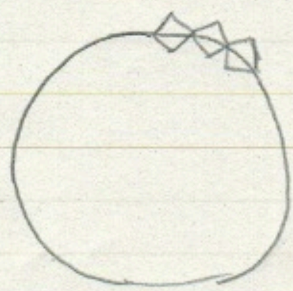
Homework #1 SOLUTION

1 a) If the entire wafer was usable, then the no. of dies will be $\frac{\pi w r^2}{de^2}$

However, because of the shape, the dies around the circumference are rendered useless.

Some of the incomplete edges have lost a lot of their die area, while some have lost only a little. So lets say, on an average, an imperfect die would have lost half of its area.

To get an estimate, lets circumscribe the wafer with dies half inside, half outside; the count of such dies would be a good estimate of the no. of dies lost due to shape curtailment.



dies on the edge :- $\frac{\text{Circumference of the circle}}{\text{diagonal of the die}}$

$$:- \frac{2\pi w r}{\sqrt{2 de^2}} = \frac{\sqrt{2}\,\pi w r}{de}$$

$$\boxed{\#\text{ of physically intact dies} = \frac{\pi w r^2}{de^2} - \frac{\sqrt{2}\,\pi w r}{de}}$$

(b) Minimum ratio $\frac{w_r}{d_e}$ for which one useful die is obtained:-

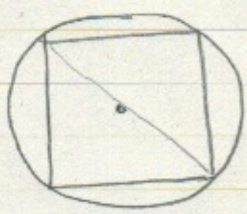
Method 1 There are two ways to do this. You could use the result obtained in part (a) and solve the equation

$$\pi \left(\frac{w_r}{d_e}\right)^2 - \sqrt{2}\,\pi \left(\frac{w_r}{d_e}\right) = 1 \quad \left\{ \text{For one useful die} \right.$$

When you solve this equation, you obtain

$$\left(\frac{w_r}{d_e}\right) = 1.6117$$

Method 2

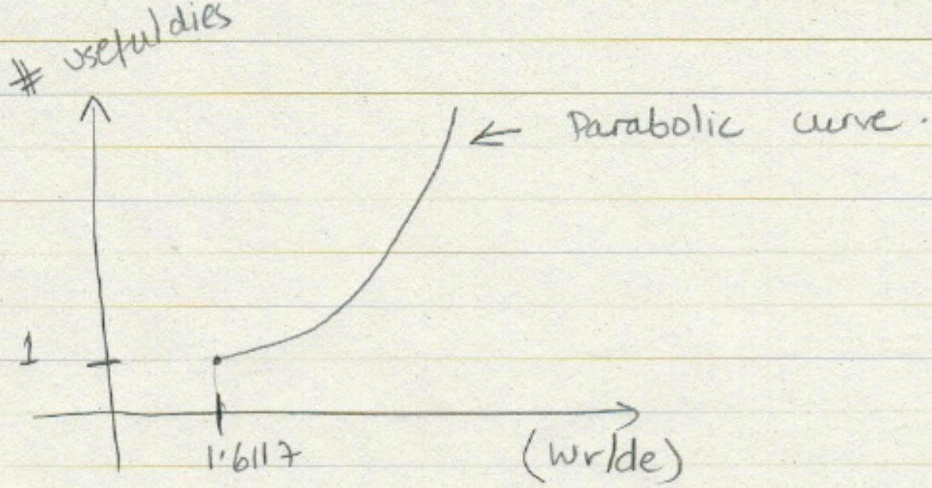


$$2 w_r = \sqrt{2}\, d_e$$

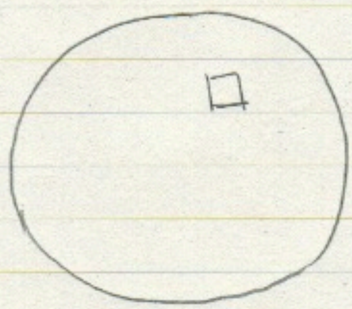
$$\frac{w_r}{d_e} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} = 0.7$$

Note:- The two methods give different results simply because part (a) was obtained under the "assumption" that the dies around the circumference are 50% usable, that is, we circumscribed the circumference with dies that were half inside the wafer and half outside the wafer. Of course, this assumption works better when the # of dies is much much greater than 1.

(c)



(d) Losses due to defects in the wafer



Probability that a good die is missed by the defect is $\left(1 - \frac{de^2}{\pi w r^2}\right)$

Probability that the die is missed by "n" defects is $= \left(1 - \frac{de^2}{\pi w r^2}\right)^n$

$n = \pi w r^2 Dd$

$\therefore$ Probability that the die is missed by "n" defects is $= \left(1 - \frac{de^2}{\pi w r^2}\right)^{\pi w r^2 Dd}$

Yield of good dies = (# physically intact dies per wafer) * (Probability of a die being defect Free)

$$Y = \left[\frac{\pi W_r^2}{de^2} - \sqrt{2}\,\pi\,\frac{W_r}{de}\right]\left[1 - \frac{de}{\pi W_r^2}\right]^{\pi W_r^2 D_d}$$

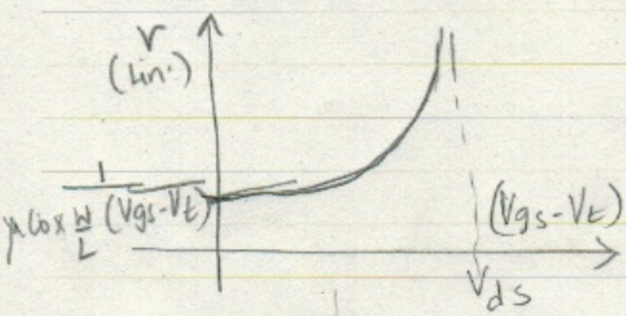
(2) $$I_D = \mu\left(\frac{W}{L}\right) C_{ox}\left[(V_{gs}-V_t)V_{ds} - \frac{V_{ds}^2}{2}\right] \quad \text{Linear}$$

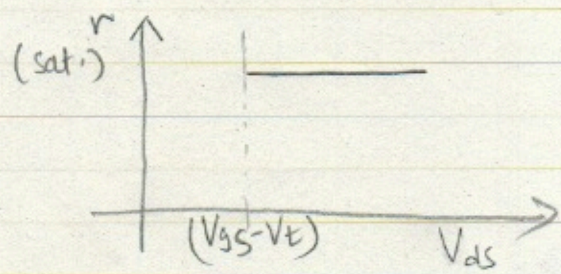
$$I_D = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)\left[V_{gs}-V_t\right]^2\left[1+\lambda V_{ds}\right] \quad \text{Saturation}$$

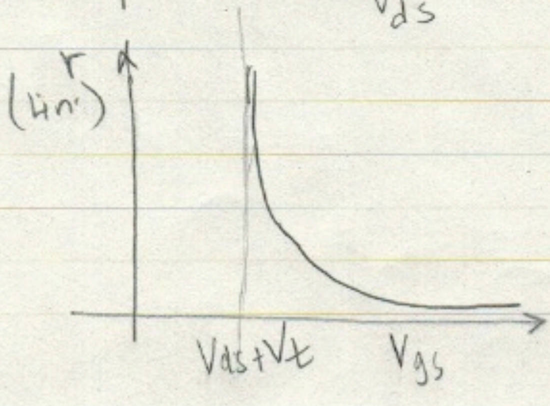
$$r = \left(\frac{\partial I_D}{\partial V_d}\right)^{-1}$$

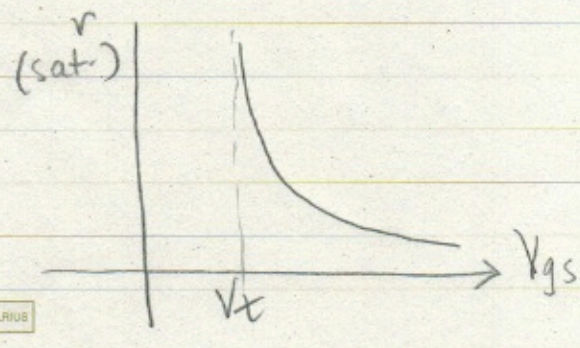
Linear: $$\frac{1}{\mu C_{ox}\left(\frac{W}{L}\right)(V_{gs}-V_t-V_{ds})}$$

Saturation: $$\frac{1}{\mu C_{ox}\left(\frac{W}{L}\right)\left[V_{gs}-V_t\right]^2\lambda}$$







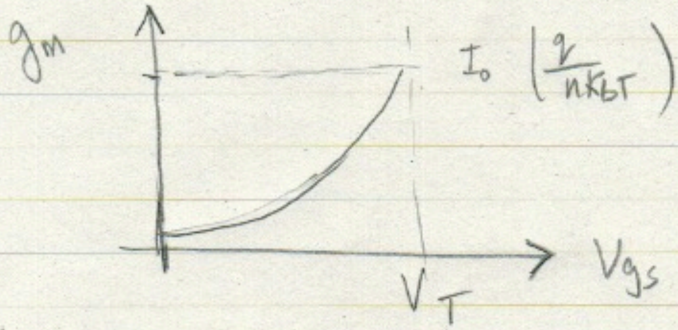


Problem 3

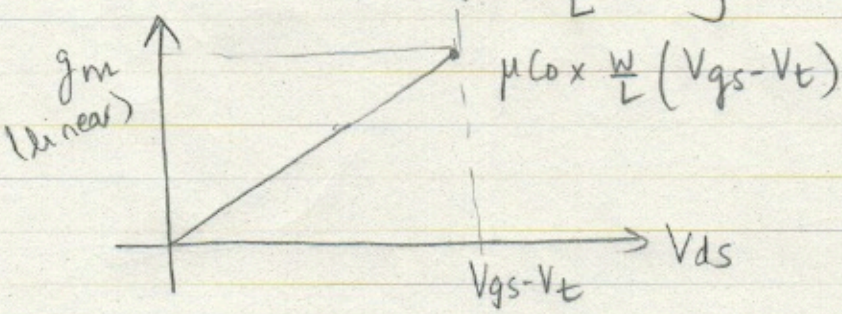
(a) $I_D = I_o \exp \dfrac{q(V_{gs}-V_T)}{n k_B T}$ For $V_{ds} >> k_B T$

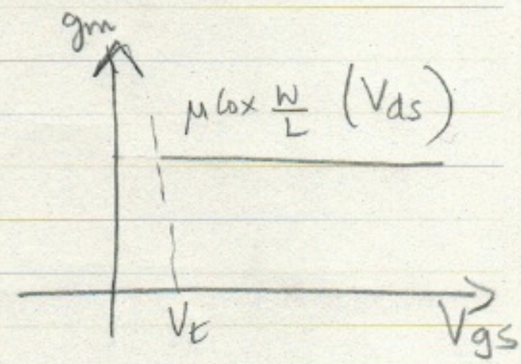
$$g_m = \frac{\partial I_D}{\partial V_g}$$

$$g_m = I_o \left(\frac{q}{n k_B T}\right) \exp\left(\frac{q(V_{gs}-V_T)}{n k_B T}\right)$$

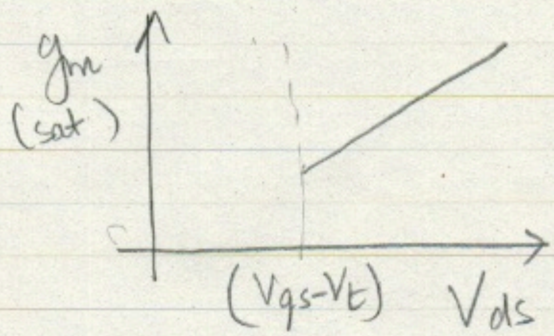


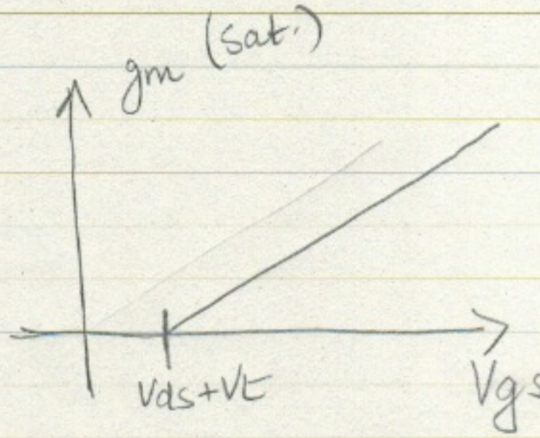
(b) (linear)
$g_m = \mu_n C_{ox} \left(\frac{W}{L}\right) \left[V_{ds} \right]$





$$\underset{\text{(saturation)}}{g_m} = \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{gs}-V_t)(1+\lambda V_{ds})$$





In all of the previous plots, it was important to note on the Vgs or Vds axis, the region where linear or saturation region holds true

Most of the students probably got the shape of the plots right, but did not get the operation region correctly