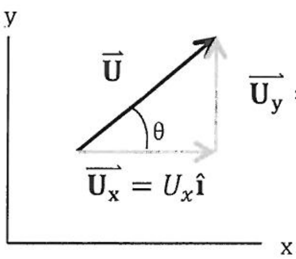
STATICS TEST 1 REVIEW

UNITS

	SI Units	US Units
mass units:	kg	slug
force units:	N	lb
gravity:	9.81m/s^2	32.2 ft/s^2
weight force:	W=m*g	W= pounds or slug*32.2 ft/s^2

CHAPTER 2 - Vectors

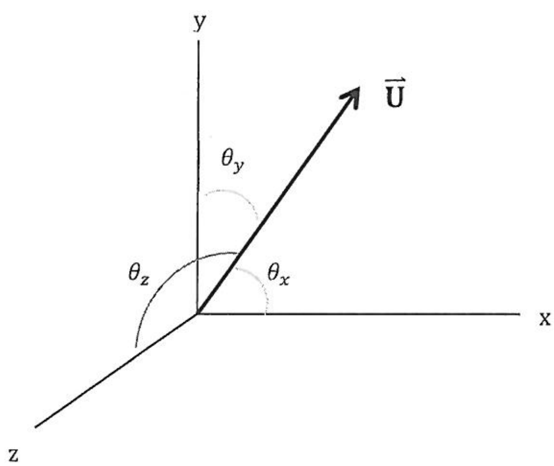


$$|\mathbf{U}| = \sqrt{U_x^2 + U_y^2}$$

$$\vec{\mathbf{U}} + \vec{\mathbf{V}} = (U_x + V_x)\hat{\mathbf{i}} + (U_y + V_y)\hat{\mathbf{j}}$$

$$a\vec{\mathbf{U}} = a(U_x\hat{\mathbf{i}} + U_y\hat{\mathbf{j}}) = aU_x\hat{\mathbf{i}} + aU_y\hat{\mathbf{j}}$$

$$U_x = |\vec{\mathbf{U}}| \cos\theta$$
$$U_y = |\vec{\mathbf{U}}| \sin\theta$$



$$\vec{\mathbf{U}} = U_x\hat{\mathbf{i}} + U_y\hat{\mathbf{j}} + U_z\hat{\mathbf{k}}$$

$$|\vec{\mathbf{U}}| = \sqrt{U_x^2 + U_y^2 + U_z^2}$$

Direction Cosines:

$$U_x = |\vec{\mathbf{U}}| \cos\theta_x$$
$$U_y = |\vec{\mathbf{U}}| \cos\theta_y$$
$$U_z = |\vec{\mathbf{U}}| \cos\theta_z$$

$$\vec{\mathbf{U}} = |\vec{\mathbf{U}}|\vec{\mathbf{e}}$$
where $\vec{\mathbf{e}}$ is a unit vector in the direction of $\vec{\mathbf{U}}$.

$$U_x = |\vec{\mathbf{U}}|\vec{\mathbf{e}}_x$$
$$U_x = |\vec{\mathbf{U}}|\vec{\mathbf{e}}_y$$
$$U_x = |\vec{\mathbf{U}}|\vec{\mathbf{e}}_z$$

$$\sqrt{\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z} = 1$$

squaring both sides...

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

POSITION VECTORS

For a position vector pointing from A to B:

$$\overrightarrow{r_{AB}} = (x_B - x_A)\hat{\mathbf{i}} + (y_B - y_A)\hat{\mathbf{j}} + (z_B - z_A)\hat{\mathbf{k}}$$

UNIT VECTORS

For a unit vector in the direction of $\overrightarrow{r_{AB}}$,

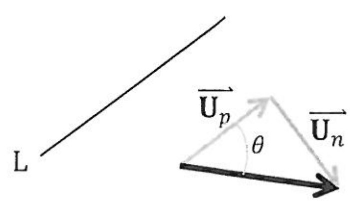
$$\overrightarrow{\mathbf{e}_{AB}} = \frac{\overrightarrow{\mathbf{r}_{AB}}}{|\overrightarrow{\mathbf{r}_{AB}}|}$$

A vector in the direction of $\overrightarrow{r_{AB}}$ is,

$$\vec{\mathbf{U}} = |\vec{\mathbf{U}}|\overrightarrow{\mathbf{e}_{AB}}$$

DOT PRODUCTS – Finding Vector Components of Forces ***Parallel*** to a Line

$$\vec{\mathbf{U}} \cdot \vec{\mathbf{V}} = |\vec{\mathbf{U}}||\vec{\mathbf{V}}| \cos\theta = U_x V_x + U_y V_y + U_z V_z$$



$$\overrightarrow{\mathbf{U}_p} = (\vec{\mathbf{e}} \cdot \vec{\mathbf{U}})\vec{\mathbf{e}}$$

is the portion of $\vec{\mathbf{U}}$ that is parallel to a line, L. $\vec{\mathbf{e}}$ is the unit vector along line L.

$$\overrightarrow{\mathbf{U}_n} = \vec{\mathbf{U}} - \overrightarrow{\mathbf{U}_p}$$

is the portion of vector $\vec{\mathbf{U}}$ that is normal to line, L

CROSS-PRODUCTS – Finding Vector Components of Forces ***Perpendicular*** to a Line

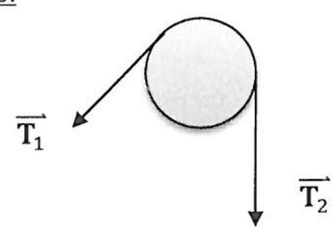
$$\vec{\mathbf{U}} \times \vec{\mathbf{V}} = |\vec{\mathbf{U}}||\vec{\mathbf{V}}| \sin\theta\, \vec{\mathbf{e}} = (U_y V_z - U_z V_y)\hat{\mathbf{i}} - (U_x V_z - U_z V_x)\hat{\mathbf{j}} + (U_x V_y - U_y V_x)\hat{\mathbf{k}}$$
$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ U_x & U_y & U_z \\ V_x & V_y & V_z \end{vmatrix}$$

CHAPTER 3 - Equilibrium, FBDs, Springs, Pulleys, Tension

Some important relationships:

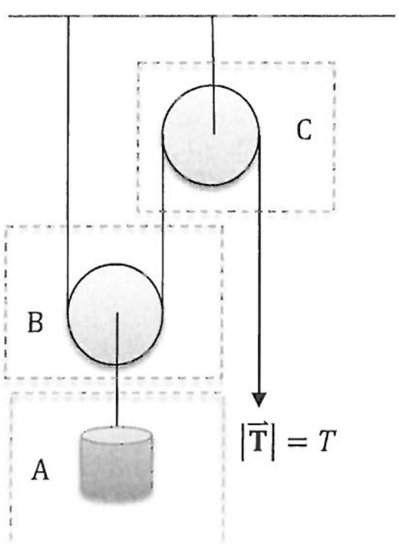
Springs: k = spring constant
L_o = initial spring length
L = stretched or compressed length after applying a force
$|\vec{\mathbf{F}}| = k|L - L_o|$

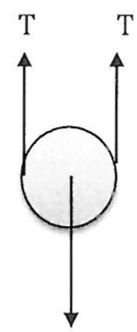
Pulleys:



$$\boxed{|\vec{\mathbf{T}}_1| = |\vec{\mathbf{T}}_2|}$$

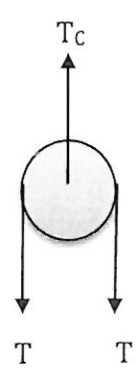
Free-Body Diagrams: When you "cut" a portion of the system that is under tension, the tension vector points *away* from the attachment point.





Free Body Diagram at B

$T_{BA} = W_A = m_A * g$



Free Body Diagram at C

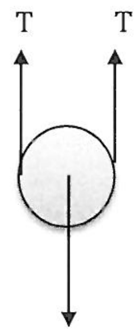
EQUILIBRIUM

If a system is in equilibrium (static or at a constant velocity), we can say the sum of the forces acting on the system equals zero. This also implies that the forces acting in every direction also equal zero.

$$\sum \vec{\mathbf{F}} = 0$$

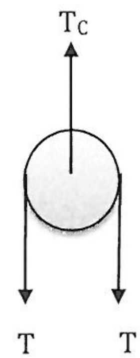
$$\sum \overrightarrow{\mathbf{F}_x} = 0 \qquad \sum \overrightarrow{\mathbf{F}_y} = 0 \qquad \sum \overrightarrow{\mathbf{F}_z} = 0$$

So applying this equilibrium idea to the FBDs from the above pulley system allows us to solve for tensions in cables.



Free Body Diagram at B

$T_{AB} = W_A = m_A{*}g$



Free Body Diagram at C

So from the FBD at B, $\sum \overrightarrow{\mathbf{F}_y} = 0 = 2T - m_A g$ and $T = \frac{m_A g}{2}$

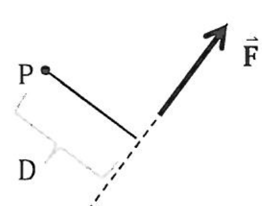
From the FBD at C, $\sum \overrightarrow{\mathbf{F}_y} = 0 = T_C - 2T$ and $T_C = 2T = m_A g$

CHAPTER 4 – Moments

The moment caused by a force applied to a system, point, or line is defined as the tendency for *rotation* to occur about the system, point, or line due to the force.

MOMENTS ABOUT POINTS

$|\overrightarrow{\mathbf{M}_\mathbf{P}}| = D|\vec{\mathbf{F}}|$ is the magnitude of a moment about point P. Where D is the perpendicular distance between P and the line of action of $\vec{\mathbf{F}}$.

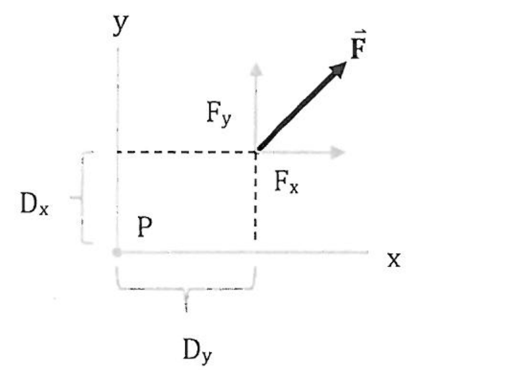


<u>**The direction of a moment**</u> about a point or line is defined as:

- Positive (+) if the rotation is counterclockwise
- Negative (-) if the rotation is clockwise

Use the right-hand-rule to check your answer. Point your fingers from point P to the line of action of $\vec{\mathbf{F}}$. Then curl your fingers in the direction of $\vec{\mathbf{F}}$. If your thumb points out of the page, the moment is positive (your fingers curled counterclockwise). If your thumb points into the page, the moment is negative (your fingers curled clockwise).

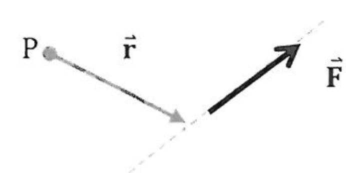
If the perpendicular distance D is difficult to find, in 2D, it can be easier to break $\vec{\mathbf{F}}$ into its components and sum the moments caused by each component around a point.



$$\sum \left|\overrightarrow{\mathbf{M}_P}\right| = -D_x F_x + D_y F_y$$

<u>THE MOMENT **VECTOR** ABOUT A POINT</u>

The moment is actually a vector with direction and magnitude.



$$\overrightarrow{\mathbf{M}_\mathbf{P}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$$

**Note that $\vec{\mathbf{r}}$ must originate at the point you are trying to find the moment about, but can point toward *any point* along the line of action of $\vec{\mathbf{F}}$.

Right-Hand-Rule (RHR):
Point your fingers from P to the line of action of $\vec{\mathbf{F}}$. Curl your fingers in the direction of $\vec{\mathbf{F}}$. Your thumb points in the direction of the moment vector, $\overrightarrow{\mathbf{M_P}}$. Your fingers curl in the direction of the rotation.

MOMENT VECTOR ABOUT A LINE

The moment of a force about a line is the portion of the moment about a point on that line, that is parallel to the line.

$$\overrightarrow{\mathbf{M_L}} = \left(\overrightarrow{\mathbf{e}_L} \cdot \overrightarrow{\mathbf{M_P}}\right)\overrightarrow{\mathbf{e}_L} = \left(\overrightarrow{\mathbf{e}_L} \cdot (\vec{\mathbf{r}} \times \vec{\mathbf{F}})\right)\overrightarrow{\mathbf{e}_L}$$

L is a line you are finding the moment about, caused by $\vec{\mathbf{F}}$
P is any point on line L
$\vec{\mathbf{r}}$ is the position vector from P to *anywhere* on the line of action of $\vec{\mathbf{F}}$
$\overrightarrow{\mathbf{e}_L}$ is the unit vector along line L (direction does not matter)

DETERMINING DIRECTION OF $\overrightarrow{\mathbf{M_L}}$

We often have complex 3D systems when calculating $\overrightarrow{\mathbf{M_L}}$. So to determine the direction of rotation about $\overrightarrow{\mathbf{M_L}}$, we use a little bit different method than before.

Process:

1. Calculate $\left(\overrightarrow{\mathbf{e}_L} \cdot (\vec{\mathbf{r}} \times \vec{\mathbf{F}})\right)$ which will be a scalar value

2. If that number is positive (+), point your thumb of your right hand in the ***same*** direction of $\overrightarrow{\mathbf{e}_L}$ that you used.

    a. Curl your fingers around the line. This is the direction of the moment about line L caused by $\vec{\mathbf{F}}$.

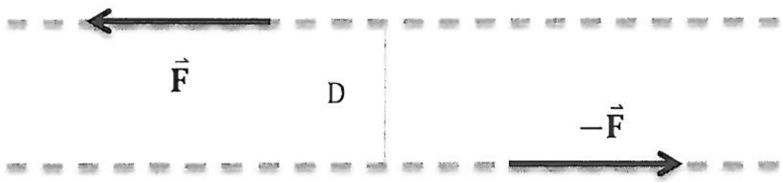
3. If $\left(\overrightarrow{\mathbf{e}_L} \cdot (\vec{\mathbf{r}} \times \vec{\mathbf{F}})\right)$ is negative (-), point your thumb of your right hand in the ***opposite*** direction of $\overrightarrow{\mathbf{e}_L}$ that you used.

    a. Curl your fingers around the line. This is the direction of the moment about line L caused by $\vec{\mathbf{F}}$.

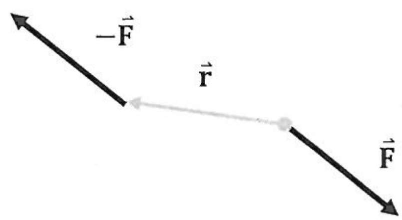
COUPLES

A couple is a pair of forces that are equal in magnitude, but opposite in direction.

$|\vec{\mathbf{M}}| = D|\vec{\mathbf{F}}|$



Where D is the perpendicular distance between the lines of action of $\vec{\mathbf{F}}$ and -$\vec{\mathbf{F}}$.

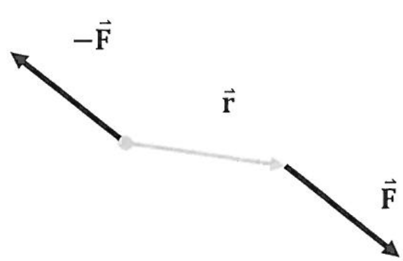


We can also use the position vector between $\vec{\mathbf{F}}$ and -$\vec{\mathbf{F}}$ and take the cross product to find the moment caused by the force couple.

$$\vec{\mathbf{M}} = \vec{\mathbf{r}} \times (-\vec{\mathbf{F}})$$

The origin of $\vec{\mathbf{r}}$ is on the line of action of one of the forces. So the moment due to the force on the right is zero. This is a good method to use...makes your calculations shorter.

The moment is negative because if we use the right hand rule, our fingers curl clockwise and our thumb points into the page.



**Note we get the same result if we had defined $\vec{\mathbf{r}}$ to point the opposite direction.

Exam 2 – Review
Chapters 5 & 6

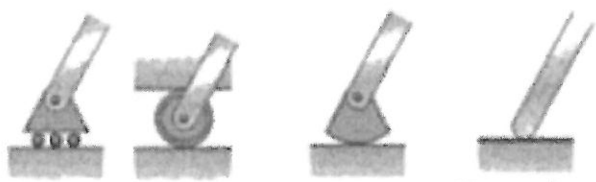
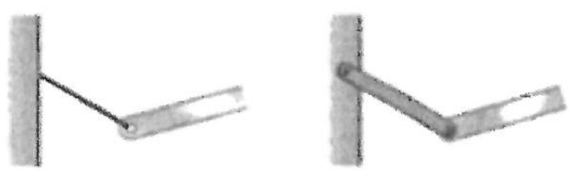
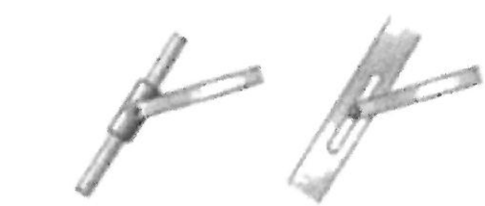
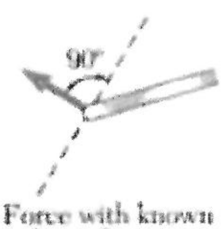
5.1
2-Dimensional Supports

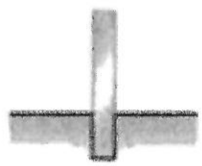
In 2-D, we have 3 equilibrium equations to analyze a system:

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_{any\ point} = 0$$

This means we can solve for 3 unknown forces or force couples.

Types of 2-D supports and the reactions at those supports:

Support or Connection	Reaction
 Rollers, Rocker, Frictionless surface	 Force normal to the contact surface
 Short cable, Short link	 Force with known line of action
 Collar on frictionless rod, Frictionless pin in slot	 Force with known line of action

 Frictionless pin or hinge  Rough surface	 Forces in x and y directions
 Fixed support	 Forces in x and y and a moment

<u>Solving For Unknown Reactions at Supports:</u>

1. FBD about the entire structure (replacing supports with appropriate reactions).
2. Apply equilibrium equations and solve.

5.2
<u>Statically Indeterminate Objects</u>

<u>Redundant Supports:</u> If a structure has more supports than the minimum necessary to maintain equilibrium (typically 3 in a 2D system), then the system has redundancy.

Degree of Redundancy = (# of support reactions) – (# of independent equations)

**Typically in a 2D system, the # of independent equations = 3.

<u>Improper Supports:</u> The system does not have enough support to maintain equilibrium.

Example: If a structure only has a total of 2 reactions at all supports, this is less than the 3 required to maintain equilibrium.

5.3
3-D Supports

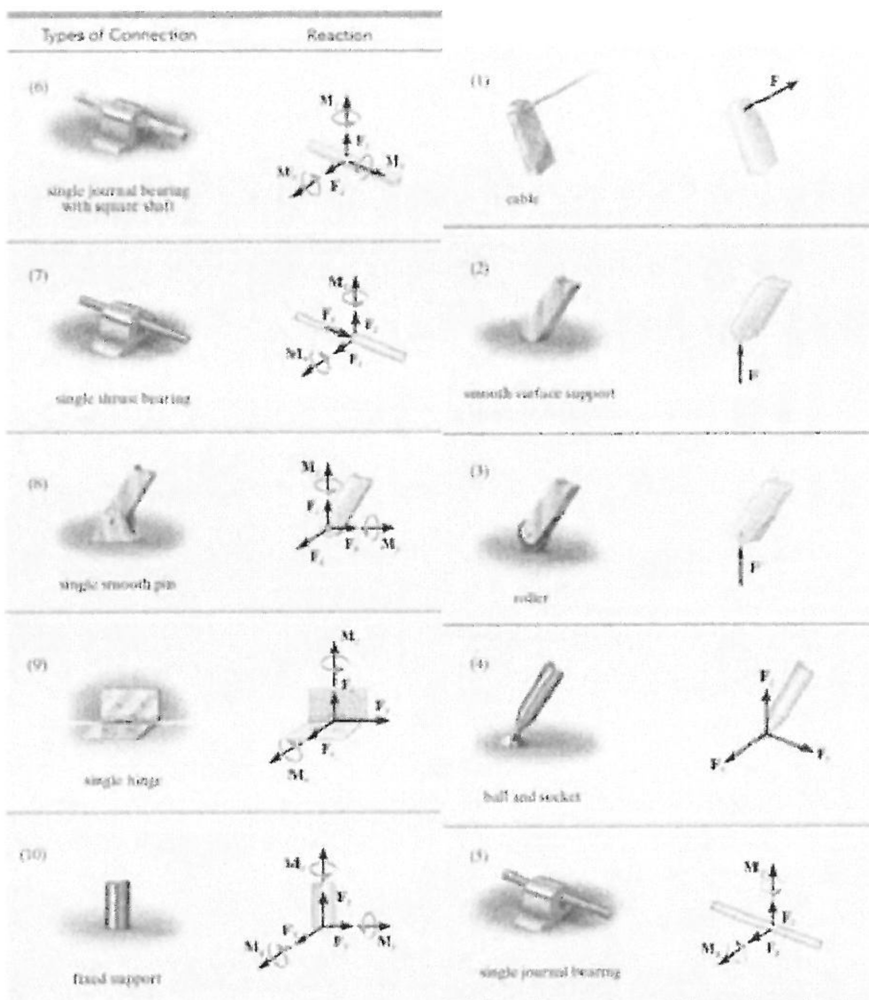
In 3D systems, we have 6 equilibrium equations available to solve for unknown forces and support reactions.

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

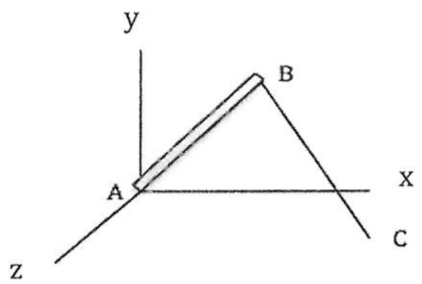
This means we can solve for 6 unknown forces or force couples.

Types of 3-D supports and the reactions at those supports:



This is a great spot to remember how to calculate the moment of a force about a point. If the perpendicular distance between a force and the point is easy to find, then we can use: $|M| = D|F|$

But this is a specific instance. The following method ALWAYS works to calculate a moment about a point:



$$\overrightarrow{M_A} = \overrightarrow{r_{AB}} \times \overrightarrow{F_{BC}} \text{ or we can use, } \overrightarrow{M_A} = \overrightarrow{r_{AC}} \times \overrightarrow{F_{BC}}$$

The position vector is always pointing <u>from</u> the point we are calculating the moment about, <u>to</u> the force line of action. If we only have a magnitude of a force, the force vector is calculated by multiplying the magnitude by the *unit vector* in the same direction.

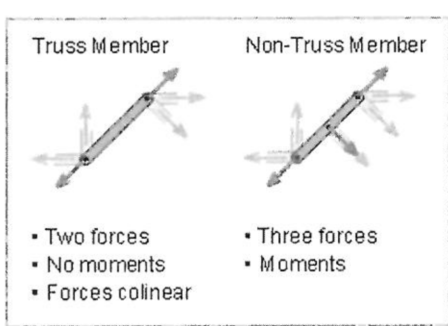
<u>Solving For Unknown Reactions at Supports:</u>

1. FBD about the entire structure (replacing supports with appropriate reactions).
2. Apply equilibrium equations and solve.

Note: When summing the moments about a point in a more complex system, *first* calculate the moment vector using the cross product. *Then*, sum moments in the $\hat{\imath}, \hat{\jmath}, and\ \hat{k}$ directions.

5.4
<u>2-Force Members</u>

If a solid member of a structure has forces acting on it in equal and opposite directions and no other forces act on that member, it is a two-force member.



In truss calculations and other complex structures, replacing tension or compression forces with 2-forces rather than x and y components can simplify your calculations.

6.1-6.3

Trusses

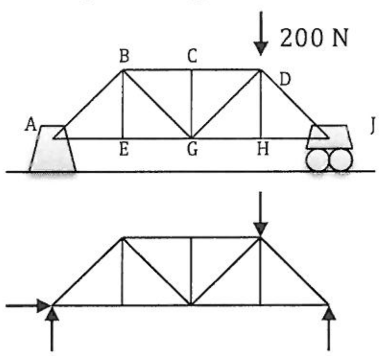
Trusses are formed by connecting multiple 2-force members together. Bridges and roof trusses are examples of these structures. We can solve for the tension or compression in truss members using the method of joints or the method of sections...or a combination of both methods.

Method of Joints and Method of Sections:

Both methods allow us to solve for tension or compression in members within a structure. Both start with the same first 2 steps...

Step 1: FBD about entire structure.
Step 2: Apply equilibrium to solve for the reactions at the supports

Example: Length AE=EG=BC=GH=CD=HJ=EB=CG=DH=5m



$$\sum F_x = A_x = 0$$

$$\sum F_y = A_y + J_y - 200N = 0$$

$$\sum M_A = (-200N)(15m) + (J_y)(20m) = 0$$

$$J_y = 150N$$

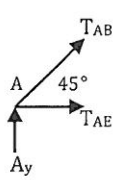
$$A_y = 50N$$

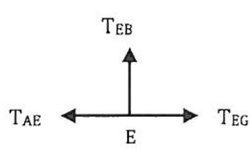
Note: clockwise moments are (-), counterclockwise moments are (+).

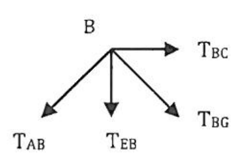
Solve for tension or compression in BC, BG and EG.

Method 1: Method of Joints.

Look at one and only one joint at a time. Apply equilibrium equations and solve for tension members. Draw forces as tensions. If the answer is negative, the force is in compression.

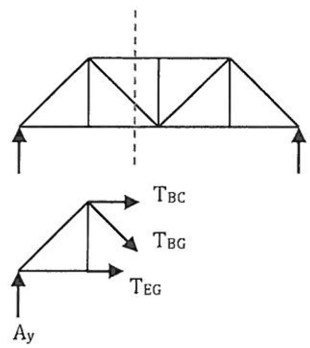






Method 2: Method of Sections.

Cut the entire structure (top to bottom) through tension members. Replace all cut members with tension forces. Apply equilibrium equations (including moments about a point in the structure). All outside forces that act on your remaining section are included in your calculations!

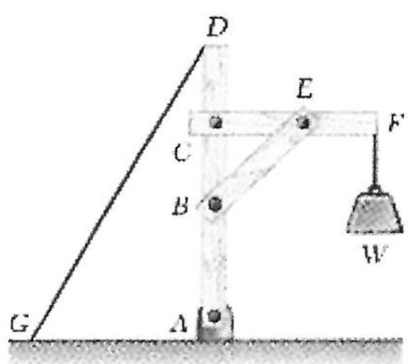


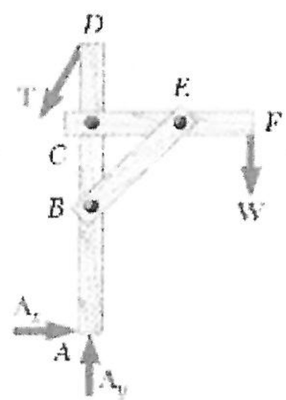
Either method works....but often the method of sections is a shorter way to solve the problem when looking at tension or compression members in the center of your structure.

6.5
Frames and Machines

If a structure is formed by members that are not all 2-force members, the structure is a frame (designed to support), or a machine (designed to apply forces).

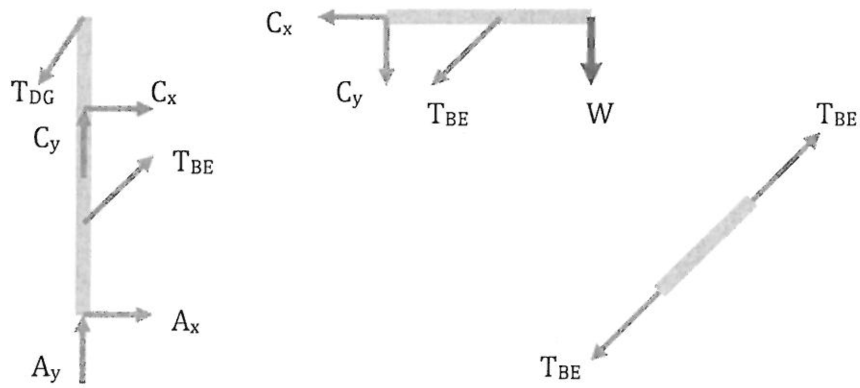
We can analyze an entire structure (including connection points) by breaking up the structure into individual members.





Member BE is a 2-force member, but CF is a 3-force member and AD is a 4-force member. So this structure is a frame used to support W.

We can solve for all of the reactions at each joint and the tension in the 2-force member. If any of these reactions are negative, it means the reaction points in the opposite direction that the reaction was originally drawn.



Now, apply equilibrium equations to each member...including summing moments about a point on that member.

At connection points shared by 2 members, the forces acting on one member are *equal and opposite* of those acting on the other member at that connection point.

**If an applied force acts at a joint shared by 2 members, we include that applied force on just ONE of the separated members.

REVIEW
Chapters 7 and 10

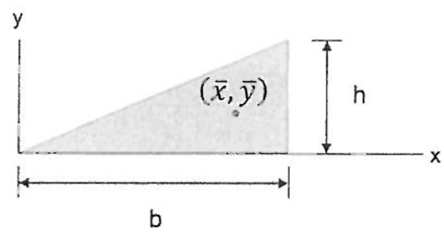
Section 7.1 – Centroids:

The centroid of any area is listed as a position $(\bar{x},\ \bar{y})$. If we define a small area of any shape as dA, then we can calculate the centroid using,

$$\bar{x} = \frac{\int x dA}{\int dA}, and\ \bar{y} = \frac{\int y dA}{\int dA}$$

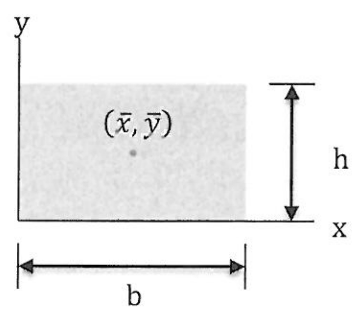
For common shapes, we can calculate these positions relatively simply:

A triangle:



$$\bar{x} = \left(\frac{2}{3}\right) b, and\ \bar{y} = \left(\frac{1}{3}\right) h$$

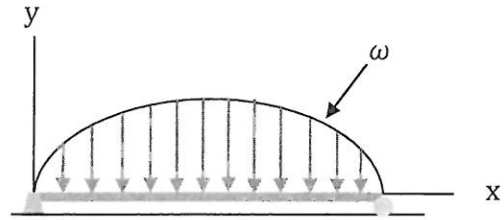
A rectangle:



$$\bar{x} = \left(\frac{1}{2}\right) b, and\ \bar{y} = \left(\frac{1}{2}\right) h$$

Section 7.3 – Distributed Loads:

If we have a load distributed, ω, across some length, x, we can calculate a "resultant" load that is a single point load representing the entire distributed load. This allows us to calculate support reactions and other calculations about the structure a bit easier.



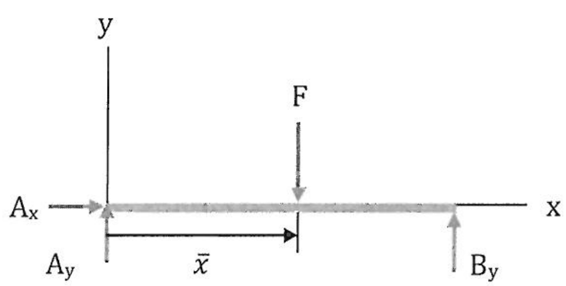
$$\boldsymbol{F_{resultant} = \int \omega dx = \int dA = A}$$

Yes…this means the resultant force equals the area under the function curve!!

$$\boldsymbol{M_{origin} = \bar{x}F = \int x\omega dx}$$

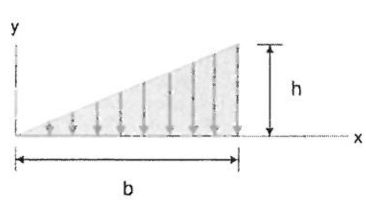
$$\boldsymbol{\bar{x} = \frac{M}{F} = \frac{\int x\omega dx}{\int \omega dx} = \frac{\int x dA}{\int dA}}$$

So we can replace the entire distributed force with our calculated F at the centroid.



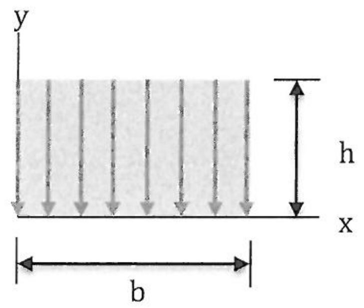
Now, we can calculate the reactions at our supports by applying equilibrium equations to our entire system.

For "triangular" and "rectangular" distributed loads, our calculation for F and the centroid are simplified.



$$F = \text{"area"} = \left(\frac{1}{2}\right) bh$$

$$\bar{x} = \left(\frac{2}{3}\right) b$$



$$F = \text{"area"} = bh$$

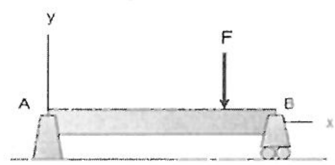
$$\bar{x} = \left(\frac{1}{2}\right) b$$

We then apply equilibrium equations to solve for support reactions, etc.

Section 10.1 – Axial Force, Shear Force, and Bending Moment

Any structural member that is subjected to a load will have internal forces acting along the length of that member. A member will have an axial force, shear force, and bending moment.

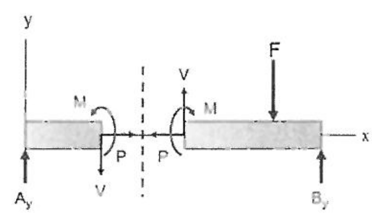
When solving a problem asking for these internal forces, follow these steps:



Step 1: Convert any distributed loads to a resultant point load and complete a FBD about the entire structure.

Step 2: Solve for your support reactions.

Step 3: Cut your structural member and replace the cut edge with the appropriate shear, axial, and bending moment forces. ***If your load was a distributed load and your cut occurs in the middle of that distributed load, then you must replace your point load with the original distributed load and calculate a new resultant force, F.*

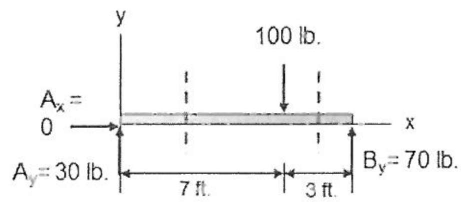


Step 4: Apply equilibrium equations to *either* the left or right portion of the member to solve for V, P, and the internal moment.

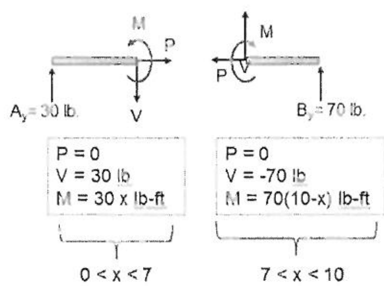
Section 10.2 – Shear Force and Bending Moment Diagrams

Okay, so now we can calculate the internal shear force, bending moment, and axial force at any point along a structural member. But we don't want to have to calculate these values for every point...rather, we would like to be able to graph our results to show our V and M in terms of x (distance along our structural member).

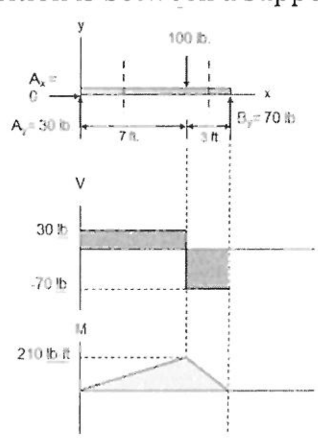
Step 1: FBD about the entire structure to solve for any support reactions.



Step 2: Cut your structural member on either side of any applied force...at an arbitrary point, x. This will allow you to solve for the shear force and moment in terms of x within a certain range of x.

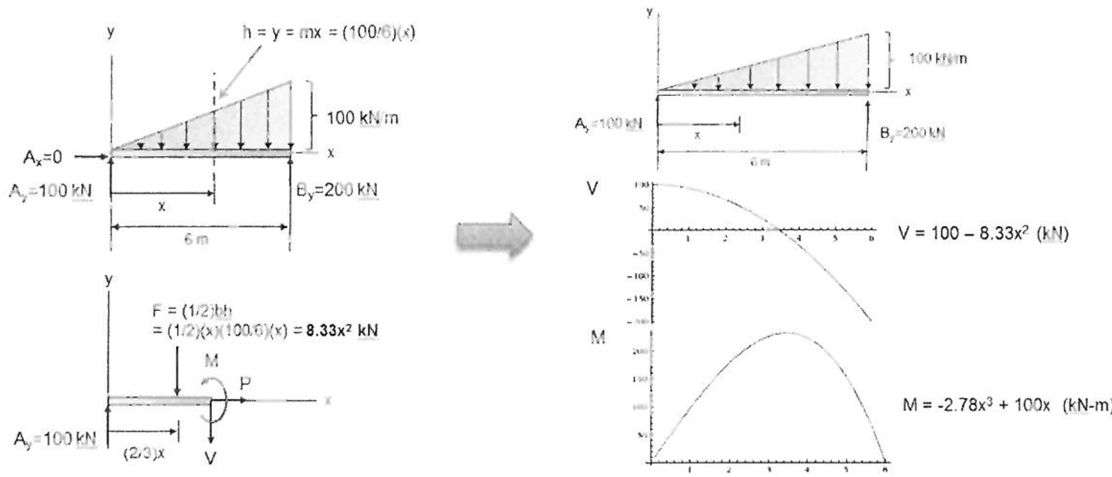


Step 3: Solve for the shear force and moment in terms of x...don't cut at a specific value of x...just be sure the position is between a support and any applied force.



Step 4: Draw your functions of V and M.

**Note: If a distributed load acts on a beam, cut the beam in the middle of the distributed load (at an arbitrary point, x). Replace the original distributed load with a scaled-down version of the load and solve in terms of x.



10.3 – Relationships Between Distributed Load, Shear Force, and Bending Moment

A distributed load, shear force function, and moment function in terms of x are all related to one another...kind of makes sense. Below are those relationships.

$$\boxed{\frac{dV}{dx} = -\omega}$$

$$\boxed{\int_{V_A}^{V_B} dV = -\int_{x_A}^{x_B} \omega dx}$$

$$\boxed{V_B - V_A = -\int_{x_A}^{x_B} \omega dx}$$

$$\boxed{\frac{dM}{dx} = V}$$

$$\boxed{\int_{M_A}^{M_B} dM = \int_{x_A}^{x_B} V dx}$$

$$\boxed{M_B - M_A = \int_{x_A}^{x_B} V dx}$$

Be sure you understand that taking the integral of a function and evaluating that integral over the bounds of the function gives you the AREA between the function and the x-axis.

The area over which a force is distributed is analogous to the resultant force.

$$F_{resultant} = \int \omega dx = \int dA = A$$

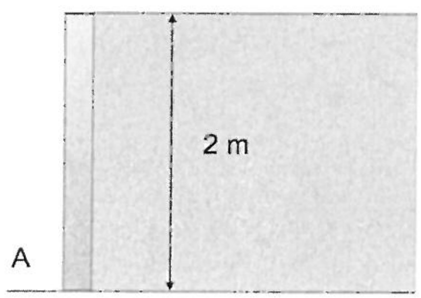
This will be an important concept on the final!

10.7 – Pressure and Center of Pressure (EXTRA CREDIT MATERIAL)

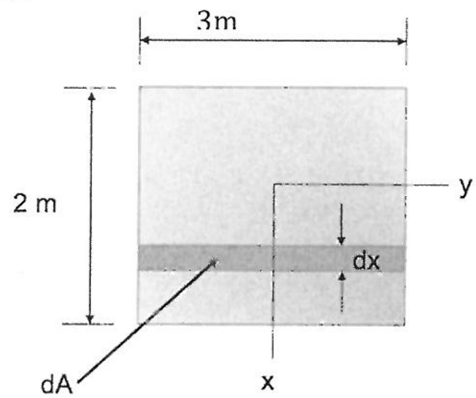
Pressure is a force that is distributed over a contact area. So we can solve pressure problems much like distributed force problems. We talked mainly about the distributed pressure force at depths under water.

Under water, we have 2 forces acting on objects, atmospheric pressure, P_{atm}, and the pressure due to the "weight" of the water, $P = \rho g x = \gamma x$. In this relationship, ρ is the density of water, $\gamma = \rho g$ is the weight density of water or just the density multiplied by the force of gravity. x is the distance "below" the surface of the water (positive x is going down).

**With vertical objects that are exposed to air on one side, P_{atm} acts on both sides of the object and therefore cancels. So with vertical submerged objects, we just have the pressure of the water acting on that object.



Side View of Submerged Gate



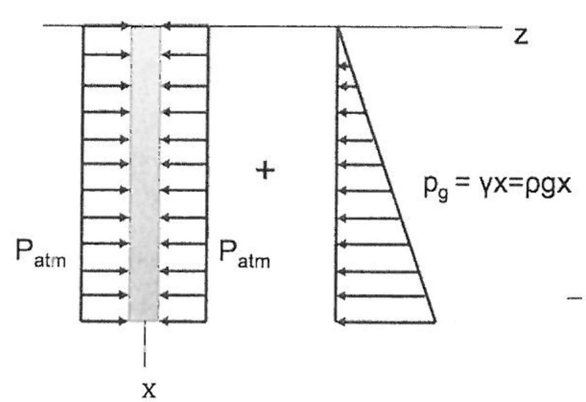
Front View of Submerged Gate

So we can solve for the resultant force acting on an object by integrating the pressure function (calculating the "area" of the pressure distribution). *Note: In this example, we have neglected the weight of the gate.*

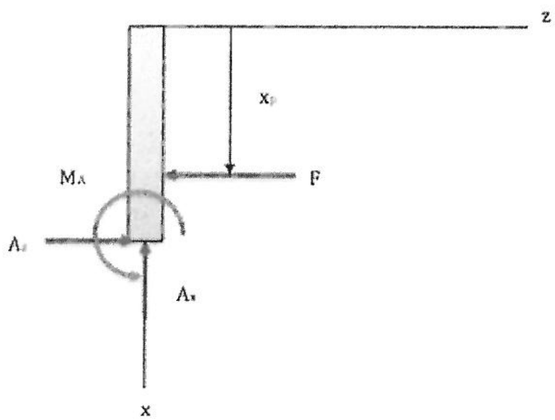
$$F = \int p dA = \int_0^2 \rho g x(3m)dx$$

$$M = \int x p dA = \int_0^2 x(\rho g x)(3m)dx$$

$$center\ of\ pressure = x_p = \frac{M}{F}$$

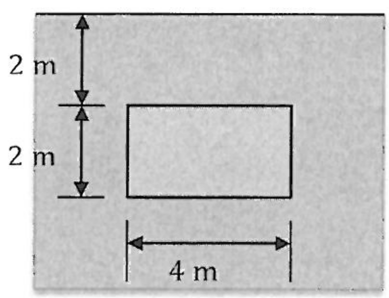


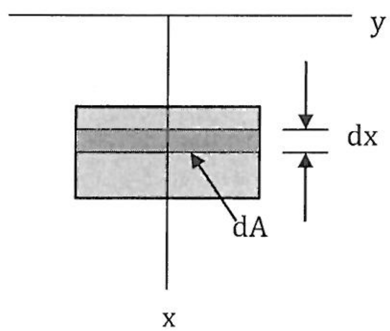
A was a fixed support, so we now replace the support with the appropriate reactions. We have solved for a resultant force, F, and the center of pressure, x_p. We can now apply equilibrium equations to solve for our support reactions, A_x, A_z, and M_A.



If an object is not exposed to the air...it is fully submerged, then atmospheric pressure distributions do not cancel. In this instance, $P = P_{atm} + \rho g x$.

Say we wanted to find the resultant force and the center of pressure of the following submerged plate:





$$F = \int p dA = \int_2^4 (P_{atm} + \rho g x)(4\ m)dx$$

$$M = \int x p dA = \int_2^4 x(P_{atm} + \rho g x)(4m)dx$$

$$x_p = \frac{M}{F}$$

$\Rightarrow$ *The center of pressure does not coincide with the center of mass here.*