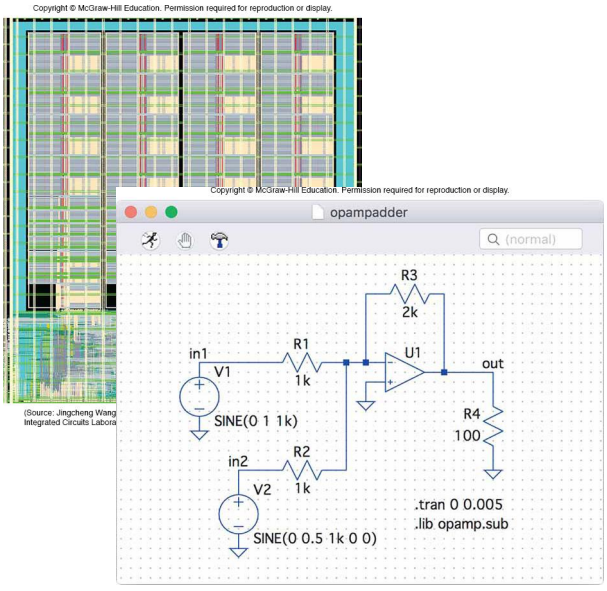
Chapter 4. Node & Mesh Analysis Methods

1. Node Analysis
2. Mesh Analysis

Circuit Analysis

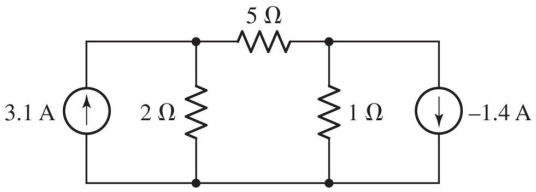
- As circuits get more complicated, we need an organized method of applying KVL, KCL, and Ohm's Law
  - Nodal analysis assigns voltages to each node, and then we apply KCL.
  - Mesh analysis assigns currents to each mesh, and then we apply KVL.

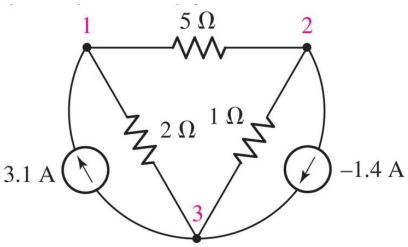
Deep learning processor with 2×10^7 transistors



Nodal Analysis Method

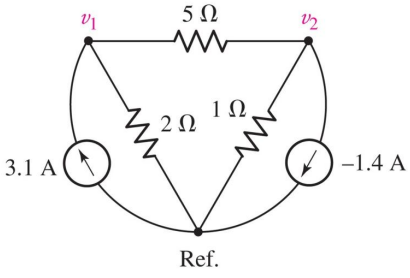
1. Identify nodes in the circuit.
   - v 3 nodes in the circuit
2. Choose a reference node.
   - v 가장 많은 branch를 연결하는 node를 기준 node로 선택
3. Assign node voltages to all node.





Nodal Analysis Method

4. Compute the branch voltage and branch current in terms of the node voltages.
5. Apply KCL to each node.



Node-1:

$$\frac{v_1}{2} + \frac{v_1 - v_2}{5} = 3.1$$

Node-2:

$$\frac{v_1 - v_2}{5} = v_2 - 1.4$$

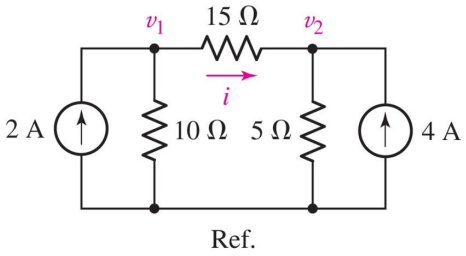
$$\begin{bmatrix} 0.7 & -0.2 \\ 0.2 & -1.2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 3.1 \\ -1.4 \end{bmatrix}$$

$$v_1 = \frac{\begin{vmatrix} 3.1 & -0.2 \\ -1.4 & -1.2 \end{vmatrix}}{\begin{vmatrix} 0.7 & -0.2 \\ 0.2 & -1.2 \end{vmatrix}} = \frac{-4}{-0.8} = 5,\ v_2 = \frac{\begin{vmatrix} 0.7 & 3.1 \\ 0.2 & -1.4 \end{vmatrix}}{\begin{vmatrix} 0.7 & -0.2 \\ 0.2 & -1.2 \end{vmatrix}} = 2$$

Nodal Analysis Method

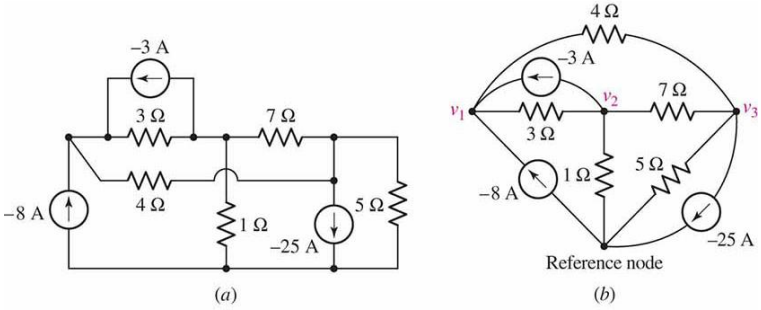
- Example 4.1 Find i.
  - Node-1: $\frac{v_1}{10} + \frac{v_1 - v_2}{15} = 2$
  - Node-2: $\frac{v_2}{5} = \frac{v_1 - v_2}{15} + 4$
  - $i = \frac{v_1 - v_2}{15}$

$$\begin{bmatrix} 5 & -2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 60 \\ 60 \end{bmatrix}$$



Nodal Analysis Method

- Example 4.2 Find node voltages.

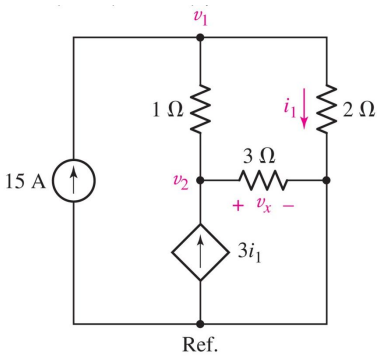


$$\begin{bmatrix} .5833 & -.3333 & -.25 \\ -.3333 & 1.4762 & -.1429 \\ -.25 & -.1429 & .5929 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -11 \\ 3 \\ 25 \end{bmatrix}$$

Nodal Analysis Method

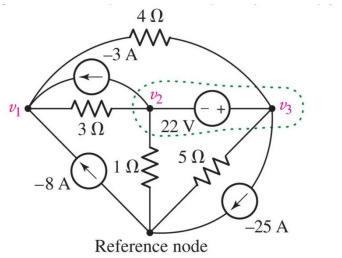
- Circuit with dependent sources
- Example 4.3 Find the power supplied by the dependent source.
  - Since the node analysis is based on KCL, dependent current source is easy to handle.
  - Node-1: $v_1 - v_2 + \frac{v_1}{2} = 15$
  - Node-2: $v_2 - v_1 + \frac{v_2}{3} = 3i_1$
  - 보조 방정식: $i_1 = \frac{v_1}{2}$

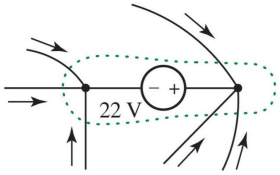
$$\begin{bmatrix} 3 & -2 \\ -15 & 8 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 30 \\ 0 \end{bmatrix}$$



Nodal Analysis Method

- When a circuit has a voltage source:
  - Hard to determine the branch current on the voltage source.
  - One way is to assign a branch current, introducing a new variable.
  - The other is to introduce the supernode and apply KCL on the supernode.
    - † Supernode는 전압원 branch의 양쪽 node를 하나의 node로 구성한 가상의 node





Nodal Analysis Method: Supernode

- Node-2:

$$\frac{v_2-v_1}{0.5}+\frac{v_2-v_3}{2}=14$$

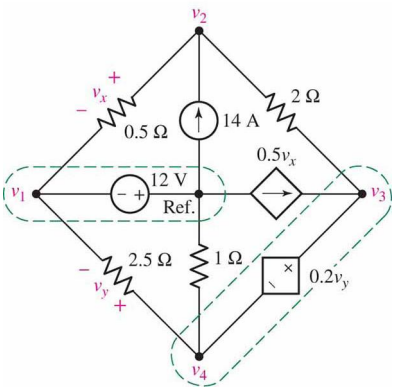
- 3-4 supernode:

$$\frac{v_3-v_2}{2}+v_4+\frac{v_4-v_1}{2.5}=0.5v_x$$

- 보조 방정식:

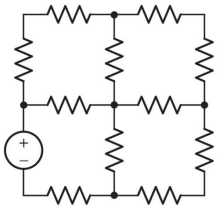
$$v_3-v_4=0.2v_y,\ v_y=v_4-v_1,\ \text{and}\ v_x=v_2-v_1$$

$$\begin{bmatrix}-2 & 2.5 & -0.5 & 0\\ 0.1 & -1 & 0.5 & 1.4\\ 1 & 0 & 0 & 0\\ 0.2 & 0 & 1 & -1.2\end{bmatrix}\begin{bmatrix}v_1\\ v_2\\ v_3\\ v_4\end{bmatrix}=\begin{bmatrix}14\\ 0\\ -12\\ 0\end{bmatrix}$$



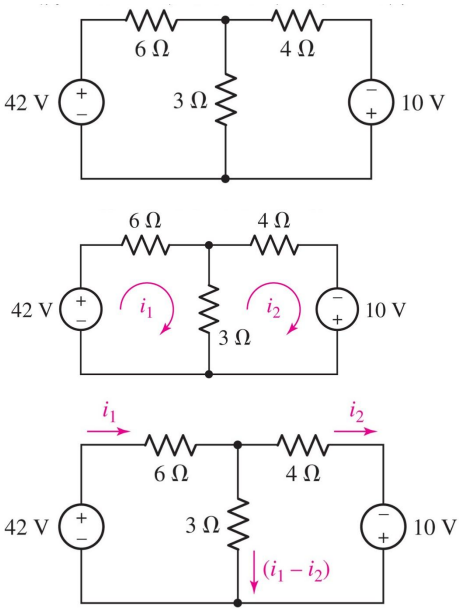
Mesh Analysis

- A mesh is a loop which does not contain any other loops within it.
  - In mesh analysis, we assign mesh currents and solve using KVL.
  - This circuit has four meshes:



Mesh Analysis

1. Identify meshes in the circuit.
2. Assign mesh currents to all mesh.
3. Express the branch current and branch voltage in terms of mesh currents.
   - Branch current와 mesh current 간의 차이점과 관계를 정확하게 이해해야 한다.
4. Apply KVL to each mesh.



Mesh Analysis

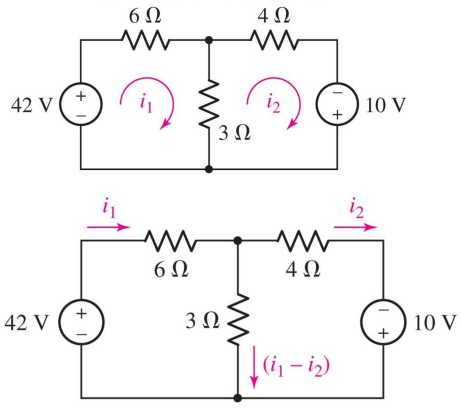
- Mesh-1

$$-42 + 6i_1 + 3(i_1 - i_2) = 0$$

- Mesh-2

$$-3(i_1 - i_2) + 4i_2 - 10 = 0$$

$$\begin{bmatrix} 9 & -3 \\ -3 & 7 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 42 \\ 10 \end{bmatrix}$$



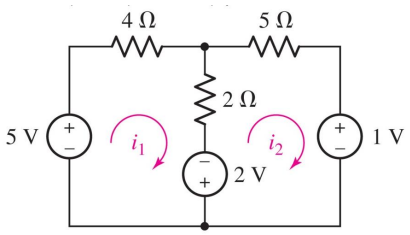
Mesh Analysis

- Example 4.7 Find the power supplied by the $2[V]$ source.
  - v Mesh-1

    $4i_1 + 2(i_1 - i_2) = 2 + 5$
  - v Mesh-2

    $2(i_2 - i_1) + 5i_2 + 1 = -2$

$$\begin{bmatrix} 6 & -2 \\ -2 & 7 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \end{bmatrix}$$



Mesh Analysis

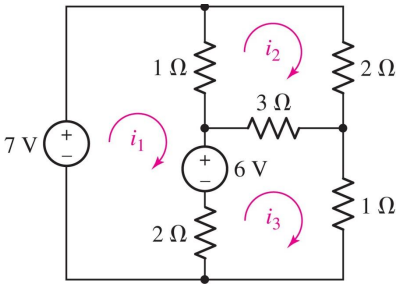
- Example 4.8 Find mesh currents.
  - v Mesh-1

    $(i_1 - i_2) + 6 + 2(i_1 - i_3) = 7$
  - v Mesh-2

    $2i_2 + 3(i_2 - i_3) + (i_2 - i_1) = 0$
  - v Mesh-3

    $3(i_3 - i_2) + i_3 + 2(i_3 - i_1) - 6 = 0$

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$



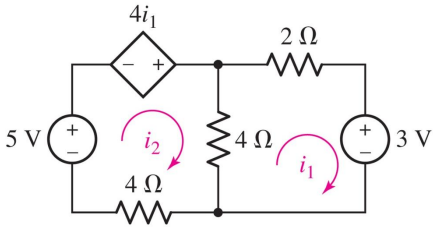
Mesh Analysis

- Mesh analysis on a circuit with dependent sources
- Example 4.9 Find i_1.
  - v Mesh-1

    $4(i_1 - i_2) + 2i_1 + 3 = 0$
  - v Mesh-2

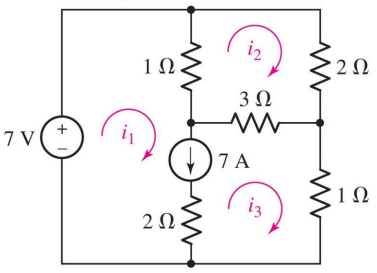
    $4(i_2 - i_1) + 4i_2 = 5 + 4i_1$

$$\begin{bmatrix} 8 & -8 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$



Mesh Analysis: Supermesh

- Mesh analysis on a circuit with current sources
  - v Hard to determine the branch voltage on the current source.
  - v One way is to assign a branch voltage, introducing a new variable.
  - v The other is to introduce the supermesh and apply KVL on the supermesh.
    - † Supermesh는 전류원 branch를 공유하는 한 쌍의 mesh를 하나의 mesh로 구성한 가상의 mesh



Mesh Analysis: Supermesh

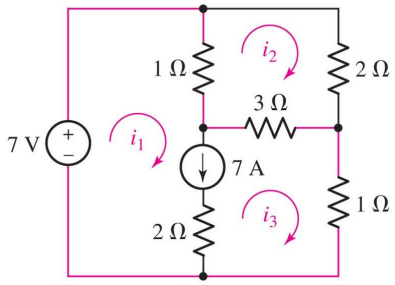
- Example 4.11 Find mesh currents.
  - Supermesh-1&3

    $(i_1 - i_2) + 3(i_3 - i_2) + i_3 = 7$
  - Mesh-2

    $2i_2 + 3(i_2 - i_3) + (i_2 - i_1) = 0$
  - 보조 방정식

    $i_1 - i_3 = 7$

$$\begin{bmatrix} 1 & -4 & 4 \\ -1 & 6 & -3 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \\ 7 \end{bmatrix}$$

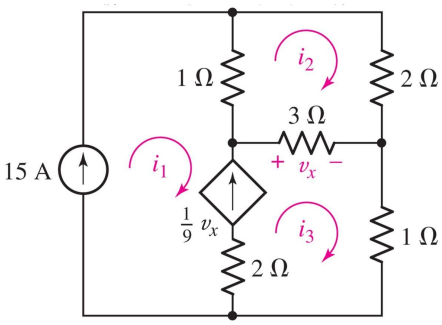


Mesh Analysis: Supermesh

- Example 4.12 Find mesh currents.
  - There is no way to compute branch voltages on current sources.
  - Mesh-2

    $2i_2 + 3(i_2 - i_3) + (i_2 - i_1) = 0$
  - 보조 방정식

    $i_1 = 15$, $\frac{v_x}{9} = i_3 - i_1$, and $v_x = 3(i_3 - i_2)$



Node or Mesh: How to choose?

- 선택 기준
  - Fewer equations
    - Circuit with voltage sources: Mesh analysis
    - Circuit with current sources: Node analysis
  - Your preference