Computer Project

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1 Introduction

The governing equation for general 2-D steady state heat conduction process is given by

GDE :

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{1}$$

The problem is to solve numerically for the temperature distribution in the interior of the given system with specified Boundary Conditions as follows

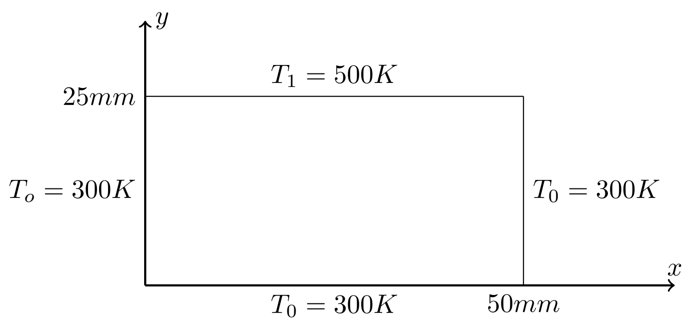


Figure 1: Rectangular slab with Dirichlet Boundary Condition.

As per given by the problem statement the Boundary Condition of the system is given as follows:

$$T(0, y) = 300K$$
$$T(x, 0) = 300K$$
$$T(5, y) = 300K$$
$$T(x, 2.5) = 500K$$

2 Analytical Solution

The analytical solution to the problem specified can be given as follows:

$$\frac{T(x,y) - T_o}{T_1 - T_o} = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1-(-1)^n}{n} \sin \frac{n\pi x}{L} \frac{\sinh \frac{n\pi y}{L}}{\sinh \frac{n\pi W}{L}} \tag{2}$$

3 Numerical Solution

For numerical analysis of the problem provided we first find the FDE for the given GDE using "central difference approximation" in both x and y.

The discretized equation is as follows:

$$\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{(\Delta x)^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{(\Delta y)^2} = 0 \tag{3}$$

Now substituting the value

$$\lambda = \frac{\Delta x}{\Delta y}$$

we get the following equation

$$T_{i+1,j} - 2(1 + \lambda^2)T_{i,j} + T_{i-1,j} + \lambda^2(T_{i,j+1} + T_{i,j-1}) = 0 \tag{4}$$

again making a substitution like

$$\gamma = -2(1 + \lambda^2)$$

we get the equation as follows:

$$T_{i+1,j} + \gamma T_{i,j} + T_{i-1,j} + \lambda^2(T_{i,j+1} + T_{i,j-1}) = 0 \tag{5}$$

Solving this equation numerically by using "Line by Line Gauss Seidel" and solving the equation using iterations and TDMA.

For iterations eq. (6) can be visualized as follows:

$$T^{k+1}_{i+1,j} + \gamma T^{k+1}_{i,j} + T^{k+1}_{i-1,j} = -\lambda^2(T^{k}_{i,j+1} + T^{k+1}_{i,j-1}) \tag{6}$$

The above equation is for the computation of the temperature distribution from bottom to top, we can also go from top to bottom using the formula:

$$T^{k+1}_{i+1,j} + \gamma T^{k+1}_{i,j} + T^{k+1}_{i-1,j} = -\lambda^2(T^{k+1}_{i,j+1} + T^{k}_{i,j-1}) \tag{7}$$

where k and k+1 denotes the iteration number.

The above two equations eq.(7) and eq(8) will generate TDM and we can solve it using TDMA line by line and then iteratively converge the numerical solution using the line by line gauss seidel method.

As given in the problem statement we will discretize the space into 60*40 grid points.

For the given discretization we have

$$\lambda^2 = 1.7477$$

$$\gamma = -5.4954$$

We will be using eq.(8) for solving the system numerically in our code.

For a point interior we have Eq.(8) to be used but for a point involving boundary we will have following results:

1. for a leftmost interior point

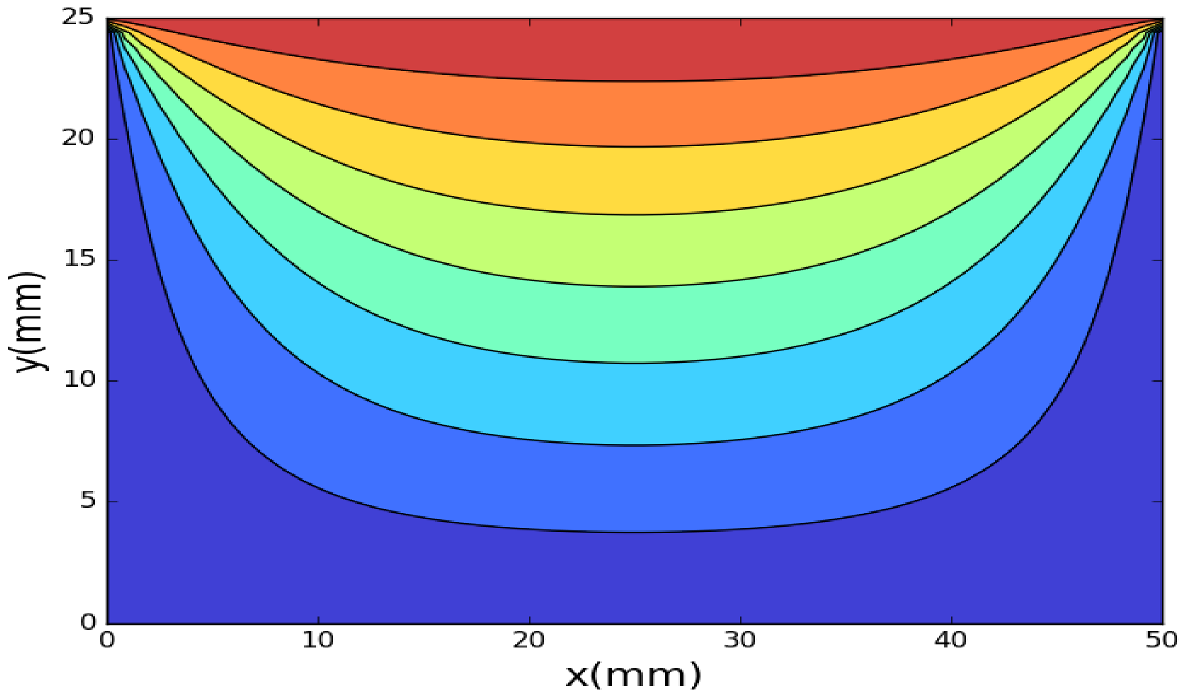
$$T^{k+1}_{i+1,j} + \gamma T^{k+1}_{i,j} = T^{k+1}_{i-1,j} - \lambda^2(T^{k+1}_{i,j+1} + T^{k}_{i,j-1}) \tag{8}$$

2. for a rightmost interior point

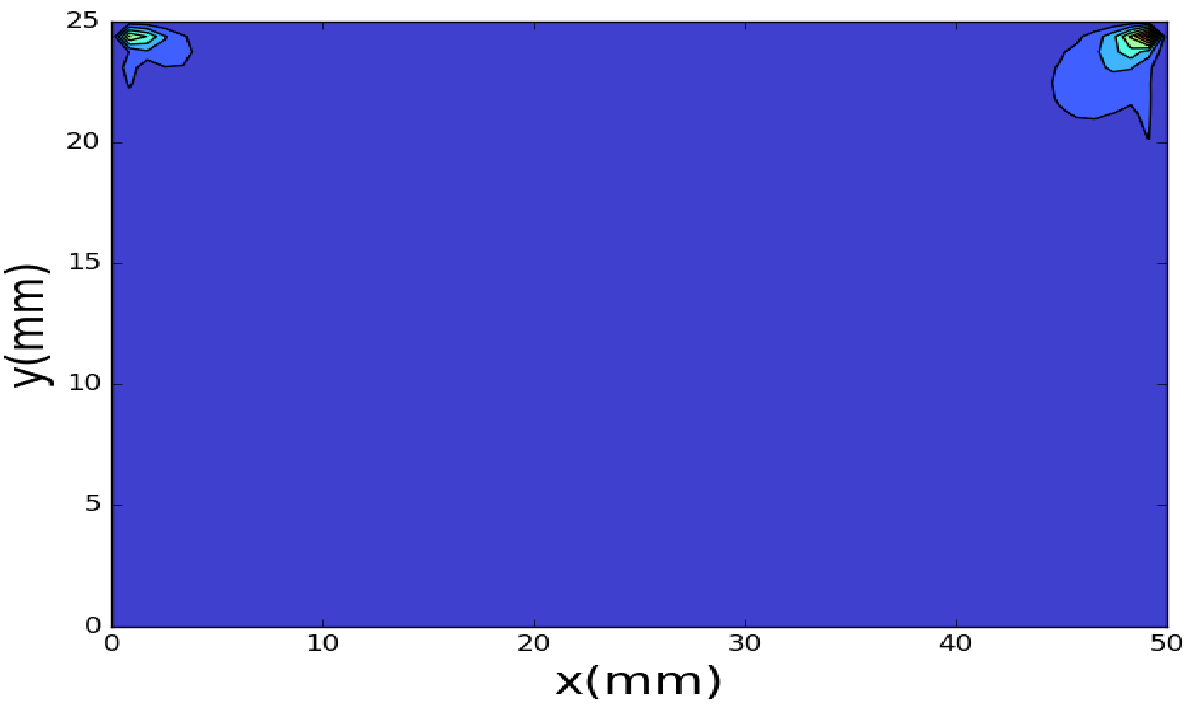
$$T^{k+1}_{i-1,j} + \gamma T^{k+1}_{i,j} = T^{k+1}_{i+1,j} - \lambda^2(T^{k+1}_{i,j+1} + T^{k}_{i,j-1}) \tag{9}$$

4 Results and Discussion

1. Isotherms or Temperature contours

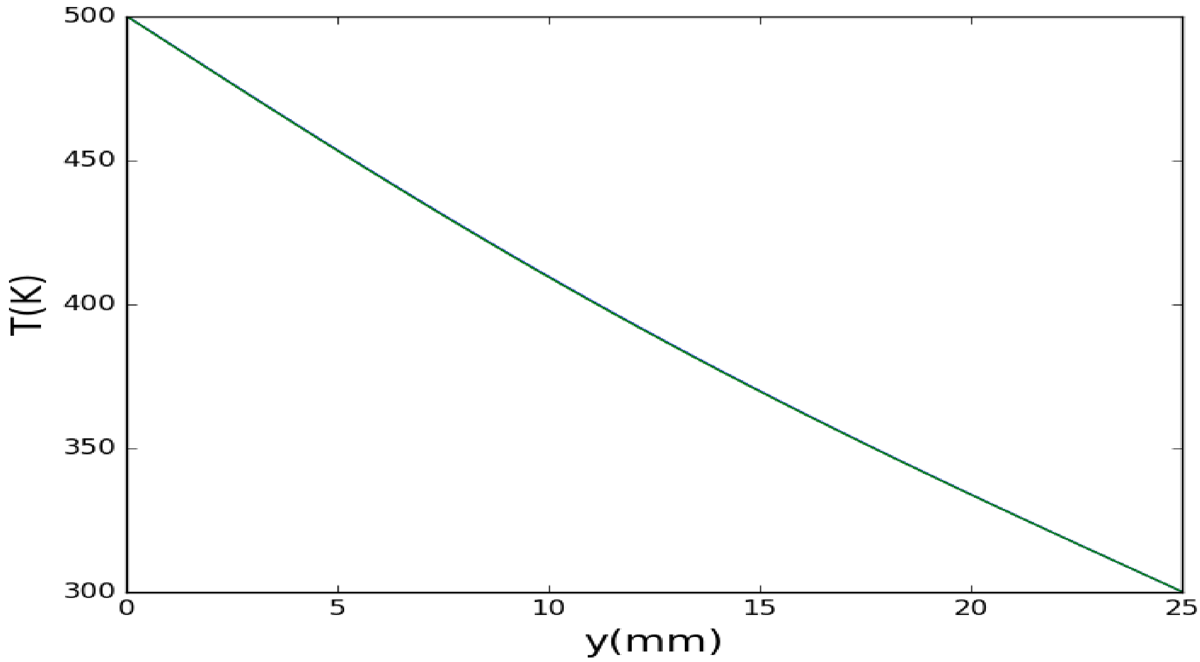


2. Error contours for Analytical and Numerical Solution



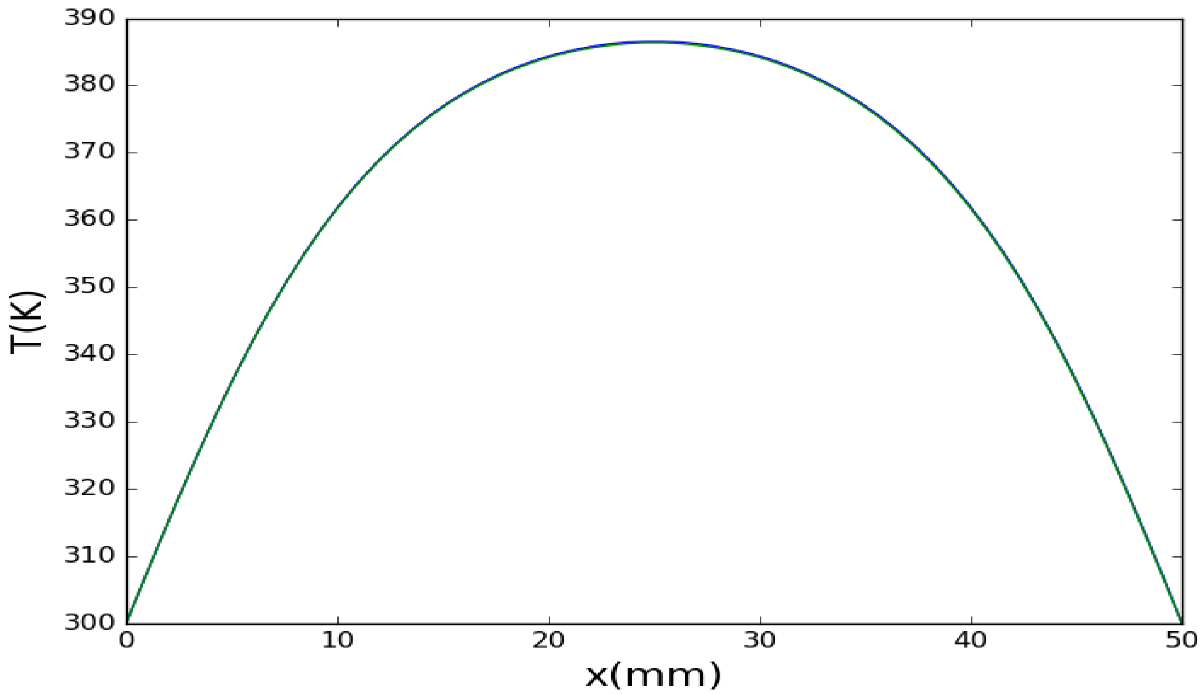
As we can see error creeps in at larger scale at corner points so we need to get correction at corner points to get better results.

3. Both Analytical and Numerically generated temperature variation (T versus y) along the central line of the slab (x=constant)



we are getting an overlap of two solution results and we can see linearity of the graph.

4. Both Analytical and Numerically generated temperature variation (T versus x) along the central line of the slab (y=constant)



we can again see the overlap and parabolic behavior of the graph.