



Proof-Writing Rubric

Communicating mathematical ideas clearly in writing is a skill that takes practice. Writing a complete solution also challenges you to place the problem in context and develop a deep understanding.

Our goals in evaluating your written solutions are to

1. provide useful feedback so that you may improve your mathematical writing skills, and
2. provide opportunities to engage with challenging problems and practice problem-solving.

Communication Score: 3pts

Attribute	Criteria	Excellent (3pts)	Very Good (2pts)	Good (1pt)	Needs Improvement (0pts)
Communication Score	Submission is well articulated using proper notation (e.g. all variables are defined) prose are used to help the reader navigate (e.g. solution is organized, uses full sentences, computations are introduced, results are clearly articulated), and there is no extraneous detail (e.g. no irrelevant results or computations). The Communication Score may not exceed the Mathematics Score.	The submission is clearly communicated and follows all of the communication criteria (notation, prose, extraneous detail).	The submission is clearly communicated and follows most of the communication criteria (notation, prose, extraneous detail).	The submission is unclearly communicated and follows some of the communication criteria (notation, prose, extraneous detail).	The submission is unclearly communicated and follows few of the communication criteria (notation, prose, extraneous detail).

Mathematics Score: 4 pts

Attribute	Criteria	Excellent (4pts)	Very Good (3pts)	Good (2pt)	Needs Improvement (0-1pts)
Mathematics Score	Submission is mathematically correct (e.g. no false statements, calculations are accurate, understanding of theory is evident), contains an appropriate amount of detail (e.g. assumptions of theorems are checked, claims are justified), and solves the problem .	Submission follows all of the mathematics criteria (correct, appropriate amount of detail, solves the problem).	Submission follows most of the mathematics criteria (correct, appropriate amount of detail, solves the problem).	Submission follows some of the mathematics criteria (correct, appropriate amount of detail, solves the problem).	Submission follows few of the mathematics criteria (correct, appropriate amount of detail, solves the problem).

Proof-Writing Checklist

To make sure your solution is complete, it may help to ask yourself the following questions:

Communication Score

Notation

- Are all variables defined? (e.g., did you say what x represents?)
- Am I using the proper notation? (e.g., a finite set versus a span.)

Prose

- Is my solution written in full sentences and organized in paragraphs (or does it look like a string of implications and computations)?
- Are my computations introduced so that the reader knows what I am about to do? (e.g., before jumping into a matrix algebra computation, we need to tell the reader how this algebra ties back to the problem at hand.)
- Do I have an introductory and concluding sentence so that it's clear what I am about to do and what my result is?

Extraneous Detail

- Have I included details that are irrelevant and do not contribute to my result? (e.g. scrap work that isn't necessary to the problem)
- Is my explanation concise, or can I communicate this in an easier way?

Mathematics Score

Correctness

- Is the terminology I am using accurate? (content vocabulary & grammar)
- If I used a theorem, am I using it correctly? (not misusing a theorem)
- Are all of my statements logically correct? (no false statements.)

Appropriate Amount of Detail (no assertions)

- If a fellow classmate who knows a little less than me was reading my solution, would they understand all steps, or should I provide more details?
- If I used a theorem, am I explaining why all assumptions are satisfied?

Solves the Problem

- Is my proof missing any parts? (no omitted sections.)
- Have I reread the prompt to make sure that I read it correctly and didn't miss any parts if it is a multipart question?
- Have I double-checked to make sure that my single PDF contains all of my work, and I haven't accidentally missed a page?

Proof-Writing Tips:

I strongly recommend first working out your thoughts on scrap paper, then writing a good copy (the way you initially solve a problem usually isn't the most optimal way to explain it to others).

Your "audience" should be another student in the class who knows a little less than you. They should be able to understand what you wrote during the first read through; if they need to read it several times then it is not communicated clearly.

- For example, suppose you claim that a statement is false and provide an explicit counterexample. You must clearly explain why the given counterexample satisfies the hypotheses of the statement, and why it does not satisfy the conclusion. (Even if the counterexample works, if you have not provided this justification, you will not receive full points.)
- Suppose a prompt asks you, "Is P true? Explain." If you just answer "Yes" or "No" without proper explanation, you will not receive any points.
- Be careful about notation. For example, $\{<1,0,0>,<0,1,0>\}$ is a finite set with two vectors. On the other hand, $\text{span}\{<1,0,0>,<0,1,0>\}$ is an infinite set. Do not write a finite set when you mean to write a span, and vice versa.
- If you are doing a matrix algebra computation, be sure to first clearly explain to the reader what you are doing and why you are doing it. There are often multiple ways to solve a problem, so it's important that you let your reader know what you're about to do. (e.g. "To show this set spans U I need to show that this system is consistent because... . To show this system is consistent, I need to ensure the rank equals.... *Now it's fine to do your matrix algebra because we know what your goal is and why.*) If you just show a bunch of matrix algebra with little or no explanation, you will not receive many points (because it can be terribly confusing to read and understand!).

Error Types:

Here are the errors we've seen so far in class. Keep the three "Writing Clarity" error types in mind when you proof-read your work on tests.

Mathematical Errors:

- False Implication
- False Statement (a statement that's false, but it's not an implication)
- Content Vocabulary & Grammar (misuse of definition or vocabulary)
- Assertion (result stated without proper justification)
- Omitted Section (e.g., a case is missing)
- Misusing a theorem (e.g., applying the converse of a theorem)
- Wrong Method (e.g., if the prompt says to use a certain method and they don't)
- Wrong Problem (e.g., told to prove the converse, but prove something else)

Writing Clarity Errors:

- Notation
- Rhetorical Vocabulary & Grammar (prose is poorly written)
- Extraneous Detail (attempted or proved irrelevant results)