1. (a) As the man continues to remain at the same place with respect to the gym, it is obvious that his net displacement is zero.

(b) In 25 min, the average velocity is

$$v_{\text{avg}} = \frac{(x_2 - x_1)}{(t_2 - t_1)} = \frac{0.0 - 0.0}{25 - 0.0} = 0.0.$$

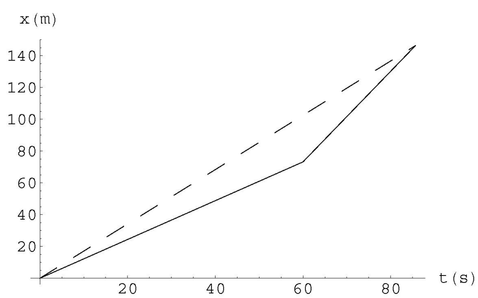
2. (a) Using the fact that time = distance/velocity while the velocity is constant, we find

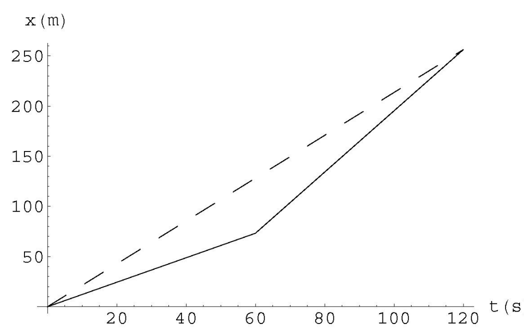
$$v_{\text{avg}} = \frac{73.2\ \text{m} + 73.2\ \text{m}}{\frac{73.2\ \text{m}}{1.22\ \text{m/s}} + \frac{73.2\ \text{m}}{2.85\ \text{m}}} = 1.71\ \text{m/s}.$$

(b) Using the fact that distance = vt while the velocity v is constant, we find

$$v_{\text{avg}} = \frac{(1.22\ \text{m/s})(60\ \text{s}) + (3.05\ \text{m/s})(60\ \text{s})}{120\ \text{s}} = 2.14\ \text{m/s}.$$

The graphs are shown below (with meters and seconds understood). The first consists of two (solid) line segments, the first having a slope of 1.22 and the second having a slope of 3.05. The slope of the dashed line represents the average velocity (in both graphs). The second graph also consists of two (solid) line segments, having the same slopes as before ð the main difference (compared to the first graph) being that the stage involving higher-speed motion lasts much longer.





3. Rachel's displacement in reaching the gymnasium is

$$x - x_0 = 2.80 \text{ km} - 0.00 \text{ km} = 2.80 \text{ km}.$$

and her initial speed is $v_0 = 6.00$ km/h. The time taken for her to reach the gymnasium is

$$t_1 = \frac{x - x_0}{v_0} = \frac{2.80 \text{ km}}{6.00 \text{ km/h}} = 0.467 \text{ h} = 28.0 \text{ min}$$

and the time taken for her to reach back home from gymnasium is

$$t_2 = \frac{2.80 \text{ km}}{7.70 \text{ km/h}} = 0.364 \text{ h} = 21.82 \text{ min}.$$

Thus, in 28.0 + 21.82 = 49.82 min, she returns back to home and stops.

During the 7-minute time interval (between $28.0 - 35.0$ min), the return distance traveled by Rachel is

$$(7.70 \text{ km/h})\left(\frac{7.00 \text{ min}}{60 \text{ min}}\right) = 0.90 \text{ km}$$

(a) The magnitude of her average velocity during the time interval $0.00 - 35.0$ min is

$$\frac{\text{displacement}}{\text{time}} = \frac{2.80 \text{ km} - 0.89 \text{ km}}{(35.0/60.0) \text{ h}} = 3.26 \text{ km/h}$$

(b) During the time interval $0.00 - 35.0$ min, the distance traveled by Rachel is

$$2.80 \text{ km} + (7.70 \text{ km/h})\left(\frac{7.00 \text{ min}}{60 \text{ min}}\right) = 2.80 \text{ km} + 0.90 \text{ km} = 3.70 \text{ km}$$

Her average speed is $\dfrac{\text{distance}}{\text{time}} = \dfrac{3.70 \text{ km}}{(35.0/60.0) \text{ h}} = 6.34$ km/h.

4. Average speed, as opposed to average velocity, relates to the total distance, as opposed to the net displacement. The distance D up the hill is, of course, the same as the distance down the hill, and since the speed is constant (during each stage of the motion) we have speed = D/t. Thus, the average speed is

$$\frac{D_{\text{up}} + D_{\text{down}}}{t_{\text{up}} + t_{\text{down}}} = \frac{2D}{\dfrac{D}{v_{\text{up}}} + \dfrac{D}{v_{\text{down}}}}$$

which, after canceling D and plugging in v_{up} = 35 km/h and v_{down} = 60 km/h, yields 44 km/h for the average speed.

5. **THINK** In this one-dimensional kinematics problem, weĉre given the position function $x(t)$, and asked to calculate the position and velocity of the object at a later time.

EXPRESS The position function is given as $x(t) = (3\ \text{m/s})t$ ï $(4\ \text{m/s}^2)t^2 + (1\ \text{m/s}^3)t^3$. The position of the object at some instant t_0 is simply given by $x(t_0)$. For the time interval $t_1 \le t \le t_2$, the displacement is $\Delta x = x(t_2) - x(t_1)$. Similarly, using Eq. 2-2, the average velocity for this time interval is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}.$$

ANALYZE

(a) Plugging in $t = 1$ s into $x(t)$ yields

$$x(1\ \text{s}) = (3\ \text{m/s})(1\ \text{s}) \text{ ï } (4\ \text{m/s}^2)(1\ \text{s})^2 + (1\ \text{m/s}^3)(1\ \text{s})^3 = 0.$$

(b) With $t = 2$ s we get $x(2\ \text{s}) = (3\ \text{m/s})(2\ \text{s})$ ï $(4\ \text{m/s}^2)\ (2\ \text{s})^2 + (1\ \text{m/s}^3)(2\ \text{s})^3 =$ ï 2 m.

(c) With $t = 3$ s we have $x\ (3\ \text{s}) = (3\ \text{m/s})(3\ \text{s})$ ï $(4\ \text{m/s}^2)\ (3\ \text{s})^2 + (1\ \text{m/s}^3)(3\ \text{s})^3 = 0$ m.

(d) Similarly, plugging in $t = 4$ s gives

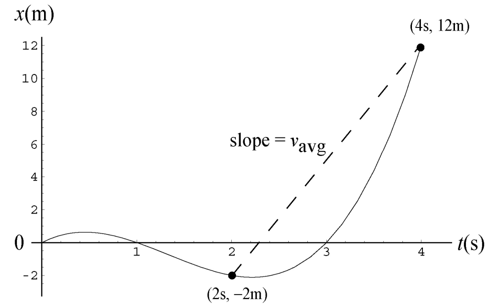
$$x(4\ \text{s}) = (3\ \text{m/s})(4\ \text{s}) \text{ ï } (4\ \text{m/s}^2)(4\ \text{s})^2 + (1\ \text{m/s}^3)\ (4\ \text{s})^3 = 12\ \text{m}.$$

(e) The position at $t = 0$ is $x = 0$. Thus, the displacement between $t = 0$ and $t = 4$ s is $\Delta x = x(4\ \text{s}) - x(0) = 12\ \text{m} - 0 = 12\ \text{m}$.

(f) The position at $t = 2$ s is subtracted from the position at $t = 4$ s to give the displacement: $\Delta x = x(4\ \text{s}) - x(2\ \text{s}) = 12\ \text{m} - (-2\ \text{m}) = 14\ \text{m}$. Thus, the average velocity is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{14\ \text{m}}{2\ \text{s}} = 7\ \text{m/s}.$$

(g) The position of the object for the interval $0 \le t \le 4$ is plotted below. The straight line drawn from the point at $(t, x) = (2$ s, ï 2 m$)$ to $(4$ s, 12 m$)$ would represent the average velocity, answer for part (f).



LEARN Our graphical representation illustrates once again that the average velocity for a time interval depends only on the net displacement between the starting and ending points.

6. Huberõs speed is

$$v_0 = (200\text{ m})/(6.509\text{ s}) = 30.72\text{ m/s} = 110.6\text{ km/h},$$

where we have used the conversion factor 1 m/s = 3.6 km/h. Since Whittingham beat Huber by 19.0 km/h, his speed is v_1 = (110.6 km/h + 19.0 km/h) = 129.6 km/h, or 36 m/s (1 km/h = 0.2778 m/s). Thus, using Eq. 2-2, the time through a distance of 200 m for Whittingham is

$$\Delta t = \frac{\Delta x}{v_1} = \frac{200\text{ m}}{36\text{ m/s}} = 5.554\text{ s}.$$

7. The velocity of approach of both cars (car A and car B as shown in the following figure) that move toward each other is

$$25 - (-16) = 41 \text{ km/h}.$$

The time taken to cover 40 km separation between cars is

$$t = \left(\frac{40}{41}\right)\text{h}$$

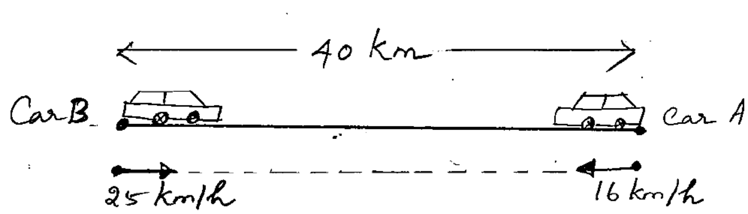
(a) The total distance covered by the pigeon is

$$36 \times \left(\frac{40}{41}\right) \approx 35\,\text{km}.$$

(b) In time t, the first car (car A) moves by a distance

$$\left(\frac{40}{41}\right) \times 16 = 15.61 \approx 16 \text{ km}.$$

Therefore, the displacement of the bird with respect to the first car (car A) is about 16 km.



8. The amount of time it takes for each person to move a distance L with speed v_s is $\Delta t = L / v_s$. With each additional person, the depth increases by one body depth d

(a) The rate of increase of the layer of people is

$$R = \frac{d}{\Delta t} = \frac{d}{L / v_s} = \frac{d v_s}{L} = \frac{(0.25\ \text{m})(3.50\ \text{m/s})}{1.75\ \text{m}} = 0.50\ \text{m/s}$$

(b) The amount of time required to reach a depth of $D = 5.0$ m is

$$t = \frac{D}{R} = \frac{5.0\ \text{m}}{0.50\ \text{m/s}} = 10\ \text{s}$$

9. Converting to seconds, the running times are t_1 = 147.95 s and t_2 = 148.15 s, respectively. If the runners were equally fast, then

$$s_{\text{avg}_1} = s_{\text{avg}_2} \quad \Rightarrow \quad \frac{L_1}{t_1} = \frac{L_2}{t_2}.$$

From this we obtain

$$L_2 - L_1 = \left(\frac{t_2}{t_1} - 1\right) L_1 = \left(\frac{148.15}{147.95} - 1\right) L_1 = 0.00135 L_1 \approx 1.4 \text{ m}$$

where we set $L_1 \approx 1000$ m in the last step. Thus, if L_1 and L_2 are no different than about 1.4 m, then runner 1 is indeed faster than runner 2. However, if L_1 is shorter than L_2 by more than 1.4 m, then runner 2 would actually be faster.

10. Let v_w be the speed of the wind and v_c be the speed of the car.

(a) Suppose during time interval t_1, the car moves in the same direction as the wind. Then the effective speed of the car is given by $v_{eff,1} = v_c + v_w$, and the distance traveled is $d = v_{eff,1} t_1 = (v_c + v_w) t_1$. On the other hand, for the return trip during time interval t_2, the car moves in the opposite direction of the wind and the effective speed would be $v_{eff,2} = v_c - v_w$. The distance traveled is $d = v_{eff,2} t_2 = (v_c - v_w) t_2$. The two expressions can be rewritten as

$$v_c + v_w = \frac{d}{t_1} \quad \text{and} \quad v_c - v_w = \frac{d}{t_2}$$

Adding the two equations and dividing by two, we obtain $v_c = \frac{1}{2}\left(\frac{d}{t_1} + \frac{d}{t_2}\right)$. Thus, method 1 gives the car's speed v_c a in windless situation.

(b) If method 2 is used, the result would be

$$v_c' = \frac{d}{(t_1 + t_2)/2} = \frac{2d}{t_1 + t_2} = \frac{2d}{\dfrac{d}{v_c + v_w} + \dfrac{d}{v_c - v_w}} = \frac{v_c^2 - v_w^2}{v_c} = v_c \left[1 - \left(\frac{v_w}{v_c}\right)^2\right].$$

The fractional difference is

$$\frac{v_c - v_c'}{v_c} = \left(\frac{v_w}{v_c}\right)^2 = (0.0240)^2 = 5.76 \times 10^{-4}.$$

11. In 150 s, the pickup vehicle moves a distance of $150.0 \times 15.00 = 2250$ m. The total distance covered by the scooter is

$$1500 \text{ m} + 2250 \text{ m} = 3750 \text{ m}.$$

The speed at which the scooterist should chase the pickup vehicle is

$$v_0 = \frac{3750}{150.0} = 25.00 \text{ m/s}.$$

12. (a) Let the fast and the slow cars be separated by a distance d at t = 0. If during the time interval $t = L/v_s = (12.0\text{ m})/(5.0\text{ m/s}) = 2.40\text{ s}$ in which the slow car has moved a distance of $L = 12.0\text{ m}$, the fast car moves a distance of $vt = d + L$ to join the line of slow cars, then the shock wave would remain stationary. The condition implies a separation of

$$d = vt - L = (25\text{ m/s})(2.4\text{ s}) - 12.0\text{ m} = 48.0\text{ m}.$$

(b) Let the initial separation at $t = 0$ be $d = 96.0\text{ m}$. At a later time t, the slow and the fast cars have traveled $x = v_s t$ and the fast car joins the line by moving a distance $d + x$. From

$$t = \frac{x}{v_s} = \frac{d + x}{v},$$

we get

$$x = \frac{v_s}{v - v_s} d = \frac{5.00\text{ m/s}}{25.0\text{ m/s} - 5.00\text{ m/s}}(96.0\text{ m}) = 24.0\text{ m},$$

which in turn gives $t = (24.0\text{ m})/(5.00\text{ m/s}) = 4.80\text{ s}$. Since the rear of the slow-car pack has moved a distance of

$$\Delta x = x - L = 24.0\text{ m} - 12.0\text{ m} = 12.0\text{ m}\text{ downstream},$$

the speed of the rear of the slow-car pack, or equivalently, the speed of the shock wave, is

$$v_{\text{shock}} = \frac{\Delta x}{t} = \frac{12.0\text{ m}}{4.80\text{ s}} = 2.50\text{ m/s}.$$

(c) Since $x > L$, the direction of the shock wave is downstream.

13. (a) Denoting the travel time and distance from San Antonio to Houston as T and D, respectively, the average speed is

$$s_{\text{avg1}} = \frac{D}{T} = \frac{(55\text{ km/h})(T/2) + (90\text{ km/h})(T/2)}{T} = 72.5\text{ km/h}$$

which should be rounded to 73 km/h.

(b) Using the fact that time = distance/speed while the speed is constant, we find

$$s_{\text{avg2}} = \frac{D}{T} = \frac{D}{\frac{D/2}{55\text{ km/h}} + \frac{D/2}{90\text{ km/h}}} = 68.3\text{ km/h}$$

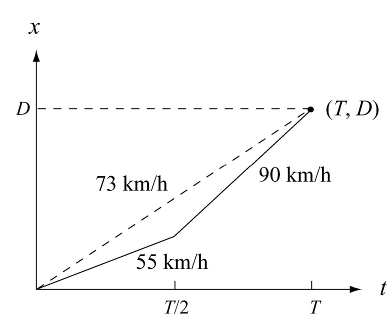
which should be rounded to 68 km/h.

(c) The total distance traveled ($2D$) must not be confused with the net displacement (zero). We obtain for the two-way trip

$$s_{\text{avg}} = \frac{2D}{\frac{D}{72.5\text{ km/h}} + \frac{D}{68.3\text{ km/h}}} = 70\text{ km/h.}$$

(d) Since the net displacement vanishes, the average velocity for the trip in its entirety is zero.

(e) In asking for a *sketch*, the problem is allowing the student to arbitrarily set the distance D (the intent is *not* to make the student go to an atlas to look it up); the student can just as easily arbitrarily set T instead of D, as will be clear in the following discussion. We briefly describe the graph (with kilometers-per-hour understood for the slopes): two contiguous line segments, the first having a slope of 55 and connecting the origin to $(t_1, x_1) = (T/2, 55T/2)$ and the second having a slope of 90 and connecting (t_1, x_1) to (T, D) where $D = (55 + 90)T/2$. The average velocity, from the graphical point of view, is the slope of a line drawn from the origin to (T, D). The graph (not drawn to scale) is depicted below:



14. Using the general property $\frac{d}{dx}\exp(bx) = b\exp(bx)$, we write

$$v = \frac{dx}{dt} = \left(\frac{d(19t)}{dt}\right)\cdot e^{-t} + (19t)\cdot\left(\frac{de^{-t}}{dt}\right) .$$

If a concern develops about the appearance of an argument of the exponential (ï t) apparently having units, then an explicit factor of $1/T$ where T = 1 second can be inserted and carried through the computation (which does not change our answer). The result of this differentiation is

$$v = 16(1 - t)e^{-t}$$

with t and v in SI units (s and m/s, respectively). We see that this function is zero when t = 1 s. Now that we know *when* it stops, we find out *where* it stops by plugging our result t = 1 into the given function $x = 16te^{-t}$ with x in meters. Therefore, we find x = 5.9 m.

15. We have $x = 18t + 5.0t^2$.

(a) Instantaneous velocity is calculated as

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \frac{d}{dt}(18t + 5.0t^2)$$

That is,

$$v = 18 + 10.0t \qquad (1)$$

At t = 2.0 s, the instantaneous velocity is

$$v = 18 + (10 \times 2.0) = 18 + 20 = 38 \text{ m/s.}$$

(b) Let x_1 be the distance of the particle at t = 2.0 s, which is calculated as

$$x_1 = 18(2.0) + 5.0(2.0)^2 = 36 + 20 = 56 \text{ m.}$$

Let x_2 be the distance of the particle at t = 3.0 s, which is calculated as

$$x_2 = 18(3.0) + 5.0(3.0)^2 = 54 + 45 = 99 \text{ m.}$$

Therefore, the average velocity of the particle between t = 2 s and t = 3 s is calculated as

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{(x_2 - x_1)}{(t_2 - t_1)} = \frac{99 - 56}{3.0 - 2.0} = \frac{43}{1.0} = 43 \text{ m/s.}$$

16. We use the functional notation $x(t)$, $v(t)$, and $a(t)$ in this solution, where the latter two quantities are obtained by differentiation:

$$v(t) = \frac{dx(t)}{dt} = -12t \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -12$$

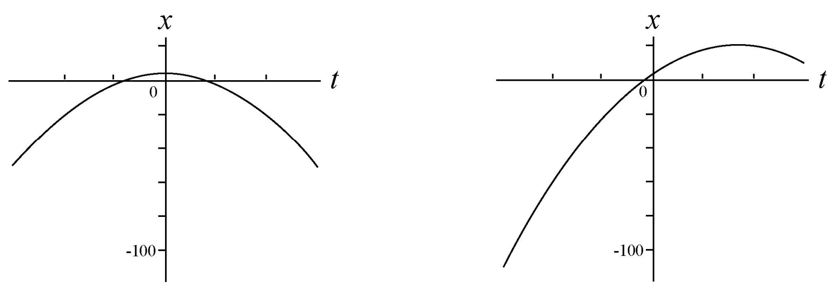
with SI units understood.

(a) From $v(t) = 0$ we find it is (momentarily) at rest at $t = 0$.

(b) We obtain $x(0) = 4.0$ m.

(c) and (d) Requiring $x(t) = 0$ in the expression $x(t) = 4.0$ ï $6.0t^2$ leads to $t = \pm 0.82$ s for the times when the particle can be found passing through the origin.

(e) We show both the asked-for graph (on the left) as well as the ñshiftedò graph that is relevant to part (f). In both cases, the time axis is given by ï $3 \le t \le 3$ (SI units understood).



(f) We arrived at the graph on the right (shown above) by adding $20t$ to the $x(t)$ expression.

(g) Examining where the slopes of the graphs become zero, it is clear that the shift causes the $v = 0$ point to correspond to a larger value of x (the top of the second curve shown in part (e) is higher than that of the first).

17. We use Eq. 2-2 for average velocity and Eq. 2-4 for instantaneous velocity, and work with distances in centimeters and times in seconds.

(a) We plug into the given equation for x for t = 2.00 s and t = 3.00 s and obtain x_2 = 21.75 cm and x_3 = 50.25 cm, respectively. The average velocity during the time interval $2.00 \le t \le 3.00$ s is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{50.25\text{ cm} - 21.75\text{ cm}}{3.00\text{ s} - 2.00\text{ s}}$$

which yields v_{avg} = 28.5 cm/s.

(b) The instantaneous velocity is $v = \frac{dx}{dt} = 4.5t^2$, which, at time t = 2.00 s, yields v = $(4.5)(2.00)^2$ = 18.0 cm/s.

(c) At t = 3.00 s, the instantaneous velocity is $v = (4.5)(3.00)^2$ = 40.5 cm/s.

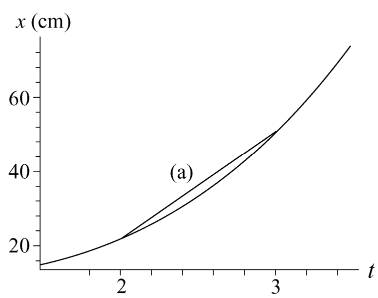
(d) At t = 2.50 s, the instantaneous velocity is $v = (4.5)(2.50)^2$ = 28.1 cm/s.

(e) Let t_m stand for the moment when the particle is midway between x_2 and x_3 (that is, when the particle is at $x_m = (x_2 + x_3)/2$ = 36 cm). Therefore,

$$x_m = 9.75 + 1.5t_m^3 \quad \Rightarrow \quad t_m = 2.596$$

in seconds. Thus, the instantaneous speed at this time is $v = 4.5(2.596)^2$ = 30.3 cm/s.

(f) The answer to part (a) is given by the slope of the straight line between t = 2 and t = 3 in this x-vs-t plot. The answers to parts (b), (c), (d), and (e) correspond to the slopes of tangent lines (not shown but easily imagined) to the curve at the appropriate points.



18. (a) Taking derivatives of $x(t) = 12t^2 - 2t^3$ we obtain the velocity and the acceleration functions:

$$v(t) = 24t - 6t^2 \quad \text{and} \quad a(t) = 24 - 12t$$

with length in meters and time in seconds. Plugging in the value $t = 3.5$ s yields $x(3.5) = 61\ \text{m}$.

(b) Similarly, plugging in the value $t = 3.5$ s yields $v(3.5) = 11$ m/s.

(c) For $t = 3.5$ s, $a(3.5) = -18\ \text{m/s}^2$.

(d) At the maximum x, we must have $v = 0$; eliminating the $t = 0$ root, the velocity equation reveals $t = 24/6 = 4$ s for the time of maximum x. Plugging $t = 4$ into the equation for x leads to $x = 64$ m for the largest x value reached by the particle.

(e) From (d), we see that the x reaches its maximum at $t = 4.0$ s.

(f) A maximum v requires $a = 0$, which occurs when $t = 24/12 = 2.0$ s. This, inserted into the velocity equation, gives $v_{max} = 24$ m/s.

(g) From (f), we see that the maximum of v occurs at $t = 24/12 = 2.0$ s.

(h) In part (e), the particle was (momentarily) motionless at $t = 4$ s. The acceleration at that time is readily found to be $24 - 12(4) = -24\ \text{m/s}^2$.

(i) The *average velocity* is defined by Eq. 2-2, so we see that the values of x at $t = 0$ and $t = 3$ s are needed; these are, respectively, $x = 0$ and $x = 54$ m (found in part (a)). Thus,

$$v_{avg} = \frac{54-0}{3-0} = 18\ \text{m/s}.$$

19. **THINK** In this one-dimensional kinematics problem, weôre given the speed of a particle at two instants and asked to calculate its average acceleration.

EXPRESS We represent the initial direction of motion as the +*x* direction. The average acceleration over a time interval $t_1 \le t \le t_2$ is given by Eq. 2-7:

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{v(t_2) - v(t_1)}{t_2 - t_1}.$$

ANALYZE Let v_1 = +18 m/s at $t_1 = 0$ and v_2 = ï 30 m/s at t_2 = 2.4 s. Using Eq. 2-7 we find

$$a_{\text{avg}} = \frac{v(t_2) - v(t_1)}{t_2 - t_1} = \frac{(-30\text{ m/s}) - (+1\text{ m/s})}{2.4\text{ s} - 0} = -20\text{ m/s}^2.$$

LEARN The average acceleration has magnitude 20 m/s^2 and is in the opposite direction to the particleôs initial velocity. This makes sense because the velocity of the particle is decreasing over the time interval. With $t_1 = 0$, the velocity of the particle as a function of time can be written as

$$v = v_0 + at = (18\text{ m/s}) - (20\text{ m/s}^2)t.$$

20. The position of a particle is $x = 25t - 6t^3$.

(a) The particle's velocity is

$$v = \frac{dx}{dt} = (25 - 18t^2)\,\text{m/s}.$$

If $v = 0\,\text{m/s}$, we have

$$18t^2 = 25.$$

Therefore,

$$t^2 = \frac{25}{18} \Rightarrow t = \pm\left(\frac{5}{3\sqrt{2}}\right)\text{s} = \pm 1.2\text{ s}.$$

That is, the particle's velocity is zero when $t = \pm 1.2$ s.

(b) The instantaneous acceleration of the particle is

$$a = \frac{dv}{dt} = \frac{d^2}{dt^2}(x) = (0 - 36t)\,\text{m/s}^2.$$

That is,

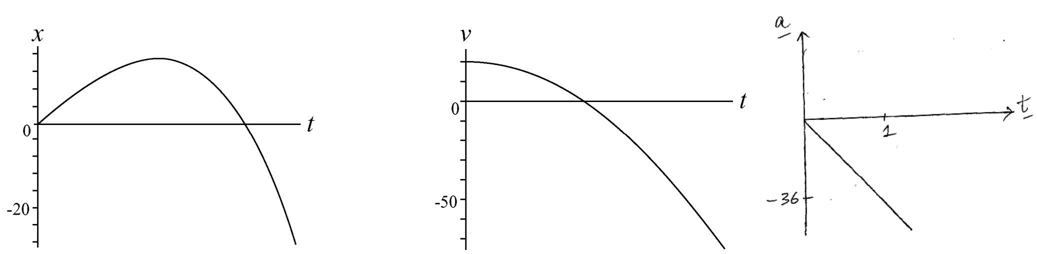
$$a = 0\text{ m/s}^2 \quad \text{or} \quad -36t = 0 \Rightarrow t = 0\text{ s}.$$

Therefore, at $t = 0$ s, the acceleration of the particle is zero.

(c) It is obvious that $a(t) =$ ï $36t$ is negative for $t > 0$.

(d) The acceleration $a(t) =$ ï $36t$ is positive for $t < 0$.

(e) The graphs are shown in Figs. (a), (b) and (c) as follows (SI units are understood):



21. The initial velocity of the car is

$$v_0 = 130 \text{ km/h} = \frac{130 \times 5.00}{18.0} \text{ m/s} = \frac{650}{18.0} \text{ m/s} = 36.1 \text{ m/s}.$$

The displacement of the car is

$$x \text{ ï } x_0 = 210 \text{ m ï } 0.00 \text{ m} = 210 \text{ m}.$$

(a) Using one of the equations of motion,

$$v^2 = v_0^2 + 2a(x - x_0)\ ,$$

we get the acceleration, a, as

$$a = \frac{v^2 - v_0^2}{2(x \text{ ï } x_0)}.$$

Substituting the values in this equation, we get the uniform retardation, that is, uniform deceleration of the car as follows:

$$a = \frac{v^2 - v_0^2}{2(x \text{ ï } x_0)} = \frac{0^2 - (36.1)^2}{2(210)} = -\frac{1303.21}{420} = -3.10 \text{ m/s}^2.$$

Thus, the magnitude is 3.10 m/s^2.

(b) Using one of the equations of motion, $v = v_0 + at$, we calculate the time taken by the car to stop as follows:

$$t = \frac{v - v_0}{a} = \frac{0 - 36.1}{-3.1} = 11.64 \text{ s} \approx 11.6 \text{ s}.$$

22. In this solution, we make use of the notation $x(t)$ for the value of x at a particular t. The notations $v(t)$ and $a(t)$ have similar meanings.

(a) Since the unit of ct^2 is that of length, the unit of c must be that of length/time^2, or m/s^2 in the SI system.

(b) Since bt^3 has a unit of length, b must have a unit of length/time^3, or m/s^3.

(c) When the particle reaches its maximum (or its minimum) coordinate its velocity is zero. Since the velocity is given by $v = dx/dt = 2ct$ ï $3bt^2$, $v = 0$ occurs for $t = 0$ and for

$$t = \frac{2c}{3b} = \frac{2(4.0\ \text{m/s}^2)}{3(2.0\ \text{m/s}^3)} = 1.3\ \text{s}\ .$$

For $t = 0$, $x = x_0 = 0$ and for $t = 1.3$ s, $x = 2.3\ \text{m} > x_0$. Since we seek the maximum, we reject the first root ($t = 0$) and accept the second ($t = 1$s).

(d) In the first 4 s the particle moves from the origin to $x = 1.0$ m, turns around, and goes back to

$$x(4\ \text{s}) = (4.0\ \text{m/s}^2)(4.0\ \text{s})^2 - (2.0\ \text{m/s}^3)(4.0\ \text{s})^3 = -64\ \text{m}\ .$$

The total path length it travels is 1.3 m + 1.3 m + 64 m = 67 m.

(e) Its displacement is $\Delta x = x_2$ ï x_1, where $x_1 = 0$ and $x_2 =$ ï 64 m. Thus, $\Delta x = -64$ m .

The velocity is given by $v = 2ct$ ï $3bt^2 = (8.0\ \text{m/s}^2)t$ ï $(6.0\ \text{m/s}^3)t^2$.

(f) Plugging in $t = 1$ s, we obtain
$v\ (1.0) = 2.0$ m/s

(g) Similarly, $v\ (2.0) = -8.0$ m/s

(h) $v\ (3.0) = -30$ m/s

(i) $v\ (4.0) = -64$ m/s

The acceleration is given by $a = dv/dt = 2c$ ï $6b = 8.0\ \text{m/s}^2$ ï $(12.0\ \text{m/s}^3)t$.

(j) Plugging in $t = 1$ s, we obtain
$a\ (1.0) = -4.0\ \text{m/s}^2$

(k) $a\ (2.0) = -16$ m

(l) $a\ (3.0) = -28$ m

(m) $a\ (4.0) = -40$ m

23. It is given that $v_0 = 0$. Let the constant acceleration be a. Distance covered during the fifth second is the distance moved between t = 4.00 s and t = 5.00 s. The velocity of the object at t = 4.00 s is used as the initial velocity for its further motion at 4.00 s is

$$v_4 = v_0 + at \Rightarrow v_4 = (4.00)a.$$

Therefore, the distance covered between t = 4.00 s and t = 5.00 s is

$$(x-x_0)_{45} = v_4 t + \frac{1}{2}at^2 = (4.00)a + \frac{1}{2}a = (4.50)a.$$

Given that t = 1.00 s between t = 4.00 s and t = 5.00 s. The distance covered up to 5.00 s is

$$(x-x_0)_5 = v_0 t + \frac{1}{2}at^2 = 0 + \left[\frac{1}{2}(a)(5.00)^2\right] = \frac{1}{2}a(25.0) = (12.5)a.$$

Given that t = 5.00 s, the required ratio is

$$\frac{(x-x_0)_{45}}{(x-x_0)_5} = \frac{(4.50)a}{(12.5)a} = 0.36.$$

24. In this problem we are given the initial and final speeds, and the displacement, and are asked to find the acceleration. We use the constant-acceleration equation given in Eq. 2-16, $v^2 = v_0^2 + 2a(x$ ï $x_0)$.

(a) Given that $v_0 = 0$, $v = 1.6$ m/s, and $\Delta x = 5.0\,\mu$m, the acceleration of the spores during the launch is

$$a = \frac{v^2 - v_0^2}{2x} = \frac{(1.6\ \text{m/s})^2}{2(5.0\times10^{-6}\ \text{m})} = 2.56\times10^5\ \text{m/s}^2 = 2.6\times10^4\, g$$

(b) During the speed-reduction stage, the acceleration is

$$a = \frac{v^2 - v_0^2}{2x} = \frac{0 - (1.6\ \text{m/s})^2}{2(1.0\times10^{-3}\ \text{m})} = -1.28\times10^3\ \text{m/s}^2 = -1.3\times10^2\, g$$

The negative sign means that the spores are decelerating.

25. From the equation of motion

$$v = v_0 + at\,,$$

we get

$$2.8 = v_0 + 2.5a \tag{1}$$

Using another equation of motion, $v^2 - v_0^2 = 2a(x - x_0)$, we get

$$(2.8)^2 - v_0^2 = 2a(8.0 - 2.0) \quad \Rightarrow \quad (2.8 + v_0)(2.8 - v_0) = 12a \tag{2}$$

Dividing Eq. (2) by Eq. (1) gives

$$\frac{(2.8 + v_0)(2.8 - v_0)}{(2.8 - v_0)} = \frac{12a}{2.5a}$$

which leads to

$$2.8 + v_0 = \frac{12}{2.5} = 4.8 \quad \Rightarrow \quad v_0 = \frac{4.8}{2.8} = 2.0 \text{ m/s}.$$

Substituting the value of v_0 in Eq. (1), we get the acceleration during the given time interval as follows:

$$\begin{aligned} & 2.8 = v_0 + 2.5a \\ \Rightarrow\ & 2.8 - 2 = 2.5a \\ \Rightarrow\ & a = \frac{0.8}{2.5} = \frac{8 \times 10}{10 \times 25} = 0.32\,\text{m/s}^2. \end{aligned}$$

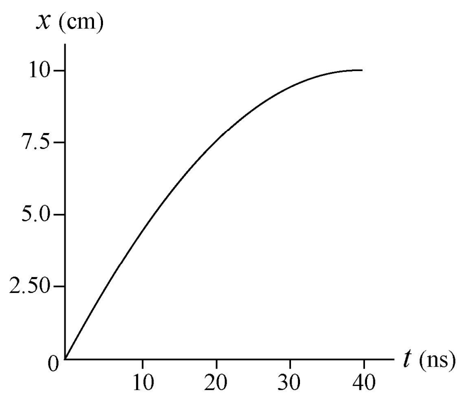
26. The constant-acceleration condition permits the use of Table 2-1.

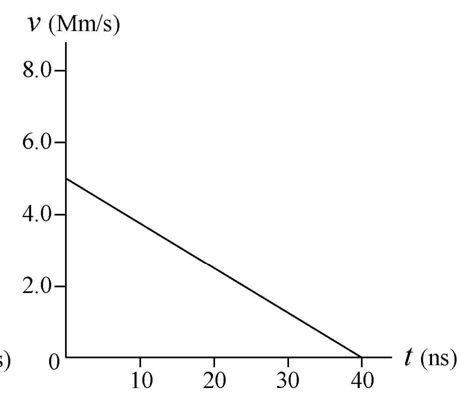
(a) Setting $v = 0$ and $x_0 = 0$ in $v^2 = v_0^2 + 2a(x - x_0)$, we find

$$x = -\frac{1}{2}\frac{v_0^2}{a} = -\frac{1}{2}\frac{(6.00 \times 10^6)^2}{-1.25 \times 10^{14}} = 0.144 \text{ m} .$$

Since the muon is slowing, the initial velocity and the acceleration must have opposite signs.

(b) Below are the time plots of the position x and velocity v of the muon from the moment it enters the field to the time it stops. The computation in part (a) made no reference to t, so that other equations from Table 2-1 (such as $v = v_0 + at$ and $x = v_0 t + \frac{1}{2}at^2$) are used in making these plots.





27. We have $v_0 = 0$; $(x - x_0) = 2.0$ cm; $t = (5.0 \times 10^{-3})$ s. Substituting these values in the equation of motion

$$(x - x_0) = v_0 t + \frac{1}{2} a t^2 ,$$

we get

$$\left(\frac{2.00}{100}\right)\mathrm{m} = 0 + \left[\frac{1}{2.00} \times a(5.00 \times 10^{-3})^2\right]$$

The magnitude of the given acceleration is obtained as follows:

$$a = \left(\frac{4.00}{100 \times 25.0 \times 10^{-6}}\right)\mathrm{m/s^2} = \left(\frac{4.00}{25.0} \times 10^4\right)\mathrm{m/s^2} = (1.6 \times 10^3)\,\mathrm{m/s^2}.$$

28. We take $+x$ in the direction of motion, so v_0 = +27.2 m/s and a = ï 4.92 m/s^2. We also take x_0 = 0.

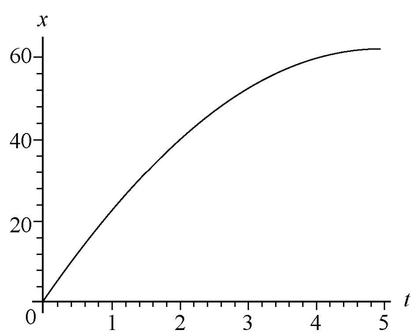
(a) The time to come to a halt is found using Eq. 2-11:

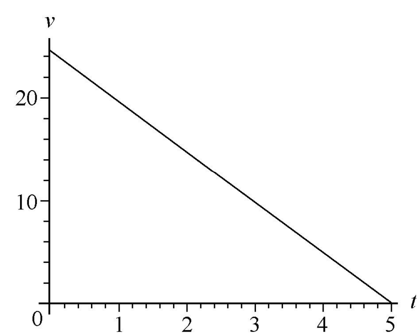
$$0 = v_0 + at \Rightarrow t = \frac{27.2 \text{ m/s}}{4.92 \text{ m/s}^2} = 5.53 \text{ s}$$

(b) Although several of the equations in Table 2-1 will yield the result, we choose Eq. 2-16 (since it does not depend on our answer to part (a)).

$$0 = v_0^2 + 2ax \Rightarrow x = -\frac{(27.2 \text{ m/s})^2}{2(-4.92 \text{ m/s}^2)} = 75.2 \text{ m}$$

(c) Using these results, we plot $v_0 t + \frac{1}{2}at^2$ (the x graph, shown next, on the left) and $v_0 + at$ (the v graph, on the right) over $0 \le t \le 5$ s, with SI units understood.





29. We assume the periods of acceleration (duration t_1) and deceleration (duration t_2) are periods of constant a so that Table 2-1 can be used. Taking the direction of motion to be $+x$ then a_1 = +1.22 m/s^2 and a_2 = ï 1.22 m/s^2. We use SI units so the velocity at t = t_1 is v = 305/60 = 5.08 m/s.

(a) We denote Δx as the distance moved during t_1, and use Eq. 2-16:

$$v^2 = v_0^2 + 2a_1\Delta x \quad \Rightarrow \quad \Delta x = \frac{(5.08\text{ m/s})^2}{2(1.22\text{ m/s}^2)} = 10.59\text{ m} \approx 10.6\text{ m}.$$

(b) Using Eq. 2-11, we have

$$t_1 = \frac{v - v_0}{a_1} = \frac{5.08\text{ m/s}}{1.22\text{ m/s}^2} = 4.17\text{ s}.$$

The deceleration time t_2 turns out to be the same so that $t_1 + t_2$ = 8.33 s. The distances traveled during t_1 and t_2 are the same so that they total to 2(10.59 m) = 21.18 m. This implies that for a distance of 190 m ï 21.18 m = 168.82 m, the elevator is traveling at constant velocity. This time of constant velocity motion is

$$t_3 = \frac{168.82\text{ m}}{5.08\text{ m/s}} = 33.21\text{ s}.$$

Therefore, the total time is 8.33 s + 33.21 s ≈ 41.5 s.

30. We choose the positive direction to be that of the initial velocity of the car (implying that $a < 0$ since it is slowing down). We assume the acceleration is constant and use Table 2-1.

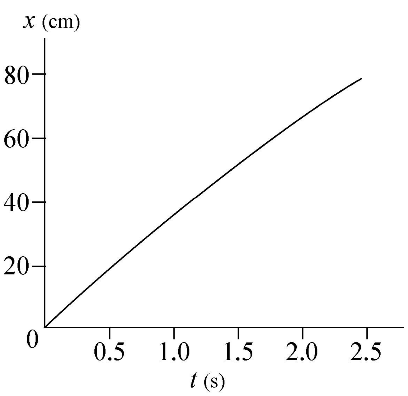
(a) Substituting v_0 = 146 km/h = 40.6 m/s, v = 90 km/h = 25 m/s, and a = ï 5.2 m/s^2 into $v = v_0 + at$, we obtain

$$t = \frac{25\text{ m/s} - 40.6\text{ m/s}}{-5.2\text{ m/s}^2} = 3.0\text{ s}.$$

(b) We take the car to be at $x = 0$ when the brakes are applied (at time $t = 0$). Thus, the coordinate of the car as a function of time is given by

- $x = (30.6\text{ m/s})t + \frac{1}{2}(-5.2\text{ m/s}^2)t^2$

in SI units. This function is plotted from $t = 0$ to $t = 2.5$ s on the graph to the right. We have not shown the v-vs-t graph here; it is a descending straight line from v_0 to v.



31. We have the initial velocity of the rocket, $v_0 = 0$; acceleration of the rocket, $a = 10.0\ \text{m/s}^2$. Referring to the following figure, which depicts the situation, the velocity at A is calculated as follows: From the equation of motion,

$$v^2 = v_0^2 + 2a(x - x_0)\ ,$$

we get

$$v^2 - v_0^2 = 2a(x - x_0)$$

$$\Rightarrow v^2 - 0^2 = 2(10.0\,\text{m/s}^2 \times 500\,\text{m})$$

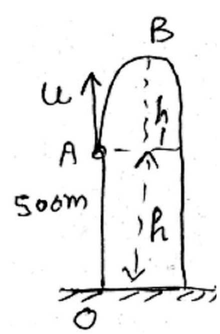
$$\Rightarrow v^2 = 10,000 \Rightarrow v = 100\ \text{m/s}.$$

After the engine of the rocket cuts off, the rocket rises against gravity with initial velocity of 100 m/s. Therefore,

$$h_1 = \frac{u^2}{2g} = \frac{100 \times 100}{2 \times 9.8} = 510.2\ \text{m}.$$

Therefore, the maximum altitude the rocket reaches is

$$H = h + h_1 = 500\ \text{m} + 510.2\ \text{m} = 1010.2\,\text{m} = 1.01\,\text{km}.$$



32. The acceleration is found from Eq. 2-11 (or, suitably interpreted, Eq. 2-7).

$$a = \frac{\Delta v}{\Delta t} = \frac{(1020 \text{ km/h})\left(\dfrac{1000 \text{ m/km}}{3600 \text{ s/h}}\right)}{1.4 \text{ s}} = 202.4 \text{ m/s}^2 \ .$$

In terms of the gravitational acceleration g, this is expressed as a multiple of 9.8 m/s^2 as follows:

$$a = \left(\frac{202.4 \text{ m/s}^2}{9.8 \text{ m/s}^2}\right) g = 21g \ .$$

33. The position of the stone is given by

$$y = 60\text{ m} - (20\text{ m/s})t - \frac{1}{2}(9.8\text{ m/s}^2)t^2$$

Solving the quadratic equation with $y = 0$, we obtain $t = +2.01\text{ s or } -6.09\text{ s}$. Since a negative value implies that the stone reaches the ground before it is released, we consider the positive value, that is, +2.0 s.

34. Let d be the 220 m distance between the cars at $t = 0$, and v_1 be the 20 km/h = 50/9 m/s speed (corresponding to a passing point of x_1 = 44.5 m) and v_2 be the 40 km/h =100/9 m/s speed (corresponding to a passing point of x_2 = 77.9 m) of the red car. We have two equations (based on Eq. 2-17):

$$d \text{ ï } x_1 = v_o t_1 + \frac{1}{2} a t_1^2 \qquad \text{where } t_1 = x_1 / v_1$$

$$d \text{ ï } x_2 = v_o t_2 + \frac{1}{2} a t_2^2 \qquad \text{where } t_2 = x_2 / v_2$$

We simultaneously solve these equations and obtain the following results:

(a) The initial velocity of the green car is v_o = -8.7 m/s. The negative indicates that the green car initially moves toward the green car.

(b) The corresponding acceleration of the car is a = -3.3 m/s^2.

35. Using $v^2 = v_0^2 + 2a(x - x_0)$, we find the acceleration of the particle to be

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(6.00 \times 10^7 \text{ m/s})^2 - (4.00 \times 10^5 \text{ m/s})^2}{2(0.0300 \text{ m})} = 6.00 \times 10^{16} \text{ m/s}^2.$$

The time interval at which the electron accelerates is obtained as follows: From the equation of motion, $v = v_0 + at$, we have the required time interval as

$$t = \frac{v - v_0}{a} = \frac{6.00 \times 10^7 \text{ m/s} - 4.00 \times 10^5 \text{ m/s}}{6.00 \times 10^{16} \text{ m/s}^2} = 9.93 \times 10^{-10} \text{s} = 0.993 \text{ ns}.$$

36. (a) Equation 2-15 is used for part 1 of the trip:

$$\Delta x_1 = v_{o1}\, t_1 + \tfrac{1}{2}\, a_1\, t_1^{\,2}$$

where $v_{o1} = 0$, $a_1 = 2.75\ \text{m/s}^2$ and $\Delta x_1 = \frac{900}{4}$ m = 225 m. Solving for t_1, we obtain

$$t_1 = \sqrt{\frac{2(\Delta x_1)}{a_1}} = \sqrt{\frac{2(225\ \text{m})}{2.75\ \text{m/s}^2}} = 12.79\ \text{s}$$

The speed attained during part 1 of the trip is

$$v_{1f} = a_1 t_1 = \sqrt{2a_1(\Delta x_1)} = \sqrt{2(2.75\ \text{m/s}^2)(225\ \text{m})} = 35.2\ \text{m/s}.$$

With $v_{20} = v_{1f}$, the time it takes for the car to come to rest with a constant acceleration

of $a_2 = -0.917\ \text{m/s}^2$ is given by $v_{2f} = v_{20} + a_2 t_2 = 0,$ or

$$t_2 = \frac{v_{2f} - v_{20}}{a_2} = \frac{0 - 35.2\ \text{m/s}}{-0.917\ \text{m/s}^2} = 38.36\ \text{s}$$

The total travel time is $t = t_1 + t_2$ = 12.79 s + 38.36 s = 51.2 s.

(b) The maximum speed is attained at the end of the first part of the trip: $v_{1f} = 35.2\ \text{m/s}.$

37. (a) From the figure, we see that x_0 = ï 2.0 m. From Table 2-1, we can apply

$$x \text{ ï } x_0 = v_0 t + \tfrac{1}{2} a t^2$$

with t = 1.0 s, and then again with t = 2.0 s. This yields two equations for the two unknowns, v_0 and a:

$$0.0-(-2.0\text{ m}) = v_0(1.0\text{ s}) + \frac{1}{2}a(1.0\text{ s})^2$$

$$6.0\text{ m}-(-2.0\text{ m}) = v_0\ (2.0\text{ s}) + \frac{1}{2}a(2.0\text{ s})^2\ .$$

Solving these simultaneous equations yields the results v_0 = 0 and a = 4.0 m/s^2.

(b) The fact that the answer is positive tells us that the acceleration vector points in the $+x$ direction.

38. We assume the train accelerates from rest ($v_0 = 0$ and $x_0 = 0$) at $a_1 = +1.34 \text{ m/s}^2$ until the midway point and then decelerates at $a_2 = -1.34 \text{ m/s}^2$ until it comes to a stop $(v_2 = 0)$ at the next station. The velocity at the midpoint is v_1, which occurs at x_1 = 806/2 = 403m.

(a) Equation 2-16 leads to

$$v_1^2 = v_0^2 + 2a_1x_1 \;\;\Rightarrow\;\; v_1 = \sqrt{2\left(1.34 \text{ m/s}^2\right)\left(440 \text{ m}\right)} = 34.3 \text{ m/s}.$$

왜 403m가 아니라 440m일까?

(b) The time t_1 for the accelerating stage is (using Eq. 2-15)

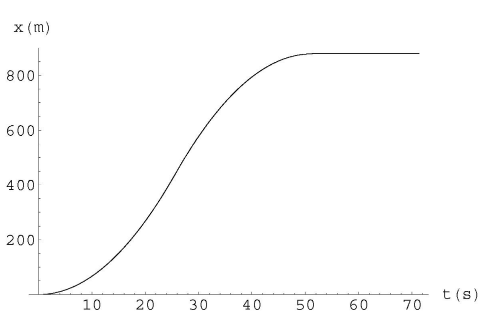
$$x_1 = v_0t_1 + \frac{1}{2}a_1t_1^2 \;\;\Rightarrow\;\; t_1 = \sqrt{\frac{2\left(440 \text{ m}\right)}{1.34 \text{ m/s}^2}} = 25.6 \text{ s}.$$

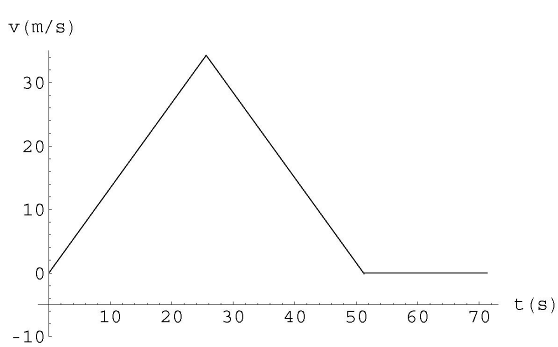
Since the time interval for the decelerating stage turns out to be the same, we double this result and obtain t = 51.2 s for the travel time between stations.

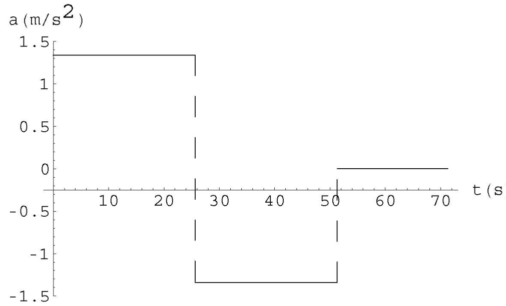
(c) With a ñdead timeò of 20 s, we have $T = t + 20$ = 71.2 s for the total time between start-ups. Thus, Eq. 2-2 gives

$$v_{\text{avg}} = \frac{880 \text{ m}}{71.2 \text{ s}} = 12.4 \text{ m/s}.$$

(d) The graphs for x, v and a as a function of t are shown below. The third graph, $a(t)$, consists of three horizontal ñstepsò ð one at 1.34 m/s^2 during 0 < t < 24.53 s, and the next at ï 1.34 m/s^2 during 24.53 s < t < 49.1 s and the last at zero during the ñdead timeò 49.1 s < t < 69.1 s).






a(m/s2)
1.5
1
0.5
0
-0.5
-1
-1.5
10
20
30
40
50
60
70
t(s)

39. Let t be the time taken by the traffic cop to overtake the speeding car. The total time (reaction plus chase) in which the car moves is (t + 1.0) s. Using the equation of motion

$$(x - x_0) = v_0 t + \frac{1}{2} a t^2,$$

we get

$$46(t+1) = 0(t) + \left(\frac{1}{2} \times 4 \times t^2\right)$$
$$\Rightarrow 46t + 46 = 2t^2$$
$$\Rightarrow t^2 - 46t - 46 = 0,$$

which is simplified further to get the time that it takes to the cop to overtake the speeding car as follows:

$$t = \frac{46 \pm \sqrt{2116 + 368}}{4} = \frac{46 \pm \sqrt{2484}}{4} = \frac{46 \pm 49.84}{4} = \frac{95.84}{4} = 23.96 \approx 24 \text{ s}.$$

40. We take the direction of motion as $+x$, so $a =$ ï $5.18\ \text{m/s}^2$, and we use SI units, so $v_0 = 55(1000/3600) = 15.28$ m/s.

(a) The velocity is constant during the reaction time T, so the distance traveled during it is

$$d_r = v_0 T \text{ ï } (15.28\ \text{m/s})\ (0.75\ \text{s}) = 11.46\ \text{m}.$$

We use Eq. 2-16 (with $v = 0$) to find the distance d_b traveled during braking:

$$v^2 = v_0^2 + 2ad_b \;\Rightarrow\; d_b = -\frac{(15.28\ \text{m/s})^2}{2\left(-5.18\ \text{m/s}^2\right)}$$

which yields $d_b = 22.53$ m. Thus, the total distance is $d_r + d_b = 34.0$ m, which means that the driver *is* able to stop in time. And if the driver were to continue at v_0, the car would enter the intersection in $t = (40\ \text{m})/(15.28\ \text{m/s}) = 2.6$ s, which is (barely) enough time to enter the intersection before the light turns, which many people would consider an acceptable situation.

(b) In this case, the total distance to stop (found in part (a) to be 34 m) is greater than the distance to the intersection, so the driver cannot stop without the front end of the car being a couple of meters into the intersection. And the time to reach it at constant speed is 32/15.28 = 2.1 s, which is too long (the light turns in 1.8 s). The driver is caught between a rock and a hard place.

41. (a) Using the equation of motion $v = v_0 + at$, we get the acceleration of the jet when it stops in 3.0 s as follows:

$$0 = 64 \text{ m/s} + a(3.0 \text{ s}) \quad \Rightarrow \quad a = \left(-\frac{64}{3}\right)\text{m/s}^2.$$

The magnitude of the acceleration is about 21 m/s^2.

(b) Using the equation of motion

$$(x - x_0) = v_0 t + \frac{1}{2} a t^2,$$

we get final position of the jet when it touches the position at $x_i = 0$ as follows:

$$x = v_0 t + \frac{1}{2} a t^2 = (64 \text{ m/s})(3.0 \text{ s}) + \frac{1}{2}(-21.33 \text{ m/s}^2)(3.0 \text{ s})^2 = 192 \text{ m} - 96 \text{ m} = 96 \text{ m}.$$

42. (a) Note that 120 km/h is equivalent to 33.3 m/s. During a two-second interval, you travel 66.67 m. The decelerating police car travels (using Eq. 2-15)

$$\Delta x_p = (33.3\ \text{m/s})(2.0\ \text{s}) - \frac{1}{2}(5.0\ \text{m/s}^2)(2.0\ \text{s})^2 = 56.67\ \text{m}$$

which is 10.0 m less than that by your car. The initial distance between cars was 25 m, this means the gap between cars is now 15.0 m.

(b) First, we add 0.4 s to the considerations of part (a). During the 2.4 s interval, you travel 80.0 m, while the decelerating police car travels

$$\Delta x_p = (33.3\ \text{m/s})(2.4\ \text{s}) - \frac{1}{2}(5.0\ \text{m/s}^2)(2.4\ \text{s})^2 = 65.6\ \text{m}$$

which is 14.4 m less than that by your car.

The initial distance between cars of 25 m has now become 25 m ï 14.4 m = 10.6 m. The speed of the police car at the instant you begin to brake is 33.3 m/s ï (5.00 m/s^2)(2.4 s) = 21.33 m/s. Collision occurs at time t when x_{you} = x_{police} (we choose coordinates such that your position is x = 0 and the police carôs position is x = 10.6 m at t = 0). Eq. 2-15 becomes, for each car:

x_{police} = 10.6 m + (21.3 m/s)t ï $\frac{1}{2}$ (5.00 m/s^2)t^2

x_{you} = (33.3 m/2)t ï $\frac{1}{2}$ (5.00 m/s^2)t^2 .

Subtracting equations, we find

10.6 m = (33.3 m/s ï 21.3 m/s)t $\Rightarrow$ t = 0.883 s

At that time your speed is (33.3 m/s) ï (5.00 m/s^2)(0.883 s) $\approx$ 28.9 m/s, or 104 km/h.

43. In this solution we elect to wait until the last step to convert to SI units. Constant acceleration is indicated, so use of Table 2-1 is permitted. We start with Eq. 2-17 and denote the trainõs initial velocity as v_t and the locomotiveõs velocity as v_ℓ (which is also the final velocity of the train, if the rear-end collision is barely avoided). We note that the distance Δx consists of the original gap between them, D, as well as the forward distance traveled during this time by the locomotive $v_\ell t$. Therefore,

$$\frac{v_t + v_\ell}{2} = \frac{\Delta x}{t} = \frac{D + v_\ell t}{t} = \frac{D}{t} + v_\ell .$$

We now use Eq. 2-11 to eliminate time from the equation. Thus,

$$\frac{v_t + v_\ell}{2} = \frac{D}{(v_\ell - v_t)/a} + v_\ell$$

which leads to

$$a = \left(\frac{v_t + v_\ell}{2} - v_\ell\right)\left(\frac{v_\ell - v_t}{D}\right) = -\frac{1}{2D}(v_\ell - v_t)^2 .$$

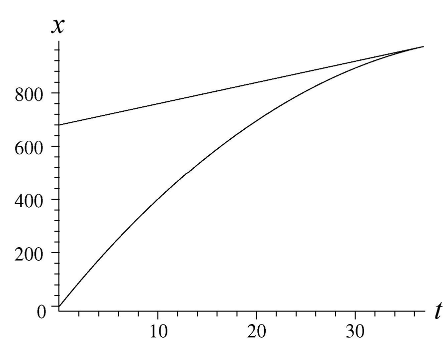
Hence,

$$a = -\frac{1}{2(0.676\ \text{km})}\left(29\frac{\text{km}}{\text{h}} - 161\frac{\text{km}}{\text{h}}\right)^2 = -12888\ \text{km}/\text{h}^2$$

which we convert as follows:

$$a = \left(-12888\ \text{km}/\text{h}^2\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)^2 = -0.994\ \text{m}/\text{s}^2$$

so that its *magnitude* is $|a| = 0.994\ \text{m/s}^2$. A graph is shown here for the case where a collision is just avoided (x along the vertical axis is in meters and t along the horizontal axis is in seconds). The top (straight) line shows the motion of the locomotive and the bottom curve shows the motion of the passenger train.



The other case (where the collision is not quite avoided) would be similar except that the slope of the bottom curve would be greater than that of the top line at the point where they meet.

44. We neglect air resistance, which justifies setting a = ï g = ï 9.8 m/s^2 (taking *down* as the ï y direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the y axis.

(a) Using $y = v_0 t - \frac{1}{2} g t^2$, with y = 0.558 m and t = 0.200 s, we find

$$v_0 = \frac{y + gt^2/2}{t} = \frac{0.558\text{ m} + (9.8\text{ m/s}^2)(0.200\text{ s})^2/2}{0.200\text{ s}} = 3.77\text{ m/s}.$$

(b) The velocity at y = 0.558 m is

$$v = v_0 - gt = 3.77\text{ m/s} - (9.8\text{ m/s}^2)(0.200\text{ s}) = 1.81\text{ m/s}.$$

(b) Using $v^2 = v_0^2 - 2gy$ (with different values for y and v than before), we solve for the value of y corresponding to maximum height (where v = 0).

$$y = \frac{v_0^2}{2g} = \frac{(3.77\text{ m/s})^2}{2(9.8\text{ m/s}^2)} = 0.725\text{ m}.$$

Thus, the armadillo goes 0.725 ï 0.558 = 0.167 m higher.

45. Let the journey last for t seconds from the top of the tower. The height of the tower is given by

$$H = \frac{1}{2}gt^2.$$

The distance covered during the tth second is

$$\Delta y = \frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2 = g\left(t - \frac{1}{2}\right)$$

Since $\Delta y = 9H/25$, we have

$$g\left(t - \frac{1}{2}\right) = \frac{9}{25}H = \frac{9}{25}\left(\frac{1}{2}gt^2\right) = \frac{9}{50}gt^2$$

which can be simplified to give

$$9t^2 - 50t + 25 = (9t-5)(t-5) = 0$$

Thus, we find t = 5.0 s. Note that the other solution $t = (5/9)\,\text{s}$ is not possible since $t > 0.5$. We conclude that

$$H = \frac{1}{2}gt^2 = \frac{1}{2}(9.8\ \text{m/s}^2)(5.0\ \text{s})^2 = 122.5\,\text{m}.$$

46. Neglect of air resistance justifies setting a = ï g = ï 9.8 m/s^2 (where *down* is our ï y direction) for the duration of the fall. This is constant acceleration motion, and we may use Table 2-1 (with Δy replacing Δx).

(a) Using Eq. 2-16 and taking the negative root (since the final velocity is downward), we have

$$v = -\sqrt{v_0^2 - 2g\Delta y} = -\sqrt{0 - 2(9.8\ \text{m/s}^2)(-1800\ \text{m})} = -188\ \text{m/s}\,.$$

Its magnitude is therefore 188 m/s.

(b) No, but it is hard to make a convincing case without more analysis. We estimate the mass of a raindrop to be about a gram or less, so that its mass and speed (from part (a)) would be less than that of a typical bullet, which is good news. But the fact that one is dealing with *many* raindrops leads us to suspect that this scenario poses an unhealthy situation. If we factor in air resistance, the final speed is smaller, of course, and we return to the relatively healthy situation with which we are familiar.

47. As the balloon is ascending at a velocity of 14 m/s, the initial velocity of the packet is v_0 = 14 m/s. The net displacement traveled by the packet is given by $S = -98$ m. As the packet reaches the ground, let v be the velocity of the packet. Here, $a = -g = -9.8$ m/s^2.

(a) As the packet reaches the ground, the velocity of the packet is calculated as follows: From the equation of motion

$$v^2 = v_0^2 + 2a(x - x_0),$$

we have

$$v^2 = (14)^2 + [2 \times -9.8 \times 98]$$

(b) That is, $v = -46.008 \text{ m/s} \approx -46 \text{ m/s}.$ We choose the negative root because the velocity is downward.

The time taken by the packet to reach the ground is obtained as follows:

$$v = v_0 + at$$
$$-46.008 = 14 - 9.8t$$
$$\Rightarrow t = \frac{60.08}{9.8} = 6.1 \text{ m/s}.$$

48. We neglect air resistance, which justifies setting a = ï g = ï 9.8 m/s^2 (taking *down* as the ï y direction) for the duration of the fall. This is constant acceleration motion, which justifies the use of Table 2-1 (with Δy replacing Δx).

(a) Noting that Δy = y ï y_0 = ï 30 m, we apply Eq. 2-15 and the quadratic formula (Appendix E) to compute t:

$$\Delta y = v_0 t - \frac{1}{2} g t^2 \quad \Rightarrow \quad t = \frac{v_0 \pm \sqrt{v_0^2 - 2g\Delta y}}{g}$$

which (with v_0 = ï 15 m/s since it is downward) leads, upon choosing the positive root (so that $t > 0$), to the result:

$$t = \frac{-15 \text{ m/s} + \sqrt{(-15 \text{ m/s})^2 - 2(9.8 \text{ m/s}^2)(-30 \text{ m})}}{9.8 \text{ m/s}^2} = 1.38 \text{ s}.$$

(b) Enough information is now known that any of the equations in Table 2-1 can be used to obtain v; however, the one equation that does *not* use our result from part (a) is Eq. 2-16:

$$v = \sqrt{v_0^2 - 2g\Delta y} = 28.5 \text{ m/s}$$

where the positive root has been chosen in order to give *speed* (which is the magnitude of the velocity vector).

49. **THINK** In this problem a package is dropped from a hot-air balloon which is ascending vertically upward. We analyze the motion of the package under the influence of gravity.

EXPRESS We neglect air resistance, which justifies setting a = ïg = ï9.8 m/s^2 (taking *down* as the ïy direction) for the duration of the motion. This allows us to use Table 2-1 (with Δy replacing Δx):

$$v = v_0 - gt \qquad (2-11)$$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2 \qquad (2-15)$$

$$v^2 = v_0^2 - 2g(y - y_0) \qquad (2-16)$$

We place the coordinate origin on the ground and note that the initial velocity of the package is the same as the velocity of the balloon, v_0 = +12 m/s and that its initial coordinate is y_0 = +80 m. The time it takes for the package to hit the ground can be found by solving Eq. 2-15 with y = 0.

ANALYZE (a) We solve $0 = y = y_0 + v_0 t - \frac{1}{2} g t^2$ for time using the quadratic formula (choosing the positive root to yield a positive value for t):

$$t = \frac{v_0 + \sqrt{v_0^2 + 2gy_0}}{g} = \frac{12\ \text{m/s} + \sqrt{(12\ \text{m/s})^2 + 2\left(9.8\ \text{m/s}^2\right)\left(80\ \text{m}\right)}}{9.8\ \text{m/s}^2} = 5.4\ \text{s}\,.$$

(b) The speed of the package when it hits the ground can be calculated using Eq. 2-11. The result is

$$v = v_0 - gt = 12\ \text{m/s} - (9.8\ \text{m/s}^2)(5.447\ \text{s}) = -41.38\ \text{m/s}\,.$$

Its final *speed* is about 41 m/s.

LEARN Our answers can be readily verified by using Eq. 2-16 which was not used in either (a) or (b). The equation leads to

$$v = -\sqrt{v_0^2 - 2g(y - y_0)} = -\sqrt{(12\ \text{m/s})^2 - 2(9.8\ \text{m/s}^2)(0 - 80\ \text{m})} = -41.38\ \text{m/s}$$

which agrees with that calculated in (b).

50. (a) The horizontal component of the velocity of the skier remains unchanged, that is,

$$v_x = v_0 = 29.4 \text{ m/s}.$$

(b) The vertical component of the velocity of the skier changes with time. Using the equation of motion

$$v = v_0 + at,$$

we get

$$v_y = 0 + gt = 0 + (-9.8 \times 3) = -29.4 \text{ m/s}.$$

Hence, we conclude that the magnitude of both horizontal and vertical components of the skierôs velocity are same.

51. For the displacement during the acceleration, we have

$$(x - x_0) = -\frac{1}{2}\times 39.2 = -19.6\,\text{m}.$$

From the equation of motion

$$v^2 = v_0^2 + 2a(x - x_0),$$

we have

$$v^2 = 0 + (2\times(-9.8)\times(-19.6))$$
$$\Rightarrow v = -19.6\,\text{m/s},$$

where v is the velocity of the melon, which makes the melon to hit the ground. We take the negative root because the melon is moving downward.

52. The full extent of the boltôs fall is given by

$$y - y_0 = \text{ï}\,\frac{1}{2}\,g\,t^2$$

where y ï y_0 = ï 100 m (if upward is chosen as the positive y direction). Thus the time for the full fall is found to be t = 4.52 s. The first 80% of its free-fall distance is given by ï 80 = ï $g\,\tau^2/2$, which requires time τ = 4.04 s.

(a) Thus, the final 20% of its fall takes t ï τ = 0.48 s.

(b) We can find that speed using $v = -g\tau$. Therefore, $|v|$ = 40 m/s, approximately.

(c) Similarly, $v_{final} = -\,g\,t \quad \Rightarrow \quad |v_{final}| = 44$ m/s.

53. **THINK** This problem involves two objects: a key dropped from a bridge, and a boat moving at a constant speed. We look for conditions such that the key will fall into the boat.

EXPRESS The speed of the boat is constant, given by $v_b = d/t$, where d is the distance of the boat from the bridge when the key is dropped (12 m) and t is the time the key takes in falling.

To calculate t, we take the time to be zero at the instant the key is dropped, we compute the time t when $y = 0$ using $y = y_0 + v_0 t - \frac{1}{2}gt^2$, with $y_0 = 45$ m. Once t is known, the speed of the boat can be readily calculated.

ANALYZE Since the initial velocity of the key is zero, the coordinate of the key is given by $y_0 = \frac{1}{2}gt^2$. Thus, the time it takes for the key to drop into the boat is

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(45\text{ m})}{9.8\text{ m/s}^2}} = 3.03\text{ s}.$$

Therefore, the speed of the boat is $v_b = \dfrac{12\text{ m}}{3.03\text{ s}} = 4.0$ m/s.

LEARN From the general expression

$$v_b = \frac{d}{t} = \frac{d}{\sqrt{2y_0/g}} = d\sqrt{\frac{g}{2y_0}},$$

we see that $v_b \sim 1/\sqrt{y_0}$. This agrees with our intuition that the lower the height from which the key is dropped, the greater the speed of the boat in order to catch it.

54. (a) We neglect air resistance, which justifies setting a = ï g = ï 9.8 m/s^2 (taking *down* as the ï y direction) for the duration of the motion. We are allowed to use Eq. 2-15 (with Δy replacing Δx) because this is constant acceleration motion. We use primed variables (except t) with the first stone, which has zero initial velocity, and unprimed variables with the second stone (with initial downward velocity ï v_0, so that v_0 is being used for the initial *speed*). SI units are used throughout.

$$\Delta y' = 0(t) - \frac{1}{2}gt^2$$

$$\Delta y = (-v_0)(t-1) - \frac{1}{2}g(t-1)^2$$

Since the problem indicates $\Delta y'$ = Δy = ï 53.6 m, we solve the first equation for t:

$$t = \sqrt{\frac{2(-\Delta y')}{g}} = \sqrt{\frac{2(53.6\text{ m})}{9.8\text{ m/s}^2}} = 3.307\text{ s}$$

Using this result to solve the second equation for the initial speed of the second stone:

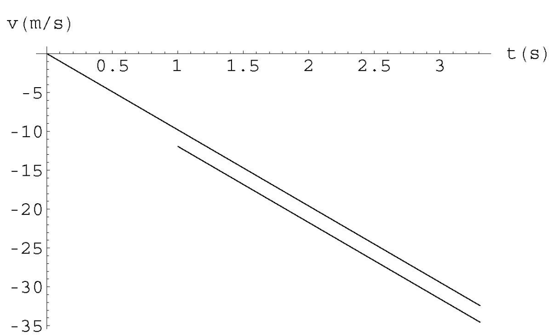
$$-53.6\text{ m} = (-v_0)(2.307\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(2.307\text{ s})^2$$

we obtain v_0 = 11.9 m/s.

(b) The velocity of the stones are given by

$$v'_y = \frac{d(\Delta y')}{dt} = -gt, \qquad v_y = \frac{d(\Delta y)}{dt} = -v_0 - g(t-1)$$

The plot is shown below:



55. **THINK** The free-falling moist-clay ball strikes the ground with a non-zero speed, and it undergoes deceleration before coming to rest.

EXPRESS During contact with the ground its average acceleration is given by $a_{\text{avg}} = \dfrac{\Delta v}{\Delta t}$, where Δv is the change in its velocity during contact with the ground and $\Delta t = 20.0\times 10^{-3}$ s is the duration of contact. Thus, we must first find the velocity of the ball just before it hits the ground (y = 0).

ANALYZE

(a) Now, to find the velocity just *before* contact, we take t = 0 to be when it is dropped. Using Eq. 2-16 with $y_0 = 15.0$ m, we obtain

$$v = -\sqrt{v_0^2 - 2g(y - y_0)} = -\sqrt{0 - 2(9.8\text{ m/s}^2)(0 - 15\text{ m})} = -17.15\text{ m/s}$$

where the negative sign is chosen since the ball is traveling downward at the moment of contact. Consequently, the average acceleration during contact with the ground is

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{0 - (-17.1\text{ m/s})}{20.0 \times 10^{-3}\text{ s}} = 857\text{ m/s}^2.$$

(b) The fact that the result is positive indicates that this acceleration vector points upward.

LEARN Since Δt is very small, it is not surprising to have a very large acceleration to stop the motion of the ball. In later chapters, we shall see that the acceleration is directly related to the magnitude and direction of the force exerted by the ground on the ball during the course of collision.

56. We use Eq. 2-16,

$$v_B^2 = v_A^2 + 2a(y_B - y_A),$$

with $a = -9.8$ m/s^2, $y_B - y_A = 0.40$ m, and $v_B = \frac{1}{3} v_A$. It is then straightforward to solve: $v_A = 3.0$ m/s, approximately.

57. The average acceleration during contact with the floor is a_{avg} = (v_2 ï v_1) / Δt, where v_1 is its velocity just before striking the floor, v_2 is its velocity just as it leaves the floor, and Δt is the duration of contact with the floor ($12 \times 10^{\text{ï}3}$ s).

(a) Taking the y axis to be positively upward and placing the origin at the point where the ball is dropped, we first find the velocity just before striking the floor, using $v_1^2 = v_0^2 - 2gy$. With v_0 = 0 and y = ï 4.00 m, the result is

$$v_1 = -\sqrt{-2gy} = -\sqrt{-2(9.8\text{ m/s}^2)(-4.00\text{ m})} = -8.85\text{ m/s}$$

where the negative root is chosen because the ball is traveling downward. To find the velocity just after hitting the floor (as it ascends without air friction to a height of 2.00 m), we use $v^2 = v_2^2 - 2g(y - y_0)$ with v = 0, y = ï 2.00 m (it ends up two meters *below* its initial drop height), and y_0 = ï 4.00 m. Therefore,

$$v_2 = \sqrt{2g(y - y_0)} = \sqrt{2(9.8\text{ m/s}^2)(-2.00\text{ m} + 4.00\text{ m})} = 6.26\text{ m/s} .$$

Consequently, the average acceleration is

$$a_{\text{avg}} = \frac{v_2 - v_1}{\Delta t} = \frac{6.26\text{ m/s} - (-\,8.85\text{ m/s})}{12.0 \times 10^{-3}\text{ s}} = 1.26 \times 10^3\ \text{m/s}^2.$$

(b) The positive nature of the result indicates that the acceleration vector points upward. In a later chapter, this will be directly related to the magnitude and direction of the force exerted by the ground on the ball during the collision.

58. We choose *down* as the $+y$ direction and set the coordinate origin at the point where it was dropped (which is when we start the clock). We denote the 1.00 s duration mentioned in the problem as t ï t' where t is the value of time when it lands and t' is one second prior to that. The corresponding distance is y ï $y' = 0.60h$, where y denotes the location of the ground. In these terms, y is the same as h, so we have h ï $y' = 0.60h$ or $0.40h = y'$.

(a) We find t' and t from Eq. 2-15 (with $v_0 = 0$):

$$y' = \frac{1}{2}gt'^2 \;\Rightarrow\; t' = \sqrt{\frac{2y'}{g}}$$

$$y = \frac{1}{2}gt^2 \;\Rightarrow\; t = \sqrt{\frac{2y}{g}} \;.$$

Plugging in $y = h$ and $y' = 0.40h$, and dividing these two equations, we obtain

$$\frac{t'}{t} = \sqrt{\frac{2(0.40h)/g}{2h/g}} = \sqrt{0.40} \;.$$

Letting $t' = t$ ï 1.00 (SI units understood) and cross-multiplying, we find

$$t - 1.00 = t\sqrt{0.40} \;\Rightarrow\; t = \frac{1.00}{1 - \sqrt{0.40}}$$

which yields $t = 2.72$ s.

(b) Plugging this result into $y = \frac{1}{2}gt^2$ we find $h = 36$ m.

(c) In our approach, we did not use the quadratic formula, but we did ñchoose a rootò when we assumed (in the last calculation in part (a)) that $\sqrt{0.40} = +0.632$ instead of ï0.707. If we had instead let $\sqrt{0.40}$ = ï0.632 then our answer for t would have been roughly 0.6 s, which would imply that $t' = t$ ï 1 would equal a negative number (indicating a time *before* it was dropped), which certainly does not fit with the physical situation described in the problem.

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59. We neglect air resistance, which justifies setting a = ï g = ï 9.8 m/s^2 (taking *down* as the ï y direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the y-axis.

(a) The time drop 1 leaves the nozzle is taken as t = 0 and its time of landing on the floor t_1 can be computed from Eq. 2-15, with v_0 = 0 and y_1 = ï 2.00 m.

$$y_1 = -\frac{1}{2} g t_1^2 \quad \Rightarrow \quad t_1 = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{-2(-2.00 \text{ m})}{9.8 \text{ m/s}^2}} = 0.639 \text{ s} .$$

At that moment, the fourth drop begins to fall, and from the regularity of the dripping we conclude that drop 2 leaves the nozzle at t = 0.639/3 = 0.213 s and drop 3 leaves the nozzle at t = 2(0.213 s) = 0.426 s. Therefore, the time in free fall (up to the moment drop 1 lands) for drop 2 is t_2 = t_1 ï 0.213 s = 0.426 s. Its position at the moment drop 1 strikes the floor is

$$y_2 = -\frac{1}{2} g t_2^2 = -\frac{1}{2}(9.8 \text{ m/s}^2)(0.426 \text{ s})^2 = -0.889 \text{ m},$$

or about 89 cm below the nozzle.

(b) The time in free fall (up to the moment drop 1 lands) for drop 3 is t_3 = t_1 ï 0.426 s = 0.213 s. Its position at the moment drop 1 strikes the floor is

$$y_3 = -\frac{1}{2} g t_3^2 = -\frac{1}{2}(9.8 \text{ m/s}^2)(0.213 \text{ s})^2 = -0.222 \text{ m},$$

or about 22 cm below the nozzle.

60. To find the ñlaunchò velocity of the rock, we apply Eq. 2-11 to the maximum height (where the speed is momentarily zero)

$$v = v_0 - gt \quad \Rightarrow \quad 0 = v_0 - \left(9.8 \text{ m/s}^2\right)\left(2.5 \text{ s}\right)$$

so that v_0 = 24.5 m/s (with $+y$ up). Now we use Eq. 2-15 to find the height of the tower (taking $y_0 = 0$ at the ground level)

$$y - y_0 = v_0 t + \frac{1}{2}at^2 \quad \Rightarrow \quad y - 0 = \left(24.5 \text{ m/s}\right)\left(1.5 \text{ s}\right) - \frac{1}{2}\left(9.8 \text{ m/s}^2\right)\left(1.5 \text{ s}\right)^2 .$$

Thus, we obtain y = 26 m.

61. We choose *down* as the $+y$ direction and place the coordinate origin at the top of the building (which has height H). During its fall, the ball passes (with velocity v_1) the top of the window (which is at y_1) at time t_1, and passes the bottom (which is at y_2) at time t_2. We are told y_2 ï y_1 = 1.20 m and t_2 ï t_1 = 0.125 s. Using Eq. 2-15 we have

$$y_2 - y_1 = v_1(t_2 - t_1) + \frac{1}{2}g(t_2 - t_1)^2$$

which yields

$$v_1 = \frac{1.20\ \text{m} - \frac{1}{2}(9.8\ \text{m/s}^2)(0.125\ \text{s})^2}{0.125\ \text{s}} = 8.99\ \text{m/s}.$$

From this, Eq. 2-16 (with $v_0 = 0$) reveals the value of y_1:

$$v_1^2 = 2gy_1 \quad \Rightarrow \quad y_1 = \frac{(8.99\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)} = 4.12\ \text{m}.$$

It reaches the ground ($y_3 = H$) at t_3. Because of the symmetry expressed in the problem (ñupward flight is a reverse of the fallô) we know that t_3 ï t_2 = 2.00/2 = 1.00 s. And this means t_3 ï t_1 = 1.00 s + 0.125 s = 1.125 s. Now Eq. 2-15 produces

$$y_3 - y_1 = v_1(t_3 - t_1) + \frac{1}{2}g(t_3 - t_1)^2$$

$$y_3 - 4.12\ \text{m} = (8.99\ \text{m/s})(1.125\ \text{s}) + \frac{1}{2}(9.8\ \text{m/s}^2)(1.125\ \text{s})^2$$

which yields $y_3 = H = 20.4$ m.

62. The height reached by the player is y = 0.78 m (where we have taken the origin of the y axis at the floor and $+y$ to be upward).

(a) The initial velocity v_0 of the player is

$$v_0 = \sqrt{2gy} = \sqrt{2(9.8\ \text{m/s}^2)(0.78\ \text{m})} = 3.91\ \text{m/s}\ .$$

This is a consequence of Eq. 2-16 where velocity v vanishes. As the player reaches y_1 = 0.78 m ï 0.15 m = 0.63 m, his speed v_1 satisfies $v_0^2 - v_1^2 = 2gy_1$, which yields

$$v_1 = \sqrt{v_0^2 - 2gy_1} = \sqrt{(3.91\ \text{m/s})^2 - 2(9.80\ \text{m/s}^2)(0.63\ \text{m})} = 1.71\ \text{m/s}\ .$$

The time t_1 that the player spends *ascending* in the top Δy_1 = 0.15 m of the jump can now be found from Eq. 2-17:

$$\Delta y_1 = \frac{1}{2}\left(v_1 + v\right)t_1 \;\Rightarrow\; t_1 = \frac{2\left(0.15\ \text{m}\right)}{1.71\ \text{m/s} + 0} = 0.175\ \text{s}$$

which means that the total time spent in that top 15 cm (both ascending and descending) is 2(0.175 s) = 0.35 s = 350 ms.

(b) The time t_2 when the player reaches a height of 0.15 m is found from Eq. 2-15:

$$0.15\ \text{m} = v_0 t_2 - \frac{1}{2}\, g t_2^2 = (3.91\ \text{m/s})t_2 - \frac{1}{2}(9.8\ \text{m/s}^2)t_2^2\ ,$$

which yields (using the quadratic formula, taking the smaller of the two positive roots) t_2 = 0.0404 s = 40.4 ms, which implies that the total time spent in that bottom 15 cm (both ascending and descending) is 2(40.4 ms) = 80.8 ms, or about 81 ms.

63. The time t the pot spends passing in front of the window of length L = 2.0 m is 0.25 s each way. We use v for its velocity as it passes the top of the window (going up). Then, with $a = -g = -9.8\ \text{m/s}^2$ (taking *down* to be the $-y$ direction), Eq. 2-18 yields

$$L = vt - \frac{1}{2}gt^2 \quad \Rightarrow \quad v = \frac{L}{t} - \frac{1}{2}gt.$$

The distance H the pot goes above the top of the window is therefore (using Eq. 2-16 with the *final velocity* being zero to indicate the highest point)

$$H = \frac{v^2}{2g} = \frac{\left(L/t - gt/2\right)^2}{2g} = \frac{\left(2.00\ \text{m}/0.25\ \text{s} - (9.80\ \text{m/s}^2)(0.25\ \text{s})/2\right)^2}{2(9.80\ \text{m/s}^2)} = 2.34\ \text{m}.$$

64. Using the equation of motion

$$(x - x_0) = v_0 t + \frac{1}{2} a t^2,$$

we get the rockôs displacement during the fall as follows:

$$\begin{aligned} h = (x - x_0) &= v_0 t + \frac{1}{2} a t^2 \\ &= -10(3) + \left[\frac{1}{2}(-9.8)(3)^2\right] \qquad \text{(acceleration here is the acceleration due to gravity, } -9.8\,\text{m/s}^2\text{)} \\ &= -74\,\text{m}. \end{aligned}$$

Thus the initial height is 74 m.

65. The key idea here is that the speed of the head (and the torso as well) at any given time can be calculated by finding the area on the graph of the headôs acceleration versus time, as shown in Eq. 2-26:

$$v_1 - v_0 = \begin{pmatrix} \text{area between the acceleration curve} \\ \text{and the time axis, from } t_0 \text{ to } t_1 \end{pmatrix}$$

(a) From Fig. 2.15a, we see that the head begins to accelerate from rest ($v_0 = 0$) at t_0 = 110 ms and reaches a maximum value of 90 m/s^2 at t_1 = 160 ms. The area of this region is

$$\text{area} = \frac{1}{2}(160 - 110)\times 10^{-3}\text{s}\cdot\left(90\ \text{m/s}^2\right) = 2.25\ \text{m/s}$$

which is equal to v_1, the speed at t_1.

(b) To compute the speed of the torso at t_1=160 ms, we divide the area into 4 regions: From 0 to 40 ms, region A has zero area. From 40 ms to 100 ms, region B has the shape of a triangle with area

$$\text{area}_\text{B} = \frac{1}{2}(0.0600\ \text{s})(50.0\ \text{m/s}^2) = 1.50\ \text{m/s}\ .$$

From 100 to 120 ms, region C has the shape of a rectangle with area

$$\text{area}_\text{C} = (0.0200\ \text{s})\ (50.0\ \text{m/s}^2) = 1.00\ \text{m/s}.$$

From 110 to 160 ms, region D has the shape of a trapezoid with area

$$\text{area}_\text{D} = \frac{1}{2}(0.0400\ \text{s})\ (50.0 + 20.0)\ \text{m/s}^2 = 1.40\ \text{m/s}.$$

Substituting these values into Eq. 2-26, with $v_0 = 0$ then gives

$$v_1 - 0 = 0 + 1.50\ \text{m/s} + 1.00\ \text{m/s} + 1.40\ \text{m/s} = 3.90\ \text{m/s},$$

or $v_1 = 3.90$ m/s.

66. The key idea here is that the position of an object at any given time can be calculated by finding the area on the graph of the objectôs velocity versus time, as shown in Eq. 2-30:

$$x_1 - x_0 = \begin{pmatrix} \text{area between the velocity curve} \\ \text{and the time axis, from } t_0 \text{ to } t_1 \end{pmatrix}.$$

(a) To compute the position of the fist at t = 50 ms, we divide the area in Fig. 2-37 into two regions. From 0 to 10 ms, region A has the shape of a triangle with area

$$\text{area}_\text{A} = \frac{1}{2}(0.010\text{ s})\,(2\text{ m/s}) = 0.01\text{ m}.$$

From 10 to 50 ms, region B has the shape of a trapezoid with area

$$\text{area}_\text{B} = \frac{1}{2}(0.040\text{ s})\,(2+4)\text{ m/s} = 0.12\text{ m}.$$

Substituting these values into Eq. 2-30 with $x_0 = 0$ then gives

$$x_1 - 0 = 0 + 0.01\text{ m} + 0.12\text{ m} = 0.13\text{ m},$$

or $x_1 = 0.13$ m.

(b) The speed of the fist reaches a maximum at t_1 = 120 ms. From 50 to 90 ms, region *C* has the shape of a trapezoid with area

$$\text{area}_\text{C} = \frac{1}{2}(0.040\text{ s})\,(4+5)\text{ m/s} = 0.18\text{ m}.$$

From 90 to 120 ms, region D has the shape of a trapezoid with area

$$\text{area}_\text{D} = \frac{1}{2}(0.030\text{ s})\,(5+7.5)\text{ m/s} = 0.19\text{ m}.$$

Substituting these values into Eq. 2-30, with $x_0 = 0$ then gives

$$x_1 - 0 = 0 + 0.01\text{ m} + 0.12\text{ m} + 0.18\text{ m} + 0.19\text{ m} = 0.50\text{ m},$$

or $x_1 = 0.50$ m.

67. The problem is solved using Eq. 2-31:

$$v_1 - v_0 = \begin{pmatrix} \text{area between the acceleration curve} \\ \text{and the time axis, from } t_0 \text{ to } t_1 \end{pmatrix}$$

To compute the speed of the unhelmeted, bare head at t_1 = 7.0 ms, we divide the area under the a vs. t graph into 4 regions: From 0 to 2 ms, region A has the shape of a triangle with area

$$\text{area}_A = \frac{1}{2}(0.0020 \text{ s})\,(120 \text{ m/s}^2) = 0.12 \text{ m/s}.$$

From 2 ms to 4 ms, region B has the shape of a trapezoid with area

$$\text{area}_B = \frac{1}{2}(0.0020 \text{ s})\,(120 + 140) \text{ m/s}^2 = 0.26 \text{ m/s}.$$

From 4 to 6 ms, region C has the shape of a trapezoid with area

$$\text{area}_C = \frac{1}{2}(0.0020 \text{ s})\,(140 + 200) \text{ m/s}^2 = 0.34 \text{ m/s}.$$

From 6 to 7 ms, region D has the shape of a triangle with area

$$\text{area}_D = \frac{1}{2}(0.0010 \text{ s})\,(200 \text{ m/s}^2) = 0.10 \text{ m/s}.$$

Substituting these values into Eq. 2-31, with v_0=0 then gives

$$v_{\text{unhelmeted}} = 0.12 \text{ m/s} + 0.26 \text{ m/s} + 0.34 \text{ m/s} + 0.10 \text{ m/s} = 0.82 \text{ m/s}.$$

Carrying out similar calculations for the helmeted head, we have the following results: From 0 to 3 ms, region A has the shape of a triangle with area

$$\text{area}_A = \frac{1}{2}(0.0030 \text{ s})\,(40 \text{ m/s}^2) = 0.060 \text{ m/s}.$$

From 3 ms to 4 ms, region B has the shape of a rectangle with area

$$\text{area}_B = (0.0010 \text{ s})\,(40 \text{ m/s}^2) = 0.040 \text{ m/s}.$$

From 4 to 6 ms, region C has the shape of a trapezoid with area

$$\text{area}_C = \frac{1}{2}(0.0020 \text{ s})\,(40 + 80) \text{ m/s}^2 = 0.12 \text{ m/s}.$$

From 6 to 7 ms, region D has the shape of a triangle with area

$$\text{area}_D = \frac{1}{2}(0.0010 \text{ s})\,(80 \text{ m/s}^2) = 0.040 \text{ m/s}.$$

Substituting these values into Eq. 2-31, with $v_0 = 0$ then gives

$$v_{\text{helmeted}} = 0.060 \text{ m/s} + 0.040 \text{ m/s} + 0.12 \text{ m/s} + 0.040 \text{ m/s} = 0.26 \text{ m/s}.$$

Thus, the difference in the speed is

$$\Delta v = v_{\text{unhelmeted}} - v_{\text{helmeted}} = 0.82 \text{ m/s} - 0.26 \text{ m/s} = 0.56 \text{ m/s}.$$

68. This problem can be solved by noting that velocity can be determined by the graphical integration of acceleration versus time. The speed of the tongue of the salamander is simply equal to the area under the acceleration curve:

$$v = \text{area} = \frac{1}{2}(10^{-2}\ \text{s})(100\ \text{m/s}^2) + \frac{1}{2}(10^{-2}\ \text{s})(100\ \text{m/s}^2 + 400\ \text{m/s}^2) + \frac{1}{2}(10^{-2}\ \text{s})(400\ \text{m/s}^2)$$
$$= 5.0\ \text{m/s}.$$

69. Since $v = dx/dt$ (Eq. 2-4), then $\Delta x = \int v\,dt$, which corresponds to the area under the v vs t graph. Dividing the total area A into rectangular (base × height) and triangular $\left(\frac{1}{2}\,\text{base} \times \text{height}\right)$ areas, we have

$$\begin{aligned} A &= A_{0<t<2} + A_{2<t<10} + A_{10<t<12} + A_{12<t<16} \\ &= \frac{1}{2}(2)(8) + (8)(8) + \left((2)(4) + \frac{1}{2}(2)(4)\right) + (4)(4) \end{aligned}$$

with SI units understood. In this way, we obtain $\Delta x = 100$ m.

70. To solve this problem, we note that velocity is equal to the time derivative of a position function, as well as the time integral of an acceleration function, with the integration constant being the initial velocity. Thus, the velocity of particle 1 can be written as

$$v_1 = \frac{dx_1}{dt} = \frac{d}{dt}\left(6.00t^2 + 3.00t + 2.00\right) = 12.0t + 3.00 .$$

Similarly, the velocity of particle 2 is

$$v_2 = v_{20} + \int a_2 dt = 15.0 + \int (-8.00t) dt = 20.0 - 4.00t^2 .$$

The condition that $v_1 = v_2$ implies

$$12.0t + 3.00 = 15.0 - 4.00t^2 \quad \Rightarrow \quad 4.00t^2 + 12.0t - 12.0 = 0$$

which can be solved with the quadratic equation to give $t = 0.790\,\text{s}$. Thus, the velocity at this time is $v_1 = v_2 = 12.0(0.790) + 3.00 = 12.5$ m/s.