

Time Complexity: Lecture 10 marks.

① #include <stdio.h>

```
int main () {
    int n1, n2, result;
    n1 = 10;
    n2 = 20;
    result = n1 + n2;
    result = 0;
}
```

Operation: 3 assignments  
Total: 4 assignments

But complexity  $O(1)$ .  
Because input is constant operation.  
 $O(1)$  constant time complexity.

Complexity depends on total no. of assignments.

int main ()

```
int n, result;
scanf ("%d", &n);
result = n * (n+1) / 2;
printf ("result = %d\n", result);
return 0;
```

Operation:

1 assignment + 1 multiplication  
Total = 2 assignments

3

total complexity  $O(1)$ .

Complexity of  $(x \cdot x)$  is  $O(1)$ .

Complexity of  $(1 \cdot n)$  is  $O(n)$ .

③

Input validation  
(gate)

After tests are done then complexity is  $O(1)$ .

Forward bias current - must be increased requirement.

resistor frequency requirement.

Received message.

(3) #include <stdio.h>

int main()

{ int i, n, result;

scanf("%d", &n);

result = 0;

for(i=1; i<=n; i++) {

    result = result + i;

}

printf("result=%d\n", result);

return 0;

}

Operation:  $n = n + 1$  ক্রিয়াকার

$n = n + 2$

$n = n + 3$

> যার assign

> যার add করুন

Operation 2

$n = 1$  তখন,

" "

" "

$n = 2$  " "

" "

$n = 3$  " "

" "

$n = 10$  " "

" "

4

6

20

অতি Complexity =  $O(2n)$

=  $2 \times O(n)$

2 একটি constant

Ignore করুন

(∴ Complexity  $O(n)$ )

যা linear complexity কারণ, operation  
আমানে  $n$ , operation করা মাত্র  
(linearly যাবে)

(4)

int main()

{ int i, n, j, count;

scanf("%d", &n);

count = 0;

for (i=0; i<n; i++)

    for (j=0; j<n; j++)

        count = count + 1

}

printf("n=%d, count=%d\n", n, count); if (n>i & i>j) not

return 0;

3 = (second step)

n	Operation	
	Count	Time
1	1	1 sec
2	4	4 sec
3	9	9 sec
10	100	100 sec

Complexity  
 $O(n^2)$

## Complexity

মুক্তি loop যদি  $n$  এর ফার নিখিল রয়ে গেল তখন  $O(n^2)$

মুক্তি nested loop  $n$  " " কিছি না রয়ে nested loop যদি  
 $i=0; i<3; i++$  এর তখন আরো comp lexith  
 ওলন্দা হল হয়ে  $O(3n)$  ফর্ম হত constant  
 ignore রয়ে  $O(n)$  হাবে।

মুক্তি complexity.

মুক্তি,  $O(1)$  (Time কম লাগে)  $O(1)$  (Time complexity কম)

$O(1) > O(\log n) > O(n) > O(n) > O(n^2) > O(n^3) > O(n^n)$

মুক্তি Time কম লাগে  $O(n^n)$ . তাই মুক্তি ধারণা Complexity.

মুক্তি  $O(1)$ ,  $O(\log n)$ ,  $O(n)$ ,  $O(n \log n)$ ,  $O(n^2)$ ,  
 $O(2^n)$ ,  $O(n!)$

$\rightarrow$  (ধারণা complexity) (মুক্তি Time লাগে)

int main()

{ int i, j, n, count;

scanf("%d", &n);

Count = 0;

for(i=0; i<n; i++){

    for(j=0; j<n; j++) {

        Count = Count + 1;

    }

    for(i=0; i<n; i++) { } এখানে মুক্তি loop  $O(n)$

    Count = Count + 1;

    printf("Count = %d\n", Count);

return 0; }

মুক্তি প্রস্তুত করে নিখিল রয়ে  
 তার complexity  $O(n^2)$

$O(n^2)$

$$\begin{aligned} \text{Total} &= O(n^2) + O(n) = O(n^2 + n) \\ &\quad (\text{কমপ্লিকেশন লিখতে হবে}) \\ &= O(n^2) \end{aligned}$$

- $O(n^4 + n^3 + n^2 + n)$  ഇതു Time Complexity =  $O(n^4)$  ആണ്,
- $O(n! + n^3 + n^2 + n)$  ഇതു Time Complexity =  $O(n!)$  ആണ്,  
(Because  $n^3$  തോറെ  $n!$  കുക്കിട്ടും വളരെ കൂടുതലും)
- $O(4n^4 + 2n^3 + 3n^2 + n)$  ഇതു Time Complexity =  $O(n^4)$  ആണ്

### Space Complexity

- \* linear search is an algorithm. and algorithm is a strategy. complexity  $\propto$  Big  $O(n)$ . \* easy to implement.

Insertion :

- \* Insertion sort is a sorting algorithm. problem is:  
sort - best case  $O(n^2)$ ,  $O(n)$ .  
worst case  $O(n^2)$  (when list sort യാരോ almost)

- \* Ascending order കുറഞ്ഞ ദിശയിൽ sort ചെയ്യുന്നത്  $O(n^2)$

- \* outer loop starts from 2nd element.. times( $k+1$ ) if starts from K.

\* Binary search :

$$\text{complexity } O(\log_2 n) = \log_2 15 = 4 .$$

for linear search :  $O(n) = 15$

## in) Binary search

$$\text{mid} = \frac{0+13}{2} = 6.5 \approx 6 \quad (\text{as integers show } \text{pos})$$

Write a program to sort <sup>of an</sup> 1<sup>st</sup> half of array in ascending order and 2<sup>nd</sup> half of the array in descending order.

ptinsqms) 9302

initially zero, starting from 0 to 1000. Then update. complexity O(n^2)

middle position to 0 to 1000. then sort. complexity O(n^2)

mid const to 0 to 1000. then sort. complexity O(n^2)

Binary search

$\text{P} = \frac{\text{E} + \text{B}}{2}$  =  $(\frac{\text{E} + \text{B}}{2})$  O

## Insertion sort code

```
#include <stdio.h>
void insertionsort (int A[], int length) {
    for (int j=1; j<length; j++) {
        int key = A[j];
        int i = j-1;
        while (i>=0 && A[i]>key) {
            A[i+1] = A[i];
            i--;
        }
        A[i+1] = key;
    }
}

void descendingorder (int A[], int length) {
    for (int j=1; j<length; j++) {
        int key = A[j];
        int i = j-1;
        while (i>=0 && A[i]>key) {
            A[i+1] = key;
        }
    }
}

int main() {
    int A[] = {2, 4, 5, 9, 7, 1};
    int length = sizeof(A) / sizeof(A[0]);
    printf("Original Array: ");
}
```

(char) type initialized

---

```

for (int i=0; i< length; i++){
    printf("%d", A[i]);
}
printf("\n");
insertsort(A, length);
printf("sorted array in ascending:");
for (int i=0; i< length; i++){
    printf("%d", A[i]);
}
printf("\n");
descendingorder(A, length);
printf("sorted array (Descending):");
for (int i=0; i< length; i++){
    printf("%d", A[i]);
}
printf("\n");
return 0;
}

```

## Space Complexity

```
#include <stdio.h>
int main()
{
    int n;
    scanf("%d", &n);
    if (n % 2 == 0)
        printf("%d is even number.\n", n);
    else
        printf("%d is odd number.\n", n);
    return 0;
}
```

Time complexity  
 $O(1)$

Space complexity

$O(1)$   
জো ক্ষেত্র মান থাকে  
হোম নথেন আছে  
ক্ষেত্রটি variable  
নে করেছি।

```
#include <stdio.h>
int main()
{
    int i, n, even[101];
    for (i=0; i<101; i++)
        even[i] = 0;
    scanf("%d", &n);
    if (even[n])
        printf("%d\n");
    else
        printf("%d\n");
    return 0;
}
```

$O(n)$   
জো  $n$  এর সংখ্যা হলো  
কিমুল

```

for(i=0; i<n; i++) {
    for(j=0; j<n; j++) {
        S[i][j] = a[i][j] + b[i][j];
    }
}

```

Space complexity  $O(n^2)$

$O(n^2)$

Linear Search

```

int linear_search (int A[], int n, int x)
{
    int i;
    for(i=0; i<n; i++) {
        if(A[i] == x) {
            return i;
        }
    }
    return -1;
}

```

60	71	88	10	11
----	----	----	----	----

10 minutes

worst complexity =  $O(n)$

variable use  
space complexity  $O(1)$ .

Linear search  $\Leftrightarrow$  n মতো অস্থায় খোঁজা।

## Binary Search

কার্তঃ ① কেই মাবিটি হোট হবে বড় অস্বা কড় হয়ে হোট আজাণি  
শাব্দতে ইচ্ছা।

② এটি middle point হিসেবে আর্চ করে  $\Rightarrow \frac{\text{high} + \text{low}}{2}$

1	4	6	7	10	13	22	23	30	35	39	46	49	50	52	55	61	67	70	71
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

- $\log_2 n = \square$  for Binary Search যদি  $n$  অস্বা
- Column এর আছে তাহলে Operation হয়ে  $\square$  কান্তি।

Example :

$$\bullet \log_2 16 = 4 \text{ বা, } 2^4 = 16$$

Here,

Operation হোকার,  $n=16$ . মানে 16 টি ইয়ের একটি আবি আজন।

$$\bullet \log_2 32 = 5 \text{ বা } 2^5 = 32$$

32 ইয়ের একটি আবি আজনে Operation হোতে 5 কান্তি। পিতৃ linear search এ operation হোতে 32 কান্তি।

অন্তর্ভুক্ত Binary Search এর time complexity  $O(\log n)$ .

```
int binary_search (int A[], int n, int x)
```

```
{
```

```
    int left, right, mid;
```

```
    left = 0;
```

```
    right = n - 1;
```

```
    while (left <= right) {
```

```
        mid = ((left + right) / 2);
```

```
        if (A[mid] == x) {
```

```
            return mid;
```

```
        if (A[mid] < x) {
```

```
            left = mid + 1; }
```

best  $O(1)$

Avg  $O(\log n)$

worst  $O(\log n)$

```

    else {
        right = mid - 1;
    }
}
return -1;
}

```

## ଅନ୍ତରକାଳ ଶର୍ତ୍ତ (Selection Sort)

10	5	2	8	7
----	---	---	---	---



2	5	7	8	10
---	---	---	---	----

### Implementation code:

```

void selection_sort (int A[], int n)
{
    int i, j, index_min, temp;

    for(i=0; i < n-1; i++) {
        index_min = i;
        for(j=i+1; j < n; j++) {
            if (A[j] < A[index_min]) {
                index_min = j;
            }
        }
        temp = A[i];
        A[i] = A[index_min];
        A[index_min] = temp;
    }
}

```

if ( $\text{index\_min} \neq i$ ) {

$\text{temp} = A[i];$       // swap

$A[i] = A[\text{index\_min}];$

$A[\text{index\_min}] = \text{temp};$

}

}      // if ( $i < \text{index\_min} \neq 0 \neq i$ )

3      // if ( $i + 1 - \text{index\_min} > 0 \neq i$ )

}      // if ( $[i+1] < [i] \neq i$ )

;  $[i+1] \neq i$  = good

### বাবল সোর্ট (Bubble sort)

10	5	2	8	7
----	---	---	---	---

5 10 2 8 7

5 2 10 8 7

5 2 8 10 7

5 2 8 7 10

2 5 8 17 6

2 5 7 18 6

2 5 17 6

2 15 6

এর নিয়ম ১ম রাখি অর্থে হোট কর তুলা হবে এবং

আজাতে হতে ।

বাবল সোর্ট মাঝে গুরুত্বপূর্ণ পয়েন্ট হচ্ছে তাই এটি আরও নয় ।

Complexity  $O(n^2)$

কাবল ২টি 100 P টালে

Space Complexity  $O(1)$

constant কাবল প্রতি array  
assignment এবং বাবল এতে সেবন  
দিয়েই কাছে ব্যবহৃত ।

## Code implementation:

```

void bubble_sort(int A[], int n)
{
    int i, j, temp;
    for(i=0; i<n; i++)
    {
        for(j=0; j<n-i-1; j++)
        {
            if(A[j] > A[j+1])
            {
                temp = A[j];
                A[j] = A[j+1];
                A[j+1] = temp;
            }
        }
    }
}

```

## इन्शेन्शन सॉर्ट (Insertion sort)

1	2	3	4	5
---	---	---	---	---

रहात होता रुदः -

चुक्की अद्य चुक्की  
प्रयुक्ति बदला (Change)  
आहे (Best Case)

तरुण गतिर वाढ करा  
गति Time Complexity

हा Best case

1	2	3	4	5
---	---	---	---	---

प्रयोग घेणाऱ्या चलवे नव्हा  
Change उत्तम गति आहे.  
Time complexity उत्तम  
 $O(n^2)$ .

∴ Best case Time Complexity  $O(n)$

1001 → 12

	sorted	unsorted	
0	1	2	3
43	31	26	20
	12		

$n=5$ .

$\text{temp} = 31 \xrightarrow{\text{swap}} 1$

0	1	2	3	4
31	43	26	20	12

Sorting ← → unsorting  
 $\text{temp} = 26$

0	1	2	3	4
26	31	43	20	12

$\text{temp} = 20$

26	20	31	43	12
----	----	----	----	----

$\text{temp} = 12$

12	26	20	31	43
----	----	----	----	----

## Quick sort

1	5	6	3	8	4	7	2
1	2	6	3	8	4	7	5
1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8

i = 0 index      11111111

j = 3 value      11111111

tarig  $\Rightarrow$  111A 00000000

i++ j

11111111

void quick\_sort(int A[], int low, int high)

{

if ( $low > high$ ) {

return;

}

int p;

p = partition (A, low, high);

quick\_sort (A, low, p - 1);

quick\_sort (A, p + 1, high);

}

## (Quick sort algorithm)

Let, pivot = any number from  $A[1:n]$ .

{ Lower value =  $i$

higher value =  $j$

while ( $i < j$ )

{ while  $A[i] \leq \text{pivot}$

{  $i++$ ;

}

while  $A[j] > \text{pivot}$

{  $j--$ ;

}

swap  $\leftrightarrow$  if  $i < j$

if  $i < j$

{

swap ( $a[i], a[j]$ )

}

if  $i \geq j$  (order)

{

swap ( $\text{pivot}$  and  $a[j]$ )

}

return  $j$ ;

}

$\begin{array}{|c|c|c|c|c|} \hline & 8 & 5 & 3 & 6 & 9 \\ \hline \end{array}$

$\begin{array}{|c|c|c|c|c|} \hline & 8 & 5 & 3 & 6 & 9 \\ \hline \end{array}$

$\begin{array}{|c|c|c|c|c|} \hline & 8 & 5 & 3 & 6 & 9 \\ \hline \end{array}$

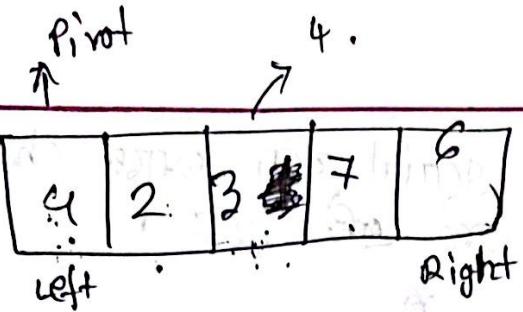
$\begin{array}{|c|c|c|c|c|} \hline & 8 & 5 & 3 & 6 & 9 \\ \hline \end{array}$

Complexity:

Avg:  $O(n \log n)$

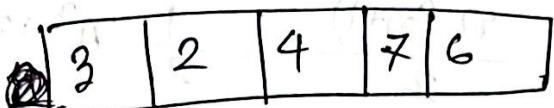
best:  $O(n \log n)$

worst:  $O(n^2)$



~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ ~~6~~

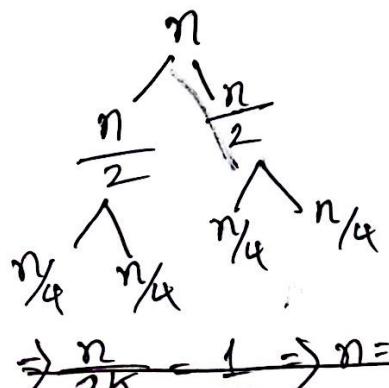
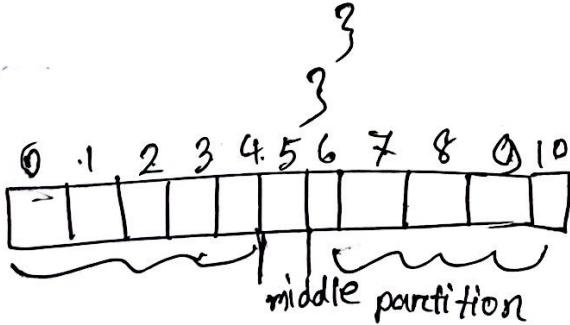
find the exact position  
for first pivot.



=====

Algorithm : Quicksort ( $A, l, h$ ) do now :

{ if ( $l < h$ )  
 {  
 }  
 } = partition ( $A, l, h$ );  
 Quicksort ( $A, l, i-1$ );  
 Quicksort ( $A, i+1, h$ );



$$\Rightarrow \frac{n}{2^k} = 1 \Rightarrow n = 2^k \Rightarrow \log n = \log 2^k \Rightarrow \log n = k \log 2 \Rightarrow k = \log n$$

$\frac{n}{2^k} = 1 \Rightarrow n = 2^k \Rightarrow \log n = \log 2^k \Rightarrow \log n = k \log_2$

Computer  
Binary log,  $\log_2$   
 $\log$ ,  $\log_2$   
 $\log_2$ ,  $\log$

Pivot

worst case

1	2	3	4	5
---	---	---	---	---

$\leftarrow j \rightarrow i$  swap

genial emr ओरे change  
25. या अब को 25!

n  
n-1  
n-2  
n-3

$$\Rightarrow \frac{n(n+1)}{2} = \frac{1}{2} n^2 + n = O(n^2)$$

∴ worst case  $O(n^2)$

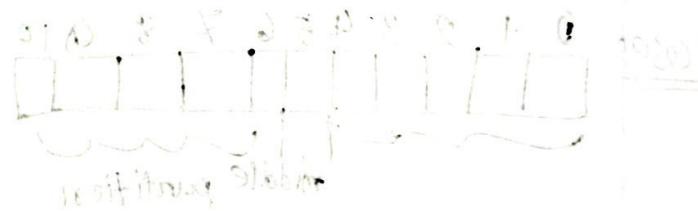
Average "  $O(n \log n)$

Best case  $O(n \log n)$

(A) partition

(B) breaking

(C) merging



(re partition)

1 = ~~10~~

2 3 4 5 6 7 8 9 10

2 3 4 5 6 7 8 9 10

2 3 4 5 6 7 8 9 10

2 3 4 5 6 7 8 9 10

2 3 4 5 6 7 8 9 10

2 3 4 5 6 7 8 9 10