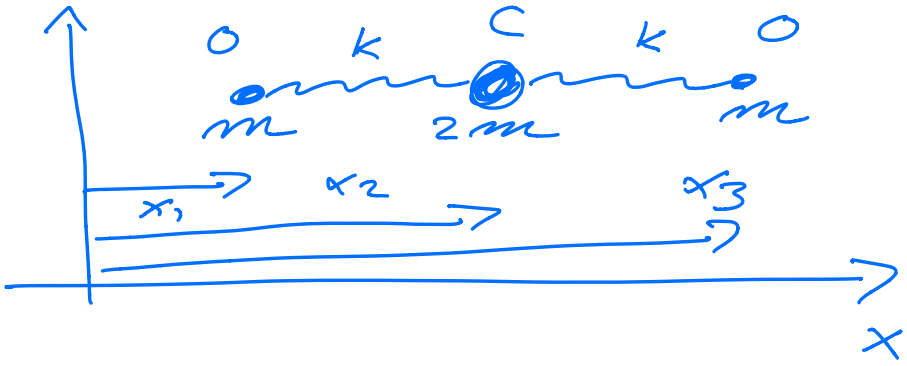
PHY 321 APRIL 21

Linear chain (CO_2)



$$\ell = 0$$

$$\mathcal{L} = \frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} m \dot{x}_2^2 + \frac{1}{2} m \dot{x}_3^2 - \frac{k}{2}(x_2 - x_1)^2 - \frac{k}{2}(x_3 - x_2)^2$$

$$m \ddot{x}_1 = -k(x_1 - x_2)$$

$$2m \ddot{x}_2 = -k(x_2 - x_1) - k(x_2 - x_3) = -k[2x_2 - x_1 - x_3]$$

$$m \ddot{x}_3 = -k[x_3 - x_2]$$

$$q_1 = x_1 - x_2$$

$$q_3 = x_3 - x_2$$

$$\vec{R} = \frac{\sum_i m_i \vec{r}_i}{\sum m_i}$$

$$X = \frac{x_1 m + 2m x_2 + x_3 m}{4m}$$

$$= \frac{x_1 + 2x_2 + x_3}{4}$$

$$x_2 = (4X - x_1 - x_3)/2$$

$$x_1 = (3q_1 - q_3 + 4X)/2$$

$$x_3 = (3q_3 - q_1 + 4X)/2$$

$$\mathcal{L} = K - V$$

$$V = \frac{k}{2}(q_1^2 + q_3^2)$$

$$K = \frac{3m}{8}(\dot{q}_1^2 + \dot{q}_3^2) - \frac{m\dot{q}_1\dot{q}_3}{4} + 2m\dot{X}^2$$

$$\frac{\partial \mathcal{L}}{\partial q_i} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} = 0$$

$$\frac{\partial \mathcal{L}}{\partial X} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{X}} = 0$$

$$\| \qquad d(4m\dot{X}) = 0$$

$0 = \frac{d}{dt}(\text{[illegible]}) = 0$

$$4m\ddot{X} = 0$$

$\dot{X}$ is a constant.

$$4m\dot{X} = P$$

Total linear momentum.

$$q_1: \quad -\frac{3}{4}m\ddot{q}_1 + \frac{m}{4}\ddot{q}_3 = -kq_1$$

$$q_3: \quad \frac{3}{4}m\ddot{q}_3 - \frac{1}{4}m\ddot{q}_1 = -kq_3$$

$$q_1 = Ae^{i\omega t}$$

$$q_3 = Be^{i\omega t}$$

set $B = 1$ $\quad A/B$

$$\omega_0^2 = k/m$$

$$-\frac{3}{4}\frac{A}{B}\omega^2 + \frac{1}{4}\omega^2 = -\omega_0^2 A/B$$

$$-\frac{3}{4}\omega^2 + \frac{1}{4}\frac{A}{B}\omega^2 = -\omega_0^2$$

$$A = \frac{\omega^2}{?}$$

B $3\omega - 4\omega_0$

$$\omega^4 - 3\omega_0^2\omega^2 + 2\omega_0^2 = 0$$

$$x^2 = \omega^4$$

(i) $\omega = \omega_0 \Rightarrow A = -B$

(ii) $\omega = \omega_0\sqrt{2} \Rightarrow A = B$