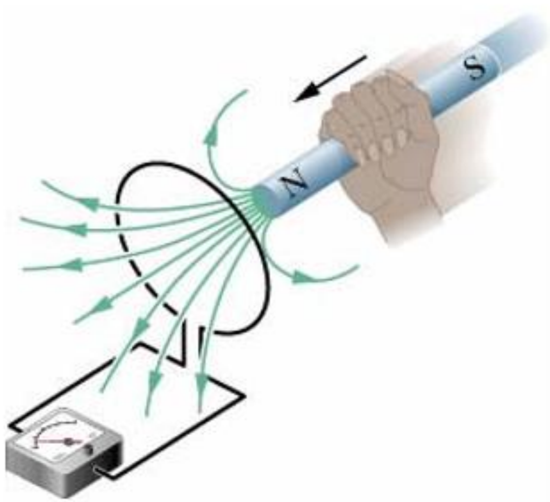
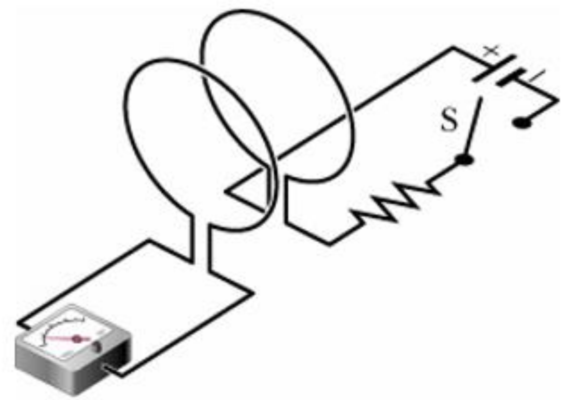
Chapter 30
Induction and Inductance

Chap. 30-1 Faraday's Law and Lenz's Law

Faraday의 두 가지 실험



도선 주위에서 자석을 움직여
자기장을 변화시킴



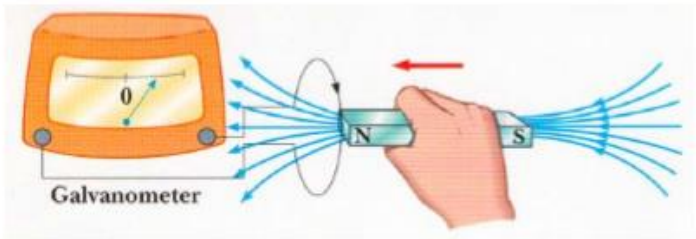
자석 대신 다른 도선 고리를 두고,
전류를 조절하여 자기장을 변화시킴

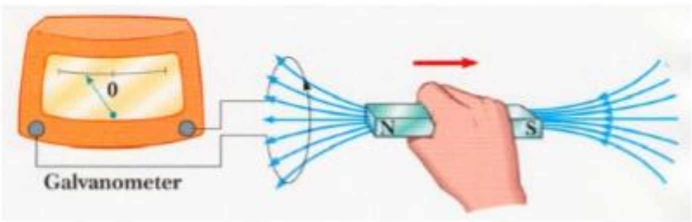
➡ **자기유도 (Magnetic induction)**

➡ **유도전류 (유도기전력) 발생**

Chap. 30-1 Faraday's Law and Lenz's Law

Faraday의 유도효과





도선 고리를 지나는 자기력선의 수가 시간에 따라 변하면 고리에 기전력(ε: electromotive force)이 생긴다

"Electric current is induced by the change of the Magnetic Flux"

Magnetic Flux $\Phi_B = \int B \cdot d\vec{A}$ *[Unit] weber (Wb),* $1\ Wb = 1\ T\cdot m^2$

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Faraday 법칙

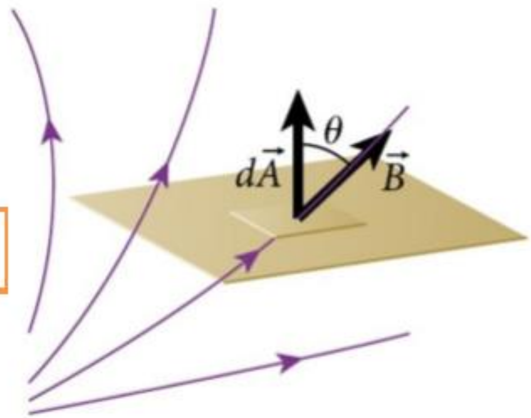
Chap. 30-1 Faraday's Law and Lenz's Law

Magnetic flux

- **자기다발**

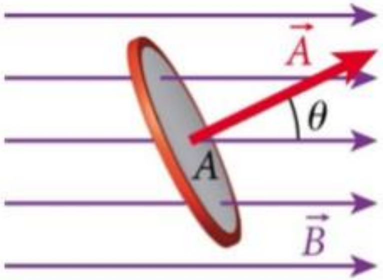
$$\Phi_B = \int_A \vec{B} \cdot d\vec{A}$$

특정한 곡면에 대해 정의된다.



- 균일한 자기장에 놓인 평면의 경우

$$\Phi_B = BA\cos\theta$$



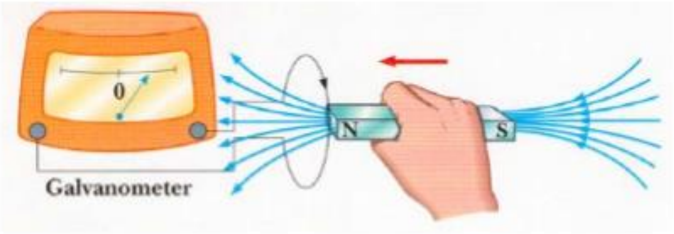
- 자기다발의 단위 : Wb (weber) $[\Phi_B] = [B][A] = \mathrm{T\,m^2} \equiv \mathrm{Wb}$

Chap. 30-1 Faraday's Law and Lenz's Law

Faraday의 유도법칙

고리를 지나는 자기력선의 수를 바꾸는 방법

$$\Phi_B = \int B \cdot d\vec{A}$$

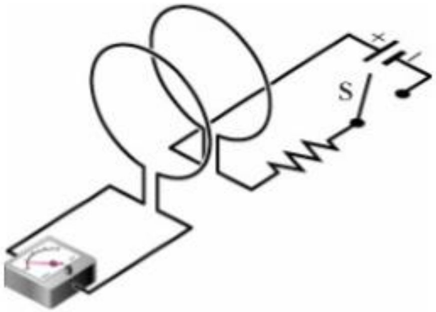


▸코일 속 자기장 세기 ***B*** 를 바꾼다. **(*B*)**

▸자기장 속에서의 코일의 넓이를 바꾼다. **(*A*)**

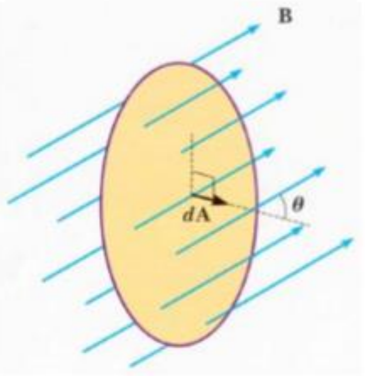
▸자기장의 방향에 대한 코일의 방향을 바꾼다. **(θ).**

▸위 셋의 복합.



For N loops $\qquad \mathcal{E} = -N\dfrac{d\Phi_B}{dt}$

$$\mathcal{E} = -\frac{d}{dt}(B \cdot A\cos\theta)$$



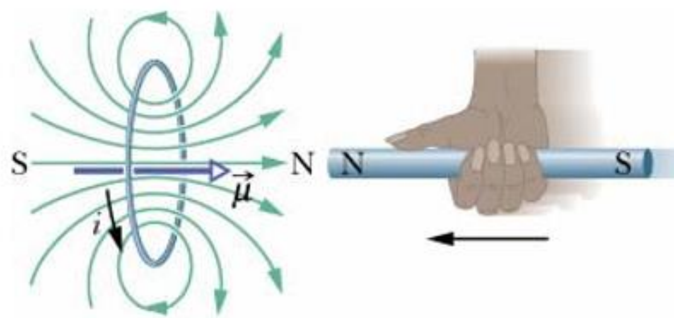
Chap. 30-1 Faraday's Law and Lenz's Law

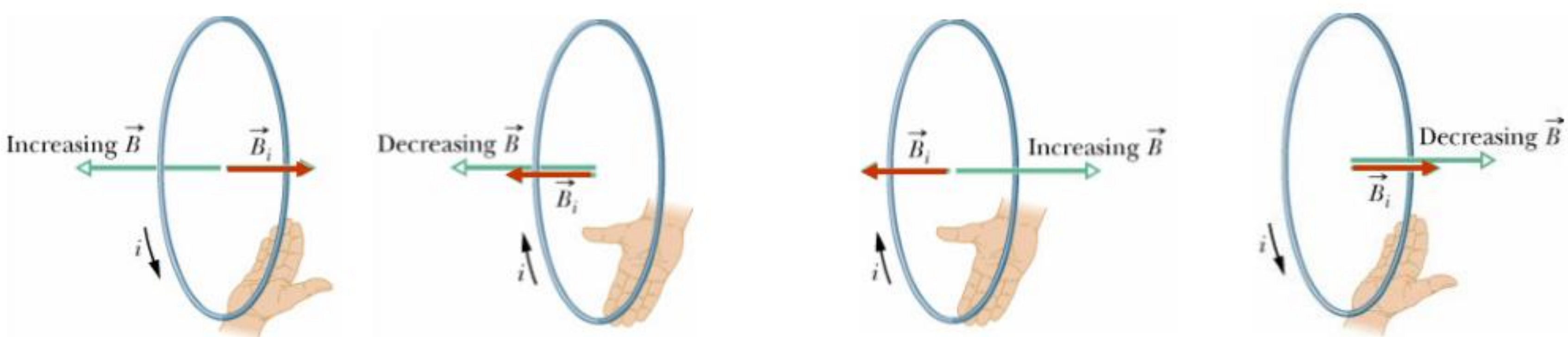
Lenz 법칙

The magnetic field due to the induced current has opposite sign to the change in the magnetic flux.

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

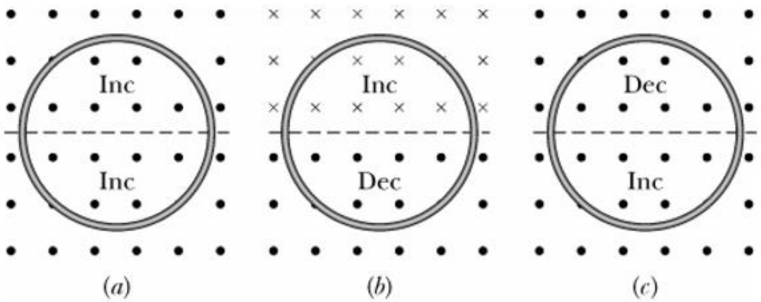
자기유도로 생기는 전류는 자기다발의 변화를 방해하는 방향으로 흐른다.





Chap. 30-1 Faraday's Law and Lenz's Law

확인문제 2.



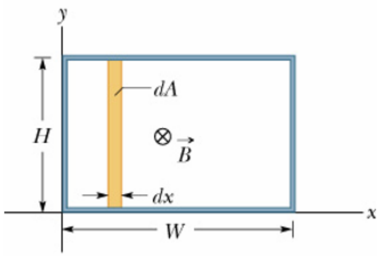
자기장이 일정한 비율로
증가/감소할 때,
고리에 유도되는 전류의
크기가 큰 순서는?

a = b > c=0

보기문제 30-3.

$$\mathrm{B} = -(4t^2x^2)\hat{z}$$

W = 3.0 m, H = 2.0 m



***t* = 0.10s** 에서 전류고리에 생기는 유도기전력의 크기와 방향은?

$$emf = -\frac{d\Phi_B}{dt}$$

$$\Phi_B = \int \mathrm{B}\cdot d\mathrm{A} = \int_0^H\int_0^W(-4t^2x^2\,\hat{\mathrm{k}})\cdot(dx\,dy\,\hat{\mathrm{k}})$$

$$= -4t^2\int_0^H dy\int_0^W x^2dx$$

$$= -\frac{4}{3}t^2HW^3,\ (H=2.0,\ W=3.0)$$

$$= -72t^2$$

$$\therefore\ \mathrm{emf} = -\frac{d}{dt}(-72t^2) = 144\,t$$

크기: emf (*t* = 0.10s) = 14.4 V

방향: 반시계 방향

Chap. 30-2 Induction and Energy Transfer

힘 F 가 한 일률 (power)

$$\Phi_B = BLx$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -BL\frac{dx}{dt} = -BLv$$

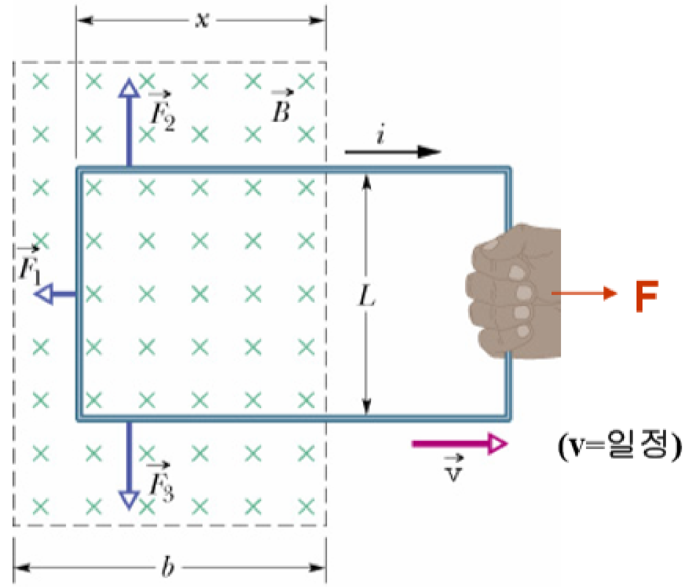
$$i = \frac{|\mathcal{E}|}{R} = \frac{BLv}{R}$$

$$F_m = i\vec{L}\times\vec{B} = iL\cdot B = \frac{B^2L^2v}{R}$$

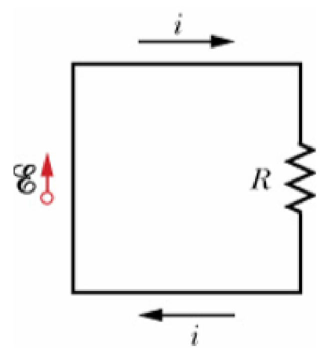
$$\vec{F}_1 = -\vec{F} \qquad \vec{F}_2 = -\vec{F}_3$$

$$\therefore F = \frac{B^2L^2v}{R}$$

일률 (Power): $P = Fv = \dfrac{B^2L^2v^2}{R}$



전류 i 에 의한 열에너지 방출률 (power)



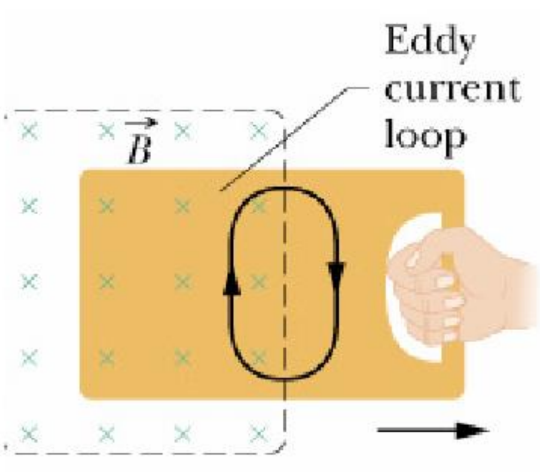
$$P = i^2R = \frac{B^2L^2v^2}{R}$$

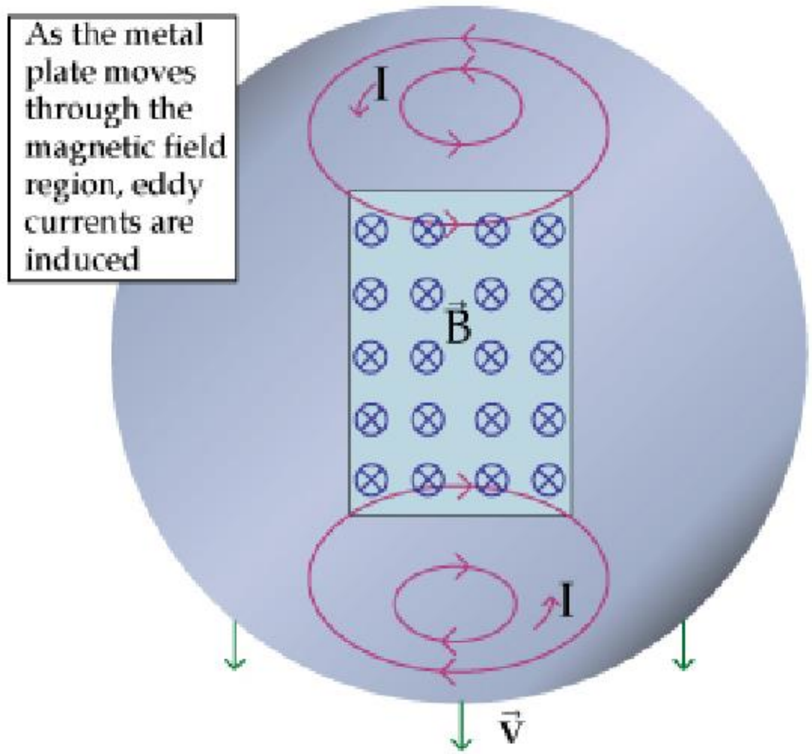
전류고리를 당길 때
힘 F가 한일은 모두
열에너지로 변환된다.

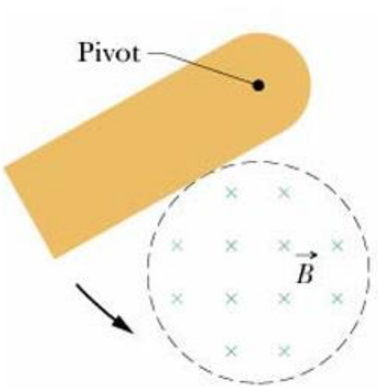
Chap. 30-2 Induction and Energy Transfer

소용돌이 전류 (Eddy current, or Foucault current)

http://www.ndt-ed.org/EducationResources/HighSchool/Electricity/eddycurrents.htm





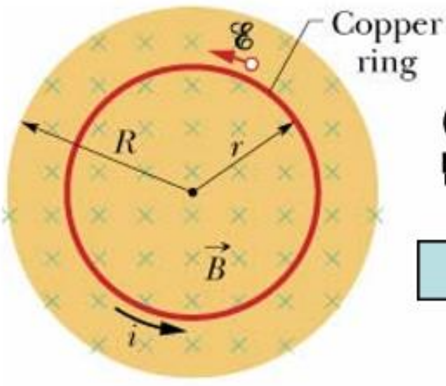


Roller coaster의 braking

Vending machines (detection of coins)

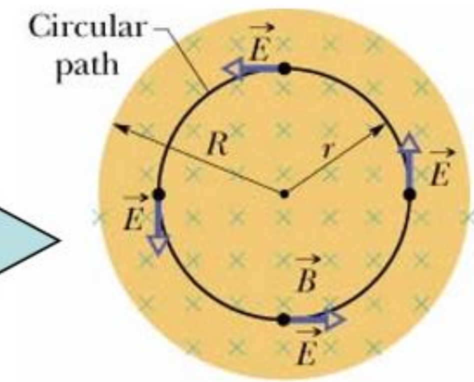
Chap. 30-3 Induced Electric Field

유도 전기장



(Faraday 법칙)
변하는 자기장은 전류고리에 유도전류를 만든다

변하는 자기장은 전기장을 만든다



Assume the magnetic field changes in time $\varepsilon = -\dfrac{d\Phi_B}{dt} \neq 0$

$W = \varepsilon\, q_0$: 전하 q_0 가 한 바퀴 돌 때 유도기전력이 한 일

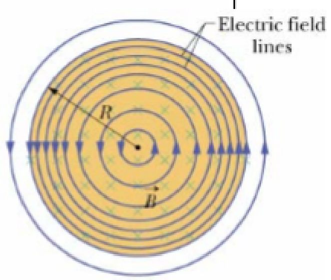
$W = \oint \vec{F} \cdot d\vec{s} = q_0 \oint \vec{E} \cdot d\vec{s}$: 전하 q_0 가 한 바퀴 돌 때 유도전기장이 한 일

⇨ $\varepsilon = \oint \vec{E} \cdot d\vec{s}$ ➡ $\boxed{\varepsilon = \oint \vec{E} \cdot d\vec{s} = -\dfrac{d\Phi_B}{dt}}$ *Faraday's Law*

Chap. 30-3 Induced Electric Field

정전기장(static E)과 유도전기장 (Induced E)이 다른 점

	정전기장	유도전기장
근원	• 전하 (쿨롱 법칙) $\oint_S \mathrm{E}_{전하} \cdot d\mathrm{A} = \frac{q}{\varepsilon_0}$	• 자기장의 변화 (파라데이 법칙) $\oint_C \mathrm{E}_{유도} \cdot d\mathrm{s} = -\frac{d\Phi_B}{dt}$
전위	• 정의됨 $\oint_C \mathrm{E}_{전하} \cdot d\mathrm{s} = 0$ $\Rightarrow V = -\int_{r_{ref}}^{r} \mathrm{E}_{전하} \cdot d\mathrm{s}$	• 정의할 수 없음 $\oint_C \mathrm{E}_{유도} \cdot d\mathrm{s} \neq 0$



양(+) 전하에서 시작하여
음(-) 전하에서 끝남.

경로에 무관 : **conservative force**

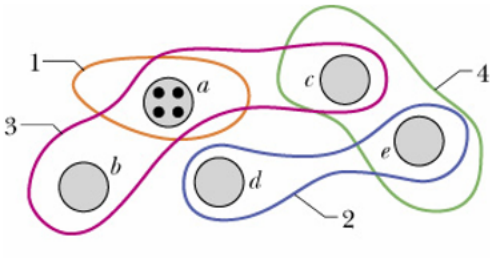
닫힌 고리에서는 **V = 0** 임.

닫힌 고리임에도 불구하고 변하는 자가
다발에서는 **V=0** 이 아님.

경로에 의존 : **nonconservative force**

→유도전기장에서는 전기퍼텐셜을
정의할 수 없다.

확인문제 4.



$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s}$$

경로 : **(1) ε, (2) 2 ε, (3) 3 ε, (4) 0** 일 때, 각 구역에 걸린 자기장의 방향은?

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(4) c 와 **e** 는 반대 방향, **(2) d**와 **e** 는 같은 방향, **(3) b**와 **c**는 **a**와 같은 방향

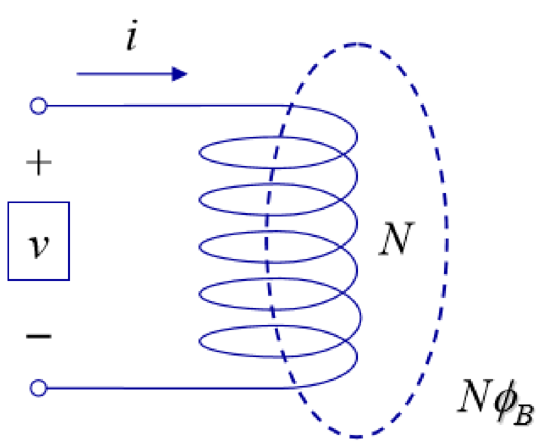
a : out

b, c : out
d, e : into

Chap. 30-4 Inductors and Inductance

유도기

- An inductor is a two-terminal device that consists of
  a coiled conducting wire wound around a core
- A current flowing through the device produces
  a magnetic flux ϕ_B forms closed loops threading its coils
- Total flux linked by N turns of coils, total flux is $\Phi = N\phi_B$
- For a linear inductor, Φ *is proportional to* i
  $\Phi = Li$ ➔ $L = \Phi / i$
- L is the inductance
- Unit: Henry (H) or (V•s/A)



유도용량

인덕터의 유도용량 (Inductance)

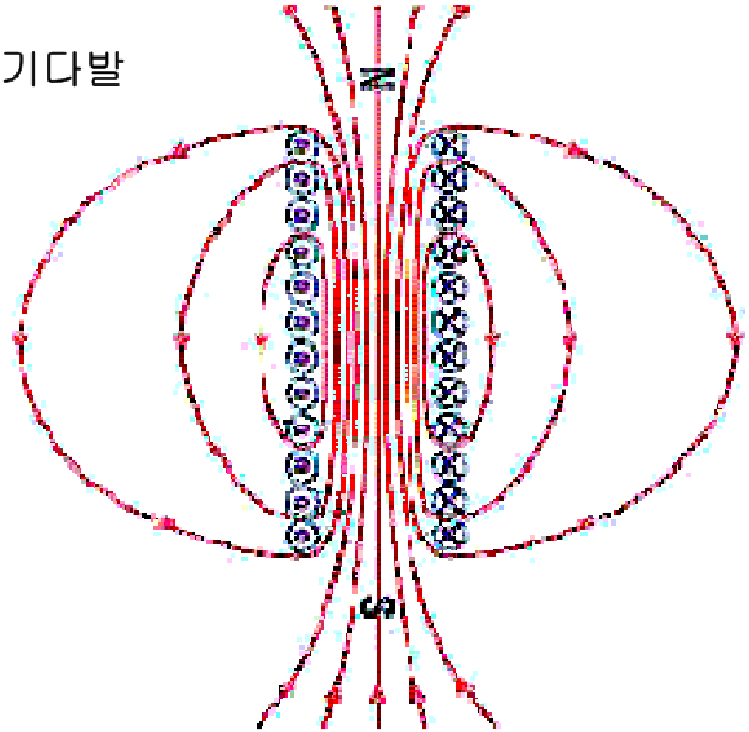
$$L \equiv \frac{N\Phi_B}{i}$$ [1 Henry = 1 H = 1 T·m²/A]

솔레노이드의 중심 부근 길이 l 인 부분의 자기다발

$$N\Phi_B = (nl)BA = (nl)(\mu_0 in)A$$

$$L \equiv \frac{N\Phi_B}{i} = \mu_0 n^2 lA$$

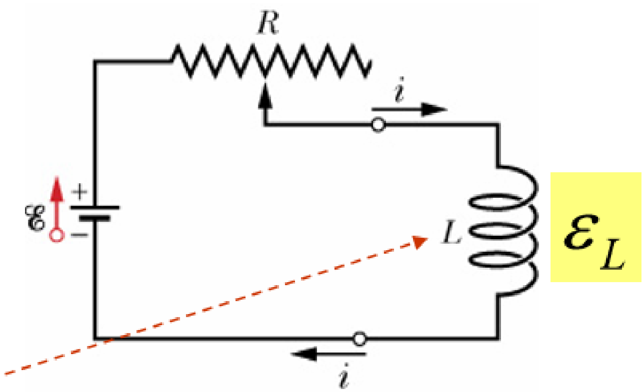
$$\boxed{\frac{L}{l} = \mu_0 n^2 A}$$: 단위 길이당 유도용량



Chap. 30-5 Self-Induction

자체 유도

B-field: $B = \mu_0 n i$



자체유도 기전력 (ε_L) :

$$\varepsilon_L = -\frac{d(N\Phi_B)}{dt}$$

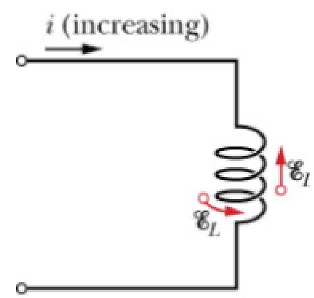
$$= -N\mu_0 n \frac{di}{dt}\cdot A$$

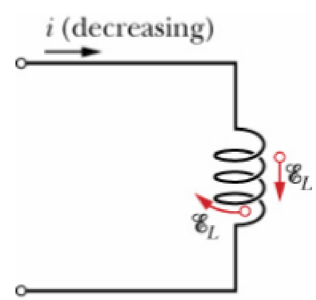
$$= -\boxed{\frac{\mu_0 N^2 A}{l}}\frac{di}{dt}$$

$$\frac{L}{l} = \mu_0 n^2 A$$

$$\varepsilon_L = -N\frac{d\Phi_B}{dt} = -L\frac{di}{dt}$$

L: Self-Inductance





ε_L : *Against the increase of the current*

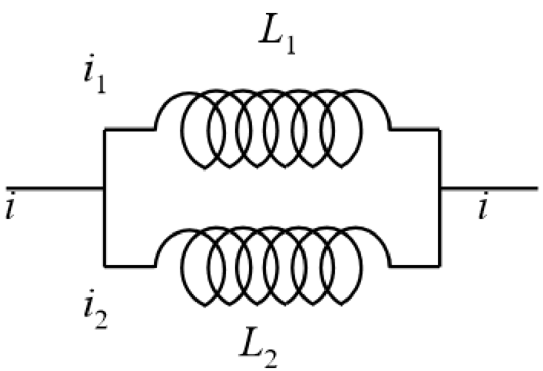
Chap. 30-5 Self-Induction

유도기의 연결

Inductance in Parallel:

$i = i_1 + i_2$

$$\varepsilon_L = -L_1 \frac{di_1}{dt} = -L_2 \frac{di_2}{dt} = -L \frac{di}{dt}$$



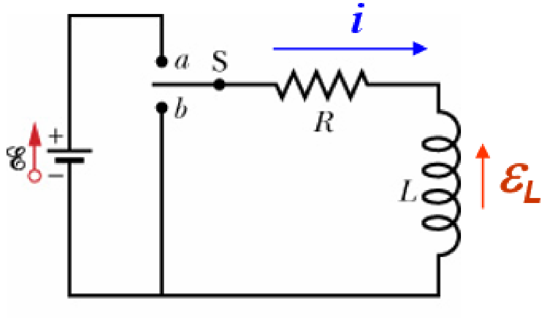
$$-\frac{\varepsilon_L}{L} = \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = -\frac{\varepsilon_L}{L_1} - \frac{\varepsilon_L}{L_2}$$

$$\Rightarrow \frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$$

(저항연결인 경우와 같음)

Inductance in Series: $L = L_1 + L_2$

RL 회로



(a) 연결시 → 전류의 증가

$$-iR-|\varepsilon_L|+\varepsilon=0$$

$$-iR-L\frac{di}{dt}+\varepsilon=0$$

$$L\frac{di}{dt}+Ri=\varepsilon$$

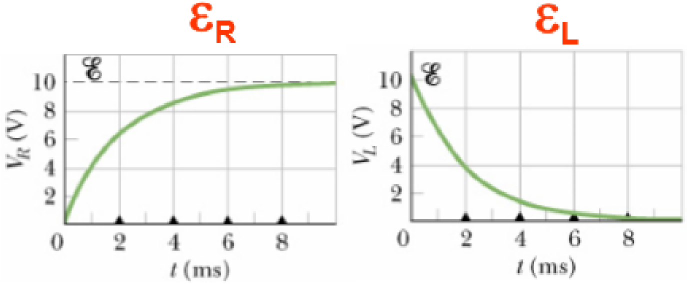
ε=0

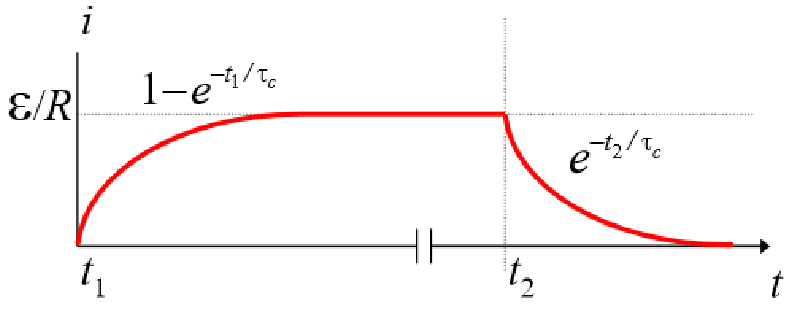
(b) 단절시 → 전류의 감소

$$L\frac{di}{dt}+Ri=0$$

$$\boxed{i=\frac{\mathcal{E}}{R}(1-e^{-t/\tau_L})}\quad ,\tau_L=\frac{L}{R}$$

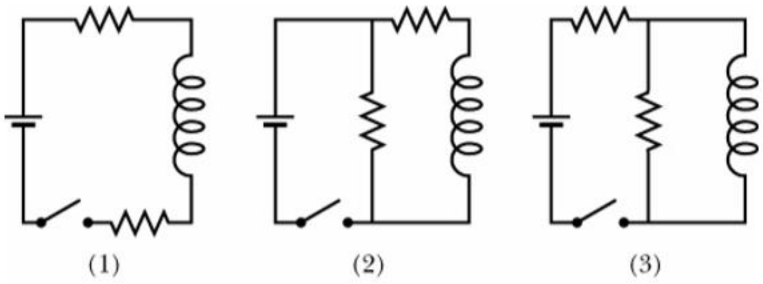
$$\boxed{i=\frac{\mathcal{E}}{R}e^{-t/\tau_L}=i_0e^{-t/\tau_L}}$$





Chap. 30-6 RL Circuits

확인문제 6.



(a) 스위치가 닫힌 직후 전류가 큰 순서는?

(b) 오랜 시간 후 전류가 큰 순서는?

(a) 2 > 3 > 1 (=0)

(b) 2 > 3 > 1

Chap. 30-7 Energy Stored in a Magnetic Field

자기장에 저장된 에너지

In LR Circuit

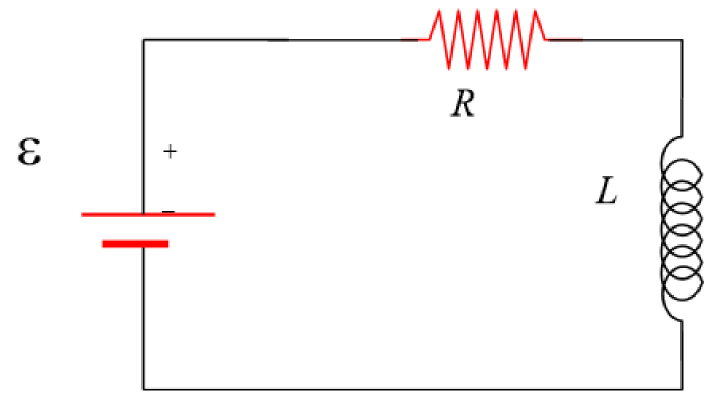
$$\mathcal{E} = iR + L\frac{di}{dt}$$

$$Power: \quad i\mathcal{E} = i^2R + Li\frac{di}{dt}$$

Inductor의 일률

$$\frac{dU_B}{dt} = Li\frac{di}{dt}$$

$$\Rightarrow \quad \boxed{U_B = \frac{1}{2}Li^2}$$



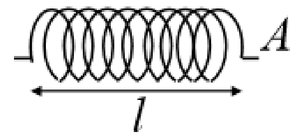
Cf) Energy stored in a capacitor

$$U_C = \frac{1}{2}CV^2$$

Chap. 30-8 Energy Density of a Magnetic Field

자기장의 에너지 밀도

In a solenoid (Magnetic Energy)



$$L = \mu_0 n^2 l \cdot A$$

$$U_B = \frac{1}{2}\left(\mu_0 n^2 l \cdot A\right) \cdot i^2 \qquad B = \mu_0 n i \Rightarrow i = \frac{B}{\mu_0 n}$$

$$U_B = \frac{B^2}{2\mu_0} l \cdot A = \frac{B^2}{2\mu_0} V_{0l}$$

$$\Rightarrow \boxed{u_B = \frac{B^2}{2\mu_0}}$$ *(Magnetic energy density)*

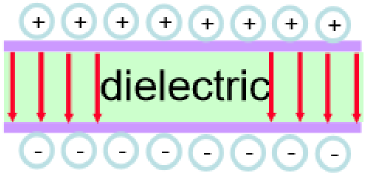
Cf) $$\boxed{u_E = \frac{1}{2}\varepsilon_0 E^2}$$ *(Electric energy density)*

Chap. 30-8 Energy Density of a Magnetic Field

How we can store the energy?

Energy stored in a capacitor ...

$$E_E(t) = \tfrac{1}{2} C v^2(t)$$



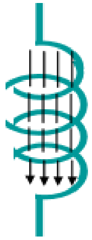
E

... energy density...

$$u_{\text{electric}} = \frac{1}{2}\varepsilon_0 E^2$$

Energy stored in an inductor ....

B



$$E_M(t) = \tfrac{1}{2} L i^2(t)$$

... energy density ...

$$u_{\text{magnetic}} = \frac{1}{2}\frac{B^2}{\mu_0}$$

Chap. 30-9 Mutual Induction

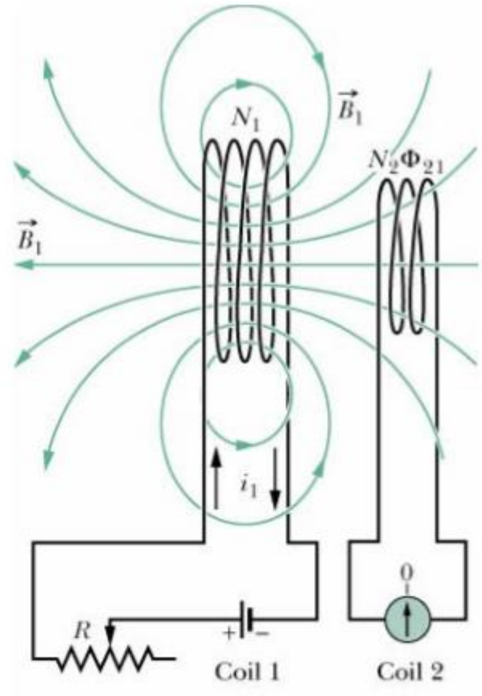
상호유도용량 (mutual inductance) : M

전류 i_1 이 흐르는 코일-1 에 의한 코일-2 의 상호유도용량

$$M_{21} \equiv \frac{N_2 \Phi_{21}}{i_1}$$

$$M_{21} i_1 = N_2 \Phi_{21} \;\Rightarrow\; M_{21} \frac{di_1}{dt} = N_2 \frac{d\Phi_{21}}{dt} = -\mathcal{E}_2$$

$$\mathcal{E}_2 = -M_{21} \frac{di_1}{dt}$$



전류 i_2 이 흐르는 코일-2 에 의한 코일-1 의 유도용량

$$\mathcal{E}_1 = -M_{12} \frac{di_2}{dt} \longrightarrow M_{12} = M_{12} \equiv M \longrightarrow$$

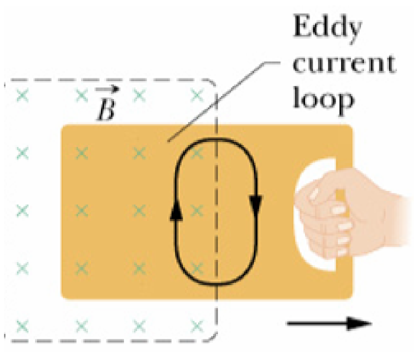
$$\mathcal{E}_1 = -M \frac{di_2}{dt}$$

$$\mathcal{E}_2 = -M \frac{di_1}{dt}$$

Summary

$$\varepsilon = \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

Faraday's Law (Lenz's Law)



Eddy current

$$\frac{L}{l} = \mu_0 n^2 A$$

: 단위 길이당 유도용량

$$\varepsilon_L = -N\frac{d\Phi_B}{dt} = -L\frac{di}{dt}$$

L: Self-Inductance

ε_L : *Against the increase of the current*

Magnetic energy

$$U_B = \frac{1}{2}Li^2$$

$$u_B = \frac{B^2}{2\mu_0}$$