#14.8

#6. $x=t,\ y=\frac{3}{2}t^2,\ z=\frac{3}{2}t^3\ ;\ 0\le t\le 2$

$$\Rightarrow\ s=\int_0^2\sqrt{1+9t^2+\left(\tfrac{9}{2}t^2\right)^2}\,dt=\int_0^2\sqrt{\frac{4+36t^2+81t^4}{4}}\,dt$$

$$=\frac{1}{2}\int_0^2\sqrt{(2+9t^2)^2}\,dt=\frac{1}{2}\int_0^2(2+9t^2)\,dt$$

$$=\frac{1}{2}\left[2t+3t^3\right]_0^2=14.$$

#16. $x=2t^4,\ y=t^2,\ z=1-t^3\ ;\ t=1$

$$\Rightarrow\ \alpha(t)=(2t^4,\ t^2,\ 1-t^3),\quad \alpha(1)=(2,1,0)$$

$$\alpha'(t)=(8t^3,\ 2t,\ -3t^2),\quad \alpha'(1)=(8,2,-3)$$

① 접선의 방정식 : $\frac{x-2}{8}=\frac{y-1}{2}=\frac{z-0}{-3}$

② 평면의 방정식 : $8(x-2)+2(y-1)-3(z-0)=0.$

#19. $x=\cos t,\ y=\sin t,\ z=t\ ;\ t=\frac{\pi}{2}$

$$\Rightarrow\ \alpha(t)=(\cos t,\ \sin t,\ t)\ ;\ \alpha\left(\tfrac{\pi}{2}\right)=\left(0,1,\tfrac{\pi}{2}\right)$$

$$\alpha'(t)=(-\sin t,\ \cos t,\ 1),\quad \alpha'\left(\tfrac{\pi}{2}\right)=(-1,0,1)$$

① 접선의 방정식 : $\frac{x-0}{-1}=\frac{z-\frac{\pi}{2}}{1},\quad y=1$

② 평면의 방정식 : $-1(x-0)+0(y-1)+1\left(z-\frac{\pi}{2}\right)=0.$

#22. $x = e^{t}$, $y = e^{-t}$, $z = t$; $t = 0$

$\Rightarrow \quad \alpha(t) = (e^{t}, e^{-t}, t)$, $\alpha(0) = (1, 1, 0)$

$\alpha'(t) = (e^{t}, -e^{-t}, 1)$, $\alpha'(0) = (1, -1, 1)$

∴ 접선의 방정식 : $\dfrac{x-1}{1} = \dfrac{y-1}{-1} = \dfrac{z-0}{1}$,

법평면의 방정식 : $(x-1) - (y-1) + z = 0$

#28. $\begin{cases} x^2 - y^2 + z^2 = 2 \\ 3x^2 + 2y^2 - z^2 = 2 \end{cases}$; $(1, 0, -1)$

$\Rightarrow \begin{cases} F_x = 2x, & F_y = -2y, & F_z = 2z \\ G_x = 6x, & G_y = 4y, & G_z = -2z \end{cases}$

$\Rightarrow (1, 0, -1)$

$\begin{cases} F_x = 2, & F_y = 0, & F_z = -2 \\ G_x = 6, & G_y = 0, & G_z = 2 \end{cases}$

$\Rightarrow \begin{vmatrix} F_y & F_z \\ G_y & G_z \end{vmatrix} : \begin{vmatrix} F_z & F_x \\ G_z & G_x \end{vmatrix} : \begin{vmatrix} F_x & F_y \\ G_x & G_y \end{vmatrix} = \begin{vmatrix} 0 & -2 \\ 0 & 2 \end{vmatrix} : \begin{vmatrix} -2 & 2 \\ 2 & 6 \end{vmatrix} : \begin{vmatrix} 2 & 0 \\ 6 & 0 \end{vmatrix}$

$= 0 : -16 : 0$

접선의 방정식 : $\begin{cases} x = 1 \\ z = -1 \end{cases}$ 법평면의 방정식 $-y = 0$

14.9

#2. $z^2 = x^2 + y^2$; $(3, 4, 5)$

$\Rightarrow \nabla F = (2x, 2y, -2z)$,

$\nabla F(3, 4, 5) = (6, 8, -10)$

법선의 방정식 $\dfrac{x-3}{6} = \dfrac{y-4}{8} = \dfrac{z-5}{-10}$

접평면의 방정식 $6(x-3) + 8(y-4) - 10(z-5) = 0$

#8. $x^2 + 4y^2 = 2z$; $(2, 1, 4)$

$\Rightarrow \nabla F = (2x, 8y, -2)$

$\nabla F \ (2, 1, 4) = (4, 8, -2)$

법선의 방정식 $\dfrac{x-2}{4} = \dfrac{y-1}{8} = \dfrac{z-4}{-2}$

접평면의 방정식 $4(x-2) + 8(y-1) - 2(z-4) = 0$.

#14. $x^{\frac{1}{2}} + y^{\frac{1}{2}} + z^{\frac{1}{2}} = 9$; $(4, 9, 16)$

$\Rightarrow \nabla F = (\frac{1}{2}x^{-\frac{1}{2}}, \frac{1}{2}y^{-\frac{1}{2}}, \frac{1}{2}z^{-\frac{1}{2}})$

$\nabla F \ (4, 9, 16) = (\frac{1}{4}, \frac{1}{6}, \frac{1}{8})$

법선의 방정식 $\dfrac{x-4}{6} = \dfrac{y-9}{4} = \dfrac{z-16}{3}$

접평면의 방정식 $6(x-4) + 4(y-9) + 3(z-16) = 0$

14. 10

#23. $\sin xy$; $(1, \pi)$

$\Rightarrow f(1,\pi) = \sin\pi = 0$

$f_x = y\cos(xy)$, $f_x(1,\pi) = \pi\cos\pi = -\pi$

$f_y = x\cos(xy)$, $f_y(1,\pi) = \cos\pi = -1$

$f_{xx} = -y^2\sin(xy)$, $f_{xx}(1,\pi) = -\pi^2\sin\pi = 0$

$f_{xy} = \cos(xy) - xy\sin(xy)$, $f_{xy}(1,\pi) = \cos\pi - \pi\sin\pi = -1$

$f_{yy} = -x^2\sin(xy)$, $f_{yy}(1,\pi) = -\sin\pi = 0$.

$$\begin{aligned}\therefore \sin(xy) &= f(1,\pi) + (x-1)f_x(1,\pi) + (y-\pi)f_y(1,\pi)\\ &\quad + \tfrac{1}{2}\{(x-1)^2 f_{xx}(1,\pi) + 2(x-1)(y-\pi)f_{xy}(1,\pi)\\ &\quad + (y-\pi)^2 f_{yy}(1,\pi)\} + \cdots\\ &= 0 + (-\pi)(x-1) + (-1)(y-\pi)\\ &\quad + \tfrac{1}{2}\{0(x-1)^2 + 2(-1)(x-1)(y-\pi) + 0(y-\pi)^2\}\\ &= -\pi(x-1) - (y-\pi) + \tfrac{1}{2}\{-2(x-1)(y-\pi)\}\\ &= -\pi(x-1) - (y-\pi) - (x-1)(y-\pi)\end{aligned}$$

#25. $f(x,y) = \ln(1+xy)$; $(2,3)$

$\Rightarrow f(2,3) = \ln(1+6) = \ln 7$

$f_x = \dfrac{y}{1+xy}$, $f_x(2,3) = \dfrac{3}{7}$

$f_y = \dfrac{x}{1+xy}$, $f_y(2,3) = \dfrac{2}{7}$

$f_{xx} = \dfrac{-y^2}{(1+xy)^2}$, $f_{xx}(2,3) = -\dfrac{9}{49}$

$f_{xy} = \dfrac{1}{(1+xy)^2}$, $f_{xy}(2,3) = \dfrac{1}{49}$

$f_{yy} = \dfrac{-x^2}{(1+xy)^2}$, $f_{yy}(2,3) = -\dfrac{4}{49}$

$$\ln(1+xy) = f(2,3) + (x-2)f_x(2,3) + (y-3)f_y(2,3)$$
$$+ \frac{1}{2}\{ (x-2)^2 f_{xx}(2,3) + 2(x-2)(y-3)f_{xy}(2,3)$$
$$+ (y-3)^2 f_{yy}(2,3) \} + \cdots$$
$$= \ln 7 + \frac{3}{7}(x-2) + \frac{2}{7}(y-3)$$
$$+ \frac{1}{2}\{ \left(\frac{-9}{49}\right)(x-2)^2 + 2(x-2)(y-3)\left(\frac{1}{49}\right) + (y-3)^2\left(-\frac{4}{49}\right) \}$$

#33. e^{xy} ; $(1,1)$

$\Rightarrow f(1,1) = e$

$f_x = ye^{xy}$, $f_x(1,1) = e$, $f_y = xe^{xy}$, $f_y(1,1) = e$

$f_{xx} = y^2e^{xy}$, $f_{xx}(1,1) = e$, $f_{xy} = e^{xy} + xye^{xy}$, $f_{xy}(1,1) = 2e$

$f_{yy} = x^2e^{xy}$, $f_{yy}(1,1) = e$

$$\therefore e^{xy} = f(1,1) + (x-1)f_x(1,1) + (y-1)f_y(1,1)$$
$$+ \frac{1}{2}\{ (x-1)^2 f_{xx}(1,1) + 2(x-1)(y-1)f_{xy}(1,1) + (y-1)^2 f_{yy}(1,1) \}$$
$$= e + e(x-1) + e(y-1) + \frac{1}{2}\{ e(x-1)^2 + 2(2e)(x-1)(y-1)$$
$$+ e(y-1)^2 \}.$$

14.11

#5. $f(x,y) = x + y\sin x$

$\Rightarrow f_x = 1 + y\cos x = 0$

$f_y = \sin x = 0$

$x = 2n\pi \Rightarrow \cos 2n\pi = 1 \Rightarrow y = -1$

$x = (2n+1)\pi \Rightarrow \cos(2n+1)\pi = -1 \Rightarrow y = 1$

$\therefore$ 임계점 : $(2n\pi, -1)$, $((2n+1)\pi, 1)$

#17. $f(x,y) = x^2 + 2xy + 2y^2 - 6y$

$\Rightarrow f_x = 2x + 2y = 0$

$f_y = 2x + 4y - 6 = 0$

∴ 임계점 : $(-3, 3)$

$$|H(x,y)| = \begin{vmatrix} 2 & 2 \\ 2 & 4 \end{vmatrix} = 4 > 0$$

$f_{xx}(-3, 3) = 2 > 0$ ∴ $(-3, 3)$에서 극소값을 갖는다.

#23. $f(x,y) = xy + \frac{1}{x} + \frac{1}{y}$

$\Rightarrow f_x = y - \frac{1}{x^2} = 0 \Rightarrow y = \frac{1}{x^2}$

$f_y = x - \frac{1}{y^2} = 0 \Rightarrow x = \frac{1}{y^2} \Rightarrow y = y^4$

∴ 임계점은 $(0,0)$ & $(1,1)$. 하지만 $(0,0)$ 에서는 함수가 정의되지 않으므로 제외시킨다.

$$|H(x,y)| = \begin{vmatrix} \frac{2}{x^3} & 1 \\ 1 & \frac{2}{y^3} \end{vmatrix} \Rightarrow |H(1,1)| = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4 - 1 > 0$$

$f_{xx}(1,1) = 2 > 0$ 이므로 $(1,1)$ 에서 극소.

#33. $f(x,y) = e^{-(x^2+y^2)}$

$\Rightarrow f_x = -2xe^{-(x^2+y^2)} = 0$

$f_y = -2ye^{-(x^2+y^2)} = 0$ 〉 임계점 $(0,0)$

$$|H(x,y)| = \begin{vmatrix} -2e^{-(x^2+y^2)} + 4x^2e^{-(x^2+y^2)} & 4xye^{-(x^2+y^2)} \\ 4xye^{-(x^2+y^2)} & -2e^{-(x^2+y^2)} + 4y^2e^{-(x^2+y^2)} \end{vmatrix}$$

$$|H(0,0)| = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix} = 4 > 0$$

$f_{xx}(0,0) = -2 < 0$ ∴ $(0,0)$ 에서 극대.

#37. $d = \sqrt{x^2+y^2+z^2}$

$$= \sqrt{x^2+y^2+\left(\frac{7}{2}-\frac{1}{2}y-\frac{1}{3}x\right)^2}$$

$$f(x,y) = x^2+y^2+\left(\frac{7}{2}-\frac{1}{2}y-\frac{1}{3}x\right)^2$$

$$f_x = 2x - \frac{7}{3} + \frac{2}{9}x + \frac{y}{3} = 0$$

$$f_y = 2y - \frac{7}{2} + \frac{x}{3} + \frac{y}{2} = 0$$

임계점 $\left(\frac{42}{49}, \frac{63}{49}\right) = \left(\frac{6}{7}, \frac{9}{7}\right)$

$$|H(x,y)| = \begin{vmatrix} \frac{20}{9} & \frac{1}{3} \\ \frac{1}{3} & \frac{5}{2} \end{vmatrix} > 0$$

$\therefore$ 최소값 9. 최단거리 $\sqrt{9} = 3$.

#43 $V = 8xyz$. $x^2+3y^2+9z^2=9$

$$V = 8xy\sqrt{1-\frac{x^2}{9}-\frac{y^2}{3}} = 8\sqrt{x^2y^2-\frac{x^4y^2}{9}-\frac{x^2y^4}{3}}$$

$$f(x,y) = x^2y^2 - \frac{x^4y^2}{9} - \frac{x^2y^4}{3}$$

$$f_x = xy^2\left(2-\frac{4}{9}x^2-\frac{2}{3}y^2\right) = 0$$

$$f_y = x^2y\left(2-\frac{2}{9}x^2-\frac{4}{3}y^2\right) = 0$$

$x^2 = 3$, $y^2 = 1$. V 의 최대값 : 8.

15.1

#1. $\iint_R (x+y)^2\, dA = f(-\frac{1}{2}, \frac{1}{2})\cdot 1 + f(-\frac{1}{2}, \frac{3}{2})\cdot 1 + f(\frac{1}{2}, \frac{1}{2})\cdot 1 + f(\frac{1}{2}, \frac{3}{2})\cdot 1$

$= 0+1+1+4 = 6.$

#6. $\iint_R y\, dA = \int_0^2\int_0^2 y\, dxdy = \int_0^2 [xy]_0^2\, dy = \int_0^2 2y\, dy = y^2\Big|_0^2 = 4$

#11. $\iint_R ([x]+[y])\, dA$, $f(x,y) = [x]+[y]$

$S = f(0,0)\cdot 1 + f(0,1)\cdot 1 + f(1,0)\cdot 1 + f(1,1)\cdot 1$

$= 0+1+1+2 = 4.$

15.2

#13. $\int_1^3\int_0^y (x+y)\, dxdy = \int_1^3 [\frac{1}{2}x^2+yx]_0^y\, dy = \int_1^3 (\frac{1}{2}y^2+y^2)\, dy = [\frac{1}{2}y^3]_1^3 = 13$

#21. $\int_0^\pi\int_0^{y^2} \sin\frac{x}{y}\, dxdy$

$= \int_0^\pi [-y\cos\frac{x}{y}]_0^{y^2}\, dy = \int_0^\pi (-y\cos y + y)\, dy$

$= -[\, y\sin y\big|_0^\pi - \int_0^\pi \sin y\, dy\,] + \frac{1}{2}y^2\Big|_0^\pi$

$= -[\cos y]_0^\pi + \frac{1}{2}\pi^2 = 1+1+\frac{1}{2}\pi^2 = 2+\frac{1}{2}\pi^2$

#27. $\int_0^1\int_0^{2x} f(x,y)\, dxdy$ $\Rightarrow$ 문제가 잘못 됨.

$\Rightarrow \int_0^1\int_0^{2y} f(x,y)\, dxdy = \int_0^2\int_{\frac{1}{2}x}^1 f(x,y)\, dydx$

#30. $\int_0^1 \int_y^1 \sin(x^2)\, dx dy$

$= \int_0^1 \int_0^x \sin(x^2)\, dy dx = \int_0^1 [y \sin(x^2)]_0^x\, dx$

$= \int_0^1 x \sin(x^2)\, dx = [-\frac{1}{2}\cos(x^2)]_0^1 = -\frac{1}{2}\cos 1 + \frac{1}{2}$

Q43. $y = xe^{-x}$, $y = x$. $x = 2$

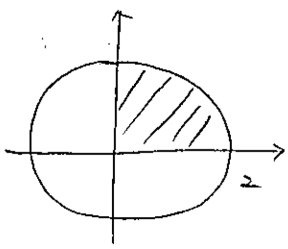
$\Rightarrow \int_0^2 \int_{xe^{-x}}^{x} dy dx = \int_0^2 (x - xe^{-x})\, dx$

$= \int_0^2 x\, dx - \int_0^2 x e^{-x}\, dx = 2 - (-3e^{-2} + 1) = 1 + 3e^{-2}$

15.3

#5. $R = \{(x, y) \mid 0 \le x \le 2,\ 0 \le y \le \sqrt{4 - x^2}\}$

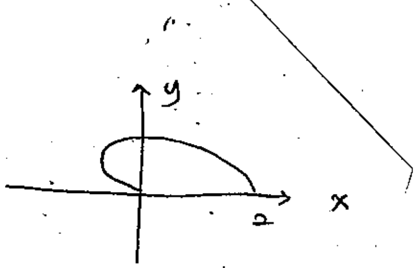
$\Rightarrow$



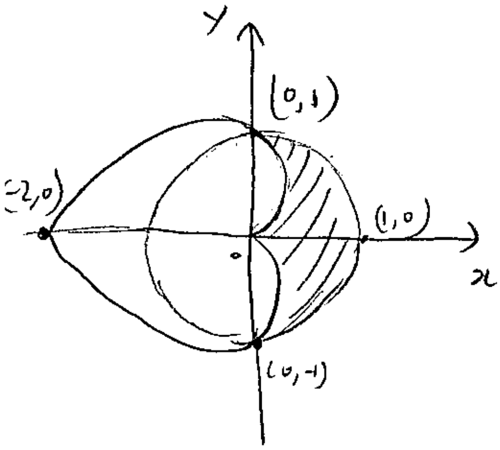
$0 < r < 2$ $\qquad x = r\sin\theta$

$0 < \theta < \frac{\pi}{2}$, $\qquad y = r\cos\theta$

#10. $R = \{(r, \theta) \mid 0 \le \theta \le \pi,\ 0 \le r \le 1 + \cos\theta\}$



#12. $r=1$ 내부, $r=1-\cos\theta$ 외부



$$R=\left\{ (r,\theta) \;\middle|\; -\frac{\pi}{2}\le\theta\le\frac{\pi}{2},\ 1-\cos\theta\le r\le 1 \right\}$$

$$A=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{1-\cos\theta}^{1} r\,dr\,d\theta$$

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1}{2}\left(1-(1-\cos\theta)^2\right)d\theta$$

$$=\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(2\cos\theta-\cos^2\theta\right)d\theta$$

$$=\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(2\cos\theta-\frac{1+\cos2\theta}{2}\right)d\theta$$

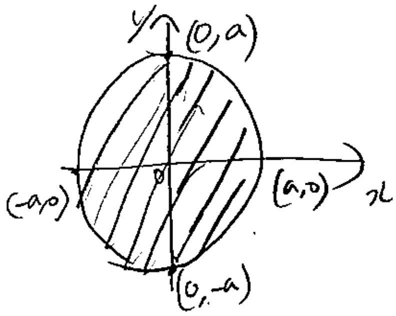
$$=\left[\sin\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}-\frac{1}{4}\left[\theta+\frac{1}{2}\sin2\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$=1-(-1)-\frac{1}{4}\left(\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right)$$

$$=2-\frac{\pi}{4}$$

#23

$f(x,y) = e^{-(x^2+y^2)}$, $R = \{ (x,y) \mid x^2+y^2 \le a^2,\ a \ge 0 \} = \{ (r,\theta) \mid 0 \le r \le a,\ 0 \le \theta \le 2\pi \}$



$$\int_0^{2\pi} \int_0^a e^{-r^2} r\,dr\,d\theta$$

$$= \int_0^{2\pi} \left[-\frac{1}{2} e^{-r^2} \right]_0^a d\theta$$

$$= \int_0^{2\pi} \left(-\frac{1}{2} e^{-a^2} + \frac{1}{2} \right) d\theta$$

$$= \left(-\frac{1}{2} e^{-a^2} + \frac{1}{2} \right) 2\pi$$