CS 460 MP-1
RUSHI PATEL (RAP295)
SMITKUMAR PATEL (SHP109)

We can always bring the one sphere on the origin and subtract all the coordinates from other sphere. Therefore, the first equation will be $x^2 + y^2 + z^2 = {r_1}^2$.

1) $x^2 + y^2 + z^2 = {r_1}^2$

   $(x-a)^2 + (y-b)^2 + (z-c)^2 = {r_2}^2$

   $(x-m)^2 + (y-n)^2 + (z-o)^2 = {r_3}^2$

   $(x-p)^2 + (y-q)^2 + (z-s)^2 = {r_4}^2$

$\boldsymbol{x^2 + y^2 + z^2 = {r_1}^2}$ **-------- (1)**

$\boldsymbol{x^2 - 2ax + a^2 + y^2 - 2by + b^2 + z^2 - 2cz + c^2 = {r_2}^2}$ **--------- (2)**

$\boldsymbol{x^2 - 2mx + m^2 + y^2 - 2ny + n^2 + z^2 - 2oz + o^2 = {r_3}^2}$ **---------- (3)**

$\boldsymbol{x^2 - 2px + p^2 + y^2 - 2qy + q^2 + z^2 - 2sz + s^2 = {r_4}^2}$ **---------- (4)**

Solving (1) and (2)

➔ $-2ax + a^2 - 2by + b^2 - 2cz + c^2 = {r_2}^2 - {r_1}^2$
$\boldsymbol{2ax + 2by + 2cz = {r_1}^2 - {r_2}^2 + a^2 + b^2 + c^2}$ **--------------- (5)**

Solving (2) and (3)

➔ $2mx - 2ax + a^2 - m^2 + 2ny - 2by + b^2 - n^2 + 2oz - 2zc + c^2 - o^2 = {r_2}^2 - {r_3}^2$
$\boldsymbol{(2m - 2a)*x + (2n - 2b)*y + (2o - 2c)*z = {r_2}^2 - {r_3}^2 - a^2 + m^2 - b^2 + n^2 - c^2 + o^2}$ **------------- (6)**

Solving (1) and (3)

➔ $-2mx + \boldsymbol{m^2} - 2ny + \boldsymbol{n^2 - 2oz + o^2} = {r_3}^2 - {r_1}^2$
$-2mx - 2ny - 2oz = {r_3}^2 - {r_1}^2 - \boldsymbol{m^2 - n^2 - o^2}$
$\boldsymbol{2mx + 2ny + 2oz = {r_1}^2 - {r_3}^2 + m^2 + n^2 + o^2}$ **------------- (7)**

Putting (5), (6) and (7) in matrix as below:

$$\left(\begin{bmatrix} 2a & 2b & 2c \\ 2m-2a & 2n-2b & 2o-2c \\ 2m & 2n & 2o \end{bmatrix} \middle| \begin{matrix} {r_1}^2 - {r_2}^2 + a^2 + b^2 + c^2 \\ {r_2}^2 - {r_3}^2 - a^2 + m^2 - b^2 + n^2 - c^2 + o^2 \\ {r_1}^2 - {r_3}^2 + m^2 + n^2 + o^2 \end{matrix} \right)$$

Solving (1) and (4)

➔ $x^2 - 2xp + p^2 + y^2 - 2yq + q^2 + z^2 - 2sz + s^2 = {r_4}^2$

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-2xp + p^2 - 2yq + q^2 – 2sz + s^2 = ${r_4}^2 - {r_1}^2$
-2xp - 2yq – 2sz = ${r_4}^2 - {r_1}^2$ - p^2 - q^2 - s^2
2xp + 2yq + 2sz = $\boldsymbol{{r_1}^2 - {r_4}^2 + p^2 + q^2 + s^2}$ ------ (8)

$$\left(\begin{bmatrix} 2a & 2b & 2c \\ 2m & 2n & 2o \\ 2p & 2q & 2s \end{bmatrix}\middle| \begin{matrix} {r_1}^2 - {r_2}^2 + a^2 + b^2 + c^2 \\ {r_1}^2 - {r_3}^2 + m^2 + n^2 + o^2 \\ {r_1}^2 - {r_4}^2 + p^2 + q^2 + s^2 \end{matrix}\right)$$

From Equation (2) and (7)

➔ x^2 – 2ax + a^2+ y^2- 2by + b^2+ z^2 – 2cz + c^2 = ${r_2}^2$
➔ x^2 – 2xp + p^2+ y^2- 2yq + q^2+ z^2 – 2sz + s^2 = ${r_4}^2$

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➔ -2ax + 2xp + a^2- p^2 – 2by + 2qy + b^2 - q^2 - 2cz + 2sz + c^2 – s^2 = ${r_2}^2 - {r_4}^2$
(-2a + 2p)*x + $\boldsymbol{a^2}$- $\boldsymbol{p^2}$ + (-2b + 2q)*y + $\boldsymbol{b^2}$ - $\boldsymbol{q^2}$ - (2c + 2s)*z + $\boldsymbol{c^2 - s^2}$ = $\boldsymbol{{r_2}^2 - {r_4}^2}$

To check the error or find area:

Plug value of the x, y, and z and check the equation (1), (2), (3), and (4) satisfied or not. If it satisfied then the error is zero. If not, we can add or subtract the value to make it satisfied and that value will be error.