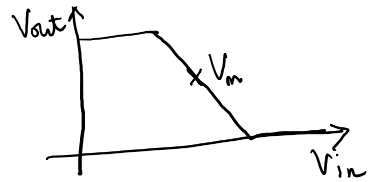
Calculating the gain of the transistor for noise margin.



$$\boxed{g = \frac{dV_{out}}{dV_{in}}}$$

Note:- Around V_m, both NFET and PFET are saturated.

$I_{PFET} = I_{NFET}$ always in D.C

$$I_{NFET} = \frac{1}{2}\beta_n\left[V_{in} - V_{TN}\right]^2\left[1 + \lambda_n V_{out}\right]$$

$$I_{PFET} = \frac{1}{2}\beta_p\left[V_{DD} - V_{IN} - |V_{TP}|\right]^2\left[1 + |\lambda_p|\left(V_{DD} - V_{out}\right)\right]$$

Lets take $\frac{dV_{out}}{dV_{in}}$ For NFET First

$$\frac{1}{2}\beta_n\left[V_{in} - V_{TN}\right]^2\left[\lambda_n\frac{dV_{out}}{dV_{in}}\right] + \beta_n\left[1 + \lambda_n V_{out}\right]\left[V_{in} - V_{TN}\right]$$

Now taking $\frac{dV_{out}}{dV_{in}}$ For PFET

$$\frac{1}{2}\beta_p\left[V_{DD} - V_{IN} - |V_{TP}|\right]^2\left[-|\lambda_p|\frac{dV_{out}}{dV_{in}}\right] - \beta_p\left[1 + |\lambda_p|\left(V_{DD} - V_{out}\right)\right]\left[V_{DD} - V_{in} - |V_{TP}|\right]$$

Now use $V_{in} \approx V_m$.

Substituting this in $\frac{dV_{out}}{dV_{in}}$ for the NFET device

$$\underbrace{\frac{1}{2}\beta_n\left[V_m - V_{TN}\right]^2}\left[\lambda_n\frac{dV_{out}}{dV_{in}}\right] + \underbrace{\beta_n\left[V_m - V_{TN}\right]}\left[1 + \lambda_n V_{out}\right]$$

Note that this is the current of NFET/PFET at threshold

this term is equal to $\sqrt{2 I_D(V_m)\beta_n}$

$I_D(V_m)$

$$I_D(V_m)\left[\lambda_n\frac{dV_{out}}{dV_{in}}\right] + \sqrt{2\beta_n I_D(V_m)}\left[1 + \lambda_n V_{out}\right]$$

Substituting $V_{in} = V_m$ in the equation for PFET

$$\underbrace{\frac{1}{2}\beta_p\left[V_{DD} - V_m - |V_{TP}|\right]^2}_{I_D(V_m)}\left[-|\lambda_p|\frac{dV_{out}}{dV_{in}}\right] - \underbrace{\beta_p\left[V_{DD} - V_m - |V_{TP}|\right]}_{\sqrt{2 I_D(V_m)\beta_p}}\left[1 + |\lambda_p|\left(V_{DD} - V_{out}\right)\right]$$

Putting everything together, we have

neglect

neglect

$$I_D(V_m)\lambda_n\frac{dV_{out}}{dV_{in}} + \sqrt{2 I_D(V_m)\beta_n}\left[1 + \cancel{\lambda_n V_{out}}\right]$$

$$= -I_D(V_m)|\lambda_p|\frac{dV_{out}}{dV_{in}} - \sqrt{2 I_D(V_m)\beta_p}\left[1 + \cancel{|\lambda_p|(V_{DD} - V_{out})}\right]$$

$$I_D(V_m)\,\lambda_n \frac{dV_{out}}{dV_{in}} \ldots$$

$$= -I_D(V_m)\,|\lambda_p| \frac{dV_{out}}{dV_{in}} - \sqrt{2 I_D(V_m)\, \beta_p \left[1 + |\lambda_p|(V_{DD} - V_{out})\right]}$$

$$I_D(V_m) \frac{dV_{out}}{dV_{in}} \left[\lambda_n - |\lambda_p|\right] = -\sqrt{2 I_D}\left[\sqrt{\beta_n} + \sqrt{\beta_p}\right]$$

$$\frac{dV_{out}}{dV_{in}} = -\frac{\sqrt{2 I_D(V_m)}}{I_D(V_m)} \; \frac{\sqrt{\beta_n}\left[1 + \sqrt{\beta_p/\beta_n}\right]}{(\lambda_n - |\lambda_p|)}$$

$$\frac{dV_{out}}{dV_{in}} = -\sqrt{\frac{2\beta_n}{I_D(V_m)}} \; \frac{\left[1 + \sqrt{\beta_p/\beta_n}\right]}{\left[\lambda_n - |\lambda_p|\right]}$$

$$I_D(V_m) = \frac{1}{2}\beta_n (V_m - V_{Tn})^2$$

$$\boxed{\therefore \frac{dV_{out}}{dV_{in}} = -\frac{2\left(1 + \sqrt{\beta_p/\beta_n}\right)}{(V_m - V_{Tn})\left[\lambda_n - |\lambda_p|\right]}}$$

Final Answer