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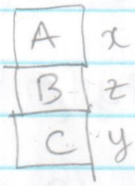
CSC384 Summer 2020 Take Home

1 a) $M_1 = \{A, B, C\}$



$above^{M_1} = \{\langle B, C\rangle, \langle A, C\rangle, \langle A, B\rangle\}$

$under^{M_1} = \{\langle C, B\rangle, \langle C, A\rangle, \langle B, A\rangle\}$

$clear^{M_1} = \{ A \}$

$ontable^{M_1} = \{ C \}$

consider $under^{M_1}(C, A)$ where C is under A, A above C or better put C is under A but A is not immediately above C, also A is above B and C and $A \neq C$ hence under(C,A) would equate false but in reality is true.

1 b) Add to existing set following Sentences

$\Rightarrow \forall x \exists y\, (Clear(x) \lor above(y, x))$

$\Rightarrow \forall x \exists y\, (ontable(x) \lor under(y, x))$

$\hookrightarrow \forall y \forall x \forall z\, ((\neg under(z, x) \land \neg above(z, y)) \land (x \,!= z\, != y))$
$\lor (ontable(x) \land (y = x)) \rightarrow under(y, x)$

No element below/under x and no element above y hence y must be directly under it

#2 Let M_1 be a structure such that

$M_1 = \{A, B\}$
$P_1^{M_1} = \{\langle A, A\rangle, \langle A, B\rangle\}$
$P_2^{M_1} = \{B\}$
$P_3^{M_1} = \{A\}$

$$\forall x[\underbrace{(\forall y(P_1(x,y) \to (P_2(y) \vee P_3(y))))}_{\text{Part 1}} \to ((\underbrace{\forall z(P_1(x,z) \to P_2(z)))}_{\text{Part 2}} \vee \underbrace{(\forall w(P_1(x,w) \to P_3(w))))}_{\text{part3}})]$$

if $x = A$ and $y = A / B$
then $P_1(x,y)$ holds
and if $y = A$ then P_3 holds and if $y = B$ then P_2 holds
hence Part 1 holds. the first part of the outer implication holds and hence, the latter must hold for tautology.

Part 2. $x = A$ and $z = A$
then part 2 does not hold since $P_1(A,A)$ holds but $P_2(A)$ does not.

Part 3. $x = A$ and $w = B$
then Part 3 also does not hold since $P_1(A,B)$ holds but $P_3(B)$ does not hold hence implication fails.

Since Part 2 AND Part 3 return false $\exists$ a structure, M_1 which disproves that this formula is a tautology.

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a) Φ is not satisfiable due to the fact if $x=A$, $y=A$, and $z=A$ or $\underline{x=y=z}$ generally then (4) results in True but (5) states $\forall x \forall y \forall z\,(between(A,A,A) \Rightarrow \neg between(A,A$ which is never True hence <u>not</u> satisfiable

<u>2 sets of sentences</u>

Φ_1 - $\forall x \forall y \forall z\,(between(x,y,z) \rightarrow between(z,y,x))$
- $\forall x \forall y \forall z \forall w\,(between(y,x,z) \rightarrow between(y,x,w) \vee between(z,x$

Φ_2 - $\forall x \forall y \forall z\,((between(x,y,z) \wedge between(y,x,z)) \rightarrow (x=y))$
- $\forall x \forall y \forall z\,((between(y,x,z) \vee between(z,y,x) \vee between(x,z,y))$

4 Convert to Clausal form.

(1) $\forall a_1 \forall a_2 \forall o \exists s (\neg(\text{permuted}(a_2) \wedge \text{subactivity}(a_1, a_2) \wedge \neg \text{occurrence\_of}(o$
$) \vee \text{occurrence\_of}(s, a_1))$

$\forall a_1 \forall a_2 \forall o \exists s ((\neg \text{permuted}(a_2) \vee \neg \text{subactivity}(a_1, a_2) \vee \neg \text{occurrence\_of}(o$
$) \vee \text{occurrence\_of}(s, a_1))$

$\forall a_1 \forall a_2 \forall o ((\neg \text{permuted}(a_2) \vee \neg \text{subactivicty}(a_1, a_2) \vee \neg \text{occurrence\_of}(o, a_2)$
$\vee \text{occurrence\_of}(g_1(a_1, a_2, o), a_1))$

$\forall a_1 \forall a_2 \forall o (\neg \text{permuted}(a_2) \vee \neg \text{subactivity}(a_1, a_2) \vee \neg \text{occurrence\_of}(o, a_2)$
$\vee \text{occurrence\_of}(g_1(a_1, a_2, o), a_1))$

Clause 1 $\neg \text{permuted}(a_2) \vee \neg \text{subactivity}(a_1 a_2) \vee \neg \text{occurrence\_of}(o, a_2)$
$\vee \text{occurrence\_of}(g_1(a_1, a_2, o), a_1)$

(2) $\forall a_1 \forall a_2 \forall a_3 (\neg(\text{subactivity}(a_1, a_2) \wedge \text{subactivity}(a_2, a_3)) \vee$
$\text{subactivity}(a_1, a_3))$

$\forall a_1 \forall a_2 \forall a_3 ((\neg \text{subactivity}(a_1, a_2) \vee \neg \text{subactivity}(a_2, a_3)) \vee$
$\text{subactivity}(a_1, a_3))$

$\forall a_1 \forall a_2 \forall a_3 (\neg \text{subactivity}(a_1, a_2) \vee \neg \text{subactivity}(a_2, a_3) \vee \text{subactivity}(a_1, a_3$

Clause 2 $\neg \text{subactivity}(a_1, a_2) \vee \neg \text{subactivity}(a_2, a_3) \vee \text{subactivity}(a_1, a_3)$

(3) $\forall o \forall f \exists s (\neg(\text{falsifies}(o, f)) \vee \text{occurrence\_of}(o, s))$
$\forall o \forall f \exists s (\neg \text{falsifies}(o, f) \vee \text{occurrence\_of}(o, s))$
$\forall o \forall f (\neg \text{falsifies}(o, f) \vee \text{occurrence\_of}(o, g_2(o, f)))$

Clause 3 $\neg \text{falsifies}(o, f) \vee \text{occurrence\_of}(o, g_2(o, f))$

(4) $\forall o \forall f\,(falsifies(o,f) \rightarrow State(f))$

$\forall o \forall f\,(\neg falsifies(o,f) \vee State(f))$

Clause (4) $\neg falsifies(o,f) \vee State(f)$

(5) $\forall o\,(\neg occurrence\_of(o,A_1) \vee falsifies(o,F_1))$

Clause (5) $\neg occurrence\_of(o,A_1) \vee falsifies(o,F_1)$

(6) $\forall o\,(\neg occurrence\_of(o,A_3) \vee falsifies(o,F_1))$

Clause (6) $\neg occurrence\_of(o,A_3) \vee falsifies(o,F_1)$

(7) $\forall s\,(\neg falsifies(s,F_2) \vee occurrence\_of(s,A_4))$

Clause (7) $\neg falsifies(s,F_2) \vee occurrence\_of(s,A_4)$

(8) $\forall o\,(\neg occurrence\_of(o,A_4) \vee falsifies(o,F_2))$

Clause (8) $\neg occurrence\_of(o,A_4) \vee falsifies(o,F_2)$

Clauses $(9 \rightarrow 15) = (9) \rightarrow (15)$ from Kb

Negation of Query

$\neg(\forall o_1\,(occurrence\_of(o_1,B) \rightarrow (\exists o_2\, falsifies(o_2,F_2))))$

$\exists o_1 \neg[\neg occurrence\_of(o_1,B) \vee (\exists o_2\, falsifies(o_2,F2))]$

$\exists o_1\,(occurrence\_of(o_1,B) \wedge (\forall o_2\, \neg falsifies(o_2,F_2)))$

$occurrence\_of(C,B) \wedge \neg falsifies(o_2,F_2)$

constant

Clause (16) $occurrence\_of(C,B)$

Clause (17) $\neg falsifies(o_2,F2)$

Resolution Proof from clauses

1. (¬permuted(a_2), ¬subactivity(a_1, a_2), ¬occurrence_of(o, a_2),
occurrence_of(g(a_1, a_2, o), a_1))

2. (¬subactivity(a_1, a_2), ¬subactivity(a_2, a_3), subactivity(a_1, a_3))

3. (¬falsifies(o, f), occurrence_of(o, g_2(o, f)))

4. (¬falsifies(o, f), state(f))

5. (¬occurrence_of(o, A), falsifies(o, F1))

6. (¬occurence_of(o, A_3), falsifies(o, F1))

7. (¬falsifies(s, F_2), occurrence_of(s, A4))

8. (¬occurence_of(O2, A4), falsifies(O2, F2))

9. occurrence_of(Occ, A)

10. subactivity(A1, A)

11. subactivity(A1, A_2)

12. subactivity(A_3, A)

13. subactivity(A4, B)

14. permuted(A)

15. permuted(B)

Query {
16. occurrence_of(C, B)
17. ¬falsifies(O2, F2)

18. R[8b, 17]{o = O2} (¬occurrence_of(O2, A4))

19. R[1d, 18]{g(a, a_2, o) = O_2, a_1 = A4} (¬permuted(a_2),
¬subactivity(A4, a_2), ¬occurrence_of(o, a_2))

20. R[19a, 15]{a_2 = B}(¬subactivity(A4, B), ¬occurrence_of(o,

21. R[20b, 16]{o = C} (¬subactivity(A4, B))

22. R[21, 13] ( )