MAT 272 Labs

This is a collection of labs to be used during the 2023 Spring semester.

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Contents

1 Calculus I Review

1. Compute the derivative of the following functions.

   (a) $y = \sin(x)$

   (b) $y = \cos(x)$

   (c) $y = x^3$

   (d) $y = 3x$

   (e) $y = e^x$

   (f) $y = \dfrac{1}{x}$

   (g) $y = \dfrac{1}{x^2}$

   (h) $y = x^\pi$

   (i) $y = \arctan(x)$

2. Compute the derivative of the following expressions using derivative rules.

   (a) $y = 2\sin(x) + \cos(x)$

   (b) $y = e^{\sin(x)}$

   (c) $y = \dfrac{x+3}{x^2 - x}$

   (d) $y = \tan(x)$

   (e) $y = \sec(x)$

   (f) $y = xe^x$

   (g) $y = (2x+1)^9$

   (h) $y = \sin(x)\cos(x^2)$

3. Find $\dfrac{dy}{dx}$ as a function of x and y for the implicit curve:

$$xy + y^2 + x^2 = 3.$$

4. Find $\dfrac{d}{dx}\left(f(x)^{100}\right)$:

5. Find $\dfrac{d}{dx}\left(\ln\left(f(x)\right)g(x)\right)$

6. Calculate these basic anti-derivatives.

(a) $\displaystyle\int \sin(x)\,dx$

(d) $\displaystyle\int dx$

(g) $\displaystyle\int \frac{1}{x^2}\,dx$

(b) $\displaystyle\int \cos(x)\,dx$

(e) $\displaystyle\int e^x\,dx$

(h) $\displaystyle\int x^\pi\,dx$

(c) $\displaystyle\int x^3\,dx$

(f) $\displaystyle\int \frac{1}{x}\,dx$

(i) $\displaystyle\int \sec(x)\tan(x)\,dx$

7. Calculate the definite integral using one of the Fundamental Theorems of Calculus.

(a) $\displaystyle\int_2^4 \left(x^2+1\right)\,dx$

(b) $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2\theta\,d\theta$

8. Find the area between the curves $f(x) = 2x$ and $g(x) = x^2$ on the interval $[0, 2]$.

9. Find the area between the curves $f(x) = \sin(x)$ and $g(x) = \cos(x)$ on the interval $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$.

10. Calculate $\dfrac{d}{dx}\left(\displaystyle\int_x^{x^2} e^{t^2}\, dt\right)$.

2 Hyperbolic Functions

1. Show that the derivative of $\sinh(x) = \cosh(x)$ using the exponential definition of the functions.

2. Find $\sinh(x + y)$ in terms of $\sinh(x)$, $\cosh(x)$, $\sinh(y)$, and $\cosh(y)$. This is analagous to the the sum of "angles" idendity for trigonometric functions. Hint: You will want to use the exponential definition of these functions.

3. Calculate the derivative of the following hyperbolic functions using basic derivative rules and the theorem(s) from the textbook.

(a) $y = \sinh(7x)$

(c) $y = \tanh\left(x^2 - 7x + 3\right)$

(b) $y = \cosh(\pi x)$

(d) $y = (\ln(x) + 1)\,\mathrm{sech}(x)$

4. Find the equation of the line tangent to the graph of $y = e^{\sinh(x)}$ at the point $(0, 1)$.

3 u-Substitution

1. Determine a u to use to integrate the given indefinite integral. Do not actually solve the integral.

   (a) $\displaystyle\int \frac{\sqrt{1-\frac{1}{x}}}{x^2}\,dx$

   (b) $\displaystyle\int \frac{e^y}{3e^y-2}\,dy$

   (c) $\displaystyle\int \theta^2 e^{\theta^3}\,d\theta$

   (d) $\displaystyle\int z\sqrt{2-7z}\,dz$

2. Evaluate the definite integral using the u-substitution with the given u.

   (a) $\displaystyle\int_0^2 2x\sqrt{1+x^2}\,dx; \quad u = 1+x^2$

(b) $\displaystyle\int_1^4 \frac{e^{1/x}}{x^2}\,dx; \quad u = 1/x$

(c) $\displaystyle\int_0^{\ln(2)} \frac{e^{2x}}{3+e^{2x}}\,dx; \quad u = 3 + e^{2x}$

(d) $\displaystyle\int_{-\pi/4}^{\pi/4} \tan(x)\,dx; \quad u = \cos(x)$

3. Evaluate the definite or indefinite integral. Some questions may require u-substitution and some may simply require some algebra manipulation.

(a) $\displaystyle\int 3x^2e^{x^3-2x^2+7} - 4xe^{x^3-2x^2+7}\,dx$

(b) $\displaystyle\int \frac{\ln(w)}{w}\,dw$

(c) $\displaystyle\int_0^{\sqrt[3]{7}} y^2e^{y^3}\,dy$

(d) $\displaystyle\int \left(\sqrt{1+\sqrt{\alpha}}\right)\,d\alpha$

(e) $\displaystyle\int_{-\pi/2}^{\pi/2} \sin^3(\theta)\cos(\theta)\,d\theta$

(f) $\displaystyle\int_0^{\pi/4} \frac{e^{\tan(x)}}{\cos^2(x)}\,dx$

4 Integration by Parts

1. Solve the following integral using the Tabular Method for repeated Integration by Parts.

   (a) $\int x^3 e^{4x}\,dx$

   (b) $\int x^2 \sin(5x)\,dx$

2. Evaluate the following integrals using integration by parts.

   (a) $\int xe^{-2x}\,dx$

   (b) $\int e^{2\theta}\cos(3\theta)\,d\theta$

(c) $\displaystyle\int x^8 \ln(x)\, dx$

(d) $\displaystyle\int_2^{e^2} x^2 \ln(x)\, dx$

(e) $\displaystyle\int_0^1 e^{2y} \sin\left(e^{2y}\right)\, dy$

(f) $\displaystyle\int_0^4 e^{\sqrt{w}}\, dw$

5 Partial Fractions and Algebraic Techniques

1. Use find the partial fraction decomposition of the following rational expressions.

   (a) $\dfrac{12x - 11}{x^2 - x}$

   (b) $\dfrac{z^2 + 20z - 15}{z^3 + 4z^2 - 5z}$

2. Evaluate the following integrals.

(a) $\displaystyle\int \frac{x^2+3}{x^3-2x^2+x}\,dx$

(b) $\displaystyle\int \frac{8}{x^3-2x^2-4x+8}\,dx$

(c) $\displaystyle\int \frac{2x+3}{x^2+4}\,dx$

(d) Use the fact that $\sec(x) = \dfrac{\cos(x)}{1-\sin^2(x)}$ and partial fractions to find $\displaystyle\int \sec\theta\,d\theta$.

(e) $\displaystyle\int \frac{\sin(\theta)}{\cos(\theta)+\cos^2(\theta)}\,d\theta.$

(f) $\displaystyle\int \frac{e^x}{e^{2x}-4e^x}\,dx.$

6 Trigonometric Integrals

1. Use an appropriate trigonometric integral to evaluate the following integrals.

   (a) $\displaystyle\int \sin^3(x)\,dx$

   (b) $\displaystyle\int \cos^3(x)\,dx$

(c) $\int \cos^4(2x)\, dx$

(d) $\int \sin^2(x)\cos^2(x)\, dx$

(e) $\displaystyle\int \sin^3(x)\cos^5(x)\,dx$

(f) $\displaystyle\int \sin^4(5x)\,dx$

(g) $\displaystyle\int 12\sec^4(x)\,dx$

(h) $\displaystyle\int \tan^3(x)\,dx$

(i) $\displaystyle\int \tan(x)\sec^3(x)\,dx$

(j) $\displaystyle\int \sqrt{\tan(x)}\sec^4(x)\,dx$

7 Trigonometric Substitutions

1. Solve the following integral using the appropriate trigonometric substitution.

   (a) $\displaystyle\int \frac{1}{\sqrt{25 - x^2}}\, dx$

   (b) $\displaystyle\int \frac{x^2}{\sqrt{1 - x^2}}\, dx$

(c) $\displaystyle\int \frac{1}{(4-x^2)^{3/2}}\,dx$

(d) $\displaystyle\int \sqrt{(81+x^2)}\,dx$

(e) $\displaystyle\int_{\frac{1}{\sqrt{3}}}^{1} \frac{1}{x^2\sqrt{1+x^2}}\,dx$

(f) $\displaystyle\int_{1}^{2} \frac{1}{x^2\sqrt{4-x^2}}\,dx$

2. A total charge Q is distributed uniformly on a line segment of length $2L$ along the y-axis. The x-component of the electric field at a point $(a, 0)$ on the x-axis is given by

$$E_x(a) = \frac{kQa}{2L} \int_{-L}^{L} \frac{1}{(a^2 + y^2)^{3/2}}\, dy,$$

where k is a physical constant and $a > 0$[1].

(a) Use a trigonometric substitution to find an explicit formula for $E_x(a)$.

(b) Let $\rho = \frac{Q}{2L}$ be the charge density on the line segement. Show that if $L \to \infty$, then $E_x(a) = \frac{2k\rho}{a}$.

[1]A detailed derivation of this can be found at `http://newb.kettering.edu/wp/experientialcalculus/wp-content/uploads/sites/15/2017/05/the-electric-field-of-a-line-of-charge.pdf`.

8 Integration Technique Review Questions

1. $\displaystyle\int te^{t^2}\ dt$
2. $\displaystyle\int te^{2t}\ dt$
3. $\displaystyle\int t^3\cos(t^2)\ dt$
4. $\displaystyle\int \ln(x^3)\ dx$
5. $\displaystyle\int \frac{dx}{\sqrt{1-x}}$
6. $\displaystyle\int \sec(5\theta)\ d\theta$
7. $\displaystyle\int \sin^2 x\ dx$
8. $\displaystyle\int \cot(3\alpha)\ d\alpha$
9. $\displaystyle\int \cos^3 x\ dx$
10. $\displaystyle\int \sec^3(2\phi)\ d\phi$
11. $\displaystyle\int \sqrt{4+7t}\,dt$
12. $\displaystyle\int \sqrt{4+7t^2}\,dt$
13. $\displaystyle\int \sqrt{1-6t^2}\,dt$
14. $\displaystyle\int t^2\sqrt[3]{1-7t^3}\,dt$
15. $\displaystyle\int t^2\sin(2t)\ dt$
16. $\displaystyle\int te^{-t^2}\ dt$
17. $\displaystyle\int \cos(\sqrt{t})\ dt$
18. $\displaystyle\int \sin^8 x\cos^5 x\ dx$
19. $\displaystyle\int e^x\cos 2x\ dx$
20. $\displaystyle\int x^5\ln x\ dx$
21. $\displaystyle\int \sin 3x\cos 4x\ dx$
22. $\displaystyle\int \frac{x}{1+2x^2}\,dx$
23. $\displaystyle\int \frac{dx}{4+x^2}$
24. $\displaystyle\int \frac{dx}{1-x^2}$
25. $\displaystyle\int \frac{2x}{(x^2+1)(x-1)^2}\,dx$
26. $\displaystyle\int \frac{4t+1}{t^2+t-2}\,dt$
27. $\displaystyle\int \frac{u^2+11}{u^2+4u+5}\,du$

Determine if the following improper integrals are convergent or divergent. If they are convergent, determine the value to which they converge.

1. $\displaystyle\int_2^\infty \frac{2x}{x^2+1}\,dx$
2. $\displaystyle\int_0^\infty xe^{-x}\,dx$
3. $\displaystyle\int_{-\infty}^0 \frac{1}{1+x^2}\,dx$
4. $\displaystyle\int_3^\infty \frac{1}{\sqrt{x}}\,dx$
5. $\displaystyle\int_0^\infty \frac{e^x}{e^{2x}+1}\,dx$
6. $\displaystyle\int_{4/\pi}^\infty \frac{1}{x^2}\sec^2\left(\frac{1}{x}\right)\ dx$

9 Volumes by Slicing – Part 1

1. Find the volume of the solid whose base is the region bounded by the semicircle $y = \sqrt{4 - x^2}$ and the x-axis and whose cross sections through the solid perpendicular to the x-axis are squares. For a 3D graph, go to `https://sagecell.sagemath.org/?q=ixkvvn`.

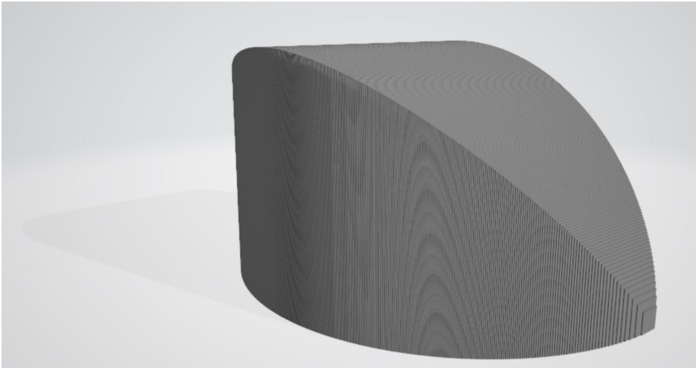

Figure 1: For a better view, go to `https://sagecell.sagemath.org/?q=rvlawp`.

2. Find the volume of the solid whose base is the region bounded by $y = x^2$ and the the line $y = 4$ and whose cross sections are equilateral triangles parallel to the x-axis.

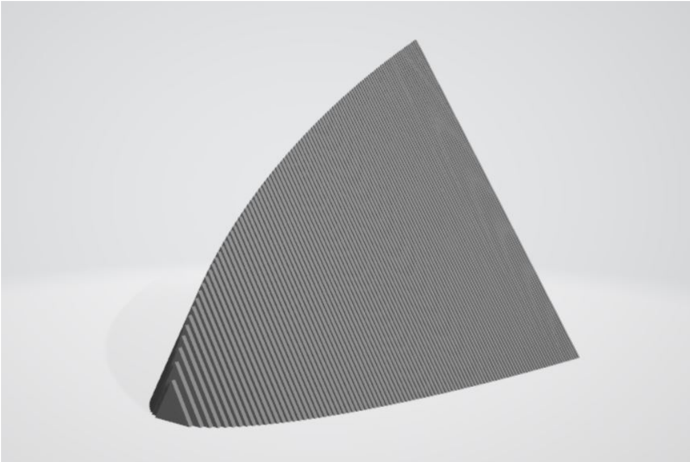

Figure 2: For a better view, go to `https://sagecell.sagemath.org/?q=rvlawp`.

3. Let R be the region bounded by the following curves. Use the disk (or washer) method to find the volume of the solid generated when R is revolved about the x-axis.

   (a) $y = e^{-x}$ and the x-axis on the interval $[0, \ln(4)]$

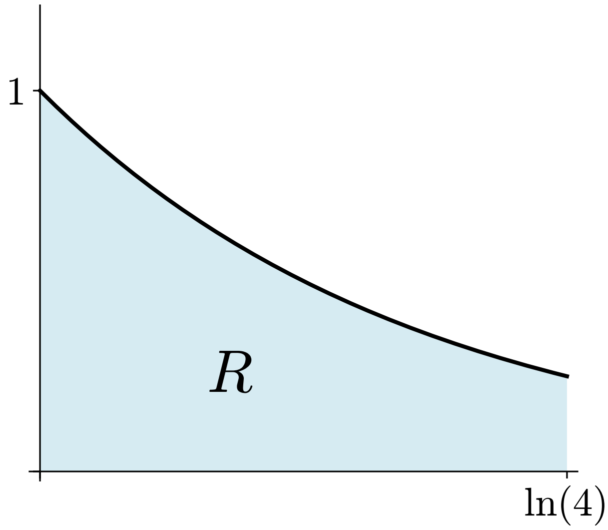

Figure 3: Region bounded by $y = e^{-x}$ and the x-axis on the interval $[0, \ln(4)]$

(b) $y = x$ and $y = \sqrt[4]{x}$

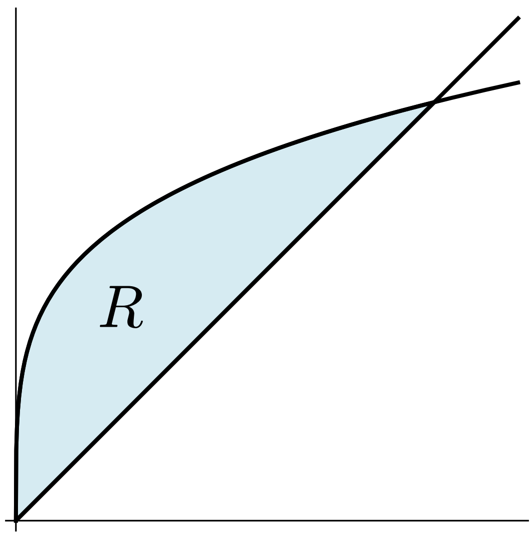

Figure 4: Region bounded by $y = \sqrt[4]{x}$ and the $y = x$

4. Let R be the region bounded by the following curves. Use the disk (or washer) method to find the volume of the solid generated when R is revolved about the y-axis.

   (a) $y = 16 - x^2$ and the x-axis

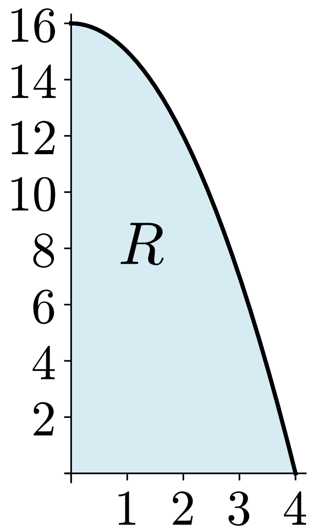

Figure 5: Region bounded by $y = 16 - x^2$ and the x-axis

(b) $y = \dfrac{x}{2}$ and $y = \sqrt{x}$

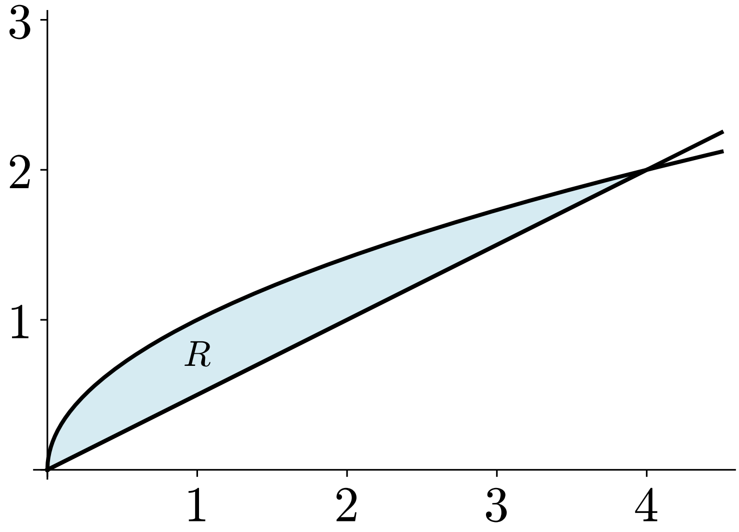

Figure 6: Region bounded by $y = 16 - x^2$ and the x-axis

10 Volumes by Slicing – Part 2

1. Use disk (or washer) method to find the volume of the solid of revolution obtain by rotating the given region, R, about the specified axis of rotation.

   (a) $y = x + 2$ and $y = x^2$ about the line $y = 5$

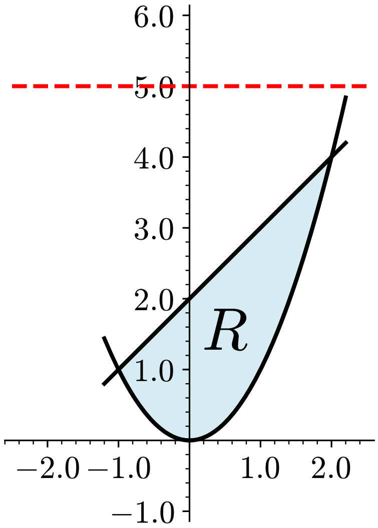

Figure 1: Region between $y = x + 2$ and $y = x^2$

(b) $y = 2$ and $y = \sqrt{x}$ about the line $x = -2$

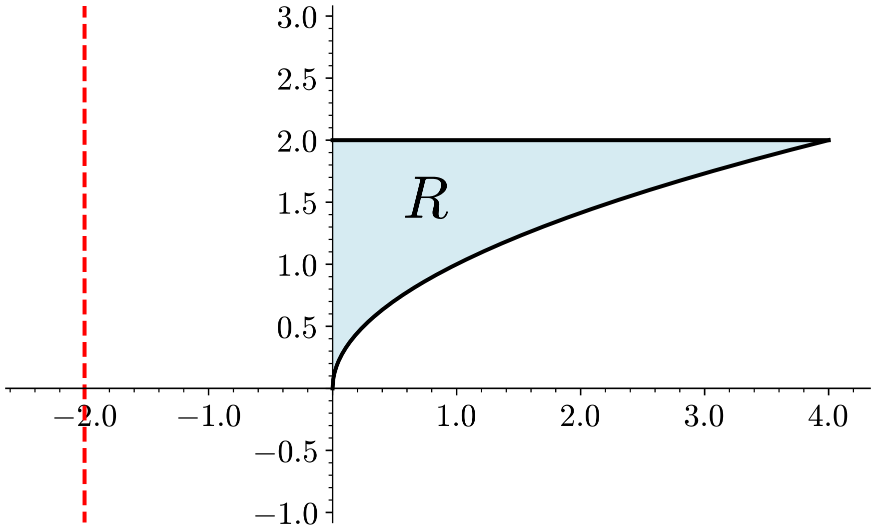

Figure 2: Region between $y = 2$ and $y = \sqrt{x}$

11 Volumes by Shells

1. Let R be the region shown in the figure below.
   (a) Draw an example of a shell created by revolving a Riemann rectangle at x_k^* in the interval $[0, \sqrt{\pi}]$ about the y-axis.

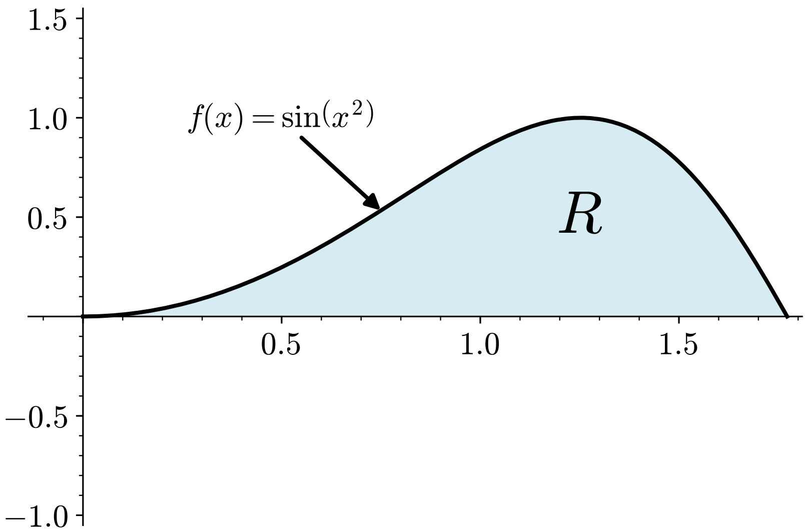

Figure 1: Region bounded between $f(x) = \sin(x^2)$ and the x-axis

   (b) Draw the image that corresponds to unraveling the shell and label it.

   (c) What is the length of the shell?

   (d) What is the height of the shell?

(e) What is the width of the shell?

(f) Use this information to construct the Riemann sum that would calculate the volume of the solid of revolution.

(g) Use your information from the previous part to construct and evaluate the definite integral that would calculate the volume of the solid of revolution.

2. Let R be the region shown in the figure below.

(a) Draw an example of a shell created by revolving a Riemann rectangle at x_k^* in the interval $[0, 2]$ about the line $x = 3$.

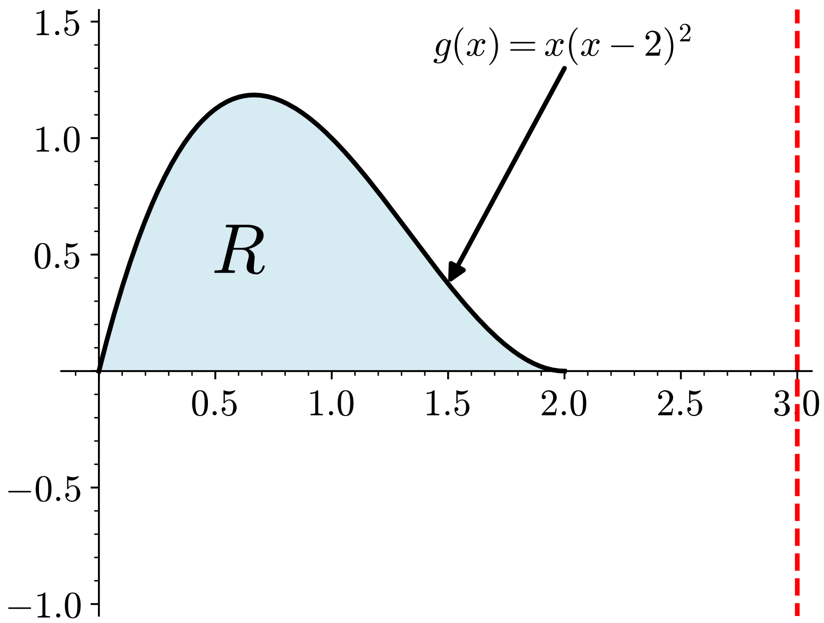

Figure 2: Region bounded between $g(x) = x(x-2)^2$ and the x axis

(b) Draw the image that corresponds to unraveling the shell and label it.

(c) What is the length of the shell?

(d) What is the height of the shell?

(e) What is the width of the shell?

(f) Use this information to construct the Riemann sum that would calculate the volume of the solid of revolution.

(g) Use your information from the previous part to construct and evaluate the definite integral that would calculate the volume of the solid of revolution.

3. Find the volume of the solid created by rotating the region bounded by $y = x^2 + 2x - 1$ and $y = 2x$ about the line $x = -4$. Draw clearly labeled picture to support your answer.

4. Find the volume of the solid created by rotating the region bounded by $y = \sin(x) + 2$ and $y = \cos(x)$ on the interval $[0, \pi]$ about the line $x = 6$.

12 Arc Length and Surface Area

1. Find the arc length of the specified function over the given domain.

   (a) $f(x) = x^2/2$; $[0,\ 2]$

   (b) $g(x) = \dfrac{1}{2}\left(e^{x} + e^{-x}\right)$; $[0,\ \ln(5)]$

(c) $h(x) = \ln(\cos x); \quad [0, \pi/4]$

(d) $k(x) = \dfrac{1}{12}x^5 + \dfrac{1}{5x^3}; \quad \left[\dfrac{1}{10}, 1\right]$

2. Find the surface area of the described solid of revolution.

   (a) The solid formed by revolving $y = 2x$ on $[0, 2]$ about the x-axis.

   (b) The solid formed by revolving $y = x^2$ on $[0, 1]$ about the y-axis.

(c) The solid formed by revolving $y = \sqrt{x}$ on $[0, 4]$ about the x-axis.

(d) The solid formed by revolving $y = \sqrt{a^2 - x^2}$ on $[-a, a]$ about the x-axis.

13 Introduction to Sequences

1. Consider the following sequence of number:

$$\{1, 3, 6, 10, 15, 21, \dots\}.$$

These are called *triangular numbers* because they are the number of vertices as pictured below.

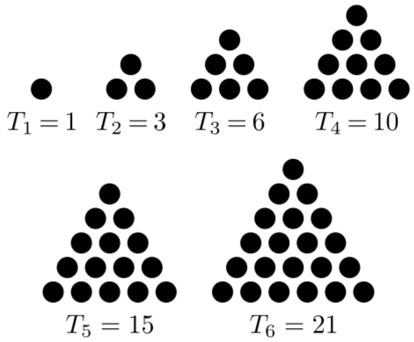

Figure 1: Triangular Numbers

(a) Write a *recursive* definition for this sequence.

(b) Find an *explicit* formula for this sequence where $n = 1$ is the first term of the sequence.

(c) Determine what T_{100} would be.

(c) ______________

2. Determine if the sequence has a least upper bound (supremum) and a greatest lower bound (infimum). If so, what are they?

(a) $\{e^{-k}\}_{k=0}^{\infty}$

(a) ______________

(b) $\{(-1)^k k\}_{k=0}^{\infty}$

(b) ______________

(c) $\left\{3 + \dfrac{1}{k^2+1}\right\}_{k=-\infty}^{\infty}$

(c) ______________

3. Match the formulas with the descriptions of the behavior of the sequence as k goes to infinity. List the first five values in the sequence as justification of your answer.

(a) $\left\{\dfrac{(-1)^n}{n+1}\right\}_{n=1}^{\infty}$

(b) $\left\{\sin\left(\dfrac{1}{k}\right)\right\}_{k=1}^{\infty}$

(c) $\left\{-4+\dfrac{(-1)^m}{m}\right\}_{m=0}^{\infty}$

(d) $\left\{\dfrac{n\cos(n)}{2n+3}\right\}_{n=12}^{\infty}$

(e) $\{n(n-1)-n\}_{n=-2}^{\infty}$

(f) $\left\{-\dfrac{p!}{(p+1)!}\right\}_{p=0}^{\infty}$

I. ______ Converges to $\frac{1}{2}$ from above and below.

II. ______ Converges to 0 through positive numbers.

III. ______ Converges to 0 from above and below.

IV. ______ Converges to 0 through negative numbers.

V. ______ Diverges to ∞.

VI. ______ Converges to -4 from above and below.

4. Opiates are drugs with a "morphine-like pharmacological action" according to the Mayo Clinic [2]. Many of these drugs have a half-life of about 5 hours. Suppose a pharmaceutical company has developed a new synthetic opiate with a half-life of 6 hours.

   (a) Complete the following table to describe the first 24 hours after taking 20mg of the new synthetic opiate at 6 hour intervals.

hour	0	6	12	18	24
amount (mg)					

   (b) Write an explicit equation that models this data of the form $N(t) = a_0(r)^{kt}$.

   (c) What is the hourly decay rate of the drug?

(c) ____________

   (d) As $t \to \infty$, what happens to the amount opiates?

[2] https://www.mayomedicallaboratories.com/test-info/drug-book/opiates.html

5. In this question, you will look at a sequence of functions, rather than a numerical sequence. Consider the function defined by $f_n(x) = \left(1 + \frac{x}{n}\right)^n$ and the sequence defined by $\{f_n(x)\}_{n=0}^{\infty}$. Go to `https://www.desmos.com/calculator/wjwfifwfnn` to help with this question.

   (a) Below are the first 5 terms of this sequence of functions:

$$1$$

$$x + 1$$

$$\frac{1}{4}x^2 + x + 1$$

$$\frac{1}{27}x^3 + \frac{1}{3}x^2 + x + 1$$

$$\frac{1}{256}x^4 + \frac{1}{16}x^3 + \frac{3}{8}x^2 + x + 1$$

   What patterns or sequences of numbers do you notice?

   (b) Use the Desmos graph that has been provided to record the different values of $f_n(1)$ to four decimal places.

n	0	1	10	100	1000	∞
$f_n(1)$						
$f_n(2)$						

   (c) What function do you hypothesize this sequence of functions converges to as $n \to \infty$? Give a justification of your answer. [Hint: $\lim_{n\to\infty}(1 + 1/n)^n = e$]

6. Determine if the sequence is bounded or unbounded, then use an appropriate test to analyze the monotonicity of the given sequence.

   (a) $\left\{\dfrac{n}{n+3}\right\}_{n=0}^{\infty}$

   (b) $\left\{\dfrac{k^3}{(k-1)!}\right\}_{k=0}^{\infty}$

   (c) $\left\{e^{-k}k^3\right\}_{k=0}^{\infty}$

14 The Integral Test

For each of the following series, determine if the series converges or diverges. You must use a test or a well-known series (i.e. geometric or telescoping) to prove convergence AND divergence. If the series is geometric or telescoping, find the value to which the series converges.

1. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{8^k}$ CONVERGES / DIVERGES.

   Proof

2. $\displaystyle\sum_{k=1}^{\infty} \frac{8}{\sqrt{k}}$ CONVERGES / DIVERGES.

   Proof

3. $\sum_{k=1}^{\infty} \frac{7}{\sqrt[3]{k+1}}$ Converges / Diverges.

Proof

4. $\sum_{k=1}^{\infty} \frac{1}{\ln(5)^k}$ Converges / Diverges.

Proof

5. $\displaystyle\sum_{k=1}^{\infty} k^2 e^{-k}$ Converges / Diverges.

Proof

6. $\displaystyle\sum_{k=1}^{\infty} \frac{3k}{k^2+4}$ Converges / Diverges.

Proof

7. Which of the following is required condition for applying the integral test to the sequence $\{a_k\}_k$, where $a_k = f(k)$.

   I. $f(k)$ is everywhere positive
   II. $f(k)$ is eventually monotonically decreasing
   III. $f(k)$ is eventually always continuous

   A. I only
   B. II only
   C. III
   D. I & II only
   E. I & III only
   F. II & III only
   G. I, II, & II

8. Which of the following statements is false?

   A. $\displaystyle\sum_k \frac{1}{k^p}$ converges if $p > 1$ and diverges otherwise.
   B. If a_k and $f(k)$ satisfy the requirements of the Integral Test, and if $\displaystyle\int_1^\infty f(k)\,dk$ converges, then $\displaystyle\sum_{k=1}^\infty a_k = \int_1^\infty f(k)\,dk$.
   C. $\displaystyle\sum_{k=2}^\infty \frac{1}{k(\ln k)^p}$ converges if $p > 1$.
   D. The integral test does not apply to divergent sequences.

9. Which of the following sequences DO NOT meet the conditions of the Integral Test?

   I. $\{k(\sin(k) + 1)\}_k$
   II. $\left\{\dfrac{1}{k^p + p}\right\}_k$
   III. $\left\{\dfrac{1}{k\sqrt{k}}\right\}_k$

   A. I only
   B. II only
   C. III only
   D. I & II only
   E. I & III only
   F. II & III only
   G. I, II, & II

15 The Comparison Tests

1. For each of the following, determine if the series converges or diverges, then use the direct comparison test to prove your answer.

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{2^k + k}$ Converges / Diverges.

   Proof

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+1)^2}$ Converges / Diverges.

   Proof

(c) $\displaystyle\sum_{\theta=0}^{\infty} \frac{1+\cos(\theta)}{10^{\theta}}$ CONVERGES / DIVERGES.

Proof

(d) $\displaystyle\sum_{k=0}^{\infty} \frac{k!}{(k+1)!}$ CONVERGES / DIVERGES.

Proof

2. For each of the following, determine if the series converges or diverges, then use the limit comparison test to prove your answer.

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{k-2}{k\sqrt{k}}$ CONVERGES / DIVERGES.

   Proof

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt[n]{e}}{n}$ CONVERGES / DIVERGES.

   Proof

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$ CONVERGES / DIVERGES.

Proof

(d) $\displaystyle\sum_{k=1}^{\infty} \frac{k^5}{k^6 - 2}$ CONVERGES / DIVERGES.

Proof

16 The Ratio and Root Tests

1. For each of the following, use the ratio test to determine if the series converges or diverges.

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k!}$ CONVERGES / DIVERGES.

   Proof

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{(n+1)!}$ CONVERGES / DIVERGES.

   Proof

(c) $\displaystyle\sum_{k=0}^{\infty} k^4 2^{-k}$ Converges / Diverges.

Proof

(d) $\displaystyle\sum_{k=0}^{\infty} \frac{(k!)^2}{(2k)!}$ Converges / Diverges.

Proof

2. For each of the following, use the root test to determine if the series converges or diverges.

(a) $\displaystyle\sum_{k=1}^{\infty}\left(\frac{k^2-2k+3}{7k^3+k-111}\right)^k$ CONVERGES / DIVERGES.

Proof

(b) $\displaystyle\sum_{n=1}^{\infty}\left(1+\frac{2}{n}\right)^{n^2}$ CONVERGES / DIVERGES.

Proof

(c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+2}\right)^{3n^2}$ CONVERGES / DIVERGES.

Proof

(d) $\displaystyle\sum_{k=1}^{\infty} \left(\sqrt[k]{k} - 1\right)^{5k}$ CONVERGES / DIVERGES.

Proof

17 Alternating Series Test

1. For each of the following, determine if the series converges absolutely, conditionally, or diverges.

   (a) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$ Abs. / Cond. / Diverges.

   Proof

   (b) $\displaystyle\sum_{n=1}^{\infty} \left((-1)^n \left(\frac{n}{3+n} \right)^n \right)$ Abs. / Cond. / Diverges.

   Proof

(c) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{k^2+10}$ ABS. / COND. / DIVERGES.

Proof

(d) $\displaystyle\sum_{k=0}^{\infty} \left((-1)^{k+1} \frac{(k!)^3}{(3k)!}\right)$ ABS. / COND. / DIVERGES.

Proof

(e) $\displaystyle\sum_{k=0}^{\infty}\left((-1)^k\left(\frac{k^2-2k+3}{7k^3+k-111}\right)\right)$ Abs. / Cond. / Diverges.

Proof

(f) $\displaystyle\sum_{n=1}^{\infty}\left(-\frac{1}{e}\right)^n$ Abs. / Cond. / Diverges.

Proof

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\ln(n)}$ ABS. / COND. / DIVERGES.

Proof

(h) $\displaystyle\sum_{k=0}^{\infty} \left(-\frac{2k+\cos((k+1)\pi)}{k+1}\right)^{k}$ ABS. / COND. / DIVERGES.

Proof

18 Convergence Test Review

1. Give an example of a series that satisfies the given criteria.
   (a) A series that is absolutely convergent.

   (b) A series that is conditionally convergent.

   (c) A series that is divergent, but the limit of the summand goes to zero.

   (d) An alternating series that DOES NOT contain $(-1)^k$. [Hint: think about your trigonometric functions.]

2. Explain why the series $\displaystyle\sum_{k=1}^{\infty} \frac{\sqrt[k]{k}}{k}$ is divergent.

3. Indicate if the given series converges or diverges by circling your choice. You must provide proof of your claim by correctly using one of the series tests.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{3^k}$ Converges / Diverges.

Proof

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k}$ Converges / Diverges.

Proof

(c) $\displaystyle\sum_{k=0}^{\infty} \frac{1}{2^k + \sin(k)}$ Converges / Diverges.

Proof

(d) $\displaystyle\sum_{k=0}^{\infty} \frac{1 \cdot 4 \cdot 7 \cdots (3k+1)}{100^k}$ CONVERGES / DIVERGES.

Proof

(e) $\displaystyle\sum_{k=1}^{\infty} \left(-2ke^{-k^2}\right)$ CONVERGES / DIVERGES.

Proof

(f) $\displaystyle\sum_{k=1}^{\infty} \frac{(k+1)^k}{(2k)^k}$ CONVERGES / DIVERGES.

Proof

4. Indicate if the given series converges absolutely, converges conditionally, or diverges. Prove your claim.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{(-2)^k}{1+3^k}$ ABSOLUTELY / CONDITIONALLY / DIVERGES.

Proof

(b) $\displaystyle\sum_{k=1}^{\infty} \left((-1)^k \frac{\sqrt{k}}{3k-1} \right)$ ABSOLUTELY / CONDITIONALLY / DIVERGES.

Proof