(M518 Team D)

Exploring Boundary Feedback Control Using Proper Orthogonal Decomposition

Reduced Order Modeling of Heat Transfer: A Study on the 1D Heat Equation

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Contents

1 Introduction

1.1 Description

This project will focus on developing a reduced-order modeling (ROM) framework using Proper Orthogonal Decomposition (POD) to implement boundary feedback control for the 1D heat equation. The benchmark case will demonstrate how POD-based models can be used to reduce computational complexity and enable feedback control in a simplified system. The core steps will involve discretizing the spatial and time domains using different numerical methods (such as Runge-Kutta 4th order and Euler schemes), running simulations without control, and then applying POD to extract dominant modes from simulation snapshots, as seen in [3]. A feedback control law will be implemented to adjust boundary conditions dynamically, showing the effectiveness of control in real-time. Finally, visualization of the system's evolution and the impact of control will provide insights into the method's efficiency. Time permitting, the study will also compare various discretization techniques to evaluate the performance of the control strategies.

1.2 Background

Reduced-Order Modeling (ROM) is a mathematical approach that seeks to reduce the complexity of high-dimensional systems by creating low-dimensional representations that capture the essential dynamics. This technique is particularly useful in systems that are governed by partial differential equations (PDEs), such as fluid dynamics and heat transfer. By reducing the dimensionality of the system, ROM enables efficient computations, making real-time simulations and control feasible for complex systems [1].

Proper Orthogonal Decomposition (POD) is a popular ROM technique that identifies dominant patterns or modes within data, usually through a set of simulation snapshots. These modes provide a reduced basis for approximating the system's state while preserving the key dynamics. POD has been widely used in various fields, from fluid dynamics to structural mechanics, due to its ability to retain the most energetically significant modes [2]. For control purposes, POD enables the simplification of the control problem by focusing on these dominant modes rather than the full-order system, which is computationally expensive [5].

Boundary feedback control refers to a strategy where control inputs are applied through the boundaries of the domain in question, which, for the 1D heat equation, means adjusting the boundary temperature to drive the system towards a desired state. In the context of PDEs, boundary control allows for the stabilization of unstable systems or the regulation of temperature or other quantities. The challenge with boundary control lies in managing the complexity of the governing equations while ensuring real-time response and stability.

The 1D heat equation serves as a classical test case for such control strategies, describing how heat diffuses through a given medium over time. Its simplicity allows for clear demonstrations of control effectiveness while also serving as a foundation for more complex systems. Discretizing this equation for numerical simulations is a well-established method, with techniques like the Euler method and higher-order Runge-Kutta methods providing different trade-offs between accuracy and computational cost [4].

1.3 Significance of the Study

The significance of studying boundary feedback control of the 1D heat equation using Reduced-Order Modeling (ROM) and Proper Orthogonal Decomposition (POD) lies in its potential to enhance computational efficiency in real-time control applications. Many physical systems, such as temperature regulation in materials or thermal management in engineering systems, can be modeled by the heat equation. However, full-order models are often too complex for practical control in real time. By applying ROM and POD, it is possible to significantly reduce the computational burden while maintaining accuracy. This enables faster simulations and the real-time implementation of feedback control strategies, making it a useful approach for systems that require dynamic regulation. Furthermore, understanding the performance of various numerical methods in conjunction with boundary control provides insights into improving the stability, robustness, and efficiency of control systems across a range of applications.

1.4 Qualifications and Experience of Group Members

1.4.1 James Rogers

I have a few years of experience working with Monte Carlo simulations in nuclear physics and conducting data analysis for neutron scattering experiments (UT and Oak Ridge National Laboratory 2022-2024). In my work, I primarily use Python for quick coding and simulations, while leveraging C++ for more computationally intensive data analysis tasks. Additionally, I have some experience with high-performance crystallization evolution simulations using Fortran, which I developed at ORNL in 2024. My projects and code are available on my GitHub: https://github.com/smjim. I believe my contribution will be in applying feedback control laws, and especially visualization of the POD/ ROM results.

1.4.2 Eben Acquah

My research is in the broad area of nonlinear dynamics and incorporates both computation and analysis. Much of my concern is with proposed hypotheses, simulated scenarios, therapeutic inventions, and the development of advanced drugs for human health, but I also study simpler ordinary and partial differential equations. I use MATLAB, Python, and Maple for coding, simulations, and data analysis tasks.
I desire to study systems that change over time, such as the spread of a disease or the movement of a population, and analyze the behavior of these systems using mathematical and computational methods, and develop models to better understand and predict their behavior. This help to better understand the factors that contribute to the spread of diseases and inform public health policies and interventions.

1.4.3 Anil Sapkota

1.4.4 Ban Hemanta

I am an electrical and electronics engineer with a strong academic and commercial foundation. My undergraduate research focused on designing Analog and Mixed-Signal Integrated Circuits. My journey has included hands-on experience in renewable energy, telecommunications, and smart systems engineering. My research included developing a smart energy meter using NB-IoT technology, in which I assessed the impact of IoT protocols on system performance and analyzed deployment challenges related to packet loss and signal strength. I conducted a reliability analysis of the finite blocklength regime in mobile edge computing. Additionally, I analyzed cybersecurity issues, security controls, and countermeasures in advanced metering infrastructures and worked on the projects for monitoring of power distribution system in low voltage level. I designed a distribution system state estimation algorithm to identify data inaccuracies related to energy readings data. I also have several years of experience in commercial industry. My contribution to this project will focus on visualization and comparison in the report.

1.4.5 Materials Available

We all have experience with Python and C++, so there will likely be a github repository containing all of our progress. As UT students, we also have access to the UT-ISAAC computing cluster in case it is necessary to run high performance simulations or applications. Some codes are available to exemplify the initial system behavior we wish to mimick, but we plan to write our own implementations from scratch.

2 Problem Description

2.1 Problem

The main problem to be studied is the development of a reduced-order modeling (ROM) framework using Proper Orthogonal Decomposition (POD) to implement boundary feedback control for the 1D heat equation. This framework aims to simplify the computational complexity associated with traditional modeling approaches while maintaining accuracy in representing the system dynamics.

2.2 Questions

This project aims to address the following questions:

1. How can POD models be effectively utilized to reduce computational complexity in simulating the 1D heat equation?
2. In what ways can POD facilitate the design and implementation of feedback control strategies for real-time applications?

2.3 Expected Impact

The proposed work is expected to be a good learning exercise, as its application has the potential to enhance the efficiency and effectiveness of control strategies in thermal systems. By demonstrating the capability of POD to reduce complexity and enable real-time feedback control, utilization of this research could allow for improved thermal management solutions in several domains, including material processing, energy systems, and engineering design. Ultimately, understanding the mechanisms behind boundary control, ROM, and POD may contribute to more robust and responsive control systems.

3 Methodology

3.1 Description

In this project, we will approach the problem of boundary feedback control for the 1D heat equation by developing a robust reduced-order modeling (ROM) framework using Proper Orthogonal Decomposition (POD). We will conduct benchmark cases to validate our model and systematically explore various tasks to refine our analysis methods. This includes assessing the effectiveness of different numerical discretization techniques and evaluating their impact on control strategies. By establishing a foundational understanding through these cases, we aim to create a model that can efficiently implement feedback control in real-time scenarios.

3.2 Tasks

1. **Run Simulations without Control**: Establish baseline behavior of the system under the 1D heat equation without feedback control to understand its dynamics.

2. **Apply POD to Extract Dominant Modes**: Utilize POD to analyze simulation snapshots and identify key modes that represent the system's behavior.

3. **Design Feedback Control Law**: Develop a feedback control law that dynamically adjusts the boundary conditions based on the identified modes to achieve desired system behavior.

4. **Visualize System Evolution**: Create visualizations to depict the system's evolution over time, showcasing the impact of the control law on the system dynamics.

5. **Compare Different Discretization Techniques**: Evaluate the performance of various numerical methods (e.g., RK4, Euler) to understand their influence on the model's accuracy and computational efficiency.

3.2.1 Deliverables for Each Task

- **Simulations**: Set of baseline simulation results, documented behavior of the system.

- **POD Analysis**: Report on dominant modes extracted, including graphical representation of modes.

- **Control Law Design**: Documentation of the feedback control law, including theoretical justification and expected outcomes.

- **Visualization**: Interactive visualizations or plots illustrating system behavior under various control scenarios.

- **Comparison Study**: A comparative analysis report detailing findings on discretization techniques and their implications for model performance.

3.2.2 Methods

- **Discretization of Time and Spatial Domains**: Implement numerical methods such as the Runge-Kutta 4th order (RK4) and Euler methods to discretize the heat equation. Evaluate their accuracy and computational cost.

- **Implementation of POD with ROM**: Utilize MATLAB or Python libraries (like `numpy`, `scipy`, or `sklearn`) to perform POD. This will involve capturing the snapshots of the system and constructing the reduced basis from these modes.

- **Designing Feedback Control**: Apply control theory principles to develop the feedback control law. This could include using Lyapunov stability criteria to ensure robustness and performance.

- **Software for Simulation and Visualization**: Use Python (with libraries like Matplotlib for visualization) for running simulations and analysis. Additionally, tools like ParaView or MATLAB can be utilized for advanced visualizations.

3.2.3 Efficacy of Approaches

- **Effectiveness of POD**: While POD is a powerful method for mode extraction, it may not always capture nonlinear dynamics effectively. Alternatives such as Dynamic Mode Decomposition (DMD) could be considered for systems with significant nonlinear behavior.

- **Pitfalls and Limitations**: Possible pitfalls include:

  - Sensitivity of POD to noise in the data, which can affect mode extraction.
  - Computational challenges in real-time implementation of feedback control, especially for larger systems.
  - Balancing accuracy and computational efficiency when selecting discretization techniques, as finer grids may lead to longer computation times without significantly improving results.

4 Deliverables

The deliverables for this project will be constructed collaboratively by the team of four members, with each member responsible for specific tasks. Each deliverable will be documented and presented as part of the overall project report. The expected deliverables are as follows:

1. **Simulation Report (Anil Sapkota)**:
    - Conduct baseline simulations of the 1D heat equation without control.
    - Document the simulation setup, including parameters used and initial conditions.
    - Provide graphical results that illustrate the system's behavior over time.
2. **POD Analysis Report (James Rogers)**:
    - Apply Proper Orthogonal Decomposition to the simulation snapshots.
    - Identify and extract dominant modes that represent the system dynamics.
    - Compile results in a report detailing the modes, including visualizations and interpretation of their significance.
3. **Feedback Control Design Document (Eben Acquah)**:
    - Develop the feedback control law for dynamically adjusting boundary conditions based on POD results.
    - Document the design process, theoretical foundations, and expected performance of the control law.
    - Provide simulations demonstrating the impact of the control law on system behavior.
4. **Visualization and Comparison Report (Ban Hemanta)**:
    - Create visualizations to depict the system's evolution with and without control.
    - Compare the performance of different discretization techniques (e.g., RK4, Euler) in terms of accuracy and computational efficiency.
    - Compile findings into a comprehensive report that summarizes the impact of feedback control and discretization methods on the model's performance.

Each deliverable will include methodology, results, and discussion sections to ensure that the project progresses cohesively and all aspects of the research are well-documented. Regular meetings will be held to integrate findings and ensure consistency across the deliverables.

5 Time Table

1. **Complete by October 4:** Obtain project starting point, commence individual work.
2. **Check in October 11:** Group meeting to ensure each project is progressing
3. **Check in October 25:** Group meeting to ensure each deliverable is close to complete
4. **Submit by November 1:** Rough draft of team report (combination of each team member's deliverables), Expect 60% progress towards project completion.
5. **Submit by December 3:** Final version of team report
6. **Complete by December 11:** Class presentation of report

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