PHY 321 MARCH 22

$x, y \to r, \phi,$ $\quad r \in [0, \infty)$
$\phi \in [0, 2\pi)$

$$\frac{d\phi}{dt} = \dot{\phi} = \frac{L}{\mu r^2}$$

$$\mu \cdot \ddot{r} = \mu \frac{d^2 r}{dt^2} = F(r) + \underbrace{\frac{L^2}{\mu r^3}}$$

kinetic energy

$$K = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\phi}^2)$$

$$E = K + V(r)$$

Example $\quad V(r) = \frac{1}{2} k r^2$

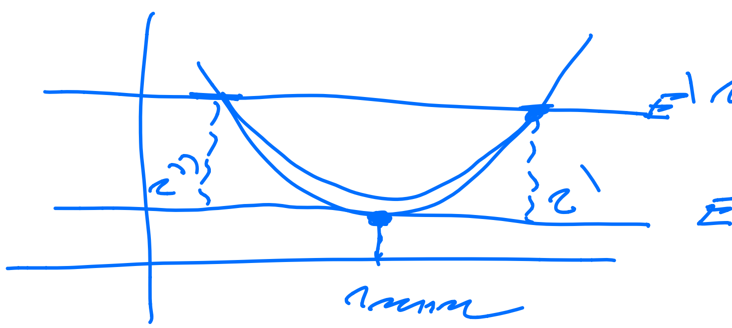
$r = \sqrt{x^2+y^2}$ $\qquad x = r\cos\phi$
$y = r\sin\phi$

$$E = \underbrace{\frac{1}{2}\mu \dot{r}^2}_{\text{radial velocity}} + \underbrace{\frac{1}{2}\mu r^2 \dot{\phi}^2}_{\text{angular velocity}} + \frac{1}{2} k r^2$$

$$= \frac{1}{2}\mu\dot{r}^2 + \underbrace{V_{eff}(r)}_{\frac{1}{2}\mu r^2\dot{\phi}^2 + \frac{1}{2}kr^2}$$

$$\mu \ddot{r} = -\frac{d\, V_{eff}(r)}{dr} = -kr + \frac{L^2}{\mu r^3}$$

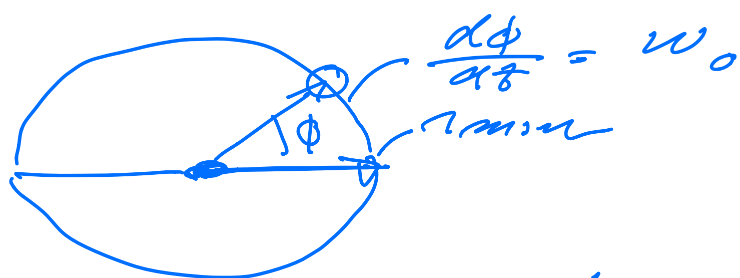
$$\left.\frac{dV_{eff}}{dr}\right|_{r_{min}} = 0$$



$$r_{min} = \left(\frac{L^2}{k\mu}\right)^{1/4}$$

$\ddot{r} = 0$? $\quad \frac{dr}{dt}$ = constant

$$\dot{\phi} = \frac{d\phi}{dt} = \sqrt{k/\mu} = \omega_0$$



$\underline{r_{min}}$ gives circular orbit.

x_0, y_0 determined by r_{min}

$$r_{min} = \sqrt{x_0^2 + y_0^2}$$

$y_0 = 0 \ \wedge \ x_0 = 1\,mm$

circular motion:

$v_{x0} \wedge v_{y0}$. How do we fix these to get circular motion?

$$\mu \frac{d^2\vec{r}}{dt^2} = -k\vec{r} \Rightarrow$$

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = -\omega_0^2\vec{r}$$

magnitude of $|\vec{a}| = \omega_0^2|\vec{r}|$

$$= \omega_0^2 r$$

$$v^2/r = \omega_0^2 r \Rightarrow$$

$$v^2 = \omega_0^2 r^2 \Rightarrow$$

$$v = \pm\omega_0 r$$

$r = 1\,mm$