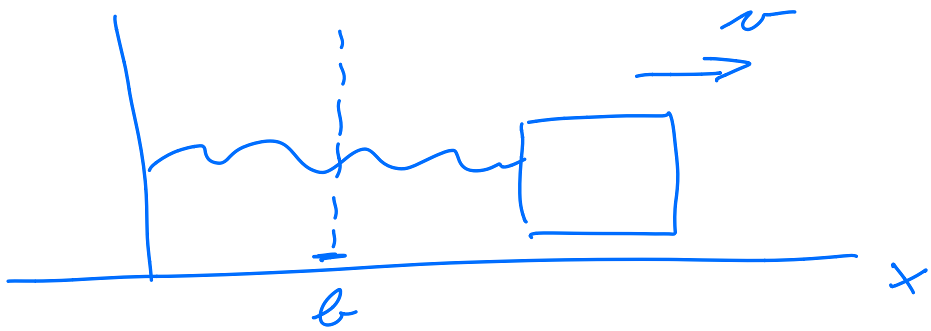
PHY 321 FEBRUARY 22

Prelude to oscillations



$$F(x) = -k(x-b)$$

Energy conserving force

$$E = K + V(x) = \text{constant}$$

$$V(x) - V(x_0) = -\int_{x_0}^{x} F(x')\,dx'$$

$$= +\int_{x_0}^{x} (x'-b)\,k\,dx'$$

$$= \underbrace{\frac{1}{2}k(x-b)^2}_{V(x)} - \underbrace{\frac{1}{2}k(x_0-b)^2}_{V(x_0)}$$

We can change $V(x_0)$ as we wish. In equilibrium $x = b$

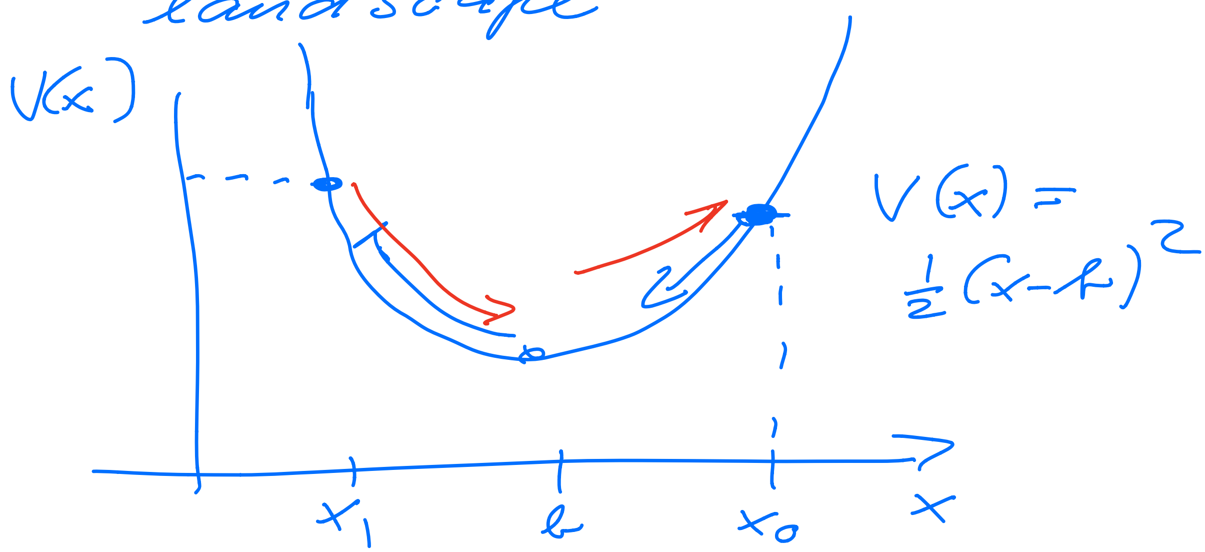
$V(x=b)=0$

$$V(b) - V(x_0) = 0 - \frac{1}{2}k(x_0-b)^2$$

$$V(x_0) = \frac{1}{2}k(x_0-b)^2$$

$$V(x) = -\int F(x)\,dx + C$$

plot of potential energy landscape



$\underline{\text{constant}}$ $\underline{E_0} = \frac{1}{2}k(x_0-b)^2 \Rightarrow$

$\underline{v_0 = 0}$ $\quad K = E_0 - \frac{1}{2}k(x_0-b)^2$

we know that

$$\frac{dV}{dx} = -F_x$$

when motion starts

$$\frac{dV}{dx} > 0 \quad (x_0 > b)$$

$$\frac{dV}{dx} = k(x_0 - b) \Rightarrow$$

$$F = -k(x-b)$$

negative when $x = x_0 > b$

The object accelerates towards $x = b$. at $x = b$

$$\frac{dV}{dx} = 0$$

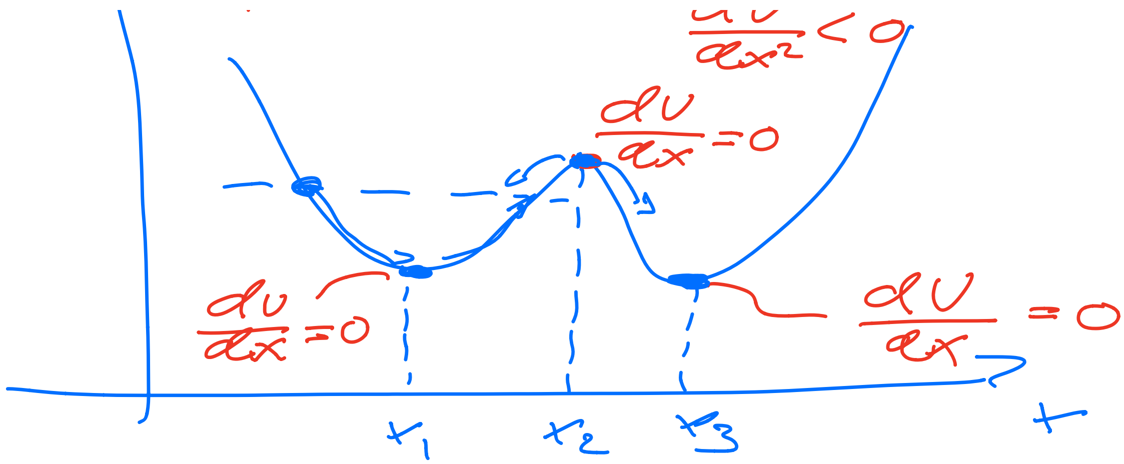
The motion continues but with $x < b$ and $F > 0$, the force acts in the positive x-direction?

$x = b$ is a so-called equilibrium point;

$$F(b) = -\frac{dV}{dx}\Big|_{x=b} = 0$$

Ex 4 in hw 6

$V(x)$



at x_1 and x_3 $\quad \frac{dU}{dx} = 0$

$$\frac{d^2U}{dx^2} > 0 \quad (\text{minimum})$$

$$V(x) = -k\frac{x^2}{2} + \alpha\frac{x^4}{4}$$

$$k = \alpha = 1$$

$$\frac{dV}{dx} = 0 = -kx + \alpha x^4$$

$$x_{min} = \pm\sqrt{k/\alpha}$$

$$F(x) = -kx + \alpha x^3$$

at the minima :

$$\frac{d^2V}{dx^2} > 0 \Rightarrow \frac{dF}{dx} < 0$$

if we give the object/particle a small velocity, it starts

the positive

moving in the positive
x-direction, the force
acting on it will try
to move it back again,

at x_2, for any perturbation
it will not remain

$$\frac{dV}{dx} = 0 \quad \text{but} \quad \frac{d^2V}{dx^2} < 0$$

$$\Rightarrow \quad \frac{dF}{dx} > 0$$

the force will increase
and brings the object
further away from
$x = x_2$, unstable
equilibrium point.

- Small oscillations/perturbations around
equilibrium $x = b$

Taylor

$$V(x) = V(b) + (x-b)\frac{dV}{dx}\bigg|_{x=b} + \frac{(x-b)^2}{2}\frac{d^2V}{dx^2}\bigg|_{x=b}$$

$$+ O\left((x-b)^3\right)$$

at equilibrium $\left.\frac{dV}{dx}\right|_{x=b} = 0$

$$V(x) = V(b) + \frac{(x-b)^2}{2} \left.\frac{d^2V}{dx^2}\right|_{x=b}$$

$$\frac{d^2V}{dx^2} = k \qquad k > 0$$

$\downarrow$

minimum then $\boxed{\frac{d^2V}{dx^2} > 0}$

$$V(x) = V(b) + \frac{1}{2} k (x-b)^2$$

$$-\frac{dV}{dx} = -k(x-b) = F(x)$$

$$b = 0$$

$$F(x) = -kx$$

$$m \frac{d^2x}{dt^2} = -kx \qquad \text{mass}/\text{time}^2$$

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = -\frac{k}{m} x$$

$$\frac{dx}{dt} = v(t)$$

dt

ω_0 = natural frequency

$= \sqrt{k/m}$

$[\omega_0] = 1/\text{time}$

$$\frac{d^2x}{dt^2} = -\omega_0^2 x$$

$$x(t) = A\cos(\omega_0 \cdot t) + B\sin(\omega_0 t)$$

$$\frac{dx}{dt} = -A\omega_0 \sin(\omega_0 t) + B\omega_0\cos(\omega_0 t)$$

$$\frac{d^2x}{dt^2} = -A\omega_0^2\cos(\omega_0 t) - B\omega_0^2\sin(\omega_0 t)$$

$$= -\omega_0^2 x(t)$$

A & B are determined by x_0 and v_0.