PHY 321 FEB 24

$$V(x) = \frac{1}{2} k x^2$$

$$F(x) = -\frac{d}{dx} V(x) = -kx$$

Equations of motion (EoM)

$$m \frac{d^2 x}{dt^2} = -kx$$

$$\omega_0 = \sqrt{k/m}$$

$$\frac{d^2 x}{dt^2} = \frac{dv}{dt} = -\omega_0^2 x$$

$$\frac{dx}{dt} = v(t)$$

$$x(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$$

Defined by initial conditions

Example $x(t_0 = 0) = x_0$

$\underline{v(t_0 = 0) = 0}$

$$x(0) = \boxed{x_0 = A}$$

$$\frac{dx}{dt} = -A\omega_0 \sin(\omega_0 t) + B\omega_0 \cos(\omega_0 t)$$

$$= v(t)$$

$$v(t=0) = 0 = B\omega_0$$

$$\omega_0 \neq 0$$

$$\Rightarrow B = 0$$

$$\Rightarrow \quad x(t) = x_0 \cos(\omega_0 t)$$

$\omega_0 \cdot T = 2\pi$, solution repeats itself $\Rightarrow$

Period: $T = 2\pi/\omega_0$

Math manipulations

(Taylor S.2)

$$e^{\pm i\omega t} = \cos(\omega t) \pm i \sin(\omega t)$$

$$\cos(\omega t) = \frac{1}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)$$

$$\sin(\omega t) = \frac{1}{2i}\left(e^{i\omega t} - e^{-i\omega t}\right)$$

$$x(t) = \boxed{C_1 e^{i\omega t} + C_2 e^{-i\omega t}}$$

$$= \boxed{A \cos(\omega t) + B \sin(\omega t)}$$

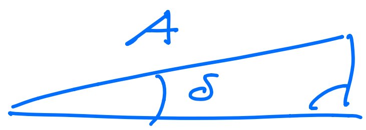
B_1 B_2

$(C_1 + C_2) \cos(\omega t) +$

$$= (C_1 + C_2)\cos(\omega t) +$$
$$i(C_1 - C_2)\sin(\omega t)$$

$$A = C_1 + C_2 \qquad B = i(C_1 - C_2)$$

New way:



$B_2 = A\,\underline{\sin\delta}$

$B_1 = A\cos\delta$

$$A = \sqrt{B_1^2 + B_2^2}$$

$$x(t) = A\left[\frac{B_1}{A}\cos(\omega t) + \frac{B_2}{A}\sin(\omega t)\right]$$

$$A\left[\cos\delta\cos(\omega t) + \sin\delta\sin(\omega t)\right]$$

$$= A\cos(\omega t - \delta)$$

phase shift.

Energy

$$x(t) = A\cos(\omega_0 t - \delta)$$
$$\omega_0 = \sqrt{k/m}$$

$$V(x) = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega_0 t - \delta)$$

$$\frac{dx}{dt} = \dot{x}(t) = -\omega_0 A\sin(\omega_0 t - \delta)$$

$$\frac{dx}{dt} = v(t) = -\omega_0 A \sin(\omega_0 t - \delta)$$

$$K = \frac{1}{2} m v^2 = \frac{1}{2} \cancel{m} \underbrace{\omega_0^2}_{k/\cancel{m}} A^2 \sin^2(\omega_0 t - \delta)$$

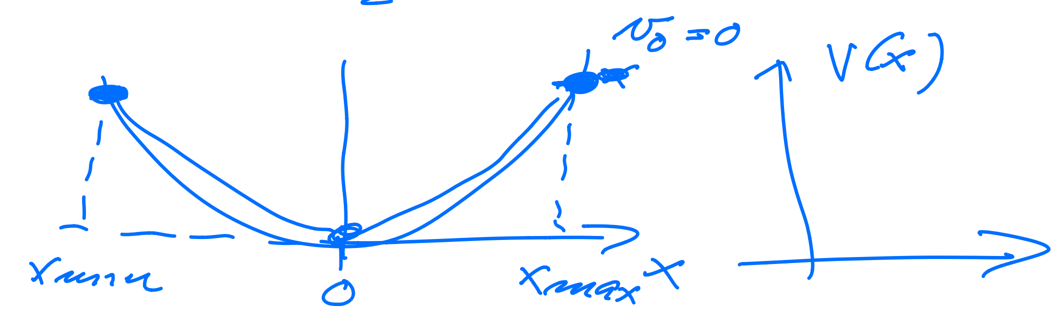
$$\omega_0^2 = k/m$$

$$E = V(x) + K = \frac{1}{2} k A^2 \left(\cos^2(\omega_0 t - \delta) + \sin^2(\omega_0 t - \delta) \right)$$

$$= \frac{1}{2} k A^2$$

<u>Example</u> using E, how can we find $x(t)$?

$$V(x) = \frac{1}{2} k x^2$$



$$|x_{min}| = |x_{max}|$$

$$x = x_{max} = x_0$$

$$V(x) = \frac{1}{2} k x_0^2 = E$$

$$E = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2 + V(x) = \frac{1}{2} k x_0^2$$

$$\frac{1}{2} m \left(\frac{dx}{dt}\right)^2 + \frac{1}{2} k x^2$$

$$= \frac{1}{2} m \left(\frac{dx}{dt}\right)^2 + \frac{1}{2} k x^2$$

$$\frac{dx}{dt} = \pm \sqrt{\frac{2}{m}\left(E - \frac{1}{2}kx^2\right)}$$

$$\frac{1}{2}kx_0^2$$

$$k/m = \omega_0^2$$

$$\frac{dx}{dt} = \pm\, \omega_0 \sqrt{x_0^2 - x^2}$$

$$dt = \frac{dx}{\omega_0 \sqrt{x_0^2 - x^2}}$$

separation of variables

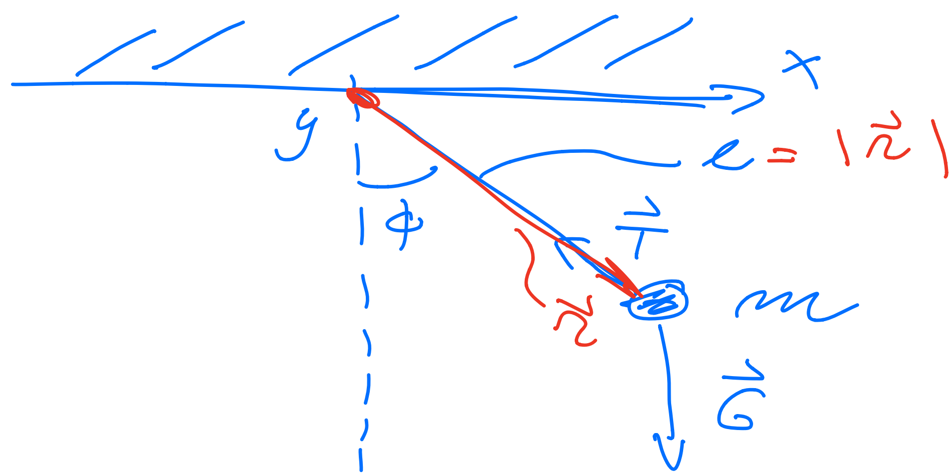
$$t_0 = 0 \qquad \cancel{t_0 =} \; x(t_0) = x_0$$

$$t = \int_0^t dt' = \int_{x_0}^{x} \frac{dx'}{\omega_0 \sqrt{x_0^2 - x'^2}} \Rightarrow$$

$$x(t) = x_0 \sin(\omega_0 t)$$

Beyond 1-dim and simple potentials, do not use.

Example: mathematical pendulum and EoM



$$\vec{T} = \underline{T \sin\phi\, \vec{e}_1} + \underline{T\cos\phi\, \vec{e}_2}$$

$$\vec{G} = -\underline{m \cdot g\, \vec{e}_2}$$

$$\vec{F}^{tot} = \vec{T} + \vec{G}$$

EOM : $\frac{d^2\phi}{dt} = -g/l \sin\phi$

$$\omega_0 = \sqrt{g/l}$$

$$\boxed{\frac{d^2\phi}{dt} = -\omega_0^2 \sin\phi}$$

(HW1, Ex1)

$\phi \ll 1$

$\sin\phi \simeq \phi$

Harmonic oscillator

$$\frac{d^2\phi}{dt^2} = -\omega_0^2\, \phi$$

$$\left(\frac{d\vec{x}}{dt^2} = -\omega_0^2 x \quad ; \quad \omega_0 = \sqrt{k/m} \right)$$

$$\vec{r} = l \sin\phi\, \vec{e}_1 + l\cos\phi\, \vec{e}_2$$

$$F_y^{tot} = T\cos\phi - mg$$

$$F_x^{tot} = T\sin\phi$$

$$\Rightarrow \frac{d\vec{r}}{dt} \;, \quad \underline{\frac{d^2\vec{r}}{dt^2}}$$