HW-3

Chi Zhang

14/10/2015

Problem 1

a

$$Y = \overline{(B + A) \cdot C \cdot D} \tag{1}$$

b

As from the figure, suppose β_N denodes β for NMOS used in this design, β_n denodes β for NMOS in equivalent generic design. And β_P and β_p respectively. Thus we have $\beta_P = 2\beta_p$ and $\beta_N = 3\beta_n$. Thus for MOSFET really used in this design

$$\begin{aligned} \frac{W}{L}_{NMOS} &= 3 \\ \frac{W}{L}_{PMOS} &= 4 \end{aligned} \tag{2}$$

c

Suppose internal capacitance for each NMOS is C_n, and C_p for PMOS respectively.

Initial State	Final State	Delay
$A_1B_0C_1D_0$	$A_1B_0C_1D_1$	$t_{pHL} \propto 3R_NC_N + R_NC_Y$
$A_0B_1C_1D_0$	$A_0B_1C_1D_1$	
$A_0B_1C_1D_1$	$A_0B_0C_1D_1$	$t_{pLH} \propto R_PC_P + R_PC_Y$

Problem 2

a

$$\begin{aligned} E_{total} &= E_r + uE_r + ... + u^N E_r \\ &= E_r \frac{u^{\frac{ln\frac{C_L}{C_1}}{lnu}+1} - 1}{u-1} \end{aligned} \tag{3}$$

b

For each inverter, there is $t_{pj} = u\tau_r$. Thus for each step, Energy-Delay Product is $EDP_j = ju^j E_r u\tau_r = ju^{j+1} E_r \tau_r = ju^{j+1} EDP_{ref}$. Thus,

$$\begin{aligned} EDP_{chain} &= EDP_{ref} + u^2 EDP_{ref} + 2u^3 EDP_{ref} + ... + Nu^{N+1} EDP_{ref} \\ &= EDP_{ref}(1 + \frac{u^2[(\frac{lnC_L/C_1}{lnu}(u-1)-1)u^{\frac{lnC_L/C_1}{lnu}}+1]}{(u-1)^2}) \end{aligned} \tag{4}$$

Thus, normalized Energy-Delay Product is,

$$\frac{EDP_{chain}}{EDP_{ref}} = 1 + \frac{u^2[(\frac{lnC_L/C_1}{lnu}(u-1)-1)u^{\frac{lnC_L/C_1}{lnu}}+1]}{(u-1)^2} \tag{5}$$

For optimal 'u' of minimum normalized EDP,

$$\begin{aligned} 0 = &\frac{u^2[u^{\frac{lnC_L/C_1}{lnu}}(\frac{lnC_L/C_1}{lnu} - \frac{(u-1)lnC_L/C_1}{uln^2u}) + (\frac{u^{\frac{lnC_L/C_1}{lnu}-1}lnC_L/C_1}{lnu} - \frac{u^{\frac{lnC_L/C_1}{lnu}}lnC_L/C_1}{ulnu})(\frac{(u-1)lnC_L/C_1}{lnu}-1)]}{(u-1)^2} \\ &+ \frac{2u[u^{\frac{lnC_L/C_1}{lnu}}(\frac{(u-1)lnC_L/C_1}{lnu}-1)]}{(u-1)^2} \\ &+ \frac{2u^2[u^{\frac{lnC_L/C_1}{lnu}}(\frac{(u-1)lnC_L/C_1}{lnu}-1)]}{(u-1)^3} \end{aligned} \tag{6}$$

c

For condition of (b), $u \approx 3.996934$. For optimal 'u' for minimum delay, $u = e \approx 2.718282$.

Problem 3