2021 Numerical Analysis Midterm.

Open book and notes. Answer in English. Show the intermediate processes for the partial credits. Compute up to 4 digits after the decimal points. You may use a calculator.

1. (10pts) Write a Fortran function Frob(A, N) that computes the Frobenius norm of a matrix A.

The Frobenius norm of A is

$$\| A \|_F = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} (a_{i,j})^2}$$

2. (15 pts)

$f(x) = \exp(-x^2) - \sin(x)$, $0 < x < 1$

It has one root in [0, 1].

a) Perform 3 steps of bisection method to find an estimate of the root.

b) $x_0 = 0.2$, $x_1 = 0.5$, Perform 2 steps of secant method.

c) With $x_0 = 0.5$, perform 2 steps of Newton method.

3. (15pts) For the following system of equations perform 2 steps of Newton's method.

$$f(x,y) = x^2 + 4y^2 - 16 = 0$$
$$g(x,y) = xy^2 - 4 \ = 0$$

Use (0.9, 1.8) as the initial guess.