PHYS 321 MARCH 10

Numerical digression

– Forces depend only on $\vec{r}$ / Energy conserving forces

Euler-Cromer

Velocity-Verlet

$$\frac{d^2x}{dt^2} = a(x) = \frac{F(x)}{m}$$

$$\frac{dx}{dt} = v \quad \wedge \quad \frac{dv}{dt} = a$$

Euler-Cromer:

$$v_{i+1} = v_i + \Delta t\, a_i \quad (\Delta t^2)$$

$$a_i = a(t_i)$$

$$x_{i+1} = x_i + \Delta t\, v_{i+1} \quad (\Delta t^2)$$

Velocity-Verlet

$$\underline{x_{i+1}} = x_i + \Delta t\, v_i + \frac{\Delta t^2}{2} a_i$$

$\rightarrow a_{i+1}$

$$v_{i+1} = v_i + \frac{\Delta t}{2}(a_{i+1} + a_i)$$

Truncation error (Δt)

- Time-dependent forces

Runge-Kutta methods

$$\frac{dy}{dt} = \boxed{f(t,y)} \quad \text{(known expression)}$$

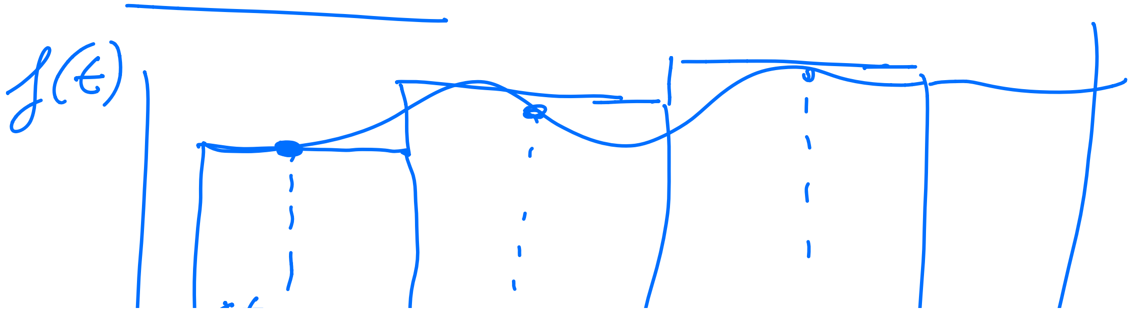
$$y(t) = y(t_0) + \int_{t_0}^{t} f(t',y)\,dt'$$

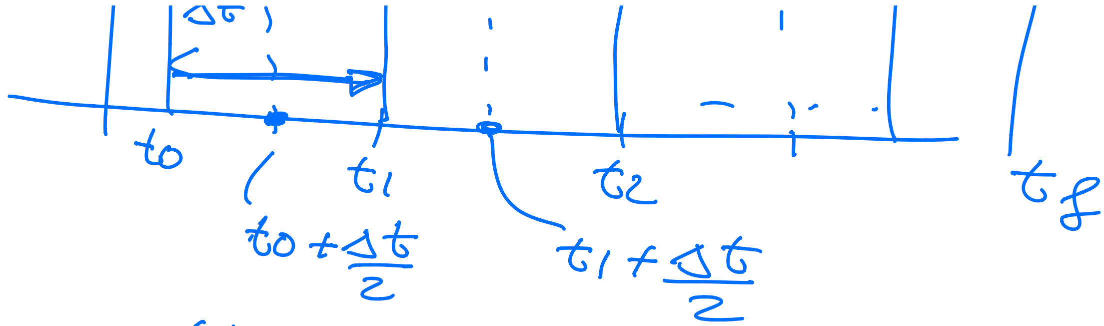
$$y_{i+1} = y_i + \int_{t_i}^{t_{i+1}} f(t,y)\,dt$$

RK-family deal with approximations to

$$\int_{t_i}^{t_{i+1}} f(t,y)\,dt$$

RK 2





$$\int_{t_i}^{t_{i+1}} f(t,y)\,dt = \Delta t \cdot f\left(t_i + \frac{\Delta t}{2}, y_{i+\frac{1}{2}}\right) + O(\Delta t^3)$$

$$y_{i+1} = \underline{y_i} + \Delta t\, f(t_{i+1/2}, y_{i+1/2})$$

$$(\Delta t^3)$$

$i = 0$

$$y_1 = \boxed{y_0} + \Delta t\, f\left(t_0 + \frac{\Delta t}{2}, y_{0+1/2}\right)$$

?

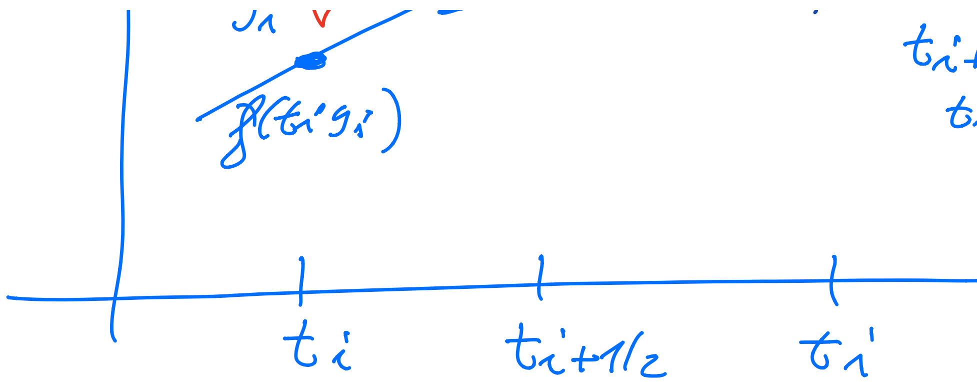
$$y_{i+1/2} \approx y_i + \frac{\Delta t}{2} f(t_i, y_i)$$

$$K_1 = \Delta t\, f(t_i, y_i)$$

$$\underline{K_2} = \underline{\Delta t}\, f(t_{i+1/2}, \underline{y_{i+1/2}})$$

$$\underline{y_{i+1}} = y_i + K_2$$

$$(\Delta t^3)$$



$$t_{i+1/2} = t_i + \frac{\Delta t}{2}$$

RK4

$$\int_{t_i}^{t_{i+1}} f(t,y)\,dt = \frac{\Delta t}{6}\Big(f(t_i, y_i) + 4 f(t_{i+1/2}, y_{i+1/2}) + f(t_{i+1}, y_{i+1})\Big) + O(\Delta t^5)$$

$2f(\text{--})$

Simpson's Rule

$$k_1 = \Delta t\, f(t_i, y_i)$$

$$k_2 = \Delta t\, f\left(t_{i+1/2},\ y_i + \frac{k_1}{2}\right)$$

$$k_3 = \Delta t\, f\left(t_{i+1/2},\ y_i + \frac{k_2}{2}\right)$$

$$k_4 = \Delta t\, f(t_{i+1},\ y_i + k_3)$$

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

RK4