by span$\{v_1, \ldots, v_k\}$ or span$\{S\}$

ex. Show that span$\left(\begin{bmatrix}2\\-1\end{bmatrix}, \begin{bmatrix}1\\3\end{bmatrix}\right) = \mathbb{R}^2$

sol. $\left[\begin{array}{cc|c} 2 & 1 & a \\ -1 & 3 & b \end{array}\right] \to \left[\begin{array}{cc|c} -1 & 3 & b \\ 2 & 1 & a \end{array}\right]$

$\to \begin{bmatrix} -1 & 3 & b \\ 0 & 7 & a+2b \end{bmatrix} \to y = \frac{a+2b}{7}, \; x = \frac{3a-b}{7}$

$\left(\frac{3a-b}{7}\right)\begin{bmatrix}2\\-1\end{bmatrix} + \frac{a+2b}{7}\begin{bmatrix}1\\3\end{bmatrix} = \begin{bmatrix}a\\b\end{bmatrix}$

Rem. $\mathbb{R}^2 = \text{span}\left(\begin{bmatrix}2\\-1\end{bmatrix}, \begin{bmatrix}1\\3\end{bmatrix}, \begin{bmatrix}5\\7\end{bmatrix}\right)$

If $x\begin{bmatrix}2\\-1\end{bmatrix} + y\begin{bmatrix}1\\3\end{bmatrix} = \begin{bmatrix}a\\b\end{bmatrix}$, then

$x\begin{bmatrix}2\\-1\end{bmatrix} + y\begin{bmatrix}1\\3\end{bmatrix} + 0\begin{bmatrix}5\\7\end{bmatrix} = \begin{bmatrix}a\\b\end{bmatrix}$

Any set of vectors containing a spanning set will also be a spanning set.