Boolean Algebra

- Precedence of Boolean operators
  1) not (1st)
  (2) AND (2nd)
  (3) OR (3rd)

- Rules for Evaluating Boolean expression
  1) evaluate left to Right
  2) Evaluate in Parenthesis first
  3) NOT
  4) AND
  5) OR

Q) $X=1,\ y=0,\ z=1$ then $F = Z + Z'Y + YX'$

Sol) Substituting value $F = 1 + 1'.0 + 0.1'$
$= 1 + 0.0 + 0.0$
$= 1 + 0 + 0 = 1$
$F = 1$ Ans

Logic GATES

1) Not gate

A	B
0	1
1	0

2) AND Gate

A	B	F
0	0	0
0	1	0
1	0	0
1	1	1

3) OR gate

A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

Laws and Rules of Boolean Algebra

1.) Commutative law

(a) $A+B=B+A$
(b) $A\cdot B=B\cdot A$

2) Associative Law: (a) $A+(B+C)=(A+B)+C$
(b) $A\cdot(B\cdot C)=(A\cdot B)\cdot C$

3) Distributive law: $A(B+C)=AB+AC$
$A+(B\cdot C)=(A+B)\cdot(A+C)$

4) Identity law:- $A+0=A$ (OR)
(b) $A\cdot 1=A$ (AND)

5) Inverse law / complementary: (a) $A+A'=1$
(b) $A\cdot A'=0$.

6) Idempotent law: a) $A+A=A$
(b) $A\cdot A=A$.

7) Involution Law:- $A''=A$

8) Absorption Law or theorem of Redundancy:-
(a) $A+A\cdot B=A$. [Proof: $A(1+B) \Rightarrow A\cdot 1$
$=A$. Hence Proved]

b) $A(A+B)=A$. [Proof: $A\cdot A+A\cdot B$
$=A+AB$
$=A(1+B)$
$=A\cdot 1=A$ Hence Proved]

9) Principle of Duality:- (a) change 0 and 1 (vice versa)
(b) interchange AND & OR.

Q: Write dual of $(X+0)\cdot(Y\cdot 1\cdot X')$

Ans) $(X\cdot 1)+(Y+0+X')$.

Q. write dual forms $XY + X'Z + YZ = XY + X'Z$

Sol: $(X+Y).(X'+Z).(Y+Z) = (X+Y).(X'+Z)$

10/1 De-Morgan's law: (a) $\overline{(A+B)} = \bar{A}.\bar{B}$

(b) $\overline{A.B} = \bar{A} + \bar{B}$

Q) find complement.

1) $X+YZ$

Ans) $\overline{(X+YZ)}$

$= \bar{X}.\overline{YZ}$

$= \bar{X}.(\bar{Y}+\bar{Z}).$

Q)(2) Simplify & write in SOP

$F_1 = \overline{\overline{X.\bar{Y}}.\overline{(\bar{Y}.Z)}}$

$F_1 = (X.\bar{Y}) + (\bar{Y}.Z)$

$\boxed{\cancel{F_1 = \bar{Y}(X + Z)}}$

$\boxed{F_1 = (X.\bar{Y}) + (\bar{Y}.Z)}$ Ans

(1) Q Verify →

$XY' + Y'Z = XY'Z + XY'Z' + X'Y'Z$

Sol: taking RHS $= XY'Z + XY'Z' + X'Y'Z$

$= XY'(Z+Z') + X'Y'Z$

$= XY'.1 + X'Y'Z$

$= XY' + X'Y'Z$

$= Y'(X+X'Z)$ ← distributive property

$= Y'(X+X')(X+Z)$

$= Y'(X+Z) = XY' + Y'Z$ Hence Proved.

(2) Q. Verify, $A + A'B = A+B$

LHS AL. distributive law

$LHS = (A+A')(A+B)$

$= 1.(A+B)$

$= A+B = RHS$ Hence Proved.

Sum of Products (SOP) (Σ)

e.g. $AB + CD$

e.g. $A'C + B'D$

Product of Sums (POS) (Π)

e.g. $(A+B+C).(X+Y)$

e.g. $(X'+W+Y').(Y'+Z+X)$

$\boxed{\text{Minterm}}$ (for SOP)

0 → complemented value
1 → non-complemented

Q1) Find minterm and SOP expression $f(X,Y) = \Sigma(0,2)$

$2^2 = 4$

decimal Equivalent	X	Y	output	Minterm
0	0	0	1	$X'Y'$
1	0	1	0	
2	1	0	1	XY'
3	1	1	0	

$= X'Y' + XY'$ Ans

$\boxed{\text{Maxterm}}$ (POS expression)

1 → complemented
0 → noncomplemented

Q1) $f(X,Y) = \Pi(1,2)$

$2^2 = 4$

deci eq.	X	Y	output	Maxterm
0	0	0	1	
1	0	1	0	$X+Y'$
2	1	0	0	$X'+Y$
3	1	1	1	

$= (X+Y').(X'+Y)$ Ans

① Q find minterm $AB'C + A'BC' + AB'C' + ABC$

Sol) $001 + 010 + 100 + 111$

$\Rightarrow 2^0 + 2^1 + 2^2 + 2^2 + 2^1 + 2^0$

$= 1 + 2 + 4 + (4 + 2 + 1)$

$= \Sigma(1, 2, 4, 7)$. Ans.

Q 2) find Maxterms $(A+B+C).(A+B'+C')(A'+B+C').((A'+B'+C)$

1 → compl.

$= (000).(011).(101).(110)$

$= 2^0 . (2^1 + 2^0) . (2^2 + 2^0) . (2^2 + 2^1)$

$= (1) . (3) (4+1) . (4+2)$

$\Pi(1, 3, 5, 6)$ Ans.

$\boxed{\text{Canonical forms}}$

1) Canonical Sum of Products (SOP) form (expand)

Q $f = a + b'c$ as sum of minterms

$f = a(b+b')(c+c') + (a+a')b'c$ (adding missing terms expanding)

$= (ab + ab')(c+c') + b'c(a+a')$

$= abc + abc' + ab'c + ab'c' + ab'c + a'b'c$

$= \cancel{ab}(c+c') + ab'(c+c') + \cancel{b'c}(a+a')$

$= ab + ab' + b'c$

$= abc + abc' + ab'c + ab'c' + a'b'c$

0 → comp.

minterms	Binary Equ.	Decimal Eq
abc	111	7
abc'	110	6
ab'c	101	5
ab'c'	100	4
a'b'c	001	1

$\Sigma(1, 4, 5, 6, 7) = m_1 + m_4 + m_5 + m_6 + m_7$.

Q/ Canonical Expansion of POS :

Q(1) $A.B+C$ find max. term

Sol
$AB+C$
$= (A+C).(B+C)$ distribut
$= (A+C+B.B').(B+C+A.A')$ distributive
$= (A+C+B)(A+C+B').(B+C+A).(B+C+A')$
$= (A+B+C).(A+B'+C).(A'+B+C)$ Removing Repeating
Ans.

Q/1 (2): $(A'+B+C).(B'+C+D')(A+B'+C'+D)$ in POS form

1st $= (A'+B+C+D.D')$
$= (A'+B+C+D).(A'+B+C+D')$ (1) distribution

2nd $= (B'+C+D')$
$= (B'+C+D'+A.A')$
$= (B'+C+D'+A) \cdot (B'+C+D'+A')$ -(2)

3rd → complete (3)

now. multiplying (1),(2),(3)

$f = (A'+B+C+D).(A'+B+C+D').(A+B'+C+D').(A'+B'+C+D')$
$.(A+B'+C'+D).$ A

Short hand for Minterm
0→Com

1) $X\,Y'Z$
⇒ $1\,0\,1$
⇒ 2^2+2^0
$= 4+1 = m_5$

Short hand for Maxterm
1→comb

Q. $(W'+X'+Y)$
⇒ (110)
$= 2^2+2^1$
$= 4+2 = m_6$.

1 Q - Conversion b/w canonical forms

If SOP is $\Sigma(1,4,5,6,7)$ it's complement

POS: $f(a,b,c) = \Pi(0,2,3)$

AND LAWS	OR LAWS
$A \cdot 0 = 0$	$A + 0 = A$
$A \cdot 1 = A$	$A + 1 = 1$
$A \cdot A = A$	$A + A = A$
$A \cdot \bar{A} = 0$	$A + \bar{A} = 1$

K Maps

Q Karnaugh Map (K-Map) :- The K-map is a graphical representation that provides a systematic method for symplyfying the Boolean expression.

Q Two variable K-map: for n variable K-map 2^n cells are required. ∴ for 2-variable K-map $2^2 = 4$ cells are required.

A \ B	0	1
0	00	01
1	10	11

A \ B	$\bar{B}$	B
$\bar{A}$	$\bar{A}\bar{B}$	$\bar{A}B$
A	$A\bar{B}$	AB

A \ B	0	1
0	0	1
1	2	3

value of binary

Q Three variable K-maps

$2^3 = 8$ cells are required.

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

easily

AB \ C	0	1
00	0	1
01	2	3
11	4	5
10	6	7

Q four variable K-map

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

Q. Plot the boolean expression (SOP form)

$Y = \bar{A}B\bar{C}\bar{D} + A\bar{B}C\bar{D} + \bar{A}BC\bar{D} + AB\bar{C}D$

$\quad\;\; 0100 \quad\; 1010 \quad\; 0110 \quad 1101$

$Y = m_4 \quad m_{10} \quad m_6 \quad m_{13}$

$Y = \Sigma m(4, 6, 10, 13)$

AB \ CD	00	01	11	10
00	0	1	3	2
01	1 (4)	5	7	1 (6)
11	12	1 (13)	15	14
10	8	9	11	1 (10)

other way to fill

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$				
$\bar{A}B$	1			1
AB		1		
$A\bar{B}$				1

Q. Grouping of cells for Simplication:

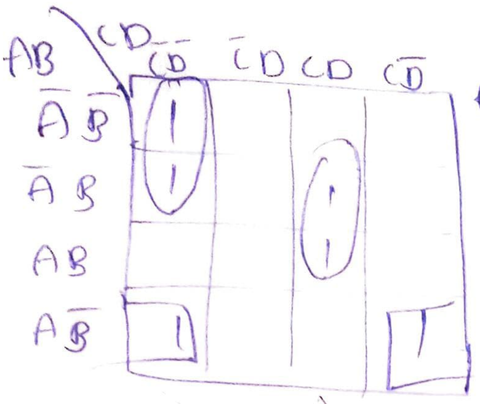
→ Adjacent cells which have 1's can be grouped together in 2's ~~form~~ power i.e. $2^0 = 1$

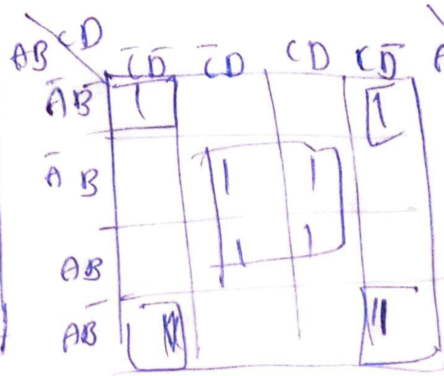
$2^1 = 2$ adjacent cell can be grouped (Pair)

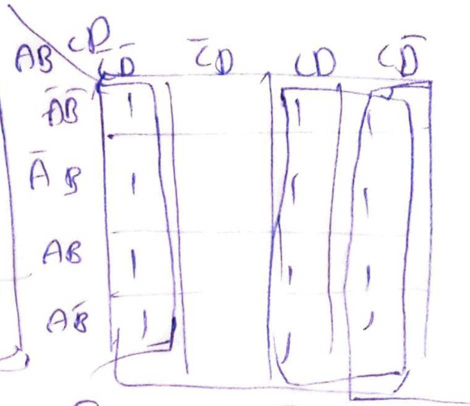
$2^2 = 4$ " " " " (Quads)

$2^3 = 8$ " " " " (Octats)

$2^4 = 16$ " " " "







$(\bar{A}\bar{B}+\bar{A}B).\bar{C}\bar{D}$

$\bar{A}(\bar{B}+B).\bar{C}\bar{D}$

$\bar{A}\bar{C}\bar{D}$

BCD

$A\bar{B}.(\bar{C}+C)\bar{D}$

$= A\bar{B}\bar{D}$

$F = \bar{A}\bar{C}\bar{D} + BCD + A\bar{B}\bar{D}$

BD

$\bar{A}\bar{B}\bar{D}$

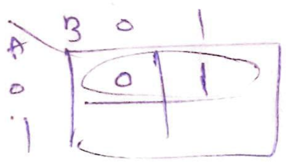
$A\bar{B}\bar{D}$

$F = BD + \bar{A}\bar{B}\bar{D} + A\bar{B}\bar{D}$

$F = C + \bar{D}$ Ans

Q Rules followed for K-map Simplification:

1) Groups do not include any cell containing a zero



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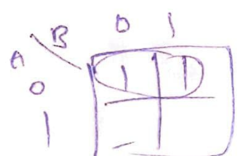

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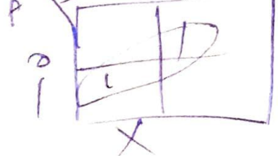
2) Groups may be horizontal or vertical, but not diagonal.



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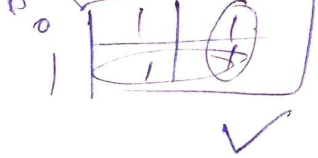

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3) groups must contain 1, 2, 4, 8 or 2^n cells

4) Each gp should be as large as possible.