

Thm. Let  $v_1, \dots, v_m$  be  
 row vectors in  $\mathbb{R}^n$  and  
 let  $A$  be the  $m \times n$  matrix  
 $\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{bmatrix}$ . Then  $v_1, v_2, \dots, v_m$  are  
 lin dep iff  $\text{rank}(A) < m$

ex.  $[1, 2, 0]$ ,  $[1, 1, -1]$ ,  $[1, 4, 2]$

$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & -1 \\ 1 & 4 & 2 \end{bmatrix} \xrightarrow{\substack{R_2' = R_2 - R_1 \\ R_3' = R_3 - R_1}} \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & -1 \\ 0 & 2 & 2 \end{bmatrix}$$

$$\xrightarrow{R_3'' = R_3' + 2R_2'} \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} 0 &= R_3'' = R_3' + 2R_2' = (R_3 - R_1) + 2(R_2 - R_1) \\ &= -3R_1 + 2R_2 + R_3 \end{aligned}$$

Thm. A set of  $m$  vectors in  
 $\mathbb{R}^n$  is lin dep if  $m > n$ .