CS 460 MP-1
RUSHI PATEL (RAP295)
SMITKUMAR PATEL (SHP109)

2) KALMAN FILTER 2D IMPLEMENTATION:

$\hat{\boldsymbol{x}}_{\boldsymbol{k}}^{-} = \boldsymbol{A}\hat{\boldsymbol{x}}_{\boldsymbol{k-1}}^{-} + \boldsymbol{B}\hat{\boldsymbol{u}}_{\boldsymbol{k-1}}^{-}$**: (Time Update Stage → Project the state ahead)**

→ $\hat{x}_k^- = \begin{pmatrix} 1 & \Delta T \\ 0 & 1 \end{pmatrix} \hat{x}_{k-1}^- + \begin{pmatrix} 0 & \frac{\Delta T^2}{2} \\ 0 & \Delta T \end{pmatrix} \hat{u}_{k-1}^-$

- We can guess any points for this 2*1 matrix: $\hat{\boldsymbol{x}}_{\boldsymbol{k-1}}^{-}$ and for this question $\hat{\boldsymbol{u}}_{\boldsymbol{k-1}}^{-}$ is given.

$\boldsymbol{P}_{\boldsymbol{k}}^{-} = \boldsymbol{A}\boldsymbol{P}_{\boldsymbol{k-1}}\boldsymbol{A}^{\boldsymbol{T}} + \boldsymbol{Q}$**: (Time Update Stage → Project the error covariance ahead, initiate estimates for** $\hat{\boldsymbol{x}}_{\boldsymbol{k-1}}^{-}$ ***and*** $\boldsymbol{P}_{\boldsymbol{k-1}}$ **)**

→ $\boldsymbol{P}_{\boldsymbol{k-1}}$ ***will be scalar*** $*$ $\boldsymbol{I}$ $(\boldsymbol{Identity\ matrix})$ $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$ **. We have to guess the scalar.**

→ $\begin{pmatrix} \mathbf{1} & \mathbf{\Delta T} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} \boldsymbol{P}_{\boldsymbol{k-1}} \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{\Delta T} & \mathbf{1} \end{pmatrix}$ + **Q**. Q is given in this problem.

After getting $\boldsymbol{P}_{\boldsymbol{k}}^{-}$ using above equation we now will find Kalman Gain.

$$\boldsymbol{K}_{\boldsymbol{k}} = \frac{\boldsymbol{P}_{\boldsymbol{k}}^{-}\boldsymbol{H}^{\boldsymbol{T}}}{\boldsymbol{H}\boldsymbol{P}_{\boldsymbol{k}}^{-}\boldsymbol{H}^{\boldsymbol{T}} + \boldsymbol{R}}$$

(Measurement Update (“Correct”) Stage → Computing the Kalman Gain)

→ At this point, we will know the value of $\boldsymbol{P}_{\boldsymbol{k}}^{-}$ using above equations and **H** will be Identity Matrix $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$ and value of **R** is given in this problem.

After finding Kalman gain, we will find **new** $\hat{\boldsymbol{x}}_{\boldsymbol{k}}$

$\boldsymbol{New}\ \hat{\boldsymbol{x}}_{\boldsymbol{k}} = \hat{\boldsymbol{x}}_{\boldsymbol{k}}^{-} + \boldsymbol{K}_{\boldsymbol{k}}(\boldsymbol{z}_{\boldsymbol{k}} - \boldsymbol{H}\hat{\boldsymbol{x}}_{\boldsymbol{k}}^{-})$ **: (Measurement Update (“Correct”) Stage → Updating estimate with measurement** $\boldsymbol{z}_{\boldsymbol{k}}$**), Value of** $\boldsymbol{z}_{\boldsymbol{k}}$ **is given in this problem.**

Now, we will find new $\boldsymbol{P}_{\boldsymbol{k}}^{-}$**: ((Measurement Update (“Correct”) Stage → Updating the error covariance)**

$$\boldsymbol{New}\ \boldsymbol{P}_{\boldsymbol{k}} = (\boldsymbol{I} - \boldsymbol{K}_{\boldsymbol{k}}\boldsymbol{H})\ \boldsymbol{P}_{\boldsymbol{k}}^{-}$$

→ Now, for the next iteration our new $\hat{\boldsymbol{x}}_{\boldsymbol{k}}$ **and** $\boldsymbol{P}_{\boldsymbol{k}}$value will be used for $\hat{\boldsymbol{x}}_{\boldsymbol{k}}^{-}$ **and** $\boldsymbol{P}_{\boldsymbol{k}}^{-}$ respectively. Continue this process to find further values. Store the data of $\hat{\boldsymbol{x}}_{\boldsymbol{k}}$ and plot $\hat{\boldsymbol{x}}_{\boldsymbol{k}}$ and $\boldsymbol{z}_{\boldsymbol{k}}$ ***using matplotlib***.