

WORKED-OUT EXAMPLES

Example 22.1: Find the ratio of populations of the two states in a He-Ne laser that produces light of wavelength 6328 Å at 27°C.

Solution: The ratio of population is given by $\frac{N_2}{N_1} = e^{-(E_2 - E_1)/kT}$

$$E_2 - E_1 = \frac{12400}{6328} \text{ eV} = 1.96 \text{ eV}$$

$$\therefore \frac{N_2}{N_1} = \exp \left[\frac{-1.96 \text{ eV}}{(8.61 \times 10^{-5} \text{ eV})(300 \text{ K})} \right] = e^{-75.88} = 1.1 \times 10^{-33}$$

Example 22.2: The wavelength of emission is 6000 Å and the coefficient of spontaneous emission is 10%/s. Determine the coefficient for the stimulated emission.

Solution: The coefficient for stimulated emission is given by

$$B_{21} = \frac{c^3}{8\pi h\nu^3 \mu^3} A_{21} = \frac{\lambda^3}{8\pi h} A_{21} \quad (\text{Taking } \mu = 1)$$

$$\therefore B_{21} = \frac{(6000 \times 10^{-10})^3}{8\pi \times 6.626 \times 10^{-34} \text{ J s}} \left(10^6 / \text{s} \right) = 1.3 \times 10^{19} \text{ m}^3/\text{kg}.$$

Example 22.3: At what temperature are the rates of spontaneous and stimulated emission equal? Assume $\lambda = 5000 \text{ Å}$.

Solution: If the rates of spontaneous and stimulated emission are equal, then

$$R_1 = \left[\frac{1}{e^{h\nu/kT} - 1} \right] = 1 \quad \text{or} \quad e^{h\nu/kT} = 2$$

As $\lambda = 5000 \text{ Å}$, $\nu = c/\lambda = 6 \times 10^{14} \text{ Hz}$ and

$$\frac{h\nu}{kT} = \frac{6.626 \times 10^{-34} \text{ J s} (6 \times 10^{14} / \text{s})}{(1.38 \times 10^{-23} \text{ J/K}) T} = \frac{28.8 \times 10^3}{T} \text{ K}$$

$$e^{h\nu/kT} = \exp \left[\frac{28.8 \times 10^3}{T} \text{ K} \right] = 2$$

$$\text{or} \quad \frac{28.8 \times 10^3}{T} \text{ K} = \ln 2 = 0.693$$

$$\therefore T = \frac{28.8 \times 10^3}{0.693} \text{ K} = 41,558 \text{ K}.$$

Example 22.4: The length of a laser tube is 150 mm and the gain factor of the laser material is 0.0005/cm. If one of the cavity mirrors reflects 100% light that is incident on it, what is the required reflectance of the other cavity mirror?

Solution:

\therefore It means that the second mirror is 100% reflective.

Example 22.5: The length of the cavity is 30 cm and the gain factor of the He-Ne laser is 0.0005/cm. What is the required reflectance of the other cavity mirror?

Solution: The separate losses are:

Number of modes

1. Explain with neat diagram, the various modes of a laser.

2. What is population inversion between two energy levels?

3. What do you understand by the term 'spontaneous emission'?

4. Discuss the four-level laser system.

5. What do you understand by the term 'stimulated emission'?

6. Why is the optical density of a laser medium important?

7. What are the essential conditions for lasing?

8. Describe the working of a laser.

9. Explain the principle of a laser.

10. In helium-neon laser, what is the role of helium?

11. In helium-neon laser, what is the role of neon?

12. What is the reason for the low efficiency of a laser?

13. With the help of a diagram, explain the operation of a laser.

14. Explain in brief the concept of a laser.

15. What is population inversion?

16. What do you mean by a laser?

17. What is the role of a laser in a communication system?

18. Write the rate equation for a laser.

19. Why heterojunction is used in a laser diode?

20. Discuss the absorption of light in a laser medium.

Solution:

$$\gamma_{th} = \frac{1}{2L} \ln \frac{1}{r_1 r_2}$$

$$\therefore r_2 = \frac{1}{r_1 e^{2L\gamma}} = \frac{1}{1 \times e^{2 \times 15 \times 0.0005}} = 0.985$$

It means that the second mirror should have a reflectance of 98.5%.

Example 22.5: The half-width of the gain profile of a He-Ne laser material is 2×10^{-3} nm. The length of the cavity is 30 cm, how many longitudinal modes can be excited? The emission wavelength of He-Ne laser is 6328 Å.

Solution: The separation between successive longitudinal modes is given by

$$\Delta\lambda = \frac{\lambda^2}{2L} = \frac{(6328 \times 10^{-10} \text{ m})^2}{2(30 \times 10^{-2} \text{ m})} = 0.66 \times 10^{-3} \text{ nm}$$

Number of modes

$$N = \frac{\delta\lambda}{\Delta\lambda} = \frac{2 \times 10^{-3} \text{ nm}}{0.66 \times 10^{-3} \text{ nm}} = 3.$$

QUESTIONS

1. Explain with neat diagram absorption, spontaneous emission and stimulated emission of radiation.
2. What is population inversion? Explain why laser action cannot occur without population inversion between atomic levels?
3. What do you understand by a negative temperature state? How can it be achieved?
4. Discuss the four-level (pumping) scheme for laser action.
5. What do you understand by an optical resonant cavity? Explain.
6. Why is the optical resonator required in lasers? Illustrate your answer with neat sketches.
7. What are the essential components of a laser? Explain their functions briefly.
8. Describe the working of solid state ruby laser.
9. Explain the principle and working of a He-Ne laser.
10. In helium – neon laser lasing is through neon gas. What is then the role of helium gas?
11. In helium – neon laser why is it necessary to use narrow tubes?
12. What is the reason for monochromaticity of laser beam?
13. With the help of energy band diagram discuss the working of a semiconductor laser.
14. Explain in brief the characteristics of a laser beam.
15. What is population inversion? Describe the construction and working of He-Ne laser. (GNDU, Amritsar, 2010)
16. What do you mean by non-radiative decay? (GNDU, Amritsar, 2010)
17. What is the role of He in He-Ne Laser? (GNDU, Amritsar, 2010)
18. Write the rate equation for three level laser system. (GNDU, Amritsar, 2010)
19. Why heterojunction semiconductor lasers are preferred? (GNDU, Amritsar, 2010)
20. Discuss the absorption and amplification of a parallel beam light passing through a medium. (GNDU, Amritsar, 2010)
21. Explain the concept of the Longitudinal and Transverse modes. (GNDU, Amritsar, 2010)

22. Explain the terms Doppler and line broadening. (GNDU, Amritsar, 2010)
23. Discuss the working, construction and applications of CO_2 Laser with neat labeled diagrams. (GNDU, Amritsar, 2010)
24. Write the requirements for producing high inversion density for Q-Switching. (GNDU, Amritsar, 2010)
25. Draw the electronic energy levels diagrams showing the output wavelength / frequency of Ruby and Nd:YAG Lasers? (GNDU, Amritsar, 2010)

PROBLEMS FOR PRACTICE

- A pulsed laser is constructed with a ruby crystal as the active element. The ruby rod contains typically a total of 3×10^{19} Cr^{3+} ions. If the laser emits light at 6943 \AA wavelength, find
 - the energy of emitted photon (in eV)
 - the total energy available per laser pulse (assuming total population inversion)
 [Ans : (a) $E = 1.79 \text{ eV}$; (b) 8.6 J]
- Find the relative populations of the two states in a ruby laser that produces a light beam of wavelength 6943 \AA at 300 K . [Ans : 8×10^{-31}]
- Find the ratio of populations of the two states in a He-Ne laser that produces light of wavelength 6328 \AA at 27°C . [Ans : 1.1×10^{-33}]
- The He-Ne system is capable of lasing at several different IR wavelengths, the prominent one being $3.3913 \mu\text{m}$. Determine the energy difference (in eV) between the upper and lower levels for this wavelength. [Ans : $E = 0.37 \text{ eV}$]
- The CO_2 laser is one of the most powerful lasers. The energy difference between the two laser levels is 0.117 eV . Determine the frequency and wavelength of the radiation. [Ans : $\lambda = 10.5 \mu\text{m}$, $\nu = 2.9 \times 10^{13} \text{ Hz}$]
- A laser beam can be focused on an area equal to the square of its wavelength. For a He-Ne laser, the wavelength of emitted light is 6328 \AA . If the laser radiates energy at the rate of 1 mW , find out the intensity of the focused beam. [Ans : $I = 2.5 \times 10^{15} \text{ W/m}^2$]
- Compute the Doppler broadening for the 6328 \AA laser transition in the He-Ne Laser, assuming a single isotope of Ne^{20} and that the laser operate at a discharge-bore temperature of 373 K . (GNDU, Amritsar, 2010)

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CHAPTER

INTRODUCTION

Images of objects are generally obtained by the photographic method. In this method, light reflected from a three-dimensional object is recorded on a photographic film where a two-dimensional image is formed. A negative is first developed from the film and then a positive is obtained from the negative. A positive print is a two-dimensional record of light intensity received from the object. It, thus, contains information about the square of the amplitude of the light wave that produced the image but information about the phase of the wave is not recorded and is lost.

In 1948 Dennis Gabor outlined a two-step lensless imaging process. It is a radically new technique of photographing the objects and is known as **front wave reconstruction**. The technique is also called

WORKED-OUT EXAMPLES

Example 24.1: In an optical fibre, the core material has refractive index 1.43 and refractive index of clad material is 1.4. Find the propagation angle.

Solution:

$$\cos \theta_c = \frac{n_2}{n_1} = \frac{1.40}{1.43} = 0.979$$

Therefore, propagation angle $\theta_c = \cos^{-1}(0.979) = 11.8^\circ$

Example 24.2: In an optical fibre, the core material has refractive index 1.6 and refractive index of clad material is 1.3. What is the value of critical angle? Also calculate the value of angle of acceptance cone.

Solution: Critical angle is given by $\sin \phi_c = \frac{n_2}{n_1} = \frac{1.3}{1.6} = 0.8125$

$$\therefore \phi_c = 54.3^\circ$$

$$\begin{aligned} \text{Acceptance angle } \theta_0 &= \sin^{-1} \left[\sqrt{n_1^2 - n_2^2} \right] = \sin^{-1} \left[\sqrt{1.6^2 - 1.3^2} \right] \\ &= \sin^{-1}(0.87) \\ &= 60.5^\circ \end{aligned}$$

Angle of acceptance cone $= 2\theta_0 = 121^\circ$

Example 24.3: Calculate the numerical aperture and acceptance angle of an optical fibre from the following data:

$$\mu_1(\text{core}) = 1.55 \text{ and } \mu_2(\text{cladding}) = 1.50$$

Solution:

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{1.55^2 - 1.50^2} = \sqrt{0.153} = 0.391.$$

Acceptance angle

$$\theta_0 = \sin^{-1} \left[\sqrt{n_1^2 - n_2^2} \right] = \sin^{-1} \left[\sqrt{1.55^2 - 1.50^2} \right] = 23.02^\circ$$

Example 24.4: What is the numerical aperture of an optical fibre cable with a clad index of 1.378 and a core index of 1.546?

Solution:

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{1.546^2 - 1.378^2} = \sqrt{0.491} = 0.70$$

Example 24.5: A fibre cable has an acceptance angle of 30° and a core index of refraction of 1.4. Calculate the refractive index of the cladding.

Solution:

$$\sin \theta_0 = \sqrt{n_1^2 - n_2^2}$$

$$\therefore \sin^2 \theta_0 = n_1^2 - n_2^2$$

$$n_2^2 = n_1^2 - \sin^2 \theta_0 = (1.4)^2 - \sin^2 30^\circ = 1.96 - 0.25$$

$$= 1.71$$

$$n_2 = 1.308$$

Example 24.6: Calculate the angle of acceptance of a given optical fibre, if the refractive indices of the core and the cladding are 1.563 and 1.498 respectively.

Solution:

$$\sin \theta_o = \sqrt{n_1^2 - n_2^2} = \sqrt{(1.563)^2 - (1.498)^2} = 0.4461$$

$$\theta_o = \sin^{-1}(0.4461) = 26.49^\circ$$

Example 24.7: Calculate the fractional index change for a given optical fibre if the refractive indices of the core and the cladding are 1.563 and 1.498 respectively.

Solution: Fractional index change $\Delta = \frac{n_1 - n_2}{n_1} = \frac{1.563 - 1.498}{1.563} = \frac{0.065}{1.563} = 0.0415$

Example 24.8: Calculate the refractive indices of the core and the cladding material of a fiber from the following data:

Numerical aperture (NA) = 0.22 and $\Delta = 0.012$
where Δ is the fractional refractive index change.

Solution:

$$NA = n_1 \sqrt{2\Delta}$$

$$0.22 = n_1 \sqrt{2 \times 0.012} = 0.155 n_1$$

$$\therefore n_1 = \frac{0.22}{0.155} = 1.42$$

$$\Delta = \frac{n_1 - n_2}{n_1} \therefore \frac{1.42 - n_2}{1.42} = 0.012 \therefore n_2 = 1.42 - 1.42 \times 0.012 = 1.403$$

Example 24.9: Find the fractional refractive index and numerical aperture for an optical fibre with refractive indices of core and cladding as 1.5 and 1.49 respectively.

Solution:

$$\Delta = \frac{n_1 - n_2}{n_1} = \frac{1.5 - 1.49}{1.5} = 0.0067$$

$$NA = n_1 \sqrt{2\Delta} = 1.5 \sqrt{2 \times 0.0067} = 0.174$$

Example 24.10: A step-index fibre is made with a core of refractive index 1.52, a diameter of 29 μm and a fractional difference index of 0.0007. It is operated at a wavelength of 1.3 μm . Find the V-number and the number of modes that the fibre will support.

Solution:

$$V = \frac{\pi d}{\lambda_o} n_1 \sqrt{2\Delta} = \frac{3.143 \times 29 \times 10^{-6} \text{ m}}{1.3 \times 10^{-6} \text{ m}} \times 1.52 \sqrt{2 \times 0.0007} = 4.049$$

$$\therefore \text{Number of modes, } N = \frac{1}{2} V^2 = \frac{1}{2} (4.049)^2 = 8 \text{ modes}$$

Example 24.11: A step-index fibre is with a core of refractive index 1.55 and cladding of refractive index 1.51. Compute the intermodal dispersion per kilometer of length of the fibre and the total dispersion in a 15 km length of the fibre.

Solution:

$$\Delta t = \frac{n_1 L}{c} \left[\frac{n_1}{n_2} - 1 \right] = \frac{1.55 \times 10^3 \text{ m}}{3 \times 10^8 \text{ m/s}} \left[\frac{1.55}{1.51} - 1 \right] = 138 \text{ ns/km.}$$

$$\text{Total dispersion for 15 km length} = \Delta t \times 15 \text{ km} = (138 \text{ ns/km}) \times 15 \text{ km} = 2.07 \mu\text{s.}$$

Example 24.12: Optical power of 1 mW is launched into an optical fibre of length 100 m. If the power emerging from the other end is 0.3 mW, calculate the fibre attenuation.

Solution: Attenuation, $\alpha = \frac{10}{L} \log \frac{P_i}{P_o} = \frac{10}{0.1 \text{ km}} \log \frac{1 \text{ mW}}{0.3 \text{ mW}} = 52.3 \text{ dB/km}$

Example 24.13: What is the attenuation in dB/km, if 15% of the power fed at the launching end of 0.5 km fibre is lost during propagation?

Solution: Attenuation, $\alpha = \frac{10}{L} \log \frac{P_i}{P_o} = \frac{10}{0.5 \text{ km}} \log \frac{1}{0.15} = 16.48 \text{ dB/km}$

QUESTIONS

1. Explain the phenomenon of total internal reflection of light. How is it used in fiber optic communications?
2. What is meant by critical propagation angle of an optical fibre? Obtain an expression for the critical propagation angle.
3. What is meant by critical angle of an optical fibre? Obtain an expression for the critical angle.
4. What is an optical fibre? What is the principle involved in its working?
5. Explain the following terms:
 - (i) critical angle
 - (ii) Acceptance cone
 - (iii) Numerical aperture
6. Deduce an expression for acceptance angle of an optical fibre.
7. Using ray theory, derive the condition for transmission of light within an optical fibre.
8. With the help of a ray diagram, show how optical fibres can guide light waves.
9. Derive an expression for angle of acceptance of fibre in terms of refractive index of the core and the cladding of an optical fibre. What is meant by acceptance cone?
10. What is meant by acceptance of angle for an optical fibre? Show how it is related to numerical aperture.
11. What do you understand by the terms acceptance angle and acceptance cone? Derive an expression for acceptance angle in terms of refractive indices of the core and the cladding.
12. Derive an expression for acceptance angle and numerical aperture for step index fibre with the help of suitable diagram.
13. Explain what you understand by acceptance angle and numerical aperture.
14. Derive an expression for N.A. for S.I. fibre in terms of refractive index of the core and relative refractive index difference between the core and the cladding.
15. Derive an expression for numerical aperture of a step-index fibre in terms of Δ .
16. Define the relative refractive index difference of an optical fibre. Show how it is related to numerical aperture.
17. Classify the fibres on the basis of refractive index profile, on the basis of modes and on the basis of single mode, multimode fibre.

22. What are the different types of attenuation losses in an optical fibre? Discuss the absorption losses.
23. Describe various mechanisms of attenuation in optical fibres.
24. Draw the diagram for an optical fibre link and explain the function of each block.
25. List the main components of optical communication system. Describe the basic optical communication system.
26. Explain optical communication through block diagram. For long distance communication whether (i) mono-mode or multimode and (ii) step index or graded index fibre, which are preferable and why?
27. Discuss the advantages and disadvantages of optical fibres over conventional communication transmission media.
28. Explain with basic principle, the construction and working of any one type of optical fibre sensor.
29. Discuss any one application of an optical fibre as a sensor.

PROBLEMS FOR PRACTICE

1. An optical fibre has a core material of refractive index of 1.55 and cladding material of refractive index 1.50. The light is launched into the fibre from air. Calculate its numerical aperture.
2. The numerical aperture of an optical fibre is 0.39. If the difference in the refractive indices of the material of its core and the cladding is 0.05, calculate the refractive index of material of the core.
3. An optical fibre has an acceptance angle 26.80° . Calculate its numerical aperture. (Ans: 0.4508)
4. An optical fibre refractive indices of core and cladding are 1.53 and 1.42 respectively. Calculate its critical angle. (Ans: 68.14°)
5. Consider a fibre having a core of index 1.48, a cladding of index 1.46 and has a core diameter of $30\text{ }\mu\text{m}$. Show that all rays making an angle less than 9.43° with the axis will propagate through the fibre.
6. A step-index fibre is made with a core of index 1.54, a cladding of index 1.50 and has a core diameter of $50\text{ }\mu\text{m}$. It is operated at a wavelength of $1.3\text{ }\mu\text{m}$. Find the V- number and the number of modes that the fibre will support. (Ans: 42.15, 888)
7. Using a step index fibre with $n_1=1.48$ and $n_2=1.46$ and the core radius $a=30\text{ }\mu\text{m}$. Calculate the number of total internal reflections that will occur on its propagation in a length of 1 km fibre.
8. A step-index fibre has a core refractive index of 1.44 and the cladding refractive index of 1.41. Find (i) the numerical aperture, (ii) the relative refractive index difference, and the acceptance angle. (Ans: 0.292, 0.021, 33.96°)
9. An optical fibre has a numerical aperture of 0.20 and a cladding refractive index of 1.59. Find the acceptance angle for the fibre in water which has a refractive index of 1.33. (Ans: $8^\circ 39'$)
10. Compute the cut-off parameter and the number of modes supported by a fibre which has a core refractive index of 1.54 and the cladding refractive index of 1.50. The radius of the core is $25\text{ }\mu\text{m}$ and operating wavelength is 1300 nm . (Ans: 42.15, 888)
11. Find the numerical aperture and acceptance angle of a fibre of core index 1.4 and $\Delta = 0.02$. (Ans: 0.28, 32.52°)

$$N.A = \sqrt{n^2 - n_c^2}$$

12. Compute the total dispersion in 10 km length of a step index fibre, which has a core refractive index of 1.55 and the relative refractive index difference of 0.026. (Ans: 138ns)
13. Consider a bare step index fibre having a refractive index of 1.46. The radius of the fibre is 50 μm . Compute the pulse dispersion per km. (Ans: 2238ns)
14. Compute the cut-off parameter and the number of modes supported by a fibre, which has a core refractive index of 1.47 and the cladding refractive index of 1.45. The radius of the core is 50 μm and operating wavelength is 850 nm. (Ans: 44.64, 996)
15. A step-index fibre has a normalized frequency $V = 26.6$ at 1300 nm wavelength. If the core radius is 25 μm , calculate the numerical aperture. (Ans: 0.22)
16. Find the core radius necessary for single mode operation at 820 nm of a step index fibre, which has a core refractive index of 1.480 and the cladding refractive index of 1.478. (Ans: 4.08 μm)
17. A signal of 100mW is injected into a fibre. The outcoming signal from the other end is 40 mW. What is the loss in dB? (Ans: 3.98 dB)
18. A communication system uses a 10 km fibre having a fibre loss of 2.5 dB/km. Find the input power if the output power is 1.265 μW . (Ans: 400 μW)
19. A fibre length 100m has power input 10 μW and power output 8.8 μW . Find the power loss in dB/km. (Ans: 5.55 dB/km)
20. When the mean optical power launched into a 8 km length fibre is 120 μW , the mean optical power at the fibre output is 3 μW . Determine
 - (i) the overall signal attenuation in dB through the fibre,
 - (ii) the signal attenuation per km for the fibre,
 - (iii) the overall signal attenuation for a 10 km optical link using the same fibre,
 - (iv) the numerical input/output ratio.
 (Ans: 16 dB, 2.0 dB/km, 20 dB, 100)

Example 5.1

Calculate the numerical aperture and hence the acceptance angle for an optical fibre given that refractive indices of the core and the cladding are 1.45 and 1.40 respectively.

Solution. As numerical aperture

$$(NA) = \sqrt{n_1^2 - n_2^2}$$

$$n_1 = 1.45$$

$$n_2 = 1.40$$

$$NA = \sqrt{(1.45)^2 - (1.40)^2}$$

$$= \sqrt{0.1425}$$

$$= 0.3775$$

Acceptance angle θ_m is given by

$$\theta_m = \sin^{-1} [\sqrt{n_1^2 - n_2^2}]$$

$$= \sin^{-1} (NA)$$

$$= \sin^{-1} (0.3775)$$

$$= 22.18^\circ$$

Example 5.2

A glass clad fibre is made with core glass of refractive index 1.5 and the cladding is doped to give a fractional index difference of 0.0005.

Find :

(a) the cladding index.

(b) the critical internal reflection angle.

(c) the external critical acceptance angle.

(d) the numerical aperture.

Solution. Given that

$$n_1 = 1.5 \text{ and } \Delta = 0.0005$$

(a) Let n_2 be the refractive index of cladding

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$0.0005 = \frac{1.5 - n_2}{1.5}$$

$$n_2 = 1.5 - 1.5 \times 0.0005 = 1.49925$$

(b) Let ϕ_c be the critical internal reflection angle

$$\sin \phi_c = \frac{n_2}{n_1}$$

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$$\begin{aligned}
 \therefore \phi_c &= \sin^{-1} \left[\frac{n_2}{n_1} \right] \\
 &= \sin^{-1} \left[\frac{1.4925}{1.5} \right] = \sin^{-1} (0.9995) \\
 &= 88.2^\circ
 \end{aligned}$$

(c) Let θ_m be the external critical acceptance angle

$$\sin \theta_m = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \text{ where } n_0 = 1$$

$$\begin{aligned}
 \therefore \theta_m &= \sin^{-1} [\sqrt{n_1^2 - n_2^2}] \\
 &= \sin^{-1} [(1.5)^2 - (1.49925)^2]^{1/2} \\
 &= \sin^{-1} (0.0474) \\
 &= 2.72^\circ
 \end{aligned}$$

(d) Numerical Aperture

$$\begin{aligned}
 (\text{NA}) &= n_1 \sqrt{2\Delta} \\
 &= 1.5 \sqrt{2 \times 0.0005} \\
 &= 1.5 \sqrt{0.0010} \\
 &= 0.0474
 \end{aligned}$$

Example 5.3

An optical fibre has a NA of 0.20 and a cladding refractive index of 1.59. Determine acceptance angle for the fibre in water which has a refractive index of 1.33.

Solution. Numerical aperture (NA)

$$= \sqrt{n_1^2 - n_2^2} \text{ when } n_0 = 1 \text{ (air)}$$

$$0.20 = \sqrt{n_1^2 - n_2^2}$$

$$\therefore n_1 = \sqrt{(0.20)^2 + (1.59)^2} = 1.6025$$

In water

$$n_0 = 1.33$$

$$\begin{aligned}
 \text{NA} &= \frac{\sqrt{n_1^2 - n_2^2}}{n_0} = \frac{\sqrt{(1.6025)^2 - (1.59)^2}}{1.33} \\
 &= 0.15
 \end{aligned}$$

\therefore Acceptance angle

$$\theta_m = \sin^{-1} (\text{NA}) = \sin^{-1} (0.15) = 8.6^\circ$$

Example 5.4

An optic fibre is made of glass with a refractive index of 1.55 and is clad with another glass with a refractive index of 1.51. Launching takes place from air :

- (a) What numerical aperture does the fibre have ?
- (b) What is the acceptance angle ?

Solution. (a) The normalized difference between the indices is

$$\Delta = \frac{n_1 - n_2}{n_1} = \frac{1.55 - 1.51}{1.55} = 0.0258$$

The numerical aperture is

$$NA \approx n_1 \sqrt{2\Delta} = 1.55 \sqrt{2 \times 0.0258} = 0.352$$

- (b) The acceptance angle is

$$\theta_m = \sin^{-1}(NA) = \sin^{-1}(0.352) = 20.6$$

5.3 MODES OF PROPAGATION

The important characteristic of the fibre, which depends on its size, is its *mode of operation*. The term "mode" as used here refers to mathematical and physical descriptions of the propagation of energy through a medium. A fibre can provide a path for one light ray or for thousands of light rays. From this characteristic, optical fibres are classified

$$\lambda_c = \frac{\lambda V}{2.405}$$

...(15)

Example 5.5

The core diameter of multimode step index ^{radius} is 60 μm . The difference in refractive indices is 0.013. The core refractive index is 1.46. Determine the number of guided modes when the operating wavelength is 0.75 μm .

Solution. As V-number is given by

$$\begin{aligned} V &= \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2} \\ &= \frac{2 \times 3.14 \times 30 \mu\text{m}}{0.75 \mu\text{m}} \sqrt{(1.46)^2 - (1.447)^2} \\ &= \frac{188.4 \times 0.0378}{0.75} = 9.493 \end{aligned}$$

Number of guided modes :

$$N \approx \frac{V^2}{2} = \frac{90.118}{2} = 45.06.$$

Example 5.6

A single-mode fibre is made with a core diameter of 10 μm and is coupled to a laser diode that produces 1.3 μm light. Its core glass has a refractive index of 1.55.

- Find the maximum value required for the normalized index difference.
- Find the refractive index required for the cladding glass.
- Find the fibre acceptance angle.

Solution. (a) As

$$V = \frac{2\pi a}{\lambda} \quad (\text{NA})$$

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Given that

$$2a = 10 \mu\text{m}, \lambda = 1.3 \mu\text{m}$$

$$\therefore (\text{NA})_{\text{max}} = \frac{V_{(\text{max})} \lambda}{2\pi a} = \frac{2.405 \times 1.3 \mu\text{m}}{3.14 \times 10 \mu\text{m}} = 0.0995$$

Normalized index difference

$$\Delta = \frac{1}{2} \left(\frac{\text{NA}}{n_1} \right)^2 = \frac{1}{2} \left(\frac{0.0995}{1.55} \right)^2 = 0.00206.$$

(b) The cladding index required is

$$n_2 = n_1 (1 - \Delta) = 1.55 (1 - 0.00206) = 1.547$$

(c) The maximum acceptance angle for the fibre is

$$\theta_m = \sin^{-1} (\text{NA})_{\text{max}} = \sin^{-1} (0.0995) = 5.710.$$

5.3.2 Graded Index Fibre

In graded index fibres, the refractive index of the core varies gradually as a function of radial distance from the fibre centre. Graded index fibres may be made with a variety of