Chapter 3. Voltage & Current laws

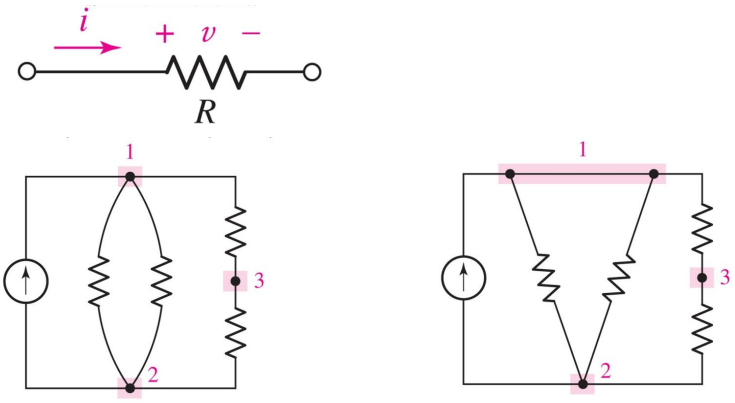
1. Network topology: Nodes, paths, loops, and branches
2. KCL / KVL
3. Single-loop / -node-pair circuits
4. Series & parallel connections
5. Voltage & current division

Network topology: Nodes, paths, loops, branches

- Terminologies based on graph theory
  - Every circuit element can be represented by a branch with two terminals.
  - Node (마디): A point at which two or more circuit elements have a common connection.
    - 두 개 이상의 회로소자는 node로 연결된다.
    - Every circuit element has a pair of nodes at each of its ends.
  - Branch (가지): A single path in a network, composed of one circuit element and nodes at each end.
  - A path is a sequence of nodes/branches.
  - A loop is a closed path
    - 한 node에서 시작하여 node 반복 없이 시작 node로 다시 돌아오는 폐경로.

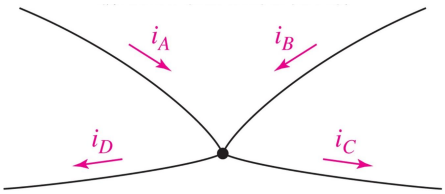
Nodes, paths, loops, branches

- Example



KCL, Kirchhoff's Current Law

- KCL: Algebraic sum of branch currents entering any node is zero.

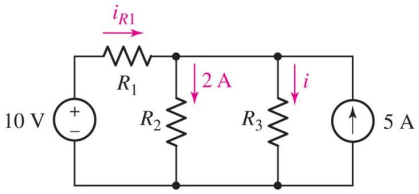


$$i_A + i_B + (-i_C) + (-i_D) = 0$$

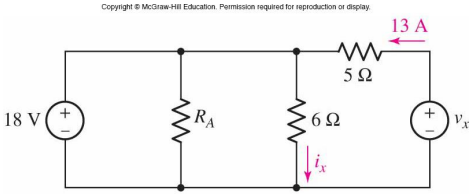
$$i_A + i_B = i_C + i_D$$

KCL

- Example 3.1 Find the current through resistor R_3, if the voltage source supplies a current of 3 $[A]$.

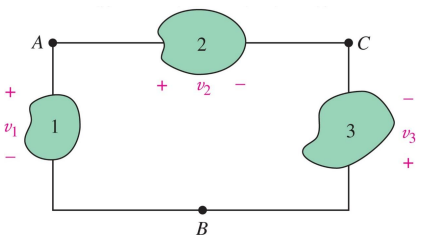


- Practice 3.1 Find R_A, when $i_x = 3[A]$ and the voltage source delivers $8[A]$ current.



KVL, Kirchhoff's Voltage Law

- KVL: Algebraic sum of branch voltages around any closed path is zero.



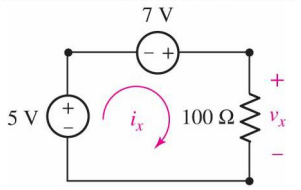
Starting from the node-A, add-up branch voltages while checking the voltage polarities:

$$v_2 + (-v_3) + (-v_1) = 0$$

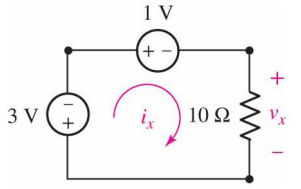
$$v_2 = v_1 + v_3$$

KVL

• Example 3.2 Find v_x and i_x.

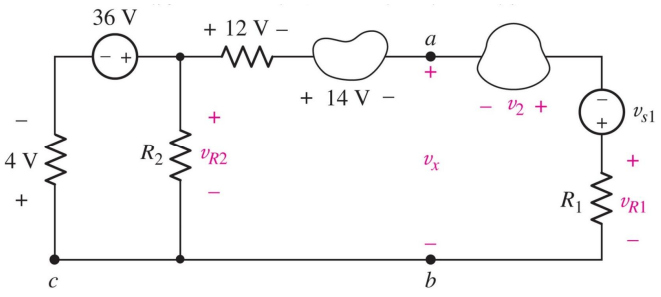


• Practice 3.2 Find v_x and i_x.

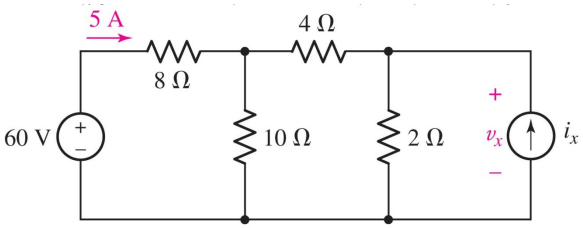


KVL

• Example 3.3 Find v_{R2} and v_x.

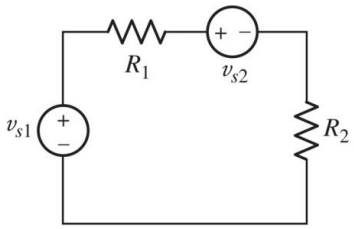


• Example 3.4 Find v_x and i_x.



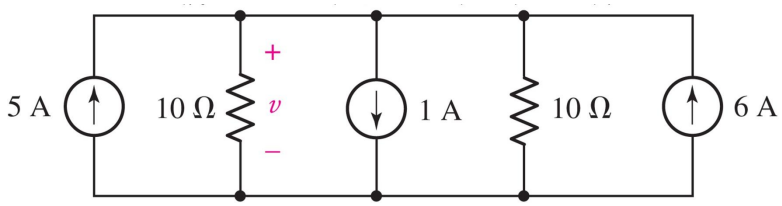
Series Connection 직렬 연결

- All elements in a circuit that carry the same current are said to be connected in series.
  - 직렬 연결된 소자는 동일한 전류 값을 공유한다.
  - 하나의 node를 두 개의 branch(혹은 회로소자) 만이 공유할 때 두 소자는 직렬연결 된다.



Parallel Connection 병렬 연결

- Elements in a circuit having a common voltage across them are said to be connected in parallel.
  - 병렬 연결된 소자는 동일한 전압 값을 공유한다.
  - 한 쌍의 node를 공유하는 두 개 이상의 branch는 병렬연결 된다.



감전 사고

Physiological reactions to current levels in humans

Physiological reaction	Current
Barely perceptive	3 ~ 5 [mA]
Extreme pain	30 ~ 50 [mA]
Muscle paralysis	50 ~ 70 [mA]
Heart stoppage	500 [mA]

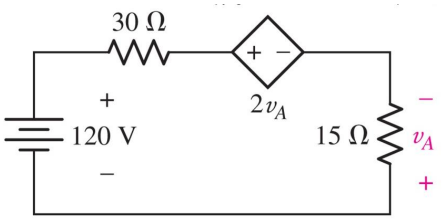
감전에 의한 인체의 위험도
1. 전류의 크기
2. 통전 시간
3. 통전 경로
4. 전원의 종류

- 감전의 위험도 ∝ (접촉전압의 크기) × (감전시간)
- 허용 접촉 전압
  - ✓ 일반 환경: 50 [V]
  - ✓ 젖어 있는 상태: 25 [V]
  - ✓ 물속: 2.5[V] 이하
- 평균 인체 저항: 500 ~ 5,000 [Ω]

Single-loop circuit

∨ A circuit where all circuit elements (or branches) are series-connected.

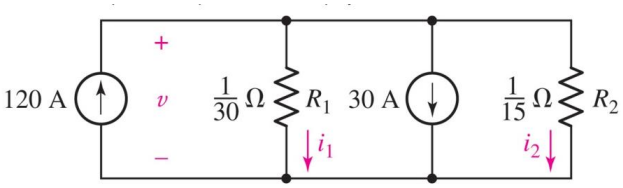
• Example 3.5 Compute powers.



Single-node-pair circuit

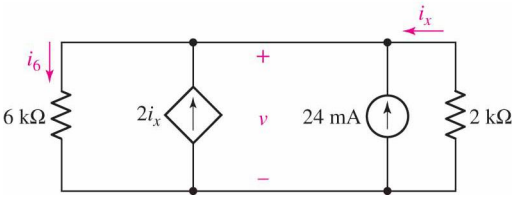
∨ A circuit where all branches share a pair of nodes.

• Example 3.6 Compute all circuit variables.

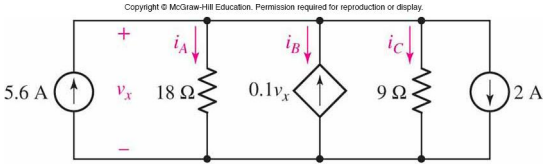


Single-node-pair circuit

• Example 3.7 Find the voltage, v.

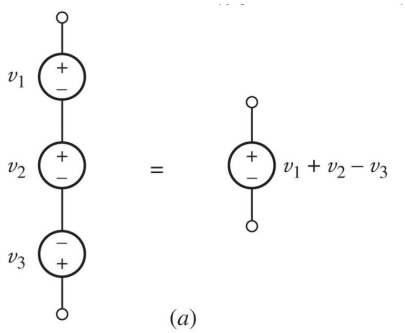


• Practice 3.8 Find i_A.

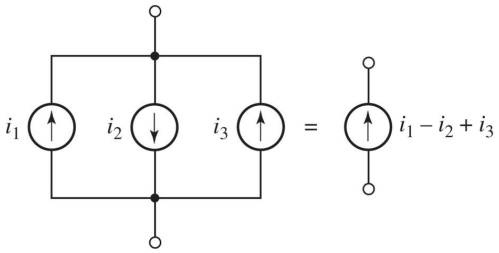


Series & parallel connection of sources

- Voltage sources connected in series can be combined into an equivalent voltage source:

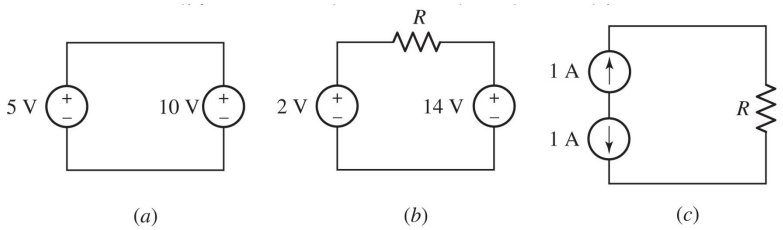


- Current sources connected in parallel can be combined into an equivalent current source:



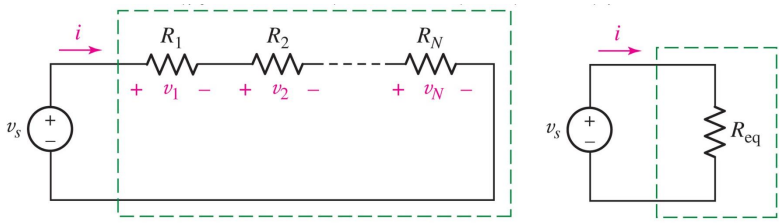
Impossible circuits

- Parallel-connected voltage sources / series-connected current sources



❖ 어떤 전자기기의 전원부는 1.5[V] battery 4개가 병렬연결 되도록 구성되어 있다. 찾아보니, 새 건전지가 2개밖에 없어서, 나머지 2개는 사용하던 건전지를 사용하였다. 어떤 문제점이 있을까?

Series connection of resistors

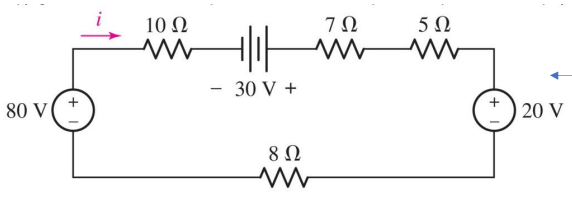


KVL: $v_1 + v_2 + \cdots + v_N = v_s$, $(R_1 + R_2 + \cdots + R_N)i = R_{eq}i = v_s$

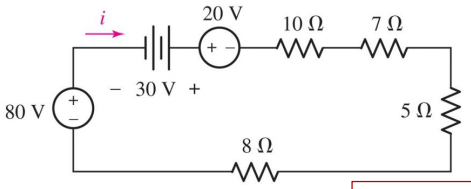
- 등가 저항 (equivalent resistance)
  - ✓ Terminal a-b에서 바라본 등가저항
  - ✓ 동일한 전류-전압 특성

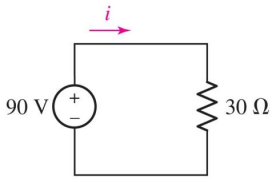
Series connection of resistors

- Example 3.11 Find i and the power supplied by the 80 $[V]$ voltage source.



Single-loop circuit에서는 회로소자의 상대적인 위치를 바꾸더라도 회로변수의 값은 변하지 않는다.

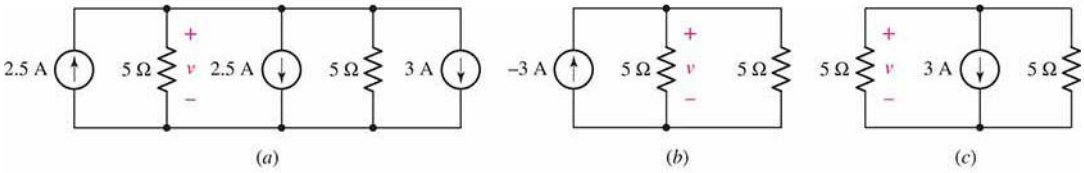




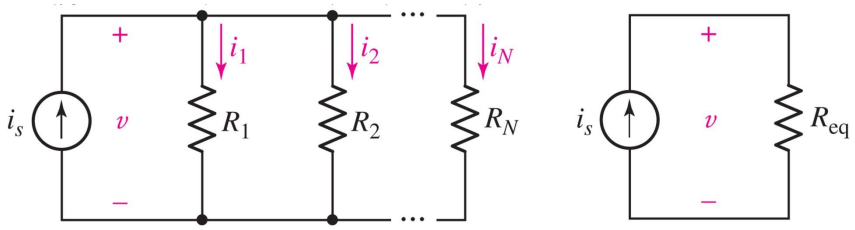
직렬 연결된 전압원과 저항 소자의 결합

Series connection of resistors

• Example 3.12 Find the power supplied by the voltage source.



Parallel connection of resistors



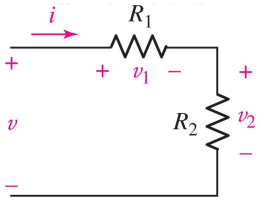
KCL: $i_1 + i_2 + \cdots + i_N = i_s$, $\left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}\right)v = \frac{1}{R_{eq}}v = i_s$

• Two resistors in parallel

$\vee \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$, $R_{eq} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}$

Voltage division

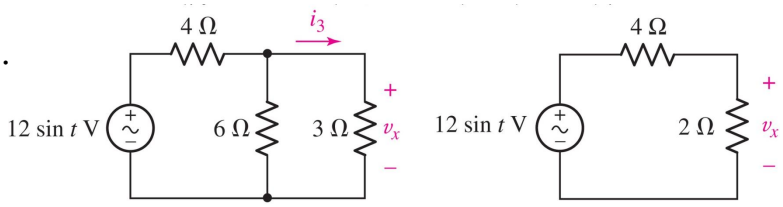
- Resistors in series "share" the applied voltage.



$v_1 = R_1 i,\ v_2 = R_2 i,\ v_1 + v_2 = v,$ and $i = \frac{v}{R_1+R_2}$

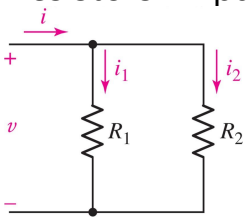
Thus, $v_1 = \frac{R_1}{R_1+R_2} v$ and $v_2 = \frac{R_2}{R_1+R_2} v$

- Example 3.13 Find v_x.



Current division

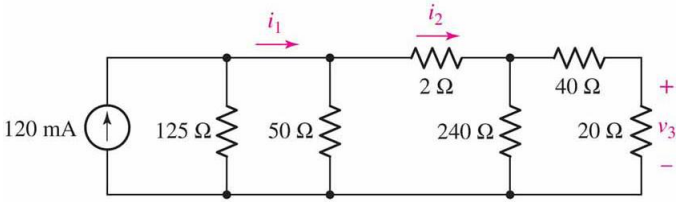
- Resistors in parallel "share" current through them.



$i_1 = \frac{v}{R_1},\ i_2 = \frac{v}{R_2},\ i_1 + i_2 = i,$ and $v = (R_1 \parallel R_2)i = \frac{R_1 R_2}{R_1+R_2} i$

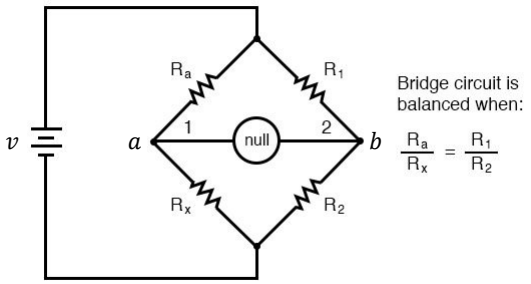
Thus, $i_1 = \frac{R_2}{R_1+R_2} i$ and $i_2 = \frac{R_1}{R_1+R_2} i$

- Practice 3.14 Find v_3.



Wheatstone bridge

- How to measure the resistance of medium range: $1 \sim 10^6[\Omega]$.
  - 4 resistors, R_1, R_2, R_a, known, while R_x unknown.
  - dc voltage source and current detector.



When $i_{ab} = 0$,

(1) $i_a = i_x$ and $i_1 = i_2$

(2) $R_a i_a = R_1 i_1$ and $R_x i_x = R_2 i_2$

(3) $\frac{R_x}{R_2} = \frac{i_2}{i_x} = \frac{i_1}{i_a} = \frac{R_a}{R_1}$

$R_x = \frac{R_2}{R_1} R_a$

Ground, 접지

- A point with zero electrical potential.
  - Reference to all other voltages