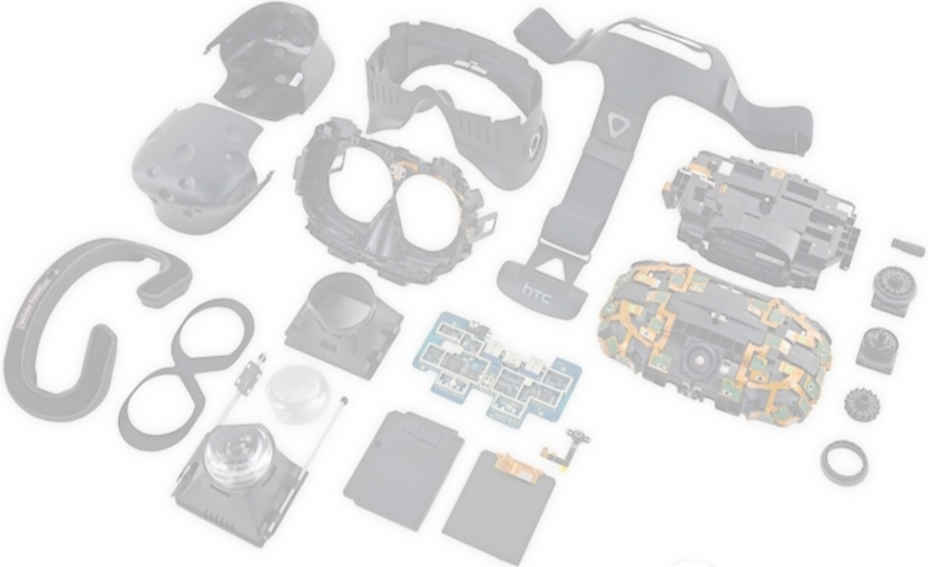
Pose Tracking I



Gordon Wetzstein
Stanford University

EE 267 Virtual Reality

Lecture 11

stanford.edu/class/ee267/

Overview

- overview of positional tracking
- camera-based tracking
- HTC's Lighthouse
- VRduino – an Arduino for VR, specifically designed for EE 267 by Keenan Molner
- pose tracking with VRduino using homographies

What are we tracking?

- Goal: track pose of headset, controller, …
- What is a pose?
  - 3D position of the tracked object
  - 3D orientation of the tracked object, e.g. using quaternions or Euler angles
- Why? So we can map the movement of our head to the motion of the camera in a virtual environment – motion parallax!

Overview of Positional Tracking

"inside-out tracking": camera or sensor is located on HMD, no need for other external devices to do tracking

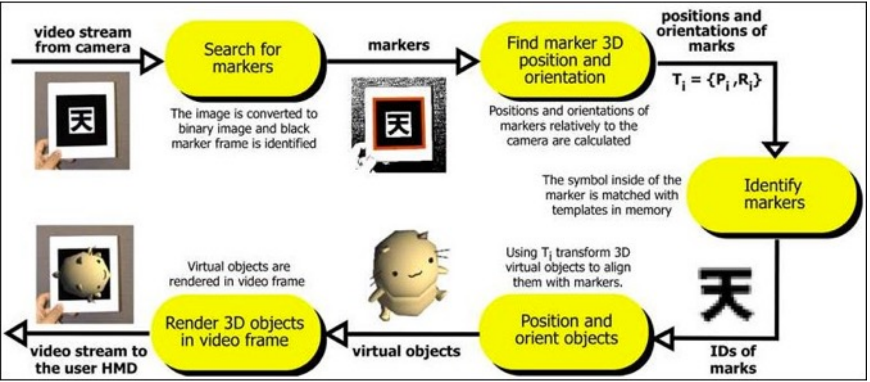
- simultaneous localization and mapping (SLAM) – classic computer & robotic vision problem (beyond this class)

"outside-in tracking": external sensors, cameras, or markers are required (i.e. tracking constrained to specific area)

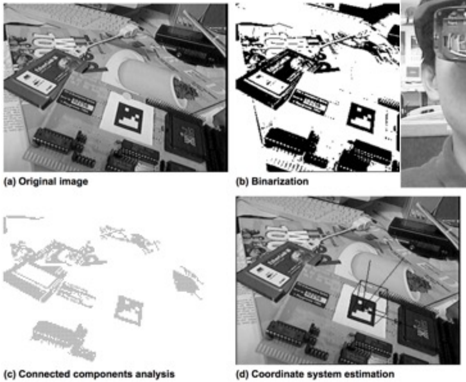
- used by most VR headsets right now, but everyone is feverishly working on insight-out tracking!

Marker-based Tracking

- seminal papers by Rekimoto 1998 and Kato & Billinghurst 1999
- widely adopted after introduced by ARToolKit



Kato, Billinghurst - ARToolKit



Rekimoto - Matrix

Marker-based Tracking

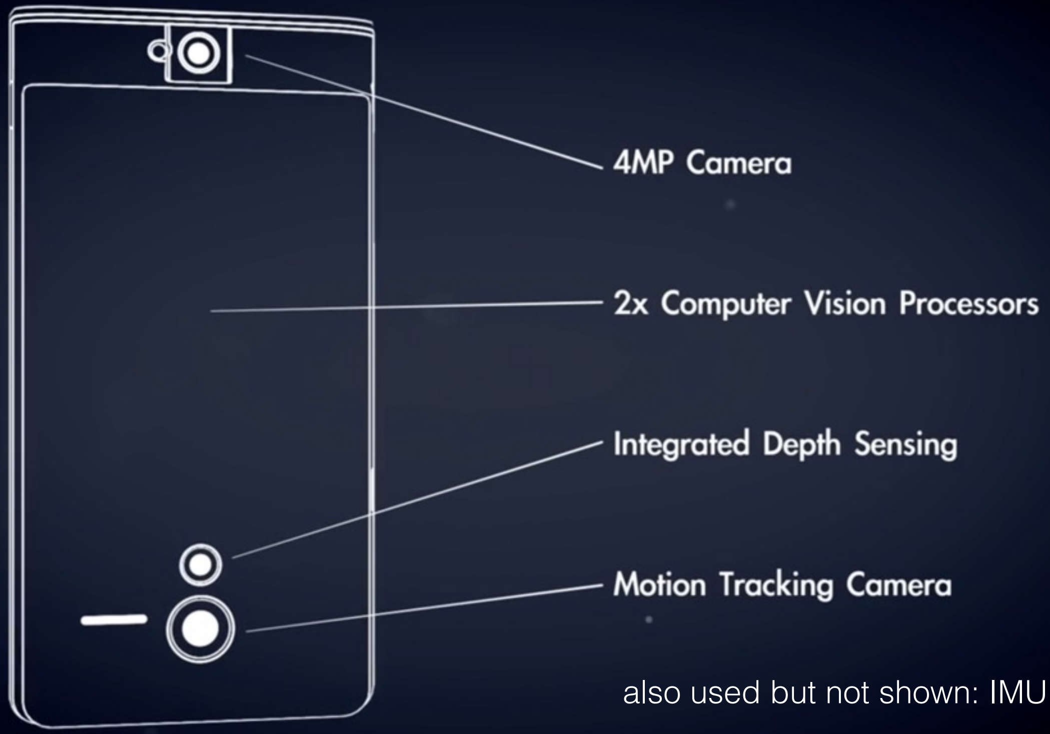
ARToolKit

OpenCV marker tracking

Inside-out Tracking

Google's Project Tango

Google’s Project Tango



also used but not shown: IMU

problem: SLAM via sensor fusion

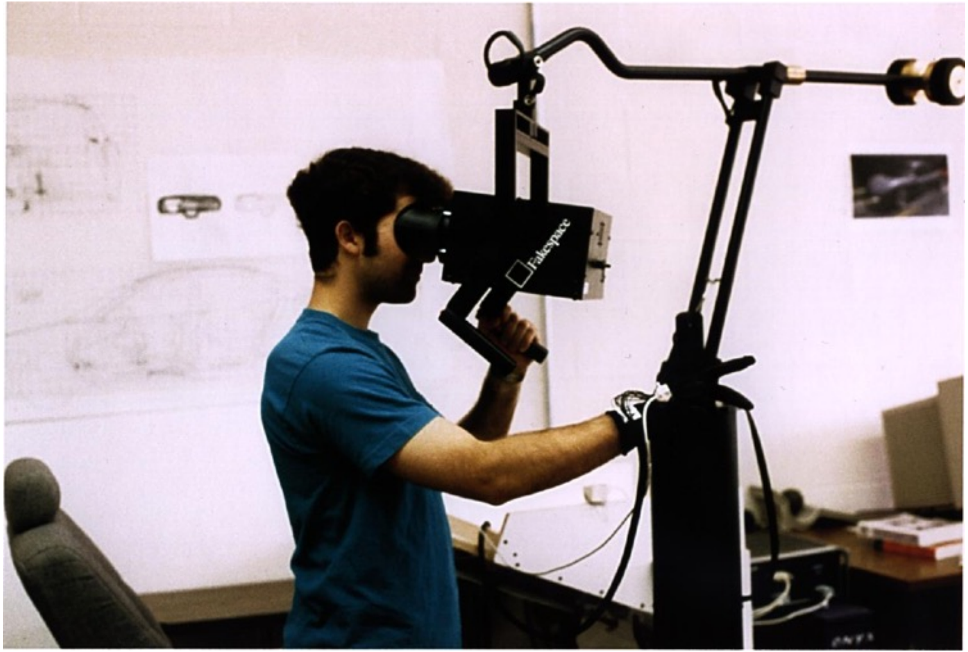
Inside-out Tracking

- marker-less inside-out tracking used by Microsoft HoloLens, Oculus Quest, Magic Leap, …
- eventually required by all untethered VR/AR systems
- if you need it for your own HMD, consider using Intel's RealSense (small & has SDK)
- if you want to learn more about SLAM, take a 3D computer vision or robotic vision class, e.g. Stanford CS231A

“Outside-in Tracking”

- mechanical tracking
- ultra-sonic tracking
- magnetic tracking
- optical tracking
- GPS
- WIFI positioning
- marker tracking
- ...

Positional Tracking - Mechanical



some mechanical linkage, e.g.

- fakespace BOOM
- microscribe

Positional Tracking - Mechanical

pros:

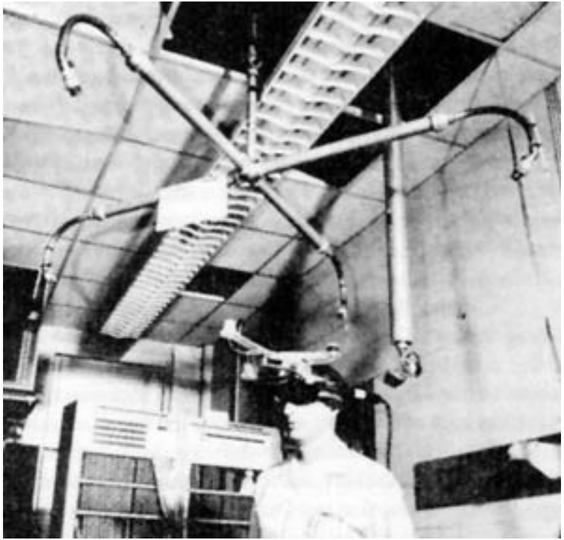
- super low latency
- very accurate

cons:

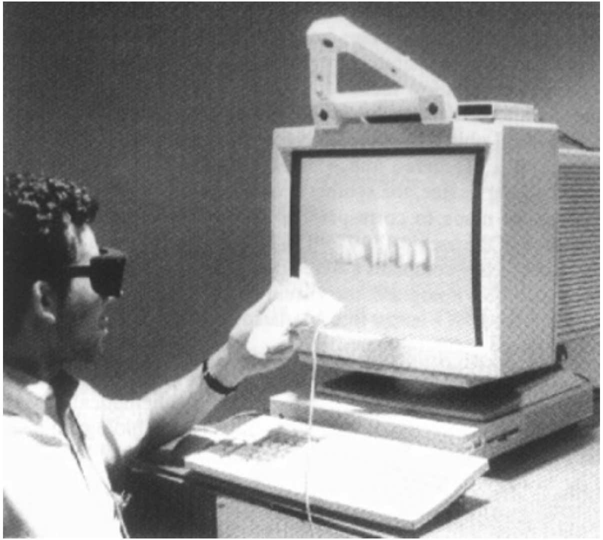
- cumbersome
- “wired” by design

Positional Tracking – Ultra-sonic

- 1 transmitter, 3 receivers → triangulation



Ivan Sutherland's "Ultimate Display"



Logitech 6DOF

Positional Tracking – Ultra-sonic

pros:

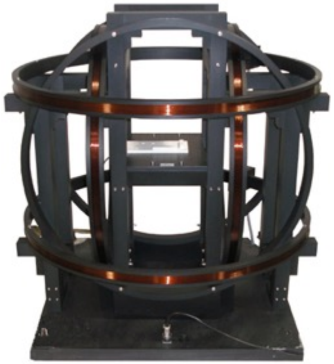
- can be light, small, inexpensive

cons:

- line-of-sight constraints
- susceptible to acoustic interference
- low update rates

Positional Tracking - Magnetic

- reasonably good accuracy
- position and orientation
- 3 axis magnetometer in sensors
- need magnetic field generator (AC, DC, …), e.g. Helmholtz coil
- magnetic field has to oscillate and be sync'ed with magnetometers



3 axis Helmholtz coil
www.directvacuum.com

Positional Tracking - Magnetic

3 axis Helmholtz coil
www.directvacuum.com

pros:

- small, low cost, low latency sensors
- no line-of-sight constraints

cons:

- somewhat small working volume
- susceptible to distortions of magnetic field
- not sure how easy it is to do this untethered (need to sync)

Positional Tracking - Magnetic

Magic Leap One controller tracking:

- magnetic field generator in controller
- magnetometer in headset

https://www.ifixit.com/Teardown/Magic+Leap+One+Teardown/112245

Positional Tracking - Optical

- track active (near IR) LEDs with cameras

OR

- track passive retro-reflectors with IR illumination around camera

- both Oculus Rift and HTC Vive come with optical tracking

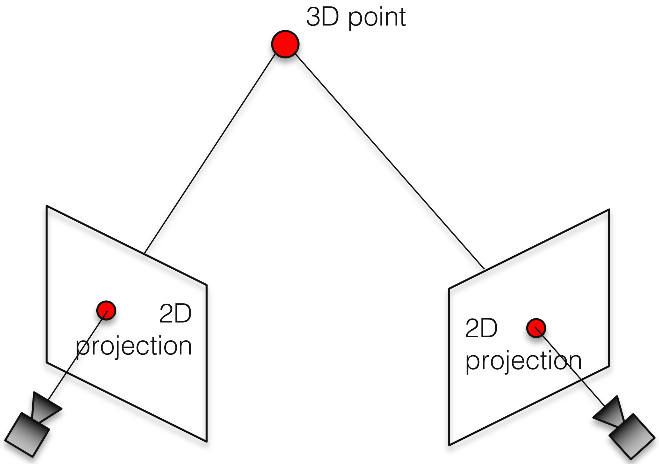
Oculus Rift
https://www.ifixit.com/Teardown/Oculus+Rift+CV1+Teardown/60612

http://steam3.com/make-magic-2015/

Understanding Pose Estimation - Triangulation

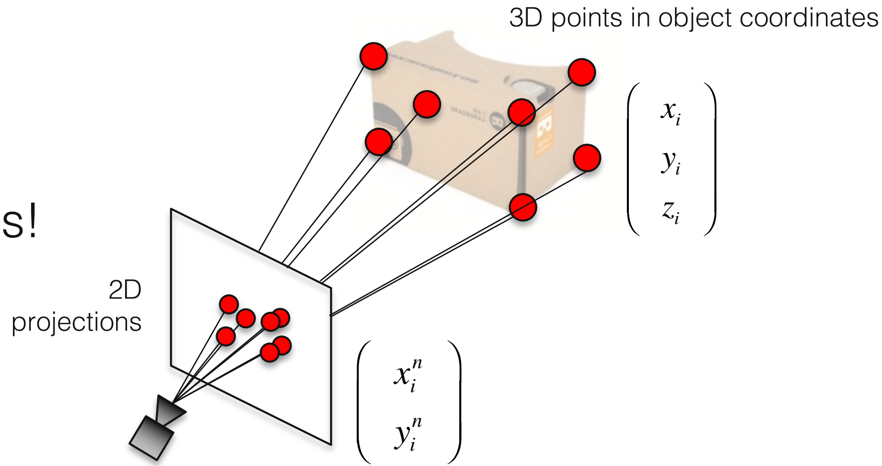
- for tracking individual 3D points, multi-camera setups usually use <u>triangulation</u>

- this does not give us the pose (rotation & translation) of camera or object yet



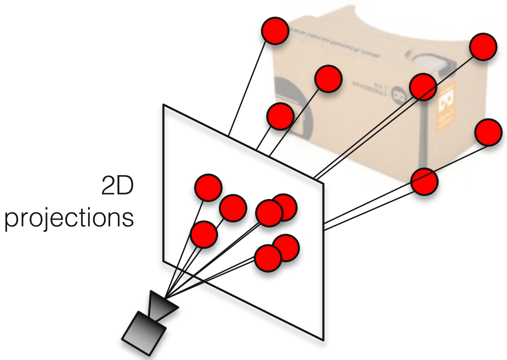
Understanding Pose Estimation

- for pose estimation, need to track multiple points with known relative 3D coordinates!



Understanding Pose Estimation

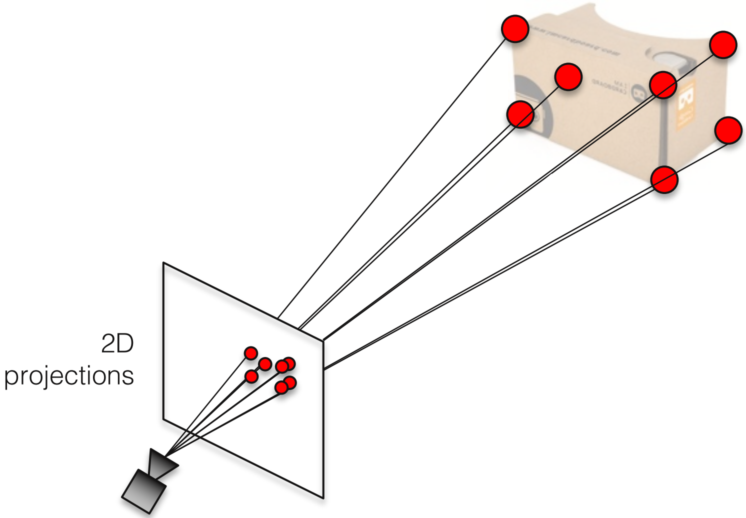
- when object is closer, projection is bigger



Understanding Pose Estimation

- when object is father, projection is smaller

... and so on ...

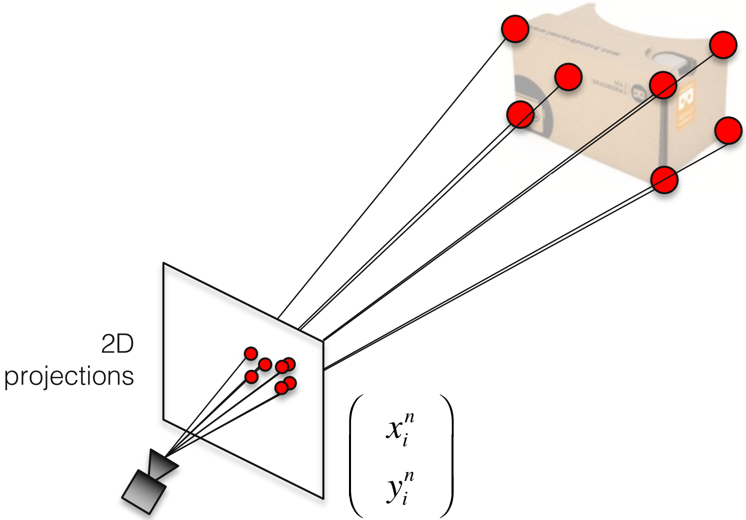


Estimating 6-DoF pose from 2D projections is known as the

Perspective-n-point problem!

Understanding Pose Estimation

1. how to get projected 2D coordinates?
2. image formation
3. estimate pose with linear homography method
4. estimate pose with nonlinear Levenberg-Marquardt method (next class)



$$\begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}$$

$$\begin{pmatrix} x_i^n \\ y_i^n \end{pmatrix}$$

Understanding Pose Estimation

1. how to get projected 2D coordinates?
2. image formation
3. estimate pose with linear homography method
4. estimate pose with nonlinear Levenberg-Marquardt method (next class)

- HTC Lighthouse
- VRduino

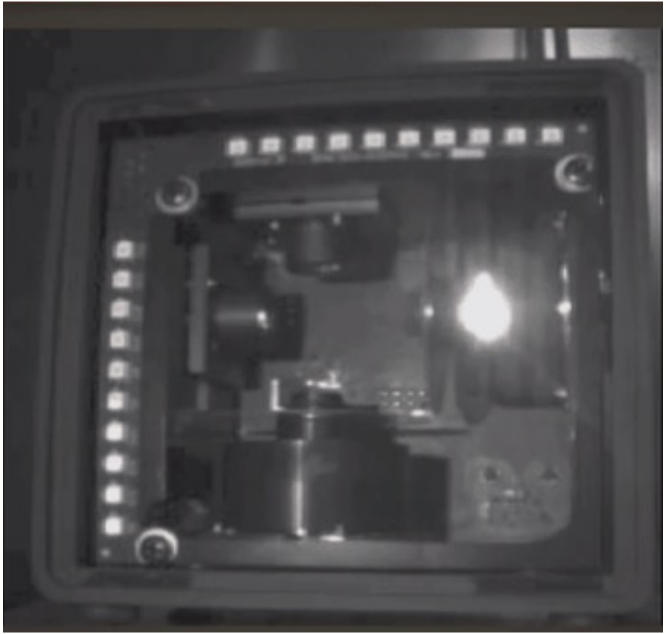
HTC Lighthouse

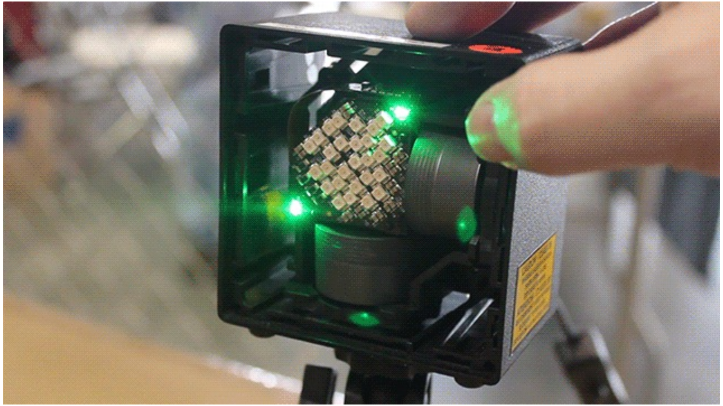
https://www.youtube.com/watch?v=J54dotTt7k0

HTC Lighthouse

https://www.youtube.com/watch?v=J54dotTt7k0

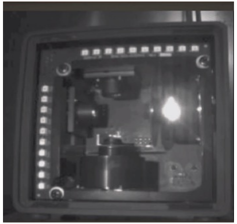
HTC Lighthouse – Base Station





http://gizmodo.com/this-is-how-valve-s-amazing-lighthouse-tracking-technol-1705356768

HTC Lighthouse – Base Station



<u>important specs</u>:

- runs at 60 Hz
    - i.e. horizontal & vertical update combined 60 Hz
    - broadband sync pulses in between each laser sweep (i.e. at 120 Hz)
- each laser rotates at 60 Hz, but offset in time
- useable field of view: 120 degrees

HTC Lighthouse – Base Station

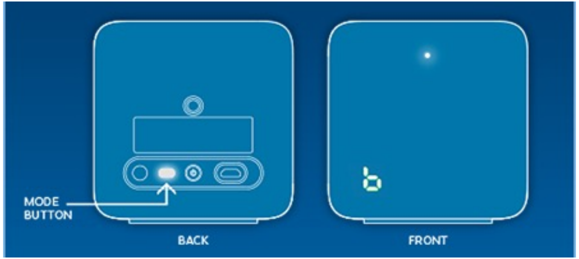
- can use up to 2 base stations simultaneously via *time-division multiplexing* (TDM)
- base station modes:

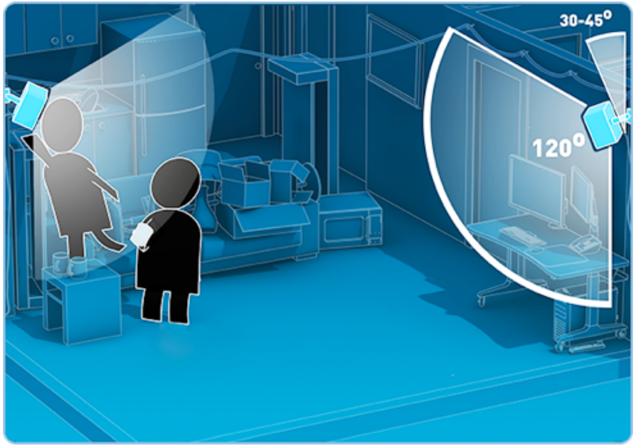
    A: TDM slave with cable sync

    B: TDM master

    C: TDM slave with optical sync

HTC





HTC Lighthouse – Base Station

- sync pulse periodically emitted (120 times per second)
- each sync pulse indicates beginning of new sweep
- length of pulse also encodes additional 3 bits of information:

Name	skip	data	axis	length (ticks)	length (µs)
j0	0	0	0	3000	62.5
k0	0	0	1	3500	72.9
j1	0	1	0	4000	83.3
k1	0	1	1	4500	93.8
j2	1	0	0	5000	104
k2	1	0	1	5500	115
j3	1	1	0	6000	125
k3	1	1	1	6500	135

- axis: horizontal or vertical sweep to follow
- skip: if 1, then laser is off for following sweep
- data: data bits of consecutive pulses yield OOTX frame

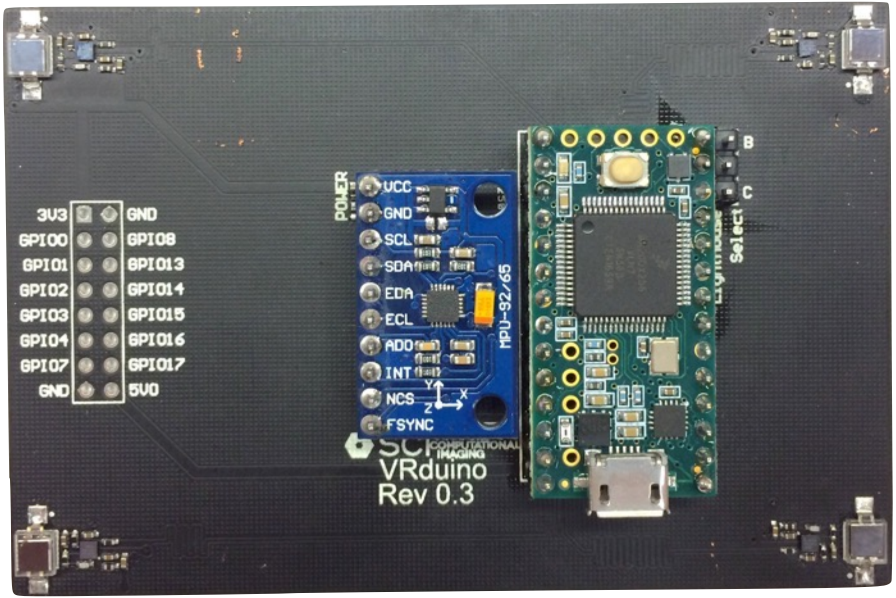
https://github.com/nairol/LighthouseRedox/blob/master/docs/Light%20Emissions.md#sync-pulse

VRduino

- in this class, we use the HTC Lighthouse base stations but implement positional tracking (i.e., pose estimation) on the VRduino

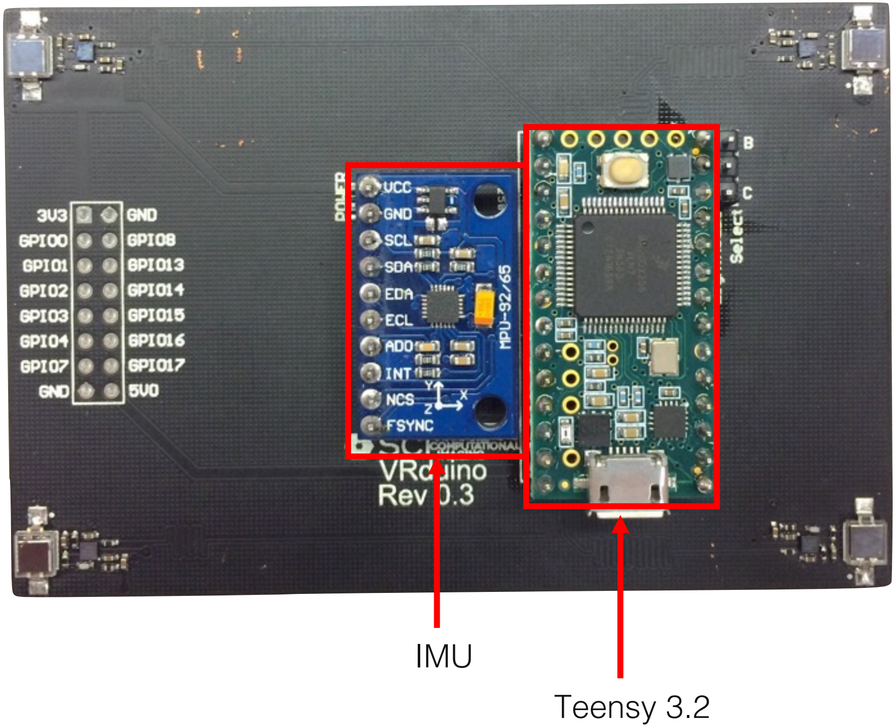
- VRduino is a shield (hardware add-on) for the Arduino Teensy 3.2; custom-designed for EE 267 by Keenan Molner

VRduino



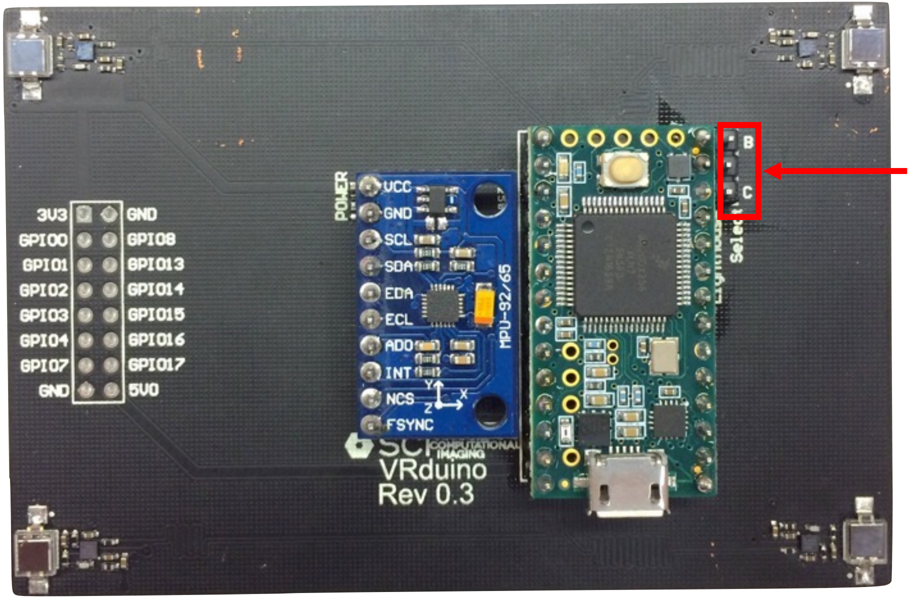
3V3
GND
GPIO0
GPIO8
GPIO1
GPIO13
GPIO2
GPIO14
GPIO3
GPIO15
GPIO4
GPIO16
GPIO7
GPIO17
GND
5V0
POWER
VCC
GND
SCL
SDA
EDA
ECL
ADD
INT
NCS
FSYNC
MPU-92/65
B
C
Select
COMPUTATIONAL IMAGING
VRduino
Rev 0.3

VRduino



3V3
GPIO0
GPIO1
GPIO2
GPIO3
GPIO4
GPIO7
GND
GND
GPIO8
GPIO13
GPIO14
GPIO15
GPIO16
GPIO17
5V0
VCC
GND
SCL
SDA
EDA
ECL
ADD
INT
NCS
FSYNC
MPU-92/65
VRduino
Rev 0.3
B
C
Select
IMU
Teensy 3.2

VRduino

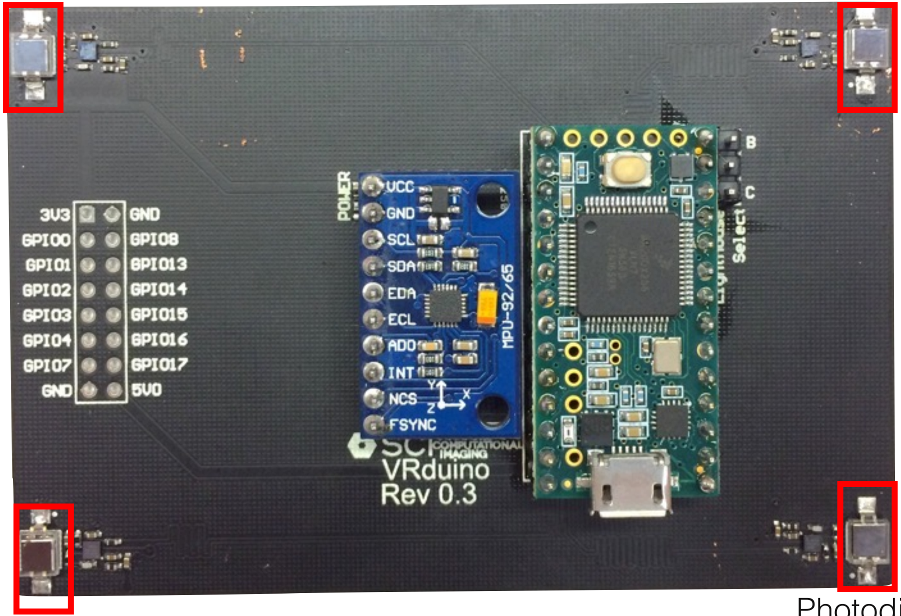


Lighthouse Select

VRduino

Photodiode 0

Photodiode 1



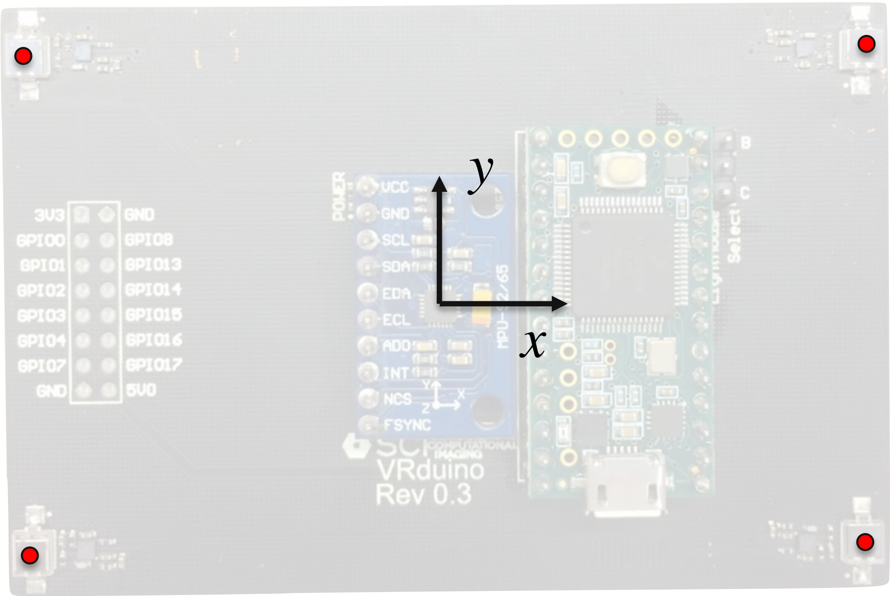
Photodiode 3

Photodiode 2

VRduino

x=-42mm, y=25mm

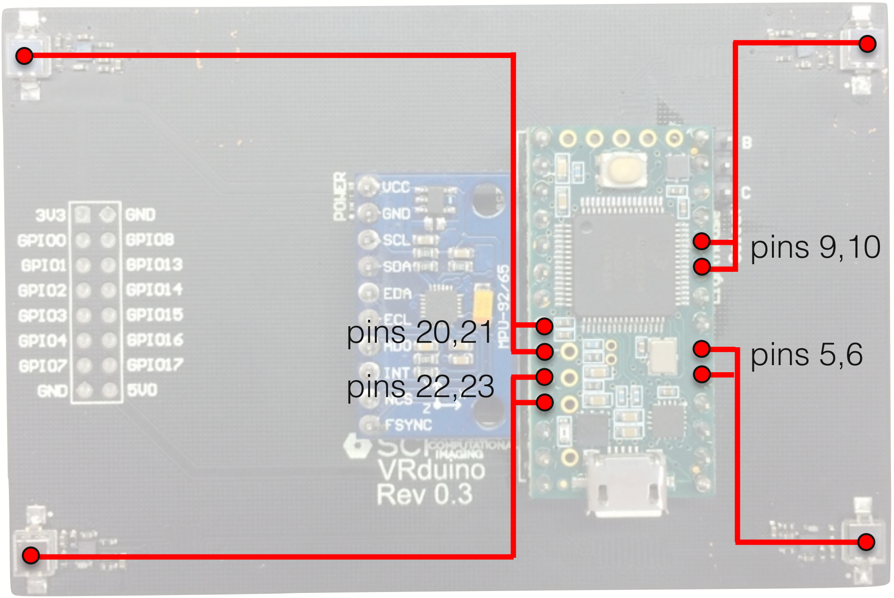
x=42mm, y=25mm



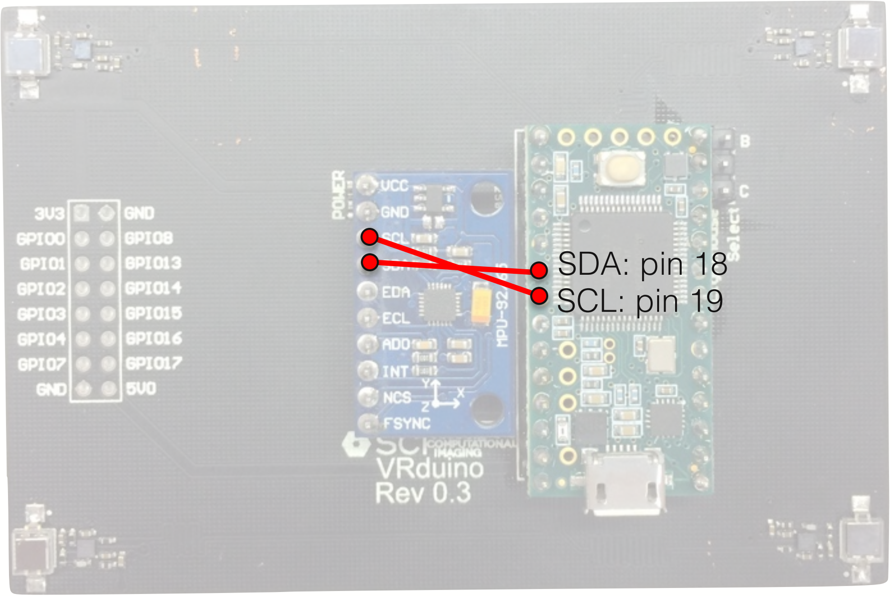
x=-42mm, y=-25mm

x=42mm, y=-25mm

VRduino

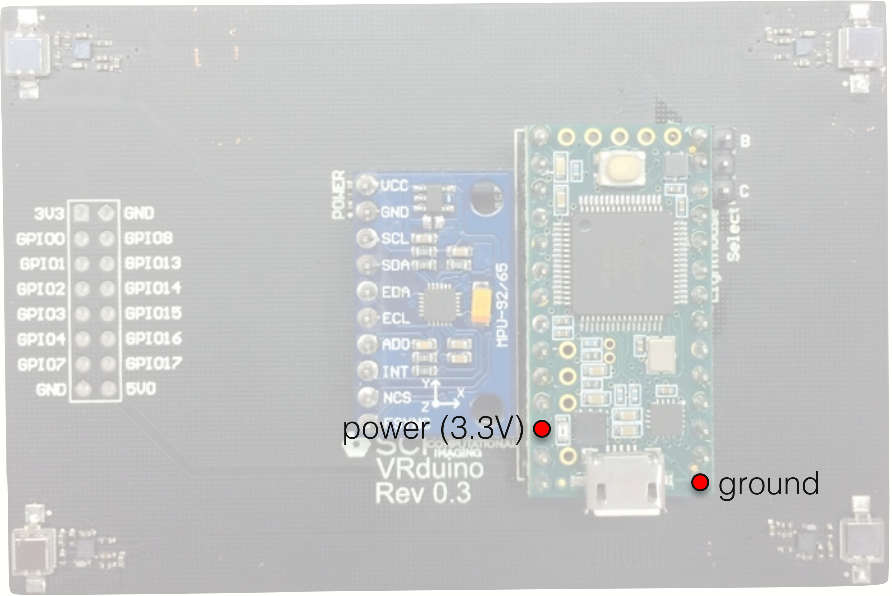

3V3
GND
GPIO0
GPIO8
GPIO1
GPIO13
GPIO2
GPIO14
GPIO3
GPIO15
GPIO4
GPIO16
GPIO7
GPIO17
GND
5V0
POWER
VCC
GND
SCL
SDA
EDA
ECL
INT
FSYNC
MPU-92/65
pins 20,21
pins 22,23
pins 9,10
pins 5,6
B
C
VRduino
Rev 0.3

VRduino



SDA: pin 18
SCL: pin 19
3V3
GND
GPIO0
GPIO8
GPIO1
GPIO13
GPIO2
GPIO14
GPIO3
GPIO15
GPIO4
GPIO16
GPIO7
GPIO17
GND
5V0
VCC
GND
SCL
EDA
ECL
ADD
INT
NCS
FSYNC
POWER
VRduino
Rev 0.3

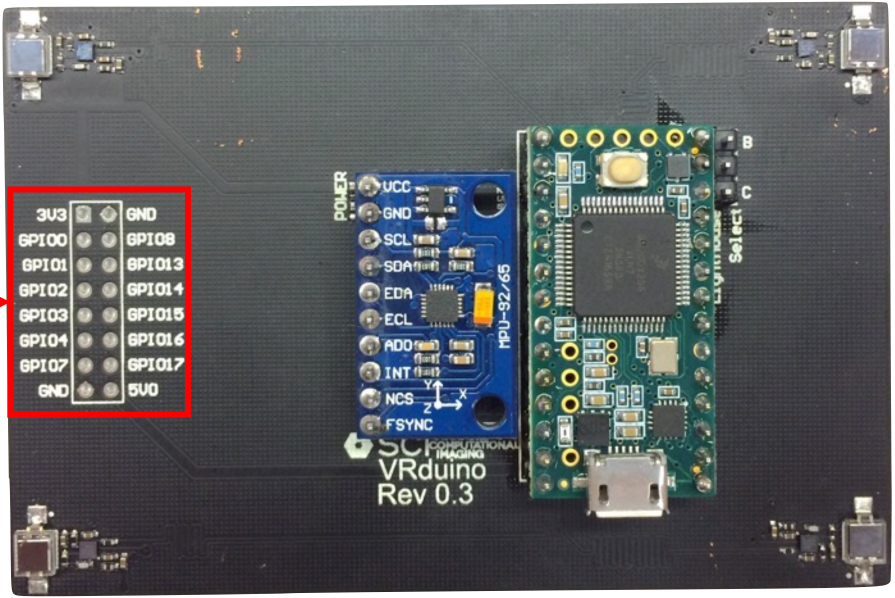
VRduino



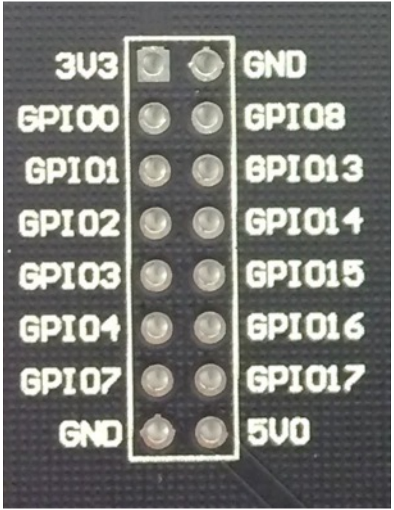
3V3
GND
GPIO0
GPIO8
GPIO1
GPIO13
GPIO2
GPIO14
GPIO3
GPIO15
GPIO4
GPIO16
GPIO7
GPIO17
GND
5V0
POWER
VCC
GND
SCL
SDA
EDA
ECL
ADO
INT
NCS
MPU-92/65
B
C
Select
power (3.3V)
ground
VRduino
Rev 0.3

VRduino

Pin Breakout



VRduino



3.3V power, **200mA MAX**	
digital R/W, Serial, cap. sense	SPI, Serial, digital R/W
digital R/W, Serial, cap. sense	digital R/W
digital R/W	SPI, analog read, digital R/W
digital R/W, PWM, CAN	SPI, analog read, digital R/W
digital R/W, PWM, CAN	I2C, analog read, digital R/W
digital R/W, Serial, SPI	I2C, analog read, digital R/W
	5V power, **500mA MAX**

For more details, see Lab Writeup

Pose Estimation with the VRduino

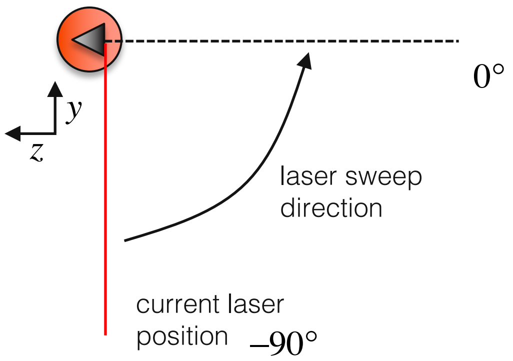
- timing of photodiodes reported in Teensy “clock ticks” relative to last sync pulse

- Teensy usually runs at 48 MHz, so 48,000,000 clock ticks per second

How to Get the 2D Coordinates?

- at time of respective sync pulse, laser is at 90° horizontally and -90° vertically
- each laser rotates 360° in 1/60 sec

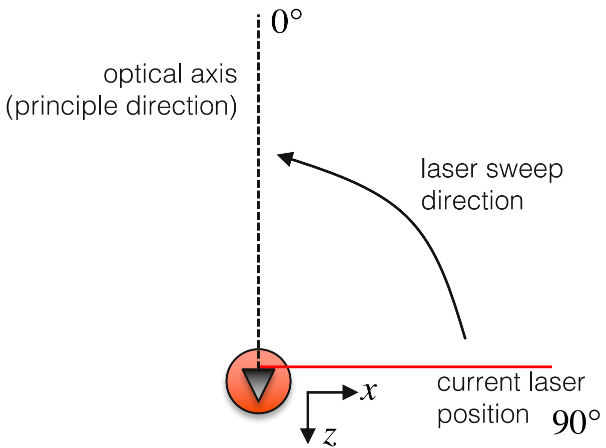
Side View



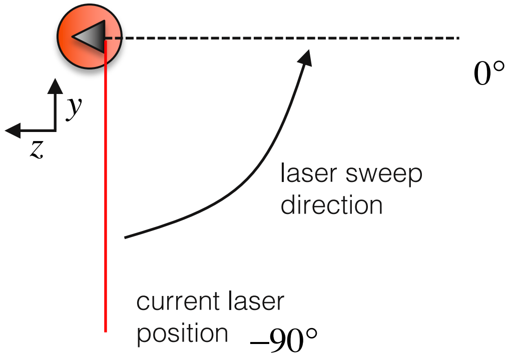
How to Get the 2D Coordinates?

- at time of respective sync pulse, laser is at 90° horizontally and -90° vertically
- each laser rotates 360° in 1/60 sec

Top View



Side View

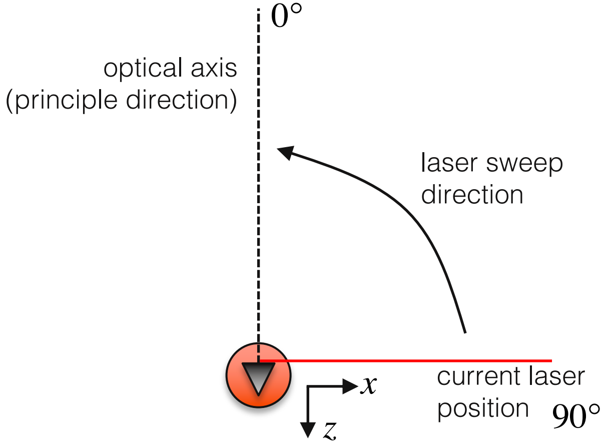


How to Get the 2D Coordinates?

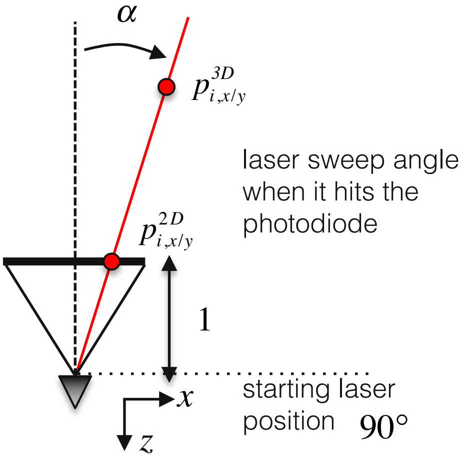
- at time of respective sync pulse, laser is at 90° horizontally and -90° vertically
- each laser rotates 360° in 1/60 sec

- convert from ticks to angle first and then to relative position on plane at unit distance

Top View



Top View



How to Get the 2D Coordinates?

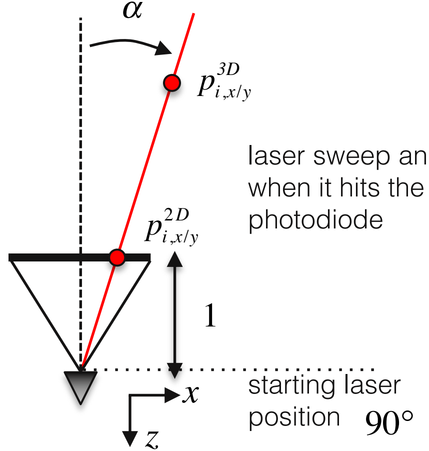
raw number of tick from photodiode

$$\Delta t\,[\mathrm{sec}] = \frac{\#\,ticks}{48{,}000{,}000\left[\frac{ticks}{\mathrm{sec}}\right]}$$

CPU speed

- convert from ticks to angle first and then to relative position on plane at unit distance

Top View



How to Get the 2D Coordinates?

- convert from ticks to angle first and then to relative position on plane at unit distance

raw number of ticks from photodiode

$$\Delta t\,[\mathrm{sec}] = \frac{\#\,ticks}{48{,}000{,}000\left[\frac{ticks}{\mathrm{sec}}\right]}$$

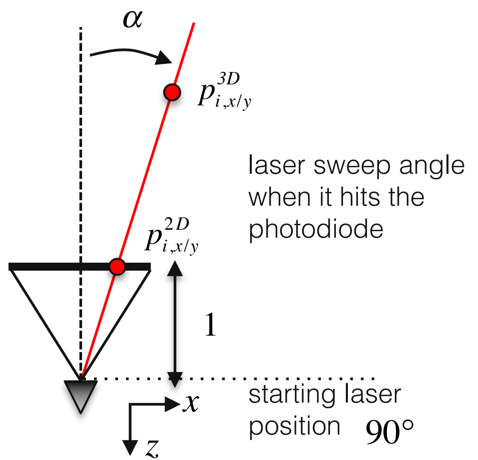
CPU speed

offset from sync pulse

$$\alpha\,[^\circ] = -\frac{\Delta t\,[\mathrm{sec}]}{\frac{1/60}{360}\left[\frac{\mathrm{sec}}{^\circ}\right]} + \frac{360}{4}\,[^\circ]$$

time per 1 revolution

Top View



How to Get the 2D Coordinates?

- convert from ticks to angle first and then to relative position on plane at unit distance

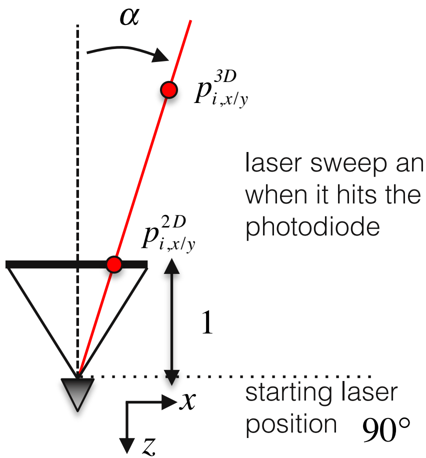
$$p^{2D}_{i,x/y} = \tan\left(\frac{\alpha}{360[^\circ]}\cdot 2\pi\right)$$

offset from sync pulse

$$\alpha[^\circ] = -\frac{\Delta t[\sec]}{\frac{1/60}{360}\left[\frac{\sec}{^\circ}\right]} + \frac{360}{4}[^\circ]$$

time per 1 revolution

Top View



How to Get the 2D Coordinates?

Horizontal Sweep

$$\alpha[\circ] = -\frac{\Delta t[\sec]}{\frac{1/60}{360}\left[\frac{\sec}{\circ}\right]} + \frac{360}{4}[\circ]$$

Vertical Sweep

$$\alpha[\circ] = \frac{\Delta t[\sec]}{\frac{1/60}{360}\left[\frac{\sec}{\circ}\right]} - \frac{360}{4}[\circ]$$

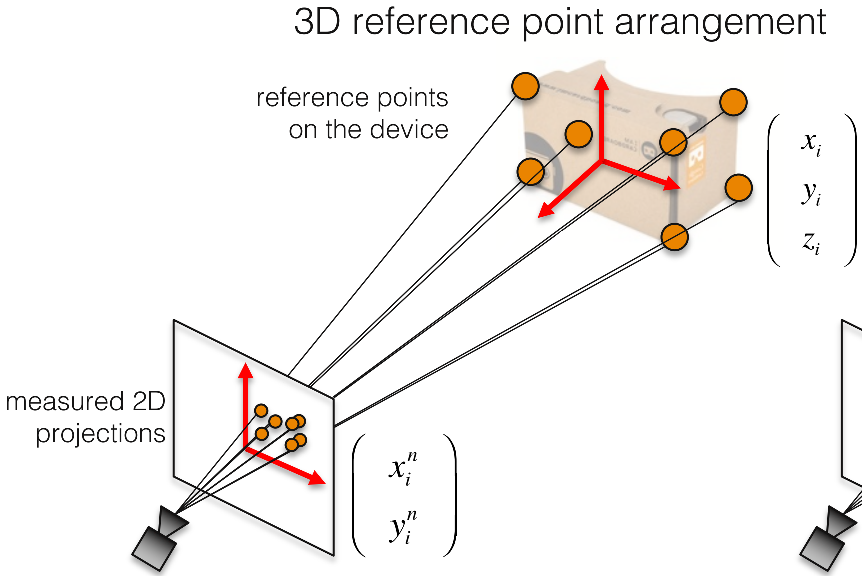
Understanding Pose Estimation

1. how to get projected 2D coordinates?
2. image formation
3. estimate pose with linear homography method
4. estimate pose with nonlinear Levenberg-Marquardt method (next class)

- how 3D points project into 2D coordinates in a camera (or a Lighthouse base station)
- very similar to graphics pipeline

Image Formation

- image formation is model for mapping 3D points in local "object" coordinate system to 2D points in "window" coordinates



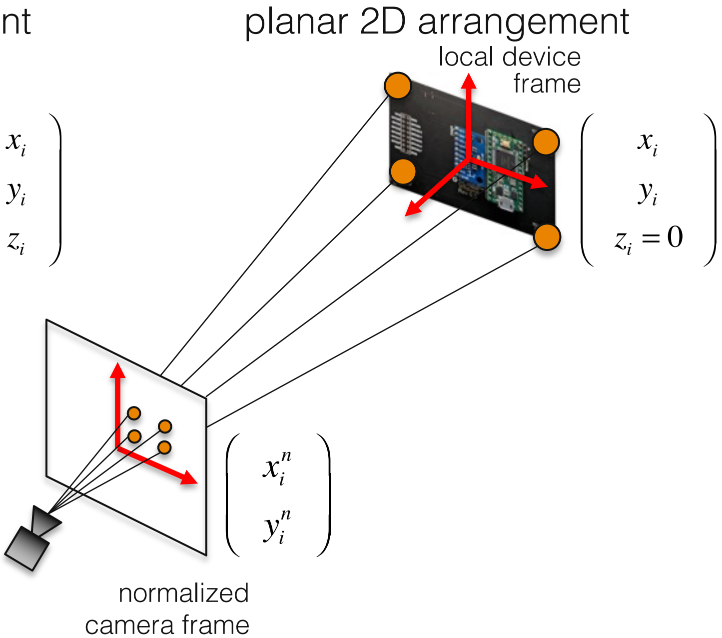


Image Formation – 3D Arrangement

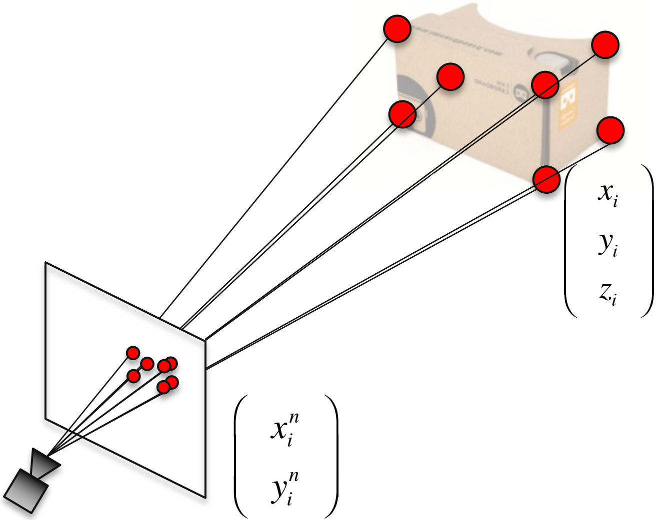
1. transform 3D point into view space:

$$\begin{pmatrix} x_i^c \\ y_i^c \\ w_i^c \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{pmatrix} \cdot \begin{pmatrix} x_i \\ y_i \\ z_i \\ 1 \end{pmatrix}$$

This is the homogeneous coordinate

"projection matrix"

"modelview matrix"
3x3 rotation matrix and translation 3x1 vector



2. perspective divide:

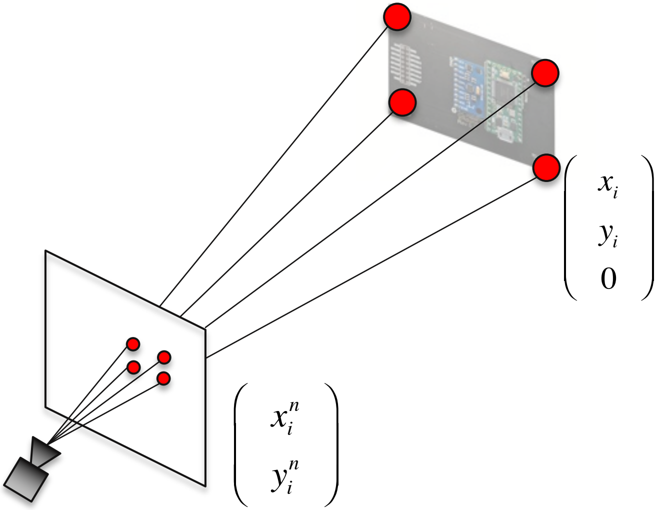
$$\begin{pmatrix} x_i^n \\ y_i^n \end{pmatrix} = \begin{pmatrix} \dfrac{x_i^c}{w_i^c} \\ \dfrac{y_i^c}{w_i^c} \end{pmatrix}$$

Image Formation – 2D Arrangement

1. transform 3D point into view space:

$$\begin{pmatrix} x_i^c \\ y_i^c \\ w_i^c \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{pmatrix} \cdot \begin{pmatrix} x_i \\ y_i \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{pmatrix} \cdot \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$



2. perspective divide:

$$\begin{pmatrix} x_i^n \\ y_i^n \end{pmatrix} = \begin{pmatrix} \dfrac{x_i^c}{w_i^c} \\ \dfrac{y_i^c}{w_i^c} \end{pmatrix}$$

Image Formation – 2D Arrangement

- all rotation matrices are orthonormal, i.e.

$$\sqrt{r_{11}^2 + r_{21}^2 + r_{31}^2} = 1$$

$$\sqrt{r_{12}^2 + r_{22}^2 + r_{32}^2} = 1$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{array}{c} \begin{array}{cc} \text{rotation R} & \text{translation T} \end{array} \\ \begin{pmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{pmatrix} \end{array}$$

The Homography Matrix

- all rotation matrices are orthonormal, i.e. $$\sqrt{r_{11}^2 + r_{21}^2 + r_{31}^2} = 1$$ $$\sqrt{r_{12}^2 + r_{22}^2 + r_{32}^2} = 1$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} \underbrace{\boxed{\begin{matrix} r_{11} & r_{12} \\ r_{21} & r_{22} \\ r_{31} & r_{32} \end{matrix}}}_{\text{rotation R}} & \underbrace{\boxed{\begin{matrix} t_x \\ t_y \\ t_z \end{matrix}}}_{\text{translation T}} \end{pmatrix} = \underbrace{\begin{pmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{pmatrix}}_{H}$$

let's call this "homography matrix"

Understanding Pose Estimation

1. how to get projected 2D coordinates?

2. image formation

3. estimate pose with linear homography method

4. estimate pose with nonlinear Levenberg-Marquardt method (next class)

- how to compute the homography matrix

- how to get position and rotation from that matrix

The Homography Matrix

Turns out that: any homography matrix has only 8 degrees of freedom – can scale matrix by *s* and get the same 3D-to-2D mapping

- image formation with scaled homography matrix sH

$$\begin{pmatrix} x_i^n \\ y_i^n \end{pmatrix} = \begin{pmatrix} \dfrac{x_i^c}{w_i^c} \\ \dfrac{y_i^c}{w_i^c} \end{pmatrix} = \begin{pmatrix} \dfrac{sh_1 x_i + sh_2 y_i + sh_3}{sh_7 x_i + sh_8 y_i + sh_9} \\ \dfrac{sh_4 x_i + sh_5 y_i + sh_6}{sh_7 x_i + sh_8 y_i + sh_9} \end{pmatrix} = \begin{pmatrix} \dfrac{\cancel{s}\left(h_1 x_i + h_2 y_i + h_3\right)}{\cancel{s}\left(h_7 x_i + h_8 y_i + h_9\right)} \\ \dfrac{\cancel{s}\left(h_4 x_i + h_5 y_i + h_6\right)}{\cancel{s}\left(h_7 x_i + h_8 y_i + h_9\right)} \end{pmatrix}$$

The Homography Matrix

- common approach: estimate a scaled version of the homography matrix, where $h_9 = 1$
- we will see later how we can get scale factor *s*

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{pmatrix} = s \boxed{\begin{pmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & 1 \end{pmatrix}}$$

estimate these 8 homography matrix elements!

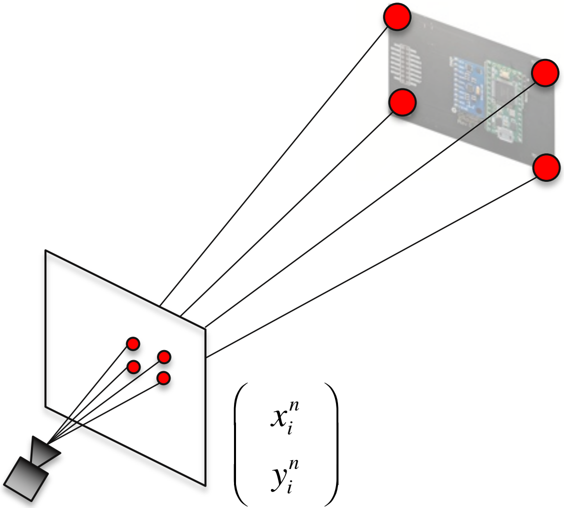
Pose Estimation via Homography

- image formation changes to

$$\begin{pmatrix} x_i^c \\ y_i^c \\ w_i^c \end{pmatrix} = s \begin{pmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

homography matrix with

8 unknowns!



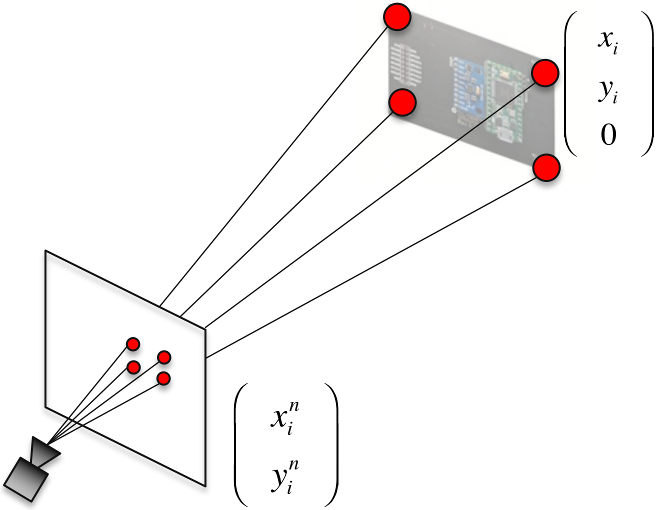
$$\begin{pmatrix} x_i \\ y_i \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x_i^n \\ y_i^n \end{pmatrix}$$

Pose Estimation via Homography

- image formation changes to

$$\begin{pmatrix} x_i^c \\ y_i^c \\ w_i^c \end{pmatrix} = s \begin{pmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$



$$\begin{pmatrix} x_i^n \\ y_i^n \end{pmatrix} = \begin{pmatrix} \frac{x_i^c}{w_i^c} \\ \frac{y_i^c}{w_i^c} \end{pmatrix} = \begin{pmatrix} \frac{h_1 x_i + h_2 y_i + h_3}{h_7 x_i + h_8 y_i + 1} \\ \frac{h_4 x_i + h_5 y_i + h_6}{h_7 x_i + h_8 y_i + 1} \end{pmatrix}$$

Pose Estimation via Homography

- multiply by denominator

$$\begin{pmatrix} x_i^n \\ y_i^n \end{pmatrix} = \begin{pmatrix} \dfrac{x_i^c}{w_i^c} \\ \dfrac{y_i^c}{w_i^c} \end{pmatrix} = \begin{pmatrix} \dfrac{h_1 x_i + h_2 y_i + h_3}{h_7 x_i + h_8 y_i + 1} \\ \dfrac{h_4 x_i + h_5 y_i + h_6}{h_7 x_i + h_8 y_i + 1} \end{pmatrix} \Rightarrow \begin{array}{l} x_i^n \left(h_7 x_i + h_8 y_i + 1\right) = h_1 x_i + h_2 y_i + h_3 \\ y_i^n \left(h_7 x_i + h_8 y_i + 1\right) = h_4 x_i + h_5 y_i + h_6 \end{array}$$

Pose Estimation via Homography

- reorder equations

$$h_1\, x_i + h_2\, y_i + h_3 - h_7\, x_i\, x_i^n - h_8\, y_i\, x_i^n = x_i^n$$
$$h_4\, x_i + h_5\, y_i + h_6 - h_7\, x_i\, y_i^n - h_8\, y_i\, y_i^n = y_i^n$$

$$x_i^n\left(h_7\, x_i + h_8\, y_i + 1\right) = h_1\, x_i + h_2\, y_i + h_3$$
$$y_i^n\left(h_7\, x_i + h_8\, y_i + 1\right) = h_4\, x_i + h_5\, y_i + h_6$$

Pose Estimation via Homography

- 8 unknowns (red) but only 2 measurements (blue) per 3D-to-2D point correspondence

$$\boxed{h_1} x_i + \boxed{h_2} y_i + \boxed{h_3} - \boxed{h_7} x_i \boxed{x_i^n} - \boxed{h_8} y_i \boxed{x_i^n} = \boxed{x_i^n}$$
$$\boxed{h_4} x_i + \boxed{h_5} y_i + \boxed{h_6} - \boxed{h_7} x_i \boxed{y_i^n} - \boxed{h_8} y_i \boxed{y_i^n} = \boxed{y_i^n}$$

- need at least 4 point correspondences to get to invertible system with 8 equations & 8 unknowns!
- VRduino has 4 photodiodes → need all 4 to compute pose

Pose Estimation via Homography

- solve Ah=b on Arduino using Matrix Math Library via `MatrixInversion` function (details in lab)

$$
\underbrace{\begin{pmatrix}
x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1 x_1^n & -y_1 x_1^n \\
0 & 0 & 0 & x_1 & y_1 & 1 & -x_1 y_1^n & -y_1 y_1^n \\
x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2 x_2^n & -y_2 x_2^n \\
0 & 0 & 0 & x_2 & y_2 & 1 & -x_2 y_2^n & -y_2 y_2^n \\
x_3 & y_3 & 1 & 0 & 0 & 0 & -x_3 x_3^n & -y_3 x_3^n \\
0 & 0 & 0 & x_3 & y_3 & 1 & -x_3 y_3^n & -y_3 y_3^n \\
x_4 & y_4 & 1 & 0 & 0 & 0 & -x_4 x_4^n & -y_4 x_4^n \\
0 & 0 & 0 & x_4 & y_4 & 1 & -x_4 y_4^n & -y_4 y_4^n
\end{pmatrix}}_{A}
\underbrace{\begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \end{pmatrix}}_{h}
=
\underbrace{\begin{pmatrix} x_1^n \\ y_1^n \\ x_2^n \\ y_2^n \\ x_3^n \\ y_3^n \\ x_4^n \\ y_4^n \end{pmatrix}}_{b}
$$

Get Position from Homography Matrix

- still need scale factor *s* to get position!

just computed this

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{pmatrix} = s \begin{pmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & 1 \end{pmatrix}$$

Get Position from Homography Matrix

- normalize homography to have approx. unit-length columns for the rotation part, such that $\sqrt{r_{11}^2 + r_{21}^2 + r_{31}^2} \approx 1, \quad \sqrt{r_{12}^2 + r_{22}^2 + r_{32}^2} \approx 1$

$$s = \frac{2}{\sqrt{h_1^2 + h_4^2 + h_7^2} + \sqrt{h_2^2 + h_5^2 + h_8^2}}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{pmatrix} = s \begin{pmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & 1 \end{pmatrix}$$

Get Position from Homography Matrix

- this gives us the position as

$$\boxed{t_x = sh_3, \quad t_y = sh_6, \quad t_z = -s}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{pmatrix} = s \begin{pmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & 1 \end{pmatrix}$$

Get Rotation from Homography Matrix

1. get normalized 1st column of 3x3 rotation matrix
2. get normalized 2nd column via orthogonalization
3. get missing 3rd column with cross product

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{pmatrix} = s \begin{pmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & 1 \end{pmatrix}$$

Get Rotation from Homography Matrix

1. get normalized 1st column of 3x3 rotation matrix

$$\tilde{r}_1 = \begin{pmatrix} h_1 \\ h_4 \\ -h_7 \end{pmatrix}, \quad r_1 = \frac{\tilde{r}_1}{\left\|\tilde{r}_1\right\|_2}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{pmatrix} = s \begin{pmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & 1 \end{pmatrix}$$

Get Rotation from Homography Matrix

2. get normalized 2nd column via orthogonalization

$$\tilde{r}_2 = \begin{pmatrix} h_2 \\ h_5 \\ -h_8 \end{pmatrix} - \left(r_1 \bullet \begin{pmatrix} h_2 \\ h_5 \\ -h_8 \end{pmatrix} \right) r_1, \quad r_2 = \frac{\tilde{r}_2}{\left\| \tilde{r}_2 \right\|_2}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{pmatrix} = s \begin{pmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & 1 \end{pmatrix}$$

Get Rotation from Homography Matrix

3. get missing 3rd column with cross product: $r_3 = r_1 \times r_2$

- r_3 this is guaranteed to be orthogonal to the other two columns

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{pmatrix} = s \begin{pmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & 1 \end{pmatrix}$$

Get Rotation from Homography Matrix

- make 3x3 rotation matrix

$$R = \left(r_1 \ r_2 \ r_3 \right) = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

- convert to quaternion or Euler angles

Get Rotation from Homography Matrix

- remember Euler angles (with yaw-pitch-roll order):

$$\underbrace{\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}}_{\mathbf{R}} = \underbrace{\begin{pmatrix} \cos(\theta_z) & -\sin(\theta_z) & 0 \\ \sin(\theta_z) & \cos(\theta_z) & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\mathbf{R}_z(\theta_z)} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_x) & -\sin(\theta_x) \\ 0 & \sin(\theta_x) & \cos(\theta_x) \end{pmatrix}}_{\mathbf{R}_x(\theta_x)} \underbrace{\begin{pmatrix} \cos(\theta_y) & 0 & \sin(\theta_y) \\ 0 & 1 & 0 \\ -\sin(\theta_y) & 0 & \cos(\theta_y) \end{pmatrix}}_{\mathbf{R}_y(\theta_y)}$$

$$= \begin{pmatrix} \cos(\theta_y)\cos(\theta_z) - \sin(\theta_x)\sin(\theta_y)\sin(\theta_z) & -\cos(\theta_x)\sin(\theta_z) & \sin(\theta_y)\cos(\theta_z) + \sin(\theta_x)\cos(\theta_y)\sin(\theta_z) \\ \cos(\theta_y)\sin(\theta_z) + \sin(\theta_x)\sin(\theta_y)\cos(\theta_z) & \cos(\theta_x)\cos(\theta_z) & \sin(\theta_y)\sin(\theta_z) - \sin(\theta_x)\cos(\theta_y)\cos(\theta_z) \\ -\cos(\theta_x)\sin(\theta_y) & \sin(\theta_x) & \cos(\theta_x)\cos(\theta_y) \end{pmatrix}$$

- get angles from 3x3 rotation matrix:

$$r_{32} = \sin(\theta_x) \qquad \Rightarrow \theta_x = \sin^{-1}(r_{32}) = \operatorname{asin}(r_{32})$$

$$\frac{r_{31}}{r_{33}} = -\frac{\cos(\theta_x)\sin(\theta_y)}{\cos(\theta_x)\cos(\theta_y)} = -\tan(\theta_y) \qquad \Rightarrow \theta_y = \tan^{-1}\left(-\frac{r_{31}}{r_{33}}\right) = \operatorname{atan2}(-r_{31}, r_{33})$$

$$\frac{r_{12}}{r_{22}} = -\frac{\cos(\theta_x)\sin(\theta_z)}{\cos(\theta_x)\cos(\theta_z)} = -\tan(\theta_z) \qquad \Rightarrow \theta_z = \tan^{-1}\left(-\frac{r_{12}}{r_{22}}\right) = \operatorname{atan2}(-r_{12}, r_{22})$$

Temporal Filter to Smooth Noise

- pose estimation is *very* sensitive to noise in the measured 2D coordinates!

  → estimated position and especially rotation may be noisy

- apply a simple temporal filter with weight α to smooth the pose at time step *k*:

$$\left(\theta_x,\theta_y,\theta_z,t_x,t_y,t_z\right)_{filtered}^{(k)} = \alpha\left(\theta_x,\theta_y,\theta_z,t_x,t_y,t_z\right)_{filtered}^{(k-1)} + (1-\alpha)\left(\theta_x,\theta_y,\theta_z,t_x,t_y,t_z\right)_{unfiltered}^{(k)}$$

- smaller α → less filtering, larger α → more smoothing

Pose Estimation via Homographies – Step-by-Step

in each loop() call of the VRduino:

1. get timings from all 4 photodiodes in "ticks"
2. convert "ticks" to degrees and then to 2D coordinates on plane at unit distance (i.e. get x_i^n, y_i^n)
3. populate matrix A using the 2D and 3D point coordinates
4. estimate homography as $h=A^{-1}b$
5. get position t_x, t_y, t_z and rotation, e.g. in Euler angles from the estimated homography
6. apply temporal filter to smooth out noise

Must read: course notes on tracking!

Understanding Pose Estimation

1. how to get projected 2D coordinates?
2. image formation
3. estimate pose with linear homography method
4. estimate pose with nonlinear Levenberg-Marquardt method (next class)

- advanced topic
- all details of this are also derived in course notes