<Sample Solution>

#1.

$u = \tan^{-1}\dfrac{2xy}{x^2-y^2}$

- $u_x = \dfrac{1}{1+\left(\dfrac{2xy}{x^2-y^2}\right)^2} \cdot \dfrac{2y(x^2-y^2)-2xy(2x)}{(x^2-y^2)^2}$

$= \dfrac{\cancel{(x^2-y^2)^2}}{(x^2-y^2)^2+(2xy)^2} \cdot \dfrac{2yx^2-2y^3-4x^2y}{\cancel{(x^2-y^2)^2}}$

$= \dfrac{-2y(x^2+y^2)}{(x^2+y^2)^2} = \dfrac{-2y}{x^2+y^2}$

- $u_{xx} = \dfrac{2y(2x)}{(x^2+y^2)^2}$

- $u_y = \dfrac{1}{1+\left(\dfrac{2xy}{x^2-y^2}\right)^2} \cdot \dfrac{2x(x^2-y^2)-2xy(-2y)}{(x^2-y^2)^2}$

$= \dfrac{\cancel{(x^2-y^2)^2}}{(x^2-y^2)^2+(2xy)^2} \cdot \dfrac{2x^3-2xy^2+4xy^2}{\cancel{(x^2-y^2)^2}}$

$= \dfrac{2x(x^2+y^2)}{(x^2+y^2)^2} = \dfrac{2x}{x^2+y^2}$

- $u_{yy} = -\dfrac{2x(2y)}{(x^2+y^2)^2}$

$\Rightarrow \quad u_{xx} + u_{yy} = 0$

#2.

지름 $= x$, 높이 $: y$. $V = \frac{\pi}{4}x^2 y$

$\log V = \log\frac{\pi}{4} + 2\log x + \log y$

$\dfrac{dV}{V} = \dfrac{2}{x}dx + \dfrac{1}{y}dy$

dx, dy가 모두 양수일 때 최대가 되므로

$\dfrac{dV}{V} = \dfrac{2(0.1)}{5} + \dfrac{0.1}{8} = \dfrac{1.6+0.5}{40} = \dfrac{2.1}{40}$

$= 5.25\%$

#3.

$x = \cos t,\ y = \sin t,\ z = \frac{1}{3}t,\quad 0 \le t \le 2\pi$

$\frac{dx}{dt} = -\sin t,\ \frac{dy}{dt} = \cos t,\ \frac{dz}{dt} = 1/3.$

$\Rightarrow \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2} = \sqrt{\sin^2 t+\cos^2 t+1/9} = \sqrt{10/9}$

$S = \int_0^{2\pi}\sqrt{10/9}\,dt = \underline{\underline{2\sqrt{10/9}\,\pi}}$

#4.

$u = f(xz, yz)$.

$u = f(s,t),\ s = xz,\ t = yz$

$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial s}\frac{\partial s}{\partial x} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial x} = f_s z + f_t \cdot 0 = z f_s$.

$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial s}\frac{\partial s}{\partial y} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial y} = f_s \cdot 0 + f_t \cdot z = z f_t$.

$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial s}\frac{\partial s}{\partial z} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial z} = f_s \cdot x + f_t y$

따라서,

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$

$= xz f_s + yz f_t = z(x f_s + y f_t)$

$= z\frac{\partial u}{\partial z}$.

#5.

$t = 1 \Rightarrow (2, 1, 0)$

$\left.\frac{dx}{dt}\right|_{t=1} = 4$

$\left.\frac{dy}{dt}\right|_{t=1} = 2$

$\left.\frac{dz}{dt}\right|_{t=1} = -3$

$\therefore 4(x-2) + 2(y-1) - 3z = 0$ //

#6.

$f = e^{-x}\log(1+y) \approx y(1-x-\frac{1}{2}y)$

$f_x = -e^{-x}\log(1+y)$, $f_y = \frac{e^{-x}}{1+y}$

$f_{xx} = e^{-x}\log(1+y)$, $f_{xy} = -\frac{e^{-x}}{1+y}$,

$f_{yy} = -\frac{e^{-x}}{(1+y)^2}$

AS $x \to 0$ & $y \to 0$

$f(x,y)=0$, $f_x=0$, $f_y=1$, $f_{xx}=0$,

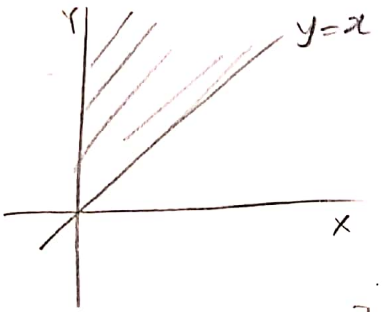
$f_{xy}=-1$, $f_{yy}=-1$.

Formula:

$f(0,0)+xf_x(0,0)+yf_y(0,0)$

$+\frac{1}{2}\{x^2 f_{xx}(0,0)+2xy\, f_{xy}(0,0)+y^2 f_{yy}(0,0)\}$

#7.



$$\frac{1}{\theta_1\theta_2}\int_0^\infty\left[\int_x^\infty \exp\left\{-\frac{x}{\theta_1}-\frac{y}{\theta_2}\right\} dy\right]dx$$

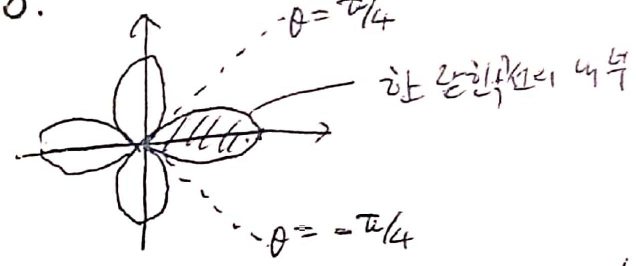
$$=\frac{1}{\theta_1\theta_2}\int_0^\infty\left[-\theta_2 \exp\left\{-\frac{x}{\theta_1}-\frac{y}{\theta_2}\right\}\right]_{y=x}^\infty dx$$

$$=\frac{1}{\theta_1}\int_0^\infty \exp\left\{-\frac{\theta_1+\theta_2}{\theta_1\theta_2}x\right\}dx$$

$$=\frac{1}{\cancel{\theta_1}}\times\frac{\cancel{\theta_1}\theta_2}{\theta_1+\theta_2}=\frac{\theta_2}{\theta_1+\theta_2}$$

$\therefore\ \theta_1=3,\ \theta_2=2 \Rightarrow \boxed{\frac{2}{5}}$ //

#8.



$D=\{(r,\theta) \mid -\pi/4 \le \theta \le \pi/4,\ 0 \le r \le \cos 2\theta\}$

$$A = 2\int_0^{\pi/4}\int_0^{\cos 2\theta} r\,dr\,d\theta$$

$$=2\int_0^{\pi/4}\frac{1}{2}\cos^2 2\theta\, d\theta$$

$$=\frac{1}{2}\int_0^{\pi/4}(1+\cos 4\theta)\,d\theta$$

$$=\frac{1}{2}\left\{\pi/4+\frac{1}{4}\sin 4\theta\Big|_0^{\pi/4}\right\}=\pi/8$$

$\therefore\ \frac{\pi}{8}\times 4 = \boxed{\frac{\pi}{2}}$ //