

Thm: A system of linear eqns with augmented matrix $[A|b]$ is consistent iff b is a linear combination of the columns of A .

$$(a) \quad \begin{array}{l} x - y = 1 \\ x + y = 3 \end{array} \Rightarrow x = 2, y = 1$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$(c) \quad \begin{array}{l} x - y = 1 \\ x - y = 3 \end{array} \quad \text{has no sol}$$

$$x \begin{bmatrix} 1 \\ 1 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} \neq \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \forall x, y.$$

Def. If $S = \{v_1, v_2, \dots, v_k\}$ is a set of vectors in \mathbb{R}^n , then the set of linear combinations of v_1, \dots, v_k is called the span of S ; and is denoted