composite

Q7 $w = f(x, y)$

$x = r\cos\theta \quad , y = r\sin\theta$

$$\left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2 = \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2$$

Soln

$$\frac{\partial w}{\partial r} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial r}$$

$$\frac{\partial w}{\partial r} = \frac{\partial f}{\partial x}\cdot\cos\theta + \frac{\partial f}{\partial y}\cdot\sin\theta \quad (1)$$

$$\frac{\partial w}{\partial \theta} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial \theta} \quad (1)$$

$$\Rightarrow \frac{\partial f}{\partial x}\cdot -r\sin\theta + \frac{\partial f}{\partial y}\cdot r\cos\theta$$

$$\frac{1}{r}\frac{\partial f}{\partial \theta} = -\frac{\partial f}{\partial x}\sin\theta + \frac{\partial f}{\partial y}\cos\theta \quad (2)$$

on sq + add both

$$\left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2 = \frac{\partial^2 f}{\partial x^2}\cos^2\theta + \frac{\partial^2 f}{\partial y^2}\sin^2\theta$$

$$- \frac{\partial^2 f}{\partial x^2}\sin^2\theta + \frac{\partial f^2}{\partial y^2}\cos^2\theta$$

Jacobian

1. Simple derivative
2. Euler's thm
3. Composite fxn
4. Jacobian

derivative → determinant

$$\iint x^2+y^2 \, dx\,dy \qquad \left[\begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array}\right.$$

$dx\,dy \to dr\,d\theta$

$r\,dr = |J|$

Defination

If u & v are fxn of two independent variables x and y, then

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

is called Jacobian of u, v w.r.t x & y

It is denoted $\frac{\partial(u,v)}{\partial(x,y)} = J\left(\frac{u,v}{x,y}\right)$

Most g/m Q [100% come]

$x = r\cos\theta$
$y = r\sin\theta$

$$\begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix}$$

$$\begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$

$r\cos^2\theta + r\sin^2\theta$
$r(1) \Rightarrow r$

$\frac{1}{r}$

① $J \cdot \frac{\partial(x,y)}{\partial(r,\theta)} = \frac{\partial(r,\theta)}{\partial(x,y)}$: No

② $\frac{\partial(x,y)}{\partial(r,\theta)} \times \frac{\partial(r,\theta)}{\partial(x,y)} = 1$

$x = r\cos\theta$ — r^2 not given

$y = r\sin\theta$ — calculate same

$z = z$

$$\begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial z} \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$u = x^2 - 2y$
$v = x + y$

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

$$\begin{vmatrix} 2x & -2 \\ 1 & 1 \end{vmatrix}$$

$2x + 2 \quad 2(x+1)$

$\Rightarrow 2x+2$

$u = x + y + z$
$uv = y + z$
$uvw = z$

$u = x + y + z$

$v = \dfrac{y+z}{x+y+z}$

$w = \dfrac{z}{xy + y^2 + 2y}$

$u = x + y + z$
$uv = y + z$
$uvw = z$

$y = uv - z$
$y = uv - uvw$

$x = u - (y + z)$
$\boxed{x = u - uv}$

$\boxed{y = uv - uvw}$

$\boxed{z = uvw}$

$$\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix}$$

$$\begin{vmatrix} 1-v & -u & 0 \\ v-vw & u-uw & -uv \\ vw & uw & uv \end{vmatrix}$$

$$\begin{vmatrix} 1-v & -u & 0 \\ v(1-w) & u(1-w) & -uv \\ vw & uw & uv \end{vmatrix}$$

$$\begin{vmatrix} 1-v & -u & 0 \\ v & u & 0 \\ vw & uw & uv \end{vmatrix}$$

$$\Rightarrow \underline{u^2 v}$$

$u = xyz$
$v = xy + yz + zx$
$w = x + y + z$

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$$

$$\begin{vmatrix} yz & xz & xy \\ y+z & x+z & y+x \\ 1 & 1 & 1 \end{vmatrix}$$

$C_2 \to C_2 - C_1$
$C_3 \to C_3 - C_2$

$$\begin{vmatrix} yz & xz & xy \\ y+z & x+z-y-z & y+x-x-z \\ 1 & 0 & 0 \end{vmatrix}$$

$$\begin{vmatrix} yz & xz-yz & xy-xz \\ y+z & x-y & y-z \\ 1 & 0 & 0 \end{vmatrix}$$

$$\begin{vmatrix} yz & z & x \\ y+z & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix}$$

expr Rj

$(x-y)(y-z)(z-x)$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} \times \frac{\partial(x,y,z)}{\partial(u,v,w)} = 1$$

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = \frac{1}{(x-y)(y-z)(z-x)}$$

Q =

$u = y^2/2x$

$v = \dfrac{x^2+y^2}{2x}$

find $\dfrac{\partial(u,v)}{\partial(x,y)}$

Q-7

$u = x^2 - 2y$

$v = x+y+z$

$w = x-2y+3z$

$\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$

① $F = xu + v - y$

$G = u^2 + vy + w$

$H = zu - v + vw$

find $\dfrac{\partial (f, g, h)}{(u w v)}$

$$\begin{vmatrix} \frac{\partial f}{\partial u} & \frac{\partial f}{\partial w} & \frac{\partial f}{\partial v} \\ \frac{\partial g}{\partial u} & \frac{\partial g}{\partial w} & \frac{\partial g}{\partial v} \\ \frac{\partial H}{\partial u} & \frac{\partial H}{\partial w} & \frac{\partial H}{\partial v} \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} x & 0 & 1 \\ 2u & 1 & y \\ z & v & -1+w \end{vmatrix}$$

$x((-1+w) - vy) + 1(2uv - z)$

$-x + xw - xvy + 2uv - z$ //

Q-1 $u = x^2 - 2y$
$v = x + y + z$
$w = x - 2y + 3z$

$\frac{\partial u}{\partial x}$ $\frac{\partial u}{\partial y}$ $\frac{\partial u}{\partial z}$

$\frac{\partial v}{\partial x}$ $\frac{\partial v}{\partial y}$ $\frac{\partial v}{\partial z}$

$\frac{\partial w}{\partial x}$ $\frac{\partial w}{\partial y}$ $\frac{\partial w}{\partial z}$

$$\Rightarrow \begin{vmatrix} 2x & -2 & 0 \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix}$$

$$\begin{vmatrix} 2x & -2 & 0 \\ 0 & 3 & -2 \\ 1 & -2 & 3 \end{vmatrix}$$

$2x(9-4) - 2(\cancel{+}2)$

$10x + 4$

Jacobian of implicit fxn

$f(x,y) = c$

If x, y, u, v are connected by implicit fxn $f(x,y,u,v)$ where u, v are implicit fxn of x & y then

$$\frac{\partial(u,v)}{\partial(x,y)} = (-1)^{n=2} \frac{\partial\left(\frac{f_1, f_2}{\partial(x,y)}\right)}{\partial\left(\frac{f_1, f_2}{u,v}\right)}$$

If $x^2 + y^2 + u^2 - v^2 = 0$

$uv + xy = 0$

$f_1 = x^2 + y^2 + u^2 - v^2$

$f_2 = uv + xy$

$$\partial\left(\frac{f_1, f_2}{x, y}\right) \begin{vmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{vmatrix}$$

$$\begin{vmatrix} 2x & 2y \\ y & x \end{vmatrix}$$

$2x^2 + 2y^2$

$$\partial\left(\frac{f_1, f_2}{u, v}\right) = \begin{vmatrix} \frac{\partial f_1}{\partial u} & \frac{\partial f_1}{\partial v} \\ \frac{\partial f_2}{\partial u} & \frac{\partial f_2}{\partial u} \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 2u & -2v \\ v & u \end{vmatrix}$$

$$2u^2 + 2v^2$$

$$\Rightarrow \frac{2x^2 + y^2}{u^2 + v^2}$$

$u = x + y + z$
$uv = y + z$
$uvw = z$

$f_1 = x + y + z - u$
$f_2 = y + z + uv$
$f_3 = z - uvw$

$\frac{\partial(f_1 f_2 f_3)}{\partial(u, v, w)}$

$(-1)^3 \frac{\partial(f_1, f_2, f_3)}{\partial(x, y, z)}$

$\frac{\partial(f_1 f_2 f_3)}{x, y, z} \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix}$

$= 1$

$\partial\left(\frac{f_1 f_2 f_3}{u, v, w}\right) = \begin{vmatrix} -1 & 0 & 0 \\ -v & -u & 0 \\ -vw & -uw & -uv \end{vmatrix}$

$+u^2v$

$U = x^2 + y^2$ ①

$V = y + z$ ②

$W = z + x^2$

$x^2 + y^2 - U = f_1$

$y + z - V = f_2$

$x^2 + z - W = f_3$

$$\begin{vmatrix} 2x & 2y & 0 \\ 0 & 1 & 2z \\ 2x & 0 & 1 \end{vmatrix}$$

$2x - 4xy \Rightarrow 2x(1-y)$

$$\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 1 & -1 \end{vmatrix}$$

$-1(1) \Rightarrow$ $= 1 + 8xyz$

$f_1(x, y, u, v) = 0$
$f_2(x, y, u, v) = 0$

$$\frac{\partial u}{\partial x} = -\left[\frac{\partial(f_1, f_2)}{\partial(x, v)} \Big/ \frac{\partial(f_1, f_2)}{\partial(u, v)}\right]$$

$$\frac{\partial x}{\partial u} = -\left[\frac{\partial(f_1, f_2)}{\partial(u, y)} \Big/ \frac{\partial(f_1, f_2)}{\partial(x, y)}\right]$$

$$\frac{\partial v}{\partial x} - \left[\frac{\partial(\partial f_1 \partial)}{\partial(u, x)} \Big/ \frac{\partial(f_1, f_2)}{\partial(u, v)}\right]$$

$x = u^2 - v^2$
$y = 2uv$

$$\frac{\partial u}{\partial x} \quad \frac{\partial(f_1 f_2)}{x v}$$

$$\begin{vmatrix} \frac{\partial f_1}{x} & \frac{\partial f_1}{\partial v} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial v} \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} -1 & -2v \\ 0 & -2v \end{vmatrix} \quad 2u$$

DATE
PAGE NO.

$$\begin{vmatrix} 2u & -2v \\ 2v & 2u \end{vmatrix}$$

$4u^2+4v^2$

$\frac{\partial v}{\partial x}$

$$\begin{vmatrix} 2u & -1 \\ 2v & +2u \end{vmatrix}$$

$-2v$

u^2-v^2-x

$2uv-y$

$$\begin{vmatrix} -1 & 2u \\ 0 & 2v \end{vmatrix}$$

(1) $u^2 + xv^2 - xy = 0$ $\quad \frac{\partial v}{\partial x}$
$u^2 + xyv + v^2 = 0$

(2) $u = x + y^2$
$v = y + z^2$
$w = z + x^2$

$\frac{\partial x}{\partial u} = ?$

$\phi = x = u + v + w$
$y = u^2 + v^2 + w^2$
$z = u^3 + v^3 + w^3$

$\frac{\partial u}{\partial x} \quad -\partial\left(\frac{f_1, f_2, f_3}{x\ v\ w}\right) \Big/ \frac{\partial f}{u}$ $\quad 6(v$

∂f_1

$$\begin{vmatrix} -1 & 1 & 1 \\ 0 & 2v & 2w \\ 0 & 3v^2 & 3w^2 \end{vmatrix}$$

$-6vw^2 + 6v^2w$
$6v^2w - 6vw^2$

$$\frac{vw}{(u-v)(u-w)}$$

$$6vw(v-w)$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2u & 2v & 2w \\ 3u^2 & 3v^2 & 3w^2 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 2u & 2(v-u) & (2w-2u) \\ 3u^2 & 3(v^2-u^2) & 3(w^2-u^2 \end{vmatrix}$$

$$6(v-u)(w-u)\begin{vmatrix} 1 & 0 & 0 \\ 2u & 2 & 2 \\ 3u^2 & 3(v+u) & 3(w+u \end{vmatrix}$$

$$6(v-u)(w-u)(w+u-v-u)$$

$$6(v-u)(w-u)(w-v)$$

$$-\left[\frac{\cancel{6}vw\cancel{(v-w)}}{\cancel{6}(v-u)(w-u)\cancel{(w-v)}}\right]$$

$$\frac{vw}{(u-v)(u-w)} \quad \text{H.P}$$

Imp

$f^n(c) = f^n(\theta x)$

$c \in (a,b)$ $\quad f^n(x) = e^x$

$c = a + \theta(b-x)$

$c = 0 + \theta(x-0)$ $\quad f^n(\theta x) = e^{\theta x}$

$\boxed{c = \theta x}$

Maclaurin's forms

But $a=0$, $b=x$

In Maclaurin theorem

when $[a,b]$

Correct

$\boxed{a=0, b=x}$

Imp Q.

$4x^2+5x+2$ in Powers $(x-1)$ using taylor

$[x-1]$ $\qquad b=x, a=$

$b-a$

$b=x$

$a=1$

DATE
PAGE NO.

$f(x) = 4x^2 + 5x + 3$ $[1, x]$
$f'(x) = 8x + 5$
$f''(x) = 8$
$f'''(x) = 0$

$f(1) = 12$
$f'(1) = 13$
$f''(1) = 8$
$f'''(1) = 0$

$4x^2 + 5x + 3 = 12 + \frac{x(13)}{1!} + \frac{x^2}{2!} \cdot 8 + 0 \; - -$

$f(x) = 2x^3 + 7x^2 + x + 6$
$f'(x) = 6x^2 + 14x + 1$
$f''(x) = 12x + 14$
$f'''(x) = 16$
$f''''(x) = 0$

$f(2) = 16 + 28 + 2 + 6$
$\Rightarrow 24 + 28$
$\Rightarrow 52$

$f'(2) = 24 + 28 + 1$
$\Rightarrow 53$

$f''(2) = 38$
$f'''(2) = 12$

52

$52 + 5(x-2) + 19(x-2)^2 + 2(x-2)^3$

Taylor's theorem for two variable

$$f(x+h, y+k) = f(x,y) + [hf_x + kf_y]$$

$$+ \frac{1}{2!}[h^2 f_{xx} + 2hk f_{xy} + k^2 f_{yy}]$$

$$+ \frac{1}{3!}[h^3 f_{xxx} + 3h^2 f_{xxy} + 3hk^2 f_{xyy}$$

$$+ k^3 f_{yyy}]$$

$$= f(x,y) + \left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right) f + \frac{1}{2!}\left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^2 f$$

$$+ \frac{1}{3!}\left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^3 f$$

$h =$
$k =$

$f(1,1$

DATE
PAGE NO.

$(1,1)$ $\quad$ (a,b)

$h = x - 1$ $\quad$ $h = x - a$
$k = y - 1$ $\quad$ $k = y - b$

$f(1,1) + [(x-1) f_x(1,1) + (y-1) f_y(1,1)]$

$+ [(x-1)^2 f_{xx}(1,1) + 2(x-1)(x-2)$
$f_{xy} f(1,1)$

$+ (y-1)^2 f_{yy}(1,1)$

2-21-21

Multiple Integrals

Double Integral

$$A=\int_1^2\int_3^4 (xy+e^y)\,dy\,dx$$

$$\int_1^2 \left[\frac{xy^2}{2}e^y\right]_3^4 dx$$

$$\int_1^2 \left[x+e^4-e^3\right] dx$$

$$\Rightarrow \left[\frac{x^2}{2}+e^4\cdot x-e^3\cdot x\right]_1^2$$

$$\Rightarrow \frac{4}{2}+2e^4-2e^3-\frac{1}{2}-e^4+e^3$$

$$B=\int_1^2\int_3^4 (xy+e^y)\,dx\,dy$$

$$\int_3^4\int_1^2 (xy+e^y)\,dy\,dx$$

$$\int_1^2 \left[\frac{xy^2}{2} + e^y\right]_3^4$$

$$\Rightarrow \int_1^2 \left(\frac{x \cdot 16}{2} + e^4 - \frac{x \cdot 9}{2} - e^3\right.$$

$$\int_1^2 \frac{7}{2}x + (e^4 - e^3)\,dx$$

$$\left[\frac{7}{2} \cdot \frac{x^2}{2} + e^4 \cdot x - e^3 x\right]_1^2$$

$$\Rightarrow \frac{7}{4} \cdot 4 + 2e^4 - 2e^3 - \frac{7}{4} - e^4 + e^3$$

$$\Rightarrow \frac{28}{4} - \frac{7}{4} + 2e^4 - 2e^3 - e^4 + e^3$$

$$\frac{21}{4} + e^4 - e^3 \text{ Ans}$$

$$\int_0^1\int_0^1 \frac{dx\,dy}{\sqrt{(1-x^2)(1-y^2)}} \qquad (1-x^2)(1-y^2)$$

$$\sqrt{1-x^2}\ \sqrt{1-}$$

$$I = \int_0^1\int_0^1 \frac{dx\,dy}{\sqrt{1-x^2}} \cdot \frac{1}{\sqrt{1-y^2}} \qquad \sin^{-1}x \Big|_0$$

$$I = \int_0^1 \frac{1}{\sqrt{1-y^2}} \left(\sin^{-1}x\right)_0^1$$

$$I = \int_0^1 \frac{dy}{\sqrt{1-y^2}} \frac{\pi}{2}$$

$$\frac{\pi}{2}\left[\sin^{-1}y\right]_0^1 \rightarrow \frac{\pi^2}{4}$$

$$\frac{\pi^2}{4}$$

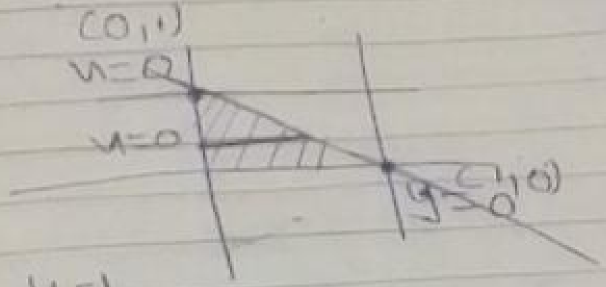
$$Q \rightarrow \int_0^2\int_0^{\sqrt{2x}} xy\,dy\,dx$$

$$\int_0^2 \left[\frac{xy^2}{2}\right]_0^{\sqrt{2x}} \rightarrow \int_0^2 x^2 \rightarrow$$

$$\int^2 \left[x \cdot \frac{2x}{2}\right]$$

$\iint e^{2x+3y}\,dx\,dy$ bounded by triangle $x=0, y=0$ and $x+y=1$

x	0	1
y	1	0



$$\int_0^1 \int_0^{y-1} e^{2x+3y}\,dx$$

$$\int_0^1 \left[\frac{e^{2x+3y}}{2}\right]_0^{y-1}$$

$$\int_0^1 \left[\frac{e^{2(y-1)+3y}}{2}\right] - \left[\frac{e^{3y}}{2}\right]$$

$$\int_0^1 \left[\frac{e^{2y-2+3y}}{2} - \frac{e^{3y}}{2}\right]$$

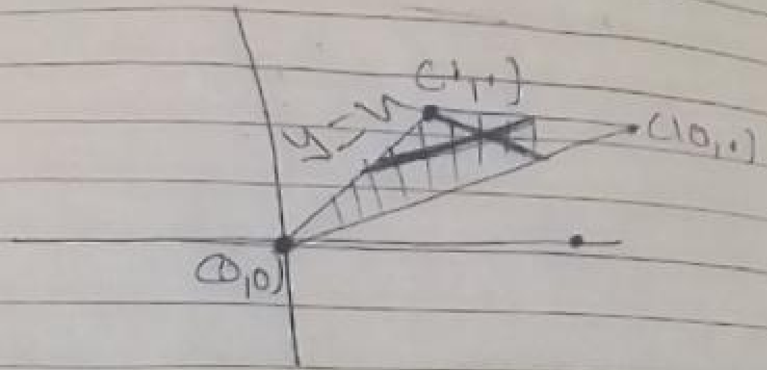
$$\Rightarrow \int_0^1 \left[\frac{e^{y-2}}{2} - \frac{e^{3y}}{2}\right] \quad \frac{1}{2}\int_0^1 e^{y-2} - e^{3y}$$

$$\Rightarrow \frac{1}{2}\left[\frac{e^{y-2}}{1} - \frac{e^{3y}}{3}\right]$$

evaluate

$$\iint (xy - y^2)\,dx\,dy$$

(0,0) (10,1) (1,1)



$$y - y_1 = m(x - x_1)$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{1}{1}(x - 0)$$

$$\boxed{y = x}$$

(0,0)(10,1)

$$y - 0 = \frac{1}{10}(x - 0)$$

$$\boxed{10y = x}$$

(1,1)

$$\int_0^1 \int_y^{10y} \sqrt{xy-y^2}\, dx\, dy$$

$$\int_0^1 \left[\frac{1}{2\sqrt{xy-y^2}} \cdot y \cdot \frac{x^2}{2} \right]_y^{10y}$$

$$\int_0^1 \left[\frac{1}{2\sqrt{10y^2-y^2}} \cdot y \cdot x^2/2 \right.$$

$$\int_0^1 \left[\frac{2}{3} (xy-y^2)^{3/2} \right]_y^{10y}$$

$$\int_0^1 \left[\frac{2}{3} \left(10y^2-y^2\right)^{3/2} - 0 \right]$$

$$\frac{5^4}{12}$$

$$\int_0^1 \left[\frac{2}{3} \left[9y^2 \right]^{3/2} \right.$$

$$(3y)^{\cancel{2} \times 3/\cancel{2}} \Rightarrow \frac{2}{3} \int_0^1 27y^3$$

$$\frac{2}{3} \int_0^1 \left[\frac{9}{\cancel{2} \times \cancel{27}} \right.$$

$$\frac{\cancel{2} \times \cancel{27}}{\cancel{12} \times 6\ 2} \rightarrow \frac{2 \times 27}{3} \left[\frac{y^4}{4} \right]_0^1$$

Application of double integrals

1 Area b/w two curves.

We will be provided eq of two curves

Draw the curves

Shade the required region

Calculate limits [either by horizontal or by vertical]

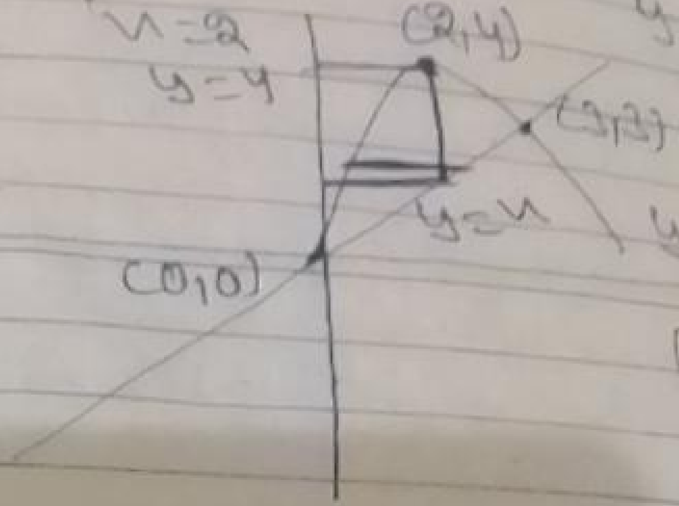
Area $= \iint dy\,dx$ or $\iint dx\,dy$

find the area b/w parabola

$y = 4x - x^2$ and a line

$y = x$

$\rightarrow y = x(4-x)$ $\quad y = 4x - \ldots$



$y = 4x - \ldots$

$y - 4 = -(\ldots$

$(x-2)^2 \ldots$

$(2,4)$

$y = x$

$y = 4x - x^2$

$x = 4x - x^2$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0, x = 3$

$y = 0 \quad y = 3$

$y = x$

$y = 4x - x^2$

$$\int_0^2 \int_x^{4x-x^2} dx\,dy$$

$$\int_0^2 \Big[y \Big]_x^{4x-x^2}$$

$\left[\frac{4x^3}{2} - \frac{x^3}{3} - \frac{x^2}{2}\right]_0^2$

$\Rightarrow \frac{16}{2} - \frac{8}{3} - \frac{4}{2}$

$\frac{48 - 16 - 24}{6} \Rightarrow \frac{48 - 40}{6}$

$\Rightarrow \frac{\cancel{8}^{4}}{\cancel{6}_{3}} \Rightarrow \frac{4}{3}$ Ans

$\frac{4 \times 9}{2} - \frac{27}{3} - \frac{9}{2}$

$\Rightarrow \frac{36 \times 3 - 27 \times 2 - 9 \times 3}{6}$

$x^2 - \frac{x^3}{3} - \frac{x^2}{2}$

$\frac{108 - 54 - 27}{6}$

$\frac{108 - 81}{6} \Rightarrow 17/6$

$\frac{4x^2}{2} - \frac{x^3}{3} - \frac{x^2}{2}$

$\frac{36}{2} - \frac{27}{3} - \frac{9}{2}$

$18 - 9 - \frac{9}{2} \Rightarrow \frac{36 - 18 - 9}{2}$

$\Rightarrow \frac{18 - 9}{2} \Rightarrow 9/2$

$36 \times 3 = 108$

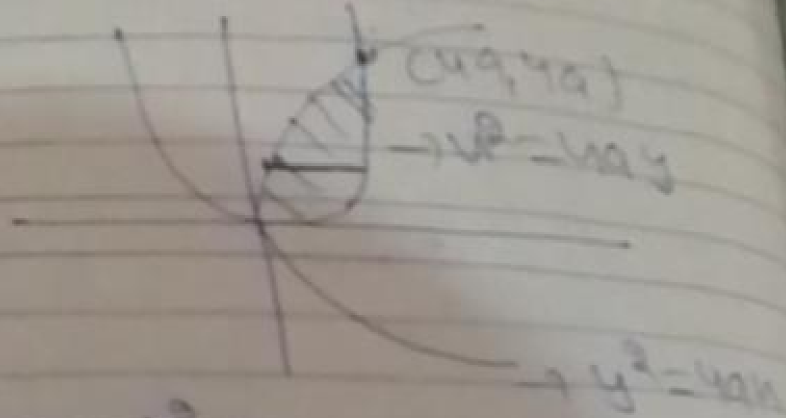
$54 + 27 = 81$

$108 - 81 = 17$

$8 \times 3 = 24$

$24 + 16 = 40$

Q→ Area b/w $y^2 = 4ax$ and $x^2 = 4ay$



$x = y^2/4a$

$x = 2\sqrt{ay}$

$$\int_0^{4a} \int_{y^2/4a}^{2\sqrt{ay}} dx\, dy$$

$$\int_0^{4a} \left[x \right]_{y^2/4a}^{2\sqrt{ay}}$$

$$\int_0^{4a} \left[2\sqrt{ay} - \frac{y^2}{4a} \right]$$

$$\left[\frac{2 \cdot 2}{3} \sqrt{a}\, y^{3/2} - \frac{y^3}{12a} \right]_0^{4a}$$

DATE
PAGE NO.

$$\left[\frac{4}{3}\cdot\sqrt{a}\,(y)^{3/2} - \frac{y^3}{12a}\right]_0^{4a}$$

$$\Rightarrow \cancel{\frac{4}{3}\cdot\sqrt{a}\cdot(2a)^{2\times 3/2}} - \frac{64a^3}{12a}$$

$$\Rightarrow \cancel{\frac{4}{3}\sqrt{a}\,4a\,4\sqrt{a}\,64}$$

$$\left[\frac{4}{3}\sqrt{a}\,(4a)^{3/2} - \frac{64a^3}{12a}\right.$$

$$\frac{4}{3}(4)^{3/2}\cdot a^2 - \frac{16a^2}{3}$$

$$\Rightarrow \frac{32}{3}a^2 - \frac{16a^2}{3}$$

$$\Rightarrow \frac{16a^2}{3}$$