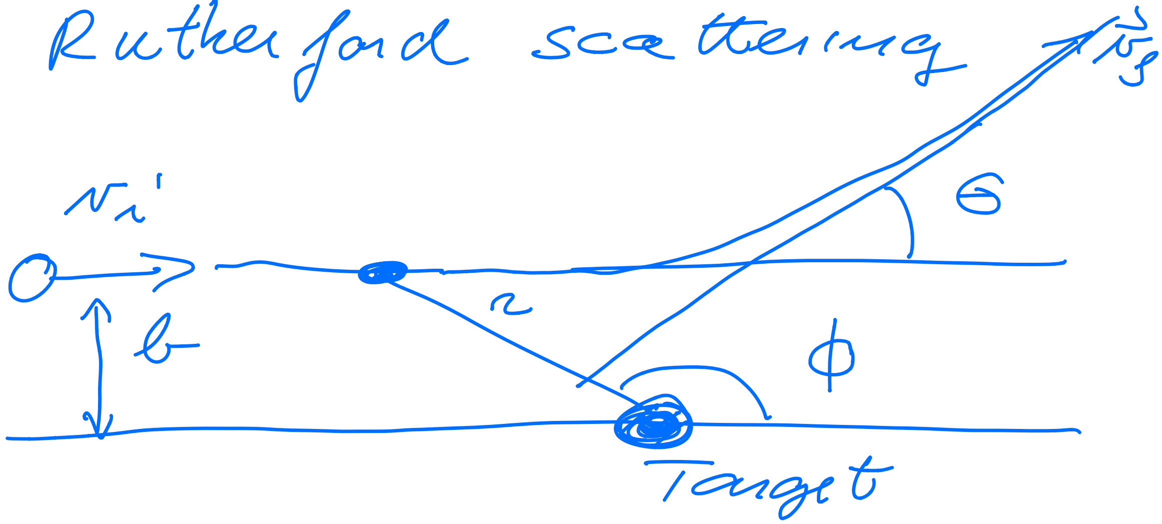
PHY 321 APRIL 7

Rutherford scattering



$$r(\phi) = \frac{c}{1+\varepsilon\cos\phi}$$

$$\frac{d\sigma}{d\Omega} = \frac{a^2}{4\sin^4\theta/2} \qquad \boxed{a = \frac{\alpha}{2E}}$$

$$F(r) = -\alpha/r^2 \qquad \alpha = G m_1 m_2$$

$$\alpha = \frac{q_1 q_2}{4\pi\varepsilon_0}$$

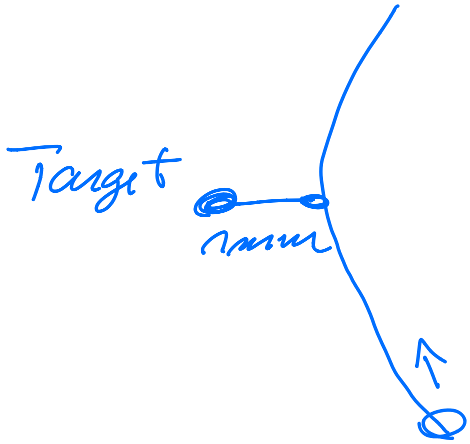
$$a = \frac{\mu\alpha}{L^2 b^2}$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16E^2\sin^4\theta/2}$$

$$\alpha = \frac{q_1 q_2}{4\pi\varepsilon_0}$$

incoming α-particles

$Z = 2$ Gold $Z = 79$

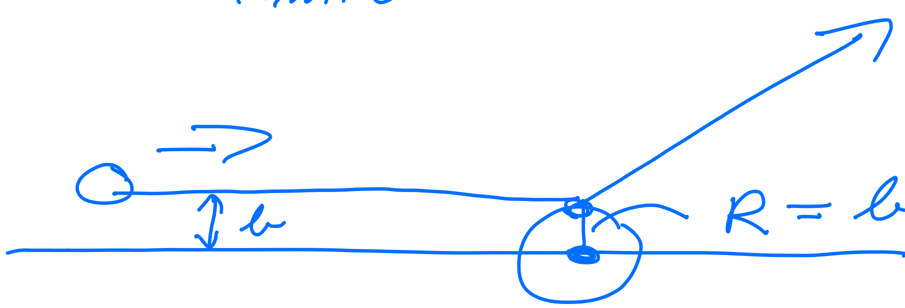


$$E = \frac{L^2}{2\mu r_{min}^2} - \frac{\alpha}{r_{min}}$$

$$E = \frac{L^2}{2\mu R^2} - \frac{\alpha}{R}$$

$$2\mu R^2 E + \frac{\alpha\, 2\mu R^2}{R} = L^2$$

$r_{min} = R$



$R = b$

$$L^2 = 2\mu R^2 \left(E + \alpha/R \right)$$

$$L = \mu \cdot v \cdot b = b\sqrt{2\mu E}$$

$$b^2 = \frac{L^2}{2\mu E} = R^2\left(\frac{E + \alpha/R}{E}\right)$$

$$b = R\sqrt{\frac{E + \alpha/R}{E}}$$

$$\sin \theta/2 = \frac{a}{\sqrt{a^2+b^2}}$$

$$\theta = 2\sin^{-1}\left[\frac{a}{\sqrt{a^2+b^2}}\right]$$

$E = 38$ keV $\quad 1\text{eV} = 10^{-19}$ J

$\sigma = 1400$ barn $\quad 1$ barn $= 10^{-28}\ m^2$

$= \pi b^2 \quad b = R$

$R = 7.5 \times 10^{-15}$ m

$z = 2 \quad Z = 79$

$$\sin \theta/2 = \frac{a}{\sqrt{a^2+b^2}} \Rightarrow$$

$\theta = 83$ degrees.

Lagrangian formalism

Top down

Equations of motions from kinetic and potential images

- Principle of least action
- Lagrangian

$$\mathcal{L} = K - V = \mathcal{L}(x, v, t)$$

- Euler-Lagrange equations
- variational calculus

Example 1

Harmonic oscillator

$$K = \frac{1}{2} m v^2$$

$$V = \frac{1}{2} k x^2$$

- Euler-Lagrange eqs.

$$\frac{\partial \mathcal{L}}{\partial x} - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial v} = 0$$

$$\frac{\partial \mathcal{L}}{\partial x} = -kx$$

$$\frac{\partial \mathcal{L}}{\partial v} = m \cdot v$$

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial v} = m \frac{dv}{dt}$$

$$-kx - m\frac{dv}{dt} = 0$$

$$= \boxed{m\frac{dv}{dt} = -kx}$$

Newton's law

$$ma = F$$

Example 2

Gravitational problem in polar coordinates

$$K = \frac{1}{2}\mu \dot{r}^2 + \frac{1}{2}(r\dot{\phi})^2\mu$$

$$V = -\alpha/r$$

$$\mathcal{L} = \frac{1}{2}\mu\dot{r}^2 + \frac{1}{2}(r\dot{\phi})^2\mu + \alpha/r = \mathcal{L}(r, \dot{r}, \phi, \dot{\phi}, t)$$

$$\frac{\partial \mathcal{L}}{\partial r} = \underline{\mu \cdot r\dot{\phi}^2 - \alpha/r^2}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{r}} = \mu \cdot \dot{r}$$

$\cup v_r$

$v_r = \dot{r} = \frac{dr}{dt}$

$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial v_r} = \mu \cdot \frac{d^2 r}{dt^2}$

$\frac{\partial \mathcal{L}}{\partial r} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial v_r} = 0$

$\mu r \dot{\phi}^2 - \alpha/r^2 - \mu \ddot{r} = 0$

$$\boxed{\mu \ddot{r} = \mu r \dot{\phi}^2 - \alpha/r^2}$$

$\mu a_r = \frac{L^2}{\mu r^3} - \alpha/r^2$

$= - \frac{d V_{eff}}{dr}$

$V_{eff}(r) = V(r) + \frac{L^2}{\mu r^2}$

$\boxed{\mu a_\phi = 0 ?}$ $\boxed{\dot{\phi} = \frac{d\phi}{dt} \propto L}$

$\frac{d\phi}{dt} = \frac{L^2}{\mu r^2}$

$\frac{\partial \mathcal{L}}{\partial \phi} = 0$

$$\overline{\partial \phi}$$

$$\frac{\partial \mathcal{L}}{\partial \dot\phi} = \mu r^2 \cdot \dot\phi \qquad \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot\phi}$$

$$= \underline{\mu r^2 \frac{d^2\phi}{dt^2}} + \boxed{\underline{\mu \dot\phi \, 2 \frac{dr}{dt} \cdot r}}$$

$$\mu r^2 \frac{d^2\phi}{dt^2} = 0$$

$$\cancel{\dot\phi = \mu r}$$

$$\mu r^2 \dot\phi = L$$

$$\frac{d\phi}{dt} = \frac{L}{\mu r^2}$$

$$\mu r^2 \frac{d^2\phi}{dt^2} + \mu \dot\phi \, 2 r \frac{dr}{dt} = 0$$

$$\mu r^2 \frac{d^2\phi}{dt^2} + \cancel{\mu}\frac{L}{\cancel{\mu} r} \frac{dr}{dt} = 0$$

$$\frac{d^2\phi}{\ \ } = -\frac{L}{\ \ }\frac{dr}{\ \ }$$

$$\boxed{\frac{}{dt^2} = \frac{}{\mu r^3}\,\frac{}{dt}}$$