Midpoint Circle Algorithm

As in the raster line algorithm, we sample at unit intervals and determine the closest pixel position to the specified circle path at each step.

For a given radius ***r*** and screen center position (x, y), we can first set up our algorithm to calculate pixel positions around a circle path centered at the coordinate origin (0,0).

Then each calculated position (x**,** y) is moved to its proper screen position by adding **xc** to **x** and **yc** to **y**.

Along the circle section from **x** = **0** to **x = y** in the first quadrant, the slope of the curve varies from 0 to -1. Therefore, we can take unit steps in the positive ***x*** direction over this octant and use a decision parameter to determine which of the two possible y positions is closer to the circle path at each step.

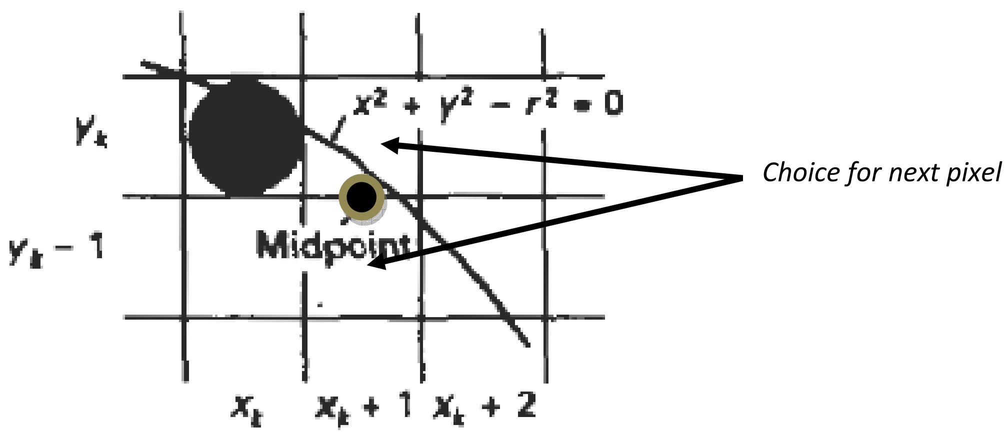
Positions in the other seven octants are then obtained by symmetry.

To apply the midpoint method, we define a circle function: $f_{circ}(x, y) = x^2 + y^2 - r^2$

The equation evaluates as follows:

$$f_{circ}(x, y) \begin{cases} < 0, & \text{if midpoint } (x, y) \text{ is inside the circle boundary} \\ = 0, & \text{if midpoint } (x, y) \text{ is on the circle boundary} \\ > 0, & \text{if midpoint } (x, y) \text{ is outside the circle boundary} \end{cases}$$

Thus, the circle function is the decision parameter in the midpoint algorithm, and we can set up incremental calculations for this function.



Assuming we have just plotted the pixel at (x_k,y_k) so we need to choose between (x_k+1,y_k) and (x_k+1,y_k-1).

Our decision variable can be defined as:

$$\begin{aligned} p_k &= f_{circ}(x_k + 1, y_k - \tfrac{1}{2}) \\ &= (x_k + 1)^2 + (y_k - \tfrac{1}{2})^2 - r^2 \end{aligned}$$

If **p_k < 0 the midpoint is inside the circle** and the pixel at **y_k is closer** to the circle **Otherwise** the **midpoint is outside and y_k-1 is closer.**

To ensure things are as efficient as possible we can do all of our calculations incrementally. Now next decision parameter X_k+2.

$$p_{k+1} = f_{circ}\left(x_k + 2, y_k + \tfrac{1}{2}\right) = (x_k + 2)^2 + \left(y_k + \tfrac{1}{2}\right)^2 - r^2$$

Or

$$p_{k+1} = f_{circ}\left(x_{k+1} + 1, y_{k+1} - \tfrac{1}{2}\right) = [(x_k + 1) + 1]^2 + \left(y_{k+1} - \tfrac{1}{2}\right)^2 - r^2$$

Now, take difference of P_{k+1} and P_k.

$$p_{k+1} = p_k + 2(x_k + 1) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1$$

Where y_{k+1} is y_k (i.e. $P_k < 0$) **or** y_k-1 (i.e. $p_k > 0$).

Then if $p_k < 0$ then the next decision variable is given as: $p_{k+1} = p_k + 2x_k + 3$

If $p_k > 0$ then the decision variable is: $p_{k+1} = p_k + 2x_k - 2y_k + 5$

The first decision variable is given as:

$$p_0 = f_{circ}(1, r - \tfrac{1}{2}) = 1 + (r - \tfrac{1}{2})^2 - r^2 = \tfrac{5}{4} - r \Rightarrow (1 - r)$$

For integer radius in the second octant the circle starts at *(0,r),* the first midpoint will be at *(1, R−1/2).*

To obtain initial parameter by evaluating the circle function at the start position (x_0, y_0) is (0,r).