Problem 4

Midterm due Nov 10, 2020 05:29 IST *Past Due*

For simplicity, suppose our rating matrix is a 2×2 matrix and we are looking for a rank-1 solution UV^T so that user and movie features U and V are both 2×1 matrices. The observed rating matrix has only a single entry:

$$Y = \begin{bmatrix} ? & 1 \\ ? & ? \end{bmatrix} \tag{7.4}$$

In order to learn user/movie features, we minimize

$$J(U,V) = \left(\frac{1}{2} \sum_{(a,i) \in D} (Y_{ai} - [UV^T]_{ai})^2 \right) + \lambda (U_1^2 + V_1^2) \tag{7.5}$$

where U_1 and V_1 are the first components of the vectors U and V respectively (if $U = [u_1, u_2]$, then $U_1 = u_1$), the set D is just the observed entries of the matrix Y , in this case just $(1,2)$.

Note that the regularization we use applies only to the first coordinate of user/movie features . We will see how things get a bit tricky with this type of partial regularization.

4. (1)

0/1 point (graded)

If we initialize $U = [u \quad 1]^T$, for some $u > 0$, what is the solution to the vector $V = [v_1 \quad v_2]^T$ as a function of λ and u?

(Enter V as a vector, enclosed in square brackets, and components separated by commas, e.g. type `[u,lambda+1]` if $V = [u \quad \lambda + 1]^T$.)

$V =$ [0,u/lambda] ✖ **Answer:** [0,1/u]

STANDARD NOTATION

Solution:

Notice that J only regularizes on the first coordinate. Thus, we only want to minimize $J(v_1, v_2) = \frac{1}{2}(1 - uv_2)^2 + \lambda(v_1^2 + u^2)$ given that $V = [v_1, v_2]^T$. We can see that J is minimized when $v_1 = 0, v_2 = \frac{1}{u}$. Therefore,

$$V = \left[0, \frac{1}{u}\right]^T. \tag{7.6}$$

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

4. (2)

1 point possible (graded)

What is the resulting value of $J(U, V)$ as a function of λ and u?

(Type `lambda` for λ).

Answer: (lambda*u^2)

STANDARD NOTATION

Solution:

Notice that J only regularizes on the first coordinate. Therefore,

$$\begin{aligned} J(U,V) &= \frac{1}{2}\sum_{(a,i)\in D}(Y_{ai}-[UV^T]_{ai})^2+\lambda(U_1^2+V_1^2) \\ &= \frac{1}{2}(1-u_1v_2)^2+\lambda(u_1^2+v_1^2) \\ &= \frac{1}{2}\left(1-u\cdot\frac{1}{u}\right)^2+\lambda(u^2+0^2) \\ &= \lambda u^2 \end{aligned}$$

Submit You have used 0 of 3 attempts

Answers are displayed within the problem

4. (3)

1/1 point (graded)

If we continue to iteratively solve for U and V, what would U and V converge to?

- (selected) U goes to $[0,1]$, V goes to $[0,\infty]$
- U goes to $[0,0]$, V goes to $[0,0]$
- U goes to $[0,1]$, V goes to $[0,0]$
- U goes to $[0,\infty]$, V goes to $[1,0]$

Solution:

The regularization error is minimized when u_1 and v_1 are 0. Over many iterations, u_1 will eventually converge to zero. The squared error term is $\frac{1}{2}(1-u_1v_2)^2$ is minized when $u_1v_2=1$. Since u_1 converges to 0, v_2 diverges to ∞.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

4. (4)

3/3 points (graded)

Not all rating matrices Y can be reproduced by UV^T when we restrict the dimensions of U and V to be 2×1.

For each matrix below, answer “Yes" or “No" according to whether it can be reproduced by such U and V of size 2×1.

$$Y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

- (selected) yes
- no

✔

$$Y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- yes
- (selected) no

✔

$$Y = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

- (selected) yes
- no

✔

Solution:

In order for matrix Y to be reproduced by UV^T we must have $[u_1, u_2]^T \times [v_1, v_2] = Y$. For the second matrix, this would require $u_1 \times v_1 = 1$, $u_1 \times v_2 = 0$, $u_2 \times v_1 = 0$, $u_2 \times v_2 = 1$, which has no solution.
The first matrix can be represented as $[1 - 1]^T \times [1 - 1]$ and the third can be represented as $[1 - 1]^T \times [11]$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

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- ? STAFF - Could you please break down the explanation of the solution of 4(1)? — 3
  I don't understand how to get to v_1 and v_2 and the solution is not clear for me. Could you please b...
- Question 4: What were your answers? — 5
- ? Unable to answer 4.3 with certainty because U_2 unconstrained — 1
  Since U_2 is missing from the objective function J(U,V), its value has no effect. In other words, any sel...
- [STAFF] My question description starts at (7.4) — 3
  The first line of text I can see after "Problem 4, Midterm due Nov 9, 2020 23:59 GMT, Bookmark this ...
- [STAFF] Problem 4.4 - U and V elements do not have region definition — 2
- 7.1 How to minimize objective function with only a single Y entry? — 3
  I'm a little confused on how we are supposed to find U and V given the Y we have, where the only en...
- 4.1 Solution to vector V — 1
  Are we looking for the solution after the first iteraction or the final result after convergence? I may b...
- Objective function — 4
- Typo in 2nd line of Q4? — 1

[Staff] Clarification on question 4 [3]
To my understanding, the loss function never depends a particular optimization variable, say u_i. So ... 1

Is there an error in equation 7.5?
Is there an error in equation 7.5? 2

Is equation 7.4, that describes the observed values accurate?
Equation 7.4, that describes the observed entries, says that Y = [[? 1] [? ?]]. There is some text a littl... 3