Chapter 24 Electrical Potential

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Chap. 24-1 Electric Potential

If we do work against a force acting on a charge, the work will be stored as a potential energy.

Work done by the force $\vec{F}$ is $W_{AB} = \int_A^B \vec{F} \cdot d\vec{r}$

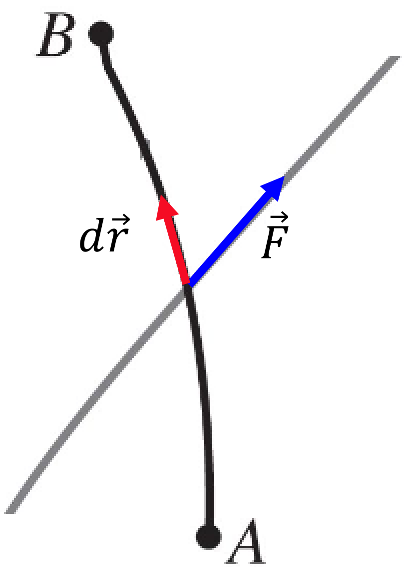
Potential energy difference is

$$\Delta U_{AB} = U_B - U_A = -W_{AB} = -\int_A^B \vec{F} \cdot d\vec{r}$$

along the path from A to B

- ΔU_{AB} is independent of path for conservative forces.
- Potential difference is

$$\Delta V_{AB} = \frac{\Delta U_{AB}}{q} = -\int_A^B \vec{E} \cdot d\vec{r}$$



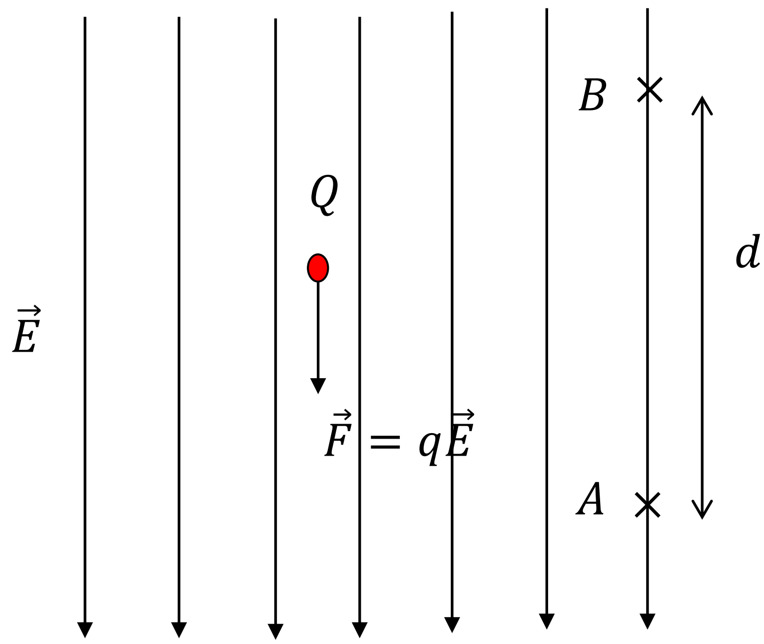
Chap. 24-1 Electric Potential

For a uniform electric field $\vec{E}$,

potential energy difference is given by

$$\Delta U_{AB} = U_B - U_A = -W_{AB} = -\int_A^B \vec{F} \cdot d\vec{r} = -qE(-\hat{y}) \cdot d\,\hat{y} = qEd\,.$$

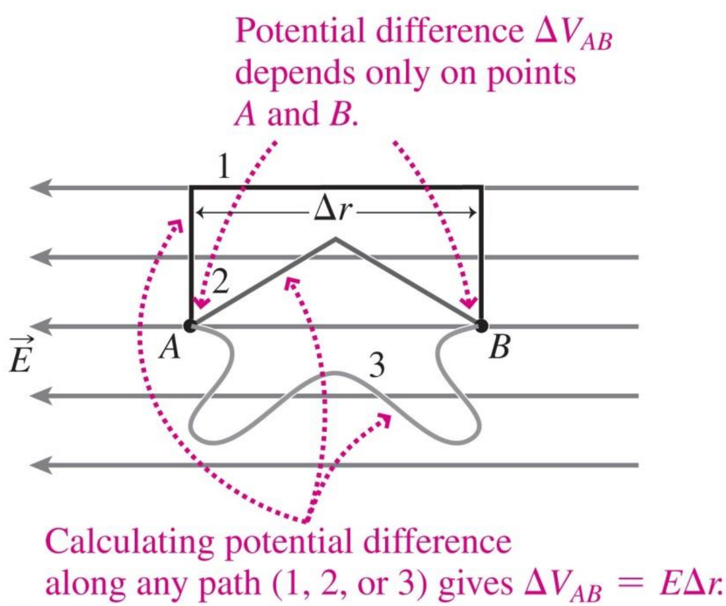
If $U_A = 0,\ U_B = qEd.$

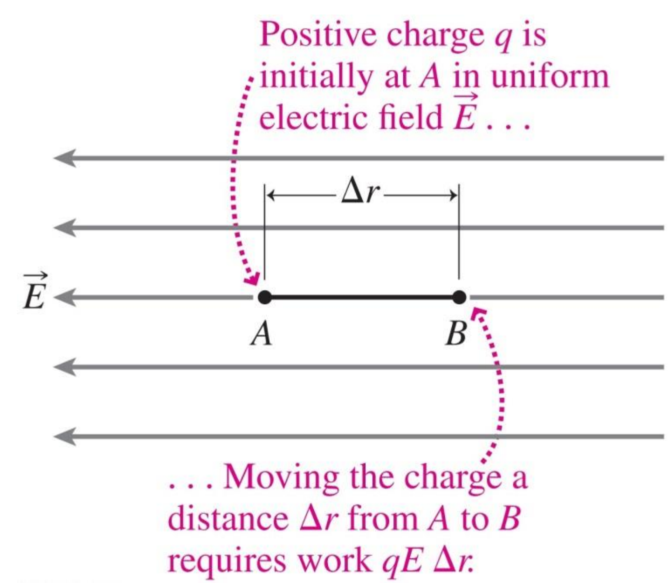


Chap. 24-1 Electric Potential

The **electric potential difference** between two points describes the energy per unit charge involved in moving charge between those two points.

- Mathematically, $\boxed{\Delta V_{AB} = \Delta U_{AB}/q = -\int_A^B \vec{E} \cdot d\vec{r},}$ where ΔV_{AB} is the potential difference between points *A* and *B*, and ΔU_{AB} is the change in potential energy of a charge *q* moved between those points.
- Potential difference is a property *of two points.*
- Since the electrostatic field is conservative, it doesn't matter what path is taken between those points.
- In a uniform field, the potential difference is $\boxed{\Delta V_{AB} = -\vec{E} \cdot \Delta\vec{r}.}$





Chap. 24-1 Electric Potential

The Volt and the Electronvolt

- The unit of electric potential difference is the **volt** (V).
  - 1 volt is 1 joule per coulomb (1 V = 1 J/C).

- The volt is *not* a unit of energy, but of energy per charge—that is, of electric potential difference.
  - A related *energy* unit is the **electronvolt** (eV), defined as the energy gained by one elementary charge *e* "falling" through a potential difference of 1 volt.
  - Therefore, 1 eV = 1.6×10^{-19} J.

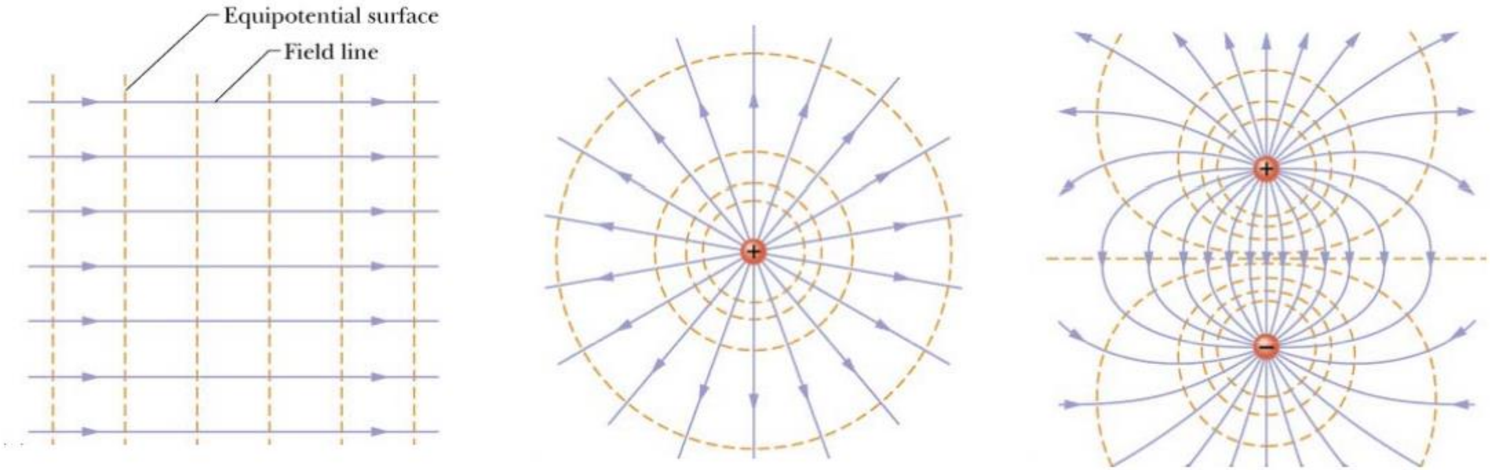
Table 22.2 Typical Potential Differences

Between human arm and leg due to heart's electrical activity	1 mV
Across biological cell membrane	80 mV
Between terminals of flashlight battery	1.5 V
Car battery	12 V
Electric outlet (depends on country)	100–240 V
Taser© (pulsed)	1200 V
Between long-distance electric transmission line and ground	365 kV
Between base of thunderstorm cloud and ground	100 MV

Chap. 24-2 Equipotential Surfaces and the Electric Field

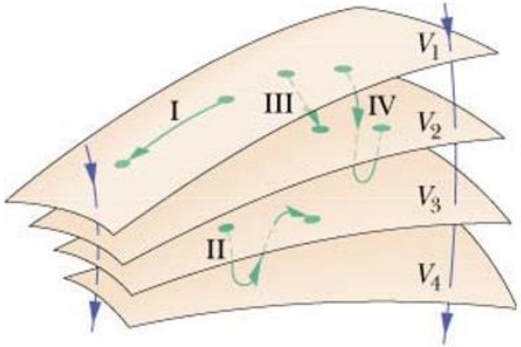
등퍼텐셜면(등전위면: Equipotential surface)

등퍼텐셜면 ≡ 전위가 같은 점들로 이루어진 면



등퍼텐셜면은 항상 전기장에 수직 (왜?)

각 이동 경로에서
전기장이 한 일 ?



$W_I = 0$

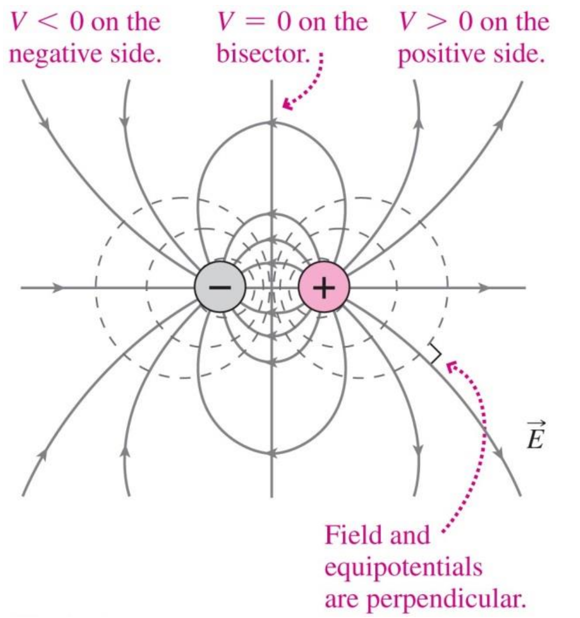
$W_{II} = 0$

$W_{III} = W_{IV}$

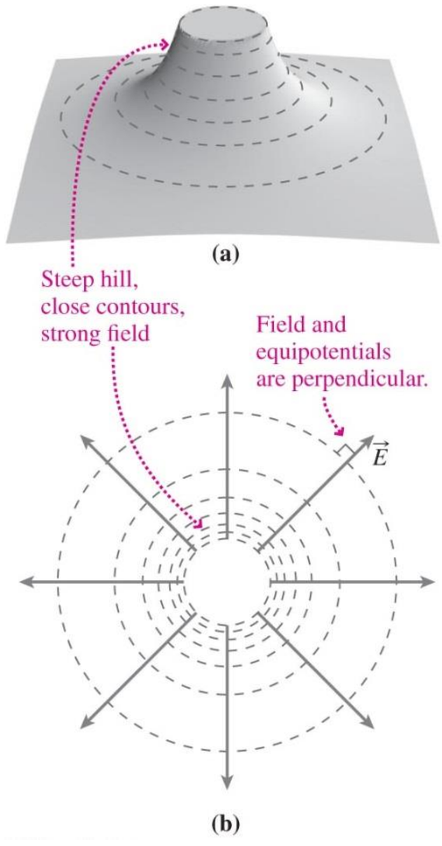
Chap. 24-2 Equipotential Surfaces and the Electric Field

- An **equipotential** is a surface on which the potential is constant.
  - In two-dimensional drawings, we represent equipotentials by curves similar to the contours of height on a map.
  - The electric field is always perpendicular to the equipotentials

The figure shows the equipotentials for a dipole.



The figure shows the equipotentials for a charged spherical shell.



Chap. 24-3 Potential due to a Charged Particle

$$V \equiv \frac{U}{q} = -\frac{W}{q} = -\int_{\infty}^{r} \vec{E} \bullet d\vec{s} \quad [\mathrm{V(volt) = J/C}]$$

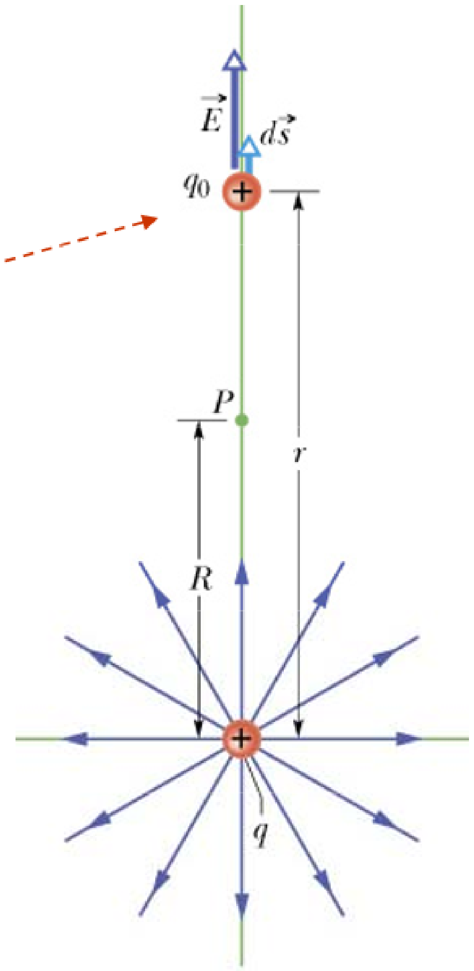
점전하 **q**에서 임의의 **r** 만큼 떨어진 위치에서의 전기장

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

점전하 **q**에서 **R** 만큼 떨어진 위치에서의 퍼텐셜 **(V)**

$$V(r) = -\int_{\infty}^{r} \vec{E} \bullet d\vec{s} = -\frac{\mathrm{q}}{4\pi\varepsilon_{\mathrm{o}}}\int_{\infty}^{r}\frac{1}{s^2}ds = \frac{1}{4\pi\varepsilon_{\mathrm{o}}}\frac{q}{r}$$

$$V(r) = \frac{1}{4\pi\varepsilon_{\mathrm{o}}}\frac{q}{r}$$



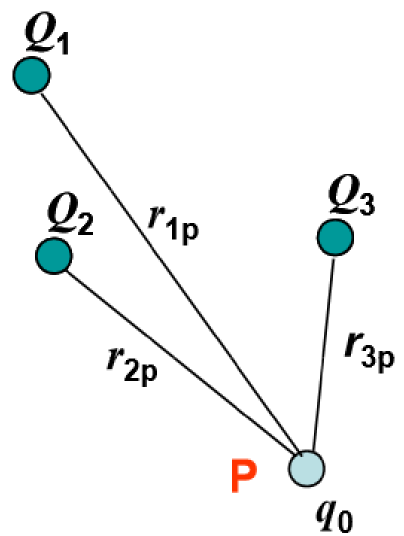
Chap. 24-3 Potential due to a Charged Particle

점전하 무리가 만드는 퍼텐셜

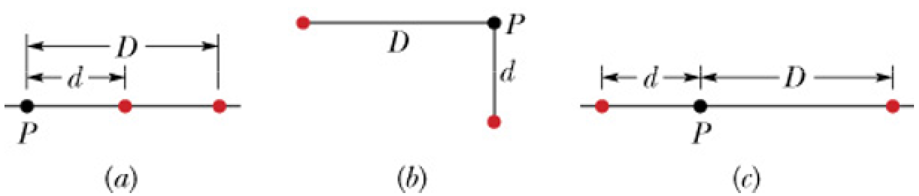
$$V=\sum_{n=1}^{N}V_n=\frac{1}{4\pi\varepsilon_o}\sum_{n=1}^{N}\frac{q}{r_n}$$

- The potential energy of an added test charge q_0 at point P is just

$$U_{of\ q_0\ at\ P}=q_0\left(k\frac{Q_1}{r_{1p}}+k\frac{Q_2}{r_{2p}}+k\frac{Q_3}{r_{3p}}\right)$$

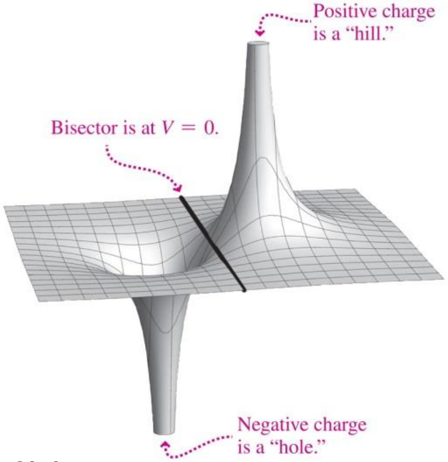


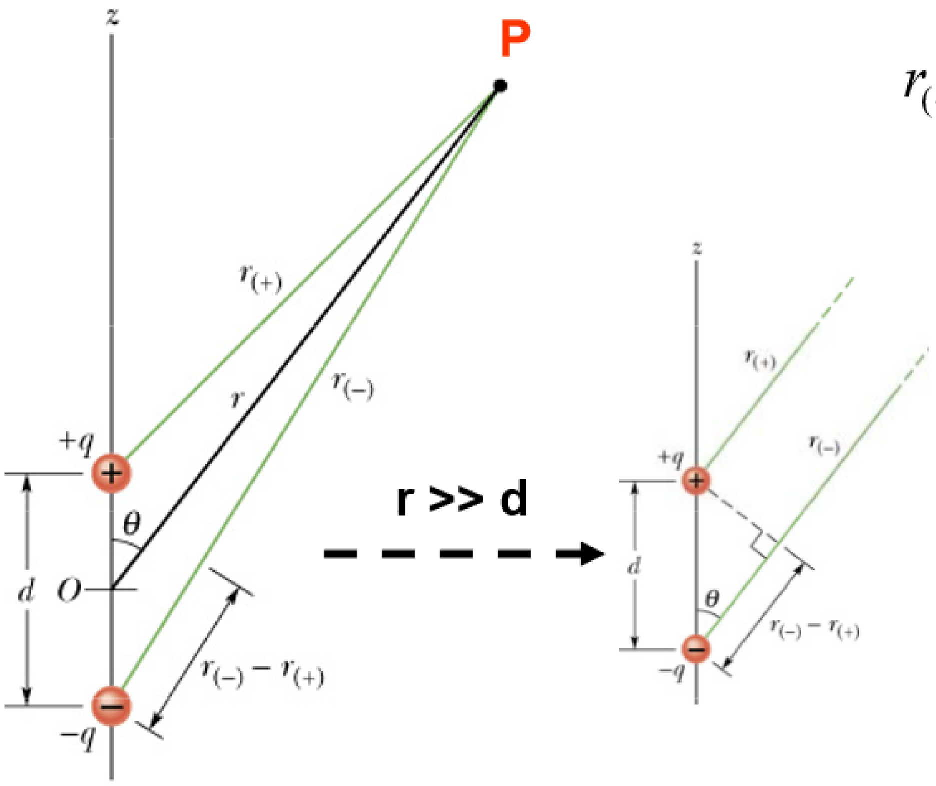
확인문제 3 : P 점에서의 알짜 전기퍼텐셜의 크기 순서는?



Chap. 24-4 Potential due to a Electric Dipole

$$V=\sum_{n=1}^{2}V_n=\frac{1}{4\pi\varepsilon_o}\left(\frac{q}{r_{(+)}}+\frac{-q}{r_{(-)}}\right)=\frac{q}{4\pi\varepsilon_o}\left(\frac{r_{(-)}-r_{(+)}}{r_{(+)}r_{(-)}}\right)$$





$$r_{(-)}-r_{(+)}\approx d\cos\theta$$

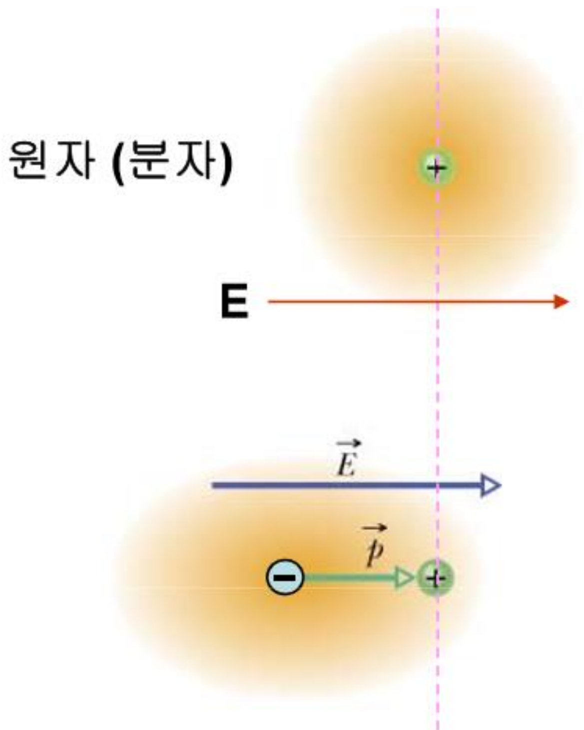
$$r_{(+)}r_{(-)}\approx r^2$$

$$V\approx\frac{q}{4\pi\varepsilon_o}\left(\frac{d\cos\theta}{r^2}\right)$$

$$=\frac{1}{4\pi\varepsilon_o}\left(\frac{p\cos\theta}{r^2}\right),\ (p=qd)$$

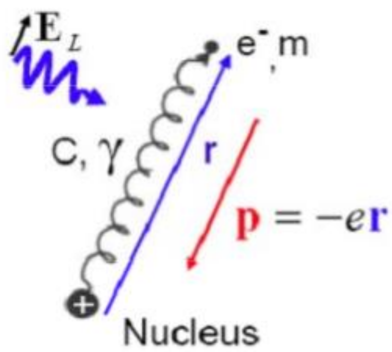
Chap. 24-4 Potential due to a Electric Dipole

유도 쌍극자 모멘트 (induced dipole moment)



E 에 의해 분극이 일어남
→ 원자 (분자)에서의
유도 쌍극자 모멘트 발생

(참고 : 물질의 굴절률 근원)



$$m\mathbf{a}_{el} = \mathbf{F}_{E,Local} + \mathbf{F}_{Damping} + \mathbf{F}_{Spring}$$ (one oscillator)

$$m\frac{d^2\mathbf{r}}{dt^2} + m\gamma\frac{d\mathbf{r}}{dt} + C\mathbf{r} = -e\mathbf{E}_L \exp(-i\omega t)$$

$$\mathbf{p} = -e\mathbf{r}$$

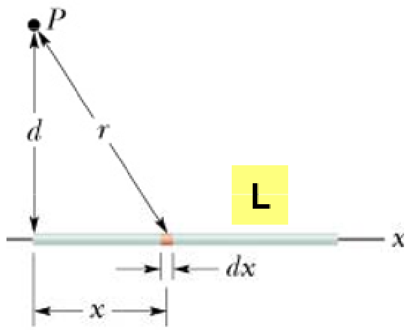
$$m\frac{d^2\mathbf{p}}{dt^2} + m\gamma\frac{d\mathbf{p}}{dt} + C\mathbf{p} = e^2\mathbf{E}_L \exp(-i\omega t)$$

Chap. 24-5 Potential due to a Continuous Charge Distribution

연속적인 전하분포가 만드는 퍼텐셜

$$dV = \frac{1}{4\pi\varepsilon_o}\frac{dq}{r} \quad \Rightarrow V = \frac{1}{4\pi\varepsilon_o}\int\frac{dq}{r}$$

선 전하

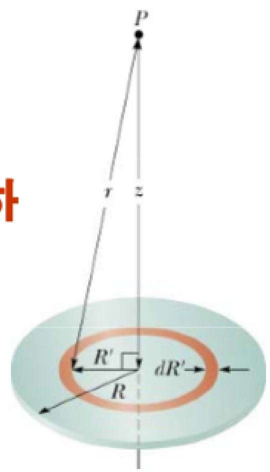


$$dq = \lambda dx$$

$$dV = \frac{1}{4\pi\varepsilon_o}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_o}\frac{\lambda dx}{\sqrt{x^2+d^2}}$$

$$V = \frac{\lambda}{4\pi\varepsilon_o}\int_0^L\frac{dx}{\sqrt{x^2+d^2}} = \frac{\lambda}{4\pi\varepsilon_o}\ln\left[\frac{L+\sqrt{L^2+d^2}}{d}\right]$$

원판 전하



$$dq = \sigma 2\pi R' dR'$$

$$dV = \frac{1}{4\pi\varepsilon_o}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_o}\frac{\sigma 2\pi R' dR'}{\sqrt{z^2+R'^2}}$$

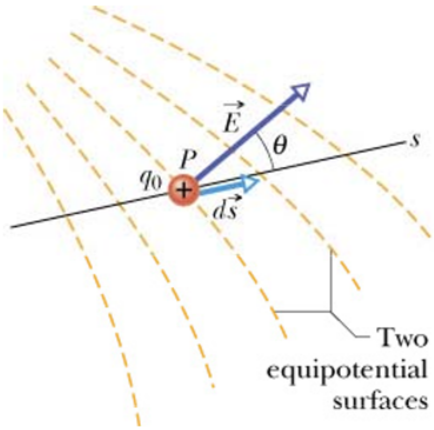
$$V = \frac{\sigma}{2\varepsilon_o}\int_0^R\frac{R' dR'}{\sqrt{z^2+R'^2}} = \frac{\sigma}{2\varepsilon_o}\left[\sqrt{z^2+R^2}-z\right]$$

Chap. 24-6 Calculating the Field from the Potential

$$V(\vec{r}) = -\int \vec{E} \cdot d\vec{r}$$

$$\vec{E} = -\nabla V(r)$$

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$



$$dV = -\vec{E} \cdot d\vec{s}$$

$$\vec{E} = -\frac{dV}{d\vec{s}} \equiv -\vec{\nabla} V = -\left[\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}\right]$$

$$\Rightarrow E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z},$$

If the E-field has only one component, E_x,

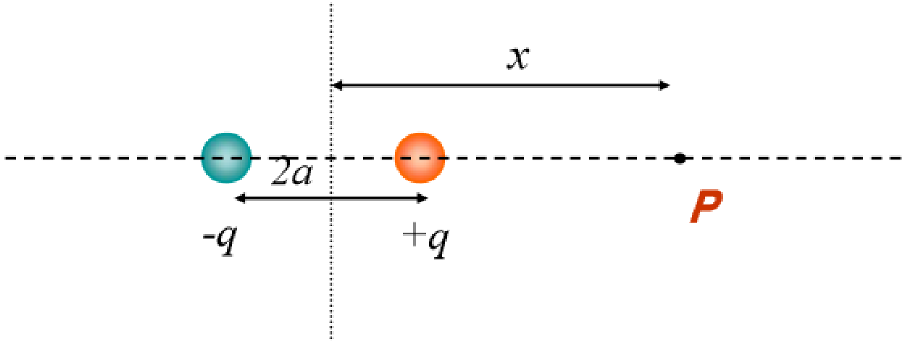
$$\vec{E} = E_x\hat{x} = -\frac{dV}{dx}\hat{x}$$

If the charge distribution has spherical symmetry, the E-field is radial, E_r,

$$\vec{E} = E_r\hat{r} = -\frac{dV}{dr}\hat{r}$$

Chap. 24-6 Calculating the Field from the Potential

쌍극자의 전기퍼텐셜과 전기장



$$V = k_e \sum \frac{q_i}{r_i} = k_e \left(\frac{q}{x-a} - \frac{q}{x+a} \right) = \frac{2k_e qa}{x^2 - a^2} \longrightarrow E_x = -\frac{dV}{dx} = k_e \frac{2k_e qa \cdot 2x}{\left(x^2 - a^2\right)^2}$$

$$\text{If } x >> a, \qquad V \approx \frac{2k_e qa}{x^2} \qquad E_x = -\frac{dV}{dx} \approx \frac{4k_e qa}{x^3}$$

Chap. 24-7 Electric Potential Energy of a System of Charged Particles

대전입자계의 전기 퍼텐셜에너지

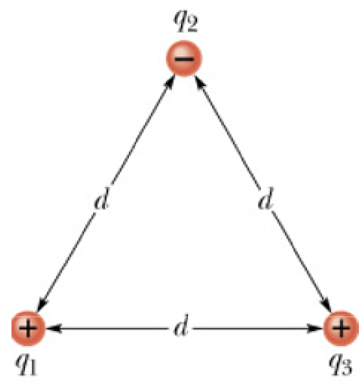
• 고정된 점전하계의 전기적 퍼텐셜(위치) 에너지

$\equiv$ 주어진 전하분포가 되게 하는데 드는 에너지 (외부에서 해야 하는 일)

$$U = W_{app} = q_2 V = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r}$$

보기문제 24-6

그림과 같은 세 점 전하의 전기적 위치 에너지?

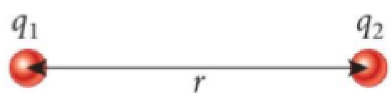


$$U = U_1 + U_2 + U_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{d} + \frac{q_1 q_3}{d} + \frac{q_2 q_3}{d}\right)$$

Chap. 24-7 Electric Potential Energy of a System of Charged Particles

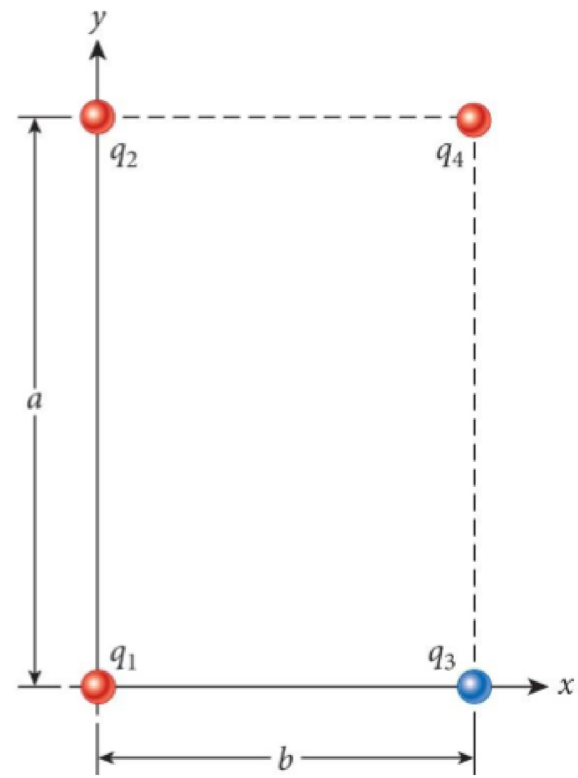
- 두 점전하

$$U = k\frac{q_1 q_2}{r}$$



- 네 점전하 – 전하를 하나씩 주어진 위치에 가져다 놓는데 필요한 에너지의 합

$$U = k\frac{q_1 q_2}{a} + \left(k\frac{q_1 q_3}{b} + k\frac{q_2 q_3}{\sqrt{a^2+b^2}}\right) + \left(k\frac{q_1 q_4}{\sqrt{a^2+b^2}} + k\frac{q_2 q_4}{b} + k\frac{q_3 q_4}{a}\right)$$



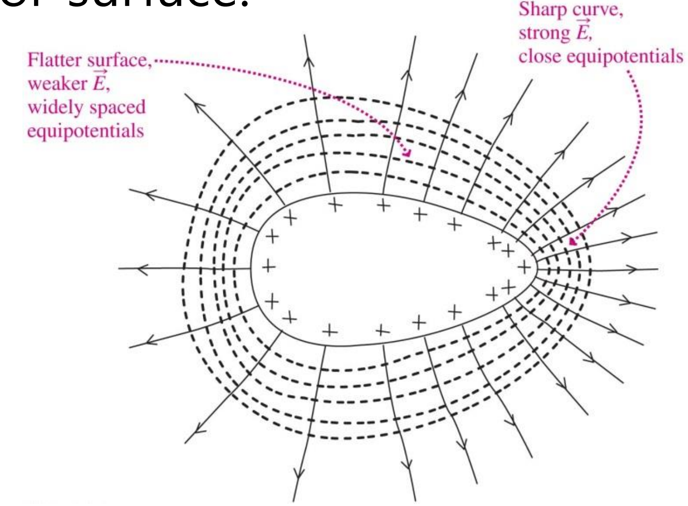
- 점전하 계의 전기 퍼텐셜에너지

$$U = k\sum_{i<j}\frac{q_i q_j}{r_{ij}} = \frac{1}{2}k\sum_{j=1}^{n}\sum_{i=1, i\neq j}^{n}\frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|}$$

Chap. 24-8 Potential of a Charged Isolated Conductor

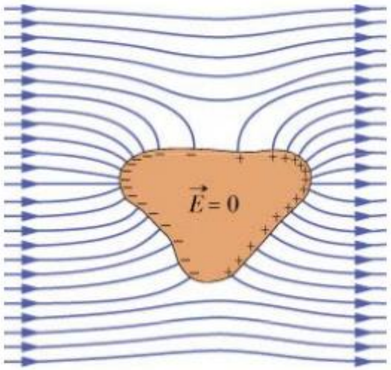
고립된 대전 도체의 퍼텐셜

- There's no electric field inside a conductor in electrostatic equilibrium.
- And even at the surface of the conductor there's no field component parallel to the surface.
- Therefore it takes no work to move charge inside or on the surface of a conductor in electrostatic equilibrium.
  - So **a conductor in electrostatic equilibrium is an equipotential.**
  - That means equipotential surfaces near a charged conductor roughly follow the shape of the conductor surface.
    - That generally makes the equipotentials closer, and therefore the electric field stronger and the charge density higher, where the conductor curves more sharply.



Chap. 24-8 Potential of a Charged Isolated Conductor

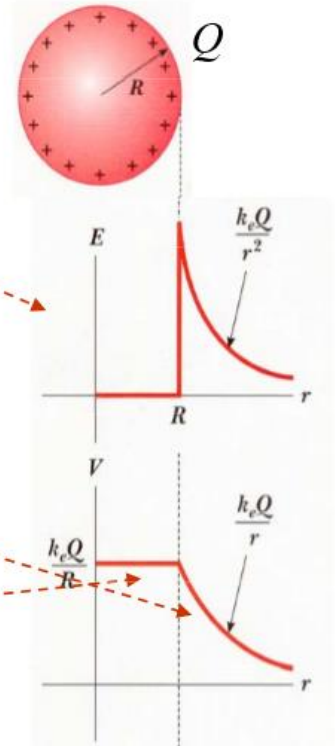
	도체 속	도체 표면
전기장 (E)	0	표면의 법선방향, $\left(\frac{\sigma}{\varepsilon_0}\right)$
전하 (q)	0	표면에 분포, σ
전위 (V)	등전위	



$$\vec{E}=k_e\frac{Q}{r^2}\hat{r} \qquad (r>R)$$
$$=0 \qquad (r<R)$$

$$(r>R) \quad V(r)=-\int_{\infty}^{r}\vec{E}\cdot d\vec{s}=\int_{r}^{\infty}\vec{E}\cdot d\vec{r}$$
$$=k_e\frac{Q}{r}$$

$$(r<R) \quad V(r)=-\int_{\infty}^{r}\vec{E}\cdot d\vec{s}=\cancelto{0}{\int_{r}^{R}\vec{E}\cdot d\vec{r}}+\int_{R}^{\infty}\vec{E}\cdot d\vec{r}$$
$$=k_e\frac{Q}{R}$$



Summary

▪ 중력퍼텐셜 – 점입자의 예

- 중력

$$F_g = -G\frac{m_1 m_2}{r^2}$$

- 중력장

$$g = \frac{F_g}{m_2} = -G\frac{m_1}{r^2}$$

- 중력 퍼텐셜에너지

$$\Delta U_g = -\int \vec{F}_g \cdot d\vec{r} = -G\frac{m_1 m_2}{r}$$

- 중력 퍼텐셜

$$\Delta V_g = \frac{\Delta U_g}{m_2} = -\int \vec{g} \cdot d\vec{r} = -G\frac{m_1}{r}$$

▪ **전기퍼텐셜** – 점전하의 예

- 전기력

$$F_e = k\frac{q_1 q_2}{r^2}$$

- 전기장

$$E = \frac{F_e}{q_2} = k\frac{q_1}{r^2}$$

- 전기 퍼텐셜에너지

$$\Delta U_e = -\int \vec{F}_e \cdot d\vec{r} = k\frac{q_1 q_2}{r^2}$$

$$(i = \infty,\ f = r)$$

- **전기 퍼텐셜**

$$\Delta V_e = \frac{\Delta U_e}{q_2} = -\int \vec{E} \cdot d\vec{r} = k\frac{q_1}{r}$$

Summary

Electric potential energy (U)

$$U = -W = -\int_{\infty}^{r} \vec{F}_q \bullet d\vec{s} = -\int_{\infty}^{r} q\vec{E} \bullet d\vec{s} \quad [\mathrm{J = N \cdot m}]$$

Electric potential (V) : 단위 전하당 U

$$V \equiv \frac{U}{q} = -\frac{W}{q} = -\int_{\infty}^{r} \vec{E} \bullet d\vec{s} \quad [\mathrm{V(volt) = J/C}]$$

점 전하 $$V = \sum_{n=1}^{N} V_n = \frac{1}{4\pi\varepsilon_o} \sum_{n=1}^{N} \frac{q}{r_n}$$

연속 전하 $$V = \frac{1}{4\pi\varepsilon_o} \int \frac{dq}{r}$$

V ←→ E $$\vec{E} = -\frac{dV}{d\vec{s}} \equiv -\vec{\nabla} V = -\left[\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}\right]$$