Homework 2 - Due 10/15

Mean variance frontier without short-selling

As we did in class, download the stock prices for Dow stocks from 2000-present, calculate the returns and keep only the stocks that have a full history.

a. Based on data only up to the end of 2022, calculate the minimum variance portfolio. Report the average and standard deviation of monthly returns of that portfolio from January 2023 to present.

b. Based on data only up to the end of 2022, calculate the minimum variance portfolio that achieves a target 1.5% monthly target return (18% annual). Report the average and standard deviation of monthly returns of that portfolio from January 2023 to present.

c. Based on data only up to the end of 2022, calculate the portfolio with the highest Sharpe ratio. Assume that the risk free rate is constant and equal to the average risk free rate over the 2000-2022 period. Data on risk free rates can be obtained from French's page, as in Homework 1. Report the average, the standard deviation and the Sharpe ratio of monthly returns of that portfolio from January 2023 to present (for the Sharpe ratio, you can calculate the average excess returns using the actual risk free rates for each month from 2023-present).

d. Calculate the average, the standard deviation and the Sharpe ratio of monthly returns of the equally weighted-portfolio from January 2023 to present - the portfolio is rebalanced at of each month, investing equal amounts in each stock.

e. Are the mean returns in a), b), c), d) statistically different from each other?

f. Redo part a. and b. assuming that portfolio weights are positive (no short-selling allowed). No closed-form solution exists to compute portfolio weights ω. Instead, the `R` function `solve.QP` from package `quadprog` delivers the numerical solution to quadratic programming problems of the form

$$\min_{\omega}(-d'\omega + 1/2\omega' D\omega) \text{ s.t. } A'\omega \geq b_0$$

- In the above, ω, d are $n \times 1$ vectors and D is an $n \times n$ matrix
- You will have to add the equality constraint $\omega_1 + ... + \omega_n = 1$ to the inequality constraints $\omega_1 \geq 0, \ldots, \omega_n \geq 0$. The argument `meq` specifies the number of equality constraints, and you need `meq = 1`
- In matrix form, A', b_0 are the $(n+1) \times n$, $(n+1) \times 1$ matrices

$$A' = \begin{pmatrix} 1 & 1 & \ldots & 1 \\ 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \ldots & 1 \end{pmatrix} \qquad b_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

- You can get A simply as `t(rbind(1,diag(N)))`
- if `ans <- quadprog::solve.Qp(Dmat=...,dvec=...,Amat=...,bvec=...,meq=1)` (with appropriate arguments) contains the answer, it will be in the form of a list, and `names(ans)` outputs the contained variables, which can be accessed with `ans$...`.

g. OPTIONAL: Plot in the same graph the mean variance frontier and the *restricted* mean variance frontier (without short sale constraints)

h. OPPTIONAL: Find the portfolio with the highest Sharpe ratio if short selling is not allowed. Describe its performance out of sample (after 2022). You can either do a grid search, or use more complex optimization routines. Sequential Quadratic Programming tends to work very well in problems with general objective functions.