2장 6절 연습문제 풀이

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1. $x = \ln 2$

$$\sinh x = \frac{e^x - e^{-x}}{2} = \frac{e^{\ln 2} - e^{-\ln 2}}{2} = \frac{2 - \frac{1}{2}}{2} = \frac{3}{4}$$
$$\cosh x = \frac{e^x + e^{-x}}{2} = \frac{e^{\ln 2} + e^{-\ln 2}}{2} = \frac{2 + \frac{1}{2}}{2} = \frac{5}{4}$$
$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{3/4}{5/4} = \frac{3}{5}.$$

3. $\sinh x = \frac{5}{12}$

$$\cosh^2 x - \sinh^2 x = 1 \Longrightarrow \cosh x = \sqrt{1 + \sinh^2 x} = \sqrt{1 + \frac{25}{144}} = \frac{13}{12}.$$

4. $\cosh x = \frac{5}{3} \Longrightarrow \text{sech } x = \frac{3}{5}$

$$\tanh x = \pm\sqrt{1 - \text{sech }^2 x} = \pm\sqrt{1 - \frac{9}{25}} = \pm\frac{4}{5}.$$

5. $\sinh x = \frac{12}{5}$

$$\text{csch } x = \frac{1}{\sinh x} = \frac{5}{12}$$
$$\cosh x = \sqrt{1 + \sinh^2 x} = \sqrt{1 + \frac{144}{25}} = \frac{13}{5}, \qquad \text{sech } x = \frac{1}{\cosh x} = \frac{5}{13}$$
$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{12/5}{13/5} = \frac{12}{13}, \qquad \coth x = \frac{1}{\tanh x} = \frac{13}{12}.$$

6. $\tanh x = -\frac{4}{5}$

$$\coth x = \frac{1}{\tanh x} = -\frac{5}{4}$$
$$\text{sech } x = \sqrt{1-\tanh^2 x} = \sqrt{1-\frac{16}{25}} = \frac{3}{5}, \qquad \cosh x = \frac{1}{\text{sech } x} = \frac{5}{3}$$
$$\sinh x = \cosh x \tanh x = \frac{5}{3}(-\frac{4}{5}) = -\frac{4}{3}, \qquad \text{csch } x = \frac{1}{\sinh x} = -\frac{3}{4}.$$

19.

$$y = 2\sinh 3x + 3\cosh 2x \Longrightarrow y' = 6\cosh 3x + 6\sinh 2x.$$

20.

$$y = \ln\cosh 3x \Longrightarrow y' = \frac{3\sinh 3x}{\cosh 3x} = 3\tanh 3x$$

21.

$$y = x\sinh x \Longrightarrow y' = \sinh x + x\cosh x.$$

22.

$$y = \sinh x\cosh 2x \Longrightarrow y' = \cosh x\cosh 2x + 2\sinh x\sinh 2x.$$

23.

$$y = \sinh^2 x \Longrightarrow y' = 2\sinh x\cosh x = \sinh 2x.$$

24.

$$\begin{aligned} y &= e^{-x}\sinh 2x + \cosh^n(\frac{x}{n}) \\ \Longrightarrow y' &= -e^{-x}\sinh 2x + 2e^{-x}\cosh 2x + n\cosh^{n-1}(\frac{x}{n})\sinh(\frac{x}{n})\frac{1}{n} \\ &= -e^{-x}\sinh 2x + 2e^{-x}\cosh 2x + \cosh^{n-1}(\frac{x}{n})\sinh(\frac{x}{n}) \end{aligned}$$

25.

$$y = x^2\ln\sinh x^2 \Longrightarrow y' = 2x\ln\sinh x^2 + x^2\frac{2x\cosh x^2}{\sinh x^2} = 2x\ln\sinh x^2 + 2x^3\tanh x^2$$

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$$\begin{aligned} y &= e^x\sinh 2x + \coth(1-x^2) \\ \Longrightarrow y' &= e^x\sinh 2x + 2e^x\cosh 2x - (-2x)\text{csch}^2(1-x^2) \\ &= e^x\sinh 2x + 2e^x\cosh 2x + 2x\text{csch}^2(1-x^2) \end{aligned}$$

27.

$$y = e^x \sinh x \Longrightarrow y' = e^x \sinh x + e^x \cosh x = e^x(\cosh x + \sinh x) = e^{2x}.$$

28.

$$\begin{aligned} y &= \sinh(\sin x) + e^{\sinh x} \\ \Longrightarrow y' &= \cosh(\sin x) \cdot \cos x + e^{\sinh x} \cosh x \end{aligned}$$

29.

$$y = \operatorname{sech}^3 \frac{3x}{2} \Longrightarrow y' = 3\operatorname{sech}^2 \frac{3x}{2} \cdot (-\operatorname{sech} \frac{3x}{2} \tanh \frac{3x}{2}) \cdot \frac{3}{2} = -\frac{9}{2} \operatorname{sech}^3 \frac{3x}{2} \tanh \frac{3x}{2}$$

31.

$$y = \sinh(\ln x^2) \longrightarrow y' = \frac{2\cosh(\ln x^2)}{x}$$

33.

$$\begin{aligned} x\cosh y + y \sinh x = 1 &\Longrightarrow \cosh y + x \sinh y \cdot y' + y' \sinh x + y \cosh x = 0 \\ &\Longrightarrow y'(x \sinh y + \sinh x) = -(\cosh y + y \cosh x) \\ &\Longrightarrow y' = -\frac{\cosh y + y \cosh x}{x \sinh y + \sinh x} \end{aligned}$$