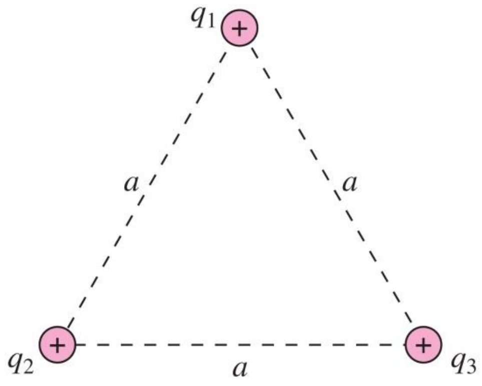
Chapter 25 Capacitance

- It takes work to assemble a distribution of electric charges
  - That work is stored as **electrostatic energy** associated with the new configuration of charges.
  - Each charge pair q_i, q_j contributes energy where r_{ij} is the distance between the charges in the final configuration.
  - The work needed to bring q_i in the presence of q_j is $U_{ij} = kq_iq_j/r_{ij}$
  - Example: Three point charges assembled to form an equilateral triangle:

$$\boxed{U_{\text{electrostatic}} = \frac{kq_1q_2}{a} + \frac{kq_1q_3}{a} + \frac{kq_2q_3}{a}}$$



Electric potential energy (U)

$$U = -W = -\int_{\infty}^{r} \vec{F}_q \bullet d\vec{s} = -\int_{\infty}^{r} q\vec{E} \bullet d\vec{s} \quad [\mathrm{J} = \mathrm{N} \cdot \mathrm{m}]$$

Electric potential (V) : 단위 전하당 U

$$V \equiv \frac{U}{q} = -\frac{W}{q} = -\int_{\infty}^{r} \vec{E} \bullet d\vec{s} \quad [\mathrm{V(volt)} = \mathrm{J/C}]$$

점 전하

$$V = \sum_{n=1}^{N} V_n = \frac{1}{4\pi\varepsilon_{\mathrm{o}}} \sum_{n=1}^{N} \frac{q}{r_n}$$

연속 전하

$$V = \frac{1}{4\pi\varepsilon_o} \int \frac{dq}{r}$$

V ←→ E

$$\vec{E} = -\frac{dV}{d\vec{s}} \equiv -\vec{\nabla} V = -\left[\frac{\partial V}{\partial x} \hat{x} + \frac{\partial V}{\partial y} \hat{y} + \frac{\partial V}{\partial z} \hat{z} \right]$$

Chap. 25-1 Capacitance

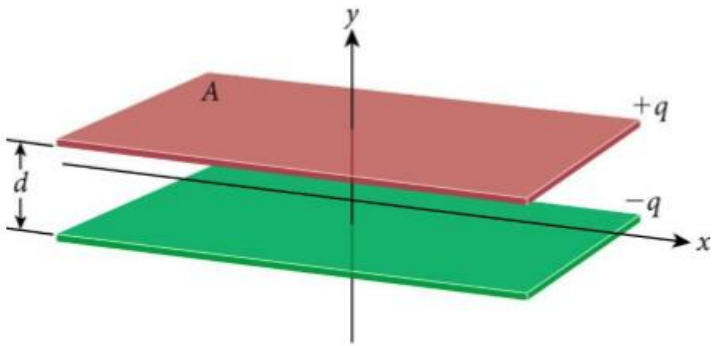
- **축전기** – 전하를 저장할 수 있는 장치
  - 기본 구조 – 절연층으로 분리된 두 금속박막

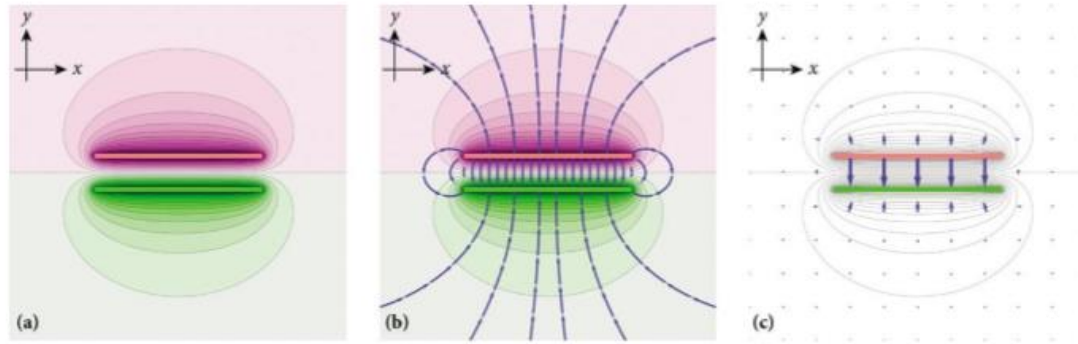
  - 매우 빠른 저장과 방출 속력을 가진다.

여러 종류의 축전기들

Chap. 25-1 Capacitance

▪ 평행판 축전기





평행판 축전기 주변의 전기퍼텐셜과 전기장

▪ **전기용량**

• 두 평행판 사이의 퍼텐셜차 ΔV는 평판에 있는 전하의 양 q에 비례한다.

$$q = C\,\Delta V$$

전기용량 (Capacitance)

축전기의 기하학적 구조에 의해 결정된다.

• 전기용량의 단위 – 패럿 (farad, F) : $$1\,\mathrm{F} = \frac{1\,\mathrm{C}}{1\,\mathrm{V}}$$

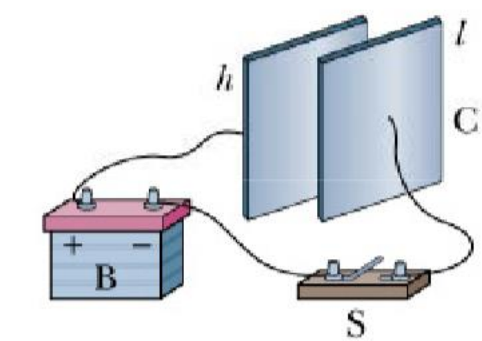
보통의 축전기에 많이 쓰이는 단위 : μF, pF

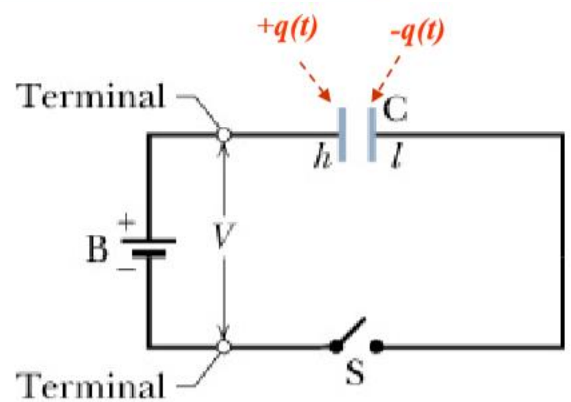
$1\ \mu\mathrm{F} = 10^{-6}\ \mathrm{F}$
$1\ p\mathrm{F} = 10^{-12}\ \mathrm{F}$
$1\ f\mathrm{F} = 10^{-15}\ \mathrm{F}$

Chap. 25-1 Capacitance

- 전기회로

	Wire	G	Galvanometer
	Capacitor	V	Voltmeter
	Resistor	A	Ammeter
	Inductor	− +	Battery
	Switch		AC source





확인문제 1. 전기용량은 증가? 감소? 그대로?

(a) q 가 2배 될 때

(b) V 가 3배 될 때

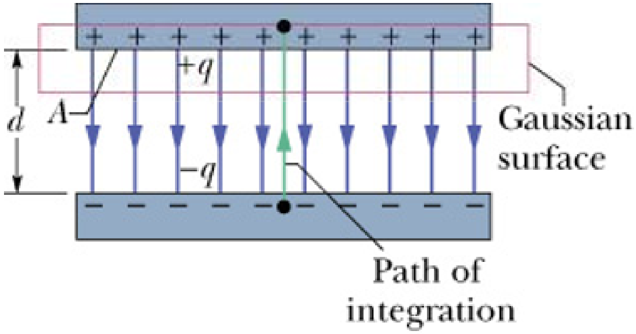
Chap. 25-2 Calculating the Capacitance

(1) 극판에 전하 q 가 있다고 가정

(2) Gauss 법칙 → E 계산 $\boxed{\varepsilon_0 \oint \mathbf{E} \cdot d\mathbf{A} = q}$

(3) E -> V 계산 $\boxed{V_f - V_i = -\int_i^f \mathbf{E} \cdot d\mathbf{s}}$

(4) C = q/V



평행판 축전기

$$\varepsilon_0 \oint \mathbf{E} \cdot d\mathbf{A} = q \implies q = \varepsilon_0 E A \implies E = \frac{q}{\varepsilon_0 A}$$

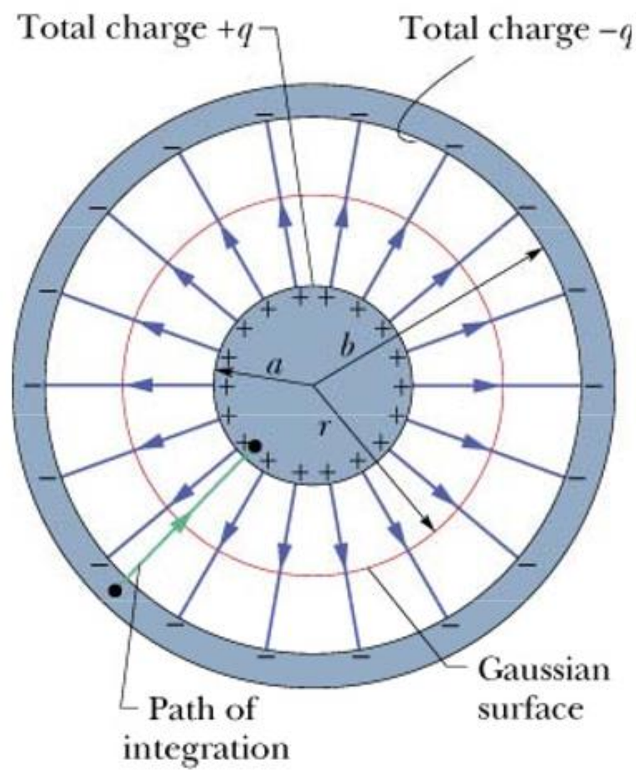
$$V = -\int_i^f \vec{E} \bullet d\vec{s} = \int_-^+ E ds = E \int_0^d ds = Ed$$

$$\boxed{C = \frac{q}{V} = \frac{\varepsilon_o A}{d}}$$

극판 사이의 유전상수와
극판의 모양 (면적과 간격)에만 의존

Chap. 25-2 Calculating the Capacitance

원통형 축전기



$$\varepsilon_0 \oint \mathbf{E} \cdot d\mathbf{A} = q$$

$$\Rightarrow q = \varepsilon_0 E A = \varepsilon_0 E (2\pi r L)$$

$$\Rightarrow E = \frac{q}{\varepsilon_0 2\pi L}\left(\frac{1}{r}\right)$$

$$V = \int_{-}^{+} E ds = \frac{q}{2\pi\varepsilon_o L}\int_a^b \frac{1}{r} dr = \frac{q}{2\pi\varepsilon_o L}\ln\left(\frac{b}{a}\right)$$

$$\boxed{C = \frac{q}{V} = 2\pi\varepsilon_o \frac{L}{\ln(b/a)}}$$

극판 사이의 유전상수
극판의 모양에만 의존

Chap. 25-2 Calculating the Capacitance

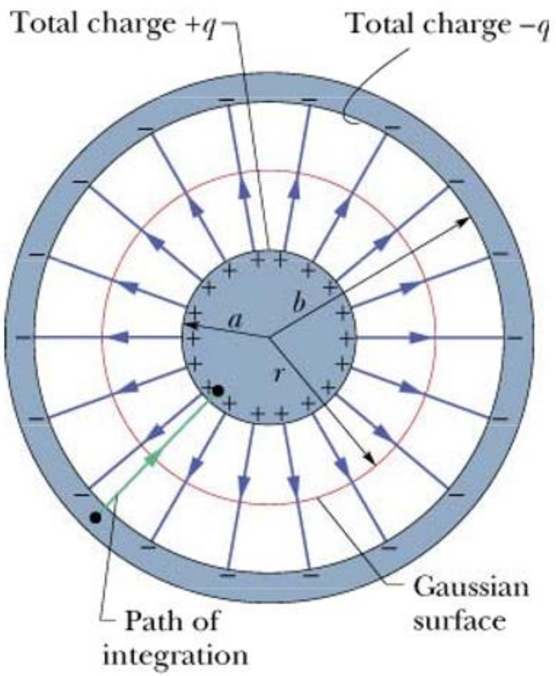
구형 축전기

$$\varepsilon_0 \oint \mathbf{E} \cdot d\mathbf{A} = q$$

$$\Rightarrow q = \varepsilon_0 E A = \varepsilon_0 E (4\pi r^2)$$

$$\Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$V = \int_-^+ E ds = \frac{q}{4\pi\varepsilon_o}\int_a^b \frac{1}{r^2} dr = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{a} - \frac{1}{b}\right)$$



$$\Rightarrow \boxed{C = \frac{q}{V} = 4\pi\varepsilon_o\left(\frac{ab}{b-a}\right)}$$

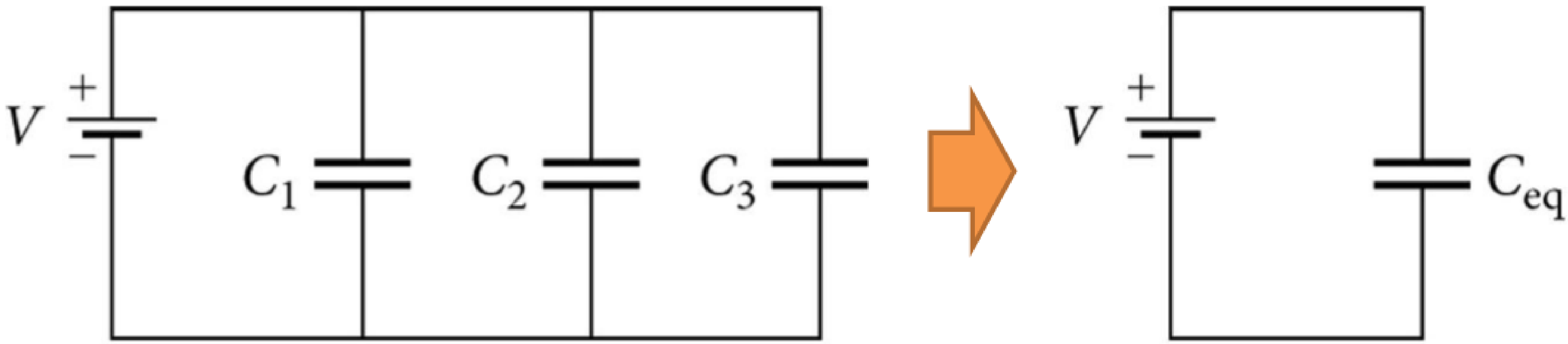
극판 사이의 유전상수
극판의 모양에만 의존

고립된 도체 구의 전기용량

$$\Rightarrow C = \frac{q}{V} = 4\pi\varepsilon_o\left(\frac{ab}{b-a}\right) \xrightarrow{b\to\infty} C = 4\pi\varepsilon_o R \quad (a = R)$$

Chap. 25-3 Capacitors in Parallel and in Series

- 축전기의 병렬연결



$$q_1 = C_1 \Delta V$$
$$q_2 = C_2 \Delta V$$
$$q_3 = C_3 \Delta V$$

$$\begin{aligned} q &= q_1 + q_2 + q_3 \\ &= C_1\Delta V + C_2\Delta V + C_2\Delta V \\ &= (C_1 + C_2 + C_3)\Delta V \end{aligned}$$

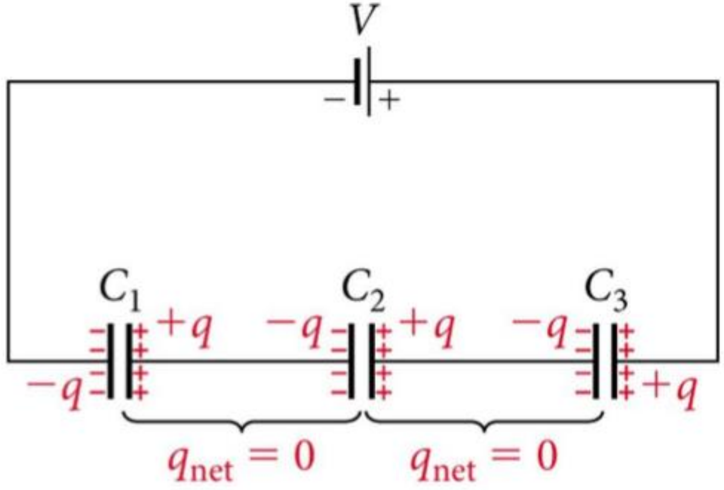
$$C_{eq} = C_1 + C_2 + C_3$$

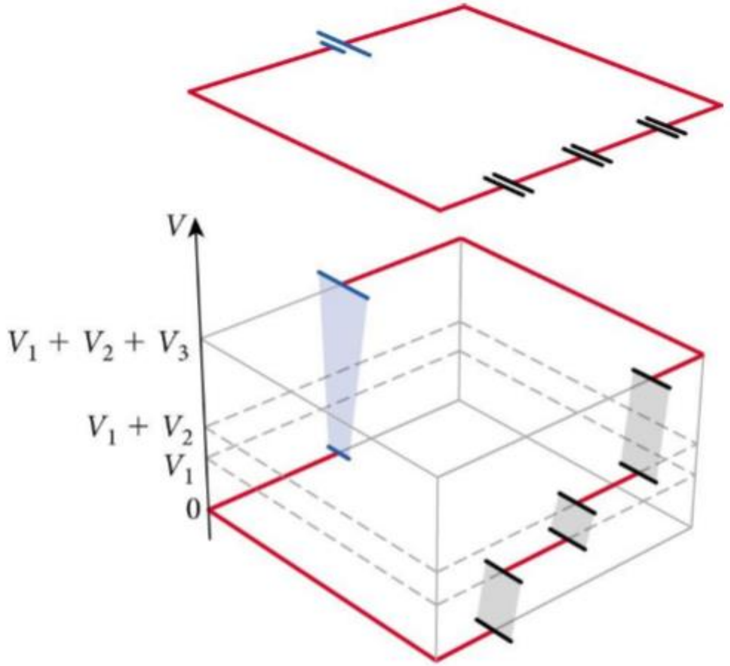
병렬연결된 각 축전기의
퍼텐셜차는 같다.

$$\boldsymbol{C_{eq} = \sum_{i=1}^{n} C_i}$$

Chap. 25-3 Capacitors in Parallel and in Series

- 축전기의 직렬연결





$$q = C_1\,\Delta V_1$$
$$q = C_2\,\Delta V_2$$
$$q = C_3\,\Delta V_3$$

직렬연결된 각 축전기의 전하는 같다.

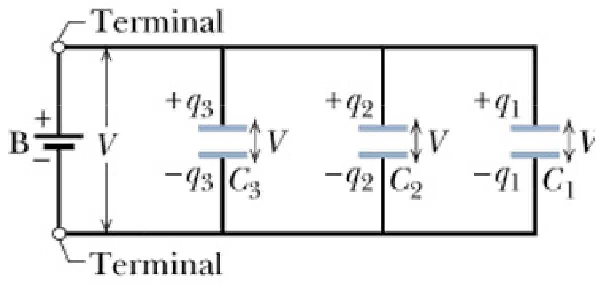
$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}$$

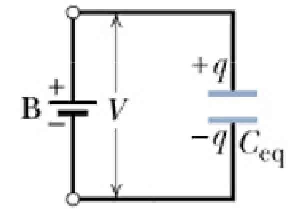
Chap. 25-3 Capacitors in Parallel and in Series

병렬연결



동일전압 : $V = V_j$

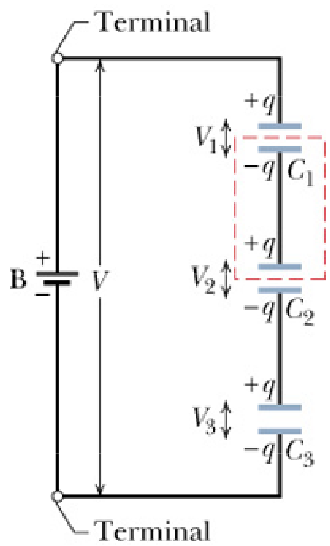
총 전하량 : $q = \sum_{j=1}^{n} q_j$



$$q = \sum_{j=1}^{n} q_j = \sum_{j=1}^{n} C_j V_j = V\left(\sum_{j=1}^{n} C_j\right) = C_{eq} V$$

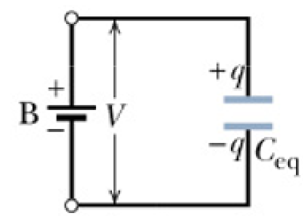
$$\boxed{C_{eq} = \sum_{j=1}^{n} C_j}$$

직렬연결



동일 전하량 : $q = q_j$

총 전압 : $V = \sum_{j=1}^{n} V_j$



$$V = \sum_{j=1}^{n} V_j = \sum_{j=1}^{n} \frac{q_j}{C_j} = q\left(\sum_{j=1}^{n} \frac{1}{C_j}\right) = \frac{q}{C_{eq}}$$

$$\boxed{\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}}$$

Chap. 25-4 Energy Stored in an Electric Field

- 축전기에 충전을 하기 위해 전지가 하는 일

$$dW = \Delta V' \, dq' = \frac{q'}{C} dq' \qquad W = \int dW = \int_0^q \frac{q'}{C} dq' = \frac{1}{2}\frac{q^2}{C}$$

- **축전기에 저장된 전기 퍼텐셜에너지**

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}q\,\Delta V$$

- **전기에너지밀도**

평행판 축전기에 저장된 에너지에 대한 해석

$$U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\left(\epsilon_0 \frac{A}{D}\right)(Ed)^2 = \frac{1}{2}\epsilon_0 E^2 \cdot Ad$$ ⇦ **에너지밀도** · 부피

$$u = \frac{1}{2}\epsilon_0 E^2$$ ⇦ **전기장이 가진 에너지밀도**

Chap. 25-4 Energy Stored in an Electric Field

축전기의 퍼텐셜에너지

축전기에 전하 q' 가 채워져 있어 두 전극의 전위차가 V' 일 때, 전하 dq' 를 더 채우는데 드는 에너지

$$dW = V'dq' = \frac{q'}{C}dq'$$

따라서, 빈 축전기에 전하를 q 까지 채우는데 드는 에너지 → 저장된 퍼텐셜에너지

$$W = \int_0^q \frac{q'}{C}dq' = \frac{q^2}{2C} = \frac{1}{2}CV^2 \quad \Rightarrow \quad \boxed{U = \frac{q^2}{2C} = \frac{1}{2}CV^2}$$

에너지 밀도 : 단위 부피당 퍼텐셜 에너지

$$u = \frac{U}{Ad} = \left(\frac{1}{2}CV^2\right)\frac{1}{Ad} = \frac{1}{2}\varepsilon_o\left(\frac{V}{d}\right)^2 \quad \Rightarrow \quad \boxed{u = \frac{1}{2}\varepsilon_0 E^2}$$

$$C = \varepsilon_0 \frac{A}{d} \qquad E = Vd$$

Chap. 25-5 Capacitor with a Dielectric

- **유전체 (dielectric)**
  - 같은 전하에 대하여 진공에서보다 더 작은 크기의 전기장이 생기는 물질
  - 유전체의 유전율, 유전상수

$$\underbrace{\epsilon}_{\text{유전율}} = \underbrace{\kappa}_{\text{유전상수}} \overbrace{\epsilon_0}^{\text{진공의 유전율}}$$

- **유전체에서의 가우스법칙**

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon}$$

- 유전체가 들어 있는 평행판 축전기

$$\Delta V = Ed = \frac{\sigma}{\epsilon} d = \frac{d}{\epsilon A} q \quad \Rightarrow \quad C = \epsilon \frac{A}{d} = \kappa \epsilon_0 \frac{A}{d} = \kappa C_{\text{vac}}$$

전기용량이 κ배 증가한다.

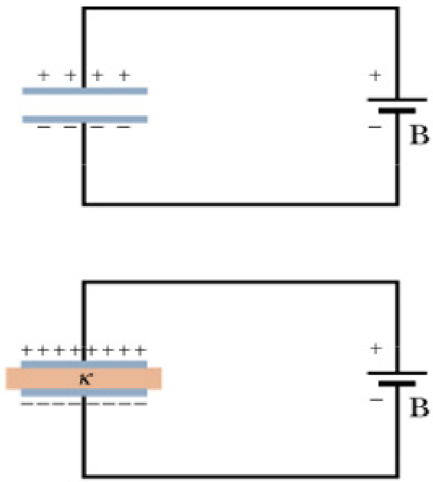
Chap. 25-5 Capacitor with a Dielectric

1837년 Michel Faraday 실험 :
평판 축전기의 전극 사이에 끼워 넣는 물질에 따라 전기용량이 달라짐.

C

κC

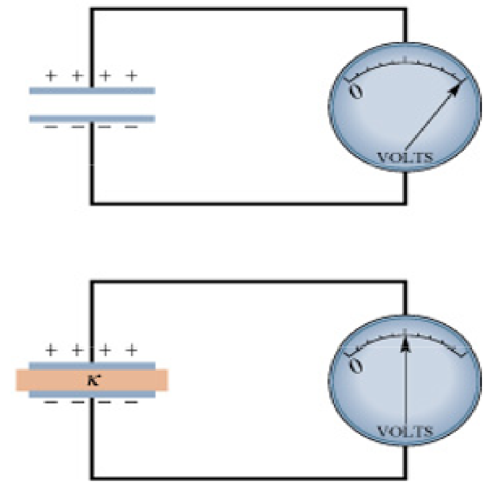
증가



$V =$ a constant

$q = CV$

증가



$q =$ a constant

$V = \dfrac{q}{C}$

감소

κ : 유전상수 (dielectric constant)

$$\varepsilon_0 \rightarrow \kappa\varepsilon_0$$

유전율 변화

$$E = \frac{\sigma}{\varepsilon_0} \rightarrow E = \frac{\sigma}{\kappa\varepsilon_0} \rightarrow E = \frac{E_0}{\kappa}$$

전기장 감소

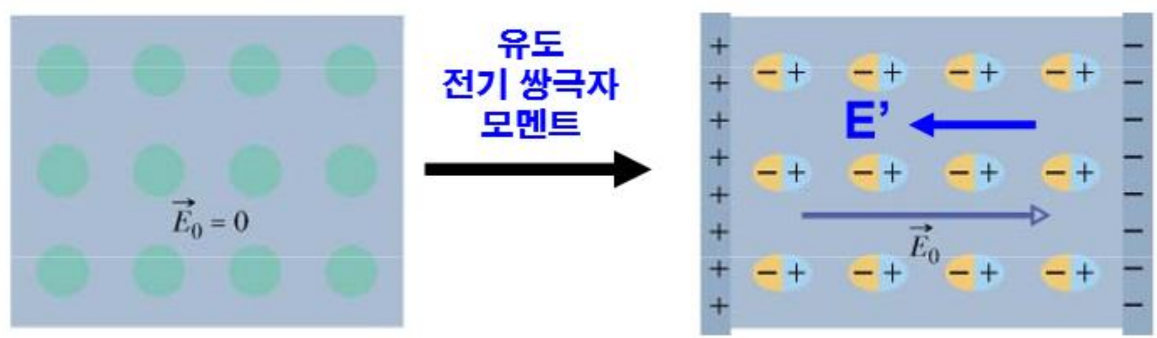
물질	유전상수	유전강도 (kV/mm)
공기(1기압)	1.00054	3
폴리스티렌	2.6	24
종이	3.5	16
파이렉스 유리	4.7	4
실리콘	12	
물(20˚C)	80.4	
스트론튬 산화 티타늄	310	8

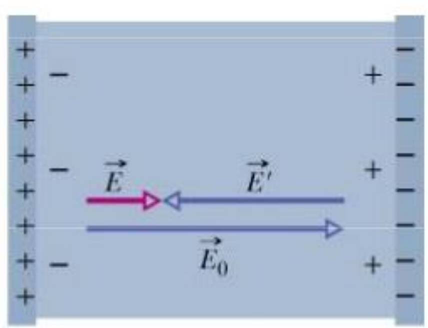
(*) 진공의 유전상수 = 1

Chap. 25-5 Capacitor with a Dielectric

극성 유전체

비극성 유전체

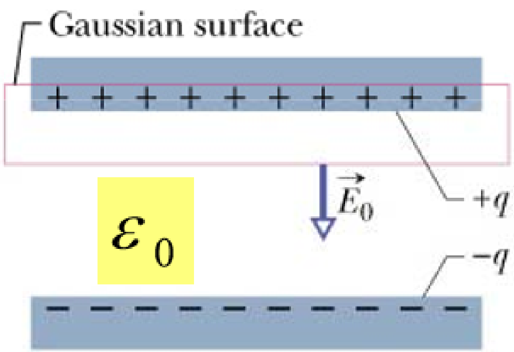


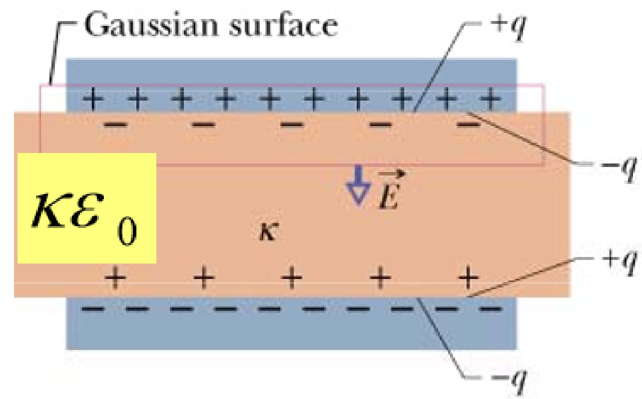


$$\vec{E} = \vec{E}_0 + \vec{E}'$$

→ 즉, 진공 중 보다
Electric field
감소

Chap. 25-6 Dielectrics and Gauss' Law





$$q = \varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 E_0 A$$

$$q - q' = \varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 E A$$

$$E_0 = \frac{q}{\varepsilon_0 A}$$

$$E = \frac{q - q'}{\varepsilon_0 A}$$

전기장 감소 $E = \dfrac{E_0}{\kappa} = \dfrac{q}{\kappa \varepsilon_0 A}$

$$q - q' = \frac{q}{\kappa}$$

$$q' = q\left(1 - \frac{1}{\kappa}\right)$$ **q 감소**

유전체에서의 Gauss 법칙 $\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q$

Chap. 25-6 Dielectrics and Gauss' Law

일반화 된 Gauss 법칙

$$\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q$$

➡ $\vec{D} = \kappa \varepsilon_0 \vec{E}$: 전기변위 (electric displacement)

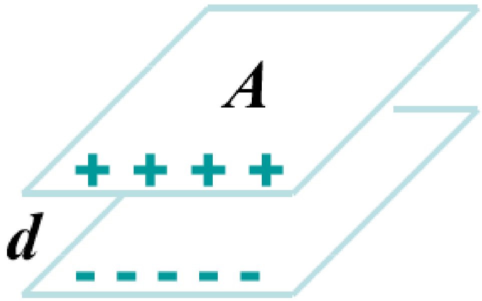
$$\oint \vec{D} \cdot d\vec{A} = q$$

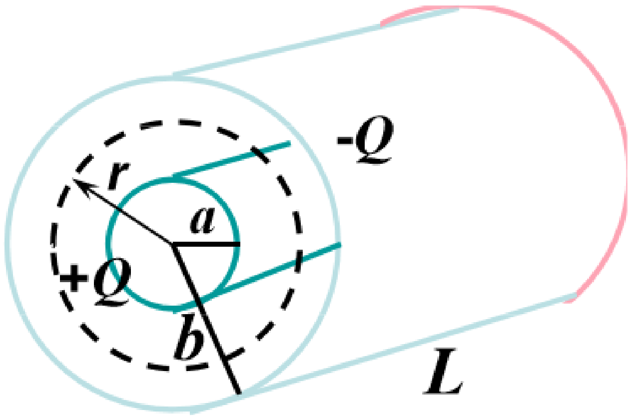
➡ 전하는 도체표면의 자유전하만 고려하면 된다.

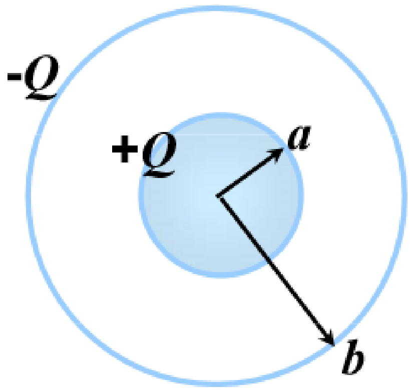
➡ κ가 상수가 아닌 (즉, 위치에 따라 다른) 일반적인 경우도 성립한다.

Summary

전기용량 (capacitance) $C \equiv \dfrac{q}{V}$







Parallel Plates

$$C \propto \frac{A}{d}$$

Cylindrical

$$C \propto \frac{L}{\ln(b/a)}$$

Spherical

$$C \propto \frac{ab}{b-a}$$

Gauss 법칙 $\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q$