PHY 321 MARCH 29

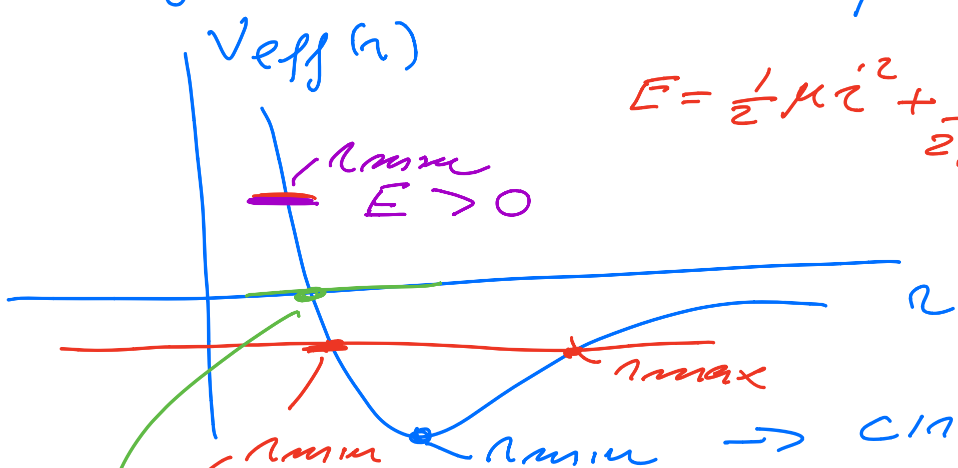
$$r(\phi) = \frac{c}{1+\varepsilon\cos\phi}$$

$$u = \frac{1}{r}$$

$$\boxed{\frac{d^2u}{d\phi^2} = -u - \frac{\mu F}{L^2u^2}}$$

$$F = -\alpha/r^2$$

$$V_{eff}(r) = V(r) + \frac{L^2}{2\mu r^2}$$



$$E = \frac{1}{2}\mu\dot{r}^2 + \frac{L^2}{2\mu r^2} + V(r)$$

$\to$ circular motion

$\varepsilon = 0$

$E < 0$

$E < 0$, ellipse

$0 < \varepsilon < 1$

$E = 0$

$\varepsilon = 1$, parabola

$E > 0$

$\varepsilon > \underline{1}$, hyperbolic

$$\underset{(i=0)}{E} = \frac{\alpha^2\mu}{2L^2}\left(\varepsilon^2 - 1\right) \quad \alpha > 0$$

$$r = \frac{c}{1+\varepsilon\cos\phi} \quad \left(F = -\alpha/r^2\right.$$

$$x = r\cos\phi \qquad y = r\sin\phi$$

$$r^2 = x^2 + y^2$$

$$r\left(1+\varepsilon\cos\phi\right) = c$$

$$= r + \varepsilon x = c$$

hyperbolic $\varepsilon > \underline{1}$

square

$$\underline{r}^2 = c^2 + \varepsilon^2 x^2 - 2c\varepsilon x = x^2 + y^2$$

$$x^2\left(\varepsilon^2 - 1\right) - y^2 - 2c\varepsilon x = -c^2$$

complete the square for x

$$\frac{\varepsilon^2 c^2}{\varepsilon^2 - 1} \qquad \delta = c\frac{\varepsilon}{\varepsilon^2 - 1}$$

$$\varepsilon^2 - 1$$

$$(\varepsilon^2-1)(x-\delta)^2 - y^2 = -\underline{c^2} + \frac{\varepsilon^2 c^2}{\varepsilon^2-1} \quad \Big| \; \frac{\varepsilon^2-1}{c^2}$$

$$\left(\underline{\alpha} = \frac{c}{\varepsilon^2-1} \qquad \underline{\beta} = \frac{c}{\sqrt{\varepsilon^2-1}} \right)$$

$$\frac{(x-\delta)^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$$

hyperbolic motion.

$E=0$	$\varepsilon = 1$	parabola
$E>0$	$\varepsilon > 1$	hyperbola
$E<0$	$0<\varepsilon<1$	ellipse
$E<0$	$\varepsilon = 0$	circle

(i) $\vec{r}_1, \vec{r}_2 \Rightarrow \vec{r}, \vec{R}$

(ii) ang mom and energy is conserved, $\vec{P} = M\vec{R}$ is conserved

(iii) COM (center of mass)

$$\vec{R} = 0$$

(iv) $$\dot{\phi} = \frac{L}{\mu r^2}$$

$$r(\phi) = \frac{c}{1+\varepsilon\cos\phi} \qquad F = -\alpha/r^2$$

Example 1

$$F = +\alpha/r^2 \qquad \alpha > 0$$

$$E = K + U = K + \alpha/r$$

$\boxed{E > 0}$? $\qquad$ $\cancel{E < 0}$?

No bound orbit, $\qquad u = \frac{1}{r}$

$$\frac{d^2u}{d\phi^2} = -u - \frac{\mu F}{L^2u^2}$$

$$F = \underline{\alpha/r^2} \implies$$

$$\frac{d^2u}{d\phi^2} = -\underline{u} - \frac{\mu\alpha}{L^2} \implies$$

$$r(\phi) = \frac{c}{\varepsilon\cos\phi - 1}$$

at r_{min} we set $\dot{r} = 0$

$$E = \frac{\alpha^2 \mu}{2 l^2}\left[\varepsilon^2 - 1\right]$$

$E > 0 \Rightarrow \underline{\varepsilon > 1} \Rightarrow$

$$\frac{(x-\delta)^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$$

hyperbolic motion.

Example 2

$F = k/r^3$ $\quad$ $k > 0$ or $k < 0$

$V = \dfrac{k}{2r^2}$ $\quad$ $F = -\dfrac{dV}{dr}$

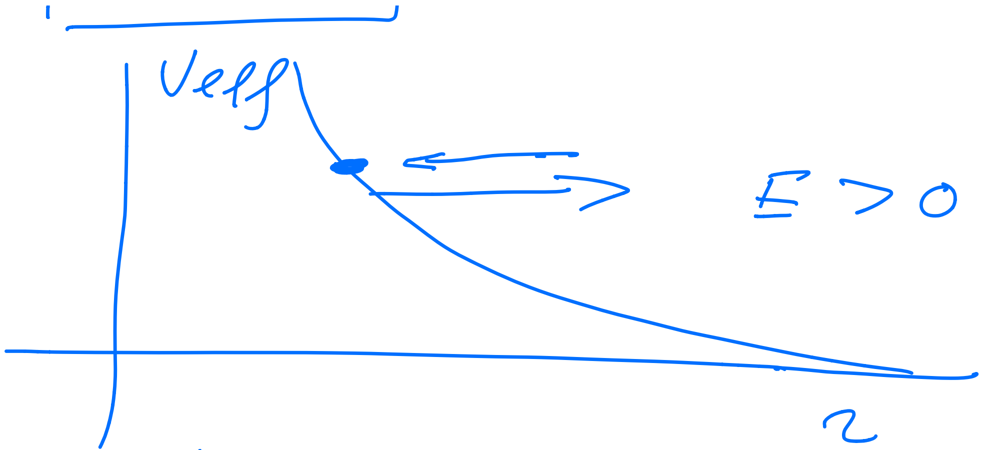
$$\underline{V_{eff}} = V(r) + \frac{l^2}{2\mu r^2}$$

$$= \frac{k + l^2/\mu}{2r^2}$$

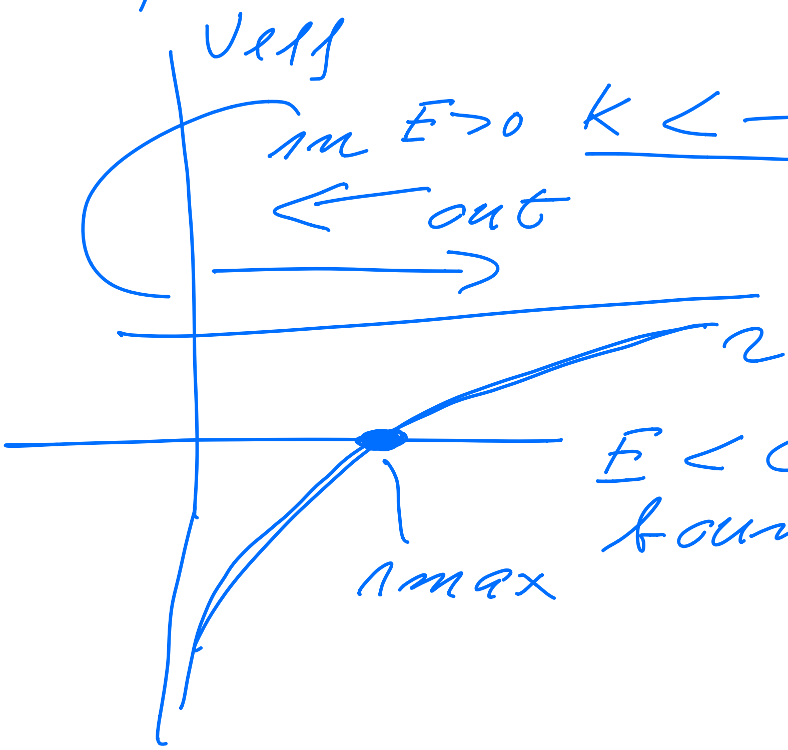
$k > -l^2/\mu$,

set at r_{min} $\dot{r} = 0$

$\boxed{E > 0}$? $\quad$ $E < 0$?



$E > 0$



$K < -L^2/m$

$E < 0$?

$E > 0$?

$E < 0$
bounded

EoM

$$\boxed{\frac{d^2u}{d\phi^2} = -u - \frac{F\mu}{L^2u^2}} \qquad \boxed{r = \frac{1}{u}}$$

$$F = K/r^3 = Ku^3$$

$$\frac{d^2u}{d\phi^2} = -\left(1 + \frac{K\mu}{L^2}\right)u$$

$d\phi^2$ $\qquad \underbrace{\qquad\qquad}_{\omega^2}$

$= - \omega^2 u$

$u = ?$ $\qquad \dfrac{d^2u}{d\phi^2} = -\omega^2 u$

$u = A\cos(\omega\phi - \delta)$

$$r(\phi) = \frac{1}{A\cos(\omega\phi - \delta)}$$

L is conserved.

$K > L^2/\mu$ then $1 + K\mu/L^2$ is always positive

The angle ϕ changes in one direction only

---

what could happen?

$\omega\phi - \delta = \pm \pi/2$ $\quad \cos = 0$

$u \Rightarrow 0$ and $r \Rightarrow \infty$

$K < -L^2/\mu \Rightarrow$

$$\frac{d^2u}{d\phi^2} = +\lambda^2 u$$

$u(\phi) = Ae^{\lambda\phi} + Be^{-\lambda\phi}$

$u(\phi) =$

$r(\phi)$ may or may not vanish, depends on A and B. if $E > 0$, then $r \to \infty$ at some value of ϕ.

if $u(\phi)$ remains bounded then the particle stay within some r_{max}

in this case $E < 0$ and bounded orbit.