Jasmine Fan and Smita Patankar

Graph Theory: Networks and the Applications

I. Introduction:

In mathematics and computer science, graph theory is the study of graphs to model pairwise relations between objects. As a part of graph theory, network theory focuses on representation of either symmetric relations or asymmetric relations between discrete objects. Network theory can be applied in disciplines including applied physics, computer science, electrical engineering, biology, economics, operations research, and sociology.

II. Terminology and Concepts

We will talk about the definitions of some terminology in graph theory in order to help the audience understand the later application of network theory.

Node, Edge,

Directed Path:

Directed Cycle:

Assortative and disassortative:

We say a hub is **assortative** when it tends to connect to the other hubs.

A **dissortative** hub avoids connecting to other hubs. If some nodes have some connections with the expected random probabilities, the hubs are neutral.

III. Application in Finance with systematic Risk:

A financial system can be modeled using network theory. Nodes in a network represent all the banks in a financial system and the edges represent links between two banks. The borrowing and lending capital translates to a network inflow or outflow. The paper defines a propagation function to model the diffusion of losses and insolvencies across such a network.

There are three different systemic risks in the financial market:

1) Informational contagion in banking systems, where depositors' expectations about the possibility of a crisis can lead to bank runs

2) Direct contagion transmitted via links, i.e., through debt/credit relationships generated on the interbank markets in banking systems

3) Common exposure to exogenous drops in the value of assets, a risk born by agents who hold the same assets or highly correlated assets.

We will talk about how the structure of the network affect the propagation of capital flow. The presence of cycle flows in a network N creates problems of indeterminacy of a propagation. If the network N is acyclic, a propagation in N is uniquely defined.

We will talk about how the following four parameters characterize a network structure:
1) the capitalization of its members: increasing capital reduces risk
2) its degree of connectivity: increasing connectivity only benefit when the shock is small
3)its degree of concentration: increases default contagion for large shocks
4) the size of the interbank exposures: increase the default risk

We will show the derivation and usage of the activation function:

$$\beta_i(\lambda_i) = \min \left(\frac{\lambda_i}{v_i}, 1\right)$$

where λ_i is the total loss from the i-th node, received from source nodes or from other nodes in $\Omega = \{w_i\}$ (the set of financial operators which are directly or indirectly connected by debt/credit relations) and v_i is the net worth of each node w_i. The variable β measures the net worth lost by a node.

We will also show the usage and derivation of the insolvency function:

$$b_i(\lambda_i) = \max \left(0, \frac{\lambda_i - v_i}{d_i}\right)$$

where d_i the sum of the loans granted to w_i by other agents in Ω. The variable b_i measures the percentage of the i-th element's debt that can't be recovered through liquidation. These equations will be further explained in the presentation, and an algorithm to put them to use will be shown. Also, visualizations will be used so that the audience will get a full grasp of what is discussed regarding graphs and connectivity.

IV. Coding examples:

We will show code and results that computes the value of of a propagation through iterated application of the activation and insolvency functions. The algorithm gives a sequence of passing financial losses, starting from source nodes and ending in the sink. When the flow out of the sources has been completely absorbed by the sink, the computation completes due to the flow conservation property being satisfied. A pair of n dimensional vectors, $\{[\beta_i], [b_i]\}$, is returned. These vectors identify the propagation for any given value of the shock vector $[b_k]$. An idiosyncratic shock, here, is defined as the insolvency of a single node, usually due to fraud. On each iteration of this algorithm, losses are passed from a set of nodes in N to their children nodes. We will give different examples of this algorithm in practice, and see how the structure of the network affects propagation.

Conclusion:

Future of the field:

How does structure affect performance and fragility of a banking system?

We will talk about how network theory benefits our future career paths and answer any questions from the audience.

References:

http://www.ecb.europa.eu/pub/pdf/other/modellingsystemicrisk012010en.pdf
http://www.webric.it/personale/mario/Mario_Eboli-Systemic%20risk%20in%20financial%20networks_a%20graph-theoretic%20approach.pdf
http://www.macroeconomics.tu-berlin.de/fileadmin/fg124/networks/Lectures/2_Networks_in_economics_and_finance.pdf
http://events.em-lyon.com/ressources/evenements/documents/cefra/Delphine%20Lautier%205%20Mai%20-Paper.pdf
http://www.cl.cam.ac.uk/teaching/1011/PrincComm/slides-lpr/graph_theory_1-11.pdf