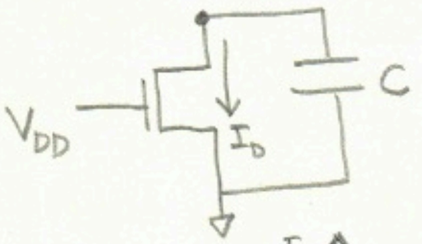
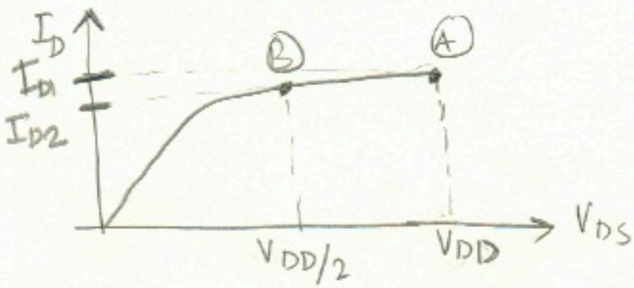
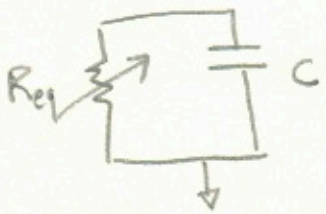
Equivalent resistance when (DIS) charging through a transistor



The transistor (n-type) turns on when its gate voltage is V_{DD}.



Represent the transistor-capacitor circuit as an R-C network as follows:-



What is R_{eq}?

R_{eq}:- Equivalent on-resistance of the transistor

$$R_{eq} = \frac{1}{2}\left[R_{on,\,initial} + R_{on,\,final}\right]$$

$R_{on,\,initial}$ $\Downarrow$ when the output is at V_{DD} [Point A in figure]

$R_{on,\,final}$ $\Downarrow$ when the output is at $\frac{V_{DD}}{2}$ [Point B in figure]

$$R_{on,\,initial} = \frac{V_{DD}}{I_{D1}} = \frac{V_{DD}}{I_{Dsat}\,(1+\lambda V_{DD})}$$

$$R_{on,\,final} = \frac{V_{DD}/2}{I_{D2}} = \frac{V_{DD}/2}{I_{Dsat}\,(1+\lambda V_{DD}/2)}$$

$$I_{Dsat} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left[(V_{GS}-V_T)\right]^2$$

so,

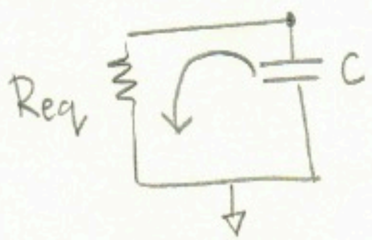
$$R_{eq} = \frac{1}{2}\left[\frac{V_{DD}}{I_{Dsat}\,(1+\lambda V_{DD})} + \frac{V_{DD}/2}{I_{Dsat}\,(1+\lambda V_{DD}/2)}\right]$$

$\downarrow$

Do some algebra when $\lambda V_{DD} << 1$

we can show

$$\boxed{R_{eq} = \frac{3V_{DD}}{4\,I_{Dsat}}\left(1-\frac{5}{6}\lambda V_{DD}\right)}$$



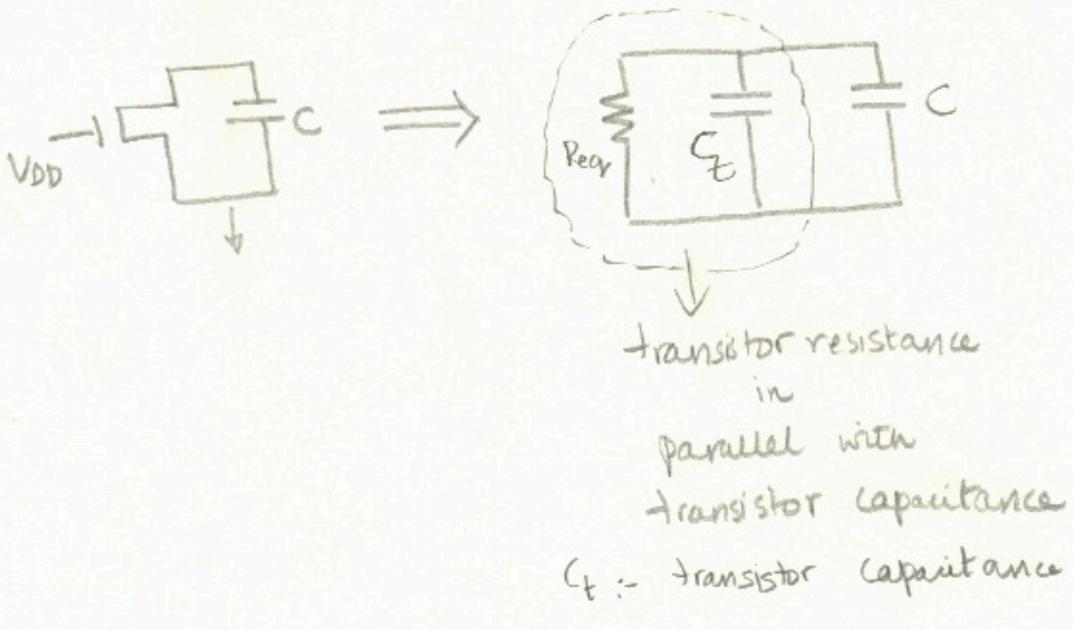
$$\tau = R_{eq}C$$
$$= \frac{3V_{DD}}{4\,I_{Dsat}}\left(1-\frac{5}{6}\lambda V_{DD}\right)$$

Increasing the transistor size increases I_{Dsat}.
Hence, it reduces Req and the time constant of charging and discharging a capacitor.

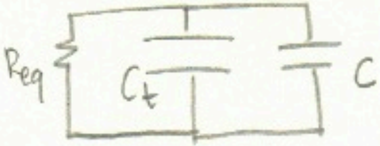
Drawback of increasing transistor size :-

of course, you're using more area on the die, which isn't good (its expensive!) but we'll see later that increasing transistor size also increases the transistor capacitance that is related with the structure of the transistor.

so in reality the equivalent circuit on the previous page should look like :-



C_t :- transistor capacitance.



time constant of discharging the capacitor will be

$$\tau = R_{eq}(C_t + C)$$

When $C_t \ll C$ then $\tau = R_{eq} C$

Both R_{eq} and C_t depend on transistor $\left(\frac{W}{L}\right)$

$\left(\frac{W}{L}\right) \uparrow \quad R_{eq} \downarrow \quad C_t \uparrow$

So one must choose $\left(\frac{W}{L}\right)$ wisely to obtain a certain 'τ' as specified by the end user.