HW-2

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Q1

a

As $V_{GS} = V_{DD} - V_{OH}$ and only when $V_{OH} \leq V_{DD} - V_{Th}$ could guarentee the transistor is turned on. And the maximum value is the output voltage before t = 0.

$$\begin{aligned} V_{OH} &= V_{DD} - V_{Th} \\ &= V_{DD} - V_{T0} - \gamma(\sqrt{2|\phi_F| + V_{OH}} - \sqrt{2|\phi_F|}) \end{aligned} \tag{1}$$

Therefore,

$$2|\phi_F| + V_{OH} + \gamma(\sqrt{2|\phi_F| + V_{OH}}) - (V_{DD} - V_{T0} + \sqrt{2|\phi_F|} + 2|\phi_F|) = 0 \tag{2}$$

So,

$$\sqrt{2|\phi_F| + V_{OH}} = -\frac{\gamma}{2} + \sqrt{\frac{\gamma^2}{4} + V_{DD} - V_{T0} + \gamma\sqrt{2|\phi_F|} + 2|\phi_F|} \tag{3}$$

Then, finally:

$$\begin{aligned} V_{OH} =& \frac{\gamma^2}{2} + V_{DD} - V_{T0} + \gamma\sqrt{2|\phi_F|} \\ &- \gamma\sqrt{\frac{\gamma^2}{4} + V_{DD} - V_{T0} + \gamma\sqrt{2|\phi_F|} + 2|\phi_F|} \\ =& V_{DD} - 0.4 \times \sqrt{V_{DD} + 0.52} - 0.04(V) \end{aligned} \tag{4}$$

c

When $t \to \infty$,

$$\begin{aligned} V_{out} &= I_{DSAT} R_{SW} \\ &= \frac{1}{2}\kappa\frac{W}{L}[V_{DD} - V_{out} - V_{T0} - \gamma(\sqrt{2|\phi_F| + V_{out}} - \sqrt{2|\phi_F|})]^2 R_{SW} \\ &= \end{aligned} \tag{5}$$

b

As informed in the question, V_{Th} is constant and is the average of its maximum and minimum, which is

$$\begin{aligned} V_{Th} &= V_{T0} + \frac{1}{2}\gamma(\sqrt{2|\phi_F| + V_{OH}} + \sqrt{2|\phi_F| + V_{out}}) - \gamma\sqrt{2|\phi_F|} \\ &= \end{aligned} \tag{6}$$

As $V_{out} = V_{OH} \to V_{OH}/2$, $V_{DS} = V_{DD} - V_{OH} \to V_{DD} - V_{OH}/2$. Thus,

$$\begin{aligned} R_{eq} &= \frac{1}{2}\left[\frac{V_{DD} - V_{OH}}{\frac{1}{2}\kappa(V_{DD} - V_{OH} - V_{Th})^2} + \frac{V_{DD} - V_{OH}/2}{\frac{1}{2}\kappa(V_{DD} - V_{OH}/2 - V_{Th})^2}\right] \\ &= \frac{V_{DD} - V_{OH}}{\kappa(V_{DD} - V_{OH} - V_{Th})^2} + \frac{V_{DD} - V_{OH}/2}{\kappa(V_{DD} - V_{OH}/2 - V_{Th})^2} \\ &= \end{aligned} \tag{7}$$

Q2

$$\begin{aligned} V_X =& V_{DD} - I_{SD}R_1 \\ =& V_{DD} + \frac{1}{2}\kappa\frac{W}{L}(V_X - V_{in} + V_{Th})^2[1 + \lambda(V_X - V_{in})]R_1 \\ =& V_{DD} + \frac{1}{2}\kappa\frac{W}{L}[V_X - V_{in} + V_{T0} + \gamma(\sqrt{2|\phi_F| + V_X} - \sqrt{2|\phi_F|})]^2 \\ & [1 + \lambda(V_X - V_{in})]R_1 \end{aligned} \tag{8}$$

a

b

$$\begin{aligned} 1.5(V) =& 2.5(V) - 0.5 \times (-30 \times 10^{-6})(A/V^2) \times \frac{W}{L} \times [1.5(V) - 0.4(V) \\ & - 0.4(V^{0.5}) \times (\sqrt{0.6 + 1.5} - \sqrt{0.6})]^2 \times (1 - 0.1(V^{-1}) \\ & \times 1.5(V)) \times 20 \times 10^3(\Omega) \end{aligned} \tag{9}$$

Thus,

$$\frac{W}{L} = 56.9 \tag{10}$$

Q3

a

$$\begin{aligned} t_{tLH} &= 0.69R_{eq}C_L \\ &= \end{aligned} \tag{11}$$

b

$$\begin{aligned} \Delta t_{pLH} &= 0.69R_{eq}C_0W_0 \\ &= \end{aligned} \tag{12}$$

c

$$
\begin{aligned}
t_{pHL} &= 0.69RC_L \\
&=
\end{aligned} \tag{13}
$$

d

With $\beta_n = \beta_p$, and $|\gamma_n| = |\gamma_p|$ with $|\lambda_n| = |\lambda_p|$ from the table coming with questions. $\frac{t_{pLH,n}}{t_{pLH,p}}$ becomes,

$$
\begin{aligned}
\frac{t_{pLH,n}}{t_{pLH,p}} &= \frac{R_{eq,n}}{R_{eq,p}} \\
&= \frac{1 - \lambda_n(V_{DD} - V_{out})}{1 - |\lambda_p|(V_{DD} - V_{out})}
\end{aligned} \tag{14}
$$

With situation mentioned above, $R_{eq,n} > R_{eq,p}$, thus it will be slower usein PFET.

Q4