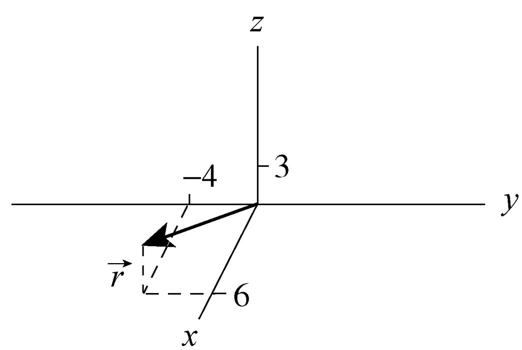
1. (a) The magnitude of $\vec{r}$ is

$$|\vec{r}| = \sqrt{(6.0\text{ m})^2 + (-4.0\text{ m})^2 + (3.0\text{ m})^2} = 7.8\,\text{m}.$$

(b) The sketch is shown in the following figure. The coordinate values are in meters.



2. (a) The position vector, according to Eq. 4-1, is $\vec{r} = (-5.0 \text{ m})\,\hat{i} + (9.0 \text{ m})\hat{j}$.

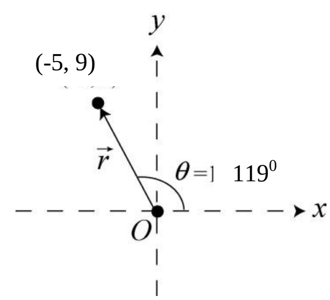
(b) The magnitude is $|\vec{r}\,| = \sqrt{x^2 + y^2 + z^2} = \sqrt{(-5.0 \text{ m})^2 + (9.0 \text{ m})^2 + (0 \text{ m})^2} = 10 \text{ m}$.

(c) Many calculators have polar ↔ rectangular conversion capabilities that make this computation more efficient than what is shown below. Noting that the vector lies in the *xy* plane and using Eq. 3-6, we obtain:

$$\theta = \tan^{-1}\left(\frac{9.0 \text{ m}}{-5.0 \text{ m}}\right) = -61° \text{ or } 119°$$

where the latter possibility (119° measured counterclockwise from the +*x* direction) is chosen since the signs of the components imply the vector is in the second quadrant

(d) The sketch is shown to the right. The vector is 119° counterclockwise from the +*x* direction.




(e) The displacement is $\Delta\vec{r} = \vec{r}\,' - \vec{r}$ where $\vec{r}$ is given in part (a) and $\vec{r}\,' = (3.0 \text{ m})\hat{i}$. Therefore, $\Delta\vec{r} = (8.0 \text{ m})\hat{i} - (9.0 \text{ m})\hat{j}$.

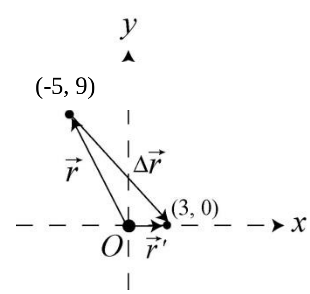
(f) The magnitude of the displacement is

$$|\Delta\vec{r}\,| = \sqrt{(8.0 \text{ m})^2 + (-9.0 \text{ m})^2} = 12 \text{ m}.$$

(g) The angle for the displacement, using Eq. 3-6, is

$$\tan^{-1}\left(\frac{8.0 \text{ m}}{-9.0 \text{ m}}\right) = -42° \text{ or } 138°$$

where we choose the former possibility (–42°, or 42° measured *clockwise* from +*x*) since the signs of the components imply the vector is in the fourth quadrant. A sketch of $\Delta\vec{r}$ is shown on the right.



3. The initial position vector $\vec{r}_o$ satisfies the displacement equation

$$\vec{r} - \vec{r}_0 = \Delta\vec{r}\,, \qquad (1)$$

where

$$\Delta\vec{r} = 2.0\hat{i} - 4.0\hat{j} + 8.0\hat{k}$$

Therefore, from Eq. (1), we have

$$\begin{aligned}(4.0\hat{j} - 5.0\hat{k})\,\mathrm{m} - (\vec{r}_0) &= (2.0\hat{i} - 4.0\hat{j} + 8.0\hat{k})\,\mathrm{m}\\ \Rightarrow \vec{r}_0 &= (4.0\,\mathrm{m})\hat{j} - (5.0\,\mathrm{m})\hat{k} - (2.0\,\mathrm{m})\hat{i} + (4.0\,\mathrm{m})\hat{j} - (8.0\,\mathrm{m})\hat{k}\\ &= -(2.0\,\mathrm{m})\hat{i} + (8.0\,\mathrm{m})\hat{j} - (13\,\mathrm{m})\hat{k}.\end{aligned}$$

4. We choose a coordinate system with origin at the clock center and $+x$ rightward (toward the ñ3:00ò position) and $+y$ upward (toward ñ12:00ò).

(a) In unit-vector notation, we have $\vec{r}_1 = (12\text{ cm})\hat{i}$ and $\vec{r}_2 = (-12\text{ cm})\hat{j}$. Thus, Eq. 4-2 gives

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (-12\text{ cm})\hat{i} + (-12\text{ cm})\hat{j}.$$

The magnitude is given by $|\Delta\vec{r}| = \sqrt{(-12\text{ cm})^2 + (-12\text{ cm})^2} = 17\text{ cm}$.

(b) Using Eq. 3-6, the angle is

$$\theta = \tan^{-1}\left(\frac{-12\text{ cm}}{-12\text{ cm}}\right) = 45° \text{ or } -135°.$$

We choose $-135°$ since the desired angle is in the third quadrant. In terms of the magnitude-angle notation, one may write

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (-12\text{ cm})\hat{i} + (-12\text{ cm})\hat{j}.$$

(c) In this case, we have $\vec{r}_1 = (-12\text{ cm})\hat{j}$ and $\vec{r}_2 = (12\text{ cm})\hat{j}$, and $\Delta\vec{r} = (24\text{ cm})\hat{j}$. Thus, $|\Delta\vec{r}| = 24\text{ cm}$.

(d) Using Eq. 3-6, the angle is given by

$$\theta = \tan^{-1}\left(\frac{24\text{ cm}}{0\text{ cm}}\right) = 90°.$$

(e) In a full-hour sweep, the hand returns to its starting position, and the displacement is zero.

(f) The corresponding angle for a full-hour sweep is also zero.

5. **THINK** This problem deals with the motion of a train in two dimensions. The entire trip consists of three parts, and we're interested in the overall average velocity.

**EXPRESS** The average velocity of the entire trip is given by Eq. 4-8, $\vec{v}_{\text{avg}} = \Delta\vec{r}/\Delta t$, where the total displacement $\Delta\vec{r} = \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3$ is the sum of three displacements (each result of a constant velocity during a given time), and $\Delta t = \Delta t_1 + \Delta t_2 + \Delta t_3$ is the total amount of time for the trip. We use a coordinate system with +*x* for East and +*y* for North.

**ANALYZE**
(a) In unit-vector notation, the first displacement is given by

$$\Delta\vec{r}_1 = \left(60.0\ \frac{\text{km}}{\text{h}}\right)\left(\frac{40.0\ \text{min}}{60\ \text{min/h}}\right)\hat{i} = (40.0\ \text{km})\hat{i}.$$

The second displacement has a magnitude of $(60.0\ \frac{\text{km}}{\text{h}})\cdot(\frac{20.0\ \text{min}}{60\ \text{min/h}}) = 20.0\ \text{km}$, and its direction is 40° north of east. Therefore,

$$\Delta\vec{r}_2 = (20.0\ \text{km})\cos(40.0°)\,\hat{i} + (20.0\ \text{km})\sin(40.0°)\,\hat{j} = (15.3\ \text{km})\,\hat{i} + (12.9\ \text{km})\,\hat{j}.$$

Similarly, the third displacement is

$$\Delta\vec{r}_3 = -\left(60.0\ \frac{\text{km}}{\text{h}}\right)\left(\frac{50.0\ \text{min}}{60\ \text{min/h}}\right)\hat{i} = (-50.0\ \text{km})\,\hat{i}.$$

Thus, the total displacement is

$$\begin{aligned}\Delta\vec{r} &= \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3 = (40.0\ \text{km})\hat{i} + (15.3\ \text{km})\,\hat{i} + (12.9\ \text{km})\,\hat{j} - (50.0\ \text{km})\,\hat{i} \\ &= (5.30\ \text{km})\,\hat{i} + (12.9\ \text{km})\,\hat{j}.\end{aligned}$$

The time for the trip is $\Delta t = (40.0 + 20.0 + 50.0)$ min = 110 min, which is equivalent to 1.83 h. Eq. 4-8 then yields

$$\vec{v}_{\text{avg}} = \frac{(5.30\ \text{km})\,\hat{i} + (12.9\ \text{km})\,\hat{j}}{1.83\ \text{h}} = (2.90\ \text{km/h})\,\hat{i} + (7.01\ \text{km/h})\,\hat{j}.$$

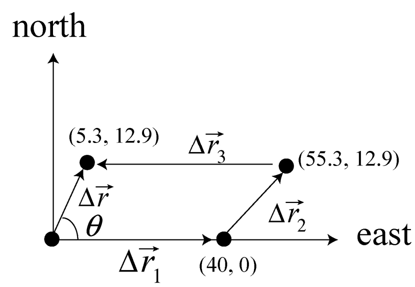
The magnitude of $\vec{v}_{\text{avg}}$ is $|\vec{v}_{\text{avg}}| = \sqrt{(2.90\text{ km/h})^2 + (7.01\text{ km/h})^2} = 7.59\text{ km/h}.$

(b) The angle is given by

$$\theta = \tan^{-1}\left(\frac{v_{\text{avg},y}}{v_{\text{avg},x}}\right) = \tan^{-1}\left(\frac{7.01\text{ km/h}}{2.90\text{ km/h}}\right) = 67.5° \ \ (\text{north of east}),$$

or $22.5°$ east of due north.

**LEARN** The displacement of the train is depicted in the figure below:




Note that the net displacement $\Delta\vec{r}$ is found by adding $\Delta\vec{r}_1$, $\Delta\vec{r}_2$ and $\Delta\vec{r}_3$ vectorially.

6. To emphasize the fact that the velocity is a function of time, we adopt the notation $v(t)$ for $dx/dt$.

(a) Equation 4-10 leads to

$$v(t) = \frac{d}{dt}\,(3.00t\,\hat{i} - 4.00t^2\,\hat{j} + 2.00\,\hat{k}) = (3.00\text{ m/s})\,\hat{i} - (8.00t\text{ m/s})\,\hat{j}$$

(b) Evaluating this result at $t$ = 3.00 s produces $\vec{v} = (3.00\,\hat{i} - 24.0\,\hat{j})$ m/s.

(c) The speed at $t$ = 3.00 s is $v = |\vec{v}| = \sqrt{(3.00\text{ m/s})^2 + (-24.0\text{ m/s})^2} = 24.2\text{ m/s}$.

(d) The angle of $\vec{v}$ at that moment is

$$\tan^{-1}\left(\frac{-24.0\text{ m/s}}{3.00\text{ m/s}}\right) = -82.9^\circ \text{ or } 97.1^\circ$$

where we choose the first possibility (82.9° measured *clockwise* from the +$x$ direction, or 277° counterclockwise from +$x$) since the signs of the components imply the vector is in the fourth quadrant.

7. Using the two equations

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

and

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t},$$

we calculate the average velocity of the particle as follows:

$$\begin{aligned}\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} &= \frac{(-3.0\hat{i} + 9.0\hat{j} - 3.0\hat{k})\,\text{m} - (6.0\hat{i} - 7.0\hat{j} + 3.0\hat{k})\,\text{m}}{10\,\text{s}} \\ &= \frac{(-9.0\hat{i} + 16.0\hat{j} - 6.0\hat{k})\,\text{m}}{10\,\text{s}} \\ &= (-0.90\hat{i} + 1.6\hat{j} - 0.60\hat{k})\,\text{m/s}.\end{aligned}$$

8. Our coordinate system has $\hat{\text{i}}$ pointed east and $\hat{\text{j}}$ pointed north. The first displacement is $\vec{r}_{AB} = (483 \text{ km})\hat{\text{i}}$ and the second is $\vec{r}_{BC} = (-966 \text{ km})\hat{\text{j}}$.

(a) The net displacement is

$$\vec{r}_{AC} = \vec{r}_{AB} + \vec{r}_{BC} = (483 \text{ km})\hat{\text{i}} - (966 \text{ km})\hat{\text{j}}$$

which yields $|\vec{r}_{AC}| = \sqrt{(483 \text{ km})^2 + (-966 \text{ km})^2} = 1.08 \times 10^3 \text{ km}$.

(b) The angle is given by

$$\theta = \tan^{-1}\left(\frac{-966 \text{ km}}{483 \text{ km}}\right) = -63.4°.$$

We observe that the angle can be alternatively expressed as 63.4° south of east, or 26.6° east of south.

(c) Dividing the magnitude of $\vec{r}_{AC}$ by the total time (2.25 h) gives

$$\vec{v}_{\text{avg}} = \frac{(483 \text{ km})\hat{\text{i}} - (966 \text{ km})\hat{\text{j}}}{2.30 \text{ h}} = (210 \text{ km/h})\hat{\text{i}} - (420 \text{ km/h})\hat{\text{j}}$$

with a magnitude $|\vec{v}_{\text{avg}}| = \sqrt{(210 \text{ km/h})^2 + (-420 \text{ km/h})^2} = 470 \text{ km/h}$.

(d) The direction of $\vec{v}_{\text{avg}}$ is 26.6° east of south, same as in part (b). In magnitude-angle notation, we would have $\vec{v}_{\text{avg}} = (470 \text{ km/h} \angle -63.4°)$.

(e) Assuming the *AB* trip was a straight one, and similarly for the *BC* trip, then $|\vec{r}_{AB}|$ is the distance traveled during the *AB* trip, and $|\vec{r}_{BC}|$ is the distance traveled during the *BC* trip. Since the average speed is the total distance divided by the total time, it equals

$$\frac{483 \text{ km} + 966 \text{ km}}{2.30 \text{ h}} = 630 \text{ km/h}.$$

9. The $(x,y)$ coordinates (in meters) of the points are $A = (15, -15)$, $B = (30, -45)$, $C = (20, -15)$, and $D = (45, 45)$. The respective times are $t_A = 0$, $t_B = 300$ s, $t_C = 600$ s, and $t_D = 900$ s. Average velocity is defined by Eq. 4-8. Each displacement $\Delta\vec{r}$ is understood to originate at point $A$.

(a) The average velocity having the least magnitude (5.0 m/600 s) is for the displacement ending at point $C$: $|\vec{v}_{\text{avg}}| = 0.0083$ m/s.

(b) The direction of $\vec{v}_{\text{avg}}$ is 0° (measured counterclockwise from the +$x$ axis).

(c) The average velocity having the greatest magnitude ($\sqrt{(15\text{ m})^2 + (30\text{ m})^2}/300\text{ s}$) is for the displacement ending at point $B$: $|\vec{v}_{avg}| = 0.11\text{ m/s}$.

(d) The direction of $\vec{v}_{avg}$ is 297° (counterclockwise from +$x$) or −63° (which is equivalent to measuring 63° *clockwise* from the +$x$ axis).

10. We differentiate $\vec{r} = 5.00t\,\hat{\mathrm{i}} + (et + ft^2)\,\hat{\mathrm{j}}$.

(a) The particle's motion is indicated by the derivative of $\vec{r}$: $\vec{v} = 5.00\,\hat{\mathrm{i}} + (e + 2ft)\,\hat{\mathrm{j}}$. The angle of its direction of motion is consequently

$$\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}[(e + 2ft)/5.00].$$

The graph indicates $\theta_o = 35.0°$, which determines the parameter $e$:

$$e = (5.00 \text{ m/s}) \tan(35.0°) = 3.50 \text{ m/s}.$$

(b) We note (from the graph) that $\theta = 0$ when $t = 14.0$ s. Thus, $e + 2ft = 0$ at that time. This determines the parameter $f$:

$$f = \frac{-e}{2t} = \frac{-3.5 \text{ m/s}}{2(14.0 \text{ s})} = -0.125 \text{ m/s}^2 .$$

11. Position of the particle is given by

$$\vec{r} = (3.00t^3 - 6.00t)\hat{i} + (7.00 - 8.00t^4)\hat{j}$$

Therefore,

$$\vec{v} = \frac{d\vec{r}}{dt} = (9.00t^2 - 6.00)\hat{i} + (-32.0t^4)\hat{j}$$

and

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = (18.0t)\hat{i} - (96.0t^2)\hat{j}$$

(a) We calculate $\vec{r}$, $\vec{v}$, and $\vec{a}$ at $t$ = 3 s as follows:

$$(\vec{r})_{t=3\,\text{s}} = (81.0 - 18.0)\hat{i} + (7.00 - 8.00 \times 81.0)\hat{j} = (63.0\hat{i} - 641\hat{j})\ \text{m}\,.$$

$$(\vec{v})_{t=3} = (9.00 \times 3.00^2 - 6.00)\hat{i} - (32.0 \times 3.00^3)\hat{j} = 75.0\hat{i} - 864\hat{j}$$

$$(\vec{a})_{t=3} = (18 \times 3)\hat{i} - (96 \times 3^2)\hat{j} = (54\hat{i} - 864\hat{j})\ \text{m/s}^2\,.$$

(b) The angle of $\vec{v}$, measured from $+x$, is either

$$\theta = \tan^{-1}\left(\frac{-864}{75}\right) = -85.0° \text{ or } 94.0°$$

where we settle on the first choice (–85.0°, which is equivalent to 275° measured counterclockwise from the $+x$ axis) since the signs of its components imply that it is in the fourth quadrant.

12. We adopt a coordinate system with $\hat{\text{i}}$ pointed east and $\hat{\text{j}}$ pointed north; the coordinate origin is the flagpole. We "translate" the given information into unit-vector notation as follows:

$$\vec{r}_o = (30.0\text{ m})\hat{\text{i}} \quad \text{and} \quad \vec{v}_o = (-10.0\text{ m/s})\hat{\text{j}}$$
$$\vec{r} = (40.0\text{ m})\hat{\text{j}} \quad \text{and} \quad \vec{v} = (10.0\text{ m/s})\hat{\text{i}}.$$

(a) Using Eq. 4-2, the displacement $\Delta\vec{r}$ is

$$\Delta\vec{r} = \vec{r} - \vec{r}_o = (-30.0\text{ m})\hat{\text{i}} + (40.0\text{ m})\hat{\text{j}}$$

with a magnitude $|\Delta\vec{r}| = \sqrt{(-30.0\text{ m})^2 + (40.0\text{ m})^2} = 50.0\text{ m}$.

(b) The direction of $\Delta\vec{r}$ is

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{40.0\text{ m}}{-30.0\text{ m}}\right) = -53.1° \text{ or } 127°.$$

Since the desired angle is in the second quadrant, we pick $127°$ ($53°$ north of due west). Note that the displacement can be written as $\Delta\vec{r} = \vec{r} - \vec{r}_o = (50.0\angle 127°)$ in terms of the magnitude-angle notation.

(c) The magnitude of $\vec{v}_{avg}$ is simply the magnitude of the displacement divided by the time ($\Delta t$ = 30.0 s). Thus, the average velocity has magnitude (50.0 m)/(30.0 s) = 1.67 m/s.

(d) Equation 4-8 shows that $\vec{v}_{avg}$ points in the same direction as $\Delta\vec{r}$, that is, $127°$ ($53°$ north of due west).

(e) Using Eq. 4-15, we have

$$\vec{a}_{avg} = \frac{\vec{v} - \vec{v}_o}{\Delta t} = (0.333\text{ m/s}^2)\hat{\text{i}} + (0.333\text{ m/s}^2)\hat{\text{j}}.$$

The magnitude of the average acceleration vector is therefore equal to $|\vec{a}_{avg}| = \sqrt{(0.333\text{ m/s}^2)^2 + (0.333\text{ m/s}^2)^2} = 0.471\text{ m/s}^2$.

(f) The direction of $\vec{a}_{avg}$ is

$$\theta = \tan^{-1}\left(\frac{0.333\text{ m/s}^2}{0.333\text{ m/s}^2}\right) = 45° \text{ or } -135°.$$

Since the desired angle is now in the first quadrant, we choose $45°$, and $\vec{a}_{avg}$ points north of due east.

13. Given the position vector $\vec{r}(t)$, the velocity and acceleration of the particle can be found by differentiating $\vec{r}(t)$ with respect to time:

$$\vec{v} = \frac{d\vec{r}}{dt}, \quad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

(a) The expression for the velocity of the object is

$$\vec{v} = \frac{d\vec{r}}{dt} = 0\hat{i} + (6t)\hat{j} + \hat{k} = (6t)\hat{j} + \hat{k}.$$

(b) The expression for the acceleration of the object is

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = \frac{d^2(\hat{i} + 3t^2\hat{j} + t\hat{k})}{dt^2} = 6\hat{j}.$$

14. We use Eq. 4-15 with $\vec{v}_1$ designating the initial velocity and $\vec{v}_2$ designating the later one.

(a) The average acceleration during the $\Delta t = 4$ s interval is

$$\vec{a}_{\text{avg}} = \frac{(-2.0\,\hat{\text{i}} - 2.0\,\hat{\text{j}} + 5.0\,\hat{\text{k}})\ \text{m/s} - (4.0\,\hat{\text{i}} - 22\,\hat{\text{j}} + 3.0\,\hat{\text{k}})\ \text{m/s}}{4\ \text{s}} = (-1.5\ \text{m/s}^2)\,\hat{\text{i}} + (0.5\,\text{m/s}^2)\,\hat{\text{k}}.$$

(b) The magnitude of $\vec{a}_{\text{avg}}$ is $\sqrt{(-1.5\ \text{m/s}^2)^2 + (0.5\ \text{m/s}^2)^2} = 1.6\,\text{m/s}^2$.

(c) Its angle in the *xz* plane (measured from the +*x* axis) is one of these possibilities:

$$\tan^{-1}\left(\frac{0.5\ \text{m/s}^2}{-1.5\ \text{m/s}^2}\right) = -18° \text{ or } 162°$$

where we settle on the second choice since the signs of its components imply that it is in the second quadrant.

15. Since the acceleration, $\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} = (-9.0\ \mathrm{m/s^2})\hat{\mathrm{i}} + (3.0\ \mathrm{m/s^2})\hat{\mathrm{j}}$, is constant in both $x$ and $y$ directions, we may use equations for the motion along each direction. This can be handled individually (for $x$ and $y$) or together with the unit-vector notation (for $\Delta\vec{r}$).

The particle started at the origin, so the coordinates of the particle at any time $t$ are given by $\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$. The velocity of the particle at any time $t$ is given by $\vec{v} = \vec{v}_0 + \vec{a}t$, where $\vec{v}_0$ is the initial velocity and $\vec{a}$ is the (constant) acceleration. Along the $x$-direction, we have

$$x(t) = v_{0x}t + \frac{1}{2}a_x t^2,\ \ v_x(t) = v_{0x} + a_x t$$

Similarly, along the $y$-direction, we get

$$y(t) = v_{0y}t + \frac{1}{2}a_y t^2,\ \ v_y(t) = v_{0y} + a_y t$$

(a) Given that $v_{0x} = 7.0\ \mathrm{m/s}$, $v_{0y} = 0$, $a_x = -9.0\ \mathrm{m/s^2}$, $a_y = +3.0\ \mathrm{m/s^2}$, the components of the velocity are

$$\begin{aligned} v_x(t) &= v_{0x} + a_x t = (7.0\ \mathrm{m/s}) - (9.0\ \mathrm{m/s^2})t \\ v_y(t) &= v_{0y} + a_y t = +(3.0\ \mathrm{m/s^2})t \end{aligned}$$

When the particle reaches its maximum $x$ coordinate at $t = t_m$, we must have $v_x = 0$. Therefore, (7.0 m/s) − (9.0 m/s$^2$)$t_m$ = 0 or $t_m$ = 0.78 s. The $y$ component of the velocity at this time is

$$v_y(t = 0.78\ \mathrm{s}) = +(3.0\ \mathrm{m/s^2})(0.78\ \mathrm{s}) = +2.3\ \mathrm{m/s}$$

Thus, $\vec{v}_m = (+2.3\ \mathrm{m/s})\hat{\mathrm{j}}$.

(b) At $t$ = 0.78 s, the components of the position are

$$\begin{aligned} x(t = 0.78\ \mathrm{s}) &= v_{0x}t + \frac{1}{2}a_x t^2 = (7.0\ \mathrm{m/s})(0.78\ \mathrm{s}) + \frac{1}{2}(-9.0\ \mathrm{m/s^2})(0.78\ \mathrm{s})^2 = 2.72\ \mathrm{m} \\ y(t = 0.78\ \mathrm{s}) &= v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(+3.0\ \mathrm{m/s^2})(0.78\ \mathrm{s})^2 = 0.91\ \mathrm{m} \end{aligned}$$

Using unit-vector notation, the results can be written as $\vec{r}_m = (2.7\ \mathrm{m})\hat{\mathrm{i}} + (0.91\ \mathrm{m})\hat{\mathrm{j}}$.

16. We make use of Eq. 4-16.

(a) The acceleration as a function of time is

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\left(6.0t - 4.0t^2\right)\hat{i} + 8.0\hat{j}\right) = \left(6.0 - 8.0t\right)\hat{i}$$

in SI units. Specifically, we find the acceleration vector at $t = 2.5\,\mathrm{s}$ to be $\left(6.0 - 8.0(2.5)\right)\hat{i} = (-14\ \mathrm{m/s^2})\hat{i}$.

(b) The equation is $\vec{a} = \left(6.0 - 8.0t\right)\hat{i} = 0$; we find $t$ = 0.75 s.

(c) Since the $y$ component of the velocity, $v_y$ = 8.0 m/s, is never zero, the velocity cannot vanish.

(d) Since speed is the magnitude of the velocity, we have

$$v = |\,\vec{v}\,| = \sqrt{\left(6.0t - 4.0t^2\right)^2 + \left(8.0\right)^2} = 10$$

in SI units (m/s). To solve for $t$, we first square both sides of the above equation, followed by some rearrangement:

$$\left(6.0t - 4.0t^2\right)^2 + 64 = 100 \quad \Rightarrow \quad \left(6.0t - 4.0t^2\right)^2 = 36$$

Taking the square root of the new expression and making further simplification lead to

$$6.0t - 4.0t^2 = \pm 6.0 \quad \Rightarrow \quad 4.0t^2 - 6.0t \pm 6.0 = 0$$

Finally, using the quadratic formula, we obtain

$$t = \frac{6.0 \pm \sqrt{36 - 4\left(4.0\right)\left(\pm 6.0\right)}}{2\left(8.0\right)}$$

where the requirement of a real positive result leads to the unique answer: $t$ = 2.2 s.

17. We find $t$ by applying the equation $v = v_0 + at$ to motion along the $y$ axis (with $v_y = 0$ characterizing $y = y_{max}$):

$$0 = (18 \text{ m/s}) + (-3.0 \text{ m/s}^2)t \quad \Rightarrow \quad t = 6.0 \text{ s}.$$

Applying the equation to the motion along the $x$ axis gives

$$v_x = (12.0 \text{ m/s}) + (6.0 \text{ m/s}^2)(6.0 \text{ s}) = 48 \text{ m/s}.$$

Therefore, in unit-vector notation, the velocity of the bike, when it reaches $y = y_{max}$, is $(48 \text{ m/s})\hat{\text{i}}$.

18. (a) We find $t$ by solving $\Delta x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$:

$$10.0 \text{ m} = 0 + (4.00 \text{ m/s})t + \frac{1}{2}(5.00 \text{ m/s}^2)t^2$$

where we have used $\Delta x$ = 10.0 m, $v_x$ = 4.00 m/s, and $a_x$ = 5.00 m/s$^2$. We use the quadratic formula and find $t$ = 1.35 s. Then, Eq. 2-11 (actually, its analog in two dimensions) applies with this value of $t$. Therefore, its velocity (when $\Delta x$ = 10.00 m) is

$$\begin{aligned}\vec{v} &= \vec{v}_0 + \vec{a}t = (4.00 \text{ m/s})\hat{i} + (5.00 \text{ m/s}^2)(1.35 \text{ s})\hat{i} + (7.00 \text{ m/s}^2)(1.35 \text{ s})\hat{j} \\ &= (10.8 \text{ m/s})\hat{i} + (9.45 \text{ m/s})\hat{j}.\end{aligned}$$

Thus, the magnitude of $\vec{v}$ is $|\vec{v}| = \sqrt{(10.8 \text{ m/s})^2 + (9.45 \text{ m/s})^2} = 14.4$ m/s.

(b) The angle of $\vec{v}$, measured from $+x$, is

$$\tan^{-1}\left(\frac{9.45 \text{ m/s}}{10.8 \text{ m/s}}\right) = 41.2°.$$

19. We make use of Eq. 4-16 and Eq. 4-10.

Using $\vec{a} = 3t\hat{i} + 4t\hat{j}$, we have (in m/s)

$$\vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a}\ dt = (5.00\hat{i} + 2.00\hat{j}) + \int_0^t (3t\hat{i} + 4t\hat{j})\ dt = \left(5.00 + 3t^2/2\right)\hat{i} + \left(2.00 + 2t^2\right)\hat{j}$$

Integrating using Eq. 4-10 then yields (in meters)

$$\begin{aligned} \vec{r}(t) = \vec{r}_0 + \int_0^t \vec{v}dt &= (20.0\hat{i} + 40.0\hat{j}) + \int_0^t [(5.00 + 3t^2/2)\hat{i} + (2.00 + 2t^2)\hat{j}]dt \\ &= (20.0\hat{i} + 40.0\hat{j}) + (5.00t + t^3/2)\hat{i} + (2.00t + 2t^3/3)\hat{j} \\ &= (20.0 + 5.00t + t^3/2)\hat{i} + (40.0 + 2.00t + 2t^3/3)\hat{j} \end{aligned}$$

(a) At $t = 4.00$ s, we have $\vec{r}(t = 4.00\,\text{s}) = (72.0\ \text{m})\hat{i} + (90.7\ \text{m})\hat{j}$.

(b) $\vec{v}(t = 4.00\,\text{s}) = (29.0\ \text{m/s})\hat{i} + (34.0\ \text{m/s})\hat{j}$. Thus, the angle between the direction of travel and +*x*, measured counterclockwise, is

$$\theta = \tan^{-1}[(34.0\,\text{m/s})/(29.0\,\text{m/s})] = 49.5°.$$

20. The acceleration is constant so that use of Table 2-1 (for both the $x$ and $y$ motions) is permitted. Where units are not shown, SI units are to be understood. Collision between particles $A$ and $B$ requires two things. First, the $y$ motion of $B$ must satisfy (using Eq. 2-15 and noting that $\theta$ is measured from the $y$ axis)

$$y = \frac{1}{2}a_y t^2 \quad \Rightarrow \quad 30\text{ m} = \frac{1}{2}\left[(0.40\text{ m/s}^2)\cos\theta\right]t^2.$$

Second, the $x$ motions of $A$ and $B$ must coincide:

$$vt = \frac{1}{2}a_x t^2 \quad \Rightarrow \quad (3.0\text{ m/s})t = \frac{1}{2}\left[(0.40\text{ m/s}^2)\sin\theta\right]t^2.$$

We eliminate a factor of $t$ in the last relationship and formally solve for time:

$$t = \frac{2v}{a_x} = \frac{2(3.0\text{ m/s})}{(0.40\text{ m/s}^2)\sin\theta}.$$

This is then plugged into the previous equation to produce

$$30\text{ m} = \frac{1}{2}\left[(0.40\text{ m/s}^2)\cos\theta\right]\left(\frac{2(3.0\text{ m/s})}{(0.40\text{ m/s}^2)\sin\theta}\right)^2$$

which, with the use of $\sin^2\theta = 1$ ï $\cos^2\theta$, simplifies to

$$30 = \frac{9.0}{0.20}\,\frac{\cos\theta}{1-\cos^2\theta} \quad \Rightarrow \quad 1-\cos^2\theta = \frac{9.0}{(0.20)(30)}\cos\theta.$$

We use the quadratic formula (choosing the positive root) to solve for $\cos\theta$:

$$\cos\theta = \frac{-1.5+\sqrt{1.5^2-4(1.0)(-1.0)}}{2} = \frac{1}{2}$$

which yields

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60°.$$

21. For the horizontal motion (i.e., along $x$-axis) of the stone, we have $x = v_{0x}t = (6.75 \text{ m/s})t$ . On the other hand, for the vertical motion (i.e., along $y$-axis) of the stone, we have

$$(y - y_0) = v_{0y}t - \frac{1}{2}gt^2 ,$$

which leads to

$$-0.050 \text{ m} = (0)t - \frac{1}{2}(9.8 \text{ m/s}^2)t^2 \quad \Rightarrow \; t = \sqrt{\frac{0.10 \text{ m}}{9.8 \text{ m/s}^2}} = 0.101 \text{ s}.$$

Substituting this value of $t$ in the equation of $x$, we get the horizontal distance between the starting point of the stone and the target as

22. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable.

(a) With the origin at the initial point (edge of table), the $y$ coordinate of the ball is given by $y = -\frac{1}{2}gt^2$. If $t$ is the time of flight and $y$ = ï 1.50 m indicates the level at which the ball hits the floor, then

$$t = \sqrt{\frac{2(-1.50\text{ m})}{-9.80\text{ m/s}^2}} = 0.553\text{s}.$$

(b) The initial (horizontal) velocity of the ball is $\vec{v} = v_0\,\hat{\text{i}}$. Since $x$ = 1.52 m is the horizontal position of its impact point with the floor, we have $x = v_0t$. Thus,

$$v_0 = \frac{x}{t} = \frac{1.52\text{ m}}{0.553\text{ s}} = 2.75\text{ m/s}.$$

23. For the horizontal ($x$-component) motion of the shell, we have $x = 285\,t$ and for the vertical motion of the shell, from one of the equations of motion

$$(x - x_0) = v_0 t + \frac{1}{2} a t^2 ,$$

we get

$$40.4 = \frac{1}{2} \times 9.8 \times t^2 \qquad (\because a = g \approx 9.8\,\text{m/s}^2)$$

$$\Rightarrow t^2 = \frac{40.4 \times 2.00}{9.8}$$

(a) The projectile remains in the air for

$$t = \frac{\sqrt{808}}{9.8} \approx 2.87\ \text{s}$$

(b) The horizontal ($x$-component) distance the shell strikes the plane from the firing point is

$$x = 285 \times 2.84 = 818\,\text{m}.$$

(c) The magnitude of the horizontal component ($x$-component) of the shellûs velocity as it strikes the ground is

$$v_x = 285\ \text{m/s}.$$

(d) The magnitude of the vertical component ($y$-component) of the shellûs velocity as it strikes the ground is

$$v_y = 9.8 \times 2.84 = 28.1\,\text{m/s}.$$

24. We use Eq. 4-26

$$R_{\max} = \left( \frac{v_0^2}{g} \sin 2\theta_0 \right)_{\max} = \frac{v_0^2}{g} = \frac{(9.50\,\text{m/s})^2}{9.80\,\text{m/s}^2} = 9.209\ \text{m} \approx 9.21\text{m}$$

to compare with Powellôs long jump; the difference from $R_{\max}$ is only $\Delta R$ =(9.21m ï 8.95m) = 0.259 m.

25. Using Eq. (4-26), the take-off speed of the jumper is

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{(9.80\ \text{m/s}^2)(77.0\ \text{m})}{\sin 2(12.0°)}} = 43.1\ \text{m/s}$$

or about 155 km/h.

26. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is the throwing point (the stoneĉs initial position). The $x$ component of its initial velocity is given by $v_{0x} = v_0 \cos\theta_0$ and the $y$ component is given by $v_{0y} = v_0 \sin\theta_0$, where $v_0$ = 18 m/s is the initial speed and $\theta_0$ = 40.0° is the launch angle.

(a) At $t$ = 1.10 s, its $x$ coordinate is

$$x = v_0 t \cos\theta_0 = (18.0 \text{ m/s})(1.10 \text{ s}) \cos 40.0° = 15.17 \text{ m} \approx 15.2 \text{ m}$$

(b) Its $y$ coordinate at that instant is

$$y = v_0 t \sin\theta_0 - \frac{1}{2} g t^2 = (18.0 \text{m/s})(1.10 \text{s}) \sin 40.0° - \frac{1}{2}\left(9.80 \text{m/s}^2\right)(1.10 \text{s})^2 = 6.80 \text{m}.$$

(c) At $t'$ = 1.80 s, its $x$ coordinate is $x = (18.0 \text{ m/s})(1.80 \text{ s}) \cos 40.0° = 24.8 \text{ m}$.

(d) Its $y$ coordinate at $t'$ is

$$y = (18.0 \text{m/s})(1.80 \text{s}) \sin 40.0° - \frac{1}{2}\left(9.80 \text{m/s}^2\right)\left(1.80 \text{s}^2\right) = 4.95 \text{m}.$$

(e) The stone hits the ground earlier than $t$ = 5.0 s. To find the time when it hits the ground solve $y = v_0 t \sin\theta_0 - \frac{1}{2} g t^2 = 0$ for $t$. We find

$$t = \frac{2 v_0}{g} \sin\theta_0 = \frac{2(18.0 \text{ m/s})}{9.8 \text{ m/s}^2} \sin 40° = 2.36 \text{ s}.$$

Its $x$ coordinate on landing is

$$x = v_0 t \cos\theta_0 = (18.0 \text{ m/s})(2.36 \text{ s}) \cos 40° = 32.6 \text{ m}.$$

(f) Assuming it stays where it lands, its vertical component at $t$ = 5.00 s is $y$ = 0.

27. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the release point. We write $\theta_0$ = ï 30.0° since the angle shown in the figure is measured clockwise from horizontal. We note that the initial speed of the decoy is the planeôs speed at the moment of release: $v_0$ = 290 km/h, which we convert to SI units: (290)(1000/3600) = 80.6 m/s.

(a) We use Eq. 4-12 to solve for the time:

$$\Delta x = (v_0 \cos\theta_0)\ t \quad \Rightarrow \quad t = \frac{700\ \text{m}}{(80.6\ \text{m/s})\cos(-30.0°)} = 10.0\ \text{s}.$$

(b) And we use Eq. 4-22 to solve for the initial height $y_0$:

$$y - y_0 = (v_0 \sin\theta_0)\ t - \frac{1}{2}gt^2 \quad \Rightarrow \quad 0 - y_0 = (-40.3\ \text{m/s})(10.0\ \text{s}) - \frac{1}{2}(9.80\ \text{m/s}^2)(10.0\ \text{s})^2$$

which yields $y_0$ = 897 m.

28. (a) Using the same coordinate system assumed in Eq. 4-22, we solve for $y = h$:

$$h = y_0 + v_0 \sin\theta_0 t - \frac{1}{2}gt^2$$

which yields $h = 51.8$ m for $y_0 = 0$, $v_0 = 42.0$ m/s, $\theta_0 = 60.0°$, and $t = 5.50$ s.

(b) The horizontal motion is steady, so $v_x = v_{0x} = v_0 \cos \theta_0$, but the vertical component of velocity varies according to Eq. 4-23. Thus, the speed at impact is

$$v = \sqrt{(v_0 \cos\theta_0)^2 + (v_0 \sin\theta_0 - gt)^2} = 27.4 \text{ m/s}.$$

(c) We use Eq. 4-24 with $v_y = 0$ and $y = H$:

$$H = \frac{(v_0 \sin\theta_0)^2}{2g} = 67.5 \text{ m}$$

29. We adopt the positive direction choices. The coordinate origin is at its initial position (where it is launched). At maximum height, we observe $v_y = 0$ and denote $v_x = v$ (which is also equal to $v_{0x}$). In this notation, we have $v_0 = 6v$. Next, we observe $v_0 \cos \theta_0 = v_{0x} = v$, so that we arrive at an equation (where $v \neq 0$ cancels) which can be solved for $\theta_0$:

$$(6v)\cos\theta_0 = v \;\Rightarrow\; \theta_0 = \cos^{-1}\left(\frac{1}{6}\right) = 80.4°.$$

30. Although we could use Eq. 4-26 to find where it lands, we choose instead to work with Eq. 4-21 and Eq. 4-22 (for the soccer ball) since these will give information about where *and when* and these are also considered more fundamental than Eq. 4-26. With $\Delta y = 0$, we have

$$\Delta y = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2 \;\Rightarrow\; t = \frac{(21.3 \text{ m/s}) \sin 45.0°}{(9.80 \text{ m/s}^2)/2} = 3.074 \text{ s.}$$

Then Eq. 4-21 yields $\Delta x = (v_0 \cos\theta_0)t = 46.3$ m. Thus, using Eq. 4-8, the player must have an average velocity of

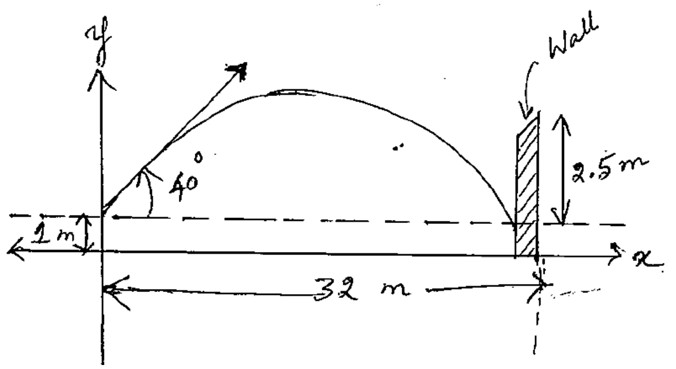
$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(46.3 \text{ m})\,\hat{\text{i}} - (55 \text{ m})\,\hat{\text{i}}}{3.07 \text{ s}} = (-2.83 \text{ m/s})\,\hat{\text{i}}$$

which means his average speed (assuming he ran in only one direction) is about 2.8 m/s.

31. The following figure depicts the situation. The horizontal range of the ball after the playerôs punt on the ball is

$$\frac{v_0^2 \sin 2\theta_0}{g} = \frac{(18)^2 \sin 80°}{9.8} \approx 32\,\text{m}.$$

The claim of the soccer player is not justified.



32. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the release point (the initial position for the ball as it begins projectile motion in the sense of §4-5), and we let $\theta_0$ be the angle of throw (shown in the figure). Since the horizontal component of the velocity of the ball is $v_x = v_0 \cos 40.0°$, the time it takes for the ball to hit the wall is

$$t = \frac{\Delta x}{v_x} = \frac{22.0 \text{ m}}{(25.0 \text{ m/s}) \cos 40.0°} = 1.15 \text{ s}.$$

(a) The vertical distance is

$$\Delta y = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2 = (25.0 \text{ m/s}) \sin 40.0°(1.15 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(1.15 \text{ s})^2 = 12.0 \text{ m}.$$

(b) The horizontal component of the velocity when it strikes the wall does not change from its initial value: $v_x = v_0 \cos 40.0° = 19.2$ m/s.

(c) The vertical component becomes (using Eq. 4-23)

$$v_y = v_0 \sin \theta_0 - gt = (25.0 \text{ m/s}) \sin 40.0° - (9.80 \text{ m/s}^2)(1.15 \text{ s}) = 4.80 \text{ m/s}.$$

(d) Since $v_y > 0$ when the ball hits the wall, it has not reached the highest point yet.

33. The following figure depicts the situation.

(a) The initial speed of the projectile is the planeĉs speed at the moment of release. Given that $y_0 = 720$ m and $y = 0$ at $t = 6.00$ s. We find $v_0$ as follows:

$$y - y_0 = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2$$

$$\Rightarrow \quad 0 - 720 \text{ m} = v_0 \sin(-38.0°)(6.00 \text{ s}) - \left[\frac{1}{2}(9.80 \text{ m/s}^2)(6.00 \text{ s})^2\right]$$

which yields $v_0$ = 147.159 $\approx 147$ m/s.

(b) The horizontal distance traveled is

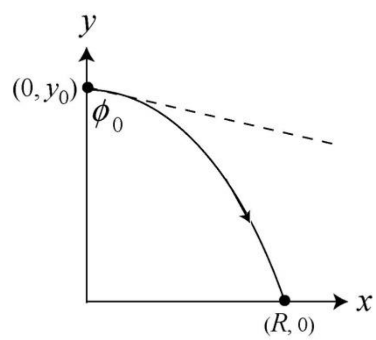
$$R = v_x t = (v_0 \cos\theta_0) t = [(147.159 \text{ m/s})\cos(-38.0°)](6.00 \text{ s}) \approx 696 \text{m}.$$

(c) The $x$-component of the velocity (just before impact) is

$$v_x = v_0 \cos\theta_0 = (147.159 \text{ m/s})\cos(-38.0°) \approx 116 \text{ m/s}.$$

(d) The $y$-component of the velocity (just before impact) is

$$v_y = v_0 \sin\theta_0 - gt = (147.159 \text{ m/s})\sin(-38.0°) - (9.80 \text{ m/s}^2)(6.00 \text{ s}) \approx -149 \text{ m/s}.$$




Note that in this projectile problem we analyzed the kinematics in the vertical and horizontal directions separately since they do not affect each other. The $x$-component of the velocity, $v_x = v_0 \cos\theta_0$, remains unchanged throughout since thereĉs no horizontal acceleration.

34. (a) Since the *y*-component of the velocity of the stone at the top of its path is zero, its speed is

$$v=\sqrt{v_x^2+v_y^2}=v_x=v_0\cos\theta_0=(30.0\text{ m/s})\cos 40.0°=22.98\text{ m/s}\approx 23.0\text{ m/s}.$$

(b) Using the fact that $v_y=0$ at the maximum height $y_{\max}$, the amount of time it takes for the stone to reach $y_{\max}$ is given by Eq. 4-23:

$$0=v_y=v_0\sin\theta_0-gt\quad\Rightarrow\quad t=\frac{v_0\sin\theta_0}{g}.$$

Substituting the above expression into Eq. 4-22, we find the maximum height to be

$$y_{\max}=(v_0\sin\theta_0)\,t-\frac{1}{2}gt^2=v_0\sin\theta_0\left(\frac{v_0\sin\theta_0}{g}\right)-\frac{1}{2}g\left(\frac{v_0\sin\theta_0}{g}\right)^2=\frac{v_0^2\sin^2\theta_0}{2g}.$$

To find the time the stone descends to $y=y_{\max}/2$, we solve the quadratic equation given in Eq. 4-22:

$$y=\frac{1}{2}y_{\max}=\frac{v_0^2\sin^2\theta_0}{4g}=(v_0\sin\theta_0)\,t-\frac{1}{2}gt^2\quad\Rightarrow\quad t_\pm=\frac{(2\pm\sqrt{2})v_0\sin\theta_0}{2g}.$$

Choosing $t=t_+$ (for descending), we have

$$v_x=v_0\cos\theta_0=(30.0\text{ m/s})\cos 40.0°=22.98\text{ m/s}$$

$$v_y=v_0\sin\theta_0-g\frac{(2+\sqrt{2})v_0\sin\theta_0}{2g}=-\frac{\sqrt{2}}{2}v_0\sin\theta_0=-\frac{\sqrt{2}}{2}(30.0\text{ m/s})\sin 40.0°=-13.64\text{ m/s}$$

Thus, the speed of the stone when $y=y_{\max}/2$ is

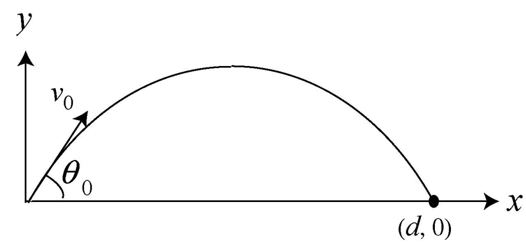
$$v=\sqrt{v_x^2+v_y^2}=\sqrt{(22.98\text{ m/s})^2+(-13.64\text{ m/s})^2}=26.72\text{ m/s}.$$

(c) The percentage difference is

$$\frac{26.72\text{ m/s}-22.98\text{ m/s}}{22.98\text{ m/s}}=0.163=16.3\%.$$

35. **THINK** This problem deals with projectile motion of a bullet. We're interested in the firing angle that allows the bullet to strike a target at some distance away.

**EXPRESS** We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the end of the rifle (the initial point for the bullet as it begins projectile motion in the sense of § 4-5), and we let $\theta_0$ be the firing angle. If the target is a distance $d$ away, then its coordinates are $x = d$, $y = 0$.




The projectile motion equations lead to

$$d = (v_0 \cos\theta_0)t, \qquad 0 = v_0 t \sin\theta_0 - \frac{1}{2}gt^2$$

where $\theta_0$ is the firing angle. The setup of the problem is shown in the figure above (scale exaggerated).

**ANALYZE** The time at which the bullet strikes the target is given by $t = d/(v_0 \cos\theta_0)$. Eliminating $t$ leads to $2v_0^2 \sin\theta_0 \cos\theta_0 - gd = 0$. Using $\sin\theta_0 \cos\theta_0 = \frac{1}{2}\sin(2\theta_0)$, we obtain

$$v_0^2 \sin\,(2\theta_0) = gd \;\Rightarrow\; \sin(2\theta_0) = \frac{gd}{v_0^2} = \frac{(9.80\text{ m/s}^2)(45.7\text{ m})}{(460\text{ m/s})^2}$$

which yields $\sin(2\theta_0) = 2.11\times10^{-3}$, or $\theta_0$ = 0.0606°. If the gun is aimed at a point a distance $\ell$ above the target, then $\tan\theta_0 = \ell/d$ so that

$$\ell = d\tan\theta_0 = (45.7\text{ m})\tan(0.0606°) = 0.0484\text{ m} = 4.84\text{ cm}.$$

**LEARN** Due to the downward gravitational acceleration, in order for the bullet to strike the target, the gun must be aimed at a point slightly above the target.

36. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the point where the ball was hit by the racquet.

(a) We want to know how high the ball is above the court when it is at $x$ = 12.0 m. First, Eq. 4-21 tells us the time it is over the fence:

$$t = \frac{x}{v_0 \cos\theta_0} = \frac{12.0 \text{ m}}{(23.6 \text{ m/s})\cos 0°} = 0.508 \text{ s}.$$

At this moment, the ball is at a height (above the court) of

$$y = y_0 + (v_0 \sin\theta_0)\,t - \frac{1}{2}gt^2 = 2.42 \text{ m} - \frac{1}{2}(9.8 \text{ m/s}^2)(0.508 \text{ s})^2 = 1.153 \text{ m}$$

which implies it does indeed clear the 0.90-m-high fence.

(b) At $t$ = 0.508 s, the center of the ball is (1.153 m ï 0.90 m) = 0.253 m above the net.

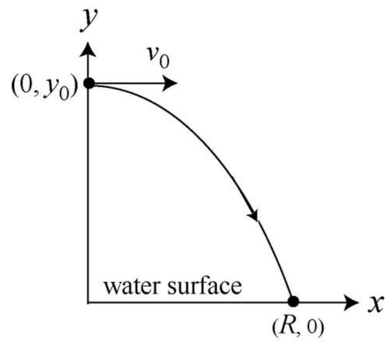
(c) Repeating the computation in part (a) with $\theta_0$ = ï5.0° results in $t$ = 0.510 s and $y = 0.0936$ m, which clearly indicates that it cannot clear the net.

(d) In the situation discussed in part (c), the distance between the top of the net and the center of the ball at $t$ = 0.510 s is 0.90 m ï 0.0936 m = 0.81 m.

37. The initial velocity has no vertical component ($\theta_0 = 0$) ð only an $x$-component. We have the equations

$$x - x_0 = v_{0x}t$$
$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2 = -\frac{1}{2}gt^2$$

where $x_0 = 0$, $v_{0x} = v_0 = +2.50\ \text{m/s}$, and $y_0$ = +12.0 m (taking the water surface to be at $y = 0$). The following figure depicts the situation.




(a) At $t = 0.90\ \text{s}$, the horizontal distance of the diver from the edge is

$$x = x_0 + v_{0x}t = 0 + (2.50\ \text{m/s})(0.90\ \text{s}) = 2.25\ \text{m}$$

(b) Similarly, using the second equation for the vertical motion, we obtain

$$y = y_0 - \frac{1}{2}gt^2 = 12.0\ \text{m} - \frac{1}{2}(9.80\ \text{m/s}^2)(0.90\ \text{s})^2 = 8.03\ \text{m}.$$

(c) At the instant the diver strikes the water surface, $y$ = 0. Solving for $t$ using the equation $y = y_0 - \frac{1}{2}gt^2 = 0$ leads to

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(12.0\ \text{m})}{9.80\ \text{m/s}^2}} = 1.564\ \text{s}.$$

During this time, the $x$-displacement of the diver is $R = x$ = (2.50 m/s)(1.564 s) = 3.91 m. Note that with $\theta_0 = 0$, the trajectory of the diver can also be written as

$$y = y_0 - \frac{gx^2}{2v_0^2}$$

38. In this projectile motion problem, we have $v_0 = v_x$ = constant, and what is plotted is $v = \sqrt{v_x^2 + v_y^2}$. We infer from the plot that at $t$ = 2.5 s, the ball reaches its maximum height, where $v_y = 0$. Therefore, we infer from the graph that $v_x$ = 19 m/s.

(a) During $t$ = 5 s, the horizontal motion is $x$ ï $x_0 = v_x t$ = 95 m.

(b) Since $\sqrt{(19\text{ m/s})^2 + v_{0y}^2} = 31\text{ m/s}$ (the first point on the graph), we find $v_{0y} = 24.5$ m/s. Thus, with $t$ = 2.5 s, we can use $y_{\max} - y_0 = v_{0y}t - \frac{1}{2}gt^2$ or $v_y^2 = 0 = v_{0y}^2 - 2g(y_{\max} - y_0)$, or $y_{\max} - y_0 = \frac{1}{2}\left(v_y + v_{0_y}\right)t$ to solve. Here we will use the latter:

$$y_{\max} - y_0 = \frac{1}{2}(v_y + v_{0y})\ t \Rightarrow y_{\max} = \frac{1}{2}(0 + 24.5\text{m/s})(2.5\text{ s}) = 31\text{ m}$$

where we have taken $y_0 = 0$ as the ground level.

39. Following the hint, we have the time-reversed problem with the ball thrown from the ground, toward the right, at 60° measured counterclockwise from a rightward axis. We see in this time-reversed situation that it is convenient to use the familiar coordinate system with $+x$ as *rightward* and with positive angles measured counterclockwise.

(a) The $x$-equation (with $x_0 = 0$ and $x = 25.0$ m) leads to

$$25.0 \text{ m} = (v_0 \cos 60.0°)(1.50 \text{ s}),$$

so that $v_0 = 33.3$ m/s. And with $y_0 = 0$, and $y = h > 0$ at $t = 1.50$ s, we have $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ where $v_{0y} = v_0 \sin 60.0°$. This leads to $h = 32.3$ m.

(b) We have

$v_x = v_{0x}$ = (33.3 m/s)cos 60.0° = 16.7 m/s
$v_y = v_{0y}$ ï $gt$ = (33.3 m/s)sin 60.0° ï (9.80 m/s$^2$)(1.50 s) = 14.2 m/s.

The magnitude of $\vec{v}$ is given by

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(16.7 \text{ m/s})^2 + (14.2 \text{ m/s})^2} = 21.9 \text{ m/s}.$$

(c) The angle is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{14.2 \text{ m/s}}{16.7 \text{ m/s}}\right) = 40.4°.$$

(d) We interpret this result (ñundoingò the time reversal) as an initial velocity (from the edge of the building) of magnitude 21.9 m/s with angle (down from leftward) of 40.4°.

40. (a) Solving the quadratic equation Eq. 4-22:

$$y - y_0 = (v_0 \sin\theta_0)\, t - \frac{1}{2}gt^2 \quad\Rightarrow\quad 0 - 2.160\text{ m} = (15.00\text{ m/s})\sin(45.00°)t - \frac{1}{2}(9.800\text{ m/s}^2)t^2$$

the total travel time of the shot in the air is found to be $t = 2.352\text{ s}$. Therefore, the horizontal distance traveled is

$$R = \left(v_0 \cos\theta_0\right)t = (15.00\text{ m/s})\cos 45.00°(2.352\text{ s}) = 24.95\text{ m}.$$

(b) Using the procedure outlined in (a) but for $\theta_0 = 42.00°$, we have

$$y - y_0 = (v_0 \sin\theta_0)\, t - \frac{1}{2}gt^2 \quad\Rightarrow\quad 0 - 2.160\text{ m} = (15.00\text{ m/s})\sin(42.00°)t - \frac{1}{2}(9.800\text{ m/s}^2)t^2$$

and the total travel time is $t = 2.245\text{ s}$. This gives

$$R = \left(v_0 \cos\theta_0\right)t = (15.00\text{ m/s})\cos 42.00°(2.245\text{ s}) = 25.02\text{ m}.$$

41. With the Archer fish set to be at the origin, the position of the insect is given by ($x$, $y$) where $x = R/2 = v_0^2 \sin 2\theta_0 / 2g$ , and $y$ corresponds to the maximum height of the parabolic trajectory: $y = y_{\max} = v_0^2 \sin^2 \theta_0 / 2g$ . From the figure, we have

$$\tan\phi = \frac{y}{x} = \frac{v_0^2 \sin^2 \theta_0 / 2g}{v_0^2 \sin 2\theta_0 / 2g} = \frac{1}{2}\tan\theta_0$$

Given that $\phi = 36.0°$ , we find the launch angle to be

$$\theta_0 = \tan^{-1}\left(2\tan\phi\right) = \tan^{-1}\left(2\tan 36.0°\right) = \tan^{-1}\left(1.453\right) = 55.46° \approx 55.5° .$$

Note that $\theta_0$ depends only on $\phi$ and is independent of $d$.

42. (a) Using the fact that the person (as the projectile) reaches the maximum height over the middle wheel located at $x = 23\text{ m} + (23/2)\text{ m} = 34.5\text{ m}$, we can deduce the initial launch speed from Eq. 4-26:

$$x = \frac{R}{2} = \frac{v_0^2 \sin 2\theta_0}{2g} \quad \Rightarrow \quad v_0 = \sqrt{\frac{2gx}{\sin 2\theta_0}} = \sqrt{\frac{2(9.8\text{ m/s}^2)(34.5\text{ m})}{\sin(2\cdot 53°)}} = 26.5\text{ m/s}.$$

Upon substituting the value to Eq. 4-25, we obtain

$$y = y_0 + x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} = 3.0\text{ m} + (23\text{ m})\tan 53° - \frac{(9.8\text{ m/s}^2)(23\text{ m})^2}{2(26.5\text{ m/s})^2(\cos 53°)^2} = 23.3\text{ m}.$$

Since the height of the wheel is $h_w = 18\text{ m}$, the clearance over the first wheel is $\Delta y = y - h_w = 23.3\text{ m} - 18\text{ m} = 5.3\text{ m}$.

(b) The height of the person when he is directly above the second wheel can be found by solving Eq. 4-24. With the second wheel located at $x = 23\text{ m} + (23/2)\text{ m} = 34.5\text{ m}$, we have

$$y = y_0 + x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} = 3.0\text{ m} + (34.5\text{ m})\tan 53° - \frac{(9.8\text{ m/s}^2)(34.5\text{ m})^2}{2(26.52\text{ m/s})^2(\cos 53°)^2}$$
$$= 25.9\text{ m}.$$

Therefore, the clearance over the second wheel is

$$\Delta y = y - h_w = 25.9\text{ m} - 18\text{ m} = 7.9\text{ m}.$$

(c) The location of the center of the net is given by

$$0 = y - y_0 = x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} \quad \Rightarrow \quad x = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{(26.52\text{ m/s})^2 \sin(2\cdot 53°)}{9.8\text{ m/s}^2} = 69\text{ m}.$$

43. Let the given velocity $\vec{v} = (8.6 \text{ m/s})\hat{i} + (7.2 \text{ m/s})\hat{j}$ be $\vec{v}_1$, as opposed to the velocity when it reaches the maximum height $\vec{v}_2$ or the velocity when it returns to the ground $\vec{v}_3$, and take $\vec{v}_0$ as the launch velocity, as usual. The origin is at its launch point on the ground.

(a) Different approaches are available, but since it will be useful (for the rest of the problem) to first find the initial $y$ velocity, that is how we will proceed. We have

$$v_{1y}^2 = v_{0y}^2 - 2g\Delta y \quad \Rightarrow \quad (7.2 \text{ m/s})^2 = v_{0y}^2 - 2(9.8 \text{ m/s}^2)(10.3 \text{ m})$$

which yields $v_{0y}$ = 15.928 m/s. Knowing that $v_{2y}$ must equal 0, we use the above equation again but now with $\Delta y = h$ for the maximum height:

$$v_{2y}^2 = v_{0y}^2 - 2gh \quad \Rightarrow \quad 0 = (15.928 \text{ m/s})^2 - 2(9.8 \text{ m/s}^2)h$$

which yields $h$ = 12.9449 m ≈ 13 m.

(b) Using $v_{0y}$ for $v_0 \sin\theta_0$ and $v_{0x}$ for $v_0 \cos\theta_0$, we have

$$0 = v_{0y}t - \frac{1}{2}gt^2, \quad R = v_{0x}t$$

which leads to $R = 2v_{0x}v_{0y}/g$. Noting that $v_{0x} = v_{1x}$ = 8.6 m/s, we substitute the values and obtain

$$R = 2(8.6 \text{ m/s})(15.928 \text{ m/s})/(9.8 \text{ m/s}^2) \approx 28 \text{ m}.$$

(c) Since $v_{3x} = v_{1x}$ = 8.6 m/s and $v_{3y} = -v_{0y}$ = −15.928 m/s, we have

$$v_3 = \sqrt{v_{3x}^2 + v_{3y}^2} = \sqrt{(8.6 \text{ m/s})^2 + (-15.928 \text{ m/s})^2} = 18 \text{ m/s}.$$

(d) The angle (measured from horizontal) for $\vec{v}_3$ is one of these possibilities:

$$\tan^{-1}\left(\frac{-15.928 \text{ m}}{8.6 \text{ m}}\right) \approx -62° \text{ or } 118°$$

where we settle on the first choice (−62°, which is equivalent to 298°) since the signs of its components imply that it is in the fourth quadrant.

44. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. Since the initial velocity is horizontal, $v_{0y}$ = 0 and $v_{0x} = v_0 = 153\ \text{km/h}$. Converting to SI units, this is $v_0$ = 42.5 m/s.

(a) With the origin at the initial point (where the ball leaves the pitcherôs hand), the $y$ coordinate of the ball is given by $y = -\frac{1}{2}gt^2$, and the $x$ coordinate is given by $x = v_0t$. From the latter equation, we have a simple proportionality between horizontal distance and time, which means the time to travel half the total distance is half the total time. Specifically, if $x$ = (18.3 m)/2, then $t$ = (18.3/2 m)/(42.5 m/s) = 0.215 s.

(b) And the time to travel the next 18.3/2 m must also be 0.215 s. It can be useful to write the horizontal equation as $\Delta x = v_0\Delta t$ in order that this result can be seen more clearly.

(c) Using the equation $y = -\frac{1}{2}gt^2$, we see that the ball has reached the height of $|-\frac{1}{2}(9.80\ \text{m/s}^2)(0.215\ \text{s})^2| = 0.227$ m at the moment the ball is halfway to the batter.

(d) The ballôs height when it reaches the batter is $|-\frac{1}{2}(9.80\ \text{m/s}^2)(0.430\ \text{s})^2| = 0.908\ \text{m}$, which, when subtracted from the previous result, implies it has fallen another 0.908 m ï 0.227 m = 0.681 m. Since the value of $y$ is not simply proportional to $t$, we do not expect equal time-intervals to correspond to equal height-changes; in a physical sense, this is due to the fact that the initial $y$-velocity for the first half of the motion is not the same as the ñinitialò $y$-velocity for the second half of the motion.

45. (a) Let $m = \frac{d_2}{d_1} = 0.600$ be the slope of the ramp, so $y = mx$ there. We choose our coordinate origin at the point of launch and use Eq. 4-25. Thus,

$$y = \tan(50.0°)x - \frac{(9.80\text{ m/s}^2)x^2}{2(10.0\text{ m/s})^2(\cos 50.0°)^2} = 0.600x$$

which yields $x = 4.99$ m. This is less than $d_1$ so the ball *does* land on the ramp.

(b) Using the value of $x$ found in part (a), we obtain $y = mx = 2.99$ m. Thus, the Pythagorean theorem yields a displacement magnitude of $\sqrt{x^2 + y^2} = 5.82$ m.

(c) The angle is, of course, the angle of the ramp: $\tan^{-1}(m) = 31.0°$.

46. Using the fact that $v_y = 0$ when the player is at the maximum height $y_{\max}$, the amount of time it takes to reach $y_{\max}$ can be solved by using Eq. 4-23:

$$0 = v_y = v_0 \sin\theta_0 - gt \quad \Rightarrow \; t_{\max} = \frac{v_0 \sin\theta_0}{g}.$$

Substituting the above expression into Eq. 4-22, we find the maximum height to be

$$y_{\max} = (v_0 \sin\theta_0)\, t_{\max} - \frac{1}{2} g t_{\max}^2 = v_0 \sin\theta_0 \left( \frac{v_0 \sin\theta_0}{g} \right) - \frac{1}{2} g \left( \frac{v_0 \sin\theta_0}{g} \right)^2 = \frac{v_0^2 \sin^2\theta_0}{2g}.$$

To find the time when the player is at $y = y_{\max}/2$, we solve the quadratic equation given in Eq. 4-22:

$$y = \frac{1}{2} y_{\max} = \frac{v_0^2 \sin^2\theta_0}{4g} = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2 \quad \Rightarrow \quad t_{\pm} = \frac{(2 \pm \sqrt{2}) v_0 \sin\theta_0}{2g}.$$

With $t = t_-$ (for ascending), the amount of time the player spends at a height $y \geq y_{\max}/2$ is

$$\Delta t = t_{\max} - t_- = \frac{v_0 \sin\theta_0}{g} - \frac{(2 - \sqrt{2}) v_0 \sin\theta_0}{2g} = \frac{v_0 \sin\theta_0}{\sqrt{2} g} = \frac{t_{\max}}{\sqrt{2}} \quad \Rightarrow \quad \frac{\Delta t}{t_{\max}} = \frac{1}{\sqrt{2}} = 0.707.$$

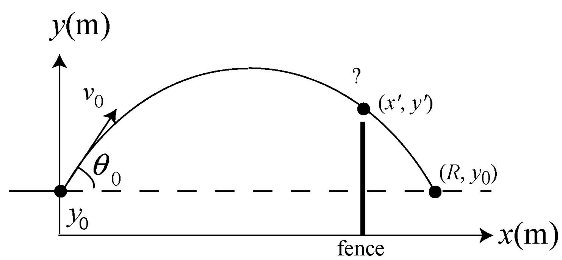
Therefore, the player spends about 70.7% of the time in the upper half of the jump. Note that the ratio $\Delta t / t_{\max}$ is independent of $v_0$ and $\theta_0$, even though $\Delta t$ and $t_{\max}$ depend on these quantities.

47. **THINK** The baseball undergoes projectile motion after being hit by the batter. Weʹd like to know if the ball clears a high fence at some distance away.

**EXPRESS** We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below impact point between bat and ball. In the absence of a fence, with $\theta_0 = 45°$, the horizontal range (same launch level) is $R = 107$ m. We want to know how high the ball is from the ground when it is at $x' = 97.5$ m, which requires knowing the initial velocity. The trajectory of the baseball can be described by Eq. 4-25:

$$y - y_0 = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2}.$$

The setup of the problem is shown in the figure below (not to scale).




**ANALYZE**

(a) We first solve for the initial speed $v_0$. Using the range information ($y = y_0$ when $x = R$) and $\theta_0$ = 45°, Eq. 4-25 gives

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{(9.8\text{ m/s}^2)(107\text{ m})}{\sin(2\cdot 45°)}} = 32.4\text{ m/s}.$$

Thus, the time at which the ball flies over the fence is:

$$x' = (v_0\cos\theta_0)t' \quad\Rightarrow\quad t' = \frac{x'}{v_0\cos\theta_0} = \frac{97.5\text{ m}}{(32.4\text{ m/s})\cos 45°} = 4.26\text{ s}.$$

At this moment, the ball is at a height (above the ground) of

$$\begin{aligned} y' &= y_0 + (v_0\sin\theta_0)t' - \frac{1}{2}gt'^2 \\ &= 1.22\text{ m} + [(32.4\text{ m/s})\sin 45°](4.26\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(4.26\text{ s})^2 \\ &= 9.88\text{ m} \end{aligned}$$

which implies it does indeed clear the 7.32 m high fence.

(b) At $t' = 4.26$ s, the center of the ball is 9.88 m ï 7.32 m = 2.56 m above the fence.

**LEARN** Using the trajectory equation above, one can show that the minimum initial velocity required to clear the fence is given by

$$y' - y_0 = (\tan\theta_0)x' - \frac{gx'^2}{2(v_0\cos\theta_0)^2},$$

or about 31.9 m/s.

48. Following the hint, we have the time-reversed problem with the ball thrown from the roof, toward the left, at 60° measured clockwise from a leftward axis. We see in this time-reversed situation that it is convenient to take $+x$ as *leftward* with positive angles measured clockwise. Lengths are in meters and time is in seconds.

(a) With $y_0$ = 20.0 m, and $y$ = 0 at $t$ = 4.50 s, we have $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ where $v_{0y} = v_0 \sin 60°$. This leads to $v_0$ = 20.3 m/s. This plugs into the $x$-equation $x - x_0 = v_{0x}t$ (with $x_0$ = 0 and $x = d$) to produce

$$d = (20.3 \text{ m/s})\cos 60°(4.50 \text{ s}) = 45.7 \text{ m}.$$

(b) We have

$$v_x = v_{0x} = (20.3 \text{ m/s})\cos 60.0° = 10.2 \text{ m/s}$$
$$v_y = v_{0y} - gt = (20.3 \text{ m/s})\sin 60.0° - (9.80 \text{m/s}^2)(4.50 \text{ s}) = -26.5 \text{ m/s}.$$

The magnitude of $\vec{v}$ is $|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(10.2 \text{ m/s})^2 + (-26.5 \text{ m/s})^2} = 28.4 \text{ m/s}.$

(c) The angle relative to horizontal is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-26.5 \text{ m/s}}{10.2 \text{ m/s}}\right) = -68.9°.$$

We may convert the result from rectangular components to magnitude-angle representation:

$$\vec{v} = (10.2, -26.5) \rightarrow (28.4 \angle -68.9°)$$

and we now interpret our result (ñundoingò the time reversal) as an initial velocity of magnitude 28.4 m/s with angle (up from rightward) of 68.9°.

49. **THINK** In this problem a football is given an initial speed and it undergoes projectile motion. Weâd like to know the smallest and greatest angles at which a field goal can be scored.

**EXPRESS** We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the point where the ball is kicked. We use $x$ and $y$ to denote the coordinates of the ball at the goalpost, and try to find the kicking angle(s) $\theta_0$ so that $y$ = 3.44 m when $x$ = 50 m. Writing the kinematic equations for projectile motion:

$$x = v_0 \cos\theta_0, \quad y = v_0 t \sin\theta_0 - \tfrac{1}{2} g t^2,$$

we see the first equation gives $t = x/v_0 \cos\theta_0$, and when this is substituted into the second the result is

$$y = x \tan\theta_0 - \frac{gx^2}{2v_0^2 \cos^2\theta_0}.$$

**ANALYZE** One may solve the above equation by trial and error: systematically trying values of $\theta_0$ until you find the two that satisfy the equation. A little manipulation, however, will give an algebraic solution: Using the trigonometric identity

$$1 / \cos^2\theta_0 = 1 + \tan^2\theta_0,$$

we obtain

$$\frac{1}{2}\frac{gx^2}{v_0^2}\tan^2\theta_0 - x\tan\theta_0 + y + \frac{1}{2}\frac{gx^2}{v_0^2} = 0$$

which is a second-order equation for $\tan\theta_0$. To simplify writing the solution, we denote

$$c = \frac{1}{2} gx^2 / v_0^2 = \frac{1}{2}\left(9.80\ \text{m/s}^2\right)\left(50\ \text{m}\right)^2 / \left(25\ \text{m/s}\right)^2 = 19.6\ \text{m}.$$

Then the second-order equation becomes $c\tan^2\theta_0$ ï $x\tan\theta_0 + y + c = 0$. Using the quadratic formula, we obtain its solution(s).

$$\tan\theta_0 = \frac{x \pm \sqrt{x^2 - 4\left(y+c\right)c}}{2c} = \frac{50\ \text{m} \pm \sqrt{(50\ \text{m})^2 - 4\left(3.44\ \text{m} + 19.6\ \text{m}\right)\left(19.6\ \text{m}\right)}}{2\left(19.6\ \text{m}\right)}.$$

The two solutions are given by $\tan\theta_0$ = 1.95 and $\tan\theta_0$ = 0.605. The corresponding (first-quadrant) angles are $\theta_0$ = 63° and $\theta_0$ = 31°. Thus,

(a) The smallest elevation angle is $\theta_0$ = 31°, and

(b) The greatest elevation angle is $\theta_0$ = 63°.

**LEARN** If kicked at any angle between 31° and 63°, the ball will travel above the cross bar on the goalposts.

50. We apply Eq. 4-21, Eq. 4-22, and Eq. 4-23.

(a) From $\Delta x = v_{0x} t$, we find $v_{0x} = 40 \text{ m} / 2 \text{ s} = 20 \text{ m/s}$.

(b) From $\Delta y = v_{0y} t - \frac{1}{2} g t^2$, we find

$$v_{0y} = \frac{\Delta y + \frac{1}{2} g t^2}{t} = \frac{58 \text{ m} + \frac{1}{2}(9.8 \text{ m/s}^2)(2.0 \text{ s})^2}{2.0 \text{ s}} = 38.8 \text{ m/s} \approx 39 \text{ m/s}.$$

(c) From $v_y = v_{0y} - g t'$ with $v_y = 0$ as the condition for maximum height, we obtain $t' = (38.8 \text{ m/s})/(9.8 \text{ m/s}^2) = 3.96 \text{ s}$. During that time the $x$-motion is constant, so

$$x' - x_0 = (20 \text{ m/s})(3.96 \text{ s}) = 79 \text{ m}.$$

51. (a) The skier jumps up at an angle of $\theta_0 = 11.3°$ up from the horizontal and thus returns to the launch level with his velocity vector 11.3° below the horizontal. With the snow surface making an angle of $\alpha = 9.0°$ (downward) with the horizontal, the angle between the slope and the velocity vector is $\phi = \theta_0 - \alpha = 11.3° - 9.0° = 2.3°$.

(b) Suppose the skier lands at a distance $d$ down the slope. Using Eq. 4-25 with $x = d\cos\alpha$ and $y = -d\sin\alpha$ (the edge of the track being the origin), we have

$$-d\sin\alpha = d\cos\alpha\tan\theta_0 - \frac{g(d\cos\alpha)^2}{2v_0^2\cos^2\theta_0}.$$

Solving for $d$, we obtain

$$d = \frac{2v_0^2\cos^2\theta_0}{g\cos^2\alpha}\left(\cos\alpha\tan\theta_0 + \sin\alpha\right) = \frac{2v_0^2\cos\theta_0}{g\cos^2\alpha}\left(\cos\alpha\sin\theta_0 + \cos\theta_0\sin\alpha\right)$$
$$= \frac{2v_0^2\cos\theta_0}{g\cos^2\alpha}\sin(\theta_0 + \alpha).$$

Substituting the values given, we find

$$d = \frac{2(10\text{ m/s})^2\cos(11.3°)}{(9.8\text{ m/s}^2)\cos^2(9.0°)}\sin(11.3° + 9.0°) = 7.117\text{ m}.$$

which gives

$$y = -d\sin\alpha = -(7.117\text{ m})\sin(9.0°) = -1.11\text{ m}.$$

Therefore, at landing the skier is approximately 1.1 m below the launch level.

(c) The time it takes for the skier to land is

$$t = \frac{x}{v_x} = \frac{d\cos\alpha}{v_0\cos\theta_0} = \frac{(7.117\text{ m})\cos(9.0°)}{(10\text{ m/s})\cos(11.3°)} = 0.72\text{ s}.$$

Using Eq. 4-23, the $x$-and $y$-components of the velocity at landing are

$$v_x = v_0\cos\theta_0 = (10\text{ m/s})\cos(11.3°) = 9.81\text{ m/s}$$
$$v_y = v_0\sin\theta_0 - gt = (10\text{ m/s})\sin(11.3°) - (9.8\text{ m/s}^2)(0.72\text{ s}) = -5.07\text{ m/s}$$

Thus, the direction of travel at landing is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-5.07\text{ m/s}}{9.81\text{ m/s}}\right) = -27.3°.$$

or 27.3° below the horizontal. The result implies that the angle between the skierôs path and the slope is $\phi = 27.3° - 9.0° = 18.3°$, or approximately 18° to two significant figures.

52. From Eq. 4-21, we find $t = x / v_{0x}$. Then Eq. 4-23 leads to

$$v_y = v_{0y} - gt = v_{0y} - \frac{gx}{v_{0x}}.$$

Since the slope of the graph is –0.500, we conclude

$$\frac{g}{v_{0x}} = \frac{1}{2} \Rightarrow v_{ox} = 19.6 \text{ m/s}.$$

And from the ñ$y$ interceptò of the graph, we find $v_{oy}$ = 5.00 m/s. Consequently,

$$\theta_o = \tan^{-1}(v_{oy} / v_{ox}) = 14.3° \approx 14°.$$

53. Let $y_0 = h_0 = 1.00$ m at $x_0 = 0$ when the ball is hit. Let $y_1 = h$ (the height of the wall) and $x_1$ describe the point where it first rises above the wall one second after being hit; similarly, $y_2 = h$ and $x_2$ describe the point where it passes back down behind the wall four seconds later. And $y_f = 1.00$ m at $x_f = R$ is where it is caught. Lengths are in meters and time is in seconds.

(a) Keeping in mind that $v_x$ is constant, we have $x_2 - x_1 = 50.0\text{ m} = v_{1x}\,(4.00\text{ s})$, which leads to $v_{1x} = 12.5$ m/s. Thus, applied to the full six seconds of motion:

$$x_f - x_0 = R = v_x(6.00\text{ s}) = 75.0\text{ m}.$$

(b) We apply $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ to the motion above the wall,

$$y_2 - y_1 = 0 = v_{1y}\left(4.00\text{ s}\right) - \frac{1}{2}g\left(4.00\text{ s}\right)^2$$

and obtain $v_{1y} = 19.6$ m/s. One second earlier, using $v_{1y} = v_{0y} - g(1.00\text{ s})$, we find $v_{0y} = 29.4\text{ m/s}$. Therefore, the velocity of the ball just after being hit is

$$\vec{v} = v_{0x}\hat{\text{i}} + v_{0y}\hat{\text{j}} = (12.5\text{ m/s})\,\hat{\text{i}} + (29.4\text{ m/s})\,\hat{\text{j}}$$

Its magnitude is $|\vec{v}| = \sqrt{(12.5\text{ m/s})^2 + (29.4\text{ m/s})^2} = 31.9\text{ m/s}.$

(c) The angle is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{29.4\text{ m/s}}{12.5\text{ m/s}}\right) = 67.0°.$$

We interpret this result as a velocity of magnitude 31.9 m/s, with angle (up from rightward) of 67.0°.

(d) During the first 1.00 s of motion, $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$ yields

$$h = 1.0\text{ m} + \left(29.4\text{ m/s}\right)\left(1.00\text{ s}\right) - \tfrac{1}{2}\left(9.8\text{ m/s}^2\right)\left(1.00\text{ s}\right)^2 = 25.5\text{ m}.$$

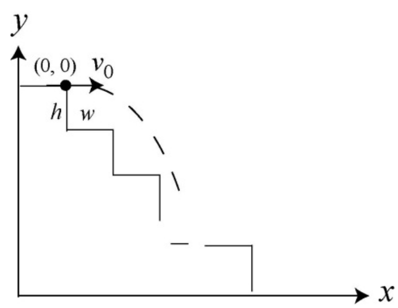
54. For $\Delta y = 0$, Eq. 4-22 leads to $t = 2v_o\sin\theta_o/g$, which immediately implies $t_{max} = 2v_o/g$ (which occurs for the ñstraight upò case: $\theta_o = 90°$). Thus,

$$\frac{1}{2}t_{max} = v_o/g \quad \Rightarrow \frac{1}{2} = \sin\theta_o.$$

Therefore, the half-maximum-time flight is at angle $\theta_o = 30.0°$. Since the least speed occurs at the top of the trajectory, which is where the velocity is simply the $x$-component of the initial velocity ($v_o\cos\theta_o = v_o\cos 30°$ for the half-maximum-time flight), then we need to refer to the graph in order to find $v_o$ ï in order that we may complete the solution. In the graph, we note that the range is 240 m when $\theta_o = 45.0°$. Equation 4-26 then leads to $v_o = 48.5$ m/s. The answer is thus (48.5 m/s)cos30.0° = 42.0 m/s.

55. **THINK** In this problem a ball rolls off the top of a stairway with an initial speed, and weôd like to know on which step it lands first.

We denote $h$ as the height of a step and $w$ as the width. To hit step $n$, the ball must fall a distance $nh$ and travel horizontally a distance between ($n$ ï 1)$w$ and $nw$. We take the origin of a coordinate system to be at the point where the ball leaves the top of the stairway, and we choose the $y$ axis to be positive in the upward direction, as shown in the following figure.




The coordinates of the ball at time $t$ are given by $x = v_{0x}t$ and $y = -\frac{1}{2}gt^2$ (since $v_{0y} = 0$).

**ANALYSE**
We equate $y$ to ï $nh$ and solve for the time to reach the level of step $n$:

$$t = \sqrt{\frac{2nh}{g}}.$$

Therefore, the $x$-coordinate is

$$x = v_{0x}\sqrt{\frac{2nh}{g}} = (1.00 \text{ m/s})\sqrt{\frac{2n(0.183 \text{ m})}{9.8 \text{ m/s}^2}} = (0.193 \text{ m})\sqrt{n}.$$

The method is to try values of $n$ until we find one for which $x/w$ is less than $n$ but greater than $n$ ï 1. For $n$ = 1, $x$ = 0.193 m and $x/w$ = 1.05, which is greater than $n$. For $n$ = 2, $x$ = 0.273 m and $x/w$ =0.273/0.183 = 1.49 which is less than 2 and greater than 1, which is also greater than $n$. For $n$ = 3, $x$ = 0.333 m and $x/w$ = 0.333/0.183 = 1.81 which is also less than 3 but also less than 2. Now, this is less than $n$ and greater than $n$ ï 1, so the ball hits the second step.

**LEARN** To check the consistency of our calculation, we can substitute $n$ = 3 into the above equations. The results are $t$ = 0.353 s, $y$ = 0.609 m and $x$ = 0.535 m. This indeed corresponds to the third step.

56. We apply Eq. 4-35 to solve for speed $v$ and Eq. 4-34 to find acceleration $a$.

(a) Since the radius of Earth is $6.37 \times 10^6$ m, the radius of the satellite orbit is

$$r = (6.37 \times 10^6 + 750 \times 10^3) \text{ m} = 7.12 \times 10^6 \text{ m}.$$

Therefore, the speed of the satellite is

$$v = \frac{2\pi r}{T} = \frac{2\pi\left(7.12 \times 10^6 \text{ m}\right)}{\left(98.0 \text{ min}\right)\left(60 \text{ s/min}\right)} = 7.61 \times 10^3 \text{ m/s}.$$

(b) The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{\left(7.61 \times 10^3 \text{ m/s}\right)^2}{7.12 \times 10^6 \text{ m}} = 8.13 \text{ m/s}^2.$$

57. The magnitude of centripetal acceleration ($a = v^2/r$) and its direction (toward the center of the circle) form the basis of this problem.

(a) If a passenger at this location experiences $\vec{a} = 1.83 \text{ m/s}^2$ east, then the center of the circle is *east* of this location. The distance is $r = v^2/a = (3.66 \text{ m/s})^2/(1.83 \text{ m/s}^2) = 7.32$ m.

(b) Thus, relative to the center, the passenger at that moment is located 7.32 m toward the west.

(c) If the direction of $\vec{a}$ experienced by the passenger is now *south*ð indicating that the center of the merry-go-round is south of him, then relative to the center, the passenger at that moment is located 7.32 m toward the north.

58. (a) The circumference is $c = 2\pi r = 2\pi(0.15 \text{ m}) = 0.94$ m.

(b) With $T = (60 \text{ s})/1100 = 0.0545$ s, the speed is $v = c/T = (0.94 \text{ m})/(0.0545 \text{ s}) = 17.3$ m/s. This is equivalent to using Eq. 4-35.

(c) The magnitude of the acceleration is $a = v^2/r = (17.3 \text{ m/s})^2/(0.15 \text{ m}) = 2.0 \times 10^3 \text{ m/s}^2$.

(d) The period of revolution is $(1100 \text{ rev/min})^{-1} = 9.1 \times 10^{-4}$ min, which becomes, in SI units, $T = 0.0545$ s, or approximately 55 ms.

59. If a particle travels along a circle or circular arc of radius $r$ at constant speed $v$, it is said to be in uniform circular motion $\vec{a}$ and has an acceleration of constant magnitude

$$a = \frac{v^2}{r} \qquad (1)$$

The direction of $\vec{a}$ is toward the center of the circle or circular arc, and $\vec{a}$ is said to be centripetal. The time for the particle to complete a circle is

$$T = \frac{2\pi r}{v} \qquad (2)$$

$T$ is called the period of revolution, or simply the period, of the motion. From Eq. (2), we calculate the velocity of the record as

$$v = \frac{2\pi r}{v} = \frac{2\pi(0.16)}{1.8} = 0.55\,\text{m/s}.$$

Using Eq. (1), we calculate the centripetal acceleration of a point P located on the edge of the record as

$$a = \frac{v^2}{r} = \frac{(0.55)^2}{0.16} = 1.89\,\text{m/s}^2 = 1.9\,\text{m/s}^2.$$

60. (a) During constant-speed circular motion, the velocity vector is perpendicular to the acceleration vector at every instant. Thus, $\vec{\mathrm{v}} \cdot \vec{a} = 0$.

(b) The acceleration in this vector, at every instant, points toward the center of the circle, whereas the position vector points from the center of the circle to the object in motion. Thus, the angle between $\vec{r}$ and $\vec{a}$ is 180° at every instant, so $\vec{r} \times \vec{a} = 0$.

61. We apply Eq. 4-35 to solve for speed $v$ and Eq. 4-34 to find centripetal acceleration $a$.

(a) $v = 2\pi r/T = 2\pi(20 \text{ km})/1.0 \text{ s} = 126 \text{ km/s} = 1.3 \times 10^5$ m/s.

(b) The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(126 \text{ km/s})^2}{20 \text{ km}} = 7.9 \times 10^5 \text{ m/s}^2.$$

(c) Clearly, both $v$ and $a$ will increase if $T$ is reduced.

62. The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(10\ \text{m/s})^2}{20\ \text{m}} = 5.0\ \text{m/s}^2.$$

63. We first note that $\vec{a}_1$ (the acceleration at $t_1$ = 2.00 s) is perpendicular to $\vec{a}_2$ (the acceleration at $t_2$=5.00 s), by taking their scalar (dot) product:

$$\vec{a}_1 \cdot \vec{a}_2 = [(6.00 \text{ m/s}^2)\hat{i} + (4.00 \text{ m/s}^2)\hat{j}] \cdot [(4.00 \text{ m/s}^2)\hat{i} + (-6.00 \text{ m/s}^2)\hat{j}] = 0.$$

Since the acceleration vectors are in the (negative) radial directions, then the two positions (at $t_1$ and $t_2$) are a quarter-circle apart (or three-quarters of a circle, depending on whether one measures clockwise or counterclockwise). A quick sketch leads to the conclusion that if the particle is moving counterclockwise (as the problem states) then it travels three-quarters of a circumference in moving from the position at time $t_1$ to the position at time $t_2$. Letting $T$ stand for the period, then $t_2 - t_1$ = 3.00 s = $3T/4$. This gives $T$ = 4.00 s. The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(6.00 \text{ m/s}^2)^2 + (4.00 \text{ m/s})^2} = 7.21 \text{ m/s}^2.$$

Using Eqs. 4-34 and 4-35, we have $a = 4\pi^2 r / T^2$, which yields

$$r = \frac{aT^2}{4\pi^2} = \frac{(7.21 \text{ m/s}^2)(4.00 \text{ s})^2}{4\pi^2} = 2.92 \text{ m}.$$

64. When traveling in circular motion with constant speed, the instantaneous acceleration vector necessarily points toward the center. Thus, the center is ñstraight upò from the cited point.

(a) Since the center is ñstraight upò from (4.00 m, 4.00 m), the $x$ coordinate of the center is 4.00 m.

(b) To find out ñhow far upò we need to know the radius. Using Eq. 4-34 we find

$$r = \frac{v^2}{a} = \frac{(5.00\ \text{m/s})^2}{12.5\ \ \text{m/s}^2} = 2.00\ \text{m}.$$

Thus, the $y$ coordinate of the center is 2.00 m + 4.00 m = 6.00 m. Thus, the center may be written as $(x, y)$ = (4.00 m, 6.00 m).

65. Since the period of a uniform circular motion is $T = 2\pi r / v$, where $r$ is the radius and $v$ is the speed, the centripetal acceleration can be written as

$$a = \frac{v^2}{r} = \frac{1}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 r}{T^2}.$$

Based on this expression, we compare the (magnitudes) of the wallet and purse accelerations, and find their ratio is the ratio of $r$ values. Therefore, $a_{\text{wallet}} = 1.50\ a_{\text{purse}}$. Thus, the wallet acceleration vector is

$$a = 1.50[(2.00\ \text{m/s}^2)\hat{i} + (4.00\ \text{m/s}^2)\hat{j}] = (3.00\ \text{m/s}^2)\hat{i} + (6.00\ \text{m/s}^2)\hat{j}.$$

66. The fact that the velocity is in the $+y$ direction and the acceleration is in the $+x$ direction at $t_1$ = 5.00 s implies that the motion is clockwise. The position corresponds to the ñ9:00 position.ò On the other hand, the position at $t_2$ = 10.0 s is in the ñ6:00 positionò since the velocity points in the $-x$ direction and the acceleration is in the $+y$ direction. The time interval $\Delta t = 10.0\ \text{s} - 5.00\ \text{s} = 5.00\ \text{s}$ is equal to 3/4 of a period:

$$5.00\ \text{s} = \frac{3}{4}T \quad \Rightarrow \quad T = 6.67\ \text{s}.$$

Equation 4-35 then yields

$$r = \frac{vT}{2\pi} = \frac{(3.00\ \text{m/s})(6.67\ \text{s})}{2\pi} = 3.18\ \text{m}.$$

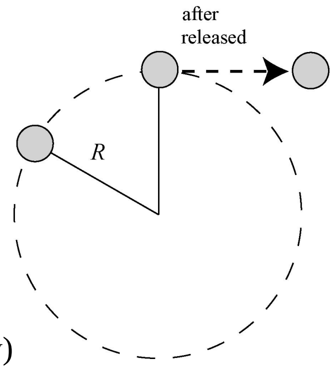
(a) The $x$ coordinate of the center of the circular path is $x = 5.00\ \text{m} + 3.18\ \text{m} = 8.18\ \text{m}.$

(b) The $y$ coordinate of the center of the circular path is $y = 6.00\ \text{m}.$

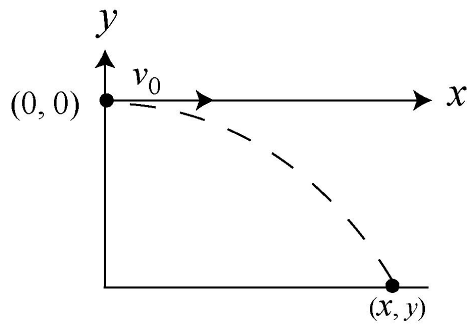
In other words, the center of the circle is at $(x,y)$ = (8.18 m, 6.00 m).

67. **THINK** In this problem we have a stone whirled in a horizontal circle. After the string breaks, the stone undergoes projectile motion.

**EXPRESS** The stone moves in a circular path (top view shown below left) initially, but undergoes projectile motion after the string breaks (side view shown below right). Since $a = v^2/R$, to calculate the centripetal acceleration of the stone, we need to know its speed during its circular motion (this is also its initial speed when it flies off). We use the kinematic equations of projectile motion (discussed in §4-6) to find that speed.



(top view)



(side view)

Taking the +$y$ direction to be upward and placing the origin at the point where the stone leaves its circular orbit, then the coordinates of the stone during its motion as a projectile are given by $x = v_0 t$ and $y = -\frac{1}{2}gt^2$ (since $v_{0y} = 0$). It hits the ground at $x = 10$ m and $y = -2.0$ m.

**ANALYZE** Formally solving the $y$-component equation for the time, we obtain $t = \sqrt{-2y/g}$, which we substitute into the first equation:

$$v_0 = x\sqrt{-\frac{g}{2y}} = (10\text{ m})\sqrt{-\frac{9.8\text{ m/s}^2}{2(-2.0\text{ m})}} = 15.7\text{ m/s}.$$

Therefore, the magnitude of the centripetal acceleration is

$$a = \frac{v_0^2}{R} = \frac{(15.7\text{ m/s})^2}{1.5\text{ m}} = 160\text{ m/s}^2.$$

**LEARN** The above equations can be combined to give $a = \dfrac{gx^2}{-2yR}$. The equation implies that the greater the centripetal acceleration, the greater the initial speed of the projectile, and the greater the distance traveled by the stone. This is precisely what we expect.

68. We note that after three seconds have elapsed ($t_2 - t_1 = 3.00$ s) the velocity (for this object in circular motion of period $T$ ) is reversed; we infer that it takes three seconds to reach the opposite side of the circle. Thus, $T = 2(3.00\text{ s}) = 6.00$ s.

(a) Using Eq. 4-35, $r = vT/2\pi$, where $v = \sqrt{(3.00\text{ m/s})^2 + (4.00\text{ m/s})^2} = 5.00\text{ m/s}$, we obtain $r = 4.77\text{ m}$. The magnitude of the object's centripetal acceleration is therefore $a = v^2/r = 5.24\text{ m/s}^2$.

(b) The average acceleration is given by Eq. 4-15:

$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{(-3.00\hat{\text{i}} - 4.00\hat{\text{j}})\text{ m/s} - (3.00\hat{\text{i}} + 4.00\hat{\text{j}})\text{ m/s}}{5.00\text{ s} - 2.00\text{ s}} = (-2.00\text{ m/s}^2)\hat{\text{i}} + (-2.67\text{ m/s}^2)\hat{\text{j}}$$

which implies $|\vec{a}_{\text{avg}}| = \sqrt{(-2.00\text{ m/s}^2)^2 + (-2.67\text{ m/s}^2)^2} = 3.33\text{ m/s}^2$.

69. We use Eq. 4-15 first using velocities relative to the truck (subscript t) and then using velocities relative to the ground (subscript g). We work with SI units, so $20\ \text{km/h} \to 5.6\ \text{m/s}$, $30\ \text{km/h} \to 8.3\ \text{m/s}$, and $45\ \text{km/h} \to 12.5\ \text{m/s}$. We choose east as the $+\hat{\text{i}}$ direction.

(a) The velocity of the cheetah (subscript c) at the end of the 2.0 s interval is (from Eq. 4-44)

$$\vec{v}_{\text{c t}} = \vec{v}_{\text{c g}} - \vec{v}_{\text{t g}} = (12.5\ \text{m/s})\ \hat{\text{i}} - (-5.6\ \text{m/s})\ \hat{\text{i}} = (18.1\ \text{m/s})\ \hat{\text{i}}$$

relative to the truck. Since the velocity of the cheetah relative to the truck at the beginning of the 2.0 s interval is $(-8.3\ \text{m/s})\hat{\text{i}}$, the (average) acceleration vector relative to the cameraman (in the truck) is

$$\vec{a}_{\text{avg}} = \frac{(18.1\ \text{m/s})\hat{\text{i}} - (-8.3\ \text{m/s})\hat{\text{i}}}{2.0\ \text{s}} = (13\ \text{m/s}^2)\hat{\text{i}},$$

or $|\vec{a}_{\text{avg}}| = 13\ \text{m/s}^2$.

(b) The direction of $\vec{a}_{\text{avg}}$ is $+\hat{\text{i}}$, or eastward.

(c) The velocity of the cheetah at the start of the 2.0 s interval is (from Eq. 4-44)

$$\vec{v}_{0\text{cg}} = \vec{v}_{0\text{ct}} + \vec{v}_{0\text{tg}} = (-8.3\ \text{m/s})\hat{\text{i}} + (-5.6\ \text{m/s})\hat{\text{i}} = (-13.9\ \text{m/s})\hat{\text{i}}$$

relative to the ground. The (average) acceleration vector relative to the crew member (on the ground) is

$$\vec{a}_{\text{avg}} = \frac{(12.5\ \text{m/s})\hat{\text{i}} - (-13.9\ \text{m/s})\hat{\text{i}}}{2.0\ \text{s}} = (13\ \text{m/s}^2)\hat{\text{i}}, \quad |\vec{a}_{\text{avg}}| = 13\ \text{m/s}^2$$

identical to the result of part (a).

(d) The direction of $\vec{a}_{\text{avg}}$ is $+\hat{\text{i}}$, or eastward.

70. We use Eq. 4-44, noting that the upstream corresponds to the $+\hat{i}$ direction.

(a) The subscript b is for the boat, w is for the water, and g is for the ground.

$$\vec{v}_{\text{bg}} = \vec{v}_{\text{bw}} + \vec{v}_{\text{wg}} = (14\text{ km/h})\,\hat{i} + (-8.2\text{ km/h})\,\hat{i} = (6\text{ km/h})\,\hat{i}.$$

Thus, the magnitude is $|\vec{v}_{\text{bg}}| = 6\ \text{km/h}$.

(b) The direction of $\vec{v}_{\text{bg}}$ is $+x$, or upstream.

(c) We use the subscript c for the child, and obtain

$$\vec{v}_{\text{c g}} = \vec{v}_{\text{c b}} + \vec{v}_{\text{b g}} = (-6\text{ km/h})\,\hat{i} + (6\text{ km/h})\,\hat{i} = (0\text{ km/h})\,\hat{i}.$$

(d) The child has no direction or magnitude for hwe velocity relative to the ground.

71. While moving in the same direction as the sidewalkôs motion (covering a distance $d$ relative to the ground in time $t_1$ = 2.50 s), Eq. 4-44 leads to

$$v_{\text{sidewalk}} + v_{\text{man running}} = \frac{d}{t_1} \ .$$

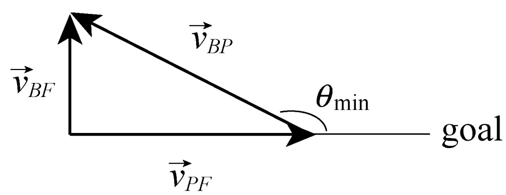
While he runs back (taking time $t_2$ = 10.0 s) we have

$$v_{\text{sidewalk}} - v_{\text{man running}} = -\frac{d}{t_2} \ .$$

Dividing these equations and solving for the desired ratio, we get $\frac{12.5}{7.5} = \frac{5}{3}$ = 1.67.

72. We denote the velocity of the player with $\vec{v}_{PF}$ and the relative velocity between the player and the ball be $\vec{v}_{BP}$. Then the velocity $\vec{v}_{BF}$ of the ball relative to the field is given by $\vec{v}_{BF} = \vec{v}_{PF} + \vec{v}_{BP}$. The smallest angle $\theta_{\min}$ corresponds to the case when $\vec{v}_{BF} \perp \vec{v}_{PF}$. Hence,

$$\theta_{\min} = 180° - \cos^{-1}\left(\frac{|\vec{v}_{PF}|}{|\vec{v}_{BP}|}\right) = 180° - \cos^{-1}\left(\frac{3.5 \text{ m/s}}{6.0 \text{ m/s}}\right) = 126°.$$



73. We denote the police and the motorist with subscripts $p$ and $m$, respectively. The coordinate system is indicated in Fig. 4-46.

(a) The velocity of the motorist with respect to the police car is

$$\vec{v}_{m\,p} = \vec{v}_m - \vec{v}_p = (-60 \text{ km/h})\hat{j} - (-80 \text{ km/h})\hat{i} = (80 \text{ km/h})\hat{i} - (60 \text{ km/h})\hat{j}.$$

(b) $\vec{v}_{m\,p}$ does happen to be along the line of sight. Referring to Fig. 4-46, we find the vector pointing from one car to another is $\vec{r} = (800 \text{ m})\hat{i} - (600 \text{ m})\hat{j}$ (from $M$ to $P$). Since the ratio of components in $\vec{r}$ is the same as in $\vec{v}_{m\,p}$, they must point the same direction.

(c) No, they remain unchanged.

74. Velocities are taken to be constant. With 18 min = 0.30 h, the velocity of the plane relative to the ground is $\vec{v}_{PG} = (55\ \text{km})/(0.30\ \text{h})\hat{j} = (183.3\ \text{km/h})\hat{j}$. In addition,

$$\vec{v}_{AG} = (42\ \text{km/h})(\cos 20°\hat{i} - \sin 20°\hat{j}) = (39.5\ \text{km/h})\hat{i} - (14.4\ \text{km/h})\hat{j}.$$

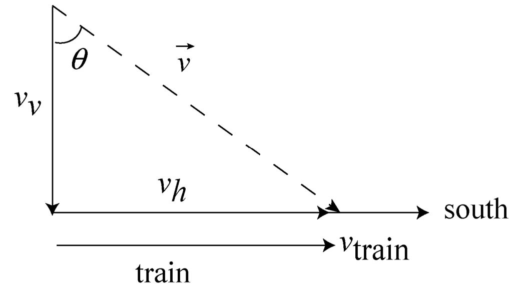
Using $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$, we have

$$\vec{v}_{PA} = \vec{v}_{PG} - \vec{v}_{AG} = -(39.5\ \text{km/h})\hat{i} + (197.7\ \text{km/h})\hat{j}$$

which implies $|\vec{v}_{PA}| = \sqrt{(-39.5\ \text{km/h})^2 + (197.7\ \text{km/h})^2} = 201.6\ \text{km/h}$, or 200 km/h (to two significant figures.)

75. **THINK** This problem deals with relative motion in two dimensions. Raindrops appear to fall vertically by an observer on a moving train.

**EXPRESS** Since the raindrops fall vertically relative to the train, the horizontal component of the velocity of a raindrop, $v_h$ = 30 m/s, must be the same as the speed of the train, i.e., $v_h = v_{\text{train}}$ (see figure).




On the other hand, if $v_v$ is the vertical component of the velocity and $\theta$ is the angle between the direction of motion and the vertical, then tan $\theta = v_h/v_v$. Knowing $v_v$ and $v_h$ allows us to determine the speed of the raindrops.

**ANALYZE** With $\theta = 70°$, we find the vertical component of the velocity to be

$$v_v = v_h/\tan\theta = (30 \text{ m/s})/\tan 70° = 10.9 \text{ m/s}.$$

Therefore, the speed of a raindrop is

$$v = \sqrt{v_h^2 + v_v^2} = \sqrt{(30 \text{ m/s})^2 + (10.9 \text{ m/s})^2} = 32 \text{ m/s}.$$

**LEARN** As long as the horizontal component of the velocity of the raindrops coincides with the speed of the train, the passenger on board will see the rain falling perfectly vertically.

76. The destination is $\vec{D}$ = 900 km $\hat{\text{j}}$ where we orient axes so that +*y* points north and +*x* points east. This takes two hours, so the (constant) velocity of the plane (relative to the ground) is $\vec{v}_{\text{pg}}$ = (450 km/h) $\hat{\text{j}}$ . This must be the vector sum of the planeÕs velocity with respect to the air which has (*x,y*) components (500cos70º, 500sin70º), and the velocity of the air (*wind*) relative to the ground $\vec{v}_{\text{ag}}$ . Thus,

$$(450 \text{ km/h})\ \hat{\text{j}} = (500 \text{ km/h}) \cos 70^\circ\ \hat{\text{i}} + (500 \text{ km/h}) \sin 70^\circ\ \hat{\text{j}} + \vec{v}_{\text{ag}}$$

which yields

$\vec{v}_{\text{ag}}$ =( ï 171 km/h)$\hat{\text{i}}$ ï ( 19.8 km/h)$\hat{\text{j}}$ .

(a) The magnitude of $\vec{v}_{\text{ag}}$ is $|\vec{v}_{\text{ag}}| = \sqrt{(-171 \text{ km/h})^2 + (-19.8 \text{ km/h})^2} = 172 \text{ km/h}.$

(b) The direction of $\vec{v}_{\text{ag}}$ is

$$\theta = \tan^{-1}\left(\frac{-19.8 \text{ km/h}}{-171 \text{ km/h}}\right) = 6.6^\circ \ \ (\text{south of west}).$$

77. **THINK** This problem deals with relative motion in two dimensions. Snowflakes falling vertically downward are seen to fall at an angle by a moving observer.

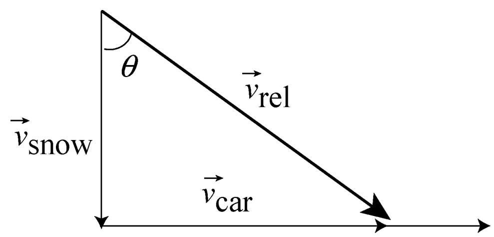
**EXPRESS** Relative to the car the velocity of the snowflakes has a vertical component of $v_v = 8.0\ \text{m/s}$ and a horizontal component of $v_h = 50\ \text{km/h} = 13.9\ \text{m/s}$.

**ANALYZE** The angle $\theta$ from the vertical is found from

$$\tan\theta = \frac{v_h}{v_v} = \frac{13.9\ \text{m/s}}{8.0\ \text{m/s}} = 1.74$$

which yields $\theta = 60°$.

**LEARN** The problem can also be solved by expressing the velocity relation in vector notation: $\vec{v}_{\text{rel}} = \vec{v}_{\text{car}} + \vec{v}_{\text{snow}}$, as shown in the figure.



78. We make use of Eq. 4-44 and Eq. 4-45.

The velocity of Jeep $P$ relative to $A$ at the instant is

$$\vec{v}_{PA} = (40.0 \text{ m/s})(\cos 60°\hat{\text{i}} + \sin 60°\hat{\text{j}}) = (20.0 \text{ m/s})\hat{\text{i}} + (34.6 \text{ m/s})\hat{\text{j}}.$$

Similarly, the velocity of Jeep $B$ relative to $A$ at the instant is

$$\vec{v}_{BA} = (25.0 \text{ m/s})(\cos 30°\hat{\text{i}} + \sin 30°\hat{\text{j}}) = (21.65 \text{ m/s})\hat{\text{i}} + (12.50 \text{ m/s})\hat{\text{j}}.$$

Thus, the velocity of $P$ relative to $B$ is

$$\begin{aligned}\vec{v}_{PB} &= \vec{v}_{PA} - \vec{v}_{BA} = (20.0 \text{ m/s})\hat{\text{i}} + (34.6 \text{ m/s})\hat{\text{j}} - \left[(21.65\text{m/s})\hat{\text{i}} + (12.50\text{m/s})\hat{\text{j}}\right] \\ &= (-1.65 \text{ m/s})\hat{\text{i}} + (22.14 \text{ m/s})\hat{\text{j}}.\end{aligned}$$

(a) The magnitude of $\vec{v}_{PB}$ is $|\vec{v}_{PB}| = \sqrt{(-1.65 \text{ m/s})^2 + (22.14 \text{ m/s})^2} = 22.2 \text{ m/s}.$

(b) The direction of $\vec{v}_{PB}$ is $\theta = \tan^{-1}[(22.14 \text{ m/s})/(-1.65 \text{ m/s})] = -85.7°$, which is $85.7^0$ north of west).

(c) The acceleration of $P$ is

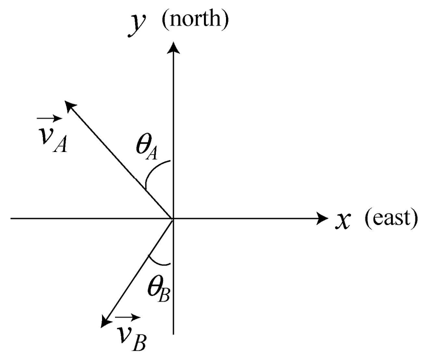
$$\vec{a}_{PA} = (0.400 \text{ m/s}^2)(\cos 60.0°\hat{\text{i}} + \sin 60.0°\hat{\text{j}}) = (0.200 \text{ m/s}^2)\hat{\text{i}} + (0.346 \text{ m/s}^2)\hat{\text{j}},$$

and $\vec{a}_{PA} = \vec{a}_{PB}$. Thus, we have $|\vec{a}_{PB}| = 0.400 \text{ m/s}^2$.

(d) The direction is 60.0° north of due east (or 30.0° east of north).

79. **THINK** This problem involves analyzing the relative motion of two ships sailing in different directions.

**EXPRESS** Given that $\theta_A = 45°$, and $\theta_B = 40°$, as defined in the figure, the velocity vectors (relative to the shore) for ships $A$ and $B$ are given by




$$\vec{v}_A = -(v_A \cos 45°)\,\hat{i} + (v_A \sin 45°)\,\hat{j}$$
$$\vec{v}_B = -(v_B \sin 40°)\,\hat{i} - (v_B \cos 40°)\,\hat{j},$$

with $v_A$ = 24 knots and $v_B$ = 28 knots. We take east as $+\hat{i}$ and north as $\hat{j}$.
The velocity of ship $A$ relative to ship $B$ is simply given by $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$.

**ANALYZE**
(a) The relative velocity is

$$\vec{v}_{A\,B} = \vec{v}_A - \vec{v}_B = (v_B \sin 40° - v_A \cos 45°)\hat{i} + (v_B \cos 40° + v_A \sin 45°)\,\hat{j}$$
$$= (1.03 \text{ knots})\hat{i} + (38.4 \text{ knots})\hat{j}$$

the magnitude of which is $|\vec{v}_{A\,B}| = \sqrt{(1.03 \text{ knots})^2 + (38.4 \text{ knots})^2} \approx 38.4$ knots.

(b) The angle $\theta_{AB}$ which $\vec{v}_{A\,B}$ makes with north is given by

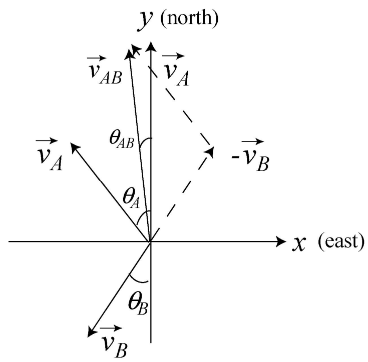
$$\theta_{AB} = \tan^{-1}\left(\frac{v_{AB,x}}{v_{AB,y}}\right) = \tan^{-1}\left(\frac{1.03 \text{ knots}}{38.4 \text{ knots}}\right) = 1.5°$$

which is to say that $\vec{v}_{A\,B}$ points 1.5° east of north.

(c) Since the two ships started at the same time, their relative velocity describes at what rate the distance between them is increasing. Because the rate is steady, we have

$$t = \frac{|\Delta r_{AB}|}{|\vec{v}_{AB}|} = \frac{160 \text{ nautical miles}}{38.4 \text{ knots}} = 4.2 \text{ h}.$$

(d) The velocity $\vec{v}_{A\,B}$ does not change with time in this problem, and $\vec{r}_{A\,B}$ is in the same direction as $\vec{v}_{A\,B}$ since they started at the same time. Reversing the points of view, we have $\vec{v}_{A\,B} = -\vec{v}_{B\,A}$ so that $\vec{r}_{A\,B} = -\vec{r}_{B\,A}$ (i.e., they are 180° opposite to each other). Hence, we conclude that $B$ stays at a bearing of 1.5° west of south relative to $A$ during the journey (neglecting the curvature of Earth).




**LEARN** The relative velocity is depicted in the figure on the right. When analyzing relative motion in two dimensions, a vector diagram such as the one shown can be very helpful.

80. This is a classic problem involving two-dimensional relative motion. We align our coordinates so that *east* corresponds to $+x$ and *north* corresponds to $+y$. We write the vector addition equation as $\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$. We have $\vec{v}_{WG} = (2.5\angle 0°)$ in the magnitude-angle notation (with the unit m/s understood), or $\vec{v}_{WG} = 2.5\hat{i}$ in unit-vector notation. We also have $\vec{v}_{BW} = (8.0\angle 120°)$ where we have been careful to phrase the angle in the "standard" way (measured counterclockwise from the $+x$ axis), or $\vec{v}_{BW} = (-4.0\hat{i} + 6.9\hat{j})$ m/s.

(a) We can solve the vector addition equation for $\vec{v}_{BG}$:

$$\vec{v}_{BG} = v_{BW} + \vec{v}_{WG} = (2.5\ \text{m/s})\hat{i} + (-4.0\hat{i} + 6.9\hat{j})\ \text{m/s} = (-1.5\ \text{m/s})\hat{i} + (6.9\ \text{m/s})\hat{j}.$$

Thus, we find $|\vec{v}_{BG}| = 7.1\ \text{m/s}$.

(b) The direction of $\vec{v}_{BG}$ is $\theta = \tan^{-1}[(6.9\ \text{m/s})/(-1.5\ \text{m/s})] = 102°$ (measured counterclockwise from the $+x$ axis), or 12° west of due north.

(c) The velocity is constant, and we apply $y - y_0 = v_y t$ in a reference frame. Thus, in the *ground* reference frame, we have $(200\ \text{m}) = (7.1\ \text{m/s})\sin(102°)t \rightarrow t = 29\ \text{s}$. Note: If a student obtains "28 s," then the student has probably neglected to take the $y$ component properly (a common mistake).

81. Here, the subscript $W$ refers to the water. Our coordinates are chosen with $+x$ being *east* and $+y$ being *north*. In these terms, the angle specifying *east* would be 0° and the angle specifying *south* would be –90° or 270°. Where the length unit is not displayed, km is to be understood.

(a) We have $\vec{v}_{A\,W} = \vec{v}_{A\,B} + \vec{v}_{B\,W}$, so that

$$\vec{v}_{A\,B} = (22 \angle -90°) - (40 \angle 37°) = (56 \angle -125°)$$

in the magnitude-angle notation (conveniently done with a vector-capable calculator in polar mode). Converting to rectangular components, we obtain

$$\vec{v}_{A\,B} = (-32\text{km/h})\,\hat{i} - (46\text{ km/h})\,\hat{j}.$$

Of course, this could have been done in unit-vector notation from the outset.

(b) Since the velocity-components are constant, integrating them to obtain the position is straightforward ($\vec{r} - \vec{r}_0 = \int \vec{v}\,dt$)

$$\vec{r} = (2.5 - 32t)\,\hat{i} + (4.0 - 46t)\,\hat{j}$$

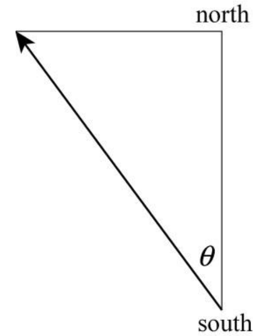
with lengths in kilometers and time in hours.

(c) The magnitude of this $\vec{r}$ is $r = \sqrt{(2.5 - 32t)^2 + (4.0 - 46t)^2}$ . We minimize this by taking a derivative and requiring it to equal zero — which leaves us with an equation for $t$

$$\frac{dr}{dt} = \frac{1}{2}\frac{6286t - 528}{\sqrt{(2.5 - 32t)^2 + (4.0 - 46t)^2}} = 0$$

which yields $t$ = 0.084 h.

(d) Plugging this value of $t$ back into the expression for the distance between the ships ($r$), we obtain $r$ = 0.2 km. Of course, the calculator offers more digits ($r$ = 0.225…), but they are not significant; in fact, the uncertainties implicit in the given data, here, should make the ship captains worry.




82. We construct a right triangle starting from the clearing on the south bank, drawing a line (200 m long) due north (*upward* in our sketch) across the river, and then a line due west (upstream, leftward in our sketch) along the north bank for a distance $(82\text{ m}) + (1.1\text{ m/s})t$, where the $t$-dependent contribution is the distance that the river will carry the boat downstream during time $t$.

The hypotenuse of this right triangle (the arrow in our sketch) also depends on $t$ and on the boatôs speed (relative to the water), and we set it equal to the Pythagorean ñsumò of the triangleôs sides:

$$(5.0)t = \sqrt{200^2 + (82 + 1.1t)^2}$$

which leads to a quadratic equation for $t$

$$46724 + 180.4t - 23.8t^2 = 0.$$

We solve for $t$ first and find a positive value: $t$ = 48.3 s.

(a) The angle between the northward (200 m) leg of the triangle and the hypotenuse (which is measured ñwest of northò) is then given by

$$\theta = \tan^{-1}\left(\frac{82 + 1.1t}{200}\right) = \tan^{-1}\left(\frac{135}{200}\right) = 34°.$$

(b) The time is $t$ = 48.3 s.