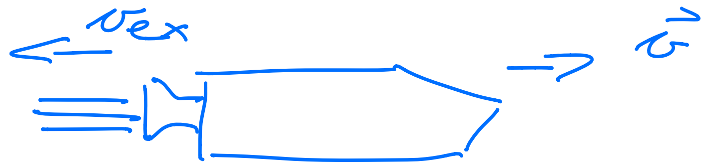
PHY 321 FEBRUARY 19

Exercise 2



conservation of linear momentum

$$M \cdot \vec{v} = (M - \Delta M)(\vec{v} + \Delta \vec{v}) + \Delta M(\vec{v} - \vec{v}_{ex})$$

$$M \cdot v = (M - \Delta M)(v + \Delta v) + \Delta M(v - v_{ex})$$

$$\Delta M \Delta v$$

$$0 = M \Delta v - \Delta M v_{ex} \Rightarrow$$

$$\frac{\Delta v}{\Delta t} = v_{ex} \frac{\Delta M}{M \Delta t} \rightarrow$$

$$\boxed{\frac{dv}{dt} = \frac{v_{ex}}{M} \frac{dM}{dt}}$$

$$dv = v_{ex} \frac{dM}{M}$$

$$\int_{v_0}^{v} dv' = v_{ex} \int_{M_0}^{M} \frac{dM'}{M'}$$

$$v_0 = 0$$

$$v = v_{ex} \ln M/M_0$$

3.11 (Ex2)

change in P in time dt

$$dP = m \cdot dv + dm\, v_{ex}$$

$$\frac{dP}{dt} = F^{ext} \Rightarrow dp = F^{ext} dt$$

$$F^{ext} dt = m \cdot dv + dm\, v_{ex}$$

$$F^{ext} = -\underline{mg}$$

$$-mg = m \cdot \frac{dv}{dt} + \frac{dm}{dt} v_{ex}$$

$$\frac{dm}{dt} = -k$$

$$\boxed{m \cdot \frac{dv}{dt} = k v_{ex} - mg}$$

$$dv = \left(\frac{k v_{ex}}{m} - g \right) dt$$

$$m = m_0 - kt$$

$$\left| dv = \left(\frac{k v_{ex}}{m} - g \right) dt \right|$$

$m_0 - kt$

Separation of variables

$$\int_{v_0}^{v} dv' = v = \int_{t_0=0}^{t} \left(\frac{k v_{ex}}{m_0 - kt} - g \right) dt$$

$$= v_{ex} \ln \left(\frac{m_0}{\underbrace{m_0 - kt}_{m}} \right) - g \cdot t$$

$$v(t) = v_{ex} \ln \left(\frac{m_0}{m} \right) - g \cdot t$$

Ex 3 (3.13 + 3.14)

$$y(t) = \int_0^t v(t')\, dt'$$

$$= v_{ex} \int_0^t \left[\ln m_0 - \ln m(t') \right] dt' - \underbrace{\int_0^t g t' \, dt'}_{-\frac{1}{2} g t^2}$$

$v_{ex} (\ln m_0) \cdot t$

$v_{ex}(\dots)$

$$- v_{ex}\int_0^t \ln m(t')\,dt'$$

$$m(t) = m_0 - kt$$

$$dt' = -dm'/k$$

$$m' = m(t')$$

$$\int_{m_0}^{m} \ln m(t')\,dt' = + \frac{1}{k}\left[m_0 \ln m_0 - m \ln m\right]$$

$$y(t) = v_{ex}t - \frac{1}{2} g t^2 + \frac{v_{ex}}{k}\left(kt \ln m_0 - m_0 \ln m_0 + m \ln m\right)$$

<u>3.14</u>

Equation of motion:

$$ma = m\cdot\frac{dv}{dt} = kv_{ex} - b\cdot v$$

$$\frac{m \cdot dv}{k v_{ex} - b \cdot v} = dt$$

Separation of variables

$$\int_{v_0}^{v} \frac{dv' \, m}{k v_{ex} - b \cdot v'} = \int_{t_0}^{t} dt'$$

$$\frac{dm}{dt} = -K$$

replace dt by $-\frac{dm}{K}$

$$\int_{t_0}^{t} dt' = -\int_{m_0}^{m} \frac{dm'}{K}$$

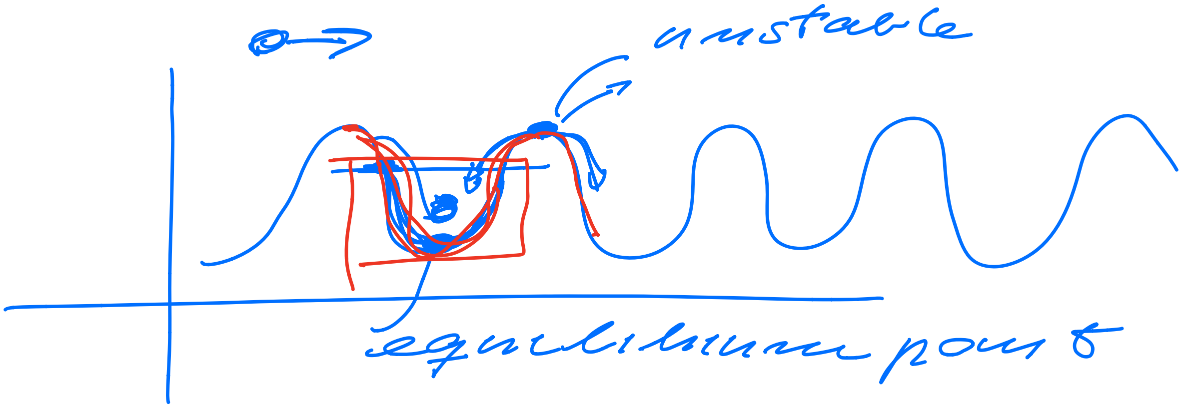
$$\int_{v_0}^{v} \frac{dv'}{k v_{ex} - b v'} = -\frac{1}{K} \int_{m_0}^{m} \frac{dm'}{m'}$$

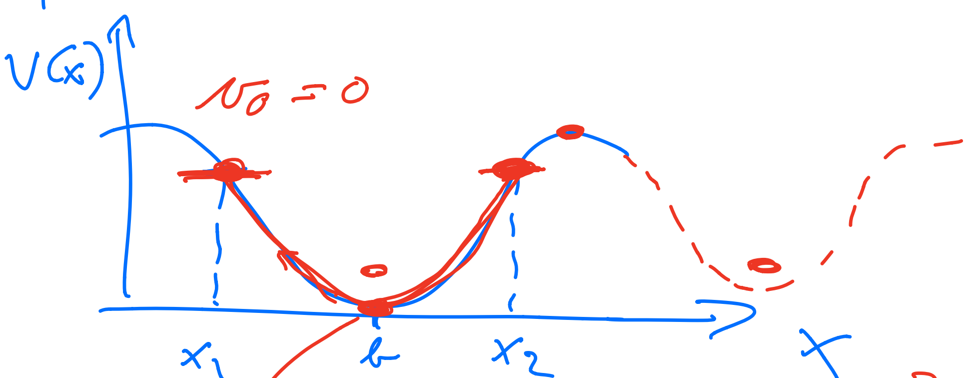
$$\frac{K}{b} \ln\left(\frac{k v_{ex} - b v}{k v_{ex}}\right) = \ln\left(\frac{m}{m_0}\right)$$

$\Rightarrow$

$$v = \frac{k v_{ex}}{b}\left[1 - \left(\frac{m}{m_0}\right)\right]$$

Separation of variables.





$$V(x) = \frac{1}{2}k(x-b)^2$$

$$E = \frac{1}{2}mv^2$$

$$E_0 = \frac{1}{2}k(x_2 - b)^2$$

$$K(x) = E_0 - V(x)$$

$$F(x) = -\frac{dV}{dx}$$

$$F(x) = -k(x - b)$$
$$x_2 > b$$