Solution

Q-1

$n = 1.6 \times 10^{19}$

$\lambda = 11500 \text{ Å} = 11500 \times 10^{-10} m$, freq $\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{11500 \times 10^{-10}} = 2.6 \times 10^{14} Hz$

Energy, $E = h\nu = \frac{hc}{\lambda} = 6.62 \times 10^{-34} \times 2.6 \times 10^{14}$

$= 1.72 \times 10^{-19} J$

Energy of pulse $= nh\nu = 1.6 \times 10^{19} \times 1.72 \times 10^{-19}$

$= 2.75 J$

Q-2

$\lambda = 6328 \text{ Å} = 6328 \times 10^{-10} m$

$E = E_2 - E_1 = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{6328 \times 10^{-10}} = 3.13 \times 10^{-19} J$

In eV, $E = \frac{3.13 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.95 eV$

Population ratio $\frac{N_2}{N_1} = e^{-\frac{(E_2 - E_1)}{KT}}$

$T = 27°C + 273 = 300K$

$K = 1.38 \times 10^{-23} J/K$

or $8.61 \times 10^{-5} eV$

$\frac{N_2}{N_1} = e^{-\left(\frac{1.96 eV}{8.61 \times 10^{-5} eV \times 300K}\right)}$

$\frac{N_2}{N_1} = e^{-75.88} = 1.11 \times 10^{-33}$

Q-3

$R_{sp} = A_{21} N_2$

$R_{st} = B_{21} N_2 u(\nu)$

As per given condition

$R_{sp} = R_{st}$

$A_{21} \cancel{N_2} = B_{21} \cancel{N_2} u(\nu) \Rightarrow \frac{A_{21}}{B_{21}} = u(\nu)$ —— (1)

Using $\frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3}$

& $u(\nu) = \frac{8\pi h \nu^3}{c^3} \frac{1}{\left(e^{\frac{h\nu}{KT}} - 1\right)}$

Eq ① becomes.

$$\cancel{\frac{8\pi h \nu^3}{c^3}} = \cancel{\frac{8\pi h \nu^3}{c^3}} \frac{1}{\left(e^{\frac{h\nu}{KT}} - 1\right)}$$

$$e^{\frac{h\nu}{KT}} - 1 = 1$$

$$e^{h\nu/KT} = 2$$

Take ln on both sides

$$\frac{h\nu}{KT} = \ln 2$$

$$\frac{hc}{\lambda KT} = \ln 2$$

$$T = \frac{hc}{\lambda K \ln 2} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{5000 \times 10^{-10} \times 1.38 \times 10^{-23} \times 0.693}$$

$$= 41570\ K.$$

Q-4 $B_{12} = 1.3 \times 10^{19}\ m/Kg$

$\lambda = 6000\,Å = 6000 \times 10^{-10}\ m$

Using $\dfrac{A_{21}}{B_{12}} = \dfrac{8\pi h \nu^3}{c^3} = \dfrac{8\pi h \cancel{c^3}}{\lambda^3 \cancel{c^3}} = \dfrac{8\pi h}{\lambda^3}$

$A_{21} = \dfrac{8\pi h}{\lambda^3} \times B_{12} = \dfrac{8 \times 3.14 \times 6.62 \times 10^{-34} \times 1.3 \times 10^{19}}{(6000)^3 \times 10^{-30}}$

$= 1 \times 10^{+6}\ /s$

$= 10^{6}\ /s.$

Q-5 $E = 0.161\,eV$

$E = 0.161 \times 1.6 \times 10^{-19}\ J$

$= 0.257 \times 10^{-19}\ J$

$E = h\nu$ or $\nu = \dfrac{E}{h} = \dfrac{0.257 \times 10^{-19}}{6.62 \times 10^{-34}}$

$= 3.88 \times 10^{13}\ Hz$

$= 0.388 \times 10^{14}\ Hz$

$\lambda = \dfrac{c}{\nu} = \dfrac{3 \times 10^{8}}{0.388 \times 10^{14}} = 7.73 \times 10^{-6}\ m$

Also $\dfrac{N_2}{N_1} = e^{-(E_2 - E_1)/KT} = e^{-\left(\frac{0.161\,eV}{8.61 \times 10^{-5} \times 300\,K}\right)}$

$= e^{-6.23} = 1.96 \times 10^{-3}$

Q-6 $\lambda = 5000\ \mathring{A} = 5000 \times 10^{-10}\ m$

diameter $= 2mm = 2 \times 10^{-3}\ m.$

$D = 4 \times 10^{8}\ m.$

Angular Spread $(\theta) \simeq \frac{\lambda}{d} = \frac{5000 \times 10^{-10}}{2 \times 10^{-3}} = 2.5 \times 10^{-4}$ rad

Areal Spread $= (\theta . D)^2$

$= (2.5 \times 10^{-4} \times 4 \times 10^{8})^2$

$= 10 \times 10^{9}\ m^2.$

Calculate size using formula Area $= \pi r^2$

Q-7 $n_1 = 1.48$ $n_2 = ?$

$\Delta = 0.004$

a) $\Delta = \frac{n_1 - n_2}{n_1} \Rightarrow n_2 = n_1 - n_1 \Delta$

$n_2 = 1.474$

b) $NA = \sqrt{n_1^2 - n_2^2} = 0.133$

c) $\theta_0 = \sin^{-1} \sqrt{n_1^2 - n_2^2} = 7.64°$

d) $C = \sin^{-1} \left(\frac{n_2}{n_1}\right) = 84.2°.$

Q-8 $n_1 = 1.45$

$d = 50\,\mu m \Rightarrow a = 25\,\mu m$ = radius

$\Delta = 0.007$

$\lambda = 1.8\,\mu m$

1) $n_2 = ?$

$$\Delta = \frac{n_1 - n_2}{n_1}$$

or $n_2 = n_1 - n_1 \Delta$

$= 1.439$

2) $NA = \sqrt{n_1^2 - n_2^2} = 0.178$

3) $\theta_0 = \sin^{-1}\sqrt{n_1^2 - n_2^2} = 10.2^\circ$

$\downarrow$

i_{max}

4) $V\text{-number} = \frac{2\pi a}{\lambda} \times NA$

No need to convert in metres

[a & λ both in micrometer so will be cancelled]

$$= \frac{2 \times 3.14 \times 25}{1.8} \times 0.178$$

$V = 15.5$

5) For step index fibre, number of modes

$$N_m = \frac{V^2}{2} = 120$$

Q-9 NA = 0.20

$n_1 = 1.60$

$i_{max} = \theta_0 = \sin^{-1}\sqrt{n_1^2 - n_2^2}$

$= \sin^{-1}(NA) = \sin^{-1}(0.20)$

$= 11.5^\circ.$

When water is launching medium

$\theta_0 = \sin^{-1}\left(\frac{\sqrt{n_1^2 - n_2^2}}{n_0}\right) = \sin^{-1}\left(\frac{0.20}{1.33}\right)$

$= 8.64^\circ$

Q-10 Number of modes in GRIN fibres

$N_m = \frac{V^2}{4} = \frac{50^2}{4} = 625$ modes

V number is normalized freq.

Q-11 $d = 200\,\mu m \Rightarrow a = 100\,\mu m$ = radius

NA = 0.30.

$\lambda = 0.90\,\mu m$

V number $= \frac{2\pi a}{\lambda} \times NA = \frac{2 \times 3.14 \times 100 \times 0.30}{0.90}$

$V = 209$

$N_m = \frac{V^2}{2} = 2184$ modes