PHY 321 MARCH 26

$$\frac{d\phi}{dt} = \dot{\phi} = \frac{L}{\mu r^2}$$

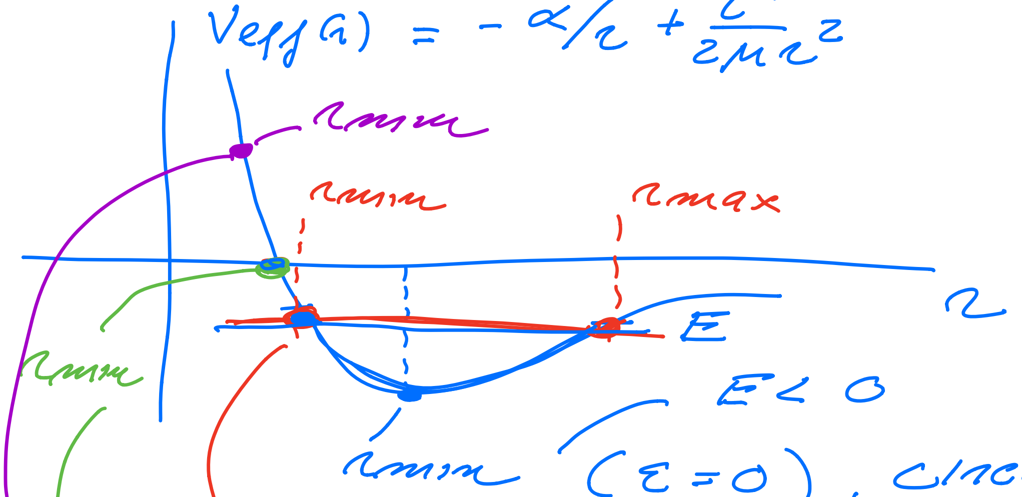
$$\mu \frac{d^2 r}{dt^2} = F(r) + \frac{L^2}{\mu r^3}$$

$$F(r) = -\alpha/r^2 \qquad \alpha = G m_1 m_2$$

$$r(\phi) = \frac{c}{1+\varepsilon\cos\phi}$$

$$c = \frac{L^2}{\alpha\mu} \qquad \varepsilon = \frac{AL^2}{\alpha\mu} \qquad \varepsilon \geq 0 \quad A \geq 0 \quad \alpha > 0$$

$$V_{eff}(r) = -\alpha/r + \frac{L^2}{2\mu r^2}$$



$(\varepsilon = 0)$, circular motion

$0 < \varepsilon < 1$, elliptical motion,

$E < 0$

$\varepsilon = 1$, parabola, $E = 0$

$\varepsilon > 1$, hyperbolic motion

$E > 0$

Energy consideration

$$E = \underbrace{\frac{1}{2}\mu \dot{r}^2 + \frac{L^2}{\mu r^2}}_{K} - \alpha/r$$

$E < 0$

$K \geq 0$

$$\frac{1}{2}\mu \dot{r}^2 + \frac{L^2}{\mu r^2} - \alpha/r < 0$$

$$\frac{1}{2}\mu \dot{r}^2 + \frac{L^2}{\mu r^2} < \alpha/r$$

at r_{min} $\quad \frac{dr}{dt} = 0$

$$r_{min} = \frac{c}{1+\varepsilon} \qquad r_{max} = \frac{c}{1-\varepsilon}$$

$$r(\phi) = \frac{c}{1+\varepsilon\cos\phi}$$

$\varepsilon = 0 \Rightarrow$ circular motion.

$0 < \varepsilon < 1$

$r(\phi)$ as function of ε when $\varepsilon < 1$. Denominator does not vanish.

$$E(r_{min}) = V_{eff}(r_{min})$$

$$\left(\frac{1}{2}\mu \dot{r}^2 = 0\right)$$

$$= -\frac{\alpha}{r_{min}} + \frac{L^2}{2\mu r_{min}^2}$$

$$= \frac{1}{2 r_{min}}\left[\frac{L^2}{\mu r_{min}} - 2\alpha\right]$$

$$r_{min} = \frac{c}{1+\varepsilon} = \frac{L^2}{\alpha\mu(1+\varepsilon)}$$

$$c = L^2/\alpha\mu$$

$$E = \frac{\alpha\mu(1+\varepsilon)}{2L^2}\left[\alpha(1+\varepsilon) - 2\alpha\right]$$

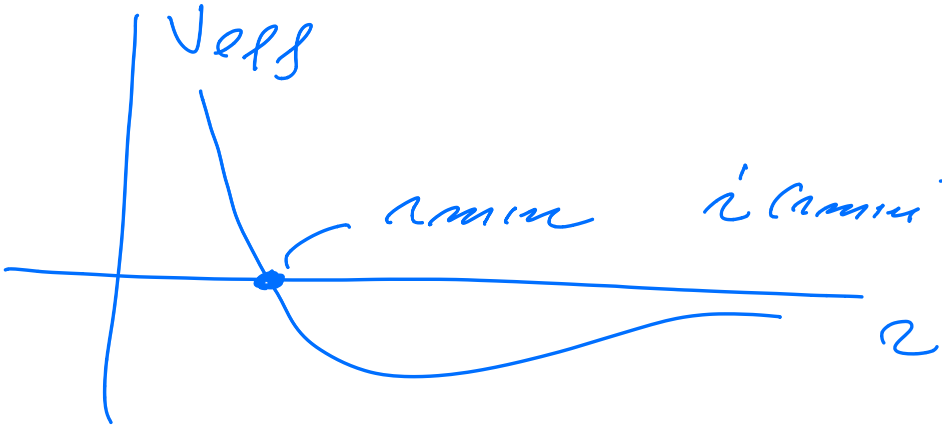
$$= \frac{\alpha^2\mu}{2L^2}\left[\varepsilon^2 - 1\right] \quad \alpha > 0$$

$$\boxed{E < 0}$$

$$\varepsilon^2 - 1 > 0 \Rightarrow$$

$$(\varepsilon - 1)(\varepsilon + 1) > 0 \quad \Rightarrow$$

$$\varepsilon < 1 \quad \wedge \quad E < 0$$



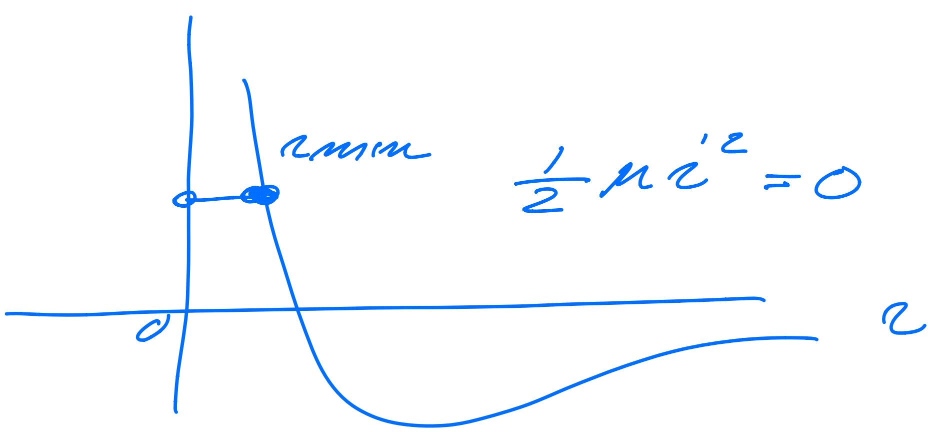
$$\dot{r}(r_{min}) = 0$$

$$V_{eff}(r_{min}) = 0 \qquad \frac{1}{2}\mu \dot{r}^2 = 0$$

$$E = \frac{L^2}{\mu r^2} - \alpha/r = 0 \quad \Rightarrow$$

$$\varepsilon^2 - 1 = 0 \quad \Rightarrow \quad \boxed{\varepsilon = +1}$$

$E > 0$, r_{min}



$$\frac{1}{2}\mu \dot{r}^2 = 0$$

$V_{eff}(r_{min}) > 0 \Rightarrow$

$\varepsilon^2 - 1 > 0 \Rightarrow$

$\varepsilon > \underline{1}$

<u>what kind of orbits</u>

$\varepsilon = 0$, circle

$0 < \varepsilon < \underline{1}$:

$$r(\underline{\phi}) = \frac{c}{1+\varepsilon\cos\phi}$$

$$\underline{r}(1+\varepsilon\underline{\cos}\phi) = c$$

$x = r \cdot \cos\phi \quad \wedge \quad y = r\sin\phi$

$r + \varepsilon x = c$

$\underline{r = c - \varepsilon x}$ square both sides

$r = \sqrt{x^2+y^2}$

$$\underline{r^2} = c^2 + \varepsilon^2 x^2 - 2x\varepsilon \cdot c = \underline{x^2+y^2}$$

$x^2(1-\varepsilon^2) + 2c\varepsilon x + y^2 = c^2$

$$x(1-\varepsilon)+2c\varepsilon x+y=c$$

Divide by $1-\varepsilon^2$

Define $(x+d)^2$ $\quad \boxed{d = \frac{c\varepsilon}{1-\varepsilon^2}}$

$$\underbrace{x^2+2\cdot dx}_{\text{complete the square}} + \frac{y^2}{1-\varepsilon^2} = \frac{c^2}{1-\varepsilon^2}$$

$$d^2-d^2$$

$$(x+d)^2 + \frac{y^2}{1-\varepsilon^2} = \frac{c^2}{1-\varepsilon^2} + d^2$$

$$= \frac{c^2}{1-\varepsilon^2}\left[1 + \frac{\varepsilon^2}{1-\varepsilon^2}\right]$$

$$= \frac{c^2}{(1-\varepsilon^2)^2} = a^2$$

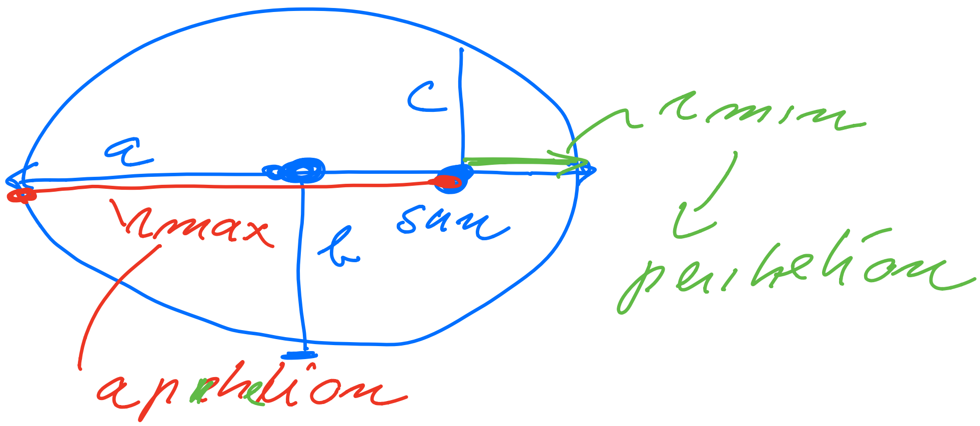
divide by a^2 $\qquad d = a\varepsilon$

$$\frac{(x+d)^2}{a^2} + \frac{y^2}{a^2(1-\varepsilon^2)} = 1$$

Def $b = a\sqrt{(1-\varepsilon^2)}$

$$\frac{(x+d)^2}{a^2} + \frac{y^2}{b^2} = 1$$

Ellipse!



Earth-Sun

aphelion $\simeq$ perihelion.

$$r_{min} = \frac{c}{1+\varepsilon} \qquad r_{max} = \frac{c}{1-\varepsilon}$$

$E = 0 \qquad \varepsilon = 1$

$$\underline{r = c - x}$$

$$r(\phi) = \frac{c}{1+\cos\phi}$$

$\phi = \pm\pi$ then r is

undefined.

$$r^2 = x^2 + y^2 = (c-x)^2 \Rightarrow$$

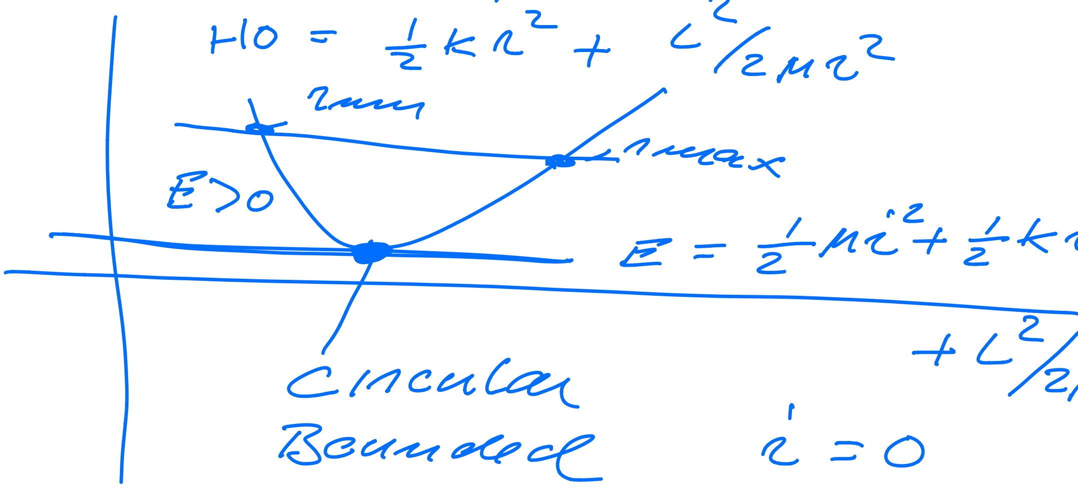
$$c^2 - 2cx + x^2 = x^2 + y^2 \Rightarrow$$

$$y^2 = c^2 - 2cx$$

parabola?

$E > 0, \quad \varepsilon > 1$

$HO = \frac{1}{2} k r^2 + L^2/2\mu r^2$



$$E = \frac{1}{2}\mu \dot{r}^2 + \frac{1}{2} k r^2 + L^2/2\mu r^2$$

$\dot{r} = 0$

$E = V_{eff}$