (lowest)
- When maximizing, cut off values lower than Alpha
- When minimizing, cut off values greater than Beta

Alpha-Beta Pruning Example



Alpha-Beta Pruning Example



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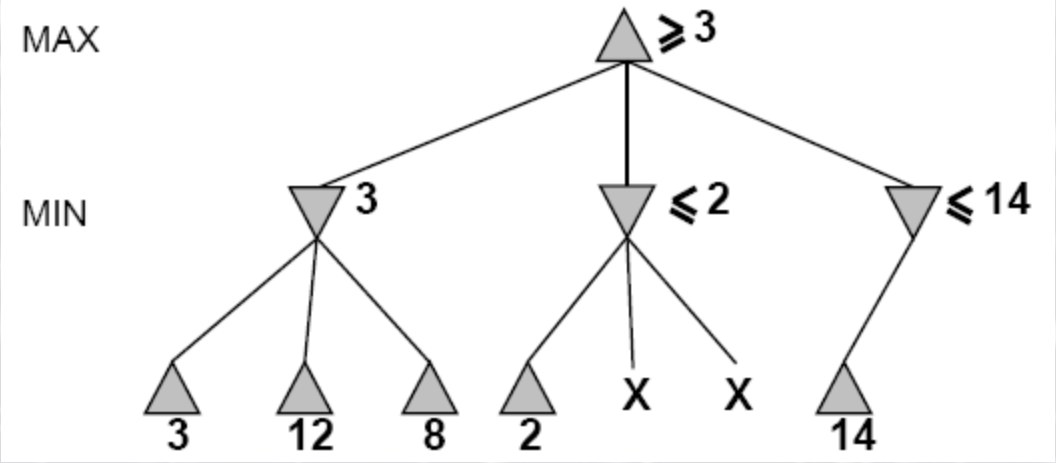
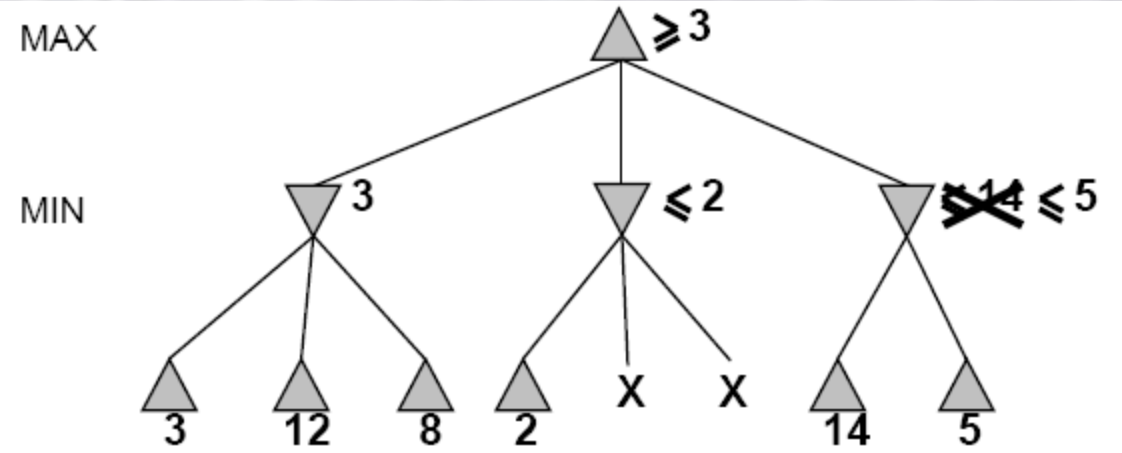
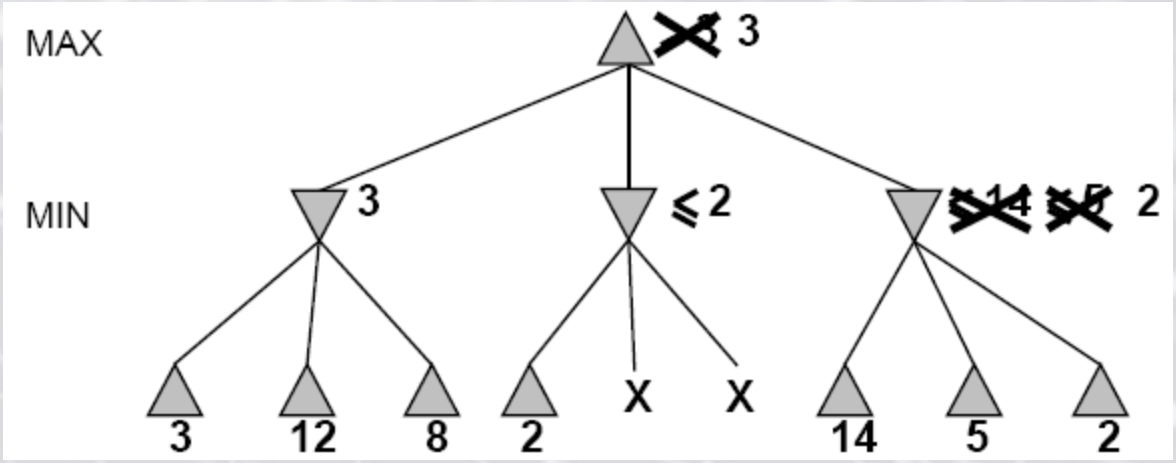


Alpha-Beta Pruning Example



Alpha-Beta Pruning Example

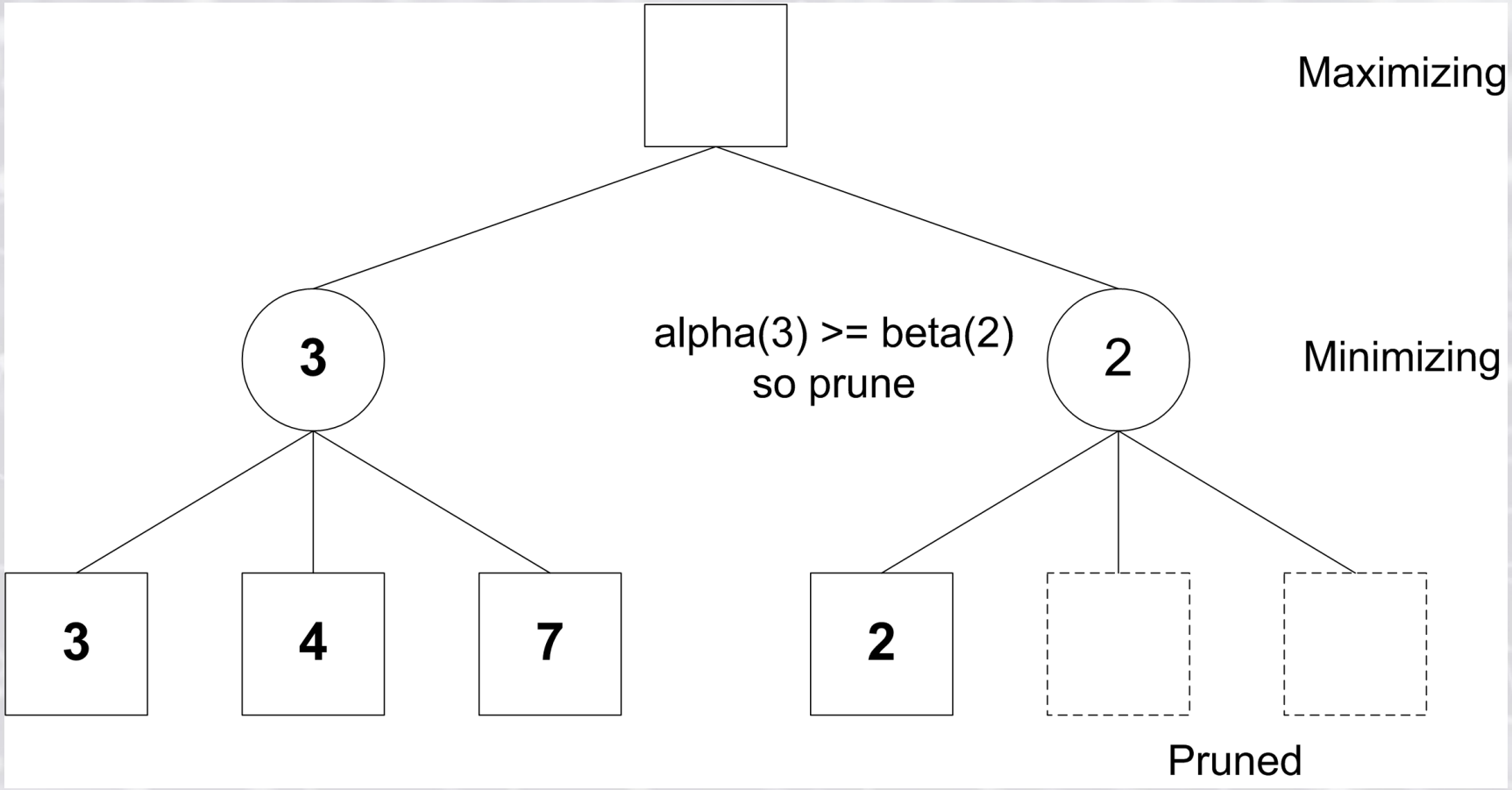


Algorithm: Minimax with Alpha-Beta Pruning

- **function** alphabeta(node, depth, α, β, Player)
- **if** depth = 0 **or** node is a terminal node
- **return** the heuristic value of node
- **if** Player = MaxPlayer
- **for each** child of node
- α := max(α, alphabeta(child, depth-1, α, β, not(Player)))
- **if** β ≤ α
- **break** *; Prune*
- **return** α
- **else**
- **for each** child of node
- β := min(β, alphabeta(child, depth-1, α, β, not(Player)))
- **if** β ≤ α **break** *; Prune*
- **return** β
- **end**
- ; Initial call
- alphabeta(origin, depth, -infinity, +infinity, MaxPlayer)

Alpha-Beta Pruning Example

From M. T. Jones, *Artificial Intelligence: A Systems Approach*



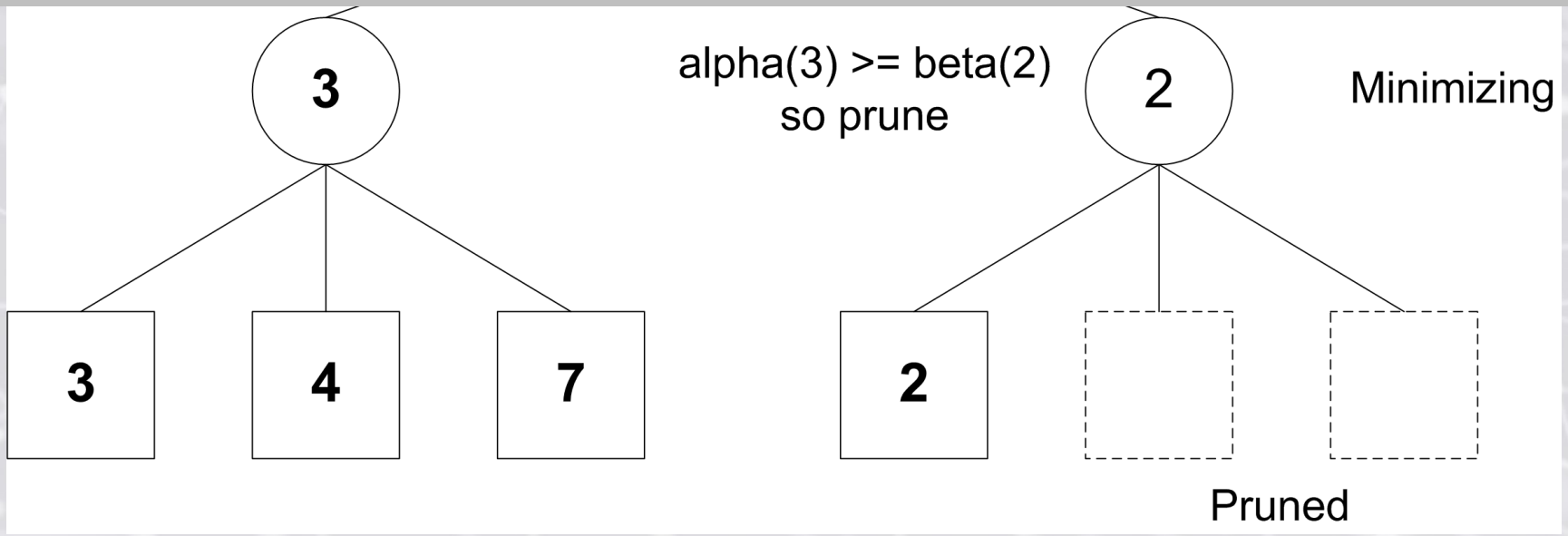
Alpha-Beta Pruning Example

From M. T. Jones, *Artificial Intelligence: A Systems Approach*

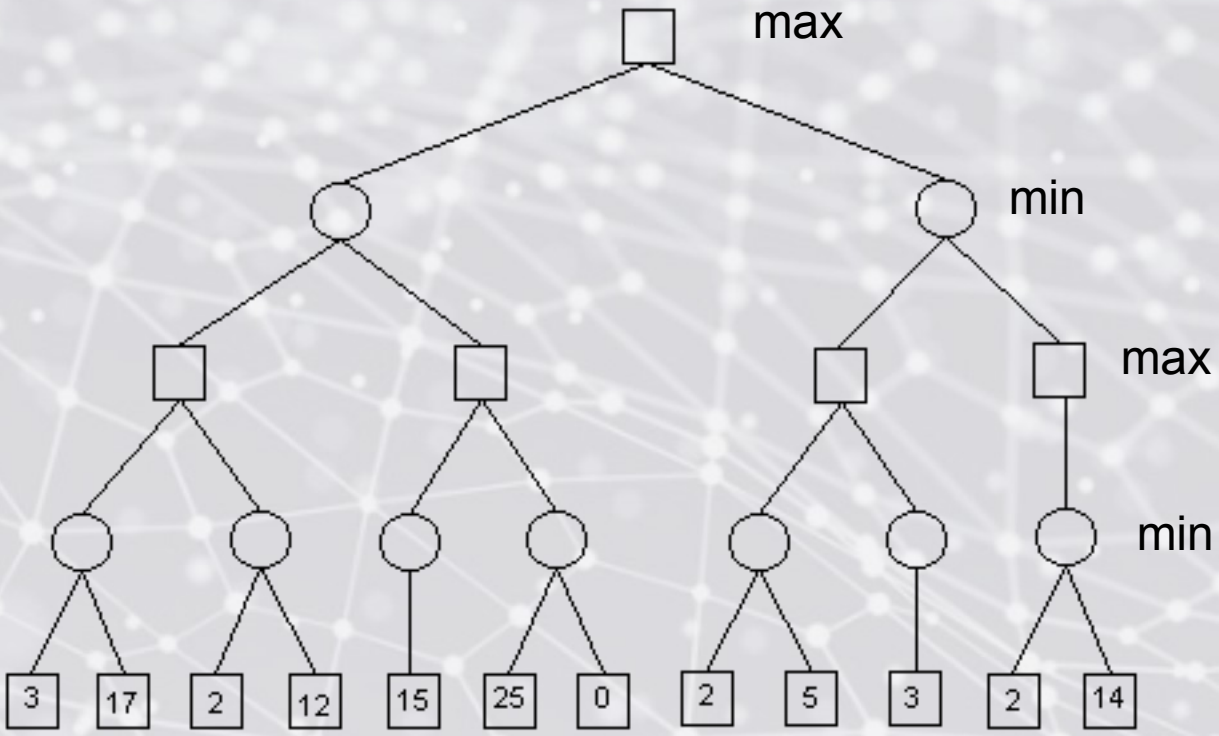
alpha = value of the best possible move you can make, that you have computed so far

beta = value of the best possible move your opponent can make, that you have computed so far

If at any time, **alpha >= beta**, then your opponent's best move can force a worse position than your best move so far, and so there is no need to further evaluate this move



Alpha-beta pruning exercise



(a) What is value at the root, using minimax alone?

(b) What nodes could have been pruned from the search using alpha-beta pruning? Show values of alpha and beta

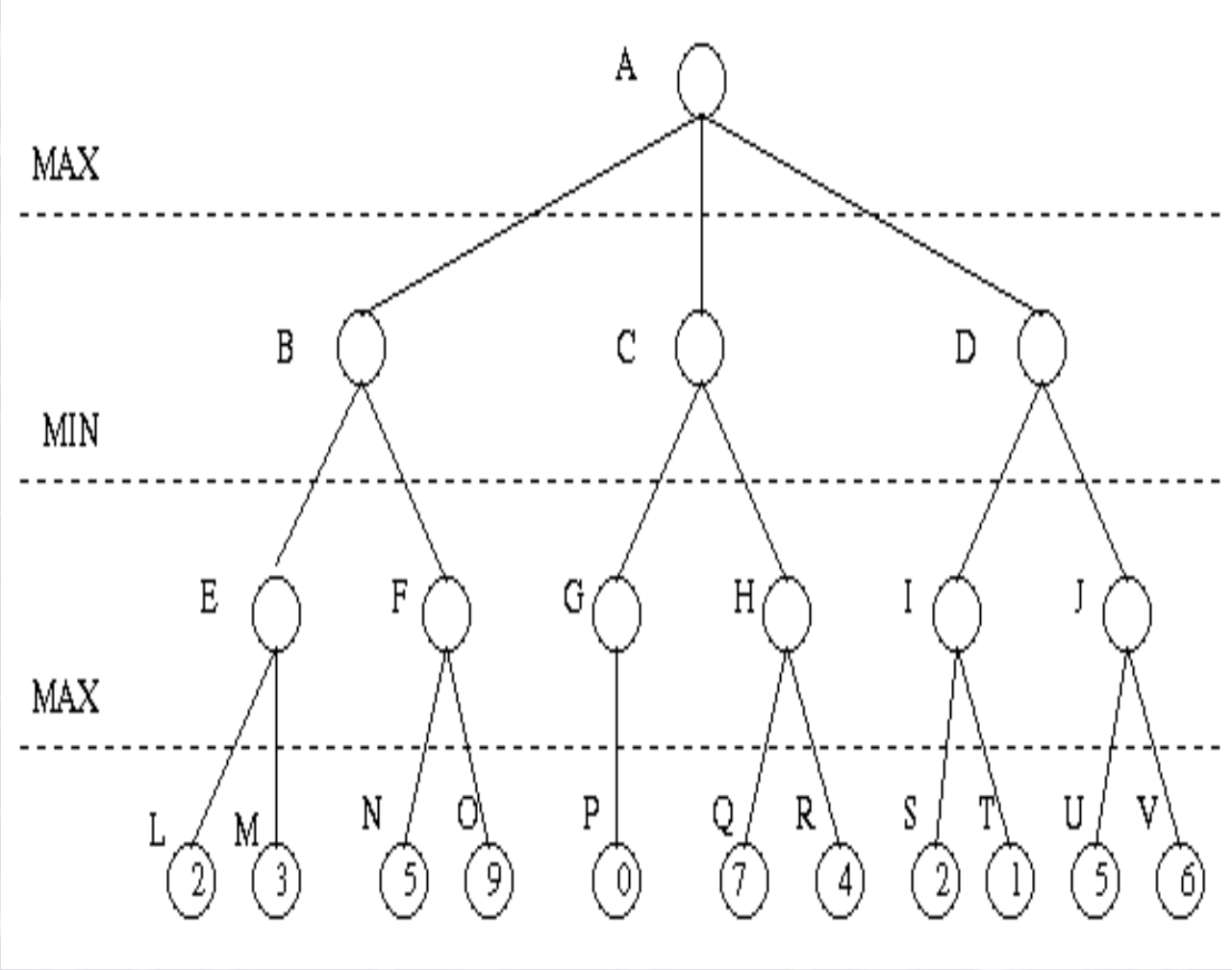
Alpha-Beta Pruning (αβ prune)

- Rules of Thumb
  - α is the best (highest) found so far along the path for Max
  - β is the best (lowest) found so far along the path for Min
  - Search below a MIN node may be alpha-pruned if the its **β** ≤ α of some MAX ancestor
  - Search below a MAX node may be **beta-pruned** if the its α≥ **β** of some MIN ancestor.

Alpha-Beta Pruning Example

1. Search below a MIN node may be alpha-pruned if the beta value is <= to the alpha value of some MAX ancestor.

2. Search below a MAX node may be beta-pruned if the alpha value is >= to the beta value of some MIN ancestor.



Alpha-Beta Pruning Example

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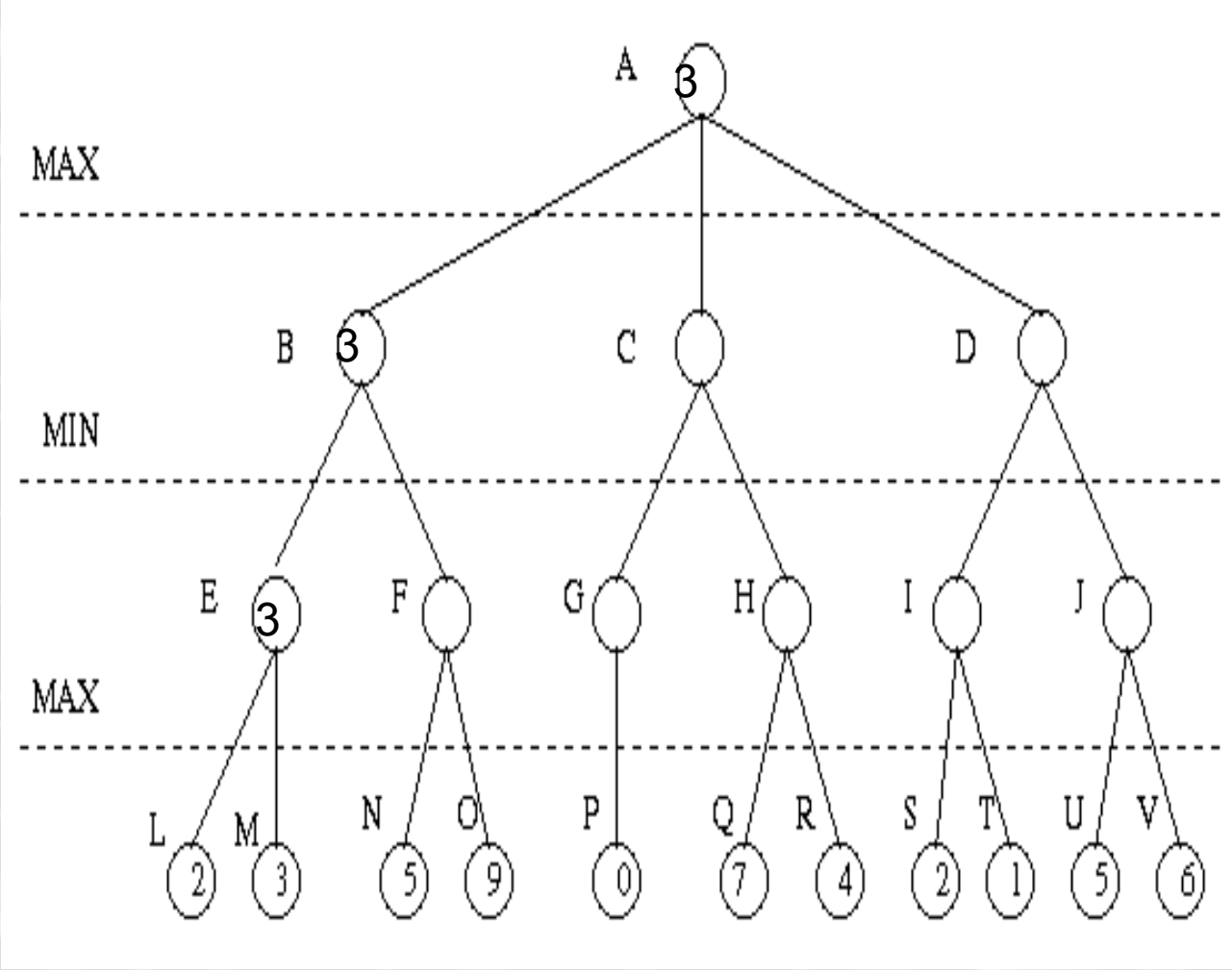

Alpha-Beta Pruning Example

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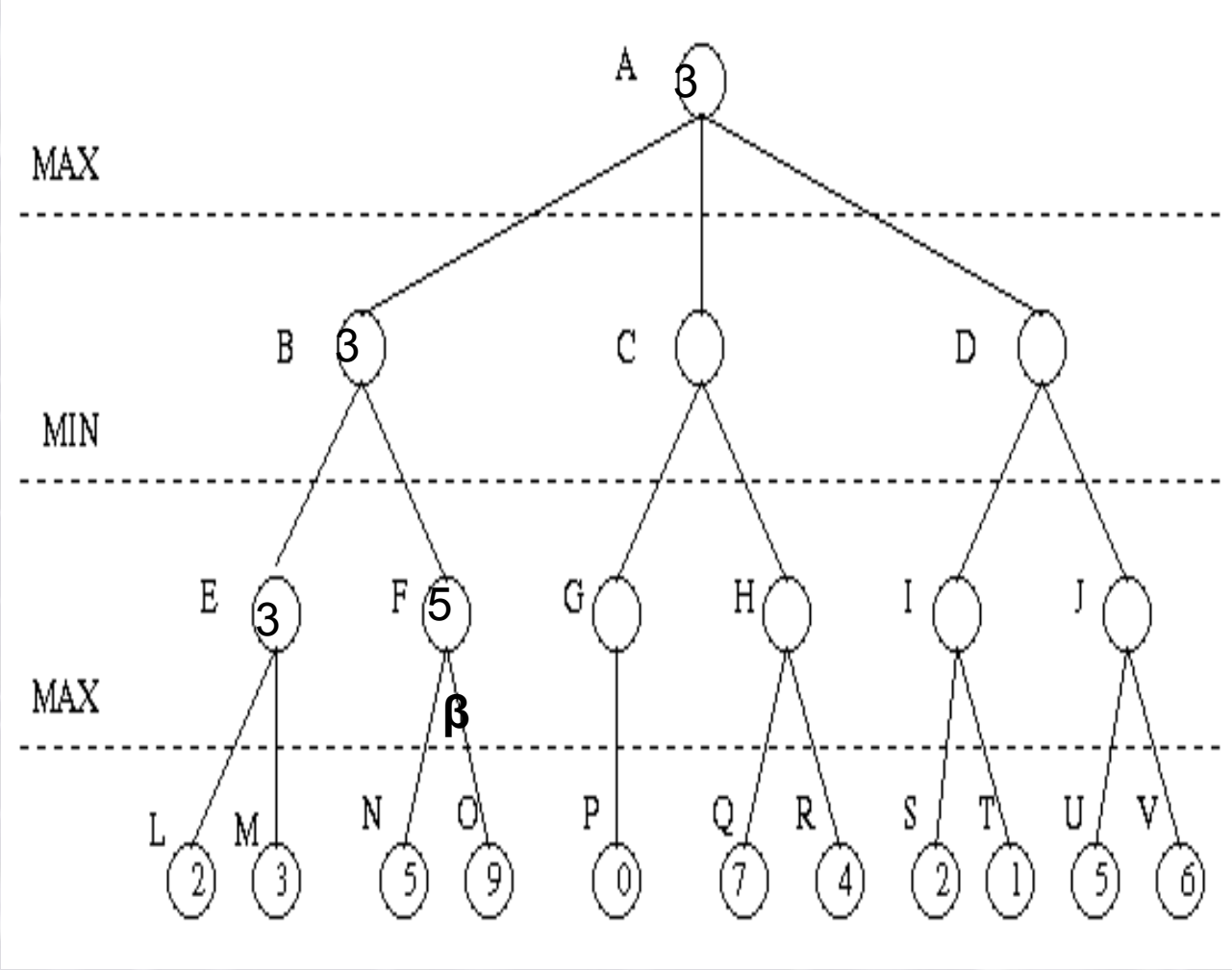

Alpha-Beta Pruning Example

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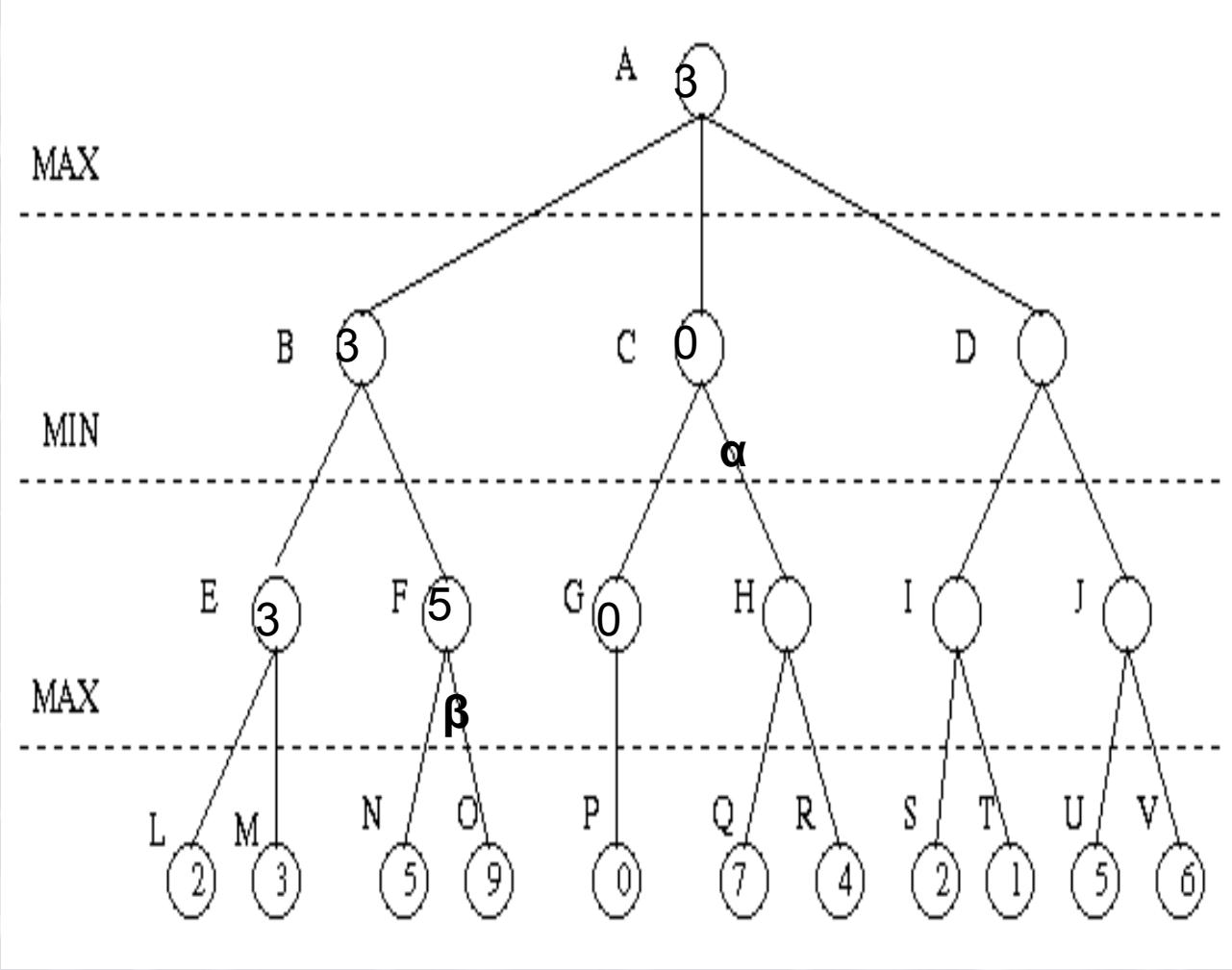

Alpha-Beta Pruning Example

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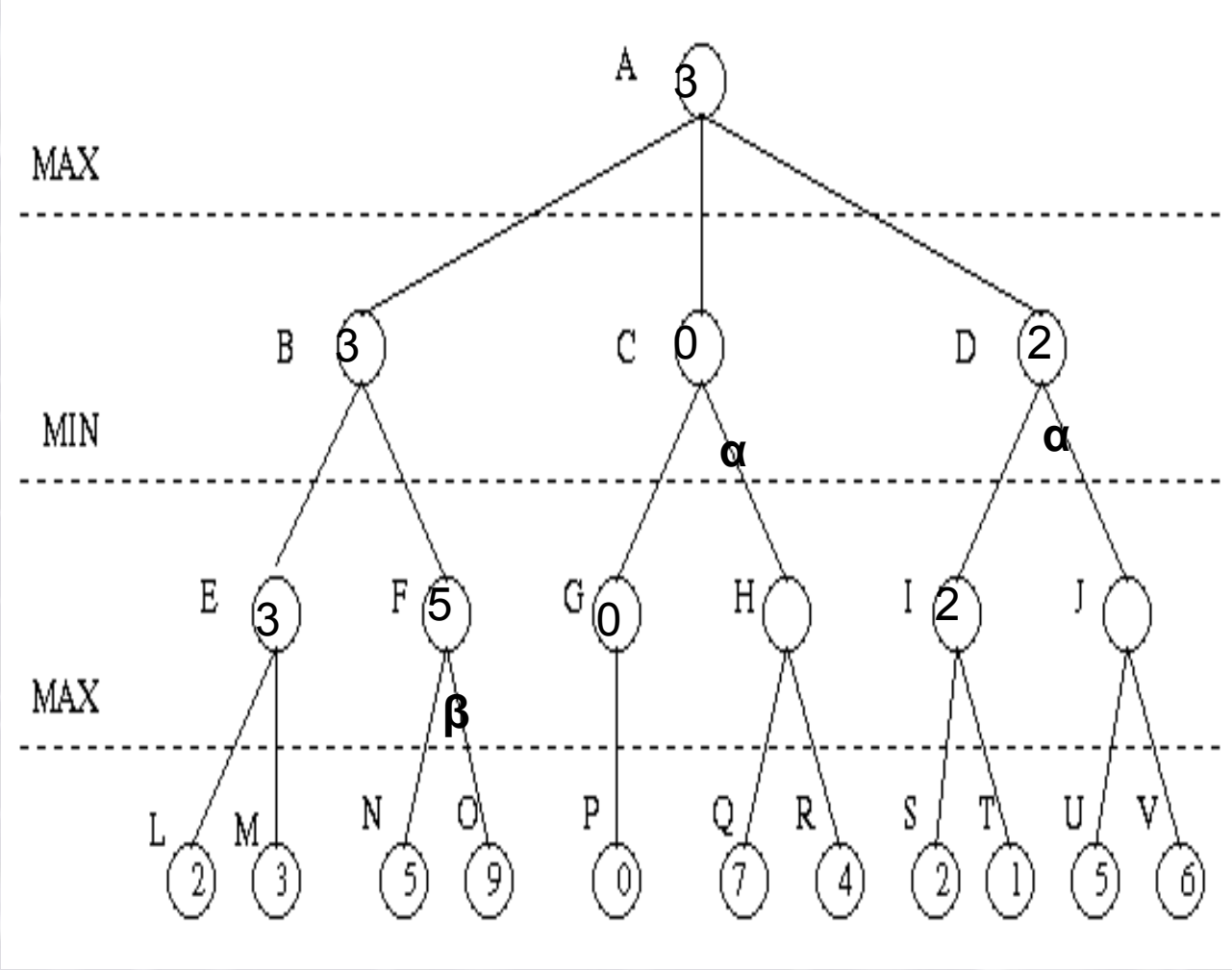
•

2. Search below a MAX node may be beta-pruned if the alpha value is >= to the beta value of some MIN ancestor.



α-β Search Algorithm

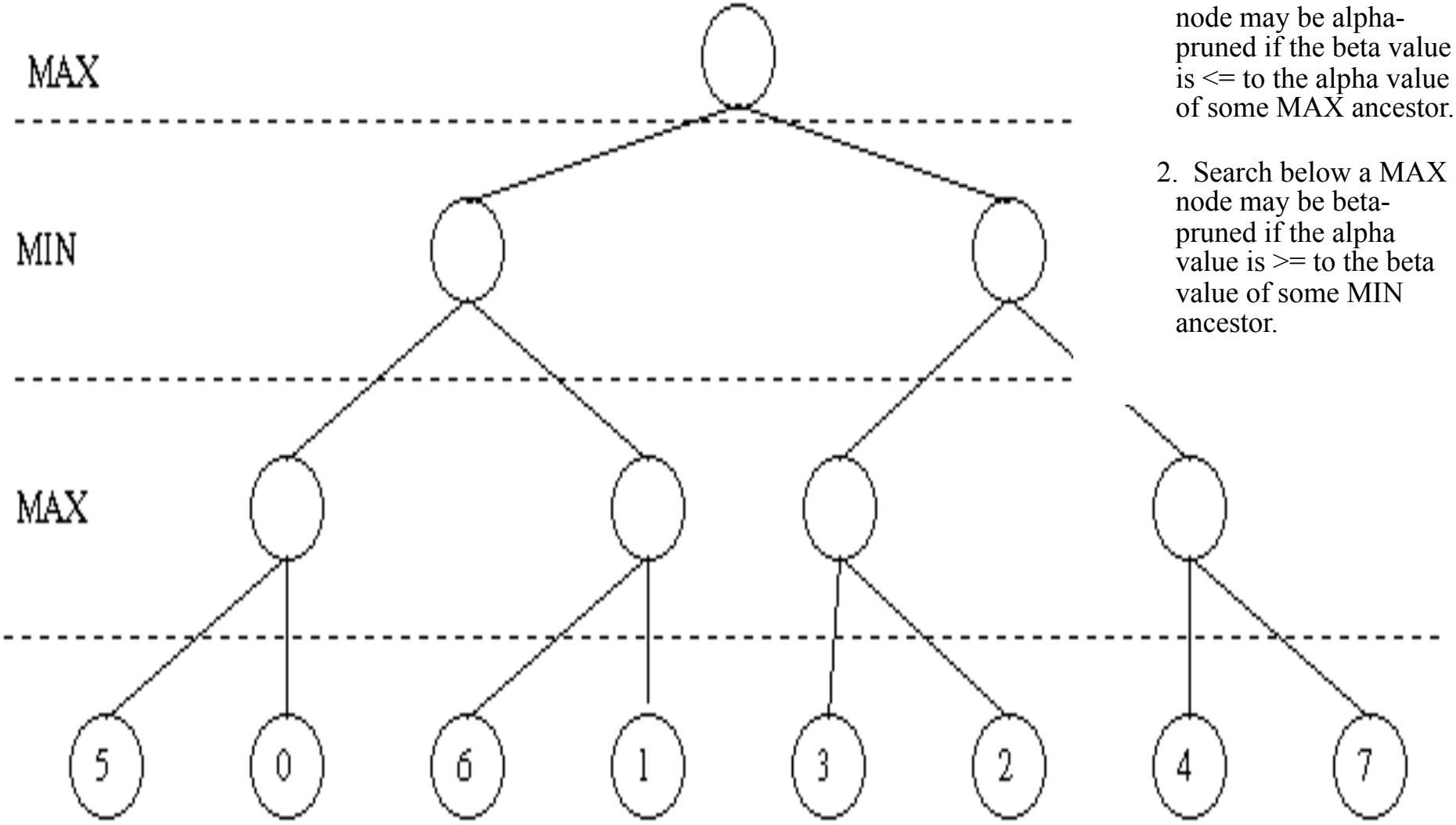
1. If terminal state, compute e(n) and return the result.
2. Otherwise, if the level is a **minimizing** level,
   - Until no more children or $\beta \leq \alpha$
     - $\upsilon_i \leftarrow \alpha - \beta$ search on a child
     - If $\upsilon_i < \beta, \beta \leftarrow \upsilon_i$.
   - Return $min(\upsilon_i)$
3. Otherwise, the level is a **maximizing** level:
   - Until no more children or $\alpha \geq \beta$
     - $\upsilon_i \leftarrow \alpha - \beta$ search on a child.
     - If $\upsilon_i > \alpha$, set $\alpha \leftarrow \upsilon_i$
   - Return $max(\upsilon_i)$

pruning

pruning

See page 170 R&N

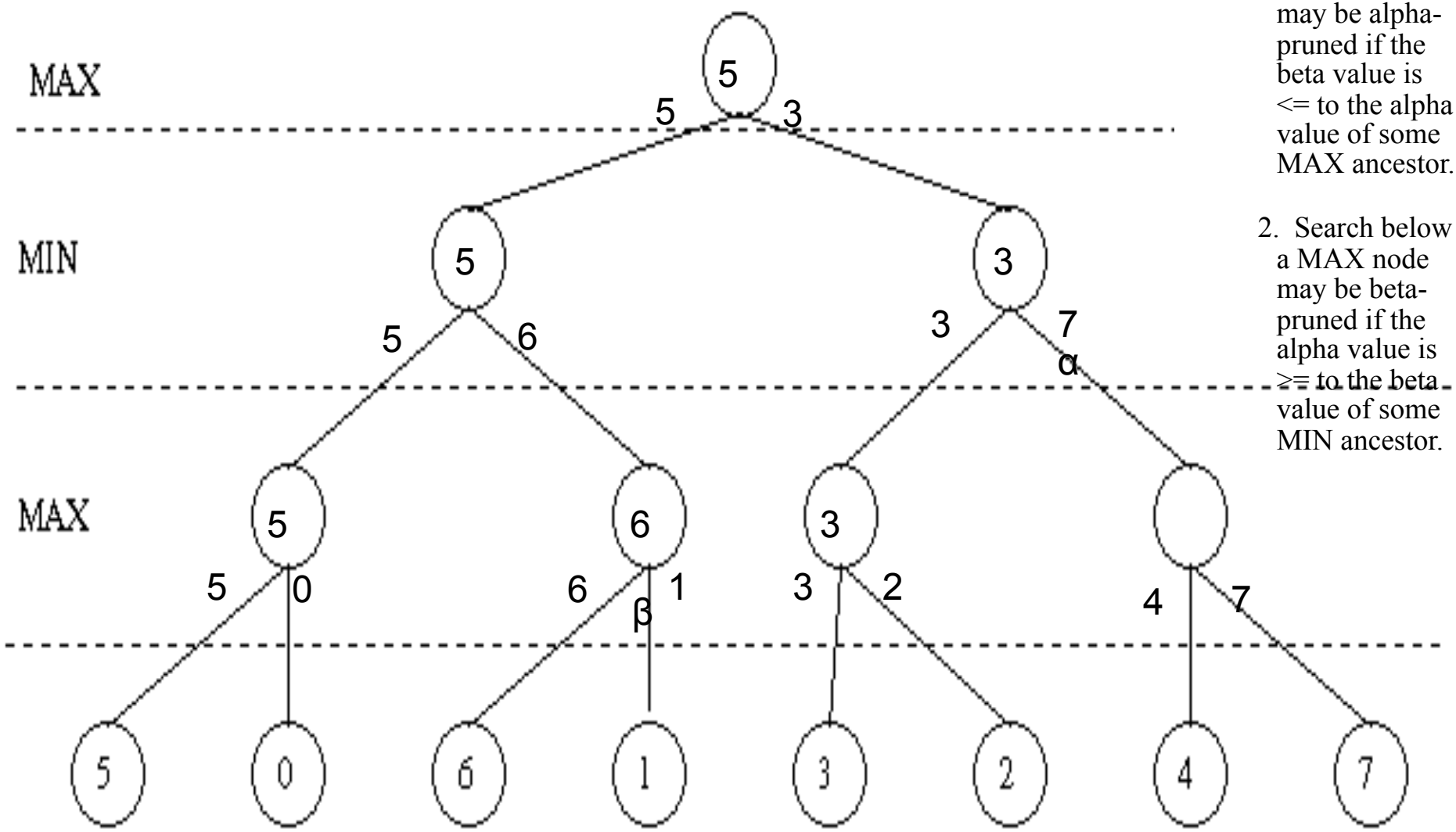
Another Example



1. Search below a MIN node may be alpha-pruned if the beta value is <= to the alpha value of some MAX ancestor.

2. Search below a MAX node may be beta-pruned if the alpha value is >= to the beta value of some MIN ancestor.

Example



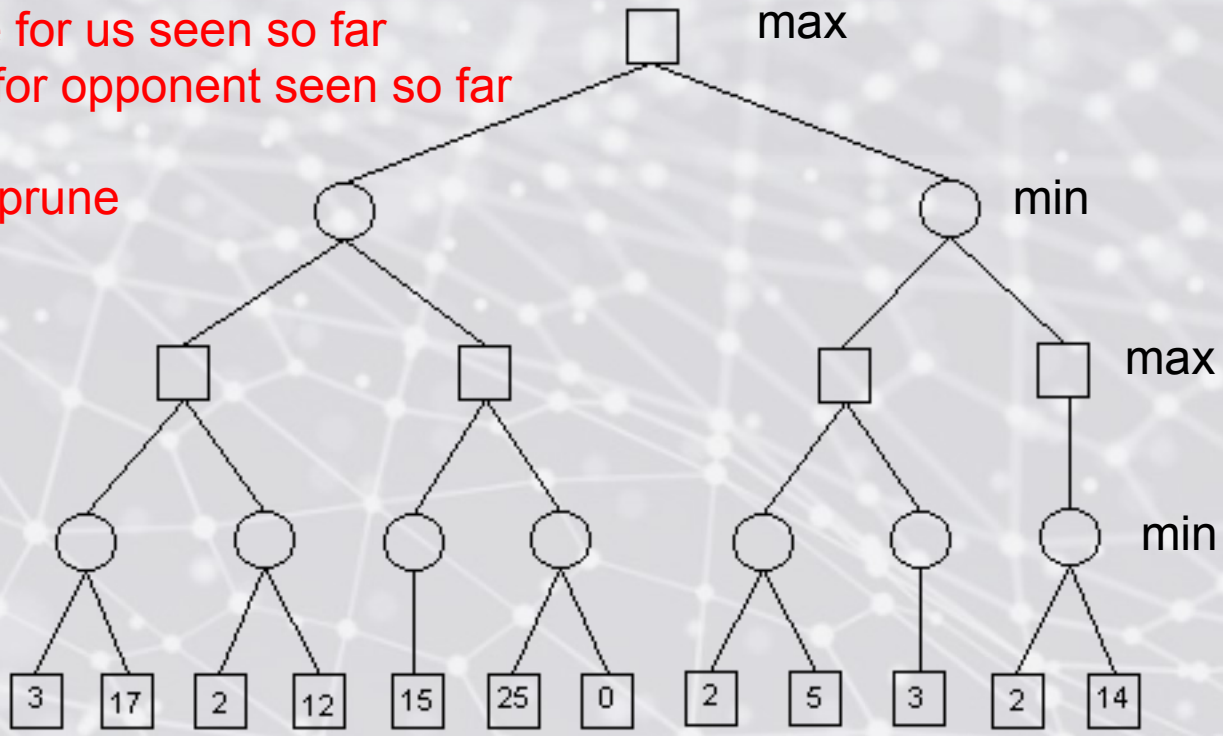
1.Search below a MIN node may be alpha-pruned if the beta value is <= to the alpha value of some MAX ancestor.

2. Search below a MAX node may be beta-pruned if the alpha value is >= to the beta value of some MIN ancestor.

Alpha-beta pruning exercise

Remember:
alpha: best move for us seen so far
beta: best move for opponent seen so far

If alpha >= beta, prune



(a) What is value at the root, using minimax alone?

(b) What nodes could have been pruned from the search using alpha-beta pruning?

Alpha-Beta Pruning

- Alpha-beta pruning effectiveness is highly dependent on move ordering.
- Under optimal ordering conditions, alpha-beta pruning needs to examine $O(b^{m/2})$ nodes to pick the best move, instead of $O(b^{m})$ for minimax.
- This means that the effective branching factor becomes root(b) instead of b – for chess about 6 instead of 35.

Alpha-Beta Pruning

- Put another way, alpha-beta can solve a tree roughly twice as deep as minimax in the same amount of time.
- If successors are examined in random order rather than best-first, the total number of nodes examined will be roughly $O(b^{3m/4})$ for moderate b.

Alpha-Beta Pruning

- Adding dynamic move-ordering schemes, such as trying the first moves that were found to be best in the past, brings the search effectiveness closer to the theoretical limit.
- In this way, the "past" could be the previous move, or it could come from previous exploration of the current move.
- One way to gain such valuable information is with iterative deepening search. (First search 1 ply and record path of best moves, then 1 ply deeper, etc.)

Alpha-Beta Pruning

- As we saw in Chapter 3, iterative deepening on an exponential game tree adds only a constant fraction to the total search time, which can be more than made up from better move ordering.
- The best moves are often called **killer moves** and to try them first is called killer move heuristic.
- To avoid **repeating states** we can use a hash table of previously seen positions (called a **transposition table** – like the explored list in GRAPH-SEARCH).

A Few More Notes on Alpha-Beta

- We assume (typically) that opponent is rational and both players have perfect information at each evaluation.
- Pruning *does not* affect the final result
- Good move ordering improves effectiveness of pruning
- With "perfect ordering", time complexity $O(b^{m/2})$
  - doubles the depth of search
  - can easily reach depth of 8 and play good chess (branching factor of 6 instead of 35)

Optimizing Minimax Search

- Use alpha-beta cutoffs
  - Evaluate most promising moves first
- Remember prior positions, reuse their backed-up values
  - Transposition table (like closed list in A*)
- Avoid generating equivalent states (e.g. 4 different first corner moves in tic tac toe)

- But, we still can't search a game like chess to the end!

Cutting Off Search

- Replace terminal test (end of game) by cutoff test (don't search deeper)
- Replace utility function (win/lose/draw) by heuristic evaluation function that estimates results on the best path below this board
  - Like A* search, good evaluation functions mean good results (and vice versa)
- Replace move generator by plausible move generator (don't consider "dumb" moves)

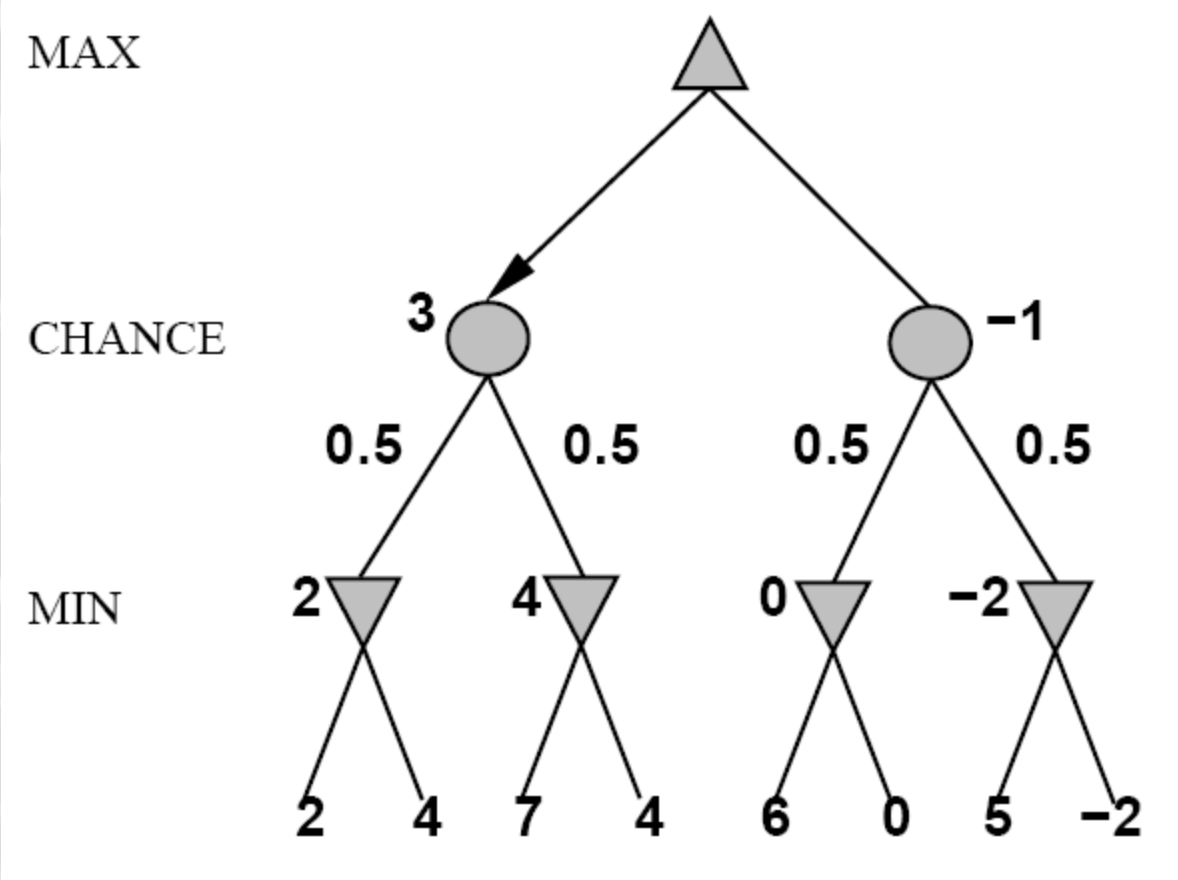
Cutting Off Search

- A more sophisticated type of cut off search applied evaluation functions only to positions that are **quiescent** (meaning: unlikely to exhibit wild swings in value in the near future).
- Non-quiescent positions can be expanded further until quiescent positions are reached. This extra search is called a **quiescent search**; sometimes it is restricted to consider only certain types of moves (e.g. capture moves).
- To mitigate the **horizon effect**, one can use **singular extension** (apply move that is clearly better than all others at a given position).

Stochastic Games

- In nondeterministic games, chance is introduced by dice, card shuffling
- How do we make correct decisions for stochastic games? We calculate expected value.

Nondeterministic Games



Algorithm for Nondeterministic Games

- Expectiminimax give perfect play
  - Just like Minimax except it has to handle chance nodes

- if state is a MAX node then
  - return highest Expectiminimax – Value of Successors(state)
- if state is a MIN node then
  - return lowest Expectiminimax – Value of Successors(state)
- if state is a CHANCE node then
  - return average Expectiminimax – Value of Successors(state)

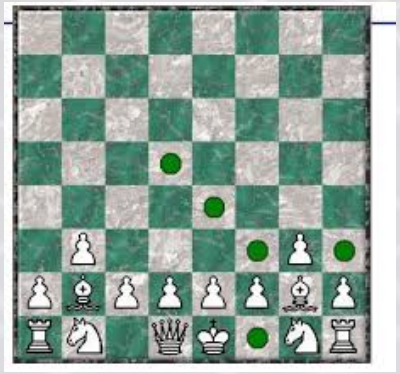
$$\textsc{Expectiminimax}(s) = \begin{cases} \textsc{Utility}(s) & \text{if } \textsc{Terminal-Test}(s) \\ \max_a \textsc{Expectiminimax}(\textsc{Result}(s,a)) & \text{if } \textsc{Player}(s) = \textsc{max} \\ \min_a \textsc{Expectiminimax}(\textsc{Result}(s,a)) & \text{if } \textsc{Player}(s) = \textsc{min} \\ \sum_r P(r)\textsc{Expectiminimax}(\textsc{Result}(s,r)) & \text{if } \textsc{Player}(s) = \textsc{chance} \end{cases}$$

Partially-Observable Games

- Chess has often been described as "war in miniature", but it lacks at least one major characteristic of real wars, namely, **partial observability**.

- **Kriegspiel** is a partially-observable variant of chess (!), in which pieces can move but are completed invisible to the opponent.

- Rules: White and Black each see a board containing only their own pieces; referee adjudicates "legal" moves, etc. Players propose moves.



Partially-Observable Games

- Recall the notion of a **belief state** (Chapter 4) – the *set of all logically possible board states*, given the complete history of percepts to date.

- Initially, White's belief state is a singleton because Blacks' pieces haven't moved yet. After White makes a move and Black responds, White's believe state contains 20 positions (why?).

- Instead of specifying a move to make for each possible move the opponent might make, we need a move for every possible percept sequence that might be received.

Kriegspiel

- For Kriegspiel, a winning strategy, or **guaranteed checkmate**, is one that, for each possible percept sequence, leads to an actual checkmate for every possible board state in the current belief state, regardless of how the opponent moves (a tall order).

- In addition to guaranteed checkmates, Kriegspiel admits of **probabilistic checkmates**.

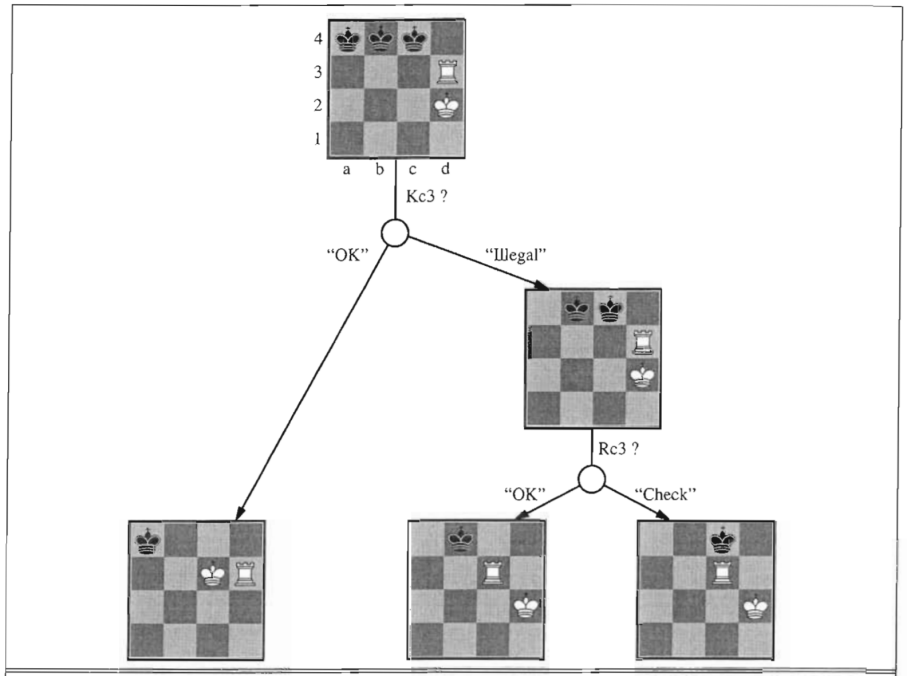


Figure 5.13 Part of a guaranteed checkmate in the KRK endgame, shown on a reduced board. In the initial belief state, Black's king is in one of three possible locations. By a combination of probing moves, the strategy narrows this down to one. Completion of the checkmate is left as an exercise.

Samuel's Checker Player

- Arthur Samuel's checkers program, written in the 1950's.
- In 1962, running on an IBM 7094, the machine defeated R. W. Nealy, a future Connecticut state checkers champion.
- One of the first machine learning programs, introducing a number of different learning techniques.

Samuel's Checker Player

Rote Learning

- When a minimax value is computed for a position, that position is stored along with its value.
- If the same position is encountered again, the value can simply be returned.
- Due to memory constraints, all the generated board positions cannot be stored, and Samuel used a set of criteria for determining which positions to actually store.

Samuel's Checker Player

Learning the evaluation function

- Comparing the static evaluation of a node with the backed-up minimax value from a lookahead search.
  - If the heuristic evaluation function were perfect, the static value of a node would be equal to the backed-up value based on a lookahead search applying the same evaluation on the frontier nodes.
  - If there's a difference between the values, the evaluation the heuristic function should be modified.

Samuel's Checker Player

Selecting terms

- Samuel's program could select which terms to actually use, from a library of possible terms.
- In addition to material, these terms attempted to measure following board features :

  center control

  advancement of the pieces

  mobility

- The program computes the correlation between the values of these different features and the overall evaluation score. If the correlation of a particular feature dropped below a certain level, the feature was replaced by another.

Deep Blue

http://www.dave-reed.com/csc550.S04/Presentations/chess/chess.ppt

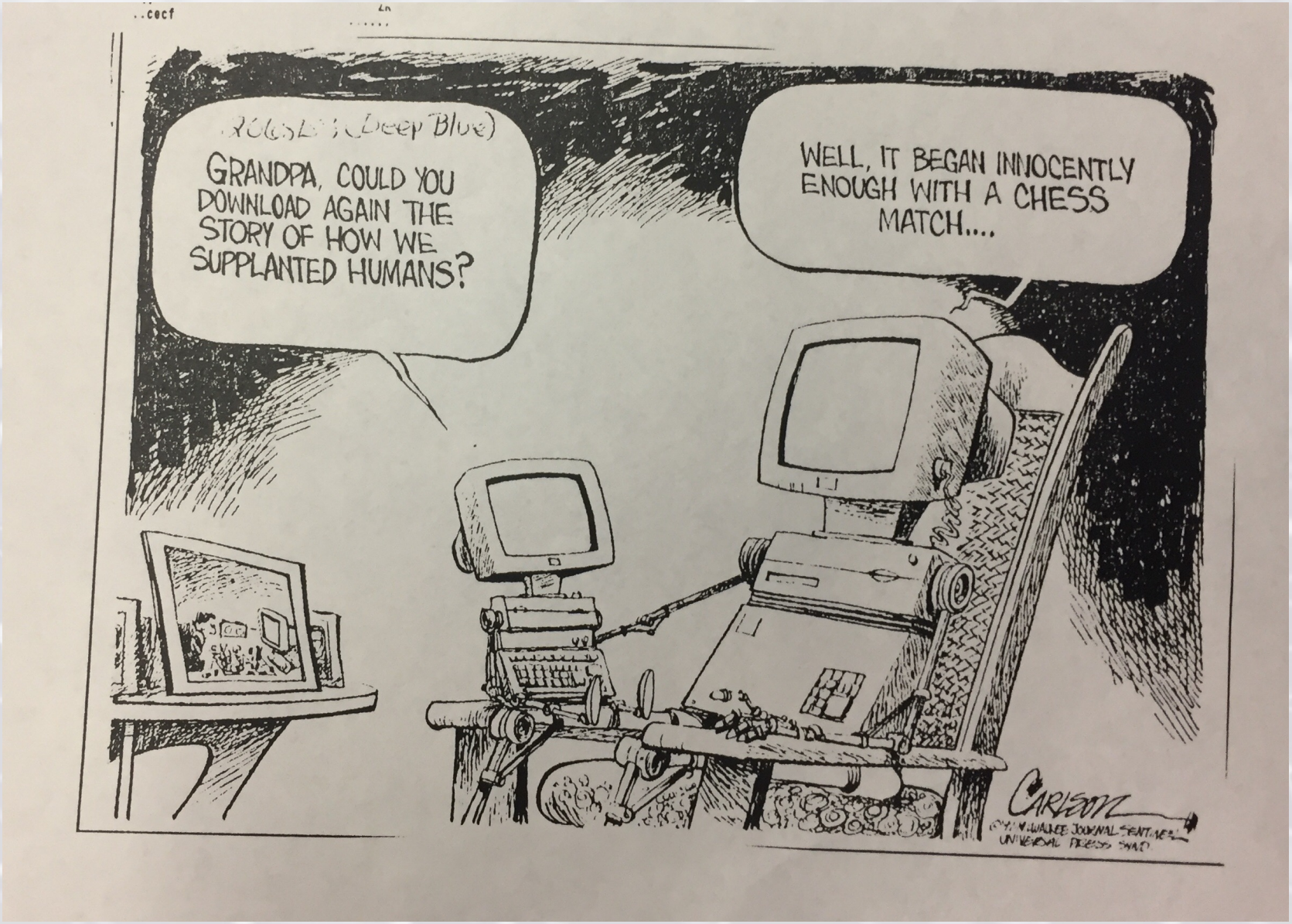
- First Created in 1997
- **Algorithm:**
  - iterative-deepening alpha-beta search, transposition table, databases including openings of grandmaster games (700,000), and endgames (all with 5 pieces or more pieces remaining)
- **Hardware:**
  - 30 IBM RS/6000 processors
    - They do: high level decision making
  - 480 custom chess processors
    - all running in parallel
    - They do :
      - deep searches into the trees
      - move generation and ordering,
      - position evaluation (over 8000 evaluation points)
- **Average performance:**
  - 126 million nodes/sec., 30 billion position/move generated, search depth: 14

From “Deep Blue (chess computer)”

http://en.wikipedia.org/wiki/Deep_Blue_(chess_computer)

On May 11, 1997, the machine won a six-game match by two wins to one with three draws against world champion Garry Kasparov.[1] Kasparov accused IBM of cheating and demanded a rematch, but IBM refused and dismantled Deep Blue.[2] Kasparov had beaten a previous version of Deep Blue in 1996.

After the loss, Kasparov said that he sometimes saw deep intelligence and creativity in the machine's moves, suggesting that during the second game, human chess players had intervened on behalf of the machine, which would be a violation of the rules. IBM denied that it cheated, saying the only human intervention occurred between games.


(Deep Blue)
GRANDPA, COULD YOU DOWNLOAD AGAIN THE STORY OF HOW WE SUPPLANTED HUMANS?
WELL, IT BEGAN INNOCENTLY ENOUGH WITH A CHESS MATCH....
CARLSON

A decade later: Topalov vs. Kramnik

http://en.wikipedia.org/wiki/Veselin_Topalov

On 14 December 2006, Topalov directly accused Kramnik of using computer assistance [from the *Fritz* chess computer] in their World Championship match.

On 14 February 2007, Topalov's manager released pictures, purporting to show cables in the ceiling of a toilet used by Kramnik during the World Championship match in Elista.

Summary

- A game can be defined by the **initial state**, the legal **actions** in each state, the **result** of each action, a **terminal test**, and a **utility function** that applies to terminal states.
- In two-player, zero-sum games with **perfect information**, the **minimax** algorithm can select optimal moves by a depth-first enumeration of the game tree.
- The **alpha-beta** search algorithm computes the same optimal move as minimax, but achieves much greater efficiency by eliminating subtrees that are probably irrelevant.

Summary

- Usually, it is not feasible to consider the whole game tree (even with alpha-beta), so we need to cut the search off at some point and apply a heuristic evaluation function that estimates the utility of a state.
- Games of chance can be handled by extension to the minimax algorithm that evaluates a **chance node** by taking the average utility of all its children, weighted by the probability of each child.