국방정보공학과 2020072306 송민경 선형대수_과제6

1. We know that $W^{\perp} = \mathrm{col}(A)^{\perp} = \mathrm{null}(A^T)$

$$A^T = \begin{bmatrix} 1 & -2 & 2 & -3 \\ 1 & 0 & 2 & -1 \\ -1 & 2 & -2 & 3 \\ 0 & 4 & 0 & 4 \\ 2 & 4 & 1 & 5 \end{bmatrix}$$ Transform the matrix to the RREF.

$$\xRightarrow[R_5 - 2R_1 \to R_5]{\substack{R_2 - R_1 \to R_2 \\ R_3 + R_1 \to R_3}} \begin{bmatrix} 1 & -2 & 2 & -3 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 4 \\ 0 & 8 & -3 & 11 \end{bmatrix} \xRightarrow[\substack{R_2 \times \frac{1}{2} \to R_2 \\ R_5 - 8R_2 \to R_5}]{R_4 - 2R_2 \to R_4} \begin{bmatrix} 1 & -2 & 2 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 3 \end{bmatrix} \xRightarrow[R_3 \times -\frac{1}{3} \to R_3]{\substack{R_3 \leftrightarrow R_5 \\ R_1 + 2R_2 \to R_1}} \begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xRightarrow{R_1 - 2R_3 \to R_1} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then, $A^T X = 0$ is,

$$\begin{bmatrix} \boxed{1} & 0 & 0 & 1 \\ 0 & \boxed{1} & 0 & 1 \\ 0 & 0 & \boxed{1} & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0.$$

The system has infinitely many solutions:

$$\begin{cases} x_1 = -x_4 = -s \\ x_2 = -x_4 = -s \\ x_3 = x_4 = s \\ x_4 = s \text{ (arbitrary)} \end{cases}$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix} s$$

Therefore, the null space has a basis formed by the set $\{(-1,-1,1,1)\}$.

(It is a basis for $W^{\perp}$)

2. $[x]_A$

$= C_1(0,1,0) + C_2\left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) + C_3\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) = (3,1,-2)$

$$\begin{cases} -\frac{1}{\sqrt{2}}C_2 + \frac{1}{\sqrt{2}}C_3 = 3 \\ C_1 = 1 \\ \frac{1}{\sqrt{2}}C_2 + \frac{1}{\sqrt{2}}C_3 = -2 \end{cases}$$

$\frac{2}{\sqrt{2}}C_3 = 1 \quad C_3 = \frac{\sqrt{2}}{2}$

$\frac{1}{\sqrt{2}}C_2 + \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{2} = -2, \quad \frac{1}{\sqrt{2}}C_2 = -\frac{5}{2}, \quad C_2 = -\frac{5\sqrt{2}}{2}$

$\Rightarrow C_1 = 1, \quad C_2 = -\frac{5\sqrt{2}}{2}, \quad C_3 = \frac{\sqrt{2}}{2}$

$\therefore [x]_A = \left(1, -\frac{5\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$[x]_B$

$= C_1(1,0,0) + C_2(0,1,2) + C_3(0,-2,1) = (3,1,-2)$

$$\begin{cases} C_1 = 3 \\ C_2 - 2C_3 = 1 \\ 2C_2 + C_3 = -2 \end{cases}$$

$2C_2 - 4C_3 = 2$

$2C_2 + C_3 = -2$

$-5C_3 = 4, \quad C_3 = -\frac{4}{5}, \quad C_2 = -\frac{3}{5}$

$C_2 - 2\times\left(-\frac{4}{5}\right) = 1$

$\Rightarrow C_1 = 3, \quad C_2 = \frac{1}{5}, \quad C_3 = -\frac{2}{5}$

$\therefore [x]_B = \left(3, -\frac{3}{5}, -\frac{4}{5}\right)$

$[x]_C$

$= C_1(1,0,0) + C_2(1,1,0) + C_3(1,1,1) = (3,1,-2)$

$$\begin{cases} C_1 + C_2 + C_3 = 3 \\ C_2 + C_3 = 1 \\ C_3 = -2 \end{cases}$$

$C_2 = 3$

$C_1 = 2$

$\Rightarrow C_1 = 2, \quad C_2 = 3, \quad C_3 = -2$

$\therefore [x]_C = (2, 3, -2)$