HW-3

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Problem 1

a

$$Y = \overline{(B + A) \cdot C \cdot D} \tag{1}$$

b

As from the figure, suppose β_N denodes β for NMOS used in this design, β_n denodes β for NMOS in equivalent generic design. And β_P and β_p respectively. Thus we have $\beta_P = 2\beta_p$ and $\beta_N = 3\beta_n$. Thus for MOSFET really used in this design

$$\begin{aligned} \frac{W}{L}_{NMOS} &= 3 \\ \frac{W}{L}_{PMOS} &= 4 \end{aligned} \tag{2}$$

c

Suppose internal capacitance for each NMOS is C_n, and C_p for PMOS respectively.

Initial State	Final State	Delay
$A_1B_0C_1D_0$	$A_1B_0C_1D_1$	$t_{pHL} \propto 3R_NC_N + R_NC_Y$
$A_0B_1C_1D_0$	$A_0B_1C_1D_1$	
$A_1B_1C_1D_0$	$A_1B_1C_1D_1$	
$A_0B_1C_1D_1$	$A_0B_0C_1D_1$	$t_{pLH} \propto R_PC_P + R_PC_Y$

Problem 2

a

$$\begin{aligned} E_{total} &= E_r + uE_r + ... + u^N E_r \\ &= E_r \frac{u^{\frac{ln \frac{C_L}{C_1}}{lnu}+1} - 1}{u-1} \end{aligned} \tag{3}$$

b

For each inverter, there is $t_{pj} = u\tau_r$. Thus for each step, Energy-Delay Product is $EDP_j = ju^j E_r u\tau_r = ju^{j+1}E_r\tau_r = ju^{j+1}EDP_{ref}$. Thus,

$$\begin{aligned} EDP_{chain} &= EDP_{ref} + u^2 EDP_{ref} + 2u^3 EDP_{ref} + ... + Nu^{N+1}EDP_{ref} \\ &= EDP_{ref}(1 + \frac{u^2[(\frac{lnC_L/C_1}{lnu}(u-1)-1)u^{\frac{lnC_L/C_1}{lnu}}+1]}{(u-1)^2}) \end{aligned} \tag{4}$$

Thus, normalized Energy-Delay Product is,

$$\frac{EDP_{chain}}{EDP_{ref}} = 1 + \frac{u^2[(\frac{lnC_L/C_1}{lnu}(u-1)-1)u^{\frac{lnC_L/C_1}{lnu}}+1]}{(u-1)^2} \tag{5}$$

For optimal 'u' of minimum normalized EDP,

$$\begin{aligned} 0 =& \frac{u^2[u^{\frac{lnC_L/C_1}{lnu}}(\frac{lnC_L/C_1}{lnu} - \frac{(u-1)lnC_L/C_1}{uln^2u}) + (\frac{u^{\frac{lnC_L/C_1}{lnu}-1}lnC_L/C_1}{lnu} - \frac{u^{\frac{lnC_L/C_1}{lnu}}lnC_L/C_1}{ulnu})(\frac{(u-1)lnC_L/C_1}{lnu}-1)]}{(u-1)^2} \\ &+ \frac{2u[u^{\frac{lnC_L/C_1}{lnu}}(\frac{(u-1)lnC_L/C_1}{lnu}-1)]}{(u-1)^2} \\ &+ \frac{2u^2[u^{\frac{lnC_L/C_1}{lnu}}(\frac{(u-1)lnC_L/C_1}{lnu}-1)]}{(u-1)^3} \end{aligned} \tag{6}$$

c

For condition of (b), $u \approx 3.996934$. For optimal 'u' for minimum delay, $u = e \approx 2.718282$.

Problem 3

With $r = 2.5$,

$$\begin{aligned}
h_0 &= \frac{C_1}{C}, g_0 = g_{NOR2} = \frac{1+2r}{1+r} = \frac{12}{7} \\
h_1 &= \frac{C_2}{C_1}, g_1 = g_{NAND2} = \frac{2+r}{1+r} = \frac{9}{7} \\
h_2 &= \frac{C_{cout}}{C_2}, g_2 = g_{NOR2} = \frac{12}{7} \\
H &= \frac{C_{out}}{C} = 10 \\
B &= b_0 = \frac{62}{27} \\
F &= GHB = \frac{6247}{72} \to f \approx 4.42703 \\
f_0 &= b_0 g_0 h_0 = \frac{248}{63} h_0 = f \to \frac{C_1}{C} \approx 1.11774 \\
f_1 &= g_1 h_1 = \frac{9}{7} h_1 = f \to \frac{C_2}{C_1} \approx 3.42222 \\
f_2 &= g_2 h_2 = \frac{12}{7} h_2 = f \to \frac{C_{out}}{C_2} \approx 2.56667
\end{aligned} \tag{7}$$

Thus,

$$\begin{aligned}
\frac{C_1}{C} &\approx 1.12 \\
\frac{C_2}{C} &\approx 3.86
\end{aligned} \tag{8}$$