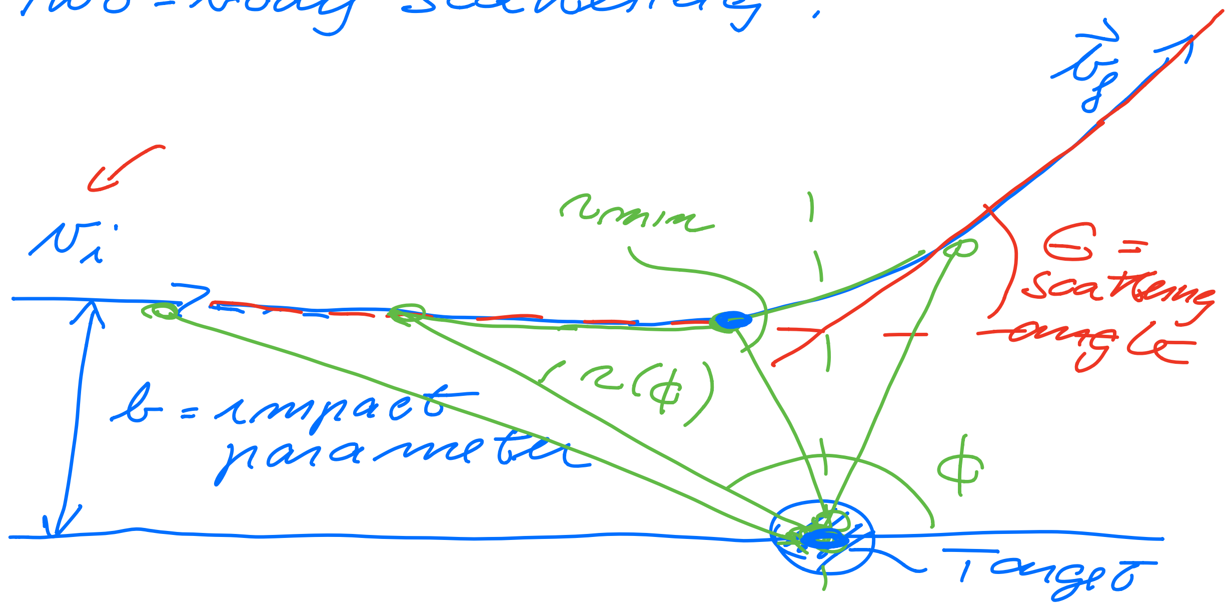
PHY 321 APRIL 5

Two-body scattering:



Expt: Θ, cross section

$$r(\phi) = \frac{c}{1+\varepsilon\cos\phi}$$

ϕ_m = projectile infinitely far away from target

$$\Theta = |\pi - 2\phi_m|$$ Taylor chapter 14.6

Number of scattered = $N_{scatter}$

$N_{incoming}$

$\underbrace{\quad\quad}_{N_{inc}} \; \rho_t \; \sigma$

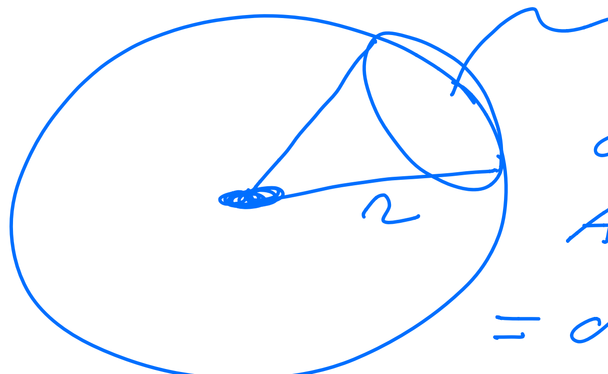
ρ_t: density of target

σ: cross section.

- symmetric (central) potential, independent of angles θ, φ

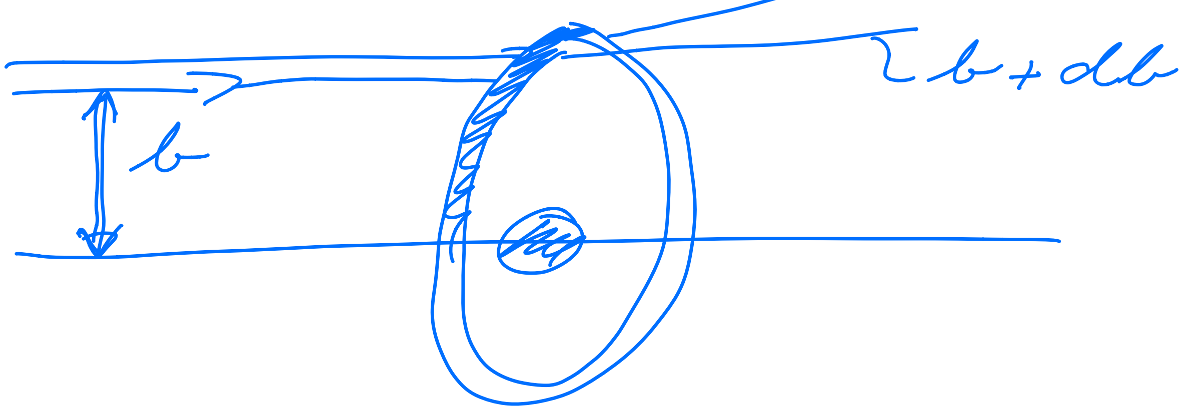
- solid angle

sphere



$d\Omega = A/r^2 = d\phi \sin\theta d\theta$

solid angle



$d\sigma = 2\pi \cdot b \cdot db$

$$\boxed{\frac{d\sigma}{d\Omega}} = \frac{2\pi \cdot b\,db}{\underbrace{d\phi}_{2\pi}\sin\theta\,d\theta}$$

differential cross section

$$= \frac{b\cdot db}{\sin\theta\,d\theta}$$

$$d\Omega = \underbrace{d\phi}\ \overline{\sin\theta\,d\phi}$$

$$\int_0^{2\pi} \underline{d\phi} \int_0^{\pi} \sin\theta\,d\phi$$

$$= 2\pi \int_0^{\pi} \sin\theta\,d\phi$$

$$\sigma = \int \frac{d\sigma}{d\Omega}\,d\Omega =$$

$$\int_0^{\pi} \sin\theta\,d\theta \int_0^{2\pi} d\phi\,\underline{\frac{d\sigma}{d\Omega}(\theta,\phi)}$$

$$\frac{d\sigma}{d\Omega}(\theta)$$

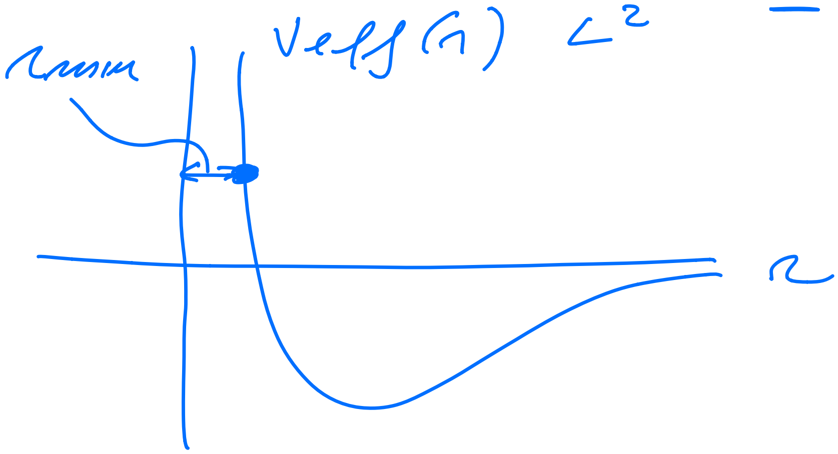
How do we relate

$\frac{d\sigma}{d\Omega}$ with $r(\phi)$?

dr

HW 9, ex 1:

$$r(\phi) = \frac{1}{\frac{\mu\alpha}{L^2} + A\cos\phi} \qquad V(r) = -\alpha/r$$



at r_{min} = no radial velocity

$$E = -\frac{\alpha}{r_{min}} + \frac{1}{2}\mu(r\dot{\phi})^2$$

$$\dot{\phi} = \frac{L}{\mu r_{min}^2}$$

$$E = -\frac{\alpha}{r_{min}} + \frac{L^2}{2\mu r_{min}^2}$$

$$\frac{2\mu}{L^2}E + \frac{2\alpha\mu}{L^2 r_{min}} = \frac{1}{r_{min}^2}$$

$$\frac{1}{r_{min}} = \frac{\mu\alpha}{L^2} + \sqrt{\left(\frac{\mu\alpha}{L^2}\right)^2 + \frac{2\mu E}{L^2}}$$

closest approach

$\phi = 0 \qquad \cos\phi = \underline{1}$

$$r_{min} = \frac{c}{1+\varepsilon} \Rightarrow$$

$$\boxed{\frac{1}{r_{min}} = \frac{\mu\alpha}{L^2} + A} \Rightarrow$$

$$A = \sqrt{\left(\frac{\mu\alpha}{L^2}\right)^2 + \frac{2\mu E}{L^2}}$$

Can we relate this to parameter - b - and θ ?

$$L = \mu r^2 \underline{\dot{\phi}}^2 = \mu r v_\phi$$

$$= \mu \cdot b \cdot v_\phi$$

$$L^2 = \mu^2 b^2 v_\phi^2 = \underline{\mu^2 b^2 v^2}$$

$$E = \frac{1}{2}\mu v^2$$

$$L^2 = 2\mu b^2 E \Rightarrow$$

$$b^2 = \frac{L^2}{2\mu E}$$

$$A = \sqrt{\left(\frac{\mu\alpha}{L^2}\right)^2 + \frac{1}{b^2}}$$

$$\boxed{r(\phi) = \frac{1}{\frac{\mu\alpha}{L^2} + A\cos\phi}}$$

angle when $r(\phi)$ is infinitely far away ($r = \pm\infty$) $\Rightarrow$

$\phi_m = ?$ $\qquad \frac{\theta}{2} = \left|\frac{\pi}{2} - \phi_m\right|$

$$-\frac{\mu\alpha}{L^2 A} = \cos\phi_m \Rightarrow$$

$$\boxed{\sin\frac{\theta}{2} = \frac{\mu\alpha}{AL^2}}$$

$$\boxed{\frac{d\sigma}{\;}} = \frac{b \cdot db}{\;}$$

$$\boxed{d\Omega} \qquad \underbrace{\sin\theta\, d\theta}_{d(\cos\theta)}$$

$$\frac{\mu\alpha}{L^2\sqrt{\left(\frac{\mu\alpha}{L^2}\right)^2+\frac{1}{b^2}}}$$

$$b^2 = \frac{L^2}{2\mu E}$$

$$a = \frac{\alpha}{2\cancel{\mu}E} = \frac{\mu\alpha}{L^2 b^2}$$

$$\sin\frac{\theta}{2} = \frac{a}{\sqrt{a^2+b^2}}$$

$$\frac{1}{2}\cos\frac{\theta}{2}\, d\theta = \frac{a\, b\, db}{(a^2+b^2)^{3/2}}$$

$$= \frac{a^3}{a^2}\,\frac{b\, db}{(a^2+b^2)^{3/2}}$$

$$\left(\sin\frac{\theta}{2} = \frac{a}{(a^2+b^2)^{1/2}}\right)$$

$$\frac{1}{2}\cos\frac{\theta}{2}\, d\theta = \frac{\sin^3\theta/2\; b\, db}{a^2}$$

$$a^{\sim}$$

$$\underline{b\,db} = \frac{a^2}{2}\frac{1}{\sin^3\theta/2}\cos\frac{\theta}{2}\,d\theta$$

$$d\sigma = 2\pi\,\underline{b\,db}$$

$$= \frac{\pi a^2}{\sin^3\theta/2}\cos\frac{\theta}{2}\,d\theta$$

$$= \frac{\pi a^2}{2\sin^4\theta/2}\,\underline{\sin\theta\,d\theta}$$

$$\frac{d\sigma}{\sin\theta\,d\theta} = \frac{d\sigma}{d\Omega} = \underline{\frac{a^2}{4\sin^4\theta/2}}$$

$$\sigma = \int_0^{\pi}\frac{d\sigma}{d\Omega}\sin\theta\,d\theta$$

$$= \int_0^{\pi}\frac{a^2}{4\sin^4\theta/2}\sin\theta\,d\theta$$

Rutherford classical cross section.