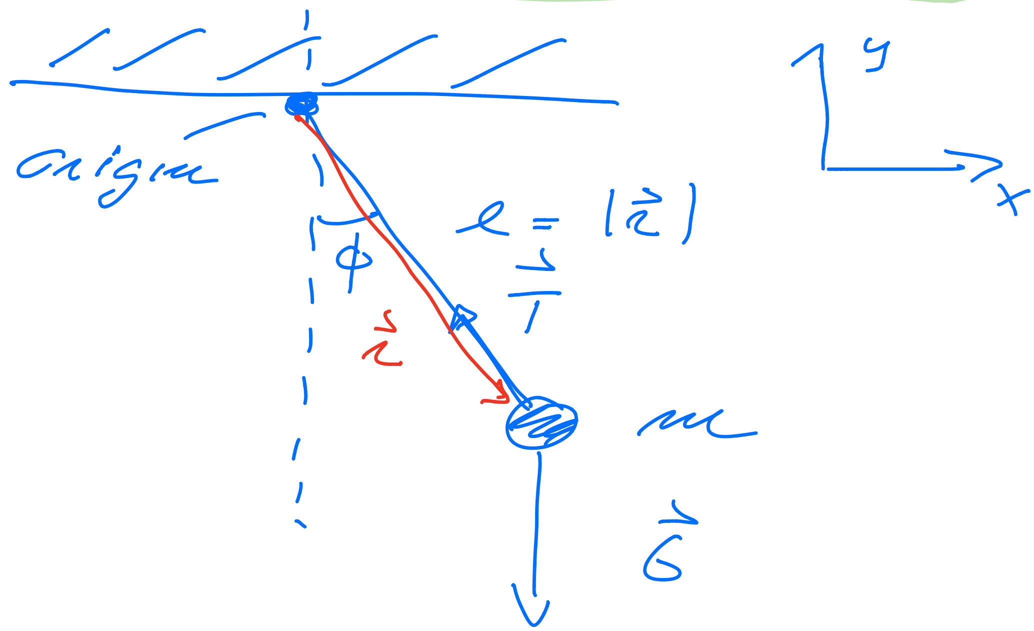
PHY321 FEBRUARY 26



$$\vec F^{net} = \vec T + \vec G = \boxed{\underbrace{T\sin\phi\,\vec e_1}_{x\text{-comp}}} + \underbrace{T\cos\phi\,\vec e_2 - mg\vec e_2}_{y\text{-comp}}$$

$$\vec r = l\sin\phi\,\vec e_1 + l\cos\phi\,\vec e_2$$

$$m\frac{d^2\vec r}{dt^2} = \vec F^{net}$$

$$\frac{d\vec r}{dt} = l\frac{d\phi}{dt}\cos\phi\,\vec e_1 - l\frac{d\phi}{dt}\sin\phi\,\vec e_2$$

$$\frac{d^2\vec r}{dt^2} = \left[l\frac{d^2\phi}{dt^2}\cos\phi - l\left(\frac{d\phi}{dt}\right)^2\sin\phi\right]\vec e_1$$

$$\frac{d\,t^2}{} \quad - \quad \frac{}{d\,t^2} \qquad \left(\frac{}{dt}\right)$$

$$+\left[-l\frac{d^2\phi}{dt^2}\sin\phi - l\left(\frac{d\phi}{dt}\right)^2\cos\phi\right]\vec{e}_2$$

$$m\frac{d^2\vec{r}}{dt^2}\,\vec{e}_1 = m\left[l\frac{d^2\phi}{dt^2}\cos\phi - \underline{l\left(\frac{d\phi}{dt}\right)^2\sin\phi}\right]$$

$$= T\cdot\sin\phi$$

$$m\frac{d^2\vec{r}}{dt^2}\,\vec{e}_2 = m\left[-l\frac{d^2\phi}{dt^2}\sin\phi - \underline{l\left(\frac{d\phi}{dt}\right)^2\cos\phi}\right]$$

$$= T\cos\phi - mg$$

$\cos\phi\,\times$

$$m\,l\frac{d^2\phi}{dt^2}\cos\phi - = \underline{\left(T + ml\left(\frac{d\phi}{dt}\right)^2\right)}\sin\phi\cos\phi$$

$\sin\phi\,\times$

$$-m\,l\frac{d^2\phi}{dt^2}\sin\phi \pm mg = \underline{\left(T + ml\left(\frac{d\phi}{dt}\right)^2\right)}\cos\phi\,\sin\phi$$

$$+\,m\,l\frac{d^2\phi}{dt^2}\underbrace{\left(\cos^2\phi + \sin^2\phi\right)}_{=1} = 0$$
$$\pm\; mg\cdot\sin\phi$$

$$\cancel{m}\,l\frac{d^2\phi}{dt^2} = -\cancel{m}g\sin\phi$$

$$\frac{d^2\phi}{} = -g/l\,\sin\phi$$

$$\overline{dt^2}$$

$$\omega_0^2 = +g/\ell$$

$$\frac{d^2\phi}{dt^2} = -\omega_0^2 \sin\phi$$

non-linear in ϕ

$\phi << 1 \quad \Rightarrow \sin\phi \simeq \phi$

$\Rightarrow$

$$\frac{d^2\phi}{dt^2} = -\omega_0^2 \phi$$

$$\phi(t) = A\cos\omega_0 t + B\sin\omega_0 t$$

—— Damping (5.4) ——

Damping : $\vec{F}^{\text{Damping}} = -b\vec{v}$

$$= -b\frac{d\vec{x}}{dt}$$

1-Dim $\quad \vec{F}^{\text{Damping}} = F = -b\frac{dx}{dt}$

$$m\frac{d^2x}{dt^2} = -b\frac{dx}{dt} - kx$$

$$\boxed{m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0}$$

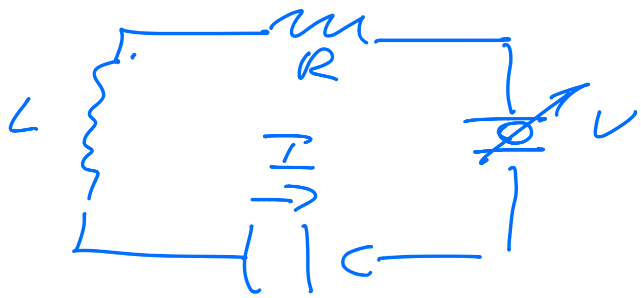
$$\vec{F}^{net} = -kx - b v_x$$
$$= -kx - b\frac{dx}{dt}$$

Pendulum

$$F^D = -\gamma \frac{d\phi}{dt}$$

$$\boxed{ml\frac{d^2\phi}{dt^2} + \gamma\frac{d\phi}{dt} + mg\sin\phi = 0}$$

RLC - circuit



$$I = \frac{dq}{dt}$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0$$
$$\ddot{q} + 2\sigma\dot{q} + q = 0$$

Scaling the equations

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + \frac{k}{m}x = 0$$

$$\frac{d^2x}{\quad} + \frac{b}{\quad}dx + \omega_n^2 x = 0$$

$\overline{dt^2} \qquad \overline{m} \; \overline{dt} \qquad \overline{\quad}$

$= k/m$

Dimensionless time

$$\tau = t \cdot \omega_0 \Rightarrow t = \tau/\omega_0$$

$$\cancel{\omega_0^2}\frac{d^2x}{d\tau^2} + \frac{b}{\omega_0 m}\,\omega_0\,\frac{dx}{d\tau} + \cancel{\omega_0^2}\,x = 0$$

$$\underset{\ddot{x}}{\frac{d^2x}{d\tau^2}} + \frac{2b}{2\omega_0 m}\,\overset{\dot{x}}{\frac{dx}{d\tau}} + x = 0$$

$$\gamma = \frac{b}{2\omega_0 m}$$

Dimensionless.

$$\Rightarrow \underline{\ddot{x} + 2\gamma\dot{x} + x} = 0$$

$$(\ddot{\phi} + 2\delta\dot{\phi} + \sin\phi)$$

solution

attempt

$$x(\tau) = e^{r\tau}$$

$$\dot{x} = r e^{r\tau}$$

$$x'' = r^2 e^{r\tau}$$

$$r^2 \cancel{e^{r\tau}} + 2\gamma r \cancel{e^{r\tau}} + \cancel{e^{r\tau}} = 0$$

$$r^2 + 2\gamma r + 1 = 0$$

auxiliary equation

$$r = -\gamma \pm \sqrt{\gamma^2 - 1}$$

$$r_1 = -\gamma \pm \sqrt{\gamma^2 - 1}$$

$$r_2 = -\gamma - \sqrt{\gamma^2 - 1}$$

$$\underbrace{e^{-\gamma\tau}}_{\text{decay term}} \underbrace{\left[C_1 e^{\tau\sqrt{\underline{\gamma^2-1}}} + C_2 e^{-\tau\sqrt{\gamma^2-1}} \right]}_{\text{oscillation}}$$

No damping $\gamma = 0$

$$\sqrt{-1} = i$$

$$x(\tau) = C_1 e^{+i\tau} + C_2 e^{-i\tau}$$

$$\tau = \omega_0 \cdot t$$

$$= C_1 e^{+i\omega_0 t} + C_2 e^{-i\omega_0 t}$$

weak damping $\gamma < \underline{1}$

Exercise 5a   Taylor 4.4

$$\vec{\nabla} \times \vec{F} = \vec{\nabla} \times (-\vec{\nabla} V(\vec{r}))$$

- paper & pencil
- symbolic package (mathematica) etc.

Exercise 4

$$V(x) = -\alpha \frac{x^2}{2} + \frac{\beta x^4}{4}$$

Min or Max point

$$\frac{dV}{dx} = 0 = -\alpha x + \beta x^3$$

Min or Max : $x = 0$ or

$$x = \pm \sqrt{\alpha / \beta}$$

$$\boxed{\frac{d^2 V}{dx^2} = -\alpha + 3\beta x^2}$$

$$\left.\frac{d^2 V}{dx^2}\right|_{x=0} = -\alpha$$

assume: $\alpha \geq 0 \Rightarrow \left.\frac{d^2 V}{dx^2}\right|_{x=0}$

$\beta \geq 0$ is a max point

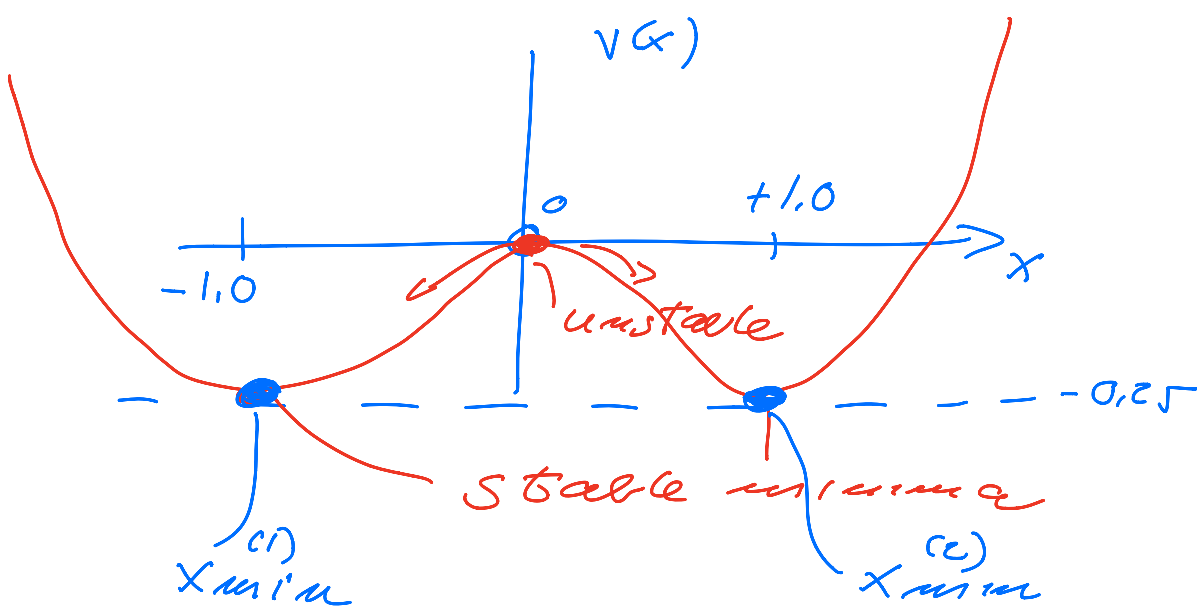
$$\left.\frac{d^2V}{dx^2}\right|_{x=\pm\sqrt{\alpha/\beta}} = -\alpha + 3\cancel{\beta}\,\alpha/\cancel{\beta}$$

$$= 2\alpha > 0$$

$x = \pm\sqrt{\alpha/\beta}$ is a min point.

$\underline{\alpha = \beta = 1}$

$x = \pm 1$ or $x = 0$



Taylor expand around xmin

$$V(x) \simeq \underline{V(x_{min})} + \cancel{(x - x_{min})\left.\frac{dV}{dx}\right|_{x_{min}}}$$

$$+\ \frac{1}{2}(x-x_{min})^2 \frac{d^2V}{dx^2}\Big|_{x_{min}} \ {}^{,,}0$$

$$\frac{d^2V}{dx^2}\Big|_{x_{min}=\sqrt{\alpha/\beta}}$$

$$= -\alpha + 3\beta x_{min}^2$$

$$= +2\alpha$$

$$F(x) = -\frac{dV}{dx} = -(x-x_{min}) \times 2\alpha$$

Hookes Law : $\boxed{F = -kx}$

$$\boxed{k = 2\alpha}$$

$$\frac{d^2V}{dx}\Big|_{x_{min}}$$

$$F(x) = -kx$$

$$x(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$$