Question

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Consider the below data:

x	:	0	1	2
$f(x)$	:	4	3	12

The value of $\int_0^2 f(x)dx$ by Trapezoidal rule will be:

This question was previously asked in

Junior Executive (ATC) Official Paper 1: Held on Nov 2018 - Shift 1

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1. 11
2. 12
3. 15
4. 9

Answer (Detailed Solution Below)

Option 1 : 11

Detailed Solution

Concept:

Trapezoidal rule states that for a function y = f(x)

x	x_0	x_1	x_2	x_3	.....	x_n
y	y_0	y_1	y_2	y_3	.....	y_n

$x_n = x_0 + nh$, where n = Number of sub-intervals

h = step-size

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \ldots + y_{n-1})] \quad \text{---(1)}$$

For a trapezoidal rule, a number of sub-intervals must be a multiple of 1.

Calculation:

$$\begin{array}{lcccc} x & : & 0 & 1 & 2 \\ f(x) & : & 4 & 3 & 12 \end{array}$$

Here: $x_0 = 4$, $x_1 = 3$, $x_2 = 12$, h = 1

From equation (1);

$$\int_0^2 f(x)dx = \frac{h}{2}[(x_0 + x_2) + 2(x_1)]$$

$$= \frac{1}{2}[(4 + 12) + 2(3)] = \frac{22}{2} = 11$$

Key Points:

Apart from the trapezoidal rule, other numerical integration methods are:

Simpson's one-third rule:

For applying this rule, the number of subintervals must be a multiple of 2.

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + y_5 + \ldots + y_{n-1}) + 2(y_2 + y_4 + y_6 + \ldots + y_{n-2})]$$

..2)

Simpson's three-eighths rule:

For applying this rule, the number of subintervals must be a multiple of 3.

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{3h}{8}[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \ldots) + 2(y_3 + y_6 + \ldots)]$$