

## 4장 2절 연습문제 풀이

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1.  $r = a \sec 2\theta; \theta = \frac{\pi}{8}$

$$\begin{aligned}\tan \psi &= \frac{r}{r'} = \frac{a \sec 2\theta}{2a \sec 2\theta \tan 2\theta} = \frac{1}{2 \tan 2\theta} \\ \theta = \frac{\pi}{8} &\implies \tan \psi = \frac{1}{2 \tan \frac{\pi}{4}} = \frac{1}{2} \\ &\implies \psi = \tan^{-1}\left(\frac{1}{2}\right).\end{aligned}$$

2.  $r = \frac{a}{1-\cos \theta}; \theta = \frac{\pi}{3}$

$$\begin{aligned}\tan \psi &= \frac{r}{r'} = \frac{\frac{a}{1-\cos \theta}}{-\frac{a \sin \theta}{(1-\cos \theta)^2}} = -\frac{1-\cos \theta}{\sin \theta} \\ \theta = \frac{\pi}{3} &\implies \tan \psi = -\frac{1-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}} \\ &\implies \psi = \frac{5\pi}{6}.\end{aligned}$$

3.  $r = a \sin \frac{\theta}{2}; \theta = \frac{\pi}{2}$ .

$$\begin{aligned}\tan \psi &= \frac{r}{r'} = \frac{a \sin \frac{\theta}{2}}{\frac{a}{2} \cos \frac{\theta}{2}} = 2 \tan \frac{\theta}{2} \\ \theta = \frac{\pi}{2} &\implies \tan \psi = 2 \tan \frac{\pi}{4} = 2 \\ &\implies \psi = \tan^{-1}(2).\end{aligned}$$

4.  $r = a \tan 2\theta; \theta = \frac{\pi}{16}.$

$$\begin{aligned}\tan \psi &= \frac{r}{r'} = \frac{a \tan 2\theta}{2a \sec^2 2\theta} \\ &= \frac{1}{2} \tan 2\theta \cos^2 2\theta = \frac{1}{2} \sin 2\theta \cos 2\theta = \frac{1}{4} \sin 2\theta \\ \theta = \frac{\pi}{4} &\implies \tan \psi = \frac{1}{4} \sin \frac{\pi}{4} = \frac{\sqrt{2}}{8} \\ &\implies \psi = \tan^{-1}\left(\frac{\sqrt{2}}{8}\right)\end{aligned}$$

5.  $r = \theta^2; \theta = \pi$

$$\begin{aligned}\tan \psi &= \frac{r}{r'} = \frac{\theta^2}{2\theta} = \frac{\theta}{2} \\ \theta = \pi &\implies \tan \psi = \frac{\pi}{2} \\ &\implies \psi = \tan^{-1} \frac{\pi}{2}.\end{aligned}$$

6.  $r = e^{3\theta}; \theta = \theta_1$

$$\begin{aligned}\tan \psi &= \frac{r}{r'} = \frac{e^{3\theta}}{3e^{3\theta}} = \frac{1}{3} \\ &\implies \psi = \tan^{-1} \frac{1}{3}.\end{aligned}$$

7.  $r = a \sin \theta$

$$\begin{aligned}\tan \psi &= \frac{r}{r'} = \frac{a \sin \theta}{a \cos \theta} = \tan \theta \\ &\implies \psi = \theta, \quad \alpha = \theta + \psi = 2\theta.\end{aligned}$$

9.  $r = \tan \theta; \theta = \frac{\pi}{4}$

$$\begin{aligned}\tan \psi &= \frac{r}{r'} = \frac{\tan \theta}{\sec^2 \theta} \\ \theta = \frac{\pi}{4} &\implies \tan \psi = 2 \implies \psi = \tan^{-1}(2) \\ \tan \alpha &= \tan(\psi + \theta) = \frac{\tan \psi + \tan \theta}{1 - \tan \psi \tan \theta} = \frac{2 + 1}{1 - 2 \cdot 1} = -3\end{aligned}$$

10.  $r = \sin 3\theta; \theta = \frac{\pi}{6}$

$$\begin{aligned}\tan \psi &= \frac{r}{r'} = \frac{\sin 3\theta}{3 \cos 3\theta} = \frac{1}{3} \tan 3\theta \\ \theta = \frac{\pi}{6} &\implies \tan \psi = \frac{1}{3} \tan \frac{\pi}{2} = \infty \implies \psi = \frac{\pi}{2} \\ \alpha &= \theta + \psi = \frac{\pi}{6} + \frac{\pi}{2} = \frac{2\pi}{3} \\ \tan \alpha &= \tan \frac{2\pi}{3} = -\sqrt{3}.\end{aligned}$$

11.  $r = \sin^2 \theta; \theta = \frac{\pi}{3}$

$$\begin{aligned}\tan \psi &= \frac{r}{r'} = \frac{\sin^2 \theta}{2 \sin \theta \cos \theta} = \frac{1}{2} \frac{\sin \theta}{\cos \theta} = \frac{1}{2} \tan \theta \\ \theta = \frac{\pi}{3} &\implies \tan \psi = \frac{1}{2} \tan \frac{\pi}{3} = \frac{\sqrt{3}}{2} \\ \tan \alpha &= \tan(\psi + \theta) = \frac{\tan \psi + \tan \theta}{1 - \tan \psi \tan \theta} = \frac{\frac{\sqrt{3}}{2} + \sqrt{3}}{1 - \frac{\sqrt{3}}{2} \cdot \sqrt{3}} = 3\sqrt{3}\end{aligned}$$

12.  $r = 1 - \cos \theta; \theta = \frac{\pi}{4}$

$$\begin{aligned}\tan \psi &= \frac{r}{r'} = \frac{1 - \cos \theta}{\sin \theta} \\ \theta = \frac{\pi}{4} &\implies \tan \psi = \frac{1 - \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2 - \sqrt{2}}{\sqrt{2}} = \sqrt{2} - 1 \\ \tan \alpha &= \tan(\psi + \theta) = \frac{\tan \psi + \tan \theta}{1 - \tan \psi \tan \theta} = \frac{\sqrt{2} - 1 + 1}{1 - (\sqrt{2} - 1)} = \frac{\sqrt{2}}{2 - \sqrt{2}}\end{aligned}$$

13.  $r = \cos 2\theta; \theta = 0$

$$\begin{aligned}\tan \psi &= \frac{r}{r'} = \frac{\cos 2\theta}{-2 \sin 2\theta} \\ \theta = 0 &\implies \tan \psi = \infty \implies \psi = -\frac{\pi}{2} \\ \alpha &= \psi + \theta = -\frac{\pi}{2} \implies \tan \alpha = -\infty\end{aligned}$$

15.  $r^2 = \cos 2\theta; \theta = \frac{\pi}{6}$

$$\begin{aligned}\tan \psi &= \frac{r}{r'} \\ r^2 &= \cos 2\theta \implies 2rr' = -2 \sin 2\theta \implies r' = -\frac{\sin 2\theta}{r} \\ &\implies \tan \psi = \frac{r}{r'} = \frac{r}{-\frac{\sin 2\theta}{r}} = -\frac{r^2}{\sin 2\theta} = -\frac{\cos 2\theta}{\sin 2\theta} \\ \theta &= \frac{\pi}{6} \implies \tan \psi = -\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}\end{aligned}$$

16.  $r\theta = 2; \theta = \pi$

$$\begin{aligned}\tan \psi &= \frac{r}{r'} = \frac{2/\theta}{-2/\theta^2} = -\theta \\ \theta &= \pi \implies \tan \psi = -\pi \\ \tan \alpha &= \tan(\psi + \theta) = \frac{\tan \psi + \tan \theta}{1 - \tan \psi \tan \theta} \\ &= \frac{-\pi + \tan \pi}{1 + \pi \tan \pi} = -\pi\end{aligned}$$

17.

$$\begin{cases} r = 1 + \cos \theta \\ r = 1 - \cos \theta \end{cases} \quad \text{그림은 각자가}$$

$$1 + \cos \theta = 1 - \cos \theta \implies \cos \theta = 0 \implies \theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

교점은  $(1, \frac{\pi}{2}), (1, \frac{3\pi}{2})$ , 극점.

18.

$$\begin{cases} r = \sin \theta \\ r = \sin 2\theta \end{cases} \quad \text{교점은 3개}$$

$$\sin \theta = \sin 2\theta \implies \sin \theta = 2 \sin \theta \cos \theta$$

$$\implies \sin \theta (2 \cos \theta - 1) = 0 \implies \sin \theta = 0 \text{ or } \cos \theta = \frac{1}{2}$$

$$\implies \theta = 0, \pi, \pm \frac{\pi}{3}$$

교점은  $(0, 0), (0, \pi), (\frac{\sqrt{3}}{2}, \frac{\pi}{3}), (-\frac{\sqrt{3}}{2}, -\frac{\pi}{3}) = (\frac{\sqrt{3}}{2}, \frac{2\pi}{3})$

19.

$$\begin{cases} r = 1 + \cos \theta \\ r = 1 - \sin \theta \end{cases} \quad \text{교점은 3개}$$

$$1 + \cos \theta = 1 - \sin \theta \implies \tan \theta = -1 \implies \theta = \frac{3\pi}{4}, \frac{7\pi}{4}$$

교점은 극점,  $(1 - \frac{\sqrt{2}}{2}, \frac{3\pi}{4}), (1 - \frac{\sqrt{2}}{2}, \frac{7\pi}{4})$

20.

$$\begin{cases} r = \tan \theta \\ r = \cot \theta \end{cases} \quad \text{교점은 4개}$$

$$\tan \theta = \cot \theta \implies \tan^2 \theta = 1$$

$$\implies \tan \theta = \pm 1$$

$$\implies \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

교점은  $(1, \frac{\pi}{4}), (-1, \frac{3\pi}{4}), (1, \frac{5\pi}{4}), (-1, \frac{7\pi}{4})$ .

21.

$$\begin{cases} r = \cos \theta \\ r^2 = 2 + \cos \theta \end{cases} \quad \text{교점은 1개}$$

$$\cos^2 \theta = 2 + \cos \theta \implies (\cos \theta - 2)(\cos \theta + 1) = 0$$

$$\cos \theta = -1 \implies \theta = \pi$$

교점은  $(-1, \pi)$ .

22.

$$\begin{cases} r = \sin 2\theta \\ r = 1 + \cos 2\theta \end{cases} \quad \text{교점은 4개}$$

$$\begin{aligned} \sin 2\theta = 1 + \cos 2\theta &\implies \sin 2\theta = 1 + 2\cos^2 \theta - 1 \\ &\implies 2\cos^2 \theta = \sin 2\theta = 2\sin \theta \cos \theta \\ &\implies \cos \theta (\cos \theta - \sin \theta) = 0 \\ &\implies \cos \theta = 0, \tan \theta = 1 \\ &\implies \theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{5\pi}{4}. \end{aligned}$$

교점은  $(0, \frac{\pi}{2}), (0, \frac{3\pi}{2}), (1, \frac{\pi}{4}), (1, \frac{5\pi}{4})$ .

23.

$$\begin{cases} r^2 \sin 2\theta = 8 \\ r = 2 \sec \theta \end{cases} \quad \text{교점은 1개}$$

$r^2 \sin 2\theta = 8$  을 직교방정식으로 고치면  $xy = 4$  이고  $r = 2 \sec \theta$  는  $x = 2$  이다. 따라서 두식의 교점의 직교좌표는  $(2, 2)$  이고 이를 극좌표로 쓰면  $(2\sqrt{2}, \frac{\pi}{4})$  이다.

24.  $\begin{cases} r = \sec \theta \tan \theta \\ r = \csc \theta \cot^2 \theta \end{cases}$  의 교점은 3개.

두식의 직교방정식은 각각  $y = x^2, y^3 = x^2$  이다.

$$\begin{aligned} \sec \theta \tan \theta = \csc \theta \cot^2 \theta &\implies \frac{\sin \theta}{\cos^2 \theta} = \frac{\cos^2 \theta}{\sin^3 \theta} \implies \sin^4 \theta - \cos^4 \theta = 0 \\ &\implies \sin^2 \theta - \cos^2 \theta = 0 \implies \cos 2\theta = 0 \\ &\implies \theta = \pm \frac{\pi}{4} \end{aligned}$$

따라서 교점은  $(0, 0), (\sqrt{2}, \pm \frac{\pi}{4})$ .

25.

$$\begin{cases} r = 6 \cos \theta \\ r = 2(1 + \cos \theta) \end{cases} \quad \text{교점은 3개}$$

교점은; 극점,  $\theta = \pm \frac{\pi}{3}$

극점에서의 교각은  $\frac{\pi}{2}$  이고 위의 그림은  $x$  축 대칭이므로  $\theta = \frac{\pi}{3}$  에서의 교각만 구해도 된다.

$$\begin{aligned} \tan \psi_1 &= \frac{r_1}{r'_1} = \frac{6 \cos \theta}{-6 \sin \theta} = -\cot \theta \\ \theta &= \frac{\pi}{3} \implies \tan \psi_1 = -\frac{1}{\sqrt{3}}. \\ \tan \psi_2 &= \frac{r_2}{r'_2} = \frac{2(1 + \cos \theta)}{-2 \cos \theta} = \frac{2(1 + \frac{\sqrt{3}}{2})}{-2 \cdot \frac{\sqrt{3}}{2}} = -\frac{2 + \sqrt{3}}{\sqrt{3}} \\ \tan \phi &= \tan |\psi_1 - \psi_2| = \frac{|\tan \psi_1 - \tan \psi_2|}{1 + \tan \psi_1 \tan \psi_2} \\ &= \frac{-\frac{1}{\sqrt{3} + \frac{2 + \sqrt{3}}{\sqrt{3}}}}{1 + (-\frac{1}{\sqrt{3}}) \cdot (-\frac{2 + \sqrt{3}}{\sqrt{3}})} = \frac{\sqrt{3}(1 + \sqrt{3})}{5 + \sqrt{3}}. \end{aligned}$$

26.

$$\begin{cases} r = a \sin \theta \\ r = a \cos \theta \end{cases} \quad \text{교점은 2개}$$

교점은 극점과  $\theta = \frac{\pi}{4}$ .

극점에서의 교각은  $\frac{\pi}{2}$ .

$$\begin{aligned} \tan \psi_1 &= \frac{r_1}{r'_1} = \frac{\sin \theta}{\cos \theta} = \tan \theta \implies \psi_1 = \theta \\ \tan \psi_2 &= \frac{r_2}{r'_2} = \frac{\cos \theta}{-\sin \theta} = -\cot \theta \\ \theta &= \frac{\pi}{4} \implies \psi_2 = \frac{3\pi}{4}, \psi_1 = \frac{\pi}{4} \\ \psi &= |\psi_1 - \psi_2| = \left| \frac{\pi}{4} - \frac{3\pi}{4} \right| = \frac{\pi}{2}. \end{aligned}$$

27.

$$\begin{cases} r = 2 \csc \theta & \Leftrightarrow y = 2 \\ r = \sec^2 \frac{\theta}{2} & \Leftrightarrow \sqrt{x^2 + y^2} + x = 2 \end{cases} \quad \text{교점은 1개}$$

교점은  $\theta = \frac{\pi}{2}$ .

$$\tan \psi_1 = \frac{r_1}{r'_1} = \frac{2 \csc \theta}{-2 \csc \theta \cot \theta} = -\tan \theta \Rightarrow \psi_1 = -\frac{\pi}{2}$$

$$\tan \psi_2 = \frac{r_2}{r'_2} = \frac{\sec^2 \frac{\theta}{2}}{\sec^2 \frac{\theta}{2} \tan \frac{\theta}{2}} = \cot \frac{\theta}{2}$$

$$\theta = \frac{\pi}{2} \Rightarrow \psi_2 = \frac{\pi}{4}, \psi_1 = -\frac{\pi}{2}$$

$$\psi = |\psi_1 - \psi_2| = \left| -\frac{\pi}{2} - \frac{\pi}{4} \right| = \frac{3\pi}{4} \text{ or } \frac{\pi}{4}$$

28. 그림그려 보세요  $\pi \pi$

$$\begin{cases} r = \tan \theta \\ r = \cot \theta \end{cases} \quad \text{교점은 5개(극점포함)}$$

교점을 구하기 위하여

$$\tan \theta = \cot \theta \Rightarrow \tan^2 \theta = 1 \Rightarrow \tan \theta = \pm 1 \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}.$$

극점에서의 교각은  $\frac{\pi}{2}$ .

$$\tan \psi_1 = \frac{r_1}{r'_1} = \frac{\tan \theta}{\sec^2 \theta} = \tan \theta$$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4} \Rightarrow \tan \psi_1 = \frac{1}{2}, \quad \theta = \frac{3\pi}{4}, \frac{7\pi}{4} \Rightarrow \tan \psi_1 = -\frac{1}{2}$$

$$\tan \psi_2 = \frac{r_2}{r'_2} = \frac{\cot \theta}{-\csc^2 \theta}$$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4} \Rightarrow \tan \psi_2 = -\frac{1}{2}, \quad \theta = \frac{3\pi}{4}, \frac{7\pi}{4} \Rightarrow \tan \psi_2 = \frac{1}{2}$$

$$\tan \phi = \left| \frac{\tan \psi_1 - \tan \psi_2}{1 + \tan \psi_1 \tan \psi_2} \right| = \frac{4}{3}$$



31.

$$\begin{cases} r = 1 + \cos \theta \\ r = 1 - \cos \theta \end{cases} \quad \text{교점은 3개}$$

교점은 극점과  $\theta = \pm \frac{\pi}{3}$  이고 극점에서의 교각은 0.

$$\tan \psi_1 = \frac{r_1}{r'_1} = \frac{1 + \cos \theta}{-\sin \theta}$$

$$\tan \psi_2 = \frac{r_2}{r'_2} = \frac{1 - \cos \theta}{\sin \theta}$$

$$\theta = \frac{\pi}{3} \implies \tan \psi_1 = -\sqrt{3}, \tan \psi_2 = \frac{1}{\sqrt{3}}.$$

$$\tan \phi = \tan |\psi_1 - \psi_2| = \frac{|\tan \psi_1 - \tan \psi_2|}{1 + \tan \psi_1 \tan \psi_2}$$

$$= \frac{|\frac{1}{\sqrt{3}} + \sqrt{3}|}{1 - \frac{1}{\sqrt{3}} \cdot \sqrt{3}} = \infty$$

$$\implies \phi = \frac{\pi}{2}.$$