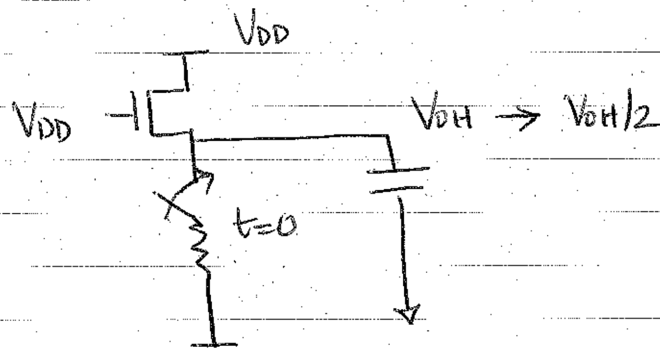
Homework #2 SOLUTIONS

1)



a) @ $t=0^+$, $V_{OH} = V_{DD} - V_{T1}$

V_{T1} = Threshold voltage at $V_{SB} = V_{OH}$

$$V_{T1} = V_{T0} + \gamma\left(\sqrt{2|\phi_F| + V_{SB}} - \sqrt{2|\phi_F|}\right)$$

$$V_{T1} = 0.43 + 0.4\left(\sqrt{2\times0.3 + V_{DD} - V_{T1}} - \sqrt{0.6}\right)$$

This is a quadratic equation in V_{T1}. When you solve this, you get two roots for V_{T1}. Select only the positive root as V_{T1} for NMOS is always positive.

$$V_{T1} = 0.735V$$

$$V_{OH} = V_{DD} - V_{T1} = 2.5 - 0.735 = 1.765V$$

$$\boxed{V_{OH} = 1.765V}$$

b) Since $V_{OH} = 1.765V$, $V_{OH}/2 = 0.88V$

V_{T2} @ $V_{SB} = 0.88V$

$$V_{T2} = V_{T0} + \gamma\left(\sqrt{2|\phi_F| + V_{SB}} - \sqrt{2|\phi_F|}\right)$$

$$V_{T2} = 0.43 + 0.4\left(\sqrt{0.6 + 0.88} - \sqrt{0.6}\right)$$

$$V_{T2} = 0.607\ V$$

$$V_{T,avg} = \frac{V_{T1} + V_{T2}}{2} = 0.671\,V$$

$$R_{avg} = \frac{1}{2}\Big(\underbrace{R_a}_{@\,V_{OH}} + \underbrace{R_b}_{@\,\frac{V_{OH}}{2}}\Big)$$

At both $V_{out} = V_{OH}$ & $\frac{V_{OH}}{2}$, NMOS device is in saturation mode of operation. So resistance must be calculated using appropriate models.

$$R_a = \frac{V_{DD} - V_{OH}}{\frac{1}{2}\beta_n\left(V_G - V_{OH} - V_{Tavg}\right)^2}$$

$$R_a = \frac{0.735}{0.5 \times 115 \times 10^{-6}\left(0.735 - 0.671\right)^2}$$

$$R_a = 3.1208 \times 10^{6}\,\Omega$$

Notice, R_a is very high. This is to be expected since for $V_{out} = V_{OH}$, the NMOS device is nearly in cut-off.

$$R_b = \frac{V_{DD} - V_{OH}/2}{\frac{1}{2}\beta_n\left(V_{DD} - \frac{V_{OH}}{2} - V_{Tavg}\right)^2}$$

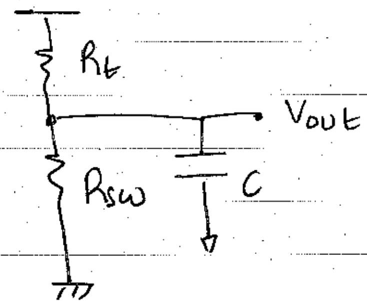
$$R_b = \frac{2.5 - 0.88}{\frac{1}{2} \times 115 \times 10^{-6}\left(2.5 - 0.88 - 0.671\right)^2}$$

$$R_b = 3.128 \times 10^{4}\,\Omega$$

$$R_{avg} = 0.5\left(3.1208 \times 10^6 + 3.128 \times 10^4\right)$$

Ans:- $\boxed{R_{avg} = 1.5760 \times 10^6 \ \Omega}$

(c) At $t \to \infty$

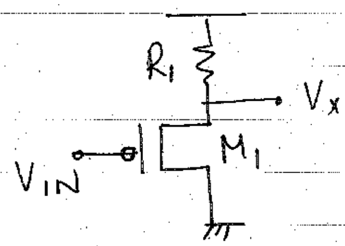


$$V_{out} = \frac{V_{DD}\, R_{sw}}{R_{sw} + R_t}$$

R_t = equivalent resistance of the transistor.

Under the condition $R_{sw} << R_t$, $V_{out} \to 0$

PROBLEM #2



$V_{SG} = V_X - V_{IN}$

$V_{SD} = V_X$

$V_{SDsat} = V_X - V_{IN} - |V_{TP}|$

$\Downarrow$

$V_{SD} > V_{SDsat}$

"M1" in saturation

$$I = \frac{V_{DD} - V_X}{R_1} = I_{SDsat}$$

$$I_{SDsat} = \frac{1}{2}\beta_P \left(V_X - V_{in} - |V_{TP}|\right)^2 \left(1 + |\lambda_P| V_X\right)$$

Equating the two, we get

$$\frac{1}{2}\beta_P \left(V_X - V_{IN} - |V_{TP}|\right)^2 \left(1 + |\lambda_P| V_X\right) = \frac{V_{DD} - V_X}{R_1}$$

* To a first order, lets ignore the channel-length modulation.

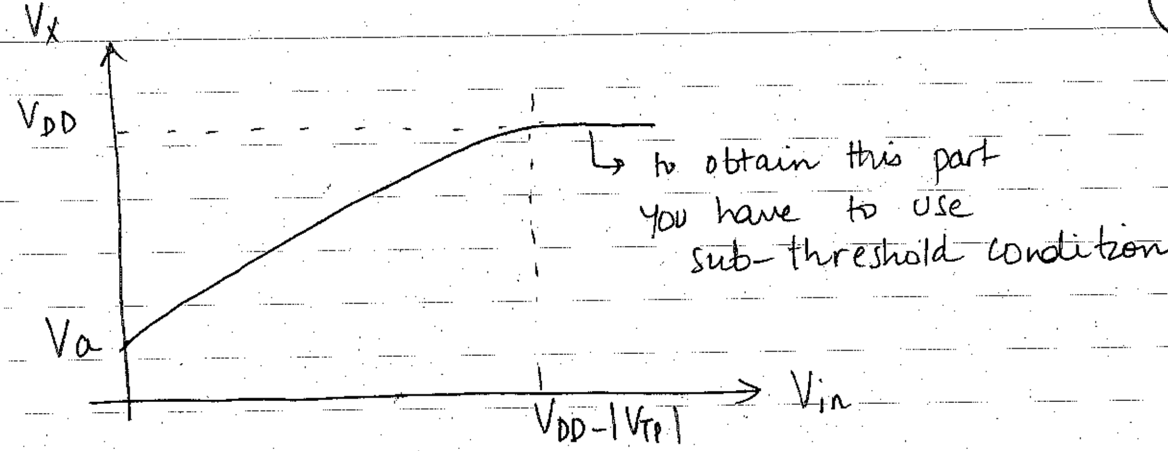
You don't have to ignore it, but it simplifies the analysis a bit.

$$V_{in} = V_X - |V_{TP}| - \underbrace{\sqrt{\frac{2}{\beta_P}\frac{(V_{DD} - V_X)}{R_1}}}_{>0}$$

So $V_{in} < V_X - |V_{TP}|$

For $V_{in} = 0 \Rightarrow V_X - |V_{TP}| = \sqrt{\frac{2}{\beta_P}\frac{(V_{DD} - V_X)}{R_1}}$

The value of V_X will be a non-zero quantity depending on the solution of the above quadratic equation in V_X.



V_a is the initial value of V_x that you can obtain for $V_{in} = 0$.

The general shape of the plot remains the same even if you were to consider body effect as long as $|\lambda_p| V_x << 1$.

Also, if the body and source are not tied together then the device will be under body effect.

(b) solution without body effect :—

$$\frac{\beta_p}{2}\left(V_x - |V_{TP}|\right)^2 \left(1 + |\lambda_p| V_x\right) = \frac{V_{DD} - V_x}{R_1}$$

$V_x = 1.5V$

$\beta_p = K_p (W/L)_p$

$$(W/L)_p = \frac{2 (2.5 - 1.5)}{30 \times 10^{-6} \quad 20 \times 10^{3} \times (1.5 - 0.4)^2 \; (1 + 0.1 \times 1.5)}$$

$\boxed{\left(\frac{W}{L}\right)_p = 2.395} \rightarrow$ without body effect

Solution w/ body effect

$$|V_{TP}| = |V_{T0}| + |\gamma| \left[\sqrt{2\phi_f - V_{SB}} - \sqrt{2\phi_f} \right]$$

$$|V_{TP}| = 0.4 + 0.4 \left[\sqrt{0.6 + V_{DD} - V_X} - \sqrt{0.6} \right]$$

$$|V_{TP}| = 0.4 + 0.4 \left[\sqrt{1.6} - \sqrt{0.6} \right]$$

$$|V_{TP}| = 0.5961\ V$$

$$\frac{\beta_p}{2} \left(V_X - |V_{TP}| \right)^2 \left(1 + |\lambda_p| V_X \right) = \frac{V_{DD} - V_X}{R_1}$$

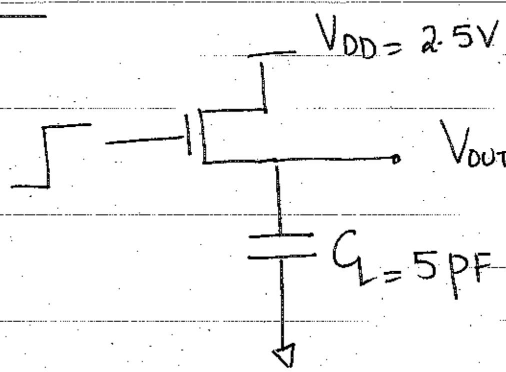
$$(W/L)_p = \frac{2\,(2.5 - 1.5)}{30\times10^{-6} \times 20 \times 10^{-3} (1.5 - 0.5961)^2 (1.15)}$$

$$= 3.5476$$

$$\boxed{\left(\frac{W}{L}\right)_p = 3.5476}$$ with body effect.

You will receive credit for either one of those solutions.

PROBLEM #3



$t_{PLH} = 0.69\, R_{eq}\, C_L$

Note :- V_{out} varies from 0 to $V_{DD} - V_T$.
To find the charging time, we calculate Resistance at the beginning of the transition and half way through the transition at the output node.

$$R_{eq} = 0.5\left(R_a + R_b\right)$$

R_a at $V_{OUT} = 0$ $\quad$ R_b at $V_{OUT} = \dfrac{V_{DD} - V_T}{2}$

$$R_a = \frac{V_{DD}}{\frac{1}{2}\beta_n \left(V_{DD} - V_{TO}\right)^2 \left(1 + \lambda V_{DD}\right)}$$

note at initial point there is no body effect. since $V_{SB} = 0$

$$R_a = \frac{2.5}{\frac{1}{2} \times 115 \times 10^{-6} \times \frac{20}{2}\left(2.5 - 0.43\right)^2 \left(1 + 0.06 \times 2.5\right)}$$

$R_a = 882.3353\ \Omega$

To calculate R_b, we will need to calculate V_T when $V_{SB} = \dfrac{V_{DD} - V_T}{2}$ due to the body effect.

#8

$$V_T = 0.43 + 0.4\left(\sqrt{0.6 + \frac{2.5 - V_T}{2}} - \sqrt{0.6}\right)$$

You get a quadratic equation in V_T, which you can solve for V_T.

$V_T = 0.6167\,V$ so $V_{DD} - V_T = \cancel{0.6}\ 2.5 - 0.6167 = 1.883\,V$

$$R_b = \frac{V_{DD} - \left(\frac{V_{DD}-V_T}{2}\right)}{\frac{1}{2}\beta_n\left(V_{DD} - \frac{V_{DD}-V_T}{2} - V_T\right)^2\left(1 + \lambda\left(\frac{V_{DD}+V_T}{2}\right)\right)}$$

$$R_b = \frac{2.5 - 1.883/2}{\frac{1}{2} \times 115 \times 10^{-6} \times \frac{20}{2}\left(2.5 - \frac{1.883}{2} - 0.6167\right)^2\left(1 + 0.06 \times \cancel{\frac{1.883}{2}}\ 1.5583\right)}$$

$$R_b = \frac{1.5585}{5.385 \times 10^{-4}} = \cancel{2894.2}\ 2794.5\ \Omega$$

$$R_{eq} = \frac{1}{2}(R_a + R_b) = \frac{1}{2}(882.3 + \cancel{2894.2}\ 2794.5) = \cancel{1888.7}\ 1838.4\ \Omega$$

$$t_{pLH} = 0.69\, R_{eq}\, C_L$$

$$t_{pLH} = 0.69 \times 1888.7 \times 5 \times 10^{-12} = \cancel{6.516 \times 10^{-9}\,s}\ 6.34\text{ ns}$$

Ans:- $\boxed{t_{pLH} = \cancel{6.516}\ 6.34\text{ ns}}$

(b) If the load capacitor scales with device width delay is Unaffected

Since $R_{eq} \propto 1/W$ while $C_L \propto W$

$t_{pLH} \Rightarrow$ constant with width

(c) $t_{pHL} = 0.69 \times 5k\Omega \times 5pF = 17.25ns$

(d) If the NFET is replaced with PFET, we compute its equivalent resistance as the output charges from $V_{out} = 0$ to $V_{out} = \frac{V_{DD}}{2}$. Note, for PFET, the O/P goes all the way up to V_{DD}. Hence, we calculate the equivalent resistance at $V_{out} = 0$ & $V_{out} = V_{DD}/2$

In class, we obtained the equivalent resistance of the PFET device as

$$R_{eq} = \frac{3}{4}\frac{V_{DD}}{I_{Dsat}}$$

$$I_{Dsat} = \frac{1}{2} \times \beta_p \times \left(V_{DD} - |V_{Tp}|\right)^2$$

Now, it is mentioned that $\beta_p = \beta_n$

$$\beta_p = 115 \times 10^{-6} \times \left(\frac{20}{2}\right)$$

$$I_{Dsat} = \frac{1}{2} \times 115 \times 10^{-6} \times \frac{20}{2} \times \left(2.5 - 0.4\right)^2$$

$I_{Dsat} = 2.5 \times 10^{-3}$ A

$R_{eq} = \frac{3}{4} \times \frac{2.5}{2.5 \times 10^{-3}} = 750\Omega$

$t_{PLH} = 0.69 \times 750 \times 5 \times 10^{-12} = \underline{2.5875 \text{ ns}}$

Hence, in this case the PFET circuit will be faster since we have the same $\underline{\beta p = \beta n}$.

NOTE:- You will receive credit even if you haven't quantified how much faster PFET charging is, if you have qualitatively explained the solution.

Qualitatively the circuit becomes faster because the PFET sees an overall larger "turn on" as opposed to the NFET. The larger drop happens because $|V_{TP}| < V_{TN}$. And second PMOS has a good pull-up to "1" so the capacitor can go all the way up to V_{DD}.

Both of these effects combined help to lower the equivalent resistance of PMOS and hence the delay for PMOS will be lower than that for NMOS.

Common mistakes

Problem 3a) When you want to calculate equivalent resistance, you need 'V_T' at the middle point of the transition at the output node.

If the o/p node goes from $\underline{0}$ to "$\underline{V_{DD}-V_T}$"

then you need to calculate "V_T" From "0" to "$\frac{V_{DD}-V_T}{2}$" as the swing.

Also the definition of equivalent resistance is $\frac{1}{2}(\underset{\text{initial}}{R_a} + \underset{\text{half way through the transition}}{R_b})$

$$R_b = \frac{V_{DS}}{\frac{1}{2}\beta\left(V_{GS}-V_T\right)^2\left(1+\lambda V_{DS}\right)}$$

Note V_{GS} will be $V_{IN} - \left(\frac{V_{DD}-V_T}{2}\right)$

V_{DS} will be $V_{DD} - \left(\frac{V_{DD}-V_T}{2}\right)$

Several students did not use the right values of V_{DS} and V_{GS}

Several students calculated "V_T" incorrectly.

But if you understood the basic concept & show steps, you will receive credit for it.