Chapter 1

BASIC MATHEMATICS

1. MENSURATION FORMULAS :

r : radius ; d = diameter ;
V = Volume S.A = surface area

(a) Circle

Perameter : $2\pi r = \pi d$, Area : $\pi r^2 = \frac{1}{4}\pi d^2$

(b) Sphere

Surface area = $4\pi r^2 = \pi d^2$, Volume = $\frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$

(c) Spherical Shell (Hollow sphere)

Surface area = $4\pi r^2 = \pi d^2$

Volume of material used = $(4\pi r^2)(dr)$, dr = thickness

(d) Cylinder

Lateral area = $2\pi rh$

V = $\pi r^2 h$

Total area = $2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

(e) Cone

Lateral area = $\pi r\sqrt{r^2+h^2}$ h = height

Total area = $\pi r\left(\sqrt{r^2+h^2}+r\right)$ V = $\frac{1}{3}\pi r^2 h$

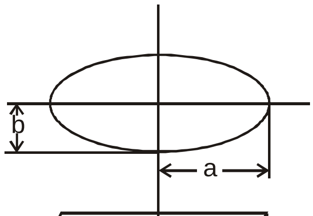
(f) Ellipse

Circumference $\approx 2\pi\sqrt{\frac{a^2+b^2}{2}}$

area = πab

a = semi major axis

b = semi minor axis

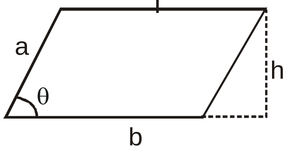


(g) Parallelogram

A = bh = ab sin θ

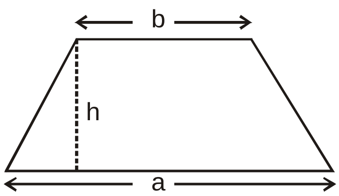
a = side ; h = height ; b = base

θ = angle between sides a and b



(h) Trapezoid

area = $\frac{h}{2}(a+b)$



a and b parallel sides
h = height

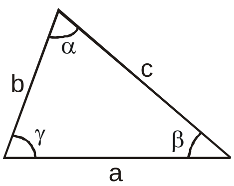
(i) Triangle

area = $\frac{bh}{2} = \frac{ab}{2}\sin\gamma = \sqrt{s(s-a)(s-b)(s-c)}$

a, b, c sides are opposite to angles α, β, γ

b = base ; h = height

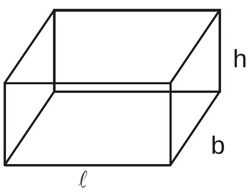
s = $\frac{1}{2}(a+b+c)$



(j) Rectangular container

lateral area = $2(\ell b + bh + h\ell)$

V = ℓ bh



side ℓ,b,h

Mathematics is the language of physics. It becomes easier to describe, understand and apply the physical principles, if one has a good knowledge of mathematics.

2. LOGARITHMS :

(i) $e \approx 2.7183$ (ii) If $e^x = y$, then $x = \log_e y = \ln y$ (iii) If $10^x = y$, then $x = \log_{10} y$

(iv) $\log_{10} y = 0.4343 \log_e y = 2.303 \log_{10} y$ (v) $\log(ab) = \log(a) + \log(b)$

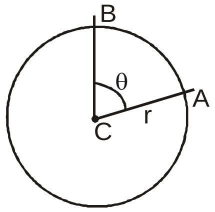
(vi) $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$ (vii) $\log a^n = n \log(a)$

3. TRIGONOMETRIC PROPERTIES :

(i) Measurement of angle & relationship between degrees & radian

In navigation and astronomy, angles are measured in degrees, but in calculus it is best to use units called radians because of they simplify later calculations.

Let ACB be a central angle in circle of radius r, as in figure.



Then the angle ACB or θ is defined in radius as -

$$\theta = \frac{\text{Arc length}}{\text{Radius}} \Rightarrow \theta = \frac{\overset{\frown}{AB}}{r}$$

If $r = 1$ then $\theta = AB$

The radian measure for a circle of unit radius of angle ABC is defined to be the length of the circular arc AB. since the circumference of the cirlce is 2π and one complete revolution of a cicle is 360°, the relation between radians and degrees is given by the following equation.

$$\pi \text{ radians} = 180°$$

ANGLE CONVERSION FORMULAS

1 degree $= \frac{\pi}{180°}$ (≈ 0.02) radian Degrees to radians : multiply by $\frac{\pi}{180°}$

1 radian $\approx$ 57 degrees Radians to degrees : multiply by $\frac{180°}{\pi}$

Ex.1 *Covert 45° to radians :* $45 \cdot \frac{\pi}{180} = \frac{\pi}{4}$ *rad*

Convert $\frac{\pi}{6}$ *rad to degrees :* $\frac{\pi}{6} \cdot \frac{180}{\pi} = 30°$

Ex.2 *Convert 30° to radians :*

Sol. $30° \times \frac{\pi}{180°} = \frac{\pi}{6}$ rad

Ex.3 *Convert* $\frac{\pi}{3}$ *rad to degrees.*

Sol. $\frac{\pi}{3} \times \frac{180}{\pi} = 60$

Standard values

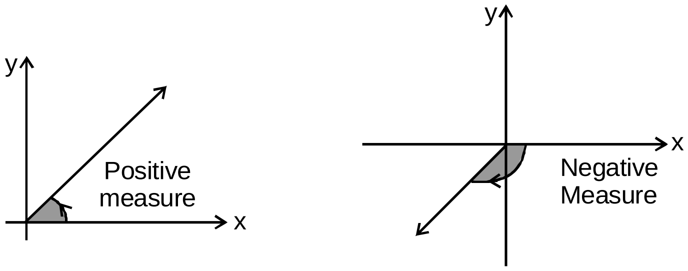
(1) $30° = \frac{\pi}{6}$rad (2) $45° = \frac{\pi}{4}$rad (3) $60° = \frac{\pi}{3}$rad

(4) $90° = \frac{\pi}{2}$rad (5) $120° = \frac{2\pi}{3}$rad (6) $135° = \frac{3\pi}{4}$rad

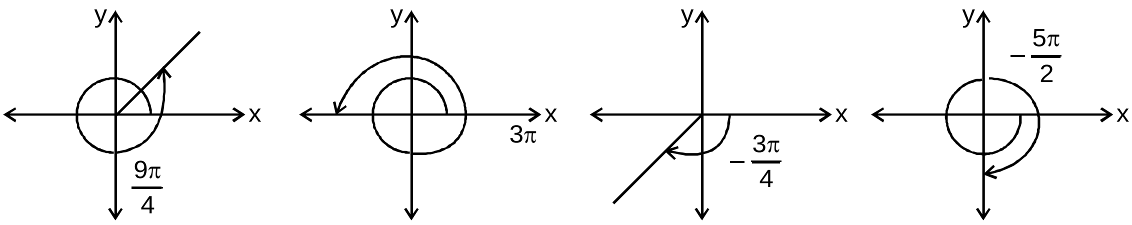
(7) 150° = $\frac{5\pi}{6}$ rad (8) 180° = π rad (9) 360° = 2π rad

(Check these values yourself to see that the satisfy the conversion formulaes)

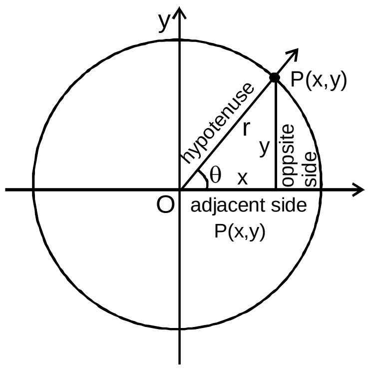
(ii) Measurement of positive & Negative Angles :



An angle in the xy-plane is said to be in standard position if its vertex lies at the origin and its initial ray lies along the positive x-axis (Fig). Angles measured counterclockwise from the positive x-axis are assigned positive measures; angles measured clockwise are assigned negative measures.



(iii) Six Basic Trigonometric Functions :



The trigonometric fucntion of a general angle θ are defined in tems of x, y and r.

Sine : $\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$ Cosecant : $\mathrm{cosec}\,\theta = \frac{hyp}{opp} = \frac{r}{y}$

Cosine: $\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$ Secant : $\sec\theta = \frac{hyp}{adj} = \frac{r}{x}$

Tangent: $\tan\theta = \frac{opp}{adj} = \frac{y}{x}$ Cotangent: $\cot\theta = \frac{adj}{opp} = \frac{x}{y}$

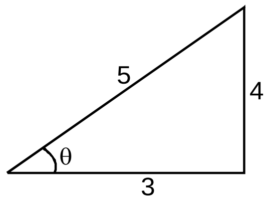
VALUES OF TRIGONOMETRIC FUNCTIONS

If the circle in (Fig. above) has radius r = 1, the equations defining sin θ and cos θ become

$\cos\theta = x$, $\sin\theta = y$

We can then calculate the values of the cosine and sine directly from the coordinates of P.

Ex.4 *Find the six trigonometric ratios from given fig. (see above)*



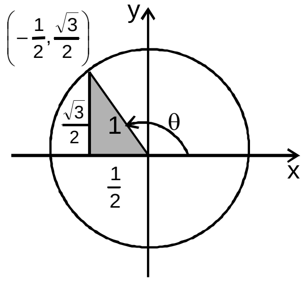
Sol. $\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5}$ $\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$ $\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{3}{4}$

$\sec\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{3}$ $\text{cosec}\,\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{4}$

Ex.5 *Find the sine and cosine of angle θ shown in the unit circle if coordinate of point p are as shown.*

Sol.

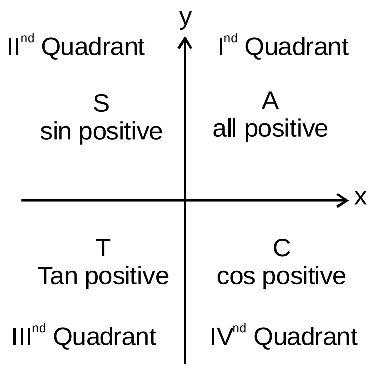


$\cos\theta$ = x-coordinate of P = $-\frac{1}{2}$ $\sin\theta$ = y-coordinate of P = $\frac{\sqrt{3}}{2}$

4. Values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ for some standard angles.

Degree	0	30	37	45	53	60	90	120	135	180
Radians	0	$\pi/6$	$37\pi/180$	$\pi/4$	$53\pi/180$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π
$\sin\theta$	0	1/2	3/5	$1/\sqrt{2}$	4/5	$\sqrt{3}/2$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	0
$\cos\theta$	1	$\sqrt{3}/2$	4/5	$1/\sqrt{2}$	3/5	1/2	0	–1/2	$-1/\sqrt{2}$	–1
$\tan\theta$	0	$1/\sqrt{3}$	3/4	1	4/3	$\sqrt{3}$	∞	$-\sqrt{3}$	–1	0

A useful rule for remembering when the basic trigonometric funcions are positive and negative is the CAST rule. If you are not very enthusiastic about CAST. You can remember it as ASTC (After school to college)



The CAST rule

RULES FOR FINDING TRIGONOMETRIC RATIO OF ANGLES GREATER THAN 90°.

Step 1 → Identify the quadrant in which angle lies.

Step 2 → (a) If angle = $(n\pi \pm \theta)$ where n is an integer. Then

(b) If angle = $\left[(2n+1)\frac{\pi}{2} + \theta\right]$ where n is in interger. Then

trigonometric function of $\left[(2n+1)\frac{\pi}{2} \pm \theta\right]$ = complimentry trignometric function of θ and sign will be decided by CAST Rule.

Ex.6 *Evaluate sin 120°*

Sol. $\sin 120° = \sin(90° + 30°) = \cos 30° = \frac{\sqrt{3}}{2}$

Aliter $\sin 120° = \sin(180° - 60°) = \sin 60° = \frac{\sqrt{3}}{2}$

Ex.7 *Evaluate cos 210°*

Sol. $\cos 210° = \cos(180° + 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

Ex.8 *$\tan 210° = \tan(180° + 30°) = \tan 30° = +\frac{1}{\sqrt{3}}$*

5. *IMPORTANT FORMULAS*

(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $1 + \tan^2\theta = \sec^2\theta$

(iii) $1 + \cot^2\theta = \mathrm{cosec}^2\theta$

(iv) $\sin 2\theta = 2\sin\theta\cos\theta$

(v) $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta$

(vi) $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

(vii) $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

(viii) $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

(ix) $\sin C - \sin D = 2\sin\left(\frac{C-D}{2}\right)\cos\left(\frac{C+D}{2}\right)$

(x) $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

(xi) $\cos C - \cos D = 2\sin\frac{D-C}{2}\sin\frac{C+D}{2}$

(xii) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

(xiii) $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

(xiv) $\sin(90° + \theta) = \cos\theta$

(xv) $\cos(90° + \theta) = -\sin\theta$

(xvi) $\tan(90° + \theta) = -\cot\theta$

(xvii) $\sin(90° - \theta) = \cos\theta$

(xviii) $\cos(90° - \theta) = \sin\theta$

(xix) $\cos(180° - \theta) = -\cos\theta$

(xx) $\sin(180° - \theta) = \sin\theta$

(xxi) $\cos(180° + \theta) = -\cos\theta$

(xxii) $\tan(180° + \theta) = \tan\theta$

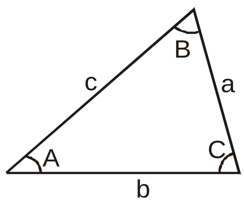
(xxiii) $\sin(-\theta) = -\sin\theta$

(xxiv) $\cos(-\theta) = \cos\theta$

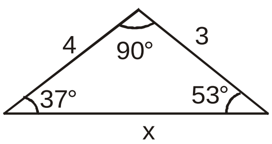
(xxv) $\tan(-\theta) = -\tan\theta$

- Sine Rule $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

- Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$



Ex.9



Find x :

Sol. $\frac{\sin 90°}{x} = \frac{\sin 53°}{4}$

$x = 5$

6. SMALL ANGLE APPROXIMATION

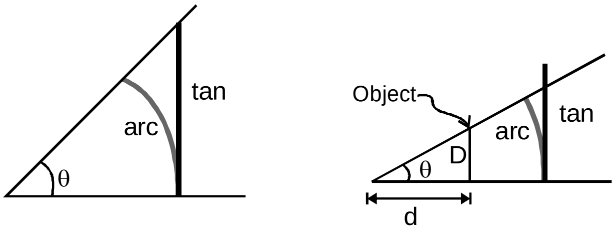
It is a useful simplification which is only approximately true for finite angles. It involves linerarization of the trigonometric functions so that, when the angle θ is measured in radians.

$\sin\theta \simeq \theta$

$\cos\theta \simeq 1$ or $\cos\theta \simeq 1 - \frac{\theta^2}{2}$ for the second - order approximation

$\tan\theta \simeq \theta$

Geometric justification



Small angle approximation. The value of the small angle X in radians is approximately equal to its tangent.

- When one angle of a right triangle is small, is hypotenuse in approximately equal in length to the leg adjacent
to the small angle, so the cosine is approximately 1.
- The short leg is approximately equal to the arc from the long leg to the hypotenuse, so the sine and tangent are
both approximated by the value of the angle in radians.

7. BINOMIAL THEOREM :

$(1 \pm x)^n = 1 \pm nx + \frac{n(n-1)x^2}{2!}$...........

$(1 \pm x)^{-n} = 1 \mp nx + \frac{n(n+1)}{2!}x^2$.........

If $x << 1$; then

$(1 \pm x)^n = 1 \pm nx$ (neglecting higher terms)

$(1 \pm x)^{-n} = 1 \pm (-n)x = 1 \mp nx$

$(1 + x)^2 = 1 + 2x + x^2$

$(1 + x)^3 = 1 + 3x + x^3 - 3x^2$

$(1 + x)^n = 1 + nx$ ....

if $x << 1$

Note : (1) When n is a positive integer, then expansion will have (n + 1) terms
(2) When n is a negative integer, expansion will have infinite terms.
(3) When n is a fraction expansion will have infinite terms.

Ex. 10 Calculate $(1001)^{1/3}$.

Sol. We can write 1001 as : $1001 = 1000\left(1+\frac{1}{1000}\right)$, so that we have

$$(1001)^{1/3} = \left[1000\left(1+\frac{1}{1000}\right)\right]^{1/3} = 10\left[1+\frac{1}{1000}\right]^{1/3}$$

$$= 10(1+0.001)^{1/3} = 10(1+\frac{1}{3}\times 0.001)$$

$= 10.003333$

Ex.11 Expand $(1+x)^{-3}$.

Sol. $$(1+x)^{-3} = 1+(-3)x+\frac{(-3)(-3-1)x^2}{2!}+\frac{(-3)(-3-1)(-3-2)}{3!}x^3+$$

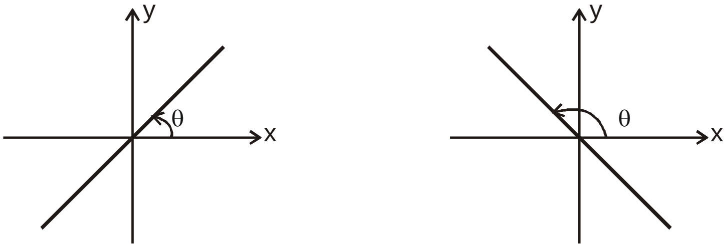
$$= 1-3x+\frac{12}{2}x^2-\frac{60}{3\times 2}x^3+.......$$

$$= 1-3x+6x^2-10x^3+.......$$

8. GRAPHS :

Following graphs and their corresponding equations are frequently used in Physics.

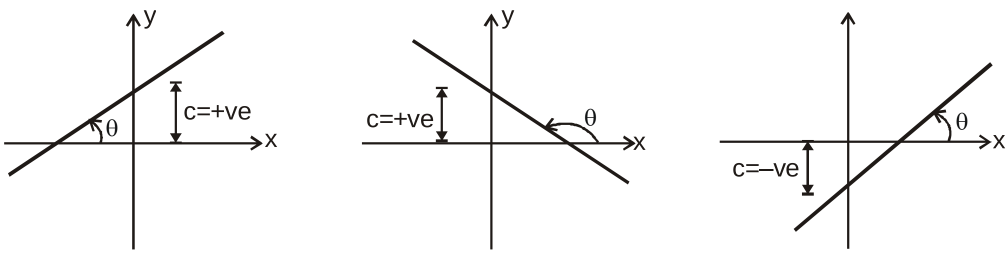
(i) y = mx, represents a straight line passing through origin. Here, m = tan θ is also called the slope of line, where θ is the angle which the line makes with positive x-axis, when drawn in anticlockwise direction from the positive x-axis towards the line.



(i) (ii)

The two possible cases are shown in figure 1.1 (i) θ < 90°. Therefore, tan θ or slope of line is positive. In fig. 1.1 (ii), 90° < θ < 180°. Therefore, tan θ or slope of line is negative.

Note : That y = mx of y ∝ x also means that value of y becomes 2 time if x is doubled. Or it becomes $\frac{1}{4}$th if x becomes $\frac{x}{4}$, and c the intercept on y-axis.

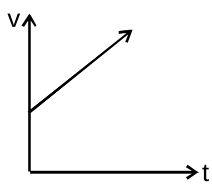


(i) (ii) (iii)

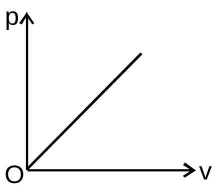
In figure (i) : slope and intercept both are positive.
In figure (ii) : slope is negative but intercept is positive and
In figure (iii) : slope is positive but intercept is negative.

Note : That in y = mx + c, y does not become two times if x is doubled

Ex.12 v = u + at



Ex.13 P = mv



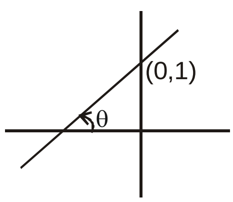
Ex.14 Draw the graph for the equation : 2y = 3x + 2

Sol. $2y = 3x + 2 \Rightarrow y = \frac{3}{2}x + 1$

$m = \frac{3}{2} > 0 \Rightarrow \theta < 90°$

$c = +1 > 0$

⇒ The line will pass through (0, 1)



$\tan\theta = \frac{3}{2}$

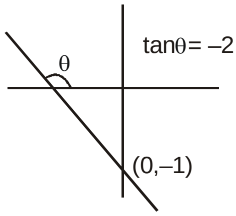
Ex.15 Draw the graph for the equation : $2y + 4x + 2 = 0$

Sol. $2y + 4x + 2 = 0 \quad \Rightarrow y = -2x - 1$

$m = -2 < 0$ i.e., $\theta > 90°$

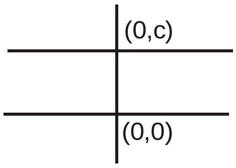
$c = -1$ i.e.,

line will pass through $(0, -1)$

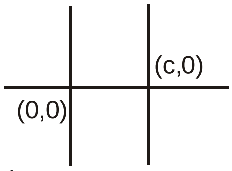


: (i) If $c = 0$ line will pass through origin.

(ii) $y = c$ will be a line parallel to x axis.



(iii) $x = c$ will be a line perpendicular to y axis



(ii) Parabola

A general quadratic equation represents a parabola.

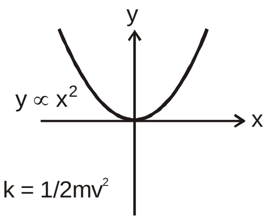
$$y = ax^2 + bx + c \qquad a \neq 0$$

if $a > 0$; It will be a opening upwards parabola.

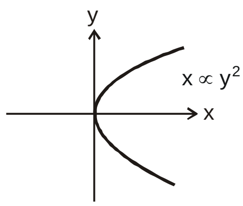
if $a < 0$; It will be a opening downwards parabola.

if $c = 0$; It will pass through origin.

$y \propto x^2$ or $y = 2x^2$, etc. represents a parabola passing through origin as shown in figure shown.



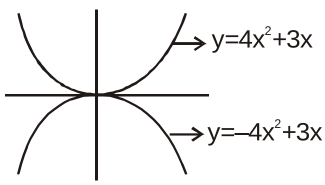
(i)

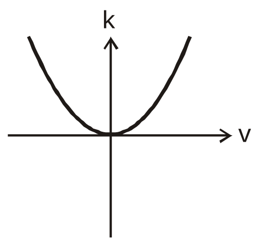


(ii)

e.g. $y = 4x^2 + 3x$

e.g. $k = \frac{1}{2}mv^2$





Note : That in the parabola $y = 2x^2$ or $y \propto x^2$, if x is doubled, y will beome four times.

Graph $x \propto y^2$ or $x = 4y^2$ is again a parabola passing through origin as shown in figure shown. In this cae if y is doubled, x will become four times.

$y = x^2 + 4$ or $x = y^2 - 6$ will represent a parabola but not passing through origin. In the first equation ($y = x^2 + 4$), if x doubled, y will not become four times.

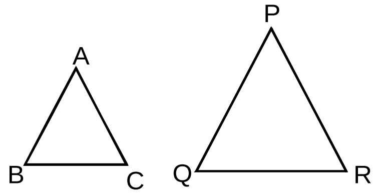
9. SIMILAR TRIANGLE

Two given triangle are said to be similar if

(1) All respective angle are same

or

(2) All respective side ratio are same.



As example, ABC, PQR are two triangle as shown in figure.

If they are similar triangle then

(1) $\angle A = \angle P$

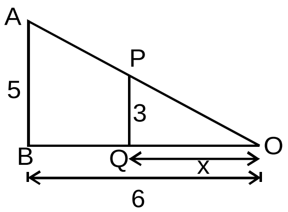
$\angle B = \angle Q$

$\angle C = \angle R$

OR

(2) $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

Ex.16



Find x :

Sol. By similar triangle concept

$$\frac{AB}{PQ} = \frac{OB}{OQ}$$

$$\frac{5}{3} = \frac{6}{x} \quad \Rightarrow \quad x = \frac{18}{5}$$

VECTOR

1. SCALAR :

In physics we deal with two type of physical quantity one is scalar and other is vector. Each scalar quantity has a magnitude and a unit.

For example mass = 4kg

Magnitude of mass = 4

and unit of mass = kg

Example of scalar quantities : mass, speed, distance etc.

Scalar quantities can be added, subtracted and multiplied by simple laws of algebra.

2. VECTOR :

Vector are the physical quantites having magnitude as well as specified direction.

For example :

Speed = 4 m/s (is a scalar)

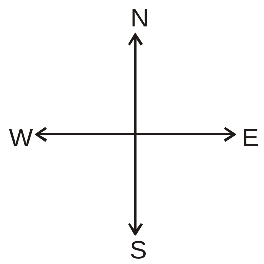
Velocity = 4 m/s toward north (is a vector)

If someone wants to reach some location then it is not sufficient to provide information about the distance of that location it is also essential to tell him about the proper direction from the initial location to the destination.

The magnitude of a vector ($\vec{A}$) is the absolute value of a vector and is indicated by $|\vec{A}|$ or A.

Example of vector quantity : Displacement, velocity, acceleration, force etc.

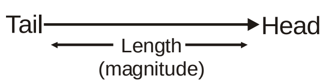
Knowledge of direction



3. GENERAL POINTS REGARDING VECTORS :

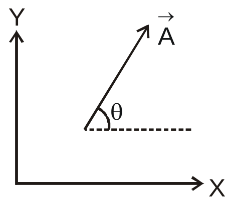
3.1 Representation of vector :

Geometrically, the vector is represented by a line with an arrow indicating the direction of vector as



Mathematically, vector is represented by $\vec{A}$.

Sometimes it is represented by bold letter **A**.

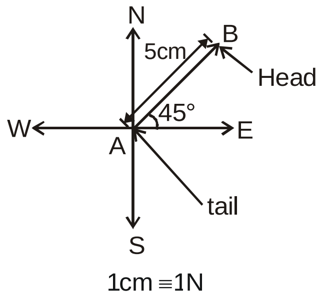


Thus, the arrow in abow figure represents a vector $\vec{A}$ in xy-plane making an angle θ with x-axis.

A representation of vector will be complete if it gives us direction and magnitude.

Symbolic form : $\vec{v}, \vec{a}, \vec{F}, \vec{s}$ used to separate a vector quantity from scalar quantities (u, i, m)

Graphical form : A vector is represented by a directed straight line, having the magnitude and direction of the quantity represented by it.
e.g. if we want to represent a force of 5 N acting 45° N of E
(i) We choose direction co-ordinates.
(ii) We choose a convenient scale like 1 cm ≡ 1 N
(iii) We draw a line of length equal in magnitude and in the direction of vector to the chosen quantity.
(iv) We put arrow in the direction of vector.



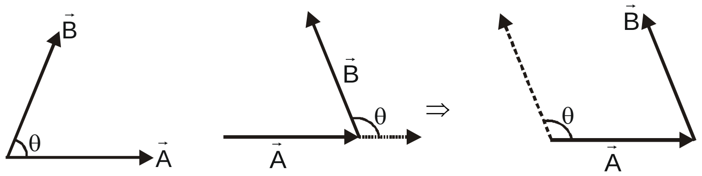
$\overrightarrow{AB}$

Magnitude of vector :

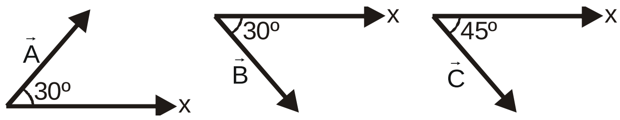
$|\overrightarrow{AB}| = 5N$

3.2 Angle between two Vectors (θ)

Angle between two vectors means smaller of the two angles between the vectors when they are placed tail to tail by displacing either of the vectors parallel to itself (i.e $0 \le \theta \le \pi$).



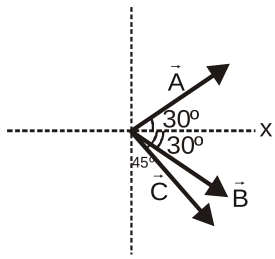
Ex.1 *Three vectors* $\vec{A}, \vec{B}, \vec{C}$ *are shown in the figure. Find angle between (i)* $\vec{A}$ *and* $\vec{B}$*, (ii)* $\vec{B}$ *and* $\vec{C}$*, (iii)* $\vec{A}$ *and* $\vec{C}$.



Sol. To find the angle between two vectors we connect the tails of the two vectors. We can shift $\vec{B}$ & $\vec{C}$ such that

tails of $\vec{A}, \vec{B}$ and $\vec{C}$ are connected as shown in figure.

Now we can easily observe that angle between $\vec{A}$ and $\vec{B}$ is 60°, $\vec{B}$ and $\vec{C}$ is 15° and between $\vec{A}$ and $\vec{C}$ is 75°.

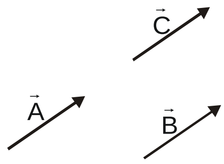


3.3 Negative of Vector

It implies vector of same magnitude but opposite in direction.

3.4 Equality of Vectors.

Vectors having equal magnitude and same direction are called equal vectors



if $|\vec{A}| = |\vec{B}| = |\vec{C}|$

and $\hat{A} = \hat{B} = \hat{C}$

then $\vec{A} = \vec{B} = \vec{C}$

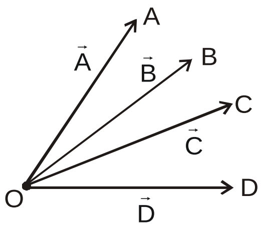
3.5 Collinear vectors :

Any two vectors are co-linear then one can be express in the term of other.

$\vec{a} = \lambda\vec{b}$ (where λ is a constant)

3.6 Co-initial vector : If two or more vector start from same point then they called co-initial vector.

e.g.



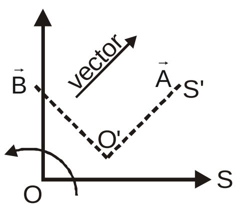
here A, B, C, D are co-initial.

3.7 Coplanar vectors :

Three (or more) vectors are called coplanar vectors if they lie in the same plane or are parallel to the same plane. Two (free) vectors are always coplanar.

Important points

- If the frame of reference is translated or rotated the vector does not change (though its components may change).



Two vectors are called equal if their magnitudes and directions are same, and they represent values of same physical quantity.

3.8 Multiplication and division of a vector by a scalar

Multiplying a vector $\vec{A}$ with a positive number λ gives a vector $(\vec{B} = \lambda\,\vec{A})$ whose magnitude become λ times but the direction is the same as that of $\vec{A}$. Multiplying a vector $\vec{A}$ by a negative number λ gives a vector $\vec{B}$ whose direction is opposite to the direction of $\vec{A}$ and whose magnitude is $-\lambda$ times $|\vec{A}|$.

The division of vector $\vec{A}$ by a non-zero scalar m is defined as multiplication of $\vec{A}$ by $\frac{1}{m}$.

At here $\vec{A}$ and $\vec{B}$ are co-linear vector

Ex.2 *A physical quantity (m = 3kg) is multiplied by a vector $\vec{a}$ such that $\vec{F} = m\vec{a}$. Find the magnitude and direction of $\vec{F}$ if*

(i) $\vec{a}$ = 3m / s² East wards

(ii) $\vec{a}$ = –4 m / s² North wards

Sol. (i) $\vec{F} = m\vec{a} = 3 \times 3ms^{-2}$ East wards

= 9 N East wards

(ii) $\vec{F} = m\vec{a} = 3 \times (-4)N$ North wards

= –12 N North wards

= 12 N South wards

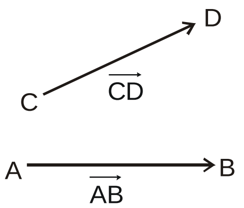
4. LAWS OF ADDITION AND SUBTRACTION OF VECTORS :

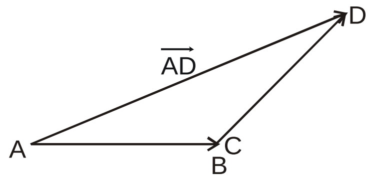
4.1 Triangle rule of addition : Steps for additing two vector representing same physical quantity by triangle law.

(i) Keep vectors s.t. tail of one vector coincides with head of other.

(ii) Join tail of first to head of the other by a line with arrow at head of the second.

(iii) This new vector is the sum of two vectors. (also called reultant)

(i)


(ii)


(iii) $\overrightarrow{AB} + \overrightarrow{CD} = \overrightarrow{AD}$

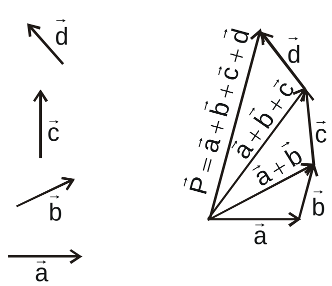
Take example here.

Q. A boy moves 4 m south and then 5 m in direction 37° E of N. Find resultant displacement.

4.2 Polygon Law of addition :

This law is used for adding more than two vectors. This is extension of triangle law of addition. We keep on arranging vectors s.t. tail of next vector lies on head of former.

When we connect the tail of first vector to head of last we get resultant of all the vectors.



Note : $\vec{P} = \left((\vec{a} + \vec{b}) + \vec{c}\right) + \vec{d} = \left((\vec{c} + \vec{a} + \vec{d})\right) + \vec{d}$ [Associative Law]

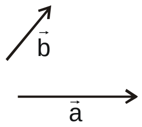
4.3 Parallelogram law of addition :

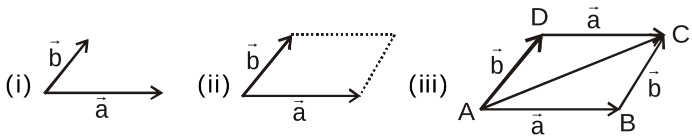
Steps :

(i) Keep two vectors such that there tails coincide.

(ii) Draw parallel vectors to both of them considering both of them as sides of a parallelogram.

(iii) Then the diagonal drawn from the point where tails coincide represents the sum of two vectors, with its tail at point of coincidence of the two vectors.





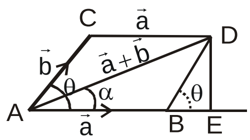
$\overrightarrow{AC} = \vec{a} + \vec{b}$

Note : $\overrightarrow{AC} = \vec{a} + \vec{b}$ and $\overrightarrow{AC} = \vec{b} + \vec{a}$ thus $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ [Cummutative Law]

Note : Angle between 2 vectors is the angle between their positive directions.

Suppose angle between these two vectors is θ, and $|\vec{a}| = a, |\vec{b}| = b$

$$
\begin{aligned}
(AD)^2 &= (AE)^2 + (DE)^2 \\
&= (AB + BE)^2 + (DE)^2 \\
&= (a + b\cos\theta)^2 + (b\sin\theta)^2 \\
&= a^2 + b^2\cos^2\theta + 2ab\cos\theta + b^2\sin^2\theta \\
&= a^2 + b^2 + 2ab\cos\theta
\end{aligned}
$$



Thus, $AD = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

or $\quad |\vec{a} + \vec{b}| = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

angle α with vector a is

$$\tan\alpha = \frac{DE}{AE} = \frac{b\sin\theta}{(a + b\cos\theta)}$$

Important points :

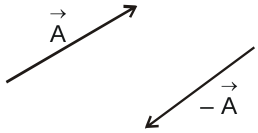
- To a vector, only a vector of same type can be added that represents the same physical quantity and the resultant
  is also a vector of the same type.
- As $R = [A^2 + B^2 + 2AB\cos\theta]^{1/2}$ so R will be maximum when, $\cos\theta = \max = 1$,
  i.e., $\theta = 0°$, i.e. vectors are like or parallel and $R_{max} = A + B$.
- $|\vec{A}| = |\vec{B}|$ and angle between them θ then R = $2A\cos\theta/2$
- $|\vec{A}| = |\vec{B}|$ and angle between them $\pi - \theta$ then R = $2A\sin\theta/2$
- The resultant will be minimum if, $\cos\theta = \min = -1$, i.e., $\theta = 180°$, i.e. vectors are antiparallel and $R_{min} = A - B$.
- If the vectors A and B are orthogonal, i.e., $\theta = 90°$, $R = \sqrt{A^2 + B^2}$

- As previously mentioned that the resultant of two vectors can have any value from (A – B) to (A + B) depending on the angle between them and the magnitude of resultant decreases as θ increases 0° to 180°.
- Minimum number of unequal coplanar vectors whose sum can be zero is three.
- The resultant of three non-coplanar vectors can never be zero, or minimum number of non coplanar vectors whose sum can be zero is four.

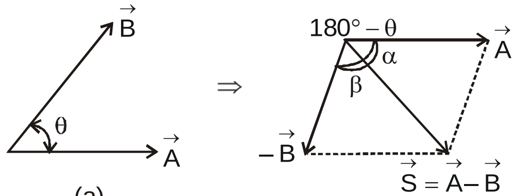
5. SUBTRACTION OF VECTOR :

Negative of a vector say $-\vec{A}$ is a vector of the same magnitude as vector $\vec{A}$ but pointing in a direction opposite to that of $\vec{A}$.

Thus, $\vec{A}-\vec{B}$ can be written as $\vec{A}+(-\vec{B})$ or $\vec{A}-\vec{B}$ is really the vector addition of $\vec{A}$ and $-\vec{B}$.



Suppose angle between two vectors $\vec{A}$ and $\vec{B}$ is θ. Then angle between $\vec{A}$ and $-\vec{B}$ will be 180° – θ as shown in figure.



(a) (b)

Magnitude of $\vec{S} = \vec{A} - \vec{B}$ will be thus given by

$$S = |\vec{A}-\vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos(180° - \theta)}$$

or $$S = \sqrt{A^2 + B^2 - 2AB\cos\theta} \qquad \ldots(i)$$

For direction of $\vec{S}$ we will either calculate angle α or β, where,

$$\tan\alpha = \frac{B\sin(180° - \theta)}{A + B\cos(180° - \theta)} = \frac{B\sin\theta}{A - B\cos\theta} \qquad \ldots(ii)$$

or $$\tan\beta = \frac{A\sin(180° - \theta)}{B + A\cos(180° - \theta)} = \frac{A\sin\theta}{B - A\cos\theta} \qquad \ldots(iii)$$

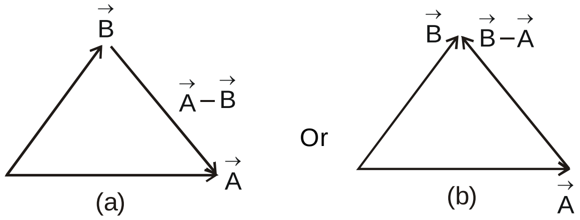
Ex.3 *Two vectors of 10 units & 5 units make an angle of 120° with each other. Find the magnitude & angle of resultant with vector of 10 unit magnitude.*

Sol. $|\vec{a}+\vec{b}| = \sqrt{a^2 + b^2 + 2ab\cos\theta} = \sqrt{100 + 25 + 2 \times 10 \times 5(-1/2)} = 5\sqrt{3}$

$$\tan\alpha = \frac{5\sin 120°}{10 + 5\cos 120°} = \frac{5\sqrt{3}}{20 - 5} = \frac{5\sqrt{3}}{5 \times 3} = \frac{1}{\sqrt{3}} \Rightarrow \alpha = 30°$$

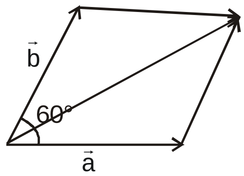
[Here shows what is angle between both vectors = 120° and not 60°]

Note : $\vec{A}-\vec{B}$ or $\vec{B}-\vec{A}$ can also be found by making triangles as shown in figure. (a) and (b)

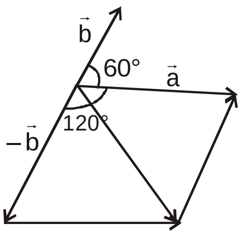


Ex.4 *Two vectors of equal magnitude 2 are at an angle of 60° to each other find magnitude of their sum & difference.*

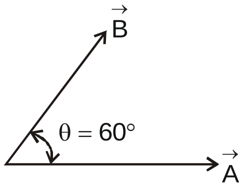
Sol. $|\vec{a}+\vec{b}| = \sqrt{2^2+2^2+2\times2\times2\cos60°} = \sqrt{4+4+4} = 2\sqrt{3}$



$|\vec{a}-\vec{b}| = \sqrt{2^2+2^2+2\times2\times2\cos120°} = \sqrt{4+4-4} = 2$



Ex.5 *Find $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$ in the diagram shown in figure. Given A = 4 units and B = 3 units.*



Sol. Addition :

$$R = \sqrt{A^2+B^2+2AB\cos\theta}$$

$$= \sqrt{16+9+2\times4\times3\cos60°} = \sqrt{37} \text{ units}$$

$$\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta} = \frac{3\sin60°}{4+3\cos60°} = 0.472$$

$\therefore$ $\alpha = \tan^{-1}(0.472) = 25.3°$

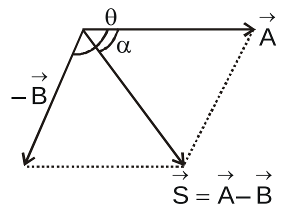
Thus, resultant of $\vec{A}$ and $\vec{B}$ is $\sqrt{37}$ units at angle 25.3° from $\vec{A}$ in the direction shown in figure.

Subtraction : $S = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$= \sqrt{16+9-2\times4\times3\cos60^\circ} = \sqrt{13}$ units

and $\tan\theta = \dfrac{B\sin\theta}{A - B\cos\theta}$

$= \dfrac{3\sin60^\circ}{4-3\cos60^\circ} = 1.04$



$\therefore \quad \alpha = \tan^{-1}(1.04) = 46.1^\circ$

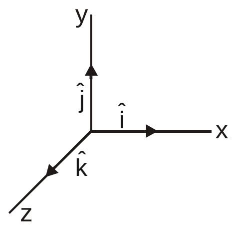
Thus, $\vec{A} - \vec{B}$ is $\sqrt{13}$ units at 46.1° from $\vec{A}$ in the direction shown in figure.

6. UNIT VECTOR AND ZERO VECTOR

Unit vector is a vector which has a unit magnitude and points in a particular direction. Any vector $(\vec{A})$ can be written as the product of unit vector $(\hat{A})$ in that direction and magnitude of the given vector.

$$\vec{A} = A\,\hat{A} \quad \text{or} \quad \hat{A} = \frac{\vec{A}}{A}$$

A unit vector has no dimensions and unit. Unit vectors along the positive x-, y-and z-axes of a rectangular coordinate system are denoted by $\hat{i}, \hat{j}$ and $\hat{k}$ respectively such that $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$.



A vector of zero magnitude is called a zero or a null vector. Its direction is arbitrary.

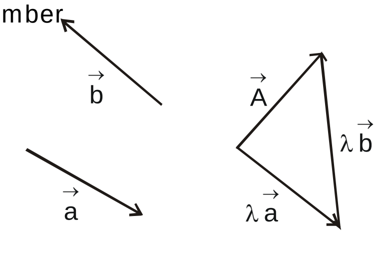
Ex.6 *A unit vector along East is defined as $\hat{i}$. A force of 10^5 dynes acts west wards. Represent the force in terms of $\hat{i}$.*

Sol. $\vec{F} = -10^5\,\hat{i}$ dynes

7. RESOLUTION OF VECTORS

If $\vec{a}$ and $\vec{b}$ be any two non-zero vectors in a plane with different directions and $\vec{A}$ be another vector in the same plane. $\vec{A}$ can be expressed as a sum of two vectors-one obtained by multiplying $\vec{a}$ by a real number and the other obtained by multiplying $\vec{b}$ by another real number.

$\vec{A} = \lambda\vec{a} + \mu\vec{b}$ (where λ and μ are real numbers)



We say that $\vec{A}$ has been resolved into two component vectors namely

$\vec{A} = \lambda\vec{a} + \mu\vec{b}$ (where λ and μ are real number)

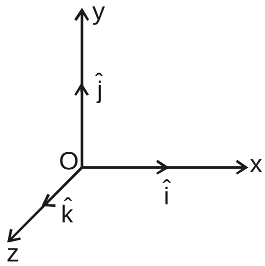
We say that $\vec{A}$ has been resolved into two component vectors namely

$\lambda\vec{a}$ and $\mu\vec{b}$

$\lambda\vec{a}$ and $\mu\vec{b}$ along $\vec{a}$ and $\vec{b}$ respectively. Hence one can resolve a given vector into two component vectors along a set of two vectors – all the three lie in the same plane.

7.1 Resolution along rectangular component :

It is convenient to resolve a general vector along axes of a rectangular coordinate system using vectors of unit magnitude, which we call as unit vectors. $\hat{i}, \hat{j}, \hat{k}$ are unit along x, y and z-axis as shown in figure below :

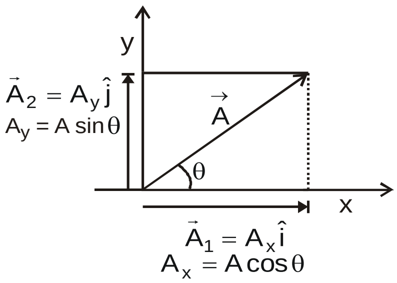


7.2 Resolution in two Dimension

Consider a vector $\vec{A}$ that lies in xy plane as shown in figure,

$$\vec{A} = \vec{A}_1 + \vec{A}_2$$

$$\vec{A}_1 = A_x\hat{i}, \ \vec{A}_2 = A_y\hat{j} \qquad \Rightarrow \qquad \vec{A} = A_x\hat{i} + A_y\hat{j}$$



The quantities A_x and A_y are called x-and y-components of the vector $\vec{A}$.

A_x is itself not a vector but $A_x\hat{i}$ is a vector and so it $A_y\hat{j}$.

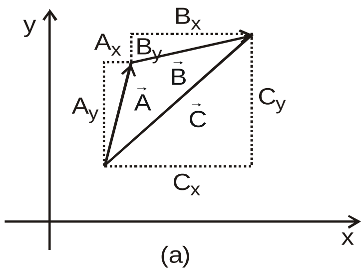
$A_x = A\cos\theta$ and $A_y = A\sin\theta$

It's clear from above equation that a component of a vector can be positive, negative or zero depending on the value of θ. A vector $\vec{A}$ can be specified in a plane by two ways :

(a) its magnitude A and the direction θ it makes with the x-axis; or

(b) its components A_x and A_y $\quad A = \sqrt{A_x^2 + A_y^2}$, $\qquad \theta = \tan^{-1}\dfrac{A_y}{A_x}$

Note : If $A = A_x \Rightarrow A_y = 0$ and if $A = A_y \Rightarrow A_x = 0$ i.e., components of a vector perpendicular to itself is always zero. The rectangular components of each vector and those of the sum $\vec{C} = \vec{A} + \vec{B}$ are shown in figure. We saw that



(a)

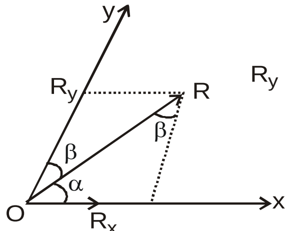
$\vec{C} = \vec{A} + \vec{B}$ is equivalent to both

$C_x = A_x + B_x$

and $C_y = A_y + B_y$

Refer figure (b)

Vector $\vec{R}$ has been resolved in two axes x and y not perpendicular to each other. Applying sine law in the triangle shown, we have



(b)

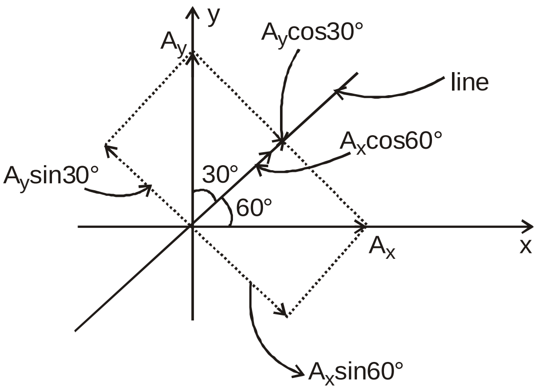
$$\frac{R}{\sin[180° - (\alpha+\beta)]} = \frac{R_x}{\sin\beta} = \frac{R_y}{\sin\alpha}$$

or $\quad R_x = \dfrac{R\sin\beta}{\sin(\alpha+\beta)} \quad$ and $\quad R_y = \dfrac{R\sin\alpha}{\sin(\alpha+\beta)}$

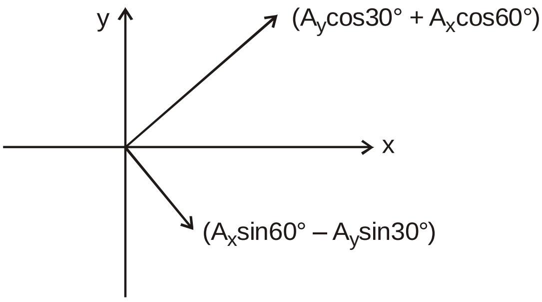
If $\alpha + \beta = 90°$, $R_x = R\sin\beta$ and $R_y = R\sin\alpha$

Ex.7 *Resolve the vector* $\mathbf{A} = A_x\hat{i} + A_y\hat{j}$ *along an perpendicular to the line which make angle 60° with x-axis.*

Sol.



$\Rightarrow$

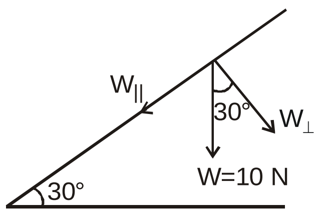


so the component along line = $|A_y\cos30° + A_x\cos60°|$

and perpendicular to line = $|A_x\sin60° – A_y\sin30°|$

Ex.8 *Resolve a weight of 10 N in two directions which are parallel and perpendicular to a slope inclined at 30° to the horizontal*

Sol. Component perpendicular to the plane



$$W_\perp = W\cos30°$$

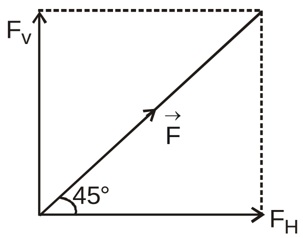
$$= (10)\frac{\sqrt{3}}{2} = 5\sqrt{3}\text{ N} \quad \text{Ans.}$$

and component parallel to the plane

$$W_{||} = W\sin 30° = (10)\left(\frac{1}{2}\right) = 5\text{ N}$$

Ex.9 *Resolve horizontally and vertically a force F = 8 N which makes an angle of 45° with the horizontal.*

Sol. Horizontal component of $\vec{F}$ is



$$F_H = F\cos 45° = (8)\left(\frac{1}{\sqrt{2}}\right) = 4\sqrt{2}\text{N}$$

and vertical component of $\vec{F}$ is

$$F_v = F\sin 45° = (8)\left(\frac{1}{\sqrt{2}}\right) = 4\sqrt{2}\text{N} \quad \text{Ans.}$$

8. PROCEDURE TO SOLVE THE VECTOR EQUATION

$$\vec{A} = \vec{B} + \vec{C} \quad \ldots(1)$$

(a) There are 6 variables in this equation which are following :

(1) Magnitude of $\vec{A}$ and its direction

(2) Magnitude of $\vec{B}$ and its direction

(3) Magnitude of $\vec{C}$ and its direction.

(b) We can solve this equation if we know the value of 4 variables [Note : two of them must be directions]

(c) If we know the two direction of any two vectors then we will put them on the same side and other on the different side.

For example

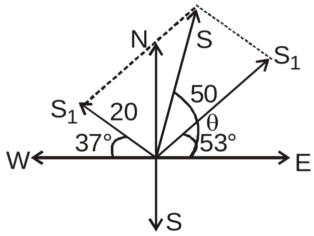
If we know the directions of $\vec{A}$ and $\vec{B}$ and $\vec{C}$'s direction is unknown then we make equation as follows:-

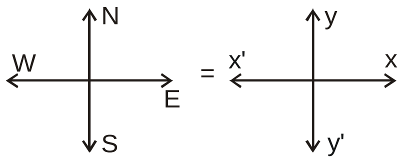
$$\vec{C} = \vec{A} - \vec{B}$$

(d) Then we make vector diagram according to the equation and resolve the vectors to know the unknown values.

Ex.10 Find the net displacement of a particle from its starting point if it undergoes two sucessive displacement given by $\vec{S}_1 = 20\text{m}$, 37° North of West, $\vec{S}_2 = 50\text{m}$, 53° North of East

Sol.





$$\vec{S} = \vec{S}_1 + \vec{S}_2$$

$S_x = S_{1x} + S_{2x}$

$= -20\cos 37° + 50\cos 53°$

$= 14$

$S_y = S_{1y} + S_{2y}$

$= 20\sin 37° + 50\sin 53°$

$= 52$

$$S = \sqrt{S_x^2 + S_y^2} = \sqrt{(14)^2 + (52)^2} = 53.85$$

Angle from west - east axis (x- axis)

$$\tan\theta = \frac{S_y}{S_x} = \frac{52}{14} = \frac{26}{7}$$

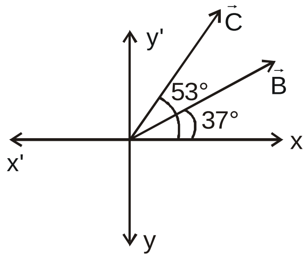
$$\theta = \tan^{-1}\left(\frac{26}{7}\right)$$

Ex.11 *Find magnitude of $\vec{B}$ and direction of $\vec{A}$. If $\vec{B}$ makes angle 37° and $\vec{C}$ makes 53° with x axis and $\vec{A}$ has magnitude equal to 10 and $\vec{C}$ has 5. (given $\vec{A} + \vec{B} + \vec{C} = 0$)*

Sol.

$$-\vec{A} = \vec{C} + \vec{B}$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$



$$\Rightarrow -\vec{A} = -A_x\hat{i} + -A_y\hat{j}$$

$A_x = -(B\cos 37° + C\cos 53°)$

$A_y = -(B\sin 37° + C\cos 53°)$

$$|\vec{A}|^2 = {A_x}^2 + {A_y}^2$$

$$A^2 = \left(B \times \frac{4}{5} + C \times \frac{3}{5}\right)^2 + \left(B \times \frac{3}{5} + C \times \frac{4}{5}\right)^2$$

$$10^2 = \left(\frac{4B}{5}+3\right)^2 + \left(\frac{3B}{5}+4\right)^2$$

$$\Rightarrow\ 100 = \frac{16}{25}B^2 + \frac{9}{25}B^2 + 25 + 2\left(\frac{3\times4}{5}+\frac{4\times3}{5}\right)B$$

$$\Rightarrow B^2 + \frac{48}{5}B - 75 = 0$$

B = 5 (magnitude can not be negative)

& Angle made by A

$$\Rightarrow A_x = -\left(\frac{20}{5} + 3\right) = -12$$

$$A_y = -\left(\frac{15}{5} + 4\right) = -7$$

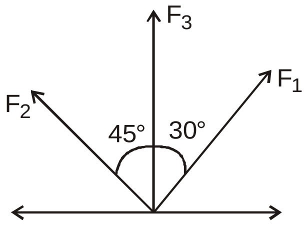
$$\tan\theta = \frac{A_y}{A_x} = \frac{-7}{-12}$$

$\theta = 180° + 25° = 205°$

Ex.12 *Find the magnitude of F_1 and F_2. If F_1, F_2 make angle 30° and 45° with F_3 and magnitude of F_3 is 10 N. (given $\vec{F}_1+\vec{F}_2 = \vec{F}_3$)*

Sol. $|\vec{F}_3| = F_1\cos30° + F_2\cos45°$

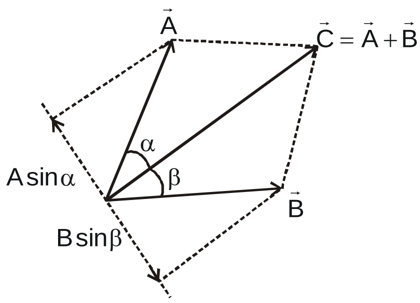
& $F_2\sin45° = F_1\sin30°$



$$\Rightarrow 10 = \frac{\sqrt{3}F_1}{2} + \frac{F_2}{\sqrt{2}},\quad \frac{F_2}{\sqrt{2}} = \frac{F_1}{2}$$

$$\Rightarrow F_1 = \frac{20}{\sqrt{3}+1} \quad \& \ F_2 = \frac{20\sqrt{2}}{\sqrt{3}+1}$$

9. SHORT - METHOD

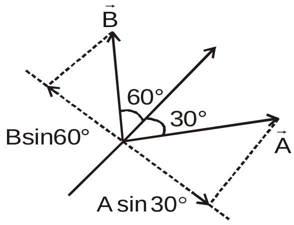


If their are two vectors $\vec{A}$ and $\vec{B}$ and their resultent make an anlge α with $\vec{A}$ and β with $\vec{B}$.

then $A\sin\alpha = \beta\sin\beta$

Means component of $\vec{A}$ perpendicular to resultant is equal in magnitude to the component of $\vec{B}$ perpendicular to resultant.

Ex.13 *If two vectors $\vec{A}$ and $\vec{B}$ make angle 30° and 45° with their resultant and $\vec{B}$ has magnitude equal to 10, then find magnitude of $\vec{A}$.*



So $\quad B \sin 60° = A \sin 30°$

$\Rightarrow \quad 10 \sin 60° = A \sin 30°$

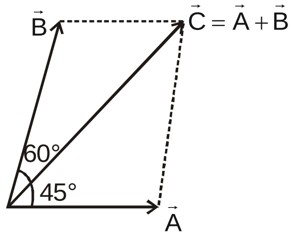
$\Rightarrow \quad A = 10\sqrt{3}$

Ex.14 *If $\vec{A}$ and $\vec{B}$ have angle between them equals to 60° and their resultant make, angle 45° with $\vec{A}$ and $\vec{A}$ have magnitude equal to 10. Then Find magnitude of $\vec{B}$.*

Sol. here $\alpha = 45°$ and $\beta = 60° - 45° = 15°$

so $A \sin \alpha = B \sin \beta$

$10 \sin 45° = B \sin 45°$



So $B = \dfrac{10}{\sqrt{2}} \sin 15°$

$$= \frac{10}{\sqrt{2}}\sqrt{\frac{1-\cos(2\times 15)}{2}} = \frac{5}{\sqrt{2}}\sqrt{2-\sqrt{3}}$$

10. ADDITION AND SUBTRACTION IN COMPONENT FORM :

Suppose there are two vectors in component form. Then the addition and subtraction between these two are

$$\mathbf{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\mathbf{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\mathbf{A} \pm \mathbf{B} = (A_x \pm B_x)\hat{i} + (A_y \pm B_y)\hat{j} + (A_z \pm B_z)\hat{k}$$

Also if we are having a third vector present in component form and this vector is added or subtracted from the addition or subtraction of above two vectors then

$$\mathbf{C} = C_x\hat{i} + C_y\hat{j} + C_z\hat{k}$$

$$\mathbf{A} \pm \mathbf{B} \pm \mathbf{C} = (A_x \pm B_x \pm C_x)\hat{i} + (A_y \pm B_y \pm C_y)\hat{j} + (A_z \pm B_z \pm C_z)\hat{k}$$

Note : Modulus of vector A is given by

$$|\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Ex.15 *Obtain the magnitude of* $2\vec{A} - 3\vec{B}$ *if*

$$\vec{A} = \hat{i} + \hat{j} - 2\hat{k} \quad \textit{and} \quad \vec{B} = 2\hat{i} - \hat{j} + \hat{k}$$

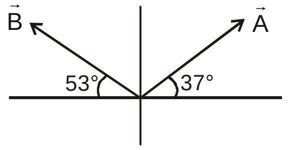
Sol. $\quad 2\vec{A} - 3\vec{B} = 2(\hat{i} + \hat{j} - 2\hat{k}) - 3(2\hat{i} - \hat{j} + \hat{k})$

$\therefore \quad$ Magnitude of $\quad 2\vec{A} - 3\vec{B} = \sqrt{(-4)^2 + (5)^2 + (-7)^2}$

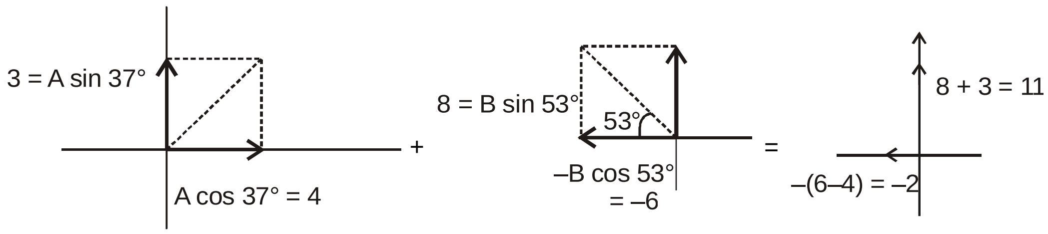
$$= \sqrt{16 + 25 + 49} = \sqrt{90} \qquad \text{Ans.}$$

Ex.16 *Find $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$ if $\vec{A}$ make angle 37° with positive x-axis and $\vec{B}$ make angle 53° with negative x-axis as shown and magnitude of $\vec{A}$ is 5 and of B is 10.*

Sol.



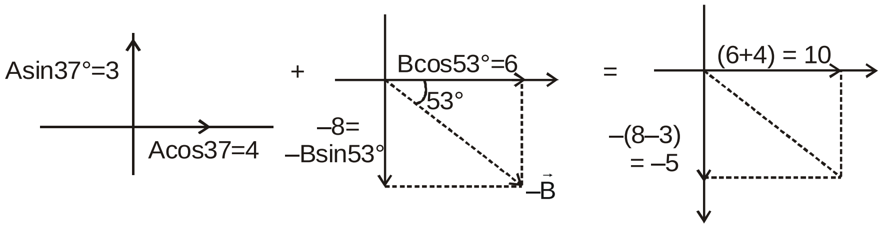
for $\vec{A}+\vec{B}$



so the magnitude of resultant will be = $\sqrt{11^2+(-2)^2}$ = $5\sqrt{5}$

and have angle θ = $\tan^{-1}\left(\frac{11}{2}\right)$ from negative x - axis towards up

for $\vec{A}-\vec{B}$



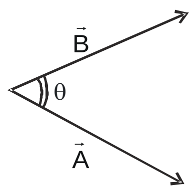
So the magnitude of resultant will be

$$= \sqrt{10^2+(-5)^2} = 5\sqrt{5}$$

and have angle $\theta = \tan^{-1}\left(\frac{5}{10}\right)$ from positive x-axis towards down.

11. MULTIPLICATION OF VECTORS (The Scalar and vector products) :

11.1 Scalar Product



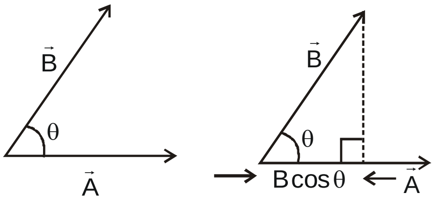
The scalar product or dot product of any two vector $\vec{A}$ and $\vec{B}$, denoted as $\vec{A}\cdot\vec{B}$ (read $\vec{A}$ dot $\vec{B}$) is defined as the product of their magnitude with cosine of angle between them. Thus,

$\vec{A}.\vec{B} = AB\cos\theta$ (here θ is the angle between the vectos)

Properties :

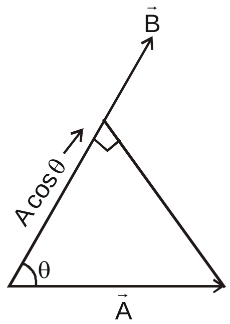
- It is always a scalar which is positive if angle between the vectors is acute (i.e.< 90°) and negative if angle between them is obtuse (i.e., 90° < q ≤ 180°)
- It is commutative i.e. $\vec{A}.\vec{B} = \vec{B}.\vec{A}$
- It is distributive, i.e. $\vec{A}.(\vec{B}+\vec{C}) = \vec{A}.\vec{B}+\vec{A}.\vec{C}$
- As by definition $\vec{A}.\vec{B}$ = AB cos θ . The angle between the vectors θ = $\cos^{-1}\left[\frac{\vec{A}.\vec{B}}{AB}\right]$
- $\vec{A}.\vec{B} = A(B\cos\theta) = B(A\cos\theta)$

Geometrically, B cos θ is the projection of $\vec{B}$ onto $\vec{A}$ and vice versa



Component of $\vec{B}$ along $\vec{A}$ = B cos θ = $\frac{\vec{A}.\vec{B}}{A}$ = $\hat{A}.\vec{B}$ (Projection of $\vec{B}$ on $\vec{A}$)

Component of $\vec{A}$ along $\vec{B}$ = A cos θ = $\frac{\vec{A}.\vec{B}}{B}$ = $\vec{A}.\hat{B}$ (Projection of $\vec{A}$ on $\vec{B}$)



- Scalar product of two vectors will be maximum when cos θ = max = 1, i.e., θ = 0°,

  i.e., vectors are parallel $\Rightarrow (\vec{A}.\vec{B})_{max} = AB$
- If the scalar product of two non-zero vectors vanishes then the vectors are perpendicular.
- The scalar product of a vector by itself is termed as self dot product and is given by

  $(\vec{A})^2 = \vec{A}.\vec{A}$ = AA cos θ = $A^2 \Rightarrow$ $A = \sqrt{\vec{A}.\vec{A}}$
- In case of unit vector $\hat{n}$,

  $\hat{n}.\hat{n}$ = 1 × 1 × cos 0° = 1 $\Rightarrow$ $\hat{n}.\hat{n} = \hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

  In case of orthogonal unit vectors, $\hat{i}$, $\hat{j}$ and $\hat{k}$; $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

  $\vec{A}.\vec{B} = (\hat{i}A_x + \hat{j}A_y + \hat{k}A_z).(\hat{i}B_x + \hat{j}B_y + \hat{k}B_z)$ = $[A_xB_x + A_yB_y + A_zB_z]$

Ex.17 *If the vectors $\vec{P} = a\hat{i} + a\hat{j} + 3\hat{k}$ and $\vec{Q} = a\hat{i} - 2\hat{j} - \hat{k}$ are perpendicular to each other. Find the value of a?*

Sol. If vectors $\vec{P}$ and $\vec{Q}$ are perpendicular

$\Rightarrow \vec{P}.\vec{Q} = 0$ $\Rightarrow (a\hat{i} + a\hat{j} + 3\hat{k}).(a\hat{i} - 2\hat{j} - \hat{k}) = 0$

$\Rightarrow a^2 - 2a - 3 = 0$ $\Rightarrow a^2 - 3a + a - 3 = 0$

$\Rightarrow a(a - 3) + 1(a - 3)$ $\Rightarrow a = -1, 3$

Ex.18 Find the component of $3\hat{i}+4\hat{j}$ *along* $\hat{i}+\hat{j}$ *?*

Sol. Component of $\vec{A}$ along $\vec{B}$ is given by $\frac{\vec{A}.\vec{B}}{B}$ hence required component

$$= \frac{(3\hat{i}+4\hat{j}).(\hat{i}+\hat{j})}{\sqrt{2}} = \frac{7}{\sqrt{2}}$$

Ex.19 Find angle between $\vec{A}=3\hat{i}+4\hat{j}$ *and* $\vec{B}=12\hat{i}+5\hat{j}$ *?*

Sol. We have $\cos\theta = \frac{\vec{A}.\vec{B}}{AB} = \frac{(3\hat{i}+4\hat{j}).(12\hat{i}+5\hat{j})}{\sqrt{3^2+4^2}\sqrt{12^2+5^2}}$

$$\cos\theta = \frac{36+20}{5\times 13} = \frac{56}{65} \qquad \theta = \cos^{-1}\left(\frac{56}{65}\right)$$

Ex.20 (i) For what value of m the vector $\vec{A}=2\hat{i}+3\hat{j}-6\hat{k}$ *is perpendicular to* $\vec{B}=3\hat{i}-m\hat{j}+6\hat{k}$

(ii) Find the component of vector $\vec{A}=2\hat{i}+3\hat{j}$ *along the direction of* $\hat{i}+\hat{j}$ *?*

Sol. (i) m = – 10 (ii) $\frac{5}{\sqrt{2}}$

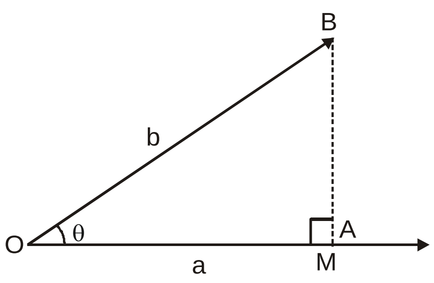
Important Note :

Components of b along and perpendicular to a.

Let $\overrightarrow{OA}$. $\overrightarrow{OB}$ represent two (non-zero) given vectors a, b respectively. Draw BM perpendicular to $\overrightarrow{OA}$

From ΔOMB, $\overrightarrow{OB} = \overrightarrow{OM} + \overrightarrow{MB}$

$\Rightarrow$ $b = \overrightarrow{OM} + \overrightarrow{MB}$



Thus $\overrightarrow{OM}$ and $\overrightarrow{MB}$ are components of b along a and perpendicular to a.

Now $\overrightarrow{OM} = (\overrightarrow{OM})\,\hat{a} = (OB\cos\theta)\,\hat{a}$

$= |b|\cos\theta\,\hat{a} = |b| \, . \, a \, . \, b \, / \, |a| \;\; |b| \; . \,\hat{a}$

$= a \, . \, b \, / \, |a| \, . \, a/|a| = (a \, . \, b) \, a \, / \, |a|^2$

$= (a \, . \, b) \, a \, / \, a^2$

$\overrightarrow{MB} = b - \overrightarrow{OM} = b - (a \, . \, b \, / \, |a|^2) \, . \, a$

Hence, components of b along a perpendicular to a are.

$(a \, . \, b/ \, |a|^2)\, a$ and $b - (a \, . \, b \, / \, |a|^2)\, a$ respectively.

Ex.21 The velocity of a particle is given by $\vec{v}=3\hat{i}+2\hat{j}+3\hat{k}$. Find the vector component of its velocity parallel to the line $\vec{l}=\hat{i}-\hat{j}+\hat{k}$.

Sol. Component of $\vec{v}$ along $\vec{l}$

$$= v\cos\theta\hat{l} = v\frac{\vec{v}.\vec{l}}{vl}\hat{l} = \frac{\vec{v}.\vec{l}}{l^2}\vec{l}$$

$$= \frac{(3\hat{i}+2\hat{j}+3\hat{k})(\hat{i}-\hat{j}+\hat{k})}{|\hat{i}-\hat{j}+\hat{k}|^2} = \frac{4}{3}(\hat{i}-\hat{j}+\hat{k})$$

11.2 Vector product

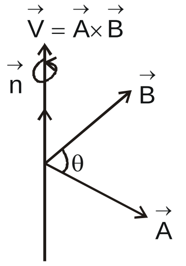
The vector product or cross product of any two vectors $\vec{A}$ and $\vec{B}$, denoted as

$\vec{A}\times\vec{B}$ (read $\vec{A}$ cross $\vec{B}$) is defined as :

$$\vec{A}\times\vec{B} = AB\sin\theta\hat{n}$$

Here θ is the angle between the vectors and the direction $\hat{n}$ is given by the right - hand - thumb rule.

Right - Hand - Thumb Rule :



To find the direction of $\hat{n}$, draw the two vectors $\vec{A}$ and $\vec{B}$ with both the tails coinciding. Now place your stretched right palm perpendicular to the plane of $\vec{A}$ and $\vec{B}$ in such a way that the fingers are along the vector $\vec{A}$ and when the fingers are closed they go towards $\vec{B}$. The direction of the thumb gives the direction of $\hat{n}$.

Properties :

- Vector product of two vectors is always a vector perpendicular to the plane containing the two vectors i.e. orthogonal to both the vectors $\vec{A}$ and $\vec{B}$, though the vectors $\vec{A}$ and $\vec{B}$ may or may not be orthogonal.
- Vector product of two vectors is not commutative i.e. $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$

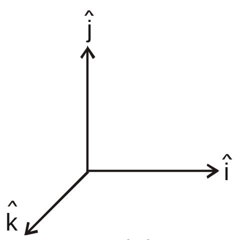
  But $|\vec{A}\times\vec{B}| = |\vec{B}\times\vec{A}| = AB\sin\theta$
- The vector product is distributive when the order of the vectors is strictly maintained i.e.

  $$\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B}+\vec{A}\times\vec{C}$$
- The magnitude of vector product of two vectors will be maximum when sin θ = max = 1. i.e. θ = 90°

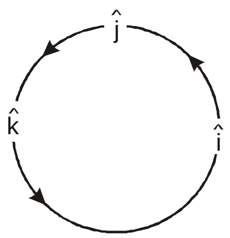
  $$|\vec{A}\times\vec{B}|_{max} = AB$$
- The magnitude of vector product of two non-zero vectors will be minimum when $|\sin\theta|$ = minimum = 0, i.e., θ = 0° or 180° and $|\vec{A}\times\vec{B}|_{min} = 0$ i.e., if the vector product of two non-zero vectors vanishes, the vectors are collinear.
- The self cross product i.e. product of a vector by itself vanishes i.e. is a null vector.

  $$\vec{A}\times\vec{A} = AA\sin 0°\hat{n} = \vec{0}$$
- In case of unit vector $\hat{n}$, $\quad \hat{n}\times\hat{n} = \vec{0} \quad \Rightarrow \quad \hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \vec{0}$
- In case of orthogonal unit vectors $\hat{i},\hat{j}$ and $\hat{k}$ in accordance with right-hand-thumb-rule,

  $$\hat{i}\times\hat{j} = \hat{k} \qquad \hat{j}\times\hat{k} = \hat{i} \qquad \hat{k}\times\hat{i} = \hat{j}$$



(A)



(B)

- In terms of components, $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

$$\vec{A}\times\vec{B} = \hat{i}(A_yB_z - A_zB_y) + \hat{j}(A_zB_x - A_xB_z) + \hat{k}(A_xB_y - A_yB_x)$$

Ex.22 *$\vec{A}$ is East wards and $\vec{B}$ is downwards. Find the direction of $\vec{A}$ × $\vec{B}$?*

Sol. Applying right hand thumb rule we find that $\vec{A}\times\vec{B}$ is along North.

Ex.23 *If $\vec{A}.\vec{B} = |\vec{A}\times\vec{B}|$, find angle between $\vec{A}$ and $\vec{B}$*

Sol. $\vec{A}.\vec{B} = |\vec{A}\times\vec{B}|$ AB cos θ = AB sin θ   tan θ = 1   ⇒   θ = 45°

Ex.24 $\vec{A}\times\vec{B} = AB\sin\theta\hat{n}$ ⇒ $\hat{n} = \dfrac{\vec{A}\times\vec{B}}{AB\sin\theta}$ *here $\hat{n}$ is perpendicular to both $\vec{A}$ and $\vec{B}$*

Ex.25 Find $\vec{A}\times\vec{B}$ if $\vec{A} = \hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{B} = 2\hat{i} - \hat{j} + 2\hat{k}$

Sol. $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 4 \\ 3 & -1 & 2 \end{vmatrix} = \hat{i}(-4-(-4)) - \hat{j}(2-12) + \hat{k}(-1-(-6)) = 10\hat{j} + 5\hat{k}$

Ex.26 *(i) $\vec{A}$ is North-East and $\vec{B}$ is down wards, find the direction of $\vec{A}\times\vec{B}$*

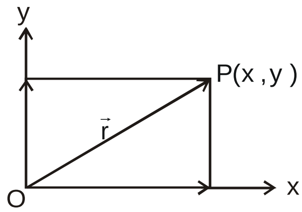
(ii) Find $\vec{B}$ × $\vec{A}$ if $\vec{A} = 3\hat{i} - 2\hat{j} + 6\hat{k}$ and $\vec{B} = \hat{i} - \hat{j} + \hat{k}$

Ans. (i) North - West.   (ii) $-4\hat{i} - 3\hat{j} + \hat{k}$

12. POSITION VECTOR :

Positin vector for a point is vector for which tail is origin & head is the given point itself.

Position vector of a point defines the position of the point w.r.t. the origin.

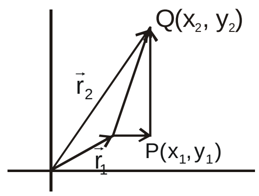


$\overrightarrow{OP} = \vec{r}$

$\vec{r} = x\hat{i} + y\hat{j}$

13. DISPLACEMENT VECTOR :

Change in position vector of particle is known as displacement vector.



$\overrightarrow{OP} = \vec{r}_1 = x_1\hat{i} + y_1\hat{j}$

$\overrightarrow{OQ} = \vec{r}_2 = x_2\hat{i} + y_2\hat{j}$

$\overrightarrow{PQ} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

Thus we can represent a vector in space starting from (x_1, y_1) & ending at (x_2, y_2) as $(x_2 - x_1)\hat{i}$ + $(y_2 - y_1)$ $\hat{j}$

CALCULUS

14. CONSTANTS : They are fixed real number which value does not change

Ex. 3, e, a, – 1, etc.

15. VARIABLE :

Somthing that is likly to vary, somthing that is subject to variation.

or

A quantity that can assume any of a set of value.

Types of variables.

(i) Independent variables : Indepedent variables is typically the variable being manipulated or change

(ii) dependent variables : The dependent variables is the object result of the independent variable being manipulated.

Ex. $y = x^2$

here y is dependent variable and x is independent variable

16. FUNCTION :

Function is a rule of relationship between two variables in which one is assumed to be dependent and the other independent variable.

The temperatures at which water boils depends on the elevation above sea level (the boiling point drops as you ascend). Here elevation above sea level is the independent & temperature is the dependent variable.

The interest paid on a cash investment depends on the length of time the investment is held. Here time is the independent and interest is the dependent variable.

In each case, the value of one variable quantity (dependent variable), which we might call y, depends on the value of another variable quantity (independent variable), which we might call x. Since the value of y is completely determined by the value of x, we say that y is a function of x and represent it mathematically as $y = f(x)$.

all possible values of independent variables (x) are called domain of function.

all possible values of dependent variable (y) are called Range of fucntion.

Think of function f as a kind machine that produces an output value f(x) in its range whenever we feed it an input value x from its domain (figure).

When we study circles, we usualy call the area A and the radius r. Since area depends on radius, we say that A is a function of r, $A = f(r)$. The eauation $A = \pi r^2$ is a rule that tells how to calculate a unique (single) output value of A for each possible input value of the radius r.

$A = f(x) = \pi r^2$. (Here the rule of relationship which describes the function may be described as square & multiply by π)

if $r = 1$ $A = \pi$

if $r = 2$ $A = 4\pi$

if $r = 3$ $A = 9\pi$

The set of all possible input values for the radius is called the domain of the function. The set of all output values of the area is the range of the function.

We usually denote functions in one of the two ways :

1. By giving a formula such as $y = x^2$ that uses a dependent variable y to denote the value of the fucntion.
2. By giving a formula such as $f(x) = x^2$ that defines a functions symbols f to name the function.
   Strictly speaking, we should call the function f and not f(x).
   y = sinx. Here the function is y since, x is the independent variable.

Ex.27 The volume V of ball (solid sphere) of radius r is given by the function $V(r) = \frac{4}{3}\pi(r)^3$
The volume of a ball of radius 3m is ?

Sol. $V(3) = \frac{4}{3}\pi(3)^3 = 36\,\pi m^3$.

Ex.28 Suppose that the function F is defined for all real numbers r by the formula.
$F(r) = 2(r - 1) + 3.$
Evaluate F at the input values 0, 2 x + 2, and F(2).

Sol. In each case we substitute the given input value for r into the formula for F:
$F(0) = 2(0 - 1) + 3 = -2 + 3 = 1$
$F(2) = 2(2 - 1) + 3 = 2 + 3 = 5$
$F(x + 2) = 2(x + 2 - 1) + 3 = 2x + 5$
$F(F(2)) = F(5) = 2(5 - 1) + 3 = 11$

Ex.29 function f(x) is defined as
$f(x) = x^2 + 3$, *Find*
f(0), f(1), f(x^2), f(x + 1) and f(f(1))

Sol.
$f(0) = 0^2 + 3 = 3$
$f(1) = 1^2 + 3 = 4$
$f(x^2) = (x^2)^2 + 3 = x^4 + 4$
$f(x + 1) = (x + 1)^2 + 3 = x^2 + 2x + 4$
$f(f(1)) = f(4) = 4^2 + 3 = 19$

17. DIFFERENTIATION

Finite difference :

The finite difference between two values of a physical is represented by Δ notation.
For example :
Difference in two values of y is written as Δy as given in the table below.

y_2	100	100	100
y_1	50	99	99.5
$\Delta y = y_2 - y_1$	50	1	0.5

Infinitey small difference :

The infinitely small difference means very-very small difference. And this difference is represented by 'd' notation insted of 'Δ'.
For example infinitely small difference in the values of y is written as 'dy'
if $y_2 = 100$ and $y_1 = 99.9999999999999.....$
then dy = 0.00000000000000..........00001

Definition of differentiation

Another name of differentiation is derivative. Suppose y is a function of x or y = f(x)

Differentiation of y with respect to x is denoted by sumbols f′ (x)

where $f'(x) = \frac{dy}{dx}$; dx is very small change in x and dy is corresponding very small change in y.

Notation : There are many ways to denote the derivative of function y = f(x), the most common notations are these :

y'	"y prime"	Nice and brief and does not name the independent variable
$\frac{dy}{dx}$	" dy by dx"	Names the variables and uses d for derivative
$\frac{df}{dx}$	" df by dx"	Emphasizes the function's name
$\frac{d}{dx}f(x)$	" d by dx of f "	Emphasizes the idea that differentiation is an operation performed on f.
$D_x f$	" dx of f "	A common operator notation
$\dot{y}$	" y dot"	One of Newton's notations, now common for time derivative i.e. dy/dt

Average rates of change :

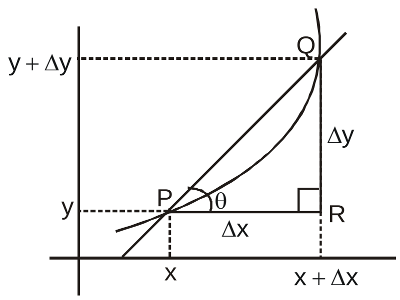
Given an arbitrary function y = f(x) we calculate the average rate of change of y with respect to x over the interval (x, x+ Δx) by dividing the change in value of y, i.e., Δy = f(x+ Δx) – f(x), by length of interval Δx over which the change occurred.

The average rate of change of y with respect to x over the interval [x, x + Δx]

$$= \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Geometrically

$$\frac{\Delta y}{\Delta x} = \frac{QR}{PR} = \tan\theta = \text{Slope of the line PQ}$$



In triangle QPR $\tan\theta = \frac{\Delta y}{\Delta x}$

therefore we can say that average rate of change of y with respect to x is equal to slope of the line joining P & Q.

The derivative of a fucntion

We know that Average rate of change of y w.r.t x is -

$$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

If the limit of this ratio exists as Δx → 0, then it is called the derivative of given function f(x) and is denoted as

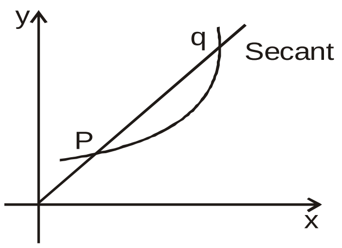
$$f'(x) = \frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

18. GEOMETRICAL MEANING OF DIFFERENTIATION :

The geometrical meaning of differentiation is very much useful in the analysis of graphs in physics. To understand the geometrical meaning of derivatives we should have knowledge of secant and tangent to a curve.

Secant and Tangent to a Curve

Secant : - A secant to a curve is a straight line, which intersects the curve at any two points.



Tangent :

A tangent is straight line, which touches the curve a particular point. Tangent is limiting case of secant which intersects the curve at two overlapping point.

In the figure - 1 shown, if value of Δx is gradually reduced then the point Q will move nearer to the point P. If the process is continuously repeated (Figure-2) value of Δx will be infinitely small and secant PQ to the given curve will become a tangent at point P.

Therefore

$$\lim_{\Delta x \to 0}\left(\frac{\Delta y}{\Delta x}\right) = \frac{dy}{dx} = \tan\theta$$

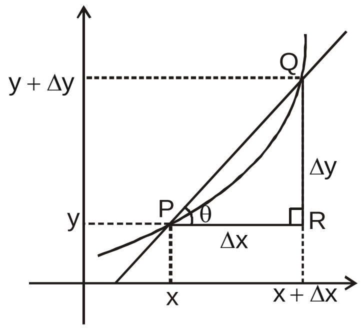


Figure-1

we can say that differentiation of y with respect to x, i.e. $\left(\frac{dy}{dx}\right)$ is equal to slope of the tangent at point P (x,y)

or $\tan\theta = \frac{dy}{dx}$

(From fig-1 the average rate change of y from x to x + Δx is identical with the slope of secant PQ)

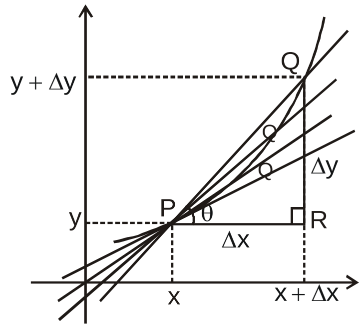


Figure-2

Rule No. 1 Derivative Of A Constant

The first rule of differentiation is that the derivative of every constant function is zero.

If c is constant, then $\frac{d}{dx}c = 0$

Ex.30 $\frac{d}{dx}(8) = 0$, $\frac{d}{dx}\left(-\frac{1}{2}\right) = 0$, $\frac{d}{dx}\left(\sqrt{3}\right) = 0$

Rule No.2 Power Rule

If n is a real number, then $\frac{d}{dx}x^n = nx^{n-1}$

To apply the power Rule, we subtract 1 from the original exponent (n) and multiply the result by n.

Ex.31

f	x	x^2	x^3	x^4	.....
f'	1	2x	$3x^2$	$4x^3$	.....

Ex.32 (i) $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}(x^{-1}) = (-1)x^{-2} = -\frac{1}{x^2}$ (ii) $\frac{d}{dx}\left(\frac{4}{x^3}\right) = 4\frac{d}{dx}(x^{-3}) = 4(-3)x^{-4} = -\frac{12}{x^4}$

Ex.33 (a) $\frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

Function defined for $x \geq 0$ derivative defined only for $x > 0$

(b) $\frac{d}{dx}(x^{1/5}) = \frac{1}{5}x^{-4/5}$

Function defined for $x \geq 0$ derivative not defined at $x = 0$

Rule No.3 The Constant Multiple Rule

If u is a differentiable function of x, and c is a constant, then $\frac{d}{dx}(cu) = c\frac{du}{dx}$

In particular, if n is a positive integer, then $\frac{d}{dx}(cx^n) = cn\,x^{n-1}$

Ex.34 *The derivative formula*

$$\frac{d}{dx}(3x^2) = 3(2x) = 6x$$

says that if we rescale the graph of $y = x^2$ by multiplying each y-coordinate by 3, then we multiply the slope at each point by 3.

Ex.35 A useful special case

The derivative of the negative of a differentiable function is the negative of the function's derivative. Rule 3 with c = – 1 gives.

$$\frac{d}{dx}(-u) = \frac{d}{dx}(-1.u) = -1.\frac{d}{dx}(u) = -\frac{d}{dx}(u)$$

Rule No.4 The Sum Rule

The derivative of the sum of two differentiable functions is the sum of their derivatives.

If u and v are differentiable functions of x, then their sum u + v is differentiable at every point where u and v are both differentiable functions in their derivatives.

$$\frac{d}{dx}(u-v) = \frac{d}{dx}[u+(-1)v] = \frac{du}{dx} + (-1)\frac{dv}{dx} = \frac{du}{dx} - \frac{dv}{dx}$$

The sum Rule also extends to sums of more than two functions, as long as there are only finite functions in the sum. If $u_1, u_2, \ldots\ldots u_n$ are differentiable at x, then so if $u_1 + u_2 + \ldots\ldots + u_n$, then

$$\frac{d}{dx}(u_1 + u_2 + \ldots\ldots + u_n) = \frac{du_1}{dx} + \frac{du_2}{dx} + \ldots\ldots + \frac{du_n}{dx}$$

Ex.36 (a) $y = x^4 + 12x$

$$\frac{dy}{dx} = \frac{d}{dx}(x^4) + \frac{d}{dx}(12x)$$

$= 4x^3 + 12$

(b) $y = x^3 + \frac{4}{3}x^2 - 5x + 1$

$$\frac{dy}{dx} = \frac{d}{dx}x^3 + \frac{d}{dx}\left(\frac{4}{3}x^2\right) - \frac{d}{dx}(5x) + \frac{d}{dx}(1)$$

$= 3x^2 + \frac{4}{3}.2x - 5 + 0$

$= 3x^2 + \frac{8}{3}x - 5$

Notice that we can differentiate any polynomial term by term, the way we differentiated the polynomials in above example.

Rule No. 5 The Product Rule

If u and v are differentiable at x, then if their product uv is considered, then $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$.

The derivative of the product uv is u times the derivative of v plus v times the derivative of u. In prime notation

$(uv)' = uv' + vu'$.

While the derivative of the sum of two functions is the sum of their derivatives, the derivative of the product of two functions is not the product of their derivatives. For instance,

$\frac{d}{dx}(x.x) = \frac{d}{dx}(x^2) = 2x,$ while $\frac{d}{dx}(x).\frac{d}{dx}(x) = 1.1 = 1$, which is wrong

Ex.37 Find the derivatives of $y = (x^2 + 1)(x^3 + 3)$

Sol. Using the product Rule with $u = x^2 + 1$ and $v = x^3 + 3$, we find

$$\frac{d}{dx}[(x^2+1)(x^3+3)] = (x^2 + 1)(3x^2) + (x^3 + 3)(2x)$$

$$= 3x^4 + 3x^2 + 2x^4 + 6x = 5x^4 + 3x^2 + 6x$$

Example can be done as well (perhaps better) by multiplying out the original expression for y and differentiating the resulting polynomial. We now check :

$y = (x^2 + 1)(x^3 + 3) = x^5 + x^3 + 3x^2 + 3$

$$\frac{dy}{dx} = 5x^4 + 3x^2 + 6x$$

This is in agreement with our first calculation.

There are times, however, when the product Rule must be used. In the following examples. We have only numerical values to work with.

Ex.38 Let y = uv be the product of the functions u and v. Find y'(2) if u(2) = 3, u'(2) = – 4, v(2) = 1, and v'(2) = 2.

Sol. From the Product Rule, in the form

$y' = (uv)' = uv' + vu'$,

we have $y'(2) = u(2)\,v'(2) + v(2)\,u'(2)$

$= (3)(2) + (1)(-4) = 6 - 4 = 2$

Rule No.6 The Quotient Rule

If u and v are differentiable at x, and $v(x) \neq 0$, then the quotient u/v is differentiable at x,

and $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Just as the derivative of the product of two differentiable functions is not the product of their derivatives, the derivative of the quotient of two functions is not the quotient of their derivatives.

Ex.39 Find the derivative of $y = \frac{t^2 - 1}{t^2 + 1}$

Sol. We apply the Quotient Rule with $u = t^2 - 1$ and $v = t^2 + 1$

$$\frac{dy}{dt} = \frac{(t^2+1)2t - (t^2-1).2t}{(t^2+1)^2} \qquad \left[\text{As } \frac{d}{dt}\left(\frac{u}{v}\right) = \frac{v(du/dt) - u(dv/dt)}{v^2}\right]$$

$$= \frac{2t^3 + 2t - 2t^3 + 2t}{(t^2+1)^2} = \frac{4t}{(t^2+1)^2}$$

Rule No. 7 Derivative Of Sine Function

$$\frac{d}{dx}(\sin x) = \cos x$$

Ex.40 (a) $y = x^2 - \sin x$: $\frac{dy}{dx} = 2x - \frac{d}{dx}(\sin x) = 2x - \cos x$ Difference Rule

(b) $y = x^2 \sin x$: $\frac{dy}{dx} = x^2\frac{d}{dx}(\sin x) + 2x \sin x$ Product Rule

$= x^2\cos x + 2x \sin x$

(c) $y = \frac{\sin x}{x}$: $\frac{dy}{dx} = \frac{x.\frac{d}{dx}(\sin x) - \sin x.1}{x^2}$ Quotient Rule

$= \frac{x\cos x - \sin x}{x^2}$

Rules No.8 Derivative Of Cosine Function

$$\frac{d}{dx}(\cos x) = -\sin x$$

Ex.41 (a) $y = 5x + \cos x$ Sum Rule

$\frac{dy}{dx} = \frac{d}{dx}(5x) + \frac{d}{dx}(\cos x) = 5 - \sin x$

(b) $y = \sin x \cos x$

$\frac{dy}{dx} = \sin x\frac{d}{dx}(\cos x) + \cos x\frac{d}{dx}(\sin x)$ Product Rule

$= \sin x(-\sin x) + \cos x(\cos x)$

$= \cos^2 x - \sin^2 x = \cos 2x$

Rule No. 9 Derivatives Of Other Trigonometric Functions

Because sin x and cos x are differentiable functions of x, the related functions

$$\tan x = \frac{\sin x}{\cos x} \quad ; \qquad \sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x} \quad ; \qquad \text{cosec } x = \frac{1}{\sin x}$$

are differentiable at every value of x at which they are defined. There derivatives, Calculated from the Quotient Rule, are given by the following formulas.

$$\frac{d}{dx}(\tan x) = \sec^2 x \quad ; \qquad \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\text{cosec}^2 x \quad ; \qquad \frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cot x$$

Ex.42 Find dy / dx if y = tan x.

Sol. $$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x \frac{d}{dx}(\sin x) - \sin x \frac{d}{dx}(\cos x)}{\cos^2 x}$$

$$= \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

Ex.43 (a) $\frac{d}{dx}(3x + \cot x) = 3 + \frac{d}{dx}(\cot x) = 3 - \text{cosec}^2 x$

(b) $\frac{d}{dx}\left(\frac{2}{\sin x}\right) = \frac{d}{dx}(2\,\text{cosec } x) = 2\frac{d}{dx}(\text{cosec } x)$

$= 2(-\text{cosec } x \cot x) = -2 \text{ cosec } x \cot x$

Rule No. 10 Derivative Of Logrithm And Exponential Functions

$$\frac{d}{dx}(\log_e x) = \frac{1}{x} \quad , \qquad \frac{d}{dx}(e^x) = e^x$$

Ex.44 $y = e^x . \log_e (x)$

$$\frac{dy}{dx} = \frac{d}{dx}(e^x).\log(x) + \frac{d}{dx}[\log_e(x)]e^x \quad \Rightarrow \quad \frac{dy}{dx} = e^x.\log_e(x) + \frac{e^x}{x}$$

Rule No. 11 Chain Rule Or 'Outside Inside' Rule

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$$

It sometime helps to think about the Chain Rule the following way. If y = f(g(x)),

$$\frac{dy}{dx} = f'[g(x)] \, . \, g'(x)$$

In words : To find dy/dx, differentiate the "outside" function f and leave the "inside" g(x) alone; then multiply by the derivative of the inside.

We now know how to differntiate sin x and $x^2 - 4$, but how do we differentiate a composite like $\sin(x^2 - 4)$?

The answer is, with the Chain Rule, which says that the derivative of the composite of two differentiable functions is the product of their derivatives evaluated at appropriate points. The Chain Rule is probably the most widely used differentiation rule in mathematics. This section describes the rule and how to use it. We begin with examples.

Ex.45 The function y = 6x – 10 = 2(3x – 5) is the composite of the functions y = 2u and u = 3x – 5. How are the derivatives of these three functions related ?

Sol. We have $\frac{dy}{dx} = 6$, $\frac{dy}{du} = 2$, $\frac{du}{dx} = 3$

Since 6 = 2 × 3 $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$

Is it an accident that $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$?

If we think of the derivative as a rate of change, our intution allows us to see that this relationship is reasonable. For y = f(u) and u = g(x), if y changes twice as fast as u and u changes three times as fast as x, then we expect y to change six times as fast as x.

Ex.46 Let us try this again on another function.

$y = 9x^4 + 6x^2 + 1 = (3x^2 + 1)^2$

is the composite $y = u^2$ and $u = 3x^2 + 1$. Calculating derivatives. We see that

$\frac{dy}{du}.\frac{du}{dx} = 2u.6x = 2(3x^2 + 1).6x = 36x^3 + 12x$

and $\frac{dy}{dx} = \frac{d}{dx}(9x^4 + 6x^2 + 1) = 36x^3 + 12x$

Once again, $\frac{dy}{du}.\frac{du}{dx} = \frac{dy}{dx}$

The derivative of the composite function f(g(x)) at x is the derivative of f at g(x) times the derivative of g at x.

Ex.47 *Find the derivation of* $y = \sqrt{x^2 + 1}$

Sol. Here y = f(g(x)), where f(u) = $\sqrt{u}$ and u = g(x) = $x^2 + 1$. Since the derivatives of f and g are

$f'(u) = \frac{1}{2\sqrt{u}}$ and $g'(x) = 2x$,

the Chain Rule gives

$$\frac{dy}{dx} = \frac{d}{dx}f(g(x)) = f'(g(x)).g'(x) = \frac{1}{2\sqrt{g(x)}}.g'(x) = \frac{1}{2\sqrt{x^2+1}}.(2x) = \frac{x}{\sqrt{x^2+1}}$$

Ex.48
$$\frac{d}{dx}\overset{\text{outside}}{\sin}\underbrace{(x^2 + x)}_{\text{Inside}} = \overset{\text{derivative of the outside}}{\cos}\underbrace{(x^2 + x)}_{\text{Inside left along}}.\underbrace{(2x + 1)}_{\text{derivative of the inside}}$$

Ex.49 (a) $\frac{d}{dx}(1 - x^2)^{1/4} = \frac{1}{4}(1 - x^2)^{-3/4}(-2x)$ $u = 1 - x^2$ and $n = 1/4$

(Function defined) on [–1, 1]

$= \frac{-x}{2(1 - x^2)^{3/4}}$ (derivative defined only on (–1, 1))

(b) $\frac{d}{dx}\sin 2x = \cos 2x \frac{d}{dx} 2x = \cos 2x .2 = 2 \cos 2x$

(c) $\frac{d}{dt}(A\sin(\omega t+\phi)) = A \cos (\omega t + \phi) \frac{d}{dt}(\omega t + \phi) = A \cos (\omega t + \phi).\, \omega = A\, \omega \cos (\omega t + \phi)$

Rull No. 12 Power Chain Rule

* If $\frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx}$

Ex.50 $\frac{d}{dx}\left(\frac{1}{3x-2}\right) = \frac{d}{dx}(3x-2)^{-1} = -1\,(3x-2)^{-2}\frac{d}{dx}(3x-2)$

$= -1\,(3x-2)^{-2}\,(3) = -\frac{3}{(3x-2)^2}$

In part (d) we could also have found the derivation with the Quotient Rule.

Ex.51 (a) $\frac{d}{dx}(Ax+B)^n$

Sol. Here $u = Ax + B$, $\frac{du}{dx} = A$

$\therefore \frac{d}{dx}(Ax+B)^n = n(Ax+B)^{n-1}.A$

(b) $\frac{d}{dx}\sin(Ax+B) = \cos(Ax+B).A$ (c) $\frac{d}{dx}\log(Ax+B) = \frac{1}{Ax+B}.A$

(d) $\frac{d}{dx}\tan(Ax+B) = \sec^2(Ax+B).A$ (e) $\frac{d}{dx}e^{(Ax+B)} = e^{(Ax+B)}.A$

Note : These results are important

19. DOUBLE DIFFERENTIATION

If f is differentiable function, then its derivative f' is also a function, so f' may have a derivative of its own, denoted by (f')'=f''. This new function f'' is called the second derivative of because it is the derivative of the derivative of f. Using Leibniz notation, we write the second derivative of y = f(x) as

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$

Another notation is $f''(x) = D_2 f(x)$.

Ex.52 If f(x) = x cos x, find f" (x)

Sol. Using the Product Rule, we have $f'(x) = x\frac{d}{dx}(\cos x) + \cos x \frac{d}{dx}(x) = -x \sin x + \cos x$

To find f" (x) we differentiate f'(x) :

$f''(x) = \frac{d}{dx}(-x\sin x + \cos x) = -x \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(-x) + \frac{d}{dx}(\cos x)$

$= -x \cos x - \sin x - \sin x = -x \cos x - 2 \sin x$

20. APPLICATION OF DERIVATIVE DIFFERENTIATION AS A RATE OF CHANGE

$\frac{dy}{dx}$ is rate of change of 'y' with respect to 'x' :

For examples :

(i) $v = \frac{dx}{dt}$ this means velocity 'v' is rate of change of displacement 'x' with respect to time 't'

(ii) $a = \frac{dv}{dt}$ this means acceleration 'a' is rate of change of velocity 'v' with respect to time 't'.

(iii) $F = \frac{dp}{dt}$ this means force 'F' is rate of change of monentum 'p' with respect to time 't'.

(iv) $\tau = \frac{dL}{dt}$ this means torque 'τ' is rate of change of angular momentum 'L' with respect to time 't'

(v) Power $= \frac{dW}{dt}$ this means power 'P' is rate of change of work 'W' with respect to time 't'

Ex.53 The area A of a circle is related to its diameter by the equation $A = \frac{\pi}{4}D^2$.

How fast is the area changing with respect to the diameter when the diameter is 10 m ?

Sol. The (instantaneous) rate of change of the area with respect to the diameter is

$$\frac{dA}{dD} = \frac{\pi}{4}2D = \frac{\pi D}{2}$$

When D =10m, the area is changing at rate (π/2) = 5π m^2/m. This mean that a small change ΔD m in the diameter would result in a changed of about 5p ΔD m^2 in the area of the circle.

Physical Example :

Ex.54 Boyle's Law state that when a sample of gas is compressed at a constant temperature, the product of the pressure and the volume remains constant : PV = C. Find the rate of change of volume with respect to pressure.

Sol. $\frac{dV}{dP} = -\frac{C}{P^2}$

Ex.55 (a) Find the average rate of change of the area of a circle with respect to its radius r as r changed from

(i) 2 to 3 *(ii) 2 to 2.5* *(iii) 2 to 2.1*

(b) Find the instantaneous rate of change when r = 2.

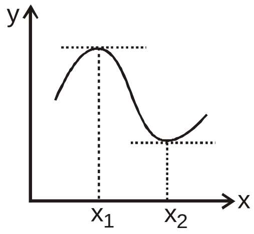
(c) Show that thre rate of change of the area of a circle with respect to its radius (at any r) is equal to the circumference of the circle. Try to explain geometrically when this is true by drawing a circle whose radius is increased by an amount Δr. How can you approximate the resulting change in area ΔA if Δr is small ?

Sol. (a) (i) 5π (ii) 4.5 π (iii) 4.1 π

(b) 4 π

(c) $\Delta A \approx 2 \pi r \Delta r$

21. MAXIMA & MINIMA

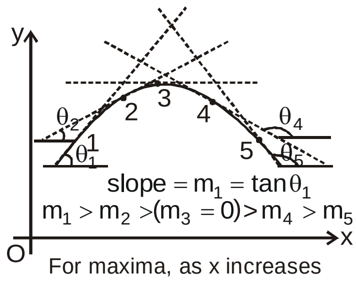


Suppose a quantity y depends on another quantity x in a manner shown in figure. It becomes maximum at x_1 and minimum at x_2. At these points the tangent to the curve is parallel to the x-axis and hence its slope is $\tan\theta = 0$. Thus, at a maxima or a minima slope

$$\Rightarrow \frac{dy}{dx} = 0$$

Maxima

Just before the maximum the slope is positive, at the maximum it is zero and just after the maximum it is negative. Thus, $\frac{dy}{dx}$ decrease at a maximum and hence the rate of change of $\frac{dy}{dx}$ is negative at a maximum i.e., $\frac{d}{dx}\left(\frac{dy}{dx}\right) < 0$ at maximum. The quantity $\frac{d}{dx}\left(\frac{dy}{dx}\right)$ is the rate of change of the slope. It is written as $\frac{d^2y}{dx^2}$. Conditions for maxima are : (a) $\frac{dy}{dx} = 0$ (b) $\frac{d^2y}{dx^2} < 0$



For maxima, as x increases the slope decreases

Minima

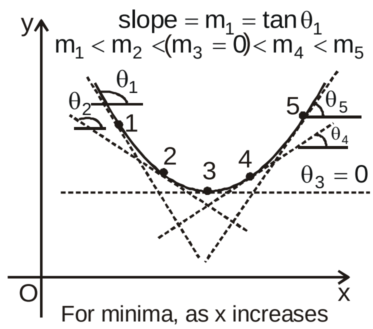
Similarly, at a minimum the slope changes from negative to positive,

Hence with the increases of x. The slope is increasing that means the rate of change of slope with respect to x is positive.

Hence $\frac{d}{dx}\left(\frac{dy}{dx}\right) > 0$

Conditions for minima are :

(a) $\frac{dy}{dx} = 0$ (b) $\frac{d^2y}{dx^2} > 0$



For minima, as x increases slope increases

Quite often it is known from the physical situation whether the quantity is a maximum or a minimum. The test on $\frac{d^2y}{dx^2}$ may then be omitted.

Ex.56 *Find maximum or minimum values of the functions :*
(A) $y = 25x^2 + 5 - 10x$ *(B) $y = 9 - (x - 3)^2$*

Sol. (A) For maximum and minimum value, we can put $\frac{dy}{dx} = 0$

or $\frac{dy}{dx} = 50x - 10 = 0 \quad \therefore \quad x = \frac{1}{5}$

Further, $\frac{d^2y}{dx^2} = 50$

or $\frac{d^2y}{dx^2}$ has positive value at x = $\frac{1}{5}$. Therefore, y has minimum value at x = $\frac{1}{5}$. Therefore, y has minimum value at x = $\frac{1}{5}$. Substituting x = $\frac{1}{5}$ in given equation, we get

$$y_{min} = 25\left(\frac{1}{5}\right)^2 + 5 - 10\left(\frac{1}{5}\right) = 4$$

(B) $y = 9 - (x-3)^2 = 9 - x^2 - 9 + 6x$

or $y = 6x - x^2$

$\therefore \quad \frac{dy}{dx} = 6 - 2x$

For minimum or maximum value of y we will substitute $\frac{dy}{dx} = 0$

or $6 - 2x = 0$

$x = 3$

To check whether value of y is maximum or minimum at x = 3 we will have to check whether $\frac{d^2y}{dx^2}$ is positive or negative.

$$\frac{d^2y}{dx^2} = -2$$

or $\frac{d^2y}{dx^2}$ is negative at x = 3. Hence, value of y is maximum. This maximum value of y is,

$$y_{max} = 9 - (3-3)^2 = 9$$

22. INTEGRATION

Definitions :

A function F(x) is a antiderivative of a function f(x) if

$$F'(x) = f(x)$$

for all x in the domain of f. The set of all antiderivatives of f is the indefinite integral of f with respect to x, denoted by

$$\int f(x)dx$$

The symbol $\int$ is an integral sign. The function f is the integrand of the integral and x is the variable of integration.

For example $f(x) = x^3$ then $f'(x) = 3x^2$

So the integral of $3x^2$ is x^3

Similarly if $f(x) = x^3 + 4$

there for general integral of $3x^2$ is $x^3 + c$ where c is a constant

One antiderivative F of a function f, the other antiderivatives of f differ from F by a constant. We indicate this in integral notation in the following way :

$$\int f(x)dx = F(x) + C \qquad .....(i)$$

The constant C is the constant of integration or arbitrary constant, Equation (1) is read, "The indefinite integral of f with respect to x is F(x) + C." When we find F(x) + C, we say that we have integrated f and evaluated the integral.

Ex.57 Evaluate $\int 2x\, dx$

Sol. $\int 2x\,dx = x^2 + C$

an antiderivative of 2x

the arbitrary constant

The formula x^2 + C generatres all the antiderivatives of the function 2x. The function $x^2 + 1$, $x^2 - \pi$, and $x^2 + \sqrt{2}$ are all antiderivatives of the function 2x, as you can check by differentiation.

Many of the indefinite integrals needed in scientific work are found by reversing derivative formulas.

Integral Formulas

	Indefinite Integral	Reversed derivated formula
1.	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$, n rational	$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$
	$\int dx = \int 1 dx = x + C$ (special case)	$\frac{d}{dx}(x) = 1$
2.	$\int \sin kx\,dx = -\frac{\cos kx}{k} + C$	$\frac{d}{dx}\left(-\frac{\cos kx}{k}\right) = \sin kx$
3.	$\int \cos kx dx = \frac{\sin kx}{k} + C$	$\frac{d}{dx}\left(\frac{\sin kx}{k}\right) = \cos kx$
4.	$\int \sec^2 x dx = \tan x + C$	$\frac{d}{dx}\tan x = \sec^2 x$
5.	$\int \mathrm{cosec}^2 x dx = -\cot x + C$	$\frac{d}{dx}(-\cot x) = \csc^2 x$
6.	$\int \sec x \tan x dx = \sec x + C$	$\frac{d}{dx}\sec x = \sec x \tan x$
7.	$\int \mathrm{cosec}\, x \cot x\, dx = -\mathrm{cosec}\, x + C$	$\frac{d}{dx}(-\csc x) = \csc x \cot x$

Ex.58 *Examples based on above formulas :*

(a) $\int dx = x + c$

(b) $\int x^5 dx = \frac{x^6}{6} + C$ Formula 1 with n = 5

(c) $\int \frac{1}{\sqrt{x}} dx = \int x^{-1/2} dx = 2x^{1/2} + C = 2\sqrt{x} + C$ Formula 1 with $n = -\frac{1}{2}$

(d) $\int \sin 2x\,dx = \frac{-\cos 2x}{2} + C$ Formula 2 with k = 2

(e) $\int \cos\frac{x}{2} dx = \int \cos\frac{1}{2} x dx = \frac{\sin(1/2)x}{1/2} + C = \int 2\sin\frac{x}{2} + C$ Formula 3 with $k = \frac{1}{2}$

Ex.59 Right : $\int x\cos x\,dx$ = x sin x + cos x + C

Reason : The derivative of the right-hand side is the integrand :

Check : $\frac{d}{dx}(x\sin x+\cos x+C)$ = x cos x + sin x – sin x + 0 = x cos x.

Wrong : $\int x\cos x\,dx$ = x sin x + C

Reason : The derivative of the right-hand side is not the integrand :

Check : $\frac{d}{dx}(x\sin x+C)$ = x cos x + sin x + 0 $\neq$ x cos x

Rule No. 1 Constant Multiple Rule

- A function is an antiderivative of a constant multiple k of a function f if and only if it is k times an antiderivative of f.

$$\int kf(x)dx = k\int f(x)dx$$

Ex.60 $\int 5x^n dx = 5\int x^n dx = \frac{5(x)^{n+1}}{n+1}+c$

Rule No.2 Sum And Difference Rule

A function is an antiderivative of a sum or difference f ± g if and only if it is the sum or difference of an antiderivative of f an antiderivative of g.

$$\int [f(x)\pm g(x)dx] = \int f(x)dx \pm \int g(x)dx$$

Ex.61 Term-by-term integration

Evaluate : $\int (x^2-2x+5)dx$

Sol. If we recognize that $(x^3/3) - x^2 + 5x$ is an antiderivative of $x^2 - 2x + 5$, we can evaluate the integral as

$$\int (x^2-2x+5)\,dx = \overbrace{\frac{x^3}{3}-x^2+5x}^{\text{antiderivative}} + \underset{\text{arbitrary constant}}{C}$$

If we do not recognize the antiderivative right away, we can generate it term by term with the sum and difference Rule :

$$\int (x^2-2x+5)\,dx = \int x^2dx - \int 2xdx + \int 5dx$$

$$= \frac{x^3}{3}+C_1-x^2+C_2+5x+C_3$$

This formula is more complicated than it needs to be. If we combine C_1, C_2 and C_3 into a single constant $C = C_1 + C_2 + C_3$, the formula simplifies to

$$\frac{x^3}{3}-x^2+5x+C$$

and still gives all the antiderivatives there are. For this reason we recommend that you go right to the final form even if you elect to integrate term by term. Write

$$\int (x^2 - 2x + 5)dx = \int x^2 dx - \int 2x dx + \int 5 dx = \frac{x^3}{3} - x^2 + 5x + C$$

Find the simplest antiderivative you can for each part add the constant at the end.

Ex.62 *We can sometimes use trigonometric identities to transform integrals we do not know how to evaluate into integrals. The inetgral formulas for $\sin^2 x$ and $\cos^2 x$ arise frequently in applications.*

(a) $\int \sin^2 x\,dx \quad = \int \frac{1-\cos 2x}{2}dx \qquad \sin^2 x = \frac{1-\cos 2x}{2}$

$$= \frac{1}{2}\int (1-\cos 2x)dx = \frac{1}{2}\int dx - \frac{1}{2}\int \cos 2x\,dx$$

$$\frac{x}{2} + \left(-\frac{1}{2}\right)\frac{\sin 2x}{2} + C = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

(b) $\int \cos^2 x\,dx \quad = \int \frac{1+\cos 2x}{2}dx \qquad \cos^2 x = \frac{1+\cos 2x}{2}$

$= \frac{x}{2} + \frac{\sin 2x}{4} + C$ As in part (a), but with a sign change

23. SOME INDEFINITE INTEGRALS (AN ARBITRARY CONSTANT SHOULD BE ADDED TO EACH OF THESE INTEGRALS.

(a) $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)}$ (provided $n \neq -1$) + C

(b) $\int \frac{1}{x}dn = \ln x + C$

(c) $\int \frac{dx}{a+bx} = \frac{1}{b}\ln(a+bx) + C$

(d) $\int e^{ax}dx = \frac{1}{a}e^{ax} + C$

(e) $\int \sin(ax+b) = \frac{-1}{a}\cos(ax+b) + C$

(f) $\int \cos(ax+b) = \frac{1}{a}\sin(ax+b) + C$

Ex.63 (a) $\int (3x+2)^3 dx = \frac{(3x+2)^4}{4\times 3} + C = \frac{(3x+2)^4}{12} + C$

(b) $\int \frac{2dx}{x} = 2\ln x + C$

(c) $\int \frac{dx}{5+2x} = \frac{1}{2}\ln(5+2x) + C$

(d) $\int \frac{dx}{3-5x} = -\frac{1}{5}\ln(3-5x) + C$

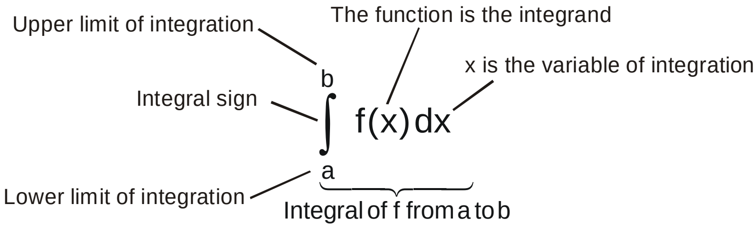
(e) $\int e^{3x}dx = \frac{1}{3}e^{3x} + C$

(f) $\int e^{-x/2}dx = -2e^{-x/2} + C$

(g) $\int \sin(3x+5)dx = -\frac{1}{3}\cos(3x+5) + C$

(h) $\int \cos(2x-5)dx = \frac{1}{2}\sin(2x-5) + C$

24. DEFINITE INTEGRATION OR INTEGRTION WITH LIMITS



$$\int_a^b f(x)\,dx = [g(x)]_a^b = g(b) - g(a)$$

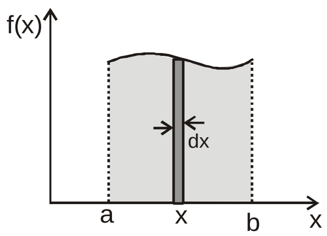
Ex.64 $\int_{-1}^{4} 3dx = 3\int_{-1}^{4} dx = 3[x]_{-1}^{4}$ = 3 [4 – (–1)] = (3) (5) = 15

$\int_{0}^{\pi/2} \sin x\,dx = [-\cos x]_0^{\pi/2}$ = $-\cos\left(\frac{\pi}{2}\right)$ + cos (0) = – 0 + 1 = 1

Ex.65 (1) $\int_0^a x^2 dx = \left[\frac{x^3}{3}\right]_0^a = \frac{a^3}{3}$ (2) $\int_3^5 x dx = \left[\frac{x^2}{2}\right]_3^5 = \frac{5^2 - 3^2}{2} = 8$ (3) $\int_0^b x^{3/2} dx = \left[\frac{x^{5/2}}{5/2}\right]_0^b = \frac{2}{5} b^{5/2}$

25. APPLICATION OF DEFINITE INTERGRAL

Calculation Of Area Of A Curve.



From graph shown in figure if we divide whole area in infinitely small strips of dx width.

We take a strip at x position of dx width.

Small area of this strip dA = f(x) dx

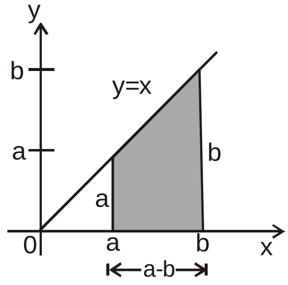
So, the total area between the curve and x-axis = sum of area of all strips = $\int_a^b f(x)dx$

Let f(x) ≥ 0 be continuous on [a,b]. The area of the region between the graph of f and the x-axis is

$$A = \int_a^b f(x)dx$$

Ex.66 *Using an area to evaluate a definite integral*

Evaluate $\int_a^b x\,dx$ *0 < a < b.*



The region in Example

Sol. We sketch the region under the curve y = x, $a \le x \le b$ (figure) and see that it is a trapezoid with height (b – a) and bases a and b. The value of the integral is the area of this trapezoid :

Thus $\int_a^b x\,dx = (b-a) \bullet \frac{a+b}{2} = \frac{b^2}{2} - \frac{a^2}{2}$

$$\int_1^{\sqrt{5}} x\,dx = \frac{(\sqrt{5})^2}{2} - \frac{(1)^2}{2} = 2$$

and so on.

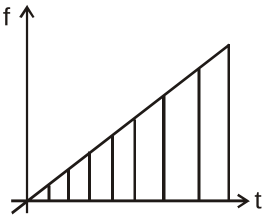
Notice that $x^2/2$ is an antiderivative of x, further evidence of a connection between antiderivatives and summation.

(i) To find impulse

$$dF = \frac{dp}{dt} \text{ so imples } = \int F.dt$$

Ex.67 If F = kt then find impulse at t = 3 sec.

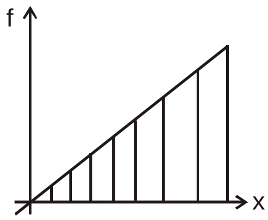
so impulse will be area under f - t curve



$$I = \int_0^3 kt\,dt = K\left[\frac{t^2}{2}\right]_0^3$$

$$\Rightarrow \quad I = \frac{9k}{2}$$

2. To calculate work done by force :



$$w = \int \vec{f}.d\vec{x}$$

So area under f - x curve will give the value of work done.

EXERCISE - I

SECTION - A : FUNCTION

1. $f(x) = \cos x + \sin x$ Find $f(\pi/2)$
2. $f(x) = 4x + 3$ Find $f(f(2))$
3. $f(x) = \log x^3$ and $g(x) = \log x$

Which of the following statement is / are true-

(a) $f(x) = g(x)$ (b) $3f(x) = g(x)$

(c) $f(x) = 3g(x)$ (d) $f(x) = (g(x))^3$

SECTION - B : DIFFERENTIATION OF ELEMENTRY FUNCTIONS

Find the derivative of given function w.r.t. corresponding independent variable.

4. $y = x^2 + x + 8$
5. $s = 5t^3 - 3t^5$
6. $y = 5\sin x$
7. $y = x^2 + \sin x$
8. $y = \tan x + \cot x$

Find the first derivative & second derivative of given functions w.r.t. corresponding independent variable.

9. $y = 6x^2 - 10x - 5x^{-2}$
10. $r = \frac{12}{\theta} - \frac{4}{\theta^3} + \frac{1}{\theta^4}$
11. $\omega = 3z^7 - 7z^3 + 21z^2$
12. $y = \sin x + \cos x$
13. $y = \ell nx + e^x$

SECTION - C : DIFFERENTIATION BY PRODUCT RULE

Find derivative of given functions w.r.t. the independent variable x.

14. $x\sin x$
15. $y = e^x \ell nx$
16. $y = (x-1)(x^2 + x + 1)$
17. $y = (x^2+1)\left(x + 5 + \frac{1}{x}\right)$
18. $y = \sin x \cos x$

SECTION - D : DIFFERENTIATION BY QUOTIENT RULE

Find derivative of given function w.r.t. the independent variable.

19. $y = \frac{\sin x}{\cos x}$
20. $y = \frac{2x+5}{3x-2}$
21. $y = \frac{\ell nx}{x}$
22. $f(t) = \frac{t^2 - 1}{t^2 + t - 2}$, find $f'(t)$
23. $z = \frac{2x+1}{x^2 - 1}$
24. $y = x^2 \cot x$

SECTION - E : DIFFERENTIATION BY CHAIN RULE

Find $\frac{dy}{dx}$ as a function of x

25. $y = (2x+1)^5$
26. $y = (4-3x)^9$
27. $y = \left(1 - \frac{x}{7}\right)^{-7}$
28. $y = \left(\frac{x}{2} - 1\right)^{-10}$
29. $y = \sin 5x$
30. $y = \sin(x) + \ln(x^2) + e^{2x}$
31. $y = 2\sin(\omega x + \phi)$ where ω and ϕ constants

SECTION - G : DIFFERENTIATION AS A RATE MEASUREMENT

32. Suppose that the radius r and area A = πr^2 of a circle are differentiable functions of t. Write an equation that relates dA / dt to dr / dt.
33. Suppose that the radius r and surface area S = $4\pi r^2$ of a sphere are differentiable functions of t. Write an equation that relates $\frac{ds}{dt}$ to $\frac{dr}{dt}$.

SECTION - H : MAXIMA & MINIMA

34. Particle's position as a function of time is given by $x = -t^2 + 4t + 4$ find the maximum value of position coordinate of particle.
35. Find the maximum and minimum values of function $2x^3 - 15x^2 + 36x + 11$

SECTION - I

Given y = f(u) and u = g(x) Find $\frac{dy}{dx}$

36. $y = 2u^3, u = 8x - 1$

37. $y = \sin u,\ \ u = 3x + 1$

38. $y = 6u - 9,\ u = (1/2)x^4$

39. $y = \cos u,\ \ u = -\frac{x}{3}$

PART - II : INTEGRATION

Find integrals of given functions

1. (a) $2x$
(b) x^2
(c) $x^2 - 2x + 1$
2. (a) $-3x^{-4}$
(b) x^{-4}
(c) $x^{-4} + 2x + 3$
3. (a) $\frac{1}{x^2}$
(b) $\frac{5}{x^2}$
(c) $2 - \frac{5}{x^2}$
4. (a) $\frac{3}{2}\sqrt{x}$
(b) $\frac{3}{2\sqrt{x}}$
(c) $\sqrt{x} + \frac{1}{\sqrt{x}}$
5. (a) $\frac{4}{3}\sqrt[3]{x}$
(b) $\frac{1}{3\sqrt[3]{x}}$
(c) $\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}$
6. (a) $\frac{1}{2}x^{-1/2}$
(b) $-\frac{1}{2}x^{-3/2}$
(c) $-\frac{3}{2}x^{-5/2}$
7. $(1 - x^2 - 3x^5)$
8. $3\sin x$
9. $\frac{1}{3x}$

Integrate by using the substitution suggested in bracket.

10. $\int \sin 3x\, dx$, (use, u = 3x)

11. $\int \sec 2t \tan 2t\, dt$, (use, u = 2t)

12. $\int_{-2}^{1} 5\, dx$

13. $\int_{-4}^{-1} \frac{\pi}{2} d\theta$

14. $\int_{-2}^{4} \left(\frac{x}{2} + 3\right) dx$

15. $\int_{\sqrt{2}}^{5\sqrt{2}} r\, dr$

16. $\int_{0}^{2\pi} \sin\theta\, d\theta$

17. $\int_{0}^{1} e^x\, dx$

Use a definite integral to find the area of the region between the given curve and the x-axis on the interval [0, b]

18. $y = 2x$

19. $y = \frac{x}{2} + 1$

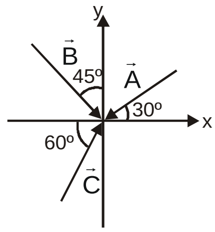
Use a definite integral to find the area of the region between the given curve and the x-axis on the interval $[0, \pi]$

20. $y = \sin x$

PART - III VECTOR

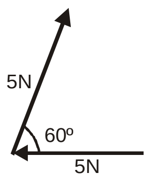
SECTION - A : DEFINITION OF VECTOR & ANGLE BETWEEN VECTORS

1. Vectors $\vec{A}, \vec{B}$ and $\vec{C}$ are shown in figure. Find angle between



(i) $\vec{A}$ and $\vec{B}$ (ii) $\vec{A}$ and $\vec{C}$ (iii) $\vec{B}$ and $\vec{C}$.

2. The forces, each numerically equal to 5 N, are acting as shown in the Figure. Find the angle between forces?



3. Rain is falling vertically down wards with a speed 5 m/s. If unit vector along upward is defined as $\hat{j}$, represent velocity of rain in vector form.

4. The vector joining the points A(1, 1, –1) and B(2, –3, 4) & pointing from A to B is

(a) $-\hat{i}+4\hat{j}-5\hat{k}$ (b) $\hat{i}+4\hat{j}+5\hat{k}$

(c) $\hat{i}-4\hat{j}+5\hat{k}$ (d) $-\hat{i}-4\hat{j}-5\hat{k}$

SECTION - B : ADDITION OF VECTORS

5. A man walks 40 m North, then 30 m East and then 40 m South. Find the displacement from the starting point?

6. Two forces $\vec{F}_1$ and $\vec{F}_2$ are acting at right angles to each other, find their resultant ?

7. A vector of magnitude 30 and direction eastwards is added with another vector of magnitude 40 and direction Northwards. Find the magnitude and direction of resultant with the east.

8. Two force of $\vec{F}_1 = 500N$ due east and $\vec{F}_2 = 250N$ due north. Find $\vec{F}_2 - \vec{F}_1$?

9. Two vectors $\vec{a}$ and $\vec{b}$ inclined at an angle θ w.r.t. each other have a resultant $\vec{c}$ which makes an angle β with $\vec{a}$. If the directions of $\vec{a}$ and $\vec{b}$ are interchanged, then the resultant will have the same

(A) magnitude

(B) direction

(C) magnitude as well as direction

(D) neither magnitude nor direction.

10. Two vectors $\vec{A}$ and $\vec{B}$ lie in a plane. Another vector $\vec{C}$ lies outside this plane. The resultant $\vec{A}+\vec{B}+\vec{C}$ of these three vectors

(A) can be zero

(B) cannot be zero

(C) lies in the plane of $\vec{A}+\vec{B}$

(D) lies in the plane of $\vec{A}-\vec{B}$

11. The vector sum of the forces of 10 N and 6 N can be

(A) 2N (B) 8N (C) 18N (D) 20N

12. A set of vectors taken in a given order gives a closed polygon. Then the resultant of these vectors is a

(A) scalar quantity (B) pseudo vector

(C) unit vector (D) null vector

13. The vector sum of two force P and Q is minimum when the angle θ between their positive directions, is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) π

14. The vector sum of two vectors $\vec{A}$ and $\vec{B}$ is maximum, then the angle θ between two vectors is

(A) 0° (B) 30° (C) 45° (D) 60°

15. Given : $\vec{C} = \vec{A} + \vec{B}$. Also, the magnitude of $\vec{A}, \vec{B}$ and $\vec{C}$ are 12, 5 and 13 units respectively. The angle between $\vec{A}$ and $\vec{B}$ is

(A) 0° (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

16. If $\vec{P}+\vec{Q}=\vec{P}-\vec{Q}$ and θ is the angle between $\vec{P}$ and $\vec{Q}$, then

(A) θ = 0° (B) θ = 90° (C) P = 0 (D) Q = 0

17. The sum and difference of two perpendicular vectors of equal lengths are

(A) of equal lengths and have an acute angle between them

(B) of equal lengths and have an obtuse angle between them

(C) also perpendicular to each other and are of different lengths

(D) also perpendicular to each other and are of equal lengths

SECTION - C : RESOLUTION OF VECTORS

18. Find the magnitude of $3\hat{i}+2\hat{j}+\hat{k}$?

19. If $\vec{A} = 3\hat{i}+4\hat{j}$ then find $\hat{A}$

20. What are the x and the y components of a 25 m

displacement at an angle of 210º with the x-axis (clockwise) ?

21. One of the rectangular components of a velocity of 60 km h^{-1} is 30 km h^{-1}. Find other rectangular component ?

22. If $0.5\hat{i}+0.8\hat{j}+C\hat{k}$ is a unit vector. Find the value of C

23. The rectangular components of a vector are (2, 2). The corresponding rectangular components of another vector are $(1, \sqrt{3})$. Find the angle between the two vectors.

24. The x and y components of a force are 2N and –3N. The force is

(A) $2\hat{i}-3\hat{j}$ (B) $2\hat{i}+3\hat{j}$ (C) $-2\hat{i}-3\hat{j}$ (D) $3\hat{i}+2\hat{j}$

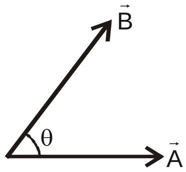
SECTION-D : PRODUCT OF VECTORS

25. If $\vec{A}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{B}=2\hat{i}+\hat{j}$ find

(a) $\vec{A}.\vec{B}$ (b) $\vec{A}\times\vec{B}$

26. If $|\vec{A}|=4$, $|\vec{B}|=3$ and θ = 60º in the figure. Find

(a) $\vec{A}.\vec{B}$ (b) $|\vec{A}\times\vec{B}|$



27. Three non-zero vectors $\vec{A},\vec{B}$ & $\vec{C}$ satisfy the relation $\vec{A}.\vec{B}=0$ & $\vec{A}.\vec{C}=0$. Then $\vec{A}$ can be parallel to:

(A) $\vec{B}$ (B) $\vec{C}$ (C) $\vec{B}.\vec{C}$ (D) $\vec{B}\times\vec{C}$

28. The magnitude of scalar product of two vectors is 8 and that of vector product is $8\sqrt{3}$. The angle between them is

(A) 30º (B) 60º (C) 120º (D) 150º

EXERCISE - II

SECTION - A : FUNCTION

1. If $f(x)=\frac{x-1}{x+1}$ then find $f\{f(x)\}$

2. If $f(x)=\begin{cases} x+2, & x<2 \\ 2x-1, & x\ge 2 \end{cases}$ Evalute f(2), f(1), and f(3)

SECTION - B : DIFFERENTIATION OF ELEMENTRY FUNCTIONS

Find the first derivative and second derivative of given functions w.r.t. the independent variable x.

3. $y=\ell nx^2+\sin x$

4. $y=\sqrt[7]{x}+\tan x$

SECTION - C : DIFFERENTIATION BY PRODUCT RULE

Find derivative of given functions w.r.t. the corresponding independent variable.

5. $y=e^x\tan x$

6. $y=x^2\sin^4 x+x\cos^{-2} x$

7. $y=\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}+1\right)$

8. $y=x^2\sin x+2x\cos x-2\sin x$

9. $y=x^2\cos x-2x\sin x-2\cos x$

10. $r=(1+\sec\theta)\sin\theta$

SECTION - D : DIFFERENTIATION BY QUOTIENT RULE

Find derivative of given functions w.r.t. the respective independent variable.

11. $y=\frac{\sin x+\cos x}{\cos x}$

12. $y=\frac{\cot x}{1+\cot x}$

13. $y=\frac{\cos x}{x}+\frac{x}{\cos x}$

14. $p=\frac{\tan q}{1+\tan q}$

SECTION - E : DIFFERENTIATION BY CHAIN RULE

Find $\frac{dy}{dx}$ as a function of x

15. $y=\sin^3 x+\sin 3x$

16. $\sin^2(x^2+1)$

17. $y=x(x^2+1)^{-1/2}$

18. $q=\sqrt{2r-r^2}$, find $\frac{dq}{dr}$

19. $y=\left(\frac{x^2}{8}+x-\frac{1}{x}\right)^4$

SECTION - F : DIFFERENTIATION AS A RATE MEASUREMENT

20. The radius r and height h of a circular cylinder are related to the cylinder's volume V by the formula $V=\pi r^2 h$.

(a) If height is increasing at a rate of 5 m/s while radius is constant, Find rate of increase of volume of cylinder.

(b) If radius is increasing at a rate of 5 m/s while height is constant, Find rate of increase of volume of cylinder.

(c) If height is increasing at a rate of 5 m/s and radius is increasing at a rate of 5 m/s. Find rate of increase of volume of cylinder.

SECTION - G : MAXIMA & MINIMA

21. Find two positive numbers x & y such that x + y = 60 and xy is maximum.

22. A sheet of area 40 m^2 in used to make an open tank with a square base, then find the dimensions of the base such that volume of this tank is maximum.

SECTION - H : MISCELLANEOUS

23. Find y'' if

(a) $y=\cos x$

(b) $y=\sec x$

24. $y=\cos u,\ u=\sin x$

25. $y=\sin u,\ u=x-\cos x$

PART - II : INTEGRATION

Find integrals of given functions

1. $\int(2x^3-5x+7)dx$

2. $\int\left(\frac{1}{5}-\frac{2}{x^3}+2x\right)dx$

3. $\int\left(\sqrt{x}+\sqrt[3]{x}\right)dx$

4. $\int x^{-3}(x+1)\,dx$

5. $\int\frac{t\sqrt{t}+\sqrt{t}}{t^2}\,dt$

6. $\int\frac{4+\sqrt{t}}{t^3}\,dt$

7. $\int\cos\theta(\tan\theta+\sec\theta)\,d\theta$

8. $\int_{\pi}^{2\pi}\theta\,d\theta$

9. $\int_{0}^{\sqrt[3]{7}}x^2\,dx$

10. $\int_{0}^{\pi}\cos x\,dx$

11. $\int_{0}^{1}\frac{dx}{3x+2}$

Use a definite integral to find the area of the region between the given curve and the x-axis on the interval [0, b]

12. $y = 3x^2$

13. $y=\sqrt{b^2-x^2}$

PART - III : VECTOR

SECTION - A : DEFINITION OF VECTOR & ANGLE BETWEEN VECTORS

1. Vector $\vec{A}$ points N–E and its magnitude is 3 kg ms^{-1} it is multiplied by the scalar λ such that λ = –4 second. Find the direction and magnitude of the new vector quantity. Does it represent the same physical quantity or not ?

2. A hall has the dimensions 10 m × 12 m × 14 m. A fly starting at one corner ends up at a diametrically opposite corner. The magnitude of its displacement is nearly

(A) 16 m (B) 17 m

(C) 18 m (D) 21 m

3. A vector is not changed if

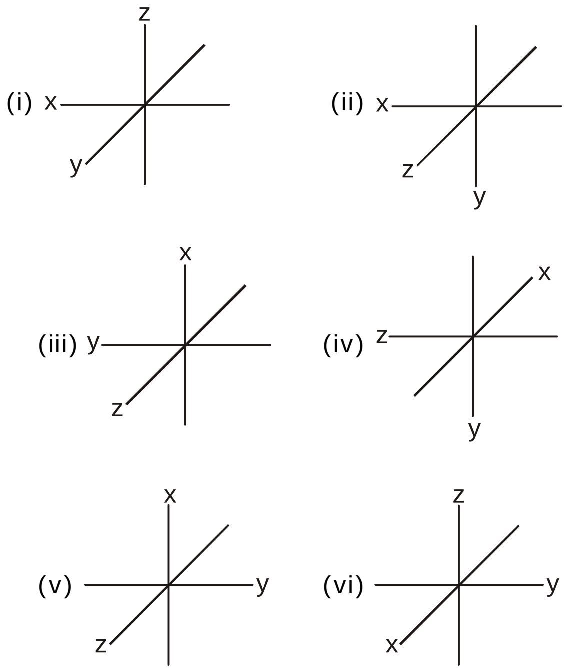
(A) it is displaced parallel to itself

(B) it is rotated through an arbitrary angle

(C) it is cross-multiplied by a unit vector

(D) it is multiplied by an arbitrary scalar

4. Which of the arrangement of axes in fig. can be labelled "right handed coordinate system" ? As usual, each axis lable indicates the positive side of the axis.



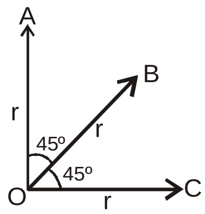
(A) (i), (ii) (B) (iii) (iv)

(C) (vi) (D) (v)

SECTION : B ADDITION OF VECTOR

5. The angle θ between directions of forces $\vec{A}$ and $\vec{B}$ is 90º where A = 8 dyne and B = 6 dyne. If the resultant $\vec{R}$ makes an angle α with $\vec{A}$ then find the value of 'α' ?

6. Find the resultant of three vectors $\vec{OA},\vec{OB}$ and $\vec{OC}$ each of magnitude r as shown in figure ?



7. If the angle between two forces increases, the magnitude of their resultant

(A) decreases

(B) increases

(C) remains unchanged

(D) first decreases and then increases

8. A car is moving on a straight road due north with a uniform speed of 50 km h^{-1} when it turns left through 90°. If the speed reamins unchanged after turning, the change in the velcoity of the car in the turning process is :

(A) zero

(B) $50\sqrt{2}kmh^{-1}S-W$ direction

(C) $50\sqrt{2}kmh^{-1}N-W$ direction

(D) $50kmh^{-1}$ due West

9. Which of the following sets of displacements might be capable of bringing a car to its returning point ?
(A) 5, 10, 30 and 50 km
(B) 5, 9, 9 and 16 km
(C) 40, 40, 90 and 200 km
(D) 10, 20, 40 and 90 km

10. When two vector $\vec{a}$ and $\vec{b}$ are added, the magnitude of the resultant vector is always
(A) greater than (a + b)
(B) less than or equal to (a + b)
(C) less than (a + b)
(D) equal to (a + b)

11. Given : $\vec{A}=2\hat{i}+3\hat{j}$ and $\vec{B}=5\hat{i}-6\hat{j}$. The magnitude of $\vec{A}+\vec{B}$ is
(A) 4 units (B) 10 units
(C) $\sqrt{58}$ units (D) $\sqrt{61}$ units

12. Given : $\vec{A}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{B}=-\hat{i}-\hat{j}+\hat{k}$. The unit vector of $\vec{A}-\vec{B}$ is

(A) $\frac{3\hat{i}+\hat{k}}{\sqrt{10}}$ (B) $\frac{3\hat{i}}{\sqrt{10}}$ (C) $\frac{\hat{k}}{\sqrt{10}}$ (D) $\frac{-3\hat{i}-\hat{k}}{\sqrt{10}}$

13. If $|\vec{A}+\vec{B}|=|\vec{A}|=|\vec{B}|$, then the angle between $\vec{A}$ and $\vec{B}$ is
(A) 0 (B) 60° (C) 90° (D) 120°

14. Given : $\vec{a}+\vec{b}+\vec{c}=0$. Cut of the three vectors $\vec{a},\vec{b}$ a nd $\vec{c}$ two are equal in magnitude. The magnitude of the third vector is $\sqrt{2}$ times that of either of the two having equal magnitude. The angles between the vectors are
(A) 90°, 135°, 135° (B) 30°, 60°, 90°
(C) 45°, 45°, 90° (D) 45°, 60°, 90°

15. Which of the following is a true statement ?
(A) A vector cannot be divided by another vector
(B) Angular displacement can either be a scalar or a vector
(C) Since addition of vectors is commutative therefore vector subtraction is also commutative
(D) The resultant of two equal forces of magnitude F acting at a point is F if the angle between the two forces is 120°.

SECTION - C : RESOLUTION OF VECTORS

16. If $\vec{A}=3\hat{i}+4\hat{j}$ and $\vec{B}=\hat{i}+\hat{j}+2\hat{k}$ then find out unit vector along $\vec{A}+\vec{B}$.

17. Vector $\vec{A}$ is of length 2 cm and is 60° above the x-axis in the first quadrant. Vector $\vec{B}$ is of length 2 cm and 60° below the x-axis in the fourth quadrant. The sum $\vec{A}+\vec{B}$ is a vector of magnitude.
(A) 2 along + y axis (B) 2 along + x-axis
(C) 1 along – x-axis (D) 2 along – x-axis

18. Six forces, 9.81 N each, acting at a point are coplanar. If the angles between neighbouring forces are equal, then the resultant is
(A) 0 N (B) 9.81 N
(C) 2 × 9.81 N (D) 3 × 9.81 N

SECTION : D PRODUCT OF VECTORS

19. If $\vec{a}=x_1\hat{i}+y_1\hat{j}$ & $\vec{b}=x_2\hat{i}+y_2\hat{j}$. The condition that would make $\vec{a}$ & $\vec{b}$ parallel to each other is ______.

20. A vector $\vec{A}$ points vertically downward & $\vec{B}$ points towards east, then the vector product $\vec{A}\times\vec{B}$ is
(A) along west (B) along east
(C) zero (D) none of above

EXERCISE - III

LEVEL - I

1. Match the statements given in column-I with statements given in column-II

Column - I	Column - II
(A) If $|\vec{A}|=|\vec{B}|$ and $|\vec{A}+\vec{B}|=|\vec{A}|$ then angle between $\vec{A}$ and $\vec{B}$ is	(p) 90°
(B) Magnitude of resultant of two forces $|\vec{F}_1|=8N$ and $|\vec{F}_2|=4N$ may be	(q) 120°
(C) Angle between $\vec{A}=2\hat{i}+2\hat{j}$ & $\vec{B}=3\hat{k}$ is	(r) 12 N
(D) Magnitude of resultant of vectors $\vec{A}=2\hat{i}+\hat{j}$ & $\vec{B}=3\hat{k}$ is	(s) $\sqrt{14}$

2. Position of particle is given by S = $t^3 - 2t^2 + 5t + 4$

(a) Find the position of particle at t = 1 sec

(b) Find the first derivative of S at t = 1 sec

(c) Find the second derivative of S t = 1 sec

3. Two forces $\vec{F}_1 = 2\hat{i}+2\hat{j}N$ amd $\vec{F}_2 = 3\hat{i}+4\hat{k}N$ are acting on a particle

(a) Find the resultant force acting on particle

(b) Find the angle between $\vec{F}_1 \& \vec{F}_2$

(c) Find the componant of force $\vec{F}_1$ along force $\vec{F}_2$

4. Statement-1 : A vector is a quantity that has both magnitude and direction and obeys the triangle law of addition.

Statement-2 : The magnitude of the resultant vector of two given vectors can never be less than the magnitude on any the given vector.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

5. Statement-1 : If the rectangular components of a force are 8 N and 6 N, then the magnitude of the force is 10 N.

Statement-2 : If $|\vec{A}|=|\vec{B}|=1$ then $|\vec{A}\times\vec{B}|^2+|\vec{A}.\vec{B}|^2=1$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Stateent-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

6. Statement-1 : If three vectors $\vec{A},\vec{B}$ and $\vec{C}$ satisfy the relation $\vec{A}.\vec{B}=0$ & $\vec{A}.\vec{C}=0$ then the vector $\vec{A}$ is parallel to $\vec{B}\times\vec{C}$.

Statement-2 : $\vec{A}\perp\vec{B}$ and $\vec{A}\perp\vec{C}$ hence A is perpendicular to plane formed by $\vec{B}$ and $\vec{C}$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

7. Statement-1 : The minimum number of vectors of unequal magnitude required to produce zero resultant is three.

Statement-2 : Three vectors of unequal magnitude which can be represeted by the three sides of a triangle taken in order, produce zero resultant.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

8. Statement-1 : The angle between the two vectors $(\hat{i}+\hat{j})$ and $(\hat{k})$ is $\frac{\pi}{2}$ radian.

Statement-2 : Angle between two vectors $\vec{A}$ and $\vec{B}$

is given by $\theta = \cos^{-1}\left(\frac{\vec{A}.\vec{B}}{AB}\right)$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

9. Statement-1 : Distance is scalar quantity.

Statement-2 : Distance is the length of path transversed.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

10. State true or false

(i) If $\vec{A}$ & $\vec{B}$ are two force vectors $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

(ii) If $\vec{A}$ & $\vec{B}$ are two force vectors then $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$

(iii) If the vector product of two non-zero vectors vanishes, the vectors are collinear.

(iv) If a function has maximum value at point P theh slope of tangent drawn on function at point P is zero.

11. Fill in the blanks

(i) The scalar product of vector $\vec{A} = 2\hat{i} + 5\hat{k}$ and $\vec{B} = 3\hat{j} + 5\hat{k}$ is ....

(ii) If $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = 7\hat{i} + 24\hat{j}$, then the vector having the same magnitude as $\vec{B}$ and parallel to $\vec{A}$ is ....

(iii) If $\vec{A} \| \vec{B}$ then $\vec{A} \times \vec{B} =$ ....

(iv) The magnitude of area of the parallelogram formed by the adjacent sides of vectors $\vec{A} = 3\hat{i} + 2\hat{j}$ a nd $\vec{B} = 2\hat{i} - 2\hat{k}$ is ....

(v) A force is represented by $2\hat{i} + 3\hat{j} + 6\hat{k}$. The magnitude of the force is ....

(vi) The unit vector along vector $\hat{i} + \hat{j} + \hat{k}$ is ....

(vii) If $\vec{A}$ is .... to $\vec{B}$, then $\vec{A}.\vec{B} = 0$

(viii) The vector $\vec{A} = \hat{i} + \hat{j}$, where $\hat{i}$ and $\hat{j}$ are unit vectors along x-axis and y-axis respectively, makes an angle of .... degree with x-axis.

(ix) If $\vec{A} + \vec{B} + \vec{C} = \vec{0}$, then $\vec{A}.(\vec{B} \times \vec{C}) =$ ....

LEVEL - II

1. If the resultant of two forces of magnitudes P and Q acting at a point at an angle of 60º is $\sqrt{7}\,Q$, then P/Q is

(A) 1 (B) 3/2 (C) 2 (D) 4

2. The resultant of two forces F_1 and F_2 is P. If F_2 is reversed, then resultant is Q. Then the value of $(P^2 + Q^2)$ in terms of F_1 and F_2 is

(A) $2(F_1^2 + F_2^2)$ (B) $F_1^2 + F_2^2$
(C) $(F_1 + F_2)^2$ (D) none of these

3. A man moves towards 3m north then 4m towards east and finally 5m towards 37º south of west. His displacement from origin is

(A) $5\sqrt{2}$ m (B) 0 m (C) 1 m (D) 12 m

4. Three forces P, Q & R are acting at a point in the plane. The angle between P & Q and Q & R are 150º & 120º respectively, then for equilibrium, forces P, Q & R are in the ratio

(A) 1 : 2 : 3 (B) 1 : 2 : $\sqrt{3}$ (C) 3 : 2 : 1 (D) $\sqrt{3}$: 2 : 1

5. A man rows a boat with a speed of 18 km/hr in northwest direction. The shoreline makes an angle of 15º south of west. Obtain the component of the velocity of the boat along the shoreline.

(A) 9 km/hr (B) $18\frac{\sqrt{3}}{2}$ km / hr
(C) 18 cos 15º km/hr (D) 18 cos 75º km/hr

6. A brid moves from point (1, – 2, 3) to (4, 2, 3). If the speed of the bird is 10m/sec, then the velocity vector of the bird is

(A) $5(\hat{i} - 2\hat{j} + 3\hat{k})$ (B) $5(4\hat{i} + 2\hat{j} + 3\hat{k})$
(C) $0.6\,\hat{i} + 0.8\,\hat{j}$ (D) $6\,\hat{i} + 8\,\hat{j}$

7. The resultant of two forces, one double the other in magnitude is perpendicular to the smaller of the two forces. The angle between the two forces is

(A) 150º (B) 90º (C) 60º (D) 120º

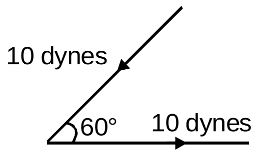
8. If the angle between the unit vectors $\hat{a}$ and $\hat{b}$ is 60º, then $|\hat{a} - \hat{b}|$ is

(A) 0 (B) 1 (C) 2 (D) 4

9. For a particle moving in a straight line, the position of the particle at time (t) is given by $x = t^3 - 6t^2 + 3t + 7$ what is the velocity of the particle when it's acceleration is zero ?

(A) – 9ms^{-1} (B) –12ms^{-1} (C) 3ms^{-1} (D) 42ms^{-1}

10. Two forces each numerically equal to 10 dynes are acting as shown in the following figure, then their resultant is -



(A) 10 dynes (B) 20 dynes
(C) $10\sqrt{3}$ dynes (D) 5 dynes

11. Two vectors $\vec{A}$ and $\vec{B}$ are such that $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$. The angle between the vectors $\vec{A}$ and $\vec{B}$ is -

(A) 0 (B) $\pi/3$ (C) $\pi/2$ (D) π

12. A particle moves through angular displacement θ on a circular path of radius r. The linear displacement will be -

(A) 2r sin (θ/2) (B) 2r cos (θ/2)
(C) 2r tan (θ/2) (D) 2r cot (θ/2)

13. The vector $\vec{P}$ makes 120° with the x-axis and vector $\vec{Q}$ makes 30° with the y-axis. What is their resultant ?

(A) P + Q (B) P – Q (C) $\sqrt{P^2 + Q^2}$ (D) $\sqrt{P^2 - Q^2}$

14. A man travels 1 mile due east, then 5 miles due south, then 2 miles due east and finally 9 miles due north, how far is he from the starting point -

(A) 3 miles (B) 5 miles
(C) 4 miles (D) between 5 and 9 miles

15. The angle that the vector $\vec{A} = 2\hat{i} + 3\hat{j}$ makes with y-axis is-

(A) tan^{-1}(3/2) (B) tan^{-1}(2/3)
(C) sin^{-1}(2/3) (D) cos^{-1}(3/2)

16. A man moves towards 3m north then 4m towards east and finally 5 m towards 37° south of west. His displacement from origin is -

(A) $5\sqrt{2}$ m (B) 0 m (C) 12 m (D) 5 m

17. If $3\hat{i} + 2\hat{j} + 8\hat{k}$ and $2\hat{i} + x\hat{j} + \hat{k}$ are at right angles that x=

(A) 7 (B) –7 (C) 5 (D) –4

18. $a_1\hat{i} + a_2\hat{j}$ is a unit vector perpendicular to $4\hat{i} - 3\hat{j}$ if -

(A) $a_1 = .6$, $a_2 = .8$ (B) $a_1 = 3$, $a_2 = 4$
(C) $a_1 = .8$, $a_2 = .6$ (D) $a_1 = 4$, $a_2 = 3$

19. If $\vec{a}$ is a vector and x is a non-zero scalar, then -

(A) $x\vec{a}$ is a vector in the direction of $\vec{a}$
(B) $x\vec{a}$ is a vector collinear to $\vec{a}$
(C) $x\vec{a}$ and $\vec{a}$ have independent directions
(D) none of these

20. Two vectors have magnitudes 3 unit and 4 unit respectively. What should be the angle between them if the magnitude of the resultant is

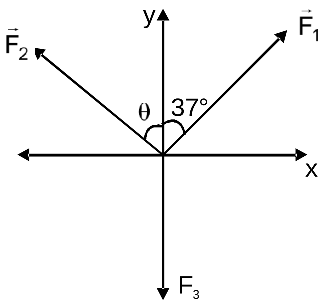
(a) 1unit,

(b) 5 unit and

(c) 7 unit.

21. When two forces of magnitude P and Q are perpendicular to each other, their resultant is of magnitude R. When they are at an angle of 180° to each other their resultant is of magnitude $\frac{R}{\sqrt{2}}$. Find the ratio of P and Q.

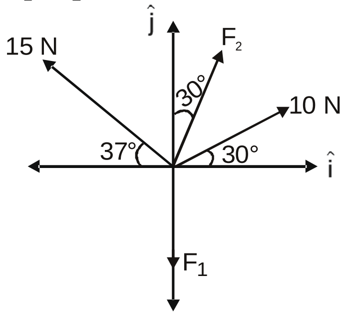
22. A body acted upon by 3 given forces is under equilibrium.



(a) If $|\vec{F}_1| = 10\text{Nt.}, |\vec{F}_2| = 6\text{Nt.}$ Find the values of $|\vec{F}_3|$ & angle (θ).

(b) Express $\vec{F}_2$ in unit vector form

23. If the four forces as shown are in equilibrium Express $\vec{F}_1$ & $\vec{F}_2$ in unit vector form.



24. A particle is acted upon by the forces $\vec{F}_1 = 2\hat{i} + a\hat{j} - 3\hat{k}$, $\vec{F}_2 = 5\hat{i} + c\hat{j} - b\hat{k}$, $\vec{F}_3 = b\hat{i} + 5\hat{j} - 7\hat{k}$, $\vec{F}_4 = c\hat{i} + 6\hat{j} - a\hat{k}$. Find the values of the constants a, b, c in order that the particle will be in equilibrium.

25. A plane body has perpendicular axes OX and OY marked on it and is acted on by following forces

5P in the direction OY

4P in the direction OX

10P in the direction OA where A is the point (3a, 4a)

15P in the direction AB where B is the point (–a, a)

Express each force in the unit vector from & calculate the magnitude & direction of sum of the vector of these forces.

26. A vector $\vec{A}$ of length 10 units makes an angle of 60° with the vector $\vec{B}$ of length 6 units. Find the magnitude of the vector difference $\vec{A} - \vec{B}$ & the angle it makes with vector $\vec{A}$.

27. (a) Calculate $\vec{r} = \vec{a} - \vec{b} + \vec{c}$ where $\vec{a} = 5\hat{i} + 4\hat{j} - 6\hat{k}$, $\vec{b} = -2\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{c} = 4\hat{i} + 3\hat{j} + 2\hat{k}$.

(b) Calculate the angle between $\vec{r}$ and the z-axis.

(c) Find the angle between $\vec{a}$ and $\vec{b}$

EXERCISE - I

PART - I

SECTION - A

1. 1 **2.** 47 **3.** (c)

SECTION - B

4. $\frac{dy}{dx}=2x+1$ **5.** $\frac{ds}{dt}=15t^2-15t^4$ **6.** $\frac{dy}{dx}=5\cos x$ **7.** $\frac{dy}{dx}=2x+\cos x$ **8.** $\sec^2 x-\operatorname{cosec}^2 x$

9. $\frac{dy}{dx}=12x-10+10x^{-3}$, $\frac{d^2y}{dx^2}=12-30x^{-4}$

10. $\frac{dr}{d\theta}=-12\theta^{-2}+12\theta^{-4}-4\theta^{-5}$, $\frac{d^2r}{d\theta^2}=24\theta^{-3}-48\theta^{-5}+20\theta^{-6}$

11. $\frac{d\omega}{dz}=21z^6-21z^2+42z$, $\frac{d^2\omega}{dz^2}=126z^5-42z+42$

12. $\frac{dy}{dx}=\cos x-\sin x$, $\frac{d^2y}{dx^2}=-\sin x-\cos x$ **13.** $\frac{dy}{dx}=\frac{1}{x}+e^x$, $\frac{d^2y}{dx^2}=-\frac{1}{x^2}+e^x$

SECTION - C

14. $\sin x+x\cos x$ **15.** $e^x\ell nx+\frac{e^x}{x}$ **16.** $\frac{dy}{dx}=3x^2$ **17.** $y'=3x^2+10x+2-\frac{1}{x^2}$

18. $\cos^2 x-\sin^2 x$

SECTION - D

19. $\sec^2 x$ **20.** $y'=\frac{-19}{(3x-2)^2}$ **21.** $\frac{1}{x^2}-\frac{\ell nx}{x^2}$ **22.** $f'(t)=\frac{t^2-2t+1}{(t^2+t-2)^2}$

23. $\frac{dz}{dx}=\frac{-2x^2-2x-2}{(x^2-1)^2}$ **24.** $\frac{dy}{dx}=-x^2\csc^2 x+2x\cot x$

SECTION - E

25. With $u=(2x+1)$

$y=u^5:\frac{dy}{dx}=\frac{dy}{du}=5u^4\,.\,2=10(2x+1)^4$

26. $\frac{dy}{dx}=-27(4-3x)^8$

27. With $u=(1-\left(\frac{x}{7}\right))$ $y=u^{-7}:\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}=-7u^{-8}.\left(-\frac{1}{7}\right)=\left(1-\frac{x}{7}\right)^{-8}$

28. $\frac{dy}{dx}=-5\left(\frac{x}{2}-1\right)^{-11}$ **29.** $5\cos 5x$ **30.** $\cos(x)+\frac{2}{x}+2e^{2x}$ **31.** $2\omega\cos(\omega x+\phi)$

SECTION - G

32. $\frac{dA}{dt}=2\pi r\frac{dr}{dt}$ **33.** $\frac{ds}{dt}=8\pi r\frac{dr}{dt}$

SECTION - H

34. 8 **35.** $y_{max}=39$, $y_{min}=38$

SECTION - I

36. $\frac{dy}{dx}=48(8x-1)^2$ **37.** $3\cos(3x+1)$ **38.** $12x^3$ **39.** $\frac{dy}{dx}=-\frac{1}{3}\sin\frac{x}{3}$

PART - II

1. (a) x^2 (b) $\frac{x^3}{3}$ (c) $\frac{x^3}{3} - x^2 + x$ **2.** (a) x^{-3} (b) $-\frac{1}{3}x^{-3}$ (c) $-\frac{1}{3}x^{-3} + x^2 + 3x$

3. (a) $-\frac{1}{x}$ (b) $-\frac{5}{x}$ (c) $2x + \frac{5}{x}$ **4.** (a) $\sqrt{x^3}$ (b) $3\sqrt{x}$ (c) $\frac{2\sqrt{x^3}}{3} + 2\sqrt{x}$

5. (a) $x^{4/3}$ (b) $\frac{x^{\frac{2}{3}}}{2}$ (c) $\frac{3x^{\frac{4}{3}}}{4} + \frac{3x^{\frac{2}{3}}}{2}$ **6.** (a) $x^{1/2}$ (b) $x^{-1/2}$ (c) $x^{-3/2}$ **7.** $x - \frac{x^3}{3} - \frac{x^6}{2} + C$

8. $-3\cos x$ **9.** $\frac{1}{3}\ell nx$ **10.** $-\frac{1}{3}\cos 3x + C$

11. $\frac{1}{2}\sec 2t + C$ **12.** 15 **13.** $\frac{3\pi}{2}$ **14.** Area = 21 **15.** 24 **16.** 0

17. e – 1 **18.** Using n subintervals of length $\Delta x = \frac{b}{n}$ and right- endpoint values : Area = $\int_0^b 2x\,dx = b^2$

19. $\frac{b^2}{4} + b = \frac{b(4+b)}{4}$ **20.** 2

PART - III

SECTION - A

1. (i) 105º, (ii) 150º, (iii) 105º **2.** 120º **3.** $\vec{V}_R = -5\hat{j}$ **4.** (C)

SECTION - B

5. $30\,\text{m East}$ **6.** $\sqrt{F_1^2 + F_2^2}$ **7.** 50, 53º with East **8.** $250\sqrt{5}$ N, $\tan^{-1}(2)$ W of N

9. (A) **10.** (B) **11.** (B) **12.** (D) **13.** (D) **14.** (A)
15. (C) **16.** (B) **17.** (D)

SECTION - C

18. $\sqrt{14}$ **19.** $\frac{3\hat{i} + 4\hat{j}}{5}$ **20.** – 25 cos 30º and + 25 sin 30º **21.** $30\sqrt{3}\,\text{km h}^{-1}$

22. $\sqrt{0.11}$ **23.** 15º **24.** (A)

SECTION - D

25. (a) 3 (b) $-\hat{i} + 2\hat{j} - \hat{k}$ **26.** (a) 6 (b) $6\sqrt{3}$ **27.** (D) **28.** (B)

PART-I

SECTION-A

1. $-\frac{1}{x}$ **2.** f(2) = 3, f(1) = 3, f(3) = 5

SECTION-B

3. $\frac{dy}{dx} = \frac{2}{x} + \cos x$, $\frac{d^2y}{dx^2} = \frac{-2}{x^2} - \sin x$ **4.** $\frac{dy}{dx} = \frac{x^{-\frac{6}{7}}}{7} + \sec^2 x \Rightarrow$ $\frac{d^2y}{dx^2} = \frac{-6}{49} \times x^{\frac{-13}{7}} + 2\tan x \sec^2 x$

SECTION-C

5. $e^x(\tan x + \sec^2 x)$ **6.** $2x\sin^4 x + 4x^2 \sin^3 x \cos x + \cos^{-2} x + 2x\cos^{-3} x \sin x$ **7.** $\frac{dy}{dx} = 1 + 2x + \frac{2}{x^3} - \frac{1}{x^2}$

EXERCISE - II

8. $x^2 \cos x$ **9.** $\frac{dy}{dx} = -x^2 \sin x$ **10.** $\frac{dr}{d\theta} = \cos\theta + \sec^2\theta$

SECTION-D

11. $\frac{dy}{dx} = \sec^2 x$ **12.** $\frac{-\csc^2 x}{(1+\cot x)^2}$ **13.** $\frac{dy}{dx} = \frac{-x\sin x - \cos x}{x^2} + \frac{x\sin x + \cos x}{\cos^2 x}$ **14.** $\frac{\sec^2 q}{(1+\tan q)^2}$

SECTION-E

15. $3\sin^2 x \cos x + 3\cos 3x$ **16.** $4x \sin(x^2+1)\cos(x^2+1)$ **17.** $\frac{1}{(x^2+1)^{3/2}}$ **18.** $\frac{1-r}{\sqrt{2r-r^2}}$

19. With $u = \left(\frac{x^2}{8}\right) + x - \left(\frac{1}{x}\right)$, $y = u^4$: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 4u^3 . \left(\frac{x}{4} + 1 + \frac{1}{x^2}\right) = 4$, $4\left(\frac{x^2}{8} + x - \frac{1}{x}\right)^3 \left(\frac{x}{4} + 1 + \frac{1}{x^2}\right)$

SECTION-F

20. (a) $\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} = 5\pi r^2$ (b) $\frac{dV}{dt} = 2\pi hr\frac{dr}{dt} = 10\pi rh$ (c) $\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + 2\pi hr\frac{dr}{dt}$ $= 5\pi r^2 + 10\pi rh$

SECTION - G

21. $x = 30$ & $y = 30$ **22.** $x = \sqrt{\frac{40}{3}}m$

SECTION-H

23. **(a)** $-\cos x$, **(b)** $2\sec^3 x - \sec x$

Given $y = f(u)$ and $u = g(x)$, find $\frac{dy}{dx}$

24. $-\sin(\sin x)\cos x$. **25.** $\frac{dy}{dx} = \cos(x - \cos x)(1 + \sin x)$

PART - II

1. $\frac{x^4}{2} - \frac{5x^2}{2} + 7x + C$ **2.** $\frac{x}{5} + \frac{1}{x^2} + x^2 + C$ **3.** $\frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C$ **4.** $-\frac{1}{x} - \frac{1}{2x^2} + C$ **5.** $2\sqrt{t} - \frac{2}{\sqrt{t}} + C$

6. $-2t^{-2} - \frac{2}{3}t^{-3/2} + C$ **7.** $-\cos\theta + \theta + C$ **8.** $\frac{3\pi^2}{2}$ **9.** $\frac{7}{3}$ **10.** 0 **11.** $\frac{1}{3}\ell n\frac{5}{2}$

12. Using n subintervals of length $\Delta x = \frac{b}{n}$ and right-end point values : Area $= \int_0^b 3x^2 dx = b^3$ **13.** $\frac{\pi b^2}{4}$

PART - III
SECTION - A

1. $\vec{B} = \lambda\vec{A} = -4 \times 3N - E$ = 12 S-W

No it does not represent the same physical quantity.

2. (D) **3.** (A) **4.** (A), (B), (C)

SECTION-B

5. 37° **6.** $r(1+\sqrt{2})$ **7.** (A) **8.** (B) **9.** (B) **10.** (B) **11.** (C)

12. (A) **13.** (D) **14.** (A) **15.** (A), (B), (D)

SECTION-C

16. $\frac{4\hat{i} + 5\hat{j} + 2\hat{k}}{\sqrt{45}}$ **17.** B **18.** (A)

SECTION- D

19. $\frac{x_1}{x_2} = \frac{y_1}{y_2}$ **20.** (D)

EXERCISE - III

LEVEL - I

1. (A) $\rightarrow$ Q, (B) $\rightarrow$ R, (C) $\rightarrow$ P, (D) $\rightarrow$ S

2. (a) 8, (b) 4, (c) 2 (b) $\cos\theta = \dfrac{\vec{F}_1.\vec{F}_2}{|\vec{F}_1||\vec{F}_2|} \Rightarrow \theta = \cos^{-1}\left(\dfrac{3}{5\sqrt{2}}\right)$ (c) $F_1 \cos\theta = \dfrac{\vec{F}_1.\vec{F}_2}{|\vec{F}_2|} = \dfrac{6}{5}$

3. (a) $\vec{F}_R = \vec{F}_1 + \vec{F}_2 = 2\hat{i} + 5\hat{j} + 4\hat{k}$

4. (C) **5.** (B) **6.** (A) **7.** (A) **8.** (A) **9.** (B)

10. **(i)** True **(ii)** False **(iii)** True **(iv)** True

11. **(i)** 25 Units. **(ii)** $15\hat{i} + 20\hat{j}$ **(iii)** Null vector **(iv)** $\sqrt{224}$ units **(v)** 7 units **(vi)** $\dfrac{1}{\sqrt{3}}\hat{i} + \dfrac{1}{\sqrt{3}}\hat{j} + \dfrac{1}{\sqrt{3}}\hat{k}$ **(vii)** Perpendicular **(viii)** 45 **(ix)** zero.

LEVEL - II

1.	C	2.	A	3.	B	4.	D	5.	A	6.	D	7.	D	8.	B
9.	A	10	A	11.	C	12.	A	13.	A	14.	B	15.	B		
16.	B	17.	B	18.	A	19.	B								

20. (a) 180°, (b) 90°, (c) 0 **21.** $2 \pm \sqrt{3}$ **22.** (a) $|\vec{F}_3| = 8$ N, $\theta = 90°$ (b) $\vec{F}_2 = -6\hat{i}$

23. $\vec{F}_1 = -(12\sqrt{3} - 1)\,\hat{j}$ & $\vec{F}_2 = (12 - 5\sqrt{3})\,\hat{i} + (12\sqrt{3} - 15)\,\hat{j}$ **24.** a = – 7, b = – 3, c = – 4

25. $5P\hat{j}, 4P\hat{i}, 6P\hat{i} + 8P\hat{j}, -12P\hat{i} - 9P\hat{j}$, $\sqrt{20}P$, $\tan^{-1}[-2]$ with the +ve x axis. **26.** $2\sqrt{19}$; $\cos^{-1}\dfrac{7}{2\sqrt{19}}$

27. (a) $11\hat{i} + 5\hat{j} - 7\hat{k}$, (b) $\cos^{-1}\left(\dfrac{-7}{\sqrt{195}}\right)$, (c) $\cos^{-1}\left(\dfrac{-20}{\sqrt{1309}}\right)$

Chapter 2

UNIT & DIMENSION

1. PHYSICAL QUANTITY

The quantites which can be measured by an instrument and by means of which we can describe the laws of physics are called physical quantities.

Types of physical quantities :

Fundamental	Derived	Supplementry

1.1 Fundamental

Although the number of physical quantities that we measure is very large, we need only a limited number of units for expressing all the physical quantities since they are interrelated with one another. So, certain physical quantities have been chosen arbitrarily and their units are used for expressing all the physical quantities, such quantities are known as Fundamental, Absolute or Base Quantities (such as length, time and mass in mechanics)

(i) All other quantites may be expressed in terms of fundamental quantities.

(ii) They are independent of each other and cannot be obtained from one another.

An international body named General Conference on Weights and Measures chose seven physical quantities as fundamental :

(1) length (2) mass (3) time (4) electric current,

(5) thermodynamic temperature (6) amount of substance

(7) luminous intensity.

Note : These are also called as absolute or base quantities.

In mechanics, we treat length, mass and time as the three basic or fundamental quantities.

1.2 Derived

Derived : Physical quantities which can be expressed as combination of base quantities are called as derived quantities.

For example : Speed, velocity, acceleration, force, momentum, pressure, energy etc.

Ex.1 *Speed* = $\frac{\text{distance}}{\text{time}} = \frac{\text{length}}{\text{time}}$

1.3 Supplementary

Supplementary : Beside the seven fundamental physical quantities two supplementary quantities are also defined, they are :

(1) Plane angle (2) Solid angle.

Note : The supplementary quantities have only units but no dimensions.

2. MAGNITUDE :

Magnitude of physical quantity = (numerical value) × (unit)

Magnitude of a physical quantity is always constant. It is independent of the type of unit.

$\Rightarrow$ numerical value $\propto \frac{1}{\text{unit}}$

or $n_1u_1 = n_2u_2$ = constant

Ex.2 *Length of a metal rod bar is unchanged whether it is measured as 2 metre or 200 cm. Observe the change in the Numerical value (from 2 to 200) as unit is changed from metre to cm.*

3. UNIT :

Measurement of any physical quantity is expressed in terms of an internationally accepted certain basic reference standard called unit.

The units for the fundamental or base quantities are called fundamental or base unit. Other physical quantities are expressed as combination of these base units and hence, called derived units.

A complete set of units, both fundamental and derived is called a system of unit.

3.1. Principle systems of Unit

There are various system in use over the world : CGS, FPS, SI (MKS) etc

Table 1 : Units of some physical quantities in different systems.

	Physical Quantity	**System**		
		CGS (Gaussian)	**MKS (SI)**	**FPS (British)**
Fundamental	Length	centimeter	meter	foot
	Mass	gram	kilogram	pound
	Time	second	second	second
Derived	Force	dyne	newton $\rightarrow$ N	poundal
	Work or Energy	erg	joule $\rightarrow$ J	ft-poundal
	Power	erg/s	watt $\rightarrow$ W	ft-poundal/s

3.2 Supplementary units :

(1) Plane angle : radian (rad)

(2) Solid angle : steradian (sr)

* The SI system is at present widely used throughout the world. In IIT JEE only SI system is followed.

3.3 Definitions of some important SI Units

(i) Metre : 1 m = 1,650, 763.73 wavelengths in vaccum, of radiation corresponding to organ-red light of krypton-86.

(ii) Second : 1 s = 9,192, 631,770 time periods of a particular from Ceasium - 133 atom.

(iii) Kilogram : 1kg = mass of 1 litre volume of water at 4°C

(iv) Ampere : It is the current which when flows through two infinitely long straight conductors of negligible cross-section placed at a distance of one metre in vacuum produces a force of 2×10^{-7} N/m between them.

(v) Kelvin : 1 K = 1/273.16 part of the thermodynamic temperature of triple point of water.

(vi) Mole : It is the amount of substance of a system which contains as many elementary particles (atoms, molecules, ions etc.) as there are atoms in 12g of carbon - 12.

(vii) Candela : It is luminous intensity in a perpendicular direction of a surface of $\left(\frac{1}{600000}\right) m^2$ of a black body at the temperature of freezing point under a pressure of 1.013×10^5 N/m^2.

(viii) Radian : It is the plane angle between two radiia of a circle which cut-off on the circumference, an arc equal in length to the radius.

(ix) Steradian : The steradian is the solid angle which having its vertex at the centre of the sphere, cut-off an area of the surface of sphere equal to that of a square with sides of length equal to the radius of the sphere.

Ex.3 *Find the SI unit of speed, acceleration*

Sol. speed = $\frac{\text{distance}}{\text{time}} = \frac{\text{meter(m)}}{\text{second(s)}}$ = m/s (called as meter per second)

acceleration = $\frac{\text{velocity}}{\text{time}} = \frac{\text{displacement / time}}{\text{time}}$

$= \frac{\text{displacement}}{(\text{time})^2} = \frac{\text{meter}}{\text{second}^2}$ = m/s² (called as meter per second square)

4. S I PREFIXES

The magnitudes of physical quantities vary order a wide range. The CGPM recommended standard prefixes for magnitude too large or too small to be expressed more compactly for certain power of 10.

Power of 10	Prefix	Symbol	Power of 10	Prefix	Symbol
10^{18}	exa	E	10^{-1}	deci	d
10^{15}	peta	P	10^{-2}	centi	c
10^{12}	tera	T	10^{-3}	milli	m
10^{9}	giga	G	10^{-6}	micro	μ
10^{6}	mega	M	10^{-9}	nano	n
10^{3}	kilo	k	10^{-12}	pico	p
10^{2}	hecto	h	10^{-15}	femto	f
10^{1}	deca	da	10^{-18}	atto	a

5. GENERAL GUIDELINES FOR USING SYMBOLS FOR SI UNITS, SOME OTHER UNITS, SOME OTHER UNITS, AND SI PREFIXES

(a) Symbols for units of physical quantities are printed/written in Roman (upright type), and not in italics

For example : 1 N is correct but 1 *N* is incorrect

(b) (i) Unit is never written with capital initial letter even if it is named after a scientist.

For example : SI unit of force is newton (correct) Newton (incorrect)

(ii) For a unit named after a scientist, the symbol is a capital letter.

But for other units, the symbol is NOT a capital letter.

For example :

force → newton (N)
energy → joule (J)
electric current → ampere (A)
temperature → kelvin (K)
frequency → hertz (Hz)

For example :

length → meter (m)
mass → kilogram (kg)
luminous intensity → candela (cd)
time → second (s)

Note : The single exception is L, for the unit litre.

(c) Symbols for units do not contain any final full stop all the end of recommended letter and remain unaltered in the plural, using only singular form of the unit.

For example :

Quantity	Correct	Incorrect
25 centimeters	25 cm	25 c m 25 cms.

(d) Use of solidus (/) is recommended only for indicating a division of one letter unit symbol by another unit symbol. Not more than one solidus is used.

For example :

Correct	Incorrect
m/s^2	m / s / s
$N\ s/m^2$	N s / m/ m
J/K mol	J / K / mol
kg/m s	kg / m / s

(e) Prefix symbols are printed in roman (upright) type without spacing between the prefix symbol and the unit symbol. Thus certain approved prefixes written very close to the unit symbol are used to indicate decimal fractions or multiples of a SI unit, when it is inconveniently small or large.

For example

megawatt	$1\ MW = 10^6\ W$
centimetre	$1\ cm = 10^{-2}\ m$
kilometre	$1\ km = 10^3\ m$
millivolt	$1\ mV = 10^{-3}\ V$
kilowatt-hour	$1\ kW\ h = 10^3\ W\ h = 3.6\ M\ J = 3.6 \times 10^6\ J$
microampere	$1\mu A = 10^{-6}\ A$
angstrom	$1 Å = 0.1 nm == 10^{-10}\ m$
nanosecond	$1\ ns = 10^{-9}\ s$
picofarad	$1\ pF = 10^{-12}\ F$
microsecond	$1\mu s = 10^{-6} s$
gigahertz	$1\ GHz = 10^9\ Hz$
micron	$1\ \mu m = 10^{-6}\ m$

The unit 'fermi', equal to a femtometre or 10^{-15} m has been used as the convenient length unit in nuclear studies.

(f) When a prefix is placed before the symbol of a unit, the combination of prefix and symbol is considered as a new symbol, for the unit, which can be raised to a positive or negative power without using brackets. These can be combined with other unit symbols to form compound unit.

For example :

Quantity	Correct	Incorrect
cm^3	$(cm)^3 = (0.01\ m)^3 = (10^{-2} m)^3 = 10^{-6}\ m^3$	$0.01\ m^3$ or $10^{-2}\ m^3$
mA^2	$(mA)^2 = (0.001\ A)^2 = (10^{-3} A)^2 = 10^{-6}\ A^2$	$0.001\ A^2$

(g) A prefix is never used alone. It is always attached to a unit symbol and written or fixed before the unit symbol.

For example :

$10^3/m^3 = 1000/m^3$ or $1000\ m^{-3}$, but not k/m^3 or $k\ m^{-3}$.

(h) Prefix symbol is written very close to the unit symbol without spacing between them, while unit symbols are written separately with spacing with units are multiplied together.

For example :

Quantity	Correct	Incorrect
1 ms^{-1}	1 metre per second	1 milli per second
1 ms	1 millisecond	1 metre second
1 C m	1 coulomb metre	1 centimetre
1 cm	1 centimetre	1 coulomb metre

(i) The use of double profixes is avoided when single prefixes are available.

For example :

Quantity	Correct	Incorrect
10^{-9} m	1 nm (nanometre)	1 mμ m (milli micrometre)
10^{-6} m	1μm (micron)	1 m m m (milli millimetre)
10^{-12} F	1 pF (picofarad)	1 μ μ F (micro microfarad)
10^9 F	1 GW (giga watt)	1 kM W (kilo megawatt)

(j) The use of a combination of unit and the symbols for unit is avoided when the physical quantity is expressed by combining two or more units.

Quantity	Correct	Incorrect
joule per mole Kelvin	J/mol K or J mol^{-1} K^{-1}	Joule / mole K or J/mol Kelvin or J/mole K
newton metre second	N m s	newton m second or N m second or N metre s or newton metre s

5.1. Characteristics of base units or standards :

(A) Well defined (B) Accessibility (C) Invariability (D) Convenience in use

5.2 Some special types of units :

1. 1 Micron (1μ) = 10^{-4} cm = 10^{-6} m (length)
2. 1 Angstrom (1 Å) = 10^{-8} cm = 10^{-10}m (length)
3. 1 fermi (1f) = 10^{-13} cm = 10^{-15} m (length)
4. 1 inch = 2.54 cm (length)
5. 1 mile = 5280 feet = 1.609 km (length)
6. 1 atmosphere = 10^5 N/m^2 = 76 torr = 76 mm of Hg pressure (pressure)
7. 1 litre = 10^{-3} m^3 = 1000 cm^3 (volume)
8. 1 carat = 0.0002 kg (weight)
9. 1 pound (Ib) = 0.4536 kg (weight)

6. DIMENSIONS

Dimensions of a physical quantity are the power to which the fundamental quantities must be raised to represent the given physical quantity.

For example, density = $\frac{\text{mass}}{\text{volume}} = \frac{\text{mass}}{(\text{length})^3}$

or density = (mass) $(\text{length})^{-3}$ ...(i)

Thus, the dimensions of density are 1 in mass and –3 in length. The dimensions of all other fundamental quantities are zero.

For convenience, the fundamental quantities are represented by one letter symbols. Generally mass is denoted by M, length by L, time by T and electric current by A.

The thermodynamic temperature, the amount of substance and the luminous intensity are denoted by the symbols of their units K, mol and cd respectively. The physical quantity that is expressed in terms of the base quantities is enclosed in square brackets.

$$[\sin\theta] = [\cos\theta] = [\tan\theta] = [e^x] = [M^0L^0T^0]$$

7. DIMENSIONAL FORMULA

It is an expression which shows how and which of the fundalmental units are required to represent the unit of physical quantity.

Different quantities with units. symbol and dimensional formula.

Quantity	Symbol	Formula	S.I. Unit	D.F.
Displacement	s	ℓ	Metre or m	M^0LT^0
Area	A	$\ell \times b$	(Metre)^2 or m^2	$M^0L^2T^0$
Volume	V	$\ell \times b \times h$	(Metre)^3 or m^3	$M^0L^3T^0$
Velocity	v	$v = \frac{\Delta s}{\Delta t}$	m/s	M^0LT^{-1}
Momentum	p	p = mv	kgm/s	MLT^{-1}
Acceleration	a	$a = \frac{\Delta v}{\Delta t}$	m/s^2	M^0LT^{-2}
Force	F	F = ma	Newton or N	MLT^{-2}
Impulse	-	F × t	N.sec	MLT^{-1}
Work	W	F . d	N . m	ML^2T^{-2}
Energy	KE or U	$K.E. = \frac{1}{2}mv^2$	Joule or J	ML^2T^{-2}
	P.E. = mgh			
Power	P	$P = \frac{W}{t}$	watt or W	ML^2T^{-3}
Density	d	d = mass/volume	kg/m^3	$ML^{-3}T^0$
Pressure	P	P = F/A	Pascal or Pa	$ML^{-1}T^{-2}$
Torque	τ	$\tau = r \times F$	N.m.	ML^2T^{-2}
Angular displacement	θ	$\theta = \frac{\text{arc}}{\text{radius}}$	radian or rad	$M^0L^0T^0$
Angular velocity	ω	$\omega = \frac{\theta}{t}$	rad/sec	$M^0L^0T^{-1}$
Angular acceleration	α	$\alpha = \frac{\Delta\omega}{\Delta t}$	rad/sec^2	$M^0L^0T^{-2}$

Moment of Inertia	I	$I = mr^2$	kg-m²	ML^2T^0
Frequency		ν or f	$f = \frac{1}{T}$	hertz or Hz $M^0L^0T^{-1}$
Stress	-	F/A	N/m²	$ML^{-1}T^{-2}$
Strain	-	$\frac{\Delta\ell}{\ell};\frac{\Delta A}{A};\frac{\Delta V}{V}$	-	$M^0L^0T^0$
Youngs modulus (Bulk modulus of rigidity)	Y	$Y = \frac{F/A}{\Delta\ell/\ell}$	N/m²	$ML^{-1}T^{-2}$
Surface tension	T	$\frac{F}{\ell}$ or $\frac{W}{A}$	$\frac{N}{m};\frac{J}{m^2}$	ML^0T^{-2}
Force constant (spring)	k	$F = kx$	N/m	ML^0T^{-2}
Coefficient of viscosity	η	$F = \eta\left(\frac{dv}{dx}\right)A$	kg/ms(poise in C.G.S.)	$ML^{-1}T^{-1}$
Gravitation constant	G	$F = \frac{Gm_1m_2}{r^2}$	$\frac{N-m^2}{kg^2}$	$M^{-1}L^3T^{-2}$
Gravitational potential	V_g	$V_g = \frac{PE}{m}$	$\frac{J}{kg}$	$M^0L^2T^{-2}$
Temperature	θ	-	Kelvin or K	$M^0L^0T^0\theta^{+1}$
Heat	Q	$Q = m \times S \times \Delta t$	Joule or Calorie	ML^2T^{-2}
Specific heat	S	$Q = m \times S \times \Delta t$	$\frac{Joule}{kg.Kelvin}$	$M^0L^2T^{-2}\theta^{-1}$
Latent heat	L	$Q = mL$	$\frac{Joule}{kg}$	$M^0L^2T^{-2}$
Coefficient of thermal conductivity	K	$Q = \frac{KA(\theta_1-\theta_2)t}{d}$	$\frac{Joule}{m\sec K}$	$MLT^{-3}\theta^{-1}$
Universal gas constant	R	$PV = nRT$	$\frac{Joule}{mol.K}$	$ML^2T^{-2}\theta^{-1}$
Mechanical equivalent of heat	J	$W = JH$	-	$M^0L^0T^0$
Charge	Q or q	$I = \frac{Q}{t}$	Coulomb or C	M^0L^0TA
Current	I	-	Ampere or A	$M^0L^0T^0A$
Electric permittivity	ε_0	$\varepsilon_0 = \frac{1}{4\pi F}.\frac{q_1q_2}{r^2}$	$\frac{(coul.)^2}{N.m^2}$ or $\frac{C^2}{N-m^2}$	$M^{-1}L^{-3}T^4A^2$
Electric potential	V	$V = \frac{\Delta W}{q}$	Joule/coul	$ML^2T^{-3}A^{-1}$
Intensity of electric field	E	$E = \frac{F}{q}$	N/coul.	$MLT^{-3}A^{-1}$

Capacitance	C	Q = CV	Farad	$M^{-1}L^{-2}T^{4}A^{2}$
Dielectric constant or relative permittivity	ε_r	$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$	-	$M^{0}L^{0}T^{0}$
Resistance	R	V = IR	Ohm	$ML^{2}T^{-3}A^{-2}$
Conductance	S	$S = \frac{1}{R}$	Mho	$M^{-1}L^{-2}T^{-3}A^{2}$
Specific resistance or resistivity	ρ	$\rho = \frac{RA}{\ell}$	Ohm × meter	$ML^{3}T^{-3}A^{-2}$
Conductivity or specific conductance	s	$\sigma = \frac{1}{\rho}$	Mho/meter	$M^{-1}L^{-3}T^{3}A^{2}$
Magnetic induction	B	$F = qvB\sin\theta$ or $F = BIL$	Tesla or weber/m²	$MT^{-2}A^{-1}$
Magnetic flux	ϕ	$e = \frac{d\phi}{dt}$	Weber	$ML^{2}T^{-2}A^{-1}$
Magnetic intensity	H	$B = \mu H$	A/m	$M^{0}L^{-1}T^{0}A$
Magnetic permeability of free space or medium	μ_0	$B = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$	$\frac{N}{amp^2}$	$MLT^{-2}A^{-2}$
Coefficient of self or Mutual inductance	L	$e = L.\frac{dI}{dt}$	Henery	$ML^{2}T^{-2}A^{-2}$
Electric dipole moment	p	$p = q \times 2\ell$	C.m.	$M^{0}LTA$
Magnetic dipole moment	M	M = NIA	amp.m²	$M^{0}L^{2}AT^{0}$

8. USE OF DIMENSIONS

Theory of dimensions have following main uses :

8.1 Conversion of units :

This is based on the fact that the product of the numerical value (n) and its corresponding unit (u) is a constant, i.e.,

$$n[u] = \text{constant}$$

or $$n_1[u_1] = n_2[u_2]$$

Suppose the dimensions of a physical quantity are a in mass, b in length and c in time. If the fundamental units in one system are M_1, L_1 and T_1 and in the other system are M_2, L_2 and T_2 respectively. Then we can write.

$$n_1[M_1^a L_1^b T_1^c] = n_2[M_2^a L_2^b T_2^c] \qquad \ldots(i)$$

Here n_1 and n_2 are the numerical values in two system of units respectively. Using Eq. (i), we can convert the numerical value of a physical quantity from one system of units into the other system.

Ex.4 *The value of gravitation constant is G = 6.67 × 10^{-11} Nm^2/kg^2 in SI units. Convert it into CGS system of units.*

Sol. The dimensional formula of G is $[M^{-1} L^3 T^{-2}]$.

Using equation number (i), i.e.,

$$n_1[M_1^{-1}L_1^3T_1^{-2}] = n_2[M_2^{-1}L_2^3T_2^{-2}]$$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^{-1}\left[\frac{L_1}{L_2}\right]^{3}\left[\frac{T_1}{T_2}\right]^{-2}$$

Here, $n_1 = 6.67 \times 10^{-11}$

$M_1 = 1$ kg, $M_2 = 1$ g $= 10^{-3}$ kg $L_1 = 1$ m, $L_2 = 1$cm $= 10^{-2}$ m, $T_1 = T_2 = 1$s

Substituting in the above equation, we get

$$n_2 = 6.67 \times 10^{-11}\left[\frac{1\text{kg}}{10^{-3}\text{kg}}\right]^{-1}\left[\frac{1\text{m}}{10^{-2}\text{m}}\right]^{3}\left[\frac{1\text{s}}{1\text{s}}\right]^{-2}$$

or $n_2 = 6.67 \times 10^{-8}$

Thus, value of G in CGS system of units is 6.67×10^{-8} dyne cm^2/g^2.

8.2 To check the dimensional correctness of a given physical equation :

Every physical equation should be dimensionally balanced. This is called the 'Principle of Homogeneity'. The dimensions of each term on both sides of an equation must be the same. On this basis we can judge whether a given equation is correct or not. But a dimensionally correct equation may or may not be physically correct.

Ex.5 *Show that the expression of the time period T of a simple pendulum of length **l** given by T = $2\pi\sqrt{\frac{l}{g}}$ is dimensionally correct.*

Sol.
$$T = 2\pi\sqrt{\frac{l}{g}}$$

Dimensionally $$[T] = \sqrt{\frac{[L]}{[LT^{-2}]}} = [T]$$

As in the above equation, the dimensions of both sides are same. The given formula is dimensionally correct.

8.3 Principle of Homogeneity of Dimensions.

This principle states that the dimensions of all the terms in a physical expression should be same. For example, in the physical expression $s = ut + \frac{1}{2}at^2$, the dimensions of s, ut and $\frac{1}{2}at^2$ all are same.

Note : The physical quantities separated by the symbols +, –, =, >, < etc., have the same dimensions.

Ex.6 *The velocity v of a particle depends upon the time t according to the equation $v = a + bt + \frac{c}{d+t}$. Write the dimensions of a, b, c and d.*

Sol. From principle of homogeneity

$[a] = [v]$

or $[a] = [LT^{-1}]$ Ans.

$[bt] = [v]$

or $$[b] = \frac{[v]}{[t]} = \frac{[LT^{-1}]}{[T]}$$

or $[b] = [LT^{-2}]$

Similarly, $[d] = [t] = [T]$ Ans.

Further, $\frac{[c]}{[d+t]} = [v]$

or $[c] = [v]\,[d + t]$

or $[c] = [LT^{-1}]\,[T]$

or $[c] = [L]$ Ans.

8.4 To establish the relation among various physical quantities :

If we know the factors on which a given physical quantity may depend, we can find a formula relating the quantity with those factors. Let us take an example.

Ex.7 *The frequency (f) of a stretched string depends upon the tension F (dimensions of force), length* ***l*** *of the string and the mass per unit length* μ *of string. Derive the formula for frequency.*

Sol. Suppose, that the frequency f depends on the tension raised to the power a, length raised to the power b and mass per unit length raised to the power c. Then.

$$f \propto [F]^a [l]^b [\mu]^c$$

or $$f = k[F]^a [l]^b [\mu]^c$$

Here, k is a dimensionless constant. Thus,

$$[f] = [F]^a\,[l]^b\,[\mu]^c$$

or $$[M^0\,L^0\,T^{-1}] = [MLT^{-2}]^a\,[L]^b\,[ML^{-1}]^c$$

or $$[M^0L^0T^{-1}] = [M^{a+c}\,L^{a+b-c}\,T^{-2a}]$$

For dimensional balance, the dimension on both sides should be same.

Thus, $a + c = 0$ ...(ii)

$a + b - c = 0$ ...(iii)

and $-2a = -1$ ...(iv)

Solving these three equations, we get

$$a = \frac{1}{2},\quad c = -\frac{1}{2}\quad \text{and } b = -1$$

Substituting these values in Eq. (i), we get

$$f = k(F)^{1/2}(l)^{-1}(\mu)^{-1/2}$$

or $$f = \frac{k}{l}\sqrt{\frac{F}{\mu}}$$

Experimentally, the value of k is found to be $\frac{1}{2}$

Hence, $$f = \frac{1}{2l}\sqrt{\frac{F}{\mu}}$$

8.5 Limitations of Dimensional Analysis

The method of dimensions has the following limitations :

(i) By this method the value of dimensionless constant can not be calculated.

(ii) By this method the equation containing trigonometrical, exponential and logarithmic terms cannot be analysed.

(iii) If a physical quantity depends on more than three factors, then relation among them cannot be established because we can have only three equations by equalising the powers of M, L and T.

EXERCISE - I

SECTION A : UNITS

1. Which of the following is not the name of a physical quantity ?
(A) kilogram (B) impulse
(C) energy (D) density

2. Light year is the unit of
(A) speed (B) mass
(C) distance (D) time

3. PARSEC is a unit of
(A) Time (B) Angle
(C) Distance (D) Velocity

4. Which of the following system of units is NOT based on the unit of mass, length and time alone
(A) FPS (B) SI
(C) CGS (D) MKS

5. In the S.I. system the unit of energy is-
(A) erg (B) calorie
(C) joule (D) electron volt

6. Unit of pressure in S.I. system is-
(A) atmosphere (B) dynes per square cm
(C) pascal (D) bar

7. In SI unit the angular acceleration has unit of-
(A) $Nmkg^{-1}$ (B) ms^{-2}
(C) $rad.s^{-2}$ (D) Nkg^{-1}

8. The SI unit of the universal gravitational constant G is
(A) $Nm\ kg^{-2}$ (B) Nm^2kg^{-2}
(C) $Nm^2\ kg^{-1}$ (D) $Nmkg^{-1}$

9. Surface tension has unit of-
(A) $Joule.m^2$ (B) $Joule.m^{-2}$
(C) Joule.m (D) $Joule.m^3$

10. The unit of intensity of magnetisation is-
(A) $Amp\ m^2$ (B) $Amp\ m^{-2}$
(C) Amp m (D) $Amp\ m^{-1}$

11. The M.K.S. units of coefficient of viscosity is-
(A) $kg\ m^{-1}s^{-1}$ (B) $kg\ m\ s^{-2}$
(C) $kg\ m^2\ s^{-1}$ (D) $kg^{-1}\ m^{-1}\ s^2$

12. The specific resistance has the unit of-
(A) ohm/m (B) ohm/m^2
(C) $ohm.m^2$ (D) ohm.m

13. The mutual inductance has unit of-
(A) Gauss (B) Weber
(C) Farad (D) Henry

14. The unit of magnetic moment is-
(A) $amp\ m^2$ (B) $amp\ m^{-2}$
(C) amp m (D) $amp\ m^{-1}$

15. The SI unit of the universal gas constant R is :
(A) $erg\ K^{-1}\ mol^{-1}$ (B) $watt\ K^{-1}\ mol^{-1}$
(C) $newton\ K^{-1}\ mol^{-1}$ (D) $joule\ K^{-1}\ mol^{-1}$

16. A physical quantity is measured and the result is expressed as nu where u is the unit used and n is the numerical value. If the result is expressed in various units then
(A) $n \propto$ size of u (B) $n \propto u^2$
(C) $n \propto \sqrt{u}$ (D) $n \propto 1/u$

17. One watt-hour is equivalent to
(A) 6.3×10^3 Joule (B) 6.3×10^{-7} Joule
(C) 3.6×10^3 Joule (D) 3.6×10^{-3} Joule

18. Which of the following statement is wrong ?
(A) Unit of K.E. is Newton-metre
(B) Unit of viscosity is poise
(C) Work and energy have same dimensions
(D) Unit of surface tension is Newton metre

SECTION : B DIMENSIONS

19. What are the dimensions of lenth in force × displacement/time
(A) –2 (B) 0 (C) 2 (D) none of these

20. The angular frequency is measured in rad s^{-1}. Its dimension in length are :
(A) – 2 (B) –1 (C) 0 (D) 2

21 $[M\ L\ T^{-1}]$ are the dimensions of-
(A) power (B) momentum
(C) force (D) couple

22. The dimensional formula for angular momentum is-
(A) ML^2T^{-2} (B) ML^2T^{-1}
(C) MLT^{-1} (D) $M^0L^2T^{-2}$

23. The dimensions of universal gravitational constant are
(A) $M^{-1}\ L^3\ T^{-2}$ (B) $M^{-1}\ L^3\ T^{-1}$
(C) $M^{-1}\ L^{-1}\ T^{-2}$ (D) $M^{-2}\ L^2\ T^{-2}$

24. The SI unit of Stefan's constant is :
(A) $Ws^{-1}\ m^{-2}\ K^{-4}$ (B) $J\ s\ m^{-1}\ K^{-1}$
(C) $J\ s^{-1}\ m^{-2}\ K^{-1}$ (D) $W\ m^{-2}\ K^{-4}$

25. What are the dimensions of Boltzmann's constant?
(A) $MLT^{-2}K^{-1}$ (B) $ML^2T^{-2}K^{-1}$
(C) M^0LT^{-2} (D) $M^0L^2T^{-2}K^{-1}$

26. Dimensions of magnetic flux density is -
(A) $M^1\ L^0\ T^{-1}A^{-1}$ (B) $M^1\ L^0\ T^{-2}A^{-1}$
(C) $M^1\ L^1\ T^{-2}A^{-1}$ (D) $M^1\ L^0\ T^{-1}A^{-2}$

27. A pair of physical quantities having the same dimensional formula is :
(A) angular momentum and torque
(B) torque and energy
(C) force and power
(D) power and angular momentum

28. Dimensions of pressure are the same as that of
(A) force per unit volume
(B) energy per unit volume
(C) force
(D) energy

29. Which one of the following has the dimensions of $ML^{-1}T^{-2}$?
(A) torque (B) surface tension
(C) viscosity (D) stress

30. The dimensions of the quantity $\frac{L}{RCV}$ are -
(A) $M^0\ L^0\ T^1A^1$ (B) $M^0\ L^0\ T^{-1}A^{-1}$
(C) $M^0\ L^0\ T^0A^1$ (D) $M^0\ L^0\ T^0A^{-1}$

31. For $10^{(at+3)}$, the dimension of a is-
(A) $M^0 L^0 T^0$ (B) $M^0 L^0 T^1$
(C) $M^0 L^0 T^{-1}$ (D) None of these

32. The pressure of 10^6 dyne/cm^2 is equivalent to
(A) 10^5 N/m^2 (B) 10^6 N/m^2
(C) 10^7 N/m^2 (D) 10^8 N/m^2

33. The SI unit of length is the meter. Suppose we adopt a new unit of length which equals to
x meters. The area 1m2 expressed in terms of the new unit has a magnitude-
(A) x (B) x^2
(C) $\frac{1}{x}$ (D) $\frac{1}{x^2}$

34. ρ = 2 g/cm^3 convert it into MKS system -
(A) $2 \times 10^{-3} \frac{kg}{m^3}$ (B) $2 \times 10^3 \frac{kg}{m^3}$
(C) $4 \times 10^3 \frac{kg}{m^3}$ (D) $2 \times 10^6 \frac{kg}{m^3}$

35. Given that v is the speed, r is radius and g is acceleration due to gravity. Which of the following is dimension less
(A) $\frac{v^2 g}{r}$ (B) $v^2 rg$ (C) $vr^2 g$ (D) $\frac{v^2}{rg}$

36. Choose the correct statement(s) :
(A) A dimensionally correct equation must be correct.
(B) A dimensionally correct equation may be correct.
(C) A dimensionally incorrect equation may be correct.
(D) A dimensionally incorrect equation may be incorrect.

37. The value of G = 6.67×10^{-11} N m^2 (kg)$^{-2}$. Its numerical value in CGS system will be :
(A) 6.67×10^{-8} (B) 6.67×10^{-6}
(C) 6.67 (D) 6.67×10^{-5}

38. The density of mercury is 13600 kg m^{-3}. Its value of CGS system will be :
(A) 13.6 g cm^{-3} (B) 1360 g cm^{-3}
(C) 136 g cm^{-3} (D) 1.36 g cm^{-3}

BASIC MATHEMATICS

39. The radius of two circles are r and 4r what will be the ratio of their Area and perimeter.

40. Internal radius of a ball is 3 cm and external radius is 4 cm. What will be the volume of the material used.

EXERCISE - II

1. Which of the following sets cannot enter into the list of fundamental quantities in any system of units ?
(A) length, mass and velocity
(B) length, time and velocity
(C) mass, time and velocity
(D) length, time and mass

2. Which of the following is not the unit of time
(A) solar day (B) parallactic second
(C) leap year (D) lunar month

3. The unit of impulse is the same as that of :
(A) moment force
(B) linear momentum
(C) rate of change of linear momentum
(D) force

4. Which of the following is not the unit of energy?
(A) watt-hour (B) electron-volt
(C) N × m (D) kg × m/sec^2

5. If the unit of length is micrometer and the unit of time is microsecond, the unit of velcoity will be :
(A) 100 m/s (B) 10 m/s
(C) micrometers (D) m/s

6. A dimensionless quantity :
(A) never has a unit (B) always has a unit
(C) may have a unit (D) does not exit

7. A unitless quantity :
(A) never has a nonzero dimension
(B) always has a nonzero dimension
(C) may have a nonzero dimension
(D) does not exit

8. If a and b are two physical quantities having different dimensions then which of the following can denote a new physical quantity
(A) a + b (B) a – b
(C) a/b (D) $e^{a/b}$ (E) sin (a/b)

9. Two physical quantities whose dimensions are not same, cannot be :
(A) multiplied with each other
(B) divided
(C) added or substracted in the same expression
(D) added together

10. Choose the correct statement(s) :
(A) All quantities may be represented dimensionally in terms of the base quantities.
(B) A base quantity cannot be represented dimensionally in terms of the rest of the base quantities.
(C) The dimension of a base quantity in other base quantities is always zero.
(D) The dimension of a derived quantity is never zero in any base quantity.

11. Planck's constant has the dimensions of :
(A) force (B) energy
(C) linear momentum (D) angular momentum

12. The velocity 'v' (in cm/s) of a particle is given in terms of time 't' (in s) by the equation

$$v = at + \frac{b}{t+c}$$

The dimensions of a, b and c are

	a	b	c		a	b	c
(A)	L^2	T	LT^2	(B)	LT^2	LT	L
(C)	LT^{-2}	L	T	(D)	L	LT	T^2

13. The position of a particle at time 't' is given by the relation

$$x(t) = \frac{V_0}{\alpha}[1-e^{-\alpha t}]$$

where V_0 is a constant and $\alpha > 0$. The dimensions of V_0 and α are respectively.
(A) $M^0L^1T^0$ and T^{-1} (B) $M^0L^1T^0$ and T^{-2}
(C) $M^0L^1T^{-1}$ and T^{-1} (D) $M^0L^1T^{-1}$ and T^{-2}

14. The time dependence of a physical quantity ?
$P = P_0 \exp(-\alpha t^2)$
where α is a constant and t is time
The constant α
(A) will be dimensionless
(B) will have dimensions of T^{-2}
(C) will have dimensions as that of P
(D) will have dimensions equal to the dimension of P multiplied by T^{-2}

15. Force F is given in terms of time t and distance x by
F = A sin C t + B cos D x
Then the dimensions of $\frac{A}{B}$ and $\frac{C}{D}$ are given by
(A) MLT^{-2}, $M^0L^0T^{-1}$ (B) MLT^{-2}, $M^0L^{-1}T^0$
(C) $M^0L^0T^0$, $M^0L^1T^{-1}$ (D) $M^0L^1T^{-1}$, $M^0L^0T^0$

16. The Van der Waal equation for 1 mole of a real gas is

$$\left(P+\frac{a}{V^2}\right)(V-b)=RT$$

where P is the pressure, V is the volume, T is the absolute temperature, R is the molar gas constant and a, b are Van dar Waal constants. The dimensions of a are the same as those of
(A) PV (B) PV^2 (C) P^2V (D) P/V

17. In above question 16, the dimensions of b are the same as those of
(A) P (B) V (C) PV (D) nRT

18. In above question 16, the dimensions of nRT are the same as those of
(A) energy (B) force
(C) pressure (D) specific heat

19. In above question 16, the dimensional formula for ab is
(A) ML^2T^{-2} (B) ML^4T^{-2} (C) ML^6T^{-2} (D) ML^8T^{-2}

20. Which pair of following quantities has dimensions different from each other.
(A) Impulse and linear momentum
(B) Plank's constant and angular momentum
(C) Moment of inertia and moment of force
(D) Young's modulus and pressure

21. If force (F) is given by $F = Pt^{-1} + \alpha t$, where t is time. The unit of P is same as that of
(A) velocity (B) displacement
(C) acceleration (D) momentum

22. The product of energy and time is called action. The dimensional formula for action is same as that for
(A) power (B) angular energy
(C) force × velocity (D) impulse × distance

23. When a wave traverses a medium, the displacement of a particle located at x at time t is given by
$$y = a \sin (bt - cx)$$
where a, b and c are constants of the wave. The dimensions of b are the same as those of
(A) wave velocity (B) amplitude
(C) wavelength (D) wave frequency

24. In the above question dimensions of $\frac{b}{c}$ are the same as those of
(A) wave velocity (B) wavelength
(C) wave amplitude (D) wave frequency

25. What is the physical quantity whose dimensions are $M L^2 T^{-2}$?
(A) kinetic energy (B) pressure
(C) momentum (D) power

26. If force, acceleration and time are taken as fundamental quantities, then the dimensions of length will be :
(A) FT^2 (B) $F^{-1} A^2 T^{-1}$ (C) FA^2T (D) AT^2

27. The dimensions $ML^{-1}T^{-2}$ can correspond to
(A) moment of a force or torque
(B) surface tension
(C) pressure
(D) co-efficient of viscosity
(useful relation are $\vec{\tau} = \vec{r} \times \vec{F}$, $S = F/l$, $F = 6 \pi \eta r v$, where symbols have usual meaning)

28. Which of the following can be a set of fundamental quantities
(A) length, velocity, time
(B) momentum, mass, velocity
(C) force, mass, velocity
(D) momentum, time, frequency

29. If area (A) velocity (v) and density (ρ) are base units, then the dimensional formula of force can be represented as
(A) $Av\rho$ (B) $Av^2\rho$ (C) $Av\rho^2$ (D) $A^2v\rho$

30. In a certain system of units, 1 unit of time is 5 sec, 1 unit of mass is 20 kg and unit of length is 10m. In this system, one unit of power will correspond to
(A) 16 watts (B) 1/16 watts
(C) 25 watts (D) none of these

31. In a book, the answer for a particular question is expressed as $b = \frac{ma}{k}\left[\sqrt{1+\frac{2kl}{ma}}\right]$ here m represents mass, a represents accelerations, l represents length. The unit of b should be
(A) m/s (B) m/s² (C) meter (D) /sec

32. $\alpha = \frac{F}{V^2}\sin(\beta t)$ (here V = velocity, F = force, t = time) : Find the dimension of α and β -
(A) $\alpha = [M^1L^1T^0]$, $\beta = [T^{-1}]$
(B) $\alpha = [M^1L^1T^{-1}]$, $\beta = [T^1]$
(C) $\alpha = [M^1L^1T^{-1}]$, $\beta = [T^{-1}]$
(D) $\alpha = [M^1L^{-1}T^0]$, $\beta = [T^{-1}]$

33. If E, M, J and G denote energy, mass, angular momentum and gravitational constant respectively, then $\frac{EJ^2}{M^5G^2}$ has the dimensions of
(A) length (B) angle (C) mass (D) time

34. The dimensions $ML^{-1}T^{-2}$ may correspond to
(A) work done by a force (B) linear momentum
(C) pressure (D) energy per unit volume

35. The velocity of water waves may dpend on their wavelength λ, the density of water ρ and the acceleration due to gravity g. The method of dimensions gives the relation between these quantities as
(A) $v^2 = k\lambda^{-1} g^{-1} \rho^{-1}$
(B) $v^2 = k\, g\, \lambda$
(C) $v^2 = k\, g\, \lambda\, \rho$
(D) $v^2 = k\, \lambda^3 g^{-1} \rho^{-1}$ where k is a dimensionless constant

36. If the unit of force is 1 kilonewton, the length is 1 km and time is 100 second, what will be the unit of mass :
(A) 1000 kg (B) 10 kg
(C) 10000 kg (D) 100 kg

37. A body moving through air at a high speed 'v' experiences a retarding force 'F' given by $F = K A d v^x$ where 'A' is the surface area of the body, 'd' is the density of air and 'K' is a numerical constant. The value of 'x' is :
(A) 1 (B) 2 (C) 3 (D) 4

38. The velocity of a freely falling body changes as $g^p h^q$ where g is acceleration due to gravity and h is the height. The values of p and q are :
(A) $1, \frac{1}{2}$ (B) $\frac{1}{2}, \frac{1}{2}$
(C) $\frac{1}{2}, 1$ (D) 1, 1

39. If the acceleration due to gravity is 10 ms^{-2} and the units of length and time are changed to kilometre and hour, respectively, the numerical value of the acceleration is :
(A) 360000 (B) 72000
(C) 36000 (D) 129600

40. If 'c' the velocity of light 'g' the acceleration due to gravity and 'P" the atmospheric pressure are fundamental units, then the dimensions of length will be
(A) c/g (B) P × c × g (C) c/P (D) c^2/g

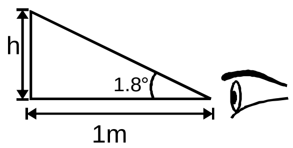
41. The units of length, velocity and force are doubled. Which of the following is the correct change in the other units ?
(A) unit of time is doubled
(B) unit of mass is doubled
(C) unit of momentum is doubled
(D) unit of energy is doubled

42. If the units of force and that of length are doubled, the unit of energy will be :
(A) 1/4 times (B) 1/2 times
(C) 2 times (D) 4 times

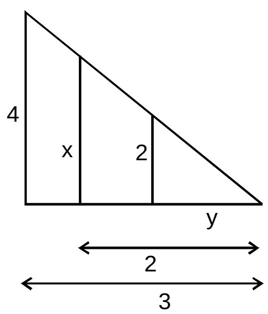
43. If the units of M, L are doubled then the unit of kinetic energy will become
(A) 2 times (B) 4 times
(C) 8 times (D) 16 times

44. Binomial
(a) $(99)^{1/2}$ (b) $(120)^{1/2}$ (c) $(126)^{1/3}$

45. A normal human eye can see an object making an angle of 1.8° at the eye. What is the approximate height of object which can be seen by an eye placed at a distance of 1 m from the eye.



46. Draw graph for following equations :
(i) $v = v_0 - at$ (ii) $x = 4t - 3$
(iii) $x = 4\ at^2$ (iv) $v = -gt$

47. 

Find x and y :

48. Which of the following is not the unit length :
(A) micron (B) light year
(C) angstrom (D) radian

49. A particle is in a uni-directional potential field where the potential energy (U) of a particle depends on the x-coordinate given by $U_x = k(1 - \cos ax)$ & k and 'a' are constants. Find the physical dimensions of 'a' & k.

50. The time period (T) of a spring mass system depends upon mass (m) & spring constant (k) & length of the spring (l) $[k = \frac{\text{Force}}{\text{length}}]$. Find the relation among, (T), (m), (l) & (k) using dimensional method.

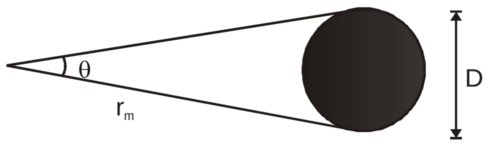
51. The equation of state for a real gas at high temperature is given by $P = \frac{nRT}{V-b} - \frac{a}{T^{1/2}V(V+b)}$ where n, P V & T are number of moles, pressure, volume & temperature respectively & R is the universal gas constant. Find the dimensions of constant 'a' in the above equation.

52. The distance moved by a particle in time t from centre of a ring under the influence of its gravity is given by x = a sinωt where a & ω are constants. If ω is found to depend on the radius of the ring (r), its mass (m) and universal gravitational constant (G), find using dimensional analysis an expression for ω in terms of r, m and G.

53. If the velocity of light c, Gravitational constant G & Plank's constant h be chosen as fundamental units, find the dimension of mass, length & time in the new system.

54. A satellite is orbiting around a planet. Its orbital velocity (v_0) is found to depend upon
(a) Radius of orbit (R)
(b) Mass of planet (M)
(c) Universal gravitation constant (G)
Using dimensional analysis find an expression relating orbital velocity (v_0) to the above physical quantities.

55. The angle subtended by the moon's diameter at a point on the earth is about 0.50°. Use this and the act that the moon is about 384000 km away to find the approximate diameter of the moon.



(A) 192000 km (B) 3350 km
(C) 1600 km (D) 1920 km

56. Use the approximation $(1 + x)^n \approx 1 + nx$, $|x| << 1$, to find approximate value for
(a) $\sqrt{99}$ (b) $\frac{1}{1.01}$

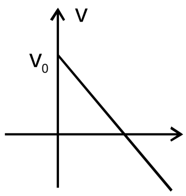
57. Use the small angle approximations to find approximate values for
(a) sin 8° and (b) tan 5°

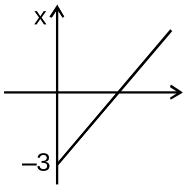
EXERCISE - I

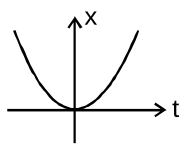
1.	A	2.	C	3.	C	4.	B	5.	C	6.	C	7.	C	8.	B
9.	B	10.	D	11.	A	12.	D	13.	D	14.	A	15.	D	16.	D
17.	C	18.	D	19.	C	20.	C	21.	B	22.	B	23.	A	24.	D
25.	B	26.	B	27.	B	28.	B	29.	D	30.	D	31.	C	32.	A
33.	D	34.	B	35.	D	36.	ABD	37.	A	38.	A	39.	$\frac{1}{16}, \frac{1}{4}$	40.	$\frac{148\pi}{3}$

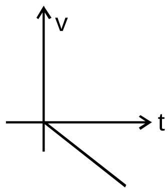
EXERCISE - II

1.	B	2.	B	3.	B	4.	D	5.	D	6.	C	7.	A	8.	C
9.	C	10.	ABC	11.	D	12.	C	13.	C	14.	B	15.	C	16.	B
17.	B	18.	A	19.	D	20.	C	21.	D	22.	D	23.	D	24.	A
25.	A	26.	D	27.	C	28.	C	29.	B	30.	A	31.	C	32.	D
33.	B	34.	CD	35.	B	36.	C	37.	B	38.	B	39.	D	40.	D
41.	C	42.	D	43.	C	44.	(a) 9.9498		(b) 10.954		(c) 5.0132	45.	π cm		

46. (i)


(ii)


(iii)


(iv)


47. $x = \frac{8}{3}, y = \frac{3}{2}$ 48. D 49. L^{-1}, ML^2T^{-2} 50. $T = a\sqrt{\frac{m}{k}}$ 51. $ML^5T^{-2}K^{1/2}$ 52. $\omega = K\sqrt{\frac{Gm}{r^3}}$

53. $[M] = [h^{1/2}.c^{1/2}.G^{-1/2}]$; $[L] = [h^{1/2}.c^{-3/2}.G^{1/2}]$; $[T] = [h^{1/2}.c^{-5/2}.G^{1/2}]$ 54. $v_0 = k\sqrt{\frac{Gm}{R}}$

55. B 56. (a) 9.95, (b) 0.99 57. 0.14, 0.09

KINEMATICS

Kinematics is such a branch of mechanics which deals with study of motion of a body without going into the cause of motion such as force or torque. It only analyses the effect caused in the motion of the body (**variations in the motional parameters**) due to applications of force or torque.

For detailed analysis of kinematics it is divided into three parts

1. Motion in one Dimension (Motion along a straight line)
   Requires only one co-ordinate
2. Motion in two Dimensions (Motion in a plane)
   Requires minimum two co-ordinates
3. Motion in three Dimensions (Motion in space)
   Requires all the three co-ordinates

MOTION IN ONE DIMENSION

Displacement

The **displacement** of a particle is defined as the difference between its final position and its initial position. We represent the displacement as Δx.

$$\Delta x = x_f - x_i$$

The subscripts *i* and *f* refer to be ***initial*** and ***final*** positions. These are not necessarily the positions from which the particle starts its motion nor where its motion ceases. The *i* and *f* designate the particular initial and final positions we are considering out of the entire motion of the object.

Note *The order : final position minus initial position . Whenever we calculate "delta" anywhere, we always take the final value minus the initial value.*

Average Velocity and Average Speed

The *average velocity* of an object travelling along the *x*–axis is defined as the ratio of its displacement to the time taken for that displacement.

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

The **average speed** of a particle is defined as the ratio of the total distance travelled to the total time taken.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\Delta t}$$

Note: that *velocity* and *speed* have *different meanings*.

If a body moves along a straight line and its direction does not changes then in this case

- Distance and displacement have same value
- Velocity and speed have also same value
- Average velocity $\le$ average speed and displacement $\le$ distance is always true. The equality of the quantities is true only when the motion is taking place on a straight line and that too only along one direction.

One-dimensional motion can be classified in two parts

(a) Non-accelerated motion **(b) Acceleration motion**

One dimensional motion

- Non-accelerated (uniform velocity)
- Accelerated (non-uniform velocity)
  - Constant acceleration (uniform acceleration)
  - Variable acceleration (non-uniform acceleration)

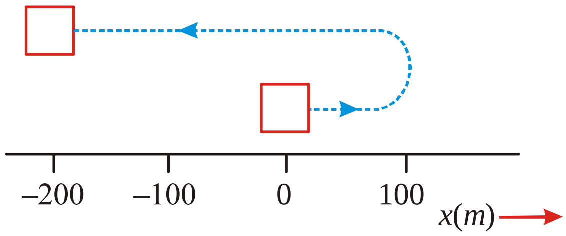
Ex. 1 A bird flies toward east at 10 *m/s* for 100 *m*. It then turns around and flies at 20 *m/s* for 15 *s*. Find

(a) its average speed

(b) its average velocity

Solution Let us take the *x* axis to point east. A sketch of the path is shown in the figure. To find the

Required quantities, we need the total time interval. The first part of the journey took Δt_1 = (100 *m*)/ (10 *m/s*) = 10 *s*, and we are given t_2 = 15 *s* for the second part. Hence the total time interval is



$\Delta t = \Delta t_1 + \Delta t_2 = 25$ s

The bird flies 100 *m* east and then (20 *m/s*) (15 *s*) = 300 m west

(a) Average speed = $\frac{Distance}{\Delta t} = \frac{100m+300m}{25s} = 16m/s$

(b) The net displacement is $x = \Delta x_1 + \Delta x_2$ = 100 *m* – 300 *m* = 200 *m*

So that $v_{av} = \frac{\Delta x}{\Delta t} = \frac{-200m}{25s} = -8$ *m/s*. The *negative sign* means that v_{av} is directed toward the west.

CAUTION

Sometimes students try to calculate the average velocity by just adding the two given velocities and dividing by two. This procedure is wrong, and it can be clearly illustrated with the following example.

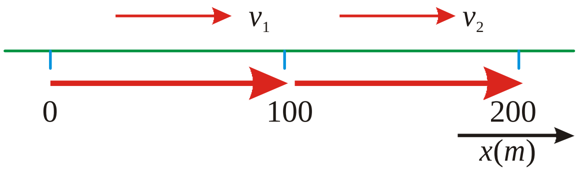
A college student drives a car 1 kilometer at 30 *kmph*. How fast must the student drive a second kilometer in order to average 60 *kmph* for the 2 minute trip.

If you believe that the average velocity is the average of the velocities, then the answer will be 60 *kmph* because $\frac{30+90}{2} = 60$ *kmph*. But the correct answer is "***not possible***". There is *no way the student can average 60 kmph for the trip!* Sixty kmph means 1 *km/min*. In order to average 60 kmph for 2 *km*, the trip must be driven in 2 *min*. But going the first kilometer at 30 *kmph* takes 2 *min*. So the driver has no time left at all to go the second kilometer.

Ex. 2 A jogger runs his first 100 *m* at 4 *m*/*s* and the second 100 *m* at 2 *m*/*s* in the same direction. What is the average velocity?

Solution A sketch of his motion is shown in figure. His net displacement

$\Delta x = \Delta x_1 + \Delta x_2 = 100\ m + 100\ m = 200\ m$



The first half took $\Delta t_1 = (100\ m)/(4\ m/s) = 25\ s$,

while the second took $\Delta t_1 = (100\ m)/(2\ m/s) = 50\ s$,

The total time interval is $\Delta t = \Delta t_1 + \Delta t_2 = 75\ s$

Therefore, his average velocity is $v_{av} = \dfrac{\Delta x}{\Delta t} = \dfrac{200m}{75s} = 2.67\ m/s$

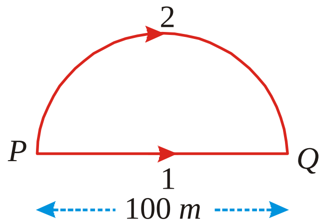
Since $2.67 \neq \dfrac{1}{2}(4+2)$, we see that the average velocity is not, in general, equal to the

average of the velocities.

Ex. 3 Ram takes path 1 (straight line) to go from *P* to *Q* and Shyam takes path 2 (semicircle).

(a) Find the distance travelled by Ram and Shyam?

(b) Find the displacement of Ram and Shyam?



Solution (a) Distance travelled by Ram = 100 *m*

Distance travelled by Shyam = $\pi(50\ m) = 50\pi\ m$

(b) Displacement of Ram = 100 *m*

Displacement of Shyam = 100 *m*

Average Acceleration is defined as the ratio of the change in velocity to the time taken.

$$a_{av} = \frac{\Delta x}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

When the time interval in the average calculation becomes too small such that $\Delta t \to 0$ then this situation represents a point. Now we can state this situation as an instant.

Instantaneous Velocity is defined as the value approached by the average velocity when the time interval for measurement becomes closer and closer to zero, i.e. $\Delta t \to 0$. Mathematically

$$v(t) = \lim_{\Delta t \to 0} v_{av} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

The *instantaneous velocity function* is the derivative with respect to the time of the displacement function.

$$v(t) = \frac{dx(t)}{dt}$$

Instantaneous Acceleration is defined analogous to the method for defining instantaneous velocity. That is, instantaneous acceleration is the value approached by the average acceleration as the time interval for the measurement becomes closer and closer to zero.

The *Instantaneous acceleration* function is the derivative with respect to time of the velocity function

$$a(t) = \frac{dv(t)}{dt}$$

Ex. 4 The position of a particle is given by $x = 40 - 5t - 5t^2$, where x is in metre and t is in seconds.

(a) Find the average velocity between 1 s and 2 s

(b) Find its instantaneous velocity at 2 s

(c) Find its average acceleration between 1 s and 2 s

(d) Find its instantaneous acceleration at 2 s

Solution (a) At $t = 1$ s $\quad x_i = 30$ m

$t = 2$ s $\quad x_f = 10$ m

$$v_{av} = \frac{x_f - x_i}{t_f - t_i} = \frac{10-30}{2-1} = -20 \ m/s$$

(b) $v = \frac{dx}{dt} = -5 - 10t$

At $t = 2$ s $\quad v = -5 - 10(2) = -25$ m/s

(c) At $t = 1$ s $\quad v_i = -5 - 10(1) = -15$ m/s

$t = 2$ s $\quad v_f = -5 - 10(1) = -25$ m/s

$$v_{av} = \frac{v_f - v_i}{t_f - t_i} = \frac{-25-(-15)}{2-1} = -10 \ m/s^2$$

(d) $a = \frac{dv}{dt} = -10$ m/s^2

Ex. 5 A particle travels half of total distance with speed v_1 and next half with speed v_2 along a straight line. Find out the average speed of the particle?

Solution Let total distance travelled by the particle be $2s$.

Time taken to travel first half $= \frac{s}{v_1}$

Time taken to travel next half $= \frac{s}{v_2}$

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{2s}{\frac{s}{v_1} + \frac{s}{v_2}} = \frac{2v_1 v_2}{v_1 + v_2}$$

UNIFORM MOTION

- definition of velocity $v = \frac{\Delta d}{\Delta t}$
- Solve for displacement in terms of velocity and time $\Delta d = v\Delta t$

Features of Uniform motion

- Uniform motion is a straight-lime motion with constant velocity.
- In uniform motion, displacement and distance are equal.
- The average and instantaneous velocities have same values in uniform motion.
- No net force is required for an object to be in uniform motion.
- The velocity in uniform motion does not depend upon the time interval
- The velocity in uniform motion is independent of the choice of origin.

UNIFORMLY ACCELERATED MOTION

- definition of acceleration $a = \frac{\Delta v}{\Delta t}$ or $a = \frac{v_2 - v_1}{\Delta t}$
- Solve for final velocity in terms of initial velocity, acceleration and time interval. $v_2 = v_1 + a\Delta t$
- displacement in terms of initial velocity, final velocity, and time interval $\Delta d = \frac{(v_1 + v_2)}{2}\Delta t$
- displacement in terms of initial velocity, acceleration, and time interval $\Delta d = v_1\Delta t + \frac{1}{2}a\Delta t^2$
- final velocity in terms of initial velocity, acceleration, and displacement $v_2^2 = v_1^2 + 2a\Delta d$
- Using derivative concept we can have $a = \frac{dv}{dt}$
- by using integration technique $\Rightarrow dv = adt \Rightarrow \int dv = \int a\,dt$.
- Velocity varies from u (initial) to v (final) $\Rightarrow \int_u^v dv = \int_0^t a\,dt \Rightarrow v - u = at$

$$\boldsymbol{v = u + at}$$

- Velocity is given by rate of change of displacement $v = \frac{dx}{dt}$ $(v = u + at)$

$$\Rightarrow dx = v\,dt \Rightarrow dx = (u + at)dt$$

- by using integration technique $\int_{x_0}^{x} dx = \int_0^t (u + at)dt$
- x_0 is the initial position $\Rightarrow x - x_0 = ut + \frac{1}{2}at^2$

$$\Rightarrow \boldsymbol{x = x_0 + ut + \frac{1}{2}at^2}$$

- if $x_0 = 0$ (origin) $\Rightarrow x = ut + \frac{1}{2}at^2$
- if $x_0 = 0$ and $u = 0$ (at rest at origin) $\Rightarrow x = \frac{1}{2}at^2$

◆ Eliminating t (time) from the above two equations we get	$v^2 = u^2 + 2as.$
◆ In addition to the above derived equations, by proper substitutions we can also have	$x = x_0 + \frac{1}{2}(u+v)t$

Problem Solving Strategy

i. Always try to make a simple sketch of the situation described

ii. Set up a co–ordinate system and clearly indicate the origin.

iii. (a) List the given quantities with appropriate signs. (by using vector analysis properties)

(b) List the unknown quantities.

iv. Find the equation that has the quantity you need as the only unknown. (This is not always possible). But you need to generate an idea about the situation.

v. It is often helpful to obtain a rough graphical solution. (Try to obtain graphs for different situations.)

vi. Solve the equation (s) to find the desired unknown(s).

Ex. 6 A car accelerates with a constant acceleration from rest to 30 *m/s* in 10 *s*. It then continues at a constant velocity. Find

(a) its acceleration

(b) how far it travels while speeding up

(c) the distance it covers while its velocity changes from 10 *m/s* to 20 *m/s*

Solution

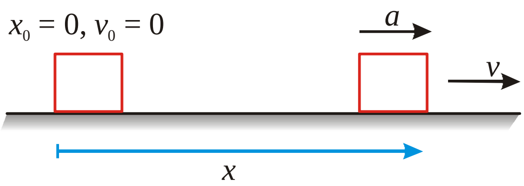


Figure (a)

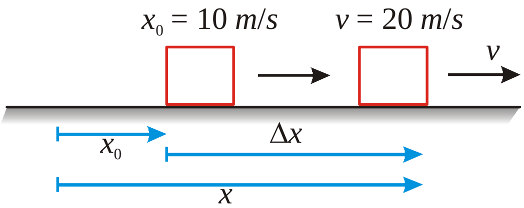


Figure (b)

(a)Given

$v_0 = 0$ $\qquad a = \frac{v - v_0}{t}$

$v = 30\ m/s$ $\qquad \Rightarrow a = \frac{30-0}{10} = +3\ m/s^2$

$t = 10s$

(b)Given

$v_0 = 0$ $\qquad x = v_0 t + 0.5at^2$

$v = 30$ m/s $\qquad \Rightarrow x = 0.5\ at^2$

$t = 10$ s $\qquad \Rightarrow x = 0.5 \times 3 \times 100 = 150\ m$

$a = 3$ m/s^2

we can also solve by using $x = x_0 + 0.5\,(v_0 + v)\,t$

$\Rightarrow \ x = 0 + 0.5\,(30 + 0) \times 10 = 150\ m$

(c) Given

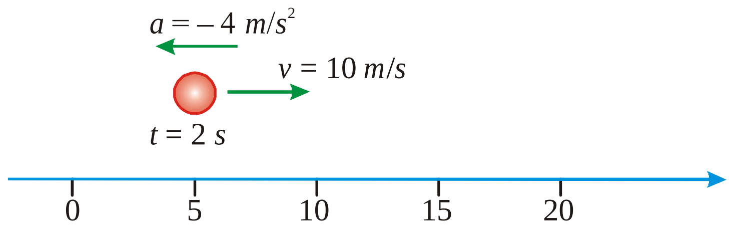
$v_0 = 10\ m/s$ $\qquad v^2 = v_0^2 + 2a\,(\Delta x)$

$v = 20\ m/s$ $\qquad \Rightarrow\ 400 = 100 + 6\,(\Delta x)$

$a = 3\ m/s^2$ $\qquad \Rightarrow\ \Delta x = 50m$

We don't neet to find the total displacement according to the situation given

Ex. 7 A particle is at $x = 5\ m$ at $t = 2\ s$ and has a velocity $v = 10\ m/s$. Its acceleration is constant at $-4\ m/s^2$. Find the initial position at $t = 0$



Solution

Given

$x = 5\ m$

$v = 10\ m/s$

$a = -4\ m/s^2$

$t = 2\ s$

by using $v = v_0 + at$

$10 = v_0 + (-4 \times 2)$

$\Rightarrow\ v_0 = 18\ m/s$

by using $x = x_0 + 0.5\,(v_0 + v)t$

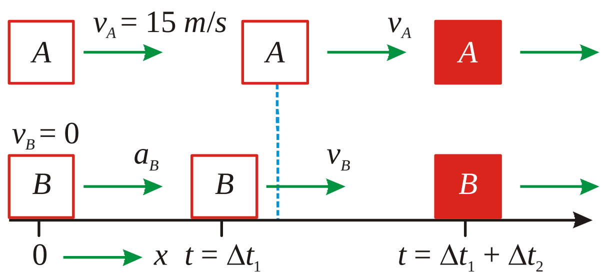
$\Rightarrow\ 5 = x_0 + 0.5\,(18 + 10) \times 2$

$\Rightarrow\ x_0 = -23\ m$

Ex. 8 Car A moves at a constant velocity 15 m/s. Another car B starts from rest just as the car A passes it. The car B accelerates at 2 m/s^2 until it reaches its maximum velocity of 20 m/s. Where and when does the car A is caught by other car B?

Solution When two particles are involved in the same problem, we use simple subscripts to distinguish the variables, as shown in figure. The motion of the car B has two phases: one at constant acceleration and other at constant velocity. In such problems it is convenient to use Δt instead of t in the equations.

The car B may or may not catch the car A during the acceleration phase. This has to be checked. We set the origin at the car B, which means $x_{0A} = x_{0B} = 0$



Acceleration Phase:

Given

$v_A = 15\ m/s$ $\quad$ $v_B = u_B + a_B t$

$a_B = 2\ m/s^2$ $\quad$ $\Rightarrow t = \frac{v_B}{a_B} = \frac{20}{2} = 10$ s

$u_B = 0$

$v_B = 20\ m/s$

At this time, the positions are given by $x = x_0 + v_0 t + \frac{1}{2} at^2$

$x_A = (15)(10) = 150\ m$

$x_B = \frac{1}{2}(2)(10)^2\ 100\ m$

The car A is still ahead

Constant velocity phase

Given

$x_{0A} = 150\ m$

$x_{0B} = 100\ m$

$v_A = 15\ m/s$

$v_B = 20\ m/s$

The cars meet when they have same position $x_A = x_B$

$x_A = 150 + 15(\Delta t_2)$

$x_B = 100 + 20(\Delta t_2)$

$\Rightarrow 150 + 15(\Delta t_2) = 100 + 20(\Delta t_2)$

$\Rightarrow \Delta t_2 = 10$ s

Also, $x_A = x_B = 300\ m$

Ex. 9 A 150 *m* long train accelerates uniformly from rest. If the front of the train passes a railway worker 50 *m* away from the station at a speed of 25 *m/s*, what will be the speed of the back part of the train as it passes the worker?

Solution $v^2 = u^2 + 2as$

$25 \times 25 = 0 + 100\ a$

$a = \frac{25}{4} m/s^2$

Now, for time taken by the back end of the train to pass the worker

we have $v'^2 = v^2 + 2al$

$\Rightarrow v'^2 = (25)^2 + 2 \times \frac{25}{4} \times 150$

$\Rightarrow v'^2 = 25 \times 25 \times 4$

$\Rightarrow v' = 50\ m/s$ **Ans.**

Ex. 10 A particle moving rectilinearly with constant acceleration is having initial velocity of 10 *m/s*. After some time, its velocity becomes 30 *m/s*. Find out velocity of the particle at the mid point of its path?

Solution: Let the total distance be $2x$.

$\therefore$ distance upto midpoint = x

Let the velocity at the mid point be v and acceleration be a.

From equations of motion

$v^2 = 10^2 + 2ax$

$30^2 = v^2 + 2ax$

Subtracting

$v^2 - 30^2 = 10^2 - v^2 \Rightarrow v^2 = 500$

$\Rightarrow v = 10\sqrt{5}$ m/s

Ex. 11 A police inspector in a jeep is chasing a pickpocket on a straight road. The jeep is going at its maximum speed *v* (assumed uniform). The pickpocket rides on the motorcycle of a waiting friend when the jeep is at a distance *d* away, and the motorcycle starts with a constant acceleration *a*. Show that the pick pocket will be caught if $v \geq \sqrt{2ad}$.

Solution: Suppose the pickpocket is caught at a time t after motorcycle starts. The distance travelled by the motorcycle during this interval is

$$s = \frac{1}{2}at^2$$

During this interval the jeep travels a distance

$s + d = vt$ From the two equations

$0.5at^2 + d = vt$ (a quadratic equation in t).

$$\Rightarrow \quad t = \frac{v \pm \sqrt{v^2 - 2ad}}{a}$$

The pickpocket will be caught if t is real and positive.

This will be possible if $v^2 \geq 2ad \Rightarrow v \geq \sqrt{2ad}$

FREE FALL

Motion that occurs solely under the influence of gravity is called ***free fall.***

In the absence of air resistance all falling bodies have the same acceleration due to gravity, regardless of their sizes or shapes.

The value of the acceleration due to gravity depends on both l*atitude* and *altitude*. It is approximately 9.8 m/s^2 near the surface of the earth. For simplicity a value of 10 m/s^2 is being used in this package.

$v = u + at \quad \Rightarrow \quad v = u - gt$

$s = s_0 + ut + 0.5\, at^2 \quad \Rightarrow \quad s = s_0 + ut - 0.5\, gt^2$

$v^2 = u^2 + 2as \quad \Rightarrow \quad v^2 = u^2 - 2gs$

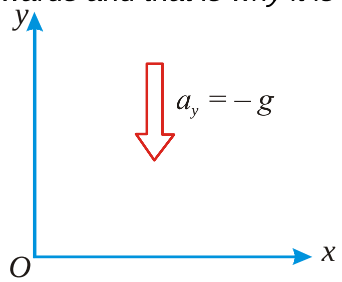
Negative sign of g is taken by considering the upward direction as always positive and downward direction as always negative.

We need to use the same kinematical equations as derived earlier for uniformly accelerated motion. ***Motion under gravity is also uniformly accelerated motion.***

The only differences is that we need to replace a by $(-g)$.

The signs of v and v_0 are determined by their directions relative to the chosen $+y$ axis.

Note : *Sign of the acceleration does not depend on whether the body is going up or coming down, because g always acts downwards and that is why it is always taken to be negative.*



At the highest point of motion the object momentarily comes to rest.

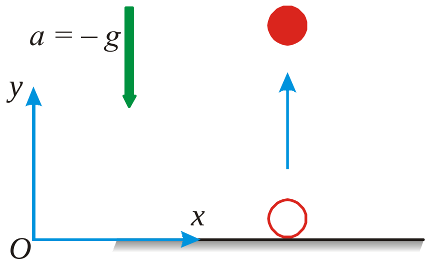
$\Rightarrow$ At h_{max} $v = 0$

But even $v = 0$ then also g is acting downwards.

⇒ **In motion under gravity acceleration never goes to zero.**

Ex. 12 A ball thrown up from the ground reaches a maximum height of 20 *m*. Find

(a) its initial velocity
(b) the time taken to reach the highest point
(c) its velocity just before hitting the ground
(d) its displacement between 0.5 and 2.5 *s*
(e) the time at which it is 15 *m* above the ground.



Solution

At the highest point the ball is instantaneously at rest, that is $v = 0$

(a) Given $\quad$ using $v^2 = u^2 + 2as$

$y_0 = 0 \quad \Rightarrow 0 = u^2 + 2(-10) \times 20$

$y = 20\ m \quad \Rightarrow u^2 = 400$

$v = 0 \quad \Rightarrow u = 20\ m/s$

u is $+ve$ because initially the motion is upwards.

(b) by using $v = u + at \quad \Rightarrow 0 = 20 - 10t \Rightarrow t = 2s$

(c) Given $\quad$ by using $v^2 = u^2 + 2a\,(\Delta s)$

$y_0 = 0 \quad \Rightarrow v^2 = (20)^2 - 2\,(10)\,(0 - 0)$

$y = 0 \quad \Rightarrow v^2 = 400 \Rightarrow v = \pm\, 20\ m/s$

$u = 20\ m/s$

At the single point $y = 0$, the velocity has two values $+u$ initially and $-u$ when it lands

$\Rightarrow v = \pm\, u$

(d) To find the displacement, $\Delta y = y_2 - y_1$

by using $s = ut + 0.5\ at^2$

$y_1 = 20(0.5) - 5\,(0.5)^2 = 8.75\ m$ and $\quad y_2 = 20(2.5) - 5\,(2.5)^2 = 18.75\ m$

Hence $\Delta y = +\,10\ m$

or at $t = 0.5$ sec.

$v = 20 - 10 \times 0.5 = 20 - 5 = 15\ m/s \quad (v = u - gt)$

$\Delta y = (15 + \Delta t) + 0.5\,(-10) \times \Delta t^2 \quad (s = ut - 0.5\ gt^2)$

$\Rightarrow \Delta y = 15 \times 2 - 5 \times 4 = 30 - 20 = 10\ m$

(e) Given $\quad$ by using $s = ut + 0.5\ at^2$

$y = 15\ m \quad 15 = 20t - 5t^2$

$y_0 = 0 \quad \Rightarrow t = 1$ sec and 3 sec.

$u = 20\ m/s$

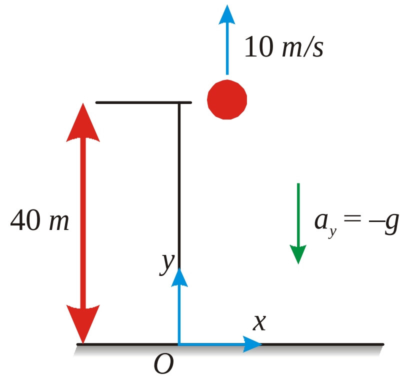
Ex. 13 A ball is thrown upward with an initial velocity of 10 *m/s* from a rooftop 40 *m* high. Find

(a) its velocity on hitting the ground. (b) the time of flight
(c) the maximum height (d) the time to return to roof level
(e) the time it is 15 *m* below the rooftop.

Solution The origin is assumed at the ground level so that all positions are positive

Given: $y_0 = 40\ m$

$u = +10\ m/s$



(a) When the ball lands, its final position coordinate is $y = 0$

by using $v^2 = u^2 + 2a\,(\Delta s)$

$\Rightarrow\ v^2 = 10^2 + 2(-10)\,(0 - 40)$

$\Rightarrow\ v^2 = 100 + 800 = 900.$

$\Rightarrow\ v = -\,30\ m/s$ ($-ve$ because final velocity is directed downwards)

(b) by using $v = u + at$

$-\,30 = 10 - 10t.$

$\Rightarrow\ 10t = 40\ \Rightarrow\ t = 4\ sec.$

or by using $\Delta y = ut + 0.5\ at^2$

$-\,40 = 10t - 5t^2$ $\Rightarrow t = 4$ sec.

(c) At the maximum height $v = 0$

by using $v^2 = u^2 + 2a\,(\Delta y)$ $\Rightarrow 0 = (10)^2 + 2\,(-10)\,(y - 40)$

Thus $y = 45\ m.$

(d) At the roof level, the final position is $y = 40\ m$

using $\Delta y = ut + 0.5at^2$

$\Rightarrow\ 0 = 10t - 5t^2$ $\Rightarrow t = 0$ or 2 sec

we pick $t = 2\ s.$

(e) $\Delta y = -\,15\ m$

$\Rightarrow -\,15 = 10t - 5t^2$ $\Rightarrow t = 3$ sec.

Ex. 14 Two balls are thrown towards each other, ball A at 20 m/s upward from the ground, and two second later ball B at 10 m/s downward from a roof 30 m high.

(a) Where and when do they meet

(b) What are their velocities on impact?

Solution Remember in this kind of problem we have to find *when* before *where.* We need to write the general

expression for the position coordinates. The coordinate system is shown in figure.

(a) Given $u_A = 20\ m/s$ $u_B = -\,10\ m/s$

If A has been in motion for time t, then B has been in motion for time $(t - 2)$.

$y_A = 20t - 5t^2$ $y_B = 30 - 10(t-2) - 5(t-2)^2$

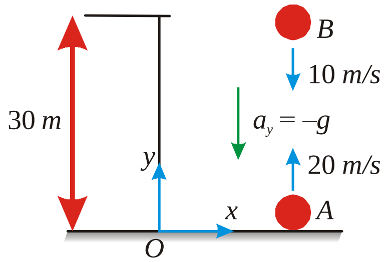
They meet when $y_A = y_B$. This condition immediately leads to $t = 3$ s. Substituting into either y_A or y_B

gives $y = 15\ m$

(b) Since $t = 3$ s, we have $v_A = 20 + (-10)\,(3) = -10$ m/s

and v_B = –10 + (–10) (3–2) = –20 *m/s*. Observe, *A* is already moving downward when it

collides with *B*.



Ex. 15 A girl is standing in an elevator that is moving upward at a velocity of 5m/s and acceleration 2m/s^2, when she drops her handbag. If she was originally holding the bag at a height of 1.5*m* above the elevator floor, how long will it take the bag to hit the floor.

Solution In such problems, generally called elevator problems we can use the concept of relative acceleration. Here we solve the problem with respect to elevator or we assume that we are observing from inside of elevator.

Imagine the situation, if you are standing in an elevator accelerating up with an acceleration *a*. If you are holding a box in your hand, it is also accelerating up with the same acceleration. If you release it free, it falls with acceleration *g* towards the elevator floor, which is coming up with acceleration *a*.

Here we can say that the approach acceleration of box towards the elevator floor is (*g + a*), and we assume that elevator floor is at rest and box is going down with respect to floor with this acceleration called reltive acceleration. Similarly if elevator is going down, we take relative acceleration (*g – a*).

In this problem, elevator is going up with a velocity 5m/s, and an acceleration 2*m/s*^{2}, when the bag is dropped. Bag's relative velocity with respect to elevator floor is zero as both at that instant have the same velocity but relative acceleration of the bag is taken as 10 + 2 = 12*m/s*^{2}.

The distance fallen by the bag is 1.5 *m*, as we are assuming elevator is at rest

Thus time required is $\sqrt{\frac{2H}{g}} = \sqrt{\frac{2\times1.5}{10}} = 0.55s$.

***Note :** If we carefully imagine the situation from outside the elevator, bag has covered the actual distance less than 1.5 m.*

Ex. 16 A particle is dropped from a tower. It is found that it travels 45 *m* in the last second of its journey. Find out the height of the tower?

Solution: Let the total time of journey be n seconds.

Using; $s_n = u + \frac{a}{2}(2n-1)$

$45 = 0 + \frac{10}{2}(2n-1)$ $\qquad n = 5$ sec

Height of tower;

$h = \frac{1}{2}gt^2 \quad = \frac{1}{2}\times 10\times 5^2 \quad = 125\ m$

DPP # 1

1. A man has to go 50 m due north, 40 m due east and 20 m due south to reach a field.
(a) What distance he has to walk to reach the field ?
(b) What is his displacement from his house to the field ?
2. A particle starts from the origin, goes along the X – axis to the point (20m, 0) and then returns along the same line to the point (- 20m, 0). Find the distance and displacement of the particle during the trip.
3. It is 260 km from Patna to Ranchi by air and 320 km by road. An aeroplane takes 30 minutes to go from Patna to Ranchi where as a delux bus takes 8 hours.
(a) Find the average speed of the plane.
(b) Find the average speed of the bus.
(c) Find the average velocity of the plane.
(d) Find the average velocity of the bus.
4. When a person leaves his home for sightseeing by his car, the meter reads 12352 km. When he returns home after two hours the reading is 12416 km.
(a) What is the average speed of the car during this period?
(b) What is the average velocity?
5. An athelete takes 2.0s to reach his maximum speed of 18.0 km/h. What is the magnitude of his average acceleration ?
6. An object having a velocity 4.0 m/s is accelerated at the rate of 1.2 m/s^2 for 5.0s. Find the distance travelled during the period of acceleration.
7. A person travelling at 43.2 km/h applies the brake giving a deceleration of 6.0 m/s^2 to his scooter. How far will it travel before stopping ?
8. A train starts from rest and moves with a constant acceleration of 2.0 m/s^2 for half a minute. The brakes are then applied and the train comes to rest in one minute. Find
(a) the total distance moved by the train,
(b) the maximum speed attained by the train and
(c) the position (s) of the train at half the maximum speed.
9. A bullet travelling with a velocity of 16 m/s penetrates a tree trunk and comes to rest in 0.4 m. Find the time taken during the retardation.
10. A bullet going with speed 350 m/s enters a concrete wall and penetrates a distance of 5.0 cm before coming to rest. Find the deceleration.
11. A particle starting from rest moves with constant acceleration. If it takes 5.0 s to reach the speed 18.0 km/h find (a) the average velocity during this period, and (b) the distance travelled by the particle during this period.
12. A driver takes 0.20s to apply the brakes after he sees a need for it. This is called the reaction time of the driver. If he is driving a car at a speed of 54 km/h and the brakes cause a deceleration of 6.0 m/s^2, find the distance travelled by the car after he sees the need to put the brakes on.
13. Complete the following table:

Car Model	Driver X Reaction time 0.20 s	Driver Y Reaction time 0.30 s
A (deceleration on hard braking = 6.0 m/s^2)	Speed = 54 km/h Breaking distance a = ………………… Total stopping distance b = ………………..	Speed = 72 km/h Breaking distance c = ………………… Total stopping distance d = ………………

B (deceleration on hard braking = 7.5 m/s^2)	Speed = 54 km/h Breaking distance e = ……………….. Total stopping distance f = ………………..	Speed = 72 km/h Breaking distance g = ……………….. Total stopping distance h = ………………..

14. A police jeep is chasing a culprit going on a motorbike. The motorbike crosses a turning at a speed of 72 km/h. The jeep follows it at a speed of 90 km/h. crossing the turning ten seconds later than the bike. Assuming that they travel at constant speeds, how far from the turning will the jeep catch up with the bike ?

15. A car travelling at 60 km/h overtakes another car travelling at 42 km/h. Assuming each car to be 5.0 m long, find the time taken during the overtake and the total road distance used for the overtake.

16. A ball is projected vertically upward with a speed of 50 m/s. Find (a) the maximum height, (b) the time to reach the maximum height, (c) the speed at half the maximum height. Take g =10 m/s^2.

17. A ball is dropped from a balloon going up at a speed of 7 m/s. If the balloon was at a height 60 m at the time of dropping the ball, how long will the ball take in reaching the ground ?

18. A stone is thrown vertically upward with a speed of 28 m/s. (a) Find the maximum height reached by the stone. (b) Find its velocity one second before it reaches the maximum height. (c) Does the answer of part (b) change if the initial speed is more than 28 m/s such as 40 m/s or 80 m/s ?

19. A person sitting on the top of a tall building is dropping balls at regular intervals of one second. Find the positions of the 3rd, 4th and 5th ball when the 6th ball is being dropped.

20. A healthy youngman standing at a distance of 7 m from a 11.8 m high building sees a kid slipping from the top floor. With what speed (assumed uniform) should he run to catch the kid at the arms height (1.8 m) ?

21. An NCC parade is going at a uniform speed of 6 km/h through a place under a berry tree on which a bird is sitting at a height of 12.1 m. At a particular instant the bird drops a berry. Which cadet (give the distance from the tree at the instant) will receive the berry on his uniform ?

22. A ball is dropped from a height. If it takes 0.200s to cross the last 6.00 m before hitting the ground, find the height from which it was dropped. Take g =10 m/s^2 .

23. A ball is dropped from a height of 5 m onto a sandy floor and penetrates the sand up to 10 cm before coming to rest. Find the retardation of the ball in sand assuming it to be uniform.

24. An elevator is descending with uniform acceleration. To measure the acceleration, a person in the elevator drops a coin at the moment the elevator starts. The coin is 6 ft above the floor of the elevator at the time it is dropped. The person observes that the coin strikes the floor in 1second. Calculate from these data the acceleration of the elevator.

25. The position of a particle moving on x-axis is given by $x = 4t^3 + 3t^2 + 6t + 4$. Find
(a) The velocity and acceleration of particle at t = 5 s.
(b) The average velocity and average acceleration during the interval t = 0 to t = 5 s, $x = 4t^3 + 3t^2 + 6t + 4$
(a) The velocity and acceleration of particle at t = 5 s.
(b) The average velocity and average acceleration during the interval t = 0 to t = 5 s.

MOTION, THEIR GRAPHS AND MEANING OF SLOPE

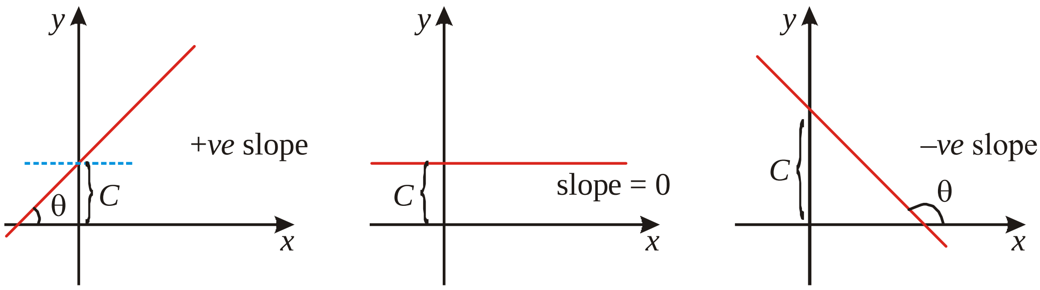
If θ is the angle at which a straight line is inclined to the positive direction of x-axis, & $0° \le \theta < 180°$, ***then the slope of the line, denoted by m, is defined by m = tan***θ. If θ is 90°, m does not exists, because the line is parallel to the y-axis. If $\theta = 0$, then m = 0 & the line is parallel to the x-axis.

Graph having $\theta = 90°$ (slope m = tan 90° = ∞) is not possible.

Slope - intercept form: $y = mx + c$ is the equation of a straight line whose slope is m with respect to x-axis & which makes an intercept c on the y-axis. Similarly, $x = my + c$ is the equation of a straight line whose slope is m with respect to y-axis and makes an intercept of c on x-axis.

Slope = m = tan θ = $\dfrac{\Delta y}{\Delta x} = \dfrac{dy}{dx}$ (where Δx becomes very small)

Average slope | Instantaneous slope

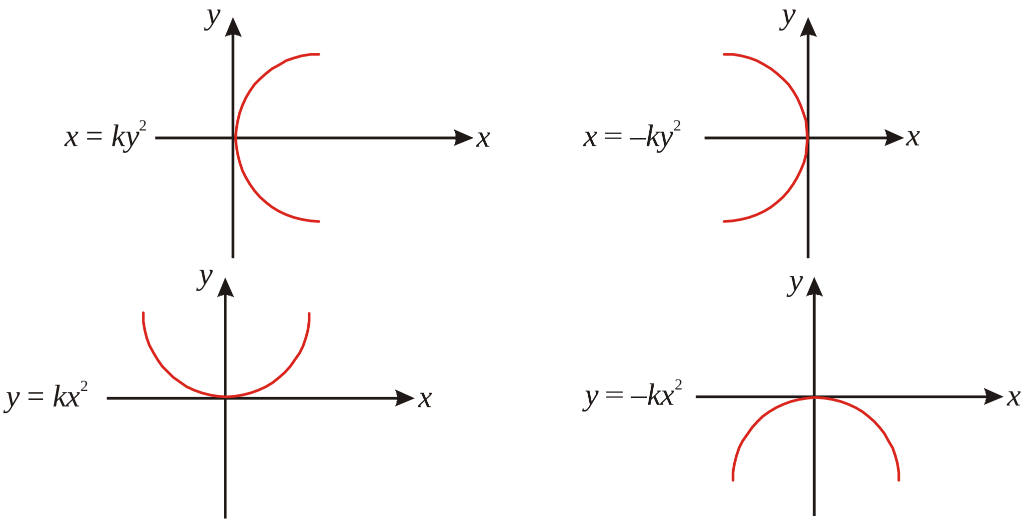


In case of a straight line graph, slope of the curve always remains constant.

Graphs having variable slope

All the given graphs are not straight line because change in x does not produces a constnat proportionate change in y always.

Any quadratic equation gives a graph of variable slope also known as parabola

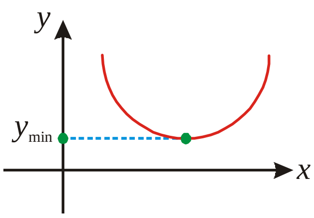


Where k is a positive constant.

Equation of parabola:

Case (i): $y = ax^2 + bx + c$

For $a > 0$



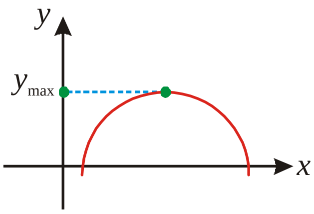
The nature of the parabola will be like that of the of nature $y = kx^2$
Minimum value of y exists at the vertex of the parabola.

$$y_{min} = \frac{-D}{4a} \text{ where } D = b^2 - 4ac$$

Coordinates of vertex $= \left(\frac{-b}{2a}, \frac{D}{4a}\right)$

Case (ii) : $a < 0$
The nature of the parabola will be like that of the nature of $y = -kx^2$
Maximum value of y exists at the vertex of parabola.

$$y_{max} = \frac{D}{4a} \text{ where } D = b^2 - 4ac$$



Graphical Interpretation of Displacement, Velocity and Acceleration

There are two important aspects related with the graphical analysis of motion.

(*i*) Slope of the graph (which gives rate of change $\frac{\Delta y}{\Delta x}$ or dy/dx). Average as well as instantaneous.

(*ii*) area under the graph.

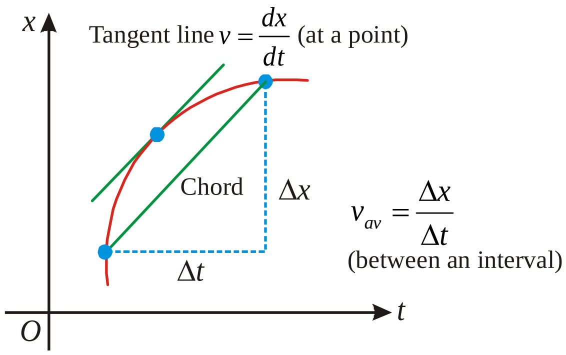
- If you need to know about the quantity which the slope with respect to x-axis of the given graph will give, divide the units of quantity on y-axis by the units of quantity on x-axis. The resultant unit is the unit of the quantity represented by the slope.
- For the area under the graph concept you need to multiply the units of the two quantities represented on x and y axes respectively. Resultant unit is the unit of the quantity represented by the area under the graph.

Average Velocity

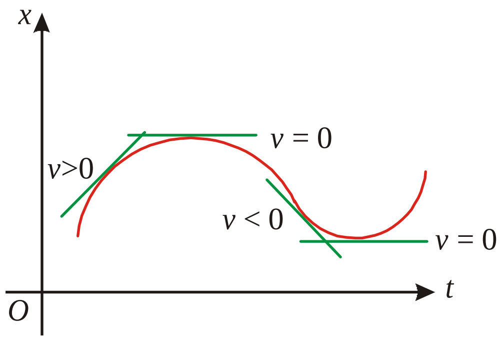
The *average velocity* between two points in a given time interval can be obtained from a *displacement* versus *time* graph by computing the *slope* of the *straight line* joining the coordinates of the two points.

Instantaneous Velocity (or velocity at a particular instant)

The *instantaneous velocity* at time t is the *slope* of the *tangent line* drawn to the *position* versus *time* graph at that time.



(*i*)



(*ii*)

$v = \frac{dx}{dt}$ = instantaneous velocity as the slope of the given graph changes at every point.

$\Rightarrow$ slope is variable

$\Rightarrow$ velocity is variable

In the graph (*ii*) we can show that sign of the slope gives the nature of velocity.

$m > 0 \;\Rightarrow\; v > 0$ $\Rightarrow$ velocity is in +ve or forward direction

$m = 0 \;\Rightarrow\; v = 0$

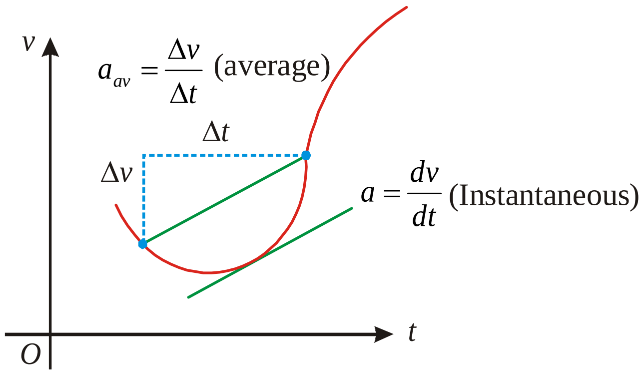
$m < 0 \;\Rightarrow\; v < 0$ $\Rightarrow$ velocity is in opposite or backward or –ve direction.

Average Acceleration

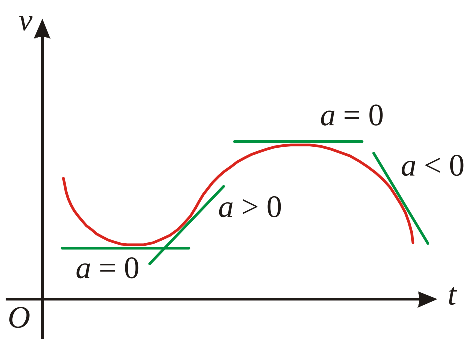
The average acceleration between two points in a time interval is equal to the slope of the chord connecting the points on a *velocity* versus *time* graph.

Instantaneous Acceleration

The *instantaneous acceleration* at time *t* is the *slope* of the *tangent* drawn to the *velocity* versus *time* graph.



(*i*)



(*ii*)

$a = \frac{dv}{dt}$ = instantaneous acceleration as the slope of the given graph changes at every point.

$\Rightarrow$ slope is variable $\Rightarrow$acceleration is variable

In the graph (*ii*) we can show that sign of the slope gives the nature of acceleration.

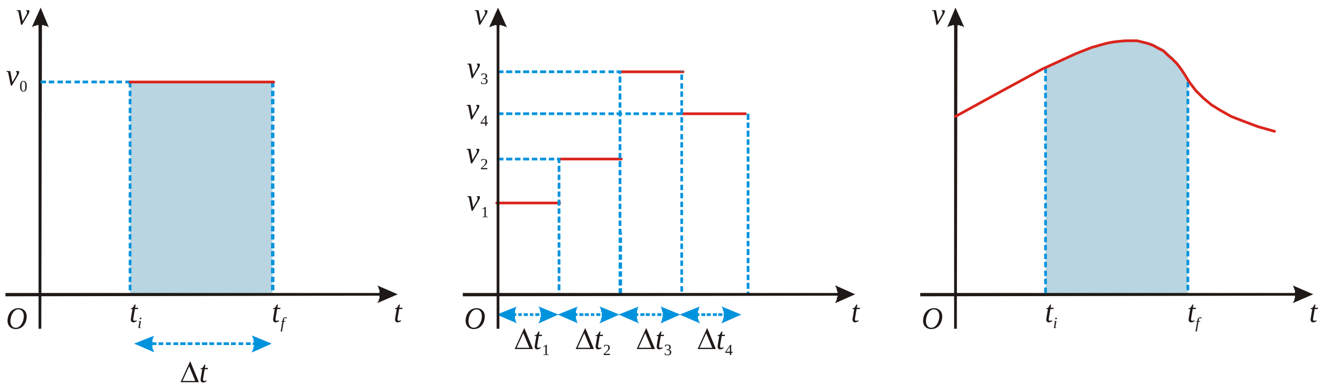
$m > 0 \;\Rightarrow\; a > 0$ $\Rightarrow$ velocity is increasing

$m = 0 \;\Rightarrow a = 0$ $\Rightarrow$ velocity is constant

$m < 0 \;\Rightarrow\; a < 0$ $\Rightarrow$ velocity is decreasing

Displacement from Velocity Time Graphs

Given a *velocity* versus *time* graph, the *displacement* during an interval between time t_i and t_f is the *area bounded by the velocity curve* and the two vertical lines $t = t_i$ and $t = t_f$, as shown in the Fig.



(a) **(b)** **(c)**

(a) The area under the *v*-versus-*t* curve is the displacement $\Delta x = v_0 \Delta t$

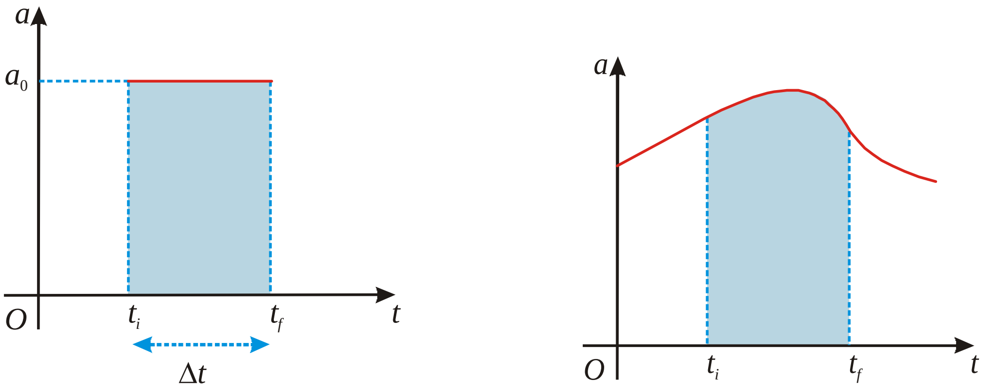
(b) For each segment of motion, the velocity is a different constant. The displacement Δx_1 during the

i^{th} interval is the area $v_1 \Delta t_1$. So the total displacement is $\Delta x = v_1\Delta t_1 + v_2\Delta t_2 + v_3\Delta t_3 + v_4\Delta t_4$

(c) When *v* versus *t* graph is a smooth complex curve, the area under the curve may be obtained using integration.

Velocity from Acceleration Time Graphs

Given an *acceleration*–versus–*time* graph, the change in velocity between $t = t_i$ and $t = t_f$ is the ***area bounded by the acceleration curve*** and the vertical lines $t = t_i$ and $t = t_f$.



(*a*) (*b*)

(a) The area under *a* – *t* graph is change in velocity $\Delta v = (a_0 \Delta t)$

(b) When *a* – *t* graph is a smooth complex curve, the area under the curve can be obtained using

integration.

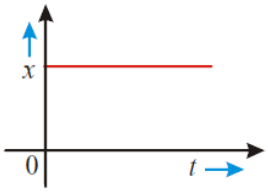
Position vs Time Graph

1 Zero Velocity

As position of particle does not change with time, so the body is at rest.

Slope = $\frac{dx}{dt} = \tan\theta = \tan 0^{\circ} = 0$

Velocity of particle is zero

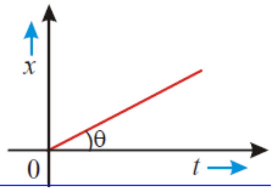


2 Uniform Velocity

Here $\tan\theta$ is constant

$\Rightarrow$ $\tan\theta = \frac{dx}{dt}$ is constant

$\Rightarrow$ velocity of particle is constant.



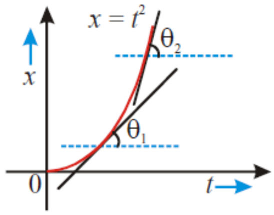
3 Non uniform velocity (increasing with time)

As time is increasing, θ is also increasing.

$\Rightarrow$ $\frac{dx}{dt} = \tan\theta$ is also increasing

Hence, velocity of particle is increasing.

$\Rightarrow$ **motion is positively accelerated**



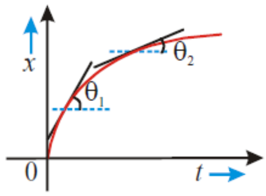
4 Non uniform velocity (decreasing with time)

As time increases, θ decreases.

$\Rightarrow$ $\frac{dx}{dt} = \tan\theta$ also decreases.

Hence, velocity of particle is decreasing.

$\Rightarrow$ **motion is negatively accelerated**



Velocity vs Time Graph

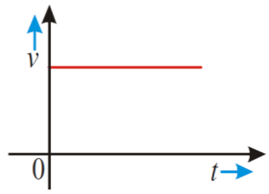
1 Zero acceleration

Velocity is constant.

$$\tan\theta = 0 \Rightarrow \frac{dv}{dt} = 0$$

Hence, acceleration is zero.

Average as well as instantaneous both

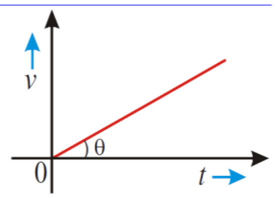


2 Uniform acceleration

$\tan\theta$ is constant.

$$\Rightarrow \frac{dv}{dt} = \text{constant}$$

Hence, it shows constant acceleration.



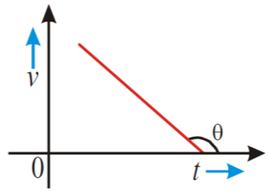
3 Uniform retardation

Since $\theta > 90^\circ$

$\Rightarrow$ $\tan\theta$ is constant and negative.

$$\Rightarrow \frac{dv}{dt} = \text{negative constant}$$

Hence, it shows constant retardation.



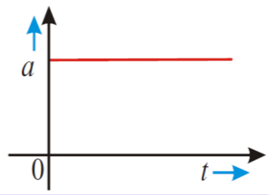
Acceleration vs Time Graph

1 Constant acceleration

$\tan\theta = 0$

$$\Rightarrow \frac{da}{dt} = 0$$

Hence, acceleration is constant.



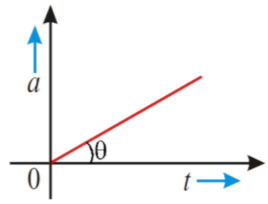
2 Uniformly increasing acceleration

θ is constant.

$0^\circ < \theta < 90^\circ \Rightarrow \tan\theta > 0$

$$\Rightarrow \frac{da}{dt} = \tan\theta = \text{constant} > 0$$

Hence, acceleration is uniformly increasing with time.



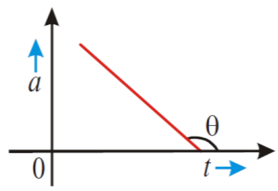
3 Uniformly decreasing acceleration

Since $\theta > 90^\circ$

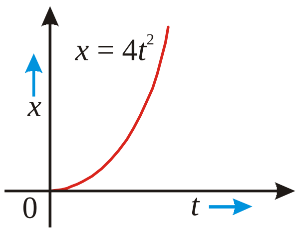
$\Rightarrow$ $\tan\theta$ is constant and negative.

$$\Rightarrow \frac{da}{dt} = \text{negative constant}$$

Hence, acceleration is uniformly decreasing with time

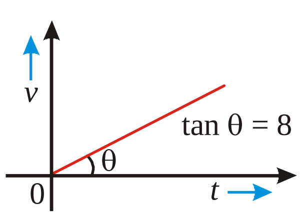


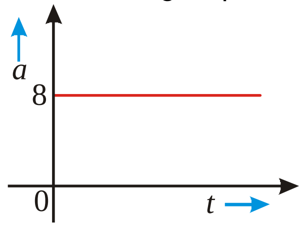
Ex. 17 The displacement vs time graph of a particle moving along a straight line is shown in the figure. Draw velocity *vs* time and acceleration vs time graph.



Solution: $x = 4t^2$

Hence, velocity-time graph is a straight line having slope tan = 8.

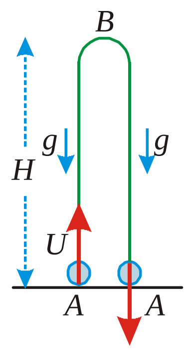




$\Rightarrow v = \frac{dx}{dt} = 8t$ $\qquad a = \frac{dv}{dt} = 8$

Important Points to Remember

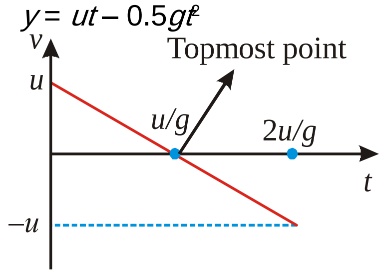
- For uniformly accelerated motion ($a \neq 0$), **x–t graph is a parabola (opening upwards if $a > 0$ and opening downwards if $a < 0$).** The slope of tangent at any point of the parabola gives the velocity at that instant.
- For uniformly accelerated motion ($a \neq 0$), **v–t graph is a straight line** whose constant slope gives the acceleration of the particle.
- In general, the slope of tangent in x–t graph is velocity and the slope of tangent in v–t graph is the acceleration.
- The area under a–t graph gives the change in velocity.
- The area between the v–t graph gives the distance travelled by the particle, if we take all areas as positive.
- Area under v–t graph gives displacement, if areas below the t-axis are taken negative.

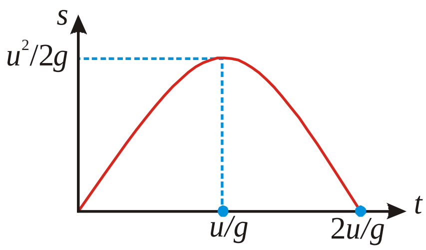


For Vertical motion under gravity

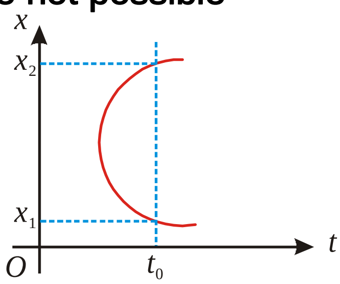
A particle has been thrown from $Y = 0$ with initial velocity U upwards.
The general equations are

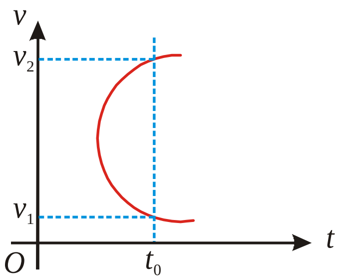
1. $v = u - gt$
2. $y = ut - 0.5gt^2$





Graphs which are not possible





- ***Displacement time graph or velocity time graph having infinite slope.***
  Infinite slope of displacement time graph means infinite velocity, which is not possible.
  Infinite slope of velocity time graph means infinite acceleration, which is not possible.
- Any displacement time graph having two values of displacement at the same instant of time or velocity time graph having two values of velocity at the same instant of time are never possible.

Ex. 18 For a particle moving along x-axis, velocity-time graph is as shown in figure. Find the distance travelled and displacement of the particle?

Solution : Distance travelled = Area under v-t graph (taking all areas as $+ve$.)

$\therefore$ Distance travelled = Area of trapezium + Area of triangle

$= \frac{1}{2}(2+6)\times 8 + \frac{1}{2}\times 4\times 5$

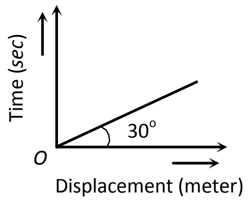
$= 32 + 10 = 42\ m$

Displacement = Area under v-t graph (taking areas below time axis as –ive.)

$\therefore$ Displacement = Area of trapezium – Area of triangle

DPP # 2

1. From the following displacement-time graph find out the velocity of a moving body

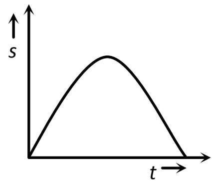


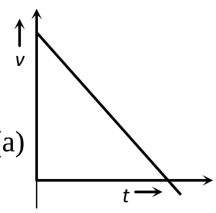
(a) $\frac{1}{\sqrt{3}}$ *m/s* (b) 3 *m/s* (c) $\sqrt{3}$ *m/s* (d) $\frac{1}{3}$

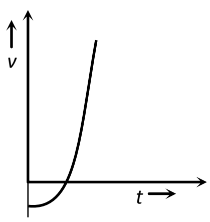
2. The acceleration–time graph of a particle moving in a straight line is as shown in figure. The velocity of the particle at time $t = 0$ is 2 m/s. The velocity after 2 seconds will be

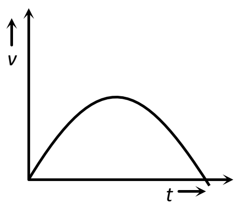
(a) 6 m/s (b) 4 m/s (c) 2 m/s (d) 8 m/s

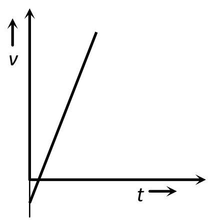
3. The graph of displacement *s*/*t* time is shown. Its corresponding velocity-time graph will be



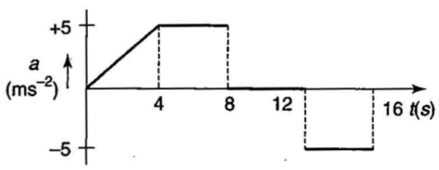
(a) 

(b) 

(c) 

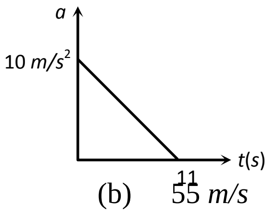
(d) 

4. The acceleration of a particle traveling along a straight line is shown in the figure. The maximum speed of the particle is



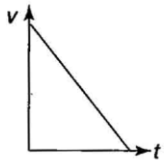
(a) 20 ms^{-1} (b) 30 ms^{-1} (c) 40 ms^{-1} (d) 60 ms^{-1}

5. A particle starts from rest. Its acceleration (a) versus time (t) is as shown in the figure. The maximum speed of the particle will be



(a) 110 m/s (b) 55 m/s (c) 550 m/s (d) 660 m/s

6. Figure gives the velocity-time graph. This shows that the body is



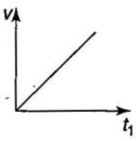
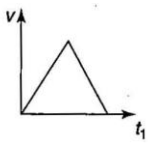
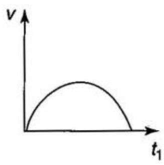
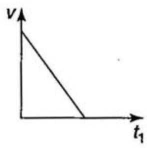
(a) starting form rest and moving with uniform velocity

(b) moving with uniform retardation

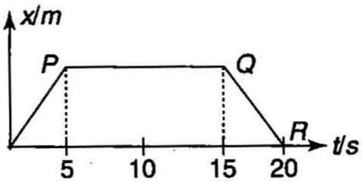
(c) moving with uniform acceleration

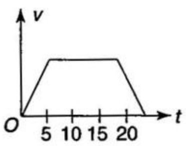
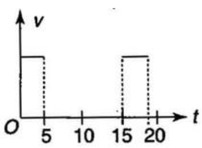
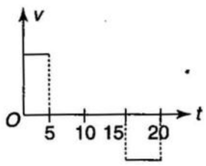
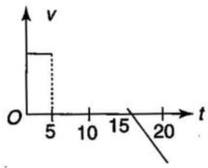
(d) having same initial and final velocity

7. A ball is thrown straight up with a velocity at $t = 0$ and returns to earth at $t - t_1$ Which graph shows the correct motion?

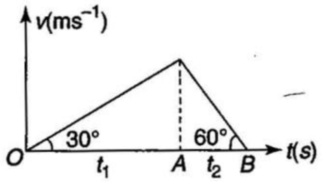
(a) 
(b) 
(c) 
(d) 

8. Figure shows the position time graph for a particle in one dimensional motion. Which of the graphs in figure represent the variation in the velocity of the particle with time?



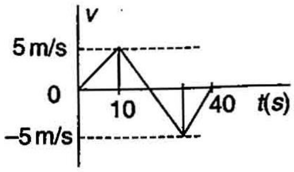
(a) 
(b) 
(c) 
(d) 

9. The velocity time graph of a body moving along a straight line is shown in figure. The ratio of the average velocities during the time t_1 and t_2 is



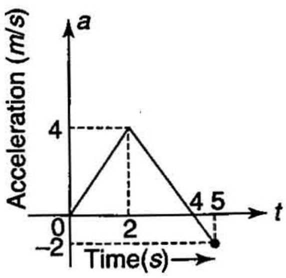
(a) 1:1 (b) 2 : 1 (c) 3 : 1 (d) 1 : 3

10. The velocity-time plot is shown in figure. Find the average speed in time interval t = 0 to t = 40 s during the period.



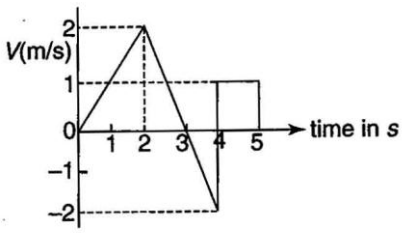
(a) zero (b) 2.5 m/s (c) 5 m/s (d) none of these

11. Figure shows the graph of acceleration of particle as a function of time. The maximum speed of the particle is (particle starts from rest)



(a) 7 m/s (b) 8 m/s (c) 4 m/s (d) 16 m/s

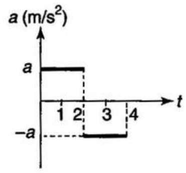
12. The velocity-time graph of a body moving along a straight line is as follows:

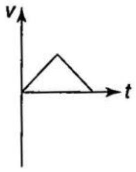
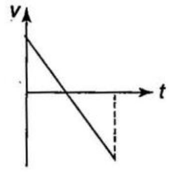
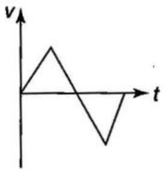
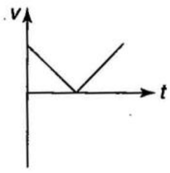


The displacement of the body in 5 s is

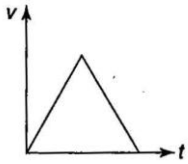
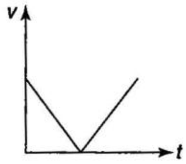
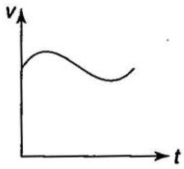
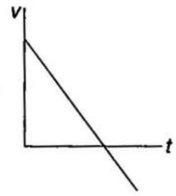
(a) 5m (b) 2m (c) 4 m (d) 3 m

13. A particle starts from rest and undergoes an acceleration as shown in figure. The velocity-time graph from figure will have a shape

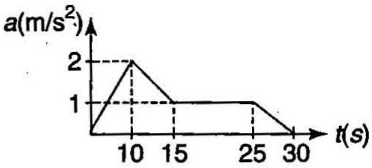


(a)  (b)  (c)  (d) 

14. Which of the following graphs show the *v*-*t* graph of a ball thrown upwards?

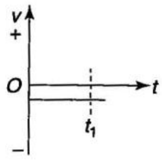
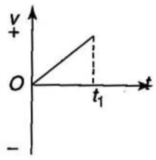
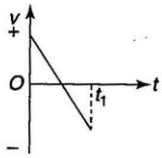
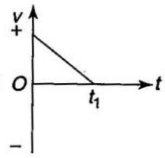
(a)  (b)  (c)  (d) 

15. In figure shown, the graph shows the variation of acceleration of body with time t. The velocity of the body at t = 0 is zero. The velocity of the body at t = 30 s is



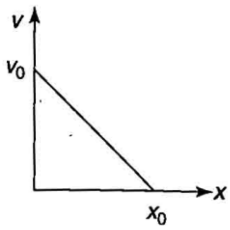
(a) 30 m/s (b) 20 m/s (c) 40 m/s (d) none

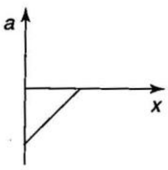
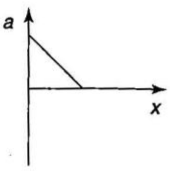
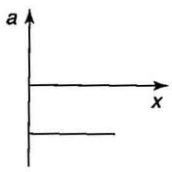
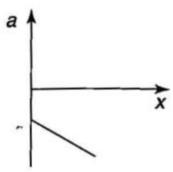
16. In which of the graphs both net displacement and velocity are negative?

(i)  (ii)  (iii)  (iv) 

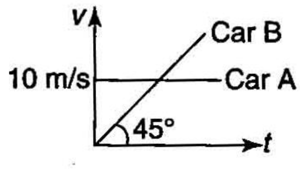
(a) Graph (iii) and (iv)
(b) Graph (i) and (ii)
(c) Graph (i) only
(d) Graph (iv) only

17. The given graph shows the variation of velocity with displacement. Which one of the graphs given below correctly represents the variation of acceleration with displacement?



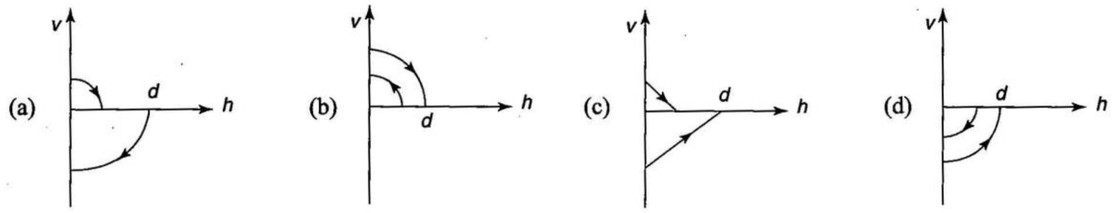
(a)  (b)  (c)  (d) 

18. Initially car *A* is 10.5 m ahead of car *B*. Both start moving at time $t = 0$ in the same direction along a straight line. The velocity-time graph of two cars is shown in figure. The time when the car *B* will catch the car *A*, will be

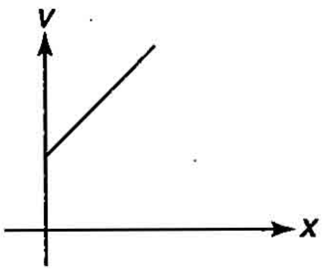


(a) f = 21 sec (b) $t = 2\sqrt{5}$ sec (c) 20 sec (d) none

19. A ball is dropped vertically from a height *d* above the ground. It hits the ground and bounces up vertically to a height *d*/2. Neglecting subsequent motion and air resistance, its velocity v varies with the height *h* above the ground is

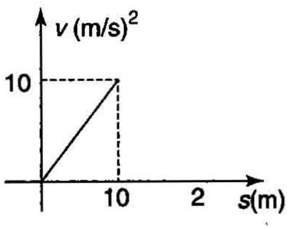


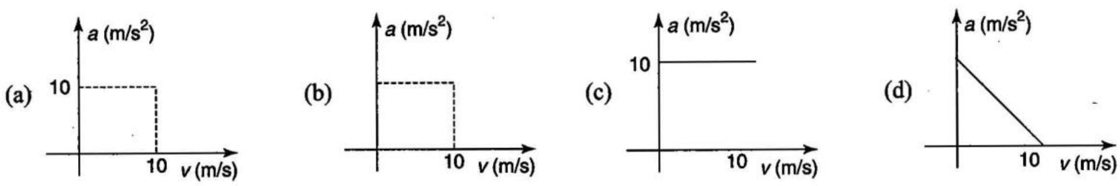
20. Graph of velocity versus displacement of a particle moving in a straight line is as shown in figure. The acceleration of the particle is



(a) constant (b) increases linearly with x
(c) increases parabolically with x (d) none of these

21. Velocity versus displacement graph of a particle moving in a straight line is shown in figure. Corresponding acceleration versus velocity graph will be

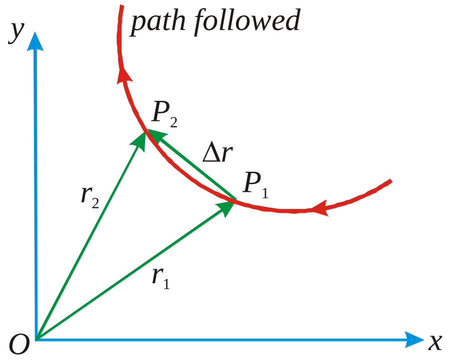




MOTION IN TWO DIMENSIONS

Whatever we have studied in the kinematics of one dimensional motion the same is applicable for motion in *two* and *three* dimensions also.

If the particle moves from P_1 at position r_1 to P_2 at position r_2, as shown in figure its displacement s given by



Initial position vector $\overrightarrow{OP_1} = \vec{r}_1$

Final position vector $\overrightarrow{OP_2} = \vec{r}_2$

change in position vector $\overrightarrow{\Delta r}$ = final – initial

$\overrightarrow{\Delta r} = \vec{r}_2 - \vec{r}_1$

$\Rightarrow \overrightarrow{\Delta r} = \vec{r}_2 - \vec{r}_1$

$\overrightarrow{\Delta r}$ is also known as displacement.

$\Rightarrow \overrightarrow{\Delta r} = \Delta x\hat{i} + \Delta y\hat{j}$

In two dimensions the position vector *r* of a particle whose coordinate are (*x*, *y*) is $r = x\hat{i} + y\hat{j}$

$\Delta\vec{r}$ is the vector that must be added to the initial position vector $\vec{r}_1$ to give the final position vector $\vec{r}_2$,

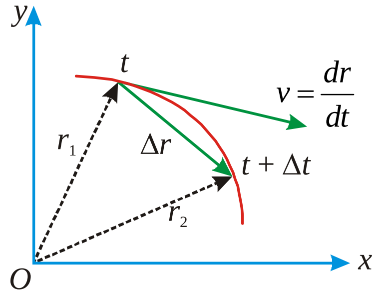
$\Rightarrow \vec{r}_2 = \vec{r}_1 + \Delta\vec{r}$

Let $p_1 \equiv (x_1, y_1)$ and $p_2 \equiv (x_2, y_2)$

$\Rightarrow$ change in *x*, $\Delta x = x_2 - x_1$ and change in *y*, $\Delta y = y_2 - y_1$

$\Delta\vec{r}$ must be taken with proper care of direction.

if for motion from p_1 to p_2, $\Delta\vec{r}$ is taken as positive then for p_2 to p_1 movement $\Delta\vec{r}$ will be negative.



The *average velocity* is defined as the ratio of the displacement over the time interval

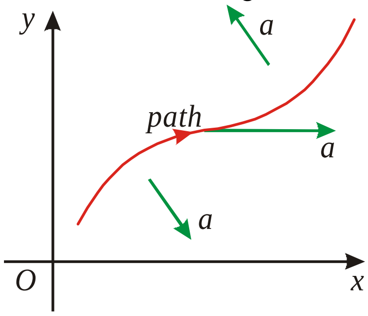
$\vec{v}_{av} = \dfrac{\Delta\vec{r}}{\Delta t}$ *(directed along the line joining initial and final position)*

instantaneous velocity is given by

$\vec{v} = \dfrac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j}$ where $v_x = \dfrac{dx}{dt}$ and $v_y = \dfrac{dy}{dt}$

The direction of *v* is along the *tangent* to the path

The *instantaneous acceleration* is the ratio of change of the velocity vector with respect to time.



Possible direction of acceleration can also be found by visualizing the tendency of movement.

Tendency of movement is always in the direction of force and acceleration is always said to be in
the direction of force.

$$\vec{a} = \frac{d\vec{v}}{dt} = a_x\hat{i} + a_y\hat{j}$$

where $a_x = \frac{dv_x}{dt}$ and $a_y = \frac{dv_y}{dt}$

***Note :** One cannot determine acceleration directly from the path of the particle. One needs to know how each component of the velocity varies as function of space and time.*

General equations of kinematics for constant acceleration in 2-D

We need to use the same kinematical equation of motion that we used before but with proper care.

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{S} = \vec{S}_0 + \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v}.\vec{v} = \vec{u}.\vec{u} = 2\vec{a}.\vec{S}$$

where $\vec{v} = v_x\hat{i} + v_y\hat{j}$ $\quad \vec{a} = a_x\hat{i} + a_y\hat{j}$ $\quad \vec{u} = u_x\hat{i} + u_y\hat{j}$ $\quad \vec{S} = x\hat{i} + y\hat{j}$

$\Rightarrow \quad v_x\hat{i} + v_y\hat{j} = (u_x\hat{i} + u_y\hat{j}) + (a_x\hat{i} + a_y\hat{j})t$ from $\vec{v} = \vec{u} + \vec{a}t$

$\Rightarrow \quad v_x\hat{i} + v_y\hat{j} = (u_x + a_xt)\hat{i} + (u_y + a_yt)\hat{j}$

$v_x = u_x + a_xt$ (one-D)

and $v_y = u_y + a_yt$ (one-D)

Similarly $S_x = S_{0x} + u_xt + 0.5\, a_xt^2$

and $S_y = S_{0y} + u_yt + 0.5\, a_yt^2$

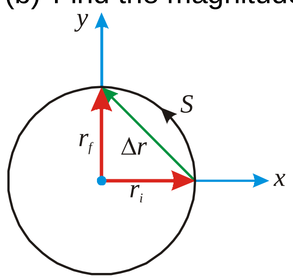
$\Rightarrow \quad \vec{S} = S_x\hat{i} + S_y\hat{j}$

Ex. 19 A particle moves one quarter of a circular path of radius 20 *m* in 10 *s*. Its initial position is given by

$\vec{r}_i$ = (20 *m*) and its final position is $\vec{r}_f$ = (20*m*) $\hat{j}$.

(a) Find its displacement and average velocity

(b) Find the magnitude of its average velocity and its average speed.



Solution (a) The displacement is given by

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i = 20\hat{j} - 20\hat{i}$$

$$\Rightarrow \Delta\vec{r} = -20\hat{i} + 20\hat{j}(m)$$

and the average velocity is

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{-20\hat{i} + 20\hat{j}}{10} = -2\hat{i} + 2\hat{j}(m/s)$$

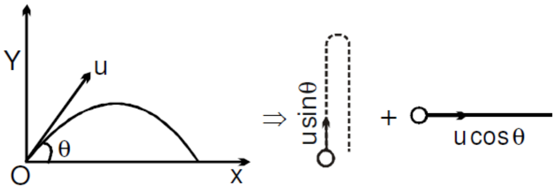
(b) The magnitude of the average velocity is

$$|\vec{v}_{av}| = v_{av} = \sqrt{(-2)^2 + (2)^2} = 2.83 m/s$$

Average speed = $\frac{\text{Total distance travelled}}{\text{Time Taken}} = \frac{\frac{1}{4}[2\pi(20)]}{10}$ = 3.14 *m*/*s*

PROJECTILE MOTION

It is the best example to understand motion in a plane. If we project a particle obliquely from the surface of earth, as shown in the figure below, then it can be considered as two perpendicular 1D motions - one along the horizontal and other along the vertical.



Assume that effect of air friction and wind resistance are negligible and value of 'acceleration due to gravity $\vec{g}$ is constant.

Take point of projection as origin and horizontal and vertical direction as +ve X and Y-axes, respectively.

For X-axis	**For Y - axis**
$u_x = u\cos\theta$,	$u_y = u\sin\theta$
$a_x = 0$,	$a_y = -g$,
$v_x = u\cos\theta$, and	$v_y = u\sin\theta - gt$, and
$x = u\cos\theta \times t$	$y = u\sin\theta\, t - \frac{1}{2}gt^2$

It is clear from above equations that horizontal component of velocity of the particle remains constant while vertical component of velocity is first decreasing, gets zero at the highest point of trajectory and then increases in the opposite direction. At the highest point, speed of the particle is minimum.

The time, which projectile takes to come back to same (initial) level is called the time of flight (T).

At initial and final points, y = 0,

So $u\sin\theta\, t - \frac{1}{2}gt^2 = 0$

$\Rightarrow$ t = 0 and $t = \frac{2u\sin\theta}{g}$ So, $T = \frac{2u\sin\theta}{g}$

Range (R) The horizontal distance covered by the projectile during its motion is said to be range of the projectile

$$R = u\cos\theta \times T = \frac{u^2\sin 2\theta}{g}$$

For a given projection speed, the range would be maximum for $\theta = 45°$.

Maximum height attained by the projectile is

$$H = \frac{u^2\sin^2\theta}{2g}$$

at maximum height the vertical component of velocity is 0.

Time of ascent = Time of descent = $\frac{u\sin\theta}{g} = \frac{T}{2}$

Speed, kinetic energy, momentum of the particle initialy decreases in a projectile motion and attains a minimum value (not equal to zero) and then again increases.

θ is the angle between $\vec{v}$ and horizontal which decreases to zero. (at top most point) and again increases in the negative direction

Ex. 20 ***A body is projected with a velocity of 30 ms^{-1} at an angle of 30° with the vertical. Find the maximum height, time of flight and the horizontal range.***

Sol. Here $u = 30\ ms^{-1}$,

Angle of projection, $\theta = 90 - 30 = 60°$

Maximum height,

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{30^2 \sin^2 60°}{2 \times 9.8} = 34.44 \text{ m}$$

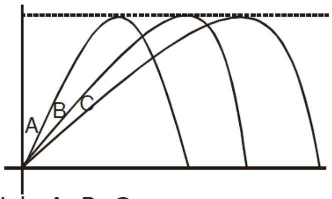
Time fo flight,

$$T = \frac{2u\sin\theta}{g} = \frac{2 \times 30 \sin 60°}{9.8} = 5.3 \text{ s}$$

Horizontal range,

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{30° \sin 120°}{9.8} = \frac{30^2 \sin 60°}{9.8} = 79.53 \text{ m.}$$

Ex. 21 ***Find out the relation between u_A, u_B, u_C (where u_A, u_B, u_C are the initial velocities of particles A, B, C, respectively)***



Sol. $\because$ H_{max} is same for all three particle A, B, C

$\Rightarrow H_{max} = \dfrac{u_y^2}{2g}$

$\Rightarrow u_y$ is same for all $\therefore u_{yA} = u_{yB} = u_{yC}$

$\Rightarrow T_A = T_B = T_C \left(\dfrac{2u_y}{g}\right)$

from figure $R_C > R_B > R_A$ $\because R = \dfrac{2u_x u_y}{g}$

$\Rightarrow u_{xC} > u_{xB} > u_{xA}$ $\Rightarrow$ $u_A < u_B < u_C$

(C) Coordinate of a particle after a given time t :

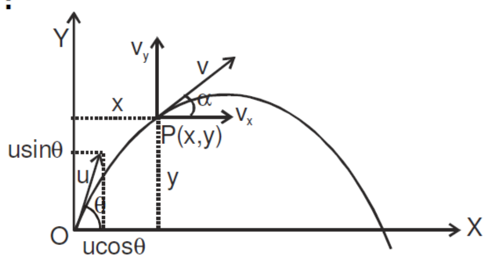
Particle reach at a point P after time t then

$x = u\cos\theta\,.t$

$y = u\sin\theta.t - \frac{1}{2}gt^2$

Position vector

$$\vec{r} = (u\cos\theta.t)\hat{i} + \left((u\sin\theta)t - \frac{1}{2}gt^2\right)\hat{j}$$



(D) Velocity and direction of motion after a given time :

After time 't' $v_x = u\cos\theta$ and $v_y = u\sin\theta - gt$

Hence resultant velocity $v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2}$

$$\tan\alpha = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta} \Rightarrow \alpha = \tan^{-1}\left(\frac{u\sin\theta - gt}{u\cos\theta}\right)$$

(E) Velocity and direction of motion at a given height :

At a height 'h', $v_x = u\cos\theta$

And $v_y = \sqrt{u^2\sin^2\theta - 2gh}$

$\therefore$ Resultant velocity

$$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{(u\cos\theta)^2 + u^2\sin^2\theta - 2gh}$$

$$v = \sqrt{u^2 - 2gh}$$

Note that this is the velocity that a particle would have at height h if it is projected vertically from ground with u.

Ex. 22 A body is projected with a velocity of 20 ms⁻¹ in a direction making an angle of 60° with the horizontal. Calculate its (i) position after 0.5 s and (ii) velocity after 0.5 s.

Sol. Here $u = 20\ ms^{-1}$, $\theta = 60°$, $t = 0.5$ s

(i) $x = (u\cos\theta)t = (20\cos60°) \times 0.5 = 5$ m

$$y = (u\sin\theta)\,t - \frac{1}{2}gt^2 = (20 \times \sin 60°) \times 0.5$$

$$- \frac{1}{2} \times 9.8 \times (0.5)^2 = 7.43 \text{ m}$$

(ii) $v_x = u\cos\theta = 20\cos 60° = 10\ ms^{-1}$

$v_y = u\sin\theta - gt = 20\sin 60° - 9.8 \times 0.5$

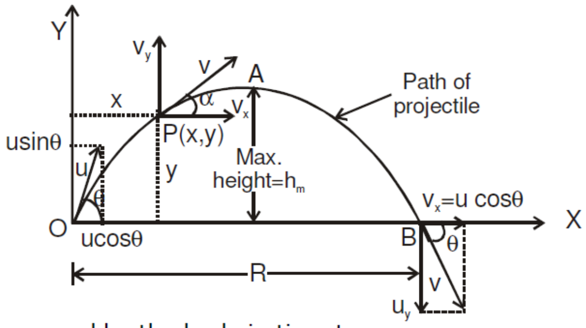
$= 12.42\ ms^{-1}$

$\therefore$ $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(10)^2 + (12.42)^2} = 15.95\ ms^{-1}$ $\quad \tan\beta = \dfrac{v_y}{v_x} = \dfrac{12.42}{10} = 1.242$

$\therefore$ $\beta = \tan^{-1} 1.242 = 51.16°$.

Equation of trajectory of a projectile.

Suppose the body reaches the point P(x, y) after time t.



$\because$ The horizontal distance covered by the body in time t,

x = Horizontal velocity × time = $u\cos\theta.\,t$

or $t = \dfrac{x}{u\cos\theta}$

For vertical motion : $u = u\sin\theta$, $a = -g$, so the vertical distance covered in time t is given by

$$s = ut + \frac{1}{2}at^2 \quad \text{or} \quad y = u\sin\theta.\frac{x}{u\cos\theta} - \frac{1}{2}g.\frac{x^2}{u^2\cos^2\theta}$$

or $\boldsymbol{y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}}$ **...(1)**

or $y = px - qx^2$, where p and q are constants.
Thus y is a quadratic function of x. Hence the trajectory of a projectile is a parabola.

From equation (1)

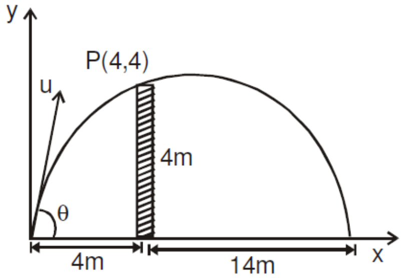
$$y = x \tan\theta \left[1 - \frac{gx\cos\theta}{2u^2\cos^2\theta\sin\theta}\right] \Rightarrow y = x\tan\theta\left[1-\frac{gx}{2u^2\cos\theta\sin\theta}\right]$$

$$\mathbf{y = x\tan\theta\left[1-\frac{x}{R}\right]} \qquad \textbf{...(2)}$$

Equation (2) is another form of trajectory equation of projectile

Ex. 23 ***A ball is thrown from ground level so as to just clear a wall 4 m high at a distance of 4 m and falls at a distance of 14 m from the wall. Find the magnitude and direction of the velocity.***

Sol. The ball passes through the point P(4, 4). So its range = 4 + 14 = 18m.
The trajectory of the ball is,
Now x = 4m, y = 4m and R = 18 m



$$\therefore \quad 4 = 4\tan\theta\left[1-\frac{4}{18}\right] = 4\tan\theta\,.\frac{7}{9}$$

or $\tan\theta = \frac{9}{7}$, $\sin\theta = \frac{9}{\sqrt{130}}$, $\cos\theta = \frac{7}{\sqrt{130}}$

or $u^2 = \frac{18\times 9.8\times 130}{2\times 9\times 7} = 182$

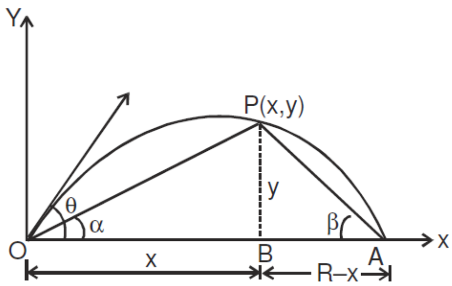
or $u = \sqrt{182} = 13.5\ \text{ms}^{-1}$

Also $\theta = \tan^{-1}(9/7) = 52.1°$

Ex. 24 ***A particle is projected over a triangle from one end of a horizontal base and grazing the vertex falls on the other end of the base. If α and β be the base angles and θ the angle of projection, prove that $\tan\theta = \tan\alpha + \tan\beta$.***

Sol. If R is the range of the particle, then from the figure we have

$$\tan\alpha + \tan\beta = \frac{y}{x} + \frac{y}{R-x} = \frac{y(R-x)+xy}{x(R-x)}$$



$$\text{or} \quad \tan\alpha + \tan\beta = \frac{y}{x}\times\frac{R}{(R-x)} \qquad ...(1)$$

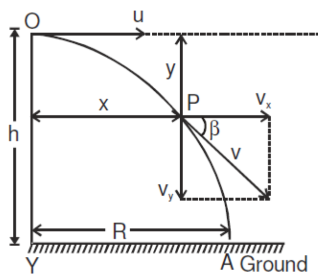
Also, the trajectrory of the particle is

$$y = x\tan\theta\left[1-\frac{x}{R}\right]$$

$$\text{or} \quad \tan\theta = \frac{y}{x}\times\frac{R}{(R-x)}$$

From equations (1) and (2), we get
$\tan\theta = \tan\alpha + \tan\beta$.

4.2 Projectile fired parallel to horizontal. As shown in shown figure suppose a body is projected horizontally with velocity u from a point O at a certain height h above the ground level. The body is under the influence of two simultaneous independent motions:
(i) Uniform horizontal velocity u.
(ii) Vertically downward accelerated motion with constant acceleration g.
Under the combined effect of the above two motions, the body moves along the path OPA.



Trajectory of the projectile. After the time t, suppose the body reaches the point P(x, y).
The horizontal distance covered by the body in time t is

$$x = ut \qquad \therefore\ t = \frac{x}{u}$$

The vertical distance travelled by the body in time t is given by

$$s = ut + \frac{1}{2}at^2$$

or $$y = 0 \times 1 + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$$

[For vertical motion, u = 0]

or $$y = \frac{1}{2}g\left(\frac{x}{u}\right)^2 = \left(\frac{g}{2u^2}\right)x^2 \qquad \left[\because t = \frac{x}{u}\right]$$

or $$y = kx^2 \ \left[\text{Here } k = \frac{g}{2u^2} = \text{a constant}\right]$$

As y is a quadratic function of x, so the trajectory of the projectile is a parabola.

Time of flight. It is the total time for which the projectile remains in its flight (from 0 to A). Let T be its time of flight.
For the vertical downward motion of the body,
we use

$$s = ut + \frac{1}{2}at^2$$

or $$h = 0 \times T + \frac{1}{2}gT^2 \qquad \text{or} \quad T = \sqrt{\frac{2h}{g}}$$

Horizontal range. It is the horizontal distance covered by the projectile during its time of flight. It is equal to OA = R. Thus R = Horizontal velocity × time of flight = u × T

or $$R = u\sqrt{\frac{2h}{g}}$$

Velocity of the projectile at any instant. At the instant t (when the body is at point P), let the velocity of the projectile be v. The velocity v has two rectangular components:
Horizontal component of velocity, $v_x = u$
Vertical component of velocity, $v_y = 0 + gt = gt$
∴ The resultant velocity at point P is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + g^2t^2}$$

If the velocity v makes an angle β with the horizontal, then

$$\tan\beta = \frac{v_y}{v_x} = \frac{gt}{u} \qquad \text{or} \quad \beta = \tan^{-1}\left(\frac{gt}{u}\right)$$

Ex. 25 ***A body is thrown horizontally from the top of a tower and strikes the ground after three seconds at an angle of 45° with the horizontal. Find the height of the tower and the speed with which the body was projected. Take g = 9.8 ms⁻².***

Sol. As shown in figure, suppose the body is thrown horizontally from the top O of a tower of height y with velocity u. The body hits the ground after 3s. Considering verticlly downward motion of the body,

$y = u_y t + \frac{1}{2}gt^2 = 0 \times 3 + \frac{1}{2} \times 9.8 \times (3)^2 = 44.1$ m [$\therefore$ Initial vertical velocity, $u_y = 0$]

Final vertical velocity,

$v_y = u_y + gt = 0 + 9.8 \times 3 = 29.4\ ms^{-1}$

Final horizontal velocity, $v_x = u$

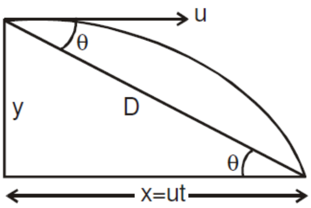
As the resultant velocity u makes an angle of 45° with the horizontal, so

$$\tan 45° = \frac{v_y}{v_x} \text{ or } 1 = \frac{29.4}{x} \quad \text{or} \quad u = 29.4\ ms^{-1}.$$

Ex. 26 ***A particle is projected horizontally with a speed u from the top of plane inclined at an angle θ with the horizontal. How far from the point of projection will the particle strike the plane?***

Sol. The horizontal distance covered in time t,

$$x = ut \text{ or } t = \frac{x}{u}$$



The vertical distance covered in time t,

$$y = 0 + \frac{1}{2}gt^2 = \frac{1}{2}g \times \frac{x^2}{u^2} \quad \text{[using (1)]}$$

Also $\frac{y}{x} = \tan\theta$ or $y = x\tan\theta$ $\therefore$ $\frac{gx^2}{2u^2} = x\tan\theta$

or $x\left(\frac{gx}{2u^2} - \tan\theta\right) = 0$

As x = 0 is not possible, so $x = \frac{2u^2\tan\theta}{g}$

The distance of the point of strike from the point of projection is

$D = \sqrt{x^2 + y^2} = \sqrt{x^2 + (x\tan\theta)^2}$

$= x\sqrt{1 + \tan^2\theta} = x\sec\theta$ or $\mathbf{D = \frac{2u^2}{g}\tan\theta\sec\theta}$

Ex. 27 ***A ball rolls off the top of a stairway with a constant horizontal velocity u. If the steps are h metre high and w meter wide, show that the ball will just hit the edge of nth step if n =*** $\frac{2hu^2}{gw^2}$

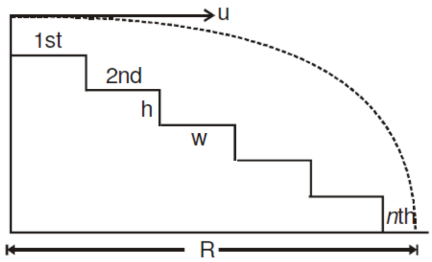
Sol. Refer to figure. For *n* th step,

net vertical displacement = nh

net horizontal displacement = nω

Let t be the time taken by the ball to reach the nth step. Then

R = ut



or $n\omega = ut$ or $t = \frac{n\omega}{u}$

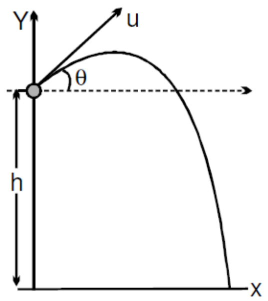
Also, $y = u_y t + \frac{1}{2}gt^2$

or $nh = 0 + \frac{1}{2}gt^2 = \frac{1}{2}g\left(\frac{n\omega}{u}\right)^2$ or $n = \frac{2hu^2}{g\omega^2}$

4.3 Projectile at an angle θ from height h

Consider the projectile as shown in the adjacent figure.

Take the point of projection as the origin the X and Y-axes as shown in figure.



For X-axis,

$u_x = u\cos\theta$

$a_x = 0$

$v_x = u\cos\theta$, and

$x = u\cos\theta \times t$

For Y-axis,

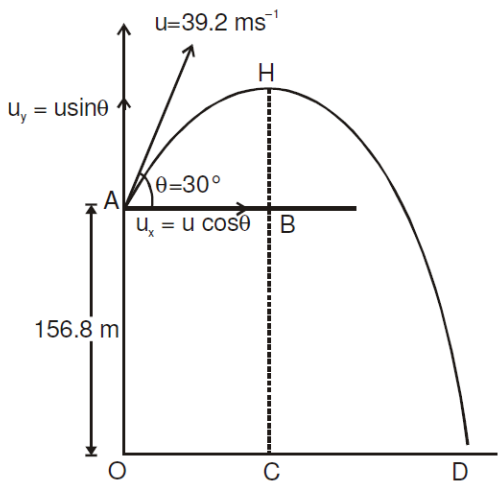
$u_y = u\sin\theta$,

$a_y = -g$,

$v_y = u\sin\theta - gt$, and $y = u\sin\theta\, t - \dfrac{gt^2}{2}$

Ex. 28 ***From the top of a tower 156.8 m high a projectile is projected with a velocity of 39.2 ms⁻¹ in a direction making an angle 30° with horizontal. Find the distance from the foot of tower where it strikes the ground and time taken to do so.***

Sol. The situation is shown



Here height of tower

OA = 156.8 m

$u = 39.2\ ms^{-1}$

$\theta = 30°$

time for which projectile remain is air = t = ?

Horizontal distance covered R = OD = ?

Now $u_x = u\cos\theta$ and

$u_y = u\sin\theta$ be the components of velocity $\vec{u}$.

Motion of projectile from O to H to D

Using equation $y = u_y t + \frac{1}{2}a_y t^2$

Here : $y = 156.8$ m ; $u_y = -u\sin\theta$

$= 39.2\sin 30°$

$a_y = 9.8\ m/s^2$; t = ?

$156.8 = -39.2 \times 0.5\,t + 4.9\,t^2$

$156.8 = -19.6\,t + 4.9\,t^2$

or $4.9t^2 - 19.6t - 156.8 = 0$

or $t^2 - 4t - 32 = 0 \Rightarrow \quad (t-8)(t+4) = 0$

We get t = 8 s; t = – 4s

t = – 4 s is not possible, thus we take t = 8s.

Now horizontal distance covered in this time

$R = u_x \times t = u\cos\theta \times t = 39.2 \times \cos 30° \times t$

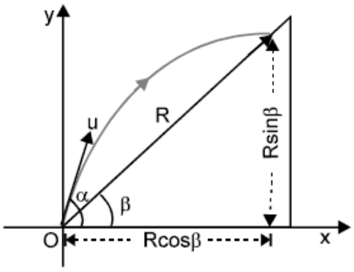
R = 271.57 m

PROJECTION ON AN INCLINED PLANE

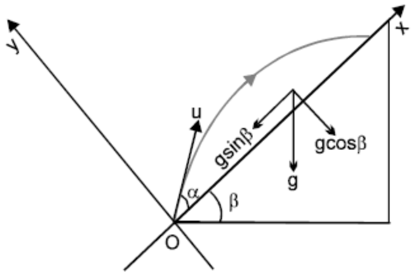
To solve the problem of projectile motion on an incline plane we can adopt two types of axis system as shown in the figures

Case (i) :

Up the incline



axis system 1



axis system 2

Here α is angle of projection with the horizontal.
In this case:

$a_x = 0$ $u_x = u\cos\alpha$

$a_y = -g$ $u_y = u\sin\alpha$

Here α is angle of projection with the inclined plane
In this case:

$a_x = -g\sin\beta$ $u_x = u\cos\alpha$

$a_y = -g\cos\beta$ $u_y = u\sin\alpha$

Time of flight (T) :

when the particle strikes the inclined plane y coordinate becomes zero

$$y = u_y t + \frac{1}{2}a_y t^2$$

$$\Rightarrow \quad 0 = u\sin\alpha T - \frac{1}{2} g\cos\beta T^2$$

$$\Rightarrow \quad T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_\perp}{g_\perp}$$

Maximum height (H) :

when half of the time is elasped y coordinate is equal to maximum height of the projectile

$$H = u\sin\alpha\left(\frac{u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2}g\sin\beta\left(\frac{u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \quad H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u^2{}_\perp}{2g_\perp}$$

Range along the inclined plane (R):

When the particle strikes the inclined plane x coordinate is equal to range of the particle

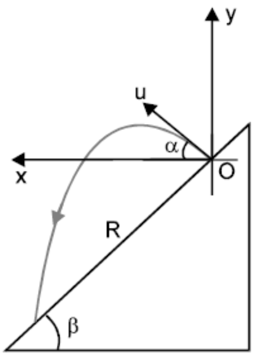
$$x = u_x t + \frac{1}{2}a_x\ t^2$$

$$\Rightarrow \quad R = u\cos\theta\left(\frac{2u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2}g\cos\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \quad R = \frac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta}$$

Case (ii) :

Down the incline

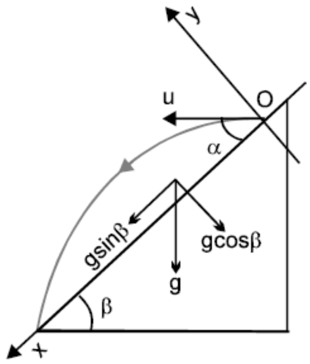


axis system 1

In this case :

$a_x = 0$ $u_x = u\cos\alpha$

$a_y = -g$ $u_y = u\sin\alpha$



axis system 2

In this case :

$a_x = g\sin\beta$ $u_x = u\cos\alpha$

$a_y = -g\cos\beta$ $u_y = u\sin\alpha$

Time of flight (T) :

when the particle strikes the inclined plane y coordinate becomes zero

$$y = u_y t + \frac{1}{2}a_y t^2$$

$$\Rightarrow \quad 0 = u\sin\alpha T - \frac{1}{2} g\cos\beta T^2$$

$$\Rightarrow \quad T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_\perp}{g_\perp}$$

Maximum height (H) :

when half of the time is elasped y coordinate is equal to maximum height of the projectile

$$H = u\sin\alpha\left(\frac{u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2}g\sin\beta\left(\frac{u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \quad H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u^2{}_\perp}{2g_\perp}$$

Range along the inclined plane (R):

When the particle strikes the inclined plane x coordinate is equal to range of the particle

$$x = u_x t + \frac{1}{2}a_x\ t^2$$

$$\Rightarrow \quad R = u\cos\theta\left(\frac{2u\sin\alpha}{g\cos\beta}\right) + \frac{1}{2}g\cos\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \quad R = \frac{2u^2\sin\alpha\cos(\alpha-\beta)}{g\cos^2\beta}$$

Table 1 : Standard results for projectile motion on an inclined plane

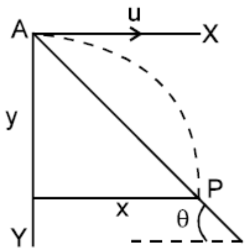
	Up the Incline	Down the Incline
Range	$\dfrac{2u^2 \sin\alpha \cos(\alpha+\beta)}{g\cos^2\beta}$	$\dfrac{2u^2 \sin\alpha \cos(\alpha-\beta)}{g\cos^2\beta}$
Time of flight	$\dfrac{2u\sin\alpha}{g\cos\beta}$	$\dfrac{2u\sin\alpha}{g\cos\beta}$
Angle of projection for maximum range	$\dfrac{\pi}{4}-\dfrac{\beta}{2}$	$\dfrac{\pi}{4}+\dfrac{\beta}{2}$
Maximum Range	$\dfrac{u^2}{g(1+\sin\beta)}$	$\dfrac{u^2}{g(1-\sin\beta)}$

Here α is the angle of projection with the incline and β is the angle of incline.

NOTE : For a given speed, the direction - which gives the maximum range of the projectile on an incline, bisects the angle between the incline and the vertical, for upward or downward projection.

Ex. 29 A particle is projected horizontally with a speed u from the top of a plane inclined at an angle θ with the horizontal. How far from the point of projection will the particle strike the plane?

Sol. Take X,Y-axes as shown in figure. Suppose that the particle strikes the plane at a point P with coordinates (x,y) . Consider the motion between A and P.



Motion in x direction :

Initial velocity = u

Acceleration = 0

$x = ut$.....................(i)

Motion in y direction :

Initial velocity = u

Acceleration = g

$$y = \frac{1}{2}gt^2 \text{.....................(ii)}$$

Eliminating t from (i) and (ii)

$$y = \frac{1}{2}g\frac{x^2}{u^2}$$

Also $y = x\tan\theta$

Thus , $\dfrac{gx^2}{gu^2} = x\tan\theta$ giving $x = 0$ or, $\dfrac{2u^2\tan\theta}{g}$

Clearly the point P corresponds to $x = \dfrac{2u^2\tan\theta}{g}$

then $y = x\tan\theta = \dfrac{2u^2\tan^2\theta}{g}$

The distance AP = $\sqrt{x^2+y^2}$

$$= \frac{2u^2}{g}\tan\theta\sqrt{1+\tan^2\theta} \qquad = \frac{2u^2}{g}\tan\theta\sec\theta$$

Ex. 30 A projectile is thrown at an angle θ with an inclined plane of inclination β as shown in figure. Find the relation between β and θ if :

(a) projectile strikes the inclined plane perpendicularly,

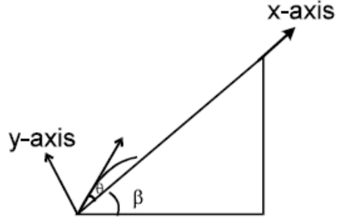
(b) projectile strikes the inclined plane horizontal.

Sol. (a) If projectile strikes perpendicularly.

$v_x = 0$ when projectile strikes

$v_x = u_x + a_x t$

$0 = u \cos\theta - g \sin\beta\, T$

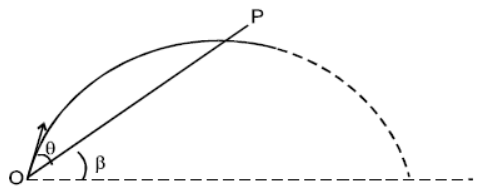


$$T = \frac{u\cos\theta}{g\sin\beta}$$

we also know that $T = \dfrac{2u\sin\theta}{g\cos\beta}$

$$\Rightarrow \quad \frac{u\cos\theta}{g\sin\beta} = \frac{2u\sin\theta}{g\cos\beta} \quad \Rightarrow \quad 2\tan\theta = \cot\beta$$

(b) If projectile strikes horizontally, then at the time of striking the projectile will be at the maximum height from the ground. Therefore :



$$t_{0P} = \frac{2u\sin\theta}{g\cos\beta}$$

$$t_{0P} = \frac{2u\sin(\theta+\beta)}{2\times g}$$

$$\Rightarrow \quad \frac{2u\sin\theta}{g\cos\beta} = \frac{2u\sin(\theta+\beta)}{2g}$$

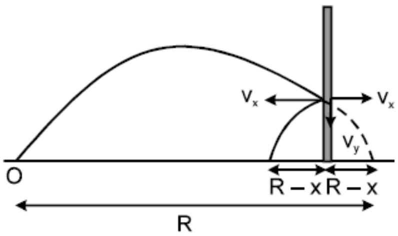
$$\Rightarrow \quad 2\sin\theta = \sin(\theta+\beta)\cos\beta\ .$$

Elastic collision of a projectile with a wall :

Suppose a projectile is projected with speed u at an angle θ from point O on the ground. Range of the projectile is R. If a wall is present in the path of the projectile at a distance x from the point O. The collision with the wall is elastic, path of the projectile changes after the collision as described below.

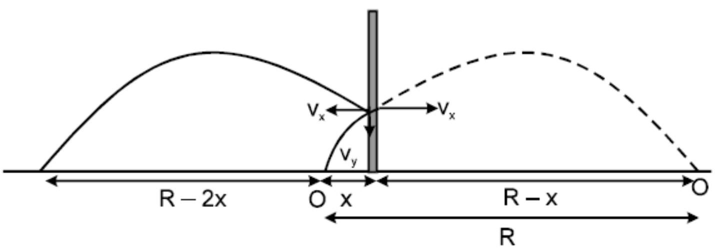
Case I : If $x \geq \dfrac{R}{2}$

Direction of x component of velocity is reversed but its magnitude remains the same and y component of velocity remains unchanged, therefore the remaining distance (R – x) is covered in the backward direction and projectile falls a distance (R – 2x) ahead of the point O as shown in figure.



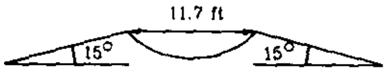
Case II : If $x < \dfrac{R}{2}$

Direction of x component of velocity is reversed but its magnitude remains the same and y component of velocity remains unchanged, therefore the remaining distance (R – x) is covered in the backward direction and projectile falls a distance (R – 2x) behind the the point O as shown in figure.

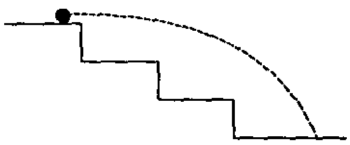


DPP # 3

1. A ball is thrown horizontally from a point 100 m above the ground with a speed of 20 m/s. Find (a) the time it takes to reach the ground, (b) the horizontal distance it travels before reaching the ground, (c) the velocity (direction and magnitude) with which it strikes the ground.
2. A ball is thrown at a speed of 40 m/s at an angle of 60° with the horizontal. Find (a) the maximum height reached and (b) the range of the ball. Take g =10 m/s^2.
3. In a soccer practice session the football is kept at the centre of the field 40 yards from the 10 ft high goalposts. A goal is attempted by kicking the football at a speed of 64 ft/s at an angle of 45° to the horizontal. Will the ball reach the goal post?
4. A popular game in Indian villages is goli which is played with small glass balls called golis. The goli of one player is situated at a distance of 2.0 m from the goli of the second player. This second player has to project his goli by keeping the thumb of the left hand at the place of his goli, holding the goli between his two middle fingers and making the throw. If the projected goli hits the goli of the first player, the second player wins. If the height from which the goli is projected is 19.6 cm from the ground and the goli is to be projected horizontally, with what speed should it be projected so that it directly hits the stationary goli without falling on the ground earlier?
5. Figure shows a 11.7 ft wide ditch, with the approach roads at an angle of 15° with the horizontal. With what minimum speed should a motorbike be moving on the road so that it safely crosses the ditch?



Assume that the length of the bike is 5 ft, and it leaves the road when the front part runs out of the approach road.
6. A person standing on the top of a cliff 171 ft high has to throw a packet to his friend standing on the ground 228 ft horizontally away. If he throws the packet directly aiming at the friend with a speed of 15.0 ft, how short will the packet fall ?
7. A ball is projected from a point on the floor with a speed of 15 m/s at an angle of 60° with the horizontal. Will it hit a vertical wall 5 m away from the point of projection and perpendicular to the plane of projection without hitting the floor ? Will the answer differ if the wall is 22 m away?
8. Find the average velocity of a projectile between the instants it crosses half the maximum height. It is projected with a speed u at an angle θ with the horizontal.
9. A bomb is dropped from a plane flying horizontally with uniform speed. Show that the bomb will explode vertically below the plane. Is the statement true if the plane flies with uniform speed but not horizontally?
10. A boy standing on a long railroad car throws a ball straight upwards. The car is moving on the horizontal road with an acceleration of 1 m/s^2 and the projection velocity in the vertical direction is 9.8 m/s. How far behind the boy will the ball fall on the car?
11. A staircase contains three steps each 10 cm high and 20 cm wide (figure). What should be the minimum horizontal velocity of a ball rolling off the uppermost plane so as to hit directly the lowest plane?



12. A person is standing on a truck moving with a constant velocity of 14.7 m/s on a horizontal road. The man throws a ball in such a way that it returns to the truck after the truck has moved 58.8 m. Find the speed and the angle of projection (a) as seen from the truck, (b) as seen from the road.
13. The benches of a gallery in a cricket stadium are 1 m wide and 1 m high. A batsman strikes the ball at a level one metre above the ground and hits a mammoth sixer. The ball starts at 35 m/s at an angle of 53° with the horizontal. The benches are perpendicular to the plane of motion and the first bench is 110 m from the batsman. On which bench will the ball hit?

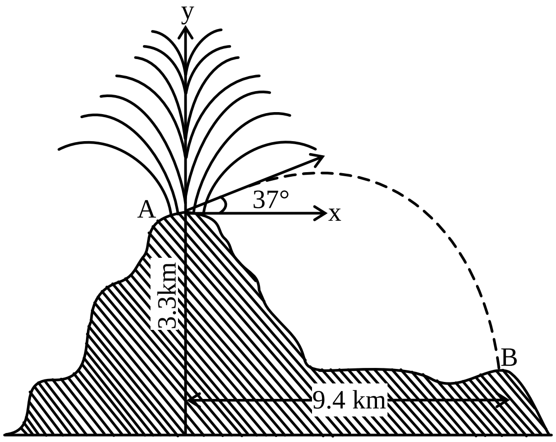
14. A man is sitting on the shore of a river. He is in the line of a 1.0 m long boat and is 5.5 m away from the centre of the boat. He wishes to throw an apple into the boat. If he can throw the apple only with a speed of 10 m/s, find the minimum and maximum angles of projection for successful shot. Assume that the point of projection and the edge of the boat are in the same horizontal level.

15. A stone is thrown horizontally. In 0.5 second after the stone began to move, the numerical value of its velocity was 1.5 times its initial velocity. Find the initial velocity of stone.

16. A dive bomber, diving at an angle of 53º with the vertical, releases a bomb at an altitude of 2400 ft. The bomb hits the ground 5.0 s after being released. (a) What is the speed of the bomber ? (b) How far did the bomb travel horizontally during its flight? (c) What were the horizontal and vertical components of its velocity just before striking the ground ?

17. A boy throws a ball so as to clear a wall of height 'h' at a distance 'x' from him. Find minimum speed of the ball to clear the wall.

18. During the volcanic eruption chunks of solid rock are blasted out of the volcano.

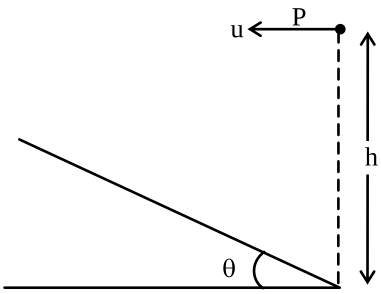


(a) At what initial speed would a volcanic object have to be ejected at 37º to the horizontal from the vent A in order to fall at B as shown in figure. (b) What is the time of flight. ($g = 9.8\ m/s^2$)

19. At a harbour enemy ship is at a distance $180\sqrt{3}$ m from the security cannon having a muzzle velocity of 60 m/s (a) To what angle must the cannon be elevated to hit the ship? (b) What is the time of flight ? (c) How far should the ship be moved away from its initial position so that it becomes beyond the range of the cannon ($g = 10\ m/s^2$) ?

20. **(a)** A particle is projected with a velocity of 29.4 m/s at an angle of 60º to the horizontal. Find the range on a plane inclined at 30º to the horizontal when projected from a point of the plane up the plane.

(b) Determine the velocity with which a stone must be projected horizontally from a point P, so that it may hit the inclined plane perpendicularly. The inclination of the plane with the horizontal is θ and P is h metre above the foot of the incline as shown in the figure.



RELATIVE MOTION

The word 'relative' is a very general term, which can be applied to physical, nonphysical, scalar or vector quantities. For example, my height is five feet and six inches while my wife's height is five feet and four inches. If I ask you how high I am relative to my wife, your answer will be two inches. What you did? You simply subtracted my wife's height from my height. The same concept is applied everywhere, whether it is a relative velocity, relative acceleration or anything else. So, from the above discussion we may now conclude that relative velocity of A with respect of B (written as $\vec{v}_{AB}$) is

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

Similarly, relative acceleration of A with respect of B is

$$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$$

If it is a one dimensional motion we can treat the vectors as scalars just by assigning the positive sign to one direction and negative to the other. So, in case of a one dimensional motion the above equations can be written as

$$v_{AB} = v_A - v_B$$

and $$a_{AB} = a_A - a_B$$

Further, we can see that

$$\vec{v}_{AB} = -\vec{v}_{BA} \quad \text{or} \quad \vec{a}_{BA} = -\vec{a}_{AB}$$

Ex. 31 Seeta is moving due east with a velocity of 1 m/s and Geeta is moving the due west with a velocity of 2 m/s. What is the velocity of Seeta with respect to Geeta?

Sol. It is a one dimensional motion. So, let us choose the east direction as positive and the west as negative. Now, given that

v_s = velocity of Seeta = 1 m/s

and v_G = velocity of Geeta = – 2m/s

Thus, v_{SG} = velocity of Seeta with respect to Geeta

$= v_S - v_G = 1 - (-2) = 3$ m/s

Hence, velocity of Seeta with respect to Geeta is 3 m/s due east.

IMPORTANT NOTE :

PROCEDURE TO SOLVE THE VECTOR EQUATION.

$\vec{A} = \vec{B} + \vec{C}$ ...(1)

(a) Their are 6 variables in this equation which are following :

(1) Magnitude of $\vec{A}$ and its direction

(2) Magnitude of $\vec{B}$ and its direction

(3) Magnitude of $\vec{C}$ and its direction.

(b) We can solve this equation if we know the value of 4 varibales [Note : two of them must be directions]

(c) If we know the two direction of any two vectors then we will put them on the same side and other on the different side.

For example

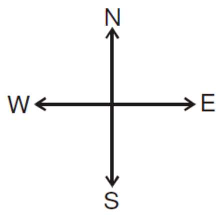
If we know the directions of $\vec{A}$ and $\vec{B}$ and $\vec{C}$'s direction is unknown then we make equation as follows : -

$$\vec{C} = \vec{A} - \vec{B}$$

(d) Then we make vector diagram according to the equation and resolve the vectors to know the unknown values.

Ex. 32 Car A has an acceleration of 2 m/s² due east and car B, 4 m/s² due north. What is the acceleration of car B with respect to car A?

Sol. It is a two dimensional motion. Therefore,



$\vec{a}_{BA}$ = acceleration of car B with respect to car A

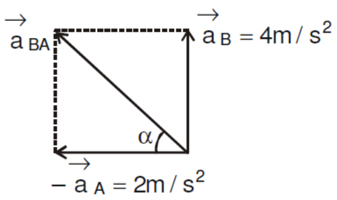
$= \vec{a}_B = -\vec{a}_A$

Here, $\vec{a}_B$ = acceleration of car B = 4 m/s^2 (due north)

and $\vec{a}_A$ = acceleration of car A = 2 m/s^2 (due east)

$$|\vec{a}_{BA}| = \sqrt{(4)^2 + (2)^2} = 2\sqrt{5}\text{m/s}^2$$

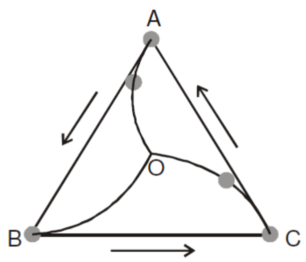
and $\alpha = \tan^{-1}\left(\frac{4}{2}\right) = \tan^{-1}(2)$



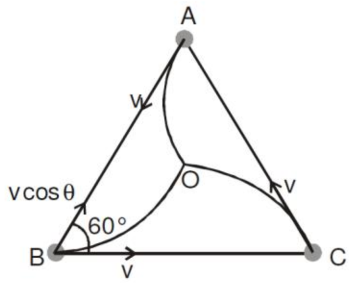
Thus, $\vec{a}_{BA}$ is $2\sqrt{5}$ m/s^2 at an angle of $\alpha = \tan^{-1}(2)$ from west towards north.

Ex. 33 Three particle A, B and C situated at the vertices of an equilateral triangle starts moving simultaneously at a constant speed "v" in the direction of adjacent particle, which falls ahead in the anti-clockwise direction. If "a" be the side of the triangle, then find the time when they meet.

Sol. Here, particle "A" follows "B", "B" follows "C" and "C" follows "A". The direction of motion of each particle keeps changing as motion of each particle is always directed towards other particle. The situation after a time "t" is shown in the figure with a possible outline of path followed by the particles before they meet.



This problem appears to be complex as the path of motion is difficult to be defined. But, it has a simple solution in component analysis. Let us consider the pair "A" and "B". The initial component of velocities in the direction of line joining the initial position of the two particles is "v" and "vcosθ" as shown in the figure here :



The component velocities are directed towards eachother. Now, considering the linear (one dimensional) motion in the direction of AB, the relative velocity of "A" with respect to "B" is :

$$v_{AB} = v_A - v_B$$

$$v_{AB} = v - (-v\cos\theta) = v + v\cos\theta$$

In equilateral triangle, $\theta = 60°$

$$v_{AB} = v + v\cos 60° = v + \frac{v}{2} = \frac{3v}{2}$$

The time taken to cover the displacement "a" i.e. the side of the triangle

$$t = \frac{2a}{3v}$$

QUESTIONS BASED ON RELATIVE MOTION ARE USUALLY OF FOLLOWING FOUR TYPES :

(a) Minimum distance between two bodies in motion
(b) River-boat problems
(c) Aircraft-wind problems
(d) Rain problems

(a) Minimum distance between two bodies in motion

When two bodies are in motion, the questions like, the minimum distance between them or the time when one body overtakes the other can be solved easily by the principle of relative motion. In these type of problems one body is assumed to be at rest and the relative motion of the other body is considered. By assuming so two body problem is converted into one body problem and the solution becomes easy. Following example will illustrate the statement.

Ex. 34 ***Car A and car B start moving simultaneously in the same direction along the line joining them. Car A with a constant acceleration a = 4 m/s², while car B moves with a constant velocity v = 1 m/s. At time t = 0, car A is 10 m behind car B. Find the time when car A overtakes car B.***

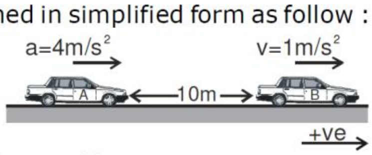
Sol. Given : $u_A = 0$, $u_B = 1$ m/s, $a_A = 4$m/s² and $a_B = 0$

Assuming car B to be at rest, we have

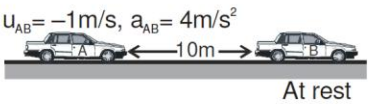
$$u_{AB} = u_A - u_B = 0 - 1 = -1 \text{ m/s}$$

$$a_{AB} = a_A - a_B = 4 - 0 = 4 \text{ m/s}^2$$

Now, the problem can be assumed in simplified form as follow :



Substituting the proper values in equation



$$s = ut + \frac{1}{2}at^2$$

we get $10 = -t + \frac{1}{2}(4)(t^2)$ or $2t^2 - t - 10 = 0$

or $t = \frac{1 \pm \sqrt{1+80}}{4} = \frac{1 \pm \sqrt{81}}{4} = \frac{1 \pm 9}{4}$ or $t = 2.5$ s and -2 s

Ignoring the negative value, the desired time is 2.5s. **Ans.**

Note : The above problem can also be solved without using the concept of relative motion as under.
At the time when A overtakes B,

$$s_A = s_B + 10$$

$\therefore$ $\frac{1}{2} \times 4 \times t^2 = 1 \times t + 10$

or $2t^2 - t - 10 = 0$

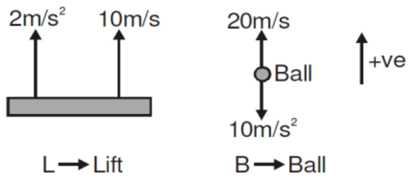
Which on solving gives $t = 2.5$ s and -2 s, the same as we found above.
As per my opinion, this approach (by taking absolute values) is more suitable in case of two body problem in one dimensional motion. Let us see one more example in support of it.

Ex. 35 ***An open lift is moving upwards with velocity 10m/s. It has an upward acceleration of 2m/s². A ball is projected upwards with velocity 20 m/s relative to ground. Find :***
(a) time when ball again meets the lift.
(b) displacement of lift and ball at that instant.
(c) distance travelled by the ball upto that instant. Take g = 10 m/s²

Sol. (a) At the time when ball again meets the lift,

$$s_L = s_B$$

$\therefore$ $10t + \frac{1}{2} \times 2 \times t^2 = 20\,t - \frac{1}{2} \times 10t^2$



Solving this equation, we get

$t = 0$ and $t = \frac{5}{3}$ s

$\therefore$ Ball will again meet the lift after $\frac{5}{3}$ s.

(b) At this instant

$$s_L = s_B = 10 \times \frac{5}{3} + \frac{1}{2} \times 2 \times \left(\frac{5}{3}\right)^2 = \frac{175}{9} \text{ m} = 19.4 \text{ m}$$

(c) For the ball $u \uparrow \downarrow a$. Therefore, we will first find t_0, the time when its velocity becomes zero.

$$t_0 = \left|\frac{u}{a}\right| = \frac{20}{10} = 2 \text{ s}$$

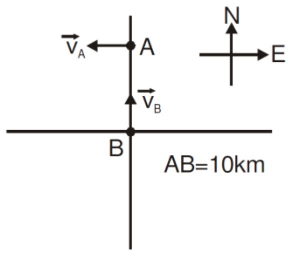
As $t\left(= \frac{5}{3}\text{s}\right) < t_0$, distance and displacement are equal

or $d = 19.4$ m **Ans.**

Concept of relative motion is more useful in two body problem in two (or three) dimensional motion. This can be understood by the following example.

Ex. 36 ***Two ships A and B are 10 km apart on a line running south to north. Ship A farther north is streaming west at 20 km/h and ship B is streaming north at 20km/h. What is their distance of closest approach and how long do they take to reach it ?***

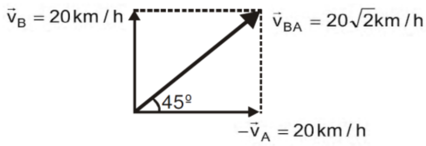
Sol. Ships A and B are moving with same speed 20 km/h in the directions shown in figure. It is a two dimensional, two body problem with zero acceleration. Let us find $\vec{v}_{BA}$



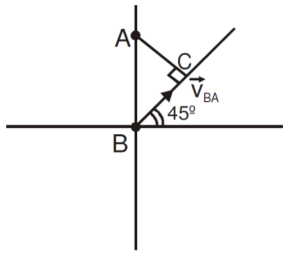
$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

Here, $|\vec{v}_{BA}| = \sqrt{(20)^2 + (20)^2}$

$$= 20\sqrt{2}\,\text{km/h}$$



i.e., $\vec{v}_{BA}$ is $20\sqrt{2}\,\text{km/h}$ at an angle of 45º from east towards north. Thus, the given problem can be simplified as :



A is at rest and B is moving with $\vec{v}_{BA}$ in the direction shown in figure. Therefore, the minimum distance between the two is

$$s_{min} = AC = AB \sin 45^\circ$$

$$= 10\left(\frac{1}{\sqrt{2}}\right)\text{km} \quad = 5\sqrt{2}\,\text{km} \quad \textbf{Ans.}$$

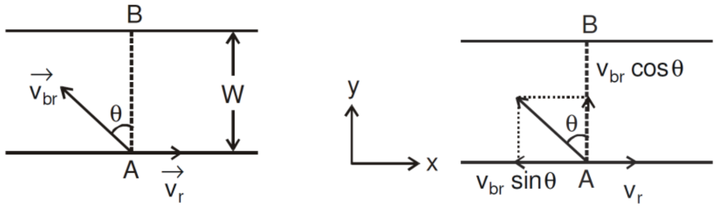
and the desired time is

$$t = \frac{BC}{|\vec{v}_{BA}|} = \frac{5\sqrt{2}}{20\sqrt{2}} \qquad (BC = AC = 5\sqrt{2}\,\text{km})$$

$$= \frac{1}{4}\text{h} = 15\,\text{min} \quad \textbf{Ans.}$$

(B) River - Boat Problems

In river-boat problems we come across the following three terms :



$\vec{v}_r$ = absolute velocity of river

$\vec{v}_{br}$ = velocity of boatman with respect to river or velocity of boatman is still water

and $\vec{v}_b$ = absolute velocity of boatman.

- Here, it is important to note that $\vec{v}_{br}$ is the velocity of boatman with which he steers and $\vec{v}_b$ is the actual velocity of boatman relative to ground.

Further, $\vec{v}_b = \vec{v}_{br} + \vec{v}_r$

Now, let us derive some standard results and their special cases.

A boatman starts from point A on one bank of a river with velocity $\vec{v}_{br}$ in the direction shown in fig. River is flowing along positive x-direction with velocity $\vec{v}_r$. Width of the river is w, then

$$\vec{v}_b = \vec{v}_{br} + \vec{v}_r$$

Therefore, $v_{bx} = v_{rx} + v_{brx} = v_r - v_{br}\sin\theta$

and $v_{by} = v_{ry} + v_{bry}$

$= 0 + v_{br}\cos\theta = v_{br}\cos\theta$

Now, time taken by the boatman to cross the river is :

$$t = \frac{w}{v_{by}} = \frac{w}{v_{br}\cos\theta}$$

or $$t = \frac{w}{v_{br}\cos\theta} \quad \text{...(i)}$$

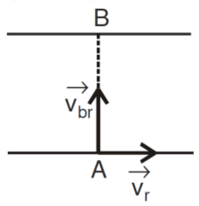
Further, displacement along x-axis when he reaches on the other bank (also called drift) is :

$$x = v_{bx}\,t = (v_r - v_{br}\sin\theta)\frac{w}{v_{br}\cos\theta}$$

or $$x = (v_r - v_{br}\sin\theta)\frac{w}{v_{br}\cos\theta} \quad \text{...(ii)}$$

Three special are :

(i) Condition when the boatman crosses the river in shortest interval of time



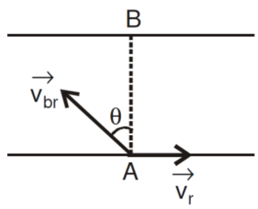
From Eq.(i) we can see that time (t) will be minimum when $\theta = 0°$, i.e., the boatman should steer his boat perpendicular to the river current.

Also, $t_{min} = \frac{w}{v_{br}}$ as $\cos\theta = 1$

(ii) Condition when the boatman wants to reach point B, i.e., at a point just opposite from where he started

In this case, the drift (x) should be zero.

$\therefore \quad x = 0$



or $$(v_r - v_{br}\sin\theta)\frac{w}{v_{br}\cos\theta} = 0$$

or $$v_r = v_{br}\sin\theta$$

or $$\sin\theta = \frac{v_r}{v_{br}} \quad \text{or } \theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$$

Hence, to reach point B the boatman should row at an angle $\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$ upstream from AB.

Further, since $\sin\theta$ not greater than 1.

So, if $v_r \geq v_{br}$, the boatman can never reach at point B. Because if $v_r = v_{br}$, $\sin\theta = 1$ or $\theta = 90°$ and it is just impossible to reach at B if $\theta = 90°$. Moreover it can be seen that $v_b = 0$ if $v_r = v_{br}$ and $\theta = 90°$. Similarly, if $v_r > v_{br}$, $\sin\theta > 1$, i.e., no such angle exists. Practically it can be realized in this manner that it is not possible to reach at B if river velocity (v_r) is too high.

(iii) Shortest path

Path length travelled by the boatman when he reaches the opposite shore is

$$s = \sqrt{w^2 + x^2}$$

Here, w = width of river is constant. So for s to be minimum modulus of x (drift) should be minimum. Now two cases are possible.

When $v_r < v_{br}$: In this case x = 0,

when $\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$ or $s_{min} = w$ at $\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$

When $v_r > v_{br}$: In this case x is minimum, where $\frac{dx}{d\theta} = 0$

or $\frac{d}{d\theta}\left\{\frac{w}{v_{br}\cos\theta}(v_r - v_{br}\sin\theta)\right\} = 0$

or $-v_{br}\cos^2\theta - (v_r - v_{br}\sin\theta)(-\sin\theta) = 0$

or $-v_{br} + v_r\sin\theta = 0$

or $\theta = \sin^{-1}\left(\frac{v_{br}}{v_r}\right)$

Now, at this angle we can find x_{min} and then s_{min} which comes out to be

$$s_{min} = w\left(\frac{v_r}{v_{br}}\right) \text{ at } \theta = \sin^{-1}\left(\frac{v_{br}}{v_r}\right)$$

Ex. 37 A man can row a boat with 4 km/h in still water. If he is crossing a river where the current is 2 km/h.

(a) In what direction will his boat be headed, if he wants to reach a point on the other bank, directly opposite to starting point?

(b) If width of the river is 4 km, how long will the man take to cross the river, with the condition in part (a)?

(c) In what direction should he head the boat if he wants to cross the river in shortest time and what is this minimum time?

(d) How long will it take him to row 2 km up the stream and then back to his starting point ?

Sol. (a) Given, that v_{br} = 4 km/h and v_r = 2 km/h

$$\therefore \theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right) = \sin^{-1}\left(\frac{2}{4}\right) = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

Hence, to reach the point directly opposite to starting point he should head the boat at an angle of 30° with AB or 90° + 30° = 120° with the river flow.

(b) Time taken by the boatman to cross the river

w = width of river = 4 km

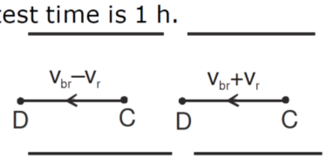
v_{br} = 4 km/h and θ = 30°

$$\therefore \quad t = \frac{4}{4\cos 30°} = \frac{2}{\sqrt{3}}h \qquad \textbf{Ans.}$$

(c) For shortest time θ = 0°

$$\text{and} \quad t_{min} = \frac{w}{v_{br}\cos 0°} = \frac{4}{4} = 1h$$

Hence, he should head his boat perpendicular to the river current for crossing the river in shortest time and this shortest time is 1 h.



(d) $t = t_{CD} + t_{DC}$

$$\text{or} \quad t = \frac{CD}{v_{db} - v_r} + \frac{DC}{v_{br} + v_r} = \frac{2}{4-2} + \frac{2}{4+2} = 1 + \frac{1}{3} = \frac{4}{3}h \qquad \textbf{Ans.}$$

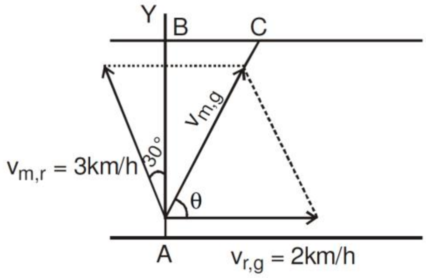
Ex. 38 ***A man can swim at a speed of 3 km/h in still water. He wants to cross a 500 m wide river flowing at 2 kh/h. He keeps himself always at an angle of 120° with the river flow while swimming.***
(a) Find the time he takes to cross the river.
(b) At what point on the opposite bank will he arrive ?

Sol. The situation is shown in figure

Here $\vec{v}_{r,g}$ = velocity of the river with respect to the ground

$\vec{v}_{m,r}$ = velocity of the man with respect to the river

$\vec{v}_{m,g}$ = velocity of the man with respect to the ground.



(a) We have

$$\vec{v}_{m,g} = \vec{v}_{m,r} + \vec{v}_{r,g} \qquad \text{...(i)}$$

Hence, the velocity with respect to the ground is along AC.
Taking y-components in equation (i),

$$\vec{v}_{m,g}\sin\theta = 3 \text{ km/h} \cos 30° + 2 \text{ km/h} \cos 90° = \frac{3\sqrt{3}}{2} \text{ km/h}$$

Time taken to cross the river

$$= \frac{\text{displacement along the Y - axis}}{\text{velocity along the Y - axis}} = \frac{1/2\text{km}}{3\sqrt{3}/2\text{km/h}} = \frac{1}{3\sqrt{3}}\text{h}$$

(b) Taking x-components in equation (i),

$$\vec{v}_{m,g}\cos\theta = -3\text{km/h} \sin 30° + 2 \text{ km/h} = \frac{1}{2}\text{km/h}$$

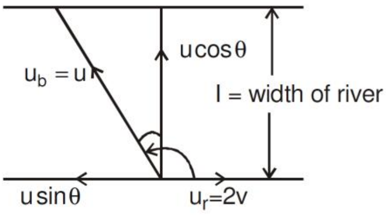
Displacement along the X-axis as the man crosses the river
= (velocity along the X-axis) (time)

$$= \left(\frac{1\text{km}}{2\text{h}}\right) \times \left(\frac{1}{3\sqrt{3}}\text{h}\right) = \frac{1}{6\sqrt{3}}\text{km}$$

Ex. 39 ***A boat moves relative to water with a velocity v and river is flowing with 2v. At what angle the boat shall move with the stream to have minimum drift?***
(A) 30° ***(B) 60°*** ***(C) 90°*** ***(D) 120°***

Sol. (D) Let boat move at angle θ to the normal as shown in

figure then time to cross the river = $\frac{1}{v\cos\theta}$



drift x = (2v − v sin θ) $\frac{1}{v\cos\theta}$ for x to be minimum

$\frac{dx}{d\theta}$ = 0 = 1 (2 sec θ tan θ − sec²θ) or sin θ = 1/2

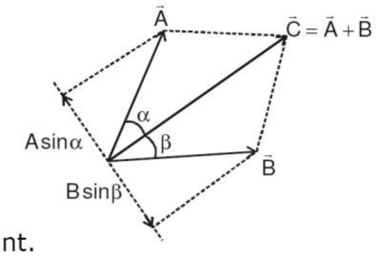
or θ = 30° and ϕ = 90 + 30 = 120°

(C) Aircraft Wind Problems

This is similar to river boat problem. The only difference is that $\vec{v}_{br}$ is replaced by $\vec{v}_{aw}$ (velocity of aircraft with respect to wind or velocity of aircraft in still air), $\vec{v}_r$ is replaced by $\vec{v}_w$ (velocity of wind) and $\vec{v}_b$ is replaced by $\vec{v}_a$ (absolute velocity of aircraft). Further, $\vec{v}_a = \vec{v}_{aw} + \vec{v}_w$. The following example will illustrate the theory.

NOTE : SHORT - TRICK

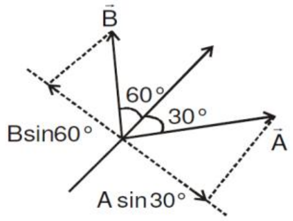
If their are two vectors $\vec{A}$ and $\vec{B}$ and their resultent make an anlge α with $\vec{A}$ and β with $\vec{B}$.



then $A \sin \alpha = \beta \sin \beta$

Means component of $\vec{A}$ perpendicular to resultant is equal in magnitude to the compopent of $\vec{B}$ also perpendicular to resultant.

Ex. 40 ***If two vectors $\vec{A}$ and $\vec{B}$ make angle 30° and 60° with their resultent and $\vec{B}$ has magnitude equal to 10, then find magnitude of $\vec{A}$.***



So $B \sin 60° = A \sin 30°$

$\Rightarrow$ $10 \sin 60° = A \sin 30°$

$\Rightarrow$ $A = 10\sqrt{3}$

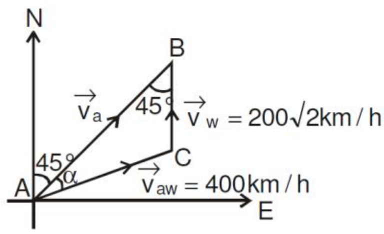
Ex. 41 ***An aircraft flies at 400 km/h in still air. A wind of $200\sqrt{2}$ km/h is blowing from the south. The pilot wishes to travel from A to a point B north east of A. Find the direction he must steer and time of his journey if AB = 1000 km.***

Sol. Given that $v_w = 200\sqrt{2}$ km/h

$v_{aw} = 400$ km/h and $\vec{v}_a$ should be along AB or in north-east direction. Thus, the direction of $\vec{v}_{aw}$ should be such as the resultant of $\vec{v}_w$ and $\vec{v}_{aw}$ is along AB or in north - east direction.

Let $\vec{v}_{aw}$ makes an angle α with AB as shown in figure.

Applying sine law in triangle ABC, we get



$$\frac{AC}{\sin 45°} = \frac{BC}{\sin \alpha}$$

or $\sin \alpha = \left(\frac{BC}{AC}\right) \sin 45° = \left(\frac{200\sqrt{2}}{400}\right)\frac{1}{\sqrt{2}} = \frac{1}{2}$

$\therefore$ $\alpha = 30°$

Therefore, the pilot should steer in a direction at an angle of $(45° + \alpha)$ or 75° from north towards east.

Further, $\dfrac{|\vec{v}_a|}{\sin(180°-45°-30°)} = \dfrac{400}{\sin 45°}$ or $|\vec{v}_a| = \dfrac{\sin 105°}{\sin 45°} \times (400)\ \dfrac{km}{h}$

$$= \left(\frac{\cos 15°}{\sin 45°}\right)(400)\ \frac{km}{h} = \left(\frac{0.9659}{0.707}\right)(400)\ \frac{km}{h}$$

$= 546.47$ km/h

$\therefore$ The time of journey from A to B is

$$t = \frac{AB}{|\vec{v}_a|} = \frac{1000}{546.47} h \Rightarrow t = 1.83\ h$$

(D) Rain Problems

In these type of problems we again come across three terms $\vec{v}_r$, $\vec{v}_m$ and $\vec{v}_{rm}$, Here,

$\vec{v}_r$ = velocity of rain

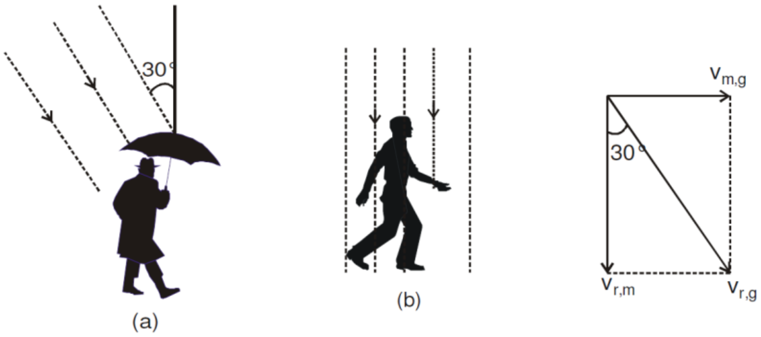
$\vec{v}_m$ = velocity of man (it may be velocity of cyclist or velocity of motorist also)

and $\vec{v}_{rm}$ = velocity of rain with respect to man.

Here, $\vec{v}_{rm}$ is the velocity of rain which appears to the man. Now, let us take one example of this.

Ex. 42 ***A man standing on a road has to hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10 km/h. He finds that raindrops are hitting his head vertically. Find the speed of raindrops with respect to (a) the road, (b) the moving man.***

Sol. When the man is at rest with respect to the ground, the rain comes to him at an angle 30° with the vertical. This is the direction of the velocity of raindrops with respect to the ground. The situation when the man runs is shown in the figure



Here $\vec{v}_{r,g}$ = velocity of the rain with respect to the ground

$\vec{v}_{m,g}$ = velocity of the man with respect to the ground and $\vec{v}_{r,m}$ = velocity of the rain with respect to the man.

We have, $\vec{v}_{r,g} = \vec{v}_{r,m} + \vec{v}_{m,g}$ ...(i)

Taking horizontal components, equation (i) gives

$v_{r,g}\sin 30° = u_{m,g}$ = 10 km/h or, v,g = $\dfrac{10\,\text{km/h}}{\sin 30°} = 20\text{km/h}$

Taking vertical components, equation (i) gives

$$v_{r,g}\cos 30° = v_{r,m} \text{ or, } v_{r,m} = (20\text{ km/h})\frac{\sqrt{3}}{2} = 10\sqrt{3}\text{ km/h.}$$

Ex. 43 ***To a man walking at the rate of 3 km/h the rain appears to fall vertically. When the increases his speed to 6 km/h it appears to meet him at an angle of 45° with vertical. Find the speed of rain.***

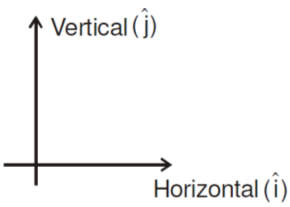
Sol. Let $\hat{i}$ and $\hat{j}$ be the unit vectors in horizontal and vertical directions respectively.

Let velocity of rain

$\vec{v}_r = a\hat{i} + b\hat{j}$ ...(i)

Then speed of rain will be

$|\vec{v}_r| = \sqrt{a^2 + b^2}$



In the first case $\vec{v}_m$ = velocity of man = $3\hat{i}$

$\therefore \quad \vec{v}_{rm} = \vec{v}_r - \vec{v}_m = (a-3)\hat{i} + b\hat{j}$

It seems to be in vertical direction. Hence,

$a - 3 = 0$ or $a = 3$

In the second case $\vec{v}_m = 6\,\hat{i}$

$\therefore \quad \vec{v}_{rm} = (a-6)\hat{i} + b\hat{j} = -3\hat{i} + b\hat{j}$

This seems to be at 45° with vertical.

Hence, $|b| = 3$

Therefore, from Eq. (ii) speed of rain is

$$|\vec{v}_r| = \sqrt{(3)^2 + (3)^2} = 3\sqrt{2}\ \text{km/h} \quad \textbf{Ans.}$$

DPP # 4

1. A river 400 m wide is flowing at a rate of 2.0 m/s. A boat is sailing at a velocity of 10 m/s with respect to the water, in a direction perpendicular to the river. (a) Find the time taken by the boat to reach the opposite bank. (b) How far from the point directly opposite to the starting point does the boat reach the opposite bank?

2. A swimmer wishes to cross a 500 m wide river flowing at 5 km/h. His speed with respect to water is 3 km/h. (a) If he heads in a direction making an angle θ with the flow, find the time he takes to cross the river. (b) Find the shortest possible time to cross the river.

3. Consider the situation of the previous problem. The man has to reach the other shore at the point directly opposite to his starting point. If he reaches the other shore somewhere else, he has to walk down to this point. Find the minimum distance that he has to walk.

4. An aeroplane has to go from a point A to another point B, 500 km away due 30° east of north. A wind is blowing due north at a speed of 20 m/s. The air-speed of the plane is 150 m/s. (a) Find the direction in which the pilot should head the plane to reach the point B. (b) Find the time taken by the plane to go from A to B.

5. Two friends A and B are standing a distance x apart in an open field and wind is blowing from A to B. A beats a drum and B hears the sound t_1 time after he sees the event. A and B interchange their positions and the experiment is repeated. This time B hears the drum t_2 time after he sees the event. Calculate the velocity of sound in still air v and the velocity of wind u. Neglect the time light takes in travelling between the friends.

6. Suppose A and B in the previous problem change their positions in such a way that the line joining them becomes perpendicular to the direction of wind while maintaining the separation x. What will be the time lag B finds between seeing and hearing the drum beating by A ?

7. Six particles situated at the corners of a regular hexagon of side a move at a constant speed v. Each particle maintains a direction towards the particle at the next corner. Calculate the time the particles will take to meet each other.

8. Men are running along a road at 15 km/h behind one another at equal intervals of 20 m. Cyclist are riding in the same direction at 25 km/h at equal intervals of 30 m. At what speed an observer travel along the road in opposite direction so that whenever he meets a runner he also meets a cyclist?

9. Two perpendicular rail tracks have two trains A & B respectively. Train A moves north with a speed of 54 km h^{-1} and train B moves west with a speed of 72 km h^{-1} . Assume that both trains starts from same point. Calculate the

(a) rate of separation of the two trains

(b) relative velocity of ground with respect to B

(c) relative velocity of A with respect to B.

10. A man is swimming in a lake in a direction of 30° East of North with a speed of 5 km/hr and a cyclist is going on a road along the lake shore towards East at a speed of 10 km/hr. In what direction and with what speed would the man appear to swim to the cyclist.

11. A motor boat has 2 throttle position on its engine. The high speed position propels the boat at 10 km hr^{-1} in still water and the low position gives half the higher speed. The boat travels from its dock downstream on a river with the throttle at low position and returns to its dock with throttle at high position. The return trip took 15 % longer time than it did for the downstream trip. Find the velocity of the water current in the river.

12. (I) A man can swim with a speed of 4 km h^{-1} in still water. How long does he take to cross a river 1 km wide if the river flows steadily at 3 km h^{-1} and he makes his strokes normal to the river current ? How far down the river does he go when he reaches the other bank ?

(II) If he keeps himself always at an angle of 120°C with the river flow while swimming.

(a) Find the time he takes to cross the river. (b) At what point on the opposite bank will he arrive ?

13. A river is flowing from west to east at a speed of 5 m/min. A man on the south bank of the river, capable of swimming at 10 m/min in still water, wants to swim across the river in shortest distance. In what direction should he swim ?

14. An airplane is flying with velocity $50\sqrt{2}$ km/hour in north-east direction. Wind is blowing at 25 km/hr from north to south. What is the resultant displacement of airplane in 2 hours ?

15. When a train has a speed of 10 m s^{-1} eastward, raindrops that are falling vertically with respect to the earth make traces that are inclined 30º to the vertical on the windows of the train.

(a) What is the horizontal component of a drop's velocity with respect to the earth ? With respect to the train ?

(b) What is the velocity of the raindrop with respect to the earth ? With respect to the train ?

16. To a man walking at 7 km/h due west, the wind appears to blow from the north-west, but when he walks at 3 km/h due west, the wind appears to blow from the north. What is the actual direction of the wind and what is its velocity ?

17. When a motorist is driving with velocity $6\hat{i}+8\hat{j}$, the wind appears to come from the direction $\hat{i}$. When he doubles his velocity the wind appears to come from the direction $\hat{i}+\hat{j}$. Then the true velocity of the wind expressed in the form of $a\hat{i}+b\hat{j}$ is ______.

18. ' n ' numbers of particles are located at the vertices of a regular polygon of ' n ' sides having the edge length ' a '. They all start moving simultaneously with equal constant speed ' v ' heading towards each other all the time. How long will the particles take to collide?

19. Two ships are 10 km apart on a line running south to north. The one further north is streaming west at 40 km/hr. The other is streaming north at 40 km/hr. What is their distance of closest approach and how long do they take to reach it?

20. A ship is sailing towards north at a speed of $\sqrt{2}$ m/s. The current is taking it towards East at the rate of 1 m/s and a sailor is climbing a vertical pole on the ship at the rate of 1 m/s. Find the velocity of the sailor in space.

EXERCISE # 1

1. A particle moves from the position of rest and attains a velocity of 30 m/sec after 10sec. The acceleration will be
(A) 9 m/sec^2 (B) 18 m/sec^2 (C) 3 m/sec^2 (D) 4 m/sec^2

2. The relation between time t and displacement x is expressed by $x = 2 - 5t + 6t^2$. What will be the initial velocity of the particle ?
(A) –5m/sec (B) –3m/sec (C) 6m/sec (D) 3m/sec

3. A particle, after starting from rest , experiences, constant acceleration for 20 seconds. If it covers a distance of S_1, in first 10 seconds and distance S_2 in next 10 sec, then
(A) $S_2 = S_1/2$ (B) $S_2 = S_1$ (C) $S_2 = 2S_1$ (D) $S_2 = 3S_1$

4. A body sliding on a smooth inclined plane requires 4sec to reach the bottom after starting from rest at the top. How much time does it take to cover one fourth the distance starting from the top
(A) 1sec (B) 2 sec (C) 0.4sec (D) 1.6 sec

5. The initial velocity of a particle is 10 m/sec and its retardation is 2 m/sec2. The distance covered in the fifth second of the motion will be
(A) 1m (B) 19m (C) 50m (D) 75m

6. A moving train is stopped by applying brakes. It stops after travelling 80m. If the speed of the train is doubled and retardation remains the same, it will cover a distance
(A) same as earlier (B) double the distance covered earlier
(C) four times the distance covered earlier (D) half the distance covered earlier

7. If u is the initial velocity of a body and a the acceleration the value of distance travelled by nth second is :
(A) $u + \frac{1}{2}$ a (2n +1) (B) $u + \frac{1}{2}$ a (2n–1) (C) $u - \frac{1}{2}$ a (2n + 1) (D) $u - \frac{1}{2}$ a(2n-–1)

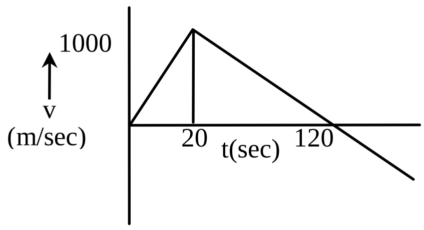
8. A body starts from rest, the ratio of distances travelled by the body during 3rd and 4th seconds is :
(A) 7/5 (B) 5/7 (C) 7/3 (D) 3/7

9. A body starting from rest and has uniform acceleration 8 m/s^2. The distance travelled by it in 5th second will be
(A) 36m (B) 40m (C) 100m (D) 200m

10. Which one of the following equations represent the motion of a body with finite constant acceleration. In these equations y denotes the position of the body at time t and a, b, and c are the constant of the motion
(A) $y = a/t + bt$ (B) $y = at$ (C) $y = at + bt^2$ (D) $y = at + bt^2 + ct^3$

11. A particle travels for 40 seconds under the influence of a constant force. If the distance travelled by the particle is S_1 in the first twenty seconds and S_2 in the next twenty second, then
(A) $S_2 = S_1$ (B) $S_2 = 2S_1$ (C) $S_2 = 3S_1$ (D) $S_2 = 4S_1$

12. A rocket is projected vertically upwards and its time-velocity graph is shown in the figure. The maximum height attained by the rocket is –



(A) 1km (B) 10km
(C) 100km (D) 60km

13. An object is released from some height. Exactly after one second, another object is released from the same height. The distance between the two objects exactly after 2 seconds of the release of second object will be:

(A) 4.9 m (B) 9.8 m (C) 19.6 m (D) 24.5 m

14. A stone is thrown vertically upwards from the top of a tower with a velocity u and it reaches the ground with a velocity 3u. The height of the tower is

(A) $3u^2/g$ (B) $4u^2/g$ (C) $6u^2/g$ (D) $9u^2/g$

15. A ball is thrown from the ground with a velocity of 80 ft/sec. Then the ball will be at a height of 96 feet above the ground after time

(A) 2 and 3sec (B) only 3 sec (C) only 2sec (D) 1 and 2 sec

16. A body is dropped from a height h under acceleration due to gravity g. If t_1 and t_2 are time intervals for its fall for first half and the second half distance, the relation between them is

(A) $t_1 = t_2$ (B) $t_1 = 2t_2$ (C) $t_1 = 2.414\, t_2$ (D) $t_1 = 4t_2$

17. A stone is dropped from a bridge and it reaches the ground in 4 seconds The height of the bridge is:

(A) 78.4m (B) 64m (C) 260m (D) 2000m

18. A rocket is launched form the earth surface so that it has an acceleration of $19.6 m/s^2$. If its engine is switched off after 5 seconds of its launch, then the maximum height attained by the rocket will be

(A) 245 m (B) 490 m (C) 980 m (D) 735 m

19. Two bodies of different masses m_a and m_b are dropped from two different heights, viz a and b. The ratio of times taken by the two to drop through these distances is

(A) a : b (B) $\frac{m_a}{m_b} : \frac{b}{a}$ (C) $\sqrt{a} : \sqrt{b}$ (D) $a^2 : b^2$

20. A body thrown up with a finite speed is caught back after 4 sec. The speed of the body with which it is thrown up is

(A) 10 m/sec (B) 20 m/sec (C) 30 m/sec (D) 40 m/sec

21. A stone is thrown vertically upwards with an initial velocity of 30 m/s. The time taken for the stone to rise to its maximum height is

(A) 0.326s (B) 3.26s (C) 30.6s (D) 3.06s

22. A ball is thrown upward and reaches a height of 64 feet , its initial velocity should be (g = 32 ft/sec2)

(A) 64 ft/sec (B) 72 ft/sec (C) 32 ft/sec (D) 4096 ft/sec

23. A body is thrown upward and reaches its maximum height. At that position-

(A) its velocity is zero and its acceleration is also zero

(B) its velocity is zero but its acceleration is maximum

(C) its acceleration is minimum

(D) its velocity is zero and its acceleration is the acceleration due to gravity

24. Two trains each of length 50 m are approaching each other on parallel rails. Their velocities are 10 m/sec and 15 m/sec. They will cross each other in -

25. A train is moving in the north at a speed 10 m/sec. Its length is 150 m. A parrot is flying parallel to the train in the south with a speed of 5 m/s. The time taken by the parrot to cross the train will be :

(A) 12 sec (B) 8 sec (C) 15 sec (D) 10 sec

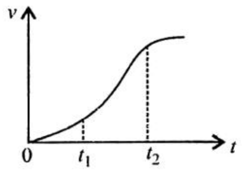
26. A thief is running away on a straight road on a jeep moving with a speed of 9 m/s. A police man chases him on a motor cycle moving at a speed of 10 m/s. If the instantaneous separation of jeep from the motor cycle is 100 m, how long will it take for the policemen to catch the thief ?

(A) 1 second (B) 19 second (C) 90 second (D) 100 second

27. A horse rider is moving towards a big mirror with velocity v. The velocity of his image with respect to him is-

(A) 0 (B) 4 v (C) 2 v (D) v

28. The velocity time graph of a particle in one–dimensional motion is shown in the figure. Which of the following formulae is correct for describing the motion of the particle over the time interval t_1 to t_2 ?



(A) $x(t_2)=x(t_1)+v(t_1)(t_2-t_1)+\left(\frac{1}{2}\right)a(t_2-t_1)^2$

(B) $v(t_2)=v(t_1)+a(t_2-t_1)$

(C) $a_{average}=\frac{(v(t_2)-v(t_1))}{(t_2-t_1)}$

(D) $V_{average}=\frac{(x(t_2)+x(t_1))}{(t_2-t_1)}$

29. A man is walking on a road with a velocity 3 km/hr. Suddenly rain starts falling. The velocity of rain is 10 km/hr in vertically downward direction. The relative velocity of the rain is -

(A) $\sqrt{13}$ km/hr (B) $\sqrt{7}$ km/hr (C) $\sqrt{109}$ km/hr (D) 13 km/hr

30. A car A is going north-east at 80 km/hr and another car B is going south-east at 60 km/hr. Then the direction of the velocity of A relative to B makes with the north an angle α such that tan α is -

(A) 1/7 (B) 3/4 (C) 4/3 (D) 3/5

31. A particle moves with constant speed v along a regular hexagon ABCDEF in same order (i.e. A to B , B to C, C to D, D to E, E to F, F to A...). Then magnitude of average velocity for its motion from A to C is –

(A) v (B) v/2 (C) $\sqrt{3}$ v/2 (D) None of these

32. A particle moves with a velocity v in a horizontal circular path. The change in its velocity for covering 60º will be -

(A) v $\sqrt{2}$ (B) v/ $\sqrt{2}$ (C) v$\sqrt{3}$ (D) v

33. A body is dropped from a height h from the state of rest. It covers a distance of 9h/25 in the last second. What is the height from which the body falls? (in meter)

(A) 12.5 (B) 1.25 (C) 125 (D) Zero

34. If the position vector of a particle is $\vec{r}=3t\hat{i}-4\hat{j}+\hat{k}$, the particle will be –

(A) moving with uniform velocity (B) stationary

(C) moving with uniform acceleration (D) insufficient data

35. A particle is executing a circular motion of radius R with a uniform speed v. After completing half the circle, the change in velocity and in speed will be respectively –

(A) zero, zero (B) 2v, zero (C) 2v, 2v (D) zero, 2v

36. A ball is thrown upward and it returns to ground describing a parabolic path. Which of the following remains constant?

(A) kinetic energy of the ball (B) speed of the ball
(C) horizontal component of velocity (D) vertical component of velocity

37. At the top of the trajectory of a projectile the direction of its velocity and acceleration are-

(A) Parallel to each other (B) inclined at an angle of 45° to the horizontal
(C) Perpendicular to each other (D) None of the above statement is correct

38. Three particles, A , B and C are projected from the same point with same initial speeds making angles 30°, 45° and 60° respectively with the horizontal. Which of the following statement is correct ?

(A) A, B and C have equal ranges
(B) ranges of A and C are equal and less than that of B
(C) ranges of A and C are equal and greater than that of B
(D) A, B and C have equal ranges

39. If a man wants to hit a target, he should point his rifle-

(A) higher than the target (B) lower than the target
(C) in the direction of the target (D) nothing can be said

40. The horizontal range covered by projectile is proportional to

(A) its velocity (B) square of its velocity
(C) sine of the angle of projection (D) square of the sine of the angle of projection

41. The horizontal range for projectile is given by

(A) $\frac{u^2 \sin^2 \theta}{g}$ (B) $\frac{u^2 \sin 2\theta}{g}$ (C) $\frac{u^2 \sin 2\theta}{2g}$ (D) $\frac{u^2 \cos 2\theta}{g}$

42. The maximum vertical height attained by a projectile is

(A) $\frac{U^2 \sin \theta}{g}$ (B) $\frac{U^2 \sin 2\theta}{g}$ (C) $\frac{U^2 \sin 2\theta}{2g}$ (D) $\frac{U^2 \sin^2 \theta}{2g}$

43. Equation of motion of a projectile is

(A) $y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$ (B) $y = x \tan \theta + \frac{gx^2}{2u^2 \cos^2 \theta}$

(C) $y = x \sin \theta - \frac{gx^2}{2u \cos^2 \theta}$ (D) $y = x \sin \theta + \frac{gx^2}{2u^2 \cos^2 \theta}$

44. A cannon on a level plane is aimed at an angle α above the horizontal and a shell is fired with a muzzle velocity v towards a vertical cliff a distance R away. The height from the bottom at which the shell strikes the side walls of the cliff is-

(A) $R \tan \alpha - \frac{1}{2} \frac{gR^2}{v_0^2 \cos^2 \alpha}$ (B) $R \tan \alpha - \frac{1}{2} \frac{gR^2}{v_0^2}$

(C) $R \sin \alpha - \frac{1}{2} \frac{gR^2}{v_0^2 \sin^2 \alpha}$ (D) $R \tan \alpha + \frac{1}{2} \frac{gR^2}{v_0^2}$

45. A player kicks up a ball at an angle θ to the horizontal. The horizontal range is maximum when θ equals-

(A) 30° (B) 45° (C) 60° (D) 90°

46.* The angle of projection of a body is 15º . The other angle for which the range is the same as the first one is equal to-
(A) 30º (B) 45º (C) 60º (D) 75º

47. A particle is projected such that the horizontal range and vertical height are the same. Then the angle of projection is-
(A) $\pi/4$ (B) $\tan^{-1}(4)$ (C) $\tan^{-1}(1)$ (D) $\pi/3$

48. A ball is thrown at an angle of 45º with the horizontal with kinetic energy E. The kinetic energy at the highest point during the flight is-
(A) Zero (B) E/2 (C) E (D) $(2)^{1/2}E$

49. A ball is thrown with initial energy 100J at an angle θ to the horizontal . If its energy at the top becomes 30 J then angle of projection-
(A) $\theta = 45º$ (B) $\theta = 30º$ (C) $\theta = \cos^{-1}(3/10)$. (D) $\theta = \cos^{-1}(3/10)^{1/2}$

50. The horizontal and vertical distances travelled by a particle in time t are given by x = 6t and $y = 8t - 5t^2$. If g = 10 m/sec², then the initial velocity of the particle is-
(A) 8 m/sec (B) 10 m/sec (C) 5 m/sec (D) zero

51. A body is thrown with a velocity of 9.8 m/s making an angle of 30º with the horizontal. It will hit the ground after a time-
(A) 3 s (B) 2 s (C) 1.5 s (D) 1 s

52. The maximum range of a projectile is 22 m. When it is thrown at an angle of 15º with the horizontal, its range will be-
(A) 22 m (B) 6 m (C) 15 m (D) 11 m

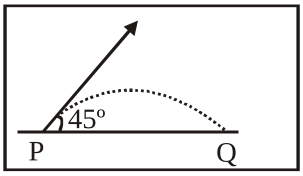
53. A boy throws a ball with a velocity v_0 at an angle α to the horizontal. At the same instant he starts running with uniform velocity to catch the ball before it hits the ground . To achieve this, he should run with a velocity of-
(A) $v_0 \cos\alpha$ (B) $v_0 \sin\alpha$ (C) $v_0 \tan\alpha$ (D) $\sqrt{v_0^2 \tan\alpha}$

54. The maximum range of a gun on a horizontal terrain is 16 km. If g = 10 m/sec² , the muzzle velocity of the shell must be-
(A) 400 m/sec (B) $160\sqrt{10}$ m/sec (C) 1600 m/sec (D) 200 m/sec

55. The range of the particle which is projected at an angle of 15º is 1.5 km. What will be the range for an angle of projection 45º
(A) 0.5 km (B) 1.5 km (C) 2.5 km (D) 3 km

56. A body is projected at an angle θ with horizontal. Another body is projected with the same velocity at an angle θ with the vertical. The ratio of the time of flights is-
(A) 1:1 (B) $\tan^2\theta$:1 (C) 1 : $\cot\theta$ (D) $\tan 2\theta$: 1

57. A projectile of mass m is fired with velocity v from the point P at an angle 45º with the horizon. The magnitude of change in momentum when it passes through the point Q on the same horizontal line on which P lies is-



(A) mv $\sqrt{2}$ (B) $\frac{1}{2}$mv
(C) Zero (D) 2 mv

58. The kinetic energy of a projectile at the highest point is-
(A) Zero (B) Maximum
(C) Minimum (D) Equal to total energy

59. The equation of a projectile is $y = \sqrt{3}\,x - \frac{gx^2}{2}$. The angle of projection is-
(A) 30º (B) 60º (C) 45º (D) None

60. The equation of projectile is $y = 16x - \frac{5x^2}{4}$. The horizontal range is-
(A) 16 m (B) 8 m (C) 3.2 m (D) 12.8 m

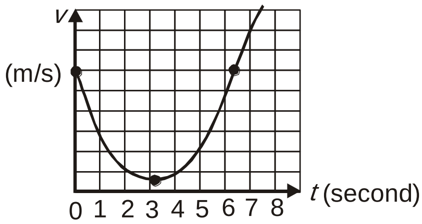
EXERCISE # 2

Chosse the correct answer

1. Equation of motion of a body moving along x-axis at an instant t second is given by $x = 40 + 12t - t^3$ m. Displacement of the particle before coming to rest is

(A) 16 m (B) 56 m (C) 24 m (D) 40 m

2. Velocity-time graph for a particle is shown in figure. Starting from $t = 0$, at what instant t, average acceleration is zero between 0 to t?



(A) 1 s (B) 3.5 s (C) 6.3 s (D) 7.3 s

3. If a particle is thrown with velocity more than 10 m/s vertically upward, then the distance travelled by the particle in last second of its ascent is

(A) g (B) $\frac{g}{2}$ (C) $\frac{g}{4}$ (D) $\frac{g}{8}$

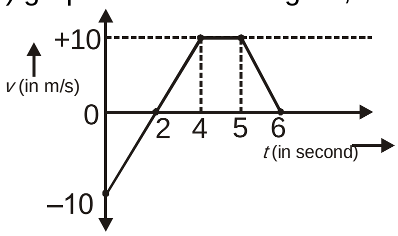
4. A car starts from rest travelling with constant acceleration. If distance covered by it in 10th second of its journey is 19m, what will be the acceleration of car?

(A) 4 m/s^2 (B) 3 m/s^2 (C) 2 m/s^2 (D) 1 m/s^2

5. A stone is dropped from the top of a tower and travels 24.5 m in the last second of its journey. The height of the tower is

(A) 44.1 m (B) 49 m (C) 78.4 m (D) 72 m

6. For the velocity ~ time ($v \sim t$) graph as shown in figure, the incorrect statement is



(A) The average velocity for the entire journey is 2.5 m/s
(B) The average acceleration from 1 s to 4 s is 5 m/s²
(C) The average speed for the first 4 s is zero
(D) The acceleration at $t = 3$ s is 5 m/s²

7. A body moves along curved path of a quarter circle. The ratio of magnitude of displacement to distance is

(A) $\frac{\pi}{2\sqrt{2}}$ (B) $\frac{\pi}{2}$ (C) $\frac{2\sqrt{2}}{\pi}$ (D) $\frac{3\pi}{2\sqrt{2}}$

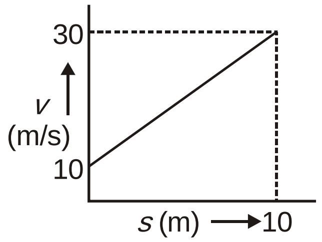
8. A ball is released from certain height h reaches ground in time T. Where will it be from the ground at time $\frac{3T}{4}$?

(A) $\frac{9h}{16}$ (B) $\frac{7h}{16}$ (C) $\frac{3h}{4}$ (D) $\frac{27h}{64}$

9. A particle moves along x-axis in such a way that its x-co-ordinate varies with time according to the equation $x = 8 - 4t + 6t^2$. The distance covered by particle between t = 0 to t = $\frac{2}{3}$ sec. is

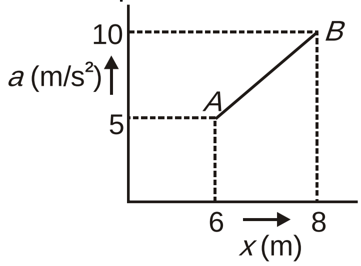
(A) Zero (B) $\frac{8}{3}$ m (C) 8 m (D) 4/3 m

10. Velocity of a particle changes with position according to following curve. Acceleration of the particle at $x = 1$ m



(A) 24 m/s² (B) 2 m/s² (C) 20 m/s² (D) 3 m/s²

11. If acceleration of a particle is varying with x according to curve. Velocity of particle at A is 2 m/s. What is the velocity of particle at point B?



(A) 4 m/s (B) $\sqrt{34}$ m/s (C) $\sqrt{28}$ m/s (D) 7 m/s

12. A particle is thrown vertically upward. It is known that distance travelled in 6th second and 7th second is same then
(A) The particle was thrown with 40 m/s (B) The particle was thrown with 70 m/s
(C) Total vertical height is 245 m (D) Total vertical height is 180 m

13. Two cars are moving in the same direction with same speed 30 km/h. They are separated from each other by 5 km. A third car moving in the same direction meets the two cars after an interval of 4 minutes. The speed of the third car is
(A) 30 km/h (B) 105 km/h (C) 140 km/h (D) 45 km/h

14. Velocity of particle starting from rest varies with position according to equation $v = \sqrt{\alpha x}$. What is distance travelled by particle in t second from start?
(A) $\frac{1}{2}\alpha t^2$ (B) $\frac{1}{4}\alpha t^2$ (C) $\frac{1}{3}\alpha t^3$ (D) $\frac{1}{6}\alpha t^3$

15. If speed of water in river is 4 m/s and speed of swimmer with respect to water is 3 m/s, then in which direction the swimmer must swim so that he will reach directly opposite end?
(A) 127° with direction of river flow (B) 90° with direction of river flow
(C) 143° with direction of river flow (D) Swimmer will never reach directly opposite end

16. Uniform circular motion is not a type of
(A) Uniform motion (B) Non-uniform motion
(C) Curvilinear motion (D) Variable accelerated motion

17. In case of an oblique projectile, the velocity is perpendicular to acceleration
(A) Once only (B) Twice (C) Never (D) Four times

18. The area under acceleration–time graph represents the
(A) Change in velocity (B) Final velocity
(C) Initial velocity (D) distance travelled

19. For a given speed of projection, maximum range of a projectile is R. What will be the maximum height attained by projectile when it is projected with double speed by making 60° with vertical?
(A) $\frac{R}{2}$ (B) $\frac{R}{4}$ (C) R (D) $\frac{3R}{2}$

20. A particle is projected with velocity $(3\hat{i} + 4\hat{j})$ m/s from horizontal. What will be the height attained by it when velocity becomes perpendicular to acceleration?
(A) 0.8 m (B) 5 m (C) 0.4 m (D) 1.6 m

21. The speed of a particle moving along a circular path is decreasing at the rate of 3 m/s². If the radius of circle is 4 m, then what will be the instantaneous acceleration of particle when its speed is 4 m/s?
(A) 4 m/s² towards center (B) 3 m/s² along tangent
(C) 5 m/s² by making 37° with tangent (D) 5 m/s² by taking 53° with tangent

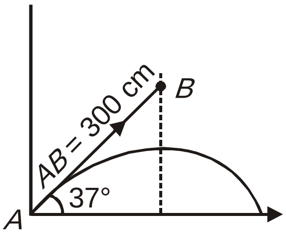
22. The speed of boat is 5 km/h in still water. It crosses a river of width 1 km along shortest possible path in 15 min. The velocity of river water is
(A) 1 km/h (B) 3 km/h (C) 4 km/h (D) 5 km/h

23. A projectile has a time of flight *T* and range *R*. If time of flight is doubled keeping angle of projection same, then what will be the new range?
(A) *R* (B) 2 *R* (C) $\frac{R}{2}$ (D) 4 *R*

24. A particle moves along the positive part of the curve $y=\frac{x^2}{2}$, where $x=\frac{t^2}{2}$. What will be velocity of particle at *t* = 2 sec.
(A) $2\hat{i}-4\hat{j}$ (B) $2\hat{i}+4\hat{j}$ (C) $4\hat{i}+2\hat{j}$ (D) $4\hat{i}-2\hat{j}$

25. Two projectiles *A* and *B* are projected with same speed at angles 30° and 60° to horizontal, then choose the wrong statement? (symbols have their usual meaning)
(A) $R_A = R_B$ (B) $H_B = 3H_A$ (C) $\sqrt{3}T_B = T_A$ (D) All of these

26. A ball *B* is at 300 cm distance from origin on a line 37° above horizontal. Another ball *A* is projected directly aiming *B* with initial velocity 700 cm/s. At the same instant *B* is released from its position. How far will *B* has fallen when it is hit by *A*?



(A) 90 cm (B) 80 cm (C) 70 cm (D) 60 cm

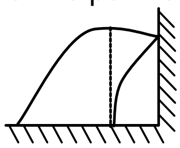
27. A particle is projected from ground at angle of 60º with horizontal. After one second its velocity is at angle of 45°. After one more second velocity become horizontal. With what speed the particle is projected? (g = 10 m/s²)
(A) 10 m/s (B) $\frac{40}{\sqrt{3}}$ m/s (C) $10\sqrt{5}$ m/s (D) 30 m/s

28. A ball is projected with momentum *p* at angle θ. What is the change in momentum till it reaches highest point?
(A) $\frac{p\sin\theta}{2}$ (B) $p\cos\theta$ (C) $p\sin\theta$ (D) $\frac{p\cos\theta}{2}$

29. A particle is projected in such a way that, taking point of projection as origin, $y = 8t - 5t^2$ and $x = 6t$. What is the range of projectile?
(A) 48 m (B) 4.8 m (C) 9.6 m (D) 24 m

30. A projectile is thrown with an initial velocity $(3\hat{i}+a\hat{j})$ m/s. If range of projectile is doubled, the maximum height is reached by it, value of *a* is
(A) 1.5 m/s (B) 6 m/s (C) 3 m/s (D) 12 m/s

32. A stone is projected from a horizontal plane. It attains maximum height 'H' and strikes a stationary smooth wall and falls on the ground vertically below the maximum height. Assume the collision to be elastic, the height of the point on the wall where stone will strike is-



(A) H/2 (B) H/4 (C) 3H/4 (D) None of these

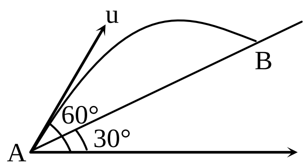
32. A ball is projected with velocity u at an angle α with horizontal plane. Its speed when it makes an angle β with the horizontal is -

(A) u cos α (B) $\frac{u}{\cos\beta}$ (C) u cos α cos β (D) $\frac{u\cos\alpha}{\cos\beta}$

33. If was calculated that a shell when fired from a gun with a certain velocity and at an angle of elevation $\frac{5\pi}{36}$ radians should strike a given target. In actual practice it was found that a hill just prevented in the trajectory. At what angle of elevation should the gun be fired to hit the target.

(A) $\frac{5\pi}{36}$ radians (B) $\frac{11\pi}{36}$ radians (C) $\frac{7\pi}{36}$ radians (D) $\frac{13\pi}{36}$ radian

34. Time taken by the projectile to reach A to B is t. Then the distance AB is equal to –



(A) $\frac{ut}{\sqrt{3}}$ (B) $\frac{\sqrt{3}\,ut}{2}$ (C) $\sqrt{3}\,ut$ (D) 2 ut

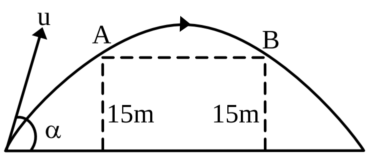
35. A body is thrown from a point with speed 50 m/s at an angle 37º with horizontal. When it has moved a horizontal distance of 80 m then its distance from point of projection is –

(A) 40 m (B) 40 $\sqrt{2}$ m (C) 40 $\sqrt{5}$ m (D) None

36. A particle moves in the plane xy with velocity V = k_1i + k_2 x j, where i and j are the unit vectors of the x and y axes, and k_1 and k_2 are constants. At the initial moment of time the particle was located at the point x = y = 0 then the equation of the particle's trajectory y (x) is –

(A) y = $\frac{k_1}{2k_2}x^2$ (B) y = $\frac{k_2}{2k_1}x^2$ (C) y = $\frac{2k_1}{k_2}x^2$ (D) y = $\frac{2k_2}{k_1}x^2$

37. A golfer standing on level ground hits a ball with a velocity of u = 52 m/s at an angle α above the horizontal. If tanα = 5/12, then the time for which the ball is at least 15m above the ground (i.e. between A and B) will be (take g = 10 m/s^2) –

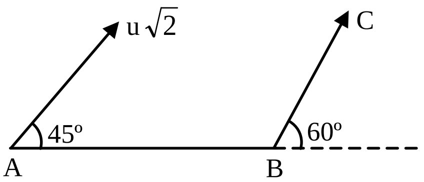


(A) 1 sec (B) 2 sec (C) 3 sec (D) 4 sec

38. A ball is projected upwards from the top of the tower with a velocity 50 ms^{-1} making an angle 30° with the horizontal. The height of tower is 70 m. After how many seconds the ball will strike the ground ?

(A) 3 sec (B) 5 sec (C) 7 sec (D) 9 sec

39. A particle is projected from a point A with velocity $u\sqrt{2}$ at an angle of 45° with horizontal as shown in figure. It strikes the plane BC at right angles. The velocity of the particle at the time of collision is :



(A) $\frac{\sqrt{3}u}{2}$ (B) $\frac{u}{2}$ (C) $\frac{2u}{\sqrt{3}}$ (D) u

40. A particle is projected from the ground with an initial speed of v at an angle θ with horizontal. The average velocity of the particle between its point of projection and highest point of trajectory is –

(A) $\frac{v}{2}\sqrt{1+2\cos^2\theta}$ (B) $\frac{v}{2}\sqrt{1+\cos^2\theta}$ (C) $\frac{v}{2}\sqrt{1+3\cos^2\theta}$ (D) $v\cos\theta$

EXERCISE-3

Medical Previous Year Questions

1. Which of the following is a one–dimensional motion? **[NEET 2001]**
(A) Landing of an aircraft (B) Earth revolving around the sun
(C) Motion of wheels of a moving trains (D) Train running on a straight track

2. A 150 m long train is moving with a uniform velocity of 45 km/h. The time taken by the train to cross a bridge of length 850 meters is **[NEET 2001]**
(A) 56 sec (B) 68 sec (C) 80 sec (D) 92 sec

3. The displacement of a particle, moving in a straight line, is given by s = $2t^2 + 2t + 4$ where s is in metres and t in seconds. The acceleration of the particle is **[NEET 2001]**
(A) 2 m/s^2 (B) 4 m/s^2 (C) 6 m/s^2 (D) 8 m/s^2

4. From the top of a tower, a particle is thrown ver-tically downwards with a velocity of 10 m/s. The ratio of the distances, covered by it in the 3rd and 2nd seconds of the motion is (Take g = 10 m/s^2)
[NEET 2002]
(A) 5 : 7 (B) 7 : 5 (C) 3 : 6 (D) 6 : 3

5. A man drops a ball downside from the roof of a tower of height 400 meters. At the same time another ball is thrown upside with a velocity 50 meter/sec from the surface of the tower, then they will meet at which height from the surface of the tower **[NEET 2003]**
(A) 100 meters (B) 320 meters (C) 80 meters (D) 240 meters

6. Two balls are dropped from heights h and $2h$ respectively from the earth surface. The ratio of time of these balls to reach the earth is **[NEET 2003]**
(A) $1 : \sqrt{2}$ (B) $\sqrt{2} : 1$ (C) 2 : 1 (D) 1 : 4

7. The acceleration due to gravity on the planet A is 9 times the acceleration due to gravity on planet *B.* A man jumps to a height of 2 m on the surface of *A.* What is the height of jump by the same person on the planet *B ?*
[NEET 2003]
(A) 18 m (B) 6 m (C) $\frac{2}{3}$ m (D) $\frac{2}{3}$ m

8. The displacement of a particle is given by $y = a + bt + ct^2 - dt^4$. The initial velocity and acceleration are respectively **[NEET 2003]**
(A) b, –4d (B) –b, 2c (C) b, 2c (D) 2c, –4d

9. A police jeep is chasing with, velocity of 45 km/h a thief in another jeep moving with velocity 153 km/h. Police fires a bullet with muzzle velocity of 180 m/s. The velocity it will strike the car of the thief is **[NEET 2004]**
(A) 150 m/s (B) 27 m/s (C) 450 m/s (D) 250 m/s

10. A body falls from a height h = 200 m (at New Delhi). The ratio of distance travelled in each 2 sec during $t = 0$ to $t = 6$ seconds of the journey is **[NEET 2004]**
(A) 1 : 4 : 9 (B) 1 : 2 : 4 (C) 1 : 3 : 5 (D) 1:2:3

11. Two boys are standing at the ends *A* and *B* of a ground where $AB = a$. The boy at *B* starts run-ning in a direction perpendicular to *AB* with velocity v_1. The boy at A starts running simul-taneously with velocity v and catches the other boy in a time t, where t is **[NEET 2005]**
(A) $a/\sqrt{v^2+v_1^2}$ (B) $\sqrt{a^2/\left(v^2-v_1^2\right)}$ (C) $a/\left(v-v_1\right)$ (D) $a/\left(v+v_1\right)$

12. The displacement x of a particle varies with time t as $x = ae^{-\alpha t} + be^{\beta t}$, where a, b, α and β are positive constants. The velocity of the particle will **[NEET 2005]**
(A) go on decreasing with time (B) be independent of α and β
(C) drop to zero when $\alpha = \beta$ (D) go on increasing with time

13. A ball is thrown vertically upward. It has a speed of 10 m/s when it has reached one half of its maxi-mum height. How high does the ball rise? (Taking g= 10 m/s^2) **[NEET 2005]**
(A) 15 m (B) 10 m (C) 20 m (D) 5 m

14. A particle moves along a straight line OX. At a time t (in seconds) the distance x (in metres) of the particle is given by $x = 40 + 12t - t^3$

How long would the particle travel before coming to rest? **[NEET 2006]**
(a) 24 m (B) 40 m (C) 56 m (D) 16 m

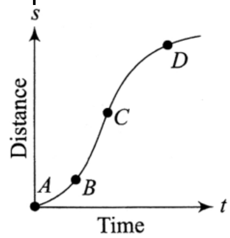
15. Two bodies *A* (of mass 1 kg) and *B* (of mass 3 kg) are dropped from heights of 16 m and 25 m, respectively. The ratio of the time taken by them to reach the ground is: **[NEET 2006]**
(A) 5/4 (B) 12/5 (C) 5/12 (D) 4/5

16. A car moves from X to 7 with a uniform speed v_u and returns to Y with a uniform speed v_d. The average speed for this round trip is: **[NEET 2007]**
(A) $\frac{2v_d v_u}{v_d + v_u}$ (B) $\sqrt{v_u v_d}$ (C) $\frac{v_d v_u}{v_d + v_u}$ (D) $\frac{v_u + v_d}{2}$

17. A particle moving along x–axis has acceleration *f*, at time *t*, given by $f = f_0\left(1 - \frac{t}{T}\right)$, where f_0 and T are constants. The particle at t=0 has zero velocity. In the time interval between $t = 0$ and the instant when $f = 0$, the particle's velocity (v_x) is: **[NEET 2007]**
(A) $f_0 T$ **(B)** $\frac{1}{2} f_0 T^2$ **(C)** $f_0 T^2$ **(D)** $\frac{1}{2} f_0 T$

18. The distance travelled by a particle starting from rest and moving with an acceleration $\frac{4}{3}$ ms^{-2}, in the third second is **[NEET 2008]**
(A) 6 m (B) 4 m (C) $\frac{10}{3} m$ (D) $\frac{19}{3} m$

19. A particle shows distance–time curve as given in this figure. The maximum instantaneous velocity of the particle is around the point **[NEET 2008]**



(A) *B* (B) C (C) *D* (D) *A*

20. A particle moves in a straight line with a con-stant acceleration. It changes its velocity from 10 ms^{-1} to 20 ms^{-1} while passing through a dis-tance 135 m in *t* second. The value of *t* is **[NEET 2008]**
(A) 10 (B) 1.8 (C) 12 (D) 9

21. A bus is moving with a speed of 10 ms^{-1} on a straight road. A scooterist wishes to overtake the bus in 100 s. If the bus is at a distance of 1 km from the scooterist, with what speed should the scooterist chase the bus? **[NEET 2009]**
(A) 20 ms^{-1} (B) 40 ms^{-1} (C) 25 ms^{-1} (D) 10 ms^{-1}

22. A particle starts its motion from rest under the action of a constant force. If the distance cov-ered in first 10 s is s_1 and that covered in the first 20 s is s_2, then **[NEET 2009]**
(A) $s_2 = 2s_1$ (B) $s_2 = 3s_1$ (C) $s_2 = 4s_1$ (D) $s_2 = s_1$

23. A ball is dropped from a high rise platform at $t = 0$ starting from rest. After 6 s another ball is thrown downwards from the same platform with a speed v. The two balls meet at $t = 18$ s. What is the value of v? (take $g = 10$ ms^{-2}) **[NEET 2010]**
(A) 74 ms^{-2} (B) 55 ms^{-1} (C) 40 ms^{-1} (D) 60 ms^{-1}

24. A particle has initial velocity $\left(3\hat{i} + 4\hat{j}\right)$ and has acceleration $\left(0.1\hat{i} + 0.3\hat{j}\right)$. Its speed after 10 s is **[NEET 2010]**
(A) 7 unit (B) $7\sqrt{2}$ unit (C) 8.5 unit (D) 10 unit

25. A particle moves a distance x in time *t* according to equation $x = (t + 5)^{-1}$. The acceleration of particle is proportional to **[NEET 2010]**
(A) $(\text{velocity})^{3/2}$ (B) $(\text{distance})^2$ (C) $(\text{distance})^{-2}$ (D) $(\text{velocity})^{2/3}$

26. A boy standing at the top of a tower of 20 m height drops a stone. Assuming g = 10 ms^{-2}, the velocity with which it hits the ground is **[NEET 2011]**

(A) 20 m/s (B) 40 m/s (C) 5 m/s (D) 10 m/s

27. A body is moving with velocity 30 m/s towards east. After 10 s its velocity becomes 40 m/s towards north. The average acceleration of the body is **[NEET 2011]**

(A) 7 m/s^2 (B) $\sqrt{7}$ m/s^2 (C) 5 m/s^2 (D) 1 m/s^2

28. A particle covers half of its total distance with speed v_1 and the rest half distance with speed v_2. Its average speed during the complete journey is **[NEET 2011]**

(A) $\frac{v_1v_2}{v_1+v_2}$ (B) $\frac{2v_1v_2}{v_1+v_2}$ (C) $\frac{2v_1^2v_2^2}{v_1^2+v_2^2}$ (D) $\frac{v_1+v_2}{2}$

29. The motion of a particle along a straight line is described by equation: **[NEET 2012]**
x = 8 + 12t – t^3 where x is in metre and *t* in second. The retardation of the particle when its velocity becomes zero is

(a) 12 ms^{-2} (B) 24 ms^{-2} (C) zero (D) 6 ms^{-2}

30. A stone falls freely under gravity. It covers distances h_1, h_2 and h_3 in the first 5 seconds, the next 5 seconds and the next 5 seconds respec-tively. The relation between h_1, h_2 and h_3 is: **[NEET 2013]**

(A) $h_1 = 2h_2 = 3h_3$ (B) $h_1 = \frac{h_2}{3} = \frac{h_3}{5}$

(C) $h_2 = 3h_1$ and $h_3 = 3h_2$ (D) $h_1 = h_2 = h_3$

31. The displacement *'x'* (in meter) of a particle of mass 'm' (in kg) moving in one dimension under the action of a force is released to time Y (in sec) by $t = \sqrt{x} + 3$. The displacement of the particle when its velocity is zero will be **[NEET 2013]**

(A) 2 m (B) 4 m (C) zero (D) 6 m

32. A particle of unit mass undergoes one–dimen-sional motion such that its velocity varies according to **[NEET 2015]**

$\upsilon(x) = \beta x^{-2n}$

where β and *n* are constants andx is the position of the particle. The acceleration of the particle as a function of x is given by

(A) $-2n\beta^2x^{-2n-1}$ (B) $-2n\beta^2x^{-4n-1}$ (C) $-2n\beta^2x^{-2n+1}$ (D) $-2n\beta^2e^{-4n+1}$

33. Two particles of mass *M* and *m* are moving in a circle of radii *R* and *r*. If their time periods are the same, what will be the ratio of their linear velocities? **[NEET 2001]**

(A) MR : mr (B) M : *m* (C) R : r (D) 1 : 1

34. The angle between the two vectors $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ will be **[NEET 2001]**

(A) zero (B) 45° (C) 90° (D) 180°

35. A particle *(A)* is dropped from a height and another particle (B) is thrown into horizontal direction with speed of 5 m/sec from the same height. The correct statement is

(A) Both particles will reach at ground simul-taneously
(B) Both particles will reach at ground with the same speed
(C) Particle *(A)* will reach at ground first with respect to particle *(B)*
(D) Particle *(B)* will reach at ground first with respect to particle *(A)*

36. From a 10 m high building a stone 'A' is dropped, and simultaneously another identical stone 'B' is thrown horizontally with an initial speed of 5 ms^{-1}. Which one of the following statement is true? **[NEET 2002]**

(A) It is not possible to calculate which one of the two stones will reach the ground first
(B) Both the stones *(A* and *B)* will reach the ground simultaneously.
(C) *'A'* stone reaches the ground earlier than 'Z?'
(D) *'B'* stone reaches the ground earlier than *'A'*

37. The vector sum of two forces is perpendicular to their vector differences. In that case, the force **[NEET 2003]**

(A) cannot be predicted (B) are equal to the each other
(C) are equal to each other in magnitude (D) are not equal to each other in magnitude

38. $\left|\vec{A}\times\vec{B}\right|=\sqrt{3}\vec{A}\cdot\vec{B}$, then the value of $\left|\vec{A}\times\vec{B}\right|$ is : **[NEET 2004]**

(A) $\left(A^2+B^2+AB\right)^{1/2}$ (B) $\left(A^2+B^2+\frac{AB}{\sqrt{3}}\right)^{1/2}$

(C) $A+B$ (D) $\left(A^2+B^2+\sqrt{3}AB\right)^{1/2}$

39. If a vector $2\hat{i}+3\hat{j}+8\hat{k}$ is perpendicular to the vector $4\hat{i}-4\hat{i}+\alpha\hat{k}$, then the value of α is: **[NEET 2005]**

(A) –1 (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) 1

40. Two boys are standing at the ends *A* and *B* of a ground, where *AB = a.* The boy at *B* starts running in a direction perpendicular to *AB* with velocity v_1. The boy at *A* starts running simul-taneously with velocity v and catches the other boy in a time t_1 where *t* is **[NEET 2005]**

(A) $\frac{a}{\sqrt{v^2+v_1^2}}$ (B) $\sqrt{\frac{a}{v^2-v_1^2}}$ (C) $\frac{a}{\left(v-v_1\right)}$ (D) $\frac{a}{\left(v+v_1\right)}$

41. A stone tied to the end of a string of 1 m long is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolutions in 44 s, what is the magnitude and direction of accelera-tion of the stone? **[NEET 2005]**

(A) $\frac{\pi^2}{4}ms^{-2}$ and direction along the radius towards the centre

(B) $\pi^2 ms^{-2}$ and direction along the radius away from centre

(C) $\pi^2 ms^{-2}$ and direction along the radius towards the centre

(D) $\pi^2 ms^{-2}$ and direction along the tangent to the circle

42. A car runs at a constant speed on a circular track of radius 100 m, taking 62.8 s for every circular loop. The average velocity and average speed for each circular loop respectively is: **[NEET 2006]**

(A) 0,0 (B) 0,10 m/s (C) 10 m/s, 10 m/s (D) 10 m/s, 0

43. For angles of projection of a projectile at angles (45° – θ) and (45° + θ*),* the horizontal ranges described by the projectile are in the ratio of: **[NEET 2006]**

(A) 1 : 1 (B) 2 : 3 (C) 1 : 2 (D) 2 : 1

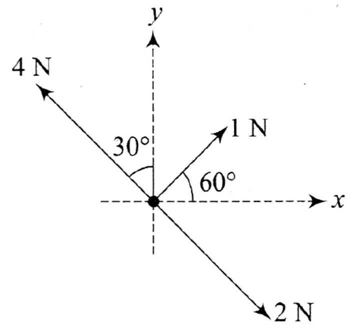
44. A particle staring from the origin (0,0) moves in a straight line in the *(x,y)* plane. Its coordinates at a later time are ($\sqrt{3}$, 3). The path of the par-ticle makes with the x–axis an angle of: **[NEET 2007]**

(A) 30° (B) 45° (C) 60° (D) 0°

45. $\vec{A}$ and $\vec{B}$ are two vectors and is the angle between them, if $\left|\vec{A}\times\vec{B}\right|$ = $\sqrt{3}\left(\vec{A}\cdot\vec{B}\right)$ the value of θ is: **[NEET 2007]**

(A) 60° (B) 45° (C) 30° (D) 90°

46. Three forces acting on a body are shown in the figure. To have the resultant force only along the *y*–direction, the magnitude of the minimum additional force needed is: **[NEET 2008]**

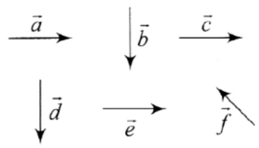


(A) 0.5 N (B) 1.5 N (C) $\frac{\sqrt{3}}{4}$ N (D) $\sqrt{3}$ N

47. A particle of mass m is projected with velocity *v* making an angle of 45° with the horizontal. When the particle lands on the level ground the magnitude of the change in its momentum will be: **[NEET 2008]**

(A) 2 *mv* (B) $mv/\sqrt{2}$ (C) $mv\sqrt{2}$ (D) *zero*

48. Six vectors through have the magnitudes and directions indicated in the figure. Which of the following statements is true? **[NEET 2010]**



(A) $\vec{b}+\vec{c}=\vec{f}$ (B) $\vec{d}+\vec{c}=\vec{f}$ (C) $\vec{d}+\vec{e}=\vec{f}$ (D) $\vec{b}+\vec{e}=\vec{f}$

49. A particle moves in a circle of radius 5 cm with constant speed and time period 0.2. The acceleration of the particle is **[NEET 2011]**

(A) 25 m/s^2 (B) 36 m/s^2 (C) 5 m/s^2 (D) 15m/s^2

50. A missile is fired for maximum range with an initial velocity of 20 m/s. If *g* =10 ms^2, the range of the missile is **[NEET 2011]**

(A) 50 m (B) 60 m (C) 20 m (D) 40 m

51. A projectile is fired at an angle of 45° with the horizontal. Elevation angle of the projectile at its highest point as seen from the point of projection is **[NEET 2011]**

(A) 60° (B) $\tan^{-1}\left(\frac{1}{2}\right)$ (C) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (D) 45^{O}

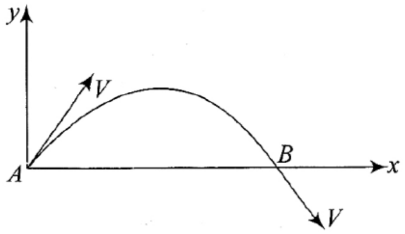
52. The horizontal range and the maximum height of a projectile are equal. The angle of projection of the projectile is **[NEET 2012]**

(A) 0 = 45° (B) $\theta = \tan^{-1}\left(\frac{1}{4}\right)$ (C) $\theta=\tan^{-1}(4)$ (D) $\theta=\tan^{-1}(2)$

53. A particle has initial velocity $\left(2\hat{i}+3\hat{j}\right)$ and acceleration $\left(0.3\hat{i}+0.2\hat{j}\right)$. The magnitude of velocity after 10 seconds will be **[NEET 2012]**

(A) 9 units (B) 9 $\sqrt{2}$ units (C) 5 $\sqrt{2}$ units (D) 5 units

54. The velocity of a projectile at the initial point A is $\left(2\hat{i}+3\hat{j}\right)$ m/s. Its velocity (in m/s) at point B is **[NEET 2013]**



(A) $-2\hat{i}-3\hat{j}$ (B) $-2\hat{i}+3\hat{j}$ (C) $2\hat{i}-3\hat{j}$ (D) $2\hat{i}+3\hat{j}$

55. A particle is moving such that its position coordinates *(x, y)* are **[NEET 2014]**
(2 m, 3 m) at time t = 0,
(6 m, 7 m) at time $t = 2s$ and
(13 m, 14 m) at time t = 5 s
Average velocity vector $\left(\vec{V}_{av}\right)$ from t=0to t = 5s is

(A) $\frac{1}{5}\left(13\hat{i}+14\hat{j}\right)$ (B) $\frac{7}{3}\left(\hat{i}+\hat{j}\right)$ (C) $2\left(\hat{i}+\hat{j}\right)$ (D) $\frac{11}{5}\left(\hat{i}+\hat{j}\right)$

56. A ship A is moving Westwards with a speed of 10 km h^{-1} and a ship B 100 km South of A is moving northwards with a speed of 10 km h^{-1}. The time after which the distance between them becomes shortest is **[NEET 2015]**

(A) 0 h (B) 5 h (C) $5\sqrt{2}$ h (D) $10\sqrt{2}$ h

57. If vectors $\vec{A}=\cos\omega t\,\hat{i}+\sin\omega t\,\hat{j}$ and $\vec{B}=\cos\frac{\omega t}{2}\hat{i}+\sin\frac{\omega t}{2}\hat{j}$ are functions of time, then the value of t at which they are orthogonal to each other is: **[NEET 2015]**

(A) $t=0$ (B) $t=\frac{\pi}{4\omega}$ (C) $t=\frac{\pi}{2\omega}$ (D) $t=\frac{\pi}{\omega}$

58. The position vector of a particle as a function of time is given by:
$\vec{R}$ = 4 $sin\,(2\pi t)\,\hat{i}$ + 4 $cos\,(2\pi t)\,\hat{j}$
Where R is in meters, t is in seconds and $\hat{i}$ and $\hat{j}$ denote unit vectors along x– and y– directions, respectively. Which one of the following state-ments is wrong for the motion of particle? **[NEET 2015 Re]**
(A) Path of the particle is a circle of radius 4 meter
(B) Acceleration vectors is along $-\vec{R}$
(C) Magnitude of acceleration vector is $\frac{v^2}{R}$ where v is the velocity of particle.
(D) Magnitude of the velocity of particle is 8 meter/second.

59. A 150 m long train is moving with a uniform velocity of 45 *km/h*. The time taken by the train to cross a bridge of length 850 *meters* is **[CBSE PMT 2001]**
(A) 56 *sec* (B) 68 *sec* (C) 80 *sec* (D) 92 *sec*

60. A car travels a distance of 2000 m. If the first half distance is covered at 40 $km/hour$ and the second half with speed v and the average speed is 48 $km/hour$ then the value of v is **[CBSE PMT 1989]**
(A) 56 km/hour (B) 60 km/hour (C) 50 km/hour (D) 48 km/hour

61. A particle moves along a straight line such that its displacement at any time t is given by $s-t^3-6t^2+3t+4$ *metres*.The velocity when the acceleration is zero is

[CBSE PMT 1994]

(A) $3 ms^{-1}$ (B) $-12\ ms^{-1}$ (C) $42\ ms^{-1}$ (D) $-9\ ms^{-1}$

62. A particle starts from rest, accelerates at 2 m/s^2 for 10s and then goes for constant speed for 30s and then decelerates at 4 m/s^2 till it stops. What is the distance travelled by it **[AIIMS 2002]**

(A) 750 m (B) 800 m (C) 700 m (D) 850 m

63. A particle is constrained to move on a straight line path. It returns to the starting point after 10 *sec*. The total distance covered by the particle during this time is 30 m. Which of the following statements about the motion of the particle is false **[CBSE PMT 2000; AFMC 2001]**

(A) Displacement of the particle is zero (B) Average speed of the particle is 3 m/s
(C) Displacement of the particle is 30 m (D) Both (a) and (b)

64. If a body starts from rest and travels 120 cm in the 6th second, then what is the acceleration **[AFMC 1997]**

(A) 0.20m/s^2 (B) 0.027 m/s^2 (C) 0.218 m/s^2 (D) 0.03 m/s^2

65. The acceleration due to gravity on the planet A is 9 times the acceleration due to gravity on planet B. A man jumps to a height of 2m on the surface of A. What is the height of jump by the same person on the planet B **[CBSE PMT 2003]**

(A) 18m (B) 6m (C) $\frac{2}{3}m$ (D) $\frac{2}{9}m$

66. From the top of a tower, a particle is thrown vertically down wards with a velocity of 10 m/s. The ratio of the distances covered by it in the 3rd and 2nd seconds of the motion is (Take g = 10 m/s^2) **[AIIMS 2000; CBSE PMT 2002]**

(A) 5: 7 (B) 7 : 5 (C) 3 : 6 (D) 6 : 3

67. Three different objects of masses m_1, m_2 and m_3 are allowed to fall from rest and from the same point 'O' along three different frictionless paths. The speeds of the three objects, on reaching the ground, will be in the ratio of **[AIIMS 2002]**

(A) $m_1 : m_2 : m_3$ (B) $m_1 : 2m_2 : 3m_3$ (C) 1 : 1 : 1 (D) $\frac{1}{m_1} : \frac{1}{m_2} : \frac{1}{m_3}$

68. A stone is thrown with an initial speed of 4.9 m/s from a bridge in vertically upward direction. It falls down in water after 2 sec. The height of the bridge is **[AFMC 1999]**

(A) 4.9 m (B) 9.8 m (C) 19.8 m (D) 24.7 m

69. At the top of the trajectory of a projectile, the directions of its velocity and acceleration are **[JIPMER 2000; AIIMS 2002]**

(A) Perpendicular to each other (B) Parallel to each other
(C) Inclined to each other at an angle of 45° (D) Antiparallel to each other

70. A stone, tied at the end of a string 80 cm long, is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 sec, what is the magnitude of acceleration of the stone **[CPMT 1996; AIIMS 2001]**

(A) 680 cm/s^2 (B) 720 cm/s^2 (C) 860 cm/s^2 (D) 990 cm/s^2

ANSWERS

DPP # 1

1. (a) 110 m (b) 50 m, $\tan^{-1}\frac{3}{4}$ north to east
2. 60 m, 20 m in the negative direction
3. (a) 520 km/h (b) 40 km/h (c) 520 km/h Patna to Ranchi
   (d) 32.5 km/h Patna to Ranchi
4. 32 km/h (b) zero
5. 2.5 m/s^2
6. 35 m
7. 12 m
8. (a) 2.7 km (b) 60 m/s (c) 225 m and 2.25 km
9. 0.05 s
10. 12.2 × 105 m/s^2
11. (a) 2.5 m/s (b) 12.5 m
12. 22 m
13. (a) 19 m (b) 22m (c) 33m (d) 39 m (e) 15 m (f) 18 m
    (g) 27 m (h) 33 m
14. 1.0 km
15. 2s, 38 m
16. (a)125 m (b) 5 s (c) 35 m/s
17. 4.3 s
18. (a) 40 m (b) 9.8 m/s (c) No
19. 44.1 m, 19.6 m and 4.9 m below the top
20. 4.9 m/s
21. 2.62 m
22. 48 m
23. 490 m/s^2
24. 20 ft/s^2
25. (a) $v_{t=5\,s}$ = 336 units, $a_{t=5\,s}$ = 126 units ; (b) $\langle v\rangle$ = 121 units, $\langle a\rangle$ = 66 units

DPP # 2

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
C	A	A	B	B	B	C	C	A	B	B	D	A	D	A
16	17	18	19	20	21									
C	A	A	A	B	A									

DPP # 3

1. (a) 4.5 s (b) 90 m (c) 49 m/s, $\theta = 66^o$ with horizontal
2. (a) 60 m (b) $80\sqrt{3}$ m
3. Yes
4. 10 m/s
5. 32 ft/s
6. 192 ft
7. Yes, Yes
8. u cosθ, horizontal in the plane of projection
10. 2m

11. 2m/s
12. (a) 19.6 m/s upward (b) 24.5 m/s at 53^o with horizontal
13. Sixth
14. Minimum angle 15^o, maximum angle 75^o but there is an interval of 53^o between 15^o and 75^o, which is not allowed for successful shot
15. 4.4 m/s
16. (a) v_0 = 667 ft/s (b) 2667 ft (c) v_x = 534 ft/s, v_y = 560 ft/s
17. $\sqrt{g\left[h+\sqrt{h^2+x^2}\right]}$
18. (a) u = 255 m/s (b) 46 s.
19. (a) θ = 30º or 60º, (b) t_1= 6 s, t_2 = 10.4 s , (c) 48.2 m
20. **(a)** 58.8 m **(b)** $\sqrt{\dfrac{2gh}{2+\cot^2\theta}}$

DPP # 4

1. (a) 40 s (b) 80 m
2. (a) $\dfrac{10\,\text{minute}}{\sin\theta}$ (b) 10 minute
3. 2/3 km
4. (a) $\sin^{-1}\left(\dfrac{1}{15}\right)$ east of the line AB (b) 50 min
5. $\dfrac{x}{2}\left(\dfrac{1}{t_1}+\dfrac{1}{t_2}\right), \dfrac{x}{2}\left(\dfrac{1}{t_1}-\dfrac{1}{t_2}\right)$
6. $\dfrac{x}{\sqrt{v^2-u^2}}$
7. 2 a/v.
8. 5 km/h
9. (a) 25 m/s or 90 km/hr (b) 20 m/s or 72 km/hr due east (c) 25 m/s or 90 km/hr at 37°N of E
10. 30º N of W at $5\sqrt{3}$ km/hr. **11.** 3 km/hr.

12. (I) **0.75 km**
(II) **(a)** $\dfrac{1}{2\sqrt{3}}$ **h** **(b)** $\dfrac{1}{2\sqrt{3}}$ **km.**
13. At an angle 30° west of north.
14. $50\sqrt{5}\,km$
15. (a) 0, 10 m/s West (b) 10 $\sqrt{3}$ m/s, 20 m/s
16. Coming from 5 km/hr, 53° N of E
17. $\left(4\hat{i}+8\hat{j}\right)$ **18.** $\dfrac{a}{v\left(1-\cos\dfrac{2\pi}{n}\right)}$
19. $\dfrac{10}{\sqrt{2}}, \dfrac{1}{8}$ hr
20. 2m/s in a direction making an angle of 60º with E, 45º with N and 60º with the vertical

EXERCISE # 1

1. (C)	2. (A)	3. (D)	4. (B)	5. (A)
6. (C)	7. (B)	8. (B)	9. (A)	10. (C)
11. (C)	12. (D)	13. (D)	14. (B)	15. (A)
16. (C)	17. (A)	18. (D)	19. (C)	20. (B)
21. (D)	22. (A)	23. (D)	24. (B)	25. (D)
26. (D)	27. (C)	28. (C)	29. (C)	30. (A)
31. (C)	32. (D)	33. (C)	34. (A)	35. (B)
36. (C)	37. (C)	38. (B)	39. (A)	40. (B)
41. (B)	42. (D)	43. (A)	44. (A)	45. (B)
46. (D)	47. (B)	48. (B)	49. (D)	50. (B)
51. (D)	52. (D)	53. (A)	54. (A)	55. (D)
56. (C)	57. (A)	58. (C)	59. (B)	60. (D)

EXERCISE # 2

1. (A)	2. (C)	3. (B)	4. (C)	5. (A)
6. (C)	7. (C)	8. (B)	9. (D)	10. (A)
11. (B)	12. (D)	13. (B)	14. (B)	15. (D)
16. (A)	17. (A)	18. (A)	19. (A)	20. (A)
21. (D)	22. (B)	23. (D)	24. (B)	25. (C)
26. (A)	27. (B)	28. (C)	29. (C)	30. (B)
31. (C)	32. (D)	33. (D)	34. (A)	35. (C)
36. (B)	37. (B)	38. (C)	39. (C)	40. (C)

EXERCISE-3

1. (D)	2. (C)	3. (B)	4. (B)	5. (C)
6. (A)	7. (A)	8. (C)	9. (A)	10. (C)
11. (B)	12. (D)	13. (B)	14. (C)	15. (D)
16. (A)	17. (D)	18. (C)	19. (B)	20. (D)
21. (A)	22. (C)	23. (A)	24. (B)	25. (A)
26. (A)	27. (C)	28. (B)	29. (A)	30. (B)
31. (C)	32. (B)	33. (C)	34. (C)	35. (A)
36. (B)	37. (C)	38. (A)	39. (C)	40. (B)
41. (C)	42. (B)	43. (A)	44. (C)	45. (A)
46. (A)	47. (C)	48. (C)	49. (C)	50. (D)
51. (B)	52. (C)	53. (C)	54. (C)	55. (D)
56. (B)	57. (D)	58. (D)	59. (C)	60. (B)
61. (D)	62. (A)	63. (C)	64. (C)	65. (D)
66. (B)	67. (C)	68. (B)	69. (A)	70. (D)

Chapter 1

NEWTON LAWS OF MOTION

1. FORCE

A pull or push which changes or tends to change the state of rest or of uniform motion or direction of motion of any object is called force. Force is the interaction between the object and the source (providing the pull or push). It is a vector quantity.

Effect of resultant force :
- may change only speed
- may change only direction of motion.
- may change both the speed and direction of motion.
- may change size and shape of a body

unit of force : newton and $\frac{kg.m}{s^2}$ (MKS System)

dyne and $\frac{g.cm}{s^2}$ (CGS System)

1 newton = 10^5 dyne

Kilogram force (kgf)

The force with which earth attracts a 1 kg body towards its centre is called kilogram force, thus

$$\text{kgf} = \frac{\text{Force in newton}}{g}$$

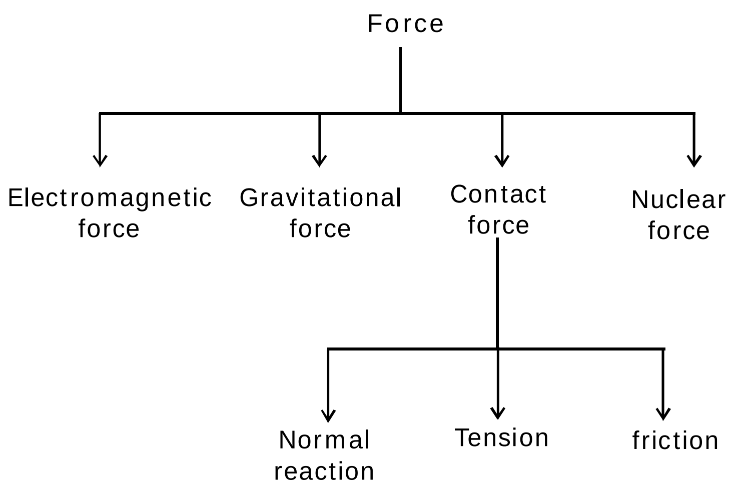
Dimensional Formula of force : [MLT^{-2}]

- For full information of force we require

$\rightarrow$ Magnitude of force

$\rightarrow$ direction of force

$\rightarrow$ point of application of the force



1.1 Electromagnetic Force

Force exerted by one particle on the other because of the electric charge on the particles is called electromagnetic force.

Following are the main characteristics of electromagnetic force
- These can be attractive or repulsive
- These are long range forces
- These depend on the nature of medium between the charged particles.
- All macroscopic force (except gravitational) which we experience as push or pull or by contact are electromagnetic, i.e., tension in a rope, the force of friction, normal reaction, muscular force, and force experienced by a deformed spring are electromagnetic forces. These are manifestations of the electromagnetic attractions are repulsions between atoms/molecules.

1.2 Gravitational force :

It acts between any two masses kept anywhere in the universe. It follows inverse square rule ($F \propto \frac{1}{\text{distance}^2}$) and is attractive in nature.

$$F = \frac{GM_1M_2}{R^2}$$

The force mg, which Earth applies on the bodies, is gravitational force.

1.3 Nuclear force :

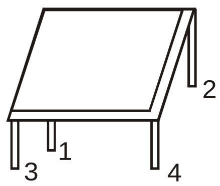
It is the strongest force. It keeps nucleons (neutrons and protons) together inside the nucleus inspite of large electric repulsion between protons. Radioactivity, fission, and fusion, etc. result because of unbalancing of nuclear forces. It acts within the nucleus that too upto a very small distance.

1.4 Contact force :

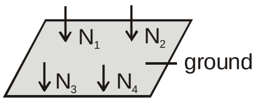
Forces which are transmitted between bodies by short range atomic molecular interactions are called contact forces. When two objects come in contact they exert contact forces on each other.

1.4.1 Normal force (N) :

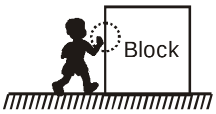
It is the component of contact force perpendicular to the surface. It measures how strongly the surfaces in contact are pressed against each other. It is the electromagnetic force. A table is placed on Earth as shown in figure.



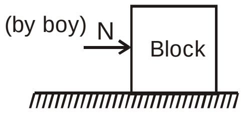
- Here table presses the earth so normal force exerted by four legs of table on earth are as shown in figure.



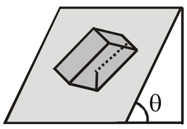
- Now a boy pushes a block kept on a frictionless surface.



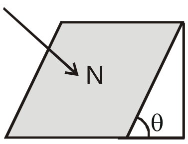
Here, force exerted by boy on block is electromagnetic interaction which arises due to similar charges appearing on finger and contact surface of block, it is normal force.



- A block is kept on inclined surface. Component of its weight presses the surface perpendicularly due to which contact force acts between surface and block.



Normal force exerted by block on the surface of inclined plane is shown in figure.

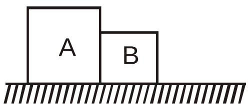


Force acts perpendicular to the surface

:

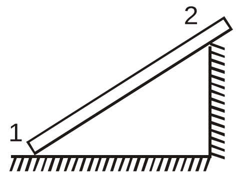
- Normal force acts in such a fashion that it tries to compress the body
- Normal is a dependent force, it comes in role when one surface presses the other.

Ex.1 *Two blocks are kept in contact on a smooth surface as shown in figure. Draw normal force exerted by A on B.*

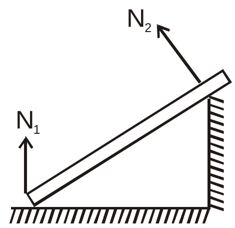


Sol. In above problem, block A does not push block B, so there is no molecular interaction between A and B. Hence normal force exerted by A on B is zero.

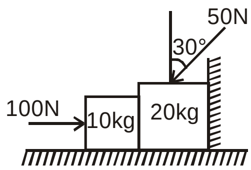
Ex.2 *Draw normal forces on the massive rod at point 1 and 2 as shown in figure.*



Sol. Normal force acts perpendicular to extended surface at point of contact.



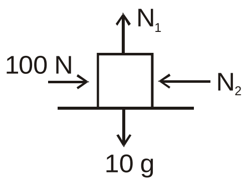
Ex.3 *Two blocks are kept in contact as shown in figure. Find*
(a) forces exerted by surfaces (floor and wall) on blocks
(b) contact force between two blcoks.



Sol. F.B.D. of 10 kg block

N_1 = 10 g = 100 N ...(1)

N_2 = 100 N ...(2)



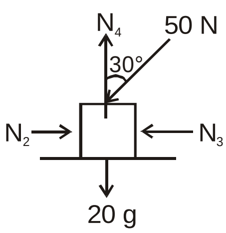
F.B.D. of 20 kg block

$N_2 = 50 \sin 30° + N_3$

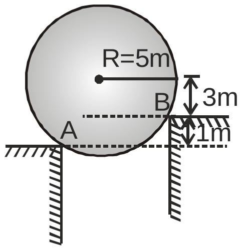
$\therefore$ N_3 = 100 – 25 = 75 N ...(3)

and $N_4 = 50 \cos 30° + 20$ g

N_4 = 243.30 N

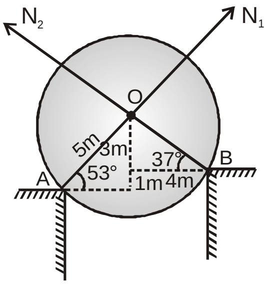


Ex.4

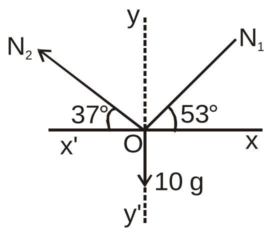


Find out the normal reaction at point A and B if the mass of sphere is 10 kg.

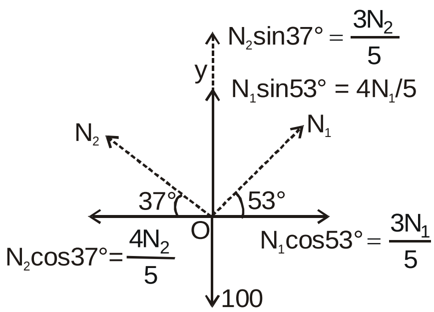
Sol.



Now F.B.D.



Now resolve the forces along x & y direction



$\because$ The body is in equilibrium so equate the force in x & y direction

In x-direction $\frac{3N_1}{5} = \frac{4N_2}{5}$ ...(1)

In y-direction $\frac{3N_2}{5} + \frac{4N_1}{5} = 100$ ...(2)

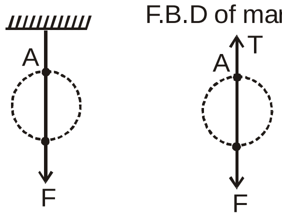
after solving above equation

$N_1 = 80$ N, $N_2 = 60$ N

1.4.2 Tension :

Tension in a string is an electromagnetic force. It arises when a string is pulled. If a massless string is not pulled, tension in it is zero. A string suspended by rigid support is pulled by a force 'F' as shown in figure, for calculating the tension at point 'A' we draw F.B.D. of marked portion of the string; Here string is massless.

F.B.D of marked portion



$\Rightarrow$ T = F

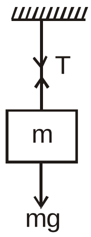
String is considered to be made of a number of small segments which attracts each other due to electromagnetic nature. The attraction force between two segments is equal and opposite due to newton's third law.

Conclusion :

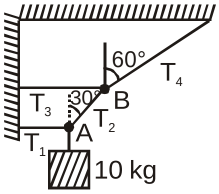
T = mg

(i) Tension always acts along the string and in such a direction that it tries to reduce the length of string

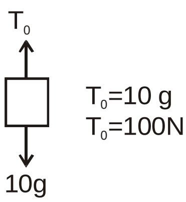
(ii) If the string is massless then the tension will be same along the string but if the string have some mass then the tension will continuously change along the string.



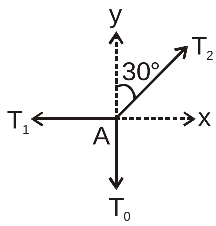
Ex.5 The system shown in figure is in equilibrium. Find the magnitude of tension in each string ; T_1, T_2, T_3 and T_4. ($g = 10\ m/s^{-2}$)



Sol. F.B.D. of block 10 kg



F.B.D. of point 'A'



$\Sigma F_y = 0$

$T_2 \cos 30° = T_0 = 100$ N

$T_2 = \frac{200}{\sqrt{3}}$N

$\Sigma F_x = 0$

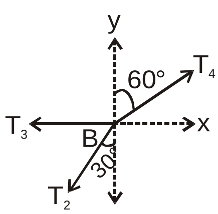
$T_1 = T_2 \sin 30° = \frac{200}{\sqrt{3}} \cdot \frac{1}{2} = \frac{100}{\sqrt{3}}$N

F.B.D of point 'B'

$\Sigma F_y = 0 \Rightarrow T_4 \cos 60° = T_2 \cos 30°$

and $\Sigma F_x = 0 \Rightarrow T_3 + T_2 \sin 30° = T_4 \sin 60°$

$\therefore\ T_3 = \frac{200}{\sqrt{3}}$N, $T_4 = 200$ N



1.4.3 Frictional force :

It is the component of contact force tangential to the surface. It opposes the relative motion (or attempted relative motion) of the two surfaces in contact. (which is explained later)

2. NEWTON'S FIRST LAW OF MOTION :

According to this law "A system will remain in its state of rest or of uniform motion unless a net external force act on it.

1st law can also be stated as "If the net external force acting on a body is zero, only then the body remains at rest."

- The word external means external to the system (object under observation), interactions within the system has not to be considered.
- The word net means the resultant of all the forces acting on the system.
- Newton's first law is nothing but Galileo's law of inertia.
- Inertia means inability of a body to change its state of motion or rest by itself.
- The property of a body that determines its resistance to a change in its motion is its mass (inertia). Greater the mass, greater the inertia.
- An external force is needed to set the system into motion, but no external force is needed to keep a body moving with constant velocity in its uniform motion.
- Newton's laws of motion are valid only in a set of frame of references, these frames of reference are known as inertial frames of reference.
- Generally, we take earth as an inertial frame of reference, but strictly speaking it is not an inertial frame.
- All frames moving uniformly with respect to an inertial frame are themselves inertial.
- We take all frames at rest or moving uniformly with respect to earth, as inertial frames.

3. NEWTON'S SECOND LAW OF MOTION :

Newton's second law states, "The rate of change of a momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts"

i.e., $\vec{F} \propto \frac{d\vec{p}}{dt}$ or $\vec{F} = k\frac{d\vec{p}}{dt}$

where k is a constant of proportionality.

$\vec{p} = m\vec{v}$, So $\vec{F} = k\frac{(dm\vec{v})}{dt}$

For a body having constant mass,

$\Rightarrow \quad \vec{F} = km\frac{d\vec{v}}{dt} = km\vec{a}$

From experiments, the value of k is found to be 1.

So, $\vec{F}_{net} = m\vec{a}$

- Force can't change the momentum along a direction normal to it, i.e., the component of velocity normal to the force doesn't change.
- Newton's 2nd law is strictly applicable to a single point particle. In case of rigid bodies or system of particles or system of rigid bodies, $\vec{F}$ refers to total external force acting on system and $\vec{a}$ refers to acceleration of centre of mass of the system. The internal forces, if any, in the system are not to be included in $\vec{F}$.
- Acceleration of a particle at any instant and at a particular location is determined by the force (net) acting on the particle at the same instant and at same location and is not in any way depending on the history of the motion of the particle.

PROBLEM SOLVING STRATEGY :

Newton's laws refer to a particle and relate the forces acting on the particle to its mass and to its acceleration. But before writing any equation from Newton's law, you should be careful about which particle you are considering. The laws are applicable to an extended body too which is nothing but collection of a large number of particles.

Follow the steps given below in writing the equations :

Step 1 : Select the body

The first step is to decide the body on which the laws of motion are to be applied. The body may be a single particle, an extended body like a block, a combination of two blocks-one kept over another or connected by a string. The only condition is that all the parts of the body or system must have the same acceleration.

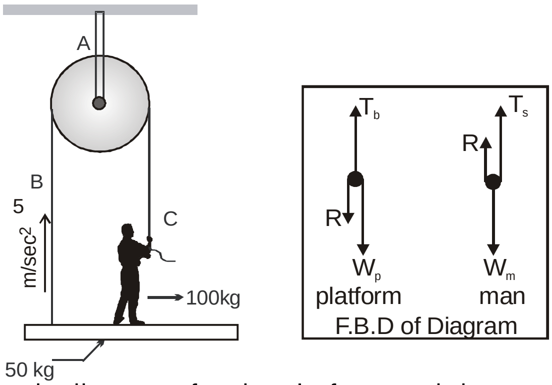
Step 2 : Identify the forces

Once the system is decided, list down all the force acting on the system due to all the objects in the environment such as inclined planes, strings, springs etc. However, any force applied by the system shouldn't be included in the list. You should also be clear about the nature and direction of these forces.

Step 3 : Make a Free-body diagram (FBD)

Make a separate diagram representing the body by a point and draw vectors representing the forces acting on the body with this point as the common origin.

This is called a free-body diagram of the body.



Look at the adjoining free-body diagrams for the platform and the man. Note that the force applied by the man on the rope hasn't been included in the FBD.

Once you get enough practice, you'd be able to identify and draw forces in the main diagram itself instead of making a separate one

Step 4 : Select axes and Write equations

When the body is in equillibrium then choose the axis in such a fashion that maximum number of force lie along the axis.

If the body is moving with some acceleration then first find out the direction of real acceleration and choose the axis one is along the real acceleration direction and other perpendicular to it.

Write the equations according to the newton's second law (F_{net} = ma) in the corresponding axis.

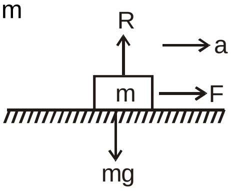
4. APPLICATIONS :

4.1 Motion of a Block on a Horizontal Smooth Surface.

Case (i) : When subjected to a horizontal pull :

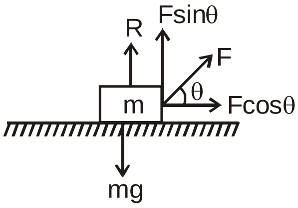
The distribution of forces on the body are shown. As there is no motion along vertical direction, hence, R = mg

For horizontal motion F = ma or a = F/m



Case (ii) : When subjected to a pull acting at an angle (θ) to the horizontal :

Now F has to be resolved into two components, F cosθ along the horizontal and F sin θ along the vertical direction.



For no motion along the vertical direction.

we have $R + F \sin\theta = mg$

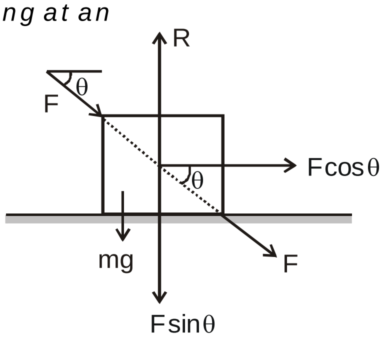
or $R = mg - F \sin\theta$

: Hence $R \neq mg$. $R < mg$

For horizontal motion

$$F \cos\theta = ma, \quad a = \frac{F\cos\theta}{m}$$

Case (iii) : When the block is subjected to a push acting at an angle θ to the horizontal : (down ward)



The force equation in this case

$R = mg + F \sin\theta$

: $R \neq mg$, $R > mg$

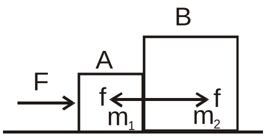
For horizontal motion

$$F \cos\theta = ma, \quad a = \frac{F\cos\theta}{m}$$

4.2 Motion of bodies in contact.

Case (i) : Two body system :

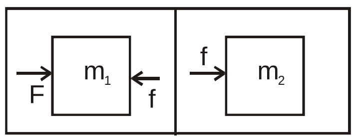
Let a force F be applied on mass m_1



Free body diagrams :

(vertical force do not cause motion, hence they have not been shown in diagram)

$$\Rightarrow \quad a = \frac{F}{m_1 + m_2} \quad \text{and } f = \frac{m_2 F}{m_1 + m_2}$$



(i) Here f is known as force of contact.

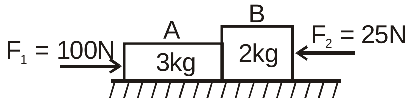
(ii) Acceleration of system can be found simply by

$$a = \frac{\text{force}}{\text{total mass}}$$

: If force F be applied on m_2, the acceleration will remain the same, but the force of contact will be different

$$\text{i.e., } f' = \frac{m_1 F}{m_1 + m_2}$$

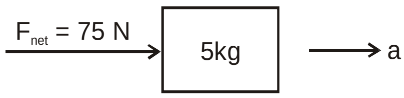
Ex.6 *Find the contact force between the 3 kg and 2kg block as shown in figure.*



Sol. Considering both blocks as a system to find the common acceleration

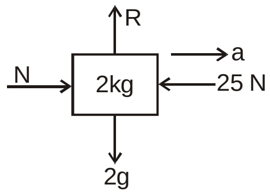
$F_{net} = F_1 - F_2 = 100 - 25 = 75$ N

common acceleration



$$a = \frac{F_{net}}{5kg} = \frac{75}{5} = 15 m/s^2$$

To find the contact force between A & B we draw F.B.D of 2 kg block

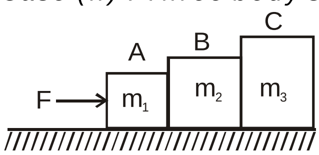


from $(\Sigma F_{net})_x = ma_x$

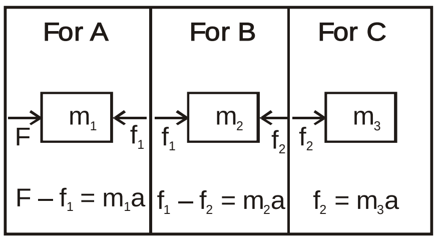
$\Rightarrow$ $N - 25 = (2)(15)$

$\Rightarrow$ $N = 55$ N

Case (ii) : Three body system :



Free body diagrams :



$\Rightarrow$ $a = \dfrac{F}{m_1 + m_2 + m_3}$

and $f_1 = \dfrac{(m_2 + m_3)F}{(m_1 + m_2 + m_3)}$

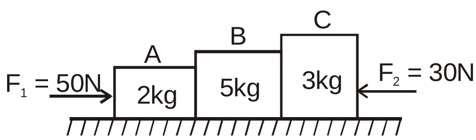
$f_2 = \dfrac{m_3F}{(m_1 + m_2 + m_3)}$

f_1 = contact force between masses m_1 and m_2

f_2 = contact force between masses m_2 and m_3

Remember : Contact forces will be different if force F will be applied on mass C

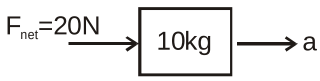
Ex.7 *Find the contact force between the block and acceleration of the blocks as shown in figure.*



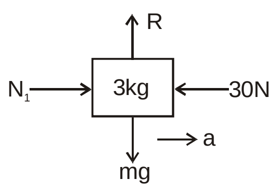
Sol. Considering all the three block as a system to find the common acceleration

$F_{net} = 50 - 30 = 20$ N

$$a = \frac{20}{10} = 2 m/s^2$$



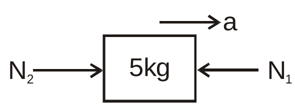
To find the contact force between B & C we draw F.B.D. of 3 kg block.



$$\left(\sum F_{net}\right)_x = ma$$

$\Rightarrow$ $N_1 - 30 = 3(2) \Rightarrow N_1 = 36$ N

To find contact force between A & B we draw F.B.D. of 5 kg block



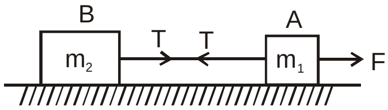
$\Rightarrow$ $N_2 - N_1 = 5a$

$N_2 = 5 \times 2 + 36 \Rightarrow N_2 = 46$ N

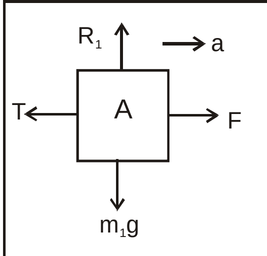
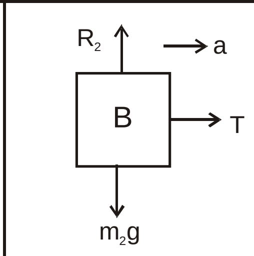
4.3 Motion of connected Bodies

Case (i) For Two Bodies :

F is the pull on body A of mass m_1. The pull of A on B is exercised as tension through the string connecting A and B. The value of tension throughout the string is T only.

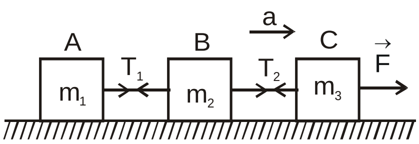


Free body diagrams :

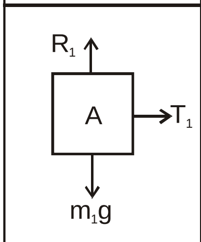
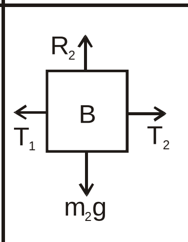
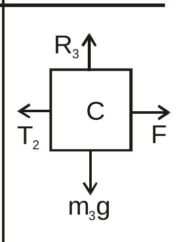
For body A	For body B
	
$R_1 = m_1g$ $F - T = m_1a$	$R_2 = m_2g$ $T = m_2a$

$$\Rightarrow \qquad a = \frac{F}{m_1 + m_2}$$

Case (ii) : For Three bodies :

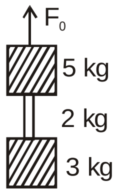


Free body diagrams :

For A	For B	For C
		
$R_1 = m_1g$ $T_1 = m_1a$	$R_2 = m_2g$ $T_2 - T_1 = m_2a$ $\Rightarrow T_2 = m_2a + T_1$ $T_2 = (m_2 + m_1)a$	$R_3 = m_3g$ $F - T_2 = m_3a$ $\Rightarrow F = m_3a + T_2$ $=m_3a+(m_1+m_2)a$ $F=(m_1+m_2+m_3)a$

$$\Rightarrow \ a = \frac{F}{m_1 + m_2 + m_3}$$

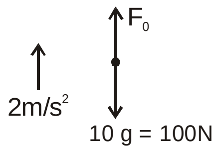
Ex.8 *A 5 kg block has a rope of mass 2 kg attached to its underside and a 3 kg block is suspended from the other end of the rope. The whole system is accelerated upward is 2 m/s² by an external force F_0.*



(a) What is F_0?
(b) What is the force on rope?
(c) What is the tension at middle point of the rope?
(g = 10 m/s²)

Sol. For calculating the value of F_0, consider two blocks with the rope as a system.
F.B.D. of whole system

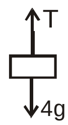
(a)



$F_0 - 100 = 10 \times 2$
$F = 120$ N ...(1)

(b) According to Newton's second law, net force on rope.
$F = ma = (2)(2) = 4$ N ...(2)

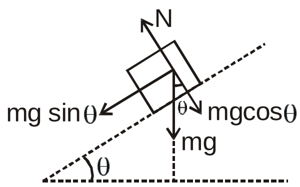
(c) For calculating tension at the middle point we draw F.B.D. of 3 kg block with half of the rope (mass 1 kg) as shown.



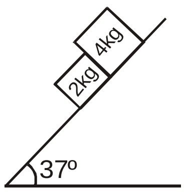
$T - 4g = 4.(2) = 48$ N

4.4 Motion of a body on a smooth inclined plane :

Natural acceleration down the plane = g sin θ
Driving force for acceleration a up the plane, F=m(a+ gsinθ)
and for an acceleration a down the plane, F=m(a – gsinθ)

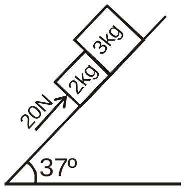


Ex.9 *Find out the contact force between the 2kg & 4kg block as shown in figure.*

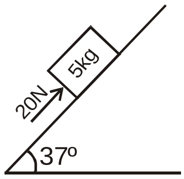


Sol. On an incline plane acceleration of the block is independent of mass. So both the blocks will move with the same acceleration (gsin 37°) so the contact force between them is zero.

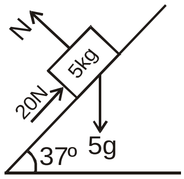
Ex.10 Find out the contact force between 2kg & 3kg block placed on the incline plane as shown in figure.



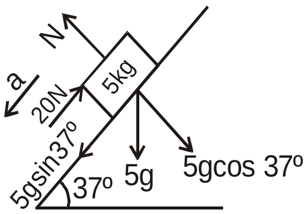
Sol. Considering both the block as a 5kg system because both will move the same acceleration.



Now show forces on the 5 kg block



$\because$ Acceleration of 5kg block is down the incline.
So choose one axis down the incline and other perpendicular to it



From Newton's second Law

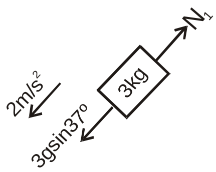
$N = 5g \cos 37^o$ ...(i)

$5g\sin 37^o - 20 = 5a$ ..(ii)

$30 - 20 = 5a$

$a = 2 m/s^2$ (down the incline)

For contact force (N_1) between 2kg & 3kg block we draw F.B.D. of 3kg block



From

$F_{net} = ma$

$\Rightarrow 3g\sin 37^o - N_1 = 3 \times 2$

$18 - N_1 = 6$

$N_1 = 12$ N

4.5 Pulley block system :

Ex.11 One end of string which passes through pulley and connected to 10 kg mass at other end is pulled by 100 N force. Find out the acceleration of 10 kg mass. ($g = 9.8$ m/s²)

Sol. Since string is pulled by 100N force.

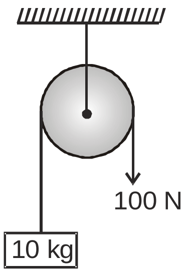
So tension in the string is 100 N.

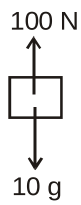
F.B.D. of 10 kg block

$100 - 10\ g = 10\ a$

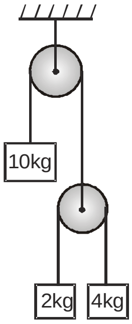
$100 - 10 \times 9.8 = 10\ a$

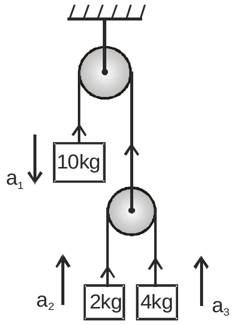
$a = 0.2\ m/s^2$





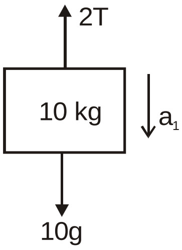
Ex.12 In the figure shown, find out acceleration of each block.





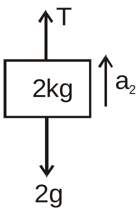
Sol. Now F.B.D. of each block and apply Newton's second law on each F.B.D

(1)



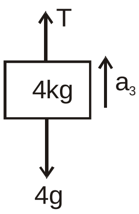
$\Rightarrow$ $10g - 2T = 10a_1$ ...(i)

(2)



$\Rightarrow$ $T - 2g = 2a_2$ ...(2)

(3)



$\Rightarrow$ $T - 4g = 4a_3$ ...(3)

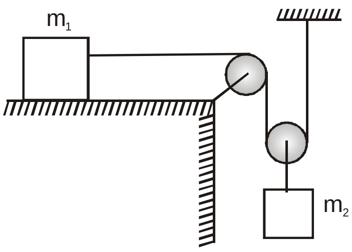
from constrain relation $2a_1 = a_2 + a_3$ ...(4)

Solving equations (1), (2), (3) and (4) we get

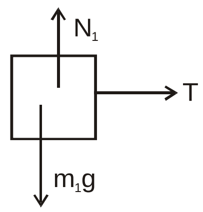
$$T = \frac{800}{23} N$$

$a_1 = 70/23\ m/s^2$ (downward), $a_2 = 170/23\ m/s^2$ (upward), $a_3 = 30/23\ m/s^2$ (downward)

Ex.13 Find the acceleration of each block in the figure shown below; in terms of their masses m_1, m_2 and g. Neglect any friction.



Sol. Let T be the tension in the string that is assumed to be massless.



For mass m_1, the FBD shows that

$N_1 = m_1g$

Where N_1 is the force applied upward by plane on the mass m_1.

If acceleration of m_1 along horizontal is a_1. then

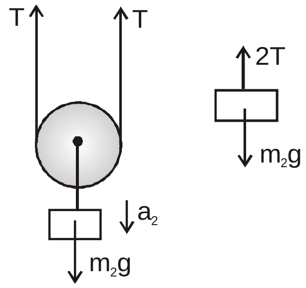
$T = m_1a_1$ ...(i)

For mass m_2, the FBD shows that

$m_2g - 2T = m_2a_2$ ...(ii)

Where a_2 is vertical acceleration of mass m_2.

Note that upward tension on m_2 is 2T applied by both sides of the string.



from constrain relation

$a_2 = \frac{a_1}{2}$

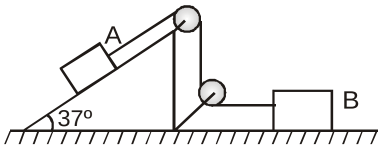
Thus, the acceleration of m_1 its twice that of m_2.

with this input, solving (i) and (ii) we find

$$a_1 = \frac{2m_2g}{4m_1 + m_2}$$

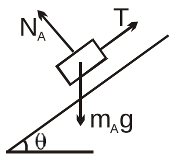
$$a_2 = \frac{m_2g}{4m_1 + m_2}$$

Ex.14 Two blocks A and B each having a mass of 20 kg, rest on frictionless surfaces as shown in the figure below. Assuming the pulleys to be light and frictionless, compute :
(a) the time required for block A, to move down by 2m on the plane, starting from rest,
(b) tension in the string, connecting the blocks.

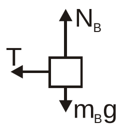


Sol.

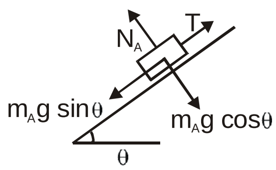
Step 1. Draw the FBDs for both the blocks. If tension in the string is T, then we have



and



Note that m_Ag, should better be resolved along and perpendicular to the plane, as the block A is moving along the plane.



Step 2. From FBDs, we write the force equations '

for block A where

$N_A = m_A g \cos\theta = 20 \times 10 \times 0.8 = 160$ N

and $m_Ag \sin\theta - T = m_A a$ ... (i)

Where 'a' is acceleration of masses of blocks A and B.

Similarly, force equations for block B are

$N_B = m_Bg = 20 \times 10 = 200$ N

and $T = m_Ba$ ...(ii)

From (i) and (ii), we obtain

$$a = \frac{m_A g\sin\theta}{m_A + m_B} = \frac{20\times 10\times 0.6}{40} = 3\ ms^{-2}$$

$$T = m_B a = 20 \times 3 = 60\ N$$

Step 3. With constant acceleration a = 3 ms^{-2}, the block A moves down the inclined plane a distance S = 2 m in time t given by

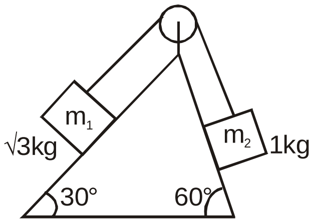
$$S = \frac{1}{2}at^2 \text{ or } t = \sqrt{\frac{2S}{a}} = \frac{2}{\sqrt{3}}\text{seconds.}$$

Ex.15 *Two blocks m_1 and m_2 are placed on a smooth inclined plane as shown in figure. If they are released from rest. Find :*

(i) acceleration of mass m_1 and m_2

(ii) tension in the string

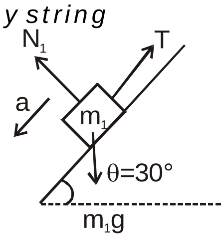
(iii) net force on pulley exerted by string



Sol. F.B.D of m_1 :

$m_1 g \sin\theta - T = m_1 a$

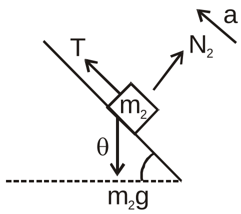
$$\frac{\sqrt{3}}{2}g - T = \sqrt{3}\,a \qquad \ldots(i)$$



F.B.D. of m_2 :

$T - m_2 g \sin\theta = m_2 a$

$$T - 1.\frac{\sqrt{3}}{2}g = 1.a \qquad \ldots(ii)$$



Adding eq. (i) and (ii) we get a = 0

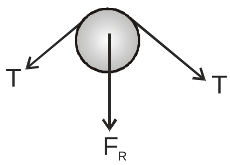
Putting this value in eq. (i) we get

$$T = \frac{\sqrt{3}g}{2},$$

F.B.D. of pulley

$F_R = \sqrt{2}\,T$

$$F_R = \sqrt{\frac{3}{2}}g$$



5. NEWTONS' 3RD LAW OF MOTION :

Statement : "To every action there is equal and opposite reaction".

But what is the meaning of action and reaction and which force is action and which force is reaction? Every force that acts on body is due to the other bodies in environment. Suppose that a body A experiences a force $\vec{F}_{AB}$ due to other body B. Also body B will experience a force $\vec{F}_{BA}$ due to A. According to Newton third law two forces are equal in magnitude and opposite in direction Mathematically we write it as

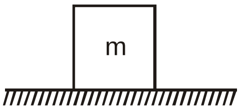
$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Here we can take either $\vec{F}_{AB}$ or $\vec{F}_{BA}$ as action force and other will be the reaction force.

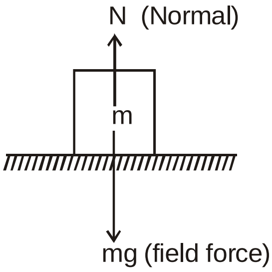
: (i) Action-Reaction pair acts on two different bodies.

(ii) Magnitude of force is same.
(iii) Direction of forces are in opposite direction.
(iv) For action-reaction pair there is no need of contact

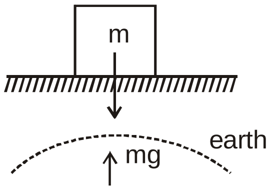
Ex.16 A block of mass 'm' is kept on the ground as shown in figure.
(i) Draw F.B.D. of block
(ii) Are forces acting on block action - reaction pair
(iii) If answer is no, draw action reaction pair.



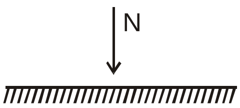
Sol. (i) F.B.D. of block



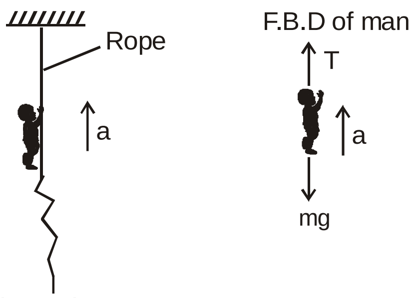
(ii) 'N' and Mg are not action - reaction pair. Since pair act on different bodies, and they are of same nature.
(iii) Pair of 'mg' of block acts on earth in opposite direction.



and pair of 'N' acts on surface as shown in figure.



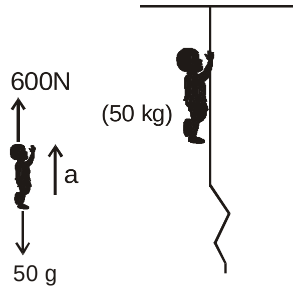
5.1 Climbing on the Rope :



Now three condition arises.
if $T > mg \Rightarrow$ man accelerates in upward direction
$T < mg \Rightarrow$ man accelerates in downward direction
$T = mg \Rightarrow$ man's acceleration is zero

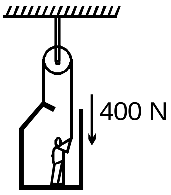
* Either climbing or decending on the rope man exerts force downward

Ex.17 If the breaking strength of string is 600N then find out the maximum acceleration of the man with which he can climb up the road



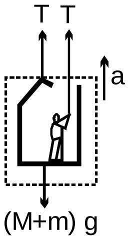
Sol. Maximum force that can be exerted on the man by the rope is 600 N.
F.B.D of man
$\Rightarrow$ $600 - 50\,g = 50\,a$
$a_{max} = 2\ m/s^2$

Ex.18 A 60 kg painter on a 15 kg platform. A rope attached to the platform and passing over an overhead pulley allows the painter to raise himself along with the platform.



(i) To get started, he pulls the rope down with a force of 400 N. Find the acceleration of the platform as well as that of the painter.
(ii) What force must he exert on the rope so as to attain an upward speed of 1 m/s in 1 s ?
(iii) What force should apply now to maintain the constant speed of 1 m/s?

Sol. The free body diagram of the painter and the platform as a system can be drawn as shown in the figure. Note that the tension in the string is equal to the force by which he pulles the rope.



(i) Applying Newton's Second Law
$2T - (M + m)g = (M + m)a$

or $a = \dfrac{2T-(M+m)g}{M+m}$

Here M = 60 kg; m = 15 kg ; T = 400 N
$g = 10\ m/s^2$

$$a = \frac{2(400)-(60+15)(10)}{60+15} = 0.67\ m/s^2$$

(ii) To attain a speed of 1 m/s in one second the acceleration a must be 1 m/s^2
Thus, the applied force is

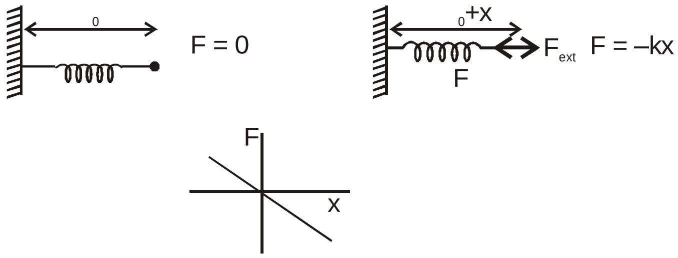
$$F = \frac{1}{2}(M + m)\,(g + a) = (60 + 15)\,(10 + 1) = 412.5\ N$$

(iii) When the painter and the platform move (upward) together with a constant speed, it is in a state of dynamic equilibrium
Thus, $2F - (M + m)\,g = 0$

$$\text{or } F = \frac{(M+m)g}{2} = \frac{(60+15)(10)}{2} = 375\ N$$

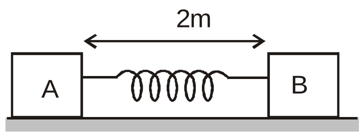
6. SPRING FORCE :

Every spring resists any attempt to change its length; when it is compressed or extended, it exerts force at its ends. The force exerted by a spring is given by $F = -kx$, where x is the change in length and k is the stiffness constant or spring constant (unit Nm^{-1})
When spring is in its natural length, spring force is zero.



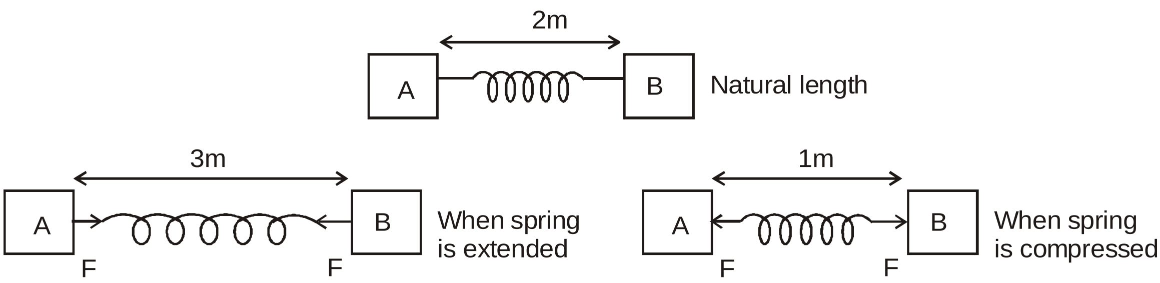
Graph between spring force v/s x

Ex.19 Two blocks are connected by a spring of natural length 2 m. The force constant of spring is 200 N/m. Find spring force in following situations.

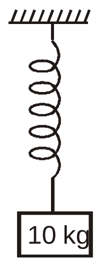


(a) If block 'A' and 'B' both are displaced by 0.5 m in same direction.
(b) If block 'A' and 'B' both are displaced by 0.5 m in opposite direction.

Sol. (a) Since both blocks are displaced by 0.5 m in same direcetion, so change in length of spring is zero. Hence, spring force is zero.
(b) In this case, change in length of spring is 1 m. So spring force is F = –Kx
= – (200). (1)
F = –200 N

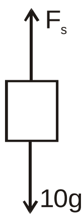


Ex.20 Force constant of a spring is 100 N/m. If a 10 kg block attached with the spring is at rest, then find extension in the spring ($g = 10\ m/s^2$)



Sol. In this situation, spring is in extended state so spring force acts in upward direction. Let x be the extension in the spring.

F.B.D. of 10 kg block :

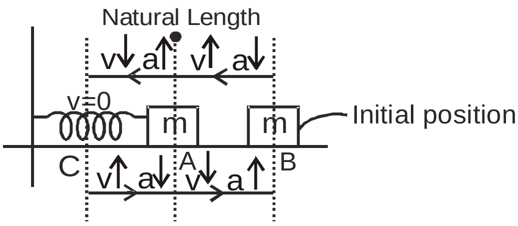


F_s = 10 g
⇒ Kx = 100
⇒ (100)x = (100)
⇒ x = 1 m

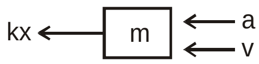
6.1 SPRING FORCE SYSTEM :

Initially the spring is in natural length at A with block m. But when the block displaced towards right then the spring is elongated and now block is released at B then the block move towards left due to spring force (kx).

Analysis of motion of block :

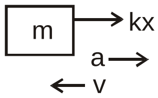


(i) From B to A speed of block increase and acceleration decreases. (due to decrease in spring force kx)

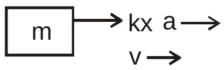


(ii) Due to inertia block crosses natural length at A.

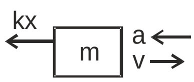
From A to C speed of the block decreases and acceleration increases.(due to increase in spring force kx)



(iii) At C the block stops momentarily at this instant and since the spring is compressed spring force is towards right and the block starts to move towards right. From C to A speed of block increases and acceleration decreases.(due to decrease in spring force kx)



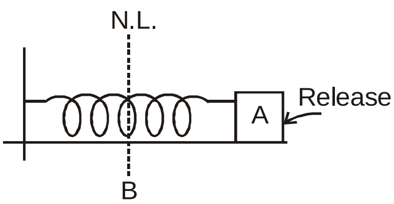
(iv) Again block crosses point A due to inertia then from A to B speed decreases and acceleration increases.



In this way block does SHM (to be expalined later) if no resistive force is acting on the block.

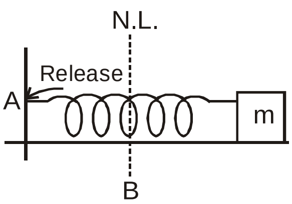
Note :

(i)



when the block A is released then it take some finite time to reach at B. i.e., spring force doesn't change instantaneously.

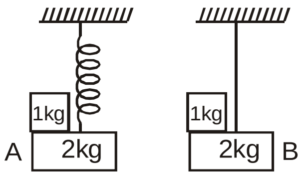
(2)



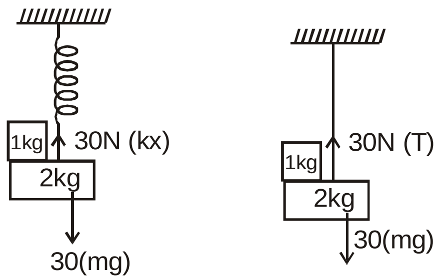
When point A of the spring is released in the above situation then the spring forces changes instantaneously and becomes zero because one end of the spring is free.

(3) In string tension may change instantaneously.

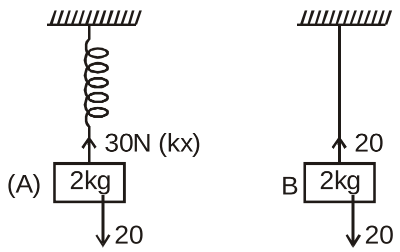
Ex.21 Find out the acceleration of 2 kg block in the figures shown at the instant 1 kg block falls from 2 kg block. (at t = 0)



Sol. F.B.D.s before fall of 1kg block

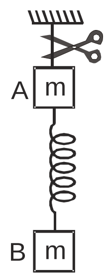


after the fall of the 1 kg block tension will change instantaneously but spring force (kx) doesn't change instantaneously. F.B.D.s just after the fall of 1 kg block



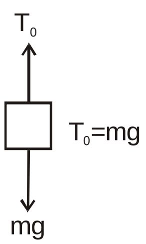
$a_A = \dfrac{30-20}{2} = 5\ m/s^2$ (upward) $\qquad a_B = 0\ m/s^2$

Ex.22 Two blocks 'A' and 'B' of same mass 'm' attached with a light spring are suspended by a string as shown in figure. Find the acceleration of block 'A' and 'B' just after the string is cut.



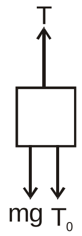
Sol. When block A and B are in equilibrium position

F.B.D of 'B'



...(i)

F.B.D of 'A'

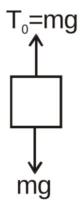


$T = mg + T_0$ .....(ii)

$T = 2\ mg$

when string is cut, tension T becomes zero. But spring does not change its shape just after cutting. So spring force acts on mass B, again draw F.B.D. of block A and B as shown in figure

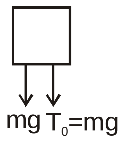
F.B.D of 'B'



$T_0 - mg = m.a_B$

$a_B = 0$

F.B.D. of 'A'

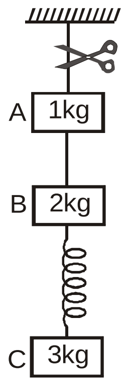


$mg + T_0 = m.\ a_A$

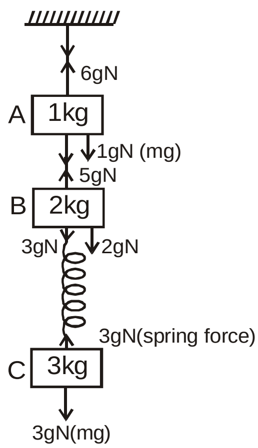
$2\ mg = m.\ a_A$

$a_A = 2g$ (downwards)

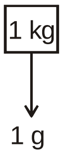
Ex.23 Find out the acceleration of 1kg, 2kg and 3kg block and tension in the string between 1 kg & 2 kg block just after cutting the string as shown in figure.



Sol. F.B.D before cutting of string



Let us assume the Tension in the string connecting blocks A & B becomes zero just after cutting the string then.

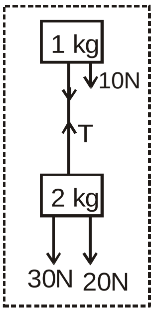


$a_1 = \frac{1g}{1} = g\ ms^{-2}$

$a_2 = \frac{5g}{2} = 2.5\ g\ ms^{-2}$

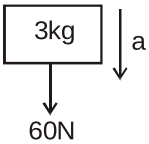
$\because \quad a_2 > a_1$ i.e., $\quad \therefore \quad T \neq 0$

If $T \neq 0$ that means string is tight and Both block A & B will have same acceleration. So it will take as a system of 3 kg mass.



System

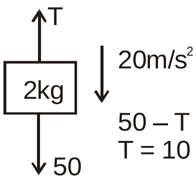
$\equiv$



Total force down ward = 10 + 30 + 20 = 60 N

Total mass = 3 kg $\Rightarrow a = \frac{60}{3} = 20\ m/s^2$

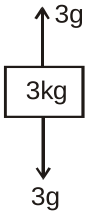
Now apply $F_{net} = ma$ at block B.



$50 - T = 2 \times 20$

$T = 10$ N

$\because$ the spring force does not change instantaneously the F.B.D of 'C'



$a_c = 0\ m/s^2$

Reference Frame :

A frame of reference is basically a coordinate system in which motion of object is analyzed. There are two types of reference frames.

(a) Inertial reference frame : Frame of reference moving with constant velocity or stationary

(b) Non-inertial reference frame : A frame of reference moving with non-zero acceleration

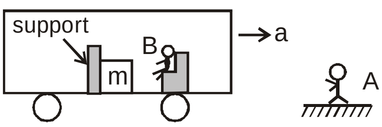
: (i) Although earth is a non inertial frame (due to rotation) but we always consider it as an inertial frame.

(ii) A body moving in circular path with constant speed is a non intertial frame (direction change cause acceleration)

7. PSEUDO FORCE :

Consider the following

example to understand the pseudo force concept



The block m in the bus is moving with constant acceleration a with respect to man A at ground. Force required for this acceleration is the normal reaction exerted by the support

So, $N = ma$ ..(i)

This block m is at rest with respect to man B who is in the bus (a non intertial frame). So the acceleration of the block with respect to man B is zero.

$N = m(0) = 0$ ..(ii)

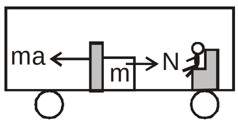
But the normal force is exerted in a non-inertial frame also. So the equation (ii) is wrong therefore we conclude that Newton's law is not valid in non-inertial frame.

If we want to apply Newton's law in non-inertial frame, then we can do so by using of the cencept pseudo force.

Pseudo force is an imaginary force, which in actual is not acting on the body. But after applying it on the body we can use Newton's laws in non-inertial frames.

This imaginary force is acting on the body only when we are solving the problem in a non-inertial frame of reference.

In the above example. The net force on the block m is zero with respect to man B after applying the pesudo force.

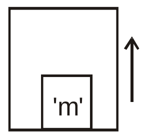


$N = ma$

:

1. Direction of pseudo force is opposite to the acceleration of frame
2. Magnitude of pseudo force is equal to mass of the body which we are analyzing multiplied by acceleration of frame
3. Point of application of pseudo force is the centre of mass of the body which we are analysing

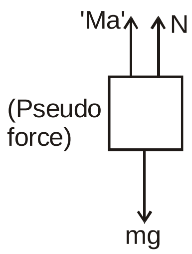
Ex.24 A box is moving upward with retardation 'a' < g, find the direction and magnitude of "pseudo force" acting on block of mass 'm' placed inside the box. Also calculate normal force exerted by surface on block



Sol. Pseudo force acts opposite to the direction of acceleration of reference frame.

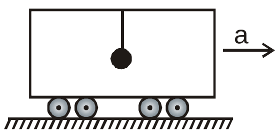
pseudo force = ma in upward direction

F.B.D of 'm' w.r.t box (non-inertial)



$N + ma = mg$
$N = mg - ma$

Ex.25 Figure shows a pendulum suspended from the roof of a car that has a constant acceleration a relative to the ground. Find the deflection of the pendulum from the vertical as observed from the ground frame and from the frame attached with the car.



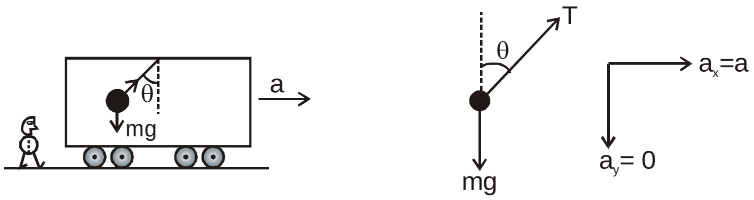
Sol. 

Figure represents free Body diagram of the bob w.r.t ground.

In an inertial frame the suspended bob has an acceleration a caused by the horizontal component of tension T.

$T \sin\theta = ma$ ...(i)

$T \cos\theta = mg$ ...(ii)

From equation (i) and (ii)

$$\tan\theta = \frac{a}{g} \quad \Rightarrow \quad \theta = \tan^{-1}\left(\frac{a}{g}\right)$$

In a non-inertial frame

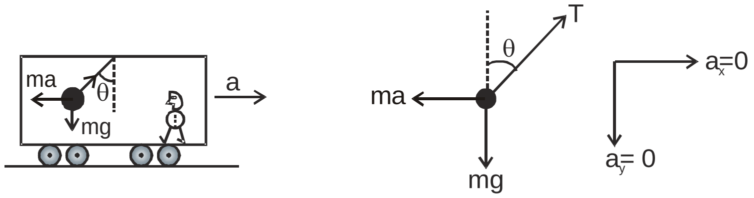


Figure represents free Body diagram of bob w.r.t car.

In the non-intertial frame of the car, the bob is in static equilibrium under the action of three froces, T, mg and ma (pseudo force)

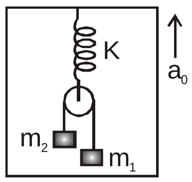
$T \sin\theta = ma$ ...(iii)

$T \cos\theta = mg$ ....(iv)

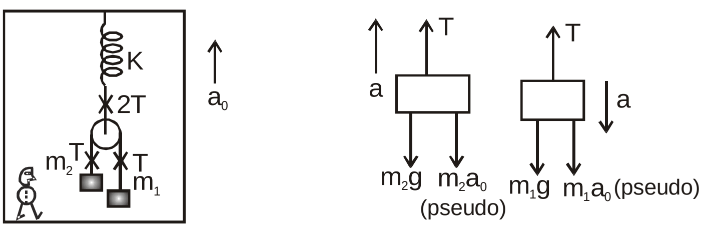
From equation (iii) and (iv)

$$\tan\theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1}\left(\frac{a}{g}\right)$$

Ex.26 A pulley with two blocks system is attached to the ceiling of a lift moving upward with an acceleration a_0. Find the deformation in the spring.



Sol. Non-Inertial Frame



Let relative to the centre of pulley, m_1 accelerates downward with a and m_2 accelerates upwards with a. Applying Newton's 2nd law.

$m_1a + m_1a_0 - T = m_1a$ ...(i)

$T - m_2g - m_2a_0 = m_2a$ ...(ii)

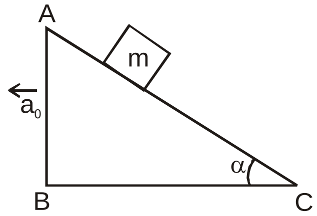
On adding (iv) and (v) we get

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)(g + a_0) \qquad \ldots(iii)$$

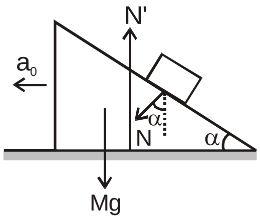
Substituting a in equation (i)

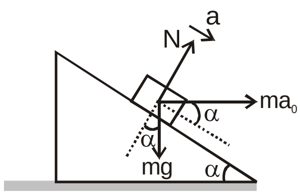
We get $T = \frac{2m_1m_2(g+a_0)}{m_1+m_2}$ $\quad \therefore\ x = \frac{F}{k} = \frac{2T}{k} = \frac{4m_1m_2(g+a_0)}{(m_1+m_2)k}$

Ex.27 *All the surfaces shown in figure are assumed to be frictionless. The block of mass m slides on the prism which in turn slides backward on the horizontal surface. Find the acceleration of the smaller block with respect to the prism.*



Sol. Let the acceleration of the prism be a_0 in the backward direction. Consider the motion of the smaller block from the frame of the prism The forces on the block are (figure)





(i) N normal force
(ii) mg downward (gravity),
(iii) ma_0 forward (Psuedo Force)
The block slides down the plane. Components of the forces parallel to the incline give

$$ma_0 \cos\alpha + mg \sin\alpha = ma$$

or, $\quad a = a_0 \cos\alpha + g \sin\alpha \qquad \ldots(i)$

Components of the forces perpendicular to the incline give

$$N + ma_0 \sin\alpha = mg \cos\alpha \qquad \ldots(ii)$$

Now consider the motion of the prism from the ground frame. No pseudo force is needed as the frame used is inertial. The forces are (figure)
(i) Mg downward
(ii) N normal to the incline (by the block)
(iii) N' upward (by the horizontal surface)
Horizontal components give,
$N \sin\alpha = Ma_0$ or $N = Ma_0 / \sin\alpha, \qquad \ldots(iii)$
Putting in (ii)

$$\frac{Ma_0}{\sin\alpha} + ma_0 \sin\alpha = mg \cos\alpha$$

or, $\quad a_0 = \frac{mg\sin\alpha\cos\alpha}{M + m\sin^2\alpha}$

From (i) $a = \frac{mg\sin\alpha\cos^2\alpha}{M+m\sin^2\alpha} + g\sin\alpha = \frac{(M+m)g\sin\alpha}{M+m\sin^2\alpha}$

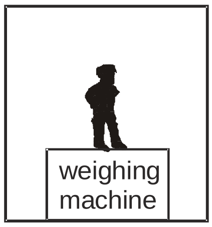
8. WEIGHING MACHING :

A weighing machine does not measure the weight but measures the force. exerted by object on its upper surface or we can say weighing machine measure normal force on the man.

8.1 Motion in a lift :

(A) If the lift is unaccelerated (v = 0 or constant)

In this case no pseudo force act on the man

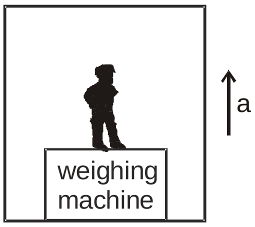


In this case the F.B.D. of the man

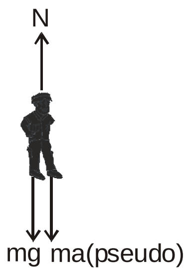
$N = mg$

In this case machine read the actual weight

(B) If the lift is accelerated upward.
(where a = constant)



F.B.D of man with respect to lift



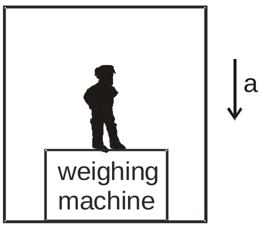
So weighing machine read

$N = m(g + a)$

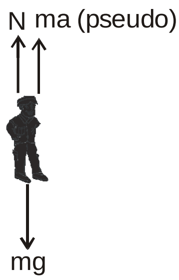
Apparent weight

N > Actual weight (mg)

(c) If the lift is accelerated down ward.



F.B.D of man with respect to lift



So weighing machine read

$N = m(g - a)$

Apparent weight

N < Actual weight (mg)

Note :

(i) If $a = g \Rightarrow N = 0$

Thus in a freely falling lift, the man will experience a state of weightlessness

(ii) If the lift is accelerated downwards such that $a > g$: So the man will be accelerated upward and will stay at the ceiling of the lift.

(iii) Apparent weight is greater than or less than actual weight only depends on the direction and magnitude of acceleration. Magnitude and direction of velocity doesn't play any roll in apparent weight.

EXERCISE-I

1. Newton's second law gives a measure of-
(A) acceleration (B) force (C) momentum (D) angular momentum
2. Newton's third law of motion leads to the law of conservation of-
(A) Angular momentum (B) Energy (C) Mass (D) Momentum
3. The newton's laws of motion are valid in-
(A) inertial frames (B) non-inertial frames (C) rotating frames (D) accelerated frames
4. The incorrect statement about newton's second law of motion is-
(A) it provides a measure of inertia (B) it provides a measure of force
(C) it relates force and acceleration (D) it relates momentum and force
5. Newton's Third law is equivalent to the-
(A) law of conservation of linear momentum (B) law of conservation of angular momentum
(C) law of conservation of energy (D) law of conservation of energy and mass
6. We can derive newton's-
(A) second and third laws from the first law (B) first and second laws from the third law
(C) third and first laws from the second law (D) all the three laws are independent of each others.
7. Ratio of intertial mass to gravitational mass is-
(A) 1 : 2 (B) 1 : 1 (C) 2 : 1 (D) no fixed number
8. A rider on horse falls back when horse starts running, all of a sudden because-
(A) rider is taken back
(B) rider is suddenly afraid of falling
(C) inertia of rest keeps the upper part of body at rest while lower part of the body moves forward with the horse
(D) none of the above
9. A man getting down a running bus, falls forward because-
(A) due to inertia of rest, road is left behind and man reaches forward
(B) due to inertia of motion upper part of body continues to be in motion in forward direction while feet come to rest as soon as they touch the road
(C) he leans forward as a matter of habit
(D) of the combined effect of all the three factors stated in (1), (2) and (3)
10. When we jump out a boat standing in water it moves-
(A) forward (B) backward (C) side ways (D) none of the above
11. On a stationary sail-boat air is blown at the sails from a fan attached to the boat, the boat will-
(A) remain stationary
(B) spin around
(C) move in a direction opposite to that in which air is blown
(D) move in the direction in which the air is blown
12. A man is at rest in the middle of a pond on perfectly smooth ice. He can get himself to the shore by making use of Newton's
(A) first law (B) second law (C) third law (D) all the laws
13. You are on a frictionless horizontal plane. How can you get off if no horizontal force is exerted by pushing against the surface-
(A) by jumping (B) by spitting or sneezing
(C) by rolling you body on the surface (D) by running on the plane

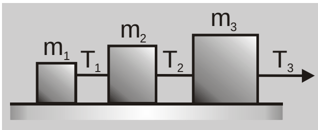
14. A boy sitting on the top most berth in the compartment of a train which is just going to stop on a railway station, drops an apple aiming at the open hand of his brother sitting vertically below his hands at a distance of about 2 m. The apple will fall-
(A) precisely on the hand of his brother
(B) slightly away from the hand of his brother in the direction of motion of the train
(C) slightly away from the hand of his brother in the direction opposite to the direction of motion of the train
(D) none of the above

15. The linear momentum P of a body varies with time and is given by the equation $P = x + yt^2$ where x and y are constants. The net force acting on the body for a one dimensional motion is proportional to-
(A) t^2 (B) a constant (C) $\frac{1}{t}$ (D) t

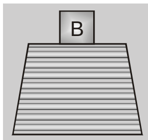
16. A body of mass 2 kg has an initial velocity of 3 m/s along OE and it is subjected to a force of 4 newtons in a direction perpendicular to OE. The distance of body from O after 4 sec. will be-
(A) 12 metres (B) 20 metres (C) 8 metres (D) 48 metres

17. A spring toy weighing 1 kg on a spring balance suddenly jumps upward. A boy standing near the toy notices that the scale of the balance reads 1.05 kg. In this process the maximum acceleration of the toy is-(g = 10m sec^{-2})
(A) 0.05 m sec^{-2} (B) 0.5 m sec^{-2} (C) 1.05 m sec^{-2} (D) 1 m sec^{-2}

18. Three block are connected as shown in fig., on a horizontal frictionless table and pulled to the right with a force T_3 = 60 N. If m_1 = 10 kg. m_2 = 20 kg. and m_3 = 30 kg. the tension T_2 is-



(A) 10 N
(B) 20 N
(C) 30 N
(D) 60 N



19. A block B is placed on a table. The force of reaction will be-
(A) downwards by the table (B) upwards by the table
(C) downwards by the block (D) upwards by the block

20. At a place where the acceleration due to gravity is 10 m sec^{-2} a force of 5 kg-wt acts on a body of mass 10 kg initially at rest. The velocity of the body after 4 second is-
(A) 5m sec^{-1} (B) 10 m sec^{-1} (C) 20 m sec^{-1} (D) 50 m sec^{-1}

21. A cricket ball of mass 150 gm is moving with a velocity of 12 m/sec. and is hit by a bat so that the ball is turned back with a velocity of 20 m/sec. The force of bat acts for 0.01 s on the ball then the average force exerted by the bat on the ball.
(A) 840 N (B) 48 N (C) 84 N (D) 480 N

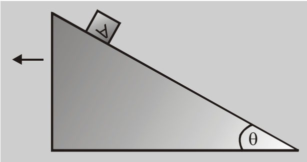
22. As an inclined plane is made slowly horizontal by reducing the value of angle θ with horizontal, the component of weight parallel to the plane of a block resting on the inclined plane-
(A) decreases (B) remains same
(C) increases (D) increases if the plane is smooth

23. A lift is ascending with an acceleration of 2 m/sec^2, what will be the apparent weight of a person of 60 kg mass in it-
(A) 720N (B) 72N (C) 48N (D) 480N

24. A man is standing on a weighing machine placed in a lift. When lift is stationary his weight is recorded as 40kg. If the lift is accelerated upwards with an acceleration of 2 m/s^2, then the weight recorded in the machine will be- (g = 10m/s^2)
(A) 32 kg (B) 40 kg (C) 42 kg (D) 48 kg

25. With what minimum acceleration can a fireman slides down a rope while breaking strength of the rope is $\frac{2}{3}$ his weight-
(A) $\frac{2}{3g}$ (B) g (C) $\frac{g}{3}$ (D) zero

26. Body A is placed on frictionless wedge making an angle θ with the horizontal. The horizontal acceleration towards left to be imparted to the wedge for the body A to freely fall vertically, is-



(A) g sin θ
(B) g cos θ
(C) g tan θ
(D) g cot θ

27. An object kept on a smooth inclined plane of 1 in ℓ can be kept stationary relative to the incline by giving a horizontal acceleration of .... to the inclined plane-

(A) $\frac{g}{\sqrt{\ell^2-1}}$ (B) $\frac{g}{\sqrt{1-\ell^2}}$ (C) $g\sqrt{\ell^2-1}$ (D) $\frac{1}{g\sqrt{1-\ell^2}}$

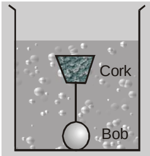
28. A block can slide on a smooth inclined plane of inclination θ, kept on the floor of a lift. When the lift is descending with a retardation a, the acceleration of the block relative to the incline is-

(A) (g + a) sin θ (B) (g – a) (C) g sin θ (D) (g – a) sin θ

29. A mass is suspended from the roof of a car by a string. While the car has a constant acceleration a, the string makes an angle of 60° with the vertical. If g = 10 m/s^2, the value of a is-

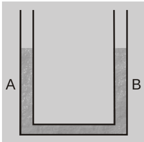
(A) $10(3)^{1/2}$ m/s² (B) $\left(\frac{10}{3}\right)^{1/2}$ m/s² (C) 5 m/s² (D) $5(3)^{1/2}$ m/s²

30. A cork and a metal bob are tied at the two ends of a string and the system is put inside a beaker full of water. The thread remains vertical as shown in the figure. If now the beaker is moved forwards the right with an acceleration a the cork will-



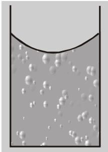
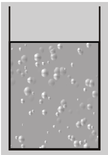
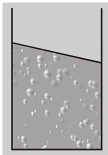
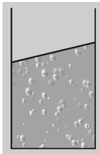
(A) remain in the previous state with string vertical
(B) will be thrown forward, string being inclined to the right
(C) will be thrown backward, string being inclined to the left
(D) the string will break up and cork will lift up the surface

31. The level of liquid in the two arms of a U-tube is same in stationary state. If the U-tube is moved to the right, with constant acceleration the level of liquid-



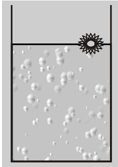
(A) in arm A will be higher as compared to B
(B) in arm A will be lower as compared to B
(C) will remain same in both the arms
(D) a gap will be created between in the liquid in the two arms

32. A beaker half-filled with water is accelerated towards the left on a straight horizontal path. The surface of the liquid inside the moving beaker is represented by the figure-

(A)  (B)  (C)  (D) 

(A) A (B) B (C) C (D) D

33. A body floats in liquid contained in a beaker. If the whole system (shown in fig.) falls under gravity then the up-thrust on the body is-



(A) 2 mg
(B) zero
(C) mg
(D) less than mg

34. The ratio of the weight of a man in a stationary lift and in a lift accelerate downwards with a uniform accleration 'a' is 3 : 2. The accleration of the lift is-

(A) $\frac{g}{3}$ (B) $\frac{g}{2}$ (C) g (D) 2g

35. A lift is moving up with an acceleration of 3.675 m/sec^2. The weight of a man-
(A) increases by 37.5% (B) decreases by 37.5% (C) increases by 137.5% (D) remains the same

36. A parrot is sitting on the floor of a closed glass cage which is in a boy's hand. If the parrot starts flying with a constant speed, the boy will feel the weight of the cage as-
(A) unchanged (B) reduced (C) increased (D) Nothing can be said

37. In the above question, if the parrot starts flying down with an accleration the boy will fell the weight of the cage as-
(A) unchanged (B) reduced (C) increased (D) Nothing can be said

38. In the above question, if the parrot starts flying upwards with an acceleration, the boy will feel the weight of cage as-
(A) unchanged (B) reduced (C) increased (D) Nothing can be said

39. The acceleration with which an object of mass 100 kg be lowered from a roof using a cord with a breaking strength of 60 kg weight without breaking the rope is-
(A) 2 m/sec^2 (B) 4 m/sec^2 (C) 6 m/sec^2 (D) 10 m/sec^2

40. A girl, of weight W, is sitting on an electric swing rotating in a vertical plane. She feels her weight to have increased by 25% as the swing goes up. What weight she would experience when the swing comes down?
(A) $\frac{3}{2W}$ (B) $\frac{5}{4W}$ (C) $\frac{3}{4W}$ (D) $\frac{W}{2}$

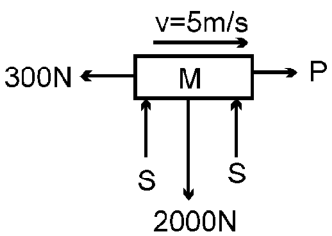
EXERCISE-II

(A) NEWTON'S LAW OF MOTION

1. A man getting down a running bus, falls forward because-
(A) due to inertia of rest, road is left behind and man reaches forward
(B) due to inertia of motion upper part of body continues to be in motion in forward direction while feet come to rest as soon as they touch the road
(C) he leans forward as a matter of habbit
(D) of the combined effect of all the three factors stated in (A), (B) and (C)

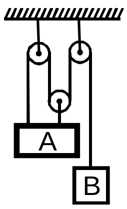
2 You are on a friction less horizontal plane. How can you get off if no horizontal force is exerted by pushing against the surface ?
(A) by jumping
(B) by spitting or sneezing
(C) by rolling your body on the surface
(D) by running on the plane

3. The forces acting on an object are shown in the fig. If the body moves horizontally at a constant speed of 5 m/s, then the values of the forces P and S are, respectively-



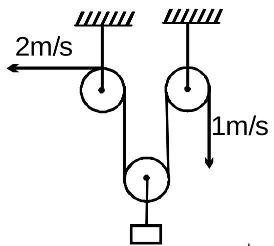
(A) 0 N, 0 N (B) 300 N, 200 N
(C) 300 N, 1000 N (D) 2000 N, 300 N

4. At a given instant, A is moving with velocity of 5 m/s upwards. What is velocity of B at the time



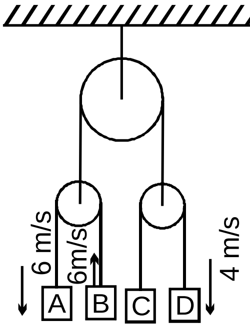
(A) 15 m/s↓ (B) 15 m/s↑ (C) 5 m/s↓ (D) 5 m/s↑

5. Find the velocity of the hanging block if the velocities of the free ends of the rope are as indicated in the figure.



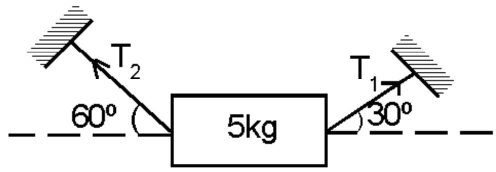
(A) 3/2 m/s↑ (B) 3/2 m/s↓
(C) 1/2 m/s↑ (D) 1/2 m/s↓

6. In the figure shown the velocity of different blocks is shown. The velocity of C is



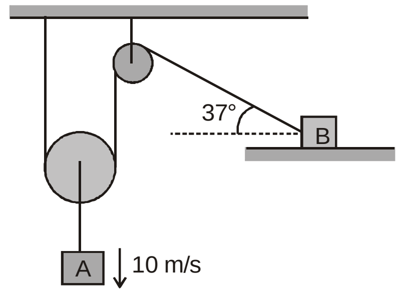
(A) 6 m/s (B) 4 m/s
(C) 0 m/s (D) none of these

7. A body of mass 5 kg is suspended by the strings making angles 60° and 30° with the horizontal -



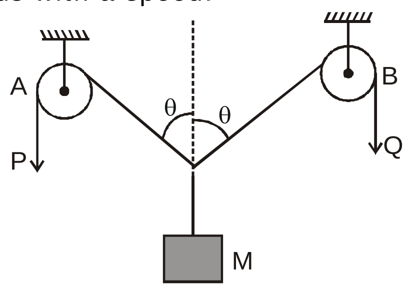
(a) T_1 = 25 N (b) T_2 = 25 N
(c) $T_1 = 25\sqrt{3}$ N (d) $T_2 = 25\sqrt{3}$ N
(A) a, b (B) a, d
(C) c, d (D) b, c

8. Find velocity of block 'B' at the instant shown in figure.



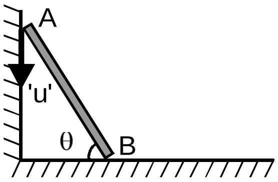
(A) 25 m/s (B) 20 m/s
(C) 22 m/s (D) 30 m/s

9. In the arrangement shown in fig. the ends P and Q of an unstretchable string move downwards with uniform speed U. Pulleys A and B are fixed. Mass M moves upwards with a speed.



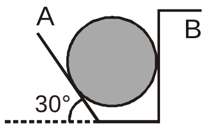
(A) 2 U cos θ (B) U cos θ
(C) $\frac{2U}{\cos\theta}$ (D) $\frac{U}{\cos\theta}$

10. The velocity of end ‘A’ of rigid rod placed between two smooth vertical walls moves with velocity ‘u’ along vertical direction. Find out the velocity of end ‘B’ of that rod, rod always remains in constant with the vertical walls.



(A) u tan 2θ (B) u cot θ
(C)u tan θ (D) 2u tan θ

11. The 50 kg homogeneous smooth sphere rests on the 30° incline A and bears against the smooth vertical wall B. Calculate the contact forces at A and B.



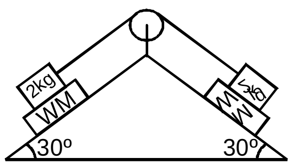
(A) $N_B = \frac{1000}{\sqrt{3}}N, \quad N_A = \frac{500}{\sqrt{3}}N$

(B) $N_A = \frac{1000}{\sqrt{3}}N, \quad N_B = \frac{500}{\sqrt{3}}N$

(C) $N_A = \frac{100}{\sqrt{3}}N, \quad N_B = \frac{500}{\sqrt{3}}N$

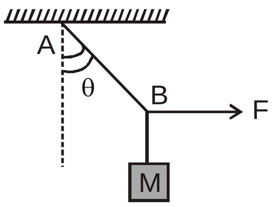
(D) $N_A = \frac{1000}{\sqrt{3}}N, \quad N_B = \frac{50}{\sqrt{3}}N$

12. Find out the reading of the weighing machine in the following cases.



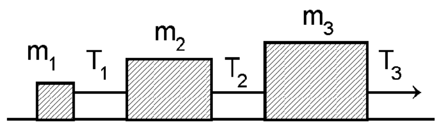
(A) $10\sqrt{3}$ (B) $10\sqrt{2}$ (C) $20\sqrt{3}$ (D) $30\sqrt{3}$

13. A mass M is suspended by a rope from a rigid support at A as shown in figure. Another rope is tied at the end B, and it is pulled horizontally with a force F. If the rope AB makes an angle θ with the vertical in equilibrium, then the tension in the string AB is :



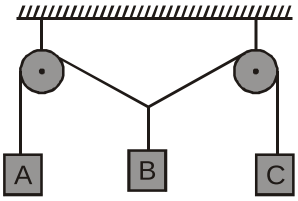
(A) F sin θ (B) F/sin θ (C) F cos θ (D) F/cos θ

14. Three block are connected as shown, on a horizontal frictionless table and pulled to the right with a force T_3 = 60 N. If m_1 = 10 kg, m_2 = 20 kg and m_3 = 30 kg, the tension T_2 is-



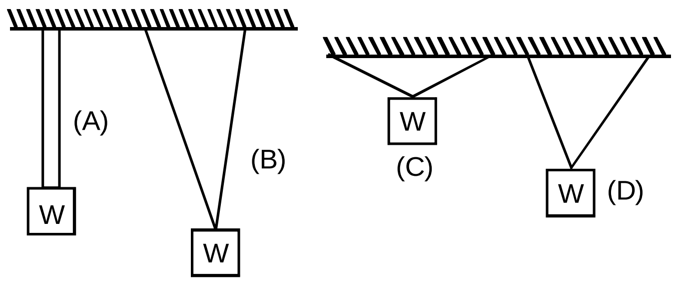
(A) 10 N (B) 20 N
(C) 30 N (D) 60 N

15. Three blocks A, B and C are suspended as shown in the figure. Mass of each blocks A and C is m. If system is in equilibrium and mass of B is M, then :



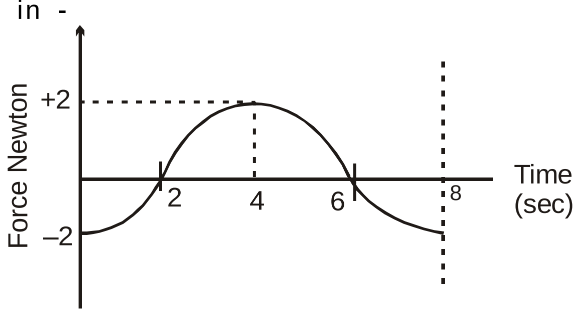
(A) M = 2m (B) M < 2 m (C) M > 2m (D) M = m

16.A weight can be hung in any of the following four ways by string of same type. In which case is the string most likely to break ?



(A) A (B) B (C) C (D) D

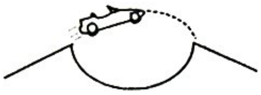
17. A force-time graph for a linear motion is shown in figure where the segments are circular. The linear momentum gained between zero and 8 seconds in -



(A) – 2π N.s (B) 0 N.s
(C) 4π N.s (D) – 6 π N.s

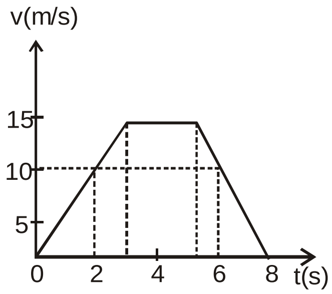
18. A particle moves in the xy plane under the action of a force F such that the value of its linear momentum (P) at any time t is, P_x = 2 cost, P_y = 2 sint. The angle θ between P and F at that time t will be -
(A) 0° (B) 30° (C) 90° (D) 180°

19.A stunt man jumps his car over a crater as shown (neglect air resistance)
(A) during the whole flight the driver experiences weightlessness
(B) during the whole flight the driver never experiences weightlessness
(C) during the whole flight the driver experiences weightlessness only at the highest point



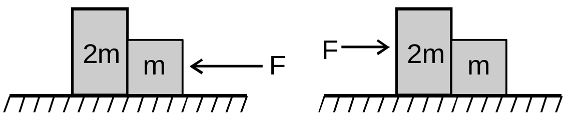
(D) the apparent weight increases during upward journey

20. A particle of mass 50 gram moves on a straight line. The variation of speed with time is shown in figure. find the force acting on the particle at t = 2, 4 and 6 seconds.



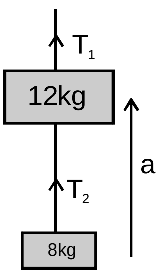
(A) 0.25 N along motion, zero, 0.25 opposite to motion
(B) 0.25 N along motion, zero, 0.25 along to motion
(C) 0.25 N opposite motion, zero, 0.25 along to motion
(D) 0.25 N opposite motion, zero, 0.25 opposite to motion

21. Two blocks are in contact on a frictionless table. One has mass m and the other 2m. A force F is applied on 2m as shown in the figure. Now the same force F is applied from the right on m. In the two cases respectively, the ratio force of contact between the two block will be :



(A) same (B) 1 : 2 (C) 2 : 1 (D) 1 : 3

22. A body of mass 8 kg is hanging another body of mass 12 kg. The combination is being pulled by a string with an acceleration of 2.2 m s^{-2}. The tension T_1 and T_2 will be respectively : (use g = 9.8 m/s^2)

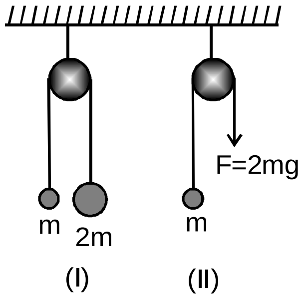


(A) 200 N, 80 N (B) 220 N, 90 N
(C) 240 N, 96 N (D) 260 N, 96 N

23. A rope of mass 5 kg is moving vertically in vertical position with an upwards force of 100 N acting at the upper end and a downwards force of 70 N acting at the lower end. The tension at midpoint of the rope is
(A) 100 N (B) 85 N (C) 75 N (D) 105 N

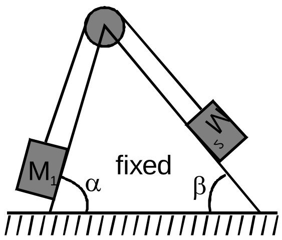
24. A particle of small mass m is joined to a very heavy body by a light string passing over a light pulley. Both bodies are free to move. The total downward force in the pulley is
(A) mg (B) 2 mg
(C) 4 mg (D) can not be determined

25. The pulley arrangements shown in figure are identical the mass of the rope being negligible. In case I, the mass m is lifted by attaching a mass 2m to the other end of the rope. In case II, the mass m is lifted by pulling the other end of the rope with cosntant downward force F = 2mg, where g is acceleration due to gravity. The acceleration of mass in case I is



(A) zero
(B) more than that in case II
(C) less than that in case II
(D) equal to that in case II

26. Two masses M_1 and M_2 are attached to the ends of a light string which passes over a massless pulley attached to the top of a double inclined smooth plane of angles of inclination α and β. The tension in the string is :



(A) $\dfrac{M_2(\sin\beta)g}{M_1+M_2}$ (B) $\dfrac{M_1(\sin\alpha)g}{M_1+M_2}$

(C) $\dfrac{M_1M_2(\sin\beta+\sin\alpha)g}{M_1+M_2}$ (D) zero

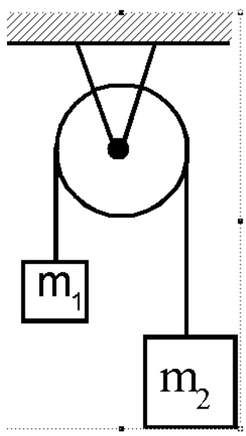
27. Two masses are hanging vertically over frictionless pulley. The acceleration of the two masses is-

(A) $\dfrac{m_1}{m_2}g$

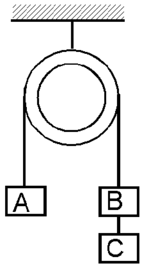
(B) $\dfrac{m_2}{m_1}g$

(C) $\left(\dfrac{m_2-m_1}{m_1+m_2}\right)g$

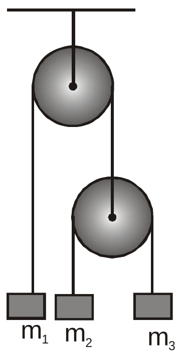
(D) $\left(\dfrac{m_1+m_2}{m_2-m_1}\right)g$



28. Three equal weights A, B, C of mass 2 kg each are hanging on a string passing over a fixed frictionless pulley as shown in the fig. The tension in the string connecting weights B and C is-
(A) zero
(B) 13 Newton
(C) 3.3 Newton
(D) 19.6 Newton



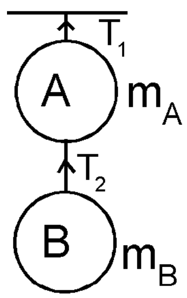
29. In the arrangement shown in figure, pulleys are massless and frictionless and threads are inextensible. The Block of mass m_1 will remain at rest, if



(A) $\frac{1}{m_1}=\frac{1}{m_2}+\frac{1}{m_3}$ (B) $m_1 = m_2 + m_3$

(C) $\frac{4}{m_1}=\frac{1}{m_2}+\frac{1}{m_3}$ (D) $\frac{1}{m_3}=\frac{2}{m_2}+\frac{3}{m_1}$

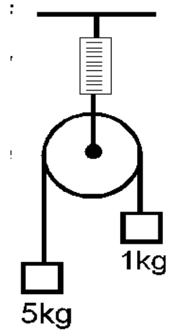
30 Two objects A and B of masses m_A and m_B are attached by strings as shown in fig. If they are given upward acceleration, then the ratio of tension $T_1 : T_2$ is -



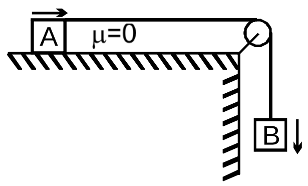
(A) $(m_A + m_B)/m_B$ (B) $(m_A + m_B)/m_A$

(C) $\frac{m_A+m_B}{m_A-m_B}$ (D) $\frac{m_A-m_B}{m_A+m_B}$

31. In the figure a smooth pulley of negligible weight is suspended by a spring balance. Weights of 1kg and 5 kg are attached to the opposite ends of a string passing over the pulley and move with acceleration because of gravity. During the motion, the spring balance reads a weight of -
(A) 6 kg
(B) less than 6 kg
(C) more than 6 kg
(D) may be more or less than 6 kg



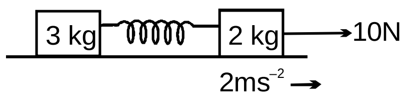
32. Both the blocks shown here are of mass m and are moving with constant velocity in direction shown in a resistive medium which exerts equal constant force on both blocks in direction opposite to the velocity. The tension in the string connecting both of them will be (Neglect friction)



(A) mg (B) mg/2 (C) mg/3 (D) mg/4

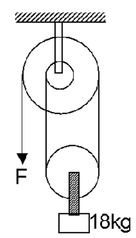
33. A monkey of mass 20 kg is holding a vertical rope. The rope can break when a mass of 25 kg is suspended from it. What is the maximum acceleration with which the monkey can climb up along the rope?
(A) 7 ms^{-2} (B) 10 ms^{-2} (C) 5 ms^{-2} (D) 2.5 ms^{-2}

34. Find the acceleration of 3 kg mass when acceleration of 2 kg mass is 2 ms^{-2} as shown in figure.



(A) 3 ms^{-2} (B) 2 ms^{-2} (C) 0.5 ms^{-2} (D) zero

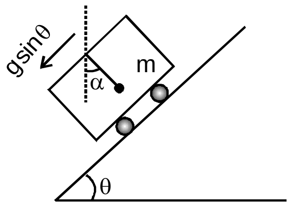
35. In the figure at the free end a force F is applied to keep the suspended mass of 18 kg at rest. The value of F is-
(A) 180 N
(B) 90 N
(C) 60 N
(D) 30 N



36. If the tension in the cable supporting an elevator is equal to the weight of the elevator, the elevator may be -
(a) going up with increasing speed
(b) going down with increasing speed
(c) going up with uniform speed
(d) going down with uniform speed
(A) a, d (B) a, b, c
(C) c, d (D) a, b

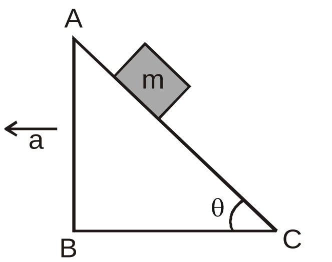
37. As an inclined plane is made slowly horizontal by reducing the value of angle θ with horizontal. The component of weight parallel to the plane of a block resting on the inclined plane-
(A) decreases
(B) remains same
(C) increases
(D) increases if the plane is smooth

38. A trolley is accelerating down an incline of angle θ with acceleration $g\sin\theta$. Which of the following is correct. (α is the constant angle made by the string with vertical)



(A) $\alpha = \theta$
(B) $\alpha = 0^\circ$
(C) Tension in the string, T = mg
(D) Tension in the string, $T = mg \sec\theta$

39. A block of mass m resting on a wedge of angle θ as shown in the figure. The wedge is given an acceleration a. What is the minimum value of a so that the mass m falls freely ?



(A) g (B) $g\cos\theta$ (C) $g\cot\theta$ (D) $g\tan\theta$

(FRICTION)

40. A block is placed on a rough floor and a horizontal force F is applied on it. The force of friction f by the floor on the block is measured for different values of F and a graph is plotted between them -
(a) The graph is a straight line of slope 45°
(b) The graph is straight line parallel to the F axis
(c) The graph is a straight line of slope 45° for small F and a straight line parallel to the F-axis for largeF.
(d) There is small kink on the graph
(A) c, d (B) a, d
(C) a, b (D) a, c

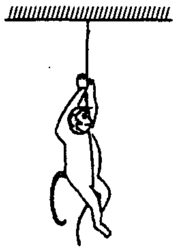
41. Mark the correct statements about the friction between two bodies -
(a) static friction is always greater than the kinetic friction
(b) coefficient of static friction is always greater than the coefficient of kinetic friction
(c) limiting friction is always greater than the kinetic friction
(d) limiting friction is never less than static friction
(A) b, c, d (B) a, b, c
(C) a, c, d (D) a, b, d

42. A block A kept on an inclined surface just begins to slide if the inclination is 30°. The block is replaced by another block B and it is found that it just begins to slide if the inclination is 40°.
(A) mass of A > mass of B
(B) mass of A < mass of B
(C) mass of A = mass of B
(D) all the three are possible

43. Two cars of unequal masses use similar tyres. If they are moving at the same initial speed, the minimum stopping distance -
(A) is smaller for the heavier car (B) is smaller for the lighter car
(C) is same for both cars
(D) depends on the volume of the car

(FRICTION)

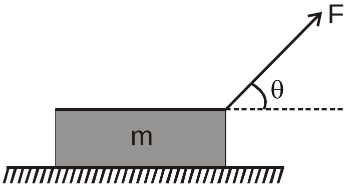
44. A monkey of mass m is climbing a rope hanging from the roof with acceleration a. The coefficient of static friction between the body of the monkey and the rope is μ. Find the direction and value of friction force on the monkey.



(A) Upward, F = m(g + a)
(B) downward, F = m(g + a)
(C) Upward, F = mg
(D) downward, F = mg

45. A body is placed on a rough inclined plane of inclination θ. As the angle θ is increased from 0° to 90° the contact force between the block and the plane
(A) remains constant
(B) first remains constant then decreases
(C) first decreases then increases
(D) first increases then decreases

46 A wooden block of mass m resting on a rough horizontal table (coefficient of friction = μ) is pulled by a force F as shown in figure. The acceleration of the block moving horizontally is :



(A) $\frac{F\cos\theta}{m}$ (B) $\frac{\mu F\sin\theta}{M}$

(C) $\frac{F}{m}(\cos\theta+\mu\sin\theta)-\mu g$ (D) none

47. A chain is lying on a rough table with a fraction 1/n of its length hanging down from the edge of the table. if it is just on the point of sliding down from the table, then the coefficient of friction between the table and the chain is -

(A) $\frac{1}{n}$ (B) $\frac{1}{(n-1)}$

(C) $\frac{1}{(n+1)}$ (D) $\frac{n-1}{(n+1)}$

48. A body of mass m moves with a velocity v on a surface whose friction coefficient is μ. If the body covers a distance s then v will be :

(A) $\sqrt{2\mu gs}$ (B) $\sqrt{\mu gs}$ (C) $\sqrt{\mu gs/2}$ (D) $\sqrt{3\mu gs}$

49. A block of mass 2kg rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.7. The frictional force on the block is-

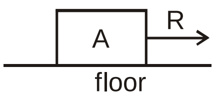
(A) 0.7 × 9.8 Newton

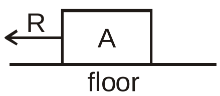
(B) 9.8 Newton

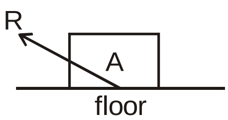
(C) 0.7 × 9.8$\sqrt{3}$ Newton

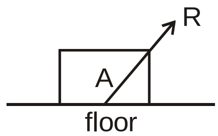
(D) 9.8 × $\sqrt{3}$ Newton

50. A box 'A' is lying on the horizontal floor of the compartment of a train running along horizontal rails from left to right. At time 't', it decelerates. Then the reaction R by the floor on the box is given best by

(A) 

(B) 

(C) 

(D) 

51. For the equilibrium of a body on an inclined plane of inclination 45°. The coefficient of static friction will be

(A) greater than one (B) less than one

(C) zero (D) less than zero

52. Starting from rest a body slides down a 45° inclined plane in twice the time it takes to slide down the same distance in the absence of friction. The co-efficient of friction between the body and the inclined plane is :

(A) 0.75 (B) 0.33 (C) 0.25 (D) 0.80

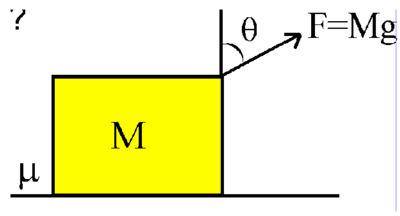
53. A block of mass M rests on a rough horizontal surface as shown. Coefficient of friction between the block and the surface is μ. A force F = Mg acting at angle θ with the vertical side of the block pulls it in which of the following cases the block can be pulled along the surface ?

(A) tan $\theta \geq \mu$

(B) tan $(\theta/2) \geq \mu$

(C) cot $\theta \geq \mu$

(D) cot $(\theta/2) \geq \mu$



54. A block moves down a smooth inclined plane of inclination θ. Its velocity on reaching the bottom is v. If it slides down a rough inclined plane of some inclination, its velocity on reaching the bottom is v/n, where n is a number greater than 0. The coefficient of friction is given by -

(A) $\mu = \tan\theta\left[1-\frac{1}{n^2}\right]$

(B) $\mu = \cot\theta\left[1-\frac{1}{n^2}\right]$

(C) $\mu = \tan\theta\left[1-\frac{1}{n^2}\right]^{1/2}$

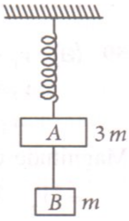
(D) $\mu = \cot\theta\left[1-\frac{1}{n^2}\right]^{1/2}$

55. A block of mass 5 kg and surface area 2 m² just begins to slide down an inclined plane when the angle of inclination is 30°. Keeping mass same, the surface area of the block is doubled. The angle at which this starts sliding down is :

(A) 30° (B) 60° (C) 15° (D) none

EXERCISE-III

1. Two blocks A and B of masses 3m and m respectively are connected by a massless and inextensible string. The whole system is suspended by a massless spring as shown in figure. The magnitudes of acceleration of A and B immediately after the string is cut, are respectively [NEET 2017]

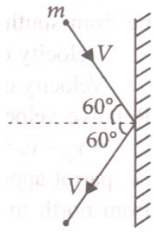


(a) $\frac{g}{3}, g$ (b) g, g (c) $\frac{g}{3}, \frac{g}{3}$ (d) $g, \frac{g}{3}$

2. One end of string of length l is connected to a particle of mass 'm' and the other end is connected to a small peg on a smooth horizontal table. If the particle moves in circle with speed 'v', the net force on the particle (directed towards centre) will be (T represents the tension in the string) [NEET 2017]

(a) $T+\frac{mv^2}{l}$ (b) $T-\frac{mv^2}{l}$ (c) Zero (d) T

3. A rigid ball of mass m strikes a rigid wall at 60° and gets reflected without loss of speed as shown in the figure. The value of impulse imparted by the wall on the ball will be [NEET-II 2016]



(a) mV (b) $2mV$ (c) $\frac{mV}{2}$ (d) $\frac{mV}{3}$

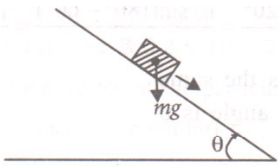
4. A car is negotiating a curved road of radius R. The road is banked at an angle θ. The coefficient of friction between the tyres of the car and the road is μ. The maximum safe velocity on this road is [NEET-I 2016]

(a) $\sqrt{\frac{g}{R}\frac{\mu_s+\tan\theta}{1-\mu_s\tan\theta}}$ (b) $\sqrt{\frac{g}{R^2}\frac{\mu_s+\tan\theta}{1-\mu_s\tan\theta}}$ (c) $\sqrt{gR^2\frac{\mu_s+\tan\theta}{1-\mu_s\tan\theta}}$ (d) $\sqrt{gR\frac{\mu_s+\tan\theta}{1-\mu_s\tan\theta}}$

5. Two stones of masses m and 2m are whirled in horizontal circles, the heavier one in a radius $\frac{r}{2}$ and the lighter one in radius r. The tangential speed of lighter stone is n times that of the value of heavier stone when they experience same centripetal forces. The value of n is [NEET 2015]

(a) 4 (b) 1 (c) 2 (d) 3

6. A plank with a box on it at one end is gradually raised about the other end. As the angle of inclination with the horizontal reaches 30°, the box starts to slip and slides 4.0 m down the plank in 4.0 s. [NEET 2015]



The coefficients of static and kinetic friction between the box and the plank will be, respectively

(a) 0.5 and 0.6 (b) 0.4 and 0.3 (c) 0.6 and 0.6 (d) 0.6 and 0.5

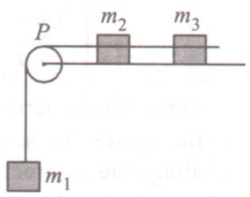
7. Three blocks A, B and C, of masses 4 kg, 2 kg and 1 kg respectively, are in contact on a frictionless surface, as shown. If a force of 14 N is applied on the 4 kg block, then the contact force between A and B is [NEET Cancelled 2015]

(a) 8 N (b) 18 N (c) 2 N (d) 6 N

8. A block A of mass m_1 rests on a horizontal table. A light string connected to it passes over a frictionless pully at the edge of table and from its other end another block B of mass m_2 is suspended. The coefficient of kinetic friction between the block and the table is μ_k. When the block A is sliding on the table, the tension in the string is [NEET Cancelled 2015]

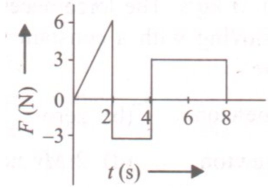
(a) $\frac{m_1 m_2(1+\mu_k)g}{(m_1+m_2)}$ (b) $\frac{m_1 m_2(1-\mu_k)g}{(m_1+m_2)}$ (c) $\frac{(m_2+\mu_k m_1)g}{(m_1+m_2)}$ (d) $\frac{(m_2-\mu_k m_1)g}{(m_1+m_2)}$

9. A system consists of three masses m_1, m_2 and m_3 connected by a string passing over a pulley P. The mass m_1 hangs freely and m_2 and m_3 are on a rough horizontal table (the coefficient of friction $= \mu$). The pulley is frictionless and of negligible mass. The downward acceleration of mass m_1 is (Assume $m_1 = m_2 = m_3 = m$) [NEET 2014]



(a) $\frac{g(1-g\mu)}{9}$ (b) $\frac{2g\mu}{3}$ (c) $\frac{g(1-2\mu)}{3}$ (d) $\frac{g(1-2\mu)}{2}$

10. The force F acting on a particle of mass m is indicated by the force-time graph shown below. The change in momentum of the particle over the time interval from zero to 8 s is [NEET 2014]

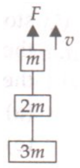


(a) 24 N s (b) 20 N s (c) 12 N s (d) 6 N s

11. A balloon with mass m is descending down with an acceleration a (where $a < g$). How much mass should be removed from it so that it starts moving up with an acceleration a? [NEET 2014]

(a) $\frac{2ma}{g+a}$ (b) $\frac{2ma}{g-a}$ (c) $\frac{ma}{g+a}$ (d) $\frac{ma}{g-a}$

12. Three blocks with masses m, 2m and 3m are connected by strings, as shown in the figure. After an upward force F is applied on block m, the masses move upward at constant speed v. What is the net force on the block of mass 2m? (g is the acceleration due to gravity) [NEET 2013]



(a) 3mg (b) 6mg (c) Zero (d) 2mg

13. An explosion breaks a rock into three parts in a horizontal plane. Two of them go off at right angles to each other. The first part of mass 1 kg moves with a speed of $12\,ms^{-1}$ and the second part of mass 2 kg moves with $8\,ms^{-1}$ speed. If the third part flies off with $4\,ms^{-1}$ speed, then its mass is [NEET 2013]

(a) 7 kg (b) 17 kg (c) 3 kg (d) 5 kg

14. The upper half of an inclined plane of inclination θ is perfectly smooth while lower half is rough. A block starting from rest at the top of the plane will again come to rest at the bottom, if the coefficient of friction between the block and lower half of the plane is given by [NEET 2013]

(a) $\mu = 2\tan\theta$ (b) $\mu = \tan\theta$ (c) $\mu = \frac{1}{\tan\theta}$ (d) $\mu = \frac{2}{\tan\theta}$

15. A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/s. A bob is suspended from the roof of the car by a light wire of length 1.0 m. The angle made by the wire with the vertical is [Karnataka NEET 2013]

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) 0°

16. A person holding a rifle (mass of person and rifle together is 100 kg) stands on a smooth surface and fires 10 shots horizontally, in 5 s. Each bullet has a mass of 10 g with a muzzle velocity of $8000 ms^{-1}$. The final velocity acquired by the person and the average force exerted on the person are [Karnataka NEET 2013]

(a) $-0.08 ms^{-1}$, 16N (b) $-0.8 ms^{-1}$, 8N (c) $-1.6 ms^{-1}$, 16N (d) $-1.6 ms^{-1}$, 8N

17. The upper half of an inclined plane with inclination f is perfectily smooth while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom if the coefficient of friction for the lower half is given by **[NEET 2013]**

(a) 2 tan ϕ (b) tan ϕ (c) 2 sin ϕ (d) 2 cos ϕ

18. A stone is dropped from a height h. It hits the ground with a certain momentum P. If the same stone is dropped from a height 100% more than the previous height, the momentum when it hits the ground will change by [NEET 2012]

(a) 68% (b) 41% (c) 200% (d) 100%

19. A person of mass 60 kg is inside a lift of mass 940 kg and presses the button on control panel. The lift starts moving upwards with an acceleration $1.0 m/s^2$. If $g = 10 ms^{-2}$, the tension in the supporting cable is [NEET 2011]

(a) 8600 N (b) 9680 N (c) 11000 N (d) 1200 N

20. A body of mass M hits normally a rigid wall with velocity V and bounces back with the same velocity. The impulse experienced by the body is [NEET 2011]

(a) MV (b) 1.5 MV (c) 2 MV (d) Zero

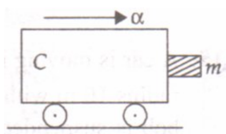
21. A conveyor belt is moving at a constant speed of $2 ms^{-1}$. A box is gently dropped on it. The coefficient of friction between them is $\mu = 0.5$. The distance that the box will move relative to belt before coming to rest on it, taking $g = 10 ms^{-2}$, is [NEET 2011]

(a) 0.4 m (b) 1.2 m (c) 0.6 m (d) Zero

22. The potential energy of a particle in a force field is U = $\frac{A}{r^2} = \frac{B}{r}$, where A and B are positive constants and r is the distance of the particle from the centre of the field. For stable equalibrium the distance of the particle is **[AIPMT 2011]**

(a) A/B (b) B/A (c) B/2A (d) 2A/B

23. A block of mass m is in contact with the cart C as shown in the figure. The coefficient of static friction between the block and the cart is μ. The acceleration α of the cart that will prevent the block from falling satisfies [NEET 2010]



(a) $\alpha > \frac{mg}{\mu}$ (b) $\alpha > \frac{g}{\mu m}$ (c) $\alpha \geq \frac{g}{\mu}$ (d) $\alpha < \frac{g}{\mu}$

24. When a horse pulls a wagon, the force that couses the horse to move forwards is the force **[AIIMS 2010]**
(a) the ground exerts on it (b) it exerts on the ground
(c) the wagon exxerts on it (d) it exerts on the wagon

25. For ordinary terrestrial experiments, the observerf in an inertial frame in the following cases is **[AIIMS 2010]**
(a) a child revolving in a giant wheel
(b) a driver in a sports car moving with a constant high speed of 200 kmh^{-1} on a straight road
(c) the pilot of an aeroplane which is taking off
(d) a cyclist negotiating a sharp curve

26. The mass of a lift is 2000 kg. When the tension in the supporting cable is 28000 N, then its acceleration is [NEET 2009]

(a) $4 ms^{-2}$ upwards (b) $4 ms^{-2}$ downwards (c) $14 ms^{-2}$ upwards (d) $30 ms^{-2}$ downwards

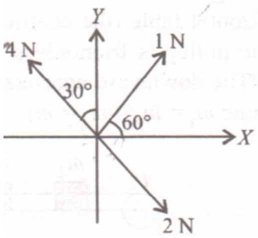
27. A body, under the action of a force $\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$, acquires an acceleration of $1\,m/s^2$. The mass of this body must be [NEET 2009]

(a) 10 kg (b) 20 kg (c) $10\sqrt{2}$ kg (d) $2\sqrt{10}$ kg

28. A roller coaster is designed such that riders experience "weightlessness" as they go round the top of a hill whose radius of curvature is 20 m. The speed of the car at the top of the hill is between [NEET 2008]

(a) 16 m/s and 17 m/s (b) 13 m/s and 14 m/s (c) 14 m/s and 15 m/s (d) 15 m/s and 16 m/s

29. Three forces acting on a body are shown in the figure. To have the resultant force only along the y- direction, the magnitude of the minimum additional force needed is [NEET 2008]

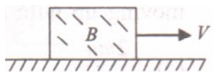


(a) $\frac{\sqrt{3}}{4}$ N (b) $\sqrt{3}$ N (c) 0.5 N (d) 1.5 N

30. Sand is being dropped on a conveyer belt at the rate of M kg/s. The force necessary to keep the beltmoving with a constant velocity of v m/s will be [NEET 2008]

(a) $\frac{Mv}{2}$ Newton (b) Zero (c) Mv newton (d) 2 Mv newton

31. A block B is pushed momentarily along a horizontal surface with an initial velocity V. If μ is the coefficient of sliding friction between B and the surface, block B will come to rest after a time [NEET 2007]

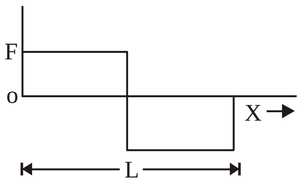


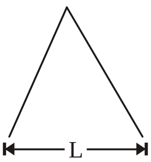
(a) $g\mu/V$ (b) g/V (c) V/g (d) $V/(g\mu)$

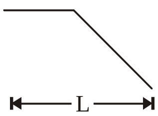
32. A 0.5 kg ball moving with a speed of 12 m/s strikes a hard wall at an angle of 30° with the wall. It is reflected with the same speed at the same angle. If the ball is in contact with the wall for 0.25 seconds, the average force acting on the wall is [NEET 2006]

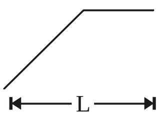
(a) 96 N (b) 48 N (c) 24 N (d) 12 N

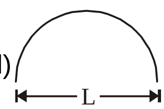
33. A person used for (F), shown in figure to move a load wiht constant velocity on a surface. Identify the correct surface profile. **[AIIMS 2006]**



(a) 

(b) 

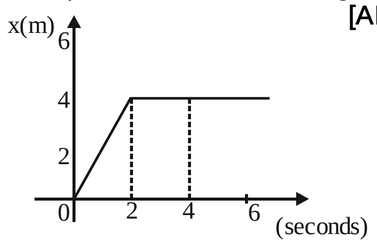
(c) 

(d) 

34. A drum of radius R and mass M rools down without slipping along an inclined palne of angle θ. The frictional force **[AIPMT 2005]**

(a) Dissipates energy as heat (b) Decreases the rotatinal motion
(c) Decreases the rotational and transltional motion (d) Converts translatinal energy to rotational energy

35. In the given figure, the position time graph of a particle of mass 0.1 kg is shown. The impulse at t = 2 sec is **[AIIMS 2005]**

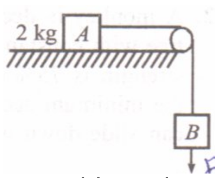


(a) 0.2 kg m sec^{-1} (b) –0.2 kg m sec^{-1} (c) 0.1 kg m sec^{-1} (d) –0.4 kg m sec^{-1}

36. A block of mass m is placed on a smooth wedge of inclination θ. The whole system is accelerated horizontally so that the block does not slip on the wedge. The force exerted by the wedge on the block will be (g is acceleration due to gravity) [NEET 2004]

(a) $mg\cos\theta$ (b) $mg\sin\theta$ (c) mg (d) $mg/\cos\theta$

37. The coefficient of static friction, μ_s, between block A of mass 2 kg and the table as shown in the figure is 0.2. What would be the maximum mass value of block B so that the two blocks do not move? The string and the pulley are assumed to be smooth and massless. $(g = 10 m/s^2)$ [NEET 2004]



(a) 2.0 kg (b) 4.0 kg (c) 0.2 kg (d) 0.4 kg

38. A man weighs 80 kg. He stands on a weighing scale in a lift which is moving upwards with a uniformacceleration of $5\ m/s^2$. What would be the reading on the scale? $(g = 10\ m/s^2)$ [NEET 2003]

(a) Zero (b) 400 N (c) 800 N (d) 1200 N

39. A monkey of mass 20 kg is holding a vertical rope. The rope will not break when a mass of 25 kg is suspended from it but will break if the mass exceeds 25 kg. What is the maximum acceleration with which the monkey can climb up along the rope? $(g = 10\ m/s^2)$ [NEET 2003]

(a) $5\ m/s^2$ (b) $10\ m/s^2$ (c) $25\ m/s^2$ (d) $2.5\ m/s^2$

40. The vector sum of two forces is perpendicular to their vector differences. In that case, the force **[CBSE PMT 2003]**

(a) Are equal to each other in magnitude (b) Are not equal to each other in magnitude

(c) Cannot be predicted (d) Are equal to each other

41. A lift of mass 1000 kg which is moving with acceleration of $1\ m/s^2$ in upward direction, then the tensiondeveloped in string which is connected to lift is [NEET 2002]

(a) 9800 N (b) 10,800 N (c) 11,000 N (d) 10,000 N

42. A block of mass 10 kg placed on rough horizontal surface having coefficient of friction m = 0.5, if ahorizontal force of 100 N acting on it then acceleration of the block will be [NEET 2002]

(a) $10\ m/s^2$ (b) $5\ m/s^2$ (c) $15\ m/s^2$ (d) $0.5\ m/s^2$

43. A force $\vec{F} = 6t^2\,\hat{i} + 4t\,\hat{j}$, is acting on a particle of mass 3 kg then what will be velocity of particle at t = 3 sec. if at t = 0, particle is at rest **[AIPMT-2002]**

(a) $18\hat{i} + 6\hat{j}$ (b) $18\hat{i} + 12\hat{j}$ (c) $12\hat{i} + 6\hat{j}$ (d) None

44. 250 N force is required to raise 75 kg mass from a pulley. If rope is pulled 12 m then the load is lifted to 3 m, the efficiency of pulley system will be [NEET 2001]

(a) 25% (b) 33.3% (c) 75% (d) 90%

45. On the horizontal surface of a truck a block of mass 1 kg is placed (m = 0.6) and truck is moving with acceleration $5\ m/sec^2$ then the frictional force on the block will be [NEET 2001]

(a) 5 N (b) 6 N (c) 5.88 N (d) 8 N

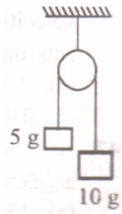
46. A cricketer catches a ball of mass 150 gm in 0.1 sec moving with speed 20 m/s, then he experiences force of [NEET 2001]
(a) 300 N (b) 30 N (c) 3 N (d) 0.3 N

47. A 1 kg stationary bomb is exploded in three parts having mass 1 : 1 : 3 respectively. Parts having same mass move in perpendicular direction with velocity 30 m/s, then the velocity of bigger part will be [NEET 2001]
(a) $10\sqrt{2}$ m/sec (b) $\frac{10}{\sqrt{2}}$ m/sec (c) $15\sqrt{2}$ m/sec (d) $\frac{15}{\sqrt{2}}$ m/sec

48. A body of mass 3 kg hits a wall at an angle of 60° and returns at the same angle. The impact time was 0.2 sec. The force exerted on the wall [NEET 2000]

(a) $150\sqrt{3}$ N (b) $50\sqrt{3}$ N (c) 100 N (d) $75\sqrt{3}$ N

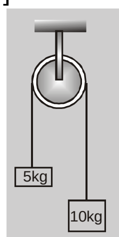
49. Two masses as shown in the figure are suspended from a massless pulley. The acceleration of the system when masses are left free is [NEET 2000]



(a) $\frac{2g}{3}$ (b) $\frac{g}{3}$ (c) $\frac{g}{9}$ (d) $\frac{g}{7}$

50. A man slips on a frictionless inclined plane leaving a bag from the same height to the ground. If the velocities of the man and bag are v_M and v_B respectively on reaching the ground, then **[AIPMT 2000]**
(a) $v_B > v_M$ (b) $v_B < v_M$ (c) $v_B = v_M$ (d) v_B and v_M can't be related

51. Two masses of 5 kg and 10 kg are connected to a pulley as shown. What will be the acceleration if the pulley is set free? [g = acceleration due to gravity] **[AIPMT 2000]**



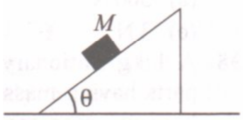
(a) g (b) $\frac{g}{2}$ (c) $\frac{g}{3}$ (d) $\frac{g}{4}$

52. If the force on a rocket, moving with a velocity of 300 m/s is 210 N, then the rate of combustion of the fuel is [NEET 1999]
(a) 0.07 kg/s (b) 1.4 kg/s (c) 0.7 kg/s (d) 10.7 kg/s

53. A mass of 1 kg is suspended by a thread. It is (i) lifted up with an acceleration $4.9\ m/s^2$, (ii) lowered with an acceleration $4.9\ m/s^2$. The ratio of the tensions is [NEET 1998]
(a) 1 : 3 (b) 1 : 2 (c) 3 : 1 (d) 2 : 1

54. A bullet is fired from a gun. The force on the bullet is given by $F = 600 - 2 \times 10^5 t$ where, F is in newton and t in seconds. The force on the bullet becomes zero as soon as it leaves the barrel. What is the average impulse imparted to the bullet? [NEET 1998]
(a) 9 N-s (b) Zero (c) 1.8 N-s (d) 0.9 N-s

55. A mass M is placed on a very smooth wedge resting on a surface without friction. Once the mass is released, the acceleration to be that M remains at rest is a where [NEET 1998]



(a) a is applied to the left and $a = g\tan\theta$ (b) a is applied to the right and $a = g\tan\theta$
(c) a is applied to the left and $a = g\sin\theta$ (d) a is applied to the left and $a = g\cos\theta$

56. A 5000 kg rocket is set for vertical firing. The exhaust speed is $800\ ms^{-1}$. To give an initial upward acceleration of $20\ ms^{-2}$, the amount of gas ejected per second to supply the needed thrust will be ($g = 10\ ms^{-2}$) [NEET 1998]
(a) $185.5\ kg\ s^{-1}$ (b) $187.5\ kg\ s^{-1}$ (c) $127.5\ kg\ s^{-1}$ (d) $137.5\ kg\ s^{-1}$

57. A force of 6 N acts on a body at rest and of mass 1 kg. During this time, the body attains a velocity of 30 m/s. The time for which the force acts on the body is [NEET 1997]
(a) 7 seconds (b) 5 seconds (c) 10 seconds (d) 8 seconds

58. A 10 N force is applied on a body produce in it an acceleration of $1\ m/s^2$. The mass of the body is [NEET 1996]
(a) 15 kg (b) 20 kg (c) 10 kg (d) 5 kg

59. A force vector applied on a mass is represented as $\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$ and accelerates with $1\ m/s^2$. What will be the mass of the body? [NEET 1996]
(a) 10 kg (b)20 kg (c) $10\sqrt{2}$ kg (d) $2\sqrt{10}$ kg

60. A man fires a bullet of mass 200 gm at a speed of 5 m/s. The gun is of one kg mass. By what velocity the gun rebounds backward? [NEET 1996]
(a) 1 m/s (b) 0.01 m/s (c) 0.1 m/s (d) 10 m/s

61. In a rocket, fuel burns at the rate of 1 kg/s. This fuel is ejected from the rocket with a velocity of 60 km/s. This exerts a force on the rocket equal to [NEET 1994]
(a) 6000 N (b) 60000 N (c) 60 N (d) 600 N

62. A block has been placed on a inclined plane with the slope angle θ, block slides down the plane at constant speed. The coefficient of kinetic friction is equal to [NEET 1993]
(a) $\sin\theta$ (b) $\cos\theta$ (c) g (d) $\tan\theta$

63. A monkey is descending from the branch of a tree with constant acceleration. If the breaking strength is 75% of the weight of the monkey, the minimum acceleration with which monkey can slide down without branch is [NEET 1993]
(a) g (b) $\frac{3g}{4}$ (c) $\frac{g}{4}$ (d) $\frac{g}{2}$

64. Consider a car moving along a straight horizontal road with a speed of 72 km/h. If the coefficient of static friction between the tyres and the road is 0.5, the shortest distance in which the car can be stopped is (taking $g = 10\ m/s^2$) [NEET 1992]
(a) 30 m (b) 40 m (c) 72 m (d) 20 m

65. Physical independence of force is a consequence of [NEET 1991]
(a) Third law of motion (b) Second law of motion
(c) First law of motion (d) All of these laws

66. A heavy uniform chain lies on horizontal table top. If the coefficient of friction between the chain and the table surface is 0.25, then the maximum fraction of the length of the chain that can hang over one edge of the table is [NEET 1991]
(a) 20% (b) 25% (c) 35% (d) 15%

67. When milk is churned, cream gets separated due to [NEET 1991]
(a) Centripetal force (b) Centrifugal force (c) Frictional force (d) Gravitational force

68. A particle of mass m is moving with a uniform velocity v_1. It is given an impulse such that its velocity becomes v_2. The impulse is equal [NEET 1990]

(a) $m[|v_2| - |v_1|]$ (b) $\frac{1}{2}m[v_2^2 - v_1^2]$ (c) $m[v_1 + v_2]$ (d) $m[v_2 - v_1]$

69. A 600 kg rocket is set for a vertical fling. If the exhaust speed is $1000\ ms^{-1}$, the mass of the gas ejected per second to supply the thrust needed to overcome the weight of rocket is [NEET 1990]

(a) $117.6\ kg\ s^{-1}$ (b) $58.6\ kg\ s^{-1}$ (c) $6\ kg\ s^{-1}$ (d) $76.4\ kg\ s^{-1}$

70. A body of mass 5 kg explodes at rest into three fragments with masses in the ratio 1 : 1 : 3. The fragments with equal masses fly in mutually perpendicular directions with speeds of 21 m/s. The velocity of heaviest fragment in m/s will be [NEET 1989]

(a) $7\sqrt{2}$ (b) $5\sqrt{2}$ (c) $3\sqrt{2}$ (d) $\sqrt{2}$

71. Starting from rest, a body slides down a 45° inclined plane in twice the time it takes to slide down the same distance in the absence of friction. The coefficient of friction between the body and the inclined plane is [NEET 1988]

(a) 0.80 (b) 0.75 (c) 0.25 (d) 0.33

EXERCISE-I

1	(B)	**2**	(D)	**3**	(A)	**4**	(A)	**5**	(A)	**6**	(C)	**7**	(B)	**8**	(C)	**9**	(B)	**10**	(B)
11	(A)	**12**	(C)	**13**	(B)	**14**	(B)	**15**	(D)	**16**	(B)	**17**	(B)	**18**	(C)	**19**	(B)	**20**	(C)
21	(D)	**22**	(A)	**23**	(A)	**24**	(D)	**25**	(C)	**26**	(D)	**27**	(A)	**28**	(A)	**29**	(A)	**30**	(B)
31	(A)	**32**	(D)	**33**	(B)	**34**	(A)	**35**	(A)	**36**	(A)	**37**	(B)	**38**	(C)	**39**	(B)	**40**	(C)

EXERCISE-II

NEWTONS'S LAW OF MOTION

1.	B	2.	B	3.	C	4.	A	5.	A	6.	B	7.	B
8.	A	9.	D	10.	C	11.	B	12.	A	13.	B	14.	C
15.	B	16.	C	17.	B	18.	C	19.	A	20.	A	21.	B
22.	C	23.	B	24.	C	25.	C	26.	C	27.	C	28.	B
29.	C	30.	A	31.	B	32.	B	33.	D	34.	B	35.	B
36.	C	37.	A	38.	A	39.	C	40.	A	41.	A	42.	D
43.	C	44.	A	45.	B	46.	C	47.	B	48.	A	49.	B
50.	C	51.	A	52.	A	53.	D	54.	A	55.	A		

EXERCISE-III

1	(A)	**2**	(D)	**3**	(A)	**4**	(D)	**5**	(C)	**6**	(D)	**7**	(D)	**8**	(A)	**9**	(C)	**10**	(C)
11	(A)	**12**	(C)	**13**	(D)	**14**	(A)	**15**	(C)	**16**	(B)	**17**	(A)	**18**	(B)	**19**	(C)	**20**	(C)
21	(A)	**22**	(D)	**23**	(C)	**24**	(A)	**25**	(D)	**26**	(A)	**27**	(C)	**28**	(C)	**29**	(C)	**30**	(C)
31	(D)	**32**	(C)	**33**	(D)	**34**	(D)	**35**	(A)	**36**	(D)	**37**	(D)	**38**	(D)	**39**	(D)	**40**	(A)
41	(B)	**42**	(B)	**43**	(A)	**44**	(C)	**45**	(A)	**46**	(B)	**47**	(A)	**48**	(A)	**49**	(B)	**50**	(C)
51	(C)	**52**	(C)	**53**	(C)	**54**	(D)	**55**	(A)	**56**	(B)	**57**	(B)	**58**	(C)	**59**	(C)	**60**	(A)
61	(B)	**62**	(D)	**63**	(C)	**64**	(B)	**65**	(C)	**66**	(A)	**67**	(B)	**68**	(D)	**69**	(C)	**70**	(A)
71	(B)																		