Chapter 7

1. (a) The speed of electron is calculated from the equation $v^2 = v_0^2 + 2a\Delta x$:

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{(1.4\times10^7\ \text{m/s})^2 + 2(2.8\times10^{15}\ \text{m/s}^2)(0.058\ \text{m})} = 2.282\times10^7\ \text{m/s}$$
$$\approx 2.3\times10^7\ \text{m/s.}$$

(b) The initial kinetic energy of electron is calculated as

$$K_i = \frac{1}{2}mv_0^2 = \frac{1}{2}(9.1\times10^{-31}\ \text{kg})(1.4\times10^7\ \text{m/s})^2 = 8.918\times10^{-17}\ \text{J.}$$

The final kinetic energy of the electron is calculated as

$$K_f = \frac{1}{2}mv^2 = \frac{1}{2}(9.1\times10^{-31}\ \text{kg})(2.282\times10^7\ \text{m/s})^2 = 23.70\times10^{-17}\ \text{J.}$$

The change in kinetic energy is $\Delta K = 23.70 \times 10^{-17}$ J $-$ 8.918×10^{-17} J $= 1.5 \times 10^{-16}$ J.

2. With speed v = 11200 m/s, we find

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(2.9\times10^5 \text{ kg})\ (11200 \text{ m/s})^2 = 1.8\times10^{13} \text{ J}.$$

3. (a) The change in kinetic energy for the meteorite would be

$$\Delta K = K_f - K_i = -K_i = -\frac{1}{2} m_i v_i^2 = -\frac{1}{2}\left(4\times10^6 \text{ kg}\right)\left(15\times10^3 \text{ m/s}\right)^2 = -5\times10^{14} \text{ J},$$

or $|\Delta K| = 5\times10^{14}$ J. The negative sign indicates that kinetic energy is lost.

(b) The energy loss in units of megatons of TNT would be

$$-\Delta K = \left(5\times10^{14} \text{ J}\right)\left(\frac{1 \text{ megaton TNT}}{4.2\times10^{15} \text{ J}}\right) = 0.1 \text{megaton TNT.}$$

(c) The number of bombs N that the meteorite impact would correspond to is found by noting that megaton = 1000 kilotons and setting up the ratio:

$$N = \frac{0.1\times1000 \text{kiloton TNT}}{13 \text{kiloton TNT}} = 8.$$

4. The work done by the applied force $\vec{F}_a$ is given by $W = \vec{F}_a \cdot \vec{d} = F_a d \cos\phi$. From the figure, we see that $W = 25\ \text{J}$ when $\phi = 0$ and $d = 5.0\ \text{cm}$. This yields the magnitude of $\vec{F}_a$:

$$F_a = \frac{W}{d} = \frac{25\ \text{J}}{0.050\ \text{m}} = 5.0\times10^2\ \text{N}.$$

(a) For $\phi = 64°$, we have $W = F_a d\cos\phi = (5.0\times10^2\ \text{N})(0.050\ \text{m})\cos 64° = 11\ \text{J}$.

(b) For $\phi = 147°$, we have $W = F_a d\cos\phi = (5.0\times10^2\ \text{N})(0.050\ \text{m})\cos 147° = -21\ \text{J}$.

5. We denote the mass of the father as m and his initial speed v_i. The initial kinetic energy of the father is

$$K_i = \frac{1}{2} K_{\text{son}}$$

and his final kinetic energy (when his speed is $v_f = v_i + 1.0$ m/s) is $K_f = K_{\text{son}}$. We use these relations along with Eq. 7-1 in our solution.

(a) We see from the above that $K_i = \frac{1}{2} K_f$, which (with SI units understood) leads to

$$\frac{1}{2} m v_i^2 = \frac{1}{2}\left[\frac{1}{2} m\,\left(v_i + 1.0\text{ m/s}\right)^2\right].$$

The mass cancels and we find a second-degree equation for v_i : $\frac{1}{2} v_i^2 - v_i - \frac{1}{2} = 0$. The positive root (from the quadratic formula) yields $v_i = 2.4$ m/s.

(b) From the first relation above $\left(K_i = \frac{1}{2} K_{\text{son}}\right)$, we have

$$\frac{1}{2} m v_i^2 = \frac{1}{2}\left(\frac{1}{2}\,(m/2)\,v_{\text{son}}^2\right)$$

and (after canceling m and one factor of 1/2) are led to $v_{\text{son}} = 2v_i = 4.8\text{ m/s}$.

6. We apply the equation $x(t) = x_0 + v_0 t + \frac{1}{2}at^2$, found in Table 2-1. Since at t = 0 s, x_0 = 0, and $v_0 = 12\ \text{m/s}$, the equation becomes (in unit of meters)

$$x(t) = 12t + \tfrac{1}{2}at^2 .$$

With $x = 10\ \text{m}$ when $t = 1.0\ \text{s}$, the acceleration is found to be $a = -4.0\ \text{m/s}^2$. The fact that $a < 0$ implies that the bead is decelerating. Thus, the position is described by $x(t) = 12t - 2.0t^2$. Differentiating x with respect to t then yields

$$v(t) = \frac{dx}{dt} = 12 - 4.0t .$$

Indeed at t =3.0 s, $v(t = 3.0) = 0$ and the bead stops momentarily. The speed at $t = 10\ \text{s}$ is $v(t = 10) = -28\ \text{m/s}$, and the corresponding kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.8\times10^{-2}\,\text{kg})(-28\ \text{m/s})^2 = 7.1\ \text{J}.$$

7. Since this involves constant-acceleration motion, we can apply the equations of Table 2-1, such as $x = v_0 t + \frac{1}{2} a t^2$ (where $x_0 = 0$). We choose to analyze the third and fifth points, obtaining

$$0.2\,\text{m} = v_0(1.0\ \text{s}) + \frac{1}{2}a\ (1.0\ \text{s})^2$$
$$0.8\,\text{m} = v_0(2.0\ \text{s}) + \frac{1}{2}a\ (2.0\ \text{s})^2.$$

Simultaneous solution of the equations leads to $v_0 = 0$ and $a = 0.40\,\text{m/s}^2$. We now have two ways to finish the problem. One is to compute force from $F = ma$ and then obtain the work from Eq. 7-7. The other is to find ΔK as a way of computing W (in accordance with Eq. 7-10). In this latter approach, we find the velocity at $t = 2.0\,\text{s}$ from $v = v_0 + at$ (so $v = 0.80\,\text{m/s}$). Thus,

$$W = \Delta K = \frac{1}{2}(3.0\,\text{kg})(0.80\,\text{m/s})^2 = 0.96\ \text{J}.$$

8. Using Eq. 7-8 (and Eq. 3-23), we find the work done by the water on the ice block:

$$W = \vec{F} \cdot \vec{d} = \left[(210\text{ N})\hat{\text{i}} - (150\text{ N})\hat{\text{j}}\right] \cdot \left[(20\text{ m})\hat{\text{i}} - (16\text{ m})\hat{\text{j}}\right] = (210\text{ N})(20\text{ m}) + (-150\text{ N})(-16\text{ m})$$

$$= 6.6 \times 10^3\text{ J}.$$

9. By the work-kinetic energy theorem,

$$W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(2.0\,\text{kg})\left((6.0\,\text{m/s})^2 - (4.0\,\text{m/s})^2\right) = 20\ \text{J}.$$

We note that the *directions* of $\vec{v}_f$ and $\vec{v}_i$ play no role in the calculation.

10. Equation 7-8 readily yields

$$W = F_x \Delta x + F_y \Delta y = (2.5\text{ N})\cos(100^\circ)(3.0\text{ m}) + (2.5\text{ N})\sin(100^\circ)(4.0\text{ m}) = 8.5\text{ J}.$$

11. From workï kinetic energy theorem, we have

$$\Delta K = W = \vec{F}\cdot\vec{d} = Fd\cos\phi\,.$$

Also we have $F = 22.0\ \text{N}$ and $d = \sqrt{(5.00\ \text{m})^2 + (-3.00\ \text{m})^2 + (4.00\ \text{m})^2} = 7.07\ \text{m}$.

(a) If the change in the particleôs kinetic energy $\Delta K = +45.0\ \text{J}$, then

$$\phi = \cos^{-1}\left(\frac{\Delta K}{Fd}\right) = \cos^{-1}\left(\frac{45.0\ \text{J}}{(22.0\ \text{N})(7.07\ \text{m})}\right) = 73.2°\,.$$

(b) If the change in the particleôs kinetic energy $\Delta K = -45.0\ \text{J}$, then

$$\phi = \cos^{-1}\left(\frac{\Delta K}{Fd}\right) = \cos^{-1}\left(\frac{-45.0\ \text{J}}{(22.0\ \text{N})(7.07\ \text{m})}\right) = 107°\,.$$

12. (a) From Eq. 7-6, $F = W/x = 3.00$ N (this is the slope of the graph).

(b) Equation 7-10 yields $K = K_i + W = 3.00$ J + 6.00 J = 9.00 J.

13. We choose $+x$ as the direction of motion (so $\vec{a}$ and $\vec{F}$ are negative-valued).

(a) Newton's second law readily yields $\vec{F} = (85\,\text{kg})(-2.0\,\text{m/s}^2)$ so that

$$F = |\vec{F}| = 1.7 \times 10^2 \text{ N}.$$

(b) From Eq. 2-16 (with $v = 0$) we have

$$0 = v_0^2 + 2a\Delta x \quad \Rightarrow \quad \Delta x = -\frac{(37\,\text{m/s})^2}{2(-2.0\,\text{m/s}^2)} = 3.4 \times 10^2 \text{ m}.$$

Alternatively, this can be worked using the work-energy theorem.

(c) Since $\vec{F}$ is opposite to the direction of motion (so the angle ϕ between $\vec{F}$ and $\vec{d} = \Delta x$ is 180°) then Eq. 7-7 gives the work done as

$$W = -F\Delta x = -5.8 \times 10^4 \text{ J}.$$

(d) In this case, Newton's second law yields $\vec{F} = (85\,\text{kg})(-4.0\,\text{m/s}^2)$ so that

$$F = |\vec{F}| = 3.4 \times 10^2 \text{ N}.$$

(e) From Eq. 2-16, we now have

$$\Delta x = -\frac{(37\,\text{m/s})^2}{2(-4.0\,\text{m/s}^2)} = 1.7 \times 10^2 \text{ m}.$$

(f) The force $\vec{F}$ is again opposite to the direction of motion (so the angle ϕ is again 180°) so that Eq. 7-7 leads to

$$W = -F\Delta x = -5.8 \times 10^4 \text{ J}.$$

The fact that this agrees with the result of part (c) provides insight into the concept of work.

14. The forces are all constant, so the total work done by them is given by $W = F_{\text{net}}\Delta x$, where F_{net} is the magnitude of the net force and Δx is the magnitude of the displacement. We add the three vectors, finding the x and y components of the net force:

$$\begin{aligned} F_{\text{net}\,x} &= -F_1 - F_2 \sin 50.0° + F_3 \cos 35.0° = -3.00\,\text{N} - (4.00\,\text{N}) \sin 50.0° + (9.00\,\text{N}) \cos 35.0° \\ &= 1.31\,\text{N} \end{aligned}$$

$$\begin{aligned} F_{\text{net}\,y} &= -F_2 \cos 50.0° + F_3 \sin 35.0° = -(4.00\,\text{N})\ \cos 50.0° + (9.00\,\text{N}) \sin 35.0° \\ &= 2.59\,\text{N}. \end{aligned}$$

The magnitude of the net force is

$$F_{\text{net}} = \sqrt{F_{\text{net}\,x}^2 + F_{\text{net}\,y}^2} = \sqrt{(1.31\ \text{N})^2 + (2.59\ \text{N})^2} = 2.90\,\text{N}.$$

The work done by the net force is

$$W = F_{\text{net}} d = (2.90\,\text{N})(4.00\,\text{m}) = 11.6\,\text{J}$$

where we have used the fact that $\vec{d} \parallel \vec{F}_{\text{net}}$ (which follows from the fact that the canister started from rest and moved horizontally under the action of horizontal forces ð the resultant effect of which is expressed by $\vec{F}_{\text{net}}$).

15. (a) The forces are constant, so the work done by any one of them is given by $W = \vec{F} \cdot \vec{d}$, where $\vec{d}$ is the displacement. Force $\vec{F}_1$ is in the direction of the displacement, so

$$W_1 = F_1 d \cos\phi_1 = (5.00\ \text{N})(3.00\ \text{m})\cos 0° = 15.0\ \text{J}.$$

Force $\vec{F}_2$ makes an angle of 120° with the displacement, so

$$W_2 = F_2 d \cos\phi_2 = (9.00\ \text{N})(3.00\ \text{m})\cos 120° = -13.5\ \text{J}.$$

Force $\vec{F}_3$ is perpendicular to the displacement, so

$$W_3 = F_3 d \cos \phi_3 = 0 \text{ since } \cos 90° = 0.$$

The net work done by the three forces is

$$W = W_1 + W_2 + W_3 = 15.0\ \text{J} - 13.5\ \text{J} + 0 = +1.50\ \text{J}.$$

(b) If no other forces do work on the box, its kinetic energy increases by 1.50 J during the displacement.

16. The change in kinetic energy can be written as

$$\Delta K = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}m(2a\Delta x) = ma\Delta x$$

where we have used $v_f^2 = v_i^2 + 2a\Delta x$ from Table 2-1. From the figure, we see that $\Delta K = (0-30)\ \text{J} = -30\ \text{J}$ when $\Delta x = +5\ \text{m}$. The acceleration can then be obtained as

$$a = \frac{\Delta K}{m\Delta x} = \frac{(-30\ \text{J})}{(7.0\ \text{kg})(5.0\ \text{m})} = -0.86\ \text{m/s}^2.$$

The negative sign indicates that the mass is decelerating. From the figure, we also see that when $x = 5\ \text{m}$ the kinetic energy becomes zero, implying that the mass comes to rest momentarily. Thus,

$$v_0^2 = v^2 - 2a\Delta x = 0 - 2(-0.86\ \text{m/s}^2)(5.0\ \text{m}) = 8.6\ \text{m}^2/\text{s}^2,$$

or $v_0 = 2.9\ \text{m/s}$. The speed of the object when $x = -3.0$ m is

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{8.6\ \text{m}^2/\text{s}^2 + 2(-0.86\,\text{m/s}^2)(-3.0\ \text{m})} = 3.7\ \text{m/s}.$$

17. We use $\vec{F}$ to denote the upward force exerted by the cable on the flood survivor. The force of the cable is toward upward and the force of gravity is *mg* toward downward. The acceleration of the survivor is $a = g/10$ upward. According to Newtonôs second law of motion, the force is given by

$$F - mg = ma \quad \Rightarrow \quad F = m(g+a) = \frac{11}{10}mg\,,$$

in the same direction as the displacement. On the other hand, the force of gravity has magnitude $F_g = mg$ and is opposite in direction to the displacement.

(a) Since the force of the cable $\vec{F}$ and the displacement $\vec{d}$ are in the same direction, the work done by $\vec{F}$ is

$$W_F = Fd = \frac{11mgd}{10} = \frac{11\,(75\text{ kg})(9.8\text{ m/s}^2)(16\text{ m})}{10} = 1.293\times10^4\text{ J} \approx 1.3\times10^4\text{ J}\,.$$

(b) The work done by gravity is

$$W_g = -F_g d = -mgd = -\,(75\text{ kg})(9.8\text{ m/s}^2)(16\text{ m}) = -1.176\times10^4\text{ J} \approx -1.2\times10^4\text{ J}$$

(c) The total work done is the sum of the two works:

$$W_{\text{net}} = W_F + W_g = 1.293\times10^4\text{ J} - 1.176\times10^4\text{ J} = 1.17\times10^3\text{ J} \approx 1.2\times10^3\text{ J}$$

Since the flood survivor started from rest, the workïkinetic energy theorem says that this is her final kinetic energy.

(d) Since $K = \frac{1}{2}mv^2$, her final speed is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.17\times10^3\text{J})}{75\text{ kg}}} = 5.6\text{ m/s}.$$

Note: For a general upward acceleration *a*, the net work done is

$$W_{\text{net}} = W_F + W_g = Fd - F_g d = m(g+a)d - mgd = mad\,.$$

Since $W_{\text{net}} = \Delta K = mv^2/2$, by the workïkinetic energy theorem, the speed of the survivor would be $v = \sqrt{2ad}$, which is independent of the mass of the survivor.

18. In both cases, there is no acceleration, so the lifting force is equal to the weight of the object.

(a) Equation 7-8 leads to $W = \vec{F} \cdot \vec{d} = (360\,\text{kN})\,(0.10\,\text{m}) = 36$ kJ.

(b) In this case, we find W = (4000 N)(0.050 m) $= 2.0 \times 10^2$ J .

19. Equation 7-15 applies, but the wording of the problem suggests that it is only necessary to examine the contribution from the rope (which would be the ñW_aò term in Eq. 7-15):

$$W_a = -(50\text{ N})(0.50\text{ m}) = -25\text{ J}$$

(the minus sign arises from the fact that the pull from the rope is anti-parallel to the direction of motion of the block). Thus, the kinetic energy would have been 25 J greater if the rope had not been attached (given the same displacement).

20. From the figure, one may write the kinetic energy (in units of J) as a function of x as

$$K = K_s - 25x = 50 - 25x\,.$$

Since $W = \Delta K = \vec{F}_x \cdot \Delta x$, the component of the force along the force along $+x$ is $F_x = dK/dx = -25$ N. The normal force on the block is $F_N = F_y$, which is related to the gravitational force by

$$mg = \sqrt{F_x^2 + (-F_y)^2}\ .$$

(Note that F_N points in the opposite direction of the component of the gravitational force.) With an initial kinetic energy $K_s = 50.0$ J and $v_0 = 5.00$ m/s, the mass of the block is

$$m = \frac{2K_s}{v_0^2} = \frac{2(50.0\ \text{J})}{(5.00\ \text{m/s})^2} = 4.00\ \text{kg}.$$

Thus, the normal force is

$$F_y = \sqrt{(mg)^2 - F_x^2} = \sqrt{(4.0\ \text{kg})^2(9.8\ \text{m/s}^2)^2 - (25\ \text{N})^2} = 30.2\ \text{N} \approx 30\ \text{N}.$$

21. **THINK** In this problem the cord is doing work on the block so that it does not undergo free fall.

EXPRESS We use F to denote the magnitude of the force of the cord on the block. This force is upward, opposite to the force of gravity (which has magnitude $F_g = Mg$), to prevent the block from undergoing free fall. The acceleration is $\vec{a} = g/4$ downward. Taking the downward direction to be positive, then Newtonôs second law yields

$$Mg - F = M\left(\frac{g}{4}\right)$$

so $F = 3Mg/4$, in the opposite direction of the displacement. On the other hand, the force of gravity $F_g = mg$ is in the same direction to the displacement.

ANALYZE (a) Since the displacement is downward, the work done by the cordôs force is, using Eq. 7-7,

$$W_F = -Fd = -\frac{3}{4}Mgd.$$

(b) Similarly, the work done by the force of gravity is $W_g = F_g d = Mgd$.

(c) The total work done on the block is simply the sum of the two works:

$$W_{\text{net}} = W_F + W_g = -\frac{3}{4}Mgd + Mgd = \frac{1}{4}Mgd\ .$$

Since the block starts from rest, we use Eq. 7-15 to conclude that this ($Mgd/4$) is the blockôs kinetic energy K at the moment it has descended the distance d.

(d) With $K = \frac{1}{2}Mv^2$, the speed is

$$v = \sqrt{\frac{2K}{M}} = \sqrt{\frac{2(Mgd/4)}{M}} = \sqrt{\frac{gd}{2}}$$

at the moment the block has descended the distance d.

LEARN For a general downward acceleration a, the force exerted by the cord is $F = m(g-a)$, so that the net work done on the block is $W_{\text{net}} = F_{\text{net}}d = mad$. The speed of the block after falling a distance d is $v = \sqrt{2ad}$. In the special case where the block hangs still, $a = 0$, $F = mg$ and $v = 0$. In our case, $a = g/4$, and $v = \sqrt{2(g/4)d} = \sqrt{gd/2}$, which agrees with that calculated in (d).

22. We use d to denote the magnitude of the spelunkerôs displacement during each stage. The mass of the spelunker is m = 85.0 kg. The work done by the lifting force is denoted W_i where i = 1, 2, 3 for the three stages. We apply the work-energy theorem, Eq. 17-15.

(a) For stage 1, $W_1 - mgd = \Delta K_1 = \frac{1}{2}mv_1^2$, where $v_1 = 5.00 \text{ m/s}$. This gives

$$W_1 = mgd + \frac{1}{2}mv_1^2 = (85.0 \text{ kg})(9.80 \text{ m/s}^2)(12.0 \text{ m}) + \frac{1}{2}(85.0 \text{ kg})(5.00 \text{ m/s})^2 = 1.11\times10^4 \text{ J}.$$

(b) For stage 2, W_2 ï $mgd = \Delta K_2 = 0$, which leads to

$$W_2 = mgd = (85.0 \text{ kg})(9.80 \text{ m/s}^2)(12.0 \text{ m}) = 1.00\times10^4 \text{ J}.$$

(c) For stage 3, $W_3 - mgd = \Delta K_3 = -\frac{1}{2}mv_1^2$. We obtain

$$W_3 = mgd - \frac{1}{2}mv_1^2 = (85.0 \text{ kg})(9.80 \text{ m/s}^2)(12.0 \text{ m}) - \frac{1}{2}(85.0 \text{ kg})(5.00 \text{ m/s})^2 = 8.93\times10^3 \text{ J}.$$

23. The fact that the applied force $\vec{F}_a$ causes the box to move up a frictionless ramp at a constant speed implies that there is no net change in the kinetic energy: $\Delta K = 0$. Thus, the work done by $\vec{F}_a$ must be equal to the negative work done by gravity: $W_a = -W_g$. Since the box is displaced vertically upward by $h = 0.150$ m, we have

$$W_a = +mgh = (3.00 \text{ kg})(9.80 \text{ m/s}^2)(0.150 \text{ m}) = 4.41 \text{ J}$$

24. (a) Using notation common to many vector-capable calculators, we have (from Eq. 7-8)

$$W = \text{dot}([23.0,0] + [0, -(3.00)(9.8)], [0.580 \angle 30.0^\circ]) = +3.03 \text{ J},$$

where ñdotò stands for dot product.

(b) Eq. 7-10 (along with Eq. 7-1) then leads to

$$v = \sqrt{2(3.03 \text{ J})/(3.00 \text{ kg})} = 1.42 \text{ m/s}$$

25. (a) The net upward force is given by

$$F + F_N - (m+M)g = (m+M)a$$

where m = 0.250 kg is the mass of the cheese, M = 900 kg is the mass of the elevator cab, F is the force from the cable, and $F_N = 3.00\ \text{N}$ is the normal force on the cheese. On the cheese alone, we have

$$F_N - mg = ma \quad \Rightarrow \quad a = \frac{3.00\ \text{N} - (0.250\ \text{kg})(9.80\ \text{m/s}^2)}{0.250\ \text{kg}} = 2.20\ \text{m/s}^2 .$$

Thus, the force from the cable is $F = (m+M)(a+g) - F_N = 1.08\times10^4\ \text{N}$, and the work done by the cable on the cab is

$$W = Fd_1 = (1.80\times10^4\ \text{N})(2.40\ \text{m}) = 2.59\times10^4\ \text{J}.$$

(b) If $W = 92.61\ \text{kJ}$ and $d_2 = 10.5\ \text{m}$, the magnitude of the normal force is

$$F_N = (m+M)g - \frac{W}{d_2} = (0.250\ \text{kg} + 900\ \text{kg})(9.80\ \text{m/s}^2) - \frac{9.261\times10^4\ \text{J}}{10.5\ \text{m}} = 2.45\ \text{N}.$$

26. If an object is attached to the springôs free end, the work W_s done on the object by the spring force when the object is moved from an initial position x_i to a final position x_f is

$$W_s = \frac{1}{2}k\,x_i^2 - \frac{1}{2}k\,x_f^2 = \frac{1}{2}k\,(x_i^2 - x_f^2)$$

Therefore, the work done to stretch the spring of spring constant 5.0×10^3 N/m which is stretched initially by 5.0 cm from the unstretched position x_i to the final position x_f is

$$\begin{aligned}W_s &= \frac{1}{2}k\,(x_i^2 - x_f^2)\\ &= \frac{1}{2}\left[(5\times10^3)\times[(10\times10^{-2})^2 - (5\times10^{-2})^2]\right]\\ &= \frac{1}{2}\times(5\times10^3)\times(75\times10^{-4})\\ &= 18.75\text{ J} \approx 19\text{ J}.\end{aligned}$$

27. From Eq. 7-25, we see that the work done by the spring force is given by

$$W_s = \frac{1}{2}k(x_i^2 - x_f^2) .$$

The fact that 360 N of force must be applied to pull the block to x = + 4.0 cm implies that the spring constant is

$$k = \frac{360 \text{ N}}{4.0 \text{ cm}} = 90 \text{ N/cm} = 9.0\times 10^3 \text{ N/m} .$$

(a) When the block moves from $x_i = +5.0$ cm to $x = +3.0$ cm, we have

$$W_s = \frac{1}{2}(9.0\times 10^3 \text{ N/m})[(0.050 \text{ m})^2 - (0.030 \text{ m})^2] = 7.2 \text{ J}.$$

(b) Moving from $x_i = +5.0$ cm to $x = -3.0$ cm, we have

$$W_s = \frac{1}{2}(9.0\times 10^3 \text{ N/m})[(0.050 \text{ m})^2 - (-0.030 \text{ m})^2] = 7.2 \text{ J}.$$

(c) Moving from $x_i = +5.0$ cm to $x = -5.0$ cm, we have

$$W_s = \frac{1}{2}(9.0\times 10^3 \text{ N/m})[(0.050 \text{ m})^2 - (-0.050 \text{ m})^2] = 0 \text{ J}.$$

(d) Moving from $x_i = +5.0$ cm to $x = -9.0$ cm, we have

$$W_s = \frac{1}{2}(9.0\times 10^3 \text{ N/m})[(0.050 \text{ m})^2 - (-0.090 \text{ m})^2] = -25 \text{ J}.$$

28. The spring constant is k = 110 N/m and the maximum elongation is x_i = 5.00 m. Using Eq. 7-25 with x_f = 0, the work is found to be

$$W = \frac{1}{2}kx_i^2 = \frac{1}{2}(110 \text{ N/m})(5.00 \text{ m})^2 = 1.38\times10^3 \text{ J.}$$

29. The work done by the spring force is given by Eq. 7-25: $W_s = \frac{1}{2}k(x_i^2 - x_f^2)$. The spring constant k can be deduced from the figure which shows the amount of work done to pull the block from 0 to x = 3.0 cm. The parabola $W_a = kx^2/2$ contains (0,0), (2.0 cm, 0.40 J) and (3.0 cm, 0.90 J). Thus, we may infer from the data that $k = 2.0\times10^3$ N/m.

(a) When the block moves from $x_i = +5.0$ cm to $x = +4.0$ cm, we have

$$W_s = \frac{1}{2}(2.0\times10^3 \text{ N/m})[(0.050 \text{ m})^2 - (0.040 \text{ m})^2] = 0.90 \text{ J}.$$

(b) Moving from $x_i = +5.0$ cm to $x = -2.0$ cm, we have

$$W_s = \frac{1}{2}(2.0\times10^3 \text{ N/m})[(0.050 \text{ m})^2 - (-0.020 \text{ m})^2] = 2.1 \text{ J}.$$

(c) Moving from $x_i = +5.0$ cm to $x = -5.0$ cm, we have

$$W_s = \frac{1}{2}(2.0\times10^3 \text{ N/m})[(0.050 \text{ m})^2 - (-0.050 \text{ m})^2] = 0 \text{ J}.$$

30. Hookeês law and the work done by a spring is discussed in the chapter. We apply the work-kinetic energy theorem, in the form of $\Delta K = W_a + W_s$, to the points in Figure 7-35 at x = 1.0 m and x = 2.0 m, respectively. The ñappliedò work W_a is that due to the constant force $\vec{P}$.

$$6\text{ J} = P(1.0\text{ m}) - \frac{1}{2}k(1.0\text{ m})^2$$
$$0 = P(2.0\text{ m}) - \frac{1}{2}k(2.0\text{ m})^2.$$

(a) Simultaneous solution leads to P = 12 N.

(b) Similarly, we find k = 12 N/m.

31. (a) As the body moves along the x axis from x_i = 3.5 m to x_f = 4.5 m the work done by the force is

$$\begin{aligned} W &= \int_{x_i}^{x_f} F_x \, dx \\ &= \int_{x_i}^{x_f} -6x \, dx \\ &= -3(x_f^2 - x_i^2) \\ &= -3\,(4.5^2 - 3.5^2) \\ &= -24 \text{ J.} \end{aligned}$$

The change in the kinetic energy, according to the workï kinetic energy theorem, is

$$W = \Delta K = \frac{1}{2} m (v_f^2 - v_i^2)$$

where v_i is the initial velocity (at x_i) and v_f is the final velocity (at x_f). From the theorem, we have

$$\begin{aligned} v_f &= \sqrt{\frac{2W}{m} + v_i^2} \\ &= \sqrt{\frac{2(-24\text{ J})}{2.5\text{ kg}} + (8.5\text{ m/s})^2} \\ &= 7.3 \text{ m/s.} \end{aligned}$$

(b) The velocity of the particle is v_f = 5.0 m/s when it is at $x = x_f$. The workï kinetic energy theorem is used to solve for x_f. The net work done on the particle is $W = -3(x_f^2 - x_i^2)$, so the theorem leads to

$$-3(x_f^2 - x_i^2) = \frac{1}{2} m (v_f^2 - v_i^2).$$

Thus,

$$x_f = \sqrt{-\frac{m}{6}\left(v_f^2 - v_i^2\right) + x_i^2} = \sqrt{-\frac{2.5\text{ kg}}{6\text{ N/m}}\left[(5.5\text{ m/s})^2 - (8.5\text{ m/s})^2\right] + (3.5\text{ m})^2} = 5.5 \text{ m.}$$

32. The work done by the spring force is given by Eq. 7-25: $W_s = \frac{1}{2}k(x_i^2 - x_f^2)$. Since $F_x = -kx$, the slope in Fig. 7-37 corresponds to the spring constant *k*. Its value is given by $k = 80\ \text{N/cm} = 8.0\times 10^3\ \text{N/m}$.

(a) When the block moves from $x_i = +8.0\ \text{cm}$ to $x = +5.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(8.0\times 10^3\ \text{N/m})[(0.080\ \text{m})^2 - (0.050\ \text{m})^2] = 15.6\ \text{J} \approx 16\ \text{J}.$$

(b) Moving from $x_i = +8.0\ \text{cm}$ to $x = -5.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(8.0\times 10^3\ \text{N/m})[(0.080\ \text{m})^2 - (-0.050\ \text{m})^2] = 15.6\ \text{J} \approx 16\ \text{J}.$$

(c) Moving from $x_i = +8.0\ \text{cm}$ to $x = -8.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(8.0\times 10^3\ \text{N/m})[(0.080\ \text{m})^2 - (-0.080\ \text{m})^2] = 0\ \text{J}.$$

(d) Moving from $x_i = +8.0\ \text{cm}$ to $x = -10.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(8.0\times 10^3\ \text{N/m})[(0.080\ \text{m})^2 - (-0.10\ \text{m})^2] = -14.4\ \text{J} \approx -14\ \text{J}.$$

33. (a) This is a situation where Eq. 7-28 applies, so we have

$$Fx = \frac{1}{2}kx^2 \Rightarrow (3.0\text{ N})\, x = \frac{1}{2}(50\text{ N/m})x^2$$

which (other than the trivial root) gives x = (3.0/25) m = 0.12 m.

(b) The work done by the applied force is $W_a = Fx$ = (3.0 N)(0.12 m) = 0.36 J.

(c) Eq. 7-28 immediately gives W_s = ï W_a = ï 0.36 J.

(d) With $K_f = K$ considered variable and $K_i = 0$, Eq. 7-27 gives $K = Fx$ ï $\frac{1}{2}kx^2$. We take the derivative of K with respect to x and set the resulting expression equal to zero, in order to find the position x_c that corresponds to a maximum value of K:

$$x_c = \frac{F}{k} = (3.0/50)\text{ m} = 0.060\text{ m}.$$

We note that x_c is also the point where the applied and spring forces ñbalance.ò

(e) At x_c we find $K = K_{max}$ = 0.090 J.

34. According to the graph the acceleration a varies linearly with the coordinate x. We may write $a = \alpha x$, where α is the slope of the graph. Numerically,

$$\alpha = \frac{24\ \text{m/s}^2}{8.0\ \text{m}} = 3.0\ \text{s}^{-2}.$$

The force on the brick is in the positive x direction and, according to Newtonôs second law, its magnitude is given by $F = ma = m\alpha x$. If x_f is the final coordinate, the work done by the force is

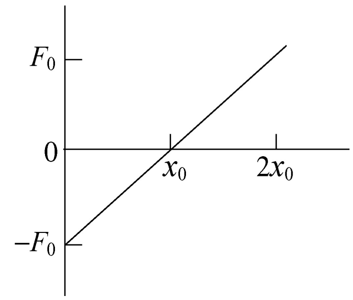
$$W = \int_0^{x_f} F\ dx = m\alpha \int_0^{x_f} x\ dx = \frac{m\alpha}{2} x_f^2 = \frac{(15\ \text{kg})(3.0\ \text{s}^{-2})}{2}(8.0\ \text{m})^2 = 1.4\times 10^3\ \text{J}.$$

35. **THINK** We have an applied force that varies with x. An integration is required to calculate the work done on the particle.

EXPRESS Given a one-dimensional force $F(x)$, the work done is simply equal to the area under the curve: $W = \int_{x_i}^{x_f} F(x)\, dx$.

ANALYZE

(a) The plot of $F(x)$ is shown to the right. Here we take x_0 to be positive. The work is negative as the object moves from $x = 0$ to $x = x_0$ and positive as it moves from $x = x_0$ to $x = 2x_0$.



Since the area of a triangle is (base)(altitude)/2, the work done from $x = 0$ to $x = x_0$ is $W_1 = -(x_0)(F_0)/2$ and the work done from $x = x_0$ to $x = 2x_0$ is

$$W_2 = (2x_0 - x_0)(F_0)/2 = (x_0)(F_0)/2$$

The total work is the sum of the two:

$$W = W_1 + W_2 = -\frac{1}{2}F_0x_0 + \frac{1}{2}F_0x_0 = 0$$

.

(b) The integral for the work is

$$W = \int_0^{2x_0} F_0\left(\frac{x}{x_0} - 1\right)dx = F_0\left(\frac{x^2}{2x_0} - x\right)\Bigg|_0^{2x_0} = 0.$$

LEARN If the particle starts out at $x = 0$ with an initial speed v_i, with a negative work $W_1 = -F_0x_0/2 < 0$, its speed at $x = x_0$ will decrease to

$$v = \sqrt{v_i^2 + \frac{2W_1}{m}} = \sqrt{v_i^2 - \frac{F_0x_0}{m}} < v_i,$$

but return to v_i again at $x = 2x_0$ with a positive work $W_2 = F_0x_0/2 > 0$.

36. From Eq. 7-32, we see that the ñareaò in the graph is equivalent to the work done. Finding that area (in terms of rectangular [length $\times$ width] and triangular $[\frac{1}{2}\ \text{base} \times \text{height}]$ areas) we obtain

$$W = W_{0<x<2} + W_{2<x<4} + W_{4<x<6} + W_{6<x<8} = (20+10+0-5)\ \text{J} = 25\ \text{J}.$$

37. (a) We first multiply the vertical axis by the mass, so that it becomes a graph of the applied force. Now, adding the triangular and rectangular ñareasò in the graph (for $0 \le x \le 4$) gives 42 J for the work done.

(b) Counting the ñareasò under the axis as negative contributions, we find (for $0 \le x \le 7$) the work to be 30 J at $x = 7.0$ m.

(c) And at $x = 9.0$ m, the work is 12 J.

(d) Equation 7-10 (along with Eq. 7-1) leads to speed $v = 6.5$ m/s at $x = 4.0$ m. Returning to the original graph (where a was plotted) we note that (since it started from rest) it has received acceleration(s) (up to this point) only in the $+x$ direction and consequently must have a velocity vector pointing in the $+x$ direction at $x = 4.0$ m.

(e) Now, using the result of part (b) and Eq. 7-10 (along with Eq. 7-1) we find the speed is 5.5 m/s at $x = 7.0$ m. Although it has experienced some deceleration during the $0 \le x \le 7$ interval, its velocity vector still points in the $+x$ direction.

(f) Finally, using the result of part (c) and Eq. 7-10 (along with Eq. 7-1) we find its speed $v = 3.5$ m/s at $x = 9.0$ m. It certainly has experienced a significant amount of deceleration during the $0 \le x \le 9$ interval; nonetheless, its velocity vector *still* points in the $+x$ direction.

38. (a) Using the work-kinetic energy theorem

$$K_f = K_i + \int_0^{2.0} (2.5 - x^2)\ dx = 0 + (2.5)(2.0) - \frac{1}{3}(2.0)^3 = 2.3 \text{ J}.$$

(b) For a variable end-point, we have K_f as a function of x, which could be differentiated to find the extremum value, but we recognize that this is equivalent to solving $F = 0$ for x:

$$F = 0 \ \Rightarrow \ 2.5 - x^2 = 0.$$

Thus, K is extremized at $x = \sqrt{2.5} \approx 1.6 \text{ m}$ and we obtain

$$K_f = K_i + \int_0^{\sqrt{2.5}} (2.5 - x^2) dx = 0 + (2.5)(\sqrt{2.5}) - \frac{1}{3}\ (\sqrt{2.5})^3 = 2.6 \text{ J}.$$

Recalling our answer for part (a), it is clear that this extreme value is a maximum.

39. Initial velocity of the particle is $5.0\hat{i}+18\hat{k}$. Hence, the initial kinetic energy of the particle is obtained as

$$\begin{aligned} K_i &= \frac{1}{2}mv_1^2 = \frac{1}{2}m(\vec{v}_1 \cdot \vec{v}_1) \\ &= \frac{1}{2}\times 0.02(25+324) \\ &= 0.01(349) = 3.49\text{ J} \end{aligned}$$

and the final kinetic energy of the particle is obtained as

$$\begin{aligned} K_f &= \frac{1}{2}mv_2^2 = \frac{1}{2}m(\vec{v}_2 \cdot \vec{v}_2) \\ &= \frac{1}{2}\times 0.02(81+484) \\ &= 0.01(565) \\ &= 5.65\text{ J.} \end{aligned}$$

Using work–kinetic energy theorem, the work done by the force, that is, the change in kinetic energy is calculated as

$$W = K_f - K_i = 5.65\text{ J} - 3.49\text{ J} = 2.16\text{ J} \approx 2.2\text{ J.}$$

40. Using Eq. 7-32, we find

$$W = \int_{0.25}^{2.25} e^{-4x^2}\, dx = 0.21 \text{ J}$$

where the result has been obtained numerically. Many modern calculators have that capability, as well as most math software packages that a great many students have access to.

41. We use workï kinetic energy theorem: To find the initial and final kinetic energies, we need the speeds. Therefore,

$$v = \frac{dx}{dt} = 4.0 - 10t + 6.0t^2$$

(in SI units). Thus, the initial speed (i.e., when t = 0 s) is v_i = 4.0 m/s and the speed at t = 6.0 s is v_f = 160 m/s. The change in kinetic energy for the object of mass m = 2.8 kg is therefore

$$\Delta K = \frac{1}{2}m(v_f^2 - v_i^2) = 3.6\times10^4 \text{ J}$$

rounded off to two significant figures.

42. We solve the problem using the work-kinetic energy theorem, which states that the change in kinetic energy is equal to the work done by the applied force, $\Delta K = W$. In our problem, the work done is $W = Fd$, where F is the tension in the cord and d is the length of the cord pulled as the cart slides from x_1 to x_2. From the figure, we have

$$d = \sqrt{x_1^2 + h^2} - \sqrt{x_2^2 + h^2} = \sqrt{(3.00\ \text{m})^2 + (1.25\ \text{m})^2} - \sqrt{(1.00\ \text{m})^2 + (1.25\ \text{m})^2}$$
$$= 3.25\ \text{m} - 1.60\ \text{m} = 1.649\ \text{m}$$

which yields $\Delta K = Fd = (28.0\ \text{N})(1.649\ \text{m}) = 46.18\ \text{J}$.

43. **THINK** This problem deals with the power and work done by a constant force.

EXPRESS The power done by a constant force F is given by $P = Fv$ and the work done by F from time t_1 to time t_2 is

$$W = \int_{t_1}^{t_2} P\, dt = \int_{t_1}^{t_2} Fv\, dt$$

Since F is the magnitude of the net force, the magnitude of the acceleration is $a = F/m$. Thus, if the initial velocity is $v_0 = 0$, then the velocity of the body as a function of time is given by $v = v_0 + at = (F/m)t$. Substituting the expression for v into the equation above, the work done during the time interval (t_1, t_2) becomes

$$W = \int_{t_1}^{t_2} (F^2/m)t\, dt = \frac{F^2}{2m}\left(t_2^2 - t_1^2\right).$$

ANALYZE

(a) For $t_1 = 0$ and $t_2 = 1.0$ s, $W = \frac{1}{2}\left(\frac{(5.0\text{ N})^2}{15\text{ kg}}\right)[(1.0\text{ s})^2 - 0] =$ 0.83 J.

(b) For $t_1 = 1.0\text{s}$, and $t_2 = 2.0$ s, $W = \frac{1}{2}\left(\frac{(5.0\text{ N})^2}{15\text{ kg}}\right)[(2.0\text{ s})^2 - (1.0\text{ s})^2] = 2.5$ J.

(c) For $t_1 = 2.0$ s and $t_2 = 3.0$ s, $W = \frac{1}{2}\left(\frac{(5.0\text{ N})^2}{15\text{ kg}}\right)[(3.0\text{ s})^2 - (2.0\text{ s})^2] = 4.2$ J.

(d) Substituting $v = (F/m)t$ into $P = Fv$ we obtain $P = F^2 t/m$ for the power at any time t. At the end of the third second, the instantaneous power is

$$P = \left(\frac{(5.0\text{ N})^2\ (3.0\text{ s})}{15\text{ kg}}\right) = 5.0\text{ W}.$$

LEARN The work done here is quadratic in t. Therefore, from the definition $P = dW/dt$ for the instantaneous power, we see that P increases linearly with t.

44. (a) Since constant speed implies $\Delta K = 0$, we require $W_a = -W_g$, by Eq. 7-15. Since W_g is the same in both cases (same weight and same path), then $W_a = 8.8\times10^2$ J just as it was in the first case.

(b) Since the speed of 1.0 m/s is constant, then 7.0 meters is traveled in 7.0 seconds. Using Eq. 7-42, and noting that average power is *the* power when the work is being done at a steady rate, we have

$$P = \frac{W}{\Delta t} = \frac{880\ \text{J}}{7.0\ \text{s}} = 1.3\times10^2\ \text{W}.$$

(c) Since the speed of 2.0 m/s is constant, 7.0 meters is traveled in 3.5 seconds. Using Eq. 7-42, with *average power* replaced by *power*, we have

$$P = \frac{W}{\Delta t} = \frac{880\ \text{J}}{3.5\ \text{s}} = 251\ \text{W} \approx 2.5\times10^2\ \text{W}.$$

45. **THINK** A block is pulled at a constant speed by a force directed at some angle with respect to the direction of motion. The quantity weøre interested in is the power, or the time rate at which work is done by the applied force.

EXPRESS The power associated with force $\vec{F}$ is given by $P = \vec{F} \cdot \vec{v} = Fv\cos\phi$, where $\vec{v}$ is the velocity of the object on which the force acts, and ϕ is the angle between $\vec{F}$ and $\vec{v}$.

ANALYZE

$$\begin{aligned} P &= \vec{F} \cdot \vec{v} = Fv\cos\phi = (125\ \text{N})(5.5\ \text{m/s})\cos 38° \\ &= 5.4\times10^{2}\ \text{W}. \end{aligned}$$

LEARN From the expression $P = Fv\cos\phi$, we see that the power is at a maximum when $\vec{F}$ and $\vec{v}$ are in the same direction ($\phi = 0$), and is zero when they are perpendicular of each other. In addition, weøre told that the block moves at a constant speed, so $\Delta K = 0$, and the net work done on it must also be zero by the work-kinetic energy theorem. Thus, the applied force here must be compensating another force (e.g., friction) for the net rate to be zero.

46. Recognizing that the force in the cable must equal the total weight (since there is no acceleration), we employ Eq. 7-47:

$$P = Fv\cos\theta = mg\left(\frac{\Delta x}{\Delta t}\right)$$

where we have used the fact that $\theta = 0°$ (both the force of the cable and the elevatorôs motion are upward). Thus,

$$P = (5.0\times10^3\ \text{kg})(9.8\ \text{m/s}^2)\left(\frac{210\ \text{m}}{23\ \text{s}}\right) = 4.5\times10^5\ \text{W}.$$

47. (a) Equation 7-8 yields

$W = F_x \Delta x + F_y \Delta y + F_z \Delta z$
= (2.00 N)(7.5 m ï 0.50 m) + (4.00 N)(12.0 m ï 0.75 m) + (6.00 N)(7.2m ï 0.20 m)
=101 J $\approx 1.0 \times 10^2$ J.

(b) Dividing this result by 12 s (see Eq. 7-42) yields P = 8.4 W.

48. (a) Since the force exerted by the spring on the mass is zero when the mass passes through the equilibrium position of the spring, the rate at which the spring is doing work on the mass at this instant is also zero.

(b) The rate is given by $P = \vec{F} \cdot \vec{v} = -Fv$, where the minus sign corresponds to the fact that $\vec{F}$ and $\vec{v}$ are anti-parallel to each other. The magnitude of the force is given by

$$F = kx = (450 \text{ N/m})(0.10 \text{ m}) = 45 \text{ N},$$

while v is obtained from conservation of energy for the spring-mass system:

$$E = K + U = 10 \text{ J} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}(0.35 \text{ kg})v^2 + \frac{1}{2}(450 \text{ N/m})(0.10 \text{ m})^2$$

which gives $v = 6.7$ m/s. Thus,

$$P = -Fv = -(45 \text{ N})(6.7 \text{ m/s}) = -3.0 \times 10^2 \text{ W}.$$

49. **THINK** We have a loaded elevator moving upward at a constant speed. The forces involved are: gravitational force on the elevator, gravitational force on the counterweight, and the force by the motor via cable.

EXPRESS The total work is the sum of the work done by gravity on the elevator, the work done by gravity on the counterweight, and the work done by the motor on the system:

$$W = W_e + W_c + W_m.$$

Since the elevator moves at constant velocity, its kinetic energy does not change and according to the work-kinetic energy theorem the total work done is zero, i.e., $W = \Delta K = 0$.

ANALYZE The elevator moves *upward* through 54 m, so the work done by gravity on it is

$$W_e = -m_e g d = -(1200 \text{ kg})(9.80 \text{ m/s}^2)(54 \text{ m}) = -6.35 \times 10^5 \text{ J}.$$

The counterweight moves *downward* the same distance, so the work done by gravity on it is

$$W_c = m_c g d = (950 \text{ kg})(9.80 \text{ m/s}^2)(54 \text{ m}) = 5.03 \times 10^5 \text{ J}.$$

Since $W = 0$, the work done by the motor on the system is

$$W_m = -W_e - W_c = 6.35 \times 10^5 \text{ J} - 5.03 \times 10^5 \text{ J} = 1.32 \times 10^5 \text{ J}.$$

This work is done in a time interval of $\Delta t = 3.0 \text{ min} = 180 \text{ s}$, so the power supplied by the motor to lift the elevator is

$$P = \frac{W_m}{\Delta t} = \frac{1.32 \times 10^5 \text{ J}}{180 \text{ s}} = 7.4 \times 10^2 \text{ W}.$$

LEARN In general, the work done by the motor is $W_m = (m_e - m_c) g d$. So when the counterweight mass balances the total mass, $m_c = m_e$, no work is required by the motor.

50. (a) Using Eq. 7-48 and Eq. 3-23, we obtain

$$P = \vec{F} \cdot \vec{v} = (4.0\text{ N})(-2.0\text{ m/s}) + (9.0\text{ N})(4.0\text{ m/s}) = 28\text{ W}.$$

(b) We again use Eq. 7-48 and Eq. 3-23, but with a one-component velocity: $\vec{v} = v\hat{\text{j}}$.

$$P = \vec{F} \cdot \vec{v} \quad \Rightarrow -15\text{ W} = (-2.0\text{ N})v.$$

which yields v = 7.5 m/s.

51. (a) The object's displacement is

$$\vec{d} = \vec{d}_f - \vec{d}_i = (-8.00\text{ m})\,\hat{i} + (6.00\text{ m})\,\hat{j} + (2.00\text{ m})\,\hat{k}.$$

Thus, Eq. 7-8 gives

$$W = \vec{F}\cdot\vec{d} = (3.00\text{ N})(-8.00\text{ m}) + (7.00\text{ N})(6.00\text{ m}) + (7.00\text{ N})(2.00\text{ m}) = 32.0\text{ J}.$$

(b) The average power is given by Eq. 7-42:

$$P_{\text{avg}} = \frac{W}{t} = \frac{32.0}{4.00} = 8.00\text{ W}.$$

(c) The distance from the coordinate origin to the initial position is

$$d_i = \sqrt{(3.00\text{ m})^2 + (-2.00\text{ m})^2 + (5.00\text{ m})^2} = 6.16\text{ m},$$

and the magnitude of the distance from the coordinate origin to the final position is

$$d_f = \sqrt{(-5.00\text{ m})^2 + (4.00\text{ m})^2 + (7.00\text{ m})^2} = 9.49\text{ m}.$$

Their scalar (dot) product is

$$\vec{d}_i\cdot\vec{d}_f = (3.00\text{ m})(-5.00\text{ m}) + (-2.00\text{ m})(4.00\text{ m}) + (5.00\text{ m})(7.00\text{ m}) = 12.0\text{ m}^2.$$

Thus, the angle between the two vectors is

$$\phi = \cos^{-1}\left(\frac{\vec{d}_i\cdot\vec{d}_f}{d_i d_f}\right) = \cos^{-1}\left(\frac{12.0}{(6.16)(9.49)}\right) = 78.2°.$$

52. According to the problem statement, the power of the car is

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = mv\frac{dv}{dt} = \text{constant}.$$

The condition implies $dt = mvdv/P$, which can be integrated to give

$$\int_0^T dt = \int_0^{v_T} \frac{mvdv}{P} \quad \Rightarrow \quad T = \frac{mv_T^2}{2P}$$

where v_T is the speed of the car at $t = T$. On the other hand, the total distance traveled can be written as

$$L = \int_0^T vdt = \int_0^{v_T} v\frac{mvdv}{P} = \frac{m}{P}\int_0^{v_T} v^2 dv = \frac{mv_T^3}{3P}.$$

By squaring the expression for L and substituting the expression for T, we obtain

$$L^2 = \left(\frac{mv_T^3}{3P}\right)^2 = \frac{8P}{9m}\left(\frac{mv_T^2}{2P}\right)^3 = \frac{8PT^3}{9m}$$

which implies that

$$PT^3 = \frac{9}{8}mL^2 = \text{constant}.$$

Differentiating the above equation gives $dPT^3 + 3PT^2dT = 0,$ or $dT = -\dfrac{T}{3P}dP.$