

## HW-2

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### Q1

**a**

As  $V_{GS} = V_{DD} - V_{OH}$  and only when  $V_{OH} \leq V_{DD} - V_{Th}$  could guarantee the transistor is turned on. And the maximum value is the output voltage before  $t = 0$ .

$$\begin{aligned} V_{OH} &= V_{DD} - V_{Th} \\ &= V_{DD} - V_{T0} - \gamma(\sqrt{2|\phi_F| + V_{OH}} - \sqrt{2|\phi_F|}) \\ &= 1.8(V) \end{aligned} \quad (1)$$

**b**

As informed in the question,  $V_{Th}$  is constant and is the average of its maximum and minimum, which is

$$\begin{aligned} V_{Th} &= V_{T0} + \frac{1}{2}\gamma(\sqrt{2|\phi_F| + V_{OH}} + \sqrt{2|\phi_F| + V_{OH}/2}) - \gamma\sqrt{2|\phi_F|} \\ &= 0.67(V) \end{aligned} \quad (2)$$

As  $V_{out} = V_{OH} \rightarrow V_{OH}/2$ ,  $V_{DS} = V_{DD} - V_{OH} \rightarrow V_{DD} - V_{OH}/2$ . Thus,

$$\begin{aligned} R_{eq} &= \frac{1}{2} \left[ \frac{V_{DD} - V_{OH}}{\frac{1}{2}\kappa(V_{DD} - V_{OH} - V_{Th})^2} + \frac{V_{DD} - V_{OH}/2}{\frac{1}{2}\kappa(V_{DD} - V_{OH}/2 - V_{Th})^2} \right] \\ &= \frac{V_{DD} - V_{OH}}{\kappa(V_{DD} - V_{OH} - V_{Th})^2} + \frac{V_{DD} - V_{OH}/2}{\kappa(V_{DD} - V_{OH}/2 - V_{Th})^2} \\ &= 6.8 \times 10^6(\Omega) \end{aligned} \quad (3)$$

**c**

When  $t \rightarrow \infty$ ,

$$\begin{aligned} V_{out} &= I_{DSAT}R_{SW} \\ &= \frac{1}{2}\kappa[V_{DD} - V_{out} - V_{T0} - \gamma(\sqrt{2|\phi_F| + V_{out}} - \sqrt{2|\phi_F|})]^2 R_{SW} \\ &= \end{aligned} \quad (4)$$

## Q2

$$\begin{aligned} V_X &= V_{DD} - I_{SD}R_1 \\ &= V_{DD} + \frac{1}{2}\kappa\frac{W}{L}(V_X - V_{in} + V_{T0})^2(1 + \lambda V_X)R_1 \end{aligned} \quad (5)$$

**a**

**b**

$$\frac{W}{L} = 1.1 \quad (6)$$

## Q3

**a**

It is clear that  $V_{X,max} = V_{DD} - V_{Th}$ . And

$$\begin{aligned} V_{Th} &= V_{T0} + \frac{1}{2}\gamma(\sqrt{2|\phi_F| + V_{X,max}/2} - \sqrt{2|\phi_F|}) \\ &= 0.53(V) \end{aligned} \quad (7)$$

Thus,  $V_{X,max} = 1.97(V)$ . As well as

$$\begin{aligned} R_{eq} &= \frac{1}{2}R_{eq,max} \\ &= \frac{V_{DD} - \frac{V_{X,max}}{2}}{\kappa\frac{W}{L}(V_{DD} - V_{X,max}/2 - V_{Th})^2[1 + \lambda(V_{DD} - V_{X,max}/2)]} \\ &= 829.7861(\Omega) \end{aligned} \quad (8)$$

Thus,

$$\begin{aligned} t_{pLH} &= R_{eq}C_L \ln \frac{V_{DD}}{V_{DD} - V_{out}} \\ &= R_{eq}C_L \ln \frac{2V_{DD}}{V_{DD} + V_{Th}} \\ &= 2.0781(ns) \end{aligned} \quad (9)$$

**b**

$$\begin{aligned} \Delta t_{pLH} &= 0.69R_{eq}C_0W_0 \\ &= t_0 \end{aligned} \quad (10)$$

**c**

$$\begin{aligned} t_{pHL} &= 0.69RC_L \\ &= 17.25(ns) \end{aligned} \quad (11)$$

**d**

With  $\beta_n = \beta_p$ , and  $|\gamma_n| = |\gamma_p|$  with  $|\lambda_n| = |\lambda_p|$  from the table coming with questions.  $\frac{t_{pLH,n}}{t_{pLH,p}}$  becomes,

$$\begin{aligned} \frac{t_{pLH,n}}{t_{pLH,p}} &= \frac{R_{eq,n}}{R_{eq,p}} \frac{\ln \frac{2V_{DD}}{V_{DD}+V_{Tn}}}{\ln \frac{2V_{DD}}{V_{DD}+V_{Tp,0}}} \\ &= \frac{(V_{DD} + V_{Tn})(V_{DD} - V_{Tp,0})^2 [1 + \frac{1}{2}\lambda_p(V_{DD} + V_{Tp,0})] \ln \frac{2V_{DD}}{V_{DD}+V_{Tn}}}{(V_{DD} + V_{Tp,0})(V_{DD} - V_{Tn})^2 [1 + \frac{1}{2}\lambda_n(V_{DD} + V_{Tn})] \ln \frac{2V_{DD}}{V_{DD}+V_{Tn}}} \\ &= 1.3001 \end{aligned} \tag{12}$$

Which means, using PMOS will make it a bit faster.

## Q4

**a**

For  $V_M = 0.5V_{DD}$ ,  $\frac{W_n}{W_p} = \frac{90}{127.50575} \approx \frac{90}{127.5}$ .

90/90	0.5576
90/100	0.5715
90/110	0.5819
90/120	0.5925
90/127.5	0.6
90/140	0.6116
90/150	0.6202
90/160	0.6283
90/170	0.6357

**b**

For high-to-low delay equals low-to-high delay,  $\frac{W_n}{W_p} = \frac{90}{135.75}$ .

**c**

((The following table need to be plotted)))

D_input(ps)	t_pHL(ps)	t_pLH(ps)
0.1	27	27
50	32	27
100	39	27
150	44	27
200	48	27
500	65	27

Quite clear that high-to-low delay is

positively related with input rise time. And in fact it is still a fuction of input.

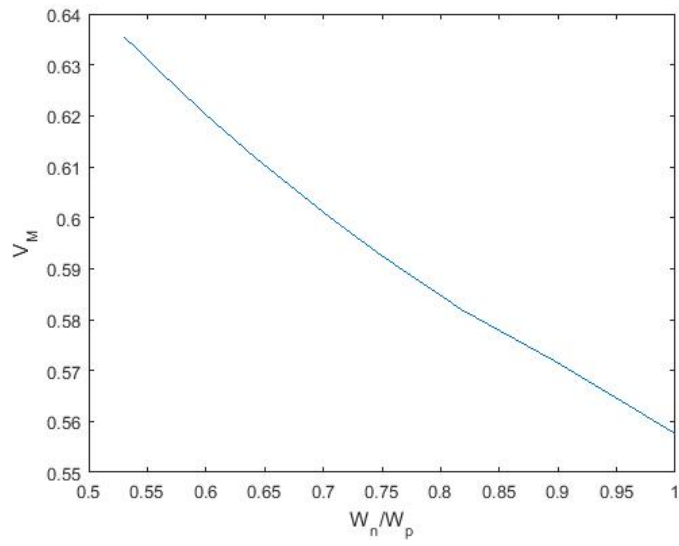


Figure 1: Q4.a

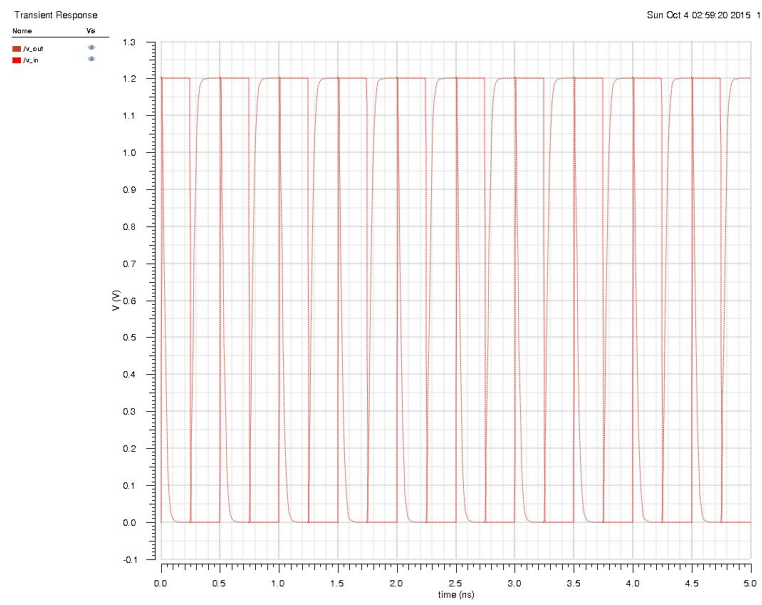


Figure 2: Q4.b

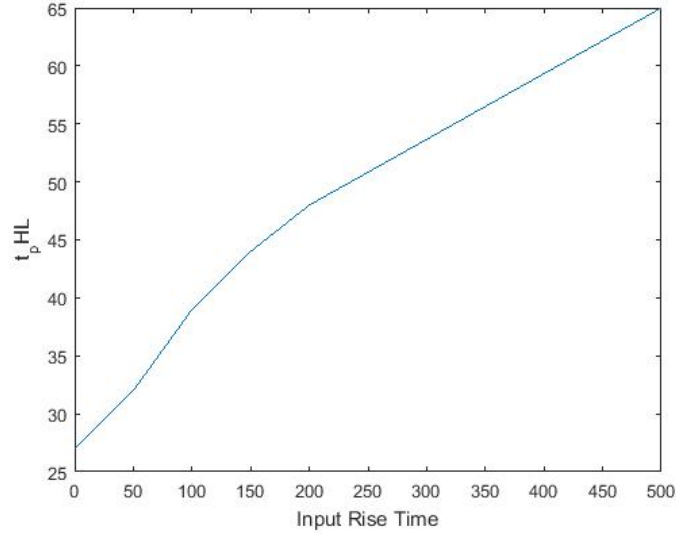


Figure 3: Q4.c

**d**

Except for high-to-low delay increased to 67ps, I saw no suspicious things. And here is my paramiters:

PMOS	L	50nm
	W	542.8nm
NMOS	L	50nm
	W	90nm
V_dd		1.2V
V_in	rise time	200ps
	fall time	10ps
	pulse width	235ps
	period	880ps
	voltage 1	0V
	voltage 2	1.2V
Cap		5fF

**e**

For what I got, it is the same as the result from part (b). And have no conflict with part(c).

$$\begin{aligned}
 \frac{t_{pHL}}{t_{pLH}} &= \frac{R_n}{R_p} \\
 &= \frac{I_{DSAT,p}(1 - \frac{7}{9}\lambda_n V_{DD})}{I_{DSAT,N}(1 - \frac{7}{9}\lambda_p V_{DD})} \\
 &= \frac{\kappa_p W_p (V_{DD} - V_{T0})^2 (1 - \frac{7}{9}\lambda_n V_{DD})}{\kappa_n W_n (V_{DD} - V_{Tn})^2 (1 - \frac{7}{9}\lambda_p V_{DD})}
 \end{aligned} \tag{13}$$

Thus the ratio has nothing to do with input rise/fall time. But, ofcourse, signal frequency become too high, output will go wild.