[12-5]

#2. $$\begin{vmatrix} a & b & c & d \\ -b & a & -d & c \\ -c & d & a & -b \\ -d & -c & b & a \end{vmatrix}\begin{vmatrix} a & -b & -c & -d \\ b & a & d & -c \\ c & -d & a & b \\ d & c & -b & a \end{vmatrix}$$

$$= (a^2+b^2+c^2+d^2)^2 \cdot (a^2+b^2+c^2+d^2)^2 = (a^2+b^2+c^2+d^2)^4$$

#3. $$\begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{vmatrix}\begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{vmatrix}$$

$$= (\cos^2\theta + \sin^2\theta)\cdot(\cos^2\alpha + \sin^2\alpha)$$

$$= 1\cdot 1 = 1.$$

#5. $$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 7 \end{bmatrix}$$

$\Rightarrow$ (i) $\mathrm{Det}(A) = 21 + 8 + 30 - 9 - 20 - 28 = 2.$

(ii) $A_{11} = \begin{vmatrix} 3 & 4 \\ 5 & 7 \end{vmatrix} = 1$ $\quad A_{12} = -\begin{vmatrix} 2 & 4 \\ 1 & 7 \end{vmatrix} = -10$ $\quad A_{13} = \begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} = 7$

$A_{21} = -\begin{vmatrix} 2 & 3 \\ 5 & 7 \end{vmatrix} = 1$ $\quad A_{22} = \begin{vmatrix} 1 & 3 \\ 1 & 7 \end{vmatrix} = 4$ $\quad A_{23} = -\begin{vmatrix} 1 & 2 \\ 1 & 5 \end{vmatrix} = -3$

$A_{31} = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = -1$ $\quad A_{32} = -\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = 2$ $\quad A_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1$

$$\Rightarrow \bar{A} = \begin{vmatrix} 1 & 1 & -1 \\ -10 & 4 & 2 \\ 7 & -3 & -1 \end{vmatrix} \qquad \therefore A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -10 & 4 & 2 \\ 7 & -3 & -1 \end{bmatrix}$$

[12-6]

#1.
$$\begin{cases} 5x + 3y + 3z = 4 \\ 2x + 6y - 3z = -2 \\ 8x - 3y + 2z = -7 \end{cases}$$

$$D = \begin{vmatrix} 5 & 3 & 3 \\ 2 & 6 & -3 \\ 8 & -3 & 2 \end{vmatrix} = 60 - 72 - 18 - 144 - 45 - 12$$
$$= -231.$$

$$D_x = \begin{vmatrix} 4 & 3 & 3 \\ -2 & 6 & -3 \\ -7 & -3 & 2 \end{vmatrix} = 48 + 63 + 18 + 126 - 36 + 12$$
$$= 231.$$

$$D_y = \begin{vmatrix} 5 & 4 & 3 \\ 2 & -2 & -3 \\ 8 & -7 & 2 \end{vmatrix} = -20 - 96 - 42 + 48 - 105 - 16$$
$$= -231$$

$$D_z = \begin{vmatrix} 5 & 3 & 4 \\ 2 & 6 & -2 \\ 8 & -3 & -7 \end{vmatrix} = -210 - 48 - 24 - 30 + 42 - 192 = -462$$

$\therefore\ x = -1.\quad y = 1.\quad z = 2.$

#3.
$$\begin{cases} x + y + z + u = 2 \\ 2x - y + 2z - u = -5 \\ 3x + 2y + 3z + 4u = 7 \\ x - 2y - 3z + 2u = 5 \end{cases}$$

$$D = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & -1 & 2 & -1 \\ 3 & 2 & 3 & 4 \\ 1 & -2 & -3 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 1 & 0 \\ 2 & -3 & 2 & -3 \\ 3 & -1 & 3 & 1 \\ 1 & -3 & -3 & 5 \end{vmatrix} = -24.$$

$$D_x = \begin{vmatrix} 2 & 1 & 1 & 1 \\ -5 & -1 & 2 & -1 \\ 7 & 2 & 3 & 4 \\ 5 & -2 & -3 & 2 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 & 0 \\ -3 & -3 & 2 & -3 \\ 4 & -1 & 3 & 1 \\ 9 & 1 & -3 & 5 \end{vmatrix} = 0.$$

$$D_y = \begin{vmatrix} 1 & 2 & 1 & 1 \\ 2 & -5 & 2 & -1 \\ 3 & 7 & 3 & 4 \\ 1 & 5 & -3 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & -9 & 0 & -3 \\ 3 & 1 & 0 & 1 \\ 1 & 3 & -4 & 1 \end{vmatrix} = -24$$

$$D_z = \begin{vmatrix} 1 & 1 & 2 & 1 \\ 2 & -1 & -5 & -1 \\ 3 & 2 & 7 & 4 \\ 1 & -2 & 5 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & -3 & -9 & -3 \\ 3 & -1 & 1 & 1 \\ 1 & -3 & 3 & 1 \end{vmatrix} = 24$$

$$D_u = \begin{vmatrix} 1 & 1 & 1 & 2 \\ 2 & -1 & 2 & -5 \\ 3 & 2 & 3 & 7 \\ 1 & -2 & -3 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & -3 & 0 & -9 \\ 3 & -1 & 0 & 1 \\ 1 & -3 & -4 & 3 \end{vmatrix} = -48$$

$\therefore\ x = 0,\quad y = 1,\quad z = -1,\quad u = 2.$