

# Kinematic Model of Wheeled Mobile Robots

B B V L Deepak<sup>1</sup>, Dayal R Parhi<sup>1</sup> and Alok Kumar Jha<sup>1</sup>

Department Of Mechanical Engineering, National Institute of Technology- Rourkela, Rourkela, India

E-mail: [deepak.bbvl@gmail.com](mailto:deepak.bbvl@gmail.com) ; [dayalparhi@yahoo.com](mailto:dayalparhi@yahoo.com)

**Abstract-** This paper deals with the structure of the kinematic models of wheeled mobile robots (WMR). A wheeled mobile robot here considered as a planer rigid body that rides on an arbitrary number of wheels. In this paper it is shown that, for a large class of possible configurations of wheels, five types of configurations can be done namely i) fixed standard wheels, ii) steerable standard wheels, iii) castor wheels, iv) Swedish wheels, and v) spherical wheels. These wheels are characterized by generic structures of the model equations. Based on the geometrical constraints of these wheels a kinematical model has been proposed and the degree of mobility, steerability and maneuverability are studied. Finally this analysis is applied to various wheeled mobile robots. Examples are presented to illustrate the models.

**Index terms-** Wheeled Mobile Robot, Kinematic Analysis, Degree of Maneuverability

## I. INTRODUCTION

A mobile robot is a self-operating machine that can move with respect to its free space. By understanding the wheel constraints placed on the robots mobility it is easier to understand the mobile robot motion. It is difficult to measure a mobile robot's position instantaneously by direct way. It is an extremely challenging task to express the exact mobile robot's position while it is in motion.

A wheeled mobile robot has been modeled as a planar rigid body that rides on an arbitrary number of wheels in order to develop a relationship between the rigid body motion [1] of the robot and the steering and drive rates of wheels. In previous research work kinematic models [2],[3][5][6] are designed for holonomic mobile platform[2] in order to provide omni-directional motions [3]by three individually driven and steered wheels. Kinematic modeling, singularity analysis, and motion control has been outlined in [3] for a mobile robot equipped with N Swedish wheels. With the help of an interactive tool [4] it is possible to solve mobile robot motion problems by understanding several well-known algorithms and techniques. In [6] it has been modeled that the wheels as a torus and proposed the use of a passive joint allowing a lateral degree of freedom for kinematic analysis of a wheeled mobile robot (WMR) moving on uneven terrain.

Most of the previous research work dealt with the kinematic models for few wheel types [3],[7]. The present research work deals with a general wheeled mobile robot (WMR), with arbitrary number of various configured wheels. Our aim is to point out the kinematic models for various WMRs. It has been shown that the variety of possible robot constructions and wheel configurations by the concepts of

degree of mobility and of degree of steerability. The set of WMR can be partitioned in to five configurations and this analysis is carried out in later sections.

## II. KINEMATICS OF VARIOUS WMR

A WMR is equipped with motors which is capable of an autonomous motion (without external human driver). These are driven by an embarked computer for its motion.

The total dimensionality of a robot chassis on the plane is three, two for position in the plane and one for orientation along the vertical axis, which is orthogonal to the plane as shown in Figure1.

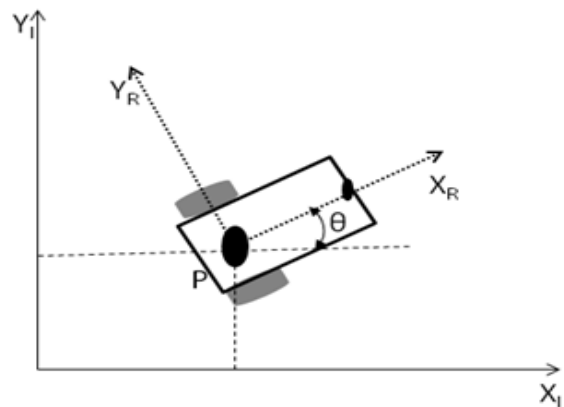


Figure1. The global and local reference planes.

The position of P in the global reference frame is specified by coordinates  $x$  and  $y$ , and the angular difference between the global and local reference frames is given by  $\theta$ .

Therefore the robot position:  $\xi_I = [x \ y \ \theta]^T$  (1)

And mapping is accomplished using the orthogonal rotation matrix:

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$

From equation (2) we can compute the robot's motion in the global reference frame with respect to its local reference frame:

$$\dot{\xi}_I = R(\theta)^{-1} \dot{\xi}_R \quad (3)$$

### A. Wheel Kinematic Constraints:

We can observe two constraints while a WMR is in motion. The first constraint is due to rolling contact as represented in Figure2. The second constraint is due to no lateral slippage – that the wheel may slide orthogonal to the wheel plane as shown in Figure3.

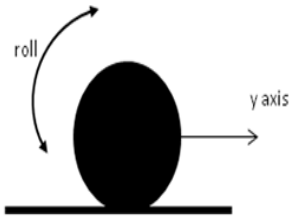


Figure2. Rolling motion

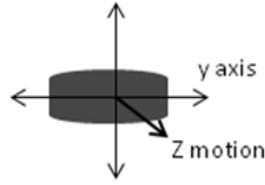


Figure3. Lateral slip

The initial stage of a kinematic model of the robot is to express constraints on the motions of individual wheels. Thereby we can compute the movement of the entire robot by combining the motions of individual wheels.

The assumptions under this analysis are, the wheel always remains in vertical during the motion of a robot and there is no sliding at the single point of contact between the ground plane and the wheel. It means that the wheel is in motion under only pure rolling conditions and rotation about the vertical axis through the contact point.

### III. WHEEL CONFIGURATIONS

Based on the geometric constraints, we can observe different configured wheels.

#### A. Fixed Standard Wheel:

There is no vertical axis of rotation or steering for the fixed standard wheel i.e. the angle between the chassis and the wheel axis is fixed. Figure4 describes a fixed standard wheel and its position relative to the robot's local reference frame. The position of the robot is then expressed in polar coordinates by distance  $l$  and angle  $\alpha$ .  $\beta$  denotes the angle of the wheel plane relative to the robot chassis. Consider a wheel radius  $r$ ; its rotational position around its horizontal axle is a function of time  $t$ :  $\phi(t)$ .

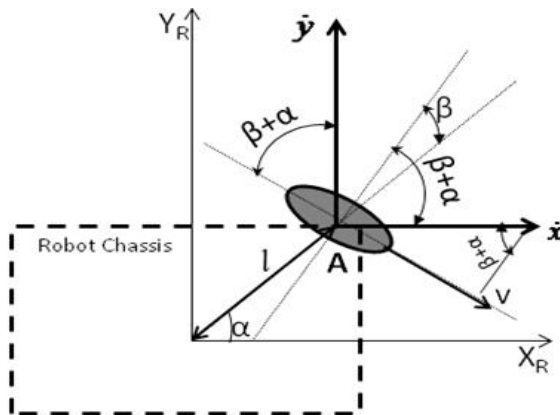


Figure4. Fixed standard wheel and its parameters

By the adequate amount of wheel spin in order to get pure rolling at the contact point, the wheel imposes that all movement along the direction of the wheel plane:

$$[\sin(\alpha + \beta) - \cos(\alpha + \beta) (-l) \cos \beta] R(\theta) \dot{\xi}_l - r \dot{\phi} = 0 \quad (4)$$

The sliding constraint for this wheel enforces the wheel's motion normal to the wheel plane must be zero:

$$[\cos(\alpha + \beta) \sin(\alpha + \beta) l \sin \beta] R(\theta) \dot{\xi}_l = 0 \quad (5)$$

Standard fixed wheels are used in differential drive configuration which is the most common WMR configuration. It is used because of their simple structure and good reliability, the nonholonomic velocity constraint (i.e. no side slip condition) limit robot motion.

#### B. Steered Standard Wheel:

The only difference between the fixed standard wheel and the steered standard wheel is that there is an additional degree of freedom: the angular orientation of the wheel to the robot chassis is no longer a single fixed value  $\beta$ , it may varies as a function of time:  $\beta(t)$  as represented in Figure5.

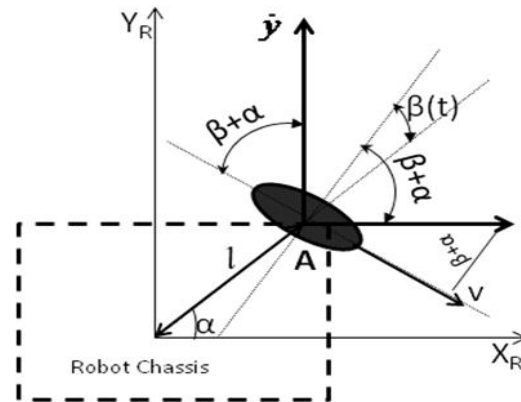


Figure 5. Steered standard wheel and its parameters.

The rolling and sliding constraints for the steered standard wheel shown in Figure6:

Motion along the wheel plane:

$$[\sin(\alpha + \beta) - \cos(\alpha + \beta) (-l) \cos \beta] R(\theta) \dot{\xi}_l - r \dot{\phi} = 0 \quad (6)$$

Motion orthogonal to the wheel plane:

$$[\cos(\alpha + \beta) \sin(\alpha + \beta) l \sin \beta] R(\theta) \dot{\xi}_l = 0 \quad (7)$$

In a steered standard wheel, the steering action does not cause a movement of the robot chassis by itself. In car like mobile robot one (tricycle) or two front wheels is made steerable. Outdoor autonomous vehicles mostly use Ackerman steering method.

#### C. Castor Wheel:

Unlike the steered standard wheel, these wheels are able to steer around a vertical axis. To specify the castor wheel's position it is required an additional parameter which is a fixed length of rigid rod 'd' connected to wheel as shown in Figure6.

For the caster wheel, the rolling constraint is similar to the fixed standard wheel.

During motion aligned with the wheel plane:

$$[\sin(\alpha + \beta) - \cos(\alpha + \beta) - l \cos \beta] R(\theta) \dot{\xi}_l - r \dot{\phi} = 0 \quad (8)$$

Because of the offset ground contact point relative to A, the constraint that there is some lateral movement would

be presented. In that appropriate rotation of the vertical axis must take place:

$$[\cos(\alpha + \beta) \sin(\alpha + \beta) d \sin \beta] R(\theta) \dot{\xi}_I + d \dot{\beta} = 0 \quad (9)$$

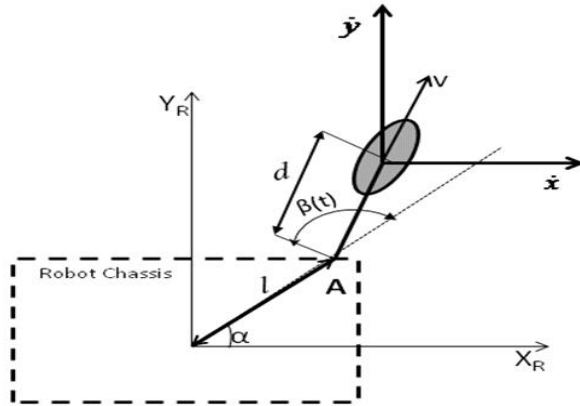


Figure6. Castor wheel and its parameters

Although the kinematics of castor wheel is somewhat complex, such wheels do not impose any real constraints on the kinematics of a robot chassis. Castor wheels are found in numerous applications including office chair, shopping cart, and material handling equipment.

#### D. Swedish Wheel

Swedish wheels are able to move omnidirectionally although these wheels have no vertical axis of rotation. Swedish wheels are configured like a fixed standard wheel with rollers attached to the wheel circumference with antiparallel axes to the main axis of the fixed wheel component. The exact angle  $\gamma$  between the main axis of the wheel and the roller axes may vary, as shown in Figure 7.

The obtained motion constraint is similar to the fixed standard wheel formula is modified by adding  $\gamma$  such that the desired direction along the rolling constraint holds is along this zero component.

$$[\sin(\alpha + \beta + \gamma) - \cos(\alpha + \beta + \gamma) (-l) \cos(\beta + \gamma)] R(\theta) \dot{\xi}_I - r \dot{\phi} \cos \gamma = 0 \quad (10)$$

Because of the free rotation  $\dot{\phi}_{sw}$  of the small rollers, the motion is not constrained in orthogonal direction.

$$[\sin(\alpha + \beta + \gamma) \cos(\alpha + \beta + \gamma) l \sin(\beta + \gamma)] R(\theta) \dot{\xi}_I - r \dot{\phi} \sin \gamma - r_{sw} \dot{\phi}_{sw} = 0 \quad (11)$$

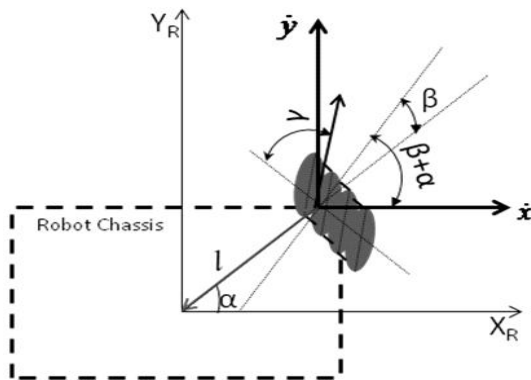


Figure7. Swedish wheel and its parameters

As well as moving forward and backward like conventional wheels, Swedish wheels allow sideways movement by spinning wheels on the front and rear axles in opposite directions.

#### E. Spherical Wheel

Spherical wheel as shown in Figure 8 has no direct constraints on motion. In this configuration there are no principal axes of rotation, and therefore no rolling or sliding constraints may exist. Therefore equation (12) simply represents the roll rate of the sphere in the direction of motion  $v_A$  of point A.

$$[\sin(\alpha + \beta) - \cos(\alpha + \beta) (-l) \cos \beta] R(\theta) \dot{\xi}_I = r \dot{\phi} \quad (12)$$

Wheel's motion normal to the wheel plane must be zero:

$$[\cos(\alpha + \beta) \sin(\alpha + \beta) l \sin \beta] R(\theta) \dot{\xi}_I = 0 \quad (13)$$

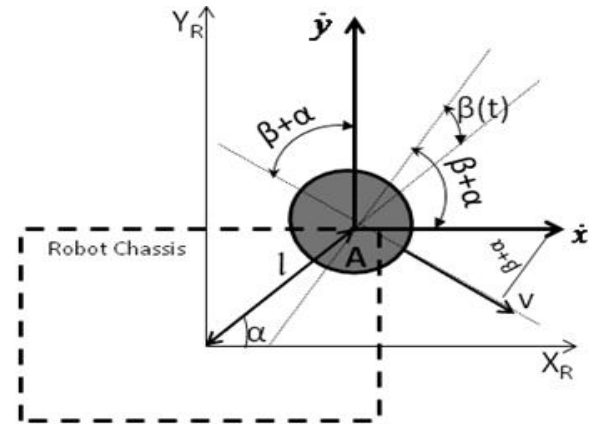


Figure8. Spherical wheel and its parameters

Spherical wheels provide smooth sliding as compared to fixed or castor wheels, can work oblique till  $15^\circ$  and one of the most important advantages is its extraordinary anti-overturn function (i.e. it avoids the typical off-centre of swivel wheels).

## IV. ROBOT KINEMATIC CONSTRAINTS

Consider a WMR having  $N$  wheels of the above five described categories. We use the five following subscripts to identify quantities relative to these 5 classes:  $f$  for fixed wheels,  $s$  for steerable standard wheels,  $c$  for castor wheels,  $sw$  for Swedish wheels and  $sp$  for spherical wheels. The numbers of wheels of each type are denoted  $N_f, N_s, N_c, N_{sw}$ , and  $N_{sp}$  with  $N_f + N_s + N_c + N_{sw} + N_{sp} = N$ . The following vectors of coordinates describe the configuration of the robot.

- Posture coordinates:  $\xi_I = [x(t) \ y(t) \ \theta(t)]^T$
- Angular coordinates:  $\beta_f, \beta_s(t), \beta_c(t), \beta_{sw}(t)$ , and  $\beta_{sp}(t)$  for the five types of wheels respectively.
- Rotational coordinates:

$[\phi_f(t) \ \phi_s(t) \ \phi_c(t) \ \phi_{sw}(t) \ \phi_{sp}(t)]^T$  for the rotation angles of the wheels around their horizontal axis.

Now it is easier to express the rolling constraints of all wheels into a single expression as shown in equation (14).

$$J_1(\beta) R(\theta) \dot{\xi}_I - J_2 \dot{\phi} = 0 \quad (14)$$

Where  $J_1(\beta)$  denotes a matrix for all wheels to their motions along their individual wheel planes, and  $J_2$  is a constant diagonal matrix NXN of all standard wheels radii.

$$J_1(\beta) = \begin{bmatrix} J_{1f} \\ J_{1s}(\beta_s) \\ J_{1c}(\beta_c) \\ J_{1sw} \\ J_{1sp} \end{bmatrix} \quad (15)$$

where  $J_{1f}$ ,  $J_{1s}$ ,  $J_{1c}$ ,  $J_{1sw}$ , and  $J_{1sp}$  are the matrices of  $(N_f \times 3)$ ,  $(N_s \times 3)$ ,  $(N_c \times 3)$ ,  $(N_{sw} \times 3)$  and  $(N_{sp} \times 3)$ , whose forms are derived from the constraints (4), (6), (8), (10) and (12).  $J_2$  is a constant  $(N \times N)$  matrix whose diagonal entries are the radii of the wheels, except for the radii of the Swedish wheels which are multiplied by  $\cos\gamma$ .

In the similar way we can formulate for the sliding constraints by combining all wheels into a single expression:

$$C_1(\beta)R(\theta)\xi_i + C_2\beta_s = 0 \quad (16)$$

$$\text{Where } C_1(\beta) = \begin{bmatrix} C_{1f} \\ C_{1s}(\beta_s) \\ C_{1c}(\beta_c) \\ C_{1sw} \\ C_{1sp} \end{bmatrix} \text{ and } C_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (17)$$

## V. MOBILE ROBOT MANEUVERABILITY

It is defined as the ability to move a robot in its environment. The basic constraint limiting mobility is that every wheel must satisfy its sliding constraint. Besides the instantaneous kinematic motion, a mobile robot is able to further manipulate its position, over time, by steering steerable wheels. The overall maneuverability of a robot is thus a combination of the mobility available based on the kinematic sliding constraints of the standard wheels, plus the additional freedom contributed by steering and spinning the steerable standard wheels.

### A. Degree of Mobility:

It is obvious from the wheel kinematic constraints described above that the castor wheel, Swedish wheel, and spherical wheel impose no kinematic constraints on the robot chassis, since these wheels can freely move in all of these cases during its internal wheel degrees of freedom. Therefore it is required to consider only fixed standard wheels and steerable standard wheels impact on robot chassis kinematics.

Consider now the  $(N_f + N_s)$  wheels of fixed and steered standard wheels. To avoid any lateral slip the motion vector  $R(\theta)\xi_i$  has to satisfy the following constraints:

$$C_{1f}R(\theta)\xi_i = 0 \quad (18)$$

$$C_1(\beta_s)R(\theta)\xi_i = 0 \quad (19)$$

$$\text{And } C_1(\beta_s) = \begin{bmatrix} C_{1f} \\ C_{1s}(\beta_s) \end{bmatrix}$$

Mathematically it represents  $R(\theta)\xi_i$  must belong to the null space of the projection matrix. Null space of  $C_1(\beta_s)$  is

the space N such that for any vector n in N

$$C_1(\beta_s).n = 0 \quad (20)$$

By using the concept of a robot's *instantaneous center of rotation* (ICR) the kinematic constraints (equations (18) and (19)) can be explained geometrically. According to this at any given instant, wheel motion must be zero along the zero motion line. When the wheel moves in a straight line its ICR is at infinity. This ICR construction illustrates how robot mobility depends on the number of constraints but not on the number of wheels.

A robot like an Ackerman vehicle in Figure9 (a) with several wheels, but it must have a single ICR. Because it's zero motion lines meet at a single point. By placing the ICR at this meet point there is a single solution for robot motion. In Figure 9(b), the bicycle like robot has two wheels  $W_1$  and  $W_2$ . Each wheel gives a constraint, or a zero motion line. The solution for the ICR is only a single point by taking the two constraints together.

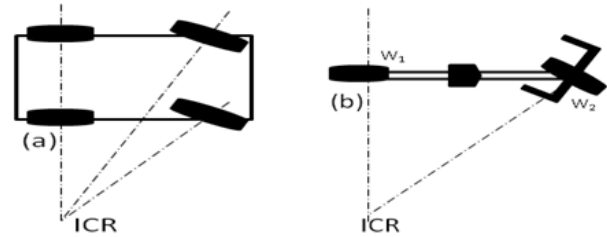


Figure9 (a) Four wheel with car like Ackerman steering (b) bicycle

Therefore kinematics of robot chassis is a function of independent constraints arising from all standard wheels. The mathematical significance of independence is corresponded to the rank of a matrix. Equation (18) represents all sliding constraints imposed by the wheels of the mobile robot. Therefore  $\text{rank}[C_{1s}(\beta_s)]$  is the number of independent constraints.

In practice, a wheeled mobile robot will have zero or more fixed standard wheels and zero or more steerable standard wheels. We can therefore identify the possible range of rank values for any robot:  $0 \leq [C_{1s}(\beta_s)] \leq 3$ .

Now we can define a robot's degree of mobility  $\delta_m$ :

$$\delta_m = \dim N[C_1(\beta_s)] = 3 - \text{rank}[C_1(\beta_s)] \quad (20)$$

### B. Degree of Steerability:

A WMR can be steered independently by equipped steered standard wheels to it. This result is indirect since the robot must move for the change in steering angle of the steerable standard wheel.

Therefore the degree of steerability can be obtained by:

$$\delta_s = \text{rank}[C_1(\beta_s)] \quad (21)$$

Greater maneuverability means more degrees of steering freedom. It can be obtained by increasing the rank of  $[C_1(\beta_s)]$ .

The range of  $\delta_s$  is  $0 \leq \delta_s \leq 2$ , since includes. A steerable standard wheel can both decrease mobility and increase steerability.



### C. Maneuverability of WMR With Examples

The overall Degrees of Freedom that a robot can manipulate called the degree of maneuverability can be defined in terms of mobility and steerability. Thus the maneuverability consists of the degrees of freedom that the robot manipulates directly through wheel velocity and the degrees of freedom that it indirectly manipulates by changing the steering configuration and moving.

$$\therefore \delta_M = \delta_m + \delta_s \quad (22)$$

1. Omni directional: Figure10 represents an omni-directional mobile robot having four wheels other than fixed and steered standard wheels. For this type of robot  $\text{rank}[C_{1s}(\beta_s)]$  and  $\text{rank}[C_1(\beta_s)]$  is zero since it has no steerable standard wheels and fixed standard wheels.

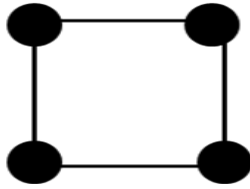


Figure10 Omnidirectional WMR

This results the degree of mobility  $\delta_m = 3$  and the degree of steerability  $\delta_s = 0$

$$\therefore \delta_M = \delta_m + \delta_s = 3$$

2. Differential: Figure11 represents a differential WMR having two wheels dependent fixed and other is castor wheels. For this type of robot  $\text{rank}[C_{1s}(\beta_s)]$  is zero and  $\text{rank}[C_1(\beta_s)]$  is one since it has no steerable standard wheels.

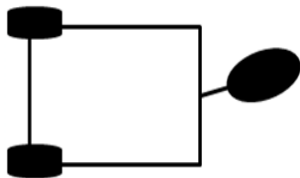


Figure11 Differential Three WMR

This results the degree of mobility  $\delta_m = 2$  and the degree of steerability  $\delta_s = 0$

$$\therefore \delta_M = \delta_m + \delta_s = 2$$

3. Omni-Steer: Figure12 represents omni-steer mobile robot having two dependent steerable wheels and two spherical wheels. For this type of robot  $\text{rank}[C_{1s}(\beta_s)]$  is one and  $\text{rank}[C_1(\beta_s)]$  is one since it has one steerable standard (two dependent) wheel.

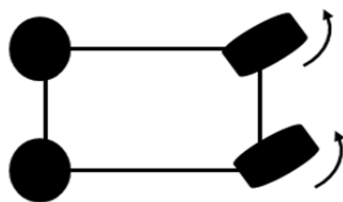


Figure12 Three wheeled omni-steer mobile robot

This results the degree of mobility  $\delta_m = 2$  and the degree of steerability  $\delta_s = 1$

$$\therefore \delta_M = \delta_m + \delta_s = 3$$

4. Tricycle: Figure13 represents Tricycle mobile robot having two fixed wheels and one steered standard wheel. Steering and power are provided through the front wheel. For this type of robot  $\text{rank}[C_{1s}(\beta_s)]$  is one and  $\text{rank}[C_1(\beta_s)]$  is two since it has one steerable standard wheel.

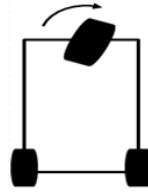


Figure13 Tricycle WMR

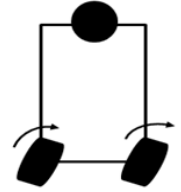


Figure14 Two steer WMR

This results the degree of mobility  $\delta_m = 1$  and the degree of steerability  $\delta_s = 1$

$$\therefore \delta_M = \delta_m + \delta_s = 2$$

5. Two steer: Figure14 represents two steer WMR having two steered standard wheels and one omni-directional wheel. For this type of robot  $\text{rank}[C_{1s}(\beta_s)]$  is two and  $\text{rank}[C_1(\beta_s)]$  is two since it has two independent steerable standard wheels.

This results the degree of mobility  $\delta_m = 1$  and the degree of steerability  $\delta_s = 2$

$$\therefore (\delta_M) = \delta_m + \delta_s = 3$$

6. Maneuverability of various configured WMR: Consider various WMR configurations as shown in Figure15 having three or four wheels with arbitrary orientations with respect to its wheel plane. The maneuverability of these WMRs can be observed in Table.1

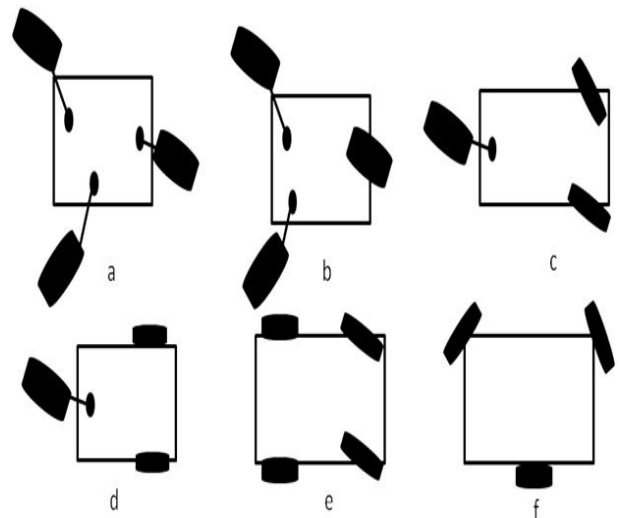


Figure15 WMRs with various configured wheels

TABLE.I  
MANEUVERABILITY OF VARIOUS WMR FOR FIGURE 15

Figure	$\delta_m$	$\delta_s$	$\delta_M$
a (neither fixed wheels nor centered orientable wheels)	3	0	3
b (one steerable standard wheel & two castor wheels)	2	1	3
c (two independent steerable standard & one castor wheels)	1	2	3
d (two dependent fixed & one castor wheels)	2	0	2
e (two dependent fixed & centered orientable wheels)	1	1	2
f (three fixed wheels)	0	0	0

#### D. Example to Find Out Wheel Velocities:

Consider an omnidirectional WMR as shown in Figure 16 having three Swedish wheels. Let the angle between the main wheel plane and the axis of rotation of the small circumferential rollers  $\gamma=0$ . Assume that the distance between each wheel and local reference frame is  $l$ .

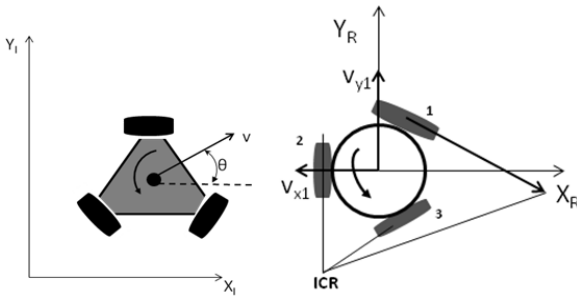


Figure16 omnidrive WMR

Figure17 Local reference frame

From "10" rolling constraint for a Swedish wheel:

$$[\sin(\alpha + \beta + \gamma) \quad -\cos(\alpha + \beta + \gamma) \quad (-l)\cos(\beta + \gamma)]R(\theta)\dot{\xi}_l - r\dot{\phi}\cos\gamma = 0 \quad (23)$$

If  $\gamma = 0^\circ$ , Equation (21) becomes:

$$[\sin(\alpha + \beta) \quad -\cos(\alpha + \beta) \quad (-l)\cos(\beta)]R(\theta)\dot{\xi}_l - r\dot{\phi} = 0 \quad (24)$$

Suppose all wheels are tangent to the robot circular body as shown in Figure23 implies  $\beta = 0$ . Then equation (24) becomes:

$$[\sin(\alpha) \quad -\cos(\alpha) \quad (-l)]R(\theta)\dot{\xi}_l - r\dot{\phi} = 0 \quad (25)$$

To map motion along the axes of the global reference frame to the robot's local reference frame mapping matrix  $R(\theta)$  is used. Let us take here  $\theta=45^\circ$  is the angular difference between the global and local reference frame.

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (26)$$

From equation (14), the rolling constraints of all wheels can be expressed. Since the robot has no steered standard wheels  $J_1(\beta_s)$  can be taken as  $J_{1f}$ . Therefore equation (14) becomes:

$$\Rightarrow \dot{\xi}_l = R(\theta)^{-1} J_{1f}^{-1} J_2 \dot{\phi} \quad (27)$$

The value of  $\alpha$  for three wheels easily computed and let them represented as  $\alpha = [\alpha_1 \quad \alpha_2 \quad \alpha_3] = [0 \quad \frac{\pi}{2} \quad -\frac{\pi}{3}]$

$$J_{1f} = \begin{bmatrix} \sin(\alpha_1) & -\cos(\alpha_1) & -l \\ \sin(\alpha_2) & -\cos(\alpha_2) & -l \\ \sin(\alpha_3) & -\cos(\alpha_3) & -l \end{bmatrix} = \begin{bmatrix} 0 & -1 & (-l) \\ 1 & 0 & (-l) \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & (-l) \end{bmatrix}$$

$J_2$  is the diagonal matrix of radii of the three wheels.

Let us take  $r_1=1$ ,  $r_2=1$  and  $r_3=1$ :

$$J_2 = \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Consider an omnidrive chassis with  $l=1$  and the wheels 1, 2, and 3 spin at speeds  $\phi_1 = 4$ ,  $\phi_2=1$  and  $\phi_3 = 2$

we can calculate

$$J_1^{-1} = \begin{bmatrix} 0.366 & 0.366 & -0.732 \\ -1.366 & 0.634 & 0.732 \\ 0.366 & -0.634 & -0.732 \end{bmatrix}$$

From the consideration

$$R(\theta)^{-1} = \begin{bmatrix} 0.7071 & -0.7071 & 0 \\ 0.7071 & 0.7071 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } \dot{\phi} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

Therefore the motion of the robot from (24):

$$\begin{aligned} \dot{\xi}_l &= R(\theta)^{-1} J_{1f}^{-1} J_2 \dot{\phi} \\ &\Rightarrow \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 2.63 \\ -2.12 \\ -0.634 \end{bmatrix} \end{aligned} \quad (28)$$

## CONCLUSION

According to the mobility restriction induced while WMR is in motion, wheels are classified into five types. The derived posture kinematic models are adequate to describe the global motion of a mobile robot. The results obtained from these kinematic models have been applied to various types of WMRs in order to get their maneuverability by calculating its degree of mobility and steerability. The present research on kinematic analysis can be applied in various fields such as the automotive industry, especially to all wheel drive electric vehicles. As a future work, it is required to develop the control algorithms and the motion experiments for real application.

## REFERENCES

- [1]Johann Borenstein and Yoram Koren, "Motion Control Analysis Of A Mobile Robot", Transactions of ASME, Journal of Dynamics, Measurement and Control, Vol. 109, No. 2, pp 73-79, November 1986.
- [2]Dong-Sung Kim, Wook Hyun Kwon, and Hong Sung Park, "Geometric Kinematics and Applications of a Mobile Robot", International Journal of Control, Automation, and Systems Vol. 1, No. 3, pp 376-384, September 2003.
- [3]Giovanni Indiveri, "Swedish Wheeled Omnidirectional Mobile Robots: Kinematics Analysis and Control", IEEE transactions on robotics, VOL. 25, NO. 1, pp 164-171, February 2009.
- [4]J.L. Guzmán, M. Berenguela, F. Rodríguez, and S. Dormidob, "An interactive tool for mobile robot motion planning", Robotics and Autonomous Systems 56, pp 396-409, 2008.
- [5]Johann Borenstein, "Control and Kinematic Design of Multi-Degree-of-Freedom Mobile Robots with Compliant Linkage", IEEE transactions on robotics and automation, vol. 1, pp 21-35, February 1995.
- [6]Nilanjan Chakraborty and Ashitava Ghosal, "Kinematics of wheeled mobile robots on uneven terrain", Mechanism and Machine Theory 39, pp 1273-1287, 2004.
- [7]Vrunda A. Joshi and Ravi N. Banavar, "Motion analysis of a spherical mobile robot", Robotica (2009), volume 27, Cambridge University Press, pp. 343-353.