PHY321 MARCH 15

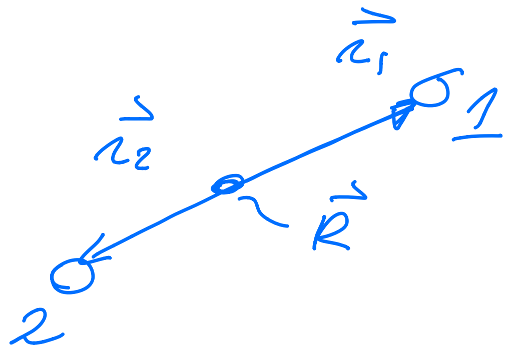
- Transformation of variables
  $\vec{r}_1, \vec{r}_2 \Rightarrow \vec{R}, \vec{r}$
- In 3-dim, we have in cartesian coordinates
  6 degrees of freedom
  $x_1, y_1, z_1 \wedge x_2\ y_2\ z_2 \Rightarrow$
  (hw 5) 6 coupled first-order diff. eq.
  $\frac{dx_1}{dt}$ $\frac{dv_{x_1}}{dt}$, $\frac{dy_1}{dt}$, $\frac{dv_{y_1}}{dt}$ ....,
- in the $\vec{R}$ and $\vec{r}$ system, we have only two equations
  $\Rightarrow$ they can be solved analytically ?
- Derive elliptical orbits
- Kepler's laws

.... much more.

Technicalities

CoM : $\vec{R} = \dfrac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$

$= \dfrac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{M}$



$\vec{r} = \vec{r}_1 - \vec{r}_2$

$\vec{r}_1 = \vec{r} + \vec{r}_2$

$M\vec{R} = m_1(\vec{r} + \vec{r}_2) + m_2 \vec{r}_2$

$$\boxed{\begin{aligned} \vec{r}_2 &= \vec{R} - \frac{m_1 \vec{r}}{M} \\ \vec{r}_1 &= \vec{R} + \frac{m_2 \vec{r}}{M} \end{aligned}}$$

$m_1 = m_2 \Rightarrow$ $\vec{r}_2 = R - \dfrac{\vec{r}}{2}$

$\vec{r}_1 = R + \vec{r}/2$

$$K = \frac{1}{2} m_1 \left[\frac{d\vec{r}_1}{dt}\right]^2 + \frac{1}{2} m_2 \left[\frac{d\vec{r}_2}{dt}\right]^2$$

$$\frac{d\vec{r}_1}{dt} = \frac{d\vec{R}}{dt} + \frac{m_2}{M}\frac{d\vec{r}}{dt}$$

$$\frac{d\vec{r}_2}{dt} = \frac{d\vec{R}}{dt} - \frac{m_1}{M}\frac{d\vec{r}}{dt}$$

$$\frac{1}{2} m_1 \left[\dot{\vec{R}}^2 + \frac{m_2^2}{M^2}\dot{\vec{r}}^2 + \cancel{2\frac{m_2}{M}\dot{\vec{r}}\dot{\vec{R}}} \right]$$

$$\left(\frac{d\vec{R}}{dt} = \dot{\vec{R}} \;,\; \frac{d\vec{r}}{dt} = \dot{\vec{r}} \right)$$

$$+ \frac{1}{2} m_2 \left[\dot{\vec{R}}^2 + \frac{m_1^2}{M^2}\dot{\vec{r}}^2 - \cancel{2\frac{m_1}{M}\dot{\vec{r}}\dot{\vec{R}}} \right]$$

$$m_1 + m_2 = M$$

$$= \frac{1}{2} M \dot{\vec{R}}^2 + \frac{1}{2}\frac{m_1 m_2}{M}\dot{\vec{r}}^2$$

$$\mu = \frac{m_1 m_2}{M}$$

$$\boxed{K = \frac{1}{2} M \dot{\vec{R}}^2 + \frac{1}{2}\mu \dot{\vec{r}}^2}$$

$\vec{P}$ = CoM momentum (HW4)

$$= M \cdot \dot{\vec{R}} = M\frac{d\vec{R}}{dt}$$

$$\frac{d\vec{P}}{dt} = M \cdot \ddot{\vec{R}} = M\frac{d^2\vec{R}}{dt^2}$$

$\overline{dt}$ $\qquad\qquad\qquad dt^2$

$= M \cdot \vec{a}_{\vec{R}}$

Equation of motion in the CoM - direction,

$\vec{F} = ?$

$$\boxed{F(\vec{r}) = \underbrace{f(\vec{r})}_{\text{scalar}}\, \vec{r}}$$

$\frac{\alpha}{r^3}\vec{r}$

$\mu \frac{d\vec{r}}{dt} = \mu \vec{p} \Rightarrow$

$$\boxed{\mu \frac{d^2\vec{r}}{dt^2} = \mu \vec{a}_{\vec{r}} = f(\vec{r}) \cdot \vec{r}}$$

$$\boxed{M \frac{d^2 R}{dt^2} = 0}$$

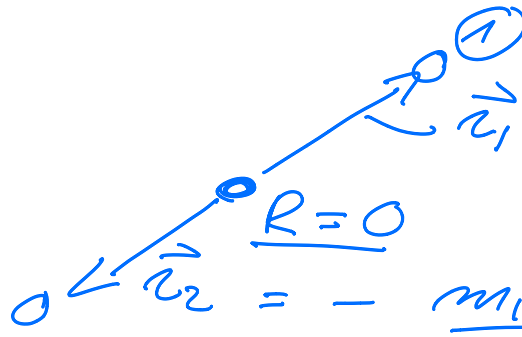
what does it mean?

Hint: conservation of ---- CoM momentum

$\vec{P} = M \frac{d\vec{R}}{dt}$ is conserved

$\vec{P} = \text{constant.}$

$\vec{R} = 0$ COM inertial frame.



$\vec{r}_1 = \frac{m_2 \vec{r}}{M}$

$\vec{r}_2 = -\frac{m_1 \vec{r}}{M}$

– Conservation of angular momentum (HW4)

$$\vec{l} = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2$$

$$\vec{p}_1 = m_1 \frac{d\vec{r}_1}{dt} \wedge \vec{p}_2 = m_2 \frac{d\vec{r}_2}{dt}$$

$$\vec{r}_1 \times \vec{p}_1 = m_1 \underbrace{\frac{m_2}{M} \vec{r} \times \frac{m_2}{M}}_{\frac{m_1 m_2^2}{M}} \frac{d\vec{r}}{dt}$$

$$\vec{r}_1 = \frac{m_2}{M} \vec{r}$$

$$\vec{r}_2 \times \vec{p}_2 = \frac{m_1^2 m_2}{M^2} \vec{r} \times \frac{d\vec{r}}{dt}$$

$$\vec{r}_2 = -\frac{m_1}{M} \vec{r}$$

$$\underline{m_1^2 m_2}$$

m^2

$$\Rightarrow \vec{\ell} = \vec{r} \times \mu \frac{d\vec{r}}{dt}$$

$$= \boxed{\mu \left(\vec{r} \times \frac{d\vec{r}}{dt} \right)} \qquad \vec{R} = 0$$

$$\ell = \underbrace{\vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2}_{\text{two-body angular momentum}}$$

effective angular momentum which acts like a 1-body object.

$$\frac{d\vec{\ell}}{dt} = \mu \left(\frac{d\vec{r}}{dt} \times \frac{d\vec{r}}{dt} \right) \equiv 0$$

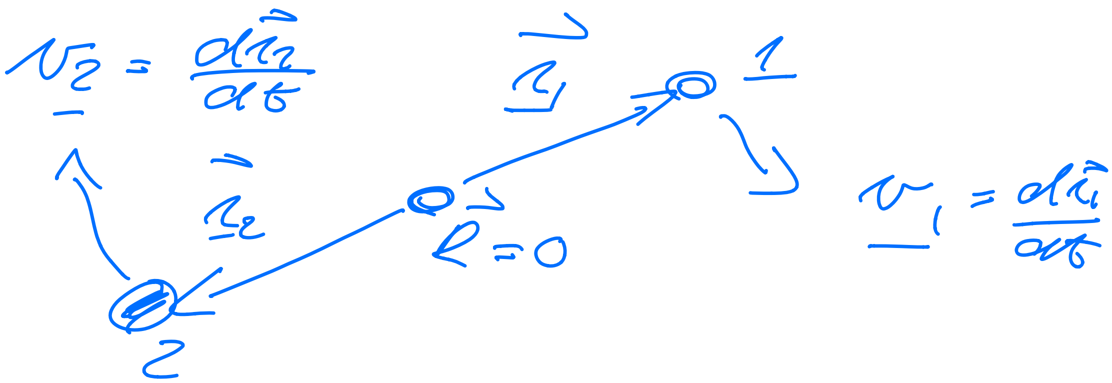
$$+ \mu \left(\vec{r} \times \frac{d^2\vec{r}}{dt^2} \right)$$

$$= \mu \left(\vec{r} \times \frac{d^2\vec{r}}{dt^2} \right)$$

$$\mu \frac{d^2 \vec{r}}{dt^2} = \underbrace{f(r)}_{\text{scalar}} \vec{r}$$

$$= 0$$

Ang mom is conserved?



$$\vec{\ell} = \text{constant.}$$