

• The Cross Product

Def. The cross product of

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \text{ and } v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

is $u \times v$ defined by

$$\begin{bmatrix} u_2 v_3 - u_3 v_2 \\ u_3 v_1 - u_1 v_3 \\ u_1 v_2 - u_2 v_1 \end{bmatrix} = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$= i \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} - j \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} + k \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$$

Rem. a) $(u \times v) \cdot u = 0$

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b) $\|u \times v\| = \|u\| \|v\| \sin \theta$

$A = \frac{1}{2} \|u \times v\|$

