PHY 321 MARCH 5

$$m \frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$\omega_0 = \sqrt{k/m} \qquad \gamma = \frac{b}{2m\omega_0}$$

$$\tau = t \cdot \omega_0$$

$$\frac{d^2x}{d\tau^2} + 2\gamma \frac{dx}{d\tau} + x = 0$$

Homogenous solution

$$x(\tau) = c_1 e^{-\gamma\tau} e^{\tau\sqrt{\gamma^2-1}} + c_2 e^{-\gamma\tau} e^{-\tau\sqrt{\gamma^2-1}}$$

(i) underdamping ; $\gamma < 1$

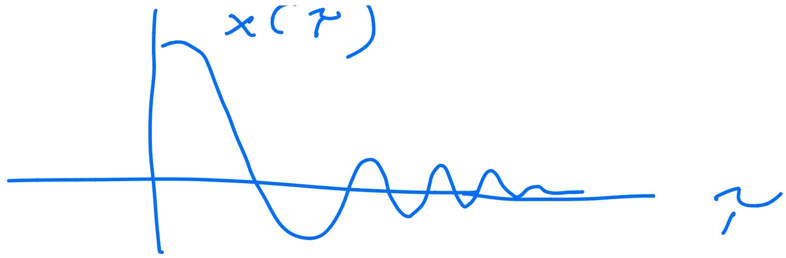
$$x(\tau) = A_1 e^{-\gamma\tau}\cos(\gamma'\tau) + A_2 e^{-\gamma\tau}\sin(\gamma'\tau)$$

$$\gamma' = \sqrt{\gamma^2-1} \qquad A_1 = c_1 + c_2$$

$$A_2 = i(c_1 - c_2)$$

Damping which decays as $e^{-\gamma\tau}$

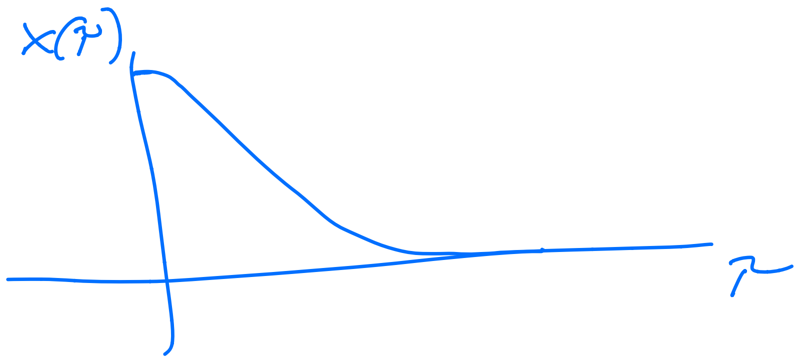
oscillating motion



A_1 and A_2 are arbitrary defined by initial conditions x_0, v_0

(ii) critical damping $\gamma = 1$

$$x(\tau) = A_1 \underline{e^{-\gamma\tau}} + A_2 \underline{\tau e^{-\gamma\tau}}$$



(iii) overdamping $\gamma > 1$

$$x(\tau) = A_1 e^{-(\gamma - \sqrt{\gamma^2 - 1})\tau} + A_2 e^{-(\gamma + \sqrt{\gamma^2 - 1})\tau}$$

Motion is damped by two exponentials (decaying) and is dominated by the fastest decaying one.

— Full model —

Add external Force

$$F^{ext}(t) = F_0 \cos(\omega t)$$

$$F^{total}(x, v, t) =$$

$$- kx - b \underbrace{\frac{dx}{dt}}_{v} + F_0 \cos(\omega t)$$

$$= m a_x = m \frac{d^2x}{dt^2}$$

$$\cancel{m} \frac{d^2x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{k}{m} x = \frac{F_0 \cos(\omega t)}{m}$$

(Mathematical pendulum:

$$\cancel{m} \frac{d^2\theta}{dt^2} + \frac{b}{m} \frac{d\theta}{dt} + mg \sin\theta$$
$$= F_0 \cos(\omega t))$$

scaling the equations:

$$\omega_0 = \sqrt{k/m} \qquad \underline{\tau = t \cdot \omega_0}$$

$$\omega = \omega_0 \tilde{\omega} \Rightarrow \tilde{\omega} = \frac{\omega}{\omega_0}$$

$$\omega_0$$

$$\cancel{\omega_0^2}\,\frac{d^2x}{d\tau^2} + \frac{b\cdot\omega_0}{\omega_0 m}\frac{dx}{d\tau} + \cancel{\omega_0^2}\,x$$

$$= \frac{F_0}{m\omega_0^2}\cos(\tilde{\omega}\tau)$$

$$\omega\cdot t = \frac{\omega_0}{\omega_0}\,\omega\cdot t$$

$$= \tilde{\omega}\tilde{\tau}$$

$$\tilde{F}_0 = \frac{F_0}{m\omega_0^2} \qquad 2\gamma = \frac{b}{2m\omega_0}$$

$$\Rightarrow$$

$$\frac{d^2x}{d\tau^2} + 2\gamma\frac{dx}{d\tau} + x = \tilde{F}_0\cos(\tilde{\omega}\tau)$$

$$x(\tau) = \underbrace{x_h(\tau)}_{\text{homogeneous solution}} + \underbrace{x_p(\tau)}_{\text{particular solution}}$$

$$A_1 e^{-\gamma\tau} e^{\tau\sqrt{\gamma^2-1}} + A_2 e^{-\gamma\tau} e^{-\tau\sqrt{\gamma^2-1}}$$

$$\underline{D}\cos(\tilde{\omega}\tau - \delta)$$

is defined by the parameter ω_0, γ and ω

$$D = \frac{\tilde{F}_0}{\sqrt{(1-\tilde{\omega}^2)^2 + 4\tilde{\omega}^2\gamma^2}}$$

$$\tan\delta = \frac{2\gamma\omega^2}{1-\tilde{\omega}^2}$$

$$\tilde{\omega} = \frac{\omega}{\omega_0} = 1 \Rightarrow$$

D has its max value

$$D = \frac{F_0}{2\gamma} \Rightarrow$$

Resonance