Project: Trading strategies analysis

Executive Summary:

This report presents backtesting results for multiple portfolio allocation strategies, using historical data for 58 stocks and the S&P 500 ETF in 2024. Strategies include Equal-Weight, Momentum (with and without transaction costs), Volatility Parity, Kelly Criterion, and Minimum-Variance allocation. We compare performance metrics such as volatility, Sharpe ratio, VaR, and maximum drawdown, in order to pick out the best strategy based on our backtesting for stock data in 2024.

Data cleanness:

We pick out 58 stocks from the dataset, that satisfies our data cleanness principle (full execution dates, no duplicated). Here is an overview:

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There are 58 types of adjusted stocks in 2024 that will be use later:
['CSCO', 'CL', 'LVMUY', 'UBER', 'MSFT', 'PINS', 'NVDA', 'HLT', 'AXP', 'TSLA', 'UBSFY', 'HMC', 'ZM', 'COST',
'RBLX', 'POAHY', 'NFLX', 'LOGI', 'TM', 'FL', 'DIS', 'ADDYY', 'AAPL', 'KO', 'CROX', 'PTON', 'FDX', 'JNJ', 'D
AL', 'TGT', 'NKE', 'MAR', 'SBUX', 'PG', 'SHOP', 'ABNB', 'NTDOY', 'AMD', 'GOOGL', 'JWN', 'CRM', 'MCD', 'HD',
'UL', 'COIN', 'ZI', 'MMM', 'V', 'HSY', 'ADBE', 'PHG', 'SQ', 'AEO', 'JPM', 'MA', 'AMZN', 'LUV', 'SPOT']
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Besides, we also use the S&P 500 ETF prices as the market index (in order to calculate the beta of each stock later).

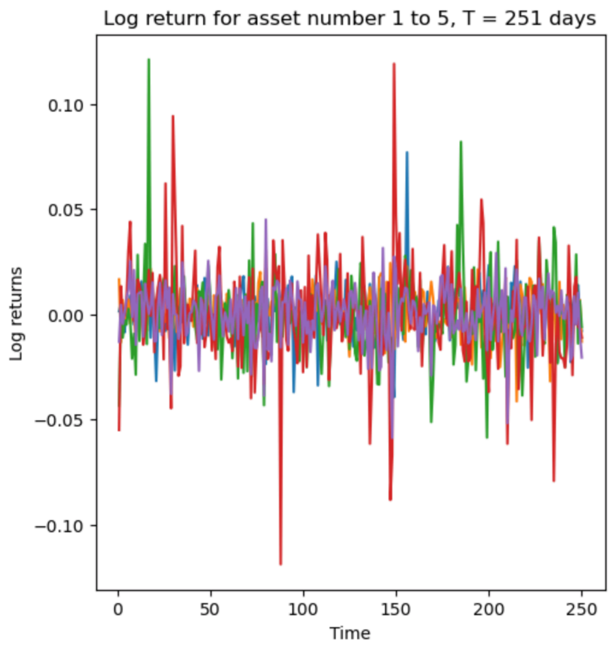
We have 251 (no duplicated) execution dates in 2024.

Basic characteristics of stocks:

First, we write functions to calculate some basic characteristics of our stocks. These include the log-returns, (rolling) sharpe ratio and beta.

1. *The daily simple/log returns:* Suppose P_t is the price of a stock at day t. Then its *daily simple return* is given by: $\frac{P_t}{P_{t-1}} - 1$. Its *daily log return* is given by: $log(\frac{P_t}{P_{t-1}})$.

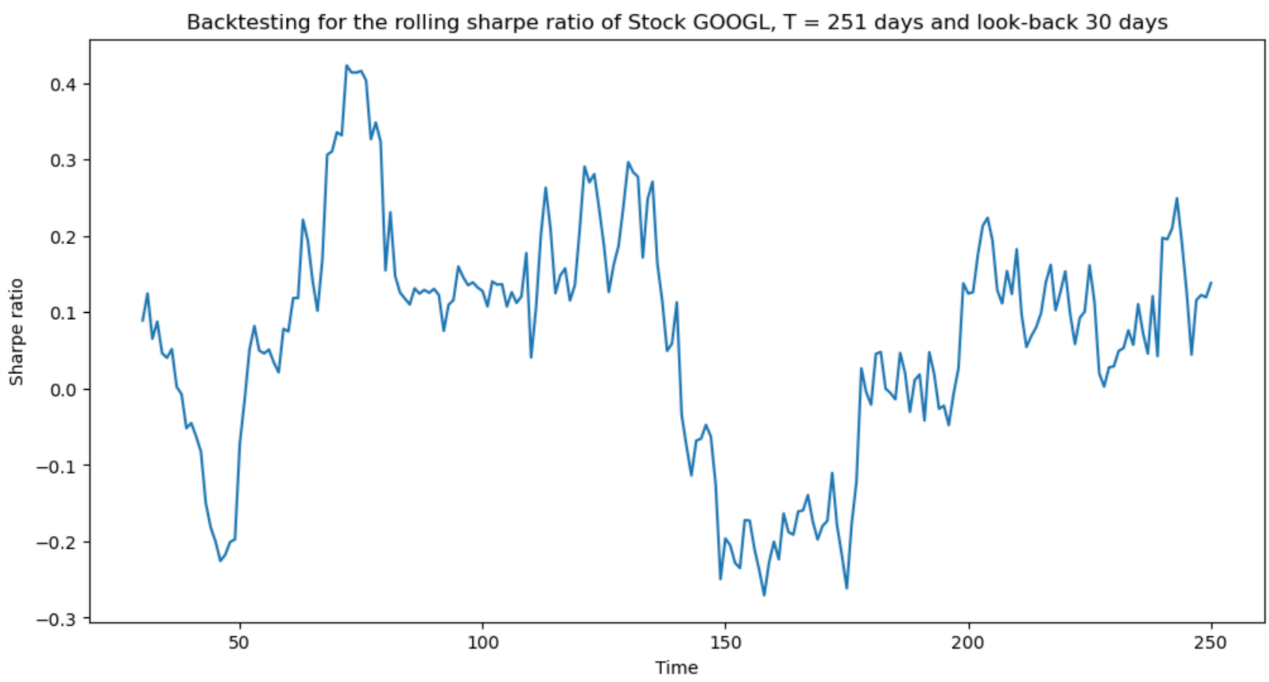
(A visualization of log-return for asset number 1-5, for the whole execution dates in 2024)



2. *The sharpe ratio (with rolling window K):* Given a series of daily log return r_t. The sharpe ratio with rolling window K on day t is given by: $\frac{\bar{r}_{(t-K):K} - r_f}{\sigma_{(t-K):K}}$, where

- $\bar{r}_{(t-K):K}$ is the mean value of log returns from day t-K to day K-1 (inclusive).
- r_f is the risk-free rate. Since we consider the daily case, it is so small that we assume it to be 0.
- $\sigma_{(t-K):K}$ is the standard deviation of the log returns from day t-K to day K-1 (inclusive).

(A visualization of the rolling sharpe ratio of the Stock GOOGL with K = 30 is given below:)



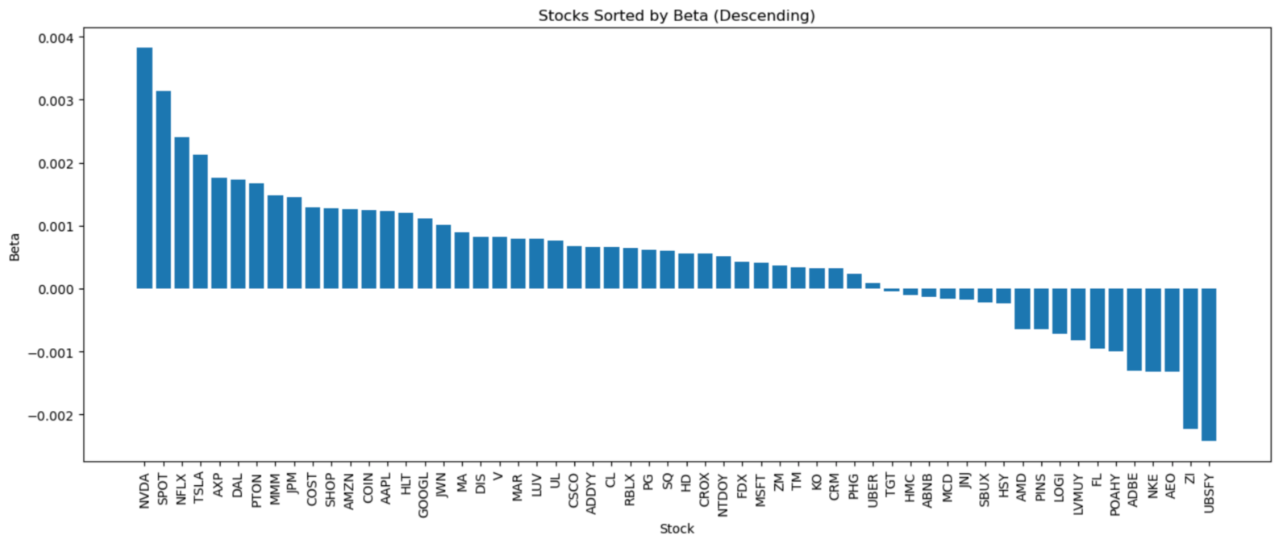
The sharpe ratio is a measure of how much excess return an investment generates for each unit of risk it takes.

3. *Beta (of a stock):* Given a series of daily log return r_t^S of the stock, as well as a series of daily log return r_t^M of the market portfolio (here we use the S&P 500 ETF). We estimate the beta of this stock by fitting the following linear regression: $r^S = \alpha + \beta r^M + \epsilon$.

When an ordinary least squares (OLS) regression is run, the formula for the slope β is given by:

$\beta = \dfrac{Cov(r^S, r^M)}{Var(r^M)}$ explicitly, which coincide with its original mathematical definition.

(A visualization of the descending ranking of the stock beta is given in the following:)



The Beta shows how sensitive the stock's returns are to the returns of the overall market.

4. Finally, we *cluster* these 58 stocks into different groups, ensuring the average correlation inside the same group is always larger than 0.7.

Given N different stocks and their correlation matrix C_{ij} ($N \times N$). The *average correlation* is calculated by the following: $\dfrac{2\sum_{i=1}^{N}\sum_{j=i+1}^{N} C_{ij}}{N(N-1)}$.

(An overview of the clustering result is given by the following:)

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Group the stock with average correlation >= 0.7:
Group1 : ['CSCO', 'TSLA', 'ZM', 'RBLX', 'PTON', 'MAR', 'SHOP', 'CRM', 'HD', 'V', 'SQ', 'MA']
Group2 : ['NVDA', 'HLT', 'AXP', 'COST', 'NFLX', 'AAPL', 'DAL', 'JWN', 'JPM', 'AMZN', 'SPOT']
Group3 : ['CL', 'ADDYY', 'KO', 'PG', 'UL', 'MMM', 'PHG']
Group4 : ['LVMUY', 'HMC', 'POAHY', 'TM', 'AEO']
Group5 : ['MSFT', 'GOOGL']
Group6 : ['SBUX', 'MCD']
Group7 : ['PINS', 'UBSFY']
Group8 : ['NKE', 'ZI']
Group9 : ['FDX']
Group10 : ['COIN']
Group11 : ['LUV']
Group12 : ['NTDOY']
Group13 : ['FL']
Group14 : ['ADBE']
Group15 : ['JNJ']
Group16 : ['CROX']
Group17 : ['HSY']
Group18 : ['UBER']
Group19 : ['TGT']
Group20 : ['LOGI']
Group21 : ['DIS']
Group22 : ['ABNB']
Group23 : ['AMD']
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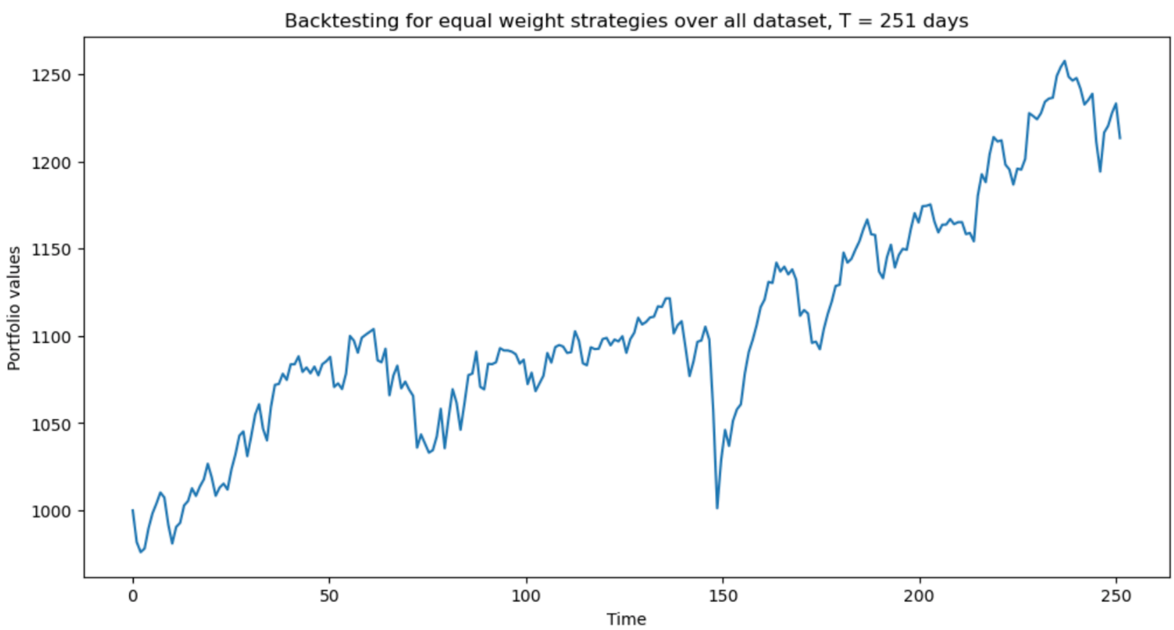
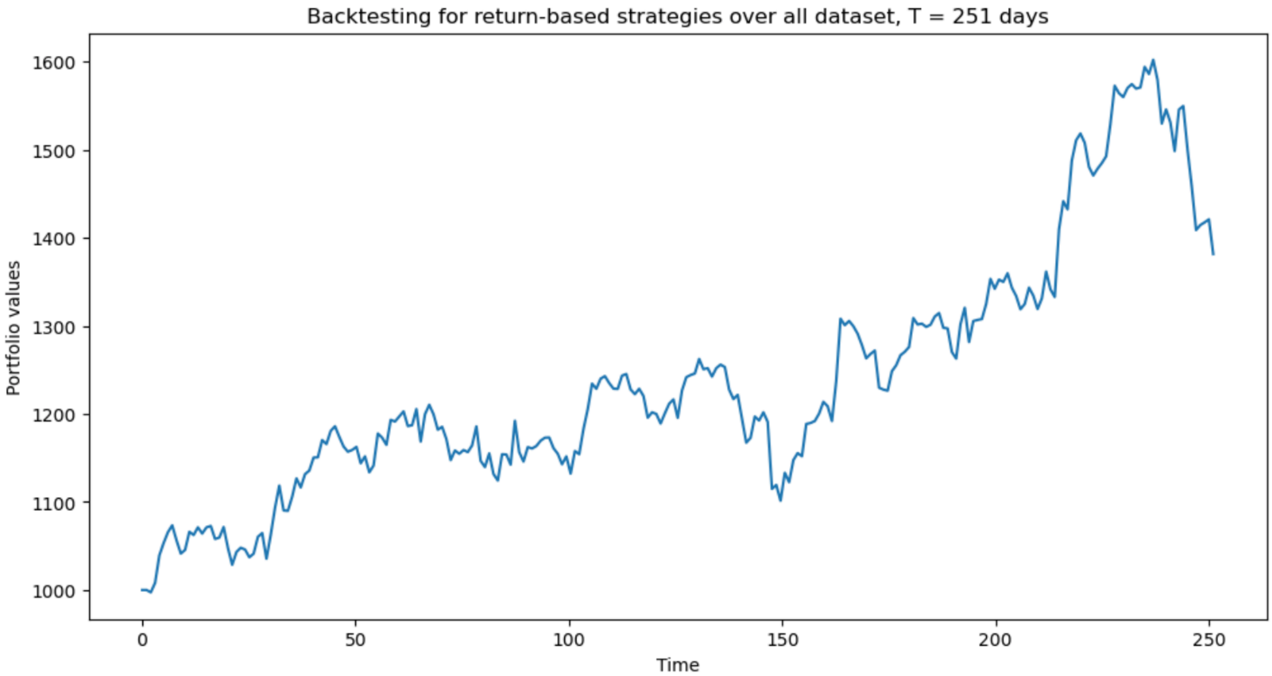
Trading strategies:

Next, we write functions to implement different trading strategies and visualize their portfolio value timeseries as outputs. Strategies include: Equal-Weight, Momentum (with and without transaction costs), Volatility Parity, Kelly Criterion, and Minimum-Variance allocation.

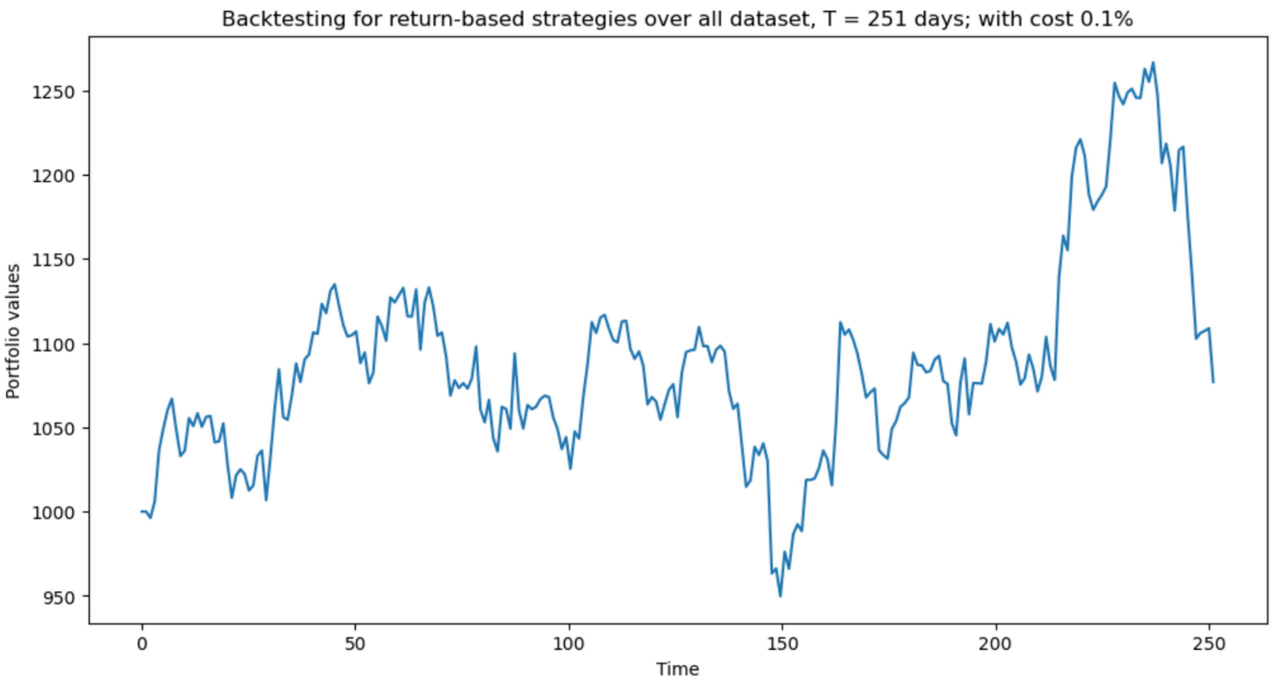
1. *Equal weight strategy:* Given prices matrix (N, T) and initial capital C, backtest a strategy where you always hold equal weights in all assets, rebalancing daily.



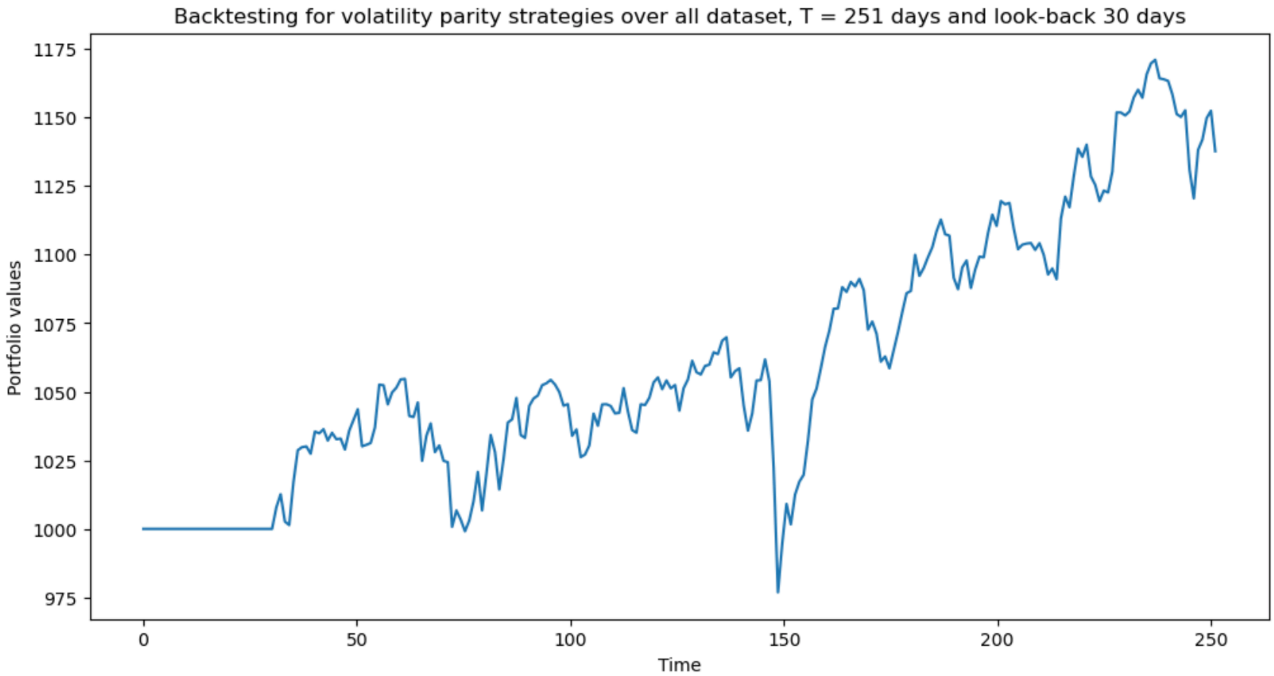
2. *Momentum Allocation Strategy:* Given prices matrix (N, T) and initial capital C, allocate to stocks proportionally to their last day's return (skip negatives, keep in cash).



3. *Momentum Allocation Strategy (with transaction cost 0.1%):* Repeat the momentum allocation strategy, but include a 0.1% transaction cost every time changing allocations. The transaction cost rate can be adjusted in the function.



4. *Volatility parity strategy:* Given prices matrix (N, T), initial capital C and a look-back period K days, we compute each stock's volatility over the last K=30 days at each trading date. Allocate inversely to volatility (higher weight to low-vol stocks), daily.



5. *Kelly strategy:* Given prices matrix (N, T), initial capital C and a look-back period K days, we compute each stock's volatility over the last K=30 days at each trading date. Use a rolling window of K = 30 days to estimate the covariance. Use the Kelly formula to determine the investing weight daily. Compute the daily portfolio values.

The Kelly formula is given by the following:

$$\max w^T\mu - 0.5w^T\Sigma w \text{ s.t } w^Te = 1$$

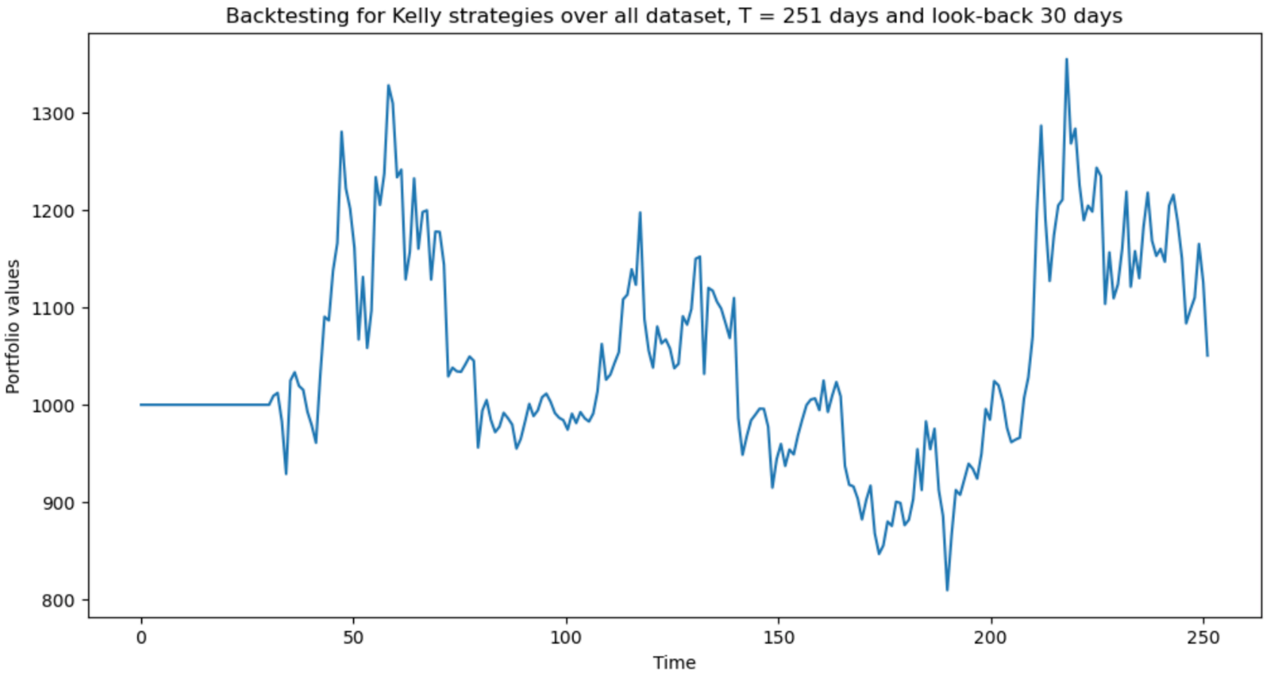
where Σ is the covariance matrix and e is the one vector of corresponding dimension.

Mathematically, the Kelly optimization problem has an explicit solution of $w = \Sigma^{-1}\mu$, with normalization.

However, in reality, we found that the original Kelly strategy performs badly. The reason is that since the Kelly weight allows for short position, and our rolling window 30 is not larger enough, the covariance matrix could not be accurate enough, and the portfolio value will have strong fluctuations. Following this idea, we use the long-only Kelly strategy which calculate weights from:

$$\max w^T\mu - 0.5w^T\Sigma w \text{ s.t } w^Te = 1 \text{ and } w \geq 0$$

It turns out that this strategy performs better. This adjustment can be seen in the "positive_weight" variable in the code.



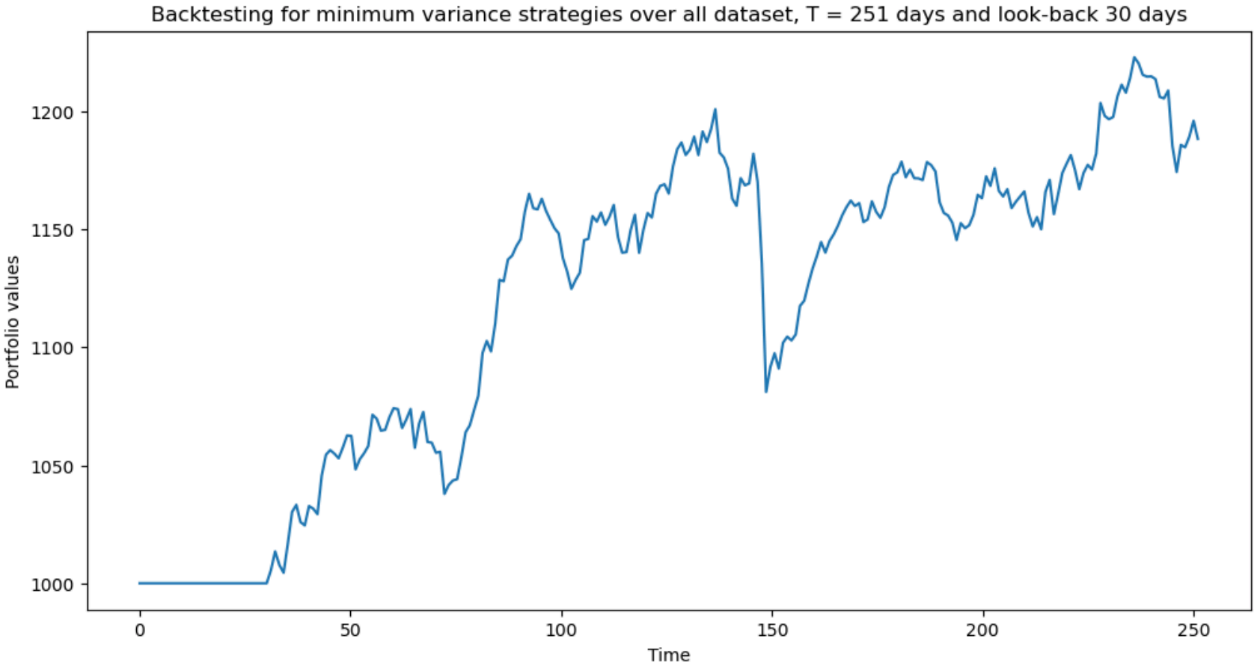
6. *Minimum variance strategy:* Given prices matrix (N, T), initial capital C and a look-back period K days, we compute each stock's volatility over the last K=30 days at each trading date. Use a rolling window of K = 30 days to estimate the covariance. Use minimum variance formula to allocate weights. Compute the daily portfolio values.

The minimum variance formula is given by: $\min w^T \Sigma w$ s.t $w^T e = 1$. It has an explicit solution of $w = \Sigma^{-1} e$, with normalization.

Similarly, we found that the original minimum variance formula performs badly, due to the allowance of short position. We use the long-only minimum variance strategy instead:

$$\min w^T \Sigma w \text{ s.t } w^T e = 1 \text{ and } w \geq 0.$$

This adjustment can be seen in the "positive_weight" variable in the code.



Strategies assessments:

Finally, we compare these 6 strategies using three metrics: the annualized sharpe ratio, the value at risk and the maximum drawdown.

1. *The annualized (average) sharpe ratio*: the sharpe ratio with look-back window K = 251 (whole year). The results are presented followings:

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Average Sharpe ratio of the following strategies:
-- Equal weight strategy: 1.3434
-- Momentum Allocation strategy: 1.4165
-- Momentum Allocation strategy with cost level 0.1%: 0.4214
-- Volatility parity strategy: 1.1437
-- Kelly strategy (long-only): 0.3832
-- Minimum variance strategy (long-only): 1.6135
Based on the Sharpe ratio, the best performed strategy is Minimum variance strategy (long-only).
The worst performed strategy is Kelly strategy (long-only).
```

2. *The Value at Risk at level α*: the maximum (daily) loss (in log return) you expect not to exceed, with probability **α = 95%**. Mathematically, it is defined as $\min\{x \in \mathbb{R} : \mathbb{P}(r \leq x) > \alpha\}$, where r is the distribution of the daily log return.

The results are presented followings:

```
Historical VaR at the confidence level α = 0.95 of :
-- Equal weight strategy: 0.0160
-- Momentum Allocation strategy: 0.0231
-- Momentum Allocation strategy with cost level 0.1%: 0.0241
-- Volatility parity strategy: 0.0129
-- Kelly strategy (long-only): 0.0666
-- Minimum variance strategy (long-only): 0.0102
Based on the historical VaR, the best performed strategy is Minimum variance strategy (long-only).
The worst performed strategy is Kelly strategy (long-only).
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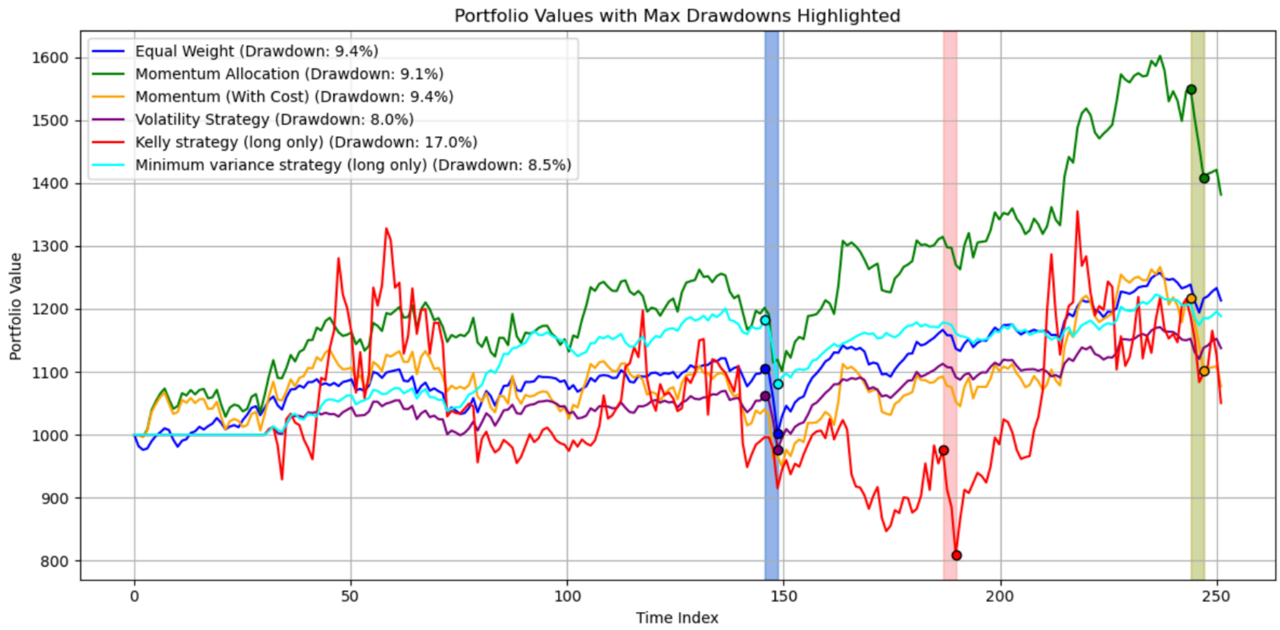
3. *The maximum drawdown*: the largest peak-to-trough loss in portfolio value.

The results are presented followings:

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Based on the maximum drawdown, the best performed strategy is Volatility parity strategy.
The worst performed strategy is Kelly strategy (long-only).
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Summary and results:

We summarize the above three metrics' ranking into one table and give results:

Strategy	Sharpe Ratio Rank	VaR (95%) Rank	Max Drawdown Rank
Equal Weight Strategy	3	3	5
Momentum Allocation Strategy	2	4	3
Momentum Allocation Strategy (Cost 0.1%)	5	5	4
Volatility Parity Strategy	4	2	1
Kelly Strategy (Long-Only)	6	6	6
Minimum Variance Strategy (Long-Only)	1	1	2

- **Minimum-Variance Strategy** delivered the best risk-adjusted performance, achieving top Sharpe ratio and lowest VaR, as its allocation favors low-volatility assets and naturally diversifies risk.
- **Volatility Parity Strategy** had the lowest maximum drawdown because it dynamically scaled exposure to risky stocks down during volatile periods, limiting large losses.
- **Momentum Strategy (No Cost)** generated strong returns by riding short-term trends, but its higher turnover led to larger drawdowns during reversals.

- **Momentum Strategy (With Costs)** underperformed significantly once transaction costs were included, highlighting the importance of execution costs in high-turnover strategies.
- **Kelly Criterion Strategy** was the most aggressive, leading to large swings in portfolio value and the weakest Sharpe ratio, as real-world returns are rarely stationary enough to justify full Kelly betting.
- **Equal-Weight Strategy** served as a balanced benchmark: reasonable performance, moderate drawdowns, but no risk targeting — leaving room for improvement versus optimized approaches.