1. Let the vector be written as $\vec{a} = a_1\hat{\text{i}} + a_2\hat{\text{j}}$. Given that

$$a_1 = \frac{1}{2}|\vec{a}| = \frac{1}{2}\sqrt{a_1^2 + a_2^2}$$

we have

$$a_1^2 = \frac{1}{4}(a_1^2 + a_2^2) \quad \Rightarrow \quad a_2 = \pm\sqrt{3}a_1.$$

Therefore, the vector is $\vec{a} = a_1\hat{\text{i}} \pm \sqrt{3}a_1\hat{\text{j}}$, which implies that

$$\tan\theta = \frac{a_2}{a_1} = \frac{\pm\sqrt{3}a_1}{a_1} = \pm\sqrt{3} \approx \pm 1.73.$$

2. (a) With $r = 15$ m and $\theta = 30°$, the x component of $\vec{r}$ is given by

$$r_x = r\cos\theta = (12\text{ m})\cos 30° = 10\text{ m}.$$

(b) Similarly, the y component is given by

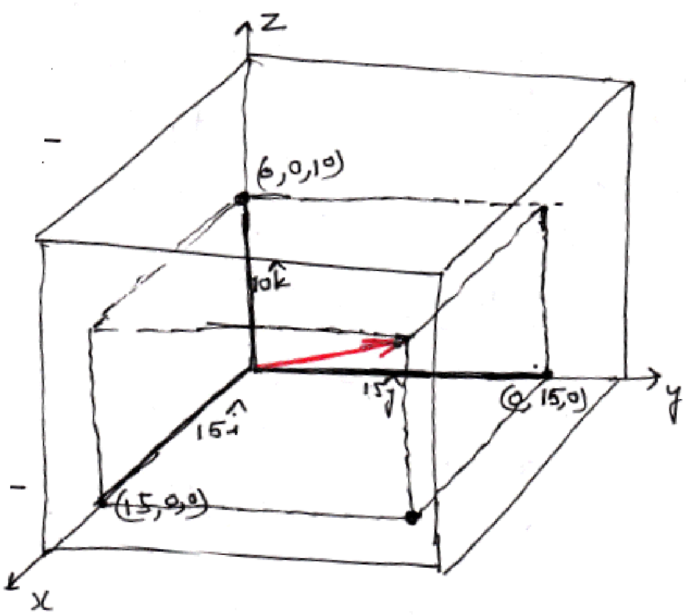
$$r_y = r\sin\theta = (12\text{ m})\sin 30° = 6.0\text{ m}.$$

3. The following figure depicts the situation. The required vector is given by

$$\vec{a} = 15\hat{i} + 15\hat{j} + 10\hat{k}$$

That is,

$$\begin{aligned}|\vec{a}| &= \sqrt{15^2 + 15^2 + 10^2}\\ &= \sqrt{225 + 225}\\ &= \sqrt{550}\\ &= 23.45207 \approx 23.\end{aligned}$$



4. The angle described by a full circle is 360° = 2π rad, which is the basis of our conversion factor.

(a) $20.0° = (20.0°)\dfrac{2\pi \text{ rad}}{360°} = 0.349 \text{ rad}$.

(b) $50.0° = (50.0°)\dfrac{2\pi \text{ rad}}{360°} = 0.873 \text{ rad}$.

(c) $100° = (100°)\dfrac{2\pi \text{ rad}}{360°} = 1.75 \text{ rad}$.

(d) $0.330 \text{ rad} = (0.330 \text{ rad})\dfrac{360°}{2\pi \text{ rad}} = 18.9°$.

(e) $2.30 \text{ rad} = (2.30 \text{ rad})\dfrac{360^0}{2\pi \text{ rad}} = 132^0$.

(f) $7.70 \text{ rad} = (7.70 \text{ rad})\dfrac{360°}{2\pi \text{ rad}} = 441°$.

5. We choose $+x$ east and $+y$ north and measure all angles in the "standard" way (positive ones counterclockwise from $+x$, negative ones clockwise). Thus, vector $\vec{d}_1$ has magnitude $d_1 = 3.66$ (with the unit meter and three significant figures assumed) and direction $\theta_1 = 90°$. Also, $\vec{d}_2$ has magnitude $d_2 = 1.83$ and direction $\theta_2 = -45°$, and vector $\vec{d}_3$ has magnitude $d_3 = 0.91$ and direction $\theta_3 = -135°$. We add the x and y components, respectively:

$$x: \; d_1 \cos\theta_1 + d_2 \cos\theta_2 + d_3 \cos\theta_3 = 0.65 \text{ m}$$

$$y: \; d_1 \sin\theta_1 + d_2 \sin\theta_2 + d_3 \sin\theta_3 = 1.7 \text{ m.}$$

(a) The magnitude of the direct displacement (the vector sum $\vec{d}_1 + \vec{d}_2 + \vec{d}_3$) is

$$\sqrt{(0.65\text{ m})^2 + (1.7\text{ m})^2} = 1.8 \text{ m.}$$

(b) The angle (understood in the sense described above) is $\tan^{-1}(1.7/0.65) = 69°$. That is, the first putt must aim in the direction 69° north of east.

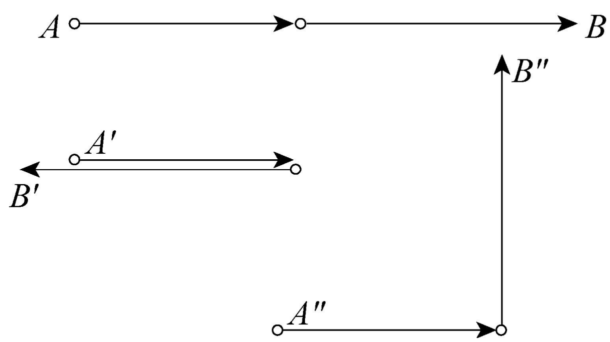
6. (a) The height is $h = d \sin\theta$, where $d = 10.5$ m and $\theta = 20.0°$. Therefore, $h = 3.59$ m.

(b) The horizontal distance is $d \cos\theta = 9.87$ m.

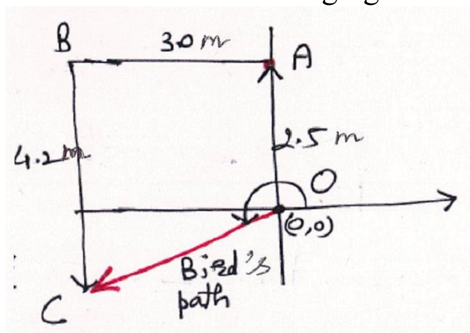
7. (a) The vectors should be parallel to achieve a resultant 7 m long (the unprimed case shown below),

(b) anti-parallel (in opposite directions) to achieve a resultant 1 m long (primed case shown),
(c) and perpendicular to achieve a resultant $\sqrt{3^2 + 4^2} = 5\,\text{m}$ long (the double-primed case shown).

In each sketch, the vectors are shown in a "head-to-tail" sketch but the resultant is not shown. The resultant would be a straight line drawn from beginning to end; the beginning is indicated by A (with or without primes, as the case may be) and the end is indicated by B.



8.(a) The vector diagram is shown in the following figure.



(b) The bird would fly along $\overrightarrow{OC}$. Now,

$$\begin{aligned}\overrightarrow{OC} &= \overrightarrow{OA}+\overrightarrow{AB}+\overrightarrow{BC}\\ &= 2.5\hat{j} - 3.0\hat{i} - 4.2\hat{j}\\ &= -3.0\hat{i} - 1.7\hat{j}\end{aligned}$$

Therefore, the distance travelled by the bird is

$$\begin{aligned}\left|\overrightarrow{OC}\right| &= \sqrt{(-3.0)^2 + (-1.7)^2}\\ &= 3.4481879\,\text{m} \approx 3.5\,\text{m}.\end{aligned}$$

(c) Unit vector in the direction $\overrightarrow{OC}$ is

$$\frac{-3.0\hat{i} - 1.7\hat{j}}{\sqrt{(-3.0)^2 + (-1.7)^2}} = \frac{-3.0\hat{i} - 1.7\hat{j}}{3.4481879}$$

We can represent the direction also in terms of $\tan\theta$. Therefore,

$$\tan\theta = \frac{-1.7}{-3.0} = \frac{17}{30}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{17}{30}\right) = 0.51554901\,\text{rad}$$

That is,

$$0.51554901 \times \frac{180°}{\pi} = 29.54°.$$

The bird flies in a straight line from the same starting point to the same final point in anticlockwise direction of angle 29.54° or approximately 30° south of due west.

9. The vectors given are

$$\vec{a} = 5.0\hat{\mathrm{i}} - 4.0\hat{\mathrm{j}} + 2.0\hat{\mathrm{k}};$$
$$\vec{b} = (-2.0m)\hat{\mathrm{i}} + (2.0m)\hat{\mathrm{j}} + (5.0m)\hat{\mathrm{k}}.$$

(a) $\vec{a} + \vec{b} = (5.0 - 2.0m)\hat{\mathrm{i}} + (-4.0 + 2.0m)\hat{\mathrm{j}} + (2.0 + 5.0m)\hat{\mathrm{k}}$.

(b) $\vec{a} - \vec{b} = (5.0 + 2.0m)\hat{\mathrm{i}} + (-4.0 - 2.0m)\hat{\mathrm{j}} + (2.0 - 5.0m)\hat{\mathrm{k}}$

(c) $\vec{a} - \vec{b} + \vec{c} = 0$ implies that

$$\begin{aligned}\vec{c} = \vec{b} - \vec{a} = -(\vec{a} - \vec{b}) &= -\{(5.0 + 2.0m)\hat{\mathrm{i}} + (-4.0 - 2.0m)\hat{\mathrm{j}} + (2.0 - 5.0m)\hat{\mathrm{k}}\} \\ &= (-5.0 - 2.0m)\hat{\mathrm{i}} + (4.0 + 2.0m)\hat{\mathrm{j}} + (-2.0 + 5.0m)\hat{\mathrm{k}}\end{aligned}$$

10. The *x, y*, and *z* components of $\vec{r} = \vec{c} + \vec{d}$ are, respectively,

(a) $r_x = c_x + d_x = 7.4 \text{ m} + 4.4 \text{ m} = 12 \text{ m}$,

(b) $r_y = c_y + d_y = -3.8 \text{ m} - 2.0 \text{ m} = -5.8 \text{ m}$, and

(c) $r_z = c_z + d_z = -6.1 \text{ m} + 3.3 \text{ m} = -2.8 \text{ m}$.

11. **THINK** This problem involves the addition of two vectors $\vec{a}$ and $\vec{b}$. We want to find the magnitude and direction of the resulting vector.

EXPRESS In two dimensions, a vector $\vec{a}$ can be written as, in unit vector notation,

$$\vec{a} = a_x\hat{\text{i}} + a_y\hat{\text{j}}.$$

Similarly, a second vector $\vec{b}$ can be expressed as $\vec{b} = b_x\hat{\text{i}} + b_y\hat{\text{j}}$. Adding the two vectors gives

$$\vec{r} = \vec{a} + \vec{b} = (a_x + b_x)\hat{\text{i}} + (a_y + b_y)\hat{\text{j}} = r_x\hat{\text{i}} + r_y\hat{\text{j}}$$

ANALYZE

(a) Given that $\vec{a} = (4.0\text{ m})\hat{\text{i}} + (3.0\text{ m})\hat{\text{j}}$ and $\vec{b} = (-13.0\text{ m})\hat{\text{i}} + (7.0\text{ m})\hat{\text{j}}$, we find the x and the y components of $\vec{r}$ to be

$$r_x = a_x + b_x = (4.0\text{ m}) + (-13\text{ m}) = -9.0\text{ m}$$
$$r_y = a_y + b_y = (3.0\text{ m}) + (7.0\text{ m}) = 10.0\text{ m}.$$

Thus $\vec{r} = (-9.0\,\text{m})\,\hat{\text{i}} + (10\,\text{m})\,\hat{\text{j}}$.

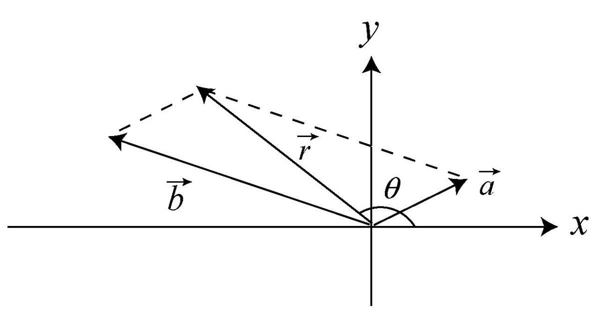
(b) The magnitude of $\vec{r}$ is $r = |\vec{r}| = \sqrt{r_x^2 + r_y^2} = \sqrt{(-9.0\text{ m})^2 + (10\text{ m})^2} = 13\text{ m}$.

(c) The angle between the resultant and the $+x$ axis is given by

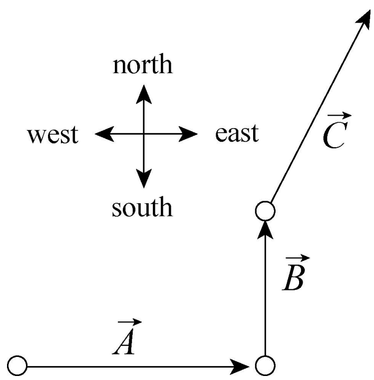
$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{10.0\text{ m}}{-9.0\text{ m}}\right) = -48^\circ \text{ or } 132^\circ.$$

Since the x component of the resultant is negative and the y component is positive, characteristic of the second quadrant, we find the angle is 132° (measured counterclockwise from $+x$ axis).

LEARN The addition of the two vectors is depicted in the figure below (not to scale). Indeed, since $r_x < 0$ and $r_y > 0$, we expect $\vec{r}$ to be in the second quadrant.



12. We label the displacement vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ (and denote the result of their vector sum as $\vec{r}$). We choose *east* as the $\hat{\mathrm{i}}$ direction (+x direction) and *north* as the $\hat{\mathrm{j}}$ direction (+y direction). We note that the angle between $\vec{C}$ and the x axis is 60°. Thus,



$$\vec{A} = (40\ \text{km})\,\hat{\mathrm{i}}$$
$$\vec{B} = (30\ \text{km})\,\hat{\mathrm{j}}$$
$$\vec{C} = (25\ \text{km})\cos(60°)\,\hat{\mathrm{i}} + (25\ \text{km})\sin(60°)\,\hat{\mathrm{j}}$$

(a) The total displacement of the car from its initial position is represented by

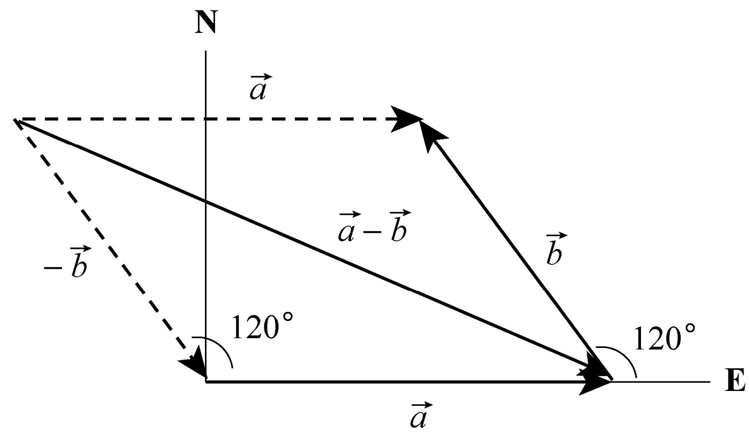
$$\vec{r} = \vec{A} + \vec{B} + \vec{C} = (62.5\ \text{km})\,\hat{\mathrm{i}} + (51.7\ \text{km})\,\hat{\mathrm{j}}$$

which means that its magnitude is

$$|\vec{r}| = \sqrt{(52.5\,\text{km})^2 + (51.7\,\text{km})^2} = 74\ \text{km}.$$

(b) The angle (counterclockwise from +x axis) is $\tan^{-1}$ (51.7 km/52.5 km) = 45°, which is to say that it points 45° *north of east*. Although the resultant $\vec{r}$ is shown in our sketch, it would be a direct line from the "tail" of $\vec{A}$ to the "head" of $\vec{C}$.

13. (a) The conditions are satisfied if the object moves, say, 1.0 m eastward, and then turns 120° counterclockwise. The situation is depicted in the figure below.



(b) Let $\vec{c}=\vec{a}-\vec{b}$. With $a=|\vec{a}|=1,\ b=|\vec{b}|=1$, using law of cosine, we obtain

$$c^2=a^2+b^2-2ab\cos(120°)=1+1-2(1)(1)(-0.5)=3$$

$$\text{Thus, } c=|\vec{c}\,|=|\vec{a}-\vec{b}\,|=\sqrt{3}\text{ m}=1.73\text{ m.}$$

14. (a) Summing the x components, we have

$$20 \text{ m} + b_x - 20 \text{ m} - 60 \text{ m} = -140 \text{ m},$$

which gives $b_x = -80 \text{ m}$.

(b) Summing the y components, we have

$$60 \text{ m} - 70 \text{ m} + c_y - 70 \text{ m} = 20 \text{ m},$$

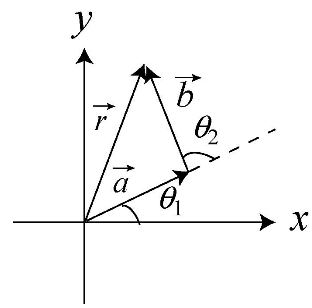
which implies $c_y = 100$ m.

(c) Using the Pythagorean theorem, the magnitude of the overall displacement is given by

$$\sqrt{(-140 \text{ m})^2 + (20 \text{ m})^2} \approx 141 \text{ m}.$$

(d) The angle is given by $\tan^{-1}(20/(-140)) = -8.1°$, (which would be 8.1° measured clockwise from the $-x$ axis, or 171.9° measured counterclockwise from the $+x$ axis).

15. **THINK** This problem involves the addition of two vectors $\vec{a}$ and $\vec{b}$ in two dimensions. We're asked to find the components, magnitude and direction of the resulting vector.



EXPRESS In two dimensions, a vector $\vec{a}$ can be written as, in unit vector notation,

$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} = (a\cos\alpha)\hat{\mathrm{i}} + (a\sin\alpha)\hat{\mathrm{j}}.$$

Similarly, a second vector $\vec{b}$ can be expressed as $\vec{b} = b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}} = (b\cos\beta)\hat{\mathrm{i}} + (b\sin\beta)\hat{\mathrm{j}}$. From the figure, we have, $\alpha = \theta_1$ and $\beta = \theta_1 + \theta_2$ (since the angles are measured from the $+x$-axis) and the resulting vector is

$$\vec{r} = \vec{a} + \vec{b} = [a\cos\theta_1 + b\cos(\theta_1+\theta_2)]\hat{\mathrm{i}} + [a\sin\theta_1 + b\sin(\theta_1+\theta_2)]\hat{\mathrm{j}} = r_x\hat{\mathrm{i}} + r_y\hat{\mathrm{j}}$$

ANALYZE

(a) Given that $a = b = 10$ m, $\theta_1 = 30°$ and $\theta_2 = 105°$, the x component of $\vec{r}$ is

$$r_x = a\cos\theta_1 + b\cos(\theta_1+\theta_2) = (10\text{ m})\cos 30° + (10\text{ m})\cos(30°+105°) = 1.59\text{ m}$$

(b) Similarly, the y component of $\vec{r}$ is

$$r_y = a\sin\theta_1 + b\sin(\theta_1+\theta_2) = (10\text{ m})\sin 30° + (10\text{ m})\sin(30°+105°) = 12.1\text{ m}.$$

(c) The magnitude of $\vec{r}$ is $r = |\vec{r}| = \sqrt{(1.59\text{ m})^2 + (12.1\text{ m})^2} = 12.2$ m.

(d) The angle between $\vec{r}$ and the $+x$-axis is

$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{12.1\text{ m}}{1.59\text{ m}}\right) = 82.5°.$$

LEARN As depicted in the figure, the resultant $\vec{r}$ lies in the first quadrant. This is what we expect. Note that the magnitude of $\vec{r}$ can also be calculated by using law of cosine ($\vec{a}, \vec{b}$ and $\vec{r}$ form an isosceles triangle):

$$r = \sqrt{a^2 + b^2 - 2ab\cos(180-\theta_2)} = \sqrt{(10\text{ m})^2 + (10\text{ m})^2 - 2(10\text{ m})(10\text{ m})\cos 75°}$$
$$= 12.2\text{ m}.$$

16. (a) $\vec{a}+\vec{b}=(3.0\,\hat{\mathrm{i}}+4.0\,\hat{\mathrm{j}})\text{ m}+(5.0\,\hat{\mathrm{i}}-2.0\,\hat{\mathrm{j}})\text{ m}=(8.0\text{ m})\,\hat{\mathrm{i}}+(2.0\text{ m})\,\hat{\mathrm{j}}.$

(b) The magnitude of $\vec{a}+\vec{b}$ is

$$|\vec{a}+\vec{b}|=\sqrt{(8.0\text{ m})^2+(2.0\text{ m})^2}=8.2\text{ m}.$$

(c) The angle between this vector and the +x axis is

$$\tan^{-1}[(2.0\text{ m})/(8.0\text{ m})] = 14°.$$

(d) $\vec{b}-\vec{a}=(5.0\,\hat{\mathrm{i}}-2.0\,\hat{\mathrm{j}})\text{ m}-(3.0\,\hat{\mathrm{i}}+4.0\,\hat{\mathrm{j}})\text{ m}=(2.0\text{ m})\,\hat{\mathrm{i}}-(6.0\text{ m})\,\hat{\mathrm{j}}.$

(e) The magnitude of the difference vector $\vec{b}-\vec{a}$ is

$$|\vec{b}-\vec{a}|=\sqrt{(2.0\text{ m})^2+(-6.0\text{ m})^2}=6.3\text{ m}.$$

(f) The angle between this vector and the +x axis is $\tan^{-1}[(-6.0\text{ m})/(2.0\text{ m})] = -72°$. The vector is 72° *clockwise* from the axis defined by $\hat{\mathrm{i}}$.

17. Many of the operations are done efficiently on most modern graphical calculators using their built-in vector manipulation and rectangular ↔ polar "shortcuts." In this solution, we employ the "traditional" methods (such as Eq. 3-6). Where the length unit is not displayed, the unit meter should be understood.

(a) Using unit-vector notation,

$$\begin{aligned}\vec{a} &= (50\text{ m})\cos(30°)\hat{\text{i}} + (50\text{ m})\sin(30°)\hat{\text{j}}\\ \vec{b} &= (50\text{ m})\cos(195°)\hat{\text{i}} + (50\text{ m})\sin(195°)\hat{\text{j}}\\ \vec{c} &= (50\text{ m})\cos(315°)\hat{\text{i}} + (50\text{ m})\sin(315°)\hat{\text{j}}\\ \vec{a}+\vec{b}+\vec{c} &= (30.4\text{ m})\hat{\text{i}} - (23.3\text{ m})\hat{\text{j}}.\end{aligned}$$

The magnitude of this result is $\sqrt{(30.4\text{ m})^2 + (-23.3\text{ m})^2} = 38\text{ m}$.

(b) The two possibilities presented by a simple calculation for the angle between the vector described in part (a) and the $+x$ direction are $\tan^{-1}[(-23.2\text{ m})/(30.4\text{ m})] = -37.5°$, and $180° + (-37.5°) = 142.5°$. The former possibility is the correct answer since the vector is in the fourth quadrant (indicated by the signs of its components). Thus, the angle is –37.5°, which is to say that it is 37.5° *clockwise* from the $+x$ axis. This is equivalent to 322.5° counterclockwise from $+x$.

(c) We find

$$\vec{a}-\vec{b}+\vec{c} = [43.3-(-48.3)+35.4]\,\hat{\text{i}} - [25-(-12.9)+(-35.4)]\,\hat{\text{j}} = (127\,\hat{\text{i}} + 2.60\,\hat{\text{j}})\text{ m}$$

in unit-vector notation. The magnitude of this result is

$$|\vec{a}-\vec{b}+\vec{c}| = \sqrt{(127\text{ m})^2 + (2.6\text{ m})^2} \approx 1.30\times10^2\text{ m}.$$

(d) The angle between the vector described in part (c) and the $+x$ axis is

$$\theta = \tan^{-1}(2.6\text{ m}/127\text{ m}) \approx 1.2°.$$

(e) Using unit-vector notation, $\vec{d}$ is given by $\vec{d} = \vec{a}+\vec{b}-\vec{c} = (-40.4\,\hat{\text{i}} + 47.4\,\hat{\text{j}})\text{ m}$, which has a magnitude of $\sqrt{(-40.4\text{ m})^2 + (47.4\text{ m})^2} = 62\text{ m}$.

(f) The two possibilities presented by a simple calculation for the angle between the vector described in part (e) and the $+x$ axis are $\tan^{-1}(47.4/(-40.4)) = -50.0°$, and

$180° + (-50.0°) = 130°$. We choose the latter possibility as the correct one since it indicates that $\vec{d}$ is in the second quadrant (indicated by the signs of its components).

18. If we wish to use Eq. 3-5 in an unmodified fashion, we should note that the angle between $\vec{C}$ and the $+x$ axis is 180° + 20.0° = 200°.

(a) The x and y components of $\vec{B}$ are given by

$$B_x = C_x - A_x = (16.0 \text{ m}) \cos 200° - (12.0 \text{ m}) \cos 40° = -24.2 \text{ m},$$
$$B_y = C_y - A_y = (16.0 \text{ m}) \sin 200° - (12.0 \text{ m}) \sin 40° = -13.2 \text{ m}.$$

Consequently, its magnitude is $|\vec{B}| = \sqrt{(-24.2 \text{ m})^2 + (-13.2 \text{ m})^2} = 27.6 \text{ m}$.

(b) The angle between $\vec{B}$ and the $+x$ axis is given by

$$\phi = \tan^{-1}\left(\frac{-13.2 \text{ m}}{-24.2 \text{ m}}\right) = 28.6°,\ 209°$$

We choose the latter possibility as the correct one since it indicates that $\vec{B}$ is in the third quadrant (indicated by the signs of its components). We note, too, that the answer can be equivalently stated as $-151°$.

19. (a) With $\hat{\text{i}}$ directed forward and $\hat{\text{j}}$ directed leftward, the resultant is $(5.00\,\hat{\text{i}} + 2.00\,\hat{\text{j}})$ m . The magnitude is given by the Pythagorean theorem: $\sqrt{(5.00\text{ m})^2 + (2.00\text{ m})^2} = 5.385$ m ≈ 5.39 m.

(b) The angle is $\tan^{-1}(2.00/5.00) \approx 21.8°$ (left of forward).

20. The desired result is the displacement vector, in units of km, $\vec{A}$ = (4.8 km), 90º (measured counterclockwise from the +*x* axis), or $\vec{A} = (4.8\ \text{km})\hat{\text{j}}$, where $\hat{\text{j}}$ is the unit vector along the positive *y* axis (north). This consists of the sum of two displacements: during the whiteout, $\vec{B} = (7.8\ \text{km}),\ 50°$, or

$$\vec{B} = (7.8\ \text{km})(\cos 50°\hat{\text{i}} + \sin 50°\ \hat{\text{j}}) = (5.01\ \text{km})\hat{\text{i}} + (5.98\ \text{km})\hat{\text{j}}$$

and the unknown $\vec{C}$. Thus, $\vec{A} = \vec{B} + \vec{C}$.

(a) The desired displacement is given by $\vec{C} = \vec{A} - \vec{B} = (-5.01\ \text{km})\ \hat{\text{i}} - (1.18\ \text{km})\ \hat{\text{j}}$. The magnitude is $\sqrt{(-5.01\ \text{km})^2 + (-1.18\ \text{km})^2} = 5.1\ \text{km}$.

(b) The angle is $\tan^{-1}[(-1.18\ \text{km})/(-5.01\ \text{km})] = 13.3°,$ south of due west.

21. Reading carefully, we see that the (x, y) specifications for each "dart" are to be interpreted as (Δx, Δy) descriptions of the corresponding displacement vectors. We combine the different parts of this problem into a single exposition.

(a) Along the x axis, we have (with the centimeter unit understood)

$$30.0 + b_x - 20.0 - 80.0 = -140,$$

which gives $b_x = -70.0$ cm.

(b) Along the y axis we have

$$40.0 - 70.0 + c_y - 70.0 = -20.0$$

which yields $c_y = 80.0$ cm.

(c) The magnitude of the final location (–140 , –20.0) is $\sqrt{(-140)^2 + (-20.0)^2} = 141$ cm.

(d) Since the displacement is in the third quadrant, the angle of the overall displacement is given by $\pi + \tan^{-1}[(-20.0)/(-140)]$ or 188° counterclockwise from the $+x$ axis (or $-172°$ counterclockwise from the $+x$ axis).

22. Angles are given in 'standard' fashion, so Eq. 3-5 applies directly. We use this to write the vectors in unit-vector notation before adding them. However, a very different-looking approach using the special capabilities of most graphical calculators can be imagined. Wherever the length unit is not displayed in the solution below, the unit meter should be understood.

(a) Allowing for the different angle units used in the problem statement, we arrive at

$$\begin{aligned}
\vec{E} &= 3.73\ \hat{\mathrm{i}} + 4.70\ \hat{\mathrm{j}} \\
\vec{F} &= 1.29\ \hat{\mathrm{i}} - 4.83\ \hat{\mathrm{j}} \\
\vec{G} &= 1.45\ \hat{\mathrm{i}} + 3.73\ \hat{\mathrm{j}} \\
\vec{H} &= -5.20\ \hat{\mathrm{i}} + 3.00\ \hat{\mathrm{j}} \\
\vec{E} + \vec{F} + \vec{G} + \vec{H} &= 1.28\ \hat{\mathrm{i}}\ + 6.60\ \hat{\mathrm{j}}.
\end{aligned}$$

(b) The magnitude of the vector sum found in part (a) is $\sqrt{(1.28\ \mathrm{m})^2 + (6.60\ \mathrm{m})^2} = 6.72\ \mathrm{m}$.

(c) Its angle measured counterclockwise from the $+x$ axis is $\tan^{-1}(6.60/1.28) = 79.0°$.

(d) Using the conversion factor $\pi\ \mathrm{rad} = 180°$, $79.0° = 1.38$ rad.

23. We have

$$\vec{b} = (3.0)\hat{i} + (4.0)\hat{j} \text{ and } \vec{a} = \hat{i} - \hat{j}$$

That is,

$$\vec{a} = \hat{i} - \hat{j}$$

$$\left|\vec{b}\right| = \sqrt{(3.0)^2 + (4.0)^2} = 5$$

$\hat{a}$ is the unit vector in the direction of $\vec{a}$, that is,

$$\frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} - \hat{j}}{\sqrt{1^2 + (-1)^2}} = \frac{\hat{i} - \hat{j}}{\sqrt{2}}$$

The vector having the same magnitude as that of $\vec{b}$ and parallel to $\vec{a}$ is

$$\left|\vec{b}\right|\left(\frac{\hat{i} - \hat{j}}{\sqrt{2}}\right) = 5\left(\frac{\hat{i} - \hat{j}}{\sqrt{2}}\right)$$

That is,

$$\frac{5}{\sqrt{2}}(\hat{i} - \hat{j})$$

24. As a vector addition problem, we express the situation (described in the problem statement) as $\vec{A} + \vec{B} = (3A)\,\hat{j}$, where $\vec{A} = A\,\hat{i}$ and $B = 6.0$ m. Since $\hat{i} \perp \hat{j}$ we may use the Pythagorean theorem to express B in terms of the magnitudes of the other two vectors:

$$B = \sqrt{(3A)^2 + A^2} \quad \Rightarrow \quad A = \frac{1}{\sqrt{10}}\,B = 1.9 \text{ m} .$$

25. Using unit-vector notation,

$$\vec{a} = (25 \text{ m})\left[-\cos(30°)\hat{\text{i}} - \sin(30°)\,\hat{\text{j}}\right] = (-21.65 \text{ m})\,\hat{\text{i}} - (12.50 \text{ m})\,\hat{\text{j}}$$
$$\vec{b} = (30 \text{ m})\,\hat{\text{j}}$$
$$\vec{c} = \vec{a} + \vec{b} = (-21.65 \text{ m})\,\hat{\text{i}} + (17.50 \text{ m})\,\hat{\text{j}}.$$

Thus, the direction from the first oasis to the second oasis is given by

$$\theta = \tan^{-1}\left(\frac{c_y}{c_x}\right) = \tan^{-1}\left(\frac{17.50 \text{ m}}{-21.65 \text{ m}}\right) = -39°$$

or $51°$ west of due north.

26. The vector equation is $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$. Expressing $\vec{B}$ and $\vec{D}$ in unit-vector notation, we have $(1.69\hat{\text{i}} + 3.63\hat{\text{j}})$ m and $(-2.87\hat{\text{i}} + 4.10\hat{\text{j}})$ m, respectively. Where the length unit is not displayed in the solution below, the unit meter should be understood.

(a) Adding corresponding components, we obtain $\vec{R} = (-3.18\ \text{m})\hat{\text{i}} + (4.72\ \text{m})\hat{\text{j}}$.

(b) Using Eq. 3-6, the magnitude is

$$|\vec{R}| = \sqrt{(-3.18\ \text{m})^2 + (4.72\ \text{m})^2} = 5.69\ \text{m}.$$

(c) The angle is

$$\theta = \tan^{-1}\left(\frac{4.72\ \text{m}}{-3.18\ \text{m}}\right) = -56.0^\circ\ \ (\text{with } -x \text{ axis}).$$

If measured counterclockwise from +*x*-axis, the angle is then $180^\circ - 56.0^\circ = 124^\circ$. Thus, converting the result to polar coordinates, we obtain

$$(-3.18, 4.72) \;\rightarrow\; (5.69 \angle 124^\circ)$$

27. Solving the simultaneous equations yields the answers:

(a) $\vec{d}_1 = 4\,\vec{d}_3 = 8\,\hat{\mathrm{i}} + 16\,\hat{\mathrm{j}}$, and

(b) $\vec{d}_2 = \vec{d}_3 = 2\,\hat{\mathrm{i}} + 4\,\hat{\mathrm{j}}$.

28. Let $\vec{A}$ represent the first part of Beetle 1's trip (0.50 m east or $0.5\,\hat{\text{i}}$) and $\vec{C}$ represent the first part of Beetle 2's trip intended voyage (1.6 m at 50º north of east). For their respective second parts: $\vec{B}$ is 0.70 m at 30º north of east and $\vec{D}$ is the unknown. The final position of Beetle 1 is

$$\vec{A}+\vec{B}=(0.5\text{ m})\hat{\text{i}}+(0.7\text{ m})(\cos 30°\ \hat{\text{i}}+\sin 30°\ \hat{\text{j}})=(1.11\text{ m})\ \hat{\text{i}}+(0.35\text{ m})\ \hat{\text{j}}.$$

The equation relating these is $\vec{A}+\vec{B}=\vec{C}+\vec{D}$, where

$$\vec{C}=(1.60\text{ m})(\cos 50.0°\hat{\text{i}}+\sin 50.0°\hat{\text{j}})=(1.03\text{ m})\hat{\text{i}}+(1.23\text{ m})\hat{\text{j}}$$

(a) We find $\vec{D}=\vec{A}+\vec{B}-\vec{C}=(0.08\text{ m})\hat{\text{i}}+(-0.88\text{ m})\hat{\text{j}}$, and the magnitude is D = 0.88 m.

(b) The angle is $\tan^{-1}(-0.88/0.08)=-85°$, which is interpreted to mean 85º south of east (or 5º east of south).

29. Let $l_0 = 2.0\text{ cm}$ be the length of each segment. The nest is located at the endpoint of segment *w*.

(a) Using unit-vector notation, the displacement vector for point *A* is

$$\begin{aligned}\vec{d}_A &= \vec{w}+\vec{v}+\vec{i}+\vec{h} = l_0(\cos 60°\hat{\text{i}}+\sin 60°\,\hat{\text{j}})+\left(l_0\,\hat{\text{j}}\right)+l_0(\cos 120°\hat{\text{i}}+\sin 120°\,\hat{\text{j}})+\left(l_0\,\hat{\text{j}}\right)\\ &= (2+\sqrt{3})l_0\,\hat{\text{j}}.\end{aligned}$$

Therefore, the magnitude of $\vec{d}_A$ is $|\vec{d}_A| = (2+\sqrt{3})(2.0\text{ cm}) = 7.5\text{ cm}$.

(b) The angle of $\vec{d}_A$ is $\theta = \tan^{-1}(d_{A,y}/d_{A,x}) = \tan^{-1}(\infty) = 90°$.

(c) Similarly, the displacement for point *B* is

$$\begin{aligned}\vec{d}_B &= \vec{w}+\vec{v}+\vec{j}+\vec{p}+\vec{o}\\ &= l_0(\cos 60°\hat{\text{i}}+\sin 60°\,\hat{\text{j}})+\left(l_0\,\hat{\text{j}}\right)+l_0(\cos 60°\hat{\text{i}}+\sin 60°\,\hat{\text{j}})+l_0(\cos 30°\hat{\text{i}}+\sin 30°\,\hat{\text{j}})+\left(l_0\,\hat{\text{i}}\right)\\ &= (2+\sqrt{3}/2)l_0\,\hat{\text{i}}+(3/2+\sqrt{3})l_0\,\hat{\text{j}}.\end{aligned}$$

Therefore, the magnitude of $\vec{d}_B$ is

$$|\vec{d}_B| = l_0\sqrt{(2+\sqrt{3}/2)^2+(3/2+\sqrt{3})^2} = (2.0\text{ cm})(4.3) = 8.6\text{ cm}.$$

(d) The direction of $\vec{d}_B$ is

$$\theta_B = \tan^{-1}\left(\frac{d_{B,y}}{d_{B,x}}\right) = \tan^{-1}\left(\frac{3/2+\sqrt{3}}{2+\sqrt{3}/2}\right) = \tan^{-1}(1.13) = 48°.$$

30. Many of the operations are done efficiently on most modern graphical calculators using their built-in vector manipulation and rectangular ↔ polar "shortcuts." In this solution, we employ the "traditional" methods (such as Eq. 3-6).

(a) The magnitude of $\vec{a}$ is $a = \sqrt{(4.0\text{ m})^2 + (-3.0\text{ m})^2} = 5.0\text{ m}$.

(b) The angle between $\vec{a}$ and the +*x* axis is $\tan^{-1}$ [(–3.0 m)/(4.0 m)] = –37°. The vector is 37° *clockwise* from the axis defined by $\hat{\text{i}}$.

(c) The magnitude of $\vec{b}$ is $b = \sqrt{(6.0\text{ m})^2 + (8.0\text{ m})^2} = 10\text{ m}$.

(d) The angle between $\vec{b}$ and the +*x* axis is $\tan^{-1}$[(8.0 m)/(6.0 m)] = 53°.

(e) $\vec{a} + \vec{b} = (4.0\text{ m} + 6.0\text{ m})\,\hat{\text{i}} + [(-3.0\text{ m}) + 8.0\text{ m}]\hat{\text{j}} = (10\text{ m})\hat{\text{i}} + (5.0\text{ m})\hat{\text{j}}$. The magnitude of this vector is $|\vec{a} + \vec{b}| = \sqrt{(10\text{ m})^2 + (5.0\text{ m})^2} = 11\text{ m}$; we round to two significant figures in our results.

(f) The angle between the vector described in part (e) and the +*x* axis is $\tan^{-1}$[(5.0 m)/(10 m)] = 27°.

(g) $\vec{b} - \vec{a} = (6.0\text{ m} - 4.0\text{ m})\,\hat{\text{i}} + [8.0\text{ m} - (-3.0\text{ m})]\,\hat{\text{j}} = (2.0\text{ m})\,\hat{\text{i}} + (11\text{ m})\,\hat{\text{j}}$. The magnitude of this vector is $|\vec{b} - \vec{a}| = \sqrt{(2.0\text{ m})^2 + (11\text{ m})^2} = 11\text{ m}$, which is, interestingly, the same result as in part (e) (exactly, not just to 2 significant figures) (this curious coincidence is made possible by the fact that $\vec{a} \perp \vec{b}$).

(h) The angle between the vector described in part (g) and the +*x* axis is $\tan^{-1}$[(11 m)/(2.0 m)] = 80°.

(i) $\vec{a} - \vec{b} = (4.0\text{ m} - 6.0\text{ m})\,\hat{\text{i}} + [(-3.0\text{ m}) - 8.0\text{ m}]\,\hat{\text{j}} = (-2.0\text{ m})\,\hat{\text{i}} + (-11\text{ m})\,\hat{\text{j}}$. The magnitude of this vector is

$$|\vec{a} - \vec{b}| = \sqrt{(-2.0\text{ m})^2 + (-11\text{ m})^2} = 11\text{ m}.$$

(j) The two possibilities presented by a simple calculation for the angle between the vector described in part (i) and the +*x* direction are $\tan^{-1}$ [(–11 m)/(–2.0 m)] = 80°, and 180° + 80° = 260°. The latter possibility is the correct answer (see part (k) for a further observation related to this result).

(k) Since $\vec{a} - \vec{b} = (-1)(\vec{b} - \vec{a})$, they point in opposite (anti-parallel) directions; the angle between them is 180°.

3.1 (a) With a = 15.0 m and θ= 56.0° we find $a_x = a \cos \theta$= 8.39 m.

(b) Similarly, $a_y = a \sin \theta$= 12.4 m.

(c) The angle relative to the new coordinate system is θ' = (56.0° – 18.0°) = 38.0°. Thus, $a'_x = a\cos\theta' = 11.8$ m.

(d) Similarly, $a'_y = a \sin \theta'$ = 9.23 m.

32. Examining the figure, we see that $\vec{a} + \vec{b} + \vec{c} = 0$, where $\vec{a} \perp \vec{b}$.

(a) $|\vec{a} \times \vec{b}| = (3.0)(4.0) = 12$ since the angle between them is 90º.

(b) Using the Right-Hand Rule, the vector $\vec{a}\times\vec{b}$ points in the $\hat{\mathrm{i}}\times\hat{\mathrm{j}}=\hat{\mathrm{k}}$, or the $+z$ direction.

(c) $|\vec{a} \times \vec{c}| = |\vec{a} \times (-\vec{a} - \vec{b})| = |-(\vec{a} \times \vec{b})| = 12.$

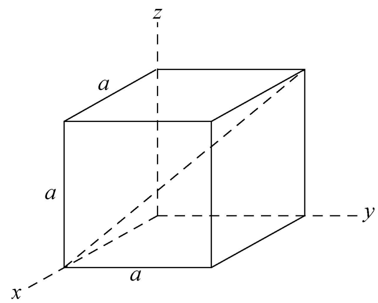
(d) The vector $-\vec{a}\times\vec{b}$ points in the $-\hat{\mathrm{i}}\times\hat{\mathrm{j}}=-\hat{\mathrm{k}}$, or the $-z$ direction.

(e) $|\vec{b} \times \vec{c}| = |\vec{b} \times (-\vec{a} - \vec{b})| = |-(\vec{b} \times \vec{a})| = |(\vec{a} \times \vec{b})| = 12.$

(f) The vector points in the $+z$ direction, as in part (a).

33. (a) As can be seen from Figure 3-30, the point diametrically opposite the origin (0,0,0) has position vector $a\ \hat{\mathrm{i}} + a\ \hat{\mathrm{j}} + a\ \hat{\mathrm{k}}$ and this is the vector along the "body diagonal."

(b) From the point (a, 0, 0), which corresponds to the position vector $a\ \hat{\mathrm{i}}$, the diametrically opposite point is (0, a, a) with the position vector $a\ \hat{\mathrm{j}} + a\ \hat{\mathrm{k}}$. Thus, the vector along the line is the difference $-a\hat{\mathrm{i}} + a\hat{\mathrm{j}} + a\hat{\mathrm{k}}$.



(c) If the starting point is (0, a, 0) with the corresponding position vector $a\ \hat{\mathrm{j}}$, the diametrically opposite point is (a, 0, a) with the position vector $a\ \hat{\mathrm{i}} + a\ \hat{\mathrm{k}}$. Thus, the vector along the line is the difference $a\ \hat{\mathrm{i}} - a\ \hat{\mathrm{j}} + a\ \hat{\mathrm{k}}$.

(d) If the starting point is (a, a, 0) with the corresponding position vector $a\ \hat{\mathrm{i}} + a\ \hat{\mathrm{j}}$, the diametrically opposite point is (0, 0, a) with the position vector $a\ \hat{\mathrm{k}}$. Thus, the vector along the line is the difference $-a\ \hat{\mathrm{i}} - a\ \hat{\mathrm{j}} + a\ \hat{\mathrm{k}}$.

(e) Consider the vector from the back lower left corner to the front upper right corner. It is $a\ \hat{\mathrm{i}} + a\ \hat{\mathrm{j}} + a\ \hat{\mathrm{k}}$. We may think of it as the sum of the vector $a\hat{\mathrm{i}}$ parallel to the x axis and the vector $a\hat{\mathrm{j}} + a\ \hat{\mathrm{k}}$ perpendicular to the x axis. The tangent of the angle between the vector and the x axis is the perpendicular component divided by the parallel component. Since the magnitude of the perpendicular component is $\sqrt{a^2 + a^2} = a\sqrt{2}$ and the magnitude of the parallel component is a, $\tan\theta = \left(a\sqrt{2}\right)/a = \sqrt{2}$. Thus $\theta = 54.7°$. The angle between the vector and each of the other two adjacent sides (the y and z axes) is the same as is the angle between any of the other diagonal vectors and any of the cube sides adjacent to them.

(f) The length of any of the diagonals is given by $\sqrt{a^2 + a^2 + a^2} = a\sqrt{3}$.

34. We apply Eq. 3-23 and Eq. 3-27.

(a) $\vec{a} \times \vec{b} = (a_x b_y - a_y b_x)\,\hat{\mathrm{k}}$ since all other terms vanish, due to the fact that neither $\vec{a}$ nor $\vec{b}$ have any z components. Consequently, we obtain $[(3.0)(4.0) - (5.0)(2.0)]\hat{\mathrm{k}} = 2.0\,\hat{\mathrm{k}}$.

(b) $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$ yields (3.0)(2.0) + (5.0)(4.0) = 26.

(c) $\vec{a} + \vec{b} = (3.0 + 2.0)\,\hat{\mathrm{i}} + (5.0 + 4.0)\,\hat{\mathrm{j}} \Rightarrow (\vec{a} + \vec{b}) \cdot \vec{b} = (5.0)\,(2.0) + (9.0)\,(4.0) = 46$.

(d) Several approaches are available. In this solution, we will construct a $\hat{b}$ unit-vector and "dot" it (take the scalar product of it) with $\vec{a}$. In this case, we make the desired unit-vector by

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{2.0\,\hat{\mathrm{i}} + 4.0\,\hat{\mathrm{j}}}{\sqrt{(2.0)^2 + (4.0)^2}}.$$

We therefore obtain

$$a_b = \vec{a} \cdot \hat{b} = \frac{(3.0)(2.0) + (5.0)(4.0)}{\sqrt{(2.0)^2 + (4.0)^2}} = 5.8.$$

35. The vector representation is as shown in the following figure. We have

$$\vec{p} = \hat{i}(3.5\cos 220^\circ + \hat{j}(3.5\sin 220^\circ)$$
$$= \hat{i}(3.5)\cos(180+40) + \hat{j}(3.5)\sin(180+40)$$
$$= (-3.5\cos 40)\hat{i} + (-3.5\sin 40)\hat{j}$$

and

$$\vec{q} = \hat{i}(6.3\cos 75^\circ) + \hat{j}(6.3\sin 75^\circ)$$

(a) Now

$$\vec{p}\times\vec{q} = \{(-3.5\cos 40)\hat{i} + (-3.5\sin 40)\hat{j}\}\times\{(6.3\cos 75)\hat{i} + (6.3\sin 75)\hat{j}\}$$
$$= \hat{k}(-3.5\times 6.3\cos 40\sin 75) - \hat{k}(-3.5\times 6.3\sin 40\cos 75) \qquad (\because \hat{i}\times\hat{i} = \hat{j}\times\hat{j} = 0;\ \hat{i}\times\hat{j} = \hat{k};\ \hat{j}\times\hat{i} = -\hat{k})$$
$$= 3.5\times 6.3\hat{k}\{\sin 40\cos 75 - \cos 40\sin 75\}$$
$$= 3.5\times 6.3\hat{k}\{\sin(40-75)\}\hat{k} = \{3.5\times 6.3\sin(-3.5)\}\hat{k}$$
$$= (-3.5\times 6.3\sin 35)\hat{k} = -(12.6473604226)\hat{k} \approx -(12.6)\hat{k}.$$

(b) Now

$$\vec{p}\cdot\vec{q} = -3.5\times 6.3\cos 40\cos 75 - 3.5\times 6.3\sin 40\sin 75$$
$$= -3.5\times 6.3\{\cos 40\cos 75 + \sin 40\sin 75\}$$
$$= -3.5\times 6.3\cos(40-75)$$
$$= -3.5\times 6.3\cos(-35)$$
$$= -3.5\times 6.3\cos 35$$
$$= -3.5\times 6.3\times 0.8191520443$$
$$= -18.0623025768$$
$$\approx -18.1.$$

36. We have

$$\vec{p}_1 = 4\hat{i} - 3\hat{j} + 5\hat{k};$$
$$\vec{p}_2 = -6\hat{i} + 3\hat{j} - 2\hat{k}.$$

Therefore,

$$\vec{p}_1 + \vec{p}_2 = -2\hat{i} + 0\hat{j} + 3\hat{k}$$

$$\vec{p}_1 \times \vec{p}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & 5 \\ -6 & 3 & -2 \end{vmatrix} = \hat{i}(6-15) - \hat{j}(-8+30) + \hat{k}(12-18)$$
$$= \hat{i}(-9) - 22\hat{j} - 6\hat{k}$$
$$= -9\hat{i} - 22\hat{j} - 6\hat{k}$$

Now,

$$(\vec{p}_1 + \vec{p}_2)\cdot(\vec{p}_1 \times 5\vec{p}_2) = 5(\vec{p}_1 + \vec{p}_2)\cdot(\vec{p}_1 \times \vec{p}_2)$$
$$= 5(-2\hat{i} + 3\hat{k})\cdot(-9\hat{i} - 22\hat{j} - 6\hat{k})$$
$$= 5(\cancel{18} + 0 - \cancel{18}) = 0$$

37. We apply Eq. 3-23 and Eq.3-27. If a vector-capable calculator is used, this makes a good exercise for getting familiar with those features. Here we briefly sketch the method.

(a) We note that $\vec{b}\times\vec{c} = -8.0\,\hat{\mathrm{i}} + 5.0\,\hat{\mathrm{j}} + 6.0\hat{\mathrm{k}}$. Thus,

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = (3.0)\,(-8.0) + (3.0)(5.0) + (-2.0)\,(6.0) = -21.$$

(b) We note that $\vec{b}+\vec{c} = 1.0\,\hat{\mathrm{i}} - 2.0\,\hat{\mathrm{j}} + 3.0\,\hat{\mathrm{k}}$. Thus,

$$\vec{a}\cdot(\vec{b}+\vec{c}) = (3.0)\,(1.0) + (3.0)\,(-2.0) + (-2.0)\,(3.0) = -9.0.$$

(c) Finally,

$$\begin{aligned}\vec{a}\times(\vec{b}+\vec{c}) &= [(3.0)(3.0)-(-2.0)(-2.0)]\,\hat{\mathrm{i}} + [(-2.0)(1.0)-(3.0)(3.0)]\,\hat{\mathrm{j}} \\ &\quad + [(3.0)(-2.0)-(3.0)(1.0)]\,\hat{\mathrm{k}} \\ &= 5.0\hat{\mathrm{i}} - 11\hat{\mathrm{j}} - 9.0\hat{\mathrm{k}}\end{aligned}$$

38. Using the fact that

$$\hat{\mathrm{i}}\times\hat{\mathrm{j}}=\hat{\mathrm{k}},\quad \hat{\mathrm{j}}\times\hat{\mathrm{k}}=\hat{\mathrm{i}},\quad \hat{\mathrm{k}}\times\hat{\mathrm{i}}=\hat{\mathrm{j}}$$

we obtain

$$\begin{aligned}2\vec{A}\times\vec{B}&=2\left(2.00\hat{\mathrm{i}}+3.00\hat{\mathrm{j}}-4.00\hat{\mathrm{k}}\right)\times\left(-3.00\hat{\mathrm{i}}+4.00\hat{\mathrm{j}}+2.00\hat{\mathrm{k}}\right)\\&=44.0\hat{\mathrm{i}}+16.0\hat{\mathrm{j}}+34.0\hat{\mathrm{k}}.\end{aligned}$$

Next, making use of

$$\begin{aligned}\hat{\mathrm{i}}\cdot\hat{\mathrm{i}}&=\hat{\mathrm{j}}\cdot\hat{\mathrm{j}}=\hat{\mathrm{k}}\cdot\hat{\mathrm{k}}=1\\\hat{\mathrm{i}}\cdot\hat{\mathrm{j}}&=\hat{\mathrm{j}}\cdot\hat{\mathrm{k}}=\hat{\mathrm{k}}\cdot\hat{\mathrm{i}}=0\end{aligned}$$

we have

$$\begin{aligned}3\vec{C}\cdot\left(2\vec{A}\times\vec{B}\right)&=3\left(7.00\hat{\mathrm{i}}-8.00\hat{\mathrm{j}}\right)\cdot\left(44.0\hat{\mathrm{i}}+16.0\hat{\mathrm{j}}+34.0\hat{\mathrm{k}}\right)\\&=3[(7.00)(44.0)+(-8.00)(16.0)+(0)(34.0)]=540.\end{aligned}$$

39. From the definition of the dot product between $\vec{A}$ and $\vec{B}$, $\vec{A}\cdot\vec{B} = AB\cos\theta$, we have

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$$

With $A = 6.00$, $B = 7.00$ and $\vec{A}\cdot\vec{B} = 14.0$, $\cos\theta = 0.333$, or $\theta = 70.5°$.

40. The displacement vectors can be written as (in meters)

$$\vec{d}_1 = (4.80 \text{ m})(\cos 63.0°\,\hat{j} + \sin 63.0°\,\hat{k}) = (2.179 \text{ m})\,\hat{j} + (4.277 \text{ m})\,\hat{k}$$
$$\vec{d}_2 = (1.40 \text{ m})(\cos 30.0°\,\hat{i} + \sin 30.0°\,\hat{k}) = (1.212 \text{ m})\,\hat{i} + (0.700 \text{ m})\,\hat{k}\,.$$

(a) The dot product of $\vec{d}_1$ and $\vec{d}_2$ is

$$\vec{d}_1 \cdot \vec{d}_2 = (2.179\,\hat{j} + 4.277\,\hat{k}) \cdot (1.212\,\hat{i} + 0.70\,\hat{k}) = (4.277\,\hat{k}) \cdot (0.700\,\hat{k}) = 2.99 \text{ m}^2.$$

(b) The cross product of $\vec{d}_1$ and $\vec{d}_2$ is

$$\begin{aligned}\vec{d}_1 \times \vec{d}_2 &= (2.179\,\hat{j} + 4.277\,\hat{k}) \times (1.212\,\hat{i} + 0.700\,\hat{k}) \\ &= (2.179)(1.212)(-\hat{k}) + (2.179)(0.700)\hat{i} + (4.277)(1.212)\hat{j} \\ &= (1.53\hat{i} + 5.19\,\hat{j} - 2.64\,\hat{k}) \text{ m}^2.\end{aligned}$$

(c) The magnitudes of $\vec{d}_1$ and $\vec{d}_2$ are

$$d_1 = \sqrt{(2.179 \text{ m})^2 + (4.277 \text{ m})^2} = 4.80 \text{ m}$$
$$d_2 = \sqrt{(1.212 \text{ m})^2 + (0.700 \text{ m})^2} = 1.40 \text{ m}.$$

Thus, the angle between the two vectors is

$$\theta = \cos^{-1}\left(\frac{\vec{d}_1 \cdot \vec{d}_2}{d_1 d_2}\right) = \cos^{-1}\left(\frac{2.99 \text{ m}^2}{(4.80 \text{ m})(1.40 \text{ m})}\right) = 63.6°.$$

41. The angle between the two vectors $\vec{a}$ and $\vec{b}$ is given by

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{ab} = \frac{(4.0\hat{\mathrm{i}} + 4.0\hat{\mathrm{j}} + 4.0\hat{\mathrm{k}})\cdot(30\hat{\mathrm{i}} + 2.0\hat{\mathrm{j}} + 4.0\hat{\mathrm{k}})}{\sqrt{(4.0)^2 + (4.0)^2 + (4.0)^2}\sqrt{(3.0)^2 + (2.0)^2 + (4.0)^2}} = \frac{12 + 8 + 16}{\sqrt{16+16+16}\sqrt{9+4+16}}$$

$$= \frac{36}{\sqrt{48}\sqrt{29}} = 3\sqrt{\frac{3}{29}}$$

Therefore,

$$\theta = \cos^{-1}\left(3\sqrt{\frac{3}{29}}\right) = 0.266\ \mathrm{rad} = 15.23^\circ \approx 15^\circ.$$

:

42. The two vectors are written as, in unit of meters,

$$\vec{d}_1 = 4.0\,\hat{\text{i}} + 5.0\,\hat{\text{j}} = d_{1x}\hat{\text{i}} + d_{1y}\hat{\text{j}}, \qquad \vec{d}_2 = -3.0\,\hat{\text{i}} + 4.0\,\hat{\text{j}} = d_{2x}\hat{\text{i}} + d_{2y}\hat{\text{j}}$$

(a) The vector (cross) product gives

$$\vec{d}_1 \times \vec{d}_2 = (d_{1x}d_{2y} - d_{1y}d_{2x})\hat{\text{k}} = [(4.0)(4.0) - (5.0)(-3.0)]\hat{\text{k}} = 31\ \hat{\text{k}}$$

(b) The scalar (dot) product gives

$$\vec{d}_1 \cdot \vec{d}_2 = d_{1x}d_{2x} + d_{1y}d_{2y} = (4.0)(-3.0) + (5.0)(4.0) = 8.0.$$

(c) $(\vec{d}_1 + \vec{d}_2)\cdot\vec{d}_2 = \vec{d}_1 \cdot \vec{d}_2 + d_2^2 = 8.0 + (-3.0)^2 + (4.0)^2 = 33.$

(d) Note that the magnitude of the d_1 vector is $\sqrt{16+25} = 6.4$. Now, the dot product is $(6.4)(5.0)\cos\theta = 8$. Dividing both sides by 32 and taking the inverse cosine yields $\theta = 75.5°$. Therefore the component of the d_1 vector along the direction of the d_2 vector is $6.4\cos\theta \approx 1.6$.

43. **THINK** In this problem we are given three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ on the xy-plane, and asked to calculate their components.

EXPRESS From the figure, we note that $\vec{c} \perp \vec{b}$, which implies that the angle between $\vec{c}$ and the $+x$ axis is $\theta + 90°$. In unit-vector notation, the three vectors can be written as

$$\vec{a} = a_x\hat{\mathrm{i}}$$
$$\vec{b} = b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}} = (b\cos\theta)\hat{\mathrm{i}} + (b\sin\theta)\hat{\mathrm{j}}$$
$$\vec{c} = c_x\hat{\mathrm{i}} + c_y\hat{\mathrm{j}} = [c\cos(\theta + 90°)]\hat{\mathrm{i}} + [c\sin(\theta + 90°)]\hat{\mathrm{j}}.$$

The above expressions allow us to evaluate the components of the vectors.

ANALYZE

(a) The x-component of $\vec{a}$ is $a_x = a \cos 0° = a = 3.00$ m.

(b) Similarly, the y-componnet of $\vec{a}$ is $a_y = a \sin 0° = 0$.

(c) The x-component of $\vec{b}$ is $b_x = b \cos 30° = (4.00 \text{ m}) \cos 30° = 3.46$ m,

(d) and the y-component is $b_y = b \sin 30° = (4.00 \text{ m}) \sin 30° = 2.00$ m.

(e) The x-component of $\vec{c}$ is $c_x = c \cos 120° = (10.0 \text{ m}) \cos 120° = -5.00$ m,

(f) and the y-component is $c_y = c \sin 30° = (10.0 \text{ m}) \sin 120° = 8.66$ m.

(g) The fact that $\vec{c} = p\vec{a} + q\vec{b}$ implies

$$\vec{c} = c_x\hat{\mathrm{i}} + c_y\hat{\mathrm{j}} = p(a_x\hat{\mathrm{i}}) + q(b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}}) = (pa_x + qb_x)\hat{\mathrm{i}} + qb_y\hat{\mathrm{j}}$$

or

$$c_x = pa_x + qb_x, \qquad c_y = qb_y\,.$$

Substituting the values found above, we have

$$-5.00 \text{ m} = p\,(3.00 \text{ m}) + q\,(3.46 \text{ m})$$
$$8.66 \text{ m} = q\,(2.00 \text{ m}).$$

Solving these equations, we find $p = -6.67$.

(h) Similarly, $q = 4.33$ (note that it's easiest to solve for q first). The numbers p and q have no units.

LEARN This exercise shows that given two (non-parallel) vectors in two dimensions, the third vector can always be written as a linear combination of the first two.

44. Applying Eq. 3-23, $\vec{F} = q\vec{v} \times \vec{B}$ (where q is a scalar) becomes

$$F_x\,\hat{\mathrm{i}} + F_y\,\hat{\mathrm{j}} + F_z\,\hat{\mathrm{k}} = q\left(v_yB_z - v_zB_y\right)\hat{\mathrm{i}} + q\left(v_zB_x - v_xB_z\right)\hat{\mathrm{j}} + q\left(v_xB_y - v_yB_x\right)\hat{\mathrm{k}}$$

which — plugging in values — leads to three equalities:

$$\begin{aligned} 4.0 &= 3\,(4.0B_z - 6.0B_y) \\ -20 &= 3\,(6.0B_x - 2.0B_z) \\ 12 &= 3\,(2.0B_y - 4.0B_x) \end{aligned}$$

Since we are told that $B_x = B_y$, the third equation leads to $B_y = -2.0$. Inserting this value into the first equation, we find $B_z = -2.7$. Thus, our answer is

$$\vec{B} = -2.0\,\hat{\mathrm{i}} - 2.0\,\hat{\mathrm{j}} - 2.7\,\hat{\mathrm{k}}.$$