Problem 8.8 *Calculate the density of diamond crystal, given that its lattice parameter 'a' is 3.5... atomic mass A = 12.*

Solution. The effective number of atoms in the diamond cubic unit cell is

$$n = \frac{1}{8} \times 8 \text{ (corner atoms)} + \frac{1}{2} \times 6 \text{ (face centered atoms)}$$
$$+ 1 \times 4 \text{ (atoms completely within the unit...)}$$
$$= 8$$

$$\text{Density} \quad \rho = \frac{\text{Mass of the unit cell}}{\text{Volume of the unit cell}}$$
$$= \frac{M \times n}{N \times V} = \frac{Mn}{N a^3}$$

where N = Avogadro's number

$$\therefore \quad \rho = \frac{8 \times 12}{6.023 \times 10^{26} \times (3.57 \times 10^{-10})^3} = 3540 \text{ kg/m}^3 = 3.54 \text{ gm/cc}$$

Problem 8.9 *Lithium crystallizes in bcc structure. Calculate the lattice constant, given that the ... weight and density for lithium are 6.94 and 530 kg/m³ respectively.*

Solution. Given $n = 2$ (for Li), $M = 6.94$, $\rho = 530 \text{ kg/m}^3$, $a = ?$

We know $$a^3 = \frac{nM}{\rho N} = \frac{2 \times 6.94}{530 \times 6.023 \times 10^{26}} = 43.50 \times 10^{-30}$$

$$\therefore \quad a = 3.517 \times 10^{-10} \text{ m} = 3.517 \text{ Å}$$

Problem 8.10 *Germanium crystallizes in diamond (form) structure with 8 atoms per unit cell. If ... constant is 5.62 Å, calculate its density.*

Solution. Given $n = 8$, $a = 5.62 \times 10^{-10}$ m, $M = 72.59$, $\rho = ?$

We know, $$a^3 = \frac{nM}{\rho N} \quad \text{or} \quad \rho = \frac{nM}{N a^3}$$

$$= \frac{8 \times 72.59}{(5.62 \times 10^{-10})^3 \times 6.023 \times 10^{26}}$$
$$= 5434.5 \text{ kg/m}^3 = 5.435 \text{ g/cc}$$

Conceptual Questions

xample 8.5 *If the density of copper is 8.98 gm/cc and has fcc structure, calculate the atomic radius of* *pper. (Atomic wt. = 63.5)*

Solution. The lattice constant 'a' for a cubic lattice is given by

$$a = \left(\frac{nM}{N\rho}\right)^{1/3}$$

here n is number of atoms per unit cell, M is the atomic weight and ρ is the density. For *fcc* ructure $n = 4$.

$$\therefore \qquad a = \left[\frac{4 \times 63.5}{6.02 \times 10^{23} \times 8.98}\right]^{1/3} = [46.985 \times 10^{-24}]^{1/3}$$

$$= 3.61 \times 10^{-8} \text{ cm} = 3.61 \text{ Å}$$

Atomic radius of copper $r = \dfrac{a}{2\sqrt{2}} = \dfrac{3.61}{2\sqrt{2}} \text{ Å} = \dfrac{3.61}{2.828} \text{ Å} = \mathbf{1.277 \text{ Å}}$

Example 8.6 *The rock salt (NaCl) has fcc structure and contains 4 molecules per unit cell.* Calculate *lattice constant for the crystal.*

Molecular weight of NaCl $= 58.45$ *kg/kmol*

Density $= 2180$ kg/m^3

Avogadro's number $= 6.02 \times 10^{26}$ $k\ mol^{-1}$

Solution. Given $n = 4$, $M = 58.45$ kg/k mol, $\rho = 2180$ kg/m^3,

$N = 6.02 \times 10^{26}$ m mol^{-1}

The lattice constant 'a' for NaCl is given by

$$a = \left(\frac{nM}{N\rho}\right)^{1/3}$$

$$= \frac{4 \times 58.45}{2180 \times 6.02 \times 10^{26}} = 5.63 \times 10^{-10} \text{ m} = 5.63 \text{ Å}.$$

Miscellaneous Solved Numerical Problems

Problem 9.1 *Find the set of Miller indices for a plane cutting of intercepts 3a,2b,4c.*

Solution. From the law of rational indices, we may write

$$3a : 2b : 4c = \frac{a}{h} : \frac{b}{k} : \frac{c}{l}$$

where h, k, l are the Miller indices.

$$\therefore \quad \frac{1}{h} : \frac{1}{k} : \frac{1}{l} = 3 : 2 : 4$$

or

$$h : k : l = \frac{1}{3} : \frac{1}{2} : \frac{1}{4}$$

Converting to smallest whole numbers having the same ratios, we have

$$h : k : l = \frac{4}{12} : \frac{6}{12} : \frac{3}{12} = 4 : 6 : 3$$

Thus, the Miller indices of the planes are 4, 6 and 3 or the plane is (463).

Problem 9.2 *Deduce the Miller indices of a plane which cuts off intercepts in the ratio 1a : 3b : –2c along the three axes.*

Solution. From the law of rational indices, we may write

$$1a : 3b : -2c = \frac{a}{h} : \frac{b}{k} : \frac{c}{l}$$

where h, k, l are the Miller indices

$$\frac{1}{h} : \frac{1}{k} : \frac{1}{l} = 1 : 3 : -2$$

or

$$h : k : l = 1 : \frac{1}{3} : -\frac{1}{2} = 6 : 2 : -3$$

Thus, $h = 6$, $k = 2$, $l = -3$.

Hence, the plane is $(62\bar{3})$.

Problem 9.3 *Find the Miller indices of a set of parallel planes which make intercepts in the ratio of 3a : 4b on the x and y axes and are parallel to the z-axis.*

Solution. The parallel planes are parallel to the z-axis, that is, their intercepts on the z-axis are infinite. From the law of rational indices, we may write

$$3a : 4b : \infty c = \frac{a}{h} : \frac{b}{k} : \frac{c}{l}$$

or

$$\frac{1}{h} : \frac{1}{k} : \frac{1}{l} = 3 : 4 : \infty$$

or

$$h : k : l = \frac{1}{3} : \frac{1}{4} : \frac{1}{\infty} = 4 : 3 : 0$$

The Miller indices are [430].

$\sqrt{2}$ $\sqrt{3}$

Example 9.6 ***Calculate the interplanar spacing for a (321) plane in a simple cubic lattice whose lattice constant is 4.2×10^{-8} cm.***

Solution. In a simple cubic lattice, the interplanar spacing d is given by

$$d = \frac{a}{\sqrt{(h^2 + k^2 + l^2)}}, \quad \text{where } (h\,k\,l) \text{ are the Miller indices.}$$

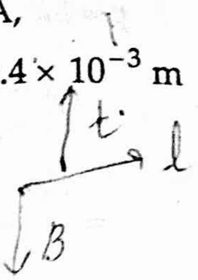
For a (321) plane $h = 3$, $k = 2$ and $l = 1$, also $a = 4.2 \times 10^{-8}$ cm

$$\therefore \quad d = \frac{4.2 \times 10^{-8}}{\sqrt{(3^2 + 2^2 + 1^2)}} = \frac{4.2 \times 10^{-8}}{3.74} = 1.12 \times 10^{-8} \text{ cm} = 1.12 \text{ Å}$$

Problem 10.4 *Hall voltage of 20 mV is found to be developed with a sample carrying a current of 15 mA placed in a transverse magnetic field of 4 kilo gauss. Calculate the concentration of charge carriers in the sample. The thickness of the sample along the magnetic field direction is 0.4 mm.*

Solution. Given Hall voltage $V_H = 20$ mV, current $I = 15$ mA,

$B = 4$ kilo gauss $= 0.4$ Tesla, thickness $d = 0.4$ mm $= 0.4 \times 10^{-3}$ m



Hall coefficient $$R_H = \frac{V_H d}{I_x B}$$

But $$R_H = \frac{1}{ne} \text{ (numerically)}$$

Then concentration $$n = \frac{I_x B}{V_B\, d\, e}$$

$$= \frac{15 \times 10^{-3} \times 0.4}{20 \times 10^{-3} \times 0.4 \times 10^{-3} \times 0.4 \times 10^{-19}} = 4.69 \times 10^{21} \text{ m}^{-3}$$

Problem 10.5 *InSb is an intrinsic semiconductor with energy gap of 0.15 eV. A Hall voltage of −4.07 mV is developed across 1 cm × 1 mm × 1 mm bar of InSb when a current $i_x = 0.1$ A passes along x-direction in the presence of magnetic induction 0.1 Wb/m² along z-direction. Calculate the Hall coefficient and density of carrier.*

Solution. $$E_H = V_H / 10^{-3}\,\text{V.m}^{-1} = -4.07\ \text{Vm}^{-1}$$

and $$R_H = \frac{E_H}{J_x B_z}$$

$$= \frac{4.07}{[(0.1/10^{-6}) \times 0.1]}\text{m}^3\text{C}^{-1} = 4.07 \times 10^{-4}\,\text{m}^3\text{C}^{-1}$$

∴ $$R_H = \frac{1}{ne}$$

$$n = \frac{1}{eR_H} = \frac{1}{4.07 \times 10^{-4} \times 1.6 \times 10^{-19}}\text{m}^{-3} = 1.54 \times 10^{22}\,\text{m}^{-3}$$

Problem 10.6 *Calculate the Hall coefficient of sodium based on free electron model. Sodium has bcc structure and the side of the cube is 4.28 Å*

Solution. Given that $a = 4.28$ Å $= 4.28 \times 10^{-10}$ m. Crystal is *bcc*

The number of electrons per unit volume for the sodium crystal is given by

$$n = \frac{2}{a^3} = \frac{2}{(4.28 \times 10^{-10})^3} = 2.55 \times 10^{28} / \text{m}^3$$

The Hall coefficient

$$R_H = \frac{1}{ne} = \frac{1}{2.55 \times 10^{28} \times 1.6 \times 10^{-19}} = 2.45 \times 10^{-9}\ \text{m}^3/\text{C}$$

...em 10.7 A germanium N-type semiconductor has donor density of $10^{21}/m^3$. Find Hall voltage when ...tic field of $0.5 Wb/m^2$ is applied and current density $= 500 A/m^2$. The thickness of the ...ductor = 3 mm

Solution. Since Hall electric field, $E_y = \frac{J_x B_z}{ne}$

∴ Hall voltage $V = \frac{E_y}{d} = \frac{J_x B_z d}{E_y}$

$$= \frac{500 \times 0.5 \times 3 \times 10^{-3}}{10^{21} \times 1.6 \times 10^{-19}} = 4.68 \times 10^{-3} V = 4.68 mV$$

...blem 10.8 A sample of semiconductor material illustrated in Fig. 10.25 has a dimension 5 mm, 2 mm ...4 mm in the x, y, z directions respectively. A 1.5 V supply results in a current i = 35 mA. With ...= 0.09 T, the high impedance voltmeter V indicates 14 mV with the polarity shown. Determine the sign of ...riers, their mobility and density of doping.

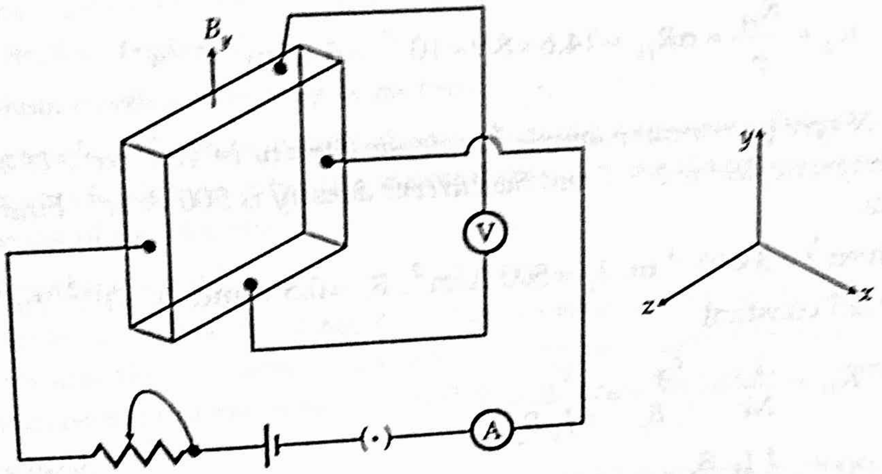


Fig. 10.25

Solution. Since the Hall voltage acts in the negative z-direction, the charge carriers are electrons and the sample is of the N-type.

The relevant equations are :

$$E_z = R_H J_x B_y$$

and $$R_H = \frac{1}{ne} \quad \text{and} \quad \mu_e = \frac{R_H}{\rho}.$$

Also $$E_z = \frac{V_z}{z}$$

Given : $V_z = 14 \times 10^{-3} V$, $z = 4 \times 10^{-3} m$

Then $$E_z = \frac{14 \times 10^{-3}}{4 \times 10^{-3}} = 3.5 V/m$$

$$J_x = \frac{i_x}{A}$$

Now given : $i_x = 35\times10^{-3}$ A and $A = 4\times2\times10^{-6} = 8\times10^{-6}\,m^2$

$$J_x = \frac{i_x}{A} = \frac{35\times10^{-3}}{8\times10^{-6}} = 4.375\times10^{-3}\,A/m^2$$

Again $$R_H = \frac{E_z}{J_z B_y}$$

Given : $E_z = 3.5$ V/m ; $J_x = 4.375\times10^{3}$ Am^2 ; $B_y = 0.09$ T ;

$$R_H = \frac{3.5}{4.375\times10^{3}\times0.09} = 8.9\times10^{-3}\,m^3/C$$

Now $$N = \frac{1}{R_H e} = \frac{1}{8.9\times10^{-3}\times1.6\times10^{-19}} = 0.7\times10^{23}/m^3.$$

Also $$\sigma = J_x/E_x = \frac{4.375\times10^{-3}\times5\times10^{3}}{1.5} - 14.6\,\Omega^{-1}m^{-1}$$

Finally $$\mu_e = \frac{R_H}{\rho} = \sigma R_H = 14.6\times8.9\times10^{-3} = 0.13\,m^3V^{-1}s^{-1}$$

__Problem 10.9__ *An N-type germanium sample has donor density of $10^{21}/m^3$. It has arranged in a Hall experiment having magnetic field of 0.5 T and the current density is 500 A/m^2. Find the Hall voltage if the sample is 3 mm wide.*

Solution. Given $t = 3\times10^{-3}$ m, $J_x = 500$ A/m^2, $B_z = 0.5$ T and $N = 10^{21}/m^3$, $e = 1.6\times10^{-19}$ C

We know, Hall constant

$$R_H = \frac{1}{Ne} = \frac{E_y}{J_x B_z} = \frac{V_H}{t\,J_x B_z}$$

$$\Rightarrow \quad V_H = \frac{t\,J_x B_z}{Ne}$$

$$= \frac{(3\times10^{-3})\times500\times0.5}{10^{21}\times(1.6\times10^{-19})}\,V = 4.69\times10^{-3}\,V = 4.69\,mV$$

__Problem 10.10__. *When 90 mA current is passed through a sodium specimen under the magnetic field 2.0 Wb/m^2, the Hall voltage is 0.09 mV. The width of the specimen is 0.04 mm. Calculate carrier concentration.*

Solution. $V_H = 0.09$ mV $= 0.09\times10^{-3}$ V, $I_x = 90$ mA $= 90\times10^{-3}$ A ;
$t = 0.04$ mm $= 0.04\times10^{-3}$ m, and $B_z = 2$ Wb/m^2

$$R_H = \frac{1}{Ne} = \frac{V_H}{t\,J_x B_z} = \frac{V_H t}{I_x B_z}$$

$$\Rightarrow \quad N = \frac{t\,I_x B_z}{V_H\,t e}$$

$$= \frac{90\times10^{-3}\times2.0}{0.09\times10^{-3}\times0.04\times10^{-3}\times1.6\times10^{-19}} = 3.1\times10^{26}/m^3$$

semiconductor. With more increase E_F reaches up to the middle of the conduction band and valence band to make the material intrinsic.

Example 11.2 *In an N-type semiconductor, the Fermi level lies 0.3 eV below the conduction band at 300 K. If the temperature is increased to 330 K, find the new position of the Fermi level.*

Solution. Given $(E_C - E_F) = 0.3$ eV or $(E_F - E_C) = -0.3$ eV at temperature 300 K

$(E_C - E_F) = ?$ at temperature 330 K

We know that
$$n_e = 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2} \exp\left(\frac{E_F - E_C}{k_B T}\right)$$

Assuming that the density of electrons in the conduction band of intrinsic semiconductor (n_e) remains unchanged by changing temperature 300 K to 330 K

$$(n_e)^{300\text{K}} = (n_e)^{330\text{K}}$$

$$\exp\left[\frac{E_F - E_C}{k_B \times 300}\right] = \exp\left[\frac{E_F - E_C}{k_B \times 330}\right]$$

$$\Rightarrow \quad \frac{0.3}{300\,\text{K}} = \frac{(E_C - E_F)_{330}}{330\,\text{K}}$$

$$(E_C - E_F) \text{ at } 330\text{K} = \frac{0.3 \times 330}{300} = 0.33 \text{ eV}$$

Thus at 330 K, the Fermi energy level lies 0.33 eV below the conduction band.

... above the acceptor

$$\Rightarrow \quad n_h = (2\, n_d)^{1/2} \left(\frac{2\,\pi\, m_h^* k_B T}{h^2} \right)^{3/4} e^{(-\Delta E / k_B T)} \qquad ...(11.22)$$

...e $\Delta E = E_a - E_V$.

...ple 11.3 *In a P-type semiconductor, the Fermi level lies 0.4 eV above the valence band. If the ...ntration of the acceptor atom is tripled, find new position of the Fermi level.*

Solution. Given P-type semiconductor, $E_F - E_V = 0.4$ eV, $n_a' = 3\, n_a$,

also $k_B T = 0.03$ eV, $(E_F' - E_V) = ?$

For a P-type semiconductor, the hole density is given by

$$n_h = n_a = 2 \left(\frac{2\pi\, m_h^* k_B T}{h^2} \right)^{3/2} e\,(E_V - E_F)/k_B T \qquad ...(i)$$

Similarly,

$$n_h' = n_a' = 3\, n_a = 2 \left(\frac{2\pi\, m_h^* k_B T}{h^2} \right)^{3/2} \exp(E_V - E_F')/k_B T \qquad ...(ii)$$

Comparing Eqs. (*i*) and (*ii*), we get

$$(E_F' - E_V) = (E_F - E_V) - k_B T \ln 3$$

$$= (0.4 - 0.03 \times 1.098) \text{ eV}$$

$$= (0.4 - 0.03294) \text{ eV}$$

$$= 0.367 \text{ eV}$$

Example 11.4 *Fermi energy of an intrinsic semiconductor is 0.6 eV. The low lying energy level in the conduction band is 0.2 eV above the Fermi level. Calculate the probability of occupation of this level by an electron at room temperature.*

Solution. Given $E_F = 0.6$ eV, then $E = (0.6+0.2)$ eV $= 0.8$ eV

The probability of occupation of an energy level by an electron is given by the F-D distribution law

$$f(E) = \frac{1}{1+\exp\left(\dfrac{E-E_F}{k_B T}\right)}$$

$$= \frac{1}{1+\exp\left[\dfrac{(0.8-0.6)\,\text{eV}\times 1.6\times 10^{-19}\,\text{J}}{1.38\times 10^{-23}\,\text{JK}^{-1}\times 300\,\text{K}}\right]}$$

$$= \frac{1}{1+\exp(7.7)} = 0.0004 = 0.04\ \%$$

EXTRINSIC SEMICONDUCTORS

... number of holes ($n_i = p_i$). But in