Quantum Walk based Qubit Mapping

Smit Chaudhary

Supervised by : Sebastian Feld Faculty of Applied Sciences Delft University of Technology Delft, The Netherlands

May 30, 2022







Outline

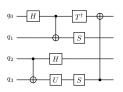
- Motivation
- 2 Methods
- Results
- 4 Conclusions



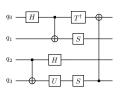
We want to map qubits on a quantum circuit (virtual qubits) to the qubits on a quantum chip (physical qubits).

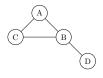


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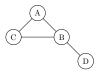






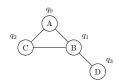






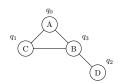










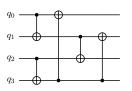


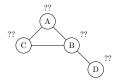


Not always possible to find a mapping. Especially with a bigger circuit.



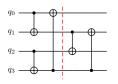
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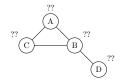






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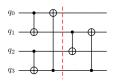


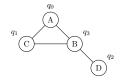


Map one part of the circuit.



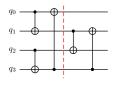
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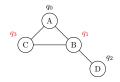




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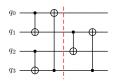
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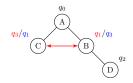




Map one part of the circuit. Map the next part of the circuit.

Not always possible to find a mapping. Especially with a bigger circuit.





Map one part of the circuit. Map the next part of the circuit. Move the qubits to go from one mapping to the next (Routing).

Outline

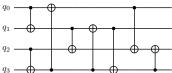
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Given: A quantum circuit to map.

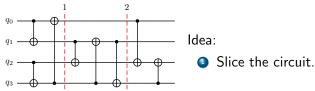


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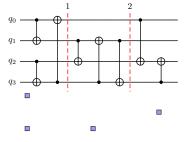


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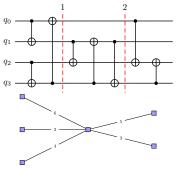


Idea:

 Slice the circuit. Get (multiple) mappings for each slice.



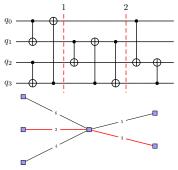
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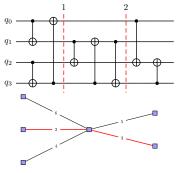
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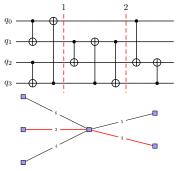
Two part problem:

Initial Mapping: Quantum Walk based search



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Two part problem:

Initial Mapping: Quantum Walk based search

Routing: Classical Optimization





It is a constraint satisfaction problem (CSP).



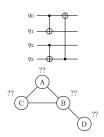
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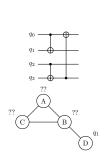
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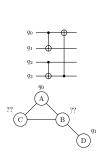
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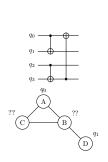
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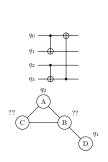


Backtrack!!



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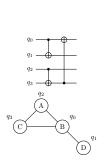


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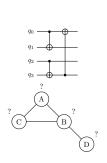


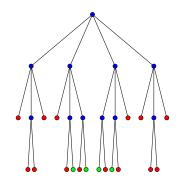
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Backtrack!!



Quantum walk speed up

Speed up this backtracking for the CSP.



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Finding a solution

(Montanaro, 2015)

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A bounded error quantum algorithm evaluates P and h

 $\mathcal{O}(\sqrt{Tn^{\frac{5}{2}}log(n)})$ times each and outputs solution.



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- Strike out previously found solutions to find all solutions.





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- If x is indeterminate, and $x \neq r$ then $D_x = \mathbb{I} 2 \ket{\psi_x} \bra{\psi_x}$

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• For the root, $D_r = \mathbb{I} - 2\ket{\psi_r}\bra{\psi_r}$

$$|\psi_r\rangle \propto \left(|r\rangle + \sqrt{n}\sum_{y,r \to y}|y\rangle\right)$$





Let A and B be the set of vertices even and odd distance from root, respectively.



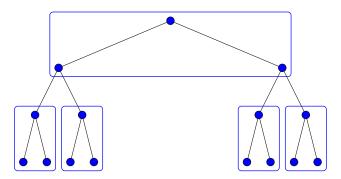
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A step of the walk is applying $R_B R_A$ where $R_A = \bigoplus_{x \in A} D_x$ and $R_B = |r\rangle \langle r| + \bigoplus_{x \in B} D_x$



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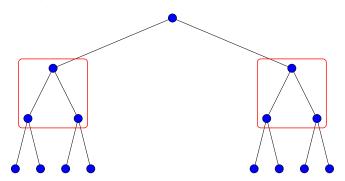
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The algorithm

(Montanaro, 2015)

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Use the subroutine K times and if the measured eigenvalue is 1 at least $\frac{3K}{8}$ times, a solution exists.





• Can use this detection procedure as a subroutine to find the solutions in the tree via binary search.



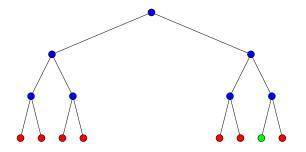
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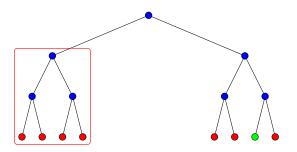


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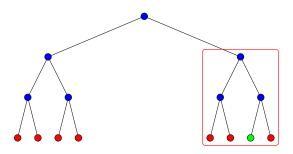


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Depth optimization



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We propose a more efficient encoding of the partial assignment $x_1 = v_1, ..., x_l = v_l \rightarrow |l\rangle |v_1\rangle ... |v_l\rangle |*\rangle ... |*\rangle$



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Saves nlog(n) qubits.

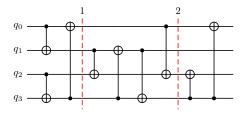


What if the detection algorithm returns "no solution exists"?



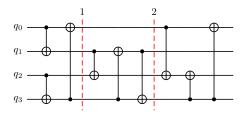
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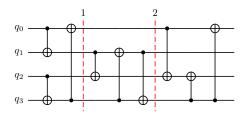
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Proposed guess for size of slice $= d_G \cdot |G_2|$

$$d_G = \frac{\text{no. of edges}}{\text{no. of possible edges}}$$
, $|G_2| = \text{no. of 2 qubit gates}$





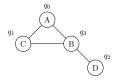
What is the metric to be optimized?

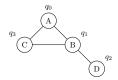


$$d_{\mathsf{map}}(\pi_1, \pi_2) = \sum_{\alpha \in \mathcal{Z}} d_{\mathsf{chip}}(\pi_1^{-1}(\alpha), \pi_2^{-1}(\alpha))$$

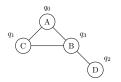


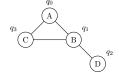
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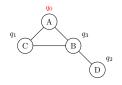
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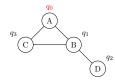


$$d_{\mathsf{map}}(\pi_1, \pi_2) = d_{\mathsf{c}}(\pi_1^{-1}(q_0), \pi_2^{-1}(q_0)) + d_{\mathsf{c}}(\pi_1^{-1}(q_1), \pi_2^{-1}(q_1)) + d_{\mathsf{c}}(\pi_1^{-1}(q_2), \pi_2^{-1}(q_2)) + d_{\mathsf{c}}(\pi_1^{-1}(q_3), \pi_2^{-1}(q_3))$$

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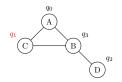
$$d_{\mathsf{map}}(\pi_1, \pi_2) = 0 + d_{\mathsf{c}}(\pi_1^{-1}(q_1), \pi_2^{-1}(q_1)) \\ + d_{\mathsf{c}}(\pi_1^{-1}(q_2), \pi_2^{-1}(q_2)) + d_{\mathsf{c}}(\pi_1^{-1}(q_3), \pi_2^{-1}(q_3))$$



$$+d_{c}(\pi_{1}^{-1}(q_{1}),\pi_{2}^{-1}(q_{1}))$$



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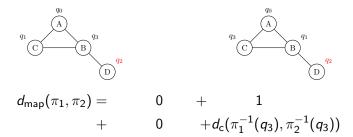


$$q_3$$
 C q_1 q_2 q_3 Q_2

$$egin{aligned} d_{\mathsf{map}}(\pi_1,\pi_2) &= 0 & + 1 \ & + d_{\mathsf{c}}(\pi_1^{-1}(q_2),\pi_2^{-1}(q_2)) & + d_{\mathsf{c}}(\pi_1^{-1}(q_3),\pi_2^{-1}(q_3)) \end{aligned}$$

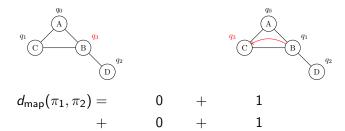


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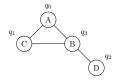


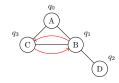
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$$d_{\mathsf{map}}(\pi_1,\pi_2) = 2$$

How to get the optimum "route"?

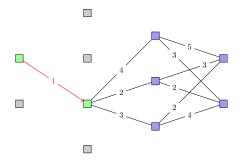


How to get the optimum "route"? Employ a simple greedy approach!



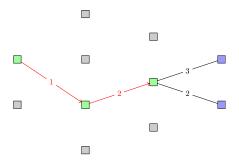
10/02

How to get the optimum "route"? Employ a simple greedy approach!





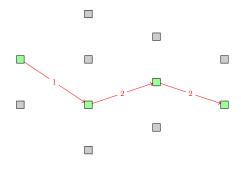
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16/22

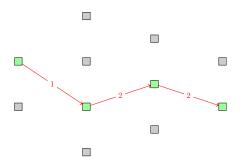
How to get the optimum "route"? Employ a simple greedy approach!





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How to get the optimum "route"? Employ a simple greedy approach!



Potentially miss global optimum but we avoid combinatorial explosion.



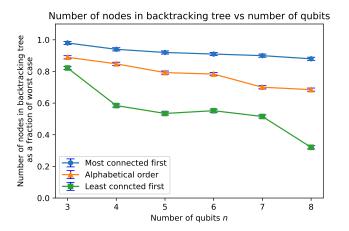
Outline

- Motivation
- 2 Methods
- Results
- 4 Conclusions





• Effect of choice of heuristic h:





Benchmarking



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Performance benchmarked against classical mappers for actual circuits (RevLib^a)



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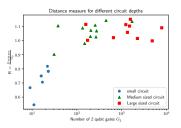
Performance detoriates for bigger circuits, upto 80% more SWAPs needed.

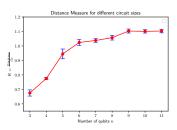
^aWille, 2008

^bLao et. al., 2022



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Outline

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 Proposed a new model for qubit mapping based on CSP and combinatorial optimization.



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- Provided a practical implementation of a quantum algorithm for (initial) qubit mapping.
- Further, optimized both the space and depth by eliminating dynamic heuristic and using a more efficient encoding.
- Benchmarked and compared the depth overhead to other classical mapping and routing algorithms.



Smit Chaudhary

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Thanks!

