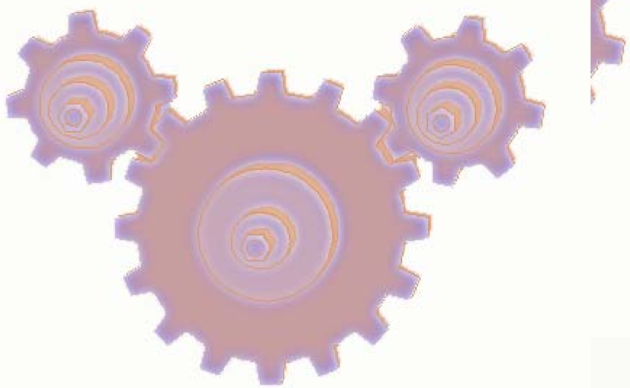
Mechanics of Materials

Chapter 4

Shear and Moment In Beams



4.1 *Introduction*

- The term beam refers to a slender bar that carries **transverse loading**; that is, the applied force are perpendicular to the bar.
- In a beam, the internal force system consist of a shear force and a bending moment acting on the cross section of the bar. The shear force and the bending moment usually vary continuously along the length of the beam.
- The internal forces give rise to **two** kinds of stresses on a transverse section of a beam: (1) normal stress that is caused by bending moment and (2) shear stress due to the shear force.
- Knowing the distribution of the shear force and the bending moment in a beam is essential for the computation of stresses and deformations. Which will be investigated in subsequent chapters.

4.2 *Supports and Loads*

- Beams are classified according to their supports. A ***simply supported beam***, shown in Fig. 4.1 (a). The **pin support** prevents displacement of the end of the beams, but not its rotation. The term **roller support** refers to a pin connection that is free to move parallel to the axis of the beam; this type of support suppresses only the transverse displacement.

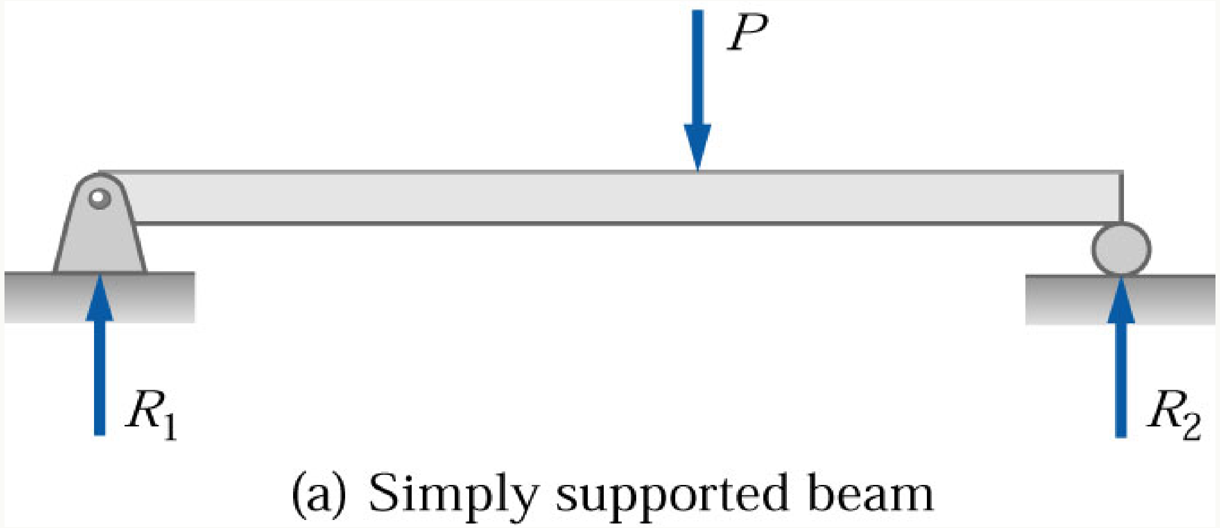
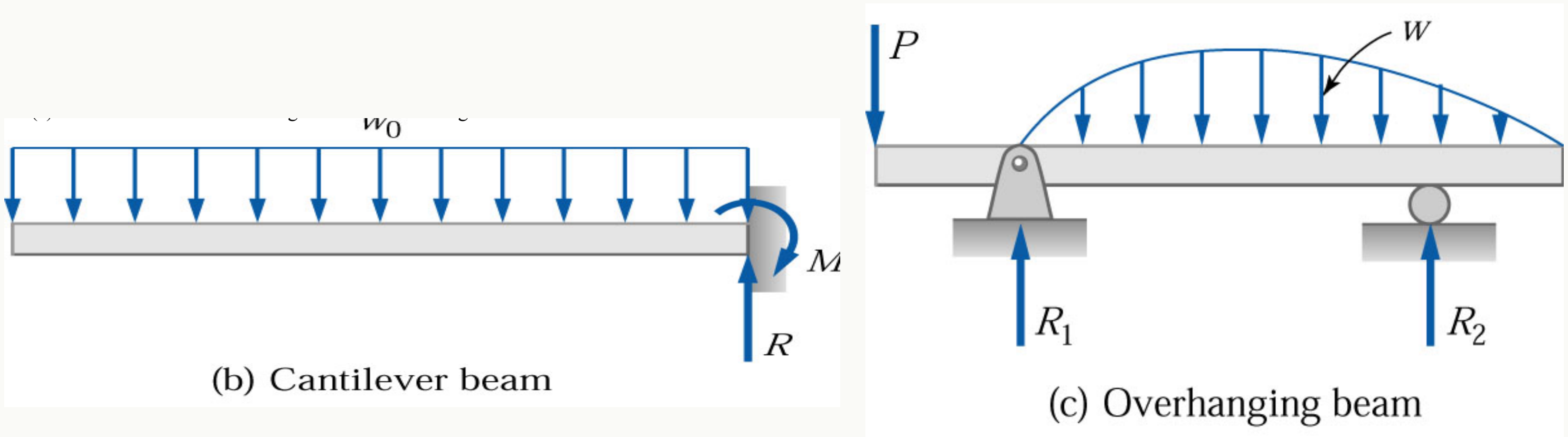


Figure 4.1 (a) Statically determinate beams.

- A ***cantilever beam*** is built into a rigid support at one end, with the other end being free, as shown in Fig.4.1(b). The built-in support prevents displacements as well as rotations of the end of the beam.
- An ***overhanging beam***, illustrated in Fig.4.1(c), is supported by a pin and a roller support, with one or both ends of the beam extending beyond the supports.
- The three types of beams are **statically determinate** because the support reactions can be found from the equilibrium equations.

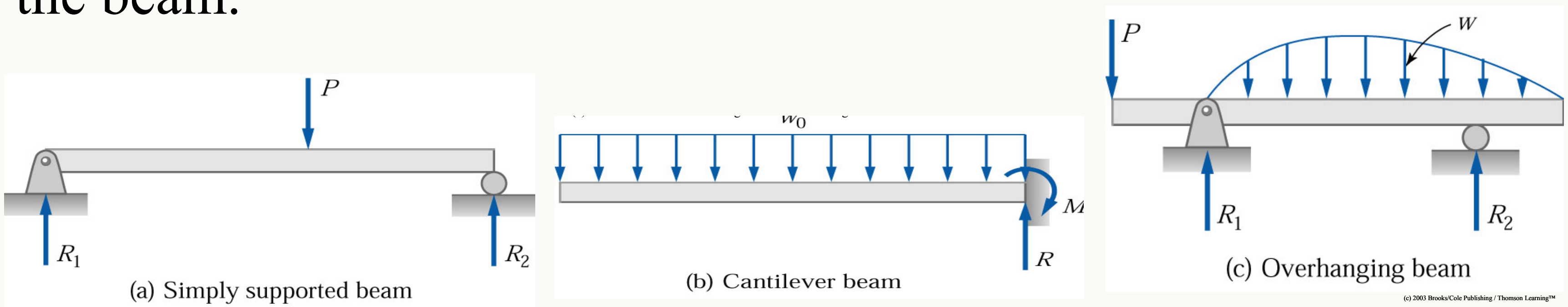


(b) Cantilever beam

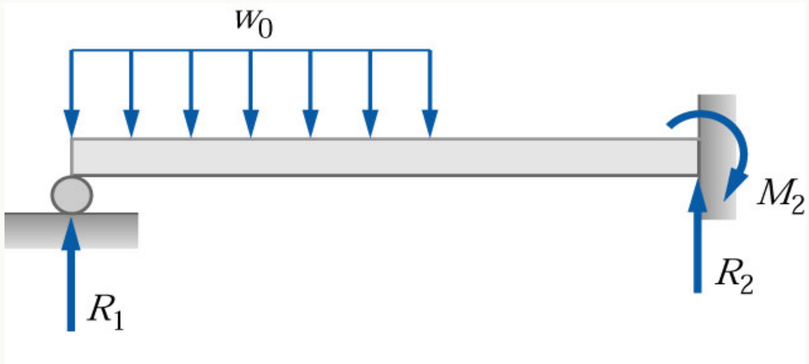
(c) Overhanging beam

Figure 4.1 Statically determinate beams

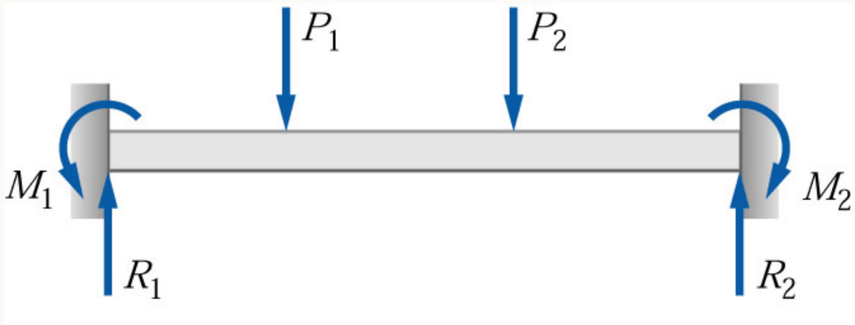
- A ***concentrated load***, such as *P* in Fig. 4.1(a). In contrast a ***distributed load*** is applied over a finite area. If the distributed load acts on a very narrow area, the load may be approximated by a *line load*.
- The **intensity *w*** of this loading is expressed as force per unit length (lb/ft, N/m, etc.) The load distribution may be uniform, as shown in Fig.4.1(b), or it may vary with distance along the beam, as in Fig.4.1(c).
- The weight of the beam is an example of distributed loading, but its magnitude is usually small compared to the loads applied to the beam.



- Figure 4.2 shows other types of beams. These beams are ***oversupported*** in the sense that each beam has at least one more reaction than is necessary for support. Such beams are **statically indeterminate**; the presence of these ***redundant supports*** requires the use of additional equations obtained by considering the deformation of the beam. The analysis of statically indeterminate beams will be discussed in Chapter 7.



(a) Propped cantilever beam



(b) Fixed or restrained beam

Figure 4.2 Statically indeterminate beams

4.3 Shear- Moment Equations and Shear-Moment Diagrams

- The determination of the internal force system acting at a *given* section of a beam : draw a free-body diagram that expose these forces and then compute the forces using equilibrium equations.
- The goal of the beam analysis —determine the shear force *V* and the bending moment *M* at *every* cross section of the beam.
- To derive the expressions for *V* and *M* in terms of the distance x measured along the beam. By plotting these expressions to scale, obtain the ***shear force and bending moment diagrams*** for the beam.
- The shear force and bending moment diagrams are convenient visual references to the internal forces in a beam; in particular, they identify the maximum values of *V* and *M*.

a. Sign conventions

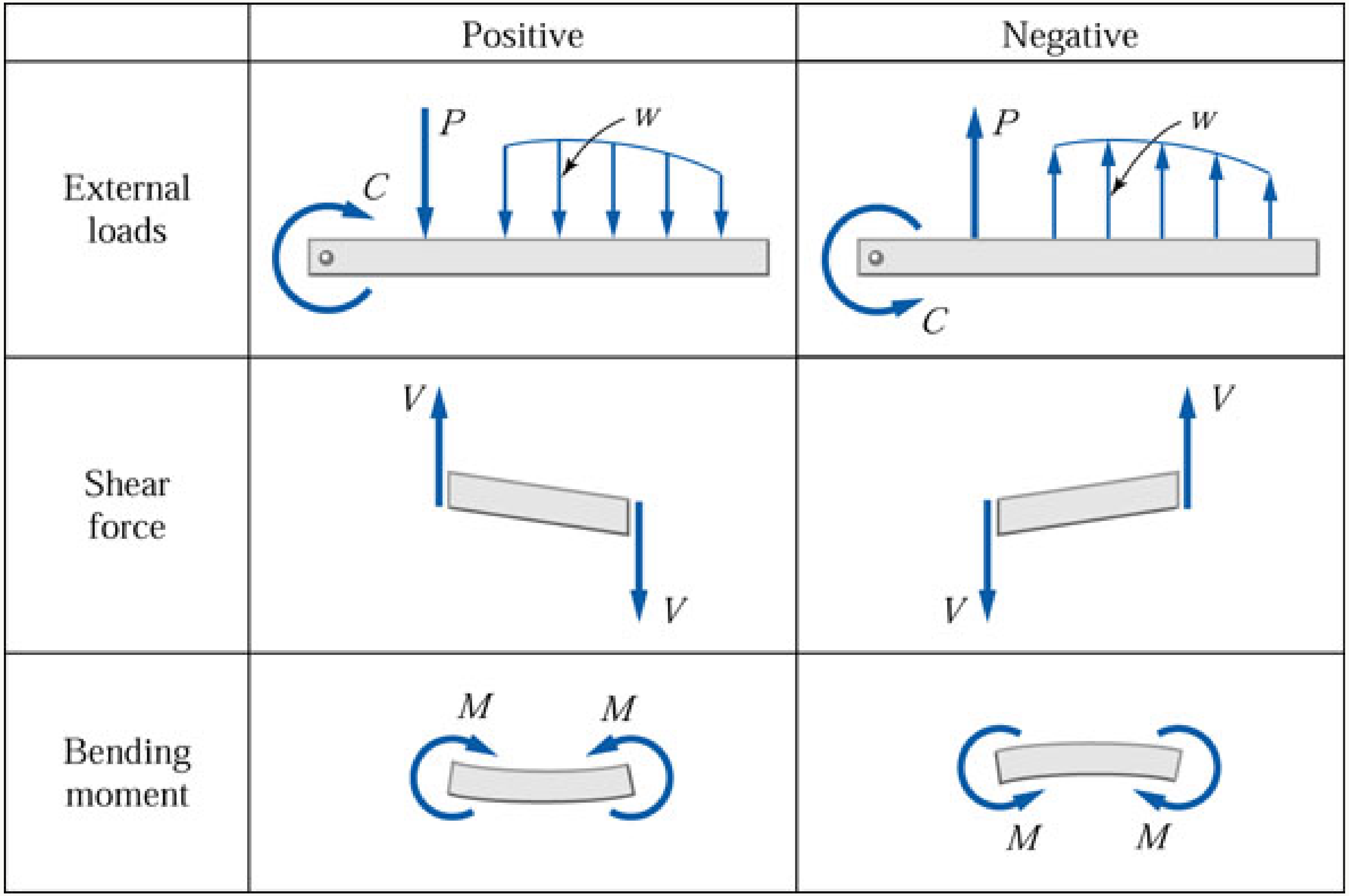


Figure 4.3 Sign conventions for external loads; shear force, and bending moment.

b. ***Procedure for determining shear force and bending moment diagrams***

- Compute the support reactions from the free-body diagram (FBD) of the entire beam.
- Divide the beam into segment so that the loading within each segment is continuous. Thus, the end-points of the segments are discontinuities of loading, including concentrated loads and couples.

■ Perform the following steps for each segment of the beam:

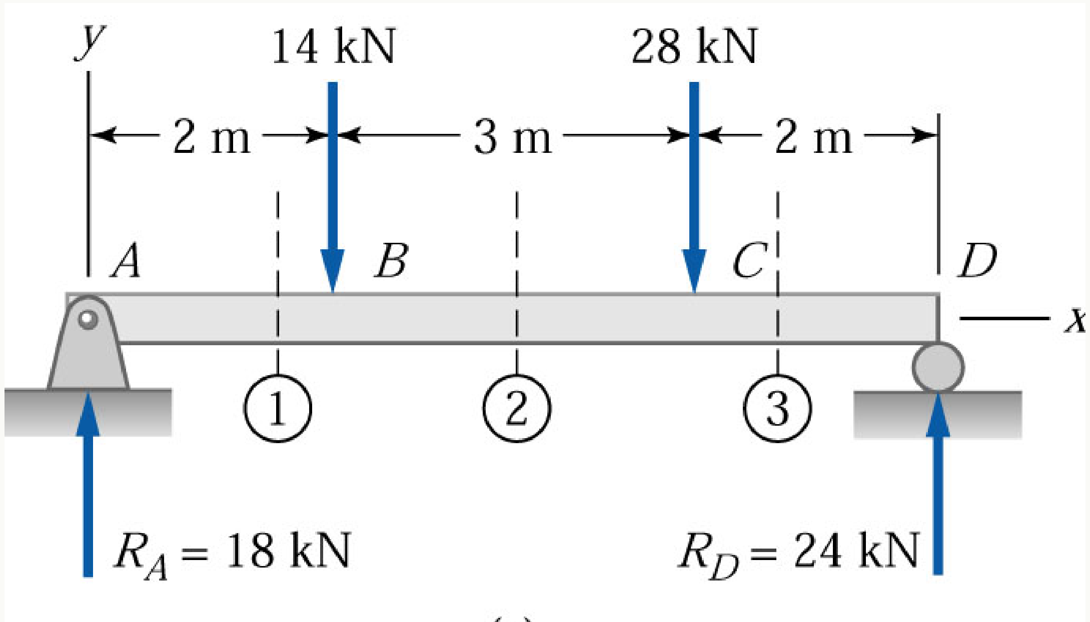
- Introduce an imaginary cutting plane within the segment, located at a distance x from the left end of the beam, that cuts the beam into two parts.
- Draw a FBD for the part of the beam lying either to the left or to the right of the cutting plane, whichever is more convenient. At the cut section, show V and M acting in their positive directions.

- Determine the expressions for *V* and *M* from the equilibrium equations obtainable from the FBD. These expressions, which are usually functions of x, are the shear force and bending moment equations for the segment.
- Plot the expressions for V and M for the segment. It is visually desirable to draw the *V*-diagram below the FBD of the entire beam, and then draw the *M*- diagram below the *V*-diagram.

❑ The bending moment and shear force diagrams of the beam are composites of the *V* and *M* diagrams of the segments. These diagrams are usually discontinuous, or have discontinuous slopes. At the end-points of the segments due to discontinuities in loading.

Sample Problem 4.1

The simply supported beam in Fig. (a) carries two concentrated loads. (1) Derive the expressions for the shear force and the bending moment for each segment of the beam. (2) Sketch the shear force and bending moment diagrams. Neglect the weight of the beam. Note that the support reactions at *A* and *D* have been computed and are shown in Fig. (a).



(a)

Solution

Part 1

The determination of the expressions for *V* and *M* for each of the three beam segments (*AB*,*BC*, and *CD*) is explained below.

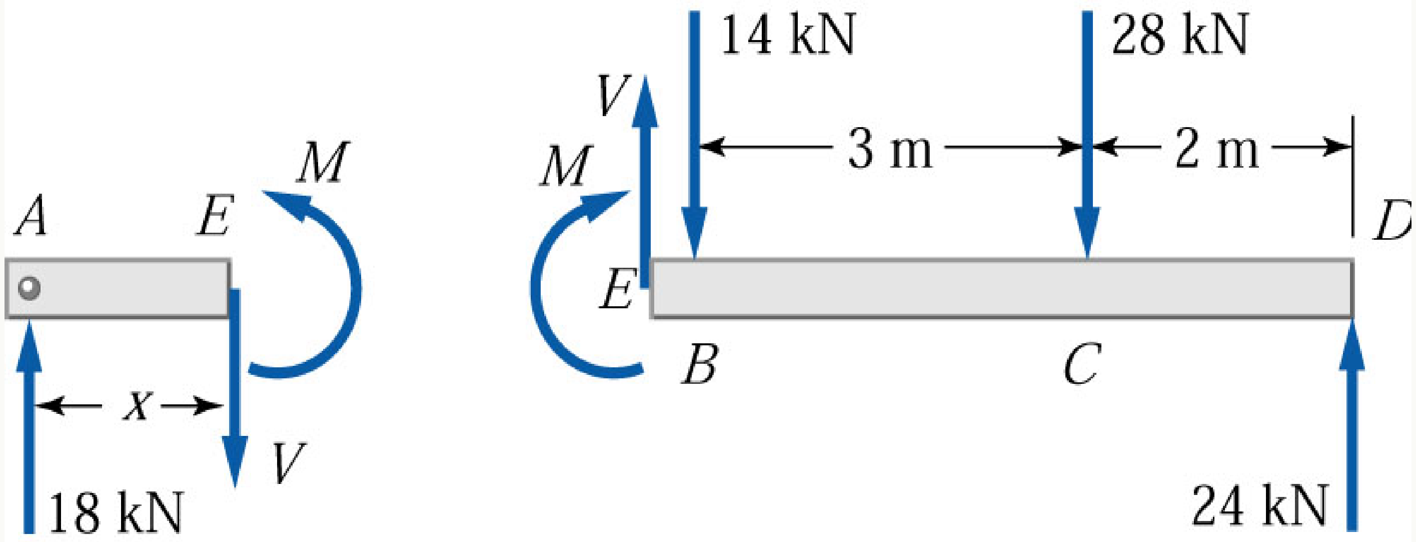
***Segment AB* (0 < x < 2 m)**

$\sum F_y = 0 + \uparrow$ $\quad$ $18 - V = 0$

$V = +18\ \text{kN}$ *Answer*

$\sum M_E = 0 + \circlearrowleft$ $\quad$ $-18x + M = 0$

$M = +18x\ \text{kN}\cdot\text{m}$ *Answer*



(b) FBDs

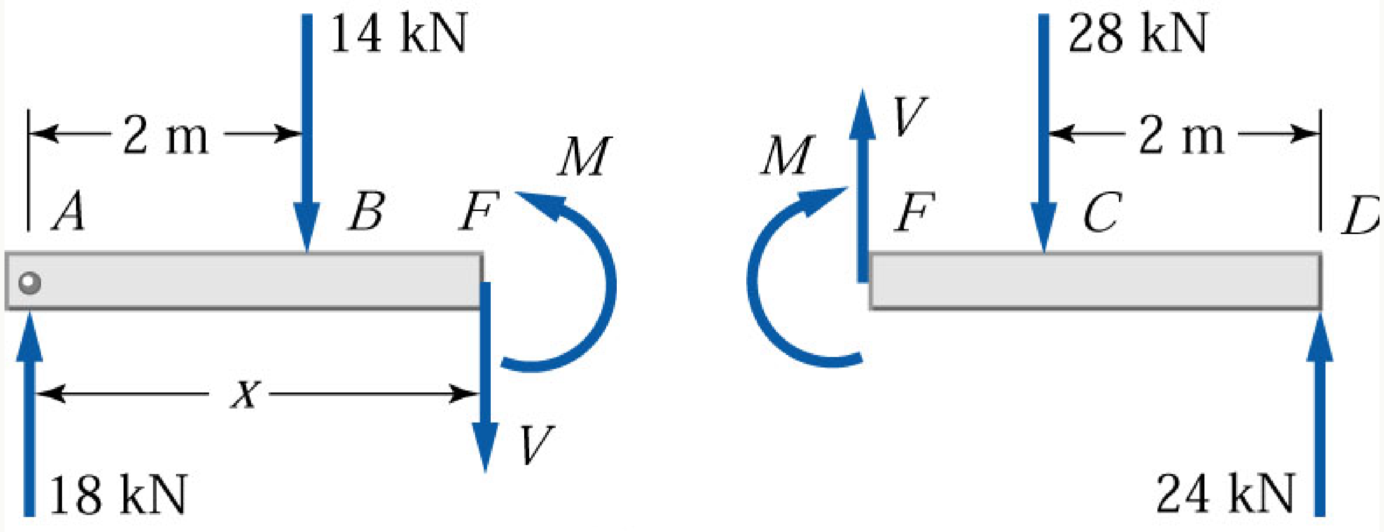
Segment AB **(2 < x < 5 m)**

$\sum F_y = 0 + \uparrow$ $\qquad 18-14-V = 0$

$V = +18-14 = +4$ kN *Answer*

$\sum M_E = 0 + \circlearrowleft$ $\qquad -18x + 14(x-2) + M = 0$

$M = +18x-14(x-2) = 4x+28$ kN· m *Answer*



(c) FBDs

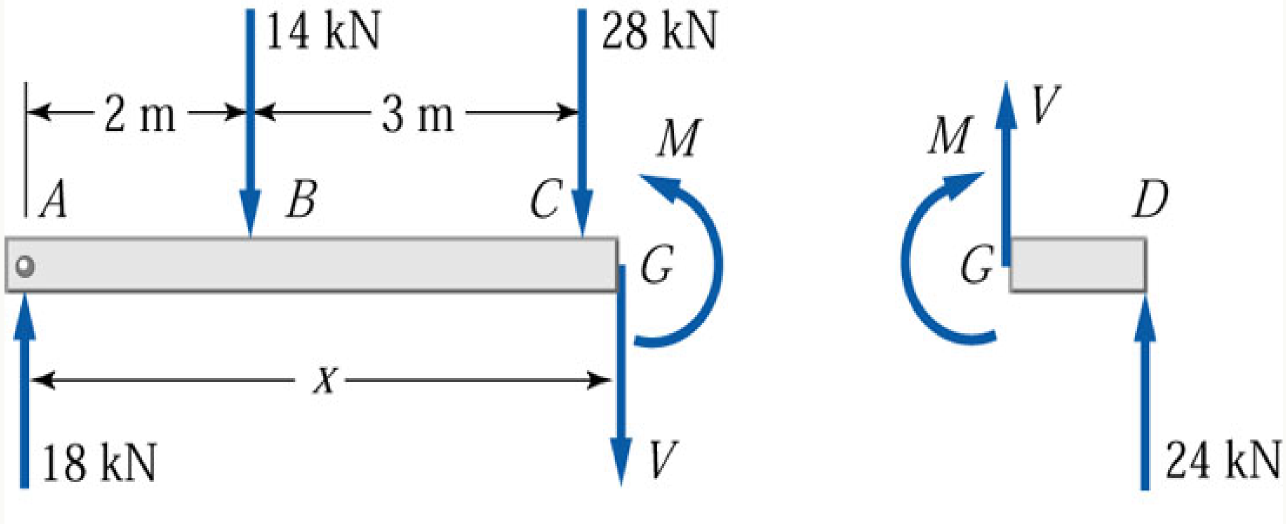
Segment CD **(5 m < x < 7 m)**

$\sum F_y = 0 + \uparrow \qquad 18\text{-}14—28\text{-}V = 0$

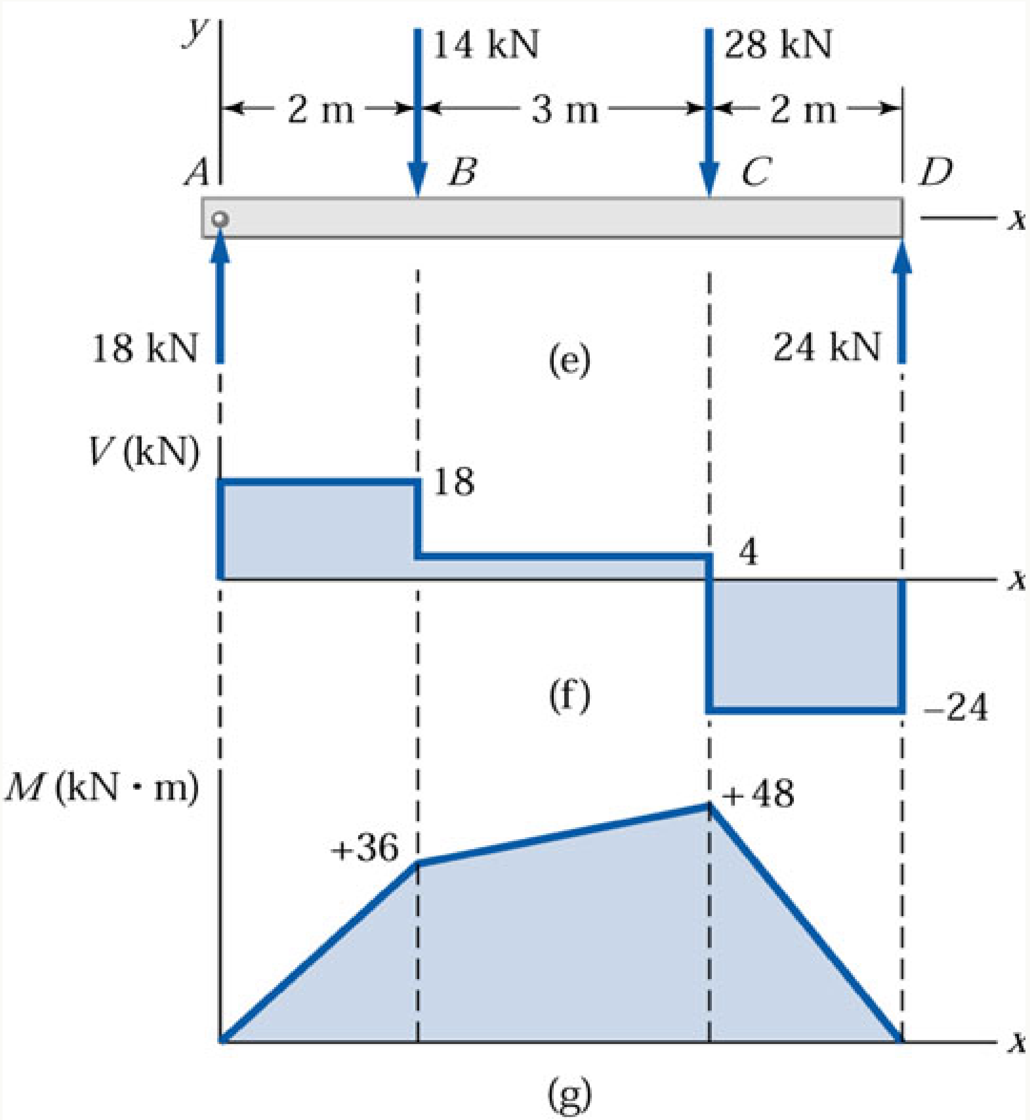
$V = +18\text{-}14\text{-}28 = \text{-}24 \text{ kN}$ *Answer*

$\sum M_G = 0 + \circlearrowleft \quad -18x + 14(x\text{-}2) + 28(x\text{-}5) + M = 0$

$M = +18x\text{-}14(x\text{-}2) – (x\text{-}5) = \text{-}24x + 168 \text{ kN}\cdot\text{m}$ *Answer*



(d) FBDs



Shear force and bending moment diagrams

Part 2

- The *V*-diagram reveals that the largest shear force in the beam is -24 kN : segment *CD*
- The *M*-diagram reveals that the maximum bending moment is +48 kN·m : the 28-kN load at *C*.
- Note that at each concentrated force the *V*-diagram **"jumps"** by an amount equal to the force.
- There is a discontinuity in the slope of the *M*-diagram at each concentrated force.

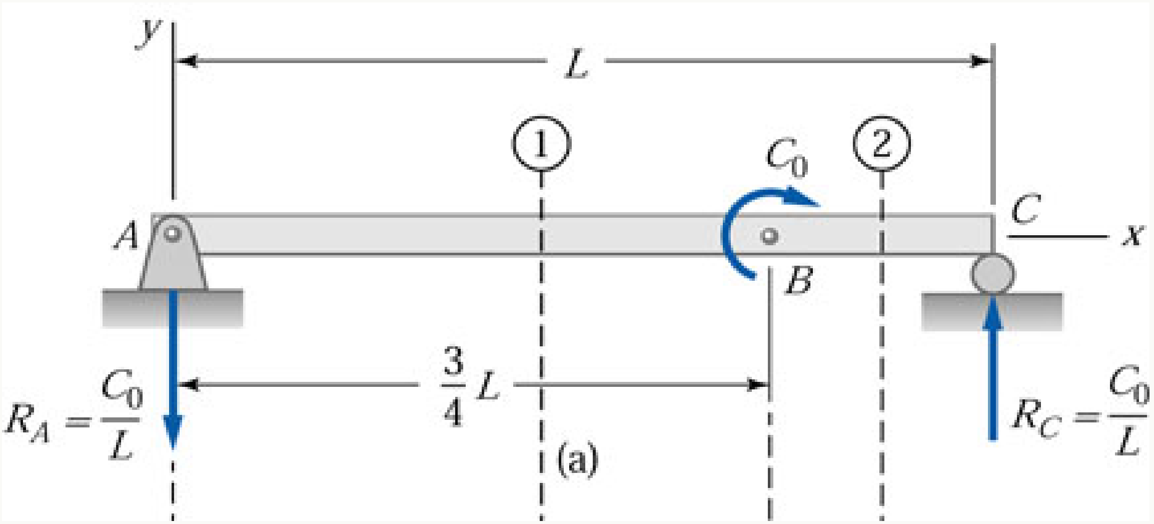
Sample problem 4.2

The simply supported beam in Fig. (a) is loaded by the clockwise couple C_0 at *B*. (1) Derive the shear and bending moment equations. And (2) draw the shear force and bending moment diagrams. Neglect the weight of the beam. The support reactions A and C have been computed, and their values are shown in Fig. (a).

Solution

Part 1

Due to the presence of the couple C_{0}, We must analyze segments *AB* and *BC* separately.



Segment AB (0<x<3*L*/4)

$$\sum F_y = 0 + \uparrow -\frac{C_0}{L} - V = 0$$

$$V = -\frac{C_0}{L} \qquad \textit{Answer}$$

$$\sum M_D = 0 + \circlearrowleft \ \frac{C_0}{L}x + M = 0$$

$$M = -\frac{C_0}{L}x \qquad \textit{Answer}$$

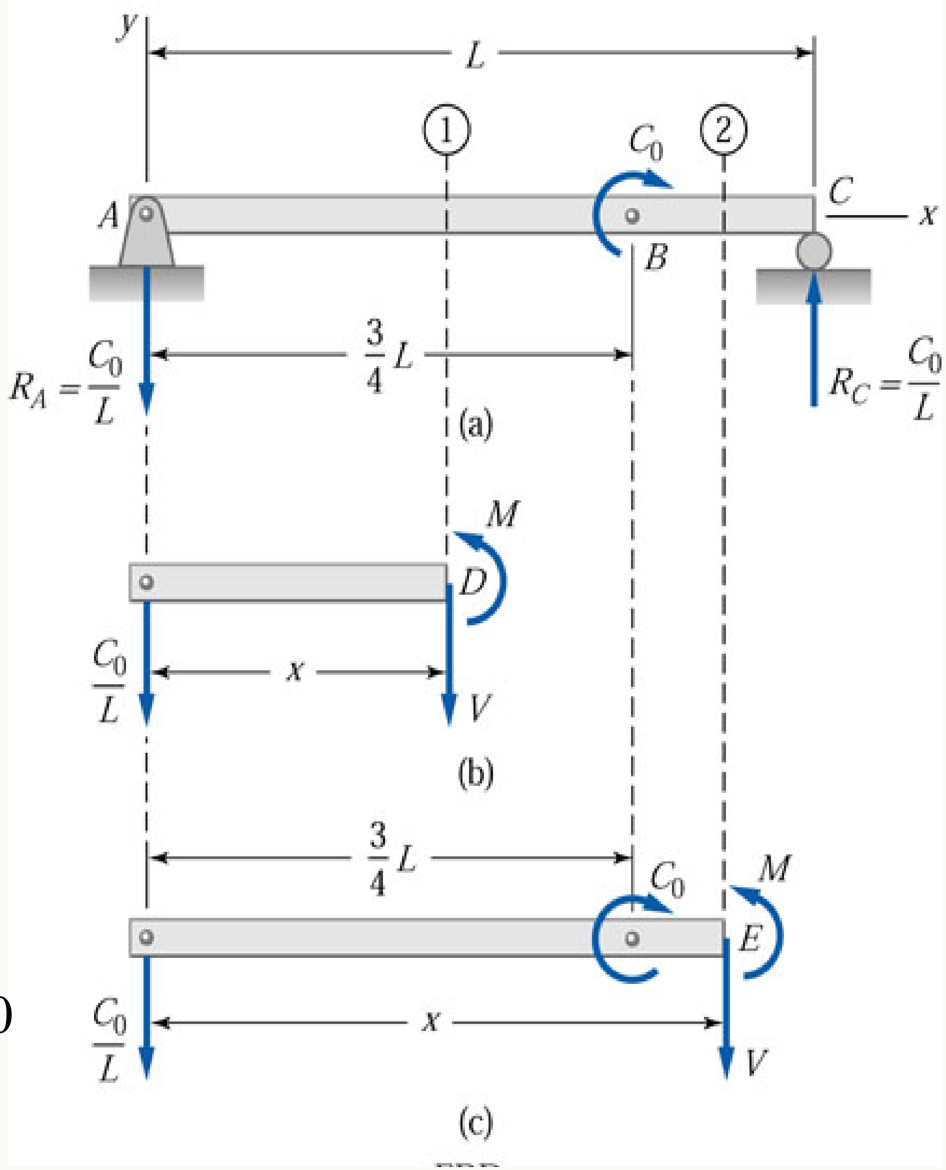
Segment BC (3*L*/4<x<L)

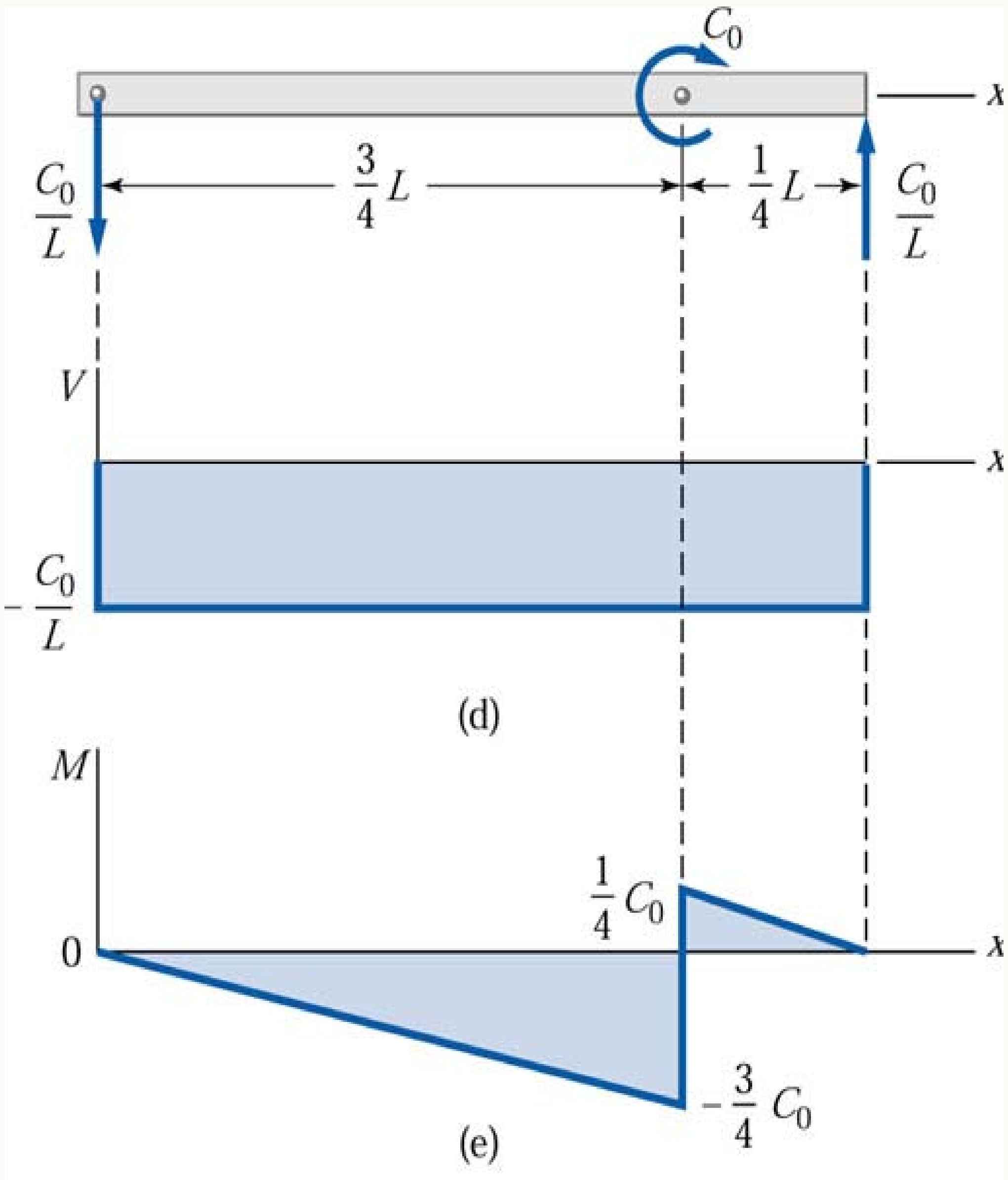
$$\sum F_y = 0 + \uparrow -\frac{C_0}{L} - V = 0$$

$$V = -\frac{C_0}{L} \qquad \textit{Answer}$$

$$\sum M_E = 0 + \circlearrowleft \ \frac{C_0}{L}x - C_0 + M = 0$$

$$M = -\frac{C_0}{L}x + C_0 \qquad \textit{Answer}$$





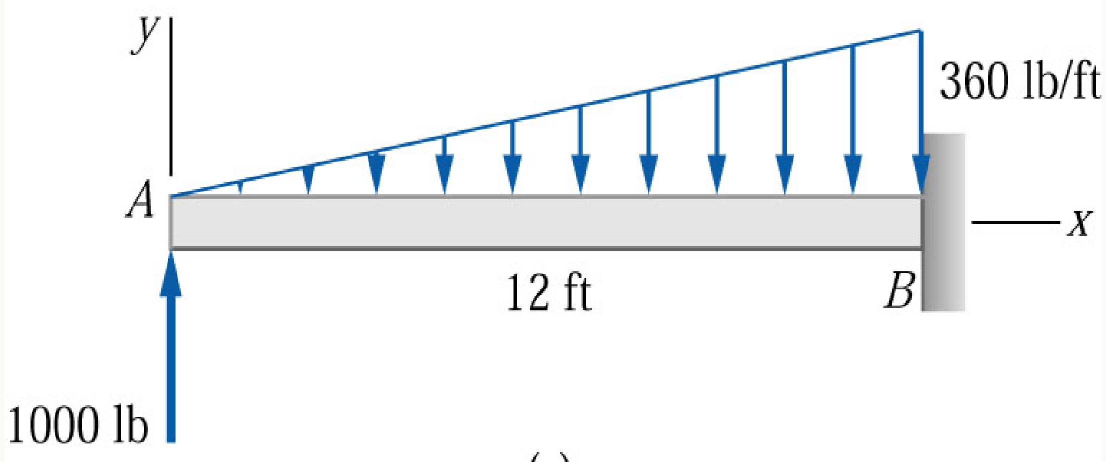
Shear force and bending moment diagrams

Part 2

From the V-diagram, the shear force is the same for all cross sections of the beam. The M-diagram shows jump of magnitude C_0 at the point of application of the couple.

***Sample Problem* 4.3**

The cantilever beam in Fig.(a) carries a triangular load. The intensity of which varies from zero at the left end to 360 lb/ft at the right end. In addition, a 1000-lb upward vertical load acts at the free end of the beam. (1) Derive the shear force and bending moment equations. And (2) draw the shear force and bending moment diagrams. Neglect the weight of the beam.

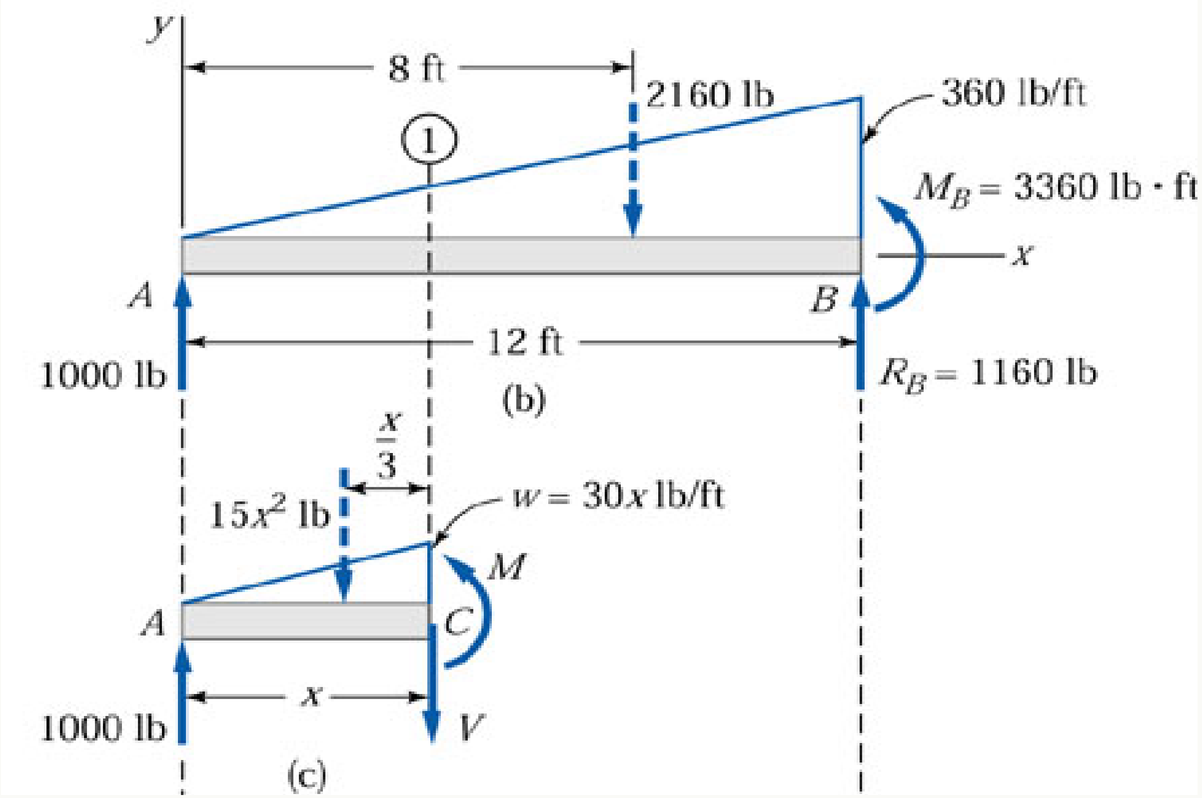


(a)

Solution

Note that the triangular load has been replaced by is resultant, which is the force 0.5 (12) (360) = 2160 lb (area under the loading diagram) acting at the centroid of the loading diagram.

- Because the loading is continuous, the beam does not have to be divided into segment.
- $w/x = 360/12$, or $w = 30x$ lb/ft.



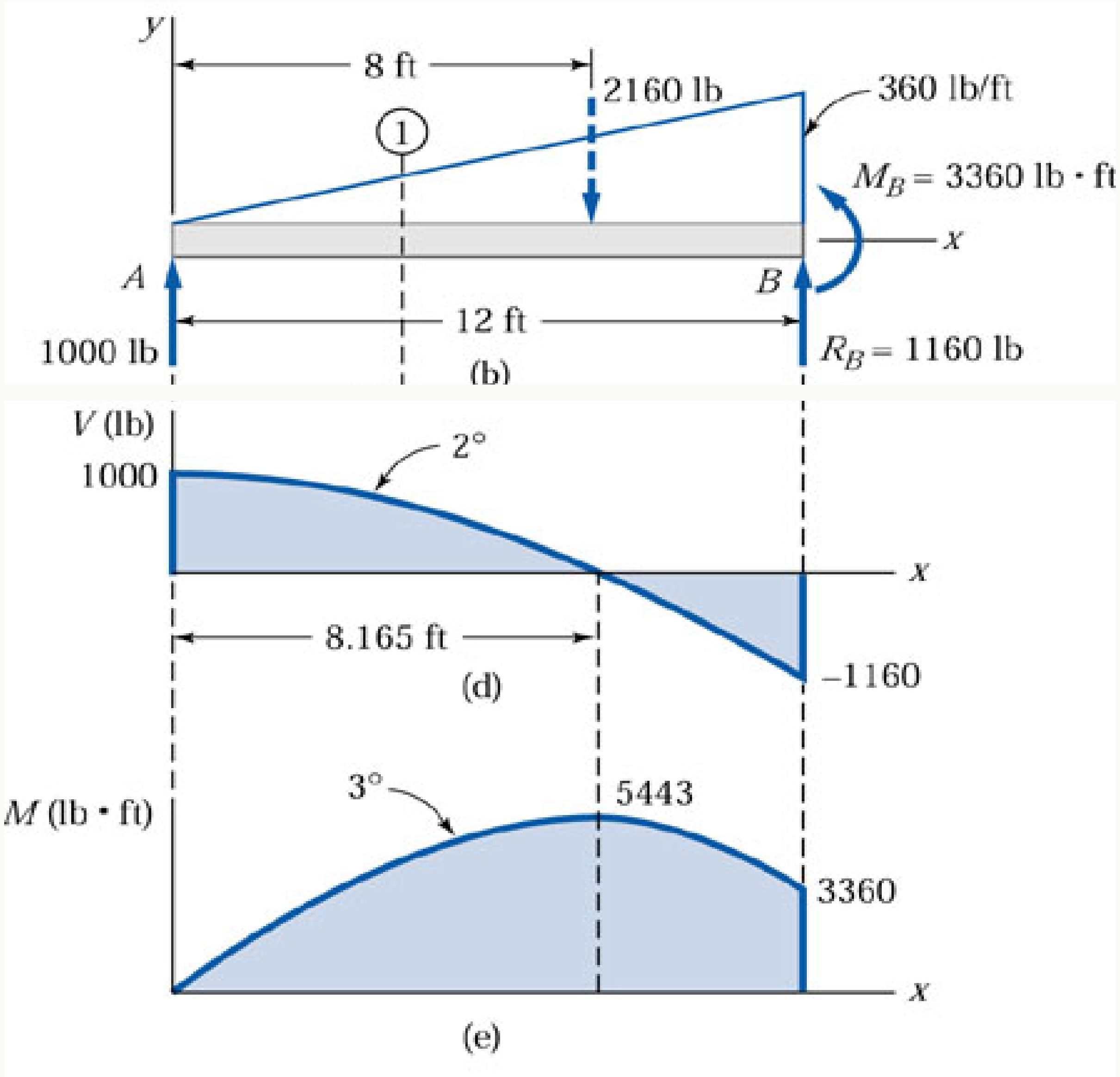
Part 1

$\sum F_y = 0 + \uparrow$ $\quad 1000 - 15x^2 - V = 0$

$V = 1000 - 15x^2$ lb *Answer*

$\sum M_C = 0 + \circlearrowleft$ $\quad -1000x + 15x^2\,(x/3) + M = 0$

$M = 1000x - 5x^3$ lb· ft *Answer*



Part 2

The location of the section where the shear force is **zero** is found from

$$V = 1000 - 15x^2 = 0$$

$$x = 8.165 \text{ ft}$$

$$\frac{dM}{dx} = 1000 - 15x^2 = 0$$

$$x = 8.165 \text{ ft}.$$

the maximum bending moment is

$M_{max} = 1000(8.165) - 5(8.165)^3 = 5443$ lb· ft