3장 8절 연습문제 풀이

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3.

$$\lim_{x\to 0}\frac{a-\sqrt{a^2-x^2}}{x}=\lim_{x\to 0}\frac{x}{x(a+\sqrt{a^2-x^2})}=\frac{1}{2a}.$$

5.

$$\lim_{x\to 0}\frac{e^x-e^{-x}}{\tan x}=\lim_{x\to 0}\frac{e^x+e^{-x}}{\sec^2 x}=2.$$

6.

$$\lim_{x\to 0}\frac{a^x-b^x}{x}=\lim_{x\to 0}\frac{a^x\ln a-b^x\ln b}{1}=\ln a-\ln b=\ln\frac{a}{b}.$$

8.

$$\lim_{x\to a}\frac{\tan 2x-\tan 2a}{x-a}=\lim_{x\to a}\frac{2\sec^2 x}{1}=2\sec^2 a.$$

10.

$$\lim_{x\to 0}\frac{\ln\sec x}{x^2}=\lim_{x\to 0}\frac{\frac{\sec x\tan x}{\sec x}}{2x}=\lim_{x\to 0}\frac{\tan x}{2x}=\lim_{x\to 0}\frac{\sec^2 x}{2}=\frac{1}{2}.$$

18.

$$\begin{aligned}\lim_{x\to 0}\frac{x-\sin^{-1}x}{x^3}&=\lim_{x\to 0}\frac{1-\frac{1}{\sqrt{1-x^2}}}{3x^2}=\lim_{x\to 0}\frac{\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}}}{3x^2}\\&=\lim_{x\to 0}\frac{\sqrt{1-x^2}-1}{3x^2\sqrt{1-x^2}}=\lim_{x\to 0}\frac{-x^2}{3x^2\sqrt{1-x^2}(\sqrt{1-x^2}+1)}\\&=-\frac{1}{6}.\end{aligned}$$

20.

$$\lim_{x\to 0}\frac{e^x+e^{-x}-2\cos x}{x\sin x}=\lim_{x\to 0}\frac{e^x-e^{-x}+2\sin x}{\sin x+x\cos x}$$
$$=\lim_{x\to 0}\frac{e^x+e^{-x}+2\cos x}{\cos x+\cos x-x\sin x}$$
$$=2.$$

22.

$$\lim_{x\to 0}\frac{1-\ln x}{e^{1/x}}=\lim_{x\to 0}\frac{-\frac{1}{x}}{-\frac{1}{x^2}e^{1/x}}=\lim_{x\to 0}\frac{x}{e^{1/x}}=0.$$

24

$$\lim_{x\to\infty}\frac{2^x}{e^{x^2}}=\lim_{x\to\infty}\frac{2^x\ln 2}{2xe^{x^2}}=\lim_{x\to\infty}\frac{\ln 2}{2x}\cdot\frac{2^x}{e^{x^2}}=0$$

25.

$$\lim_{x\to 0}x\ln x=\lim_{x\to 0}\frac{\ln x}{\frac{1}{x}}=\lim_{x\to 0}\frac{1/x}{-1/x^2}=\lim_{x\to 0}(-x)=0.$$

29.

$$\lim_{x\to 0^+}xe^{1/x}=\lim_{x\to 0^+}\frac{e^{1/x}}{1/x}=\lim_{x\to 0^+}\frac{-1/x^2e^{1/x}}{-1/x^2}=\infty.$$

31.

$$\lim_{x\to 0}x\csc x\sin^{-1}x=\lim_{x\to 0}\frac{x\sin^{-1}x}{\sin x}=\lim_{x\to 0}\frac{\sin^{-1}x+\frac{x}{\sqrt{1-x^2}}}{\cos x}=0.$$

32.

$$\lim_{x\to 0}\sin x\ln(\tan x)=\lim_{x\to 0}\frac{\ln(\tan x)}{\csc x}=\lim_{x\to 0}\frac{\frac{\sec^2 x}{\tan x}}{-\csc x\cot x}$$
$$=\lim_{x\to 0}\frac{\sec^2 x\cot x}{-\csc x\cot x}=\lim_{x\to 0}\frac{-\sin x}{\cos^2 x}$$
$$=0$$

38.

$$\lim_{x\to 1}(\frac{x}{x-1}-\frac{1}{\ln x})=\lim_{x\to 1}\frac{x\ln x-x+1}{(x-1)\ln x}=\lim_{x\to 1}\frac{\ln x}{\ln x+\frac{x-1}{x}}$$
$$=\lim_{x\to 1}\frac{\ln x}{\ln x+1-1/x}=\lim_{x\to 1}\frac{1/x}{1/x+1/x^2}=\frac{1}{2}.$$

40.

$$\lim_{x\to 1}\left(\frac{x}{\ln x}-\frac{1}{x\ln x}\right)=\lim_{x\to 1}\frac{x^2-1}{x\ln x}=\lim_{x\to 1}\frac{2x}{\ln x+1}=2.$$

43.

$$\begin{aligned}\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin x}\right)&=\lim_{x\to 0}\frac{\sin x-x}{x\sin x}=\lim_{x\to 0}\frac{\cos x-1}{\sin x+x\cos x}\\&=\lim_{x\to 0}\frac{-\sin x}{\cos x+\cos x-x\sin x}=0.\end{aligned}$$

44.

$$\lim_{x\to 0}\left(\frac{1}{x\sin x}-\frac{1}{x^2}\right)=\lim_{x\to 0}\frac{x-\sin x}{x^2\sin x}=\lim_{x\to 0}\frac{1-\cos x}{2x\sin x+x^2\cos x}\underset{\text{한번더}}{=}0.$$

46.

$$\lim_{x\to\infty}e^{\ln x-\ln(\ln x)}=\lim_{x\to\infty}\frac{x}{\ln x}=\infty$$

이므로

$$\lim_{x\to\infty}(\ln x-\ln(\ln x))=\ln(\infty)=\infty.$$

49. $Y=x^{\frac{1}{x}}\Longrightarrow \ln Y=\frac{\ln x}{x}$ 이고

$$\lim_{x\to\infty}\frac{\ln x}{x}=0$$

이므로

$$\lim_{x\to\infty}x^{\frac{1}{x}}=e^0=1.$$

50.

$$\lim_{x\to 1}\ln x^{1/(1-x)}=\lim_{x\to 1}\frac{\ln x}{1-x}=\lim_{x\to 1}\frac{1/x}{-1}=-1$$

이므로

$$\lim_{x\to 1}x^{\frac{1}{1-x}}=e^{-1}.$$

51.

$$\begin{aligned}
\lim_{x\to 0}\ln(\sin x)^x &= \lim_{x\to 0} x\ln(\sin x) = \lim_{x\to 0}\frac{\ln(\sin x)}{1/x}\\
&= \lim_{x\to 0}\frac{\frac{cosx}{\sin x}}{-1/x^2} = \lim_{x\to 0}(-\frac{x^2\cos x}{\sin x})\\
&= \lim_{x\to 0}\frac{-2x\cos x + x^2\sin x}{\cos x} = 0\\
\Longrightarrow \lim_{x\to 0}(\sin x)^x &= e^0 = 1.
\end{aligned}$$

52.

$$\begin{aligned}
\lim_{x\to 0}\ln(\sin x)^{\tan x} &= \lim_{x\to 0}\tan x\ln(\sin x) = \lim_{x\to 0}\frac{\ln(\sin x)}{\cot x}\\
&= \lim_{x\to 0}\frac{\frac{\cos x}{\sin x}}{-\csc^2 x} = \lim_{x\to 0}(-\frac{\cos x}{\csc x})\\
&= -\lim_{x\to 0}\sin x\cos x = 0\\
\Longrightarrow \lim_{x\to 0}(\sin x)^{\tan x} &= e^0 = 1.
\end{aligned}$$

53.

$$\lim_{x\to\infty}(\frac{x}{x-2})^x = \lim_{x\to\infty}(\frac{x-2}{x})^{-x} = \lim_{x\to\infty}\left[(1-\frac{2}{x})^{(-\frac{x}{2})}\right]^2 = e^2.$$

54.

$$\lim_{x\to\infty}(1+\frac{3}{x})^x = \lim_{x\to\infty}\left[(1+\frac{3}{x})^{\frac{x}{3}}\right]^3 = e^3.$$

55.

$$\begin{aligned}
\lim_{x\to 0}\ln(1+\tan x)^{\frac{1}{x}} &= \lim_{x\to 0}\frac{\ln(1+\tan x)}{x}\\
&= \lim_{x\to 0}\frac{\sec^2 x}{1+\tan x} = 1\\
\Longrightarrow \lim_{x\to 0}(1+\tan x)^{\frac{1}{x}} &= e.
\end{aligned}$$

56.

$$\begin{aligned}
\lim_{x\to 0}\ln(1-x^2)^{\cot x} &= \lim_{x\to 0}\frac{\ln(1-x^2)}{\tan x} = \lim_{x\to 0}\frac{-2x\cos^2 x}{1-x^2} = 0\\
\Longrightarrow \lim_{x\to 0}(1-x^2)^{\cot x} &= e^0 = 1.
\end{aligned}$$

57.

$$\begin{aligned}
\lim_{x\to\infty} \ln(1-\frac{1}{x^3})^x &= \lim_{x\to\infty} x\ln(1-\frac{1}{x^3}) = \lim_{x\to\infty} \frac{\ln(1-\frac{1}{x^3})}{\frac{1}{x}} \\
&= \lim_{x\to\infty} \frac{\frac{3}{x^4}\cdot\frac{1}{1-\frac{1}{x^3}}}{-\frac{1}{x^2}} = \lim_{x\to\infty} \frac{-3}{x^2(1-\frac{1}{x^3})} = 0 \\
\Longrightarrow \lim_{x\to\infty}(1-\frac{1}{x^3})^x &= e^0 = 1
\end{aligned}$$

58.

$$\lim_{x\to 0} \ln(1+2x)^{\frac{1+2x}{x}} = \lim_{x\to 0} \frac{(1+2x)\ln(1+2x)}{x} = \lim_{x\to 0} \frac{(1+2x)\frac{2}{1+2x}}{1} = 2$$
$$\Longrightarrow \lim_{x\to 0}(1+2x)^{\frac{1+2x}{x}} = e^2.$$

59.

$$\lim_{x\to\infty} \ln(e^x+x)^{\frac{1}{x}} = \lim_{x\to\infty} \frac{\ln(e^x+x)}{x} = 1$$

그러므로

$$\lim_{x\to\infty}(e^x+x)^{\frac{1}{x}} = e.$$

60. 59 번과 같은 식으로 풀면,

$$\lim_{x\to 0}(e^x+x)^{\frac{1}{x}} = e.$$

61.

$$\begin{aligned}
\lim_{x\to\infty} \ln(\cos\frac{2}{x})^{x^2} &= \lim_{x\to\infty} x^2\ln(\cos\frac{2}{x}) = \lim_{x\to\infty} \frac{\ln(\cos\frac{2}{x})}{\frac{1}{x^2}} \\
&= \lim_{x\to\infty} \frac{2\frac{\tan\frac{1}{x^2}}{x^2}}{-\frac{2}{x^3}} = \lim_{x\to\infty}(-x\tan\frac{2}{x}) \\
&= \lim_{x\to\infty}(-2\frac{\tan\frac{2}{x}}{\frac{2}{x}}) = -2 \\
\Longrightarrow \lim_{x\to\infty}(\cos\frac{2}{x})^{x^2} &= e^{-2}.
\end{aligned}$$

62.

$$\lim_{x\to 0} \ln(\sin x + \cos x)^{\cot x} = \lim_{x\to 0} \frac{\ln(\sin x + \cos x)}{\tan x} = \frac{\frac{\cos x - \sin x}{\sin x + \cos x}}{\sec^2 x} = 1$$
$$\Longrightarrow \lim_{x\to 0}(\sin x + \cos x)^{\cot x} = e.$$

63.

$$\lim_{x\to 0}(1+ax)^{b/x} = \lim_{x\to 0}\left[(1+ax)^{1/ax}\right]^{ab} = e^{ab}.$$

64.

$$\lim_{x\to\infty} \ln(1+ax)^{b/x} = \lim_{x\to\infty} \frac{b\ln(1+ax)}{x} = \lim_{x\to\infty} \frac{ab}{1+ax} = ab$$
$$\Longrightarrow \lim_{x\to\infty}(1+ax)^{b/x} = e^{ab}.$$

65.

$$\lim_{x\to\infty}(1+x)^{\ln x} = \lim_{x\to\infty}\left[(1+x)^{\frac{1}{x}}\right]^{x\ln x} = e^0 = 1.$$