1. Let V be a vector space over a field F. Then $\phi \neq W \subseteq V$ is a subspace of V iff $\alpha x + \beta y \in W$ $\forall\, x, y \in W$ & $\alpha, \beta \in F$.

2. Intersection of two subspaces is also a subspace.

3. Intersection of a family of subspaces of a vector space is also a subspace.

4. If W_1 and W_2 are subspaces of a vector space V over F, then $W_1 \cup W_2$ is a subspace of V iff either $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$.

5. Sum of two subspaces of a vector space V over a field F is also a subspace of V
$$[W_1 + W_2 = \{a+b : a \in W_1 \text{ \& } b \in W_2\}]$$

6. Let S and T be two subsets of a vector space $V(F)$. Then
   - $S \subseteq L(T) \Rightarrow L(S) \subseteq L(T)$
   - $S \subseteq T \Rightarrow L(S) \subseteq L(T)$
   - S is a subspace of V iff $S = L(S)$
   - $L(L(S)) = L(S)$.

7. If W_1 and W_2 are two subspaces of $V(F)$ then $W_1 + W_2$ is the <u>smallest</u> subspace of V containing $W_1 \cup W_2$.

8. Let $V(F)$ be a vector space. Then
   - Every non-zero singleton set is L.I.
   - A subset of V containing zero element is always L.D.
   - Any subset of L.I. is L.I.
   - Any superset of L.D set is L.D.

9. Let V_F be a vector space. Then
   - The set $\{x_1, x_2\}$ is L.D. iff x_1 and x_2 are collinear. i.e., iff one can be written as saclar multiple of other.
   - The set $\{x_1, x_2, x_3\}$ is L.D iff x_1, x_2 and x_3 are coplanar. i.e., iff one can be written as L.C of other two.

10. Vectors $(a_{11}, a_{12}, \dots, a_{1n}), (a_{21}, a_{22}, \dots, a_{2n}) \cdots (a_{n1}, a_{n2}, \dots, a_{nn})$ in F^n are L.D iff
$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = 0$$

11. Let V_F be a vector space. Then a finite subset $S = \{x_1, x_2, \dots, x_n\}$ of non-zero elements of V is L.D iff some element of S, say x_i,

can be written as linear combination of other elements of S.
Further
$$L(\{x_1, x_2, \dots, x_n\}) = L(\{x_1, x_2, \dots, x_{i-1}, x_{i+1}, \dots, x_n\})$$

12. Existence Theorem: There exist a basis for each finite dimensional vector space

13. Let $V(F)$ be a finite dimensional vector space. Then a finite set B is a basis of V iff every element of V can be uniquely written as a L.C of the elements of B.

14. Let $V(F)$ be a finitely generated vector space, then every basis of V has same number of elements.

15. Extension Theorem: Let V be a finitely generated vector space over a field F. Then any L.I subset of V can be extended to form a basis of V.

16. Let $V(F)$ be n-dimensional vector space and S be a subset of V containing n elements. Then $V = \langle S \rangle$ iff S is L.I.

17. If W is a subspace of a finite dimensional vector space $V(F)$, then
$$\dim W \leq \dim V.$$
Further, $W = V$ iff $\dim W = \dim V$

18. If W_1 and W_2 are subspaces of a finite dimensional vector space V over a field F, then
$$\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$$

19. Let $T : V \to W$ be a linear transformation, then
    - $\operatorname{Ker} T$ is a subspace of V
    - $\operatorname{Im} T$ is a subspace of W

20. Let $T : V \to W$ be a linear transformation. Then $\operatorname{Ker} T = \{0\}$ iff T is 1-1.

21. Rank-Nullity Theorem: Let $T : V \to W$ be a linear transformation and V be a finite dimensional vector space over F. Then
$$\operatorname{rank} T + \operatorname{nullity} T = \dim V.$$

22. Let V and W be vector spaces over F with dimension m and n respectively, then $\dim L(V, W) = mn$.

23. A linear operator T on a finite dimensional vector space V is invertible iff T is one-one.

24. Let V and W be finite dimensional vector spaces over F and $T : V \to W$ be a linear transformation. If B_1 and B_2 be ordered bases of V and W

respectively, then for any $v \in V$,

$$[T; B_1, B_2][v; B_1] = [T(v); B_2]$$

25. Let V be a finite dimensional vector spaces. Let A be the matrix of a linear operator $T: V \to V$ w.r.t a given ordered basis B. Then matrix A' of T w.r.t. a new ordered basis B' is given by

$$A' = P^{-1} A P$$

where P is the matrix of transformation from B' to B.