2. Sums and Products

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Homework0 due Sep 16, 2020 05:29 IST *Completed*

Summation Notation

4/4 points (graded)

Compute the following sums. Enter your input using standard notation. (Refer to the “Standard Notation" button for help with input.)

1. $\sum_{i=0}^{N} 1 =$ N+1 ✔

   $N+1$

2. $\sum_{k=1}^{K} \sum_{t=1}^{T} 1 =$ K*T ✔

   $K \cdot T$

3. $\sum_{k=1}^{K} \sum_{t=1}^{T} 0.5^k =$ T*(1-0.5^K) ✔

   $T \cdot \left(1 - 0.5^K\right)$

4. $\sum_{k=1}^{\infty} \sum_{t=1}^{T} 0.5^k =$ T ✔

STANDARD NOTATION

Submit You have used 1 of 3 attempts

Product Notation

2.0/2.0 points (graded)

The notation $\prod_{i=1}^{N} p_i$ denotes the product with N factors:

$$\prod_{i=1}^{N} p_i = p_1 p_2 \cdots p_N.$$

Compute the following products.

1. $\prod_{i=1}^{M} \frac{1}{\theta} =$ (1/theta)^M ✔

   $\left(\frac{1}{\theta}\right)^M$

2. $\prod_{k=1}^{K} \frac{k}{k+1} =$ 1/(K+1) ✔

   $\frac{1}{K+1}$

3.

$$\ln\left(\prod_{k=1}^{K} e^{k}\right) = \frac{K\cdot(K+1)}{2}$$

(K*(K+1))/2 ✔

STANDARD NOTATION

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Topic: Unit 0. Brief Prerequisite Reviews, Homework 0, and Project 0:Homework 0 / 2. Sums and Products

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Submit Button Disabled Hello, I enrolled for the course from today onwards. I am inputting correct answers however I am not able to Submit them as the op...	10
I missed the first deadline "Homework 0". Can I still enroll for this Course? Just saw that your course has started and realize that I missed the first dead line. 1) How will I be penalized from missing the first de...	2
I feel bad for not knowing the arithmetic and geometric series by heart I feel like I should know coming from a Bachelors in Economics but I had to actually look them up. :/	3
how to write a summation How can I write a summation	1
Problem in both Questions Kindly if someone explain me what to do in both questions ? I don't understand...	1
how to write a factorial? i have to display the factorial of a number in denominator. how to do that?	5
Curious for products it accepts K, M etc in the results A result like K*(K+1)/2 was accepted and marked OK in the section product , but N was not accepted for summation section	7
"Invalid Input: N not permitted in answer as a variable" Getting this error and I am unable to submit the answers. Only for this particular section, have completed the rest of the homework ...	2
Help, Not able to answer question 4	3
can not submit HELP CAN NOT ANSWER the system complain as: "Invalid Input: N not permitted in answer as a variable", but N is not a "variable" It is a fix number. sum(1) fr...	3
How to write factorial? Can someone tell me how to write factorial?	4
Product Notation question No.3 why (1+K)/2 is wrong. Ine = 1 and so it is the sum of the power	3
summation & Product notation doubt summation question 3 T*(1-(0.5)^(K-1)) and product notation question 1 1/theta I Where can i get the answers. Where am i wrong ?	4

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3. Function Properties

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Homework0 due Sep 16, 2020 05:29 IST *Completed*

Asymptotics and Trends

2.5/4.0 points (graded)

For each of the following functions $f(x)$ below :

- Find its limits $\lim_{x \to \pm\infty} f(x)$ as x approachs $\pm\infty$.
- Choose the values of x where $f(x)$ is differentiable, i.e. $f'(x)$ exists
- Choose the values of x where $f(x)$ is also strictly increasing, i.e. $f'(x) > 0$.

1. For $f(x) = \max(0, x)$:

   (If the limit diverges to infty, enter **inf** for ∞, and **-inf** for $-\infty$)

   $\lim_{x \to -\infty} f(x) =$ 0 ✔ **Answer:** 0

   0

   $\lim_{x \to +\infty} f(x) =$ inf ✔ **Answer:** inf

   inf

   Choose the intervals of x where

   $f(x)$ differentiable: $f'(x) > 0$:

   (Choose all that apply.)

☐ $x < 0$ ✔	☐ $x < 0$
☑ $x = 0$	☐ $x = 0$
☐ $x > 0$ ✔	☑ $x > 0$
✖	✔

   (Graph this function on a piece of paper!)

2. For $f(x) = \dfrac{1}{1 + e^{-x}}$:

   (Enter **inf** for ∞ and similarly **-inf** for $-\infty$ if the limit diverges to infty.)

   $\lim_{x \to -\infty} f(x) =$ 0 ✔ **Answer:** 0

   0

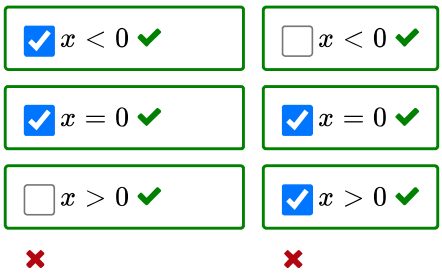
   $\lim_{x \to +\infty} f(x) =$ 1 ✔ **Answer:** 1

   1

   Choose the intervals of x where

   $f(x)$ differentiable: $f'(x) > 0$:

   (Choose all that apply.)



(Graph this function on a piece of paper!)

Solution:

See answers above.
Remark: The function $f(x) = \max(0, x)$ is also called the a **linear rectifier** and the **Sigmoid** function, and will be revisited in *Unit 3 Neural networks* as activation functions within neural networks.

Submit You have used 3 of 3 attempts

Answers are displayed within the problem

Discussion

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Topic: Unit 0. Brief Prerequisite Reviews, Homework 0, and Project 0:Homework 0 / 3. Function Properties

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- Can't submit the assignments — 1
  Why can't I submit any assignment, the button always remain greyed?
- 1/(1-e^-x) that x < 0 is one of value of x — 1
  Could any one explain that f(x) = 1/(1-e^-x) that x < 0 is one of value of x where f(x) is differentiable? I can graph this.
- Can you use online websites to plot and answer these questions? — 6
  I find it convenient to use https://www.wolframalpha.com/ or https://www.derivative-calculator.net to calculate and plot derivatives. ...
- To Staff: my submit is grayed out and homework0 shows past due. I finished it on time though. — 2
  I just checked and My progress shows my homework0 is past due. But I already completed it on time. there was 1 question that I wa...
- I missed my Homework 0 deadline — 2
  Sir/Ma'am I could not complete my Homework 0 because I am corona positive and was not in a condition to complete the homewor...
- Deerivative of sigmoid function — 5
  Since the derivative of the sigmoid function ends up being 0 at both the infinities, will considering both the intervals as having positi...
- Two equal answers — 12
  Hi, I do not understand why is there two equal answers for the intervals of x. Regards
- I couldn't figure out the intervals of x where the derivative > 0 in the max function. — 2
  Can someone explain that to me?, I'm trying to understand, I see the graph and to me is continuous but isn't differentiable at x=0. S...
- Background Question — 1
  To what extent will graphing of complex functions be important in this course? Will we be expected to draw them by hand on exams?
- Can't Attempt more than once — 7
  there are 3 attempts for these questions. I just attempted once but the submit button is disable now. Please fix this problem.
- I submitted answers for the first topic, and after that for the subsequent topics my submit button has been greyed out. Does anyone know what needs to be done, I'm unable to submit the remaining exercises — 7
  Unable to submit my answers
- Regarding differentiability for f(x)=max(0,x) — 8
  I tried to formulate this question as not to exploit the rules of MITx, but I can't understand why f(x)=max(0,x) isn't differentiable at x>...
- Unable to submit answers in this section

Unable to submit answers in this section
Filled up the answers in this section. It allows me to save but not submit. The submit button is greyed out
2

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4. Points and Vectors

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Homework0 due Sep 16, 2020 05:29 IST *Completed*

A list of n numbers can be thought of as a point or a vector in n-dimensional space. In this course, we will think of n-dimensional vectors $\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$ flexibly as points and as vectors.

Dot Products and Norm

3/3 points (graded)

Notation: In this course, we will use regular letters as symbols for numbers, vectors, matrices, planes, hyperplanes, etc. You will need to distinguish what a letter represents from the context.

Recall the dot product of a pair of vectors a and b:

$$a \cdot b \;=\; a_1 b_1 + a_2 b_2 + \cdots + a_n b_n \qquad \text{where } a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \text{ and } b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}.$$

When thinking about a and b as vectors in n-dimensional space, we can also express the dot product as

$$a \cdot b \;=\; \|a\|\|b\| \cos\alpha,$$

where α is the angle formed between the vectors a and b in n-dimensional Euclidean space. Here, $\|a\|$ refers to the length, also known as **norm**, of a:

$$\|a\| = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2}.$$

What is the length of the vector $\begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix}$?

0.5 ✔ **Answer:** 0.5

What is the length of the vector $\begin{bmatrix} -0.15 \\ 0.2 \end{bmatrix}$?

0.25 ✔ **Answer:** 0.25

What is the angle (in radians) between $\begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix}$ and $\begin{bmatrix} -0.15 \\ 0.2 \end{bmatrix}$? Choose the answer that lies between 0 and π.

(Type **pi** for the constant π. Enter an exact answer or a decimal accurate to at least 4 decimal places.)

1.5707 ✔ **Answer:** pi/2

STANDARD NOTATION

Solution:

- Plugging into the equation for norm, we get that the length of $\begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix}$ is equal to $\sqrt{0.4^2 + 0.3^2} = 0.5$. Notice

that the ratio of x:y is 3:4 so we can use 3:4:5 triangle to speed up our calculation to find the length of the vector.

- We do the same for $\begin{bmatrix} -0.15 \\ -0.2 \end{bmatrix}$.
- Using the second expression for dot product and rearranging, we get $\alpha = \cos^{-1} \frac{x \cdot y}{\|x\| \|y\|}$. Using the first expression for dot product and plugging it in we get that $\alpha = \cos^{-1} \frac{(0.4)(-0.15)+(0.3)(0.2)}{\sqrt{(0.4)^2+(0.3)^2}\sqrt{(-0.15)^2+(0.2)^2}}$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Dot Products and Orthogonality

1/1 point (graded)

Given 3-dimensional vectors $x^{(1)} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $x^{(2)} = \begin{bmatrix} a_1 \\ -a_2 \\ a_3 \end{bmatrix}$, when is $x^{(1)}$ orthogonal to $x^{(2)}$, i.e. the angle between them is $\pi/2$?

- when $2a_1 + 2a_3 = 0$
- when $a_1^2 - a_2^2 + a_3^2 = 0$
- when $a_1^2 + a_2^2 + a_3^2 = 0$

STANDARD NOTATION

Solution:

Based on the previous equations for the dot product, we find that the angle between $x^{(1)}$ and $x^{(2)}$ is:

$$\alpha = \cos^{-1} \frac{x^{(1)} \cdot x^{(2)}}{\|x^{(1)}\| \|x^{(2)}\|}$$

$$\alpha = \cos^{-1} \frac{a_1^2 - a_2^2 + a_3^2}{a_1^2 + a_2^2 + a_3^2}$$

$x^{(1)}$ is orthogonal to $x^{(2)}$ when $x^{(1)} \cdot x^{(2)} = 0$ or $a_1^2 - a_2^2 + a_3^2 = 0$.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Unit Vectors

1.0/1 point (graded)

A unit vector is a vector with length 1. The length of a vector is also called its norm.

Given any vector x, write down the unit vector pointing in the same direction as x?

(Enter **x** for the vector x and **norm(x)** for the norm $\|x\|$ of the vector x.)

(Enter **x** for the vector x, and **norm(x)** for the norm $\|x\|$ of the vector x.)

x/norm(x) ✔ **Answer:** x/norm(x)

STANDARD NOTATION

Solution:

We need to scale the vector x so that it is length 1. Right now it is length $\|x\|$ so we need to divide the vector x by $\|x\|$ in order to get the unit vector which points in the same direction.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Projections

2/2 points (graded)

Recall from linear algebra the definition of the projection of one vector onto another. As before, we have 3-dimensional vectors $x^{(1)} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $x^{(2)} = \begin{bmatrix} a_1 \\ -a_2 \\ a_3 \end{bmatrix}$.

Which of these vectors is in the same direction as the projection of $x^{(1)}$ onto $x^{(2)}$?

- ○ $x^{(1)}$
- ◉ $x^{(2)}$
- ○ $x^{(1)} + x^{(2)}$

✔

What is the signed magnitude c of the projection $p_{x^{(1)} \to x^{(2)}}$ of $x^{(1)}$ onto $x^{(2)}$? More precisely, let u be the unit vector in the direction of the correct choice above, find a number c such that $p_{x^{(1)} \to x^{(2)}} = cu$.

(Enter **a_1** for a_1, **a_2** for a_2, and **a_3** for a_3.)

$c =$

(a_1^2-1*a_2^2+a_3^2)/sqrt(a_1^2+a_2^2+a_3^2) ✔

Answer: (a_1^2-a_2^2+a_3^2)/sqrt(a_1^2+a_2^2+a_3^2)

$\frac{a_1^2 - 1 \cdot a_2^2 + a_3^2}{\sqrt{a_1^2 + a_2^2 + a_3^2}}$

STANDARD NOTATION

Solution:

- The definition of the projection of one vector onto another is the part of the first vector which points in the same direction as the second vector. Thus the projection of $x^{(1)}$ onto $x^{(2)}$ points in the direction of $x^{(2)}$
- The vector has magnitude $\|x^{(1)}\| \cos\alpha$. From our previous result $\alpha = \cos^{-1} \frac{x^{(1)} \cdot x^{(2)}}{\|x^{(1)}\| \|x^{(2)}\|}$, the projection thus has magnitude $\frac{x^{(1)} \cdot x^{(2)}}{\|x^{(2)}\|}$. Plugging in our values for $x^{(1)}$ and $x^{(2)}$ we get $\frac{a_1^2 - a_2^2 + a_3^2}{\sqrt{a_1^2 + a_2^2 + a_3^2}}$.

Hence, to find the final vector projection, we scale the unit vector in the direction of the vector projection, which is

$\frac{x^{(2)}}{||x^{(2)}||}$ by the length, $||p_{x^{(1)} \to x^{(2)}}||$. So the answer is $||p_{x^{(1)} \to x^{(2)}}|| \frac{x^{(2)}}{||x^{(2)}||}$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Discussion

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Topic: Unit 0. Brief Prerequisite Reviews, Homework 0, and Project 0:Homework 0 / 4. Points and Vectors

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can not able to submit the answer	sir after the deadline over .is there anyway to submit the answer because was not took the any session till now due to some reason. ...	1
unnecessary prerequisites	These are all totally unnecessary ,i did many ml courses these all were mentioned, but aint that much used during course	7
Can't submit the answer	I tried to submit the answer but this is disabled in all the homework, i'm on time to submit but i think this is a problem on the page. ...	2
Very annoying. Finding the c value	Even if our approach is right still the compiler shows error. Not sure what's the answer but here the box is playing with us. Got all att...	3
Why is my C incorrect for the Projections problem?	I had approached the problem by doing the following: 1. Proj (x2->x1) = u * c = u * ((x1*x2) / (x2*x2)) 2. u * ((x1*x2) / (x2*x2)) = u * ((...	2
IS it a math Challenge or HOW TO ENTER YOUR CORRECT ANSWER CHALLENGE?	IS it a math Challenge or HOW TO ENTER YOUR CORRECT ANSWER CHALLENGE?	1
Why is my c solution to the Projections problem incorrect?		5
i cant understand the c factor in projection	in found that the factor is just find the dot product between the vectors, but i cant get the answer correct, is there any error? (i just ...	10
Understanding the Various Formulas in the Wikipedia Article on Vector Projections		2
Unit vector issue...	I cant understand the question of the unit vector... can someone help me	8
annoying	So annoying! To find out Value c is very easy & straight-forward question. I believe i've typed into the correct answer of c and re-chec...	2
Same direction vs Opposite direction		4
Unable to use exponentiation operator	Am using Safari on a Mac. Also tried with Chrome. Am unable to enter (for e.g., a^2 and have to type in a*a) - this does not work for ...	2
Could not format HTML for problem. Contact course staff in the discussion forum for assistance.	Immediately before the Projections it says Could not format HTML for problem. Contact course staff in the discussion forum for assi...	4

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5. Planes

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Homework0 due Sep 16, 2020 05:29 IST *Past Due*

A hyperplane in n dimensions is a $n-1$ dimensional subspace. For instance, a hyperplane in 2-dimensional space can be any line in that space and a hyperplane in 3-dimensional space can be any plane in that space. A hyperplane separates a space into two sides.

In general, a hyperplane in n-dimensional space can be written as $\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n = 0$. For example, a hyperplane in two dimensions, which is a line, can be expressed as $Ax_1 + Bx_2 + C = 0$.

Using this representation of a plane, we can define a plane given an n-dimensional vector $\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}$ and offset θ_0.

This vector and offset combination would define the plane $\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n = 0$. One feature of this representation is that the vector θ is normal to the plane.

Number of Representations

1/1 point (graded)

Given a d-dimensional vector θ and a scalar offset θ_0 which describe a hyperplane $\mathcal{P} : \theta \cdot x + \theta_0 = 0$. How many alternative descriptions θ' and θ'_0 are there for this plane $\mathcal{P}$?

- ◯ 0
- ◯ 1
- ◉ ∞ ✔

STANDARD NOTATION

Solution:

Given a normal vector θ and an offset θ_0 that uniquely determine the plane $\theta \cdot x + \theta_0 = 0$, we can scale θ and θ_0 by $\alpha > 0$, $\alpha \in \mathbb{R}$ without changing the orientation of the plane. Notice that if we only scale the normal $\theta' = \alpha\theta$ without affecting the offset $\theta'_0 = \theta_0$, then for $\alpha > 1$ the value of the θ'_0 must decrease for $\theta' \cdot x + \theta'_0 = 0$. Thus, there is an infinite number of possible parameter vectors that can describe the plane.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Orthogonality Check

1 point possible (graded)

To check if a vector x is orthogonal to a plane $\mathcal{P}$ characterized by θ and θ_0, we check whether

- ◯ $x = \alpha\theta$ for some $\alpha \in \mathbb{R}$ ✔
- ◯ $x \cdot \theta = 0$
- ◯ $\theta + \theta_0 = 0$

$x \cdot \theta + \theta_0 = 0$

STANDARD NOTATION

Solution:

A vector x is orthogonal to the plane if and only if it is collinear with the normal vector θ of the plane.

Submit You have used 0 of 1 attempt

Answers are displayed within the problem

Perpendicular Distance to Plane

1 point possible (graded)
Given a point x in n-dimensional space and a hyperplane described by θ and θ_0, find the **signed distance between the hyperplane and** x. This is equal to the perpendicular distance between the hyperplane and x, and is positive when x is on the same side of the plane as θ points and negative when x is on the opposite side.

(Enter **theta_0** for the offset θ_0.
Enter **norm(theta)** for the norm $||\theta||$ of a vector θ.
Use * to denote the dot product of two vectors, e.g. enter **v*w** for the dot product $v \cdot w$ of the vectors v and w.)

Answer: (trans(theta)*x+theta_0)/norm(theta)

STANDARD NOTATION

Solution:

The distance from a point x_1 to a plane $\theta \cdot x + \theta_0$ is equal to $|\theta \cdot x_1 + \theta_0|/||\theta||$. If $\theta \cdot x_1 + \theta_0 > 0$, then x_1 belongs to a half-space in the direction of θ. Therefore, we can define the signed distance as:

$$d_{x_1} = \frac{\theta \cdot x_1 + \theta_0}{||\theta||}$$

Submit You have used 0 of 5 attempts

Answers are displayed within the problem

Orthogonal Projection onto Plane

1 point possible (graded)
Find an expression for the **orthogonal projection** of a point v onto a plane $\mathcal{P}$ that is characterized by θ and θ_0.
Write your answer in terms of v, θ and θ_0.

(Enter **theta_0** for the offset θ_0.
Enter **norm(theta)** for the norm $||\theta||$ of a vector θ.
Use * to denote the dot product of two vectors, e.g. enter **v*w** for the dot product $v \cdot w$ of the vectors v and w.)

Answer: v-(((trans(v)*theta)+theta_0)/(norm(theta))^2)*theta

STANDARD NOTATION

Solution:

Since $v - x$ is collinear with the normal, $v - x = \lambda\theta$ for some λ. Also, x lies in the plane, so $\theta \cdot x + \theta_0 = 0$. Solve this to get the value of λ and plug it back to find the orthogonal projection:

$$\begin{aligned}(v - \lambda\theta) \cdot \theta + \theta_0 &= 0 \\ \lambda &= \frac{v \cdot \theta + \theta_0}{\|\theta\|^2} \\ x &= v - \frac{v \cdot \theta + \theta_0}{\|\theta\|}\hat{\theta}\end{aligned}$$

Submit You have used 0 of 5 attempts

Answers are displayed within the problem

Perpendicular Distance to Plane

4/4 points (graded)

Let $\mathcal{P}_1$ be the hyperplane consisting of the set of points $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ for which $3x_1 + x_2 - 1 = 0$. (Note that this hyperplane is in fact a line, since it is 1-dimensional.)

What is the signed perpendicular distance of point $a = [-1, -1]$ from $\mathcal{P}_1$?

-1.5812 ✔ **Answer:** -5/sqrt(10)

What is the signed perpendicular distance of the origin from $\mathcal{P}_1$?

-.3162 ✔ **Answer:** -1/sqrt(10)

What is the orthogonal projection of point $a = [-1, -1]$ onto p_1?

First coordinate:

.5 ✔ **Answer:** 1/2

Second coordinate:

-.5 ✔ **Answer:** -1/2

STANDARD NOTATION

Solution:

1. For $a = [-1, -1]^T$ the signed distance is:

$$\frac{\theta \cdot a + \theta_0}{\|\theta\|} = \frac{(3)(-1) + (1)(-1) - 1}{\sqrt{(3)^2 + (1)^2}} = -\frac{5}{\sqrt{10}}$$

2. For $a = [0, 0]^T$ the signed distance is:

$$\theta \cdot 0 + \theta_0 \qquad -1 \qquad 1$$

$$\frac{}{\|\theta\|} = \frac{}{\sqrt{(3)^2+(1)^2}} = -\frac{}{\sqrt{10}}$$

3. For $a = [-1,-1]^T$ the orthogonal projection is:

$$\begin{aligned} x &= v - \frac{v\cdot\theta+\theta_0}{\|\theta\|}\hat{\theta} \\ &= [-1,-1]^T - \frac{[-1,-1]^T\cdot[3,1]^T+(-1)}{\sqrt{(3)^2+(1)^2}}[3/\sqrt{10},1/\sqrt{10}]^T \\ &= [1/2,-1/2]^T \end{aligned}$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

2. (f)

1/1 point (graded)

Consider a hyperplane in a d-dimensional space. If we project a point onto the plane, can we recover the original point from this projection?

no ✔

STANDARD NOTATION

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Topic: Unit 0. Brief Prerequisite Reviews, Homework 0, and Project 0:Homework 0 / 5. Planes

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- Grab some basics here. — 11
  For those who might be starting out cold with regards to planes and hyperplanes, do check this out: http://faculty.bard.edu/belk/ma...
  Pinned Community TA
- solution for the problems — 3
  Dear staff, since the problems are overdue. Can I get the solutions for the problems that I was not able to solve?
- To Staff: Question "What is the signed perpendicular distance of point a=[−1,−1] from P1?" — 1
- Perpendicular Distance to Plane — 3
  I don't quite get the notation. I was thinking about |s| = |(x-r)·(1/theta)theta| but it won't work since I don't have that extra point, I ...
- In the solution for Orthogonal Projection onto Plane is the last step correct? — 2
  when we plug in lambda to find x, in the denominator the l2-norm squared becomes the l2-norm. Is this correct or a mistake? If corr...
- What does "alternative descriptions" mean in the first question? — 3
  Does it mean different values or the way to interpret the equation?
- Answers to Homework 0 — 1
  Could anyone get Perpendicular Distance to Plane and the orthogonal distance to plane question right?? I know the logic but when I ...
- Could not format HTML for problem. Contact course staff in the discussion forum for assistance. — 6
  Hi, I am getting the error "Could not format HTML for problem. Contact course staff in the discussion forum for assistance." from th...

?	Any good source or recommended books to master this? Orthogonal Projection onto Plane I got some verbose equation that the grader accepted but not sure if the form was correct. And it is not something I can clearly find ...	2
	For the forth question, why does this kind of expression is not correct? I am trying to use sinθ to get the answer, but, it doesn't work? I can't paste the answer on it.	2
	Notation needs to be consistent	2
	Is the word 'subspace' really correct?	3
	2.(f) needs a title It looks like 2.(f) was part of a second question in earlier run. To be consistent with the rest of the questions on this page, it needs a t...	1

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6. Probabilty Review: Probability Density Functions

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Homework0 due Sep 16, 2020 05:29 IST *Completed*

Let X be a **continuous** random variable with probability **density** function (pdf) $f_X(x)$.

Concept Check

4/4 points (graded)

1. Is the value of $f_X(x)$ always $\in [0,1]$?

   ◯ yes

   ◉ no ✔

2. For $a < b$, $\int_a^b f_X(x)\,dx \in [0,1]$ and represents the probability that the value of X falls between a and b.

   ◉ yes ✔

   ◯ no

3. Is the value of $f_X(x)$ always non-negative?

   ◉ yes ✔

   ◯ no

4. The value of integral $\int_{-\infty}^{\infty} f_X(x)\,dx$ of $f_X(x)$ from $-\infty$ to ∞ is a finite, undetermined value.

   ◯ yes

   ◉ no ✔

Solution:

1. While probabilities are always between 0 and 1, the probability density function (PDF) is not the actual probability of observing a particular outcome. This is an important distinction from probability mass functions, the analog for discrete random variables. So the PDF can be greater than 1, but its integral, which gives the probability must always be $\in [0,1]$.

2. Yes, by definition.

3. Yes, by definition $f_X(x) \geq 0$.

4. The integral across a range (here, from $-\infty$ to ∞) is the total probability that X takes values in that range. Since this range contains all possible values any random variable can take, by definition, not only is the integral finite, but since the total probability must be 1, the integral is always 1, i.e. $\int_{-\infty}^{\infty} p_X(x)\,dx = 1$.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

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Topic: Unit 0. Brief Prerequisite Reviews, Homework 0, and Project 0:Homework 0 / 6. Probabilty Review: Probability Density Functions

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?	PDF The beginning section mentions a PDF but no PDF is attached.	5
	Is the value of PDF always ϵ[0,1] Hi everyone, PDF is presented probability, so the value of PDF is always between 0 and 1, right? Because the value of PDF is non-negativ...	9
	Maybe just me - English ambiguity Q2 I don't know whether it was me being a bit silly or due to British and American English preferences, but I read 'X falls between a and b' in...	2
	Meaning of undetermined Not to get too philosophical but this did throw me off - undetermined means "not known" in general but it can mean either unknowable...	5

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7. Univariate Gaussians

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Homework0 due Sep 16, 2020 05:29 IST *Past Due*

A univariate **Gaussian** or **normal distributions** can be completely determined by its mean and variance.

Gaussian distributions can be applied to a large numbers of problems because of the central limit theorem (CLT). The CLT posits that when a large number of **independent and identically distributed ((i.i.d.)** random variables are added, the cumulative distribution function (cdf) of their sum is approximated by the cdf of a normal distribution.

Recall the probability density function of the univariate Gaussian with mean μ and variance σ^2, $\mathcal{N}(\mu, \sigma^2)$:

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/(2\sigma^2)}.$$

Probability review: PDF of Gaussian distribution

2/2 points (graded)

In practice, it is not often that you will need to work directly with the probability density function (pdf) of Gaussian variables. Nonetheless, we will make sure we know how to manipulate the (pdf) in the next two problems.

The pdf of a Gaussian random variable X is given by

$$f_X(x) = \frac{n}{3\sqrt{2\pi}} \exp\left(-\frac{n^2(x-2)^2}{18}\right),$$

then what is the mean μ and variance σ^2 of X?
(Enter your answer in terms of n.)

$\mu =$ 2 ✔ **Answer:** 2

2

$\sigma^2 =$ 9/(n*n) ✔ **Answer:** 9/n^2

$\frac{9}{n \cdot n}$

STANDARD NOTATION

Solution:

Comparing

$$f_X(x) = \frac{n}{3\sqrt{2\pi}} \exp\left(-\frac{n^2(x-2)^2}{18}\right) = \frac{1}{(3/n)\sqrt{2\pi}} \exp\left(-\frac{(x-2)^2}{2(3/n)^2}\right)$$

with

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right),$$

yields $\mu = 2$ and $\sigma^2 = \frac{9}{n^2}$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Probability review: PDF of Gaussian distribution

1 point possible (graded)
Let $X \sim \mathcal{N}\left(\mu, \sigma^2\right)$, i.e. the pdf of X is

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right).$$

Let $Y = 2X$. Write down the pdf of the random variable Y. (Your answer should be in terms of y, σ and μ. Type **mu** for μ, **sigma** for σ.)

$f_Y(y) =$

Answer: 1/(2*sigma*sqrt(2*pi))* exp(-(y-2*mu)^2/(8*sigma^2))

STANDARD NOTATION

Solution:

If $X \sim \mathcal{N}\left(\mu, \sigma^2\right)$, then $Y = 2X \sim \mathcal{N}\left(2\mu, 4\sigma^2\right)$ by the following general properties of expectations and variance:

$$\begin{aligned} \mathbf{E}[2X] &= 2\mathbf{E}[X] \\ \mathsf{Var}[2X] &= 2^2\mathsf{Var}[X] = 4\mathsf{Var}[X]. \end{aligned}$$

Therefore,

$$f_Y(y) = \frac{1}{2\sigma\sqrt{2\pi}} \exp\left(-\frac{(y-2\mu)^2}{2\left(4\sigma^2\right)}\right).$$

Alternate solution: In general, for any continuous random variables X and any continuous monotonous (i.e. always increasing or always decreasing) function g, such that $Y = g(X)$, the pdf of Y is given by:

$$f_Y(y) = \frac{f_X(x)}{|g'(x)|} \qquad \text{where } x = g^{-1}(y).$$

In this problem, $X \sim \mathcal{N}\left(\mu, \sigma^2\right)$, $Y = g(X) = 2X$, and $g'(x) = 2$. Therefore:

$$\begin{aligned} f_Y(y) &= \frac{f_X\left(\frac{y}{2}\right)}{\left|g'\left(\frac{y}{2}\right)\right|} \\ &= \frac{1}{g'(y/2)\,\sigma\sqrt{2\pi}} \exp\left(-\frac{(y/2-\mu)^2}{2\sigma^2}\right) \\ &= \frac{1}{2\sigma\sqrt{2\pi}} \exp\left(-\frac{((y-2\mu)/2)^2}{2\sigma^2}\right) \\ &= \frac{1}{2\sigma\sqrt{2\pi}} \exp\left(-\frac{((y-2\mu))^2}{2(4)\sigma^2}\right) \end{aligned}$$

and we recover the same answer as above.

Submit You have used 0 of 3 attempts

Answers are displayed within the problem

Argmax

1 point possible (graded)
Let $f_X(x;\mu,\sigma^2)$ denote the probability density function of a normally distributed variable X with mean μ and variance σ^2. What value of x maximizes this function?

(Enter **mu** for the mean μ, and **sigma^2** for the variance σ^2.)

Answer: mu

STANDARD NOTATION

Solution:

The answer is μ, the mean of the distribution. If you look at the graph of the standardized normal distribution, you see that the maximum is at 0, its mean. Any normal distribution with different mean or variance is simply a shifted (different mean) or stretched (different variance) version of this distribution, so our result holds for any normally distributed variable. Alternatively, you can differentiate the PDF and determine the maximum, which gives you the same result.

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Answers are displayed within the problem

Maximum of pdf

1 point possible (graded)
As above, let $f_X(x;\mu,\sigma^2)$ denote the probability density function of a normally distributed variable X with mean μ and variance σ^2.

What is the maximum value of $f_X(x;\mu,\sigma^2)$?
(Enter **mu** for the mean μ, and **sigma^2** for the variance σ^2.)

Answer: 1/sqrt(2*pi*sigma^2)

STANDARD NOTATION

Solution:

From the question above, we know that the maximum value occurs when $x = \mu$. Observe the PDF of a normal variable: setting $x = \mu$ forces the exponent of e to 0, leaving us with the answer above.

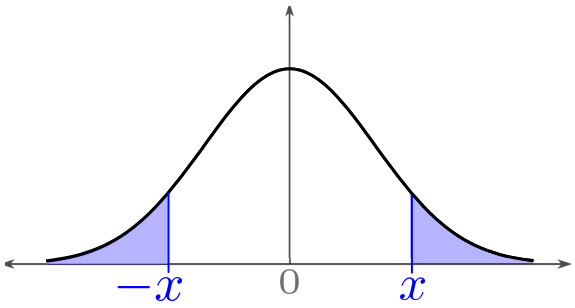
Submit You have used 0 of 3 attempts

Answers are displayed within the problem

Quantiles

1 point possible (graded)
The **quantile** of order $1-\alpha$ of a variable X, denoted by q_α (specific to a particular X), is the number such that $\mathbf{P}\left(X \leq q_\alpha\right) = 1-\alpha$.

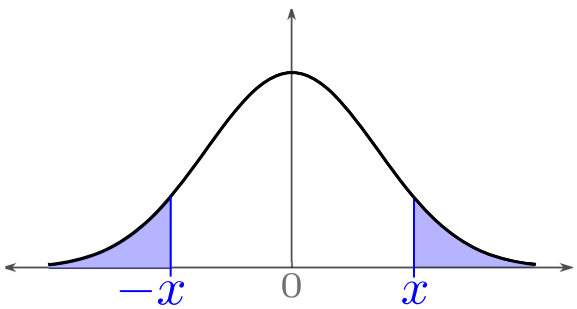
Graphed below is the pdf of the normal distribution with generic/unknown (but fixed) variance σ^2. If the total area of the two shaded regions is 0.03, then what is x?
(Choose all that apply.)



The total area of the two shaded regions is 0.03.

- [ ] $\mathbf{P}\left(|X| \leq 0.03\right)$
- [ ] $\mathbf{P}\left(|X| \leq 0.015\right)$
- [ ] 0.97
- [ ] 0.985
- [ ] $q_{0.03}$
- [x] $q_{0.015}$ ✔

Solution:



The total area of the two shaded regions equals $\mathbf{P}\left(|X| \geq x\right) = 0.03$. By symmetry, the probability in the positive tail is $\mathbf{P}\left(X \geq x\right) = 0.015$; hence $x = q_\alpha$ with $\alpha = 0.015$.
For the wrong choices:

- The first pair of choices mixed up the values of probability with the value of the variable.

- The third and fourth choices "0.97" and "0.985" are meant to play the role resembling $1-\alpha$ in this example, but these are wrong for the same reasons as the first pair of choices. In any case, to give a particular numerical value of x, the answer must depend on σ.

- The fifth choice would have been correct again if the area of one of the tails is 0.03.

Submit You have used 0 of 2 attempts

Answers are displayed within the problem

Probability

1 point possible (graded)
Let $X \sim \mathcal{N}(1,2)$, i.e., the random variable X is normally distributed with mean 1 and variance 2. What is the probability that $X \in [0.5, 2]$?

(Enter your answer accurate to at least 4 decimal places.)

$P(X \in [0.5, 2]) =$

STANDARD NOTATION

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?	Questions about these exercices 1. For the question where Y = 2X. If the underlying distribution were a univariate Gaussian distribution N(avg=0,var=1). Then the ans...	3
?	HW0 Section 7, Quantiles: Are 0.97 and 0.985 plausible values for x, since sigma is not given in the question? mu=0, P(|X|>=0.97) = 0.03 when sigma=0.447 mu=0, P(|X|>=0.985) = 0.03 when sigma=0.4566 Since the problem states that varianc...	2
?	Problem 1 Integration Hi all, I'm a bit rusty with integration and have been struggling to integrate the first PDF to get the mean and variance. I'm unsure of ...	2
	Grab some basics here! =) Here we go: - For quantile qs - https://www.youtube.com/watch?v=TzKeCv4S7nY - For normal distri qs - https://web.stanford.edu/cla... Community TA	10
?	Last Question - N(1,2) -- grader seems to be wrong Hi Staff, Can you please check if the grader is alright? I seem to have got the right answer, I verified both via tables and online calcula...	23
	Question on quantiles Confused about the quantiles question, any helpful pointer to approach the problem will be very helpful!	5
?	Is there some issue in Grading /Submit button? my already graded questions were regraded using remaining attempts. And the one that I did not ever answer is showing grayed ou...	1
	Kind of long homework isn't it? For the love of God, this is long.... for mere 1 point, this is awfully long	2
	[Staff] Probability review: PDF of Gaussian distribution The accepted answer to this question considers the combination of the standard deviations, instead of the variance when adding th...	2
?	Q1: what is meant by "in terms of n"	2

Hi, can someone hint on what is meant by "in terms of n" ? My understanding is that: 1. I infer the value of n from the expression. 2. ...

if anybody want to know what happen when you transform random variable (Probability review: PDF of Gaussian distribution) 1

https://www.khanacademy.org/math/ap-statistics/random-variables-ap/transforming-random-variables/v/impact-of-scaling-and-shif...

Accuracy to at least 4 decimals makes use of Z-score tables difficult for last question 3

Hello, Just to outline that most common Z-score tables work with an accuracy of 2 decimals on the Z-score. Thus, I couldn't obtain a ...

Q No -1 sigma^2 value

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8. (Optional Ungraded Warmup) 1D Optimization via Calculus

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(Optional) Review: 1D Optimization via Calculus

0 points possible (ungraded)
(For this problem, you are welcome to use any computational tools that would be helpful.)

Let $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 10$ defined on the interval $[-4, 4]$.

Let x_1 and x_2 be the critical points of f, and let's impose that $x_1 < x_2$. Fill in the next two boxes with the values of x_1 and x_2, respectively: (Recall that the **critical points** of f are those $x \in \mathbb{R}$ such that $f'(x) = 0$.)

$x_1 =$ Answer: -1

$x_2 =$ Answer: 3

Fill in the next two boxes with the values of $f''(x_1)$ and $f''(x_2)$, respectively:

$f''(x_1) =$ Answer: -4

$f''(x_2) =$ Answer: 4

Solution:

Observe that

$$f'(x) = x^2 - 2x - 3 = (x-3)(x+1).$$

Hence the **critical points** are $x_1 = -1$ and $x_2 = 3$. The **second derivative** is

$$f''(x) = 2x - 2$$

so that

$$f''(x_1) = -4, \quad f''(x_2) = 4.$$

Submit You have used 0 of 3 attempts

Answers are displayed within the problem

(Optional) Review: 1D Optimization via Calculus (Continued)

0 points possible (ungraded)
(For this problem, you are welcome to use any computational tools that would be helpful.)

Recall that x_1 and x_2 are the critical points of the function $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 10$.

According to the second derivative test, x_1 is a ...

◯ Local Maximum ✔

- ◯ Local Minimum
- ◯ None of the above

and x_2 is a

- ◯ Local Maximum
- ◯ Local Minimum ✔
- ◯ None of the above

At what value of x is the (global) minimum value of $f(x)$ attained on the interval $[-4, 4]$?

Answer: -4

At what value of x the (global) maximum value of $f(x)$ attained on the interval $[-4, 4]$?

Answer: -1

Solution:

The previous problem implies that f is concave at x_1 and convex at x_2, so x_1 is a **local maximum** and x_2 is a **local minimum**. To figure out the *global* extrema, we need to test the critical points as well as the endpoints: -4 and 4. We compute that

$$f(x_1) = \frac{35}{3} \approx 11.6666, \quad f(x_2) = 1$$

$$f(-4) = -\frac{46}{3} \approx -15.33333, \quad f(4) = 10/3 \approx 3.3333$$

Hence the **maximum value** of f on $[-4, 4]$ is $\frac{35}{3} \approx 11.6666$ and the **minimum value** is $-\frac{46}{3} \approx -15.33333$.

Remark: It is very important to remember to test the endpoints when doing optimization.

Submit You have used 0 of 2 attempts

Answers are displayed within the problem

(Optional)Strict Concavity

0 points possible (ungraded)

Which of the following functions are strictly concave? (Choose all that apply.) (Recall that a twice-differentiable function $f : I \to \mathbb{R}$, where I is a subset of $\mathbb{R}$, is **strictly concave** if $f''(x) < 0$ for all $x \in I$.)

- ☐ $f_1(x) = x$ on $\mathbb{R}$

- [x] $f_2(x) = -e^{-x}$ on $\mathbb{R}$ ✔
- [x] $f_3(x) = x^{0.99}$ on the interval $(0, \infty)$ ✔
- [ ] $f_4(x) = x^2$ on $\mathbb{R}$

Solution:

- $f_1(x) = x$ is **not** strictly concave because $f_1\text{''}(x) = 0$.
- $f_2(x) = -e^{-x}$ is strictly concave because $f_2\text{''}(x) = -e^{-x} < 0$ for all $x \in \mathbb{R}$.
- $f_3(x) = x^{0.99}$ is strictly concave because $f_3\text{''}(x) = (0.99)(-.01)x^{-1.01} < 0$ for all $x \in (0, \infty)$.
- $f_4(x) = x^2$ is **not** strictly concave because $f_4\text{''}(x) = 2 > 0$. In fact, this function is strictly *convex*.

Submit You have used 0 of 2 attempts

Answers are displayed within the problem

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(Optional) Review: 1D Optimization via Calculus issus I think that there is a issus on grader, The value for x1, x2 accepted by the grader are right up to the sign. I hope I'm clear enough	10
Review: 1D Optimization via Calculus f''(x) Hi, my name is Adrià and I am from Barcelona. I think there is a problem with the answers of the second derivative. Since f'(x) = x^2 -2x -...	2

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9. Gradients and Optimization

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Homework0 due Sep 16, 2020 05:29 IST *Past Due*

Multivariable Calculus Review: Simple Gradient

1 point possible (graded)

Let

$$f: \quad \mathbb{R}^d \quad \to \quad \mathbb{R}$$
$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_d \end{pmatrix} \quad \mapsto \quad f(\theta).$$

denote a **differentiable** function. The **gradient** of f is the vector-valued function

$$\nabla_\theta f: \quad \mathbb{R}^d \quad \to \quad \mathbb{R}^d$$
$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_d \end{pmatrix} \quad \mapsto \quad \left. \begin{pmatrix} \frac{\partial f}{\partial \theta_1} \\ \frac{\partial f}{\partial \theta_2} \\ \vdots \\ \frac{\partial f}{\partial \theta_d} \end{pmatrix} \right|_\theta .$$

Consider

$$f(\theta) = \theta_1^2 + \theta_2^2 .$$

Compute the gradient ∇f.

(Enter your answer as a vector, e.g., type **[2,x]** for the vector $\begin{pmatrix} 2 \\ x \end{pmatrix}$. Note the square brackets, and commas as separators. Enter **theta_i** for θ_i.)

$\nabla_\theta f(\theta) =$ [ ] **Answer:** [2*theta_1,2*theta_2]

STANDARD NOTATION

Solution:

$$f(\theta) = \theta_1^2 + \theta_2^2$$
$$\nabla f(\theta) = \left. \begin{pmatrix} \frac{\partial f}{\partial \theta_1} \\ \frac{\partial f}{\partial \theta_2} \end{pmatrix} \right|_\theta = \begin{pmatrix} 2\theta_1 \\ 2\theta_2 \end{pmatrix}.$$

Submit You have used 0 of 3 attempts

Answers are displayed within the problem

Geometric Picture of the Function

3 points possible (graded)

As above, consider $f(\theta) = \theta_1^2 + \theta_2^2$. Let us visualize $f(\theta)$ as a surface on the (θ_1, θ_2)-plane. We will use the usual horizonal plane as the (θ_1, θ_2)-plane, and the vertical axis as the $f(\theta)$-axis.

horizontal plane as the (θ_1, θ_2) plane, and the vertical axis as the $f(\theta)$ axis.

Consider the level curves $\theta_1^2 + \theta_2^2 = K$ where $K > 0$ is some fixed real number.

What is the shapes of such a curve?

- parabola
- circle ✔
- hyperbola
- line

Consider how the level curves $\theta_1^2 + \theta_2^2 = +K$ change as K increases from 0 to ∞. Does the graph (surface) of $f(\theta)$ have a global maximum, or global minimum, or neither?

- global maximum
- global minimum ✔
- neither

At each point $\theta = (\theta_1, \theta_2)$ in the (θ_1, θ_2)-plane, $f(\theta)$ decreases in the direction of...

- $\nabla_\theta f(\theta)$
- $-\nabla_\theta f(\theta)$ ✔

Solution:

The graph of $f(\theta)$ is a paraboloid that opens downwards. Its global maximum is at $\theta = (0, 0)$. We see that $\nabla_\theta f(\theta) = (2\theta_1, 2\theta_2)^T$, and hence $-\nabla_\theta f(\theta)$ points towards the origin at all points θ.

Submit You have used 0 of 2 attempts

Answers are displayed within the problem

Gradient ascent/descent methods are typical tools for maximizing/minimizing functions. Consider the function $L(x, \theta)$ where $\theta = [\theta_1, \theta_2, \ldots, \theta_n]^T$ and $x = [x_1, x_2, \ldots, x_n]^T$. Our goal is to select θ such to maximize/minimize the value of L while keeping x fixed.

Compute the Gradient

1 point possible (graded)
The gradient $\nabla_\theta L(x, \theta)$ is a vector with n components:

$$\nabla_\theta L(x, \theta) = \begin{pmatrix} \frac{\partial}{\partial \theta_1} L(x, \theta) \\ \vdots \\ \end{pmatrix}.$$

$$\left\langle \frac{\partial}{\partial \theta_n} L(x, \theta) \right\rangle$$

(Note that we are treating x as a constant and also differentiating w.r.t. to θ.)

Let

$$L(x, \theta) = \log\left(1 + \exp\left(-\theta \cdot x\right)\right).$$

(Notice that here the $\log$ function is the natural algorithm.)

Evaluate the gradient $\nabla_\theta L(x, \theta)$. Which of the following is its j^{th} component?

- $\dfrac{\exp(-\theta \cdot x)}{1 + \exp(-\theta \cdot x)}$
- $\dfrac{-x_j \exp(-\theta \cdot x)}{1 + \exp(-\theta \cdot x)}$ ✔
- $\dfrac{-x_j}{1 + \exp(-\theta \cdot x)}$

Solution:

The derivative of $\log(x) = \frac{1}{x}$ and the derivative of $e^{cx} = ce^{cx}$. Applying these rules with the chain rule gives the correct answer.

Submit You have used 0 of 1 attempt

Answers are displayed within the problem

Gradient Ascent or Descent

1 point possible (graded)
The direction of the derivative of a function gives us the direction of the largest change in the function as the independent variables vary.

In gradient ascent/descent methods, we make an educated guess about the next values of θ, with consecutive updates that will hopefully eventually converge to the global minimum of $L(x, \theta)$ (if it exists).

If

$$\theta' = \theta + \epsilon \cdot \nabla_\theta L(x, \theta)$$

where ϵ is a small positive real number, Which of the following is true?

- $L(x, \theta') > L(x, \theta)$ ✔
- $L(x, \theta') < L(x, \theta)$

STANDARD NOTATION

Solution:

Solution:

Consider the one-dimensional case. If the gradient is positive, we obtain θ' by moving from θ in the positive direction. This increases $L(x, \theta)$. If the gradient is negative, we move in the negative direction, again increasing $L(x, \theta)$. This analysis extends to higher dimensions. Note that if we used the function above to continue updating θ, we would (in theory) maximize $L(x, \theta)$. Alternatively if our update rule was $\theta' = \theta - \epsilon \cdot \nabla_\theta L(x, \theta)$, we would minimize the function. There are more complications in higher dimensions, but this is the basic idea behind stochastic gradient descent, which forms the backbone of modern machine learning.

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☑	Gradient Ascent or Descent If we dont know what the function is (convex or concave), how can we answer this question? I mean, convex and concave functions ...	3
💬	questions Questions Multivariable Calculus Review: Simple Gradient I submitted. 2 [theta_1,theta_2] or 2 * [theta_1,theta_2] was entered but w...	1
?	[Staff] Geometric Picture of the Function: typo in solution? Shouldn't the solution say it's a paraboloid is that opens **upward**... its global **minimum** is at (0,0)?	2
?	theta_i Invalid Input: \'theta_i\' not permitted in answer as a variable This is a question for STAFF.	4
?	Could not format HTML for problem. Contact course staff in the discussion forum for assistance Question to the staff - I get this message for the first few questions in this excercise set - "Could not format HTML for problem. Cont...	1
☑	[STAFF] Gradient Ascent or Descent problem	12
💬	@Staff Natural algorithm vs Natural logarithm (Notice that here the log function is the natural **logarithm**.)	2
?	@Staff can you please reset my “Compute the Gradient”?	2
💬	Confuse ... The answer seems wrong to me ---if I go up when I stop because the L is getting smaller I might a max if it is a min I should go smaller and smaller until I I start going up again . suppose L is continue	1
💬	Grab some basics here! =) Here we go - https://www.khanacademy.org/math/multivariable-calculus/multivariable-derivatives/partial-derivative-and-gradient-ar... Community TA	2
💬	Hint: Gradient Ascent or Descent Hi! Just watch again the video Loss Function, Gradient Descent, and Chain Rule from the review. After that, it should be more clear.	2
💬	Typo Error (Notice that here the log function is the natural algorithm.) >> natural logarithm	4

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10. Matrices and Vectors

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Homework0 due Sep 16, 2020 05:29 IST *Past Due*

Objectives:

- Recognize the dimensions of the product of two or more matrices.
- Understand the concept of rank of a matrix, and how it relates to the invertibility of an $n \times n$ matrix.
- (Optional) Understand the concept of **eigenvalues** and **eigenvectors** of an $n \times n$ matrix.

Matrix Vector Product 1

1 point possible (graded)

Let $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & 1 \end{bmatrix}$.

Let $g = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}$.

Can we compute $g\mathbf{A}$?

- ◯ yes ✔
- ◯ no

Solution:

The dimension of g is 1×3 and the dimension of A is 3×3. Since the number of columns in g equals the number of rows in A, the product exists.

Submit You have used 0 of 1 attempt

Answers are displayed within the problem

Matrix Vector Product 2

1 point possible (graded)

Let g and $\mathbf{A}$ be as above. Can we compute $\mathbf{A}g$?

- ◯ yes
- ◯ no ✔

Solution:

Unlike part c), the dimension of A is 3×3 and the dimension of g is 1×3. Since the number of columns in A does not equal the number of rows in g, the product does not exist. Note that this example shows that matrix multiplication is not commutative, i.e., $AB \neq BA$.

Submit You have used 0 of 1 attempt

Answers are displayed within the problem

Find the Rank

1/1 point (graded)

Let $\mathbf{B} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 4 \\ 5 & 6 & 4 \end{bmatrix}$. Determine the rank of $\mathbf{B}$. Recall that the rank of a matrix is the number of linearly independent rows or columns.

The notion of linear independence and rank is reviewed on the tab after the next one, titled *Linear Independence, Subspaces and Dimension*.

$\text{rank}\,(\mathbf{B}) =$ 2 ✔ **Answer:** 2

Solution:

Note that the first two rows of B are linearly independent since they are not multiples of each other. Now solve the system $\begin{bmatrix} 2a + b = 5c \\ a + 4b = 6c \\ 4b = 4c \end{bmatrix}$. Recall that these three vectors will be linearly independent if the only solution to this set of equations is the zero vector. Since we find that this system has the solution $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$, these vectors are not linearly independent and the rank of the matrix is 2.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Matrix Times its Inverse

1/1 point (graded)

Let $\mathbf{M}^{-1}$ denote the inverse of a matrix $\mathbf{M}$. Let $\mathbf{A}$ be as defined above. Compute $\mathbf{A}^{-1}$. What matrix does the product $\mathbf{A}\mathbf{A}^{-1}$ produce?

- (●) identity matrix
- ( ) zero matrix

✔

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Topic: Unit 0. Brief Prerequisite Reviews, Homework 0, and Project 0:Homework 0 / 10. Matrices and Vectors

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Grab some basics here! =)

Here we go - To tackle this, do consider finishing project. Using numpy will help. - https://www.mathsisfun.com/algebra/matrix-multiplyi... 4

Community TA

The matrix in last question is singular and hence it's inverse does not exist.
Doesn't that make the product A*A-1 undefined?

2

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11. Matrix Multiplication

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Homework0 due Sep 16, 2020 05:29 IST *Completed*

Matrix Multiplication

6/6 points (graded)

Let $\mathbf{A} = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & -4 \end{pmatrix}$ and let $\mathbf{B} = \begin{pmatrix} -1 & 0 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 3 \end{pmatrix}$. The dimensions of the product $\mathbf{AB}$ are:

2 ✔ **Answer:** 2 rows × 3 ✔ **Answer:** 3 columns.

More generally, let $\mathbf{A}$ be an $m \times n$ matrix and $\mathbf{B}$ be an $n \times k$ matrix. What is the size of $\mathbf{AB}$?

m ✔ **Answer:** m rows × k ✔ **Answer:** k columns.

In addition, if $\mathbf{C}$ is a $k \times j$ matrix, what is the size of $\mathbf{ABC}$?

m ✔ **Answer:** m rows × j ✔ **Answer:** j columns.

Solution:

The size of the output is the number of rows of the left matrix, and the number of columns of the right matrix. The two dimensions on the inside (columns of the left matrix, rows of the right matrix) must match.
In the first part, $\mathbf{AB}$ is 2×3.
For the second and third parts, $\mathbf{AB}$ is $m \times k$ and $\mathbf{ABC}$ is $m \times j$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Vector Inner product

1/1 point (graded)

Suppose $\mathbf{u} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$. The product $\mathbf{u}^T\mathbf{v}$ evaluates the **inner product** (also called the **dot product**) of $\mathbf{u}$ and $\mathbf{v}$, which evaluates to

$\mathbf{u}^T\mathbf{v} =$ 2 ✔ **Answer:** 2

The inner product of $\mathbf{u}$ and $\mathbf{v}$ is sometimes written as $\langle \mathbf{u}, \mathbf{v} \rangle$.

Solution:

The inner product is always a scalar (a 1×1 matrix). In this case, it evaluates to $1 \cdot -1 + 3 \cdot 1 = 2$. In general, if $\mathbf{u} = (u_1, u_2, \ldots, u_n)^T$ and $\mathbf{v} = (v_1, v_2, \ldots, v_n)^T$, then $\mathbf{u}^T\mathbf{v} = \sum_{i=1}^{n} u_i v_i$.

$$\begin{pmatrix} u_1 & \cdots & u_n \end{pmatrix} \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = (\cdot)$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Vector Outer product

4/4 points (graded)

Suppose $\mathbf{u} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$. The product $\mathbf{u}\mathbf{v}^T$ evaluates the **outer product** of $\mathbf{u}$ and $\mathbf{v}$, which is a 2×2 matrix in this case.

What is $(\mathbf{u}\mathbf{v}^T)_{1,1}$?

-1 ✔ Answer: -1

What is $(\mathbf{u}\mathbf{v}^T)_{1,2}$?

1 ✔ Answer: 1

What is $(\mathbf{u}\mathbf{v}^T)_{2,1}$?

-3 ✔ Answer: -3

What is $(\mathbf{u}\mathbf{v}^T)_{2,2}$?

3 ✔ Answer: 3

Solution:

In this case, the outer product evaluates to

$$\mathbf{u}\mathbf{v}^T = \begin{pmatrix} -1 & 1 \\ -3 & 3 \end{pmatrix}.$$

In general, if $\mathbf{u} = \begin{pmatrix} u_1 \\ \vdots \\ u_m \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$, $\mathbf{u}\mathbf{v}^T$ is an $m \times n$ matrix whose (i, j) entry is $(\mathbf{u}\mathbf{v}^T)_{i,j} = u_i v_j$.

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outer product
what do the indices outside the question manlike 1,2 or 2,1? 4

? It doesnt let me put k, why not? 4

When I write k, it gives me: Invalid Input: k not permitted in answer as a variable. why is this?

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12. Linear Independence, Subspaces and Dimension

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Vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are said to be **linearly dependent** if there exist scalars $c_1, \ldots, c_n$ such that (1) not all c_i's are zero and (2) $c_1\mathbf{v}_1 + \cdots + c_n\mathbf{v}_n = 0$.

Otherwise, they are said to be **linearly independent** : the only scalars $c_1, \ldots, c_n$ that satisfy $c_1\mathbf{v}_1 + \cdots + c_n\mathbf{v}_n = 0$ are $c_1 = \cdots = c_n = 0$.

The collection of non-zero vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n \in \mathbb{R}^m$ determines a **subspace** of $\mathbb{R}^m$, which is the set of all linear combinations $c_1\mathbf{v}_1 + \cdots + c_n\mathbf{v}_n$ over different choices of $c_1, \ldots, c_n \in \mathbb{R}$. The **dimension** of this subspace is the size of the **largest possible, linearly independent** sub-collection of the (non-zero) vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$.

Row and Column Rank (Optional)

0 points possible (ungraded)

Suppose $\mathbf{A} = \begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix}$. The rows of the matrix, $(1, 3)$ and $(2, 6)$, span a subspace of dimension

Answer: 1 . This is the **row rank** of $\mathbf{A}$.

The columns of the matrix, $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 6 \end{pmatrix}$ span a subspace of dimension

Answer: 1 . This is the **column rank** of $\mathbf{A}$.

We will be using these ideas when studying **Linear Regression**, where we will work with larger, possibly rectangular matrices.

Solution:

In both cases, the two vectors are linearly dependent.

$$2 \cdot (1, 3) - (2, 6) = (0, 0)$$

$$3\begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Submit You have used 0 of 3 attempts

Answers are displayed within the problem

Rank of a matrix (Optional)

0 points possible (ungraded)

In general, row rank is always equal to the column rank, so we simply refer to this common value as the **rank** of a matrix.

What is the largest possible rank of a 2×2 matrix?

Answer: 2

What is the largest possible rank of a 5×2 matrix?

Answer: 2

In general, what is the largest possible rank of an $m \times n$ matrix?

- ◯ m
- ◯ n
- ◯ $\min(m, n)$ ✔
- ◯ $\max(m, n)$
- ◯ None of the above

Solution:

In general, the rank of any $m \times n$ matrix can be at most $\min(m, n)$, since rank = column rank = row rank. For example, if there are five columns and three rows, the column rank cannot be larger than the largest possible row rank – the largest possible row rank for three rows is, unsurprisingly, 3. The opposite is also true if there are more rows than columns. If a matrix has two columns and six rows, then the row rank cannot exceed the column rank, which is at most 2.
In general, a matrix $\mathbf{A}$ is said to have **full rank** if $\text{rank}(\mathbf{A}) = \min(m, n)$. (note the $=$, instead of $\leq$).

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Answers are displayed within the problem

Examples of Rank (Optional)

0 points possible (ungraded)

What is the rank of $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$?

Answer: 1

What is the rank of $\begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$?

Answer: 2

What is the rank of $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$?

Answer: 0

What is the rank of $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$?

Answer: 2

/

What is the rank of $\begin{pmatrix} 1 & 1 & 0 \\ 0 & -3 & 2 \\ 0 & 0 & 1 \end{pmatrix}$?

Answer: 3

Solution:

1. The set of rows describe a subspace of dimension 1, spanned by $(1, 1)$.
2. This matrix has rank 2, since $(1, -1)$ and $(1, 0)$ are linearly independent.
3. This matrix has rank zero. By definition, the rank is equal to the number of nonzero linearly independent vectors.
4. The second and third rows are independent. However, the sum of the second and third rows are equal to the first: $(1, 0, 1) + (0, 1, 0) = (1, 1, 1)$. So this matrix has rank 2.
5. All three rows are independent. An easy way to check is to notice that this matrix is **upper triangular** , with nonzero entries along the diagonal.

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Answers are displayed within the problem

Invertibility of a matrix (Optional)

0 points possible (ungraded)
An $n \times n$ matrix $\mathbf{A}$ is invertible if and only if $\mathbf{A}$ has full rank, i.e. $\text{rank}\,(\mathbf{A}) = n$.

Which of the following matrices are invertible? Choose all that apply.

$$\mathbf{A} = \begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix}$$
$$\mathbf{B} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$
$$\mathbf{C} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$
$$\mathbf{D} = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

- ☐ $\mathbf{A}$
- ☐ $\mathbf{B}$ ✔
- ☐ $\mathbf{C}$ ✔
- ☐ $\mathbf{D}$

Solution:

We saw in a previous exercise that the rank of $\mathbf{A}$ is 1. The rank of $\mathbf{B}$ is 2, since $(1, 2)$ and $(2, 1)$ are linearly independent, since e.g. by Gaussian Elimination one obtains the reduced upper triangular matrix $\begin{pmatrix} 1 & 2 \end{pmatrix}$. In

independent, since e.g. by Gaussian Elimination one obtains the reduced upper triangular matrix $\begin{pmatrix} 1 & 2 \\ 0 & 3/2 \end{pmatrix}$. In general, an upper triangular matrix with nonzero entries along the diagonal has full rank. By the same reasoning, $\mathbf{C}$ also has full rank. Finally, $\mathbf{D}$ does not have full rank, since $(\text{row } 1) + (\text{row } 2) + (\text{row } 3) = \vec{0}$.

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Resources for understanding the concept behind rank https://en.wikipedia.org/wiki/Linear_independence https://people.math.osu.edu/costin.9/264H/Rank	2
? Is this section optional ? Is this section optional ?	3
Repeated exercise? Is it just me or does it feel like I've done the same exercise in 18.6501x?	3

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13. Determinant

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Homework0 due Sep 16, 2020 05:29 IST *Completed*

Given a matrix, $\mathbf{A}$, we denote its transpose as $\mathbf{A}^T$. The transpose of a matrix is equivalent to writing its rows as columns, or its columns as rows. Then, ${\mathbf{A}^T}_{i,j} = \mathbf{A}_{j,i}$.

Recall that the **determinant** $\det(\mathbf{A})$ of a square matrix $\mathbf{A}$ indicates whether it is invertible. For 2×2 matrices, it has the formula

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc.$$

For larger matrices, the formula is a bit more complicated.

Compute the Determinant

2/2 points (graded)

Let $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & 1 \end{bmatrix}$

1. Compute $\det(\mathbf{A}^T)$.

   $\det(\mathbf{A}^T) =$ 6 ✔ **Answer:** 6

2. Compute $\det(\mathbf{A})$.

   6 ✔ **Answer:** 6

STANDARD NOTATION

Solution:

1. First compute $\mathbf{A}^T$ by writing the first row as the first column. This gives us $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ as the first column. Repeat with rows 2 and 3 to arrive at the solution. Then compute the determinant as follows: $1(5-12) - 4(2-6) + 1(12-15) = 6$.

2. $\det(\mathbf{A}) = 1(5-12) - 2(4-6) + 3(8-5) = 6$. Notice that $\det(\mathbf{A}) = \det\mathbf{A}^T$. This is not a coincidence. In fact, this useful property holds for all matrices.

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Numpy /	5
Don't do what I did My brain saw (or assumed) the last row as 7, 8, 9, and thus I got the wrong answer. Take your time!	4
Two ways of computing determinant I find this video good for describing the two ways of computing determinants (Diagonal Method and Co-factor Method) https://www.you...	1
Do what I did. Finish project0. Use numpy. Easy Peasy compared to the hyperplane question set. =) Community TA	2

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14. Interlude: Polynomials and Geometric

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Homework0 due Sep 16, 2020 05:29 IST *Completed*

Quadratic Polynomials

1/1 point (graded)

Recall a **degree** n polynomial in $x_1, x_2, \ldots, x_k$ are all linear combinations of monomials in $x_1, x_2, \ldots, x_k$, where **monimials** in $x_1, x_2, \ldots, x_k$ are **unordered words** using $x_1, x_2, \ldots, x_k$ as the letters.

Examples:

1. A degree 2, also known as quadratic, polynomial in the 1 variable x is of the form

$$ax^2 + bx + c$$

for some numbers a, b, c. The polynomial is determined by the 3 coefficients a, b, c, and different choices of (a, b, c) result in different polynomials.
In linear algebraic terms, the space of degree 2 polynomials in 1 variable is of dimension 3 since it consists of all linear combinations of 3 linearly independent vectors x^2, x, and 1.

2. A degree 2 polynomial in 2 variables x_1, x_2 is of the form

$$ax_1^2 + bx_2^2 + cx_1x_2 + dx_1 + ex_2 + f$$

for some numbers a, b, c, d, e, f. Different choices of (a, b, c, d, e, f) result in different polynomials.
In linear algebraic terms, the space of degree 2 polynomials in 2 variables is of dimension 6 since it consists of all linear combinations of 6 linearly independent vectors x_1^2, x_2^2, x_1x_2, x_1, x_2, and 1.

Consider degree 2 polynomials in 3 variables x_1, x_2, x_3. How many coefficients are needed to completely determine such a polynomial? Equivalently, what is the dimension of the space of polynomials in 3 variables such polynomials?

Number of coefficients needed/ Dimension: 10 ✔ **Answer:** 10

What is dimension of the polynomials of degree N in K variables? (This part of the question is optional and there is no answer box for it.)

Solution:

We count the number of monomials of length $2, 1, 0$:

- The monomials of length 2 are unordered pairs of x_1, x_2, x_3, hence there are $\binom{3}{2}$ This list consists of $x_1^2, x_2^2, x_3^2, x_1x_2, x_1x_3, x_2x_3$.
- The monomials of length 1 are x_1, x_2, x_3.
- The monomial of length 0 is the constant term, i.e. 1.

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?	What is dimension of the polynomials of degree N in K variables? - proof versus heuristic patterning By writing out cases up to N=3 and K=4, I arrive at a simple combinatoric formula that matches all the cases, including the given exampl...	7
💬	Any good study resources for this area? many thanks in advance	8
💬	In the solution the number of unordered pairs seems wrong it's not 3 choose 2, but 3*4/2! = 6	1
?	what is the answer? I put my answer as 6 and it was wrong.	1
?	Intuition or luck? To solve this one I found myself calculating the outer product of vector [x,1] with itself, or [x,z,1] with itself and so on. Then I count the te...	3
💬	concept of polynomials-degrees In polynomial expression, each term is a product of constants and/or variables. Terms are separated from each other by a plus or minu...	1
?	Helpul ressource please Can you recommend a resource to help me solve this please?	2
?	[Staff] restore attemps Good day. I used my one attempt and I was wrong. But now I discovered that all my attempts are lost. I think that is a mistake. I couldn't...	3

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15. Eigenvalues, Eigenvectors and Determinants(Optional)

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Eigenvalues and Eigenvectors of a matrix (Optional)

0 points possible (ungraded)

Let $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ \frac{1}{2} & 2 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

$\mathbf{A}\mathbf{v} = \lambda_1 \mathbf{v}$, where $\lambda_1 =$

Answer: 3 .

$\mathbf{A}\mathbf{w} = \lambda_2 \mathbf{w}$, where $\lambda_2 =$

Answer: 2 .

Therefore, $\mathbf{v}$ is an eigenvector of $\mathbf{A}$ with eigenvalue λ_1, and $\mathbf{w}$ is an eigenvector of $\mathbf{A}$ with eigenvalue λ_2.

Solution:

$$\mathbf{A}\mathbf{v} = \begin{pmatrix} 3 & 0 \\ \frac{1}{2} & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \end{pmatrix} \implies \lambda_1 = 3$$

$$\mathbf{A}\mathbf{w} = \begin{pmatrix} 3 & 0 \\ \frac{1}{2} & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \implies \lambda_2 = 2$$

Submit You have used 0 of 3 attempts

Answers are displayed within the problem

Geometric Interpretation of Eigenvalues and Eigenvectors (Optional)

0 points possible (ungraded)

Let $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ \frac{1}{2} & 2 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Recall from the previous exercise that $\mathbf{v}$ and $\mathbf{w}$ are eigenvectors of $\mathbf{A}$.

Suppose $\mathbf{x} = \mathbf{v} + 2\mathbf{w} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Then $\mathbf{A}\mathbf{x} = s\mathbf{v} + t\mathbf{w}$, where:

$s =$ Answer: 3

and

$t =$ Answer: 4 .

In particular, s describes the amount that $\mathbf{A}$ stretches $\mathbf{x}$ in the direction of $\mathbf{v}$, and $\frac{t}{2}$ (note the "2" in front of $\mathbf{w}$ in $\mathbf{x}$) describes the amount that $\mathbf{A}$ stretches $\mathbf{x}$ in the direction of $\mathbf{w}$.

Solution:

We have

$$\begin{aligned}\mathbf{A}\mathbf{x} &= \mathbf{A}(\mathbf{v}+2\mathbf{w}) \\ &= \mathbf{A}\mathbf{v}+2\mathbf{A}\mathbf{w} \\ &= (3\mathbf{v})+2(2\mathbf{w}) \\ &= 3\mathbf{v}+4\mathbf{w}.\end{aligned}$$

From this, we get $s=3, t=4$.

Submit You have used 0 of 3 attempts

Answers are displayed within the problem

Determinant and Eigenvalues (optional)

0 points possible (ungraded)

What is the determinant of the matrix $\mathbf{A}=\begin{pmatrix}3 & 0\\ \frac{1}{2} & 2\end{pmatrix}$?

Answer: 6

On the other hand, what is the product of the eigenvalues λ_1, λ_2 of $\mathbf{A}$? (We already computed this in the previous exercises.)

Answer: 6

Solution:

Plugging into the formula directly gives $3\cdot 2 - 0\cdot\frac{1}{2} = 6$. On the other hand, the eigenvalues are $\lambda_1=3, \lambda_2=2$, so the product is 6. This is not a coincidence; for general $n\times n$ matrices, the **product of the eigenvalues is always equal to the determinant**.

Submit You have used 0 of 3 attempts

Answers are displayed within the problem

Trace and Eigenvalues (Optional)

0 points possible (ungraded)

Recall that the **trace** of a matrix is the sum of the diagonal entries.

What is the trace of the matrix $\mathbf{A}=\begin{pmatrix}3 & 0\\ \frac{1}{2} & 2\end{pmatrix}$?

Answer: 5

On the other hand, what is the sum of the eigenvalues λ_1, λ_2 of $\mathbf{A}$? (We already computed this in the previous exercises.)

Answer: 5

Solution:

The diagonal sum is $3+2=5$. On the other hand, the eigenvalues are $\lambda_1=3, \lambda_2=2$, so the sum is 5. Just like

the determinant, this is also not a coincidence. For general $n \times n$ matrices, the **sum of the eigenvalues is always equal to the trace of the matrix**.

Submit You have used 0 of 3 attempts

Answers are displayed within the problem

Nullspace (Optional)

0 points possible (ungraded)
If a (nonzero) vector is in the nullspace of a square matrix $\mathbf{A}$, is it an eigenvector of $\mathbf{A}$?

Select an option **Answer:** yes

Which of the following are equivalent to the statement that 0 is an eigenvalue for a given square matrix $\mathbf{A}$? (Choose all that apply.)

- [ ] There exists a nonzero solution to $\mathbf{Av} = \mathbf{0}$. ✔
- [ ] $\det(\mathbf{A}) = 0$ ✔
- [ ] $\det(\mathbf{A}) \neq 0$
- [ ] $\mathrm{NS}(\mathbf{A}) = \mathbf{0}$
- [ ] $\mathrm{NS}(\mathbf{A}) \neq \mathbf{0}$ ✔

Solution:

- If a vector $\mathbf{v}$ is in the nullspace of $\mathbf{A}$, then $\mathbf{Av} = \mathbf{0} = (0)\mathbf{v}$. So it is an eigenvector of $\mathbf{A}$ associated to the eigenvalue 0.
- If 0 is an eigenvalue for a matrix $\mathbf{A}$, then by definition, there exists a nonzero solution to $\mathbf{Av} = \mathbf{0}$; that is, $\mathrm{NS}(\mathbf{A}) \neq \mathbf{0}$, and this only happens if and only if $\det(\mathbf{A}) = 0$.

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Topic: Unit 0. Brief Prerequisite Reviews, Homework 0, and Project 0:Homework 0 / 15. Eigenvalues, Eigenvectors and Determinants(Optional)

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? hints for wrong answers — 4
Do i get hints for wrong answers to know where i went wrong?

grade — 18
how to know whether we passed homework or not?

? Null space: select an option..? — 2
If a (nonzero) vector is in the nullspace of a square matrix A, is it an eigenvector of A ? A vector in null space can qualify as eigenvector o...

? NS(A)=0 meaning

NS(A)=0 means there is no nullspace? Or does it mean the opposite? Thanks in advance

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1. Objective

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You will need to have basic knowledge of probability theory, and a sound foundation of multivariable calculus and linear algebra to follow along in this course. There will be no review of these subjects beyond this unit.

Homework 0 consists of some warmup exercises for the beginning of the course. It does **not** cover all background material you will need. Nonetheless, it can serve as rough guide on how ready you are: if you need to struggle much in these exercises, then you will need to spend substantial amount of time catching up on background material.

The following topics will allow you to understand part of the course material better, although not strictly required:

- Eigenvalues, eigenvectors, and spectral decomposition (linear algebra)
- Lagrange multipliers (multivariable calculus).

Homework 0 is due **Tuesday September 15 23:59UTC** . **Please note the UTC time zone and find the corresponding time at your location. Note that CST on EdX is Central Standard Time, NOT China Standard Time.**

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help i have just joined the course and i have missed out on a lot... is it possible to do the assignments that are past due?	1
Update from audit Hello I am now as an audit student but I would like to switch to get the certificate but there are some homeworks that are due. Is po...	2
Recommendation: MIT Opencourseware Multi-variable Calculus Hello everyone, For anyone looking to brush up or even learn some multi-variable calculus for the course, there should be a playlist ...	3
Unable to finish the exercises on time Dear Course Administrators, I was unable to finish the exercises on time, what's the penalty?	2
When is homework 0 due? There seems to be an error in the expiration date of homework 0: "Homework 0 is due next Tuesday, February 11 at 23:59 UTC.."	4
Prerequisites Will we be using any advanced topics in vector calculus like line/surface integrals? I had some exposure to those concepts in college, ...	2
Had High Hopes As a biologist who is active in ML for drug design, its frustrating to see MIT running such isolated course. what is the point of knowle...	4
Why does google translate not work? google translate does not work on the page	2
Last date for switching to verified track @staff I am currently using the audit option. Can i switch to verified course later? is there a last date for that ? Do i need to switch be...	1
Lecture videos? :) Well, that's a surprise! I wasn't expecting the course to start with a bunch of homework exercises. Are there any video lectures or int...	10
For newbies like me Hi there. If all this math seems extremely hard for you, as it is in my case, I found very good free book link: https://mml-book.github.i...	7
Progress Chart Hi - why is the progress chart disabled? Generally one can see a histogram	1
Refresh if math is still in code.	

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