PHY 3C1 MARCH 1

HW 6, ex 5

$t=0 \quad K(t_0) = K_0 = \frac{1}{2} m_E v_0^2$

$(M_E = 1)$

circular motion

E-kinetic $K(t)$ $= K_0$

$$K(t_i) = \frac{1}{2} \underline{v(t_i)^2}$$

Potential energy:

$$V(r) = \frac{4\pi^2}{r}$$

$$V(r) = \frac{G M_\odot M_E}{r}$$

$$r = \sqrt{x^2+y^2}$$

$$V(r_0) = 4\pi^2/r_0$$

$$V(r_i) = 4\pi^2/r_i$$

$$F(\vec{r}) = -\frac{G M_\odot \vec{r}}{r^3}$$

assume circular motion

$$F(r) = \frac{M_E v^2}{r} = \frac{G M_\odot M_E}{r^2}$$

$$\cancel{M_E} v^2 = \frac{G M_\odot \cancel{M_E}}{r}$$

$$\boxed{v^2 r = G M_\odot}$$

$$v = 2\pi r / 1\,yr$$

$$G M_\odot = v^2 r = 4\pi^2 (AU)^3 / yr^2$$

$$r = 1\,AU$$

—— Oscillations ——

$$m \underbrace{\frac{d^2x}{dt^2}}_{a_x} + \underbrace{b \underbrace{\frac{dx}{dt}}_{v_x}}_{\text{Damping}} + kx = 0$$

$$\omega_0^2 = k/m$$

$$\tau = \omega_0 \cdot t \qquad \gamma = \frac{b}{2m\omega_0}$$

$$\frac{d^2x}{d\tau^2} + 2\gamma \frac{dx}{d\tau} + x = 0$$

$$-\gamma\tau \qquad \tau\sqrt{\gamma^2-1}$$

$$x(\tau) = e \left[c_1 e + c_2 e^{-\tau\sqrt{\gamma^2-1}} \right]$$

no-damping

$$b = 0 \Rightarrow \gamma = 0$$

$$\sqrt{0-1} = + i$$

$$x(\tau) = c_1 e^{i\tau} + c_2 e^{-i\tau}$$

$$(\tau = \omega_0 \cdot t)$$

$$= c_1 e^{i\omega_0 t} + c_2 e^{-i\omega_0 t}$$

$$= A\cos(\omega_0 t) + B\sin(\omega_0 t)$$

weak damping $\quad \gamma < 1$

$$\gamma = \frac{b}{2m\omega_0} < 1$$

$$\boxed{\frac{b}{2m} < \omega_0}$$

$$\sqrt{\gamma^2-1} = i\sqrt{1-\gamma^2} = i\omega$$

$$x(\tau) = e^{-\gamma\tau}\left[c_1 e^{i\omega\tau} + c_2 e^{-i\omega\tau}\right]$$

$$= \boxed{e^{-\gamma\tau}\left[A_1\cos(\omega\tau) + A_2\sin(\omega\tau)\right]}$$

$A_1 = C_1 + C_2 \qquad A_2 = i(C_1 - C_2)$

Damping as function of time $x(\tau)$ when $\tau \to 0$, then $x(\tau) \to 0$

no Damping case