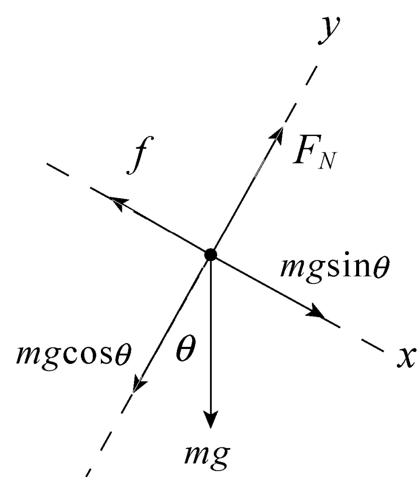
Chapter 6

1. The free-body diagram for the block is shown to the right with the various forces acting on the block placed over the rough inclined surface.



As the block has a tendency to slip down the inclined plane, the frictional force acts upward as shown in free body diagram. Because the block is on the verge of sliding, that frictional force is at its maximum magnitude ($f = \mu_s F_N$). For equilibrium of block the sum of all forces on block must be zero, that is, $\sum \vec{F} = \vec{0}$. This will be possible if the net forces along horizontal and vertical directions are separately zero.

Equating the net force along horizontal direction (parallel to inclined surface) to zero, $\sum F_x = 0$, we get

$$F + \mu_s F_N - mg\sin\theta = 0$$

Similarly, equating the net forces along vertical direction (perpendicular to inclined surface) to zero, $\sum F_y = 0$, we get

$$F_N - mg\cos\theta = 0$$

Combining the two equations gives

$$F + \mu_s mg\cos\theta = mg\sin\theta \quad \Rightarrow \quad F = mg(\sin\theta - \mu_s\cos\theta)$$

which leads to

$$F = mg(\sin\theta - \mu_s\cos\theta) = (6.0\text{ kg})(9.8\text{ m/s}^2)[\sin 60° - (0.60)\cos 60°] = 33.28\text{ N} \approx 33\text{ N}$$

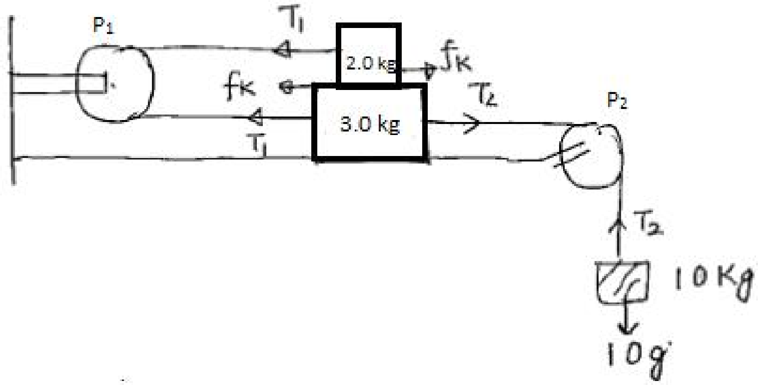
Hence, a force of magnitude F = 33 N is required if the block is on the verge of sliding down the ramp.

2. Applying Newtonôs second law to the horizontal motion, we have $F - \mu_k m g = ma$, where we have used Eq. 6-2, assuming that $F_N = mg$ (which is equivalent to assuming that the vertical force from the broom is negligible).

Eq. 2-16 relates the distance traveled and the final speed to the acceleration: $v^2 = 2a\Delta x$. This gives $a = 1.3$ m/s^2.

Returning to the force equation, we find (with $F = 25$ N and $m = 3.5$ kg) that $\mu_k = 0.60$.

3. (a) The free body diagram for the figure given in problem is drawn below indicating the various forces.



From the figure shown above and using $\sum \vec{F} = m\vec{a}$ for each mass, where $\vec{a}$ is the acceleration, we have

$$10g - T_2 = 10a \quad (1)$$

$$T_2 - T_1 - f_k = 3.0a \quad (2)$$

$$T_1 - f_k = 2.0a \quad (3)$$

where the kinetic frictional force f_k can be expressed as

$$f_k = \mu_k N = [0.3(2.0g)]$$

Adding Eqs. (1)ï (3), we get

$$10g - 2f_k = 15a$$

$$\Rightarrow a = \frac{10 \times 9.8 - 2 \times 0.3(2.0g)}{15} = \frac{98 - 11.76}{15} = 5.75 \text{ m/s}^2$$

Thus, the acceleration of both the masses is a = 5.75 m/s^2.

(b) Substituting the value of a in Eq. (3), we get

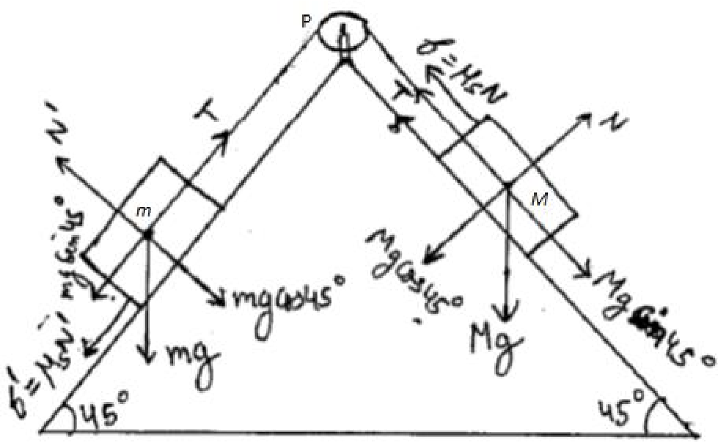
$$\begin{aligned} T_1 &= 2.0a + f_k \\ &= 2.0a + \mu_k(2.0g) \\ &= 2.0(5.75) + 0.3 \times 2 \times 9.8 \\ &= 11.5 + 5.88 \\ &= 17.38 \text{ N} \approx 17 \text{ N} \end{aligned}$$

(c) Substituting the value of a in Eq. (1), we get

$$\begin{aligned} T_2 &= 10g - 10a \\ &= 10(9.8) - 10(5.75) \\ &= 98 - 57.5 \\ &= 40.5 \text{ N} \approx 40 \text{ N.} \end{aligned}$$

Hence, the tension force in the strings is 17 N and 40 N.

4. (a) When mass m is at its minimum value, that block is on the verge of sliding up its plane and the block with mass M is on the verge of sliding down its plane, and the static frictional forces on both blocks are at their maximum values. Since the system is at rest, the net force on both blocks must be zero. The free body diagram for the two blocks is shown below.



From the above free body diagram, for the block M to be in equilibrium the net force on it should be zero. Equating net force along vertical direction (normal to the surface of inclined) on block M, we get

$$\sum F_y = 0$$
$$N - Mg\cos 45° = 0$$
$$N = Mg\cos 45° = \frac{Mg}{\sqrt{2}} \qquad (1)$$

Similarly, equating net force along horizontal direction (parallel to surface of incline) to zero we get

$$\sum F_x = 0$$
$$T + \mu_s N - Mg\sin 45° = 0$$
$$T + \mu_s N = Mg\sin 45° = \frac{Mg}{\sqrt{2}} \qquad (2)$$

From Eqs. (1) and (2), we get

$$T = \frac{Mg}{\sqrt{2}} - \mu_s\left(\frac{Mg}{\sqrt{2}}\right)$$
$$T = \frac{Mg}{\sqrt{2}}(1-\mu_s) \qquad (3)$$

For the block m, under equilibrium the net force on it will be zero. Therefore, on equating net vertical force (along direction perpendicular to incline surface) to zero, we get

$$\sum F_y = 0$$
$$N' - mg\cos 45° = 0$$
$$N' = mg\cos 45° = \frac{mg}{\sqrt{2}} \qquad (4)$$

Similarly, equating the horizontal force along direction parallel to incline surface to zero, we get

$$\sum F_x = 0$$
$$T - \mu_s N' - mg\sin 45° = 0$$
$$T = \mu_s N' + mg\sin 45°$$
$$T = \mu_s N' + \frac{mg}{\sqrt{2}} \qquad (5)$$

From Eqs. (4) and (5), we get

$$T = \mu_s \frac{mg}{\sqrt{2}} + \frac{mg}{\sqrt{2}}$$
$$T == \frac{mg}{\sqrt{2}}(1+\mu_s) \qquad (6)$$

Equations (3) and (6) give

$$\frac{Mg}{\sqrt{2}}(1-\mu_s) = \frac{mg}{\sqrt{2}}(1+\mu_s) \qquad (7)$$
$$2(1-0.28) = m(1+0.28)$$
$$1.44 = 1.28m$$
$$m = \frac{1.44}{1.28} = 1.125\text{ kg} \approx 1.12\text{ kg}$$

(b) When then mass m has its maximum value, that block is on the verge of sliding down its plane and the other block is on the verge sliding up its plane, the static frictional forces on both blocks are at their maximum values. Therefore, the directions of the frictional forces on the blocks are now reversed. Thus, we can rewrite Eq. (7) as

$$\frac{Mg}{\sqrt{2}}(1+\mu_s) = \frac{mg}{\sqrt{2}}(1-\mu_s)$$
$$\frac{2}{\sqrt{2}}(1+0.28) = \frac{m}{\sqrt{2}}(1-0.28)$$
$$2.56 = 0.72m$$
$$m = 3.56\text{ kg}$$

Hence, the minimum and maximum value of mass m for which system remains at rest are

m =1.12 kg and m = 3.56 kg.

5. In addition to the forces already shown in Fig. 6-17, a free-body diagram would include an upward normal force $\vec{F}_N$ exerted by the floor on the block, a downward $m\vec{g}$ representing the gravitational pull exerted by Earth, and an assumed-leftward $\vec{f}$ for the kinetic or static friction. We choose +*x* rightwards and +*y* upwards. We apply Newtonôs second law to these axes:

$$F - f = ma$$
$$P + F_N - mg = 0$$

where F = 6.0 N and m = 2.5 kg is the mass of the block.

(a) In this case, P = 8.0 N leads to

F_N = (2.5 kg)(9.8 m/s^2) ï 8.0 N = 16.5 N.

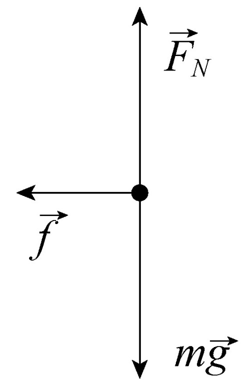
Using Eq. 6-1, this implies $f_{s,\max} = \mu_s F_N = 6.6$ N , which is larger than the 6.0 N rightward force ï so the block (which was initially at rest) does not move. Putting a = 0 into the first of our equations above yields a static friction force of $f = P$ = 6.0 N.

(b) In this case, P = 10 N, the normal force is

F_N = (2.5 kg)(9.8 m/s^2) ï 10 N = 14.5 N.

Using Eq. 6-1, this implies $f_{s,\max} = \mu_s F_N = 5.8$ N, which is less than the 6.0 N rightward force ï so the block does move. Hence, we are dealing not with static but with kinetic friction, which Eq. 6-2 reveals to be $f_k = \mu_k F_N = 3.6$ N .

(c) In this last case, P = 12 N leads to F_N = 12.5 N and thus to $f_{s,\max} = \mu_s F_N = 5.0$ N, which (as expected) is less than the 6.0 N rightward force ï so the block moves. The kinetic friction force, then, is $f_k = \mu_k F_N = 3.1$ N .



6. The free-body diagram for the player is shown to the right. $\vec{F}_N$ is the normal force of the ground on the player, $m\vec{g}$ is the force of gravity, and $\vec{f}$ is the force of friction. The force of friction is related to the normal force by $f = \mu_k F_N$.

We use Newtonôs second law applied to the vertical axis to find the normal force.

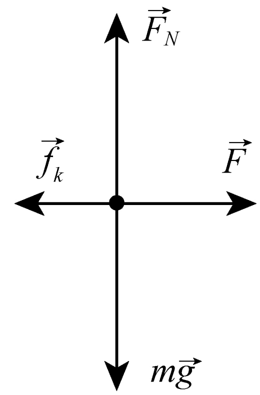
The vertical component of the acceleration is zero, so we obtain F_N ï $mg = 0$; thus, $F_N = mg$. Consequently,

$$\mu_s = \frac{485\ \text{N}}{(83\ \text{kg})(9.8\ \text{m/s}^2)} = 0.60$$

7. **THINK** A force is being applied to accelerate a crate in the presence of friction. We apply Newtonôs second law to solve for the acceleration.

EXPRESS The free-body diagram for the crate is shown to the right. We denote $\vec{F}$ as the horizontal force of the person exerted on the crate (in the +x direction), $\vec{f}_k$ is the force of kinetic friction (in the ïx direction), F_N is the vertical normal force exerted by the floor (in the +y direction), and $m\vec{g}$ is the force of gravity. The magnitude of the force of friction is given by (Eq. 6-2):

$$f_k = \mu_k F_N \, .$$



Applying Newtonôs second law to the x and y axes, we obtain

$$F - f_k = ma$$
$$F_N - mg = 0$$

respectively.

ANALYZE

(a) The second equation above yields the normal force $F_N = mg$, so that the friction is

$$f_k = \mu_k F_N = \mu_k mg = (0.30)(55 \text{ kg})(9.8 \text{ m/s}^2) = 1.6 \times 10^2 \text{ N} \, .$$

(b) The first equation becomes

$$F - \mu_k mg = ma$$

which (with F = 260 N) we solve to find

$$a = \frac{F}{m} - \mu_k g = 1.8 \text{ m/s}^2 \, .$$

LEARN For the crate to accelerate, the condition $F > f_k = \mu_k mg$ must be met. As can be seen from the equation above, the greater the value of μ_k, the smaller the acceleration with the same applied force.

8. To maintain the stone's motion, a horizontal force (in the $+x$ direction) is needed that cancels the retarding effect due to kinetic friction. Applying Newton's second to the x and y axes, we obtain

$$F - f_k = ma$$
$$F_N - mg = 0$$

respectively. The second equation yields the normal force $F_N = mg$, so that (using Eq. 6-2) the kinetic friction becomes $f_k = \mu_k mg$. Thus, the first equation becomes

$$F - \mu_k mg = ma = 0$$

where we have set $a = 0$ to be consistent with the idea that the horizontal velocity of the stone should remain constant. With $m = 20$ kg and $\mu_k = 0.80$, we find $F = 1.6 \times 10^2$ N.

9. We choose $+x$ horizontally rightwards and $+y$ upwards and observe that the 15 N force has components $F_x = F \cos \theta$ and $F_y =$ ï $F \sin \theta$.

(a) We apply Newtonôs second law to the y axis:

$$F_N - F \sin\theta - mg = 0 \;\;\Rightarrow\;\; F_N = (15\text{ N})\sin 40° + (3.5\text{ kg})(9.8\text{ m/s}^2) = 44\text{ N}.$$

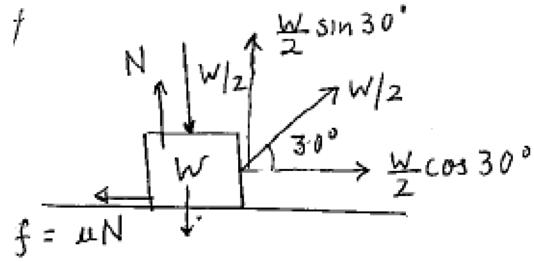
With $\mu_k = 0.25$, Eq. 6-2 leads to $f_k = 11$ N.

(b) We apply Newtonôs second law to the x axis:

$$F\cos\theta - f_k = ma \;\;\Rightarrow\;\; a = \frac{(15\text{ N})\cos 40° - 11\text{ N}}{3.5\text{ kg}} = 0.14\text{ m/s}^2.$$

Since the result is positive-valued, then the block is accelerating in the $+x$ (rightward) direction.

10. The free body diagram for the block of weight W is shown below



Since the block is on the verge of moving the sum of forces acting horizontally and vertically must be zero at that instant, that is, $\sum F_x = 0$ and $\sum F_y = 0$. Also, the value of the static frictional force must be at its maximum.

From the free body diagram shown below, equating the sum of vertical forces on the block to zero, we get

$$\sum F_y = 0$$

$$N + \frac{W}{2}\sin 30° - W - \frac{W}{2} = 0$$

$$N + \frac{W}{2}\sin 30° = W + \frac{W}{2}$$

$$N = \frac{3W}{2} - \frac{W}{4} = \frac{5W}{4} \qquad (1)$$

Similarly, equating the sum of horizontal forces on the block to zero, we get

$$\sum F_x = 0$$

$$f - \frac{W}{2}\cos 30° = 0$$

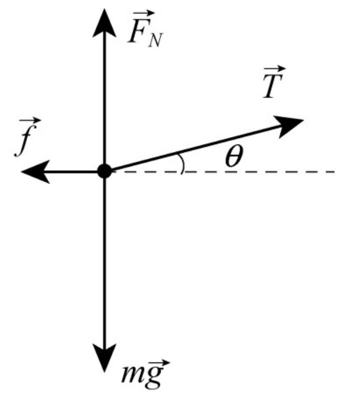
$$\frac{W}{2}\cos 30° = f$$

$$\frac{W}{2}\cos 30° = \mu N$$

$$\frac{W}{2}\cos 30° = \mu\left(\frac{5W}{4}\right)$$

$$\Rightarrow \mu = \frac{(W/2)\times 0.866}{5W/4} = \frac{2\times 0.866}{5} = 0.35.$$

11. **THINK** Since the crate is being pulled by a rope at an angle with the horizontal, we need to analyze the force components in both the x and y-directions.



The free-body diagram for the crate is shown on the right. $\vec{T}$ is the tension force of the rope on the crate, $\vec{F}_N$ is the normal force of the floor on the crate, $m\vec{g}$ is the force of gravity, and $\vec{f}$ is the force of friction. We take the $+x$ direction to be horizontal to the right and the $+y$ direction to be up. We assume the crate is motionless. The equations for the x and the y components of the force according to Newton's second law are:

$$T\cos\theta - f = 0$$
$$T\sin\theta + F_N - mg = 0$$

where $\theta = 15°$ is the angle between the rope and the horizontal. The first equation gives $f = T\cos\theta$ and the second gives $F_N = mg - T\sin\theta$. If the crate is to remain at rest, f must be less than $\mu_s F_N$, or $T\cos\theta < \mu_s(mg - T\sin\theta)$. When the tension force is sufficient to just start the crate moving, we must have

$$T\cos\theta = \mu_s(mg - T\sin\theta).$$

ANALYZE (a) We solve for the tension:

$$T = \frac{\mu_s mg}{\cos\theta + \mu_s\sin\theta} = \frac{(0.65)\,(68\text{ kg})\,(9.8\text{ m/s}^2)}{\cos 15° + 0.65\sin 15°} = 382\text{ N}.$$

(b) The second law equations for the moving crate are

$$T\cos\theta - f = ma$$
$$F_N + T\sin\theta - mg = 0.$$

Now $f = \mu_k F_N$, and the second equation gives $F_N = mg - T\sin\theta$, which yields $f = \mu_k(mg - T\sin\theta)$. This expression is substituted for f in the first equation to obtain

$$T\cos\theta - \mu_k(mg - T\sin\theta) = ma,$$

so the acceleration is

$$a = \frac{T(\cos\theta + \mu_k\sin\theta)}{m} - \mu_k g$$
$$= \frac{(304\text{ N})(\cos 15° + 0.35\sin 15°)}{68\text{ kg}} - (0.35)(9.8\text{ m/s}^2) = 1.3\text{ m/s}^2.$$

LEARN Let's check the limit where $\theta = 0$. In this case, we recover the familiar expressions: $T = \mu_s mg$ and $a = (T - \mu_k mg)/m$.

12. There is no acceleration, so the (upward) static friction forces (there are four of them, one for each thumb and one for each set of opposing fingers) equals the magnitude of the (downward) pull of gravity. Using Eq. 6-1, we have

$$4\mu_s F_N = mg = (79\text{ kg})(9.8\text{ m/s}^2)$$

which, with $\mu_s = 0.70$, yields $F_N = 2.8 \times 10^2$ N.

13. We denote the magnitude of 110 N force exerted by the worker on the crate as F. The magnitude of the static frictional force can vary between zero and $f_{s,\max} = \mu_s F_N$.

(a) In this case, application of Newtonôs second law in the vertical direction yields $F_N = mg$. Thus,

$$f_{s,\,\max} = \mu_s F_N = \mu_s mg = (0.37)(35\,\text{kg})(9.8\,\text{m/s}^2) = 1.3\times10^2\,\text{N}$$

which is greater than F.

(b) The block, which is initially at rest, stays at rest since $F < f_{s,\,\max}$. Thus, it does not move.

(c) By applying Newtonôs second law to the horizontal direction, that the magnitude of the frictional force exerted on the crate is $f_s = 1.1\times10^2$ N.

(d) Denoting the upward force exerted by the second worker as F_2, then application of Newtonôs second law in the vertical direction yields $F_N = mg$ ï F_2, which leads to

$$f_{s,\max} = \mu_s F_N = \mu_s (mg - F_2)\,.$$

In order to move the crate, F must satisfy the condition $F > f_{s,\max} = \mu_s\ (mg - F_2)$, or

$$110\,\text{N} > (0.37)\left[(35\,\text{kg})(9.8\,\text{m/s}^2) - F_2\right].$$

The minimum value of F_2 that satisfies this inequality is a value slightly bigger than 45.7 N, so we express our answer as $F_{2,\,\min} = 46$ N.

(e) In this final case, moving the crate requires a greater horizontal push from the worker than static friction (as computed in part (a)) can resist. Thus, Newtonôs law in the horizontal direction leads to

$$F + F_2 > f_{s,\,\max} \quad \Rightarrow \quad 110\ \text{N} + F_2 > 126.9\ \text{N}$$

which leads (after appropriate rounding) to $F_{2,\,\min} = 17$ N.

14. (a) Using the result obtained in Sample Problem ï ñFriction, applied force at an angle,ò the maximum angle for which static friction applies is

$$\theta_{\max} = \tan^{-1} \mu_s = \tan^{-1} 0.63 \approx 32° \ .$$

This is greater than the dip angle in the problem, so the block does not slide.

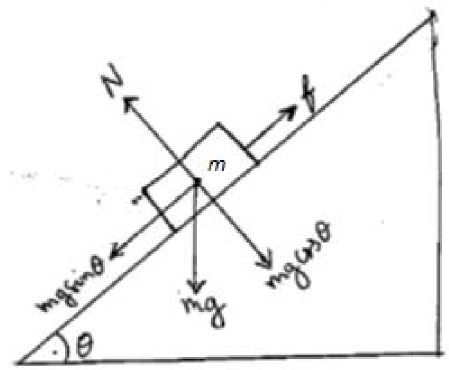
(b) Applying Newtonôs second law, we have

$$F + mg \sin \theta - f_{s,\,\max} = ma = 0$$
$$F_N - mg \cos \theta = 0.$$

Along with Eq. 6-1 ($f_{s,\,\max} = \mu_s F_N$) we have enough information to solve for F. With $\theta = 24°$ and $m = 1.5 \times 10^7$ kg, we find

$$F = mg\left(\mu_s \cos\theta - \sin\theta\right) = 2.5 \times 10^7 \text{ N}.$$

15. The free body diagram for the block is shown below



Since the block is at rest, it is in equilibrium and hence the net force on the block is zero. Using the free body diagram for the block, equating the net force along the vertical direction to zero, we get

$$\sum F_y = 0$$
$$N - mg\cos\theta = 0$$
$$N = mg\cos\theta \qquad (1)$$

Similarly, equating the net force along horizontal direction equal to zero, we get

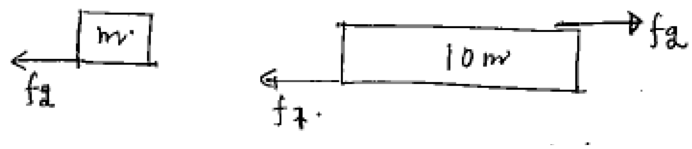
$$\sum F_x = 0$$
$$f - mg\sin\theta = 0$$
$$f = mg\sin\theta \qquad (2)$$

The net contact force is

$$\begin{aligned} F_{\text{net}} &= \sqrt{N^2 + f^2} \\ &= \sqrt{(mg\cos\theta)^2 + (mg\sin\theta)^2} \\ &= \sqrt{m^2 g^2(\cos^2\theta + \sin^2\theta)} \\ &= mg \end{aligned}$$

Hence, with m = 5.0 kg, the force exerted by the ramp on the block is mg = 49 N.

16. The force of friction at different contact surfaces is as shown in the following figures.



Here, the frictional force between small block and the upper of big block is

$$f_2 = \mu_2(mg)$$

And the force of friction between the big block and the ground is

$$f_1 = \mu_1(11mg)$$

Given that $\mu_2 > 11\mu_1$. Therefore, $f_2 > f_1$. The retardation of the upper block is

$$a_1 = \frac{f_2}{m} = \mu_2 g$$

The acceleration of the lower block is

$$a_2 = \frac{f_2 - f_1}{10\text{ m}} = \frac{(\mu_2 - 11\mu_1)g}{10}$$

The relative retardation of the upper block is

$$a_r = a_1 + a_2 = \left[\mu_2 + \left(\frac{\mu_2 - 11\mu_1}{10}\right)\right]g$$

$$= \left(\frac{11\mu_2 - 11\mu_1}{10}\right)g$$

$$= \frac{11}{10}(\mu_2 - \mu_1)g$$

(a) For minimum velocity ($v_{\min}$) of the upper block which enables it to fall off the lower one, using third kinematics relation, we get

$$0 = v_{\min}^2 - 2a_r l$$

$$v_{\min} = \sqrt{2a_r l} = \sqrt{\frac{22}{10}(\mu_2 - \mu_1)gl}$$

(b) Using first kinematics relation, we get

$$0 = v_{\min} - a_r t$$

$$\Rightarrow t = \frac{v_{\min}}{a_r} = \sqrt{\frac{20l}{11(\mu_2 - \mu_1)g}}$$

17. If the block is sliding then we compute the kinetic friction from Eq. 6-2; if it is not sliding, then we determine the extent of static friction from applying Newton's law, with zero acceleration, to the x axis (which is parallel to the incline surface). The question of whether or not it is sliding is therefore crucial, and depends on the maximum static friction force, as calculated from Eq. 6-1. The forces are resolved in the incline plane coordinate system in Figure 6-5 in the textbook. The acceleration, if there is any, is along the x axis, and we are taking uphill as $+x$. The net force along the y axis, then, is certainly zero, which provides the following relationship:

$$\sum \vec{F}_y = 0 \;\Rightarrow\; F_N = W\cos\theta$$

where $W = mg = 45$ N is the weight of the block, and $\theta = 15°$ is the incline angle. Thus, $F_N = 43.5$ N, which implies that the maximum static friction force should be

$$f_{s,\max} = (0.50)\,(43.5\text{ N}) = 21.7\text{ N}.$$

(a) For $\vec{P} = (-5.0\text{ N})\hat{i}$, Newton's second law, applied to the x axis becomes

$$f - |P| - mg\sin\theta = ma\,.$$

Here we are assuming $\vec{f}$ is pointing uphill, as shown in Figure 6-5, and if it turns out that it points downhill (which *is* a possibility), then the result for f_s will be negative. If $f = f_s$ then $a = 0$, we obtain

$$f_s = |\,P\,| + mg\sin\theta = 5.0\text{ N} + (43.5\text{ N})\sin 15° = 17\text{ N},$$

or $\vec{f}_s = (17\text{ N})\hat{i}$. This is clearly allowed since f_s is less than $f_{s,\max}$.

(b) For $\vec{P} = (-8.0\text{ N})\hat{i}$, we obtain (from the same equation) $\vec{f}_s = (20\text{ N})\hat{i}$, which is still allowed since it is less than $f_{s,\max}$.

(c) But for $\vec{P} = (-15\text{ N})\hat{i}$, we obtain (from the same equation) $f_s = 27$ N, which is not allowed since it is larger than $f_{s,\max}$. Thus, we conclude that it is the kinetic friction instead of the static friction that is relevant in this case. The result is

$$\vec{f}_k = \mu_k F_N\,\hat{i} = (0.34)(43.5\text{ N})\,\hat{i} = (15\text{ N})\,\hat{i}.$$

18. (a) We apply Newtonôs second law to the ñdownhillò direction:

$$mg\sin\theta \text{ ï } f = ma,$$

where, using Eq. 6-11,

$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta .$$

Thus, with μ_k = 0.600, we have

$$a = g(\sin\theta - \mu_k\cos\theta) = \text{ï } 3.71\ \text{m/s}^2$$

which means, since we have chosen the positive direction in the direction of motion (down the slope) then the acceleration vector points ñuphillò; it is decelerating. With $v_0 = 18.0$ m/s and $\Delta x = d$ = 30.0 m, Eq. 2-16 leads to

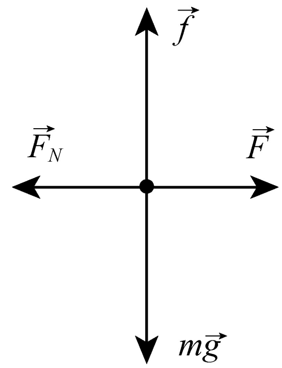
$$v = \sqrt{v_0^2 + 2ad} = \sqrt{(18\ \text{m/s})^2 + 2(-3.71\ \text{m/s}^2)(30\ \text{m})} = 10.0\ \text{m/s}.$$

(b) In this case, we find

$$a = g(\sin\theta - \mu_k\cos\theta) = (9.8\ \text{m/s}^2)\left[\sin 12° - (0.10)\cos 12°\right] = 1.08\ \text{m/s}^2$$

and the speed (when impact occurs) is

$$v = \sqrt{v_0^2 + 2ad} = \sqrt{(18\ \text{m/s})^2 + 2(+1.08\ \text{m/s}^2)(30\ \text{m})} = 19.7\ \text{m/s}.$$



19. (a) The free-body diagram for the block is shown below. $\vec{F}$ is the applied force, $\vec{F}_N$ is the normal force of the wall on the block, $\vec{f}$ is the force of friction, and $m\vec{g}$ is the force of gravity. To determine if the block falls, we find the magnitude f of the force of friction required to hold it without accelerating and also find the normal force of the wall on the block. We compare f and $\mu_s F_N$. If $f < \mu_s F_N$, the block does not slide on the wall but if $f > \mu_s F_N$, the block does slide. The horizontal component of Newton's second law is $F - F_N = 0$, so $F_N = F = 12$ N and

$$\mu_s F_N = (0.60)(12\text{ N}) = 7.2\text{ N}.$$

The vertical component is $f - mg = 0$, so $f = mg = 5.0$ N. Since $f < \mu_s F_N$ the block does not slide.

(b) Since the block does not move $f = 5.0$ N and $F_N = 12$ N. The force of the wall on the block is

$$\vec{F}_w = -F_N\,\hat{i} + f\,\hat{j} = -(12\,\text{N})\,\hat{i} + (5.0\,\text{N})\,\hat{j}$$

where the axes are as shown on Fig. 6-26 of the text.

20. Treating the two boxes as a single system of total mass $m_C + m_W = 1.0 + 3.0 = 4.0$ kg, subject to a total (leftward) friction of magnitude 2.0 N + 3.5 N = 5.5 N, we apply Newtonôs second law (with $+x$ rightward):

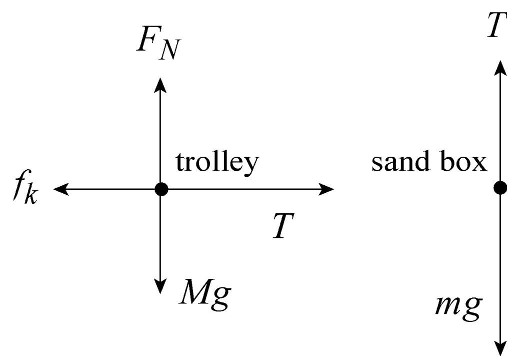
$$F - f_{\text{total}} = m_{\text{total}}\ a \quad \Rightarrow \quad 12.0\ \text{N} - 5.5\ \text{N} = (4.0\ \text{kg})a$$

which yields the acceleration $a = 1.625\ \text{m/s}^2$. We have treated F as if it were known to the nearest tenth of a Newton so that our acceleration is ñgoodò to two significant figures. Turning our attention to the larger box (the Wheaties box of mass $m_W = 3.0$ kg) we apply Newtonôs second law to find the contact force F' exerted by the Cheerios box on it.

$$F' - f_W = m_W a \quad \Rightarrow \quad F' - 3.5\ \text{N} = (3.0\ \text{kg})(1.625\ \text{m/s}^2)$$

From the above equation, we find the contact force to be $F' = 8.375\ \text{N} \approx 8.4\ \text{N}$.

21. The free-body diagrams for the sand box and the trolley are shown to the right. From the motion of the sand box, we have



$$mg - T = ma$$

where m is the mass of the sand box. Similarly, from the motion of the trolley, we have

$$T - f_k = Ma$$

where M is the mass of the trolley. With $f_k = \mu_k F_N$, we combine the equations and obtain

$$a = \frac{(m - \mu_k M)g}{m + M} = \frac{[2.0\text{ kg} - (0.040)(15\text{ kg})](9.8\text{ m/s}^2)}{2.0\text{ kg} + 15.0\text{ kg}} = 0.807\text{ m/s}^2 \approx 0.81\text{ m/s}^2$$

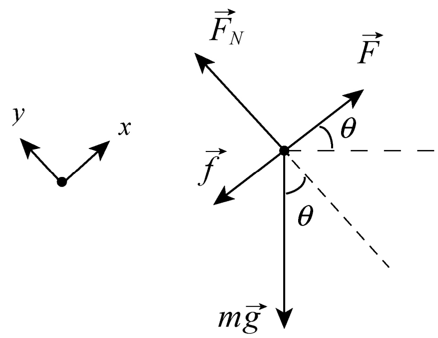
(b) The tension of the cable is calculated as

$$T = m(g - a) = mg\left(1 - \frac{m - \mu_k M}{m + M}\right) = \frac{(1 + \mu_k)mMg}{m + M} = \frac{(1 + 0.040)(2.0\text{ kg})(15\text{ kg})(9.8\text{ m/s}^2)}{2.0\text{ kg} + 15.0\text{ kg}}$$

$$= 18\text{ N}$$

22. The free-body diagram for the sled is shown below, with $\vec{F}$ being the force applied to the sled, $\vec{F}_N$ the normal force of the inclined plane on the sled, $m\vec{g}$ the force of gravity, and $\vec{f}$ the force of friction. We take the $+x$ direction to be along the inclined plane and the $+y$ direction to be in its normal direction. The equations for the x and the y components of the force according to Newtonös second law are:

$$F_x = F - f - mg\sin\theta = ma = 0$$
$$F_y = F_N - mg\cos\theta = 0$$



Now $f = \mu F_N$, and the second equation gives $F_N = mg\cos\theta$, which yields $f = \mu mg\cos\theta$. This expression is substituted for f in the first equation to obtain

$$F = mg(\sin\theta + \mu\cos\theta)$$

From the figure, we see that $F = 2.0$ N when $\mu = 0$. This implies $mg\sin\theta = 2.0$ N. Similarly, we also find $F = 5.0$ N when $\mu = 0.25$:

$$5.0\text{ N} = mg(\sin\theta + 0.25\cos\theta) = 2.0\text{ N} + 0.25mg\cos\theta$$

which yields $mg\cos\theta = 12.0$ N. Combining the two results, we get

$$\tan\theta = \frac{2}{12} = \frac{1}{6} \Rightarrow \quad \theta = 9.5°.$$

23. Let the tensions on the strings connecting m_2 and m_3 be T_{23}, and that connecting m_2 and m_1 be T_{12}, respectively. Applying Newtonôs second law (and Eq. 6-2, with $F_N = m_2 g$ in this case) to the *system* we have

$$\begin{aligned} m_3 g - T_{23} &= m_3 a \\ T_{23} - \mu_k m_2 g - T_{12} &= m_2 a \\ T_{12} - m_1 g &= m_1 a \end{aligned}$$

Adding up the three equations and using $m_1 = M, m_2 = m_3 = 2M$, we obtain

$$2Mg \text{ ï } 2\mu_k Mg \text{ ï } Mg = 5Ma\,.$$

With a = 0.500 m/s^2 this yields μ_k = 0.372. Thus, the coefficient of kinetic friction is roughly μ_k = 0.37.

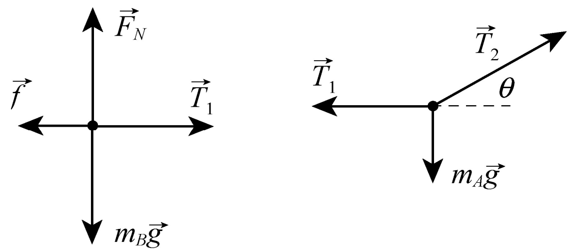
24. We find the acceleration from the slope of the graph (recall Eq. 2-11): a = 4.5 m/s^2. Thus, Newtonôs second law leads to

$$F \text{ ï } \mu_k mg = ma,$$

where F = 50.0 N is the constant horizontal force applied. With m = 4.1 kg, we arrive at μ_k =0.79.

25. **THINK** In order that the two blocks remain in equilibrium, friction must be present between block B and the surface.

EXPRESS The free-body diagrams for block B and for the knot just above block A are shown below.



$\vec{T}_1$ is the tension force of the rope pulling on block B or pulling on the knot (as the case may be), $\vec{T}_2$ is the tension force exerted by the second rope (at angle $\theta = 30°$) on the knot, $\vec{f}$ is the force of static friction exerted by the horizontal surface on block B, $\vec{F}_N$ is normal force exerted by the surface on block B, W_A is the weight of block A (W_A is the magnitude of $m_A\vec{g}$), and W_B is the weight of block B (W_B = 750 N is the magnitude of $m_B\vec{g}$).

ANALYZE For each object we take $+x$ horizontally rightward and $+y$ upward. Applying Newtonôs second law in the x and y directions for block B and then doing the same for the knot results in four equations:

$$
\begin{aligned}
T_1 - f_{s,\max} &= 0 \\
F_N - W_B &= 0 \\
T_2 \cos\theta - T_1 &= 0 \\
T_2 \sin\theta - W_A &= 0
\end{aligned}
$$

where we assume the static friction to be at its maximum value (permitting us to use Eq. 6-1). Solving these equations with $\mu_s = 0.25$, we obtain $W_A = 108\text{ N} \approx 1.1\times10^2\text{ N}$.

LEARN As expected, the maximum weight of A is proportional to the weight of B, as well as the coefficient of static friction. In addition, we see that W_A is proportional to $\tan\theta$ (the larger the angle, the greater the vertical component of T_2 that supports its weight).

26. (a) Applying Newtonôs second law to the *system* (of total mass $M = 60.0$ kg) and using Eq. 6-2 (with $F_N = Mg$ in this case) we obtain

$$F - \mu_k Mg = Ma \;\Rightarrow\; a = 0.223 \text{ m/s}^2.$$

Next, we examine the forces just on m_3 and find $F_{32} = m_3(a + \mu_k g) = 142$ N.

If the algebra steps are done more systematically, one ends up with the interesting relationship: $F_{32} = (m_3 / M)F$ (which is independent of the friction!).

(b) As remarked at the end of our solution to part (a), the result does not depend on the frictional parameters. The answer here is the same as in part (a).

27. (a) The limiting frictional force is given by $f_{s,\max} = \mu_s F_N = \mu_s mg$. Substituting the values given, we have

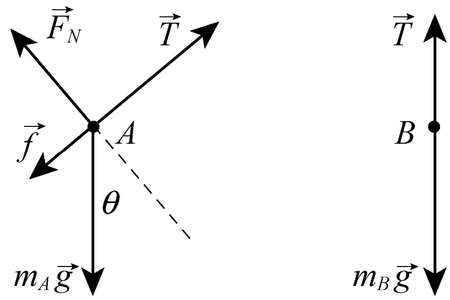
$$f_{s,\max} = (0.25)(20\text{ kg})(9.8\text{ m/s}^2) = 4.9\text{ N}.$$

Thus, the block and the trolley will not have relative motion for a force of $F_{\text{app}} = 2.0$ N, and the frictional force between them is $f_s = F_{\text{app}} = 2.0$ N.

(b) The acceleration of the trolley is given by $F_{\text{app}} = (m_{\text{trolley}} + m_{\text{block}})a$, or

$$a = \frac{F_{\text{app}}}{m_{\text{trolley}} + m_{\text{block}}} = \frac{2.0\text{ N}}{20\text{ kg} + 2.0\text{ kg}} = 0.091\text{m/s}^2.$$

28. The free-body diagrams are shown below. T is the magnitude of the tension force of the string, f is the magnitude of the force of friction on block A, F_N is the magnitude of the normal force of the plane on block A, $m_A\vec{g}$ is the force of gravity on body A (where m_A = 15 kg), and $m_B\vec{g}$ is the force of gravity on block B. θ = 30° is the angle of incline. For A we take the $+x$ to be uphill and $+y$ to be in the direction of the normal force; the positive direction is chosen *downward* for block B.



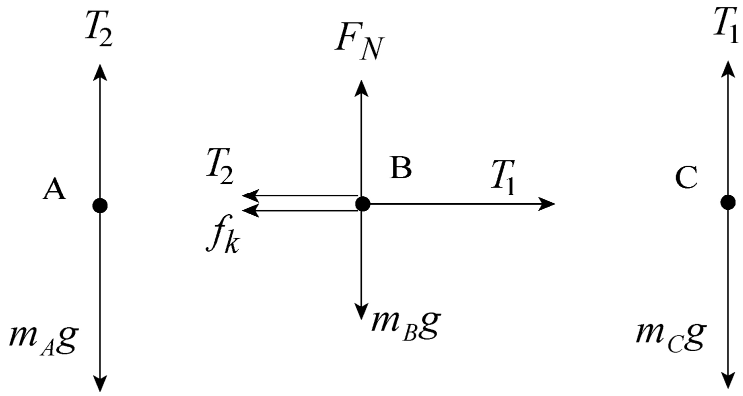
Since A is moving down the incline, the force of friction is uphill with magnitude $f_k = \mu_k F_N$ (where μ_k = 0.20). Newtonôs second law leads to

$$T - f_k + m_A g\sin\theta = m_A a = 0$$
$$F_N - m_A g\cos\theta = 0$$
$$m_B g - T = m_B a = 0$$

for the two bodies (where a = 0 is a consequence of the velocity being constant). We solve these for the mass of block B.

$$m_B = m_A\left(\sin\theta - \mu_k\cos\theta\right) = 4.9\text{ kg.}$$

29. The free-body diagrams for the three masses are shown below.



Note that $f_k = \mu_k F_N = \mu_k m_B g$. Applying Newtonôs second law, we obtain

$$\begin{aligned} m_C g - T_1 &= m_C a \\ T_1 - \mu_k m_B g - T_2 &= m_B a \\ T_2 - m_A g &= m_A a \end{aligned}$$

Adding up the three equations gives

$$(m_C - \mu_k m_B - m_A)g = (m_A + m_B + m_C)a$$

which leads to

$$a = \frac{(m_C - \mu_k m_B - m_A)g}{m_A + m_B + m_C} = \frac{[(6.0 \text{ kg} - (0.40)(3.0 \text{ kg}) - 2.0 \text{ kg}](9.8 \text{ m/s}^2)}{2.0 \text{ kg} + 3.0 \text{ kg} + 6.0 \text{ kg}} = 2.5 \text{ m/s}^2$$

30. We use the familiar horizontal and vertical axes for x and y directions, with rightward and upward positive, respectively. The rope is assumed massless so that the force exerted by the child $\vec{F}$ is identical to the tension uniformly through the rope. The x and y components of $\vec{F}$ are $F\cos\theta$ and $F\sin\theta$, respectively. The static friction force points leftward.

(a) Newton's Law applied to the y-axis, where there is presumed to be no acceleration, leads to

$$F_N + F\sin\theta - mg = 0$$

which implies that the maximum static friction is $\mu_s(mg - F\sin\theta)$. If $f_s = f_{s,\,\max}$ is assumed, then Newton's second law applied to the x axis (which also has $a = 0$ even though it is "verging" on moving) yields

$$F\cos\theta - f_s = ma \quad \Rightarrow \quad F\cos\theta - \mu_s(mg - F\sin\theta) = 0$$

which we solve, for $\theta = 42°$ and $\mu_s = 0.47$, to obtain $F = 89$ N.

(b) Solving the above equation algebraically for F, with W denoting the weight, we obtain

$$F = \frac{\mu_s W}{\cos\theta + \mu_s\sin\theta} = \frac{(0.47)(200\text{ N})}{\cos\theta + (0.47)\sin\theta} = \frac{94\text{ N}}{\cos\theta + (0.47)\sin\theta}.$$

(c) We minimize the above expression for F by working through the condition:

$$\frac{dF}{d\theta} = \frac{\mu_s W(\sin\theta - \mu_s\cos\theta)}{(\cos\theta + \mu_s\sin\theta)^2} = 0$$

which leads to the result $\theta = \tan^{-1}\mu_s = 25°$.

(d) Plugging $\theta = 25°$ into the above result for F, with $\mu_s = 0.47$ and $W = 200$ N, yields $F = 85\text{ N}$.

31. Since $\mu_1 < \mu_2$, the m_1 = 2.0 kg block would move faster than m_2 = 4.0 kg if moved separately. Here, it is in contact with 4.0 kg block, so both blocks move together with the same acceleration.

Considering both blocks taken together as a single system, we can calculate the acceleration of the block from

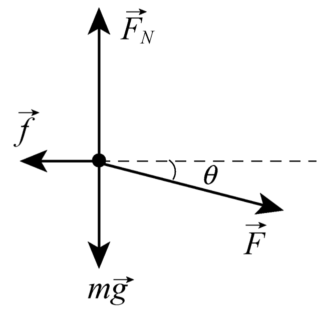
$$(m_1 + m_2)g\sin\theta - \mu_1 m_1 g\cos\theta - \mu_2 m_2 g\cos\theta = (m_1 + m_2)a$$

which gives

$$a = g\left[\sin\theta - \frac{(\mu_1 m_1 + \mu_2 m_2)\cos\theta}{m_1 + m_2}\right] = (9.8\text{ m/s}^2)\left[\sin 30° - \frac{(0.20)(2.0\text{ kg}) + (0.30)(4.0\text{ kg})\cos 30°}{2.0\text{ kg} + 4.0\text{ kg}}\right]$$
$$= 2.6\text{ m/s}^2$$

32. The free-body diagram for the block is shown below, with $\vec{F}$ being the force applied to the block, $\vec{F}_N$ the normal force of the floor on the block, $m\vec{g}$ the force of gravity, and $\vec{f}$ the force of friction. We take the +x direction to be horizontal to the right and the +y direction to be up. The equations for the x and the y components of the force according to Newtonõs second law are:

$$F_x = F\cos\theta - f = ma$$
$$F_y = F_N - F\sin\theta - mg = 0$$



Now $f = \mu_k F_N$, and the second equation gives $F_N = mg + F\sin\theta$, which yields

$$f = \mu_k (mg + F\sin\theta)\,.$$

This expression is substituted for f in the first equation to obtain

$$F\cos\theta ï\ \mu_k (mg + F\sin\theta) = ma,$$

so the acceleration is

$$a = \frac{F}{m}\left(\cos\theta - \mu_k\sin\theta\right) - \mu_k g\,.$$

From the figure, we see that $a = 3.0\text{ m/s}^2$ when $\mu_k = 0$. This implies

$$3.0\text{ m/s}^2 = \frac{F}{m}\cos\theta.$$

We also find $a = 0$ when $\mu_k = 0.20$:

$$0 = \frac{F}{m}\left(\cos\theta - (0.20)\sin\theta\right) - (0.20)(9.8\text{ m/s}^2) = 3.00\text{ m/s}^2 - 0.20\frac{F}{m}\sin\theta - 1.96\text{ m/s}^2$$
$$= 1.04\text{ m/s}^2 - 0.20\frac{F}{m}\sin\theta$$

which yields $5.2\text{ m/s}^2 = \frac{F}{m}\sin\theta$. Combining the two results, we get

$$\tan\theta = \left(\frac{5.2\text{ m/s}^2}{3.0\text{ m/s}^2}\right) = 1.73 \;\Rightarrow\; \theta = 60°.$$

33. **THINK** In this problem, the frictional force is not a constant, but instead proportional to the speed of the boat. Integration is required to solve for the speed.

EXPRESS We denote the magnitude of the frictional force αv, where $\alpha = 70\ \mathrm{N \cdot s/m}$. We take the direction of the boatôs motion to be positive. Newtonôs second law gives

$$-\alpha v = m\frac{dv}{dt}.$$

ANALYZE Thus,

$$\int_{v_0}^{v} \frac{dv}{v} = -\frac{\alpha}{m}\int_0^t dt$$

where v_0 is the velocity at time zero and v is the velocity at time t. The integrals are evaluated with the result

$$\ln\left(\frac{v}{v_0}\right) = -\frac{\alpha t}{m}$$

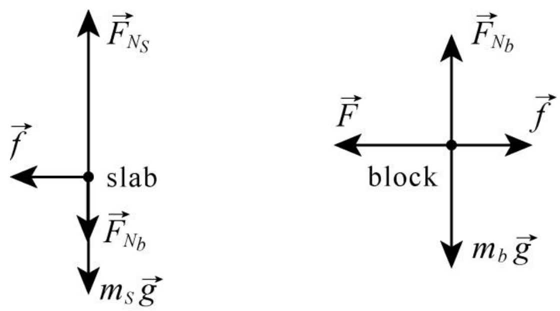
We take $v/v_0 = 0.45$ and solve for time:

$$t = -\frac{m}{\alpha}\ln\left(\frac{v}{v_0}\right) = -\frac{m}{\alpha}\ln(0.45) = -\frac{1000\ \mathrm{kg}}{70\ \mathrm{N \cdot s/m}}\ln(0.45) = 11.4\ \mathrm{s},$$

or about 11 s, to two significant digits.

LEARN The speed of the boat is given by $v(t) = v_0 e^{-\alpha t/m}$, showing exponential decay with time. The greater the value of α, the more rapidly the boat slows down.

34. The free-body diagrams for the slab and block are shown below. $\vec{F}$ is the 120 N force applied to the block, $\vec{F}_{Ns}$ is the normal force of the floor on the slab, F_{Nb} is the magnitude of the normal force between the slab and the block, $\vec{f}$ is the force of friction between the slab and the block, m_s is the mass of the slab, and m_b is the mass of the block. For both objects, we take the $+x$ direction to be to the right and the $+y$ direction to be up.



Applying Newton's second law for the x and y axes for (first) the slab and (second) the block results in four equations:

$$-f = m_s a_s$$
$$F_{Ns} - F_{Ns} - m_s g = 0$$
$$f - F = m_b a_b$$
$$F_{Nb} - m_b g = 0$$

from which we note that the maximum possible static friction magnitude would be

$$\mu_s F_{Nb} = \mu_s m_b g = (0.60)(12\text{ kg})(9.8\text{ m/s}^2) = 71\text{ N} .$$

We check to see if the block slides on the slab. Assuming it does not, then $a_s = a_b$ (which we denote simply as a) and we solve for f:

$$f = \frac{m_s F}{m_s + m_b} = \frac{(40\text{ kg})(120\text{ N})}{40\text{ kg} + 12\text{ kg}} = 92\text{ N}$$

which is greater than $f_{s,\max}$ so that we conclude the block is sliding across the slab (their accelerations are different).

(a) Using $f = \mu_k F_{Nb}$ the above equations yield

$$a_b = \frac{\mu_k m_b g - F}{m_b} = \frac{(0.40)(12\text{ kg})(9.8\text{ m/s}^2) - 120\text{ N}}{12\text{ kg}} = -6.1\text{ m/s}^2.$$

The negative sign means that the acceleration is leftward. That is, $\vec{a}_b = (-6.1\text{ m/s}^2)\hat{i}$

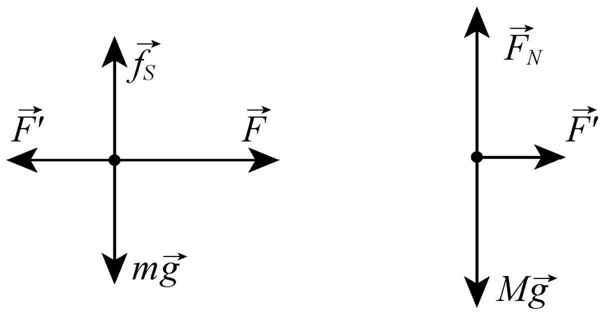
(b) We also obtain

$$a_s = -\frac{\mu_k m_b g}{m_s} = -\frac{(0.40)(12\ \text{kg})(9.8\ \text{m/s}^2)}{40\ \text{kg}} = -1.2\ \text{m/s}^2.$$ As mentioned above, this means it accelerates to the left. That is, $\vec{a}_s = (-1.2\ \text{m/s}^2)\hat{\mathrm{i}}$.

35. The free-body diagrams for the two blocks, treated individually, are shown below (first m and then M). F' is the contact force between the two blocks, and the static friction force $\vec{f}_s$ is at its maximum value (so Eq. 6-1 leads to $f_s = f_{s,\max} = \mu_s F'$ where $\mu_s = 0.33$).

Treating the two blocks together as a single system (sliding across a frictionless floor), we apply Newtonôs second law (with $+x$ rightward) to find an expression for the acceleration:

$$F = m_{\text{total}}\, a \quad \Rightarrow a = \frac{F}{m+M}$$



This is equivalent to having analyzed the two blocks individually and then combined their equations. Now, when we analyze the small block individually, we apply Newtonôs second law to the x and y axes, substitute in the above expression for a, and use Eq. 6-1.

$$F - F' = ma \quad \Rightarrow \quad F' = F - m\left(\frac{F}{m+M}\right)$$

$$f_s - mg = 0 \quad \Rightarrow \quad \mu_s F' - mg = 0$$

These expressions are combined (to eliminate F') and we arrive at

$$F = \frac{mg}{\mu_s\left(1 - \dfrac{m}{m+M}\right)}$$

$$= 562 \text{ N.}$$

36. The magnitude of the drag force is given by

$$D = \frac{1}{2} C \rho A v^2,$$

where C is the drag coefficient, which is a constant, ρ is the density of the medium (air), A is the area of cross section of the water droplet, and v is the speed of the water droplet. Let the magnitude of the drag force of the water droplet of 4.0 mm in diameter be

$$D_1 = \frac{1}{2} C \rho_1 \pi r_1^2 v^2, \qquad (\because A_1 = \pi r_1^2)$$

and let the magnitude of the drag force of the water droplet of 6.0 mm in diameter be

$$D_2 = \frac{1}{2} C \rho_2 \pi r_2^2 v^2, \qquad (\because A_2 = \pi r_2^2)$$

The ratio of the drag force acting on the drops is calculated as follows:

$$\begin{aligned}
\frac{D_1}{D_2} &= \frac{\rho_1}{\rho_2} \times \frac{\pi r_1^2}{\pi r_2^2} \times \frac{v_1^2}{v_2^2} \\
&= \frac{0.20}{0.70} \times \frac{(2.0 \times 10^{-3})^2}{(3.0 \times 10^{-3})^2} \times \frac{(10)^2}{(2.5)^2} \\
&= \frac{0.20}{0.70} \times \frac{4.0}{9.0} \times \frac{100}{6.25} \\
&= \frac{800}{393.75} \\
&= 2.03 \approx 2.0.
\end{aligned}$$

37. (a) Setting $F = D$ (for Drag force) we use Eq. 6-14 to find the wind speed v along the ground (which actually is relative to the moving stone, but we assume the stone is moving slowly enough that this does not invalidate the result):

$$v = \sqrt{\frac{2F}{C\rho A}} = \sqrt{\frac{2(157\ \text{N})}{(0.80)(1.21\ \text{kg/m}^3)(0.040\ \text{m}^2)}} = 90\ \text{m/s} = 3.2\times10^2\ \text{km/h}.$$

(b) Doubling our previous result, we find the reported speed to be 6.5×10^2 km/h.

(c) The result is not reasonable for a terrestrial storm. A category 5 hurricane has speeds on the order of 2.6×10^2 m/s.

38. a) From Table 6-1 and Eq. 6-16, we have

$$v_t = \sqrt{\frac{2F_g}{C\rho A}} \Rightarrow C\rho A = 2\frac{mg}{v_t^2}$$

where v_t = 60 m/s. We estimate the pilotôs mass at about m = 70 kg. Now, we convert v = 1300(1000/3600) ≈ 360 m/s and plug into Eq. 6-14:

$$D = \frac{1}{2}C\rho A v^2 = \frac{1}{2}\left(2\frac{mg}{v_t^2}\right)v^2 = mg\left(\frac{v}{v_t}\right)^2$$

which yields $D = (70\text{ kg})(9.8\text{ m/s}^2)(360/60)^2 \approx 2 \times 10^4$ N.

(b) We assume the mass of the ejection seat is roughly equal to the mass of the pilot. Thus, Newtonôs second law (in the horizontal direction) applied to this system of mass $2m$ gives the magnitude of acceleration:

$$|a| = \frac{D}{2m} = \frac{g}{2}\left(\frac{v}{v_t}\right)^2 = 18g\ .$$

39. For the passenger jet $D_j = \frac{1}{2}C\rho_1 A v_j^2$, and for the prop-driven transport $D_t = \frac{1}{2}C\rho_2 A v_t^2$, where ρ_1 and ρ_2 represent the air density at 15 km and 5.0 km, respectively. Thus the ratio in question is

$$\frac{D_j}{D_t} = \frac{\rho_1 v_j^2}{\rho_2 v_t^2} = \frac{\left(0.38 \text{ kg/m}^3\right)\left(1200 \text{ km/h}\right)^2}{\left(0.67 \text{ kg/m}^3\right)\left(500 \text{ km/h}\right)^2} = 2.3.\text{s}$$

40. This problem involves Newtonôs second law for motion along the slope.

(a) The force along the slope is given by

$$\begin{aligned} F_g &= mg\sin\theta - \mu F_N = mg\sin\theta - \mu mg\cos\theta = mg(\sin\theta - \mu\cos\theta) \\ &= (85.0\ \text{kg})(9.80\ \text{m/s}^2)\left[\sin 40.0^\circ - (0.03800)\cos 40.0^\circ\right] \\ &= 511\ \text{N}. \end{aligned}$$

Thus, the terminal speed of the skier is

$$v_t = \sqrt{\frac{2F_g}{C\rho A}} = \sqrt{\frac{2(511\ \text{N})}{(0.150)(1.20\ \text{kg/m}^3)(1.30\ \text{m}^2)}} = 66.1\ \text{m/s}.$$

(b) Differentiating v_t with respect to C, we obtain

$$\begin{aligned} dv_t &= -\frac{1}{2}\sqrt{\frac{2F_g}{\rho A}}\, C^{-3/2}dC = -\frac{1}{2}\sqrt{\frac{2(511\ \text{N})}{(1.20\ \text{kg/m}^3)(1.30\ \text{m}^2)}}(0.150)^{-3/2}dC \\ &= -(2.20\times 10^2\ \text{m/s})dC. \end{aligned}$$

41. Perhaps surprisingly, the equations pertaining to this situation are exactly those in Sample Problem ï ñCar in flat circular turn,ò although the logic is a little different. In the Sample Problem, the car moves along a (stationary) road, whereas in this problem the cat is stationary relative to the merry-go-around platform. But the static friction plays the same role in both cases since the bottom-most point of the car tire is instantaneously at rest with respect to the race track, just as static friction applies to the contact surface between cat and platform. Using Eq. 6-23 with Eq. 4-35, we find

$$\mu_s = (2\pi R/T\,)^2/gR = 4\pi^2 R/gT^{\,2}.$$

With T = 6.0 s and R = 6.0 m, we obtain μ_s = 0.67.

42. The magnitude of the acceleration of the car as it rounds the curve is given by v^2/R, where v is the speed of the car and R is the radius of the curve. Since the road is horizontal, only the frictional force of the road on the tires makes this acceleration possible. The horizontal component of Newtonǒs second law is $f = mv^2/R$. If F_N is the normal force of the road on the car and m is the mass of the car, the vertical component of Newtonǒs second law leads to $F_N = mg$. Thus, using Eq. 6-1, the maximum value of static friction is

$$f_{s,\max} = \mu_s F_N = \mu_s mg.$$

If the car does not slip, $f \le \mu_s mg$. This means

$$\frac{v^2}{R} \le \mu_s g \quad \Rightarrow \quad v \le \sqrt{\mu_s Rg}.$$

43. The magnitude of the acceleration of the cyclist as it rounds the curve is given by v^2/R, where v is the speed of the cyclist and R is the radius of the curve. Since the road is horizontal, only the frictional force of the road on the tires makes this acceleration possible. The horizontal component of Newtonôs second law is $f = mv^2/R$. If F_N is the normal force of the road on the bicycle and m is the mass of the bicycle and rider, the vertical component of Newtonôs second law leads to $F_N = mg$. Thus, using Eq. 6-1, the maximum value of static friction is $f_{s,\max} = \mu_s F_N = \mu_s mg$. If the bicycle does not slip, $f \le \mu_s mg$. This means

$$\frac{v^2}{R} \le \mu_s g \quad \Rightarrow \quad R \ge \frac{v^2}{\mu_s g}.$$

Consequently, the minimum radius with which a cyclist moving at 35 km/h = 9.7 m/s can round the curve without slipping is

$$R_{\min} = \frac{v^2}{\mu_s g} = \frac{(9.7\text{ m/s})^2}{(0.40)(9.8\text{ m/s}^2)} = 24\text{ m}.$$

44. With v = 96.6 km/h = 26.8 m/s, Eq. 6-17 readily yields

$$a = \frac{v^2}{R} = \frac{(26.8 \text{ m/s})^2}{7.6 \text{ m}} = 94.7 \text{ m/s}^2$$

which we express as a multiple of g:

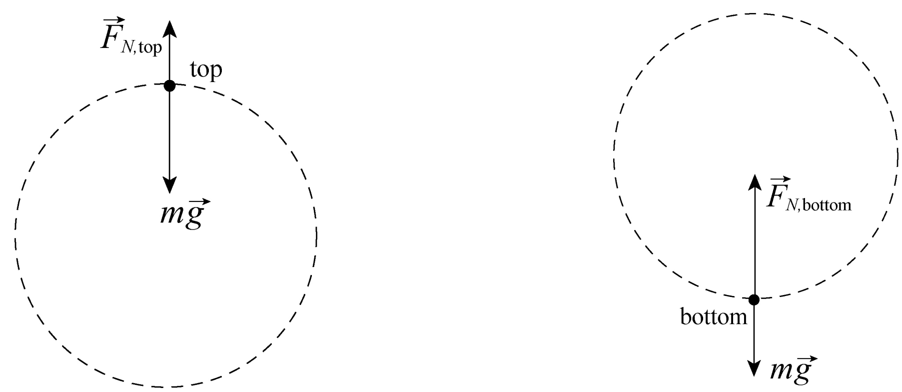
$$a = \left(\frac{a}{g}\right) g = \left(\frac{94.7 \text{ m/s}^2}{9.80 \text{ m/s}^2}\right) g = 9.7 g.$$

45. **THINK** Ferris wheel ride is a vertical circular motion. The apparent weight of the rider varies with his position.

EXPRESS The free-body diagrams of the student at the top and bottom of the Ferris wheel are shown next.

At the top (the highest point in the circular motion) the seat pushes up on the student with a force of magnitude $F_{N,\text{top}}$, while the Earth pulls down with a force of magnitude mg. Newtonôs second law for the radial direction gives

$$mg - F_{N,\text{top}} = \frac{mv^2}{R}.$$



At the bottom of the ride, $F_{N,\text{bottom}}$ is the magnitude of the upward force exerted by the seat. The net force toward the center of the circle is (choosing upward as the positive direction):

$$F_{N,\text{bottom}} - mg = \frac{mv^2}{R}.$$

The Ferris wheel is ñsteadily rotatingò so the value $F_c = mv^2/R$ is the same everywhere. The apparent weight of the student is given by F_N.

ANALYZE (a) At the top, we are told that $F_{N,\text{top}}$ = 556 N and mg = 667 N. This means that the seat is pushing up with a force that is smaller than the studentôs weight, and we say the student experiences a decrease in his ñapparent weightò at the highest point. Thus, he feels ñlight.ò

(b) From (a), we find the centripetal force to be

$$F_c = \frac{mv^2}{R} = mg - F_{N,\text{top}} = 667\ \text{N} - 556\ \text{N} = 111\ \text{N}.$$

Thus, the normal force at the bottom is

$$F_{N,\text{bottom}} = \frac{mv^2}{R} + mg = F_c + mg = 111 \text{ N} + 667 \text{ N} = 778 \text{ N}.$$

(c) If the speed is doubled,

$$F_c' = \frac{m(2v)^2}{R} = 4(111 \text{ N}) = 444 \text{ N}.$$

Therefore, at the highest point we have

$$F_{N,\text{top}}' = mg - F_c' = 667 \text{ N} - 444 \text{ N} = 223 \text{ N}.$$

(d) Similarly, the normal force at the lowest point is now found to be

$$F_{N,\text{bottom}}' = F_c' + mg = 444 \text{ N} + 667 \text{ N} = 1111 \text{ N}.$$

LEARN The apparent weight of the student is the greatest at the bottom and smallest at the top of the ride. The speed $v = \sqrt{gR}$ would result in $F_{N,\text{top}} = 0$, giving the student a sudden sensation of ñweightlessnessò at the top of the ride.

46. (a) We note that the speed 75.0 km/h in SI units is roughly 20.8 m/s. The horizontal force that keeps her from sliding must equal the centripetal force (Eq. 6-18), and the upward force on her must equal mg. Thus,

$$F_{\text{net}} = \sqrt{(mg)^2 + (m\text{v}^2/R)^2} = \sqrt{(539\ \text{N})^2 + (79.57\ \text{N})^2} = 545\ \text{N}.$$

(b) The angle is

$$\phi = \tan^{-1}[(mv^2/R)/(mg)] = \tan^{-1}[(79.57\ \text{N})/(539\ \text{N})] = 8.40^{\circ}$$

as measured from a vertical axis.

47. (a) Eq. 4-35 gives $T = 2\pi R/v = 2\pi(10\text{ m})/(5.5\text{ m/s}) = 13.7\text{ s} \approx 14\text{ s}$.

(b) The situation is similar to that of Sample Problem ï ñVertical circular loop, Diavolo,ò but with the normal force direction reversed. Adapting Eq. 6-19, we find

$$F_N = mg\left(1-\frac{v^2}{gR}\right) = (80\text{ kg})(9.8\text{ m/s}^2)\left(1-\frac{(5.5\text{ m/s})^2}{(9.8\text{ m/s}^2)(12\text{ m})}\right) = 5.8\times10^2\text{ N}$$

(c) Now we reverse both the normal force direction and the acceleration direction (from what is shown in Sample Problem ï ñVertical circular loop, Diavoloò) and adapt Eq. 6-19 accordingly. Thus we obtain

$$F_N = mg\left(1+\frac{v^2}{gR}\right) = (80\text{ kg})(9.8\text{ m/s}^2)\left(1+\frac{(5.5\text{ m/s})^2}{(9.8\text{ m/s}^2)(12\text{ m})}\right) = 986\text{ N} \approx 9.9\times10^2\text{ N}$$

48. We will start by assuming that the normal force (on the car from the rail) points up. Note that gravity points down, and the y axis is chosen positive upwards. Also, the direction to the center of the circle (the direction of centripetal acceleration) is down. Thus, Newtonôs second law leads to

$$F_N - mg = m\left(-\frac{v^2}{r}\right).$$

(a) When $v = 11$ m/s, we obtain $F_N = 4.9 \times 10^3$ N.

(b) $\vec{F}_N$ points upward.

(c) When $v = 14$ m/s, we obtain $F_N = 0$ N.

(d) There is no direction since the normal force is zero.

49. At the top of the hill, the situation is similar to that of Sample Problem ï ñVertical circular loop, Diavolo,ò but with the normal force direction reversed. Adapting Eq. 6-19, we find

$$F_N = m(g - v^2/R).$$

Since $F_N = 0$ there (as stated in the problem) then $v^2 = gR$. Later, at the bottom of the valley, we reverse both the normal force direction and the acceleration direction (from what is shown in the Sample Problem) and adapt Eq. 6-19 accordingly. Thus we obtain

$$F_N = m(g + v^2/R) = 2mg = 1568 \text{ N} \approx 1.57 \times 10^3 \text{ N}.$$

50. The centripetal force on the passenger is $F = mv^2/r$.

(a) The slope of the plot at $v = 8.30$ m/s is

$$\left.\frac{dF}{dv}\right|_{v=8.30\text{ m/s}} = \left.\frac{2mv}{r}\right|_{v=8.30\text{ m/s}} = \frac{2(85.0\text{ kg})(8.30\text{ m/s})}{3.50\text{ m}} = 403\text{ N}\cdot\text{s/m}.$$

(b) The period of the circular ride is $T = 2\pi r/v$. Thus,

$$F = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 mr}{T^2},$$

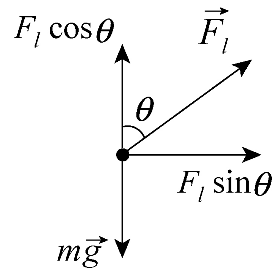
and the variation of F with respect to T while holding r constant is

$$dF = -\frac{8\pi^2 mr}{T^3}dT.$$

The slope of the plot at $T = 2.50$ s is

$$\left.\frac{dF}{dT}\right|_{T=2.50\text{ s}} = -\left.\frac{8\pi^2 mr}{T^3}\right|_{T=2.50\text{ s}} = \frac{8\pi^2(85.0\text{ kg})(3.50\text{ m})}{(2.50\text{ s})^3} = -1.50\times10^3\text{ N/s}.$$

51. **THINK** An airplane with its wings tilted at an angle is in a circular motion. Centripetal force is involved in this problem.



51. The free-body diagram for the airplane of mass m is shown to the right. We note that $\vec{F}_l$ is the force of aerodynamic lift and $\vec{a}$ points rightwards in the figure. We also note that $|\vec{a}| = v^2/R$. Applying Newtonôs law to the axes of the problem (+x rightward and +y upward) we obtain

$$F_l \sin\theta = m\frac{v^2}{R}$$
$$F_l \cos\theta = mg$$

Eliminating mass from these equations leads to $\tan\theta = \frac{v^2}{gR}$. The equation allows us to solve for the radius R.

ANALYZE With v = 600 km/h = 167 m/s and θ = 40°, we find

$$R = \frac{v^2}{g\tan\theta} = \frac{(167\text{ m/s})^2}{(9.8\text{ m/s}^2)\tan 40°} = 3390\text{ m} \approx 3.4\times10^3\text{ m}$$

LEARN Our approach to solving this problem is identical to that discussed in the Sample Problem ï ñCar in banked circular turn.ò Do you see the similarities?

52. The situation is somewhat similar to that shown in the ñloop-the-loopò example done in the textbook (see Figure 6-10) except that, instead of a downward normal force, we are dealing with the force of the boom $\vec{F}_B$ on the car ï which is capable of pointing any direction. We will assume it to be upward as we apply Newtonôs second law to the car (of total weight 6000 N): $F_B - W = ma$ where $m = W/g$ and $a = -v^2/r$. Note that the centripetal acceleration is downward (our choice for negative direction) for a body at the top of its circular trajectory.

(a) If r = 10 m and v = 5.0 m/s, we obtain $F_B = 4.5 \times 10^3$ N = 4.5 kN.

(b) The direction of $\vec{F}_B$ is up.

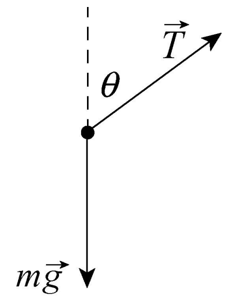
(c) If r = 10 m and v = 12 m/s, we obtain F_B = ï 2.8×10^3 N = ï 2.8 kN, or $|F_B|$ = 2.8 kN.

(d) The minus sign indicates that $\vec{F}_B$ points downward.

53. The free-body diagram (for the hand straps of mass m) is the view that a passenger might see if she was looking forward and the streetcar was curving towards the right (so $\vec{a}$ points rightwards in the figure). We note that $|\vec{a}| = v^2 / R$ where v = 16 km/h = 4.4 m/s.

Applying Newtonôs law to the axes of the problem (+x is rightward and +y is upward) we obtain

$$T \sin\theta = m\frac{v^2}{R}$$
$$T \cos\theta = mg.$$



We solve these equations for the angle:

$$\theta = \tan^{-1}\left(\frac{v^2}{Rg}\right)$$

which yields θ = 11°.

54. The centripetal force on the passenger is $F = mv^2/r$.

(a) The variation of F with respect to r while holding v constant is $dF = -\frac{mv^2}{r^2}dr$.

(b) The variation of F with respect to v while holding r constant is $dF = \frac{2mv}{r}dv$.

(c) The period of the circular ride is $T = 2\pi r/v$. Thus,

$$F = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 mr}{T^2},$$

and the variation of F with respect to T while holding r constant is

$$dF = -\frac{8\pi^2 mr}{T^3}dT = -8\pi^2 mr\left(\frac{v}{2\pi r}\right)^3 dT = -\left(\frac{mv^3}{\pi r^2}\right)dT.$$

55. We note that the period T is eight times the time between flashes ($\frac{1}{2000}$ s), so T = 0.0040 s. Combining Eq. 6-18 with Eq. 4-35 leads to

$$F = \frac{4m\pi^2 R}{T^2} = \quad = 3.3 \times 10^3 \text{ N} .$$

56. We refer the reader to Sample Problem ï ñCar in banked circular turn,ò and use the result Eq. 6-26 to calculate the banking angle:

$$\theta = \tan^{-1}\left(\frac{v^2}{gR}\right) = \tan^{-1}\left(\frac{(18.055\text{ m/s})^2}{(9.8\text{ m/s}^2)(200\text{ m})}\right) = 9.44°$$

where we have used v = 65 km/h = 18.055 m/s and R = 200 m. Now we consider a vehicle taking this banked curve at v' = 40(1000/3600) = 11 m/s. Its (horizontal) acceleration is $a' = v'^2/R$, which has components parallel the incline and perpendicular to it:

$$a_{\|} = a'\cos\theta = \frac{v'^2\cos\theta}{R}$$
$$a_{\perp} = a'\sin\theta = \frac{v'^2\sin\theta}{R}.$$

These enter Newtonôs second law as follows (choosing downhill as the +x direction and away-from-incline as +y):

$$mg\sin\theta - f_s = ma_{\|}$$
$$F_N - mg\cos\theta = ma_{\perp}$$

and we are led to

$$\frac{f_s}{F_N} = \frac{mg\sin\theta - mv'^2\cos\theta/R}{mg\cos\theta + mv'^2\sin\theta/R}.$$

We cancel the mass and plug in, obtaining f_s/F_N = 0.103. The problem implies we should set $f_s = f_{s,\max}$ so that, by Eq. 6-1, we have μ_s = 0.103.

57. For the puck to remain at rest the magnitude of the tension force T of the cord must equal the gravitational force Mg on the cylinder. The tension force supplies the centripetal force that keeps the puck in its circular orbit, so $T = mv^2/r$. Thus $Mg = mv^2/r$. We solve for the speed:

$$v = \sqrt{\frac{Mgr}{m}} = \sqrt{\frac{(2.50\text{ kg})(9.80\text{ m/s}^2)(0.250\text{ m})}{1.50\text{ kg}}} = 2.02\text{ m/s}.$$

58. (a) Using the kinematic equation given in Table 2-1, the deceleration of the car is

$$v^2 = v_0^2 + 2ad \quad \Rightarrow \quad 0 = (35\ \text{m/s})^2 + 2a(107\ \text{m})$$

which gives $a = -5.72\ \text{m/s}^2$. Thus, the force of friction required to stop by car is

$$f = m\,|a| = (1400\ \text{kg})(5.72\ \text{m/s}^2) \approx 8.0\times10^3\ \text{N}.$$

(b) The maximum possible static friction is

$$f_{s,\max} = \mu_s mg = (0.50)(1400\ \text{kg})(9.80\ \text{m/s}^2) \approx 6.9\times10^3\ \text{N}.$$

(c) If $\mu_k = 0.40$, then $f_k = \mu_k mg$ and the deceleration is $a = -\mu_k g$. Therefore, the speed of the car when it hits the wall is

$$v = \sqrt{v_0^2 + 2ad} = \sqrt{(35\ \text{m/s})^2 - 2(0.40)(9.8\ \text{m/s}^2)(107\ \text{m})} \approx 20\ \text{m/s}.$$

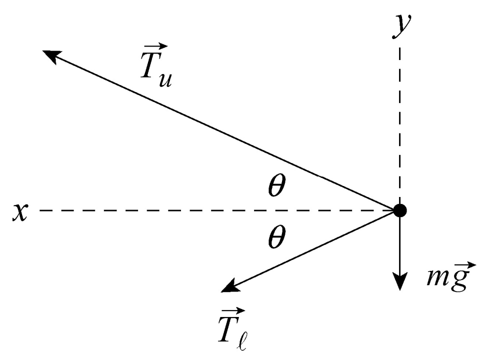
(d) The force required to keep the motion circular is

$$F_r = \frac{mv_0^2}{r} = \frac{(1400\ \text{kg})(35.0\ \text{m/s})^2}{107\ \text{m}} = 1.6\times10^4\ \text{N}.$$

(e) Since $F_r > f_{s,\max}$, no circular path is possible.

59. **THINK** As illustrated in Fig. 6-45, our system consists of a ball connected by two strings to a rotating rod. The tensions in the strings provide the source of centripetal force.

EXPRESS The free-body diagram for the ball is shown below. $\vec{T}_u$ is the tension exerted by the upper string on the ball, $\vec{T}_\ell$ is the tension in the lower string, and m is the mass of the ball. Note that the tension in the upper string is greater than the tension in the lower string. It must balance the downward pull of gravity and the force of the lower string.



We take the +x direction to be leftward (toward the center of the circular orbit) and +y upward. Since the magnitude of the acceleration is $a = v^2/R$, the x component of Newton's second law is

$$T_u \cos\theta + T_\ell \cos\theta = \frac{mv^2}{R},$$

where v is the speed of the ball and R is the radius of its orbit. The y component is

$$T_u \sin\theta - T_\ell \sin\theta - mg = 0.$$

The second equation gives the tension in the lower string: $T_\ell = T_u - mg/\sin\theta$.

ANALYZE

(a) Since the triangle is equilateral, the angle is $\theta = 30.0°$. Thus

$$T_\ell = T_u - \frac{mg}{\sin\theta} = 35.0\text{ N} - \frac{(1.34\text{ kg})(9.80\text{ m/s}^2)}{\sin 30.0°} = 8.74\text{ N}.$$

(b) The net force in the y-direction is zero. In the x-direction, the net force has magnitude

$$F_{\text{net,str}} = (T_u + T_\ell)\cos\theta = (35.0\text{ N} + 8.74\text{ N})\cos 30.0° = 37.9\text{ N}.$$

(c) The radius of the path is

$$R = L\cos\theta = (1.70\text{ m})\cos 30° = 1.47\text{ m}.$$

Using $F_{\text{net,str}} = mv^2/R$, we find the speed of the ball to be

$$v = \sqrt{\frac{RF_{\text{net,str}}}{m}} = \sqrt{\frac{(1.47\ \text{m})(37.9\ \text{N})}{1.34\ \text{kg}}} = 6.45\ \text{m/s}.$$

(d) The direction of $\vec{F}_{\text{net,str}}$ is leftward (ñradially inwardô).

LEARN The upper string, with a tension about 4 times that in the lower string ($T_u \approx 4T_\ell$), will break more easily than the lower one.