PEER-EVALUATION OF MATHEMATICAL WRITING FOR LINEAR ALGEBRA

Why are we doing this?

Communicating mathematical ideas clearly in writing is a skill that takes practice. It is challenging to learn how to write mathematics well while you are immersed in learning the concepts themselves. Evaluating the clarity of your own writing is also difficult because *you know what you mean to say*. Our goals in evaluating the mathematical writing of ours peers are to

(1) provide useful formative feedback to our peers so that they may improve their writing,
(2) discover that we face challenges that our peers are facing as they learn to write mathematics,
(3) improve our own mathematical writing by analyzing and constructively critiquing the writing of others.[1]

The Rubric

Rating	**Beginning (1)**	**Developing (2)**	**Accomplished (3)**
Notation	Some variables and/or new/non-standard are not defined; <u>consistent</u> mistakes are made with standard notation, e.g., Rn instead of $\mathbb{R}^n$; = used incorrectly	Variables are all defined; <u>few mistakes</u> are made with standard notation; = is used correctly; for example, may make mistakes with $\in$ and $\subset$	Variables are all defined; all standard notation is used correctly; new notation is efficient and helpful
Language & Clarity	Arguments are unclear or confusing; contains many spelling and grammar mistakes; overuse of symbols and equations instead of sentences	Sometimes challening to read or understand arguments, contains <u>few</u> spelling and grammar mistakes; full sentences used to clarify equations	Easy to read and understand arguments; contains <u>very few</u> spelling and grammar mistakes; use of full sentences balanced well with equations and symbols adding to the clarity;
Logic	Contains *false* statements; <u>begins with</u> or <u>assumes</u> the conclusion; quantifiers "for all", "there exists" used incorrectly; hard to understand logical connections	Follows the proof handbook; <u>most</u> logical connections clear; <u>few</u> missing/implied quantifiers; all assumptions and conclusion present	Follows proof handbook; connections between statements are clear; quantifiers "for all", "there exists" are present and used correctly; easy to follow argument
Completeness	Many details are missing; Theorems are used without checking assumptions; uses words like "obvious" to cover up missing details	Few missing details; most assumptions of Theorems used are checked; things called "obvious" really are true and easy to check	All necessary details are present; the assumptions of Theorems used are checked; nothing "obvious" is left out

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[1]Recent research has shown that the best way to improve your mathematical writing is to read, analyze, and correct proofs that contain (serious) errors. See, for instance: A. Selden and J. Selden, *Validations of Proofs Considered as Texts: Can Undergraduates Tell Whether an Argument Proves a Theorem?*. Journal for Research in Mathematics Education, Vol. 34, No. 1 (2003), pp. 4–36. (stable link).

Some Common Errors

Notation.

- Declaring a variable, but incorrectly stating where it lives, e.g., $\boldsymbol{x} \in \mathbb{R}$ when you should say that $\boldsymbol{x} \in U$, where U is a subset of $\mathbb{R}$.
- Reusing one variable for two different purposes
- Interchanging $\in$ and $\subset$
- Misuse of the zero vector $\vec{0}$ and the zero subspace $\{\vec{0}\}$
- Misuse of the number zero 0 and the empty set $\emptyset$

Language & Clarity.

- Overuse of symbols. For instance, $\therefore$ and $\because$ start to look a lot alike but have different meaning.[2]

Logic & Mathematical Errors.

Logic.

- Including false implications in a proof. For instance, writing: "Since $\{\boldsymbol{v}, \boldsymbol{w}\}$ is linearly independent, for $a = 0, b = 0$ we have $a\boldsymbol{v} + b\boldsymbol{w} = \vec{0}$" is <u>incorrect</u>. We could correct this to say: "Since $\{\boldsymbol{v}, \boldsymbol{w}\}$ is linearly independent, if $a\boldsymbol{v} + b\boldsymbol{w} = \vec{0}$ then $a = 0$ and $b = 0$."
- To prove "if P then Q", you cannot assume that Q is true – you must suppose that P is true and argue that Q is true.
- The negation of $P \Rightarrow Q$ is **not** "NOT $P \Rightarrow$ NOT Q". The negation of $P \Rightarrow Q$ is in fact the statement "P and NOT Q".
- Interchanging "there exsits = $\exists$" and "for all = $\forall$"
- Writing "Suppose" when you mean "Choose" (when proving an existence statement)
- Writing "Choose" when you should "Suppose" (when proving a "for all" statement)

Linear algebra.

- Misusing the scalar 0 and the vector $\vec{0}$ or $\mathbf{0}$.
- Using "span" incorrectly. For instance, $\{v_1, \ldots, v_k\} \neq \text{span}\{v_1, \ldots, v_k\}$, and $\text{span}\{e_1, \ldots, e_n\}$ is **not** a basis of $\mathbb{R}^n$ – the finite set $\{e_1, \ldots, e_n\}$ is a basis.

Calculus.

- Dropping "lim" from the calculation of a limit.
- Dropping "$\sum$" from manipulations involving series.
- Confusing a series $\sum_{n=1}^{\infty} a_n$ with the sequence of its terms $\{a_n\}_{n=1}^{\infty}$.

Completeness.

- Assuming that a proof must be written at a level appropriate for the instructor, and thus omitting important details. Think of proof writing as an opportunity to demonstrate your clear understanding of the topic.
- My advice: write your proof for your "future-self", i.e., write down all the details that you might forget later, or write your proof for an audience of your peers.

[2] Please, please, please **don't** use $\therefore$ and $\because$. See, it looks terrible!