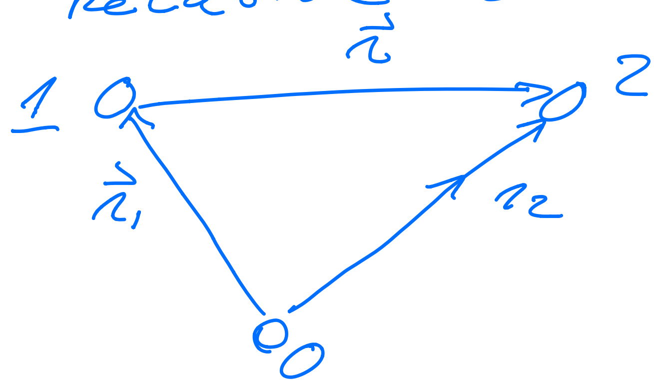
PHYS 321 MARCH 17

relative $\vec{r} = \vec{r}_1 - \vec{r}_2$



$$\vec{R} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{M}$$

$$M = m_1 + m_2$$

$$\ddot{\vec{R}} = \frac{d^2\vec{R}}{dt^2} = 0 \quad \Rightarrow$$

$$M\ddot{\vec{R}} = 0$$

$\vec{P} = M \cdot \dot{\vec{R}} \Rightarrow \vec{P}$ is conserved,

$$\vec{F}(\vec{r}) = -\vec{\nabla} V(\vec{r}) = -\vec{\nabla} V(r)$$

scalar

$$\mu = \frac{m_1 m_2}{M}$$

$$\mu \ddot{\vec{r}} = \vec{F}(\vec{r}) = -\vec{\nabla} V(r)$$

$$\vec{F}(\vec{r}) = f(r)\,\vec{r}$$

$$\vec{L} = \mu\left(\vec{r} \times \frac{d\vec{r}}{dt}\right)$$

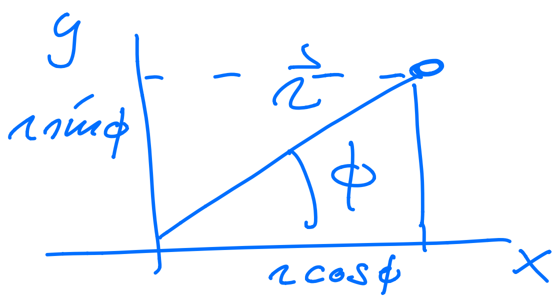
CoM-frame: $\boxed{\vec{R} = 0}$

$$\frac{d\vec{L}}{dt} = \mu\Big(\vec{r} \times \underbrace{\frac{d^2\vec{r}}{dt^2}}_{f(r)\cdot\vec{r}}\Big) = 0$$

$x \in (-\infty, \infty)$ $\quad$ $y \in (-\infty, \infty)$

$r \in [0, \infty)$ $\quad$ $\phi = [0, 2\pi]$

$x = r\cos\phi$ $\quad$ $y = r\sin\phi$



$r = \sqrt{x^2 + y^2}$

$v^2 = (\dot{x}^2 + \dot{y}^2)$

$\frac{dx}{dt} = v_x$

EoM
$$\begin{cases} \boxed{\dfrac{d\phi}{dt} = \dfrac{L}{\mu r^2}} \\ \boxed{\mu\ddot{r} = F_r + \dfrac{L^2}{\mu r^3}} \end{cases}$$

conserved = constant

<u>chain rule manipulations</u>

$\underline{d\vec{r}} = \,? = \dot{x}\vec{e}_1 + \dot{y}\vec{e}_2$

dt

$x:$ $\quad \dfrac{dx}{dt}\dfrac{dr}{dx} = \dfrac{\dot{x}\, x}{\sqrt{x^2+y^2}} = \dfrac{x}{r}\dot{x}$

$r = \sqrt{x^2+y^2}$ $\quad = \dfrac{x}{r}\dfrac{dx}{dt}$

$y:$ $\quad \dfrac{dy}{dt}\dfrac{dr}{dt} = \dfrac{y}{r}\dfrac{dy}{dt} = \dfrac{y}{r}\dot{y}$

$$a_r = \frac{d^2r}{dt^2} = \frac{x}{r}\frac{d^2x}{dt^2} + \frac{y}{r}\frac{d^2y}{dt^2} + \frac{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}{r} - \frac{\left[\frac{dr}{dt}\right]^2}{r}$$

$$= \frac{x}{r}\ddot{x} + \frac{y}{r}\ddot{y} + \frac{\dot{x}^2+\dot{y}^2}{r} - \frac{(\dot{r})^2}{r}$$

$x = r\cdot\cos\phi \qquad y = r\sin\phi$

$$\frac{dx}{dt} = \dot{x} = \underbrace{\frac{dr}{dt}}_{\dot{r}}\cos\phi - r\underbrace{\frac{d\phi}{dt}}_{\dot{\phi}}\sin\phi$$

$$\frac{dy}{dt} = \dot{y} = \frac{dr}{dt}\sin\phi +$$

$$r\frac{d\phi}{dt}\cos\phi$$

$$\left(\frac{dx}{dt}\right)^2 = \dot{r}^2\cos^2\phi + r^2\dot{\phi}^2\sin^2\phi - 2r\dot{\phi}\cos\phi\cdot\sin\phi$$

$$\left(\frac{dy}{dt}\right)^2 = \dot{r}^2\sin^2\phi + r^2\dot{\phi}^2\cos^2\phi + 2r\dot{\phi}\cos\phi\sin\phi$$

$$\dot{x}^2 + \dot{y}^2 = \dot{r}^2 + r^2\dot{\phi}^2$$

$$\frac{\dot{x}^2+\dot{y}^2}{r} - \frac{\dot{r}^2}{r} = r\cdot\dot{\phi}^2$$

$$\mu\ddot{r} = \overbrace{F_x\cdot\cos\phi + F_y\cdot\sin\phi}^{F_r} + \frac{\dot{x}^2+\dot{y}^2}{r} - \frac{\dot{r}^2}{r}$$

$$= F_r + \frac{r\dot{\phi}^2}{\frac{L^2}{\mu r^2}}$$

$$\vec{L} = \mu\left(\vec{r}\times\frac{d\vec{r}}{dt}\right)$$

$$\frac{d\vec{r}}{dt} = \left(\frac{x}{r}\frac{dx}{dt}\vec{e}_1 + \frac{y}{r}\frac{dy}{dt}\vec{e}_2\right)$$

$$\vec{r} = \underline{x\vec{e}_1 + y\vec{e}_2}$$

$$x = r\cdot\cos\phi \qquad y = r\cdot\sin\phi$$

$$L = \mu r^2 \dot{\phi}\,[\cos^2\phi + \sin^2\phi]$$

$$= \mu r^2 \dot{\phi} \quad \Rightarrow$$

$$\boxed{\begin{aligned} \frac{d\phi}{dt} &= \dot{\phi} = \frac{L}{\mu r^2} \\ \mu \cdot \ddot{r} &= F_r + \frac{L^2}{\mu r^3} \end{aligned}}$$

$$= -\frac{dV}{dr} + \frac{L^2}{\mu r^3}$$

$$V_{eff}(r) = V(r) + \frac{L^2}{2\mu r^2}$$

$$-\frac{dV_{eff}}{dr} = -\frac{dV}{dr} - \frac{L^2}{\mu r^3}$$

rewrite from kinetic energy in r and ϕ degrees of freedom

$$V(r) = \frac{1}{2} k r^2 = \frac{1}{2} k (x^2+y^2)$$

cartesian coordinates:

$$m \frac{d^2x}{dt^2} = -kx \Rightarrow x = A\cos\omega_0 t + B\sin\omega_0 t$$

$$m \frac{d^2y}{dt^2} = -ky \Rightarrow y = C\cos\omega_0 t + D\sin(\omega_0 t)$$

$$V_{eff}(r) = \frac{1}{2} k r^2 + \frac{L^2}{2\mu r^2}$$