공업수학1 과제 (5장)

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[5.1.8]

$xy' - 4y = k$ (k는 상수)

power series method에 따라,

$$y = \sum_{m=0}^{\infty} a_m x^m = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \cdots$$

$$y' = \sum_{m=1}^{\infty} m a_m x^{m-1} = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + 5a_5 x^4 + \cdots$$

$$xy' = \sum_{m=1}^{\infty} m a_m x^m = a_1 x + 2a_2 x^2 + 3a_3 x^3 + 4a_4 x^4 + 5a_5 x^5 + \cdots$$

$$\Rightarrow xy' - 4y = -4a_0 - 3a_1 x - 2a_2 x^2 - a_3 x^3 + a_5 x^5 + \cdots$$

$xy' - 4y = k$ 이므로. $a_0 = -\frac{k}{4}$, $a_1 = 0$, $a_2 = 0$, $a_3 = 0$, $a_5 = 0$

$$\therefore y = -\frac{k}{4} + a_4 x^4$$

[5.1.12]

$(1-x^2)y'' - 2xy' + 2y = 0$

power series method에 따라, $y = \sum_{m=0}^{\infty} a_m x^m$라 하면

$$y' = \sum_{m=1}^{\infty} m a_m x^{m-1}, \quad y'' = \sum_{m=2}^{\infty} m(m-1) a_m x^{m-2}$$

$$\begin{aligned}(1-x^2)y'' - 2xy' + 2y &= (1-x^2)(2a_2 + 6a_3 x + 12a_4 x^2 + \cdots) - 2x(\cancel{a_1} + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \cdots) \\ &\quad + 2(a_0 + \cancel{a_1 x} + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots) \\ &= 2a_2 + 2a_0 + 6a_3 x + (12a_4 - 4a_2)x^2 - 10a_3 x^3 - 2a_4 x^4 = 0\end{aligned}$$

$\rightarrow$ $2a_0 + 2a_2 = 0$, $a_2 = -a_0$, $a_3 = 0$, $a_4 = 0$, $a_4 = -\frac{1}{3}a_2 = -\frac{1}{3}a_0$

$$\therefore y = a_0\left(1 - x^2 - \frac{x^4}{3} + \cdots\right) + a_1 x$$

[5.3.10]

$xy'' + 2y' + 16xy = 0$

주어진 방정식을 정리하면, $y'' + \frac{2}{x}y' + 16y = 0$ 이므로

$b(x) = 2$, $c(x) = 16x^2$ 이다. $b_0 = 2$, $c_0 = 0$ 에서,

결정 방정식 : $r(r-1) + 2r = r(r+1) = 0$, $r = 0$, $r = -1$

$r = 0$ 인 경우, $y = \sum_{m=0}^{\infty} a_m x^m$ 이라 하면,

$$xy'' + 2y' + 16xy = x\sum_{m=2}^{\infty} m(m-1)a_m x^{m-2} + 2\sum_{m=1}^{\infty} m a_m x^{m-1} + 16\sum_{m=0}^{\infty} a_m x^m = 0$$

$$\Rightarrow \sum_{m=1}^{\infty} m(m+1)a_m x^{m-1} + \sum_{m=0}^{\infty} 16 a_m x^{m+1} = 0,$$

$$2a_1 + \sum_{s=0}^{\infty} \left[(s+2)(s+3)a_{s+2} + 16a_s\right] x^{s+1} = 0$$

즉, $a_1 = 0$, $a_{s+2} = \frac{-16}{(s+2)(s+3)} a_s \; (s = 0, 1, 2 \cdots)$

$$\therefore y_1 = 1 - \frac{4^2}{3!}x^2 + \frac{4^4}{5!}x^4 - \frac{4^6}{7!}x^6 + \cdots = \frac{\sin 4x}{4x}$$

계수 축소법에 의하여, $p(x) = \frac{2}{x}$ 이므로

$$u' = \frac{16x^2}{\sin^2 4x} e^{-2\ln x} = \frac{16}{\sin^2 4x}, \quad u = -4\cot 4x$$

$$\therefore y_2 = -4\cot 4x \cdot \frac{\sin 4x}{4x} = -\frac{\cos 4x}{x}$$