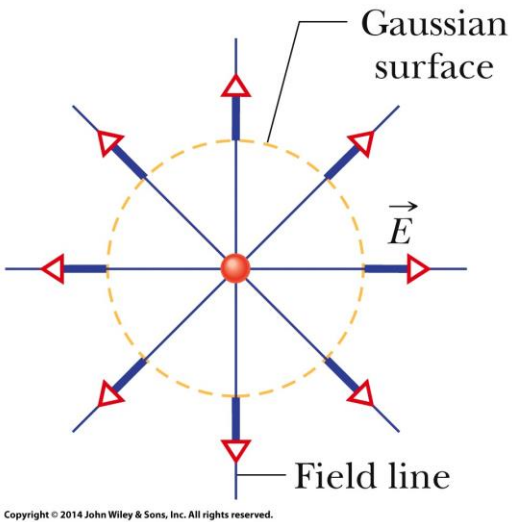
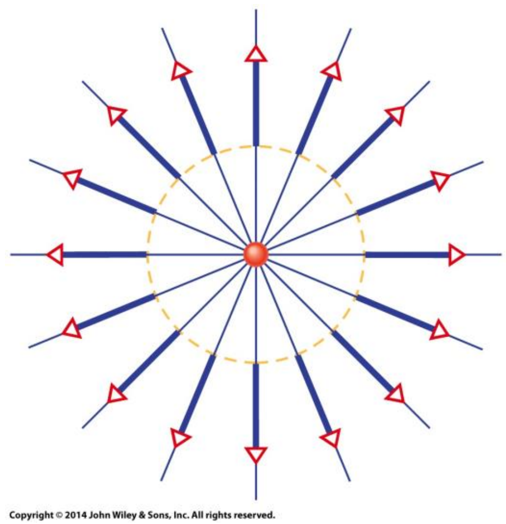
Chapter 23 Gauss' Law

Chap. 23-1 Electric Flux

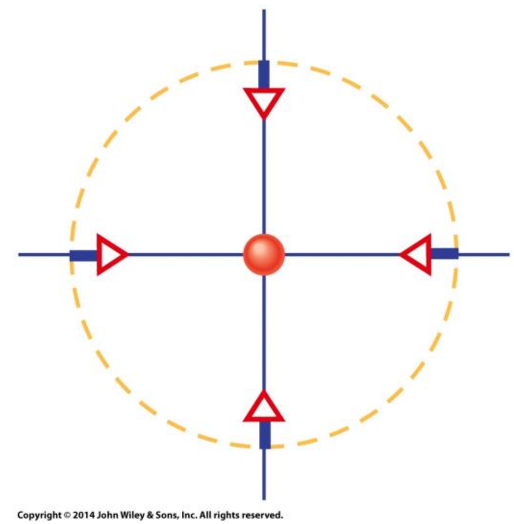
Guass' law relates the electric field at points on a (closed) Gaussian surface to the net charge enclosed by that surface.



Electric field vectors and field lines pierce an imaginary, spherical Gaussian surface that encloses a particle with charge *+Q*.



Now the enclosed particle has charge +2Q.



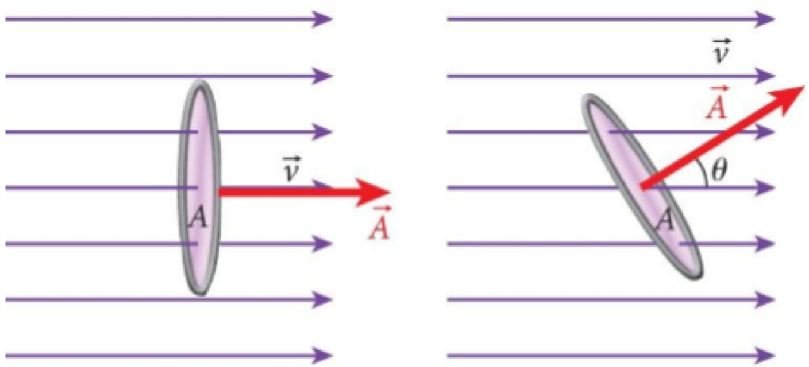
Can you tell what the enclosed charge is now?
Answer: -0.5Q

Chap. 23-1 Electric Flux

쿨롱의 법칙을 이용한 적분을 하지 않고 전기장을 구하는 방법은 없을까?

→ use flux (다발)

- 유체의 흐름에서 다발의 개념 – 단위시간당 표면 A를 통과하는 유체의 양



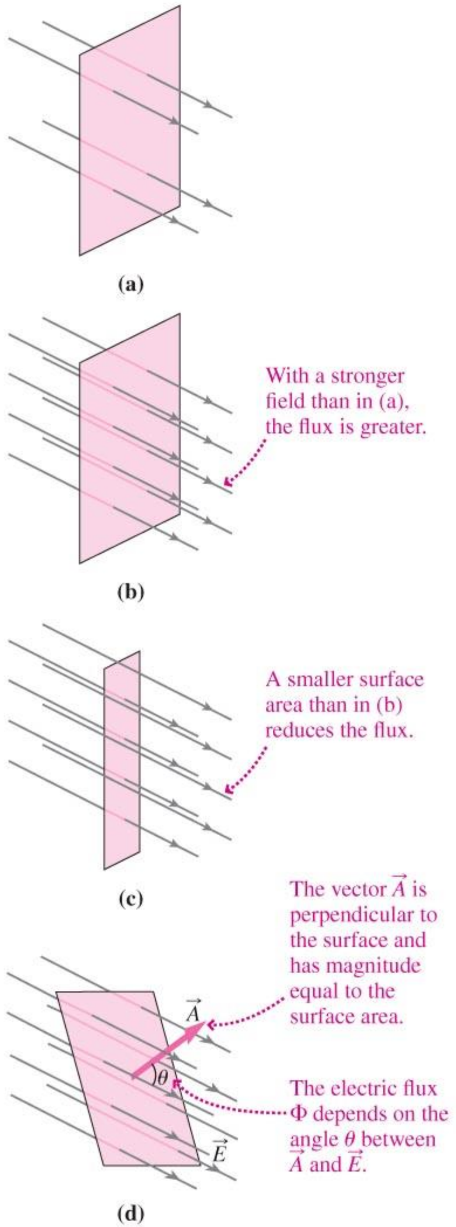
$$\Phi = \underbrace{\vec{v}}_{\text{속도}} \cdot \underbrace{\vec{A}}_{\text{면적}} = vA\cos\theta$$

Chap. 23-1 Electric Flux

- **Electric flux** quantifies the notion "number of field lines crossing a surface."
- The electric flux Φ through a flat surface in a uniform electric field depends on the field strength *E*, the surface area *A*, and the angle *θ* between the field and the surface.
- Mathematically, the flux is given by

$$\Phi = EA\cos\theta = \vec{E} \cdot \vec{A}$$

  - Here $\vec{A}$ is a vector whose magnitude is the surface area *A* and whose orien tation is normal to the surface.



Chap. 23-1 Electric Flux

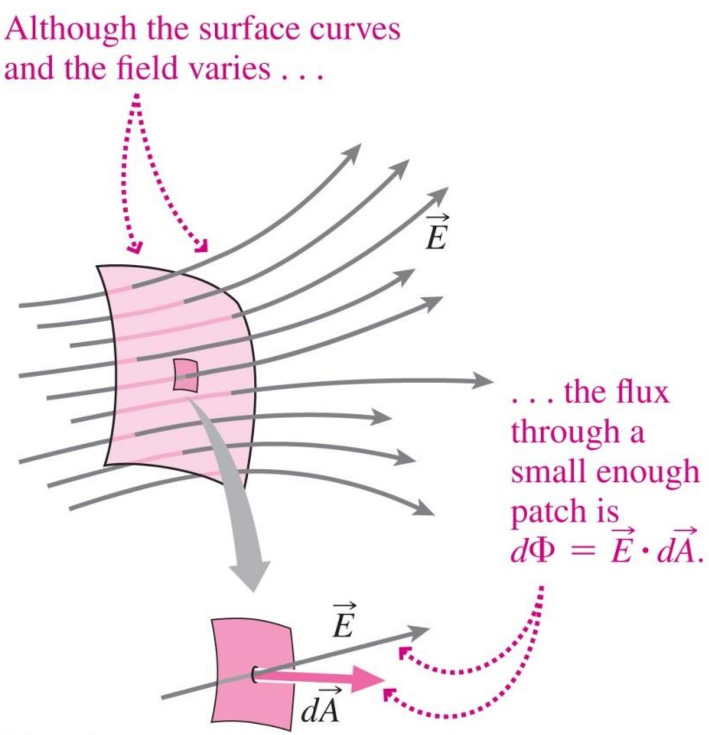
Electric Flux with Curved Surfaces and Nonuniform Fields

➢ When the surface is curved or the field is nonuniform, we calculate the flux by dividing the surface into small patches $d\vec{A}$, so small that each patch is essentially flat and the field is essentially uniform over each patch.

- We then sum the fluxes
  $d\Phi = \vec{E} \cdot d\vec{A}$ over each patch.

- In the limit of infinitely many infinitesimally small patches,
  the sum becomes a **surface integral:**

$$\Phi = \int \vec{E} \cdot d\vec{A}$$



Chap. 23-1 Electric Flux

The **total flux** through a surface is given by

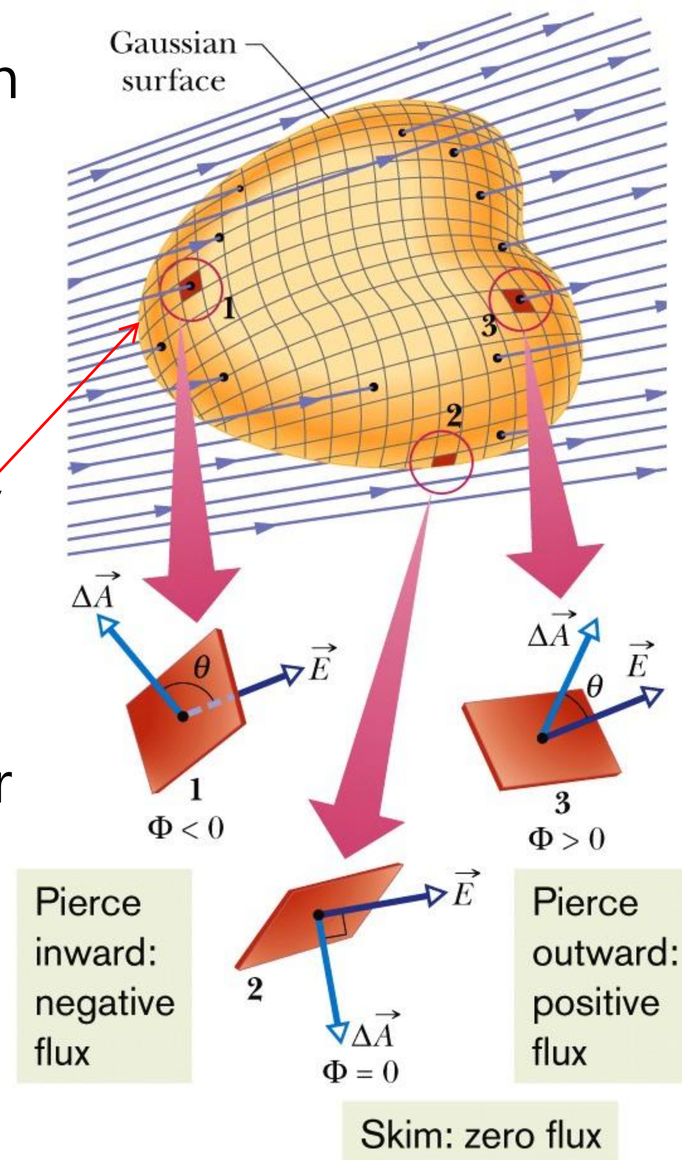
$$\Phi = \int \vec{E} \cdot d\vec{A} \qquad \text{(total flux).}$$

The **net flux** through a closed surface (which is used in Gauss' law) is given by

$$\Phi = \oint \vec{E} \cdot d\vec{A} \qquad \text{(net flux).}$$

where the integration is carried out over the entire surface.

Gauss 폐곡면
(Gaussian surface)



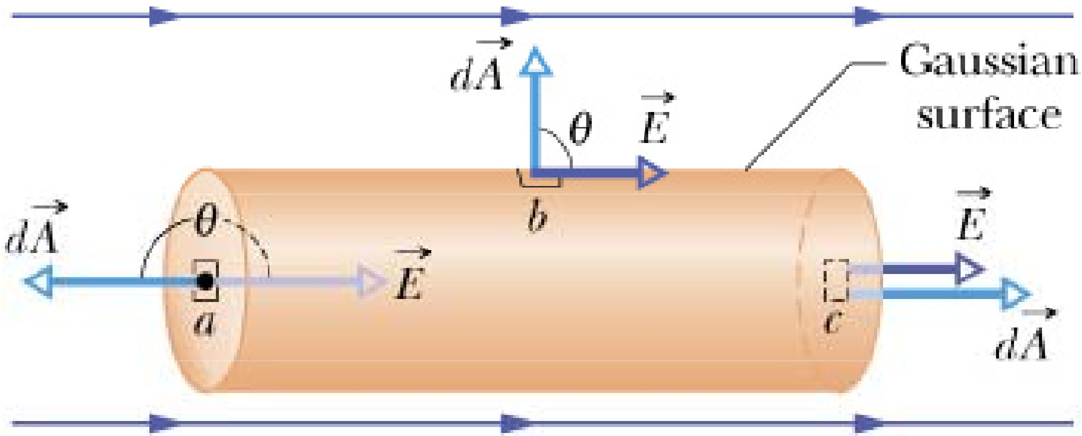
Chap. 23-1 Electric Flux

•What does this new quantity mean?

- The integral is over a CLOSED SURFACE
- Since $\vec{E} \bullet d\vec{A}$ is a SCALAR product, the electric flux is a SCALAR quantity
- The integration vector $d\vec{A}$ is normal to the surface and points OUT of the surface. $\vec{E} \bullet d\vec{A}$ is interpreted as the component of E which is NORMAL to the SURFACE
- Therefore, the electric flux through a closed surface is the sum of the normal components of the electric field all over the surface.
- The sign matters!!
  Pay attention to the direction of the normal component as it penetrates the surface… is it "out of" or "into" the surface?
- "Out of" is "+" "into" is "-"

Chap. 23-1 Electric Flux

보기문제 23-1



$$\Phi = \oint \mathbf{E} \cdot d\mathbf{A} = \int_a \mathbf{E} \cdot d\mathbf{A} + \int_b \mathbf{E} \cdot d\mathbf{A} + \int_c \mathbf{E} \cdot d\mathbf{A}$$

$$\int_a \mathbf{E} \cdot d\mathbf{A} = \int E(\cos 180^o)dA = -E\int dA = -EA$$

$$\int_b \mathbf{E} \cdot d\mathbf{A} = \int \mathrm{E}(\cos 90^o)\mathrm{dA} = 0$$

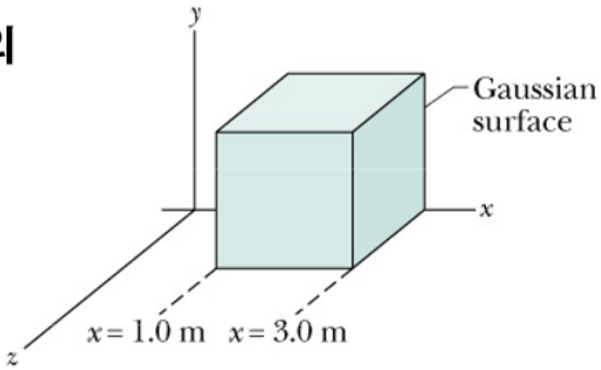
$$\int_c \mathbf{E} \cdot d\mathbf{A} = \int E(\cos 0^o)dA = EA$$

$$\Phi = -EA + 0 + EA = 0$$

보기문제 23-2

그림과 같은 정육면체의 왼쪽, 오른쪽, 위쪽 면에서의 전기장 플럭스? 단, 전기장은

$\mathbf{E} = (3.0\,x)\,i + (4.0)\,j$ (N/C).



1) 왼쪽 면

$$dA = -(dydz)\,\mathrm{i},$$

$$E = (3.0x\,\mathrm{i} + 4.0\,\mathrm{j})_{x=1.0} = (3.0)\mathrm{i} + (4.0)\mathrm{j}$$

$$\Phi = \int E \cdot dA = \int_{z=0}^{z=2}\int_{y=0}^{y=2}(3.0\mathrm{i} + 4.0\mathrm{j})\cdot(-dydz\,\mathrm{i})$$

$$= -3.0\int_{z=0}^{z=2}\int_{y=0}^{y=2} dydz = -12.0\ (\mathrm{N\cdot m^2/C})$$

2) 오른쪽 면

$$dA = dydz\,\mathrm{i},\quad E = (3.0x\,\mathrm{i} + 4.0\,\mathrm{j})_{x=3.0} = (9.0)\mathrm{i} + (4.0)\mathrm{j}$$

$$\Phi_{오른쪽} = -3\times\Phi_{왼쪽} = 36\ (\mathrm{N\cdot m^2/C})$$

3) 위쪽 면

$$dA = dzdx\,\mathrm{j},$$

$$\Phi_{위쪽} = \int_{x=1}^{x=3}\int_{z=0}^{z=2}(3.0x\,\mathrm{i} + 4.0\,\mathrm{j})\cdot(dzdx\,\mathrm{j}) = 16.0\ (\mathrm{N\cdot m^2/C})$$

Chap. 23-2 Gauss' Law

- 쿨롱의 법칙 ⇔ 가우스의 법칙

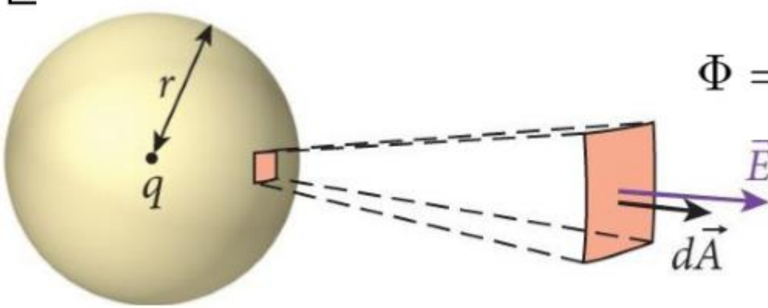
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

$$\Phi = \oint_A \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0}$$

폐곡면 A 상의 적분

A의 **내부**에 있는 전하

- 구면



$$\Phi = \oint_S \mathrm{E}\cdot d\,\mathrm{A} = E\oint_S dA = E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad \text{(쿨롱 법칙)}}$$

- Gauss's law is *always* true.
- But it's useful for calculating the electric field only in situations with sufficient symmetry:
  - Spherical symmetry
  - Line symmetry
  - Plane symmetry

Chap. 23-2 Gauss' Law

$$\varepsilon_0 \Phi = q_{enc} \quad \text{(Gauss' law).}$$

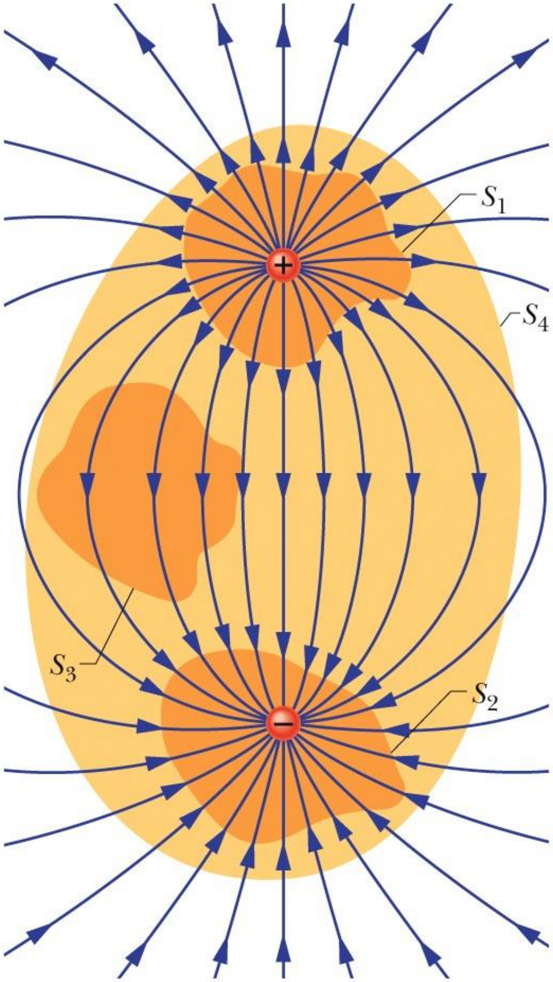
$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc} \quad \text{(Gauss' law).}$$

면 S_1 : $\Phi > 0,\ q_{enc} > 0$

면 S_2 : $\Phi < 0,\ q_{enc} < 0$

면 S_3 : $\Phi = 0,\ q_{enc} = 0$

면 S_4 : $\Phi = 0,\ q_{enc} = 0$



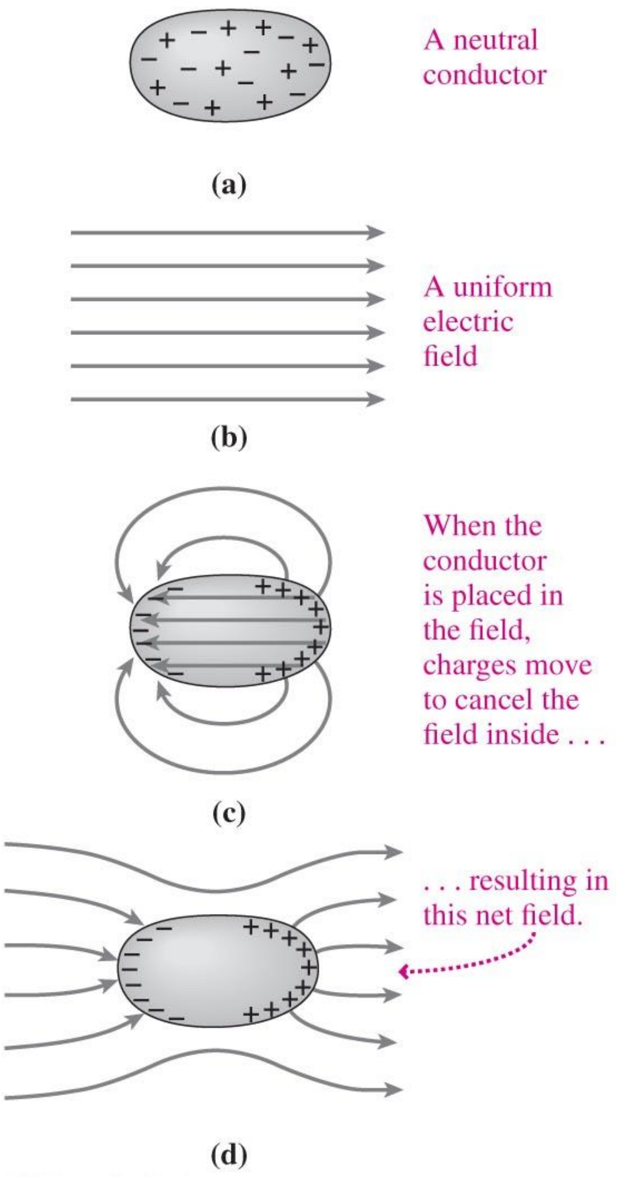
Chap. 23-2 Gauss' Law

Gauss's Law: A Problem-Solving Strategy

- **INTERPRET**: Check that your charge distribution has sufficie nt symmetry.
- **DEVELOP**: Draw a diagram and use symmetry to find the di rection of the electric field. Then draw a **Gaussian surface** on which you can evaluate the surface integral in Gauss's la w.
- **EVALUATE**:
  - Evaluate the flux $\Phi = \oint \vec{E} \cdot d\vec{A}$ over your surface. The re sult contains the unknown field strength E.
  - Evaluate the enclosed charge.
  - Equate the flux to $q_{\text{enclosed}}/\varepsilon_0$ and solve for E.
- **ASSESS**: Check that your answer makes sense, especially in comparison to charge distributions whose fields you know.

Chap. 23-3 A Charged Isolated Conductor

- Charges in conductors are free to move, and they do so in response to an applied electric field.
  - If a conductor is allowed to reach **electrostatic equilibrium**, a condition in which there is no net charge motion, then charges redistribute themselves to cancel the applied field inside the conductor.
  - Therefore **the electric field is zero inside a conductor in electrostatic equilibrium.**

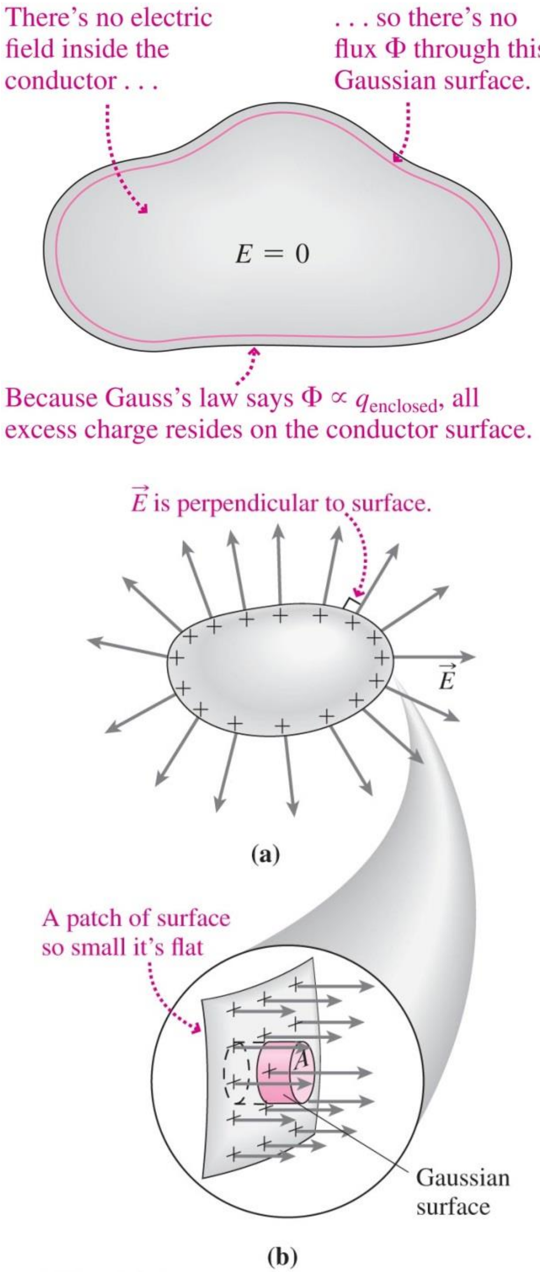


Chap. 23-3 A Charged Isolated Conductor

- Gauss's law requires that any free charge on a conductor reside on the conductor surface.

- The electric field at the surface of a charged conductor in electrostatic equilibrium is perpendicular to the surface.

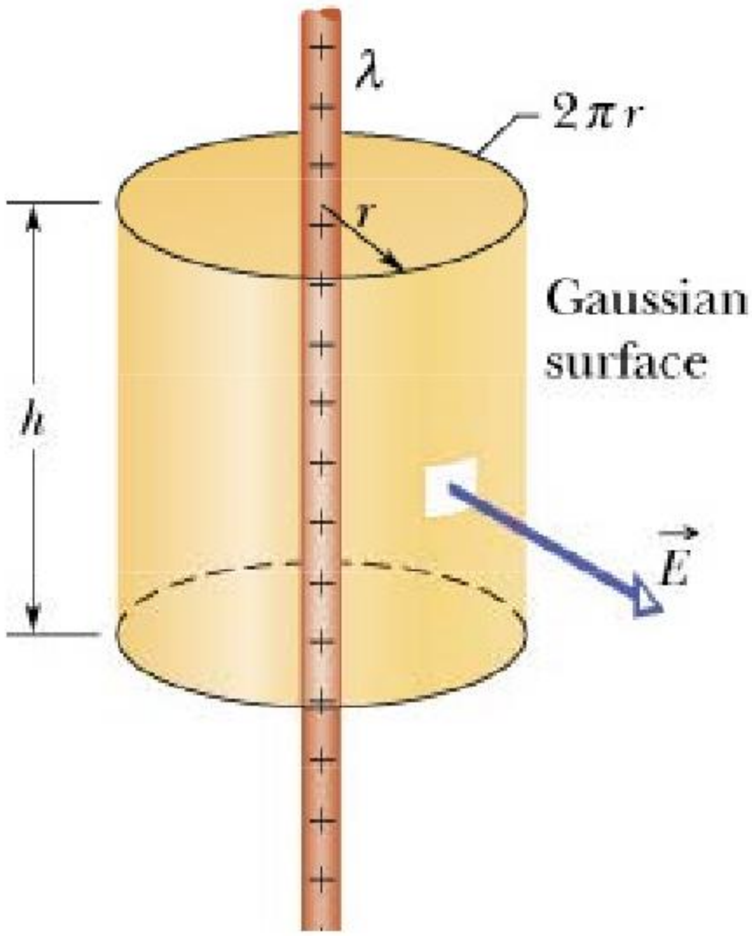
$$E = \frac{\sigma}{\varepsilon_0} \quad \text{(conducting surface).}$$



Chap. 23-4 Gauss' Law: Cylindrical Symmetry

원통대칭

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_o}$$



$$\Phi = EA\cos\theta = E(2\pi rh)$$

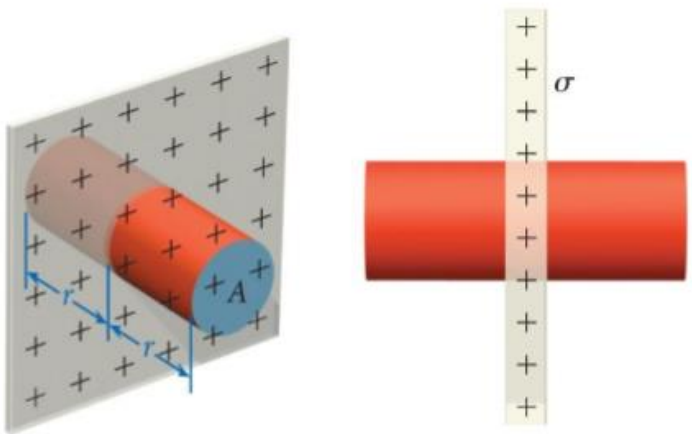
$$q_{속} = \lambda h$$

$$\therefore E = \frac{1}{2\pi\varepsilon_0}\left(\frac{\lambda}{r}\right)$$

Chap. 23-5 Gauss' Law: Planar Symmetry

면대칭

얇은 절연체판

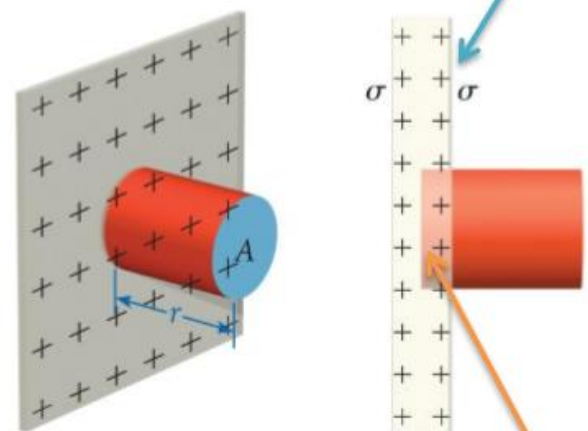


$$\oint \vec{E} \cdot d\vec{A} = EA + EA = \frac{q}{\epsilon} = \frac{\sigma A}{\epsilon}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

두 도체판

전도체는 전하가 표면에만 존재한다.



전도체는 내부의 전기장은 0이다.

$$\oint \vec{E} \cdot d\vec{A} = 0 + EA = \frac{q}{\epsilon} = \frac{\sigma A}{\epsilon}$$

$$E = \frac{\sigma}{\epsilon_0}$$

Chap. 23-6 Gauss' Law: Spherical Symmetry

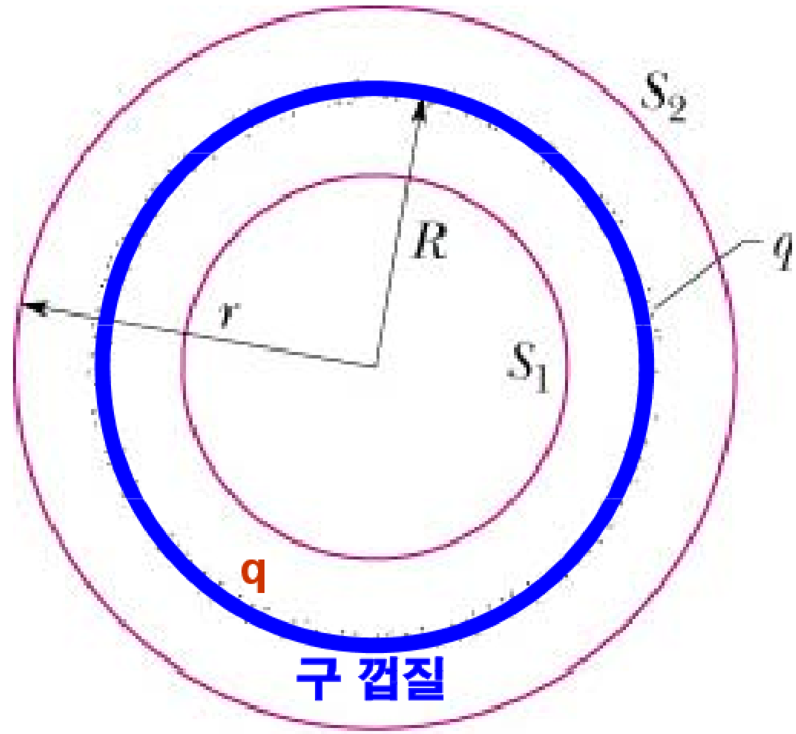
구대칭

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_o}$$

$$\Phi = 4\pi r^2 E$$

$$q_{속} = q, \quad (r > R, \text{ 공바깥}) \ (S_2)$$
$$= 0, \quad (r < R, \text{ 공속}) \ (S_1)$$

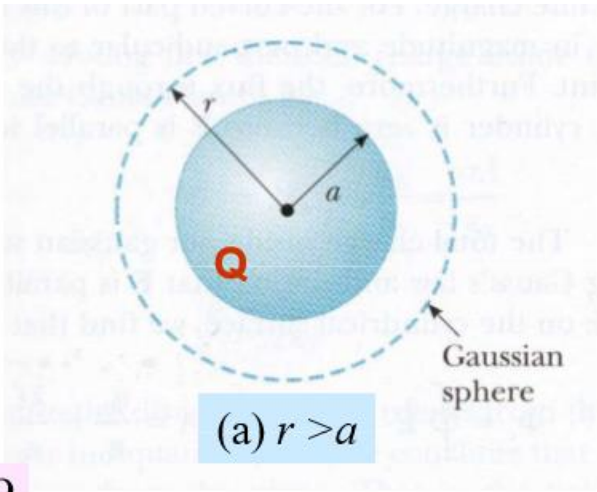
$$\therefore E = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r^2}\right), \quad (r > R, \text{ 공바깥})$$
$$= 0, \quad (r < R, \text{ 공속})$$



Chap. 23-6 Gauss' Law: Spherical Symmetry

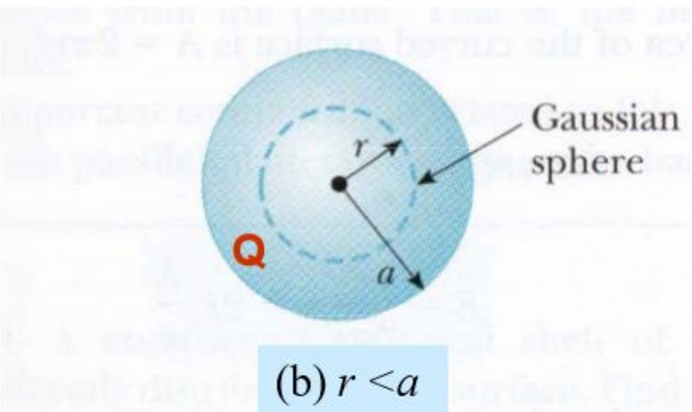
구대칭 전하분포

Charge density ρ : $\frac{4}{3}\pi a^3 \rho = Q \quad \Rightarrow \rho = \frac{3Q}{4\pi a^3}$



(a) $r > a$ $\quad \Phi_C = \oint \vec{E} \cdot d\vec{A} = \frac{q_{inc}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$

$$4\pi r^2 \cdot E = \frac{Q}{\varepsilon_0} \qquad \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = k_e \frac{Q}{r^2} \qquad \text{for } r > a$$



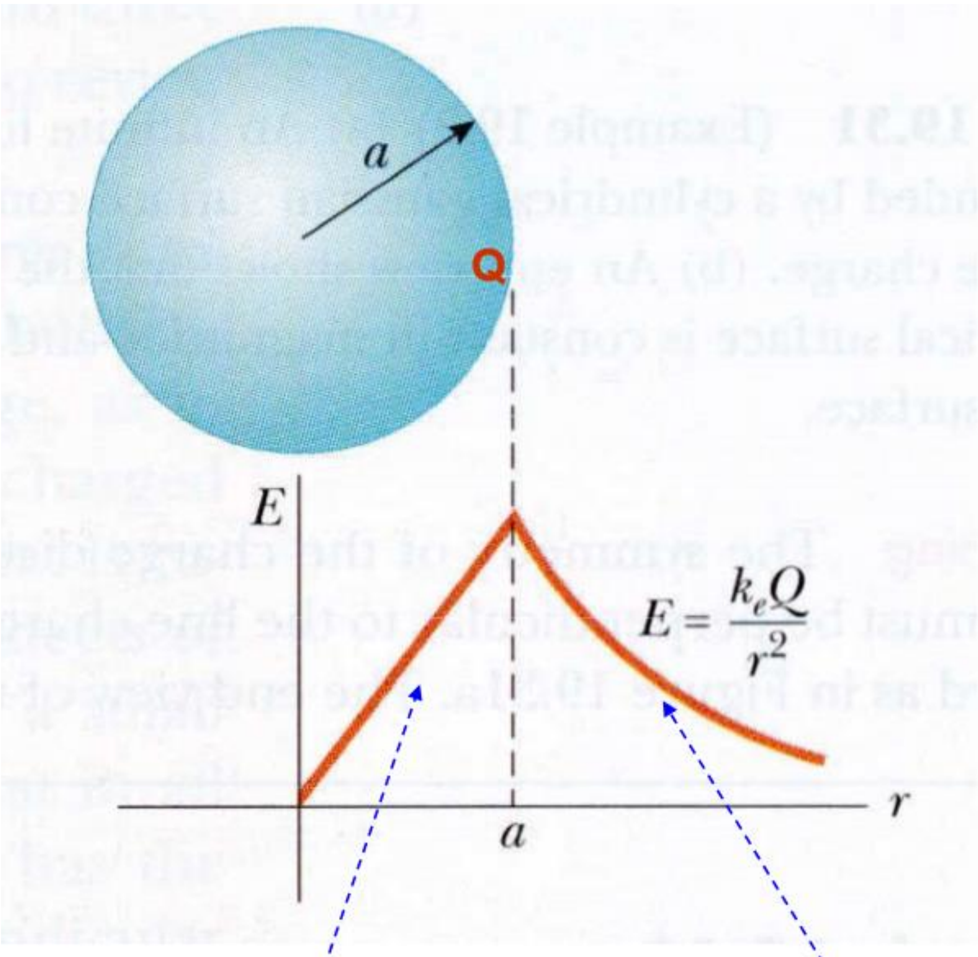
(b) $r < a$ $\quad \Phi_C = \oint \vec{E} \cdot d\vec{A} = \frac{q_{inc}}{\varepsilon_0}$

$$= \frac{1}{\varepsilon_0}\int \rho \cdot dV$$

$$= \frac{\rho}{\varepsilon_0}\frac{4\pi}{3} r^3$$

$$\left(\rho = \frac{3Q}{4\pi a^3}\right) \quad 4\pi r^2 E = \frac{Q}{\varepsilon_0}\frac{r^3}{a^3} \quad \Rightarrow E = \frac{Q}{4\pi\varepsilon_0}\frac{r}{a^3} = k_e \frac{Qr}{a^3} \qquad \text{for } r < a$$

Chap. 23-6 Gauss' Law: Spherical Symmetry



for r < a

$$\Rightarrow E = \frac{Q}{4\pi\varepsilon_0}\frac{r}{a^3} = k_e\frac{Qr}{a^3}$$

for r > a

$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = k_e\frac{Q}{r^2}$$

Summary

Gauss' Law

- Gauss' law is

$$\varepsilon_0 \Phi = q_{enc}$$ **Eq. 23-6**

- the net flux of the electric field through the surface:

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$ **Eq. 23-6**

Applications of Gauss' Law

- surface of a charged conductor

$$E = \frac{\sigma}{\varepsilon_0}$$ **Eq. 23-11**

- Within the surface *E=0.*
- Line of charge

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$ **Eq. 23-12**

- Infinite non-conducting sheet

$$E = \frac{\sigma}{2\varepsilon_0}$$ **Eq. 23-13**

- Outside a spherical shell of charge

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$ **Eq. 23-15**

- Inside a uniform spherical shell

$$E = 0$$ **Eq. 23-16**

- Inside a uniform sphere of charge

$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r.$$ **Eq. 23-20**