Chapter 8

1. The potential energy stored by the spring is given by $U = ½\, kx^2$, where k is the spring constant and x is the displacement of the end of the spring from its position when the spring is at its rest length. Thus, the energy stored in spring when Adam stretches the spring is

$$U_1 = \frac{1}{2}kx^2$$

The energy stored in the spring when John stretches the spring is

$$U_2 = \frac{1}{2}k(3x)^2 = \frac{9}{2}kx^2$$

Therefore, the ratio between the energy stored in the spring in these two stretches is

$$\frac{U_1}{U_2} = \frac{1}{9}$$

2. We use Eq. 7-12 for W_g and Eq. 8-9 for U.

(a) The displacement between the initial point and A is horizontal, so $\phi = 90.0°$ and

$$W_g = 0 \text{ (since } \cos 90.0° = 0).$$

(b) The displacement between the initial point and B has a vertical component of $h/2$ downward (same direction as $\vec{F}_g$), so we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = \frac{1}{2}mgh = \frac{1}{2}(825 \text{ kg})(9.80 \text{ m/s}^2)(50.0 \text{ m}) = 2.02\times10^5 \text{ J}.$$

(c) The displacement between the initial point and C has a vertical component of h downward (same direction as $\vec{F}_g$), so we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = mgh = (825 \text{ kg})(9.80 \text{ m/s}^2)(50.0 \text{ m}) = 4.04\times10^5 \text{ J}.$$

(d) With the reference position at C, we obtain

$$U_B = \frac{1}{2}mgh = \frac{1}{2}(825 \text{ kg})(9.80 \text{ m/s}^2)(50.0 \text{ m}) = 2.02\times10^5 \text{ J}.$$

(e) Similarly, we find

$$U_A = mgh = (825 \text{ kg})(9.80 \text{ m/s}^2)(50.0 \text{ m}) = 4.04\times10^5 \text{ J}.$$

(f) All the answers are proportional to the mass of the object. If the mass is doubled, all answers are doubled.

3. (a) Noting that the vertical displacement is 10.0 m ï 1.50 m = 8.50 m downward (same direction as $\vec{F}_g$), Eq. 7-12 yields

$$W_g = mgd\cos\phi = (2.00\ \text{kg})(9.80\ \text{m/s}^2)(8.50\ \text{m})\cos 0° = 167\ \text{J}.$$

(b) One approach (which is fairly trivial) is to use Eq. 8-1, but we feel it is instructive to instead calculate this as ΔU where $U = mgy$ (with upward understood to be the $+y$ direction). The result is

$$\Delta U = mg(y_f - y_i) = (2.00\ \text{kg})(9.80\ \text{m/s}^2)(1.50\ \text{m} - 10.0\ \text{m}) = -167\ \text{J}.$$

(c) In part (b) we used the fact that $U_i = mgy_i$ =196 J.

(d) In part (b), we also used the fact $U_f = mgy_f$ = 29 J.

(e) The computation of W_g does not use the new information (that U = 100 J at the ground), so we again obtain W_g = 167 J.

(f) As a result of Eq. 8-1, we must again find ΔU = ï W_g = ï 167 J.

(g) With this new information (that U_0 = 100 J where y = 0) we have

$$U_i = mgy_i + U_0 = 296\ \text{J}.$$

(h) With this new information (that U_0 = 100 J where y = 0) we have

$$U_f = mgy_f + U_0 = 129\ \text{J}.$$

We can check part (f) by subtracting the new U_i from this result.

4. (a) The only force that does work on the ball is the force of gravity; the force of the rod is perpendicular to the path of the ball and so does no work. In going from its initial position to the lowest point on its path, the ball moves vertically through a distance equal to the length L of the rod, so the work done by the force of gravity is

$$W = mgL = (0.382 \text{ kg})(9.80 \text{ m/s}^2)(0.498 \text{ m}) = 1.86 \text{ J} .$$

(b) In going from its initial position to the highest point on its path, the ball moves vertically through a distance equal to L, but this time the displacement is upward, opposite the direction of the force of gravity. The work done by the force of gravity is

$$W = -mgL = -(0.382 \text{ kg})(9.80 \text{ m/s}^2)(0.498 \text{ m}) = -1.86 \text{ J}.$$

(c) The final position of the ball is at the same height as its initial position. The displacement is horizontal, perpendicular to the force of gravity. The force of gravity does no work during this displacement.

(d) The force of gravity is conservative. The change in the gravitational potential energy of the ball-Earth system is the negative of the work done by gravity:

$$\Delta U = -mgL = -(0.382 \text{ kg})(9.80 \text{ m/s}^2)(0.498 \text{ m}) = -1.86 \text{ J}$$

as the ball goes to the lowest point.

(e) Continuing this line of reasoning, we find

$$\Delta U = +mgL = (0.382 \text{ kg})(9.80 \text{ m/s}^2)(0.498 \text{ m}) = 1.86 \text{ J}$$

as it goes to the highest point.

(f) Continuing this line of reasoning, we have $\Delta U = 0$ as it goes to the point at the same height.

(g) The change in the gravitational potential energy depends only on the initial and final positions of the ball, not on its speed anywhere. The change in the potential energy is the *same* since the initial and final positions are the same.

5. **THINK** As the ice flake slides down the frictionless bowl, its potential energy changes due to the work done by the gravitational force.

EXPRESS The force of gravity is constant, so the work it does is given by $W = \vec{F} \cdot \vec{d}$, where $\vec{F}$ is the force and $\vec{d}$ is the displacement. The force is vertically downward and has magnitude *mg*, where *m* is the mass of the flake, so this reduces to $W = mgh$, where h is the height from which the flake falls. The force of gravity is conservative, so the change in gravitational potential energy of the flake-Earth system is the negative of the work done: ΔU = ï W.

ANALYZE (a) The ice flake falls a distance $h = r = 22.0\ \text{cm} = 0.22\ \text{m}$. Therefore, the work done by gravity is

$$W = mgr = (2.00 \times 10^{-3}\ \text{kg})(9.8\ \text{m/s}^2)(22.0 \times 10^{-2}\ \text{m}) = 4.31 \times 10^{-3}\,\text{J}.$$

(b) The change in gravitational potential energy is ΔU = ï W = ï $4.31 \times 10^{\text{ï}3}$ J.

(c) The potential energy when the flake is at the top is greater than when it is at the bottom by $|\Delta U|$. If $U = 0$ at the bottom, then $U = + 4.31 \times 10^{\text{ï}3}$ J at the top.

(d) If $U = 0$ at the top, then U = ï $4.31 \times 10^{\text{ï}3}$ J at the bottom.

(e) All the answers are proportional to the mass of the flake. If the mass is doubled, all answers are doubled.

LEARN While the potential energy depends on the reference point (location where $U = 0$), the change in potential energy, ΔU, does not. In both (c) and (d), we find $\Delta U = -4.31 \times 10^{-3}$ J.

6. We use Eq. 7-12 for W_g and Eq. 8-9 for U.

(a) The displacement between the initial point and Q has a vertical component of h ï R downward (same direction as $\vec{F}_g$), so (with $h = 5R$) we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = 4mgR = 4(3.20\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.12\ \text{m}) = 0.13\ \text{J}.$$

(b) The displacement between the initial point and the top of the loop has a vertical component of h ï $2R$ downward (same direction as $\vec{F}_g$), so (with $h = 5R$) we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = 3mgR = 3(3.20\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.10\ \text{m}) = 0.094\ \text{J}.$$

(c) With $y = h = 5R$, at P we find

$$U = 5mgR = 5(3.20\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.10\ \text{m}) = 0.16\ \text{J}.$$

(d) With $y = R$, at Q we have

$$U = mgR = (3.20\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.10\ \text{m}) = 0.031\ \text{J}.$$

(e) With $y = 2R$, at the top of the loop, we find

$$U = 2mgR = 2(3.20\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.10\ \text{m}) = 0.063\ \text{J}.$$

(f) The new information $(v_i \neq 0)$ is not involved in any of the preceding computations; the above results are unchanged.

7. The main challenge for students in this type of problem seems to be working out the trigonometry in order to obtain the height of the ball (relative to the low point of the swing) $h = L - L\cos\theta$ (for angle θ measured from vertical as shown in Fig. 8-34). Once this relation (which we will not derive here since we have found this to be most easily illustrated at the blackboard) is established, then the principal results of this problem follow from Eq. 7-12 (for W_g) and Eq. 8-9 (for U).

(a) The vertical component of the displacement vector is downward with magnitude h, so we obtain

$$\begin{aligned} W_g &= \vec{F}_g \cdot \vec{d} = mgh = mgL(1-\cos\theta) \\ &= (5.00\ \text{kg})(9.80\ \text{m/s}^2)(2.00\ \text{m})(1-\cos 30°) = 13.1\ \text{J}. \end{aligned}$$

(b) From Eq. 8-1, we have $\Delta U = -W_g = -mgL(1 - \cos\theta) = -13.1$ J.

(c) With $y = h$, Eq. 8-9 yields $U = mgL(1 - \cos\theta) = 13.1$ J.

(d) As the angle increases, we intuitively see that the height h increases (and, less obviously, from the mathematics, we see that $\cos\theta$ decreases so that $1 - \cos\theta$ increases), so the answers to parts (a) and (c) increase, and the absolute value of the answer to part (b) also increases.

8. (a) The force of gravity is constant, so the work it does is given by $W = \vec{F} \cdot \vec{d}$, where $\vec{F}$ is the force and $\vec{d}$ is the displacement. The force is vertically downward and has magnitude mg, where m is the mass of the snowball. The expression for the work reduces to $W = mgh$, where h is the height through which the snowball drops. Thus

$$W = mgh = (1.50\ \text{kg})(9.80\ \text{m/s}^2)(11.5\ \text{m}) = 169\ \text{J}\,.$$

(b) The force of gravity is conservative, so the change in the potential energy of the snowball-Earth system is the negative of the work it does: ΔU = ï W = ï 169 J.

(c) The potential energy when it reaches the ground is less than the potential energy when it is fired by $|\Delta U|$, so U = ï 169 J when the snowball hits the ground.

9. We use Eq. 8-17, representing the conservation of mechanical energy (which neglects friction and other dissipative effects).

(a) In Problem 9-2, we found $U_A = mgh$ (with the reference position at C). Referring again to Fig. 8-29, we see that this is the same as U_0, which implies that $K_A = K_0$ and thus that $v_A = v_0 = 17.0$ m/s.

(b) In the solution to Problem 9-2, we also found $U_B = mgh/2$. In this case, we have

$$K_0 + U_0 = K_B + U_B \quad \Rightarrow \quad \frac{1}{2}mv_0^2 + mgh = \frac{1}{2}mv_B^2 + mg\left(\frac{h}{2}\right)$$

which leads to

$$v_B = \sqrt{v_0^2 + gh} = \sqrt{(17.0\text{ m/s})^2 + (9.80\text{ m/s}^2)(42.0\text{ m})} = 26.5\text{ m/s}.$$

(c) Similarly, $v_C = \sqrt{v_0^2 + 2gh} = \sqrt{(17.0\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(42.0\text{ m})} = 33.4\text{ m/s}.$

(d) To find the ñfinalò height, we set $K_f = 0$. In this case, we have

$$K_0 + U_0 = K_f + U_f \quad \Rightarrow \quad \frac{1}{2}mv_0^2 + mgh = 0 + mgh_f$$

which yields $h_f = h + \dfrac{v_0^2}{2g} = 42.0\text{ m} + \dfrac{(17.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 56.7\text{ m}.$

(e) It is evident that the above results do not depend on mass. Thus, a different mass for the coaster must lead to the same results.

10. We use Eq. 8-17, representing the conservation of mechanical energy (which neglects friction and other dissipative effects).

(a) In the solution to Problem 9-3 (to which this problem refers), we found $U_i = mgy_i$ = 196 J and $U_f = mgy_f$ = 29.0 J (assuming the reference position is at the ground). Since K_i = 0 in this case, we have

$$0 + 196\ \text{J} = K_f + 29.0\ \text{J}$$

which gives $K_f = 167\ \text{J}$ and thus leads to $v = \sqrt{\dfrac{2K_f}{m}} = \sqrt{\dfrac{2(167\ \text{J})}{2.00\ \text{kg}}} = 12.9\ \text{m/s}.$

(b) If we proceed algebraically through the calculation in part (a), we find K_f = ï ΔU = mgh where $h = y_i$ ï y_f and is positive-valued. Thus,

$$v = \sqrt{\frac{2K_f}{m}} = \sqrt{2gh}$$

as we might also have derived from the equations of Table 2-1 (particularly Eq. 2-16). The fact that the answer is independent of mass means that the answer to part (b) is identical to that of part (a), that is, $v = 12.9\ \text{m/s}$.

(c) If $K_i \neq 0$, then we find $K_f = mgh + K_i$ (where K_i is necessarily positive-valued). This represents a larger value for K_f than in the previous parts, and thus leads to a larger value for v.

11. **THINK** As the ice flake slides down the frictionless bowl, its potential energy decreases (discussed in Problem 8-5). By conservation of mechanical energy, its kinetic energy must increase.

EXPRESS If K_i is the kinetic energy of the flake at the edge of the bowl, K_f is its kinetic energy at the bottom, U_i is the gravitational potential energy of the flake-Earth system with the flake at the top, and U_f is the gravitational potential energy with it at the bottom, then

$$K_f + U_f = K_i + U_i .$$

Taking the potential energy to be zero at the bottom of the bowl, then the potential energy at the top is $U_i = mgr$ where $r = 0.220$ m is the radius of the bowl and m is the mass of the flake. $K_i = 0$ since the flake starts from rest. Since the problem asks for the speed at the bottom, we write $K_f = mv^2/2$.

ANALYZE

(a) Energy conservation leads to

$$K_f + U_f = K_i + U_i \quad \Rightarrow \quad \frac{1}{2}mv^2 + 0 = 0 + mgr .$$

The speed is $v = \sqrt{2gr} = 2.08$ m/s.

(b) Since the expression for speed is $v = \sqrt{2gr}$, which does not contain the mass of the flake, the speed would be the same, 2.08 m/s, regardless of the mass of the flake.

(c) The final kinetic energy is given by $K_f = K_i + U_i - U_f$. If K_i is greater than before, then K_f will also be greater. This means the final speed of the flake is greater.

LEARN The mechanical energy conservation principle can also be expressed as $\Delta E_{\text{mech}} = \Delta K + \Delta U = 0$, which implies $\Delta K = -\Delta U$, i.e., the increase in kinetic energy is equal to the negative of the change in potential energy.

12. We use Eq. 8-18, representing the conservation of mechanical energy. We choose the reference position for computing U to be at the ground below the cliff; it is also regarded as the ñfinalò position in our calculations.

(a) Using Eq. 8-9, the initial potential energy is given by $U_i = mgh$ where h = 11.5 m and $m = 1.50$ kg . Thus, we have

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv^2 + 0$$

which leads to the speed of the snowball at the instant before striking the ground:

$$v = \sqrt{\frac{2}{m}\left(\frac{1}{2}mv_i^2 + mgh\right)} = \sqrt{v_i^2 + 2gh} = \sqrt{(16.0\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(11.5\text{ m})} = 21.9\text{ m/s}.$$

where v_i = 14.0 m/s is the magnitude of its initial velocity (not just one component of it). Thus we find v = 21.0 m/s.

(b) As noted above, v_i is the magnitude of its initial velocity and not just one component of it; therefore, there is no dependence on launch angle. The answer is again 21.9 m/s.

(c) It is evident that the result for v in part (a) does not depend on mass. Thus, changing the mass of the snowball does not change the result for v, and we still have v = 21.9 m/s.

13. **THINK** As the marble moves vertically upward, its gravitational potential energy increases. This energy comes from the release of elastic potential energy stored in the spring.

EXPRESS We take the reference point for gravitational potential energy to be at the position of the marble when the spring is compressed. The gravitational potential energy when the marble is at the top of its motion is $U_g = mgh$. On the other had, the energy stored in the spring is $U_s = kx^2/2$. Applying mechanical energy conservation principle allows us to solve the problem.

ANALYZE (a) The height of the highest point is h = 20 m. With initial gravitational potential energy set to zero, we find

$$\Delta U_g = mgh = (5.0\times10^{-3}\text{ kg})(9.8\text{ m/s}^2)(20\text{ m}) = 0.98\text{ J}.$$

(b) Since the kinetic energy is zero at the release point and at the highest point, then conservation of mechanical energy implies $\Delta U_g + \Delta U_s = 0$, where ΔU_s is the change in the spring's elastic potential energy. Therefore, ΔU_s = ï ΔU_g = ï 0.98 J.

(c) We take the spring potential energy to be zero when the spring is relaxed. Then, our result in the previous part implies that its initial potential energy is U_s = 0.98 J. This must be $\frac{1}{2}kx^2$, where k is the spring constant and x is the initial compression. Consequently,

$$k = \frac{2U_s}{x^2} = \frac{0.98\text{ J}}{(0.080\text{ m})^2} = 3.1\times10^2\text{ N/m} = 3.1\text{ N/cm}.$$

LEARN In general, the marble has both kinetic and potential energies:

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + mgy$$

At the maximum height $y_{\max} = h,\quad v = 0$ and $mgh = kx^2/2$, or $h = \dfrac{kx^2}{2mg}$.

14. We use Eq. 8-18, representing the conservation of mechanical energy (which neglects friction and other dissipative effects).

(a) The change in potential energy is $\Delta U = mgL$ as it goes to the highest point. Thus, we have

$$\begin{gathered}\Delta K + \Delta U = 0 \\ K_{\text{top}} - K_0 + mgL = 0\end{gathered}$$

which, upon requiring $K_{\text{top}} = 0$, gives $K_0 = mgL$ and thus leads to

$$v_0 = \sqrt{\frac{2K_0}{m}} = \sqrt{2gL} = \sqrt{2(9.80\ \text{m/s}^2)(0.452\ \text{m})} = 2.98\ \text{m/s}\,.$$

(b) We also found in Problem 9-4 that the potential energy change is $\Delta U =$ ï mgL in going from the initial point to the lowest point (the bottom). Thus,

$$\begin{gathered}\Delta K + \Delta U = 0 \\ K_{\text{bottom}} - K_0 - mgL = 0\end{gathered}$$

which, with $K_0 = mgL$, leads to $K_{\text{bottom}} = 2mgL$. Therefore,

$$v_{\text{bottom}} = \sqrt{\frac{2K_{\text{bottom}}}{m}} = \sqrt{4gL} = \sqrt{4(9.80\ \text{m/s}^2)(0.452\ \text{m})} = 4.21\ \text{m/s}\,.$$

(c) Since there is no change in height (going from initial point to the rightmost point), then $\Delta U = 0$, which implies $\Delta K = 0$. Consequently, the speed is the same as what it was initially,

$$v_{\text{right}} = v_0 = 2.98\ \text{m/s}\,.$$

(d) It is evident from the above manipulations that the results do not depend on mass. Thus, a different mass for the ball must lead to the same results.

15. **THINK** The truck with failed brakes is moving up an escape ramp. In order for it to come to a complete stop, all of its kinetic energy must be converted into gravitational potential energy.

EXPRESS We ignore any work done by friction. In SI units, the initial speed of the truck just before entering the escape ramp is v_i = 130(1000/3600) = 36.1 m/s. When the truck comes to a stop, its kinetic and potential energies are $K_f = 0$ and $U_f = mgh$. We apply mechanical energy conservation to solve the problem.

ANALYZE

(a) Energy conservation implies $K_f + U_f = K_i + U_i$. With $U_i = 0$, and $K_i = \frac{1}{2}mv_i^2$, we obtain

$$\frac{1}{2}mv_i^2 + 0 = 0 + mgh \quad \Rightarrow \quad h = \frac{v_i^2}{2g} = \frac{(36.1\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)} = 66.5\ \text{m}.$$

0If L is the minimum length of the ramp, then $L\sin\theta = h$, or L sin 15° = 66.5 m so that $L = (66.5\ \text{m})/\sin 15° = 257\ \text{m}$. That is, the ramp must be about 2.6×10^2 m long if friction is negligible.

(b) The minimum length is $L = \dfrac{h}{\sin\theta} = \dfrac{v_i^2}{2g\sin\theta}$ which does not depend on the mass of the truck. Thus, the answer remains the same if the mass is reduced.

(c) If the speed is decreased, then h and L both decrease (note that h is proportional to the square of the speed and that L is proportional to h).

LEARN The greater the speed of the truck, the longer the ramp required. This length can be shortened considerably if the friction between the tires and the ramp surface is factored in.

16. We place the reference position for evaluating gravitational potential energy at the relaxed position of the spring. We use x for the spring's compression, measured positively downward (so $x > 0$ means it is compressed).

(a) With x = 0.190 m, Eq. 7-26 gives

$$W_s = -\frac{1}{2}kx^2 = -8.12 \text{ J} \approx -8.1 \text{ J}$$

for the work done by the spring force. Using Newton's third law, we see that the work done on the spring is 8.1 J.

(b) As noted above, W_s = ï 8.1 J.

(c) Energy conservation leads to

$$K_i + U_i = K_f + U_f$$

$$mgh_0 = -mgx + \frac{1}{2}kx^2$$

which (with m = 0.70 kg) yields

$$h_0 = -x + \frac{kx^2}{2mg} = -0.19 \text{ m} + \frac{(450 \text{ N/m})(0.19 \text{ m})^2}{2(0.70 \text{ kg})(9.8 \text{ m/s}^2)} = 0.99 \text{ m}$$

(d) With a new height value of 1.98 m, we solve for a new value of x using the quadratic formula (taking its positive root so that $x > 0$):

$$mgh_0' = -mgx + \frac{1}{2}kx^2$$

which yields

$$x = \frac{mg + \sqrt{(mg)^2 + 2mgkh_0'}}{k} = \frac{mg}{k}\left[1 + \sqrt{1 + \frac{2kh'}{mg}}\right]$$

$$= \frac{(0.70 \text{ kg})(9.8 \text{ m/s}^2)}{450 \text{ N/m}}\left[1 + \sqrt{1 + \frac{2(450 \text{ N/m})(1.99 \text{ m})}{(0.70 \text{ kg})(9.8 \text{ m/s}^2)}}\right] = 0.26 \text{ m}$$

17. (a) At Q the block (which is in circular motion at that point) experiences a centripetal acceleration v^2/R leftward. We find v^2 from energy conservation:

$$K_P + U_P = K_Q + U_Q \quad \Rightarrow \quad 0 + mgh = \frac{1}{2}mv^2 + mgR$$

Using the fact that $h = 5R$, we find $mv^2 = 8mgR$. Thus, the horizontal component of the net force on the block at Q is

$$F = mv^2/R = 8mg = 8(0.032 \text{ kg})(9.8 \text{ m/s}^2) = 2.5 \text{ N}.$$

The direction is to the left (in the same direction as $\vec{a}$).

(b) The downward component of the net force on the block at Q is the downward force of gravity

$$F = mg = (0.032 \text{ kg})(9.8 \text{ m/s}^2) = 0.31 \text{ N}.$$

(c) To barely make the top of the loop, the centripetal force there must equal the force of gravity:

$$\frac{mv_t^2}{R} = mg \quad \Rightarrow \quad mv_t^2 = mgR\,.$$

This requires a different value of h than what was used above.

$$\begin{aligned} K_P + U_P &= K_t + U_t \\ 0 + mgh &= \frac{1}{2}mv_t^2 + mgh_t \\ mgh &= \frac{1}{2}(mgR) + mg(2R) \end{aligned}$$

Consequently, $h = 2.5R = (2.5)(0.12 \text{ m}) = 0.30$ m.

(d) The normal force F_N, for speeds v_t greater than $\sqrt{gR}$ (which are the only possibilities for nonzero F_N ð see the solution in the previous part), obeys

$$F_N = \frac{mv_t^2}{R} - mg$$

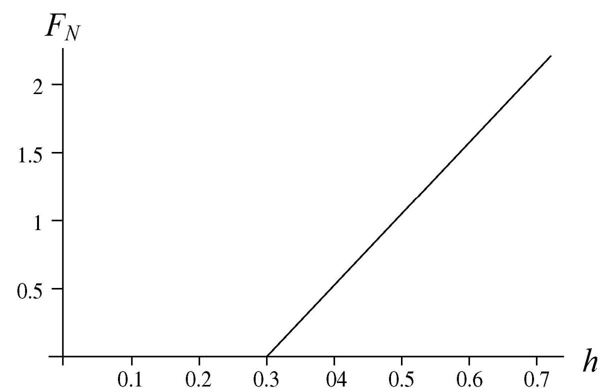
from Newton's second law. Since v_t^2 is related to h by energy conservation

$$K_P + U_P = K_t + U_t \quad \Rightarrow \quad 0 + mgh = \frac{1}{2}mv_t^2 + 2mgR$$

then the normal force, as a function for h (so long as $h \geq 2.5R$ ð see the solution in the previous part), becomes

$$F_N = \frac{2mgh}{R} - 5mg\ .$$

Thus, the graph for $h \geq 2.5R = 0.30$ m consists of a straight line of positive slope $2mg/R$ (which can be set to some convenient values for graphing purposes). Note that for $h \leq 2.5R$, the normal force is zero.



18. We use Eq. 8-18, representing the conservation of mechanical energy. The reference position for computing U is the lowest point of the swing; it is also regarded as the ñfinalò position in our calculations.

(a) The potential energy is $U = mgL(1 ï \cos\theta)$ at the position shown in Fig. 8-34 (which we consider to be the initial position). Thus, we have

$$K_i + U_i = K_f + U_f$$

$$0 + mgL(1-\cos\theta) = \frac{1}{2}mv^2 + 0$$

which leads to

$$v = \sqrt{\frac{2mgL(1-\cos\theta)}{m}} = \sqrt{2gL(1-\cos\theta)}.$$

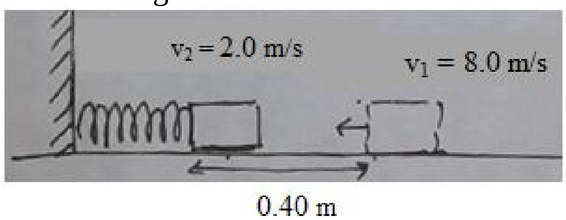
Plugging in L = 2.00 m and θ = 30.0° we find v = 2.29 m/s.

(b) It is evident that the result for v does not depend on mass. Thus, a different mass for the ball must not change the result.

19. For the first event explained (see the figure shown below), the initial velocity of block is v_1 = 8.0 m/s; the final velocity of the block is v_2 = 2.0 m/s. According to the law of conservation of energy,

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx^2$$
$$\Rightarrow mv_1^2 = mv_2^2 + kx^2$$
$$\Rightarrow k = \frac{m(v_1^2 - v_2^2)}{x^2} = \frac{1.0(8.0^2 - 2.0^2)}{(0.4)^2} = \frac{(64 - 4.0)1.0}{0.16} = 375 \text{ N/m}$$

Now, the stone is kept on the top and additional compression is applied to bring the spring below 0.50 m its natural length.



Mass of stone = 2.0 kg; k = 375 N/m; x = 0.50 m. Taking the compressed state as reference and using the law of conservation of energy, we have

$$\frac{1}{2}kx^2 = mgh$$
$$\Rightarrow h = \frac{kx^2}{2mg} = \frac{3.75 \times (0.50)^2}{2 \times 2 \times 9.8} = 2.39 \text{ m}$$

Therefore, the stone goes 2.39 ï 0.50 = 1. 9 m above the spring's rest-length height.

20. (a) We take the reference point for gravitational energy to be at the lowest point of the swing. Let θ be the angle measured from vertical. Then the height y of the pendulum ñbobò (the object at the end of the pendulum, which in this problem is the stone) is given by $L(1 \text{ ï } \cos\theta) = y$. Hence, the gravitational potential energy is

$$mgy = mgL(1 \text{ ï } \cos\theta).$$

When $\theta = 0^{\circ}$ (the string at its lowest point) we are told that its speed is 8.0 m/s; its kinetic energy there is therefore 64 J (using Eq. 7-1). At $\theta = 60^{\circ}$ its mechanical energy is

$$E_{\text{mech}} = \frac{1}{2}\, mv^2 + mgL(1 \text{ ï } \cos\theta)\ .$$

Energy conservation (since there is no friction) requires that this be equal to 64 J. Solving for the speed, we find v = 4.5 m/s.

(b) We now set the above expression again equal to 64 J (with θ being the unknown) but with zero speed (which gives the condition for the maximum point, or ñturning pointò that it reaches). This leads to $\theta_{\text{max}} = 74°$.

(c) As observed in our solution to part (a), the total mechanical energy is 64 J.

21. We use Eq. 8-18, representing the conservation of mechanical energy (which neglects friction and other dissipative effects). The reference position for computing U (and height h) is the lowest point of the swing; it is also regarded as the ñfinalò position in our calculations.

(a) Careful examination of the figure leads to the trigonometric relation $h = L$ ï $L \cos\theta$ when the angle is measured from vertical as shown. Thus, the gravitational potential energy is $U = mgL(1$ ï $\cos\theta_0)$ at the position shown in Fig. 8-34 (the initial position). Thus, we have

$$K_0 + U_0 = K_f + U_f \quad \Rightarrow \quad \frac{1}{2}mv_0^2 + mgL(1-\cos\theta_0) = \frac{1}{2}mv^2 + 0$$

which leads to

$$\begin{aligned} v &= \sqrt{\frac{2}{m}\left[\frac{1}{2}mv_0^2 + mgL(1-\cos\theta_0)\right]} = \sqrt{v_0^2 + 2gL(1-\cos\theta_0)} \\ &= \sqrt{(8.00 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(1.25 \text{ m})(1-\cos 40°)} = 8.35 \text{ m/s}. \end{aligned}$$

(b) We look for the initial speed required to barely reach the horizontal position ð described by $v_h = 0$ and $\theta = 90°$ (or $\theta =$ ï $90°$, if one prefers, but since cos(ï ϕ) = cos ϕ, the sign of the angle is not a concern).

$$K_0 + U_0 = K_h + U_h \quad \Rightarrow \quad \frac{1}{2}mv_0^2 + mgL(1-\cos\theta_0) = 0 + mgL$$

which yields

$$v_0 = \sqrt{2gL\cos\theta_0} = \sqrt{2(9.80 \text{ m/s}^2)(1.25 \text{ m})\cos 40°} = 4.33 \text{ m/s}.$$

(c) For the cord to remain straight, then the centripetal force (at the top) must be (at least) equal to gravitational force:

$$\frac{mv_t^2}{r} = mg \Rightarrow mv_t^2 = mgL$$

where we recognize that $r = L$. We plug this into the expression for the kinetic energy (at the top, where $\theta = 180°$).

$$\begin{aligned} K_0 + U_0 &= K_t + U_t \\ \frac{1}{2}mv_0^2 + mgL(1-\cos\theta_0) &= \frac{1}{2}mv_t^2 + mgL(1-\cos 180°) \\ &= \frac{1}{2}(mgL) + mg(2L) \end{aligned}$$

which leads to

$$v_0 = \sqrt{gL(3 + 2\cos\theta_0)} = \sqrt{(9.80\ \text{m/s}^2)(1.25\ \text{m})(3 + 2\cos 40°)} = 7.45\ \text{m/s}.$$

(d) The more initial potential energy there is, the less initial kinetic energy there needs to be, in order to reach the positions described in parts (b) and (c). Increasing θ_0 amounts to increasing U_0, so we see that a greater value of θ_0 leads to smaller results for v_0 in parts (b) and (c).

22. From Chapter 4, we know the height h of the skier's jump can be found from the equation $v_y^2 = 0 = v_{0\,y}^2 - 2gh$, where $v_{0y} = v_0 \sin 28°$ is the upward component of the skier's ñlaunch velocity.ò To find v_0 we use energy conservation.

(a) The skier starts at rest y = 22 m above the point of ñlaunchò so energy conservation leads to

$$mgy = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{2gy} = \sqrt{2(9.8\text{ m/s}^2)(22\text{ m})} = 21\text{ m/s}$$

which becomes the initial speed v_0 for the launch. Hence, the above equation relating h to v_0 yields

$$h = \frac{(v_0 \sin 28°)^2}{2g} = 4.8\text{ m}$$

(b) We see that all reference to mass cancels from the above computations, so a new value for the mass will yield the same result as before.

23. (a) As the string reaches its lowest point, its original potential energy $U = mgL$ (measured relative to the lowest point) is converted into kinetic energy. Thus,

$$mgL = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gL}\ .$$

With L = 1.20 m we obtain

$$v = \sqrt{2gL} = \sqrt{2(9.80\ \text{m/s}^2)(1.20\ \text{m})} = 4.85\ \text{m/s}\,.$$

(b) In this case, the total mechanical energy is shared between kinetic $\frac{1}{2}mv_b^2$ and potential mgy_b. We note that $y_b = 2r$ where $r = L$ ï d = 0.450 m. Energy conservation leads to

$$mgL = \frac{1}{2}mv_b^2 + mgy_b$$

which yields

$$v_b = \sqrt{2gL - 2g(2r)} = 2.42\ \text{m/s}$$

24. We denote m as the mass of the block, h = 0.50 m as the height from which it dropped (measured from the relaxed position of the spring), and x as the compression of the spring (measured downward so that it yields a positive value). Our reference point for the gravitational potential energy is the initial position of the block. The block drops a total distance $h + x$, and the final gravitational potential energy is ï $mg(h + x)$. The spring potential energy is $\frac{1}{2}kx^2$ in the final situation, and the kinetic energy is zero both at the beginning and end. Since energy is conserved

$$\begin{aligned} K_i + U_i &= K_f + U_f \\ 0 &= -mg(h+x) + \frac{1}{2}kx^2 \end{aligned}$$

which is a second degree equation in x. Using the quadratic formula, its solution is

$$x = \frac{mg \pm \sqrt{(mg)^2 + 2mghk}}{k}$$

Now mg = 19.6 N, h = 0.50 m, and $k = 1960 \text{ N/m}$, and we choose the positive root giving x = 0.11 m.

25. Since time does not directly enter into the energy formulations, we return to Chapter 4 (or Table 2-1 in Chapter 2) to find the change of height during this t = 6.0 s flight.

$$\Delta y = v_{0y}t - \frac{1}{2}gt^2$$

This leads to $\Delta y = -32 \text{ m}$. Therefore, $\Delta U = mg\Delta y = -318 \text{ J} \approx -3.2\times 10^{-2} \text{ J}$.

26. (a) With energy in joules and length in meters, we have

$$\Delta U = U(x) - U(0) = -\int_0^x (6x' - 12)dx'$$

Therefore, with U (0) = 27 J, we obtain $U(x)$ (written simply as U) by integrating and rearranging:

$$U = 27 + 12x - 3x^2 \ .$$

(b) We can maximize the above function by working through the $dU/dx = 0$ condition, or we can treat this as a force equilibrium situation ð which is the approach we show.

$$F = 0 \Rightarrow 6x_{eq} - 12 = 0$$

Thus, x_{eq} = 2.0 m, and the above expression for the potential energy becomes U = 39 J.

(c) Using the quadratic formula or using the polynomial solver on an appropriate calculator, we find the negative value of x for which U = 0 to be x = ï 1.6 m.

(d) Similarly, we find the positive value of x for which U = 0 to be x = 5.6 m.

27. (a) To find out whether or not the vine breaks, it is sufficient to examine it at the moment Tarzan swings through the lowest point, which is when the vine ð if it didn't break ð would have the greatest tension. Choosing upward positive, Newton's second law leads to

$$T - mg = m\frac{v^2}{r}$$

where r = 18.0 m and $m = W/g = 688/9.8 = 70.2$ kg. We find the v^2 from energy conservation (where the reference position for the potential energy is at the lowest point).

$$mgh = \frac{1}{2}mv^2 \quad \Rightarrow \; v^2 = 2gh$$

where h = 3.20 m. Combining these results, we have

$$T = mg + m\frac{2gh}{r} = mg\left(1 + \frac{2h}{r}\right)$$

which yields 933 N. Thus, the vine does not break.

(b) Rounding to an appropriate number of significant figures, we see the maximum tension is roughly 9.3×10^2 N.

28. From the slope of the graph, we find the spring constant

$$k = \frac{\Delta F}{\Delta x} = 0.10\,\mathrm{N/cm} = 10\,\mathrm{N/m}.$$

(a) Equating the potential energy of the compressed spring to the kinetic energy of the cork at the moment of release, we have

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \Rightarrow v = x\sqrt{\frac{k}{m}}$$

which yields v = 2.7 m/s for m = 0.0042 kg and x = 0.055 m.

(b) The new scenario involves some potential energy at the moment of release. With d = 0.015 m, energy conservation becomes

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{2}kd^2 \Rightarrow v = \sqrt{\frac{k}{m}\left(x^2 - d^2\right)}$$

which yields v = 2.6 m/s.

29. (a) We know that speed is maximum when kinetic energy is maximum. In the given situation, the complete potential energy of the spring is transferred to block as kinetic energy:

$$\frac{1}{2}kl^2 = \frac{1}{2}mv^2 \quad \Rightarrow \quad k = \frac{mv^2}{l^2} = \frac{(4.0\ \text{kg})(1.0\ \text{m/s})^2}{(0.20\ \text{m})^2} = 100\ \text{N/m}$$

The energy loss due to friction is

$$\Delta U_{\text{th}} = f_k(l_0 + x) = \mu_k F_N(l_0 + x) = \mu_k mg\cos\theta(l_0 + x).$$

Thus, by conservation of mechanical energy, we have

$$mg(l_0 + x)\sin\theta = \frac{1}{2}kx^2 + \mu_k mg\cos\theta(l_0 + x)$$

Solving for μ_k, we obtain

$$\begin{aligned}\mu_k &= \frac{mg(l_0 + x)\sin\theta - kx^2/2}{mg\cos\theta(l_0 + x)} = \tan\theta - \frac{kx^2}{2mg\cos\theta(l_0 + x)} \\ &= \tan 30° - \frac{(100\ \text{N/m})(0.30\ \text{m})^2}{2(4.0\ \text{kg})(9.8\ \text{m/s}^2)\cos 30°(5.0\ \text{m} + 0.30\ \text{m})} \\ &= 0.55\end{aligned}$$

(b) The distance the block moves up from the stopping point is given by

$$\frac{1}{2}kx^2 = mgl'(\sin\theta + \mu_k\cos\theta)$$

or

$$\begin{aligned}l' &= \frac{kx^2}{2mg(\sin\theta + \mu_k\cos\theta)} = \frac{(100\ \text{N/m})(0.30\ \text{m})^2}{2(4.0\ \text{kg})(9.8\ \text{m/s}^2)[\sin 30° + (0.55)\cos 30°]} \\ &= 0.118\ \text{m} \approx 0.12\ \text{m}.\end{aligned}$$

30. We take the original height of the box to be the $y = 0$ reference level and observe that, in general, the height of the box (when the box has moved a distance d downhill) is $y = -d \sin 40°$.

(a) Using the conservation of energy, we have

$$K_i + U_i = K + U \Rightarrow 0 + 0 = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kd^2.$$

Therefore, with $d = 0.10$ m, we obtain $v = 0.86$ m/s.

(b) We look for a value of $d \neq 0$ such that $K = 0$.

$$K_i + U_i = K + U \Rightarrow 0 + 0 = 0 + mgy + \frac{1}{2}kd^2.$$

Thus, we obtain $mgd \sin 40° = \frac{1}{2}kd^2$ and find $d = 0.24$ m.

(c) The uphill force is caused by the spring (Hooke's law) and has magnitude

$$kd = 25.2 \text{ N}.$$

The downhill force is the component of gravity $mg \sin 40° =$ 12.6 N. Thus, the net force on the box is (25.2 ï 12.6) N = 12.6 N uphill, with

$$a = F/m = (12.6 \text{ N})/(2.0 \text{ kg}) = 6.3 \text{ m/s}^2.$$

(d) The acceleration is up the incline.

31. Using conservation of energy, the total elastic potential energy of the spring is first converted to kinetic energy of the block and then at the greatest height to the potential energy (gravitational) of the block. Therefore,

$$E = \frac{1}{2}mv^2 = \frac{1}{2}kx^2 = mgh = (1.00\text{ kg})(9.8\text{ m/s}^2)(5.00\text{ m}) = 49.0\text{ J}.$$

(a) The spring constant is given by

$$k = \frac{2E}{x^2} = \frac{2(49.0\text{ J})}{(0.25\text{ m})^2} = 1568\text{ N/m} \approx 1.6\times10^3\text{ N/m}.$$

(b) The maximum speed is given by

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(49.0\text{ J})}{1.00\text{ kg}}} = 9.9\text{ m/s}.$$

(c) If the angle of the slope is increased, there would not be any change in the height acquired because the total energy is not dissipated in any form.

32. The work required is the change in the gravitational potential energy as a result of the chain being pulled onto the table. Dividing the hanging chain into a large number of infinitesimal segments, each of length dy, we note that the mass of a segment is $(m/L)\,dy$ and the change in potential energy of a segment when it is a distance $|y|$ below the table top is

$$dU = (m/L)g|y|\,dy = ï\,(m/L)gy\,dy$$

since y is negative-valued (we have $+y$ upward and the origin is at the tabletop). The total potential energy change is

$$\Delta U = -\frac{mg}{L}\int_{-L/4}^{0} y\,dy = \frac{1}{2}\frac{mg}{L}(L/4)^2 = mgL/32 .$$

The work required to pull the chain onto the table is therefore

$$W = \Delta U = mgL/32 = (0.016\text{ kg})(9.8\text{ m/s}^2)(0.24\text{ m})/32 = 0.0012\text{ J}.$$

33. All heights h are measured from the lower end of the incline (which is our reference position for computing gravitational potential energy mgh). Our x axis is along the incline, with $+x$ being uphill (so spring compression corresponds to $x > 0$) and its origin being at the relaxed end of the spring. The height that corresponds to the canister's initial position (with spring compressed amount x = 0.200 m) is given by $h_1 = (D+x)\sin\theta$, where $\theta = 37°$.

(a) Energy conservation leads to

$$K_1 + U_1 = K_2 + U_2 \quad \Rightarrow \quad 0 + mg(D+x)\sin\theta + \frac{1}{2}kx^2 = \frac{1}{2}mv_2^2 + mgD\sin\theta$$

which yields, using the data m = 2.00 kg and k = 170 N/m,

$$v_2 = \sqrt{2gx\sin\theta + kx^2/m} = 2.40\,\text{m/s}.$$

(b) In this case, energy conservation leads to

$$K_1 + U_1 = K_3 + U_3$$
$$0 + mg(D+x)\sin\theta + \frac{1}{2}kx^2 = \frac{1}{2}mv_3^2 + 0$$

which yields $v_3 = \sqrt{2g(D+x)\sin\theta + kx^2/m} = 4.19$ m/s.

34. Let $\vec{F}_N$ be the normal force of the ice on him and m is his mass. The net inward force is $mg\cos\theta - F_N$ and, according to Newton's second law, this must be equal to mv^2/R, where v is the speed of the boy. At the point where the boy leaves the ice $F_N = 0$, so $g\cos\theta = v^2/R$. We wish to find his speed. If the gravitational potential energy is taken to be zero when he is at the top of the ice mound, then his potential energy at the time shown is

$$U = -mgR(1 - \cos\theta).$$

He starts from rest and his kinetic energy at the time shown is $\frac{1}{2}mv^2$. Thus conservation of energy gives

$$0 = \tfrac{1}{2}mv^2 - mgR(1-\cos\theta),$$

or $v^2 = 2gR(1 - \cos\theta)$. We substitute this expression into the equation developed from the second law to obtain $g\cos\theta = 2g(1 - \cos\theta)$. This gives $\cos\theta = 2/3$. The height of the boy above the bottom of the mound is

$$h = R\cos\theta = \frac{2}{3}R = \frac{2}{3}(12.8\text{ m}) = 8.53\text{ m}.$$

35. (a) The (final) elastic potential energy is

$$U = \frac{1}{2}\, kx^2 = \frac{1}{2}\,(431 \text{ N/m})(0.210 \text{ m})^2 = 9.50 \text{ J}.$$

Ultimately this must come from the original (gravitational) energy in the system mgy (where we are measuring y from the lowest ñelevationò reached by the block, so

$$y = (d + x)\sin(30^{\circ}).$$

Thus,

$$mg(d + x)\sin(30^{\circ}) = 9.50 \text{ J} \qquad \Rightarrow \qquad d = 0.396 \text{ m}.$$

(b) The block is still accelerating (due to the component of gravity along the incline, $mg\sin(30^{\circ})$) for a few moments after coming into contact with the spring (which exerts the Hookeôs law force kx), until the Hookeôs law force is strong enough to cause the block to begin decelerating. This point is reached when

$$kx = mg\sin 30^{\circ}$$

which leads to x = 0.0364 m = 3.64 cm; this is long before the block finally stops (36.0 cm before it stops).

36. The distance the marble travels is determined by its initial speed (and the methods of Chapter 4), and the initial speed is determined (using energy conservation) by the original compression of the spring. We denote h as the height of the table, and x as the horizontal distance to the point where the marble lands. Then $x = v_0\, t$ and $h = \frac{1}{2} g t^2$ (since the vertical component of the marble's ñlaunch velocityò is zero). From these we find $x = v_0 \sqrt{2\, h/g}$. We note from this that the distance to the landing point is directly proportional to the initial speed. We denote v_{01} be the initial speed of the first shot and D_1 = (2.20 ï 0.263) m = 1.94 m be the horizontal distance to its landing point; similarly, v_{02} is the initial speed of the second shot and D = 2.20 m is the horizontal distance to its landing spot. Then

$$\frac{v_{02}}{v_{01}} = \frac{D}{D_1} \quad \Rightarrow \quad v_{02} = \frac{D}{D_1} v_{01}$$

When the spring is compressed an amount ℓ, the elastic potential energy is $\frac{1}{2} k\ell^2$. When the marble leaves the spring its kinetic energy is $\frac{1}{2} m v_0^2$. Mechanical energy is conserved: $\frac{1}{2} m v_0^2 = \frac{1}{2} k\ell^2$, and we see that the initial speed of the marble is directly proportional to the original compression of the spring. If ℓ_1 is the compression for the first shot and ℓ_2 is the compression for the second, then $v_{02} = (\ell_2/\ell_1) v_{01}$. Relating this to the previous result, we obtain

$$\ell_2 = \frac{D}{D_1}\ell_1 = \left(\frac{2.20 \text{ m}}{1.94 \text{ m}}\right)(1.10 \text{ cm}) = 1.25 \text{ cm}.$$

37. To move the hanging part of the chain upward, a force has to be applied by the machine, which is equal to the weight of chain. The weight of a small element of length *dy* in the hanging part has weight

$$\Delta F_g = (dm)g_{\text{Moon}} = (\mu dy)g_{\text{Moon}}$$

where $\mu = m_0 / L = (2.0\text{ kg})/(3.0\text{ m}) = 0.67\text{ kg/m}$ is the mass per unit length. Let dW be the amount of work done by the machine in moving a length dy up a vertical distance y. We have

$$dW = (\Delta F_g)y = \mu g_{\text{Moon}} y dy$$

Thus, the amount of work done in pulling a segment of chain of length l onto the platform is

$$W = \int_0^l \mu g_{\text{Moon}} y dy = \frac{\mu g_{\text{Moon}} l^2}{2}$$

Solving for l, we obtain

$$l = \sqrt{\frac{2W}{\mu g_{\text{Moon}}}} = \sqrt{\frac{2WL}{m g_{\text{Moon}}}} = \sqrt{\frac{2(1.0\text{ J})(3.0\text{ m})}{(2.0\text{ kg})(9.8\text{ m/s}^2/6)}} = 1.36\text{ m} \approx 1.4\text{ m}\,.$$

38. In this problem, the mechanical energy (the sum of K and U) remains constant as the particle moves.

(a) Since mechanical energy is conserved, $U_B + K_B = U_A + K_A$, the kinetic energy of the particle in region A ($3.00 \text{ m} \le x \le 4.00 \text{ m}$) is

$$K_A = U_B - U_A + K_B = 12.0 \text{ J} - 9.00 \text{ J} + 4.00 \text{ J} = 7.00 \text{ J}.$$

With $K_A = mv_A^2/2$, the speed of the particle at $x = 3.5$ m (within region A) is

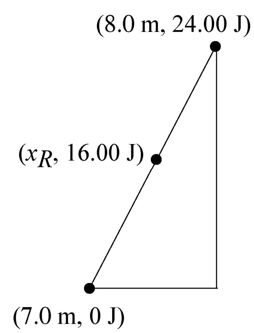
$$v_A = \sqrt{\frac{2K_A}{m}} = \sqrt{\frac{2(7.00 \text{ J})}{0.200 \text{ kg}}} = 8.37 \text{ m/s}.$$

(b) At $x = 6.5$ m, $U = 0$ and $K = U_B + K_B = 12.0 \text{ J} + 4.00 \text{ J} = 16.0 \text{ J}$ by mechanical energy conservation. Therefore, the speed at this point is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(16.0 \text{ J})}{0.200 \text{ kg}}} = 12.6 \text{ m/s}.$$

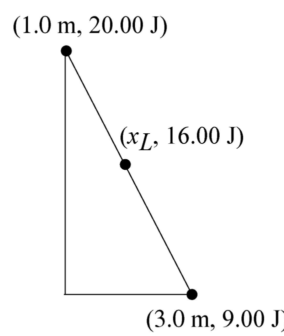
(c) At the turning point, the speed of the particle is zero. Let the position of the right turning point be x_R. From the figure shown on the right, we find x_R to be

$$\frac{16.00 \text{ J} - 0}{x_R - 7.00 \text{ m}} = \frac{24.00 \text{ J} - 16.00 \text{ J}}{8.00 \text{ m} - x_R} \quad \Rightarrow \quad x_R = 7.67 \text{ m}.$$



(d) Let the position of the left turning point be x_L. From the figure shown, we find x_L to be

$$\frac{16.00 \text{ J} - 20.00 \text{ J}}{x_L - 1.00 \text{ m}} = \frac{9.00 \text{ J} - 16.00 \text{ J}}{3.00 \text{ m} - x_L} \quad \Rightarrow \quad x_L = 1.73 \text{ m}.$$



39. From the figure, we see that at x = 4.5 m, the potential energy is U_1 = 15 J. If the speed is v = 7.0 m/s, then the kinetic energy is

$$K_1 = mv^2/2 = (0.90\text{ kg})(7.0\text{ m/s})^2/2 = 22\text{ J}.$$

The total energy is $E_1 = U_1 + K_1 = (15 + 22)\text{ J} = 37\text{ J}$.

(a) At x = 1.0 m, the potential energy is U_2 = 35 J. By energy conservation, we have K_2 = 2.0 J > 0. This means that the particle can reach there with a corresponding speed

$$v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(2.0\text{ J})}{0.90\text{ kg}}} = 2.1\text{ m/s}.$$

(b) The force acting on the particle is related to the potential energy by the negative of the slope:

$$F_x = -\frac{\Delta U}{\Delta x}$$

From the figure we have $F_x = -\frac{35\text{ J} - 15\text{ J}}{2\text{ m} - 4\text{ m}} = +10\text{ N}$.

(c) Since the magnitude $F_x > 0$, the force points in the +x direction.

(d) At x = 7.0 m, the potential energy is U_3 = 45 J, which exceeds the initial total energy E_1. Thus, the particle can never reach there. At the turning point, the kinetic energy is zero. Between x = 5 and 6 m, the potential energy is given by

$$U(x) = 15 + 30(x-5), \quad 5 \le x \le 6.$$

Thus, the turning point is found by solving $37 = 15 + 30(x-5)$, which yields

$$x = 5.7\text{ m}.$$

(e) At x = 5.0 m, the force acting on the particle is

$$F_x = -\frac{\Delta U}{\Delta x} = -\frac{(45-15)\text{ J}}{(6-5)\text{ m}} = -30\text{ N}.$$

The magnitude is $|F_x| = 30\text{ N}$.

(f) The fact that $F_x < 0$ indicated that the force points in the ï x direction.

40. (a) The force at the equilibrium position $r = r_{\text{eq}}$ is

$$F = -\left.\frac{dU}{dr}\right|_{r = r_{\text{eq}}} = 0 \quad \Rightarrow \quad -\frac{12A}{r_{\text{eq}}^{13}} + \frac{6B}{r_{\text{eq}}^{7}} = 0$$

which leads to the result

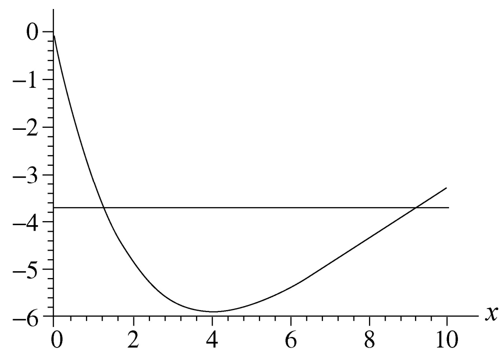
$$r_{\text{eq}} = \left(\frac{2A}{B}\right)^{\frac{1}{6}} = 1.12\left(\frac{A}{B}\right)^{\frac{1}{6}}$$

(b) This defines a minimum in the potential energy curve (as can be verified either by a graph or by taking another derivative and verifying that it is concave upward at this point), which means that for values of r slightly smaller than r_{eq} the slope of the curve is negative (so the force is positive, repulsive).

(c) And for values of r slightly larger than r_{eq} the slope of the curve must be positive (so the force is negative, attractive).

41. (a) The energy at $x = 5.0$ m is $E = K + U = 2.0 \text{ J} - 5.7 \text{ J} = -3.7$ J.

(b) A plot of the potential energy curve (SI units understood) and the energy E (the horizontal line) is shown for $0 \le x \le 10$ m.



(c) The problem asks for a graphical determination of the turning points, which are the points on the curve corresponding to the total energy computed in part (a). The result for the smallest turning point (determined, to be honest, by more careful means) is $x = 1.3$ m.

(d) And the result for the largest turning point is $x = 9.1$ m.

(e) Since $K = E - U$, then maximizing K involves finding the minimum of U. A graphical determination suggests that this occurs at $x = 4.0$ m, which plugs into the expression $E - U = -3.7 - (-4xe^{-x/4})$ to give

$$K = 2.16 \text{ J} \approx 2.2 \text{ J}.$$

Alternatively, one can measure from the graph from the minimum of the U curve up to the level representing the total energy E and thereby obtain an estimate of K at that point.

(f) As mentioned in the previous part, the minimum of the U curve occurs at $x = 4.0$ m.

(g) The force (understood to be in newtons) follows from the potential energy, using Eq. 8-20 (and Appendix E if students are unfamiliar with such derivatives).

$$F = \frac{dU}{dx} = (4 - x)e^{-x/4}$$

(h) This revisits the considerations of parts (d) and (e) (since we are returning to the minimum of $U(x)$) — but now with the advantage of having the analytic result of part (g). We see that the location that produces $F = 0$ is exactly $x = 4.0$ m.

42. Since the velocity is constant, $\vec{a} = 0$ and the horizontal component of the worker's push $F \cos \theta$ (where θ = 32°) must equal the friction force magnitude $f_k = \mu_k F_N$. Also, the vertical forces must cancel, giving:

$$F \cos \theta - \mu_k F_N = 0$$
$$F_N - F \sin \theta - mg = 0$$

which is solved to find F = 60.75 N.

(a) The work done on the block by the worker is, using Eq. 7-7,

$$W_F = Fd \cos\theta = (60.75 \text{ N})(8.4 \text{ m}) \cos(32°) = 433 \text{ J} \approx 4.3\times10^2 \text{ J}$$

(b) Since $f_k = \mu_k (mg + F \sin \theta)$, we find $\Delta E_{\text{th}} = f_k d = (51.5\,\text{N})(8.4\,\text{m}) = 4.3\times10^2\,\text{J} = W_F$.

43. (a) The vertical forces acting on the block are the normal force, upward, and the force of gravity, downward. Since the vertical component of the block's acceleration is zero, Newton's second law requires $F_N = mg$, where m is the mass of the block. Thus $f = \mu_k F_N = \mu_k mg$. The increase in thermal energy is given by $\Delta E_{th} = fd = \mu_k mgD$, where D is the distance the block moves before coming to rest. Using Eq. 8-29, we have

$$\Delta E_{th} = (0.25)(3.5\,\text{kg})(9.8\,\text{m/s}^2)(7.8\,\text{m}) = 67\,\text{J}.$$

(b) The block has its maximum kinetic energy K_{max} just as it leaves the spring and enters the region where friction acts. Therefore, the maximum kinetic energy equals the thermal energy generated in bringing the block back to rest, 67 J.

(c) The energy that appears as kinetic energy is originally in the form of potential energy in the compressed spring. Thus, $K_{max} = U_i = \frac{1}{2}kx^2$, where k is the spring constant and x is the compression. Thus,

$$x = \sqrt{\frac{2K_{max}}{k}} = \sqrt{\frac{2(67\,\text{J})}{640\,\text{N/m}}} = 0.46\,\text{m}$$

44. (a) The work is $W = Fd$ = (41.0 N)(2.00 m) = 82.0 J.

(b) The total amount of energy that has gone to thermal forms is (see Eq. 8-31 and Eq. 6-2)

$$\Delta E_{\text{th}} = \mu_k mgd = (0.600)(4.00\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) = 47.0\text{ J}.$$

If 40.0 J has gone to the block then (47.0 ï 40.0) J = 7.0 J has gone to the floor.

(c) Much of the work (82.0 J) has been ñwastedò due to the 47.0 J of thermal energy generated, but there still remains (82.0 ï 47.0) J = 35.0 J that has gone into increasing the kinetic energy of the block. (It has not gone into increasing the potential energy of the block because the floor is presumed to be horizontal.)

45. (a) The power generated by the engine is

$$P = Fv = (1000\text{ N})(6.000\text{ m/s}) = 6.000\times10^3\text{ W}$$

The work done in 10 minutes is then

$$W = Pt = (6.000\times10^3\text{ W})(10\text{ min})(60\text{ s/min}) = 3.600\times10^6\text{ J}.$$

(b) When the truck starts to climb the hill, the power needed to overcome the gravitational pull of the Earth is

$$P_1 = F_g v = (mg\sin\theta)v = (3000\text{ kg})(9.8\text{ m/s}^2)\sin 30°(6.000\text{ m/s}) = 8.820\times10^4\text{ W}$$

Similarly, the power needed to overcome friction is

$$P_2 = (\mu mg\cos\theta)v = (f\cos\theta)v = (1000\text{ N})\cos 30°(6.000\text{ m/s}) = 5.196\times10^3\text{ W}$$

Therefore, the total work done in 10 minutes is

$$W' = (P_1 + P_2)t = (8.820\times10^4\text{ W} + 5.196\times10^3\text{ W})(10\text{ min})(60\text{ s/min}) = 5.604\times10^7\text{ J}$$

or about 56 MJ.

(c) Adding up the results from (a) and (b), we find the total work done by engine in 20 minutes to be

$$W_{\text{tot}} = W + W' = 3.600\times10^6\text{ J} + 5.604\times10^7\text{ J} = 5.964\times10^7\text{ J}$$

or about 60 MJ.

46. We work this using English units (with g = 32 ft/s), but for consistency we convert the weight to pounds

$$mg = (9.0)\,\text{oz}\left(\frac{1\ \text{1b}}{16\,\text{oz}}\right) = 0.56\,\text{lb}$$

which implies $m = 0.018\ \text{lb}\cdot\text{s}^2/\text{ft}$ (which can be phrased as 0.018 slug as explained in Appendix D). And we convert the initial speed to feet-per-second

$$v_1 = (83.2\ \text{mi/h})\left(\frac{5280\ \text{ft/mi}}{3600\ \text{s/h}}\right) = 122\ \text{ft/s}$$

or a more ñdirectò conversion from Appendix D can be used. Equation 8-30 provides ΔE_{th} = ï ΔE_{mec} for the energy ñlostò in the sense of this problem. Thus,

$$\Delta E_{\text{th}} = \frac{1}{2}m(v_i^2 - v_f^2) + mg(y_i - y_f) = \frac{1}{2}(0.018)(122^2 - 110^2) + 0 = 25\ \text{ft}\cdot\text{lb}.$$

47. (a) The increase in potential energy of the block in going from *P* to *Q* is

$$mgh = \frac{1}{4} \times 9.8 \times (50 - 10)$$

$$= \frac{1}{4} \times 9.8 \times 40 = 98.0 \text{ J}$$

Kinetic energy at *Q* = Kinetic energy at *P* ï Increase in potential energy at *θ*
= 500 ï 98.0 = 402 J.

(b) Potential energy at *R* = Potential energy at *P* ï Decrease in potential energy at *R*:

$$R = 0 - mgh = -\frac{1}{4} \times 9.8 \times 10$$

$$= -\frac{1}{4} \times 9.8 = -24.5 \text{ J.}$$

(a) Kinetic energy at *R* = Kinetic energy at *P* ï Potential energy at
R = 500 ï (ï 24.5)
= 500 + 24.5
= 524.5 J

Let the speed at *R* be *V*. Therefore,

$$\frac{1}{2} mv^2 = 524.5$$

$$\Rightarrow v = \sqrt{\frac{(524.5) \times 2}{1/4}} = \sqrt{8 \times 524.5} \approx 64.8 \text{ m/s.}$$

(d) Change in the potential energy = Potential energy at *R* ï Potential energy at *Q*
= -24.5 - 98.0
= -122.5 J å 123 J

48. We use Eq. 8-31 to obtain

$$\Delta E_{\text{th}} = f_k d = (10\,\text{N})(5.9\,\text{m}) = 59\ \text{J}$$

and Eq. 7-8 to get

$$W = Fd = (2.0\,\text{N})(5.9\,\text{m}) = 12\ \text{J}.$$

Similarly, Eq. 8-31 gives

$$W = \Delta K + \Delta U + \Delta E_{\text{th}}$$
$$12 = 35 + \Delta U + 59$$

which yields ΔU = ï 82 J. By Eq. 8-1, then, the work done by gravity is W = ï ΔU = 82 J.

49. The initial energy of the child-seat system is

$$U_g = (m_{\text{child}} + m_{\text{seat}})gh .$$

On the other hand, the energy loss due to friction is

$$\Delta U_{\text{th}} = \mu_k F_N d = \mu_k (m_{\text{child}} + m_{\text{seat}})d ,$$

Thus, by conservation of mechanical energy, we have

$$U_g = \Delta U_{\text{th}} \quad \Rightarrow \quad (m_{\text{child}} + m_{\text{seat}})gh = \mu_k (m_{\text{child}} + m_{\text{seat}})d$$

leading to

$$d = \frac{h}{\mu_k} = \frac{7.0 \text{ m}}{0.30} = 23.3 \text{ m} \approx 23 \text{ m} .$$

50. Equation 8-33 provides ΔE_{th} = ï ΔE_{mec} for the energy ñlostò in the sense of this problem. Thus,

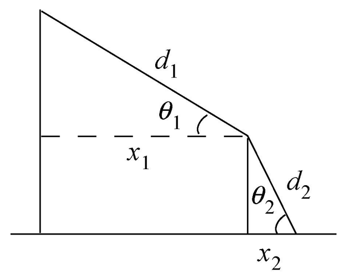
$$\begin{aligned}\Delta E_{\text{th}} &= \frac{1}{2}m(v_i^2 - v_f^2) + mg(y_i - y_f)\\ &= \frac{1}{2}(60\text{ kg})[(27\text{ m/s})^2 - (22\text{ m/s})^2] + (60\text{ kg})(9.8\text{ m/s}^2)(14\text{ m})\\ &= 1.6\times10^4\text{ J.}\end{aligned}$$

That the angle of 25° is nowhere used in this calculation is indicative of the fact that energy is a scalar quantity.

51. (a) By energy conservation, we have $K_i + U_i = f_k d + K_f + U_f$. The initial potential energy is $U_i = mgH$, and the frictional force is $f_k = \mu_k F_N = \mu_k mg\cos\theta$. The final kinetic energy of the block is $K_f = \frac{1}{2}mv_1^2$, which is equal to

$$K_f = U_i - f_k d = mgH - (\mu mg\cos\theta)d = mgH - \mu mg(5.0\text{ m}) = mg\left(H - 5.0\mu\right)$$

(b) In this case, the thermal energy due to friction is

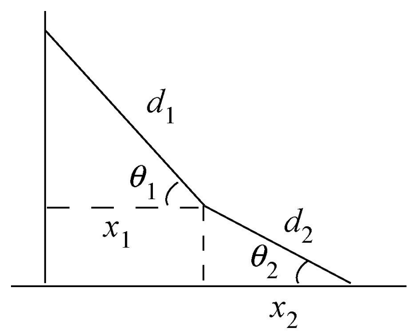


$$\begin{aligned}\Delta E_{\text{th}} &= f_{1k}d_1 + f_{2k}d_2 = (\mu mg\cos\theta_1)d_1 + (\mu mg\cos\theta_2)d_2 \\ &= \mu mg(d_1\cos\theta_1 + d_2\cos\theta_2) = \mu mg(x_1 + x_2) \\ &= \mu mg\left(5.0\text{ m}\right)\end{aligned}$$

Therefore,

$$K_f = U_i - \Delta E_{\text{th}} = mgH - \mu mg(5.0\text{ m}) = mg\left(H - 5.0\mu\right)$$

(c) Similarly, in this case, the thermal energy due to friction is



$$\begin{aligned}\Delta E_{\text{th}} &= f_{1k}d_1 + f_{2k}d_2 = (\mu mg\cos\theta_1)d_1 + (\mu mg\cos\theta_2)d_2 \\ &= \mu mg(d_1\cos\theta_1 + d_2\cos\theta_2) = \mu mg(x_1 + x_2) \\ &= \mu mg\left(5.0\text{ m}\right)\end{aligned}$$

and

$$K_f = U_i - \Delta E_{\text{th}} = mgH - \mu mg(5.0\text{ m}) = mg\left(H - 5.0\mu\right)$$

52. (a) An appropriate picture (once friction is included) for this problem is Figure 8-3 in the textbook. We apply Eq. 8-31, $\Delta E_{\text{th}} = f_k\, d$, and relate initial kinetic energy K_i to the "resting" potential energy U_r:

$$K_i + U_i = f_k d + K_r + U_r \quad \Rightarrow \quad 20.0\text{ J} + 0 = f_k d + 0 + \frac{1}{2}kd^2$$

where f_k = 10.0 N and k = 360 N/m. We solve the equation for d using the quadratic formula:

$$d = \frac{-f_k + \sqrt{f_k^2 + 2kK_i}}{k} = \frac{-(10.0\text{ N}) + \sqrt{(10.0\text{ N})^2 + 2(360\text{ N/m})(20.0\text{ J})}}{360\text{ N/m}} = 0.307\text{ m}$$

(b) The total change in thermal energy as the cookie slides back is

$$\Delta E_{\text{th}} = 2f_k\, d = 2(10.0\text{ N})(0.307\text{ m}) = 6.1\text{ J}.$$

Thus, the kinetic energy of the cookie is

$$K_f = K_i - \Delta E_{\text{th}} = 20.0\text{ J} - 6.1\text{ J} = 13.9\text{ J}.$$

53. To swim at constant velocity the swimmer must push back against the water with a force of 110 N. Relative to him the water is going at 0.22 m/s toward his rear, in the same direction as his force. Using Eq. 7-48, his power output is obtained:

$$P = \vec{F} \cdot \vec{v} = Fv = (110\,\mathrm{N})(0.22\,\mathrm{m/s}) = 24\,\mathrm{W}.$$

54. (a) Using the force analysis shown in Chapter 6, we find the normal force $F_N = mg\cos\theta$ (where mg = 267 N, or m = 27.2 kg) which means

$$f_k = \mu_k F_N = \mu_k mg \cos \theta.$$

Thus, Eq. 8-31 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgd \cos\theta = (0.10)(267 \text{ N})(6.5 \text{ m})\cos 20° = 1.63\times10^2 \text{ J}$$

or 1.6×10^2 J, to two significant digits.

(b) The potential energy change is

$$\Delta U = mg(\text{ï}\, d \sin \theta) = (267 \text{ N})(\text{ï}\, 6.5 \text{ m}) \sin 20° = \text{ï}\, 5.94 \times 10^2 \text{ J}.$$

The initial kinetic energy is

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}\left(27.2 \text{ kg}\right)(0.457 \text{ m/s})^2 = 2.85 \text{ J}.$$

Therefore, using Eq. 8-33 (with W = 0), the final kinetic energy is

$$K_f = K_i - \Delta U - \Delta E_{th} = 2.85 \text{ J} - (-5.94\times10^2 \text{ J}) - 1.63\times10^2 \text{ J} = 4.33\times10^2 \text{ J}.$$

Consequently, the final speed is

$$v_f = \sqrt{2K_f / m} = \sqrt{2(4.33\times10^2 \text{ J})/27.2 \text{ kg}} = 5.6 \text{ m/s}.$$

55. When kinetic energy is maximum, the velocity is maximum and the potential energy is minimum. Given that kinetic energy + Potential energy = 5.00 J, therefore,

KE_{max} PE_{min} = 5.00J (1)

Thus, the total mechanical energy remains same. We have $U(x) = x^4$ ï $2.00x^2$. Therefore,

$U'(x)$ ï 0 ⇒ $4x^3$ ï 4x = 0
⇒ $4x(x^2$ ï 1) = 0
⇒ $x = 0, x = \pm 1$

and

$U''(x) = 12x^2$ ï 4 $\quad$ $U''(0)$ = ï 4 < 0
$U''(\pm 1) = 12x^2$ ï 4 >0

Therefore, potential energy is minimum at ±1.

Minimum potential energy = $(\pm 1)^4 - 2(\pm 1)^2 = 1 - 2 = -1$ J.

From Eq. (1), we get

KE_{max} + (ï 1) = 5.00 J

Therefore,

KE_{max} = 6.00 J

$$\frac{1}{2}m(v_{max})^2 = 6.00$$

$$(v_{max})^2 = \frac{6.00 \times 2.00}{2} = 6.00$$

Therefore, the maximum velocity acquired by the particle is

$$v_{max} = \pm\sqrt{6} \approx \pm 2.45\,\mathrm{m/s}.$$

56. Energy conservation, as expressed by Eq. 8-33 (with $W = 0$) leads to

$$\Delta E_{\text{th}} = K_i - K_f + U_i - U_f \;\Rightarrow\; f_k d = 0 - 0 + \frac{1}{2}kx^2 - 0$$

With $f_k = \mu_k mg,$ the coefficient of kinetic friction is

$$\mu_k = \frac{kx^2}{2mgd} = \frac{(170\,\text{N/m})(0.12\,\text{m})^2}{2(2.0\,\text{kg})(9.8\,\text{m/s}^2)(0.75\,\text{m})} = 0.083\,.$$

57. (a) While going up the incline, the work done by the gravitational force is

$$W_1 = \vec{F}_g \cdot \vec{d} = -(mg\sin\theta)d = -mgh = -(6.0\text{ kg})(9.8\text{ m/s}^2)(5.0\text{ m}) = -294\text{ J}.$$

Similarly, when coming down the incline, the work done by the gravitational force is

$$W_2 = \vec{F}_g \cdot \vec{d} = +(mg\sin\theta)d = +mgh = +(6.0\text{ kg})(9.8\text{ m/s}^2)(5.0\text{ m}) = +294\text{ J}$$

Therefore, the total work done is ï 294 + 294 = 0 J, which confirms that that the gravitation is a conservative force field.

(b) The angle the incline makes with the horizontal is

$$\theta = \sin^{-1}(h/d) = \sin^{-1}(5.0\text{ m}/10.0\text{ m}) = \sin^{-1}(1/2) = 30°.$$

The work done by the man against both the gravity and the friction is

$$\begin{aligned} W &= (mg\sin\theta + \mu_k mg\cos\theta)d = mgd\cos\theta(\tan\theta + \mu_k) \\ &= (6.0\text{ kg})(9.8\text{ m/s}^2)(10\text{ m})\cos 30°\left(\tan 30° + 0.40\right) = 498\text{ J} \end{aligned}$$

or about $5.0\times10^2\text{ J}$, to two significant digits.

(c) Since friction in a non-conservative force, it opposes the motion. The total energy dissipated due to friction while going up and coming down is

$$\begin{aligned} \Delta U_{\text{th}} &= 2\mu_k F_N d = 2\mu_k(mg\cos\theta)d \\ &= 2(0.40)(6.0\text{ kg})(9.8\text{ m/s}^2)\cos 30°(10\text{ m}) = 407\text{ J} \end{aligned}$$

or about $4.1\times10^2\text{ J}$, to two significant digits.

(d) Using Newtonôs second law, we have

$$mg\sin\theta - \mu_k mg\cos\theta = ma \quad \Rightarrow \quad a = g\sin\theta(1-\mu_k\cot\theta)$$

Substituting the values given, we find the acceleration to be

$$a = g\sin\theta(1-\mu_k\cot\theta) = (9.8\text{ m/s}^2)\sin 30°\left[1-(0.40)\cot 30°\right] = 1.51\text{ m/s}^2$$

Using $v^2 = v_0^2 + 2ad$, we find the speed of the block at the bottom of the incline to be

$$v = \sqrt{v_0^2 + 2ad} = \sqrt{0 + 2(1.51\text{ m/s}^2)(10\text{ m})} = 5.49\text{ m/s} \approx 5.5\text{ m/s}.$$

58. This can be worked entirely by the methods of Chapters 2ï 6, but we will use energy methods in as many steps as possible.

(a) By a force analysis of the style done in Chapter 6, we find the normal force has magnitude $F_N = mg\cos\theta$ (where $\theta = 40°$), which means $f_k = \mu_k F_N = \mu_k mg\cos\theta$ where $\mu_k = 0.15$. Thus, Eq. 8-31 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgd\cos\theta.$$

Also, elementary trigonometry leads us to conclude that $\Delta U = mgd\sin\theta$. Eq. 8-33 (with $W = 0$ and $K_f = 0$) provides an equation for determining d:

$$K_i = \Delta U + \Delta E_{\text{th}} \quad\Rightarrow\quad \frac{1}{2}mv_i^2 = mgd(\sin\theta + \mu_k\cos\theta)$$

where $v_i = 1.4\ \text{m/s}$. Dividing by mass and rearranging, we obtain

$$d = \frac{v_i^2}{2g(\sin\theta + \mu_k\cos\theta)} = 0.13\,\text{m}.$$

(b) Now that we know where on the incline it stops ($d' = 0.13 + 0.45 = 0.58$ m from the bottom), we can use Eq. 8-33 again (with $W = 0$ and now with $K_i = 0$) to describe the final kinetic energy (at the bottom):

$$K_f = -\Delta U - \Delta E_{\text{th}}$$
$$\frac{1}{2}mv^2 = mgd'(\sin\theta - \mu_k\cos\theta)$$

which ð after dividing by the mass and rearranging ð yields

$$v = \sqrt{2gd'(\sin\theta - \mu_k\cos\theta)d} = 2.45\,\text{m/s}$$

or about 2.5 m/s.

(c) In part (a) it is clear that d increases if μ_k decreases ð both mathematically (since it is a positive term in the denominator) and intuitively (less friction ð less energy ñlostò). In part (b), there are two terms in the expression for v that imply that it should increase if μ_k were smaller: the increased value of $d' = d_0 + d$ and that last factor $\sin\theta$ ï $\mu_k\cos\theta$, which indicates that less is being subtracted from $\sin\theta$ when μ_k is less (so the factor itself increases in value).

59. (a) The maximum height reached is h. The thermal energy generated by air resistance as the stone rises to this height is $\Delta E_{th} = fh$ by Eq. 8-31. We use energy conservation in the form of Eq. 8-33 (with $W = 0$):

$$K_f + U_f + \Delta E_{\text{th}} = K_i + U_i$$

and we take the potential energy to be zero at the throwing point (ground level). The initial kinetic energy is $K_i = \frac{1}{2}mv_0^2$, the initial potential energy is $U_i = 0$, the final kinetic energy is $K_f = 0$, and the final potential energy is $U_f = wh$, where $w = mg$ is the weight of the stone. Thus, $wh + fh = \frac{1}{2}mv_0^2$, and we solve for the height:

$$h = \frac{mv_0^2}{2(w+f)} = \frac{v_0^2}{2g(1+f/w)}.$$

Numerically, we have, with $m = (5.29\text{ N})/(9.80\text{ m/s}^2) = 0.54$ kg,

$$h = \frac{(20.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)(1+0.265/5.29)} = 19.4\text{ m}.$$

(b) We notice that the force of the air is downward on the trip up and upward on the trip down, since it is opposite to the direction of motion. Over the entire trip the increase in thermal energy is $\Delta E_{th} = 2fh$. The final kinetic energy is $K_f = \frac{1}{2}mv^2$, where v is the speed of the stone just before it hits the ground. The final potential energy is $U_f = 0$. Thus, using Eq. 8-31 (with $W = 0$), we find

$$\frac{1}{2}mv^2 + 2fh = \frac{1}{2}mv_0^2.$$

We substitute the expression found for h to obtain

$$\frac{2fv_0^2}{2g(1+f/w)} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

which leads to

$$v^2 = v_0^2 - \frac{2fv_0^2}{mg(1+f/w)} = v_0^2 - \frac{2fv_0^2}{w(1+f/w)} = v_0^2\left(1 - \frac{2f}{w+f}\right) = v_0^2\,\frac{w-f}{w+f}$$

where w was substituted for mg and some algebraic manipulations were carried out. Therefore,

$$v = v_0\sqrt{\frac{w-f}{w+f}} = (20.0\text{ m/s})\sqrt{\frac{5.29\text{ N} - 0.265\text{ N}}{5.29\text{ N} + 0.265\text{ N}}} = 19.0\text{ m/s}.$$

60. We look for the distance along the incline d, which is related to the height ascended by $\Delta h = d \sin \theta$. By a force analysis of the style done in Chapter 6, we find the normal force has magnitude $F_N = mg \cos\theta$, which means $f_k = \mu_k mg \cos\theta$. Thus, Eq. 8-33 (with W = 0) leads to

$$\begin{aligned} 0 &= K_f - K_i + \Delta U + \Delta E_{\text{th}} \\ &= 0 - K_i + mgd \sin\theta + \mu_k mgd \cos\theta \end{aligned}$$

which leads to

$$d = \frac{K_i}{mg(\sin\theta + \mu_k \cos\theta)} = \frac{150}{(4.0)(9.8)(\sin 30 + .36 \cos 30)} = 4.7 \text{ m}$$

61. Energy conservation gives $mgh = \frac{1}{2}k(\Delta y)^2$, where m = 10 kg is the mass of the block, k is the spring constant, and Δy is the maximum compression. On the Moon, using energy conservation, we have

$$Mg_{\text{moon}}h_{\text{moon}} = \tfrac{1}{2}k(\Delta y)^2,$$

where M = 50 kg is the mass of the astronaut, h_{moon} is the maximum height the astronaut has reached, and $g_{\text{moon}} = g/6$. Thus,

$$\frac{1}{2}k(\Delta y)^2 = Mg_{\text{moon}}h_{\text{moon}} = mgh$$

Solving for h_{moon}, we obtain

$$h_{\text{moon}} = \left(\frac{m}{M}\right)\left(\frac{g}{g_{\text{moon}}}\right)h = \left(\frac{10.0\text{ kg}}{50.0\text{ kg}}\right)(6)(30.0\text{ m}) = 36.0\text{ m}.$$

62. We will refer to the point where it first encounters the ñrough regionò as point C (this is the point at a height h above the reference level). From Eq. 8-17, we find the speed it has at point C to be

$$v_C = \sqrt{v_A^2 - 2gh} = \sqrt{(8.0)^2 - 2(9.8)(2.0)} = 4.980 \approx 5.0 \text{ m/s}.$$

Thus, we see that its kinetic energy right at the beginning of its ñrough slideò (heading uphill towards B) is

$$K_C = \frac{1}{2} m(4.980 \text{ m/s})^2 = 12.4m$$

(with SI units understood). Note that we ñcarry alongò the mass (as if it were a known quantity); as we will see, it will cancel out, shortly. Using Eq. 8-37 (and Eq. 6-2 with $F_N = mg\cos\theta$) and $y = d\sin\theta$, we note that if $d < L$ (the block does not reach point B), this kinetic energy will turn entirely into thermal (and potential) energy

$$K_C = mgy + f_k d \quad \Rightarrow \quad 12.4m = mgd\sin\theta + \mu_k mgd\cos\theta.$$

With $\mu_k = 0.40$ and $\theta = 30^\circ$, we find $d = 1.49$ m, which is greater than L (given in the problem as 0.65 m), so our assumption that $d < L$ is incorrect. What is its kinetic energy as it reaches point B? The calculation is similar to the above, but with d replaced by L and the final v^2 term being the unknown (instead of assumed zero):

$$\frac{1}{2} m v^2 = K_C - (mgL\sin\theta + \mu_k mgL\cos\theta) .$$

This determines the speed with which it arrives at point B:

$$\begin{aligned} v_B &= \sqrt{v_C^2 - 2gL(\sin\theta + \mu_k \cos\theta)} \\ &= \sqrt{(4.98 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(0.65 \text{ m})(\sin 30^\circ + 0.4\cos 30^\circ)} = 3.7 \text{ m/s}. \end{aligned}$$

63. We observe that the last line of the problem indicates that static friction is not to be considered a factor in this problem. The friction force of magnitude f = 4400 N mentioned in the problem is kinetic friction and (as mentioned) is constant (and directed upward), and the thermal energy change associated with it is $\Delta E_{\text{th}} = fd$ (Eq. 8-31) where d = 3.7 m in part (a) (but will be replaced by x, the spring compression, in part (b)).

(a) With W = 0 and the reference level for computing $U = mgy$ set at the top of the (relaxed) spring, Eq. 8-33 leads to

$$U_i = K + \Delta E_{\text{th}} \Rightarrow v = \sqrt{2d\left(g - \frac{f}{m}\right)}$$

which yields $v = 7.4\,\text{m/s}$ for m = 1800 kg.

(b) We again utilize Eq. 8-33 (with W = 0), now relating its kinetic energy at the moment it makes contact with the spring to the system energy at the bottom-most point. Using the same reference level for computing $U = mgy$ as we did in part (a), we end up with gravitational potential energy equal to mg(ï x) at that bottom-most point, where the spring (with spring constant $k = 1.5 \times 10^5\ \text{N/m}$) is fully compressed.

$$K = mg(-x) + \frac{1}{2}kx^2 + fx$$

where $K = \frac{1}{2}mv^2 = 4.9 \times 10^4$ J using the speed found in part (a). Using the abbreviation $\xi = mg$ ï $f = 1.3 \times 10^4$ N, the quadratic formula yields

$$x = \frac{\xi \pm \sqrt{\xi^2 + 2kK}}{k} = 0.90\,\text{m}$$

where we have taken the positive root.

(c) We relate the energy at the bottom-most point to that of the highest point of rebound (a distance d' above the relaxed position of the spring). We assume $d' > x$. We now use the bottom-most point as the reference level for computing gravitational potential energy.

$$\frac{1}{2}kx^2 = mgd' + fd' \Rightarrow d' = \frac{kx^2}{2(mg + d)} = 2.8\,\text{m}.$$

(d) The non-conservative force (§8-1) is friction, and the energy term associated with it is the one that keeps track of the total distance traveled (whereas the potential energy terms, coming as they do from conservative forces, depend on positions ð but not on the paths that led to them). We assume the elevator comes to final rest at the equilibrium position of the spring, with the spring compressed an amount d_{eq} given by

$$mg = kd_{\text{eq}} \Rightarrow d_{\text{eq}} = \frac{mg}{k} = 0.12\,\text{m}.$$

In this part, we use that final-rest point as the reference level for computing gravitational potential energy, so the original $U = mgy$ becomes $mg(d_{\text{eq}} + d)$. In that final position, then, the gravitational energy is zero and the spring energy is $kd_{\text{eq}}^2/2$. Thus, Eq. 8-33 becomes

$$mg\left(d_{\text{eq}} + d\right) = \frac{1}{2}kd_{\text{eq}}^2 + fd_{\text{total}}$$

$$(1800)(9.8)(0.12 + 3.7) = \frac{1}{2}\left(1.5\times10^5\right)(0.12)^2 + (4400)\,d_{\text{total}}$$

which yields $d_{\text{total}} = 15$ m.

64. In the absence of friction, we have a simple conversion (as it moves along the inclined ramps) of energy between the kinetic form (Eq. 7-1) and the potential form (Eq. 8-9). Along the horizontal plateaus, however, there is friction that causes some of the kinetic energy to dissipate in accordance with Eq. 8-31 (along with Eq. 6-2 where μ_k = 0.50 and $F_N = mg$ in this situation). Thus, after it slides down a (vertical) distance d it has gained $K = \frac{1}{2}mv^2 = mgd$, some of which ($\Delta E_{th} = \mu_k\, mgd$) is dissipated, so that the value of kinetic energy at the end of the first plateau (just before it starts descending towards the lowest plateau) is

$$K = mgd - \mu_k mgd = \frac{1}{2}mgd\ .$$

In its descent to the lowest plateau, it gains $mgd/2$ more kinetic energy, but as it slides across it ñlosesò $\mu_k\, mgd/2$ of it. Therefore, as it starts its climb up the right ramp, it has kinetic energy equal to

$$K = \frac{1}{2}mgd + \frac{1}{2}mgd - \frac{1}{2}\mu_k mgd = \frac{3}{4}mgd\ .$$

Setting this equal to Eq. 8-9 (to find the height to which it climbs) we get $H = ¾d$. Thus, the block (momentarily) stops on the inclined ramp at the right, at a height of

$$H = 0.75d = 0.75\ (40\ \text{cm}) = 30\ \text{cm}$$

measured from the lowest plateau.

65. The initial and final kinetic energies are zero, and we set up energy conservation in the form of Eq. 8-33 (with W = 0) according to our assumptions. Certainly, it can only come to a permanent stop somewhere in the flat part, but the question is whether this occurs during its first pass through (going rightward) or its second pass through (going leftward) or its third pass through (going rightward again), and so on. If it occurs during its first pass through, then the thermal energy generated is $\Delta E_{\text{th}} = f_k d$ where $d \le L$ and $f_k = \mu_k mg$. If it occurs during its second pass through, then the total thermal energy is $\Delta E_{\text{th}} = \mu_k mg(L + d)$ where we again use the symbol d for how far through the level area it goes during that last pass (so $0 \le d \le L$). Generalizing to the n^{th} pass through, we see that

$$\Delta E_{\text{th}} = \mu_k mg[(n - 1)L + d].$$

In this way, we have

$$mgh = \mu_k mg\big((n-1)L + d\big)$$

which simplifies (when $h = L/2$ is inserted) to

$$\frac{d}{L} = 1 + \frac{1}{2\mu_k} - n.$$

The first two terms give $1 + 1/2\mu_k = 3.5,$ so that the requirement $0 \le d/L \le 1$ demands that n = 3. We arrive at the conclusion that $d/L = \frac{1}{2}$, or

$$d = \frac{1}{2}L = \frac{1}{2}(40\ \text{cm}) = 20\ \text{cm}$$

and that this occurs on its third pass through the flat region.