국방정보 공학과 2020032306 송민경 LA_HW2

Given $A = \begin{bmatrix} 1 & 1 & -1 & 0 & 2 \\ 0 & 2 & -1 & 0 & 0 \\ 1 & 2 & 0 & 1 & 2 \\ 0 & 0 & -2 & 1 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

We can find A^{-1} by applying the Gauss-Jordan process to augmented matrix $[A : I_5]$.

$$[A : I_5] = \left[\begin{array}{ccccc:ccccc} 1 & 1 & -1 & 0 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 1 & 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

<1> $R_2 \leftrightarrow R_3$; $(R_2 - R_1) \rightarrow R_2$

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 1 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

<2> $(R_3 - 2R_2) \rightarrow R_3$

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -3 & -2 & 0 & 2 & 1 & -2 & 0 & 0 \\ 0 & 0 & -2 & 1 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

<3> $R_3 \times (-\frac{1}{3}) \rightarrow R_3$, $R_4 \leftrightarrow R_5$

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{2}{3} & 0 & -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -2 & 1 & 2 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

<4> $(R_5 + 2R_3) \rightarrow R_5$, $(R_5 - \frac{7}{3}R_4) \rightarrow R_5$, $R_5 \times (\frac{1}{2}) \rightarrow R_5$

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{2}{3} & 0 & -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 & -\frac{4}{3} & -\frac{2}{3} & \frac{4}{3} & 1 & -\frac{7}{3} \end{bmatrix}$$

<5> $R_5 \times (\frac{1}{2}) \rightarrow R_5$, $(R_3 - \frac{2}{3}R_4) \rightarrow R_3$

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} & 0 & -\frac{2}{3} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} & \frac{1}{2} & -\frac{7}{6} \end{bmatrix}$$

<6> $(R_2 - R_3) \rightarrow R_2$, $(R_2 - R_4) \rightarrow R_2$

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & 0 & 0 & -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} & 0 & -\frac{2}{3} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} & \frac{1}{2} & -\frac{7}{6} \end{bmatrix}$$

<7> $(R_1 - R_2) \rightarrow R_1$, $(R_1 + R_3) \rightarrow R_1$, $(R_1 - 2R_5) \rightarrow R_1$

$$\left[\begin{array}{ccccc:ccccc} 1 & 0 & 0 & 0 & 0 & 2 & 0 & -1 & -1 & 2 \\ 0 & 1 & 0 & 0 & 0 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & 0 & 0 & -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} & 0 & -\frac{2}{3} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} & \frac{1}{2} & -\frac{7}{6} \end{array}\right] = [I_5 : A^{-1}]$$

$$\therefore\ A^{-1} = \begin{bmatrix} 2 & 0 & -1 & -1 & 2 \\ -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & -\frac{1}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} & 0 & -\frac{2}{3} \\ 0 & 0 & 0 & 0 & 1 \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} & \frac{1}{2} & -\frac{7}{6} \end{bmatrix}$$