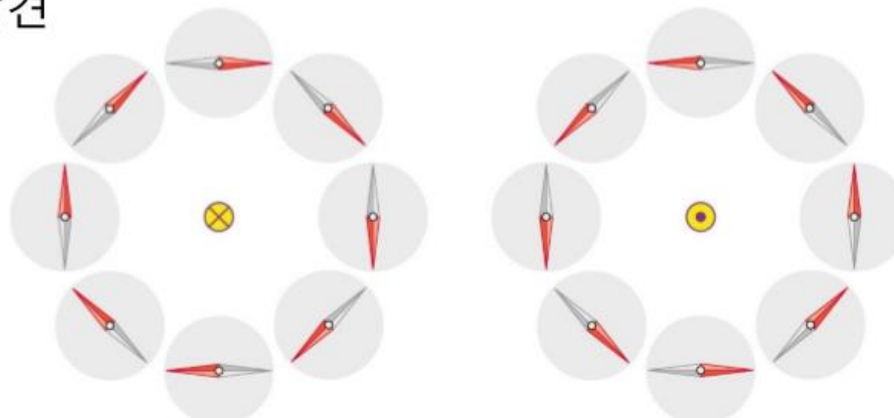
Chapter 29
Magnetic Fields due to Currents

Chap. 29-1 Magnetic Field due to a Current
Chap. 29-2 Force Between Two Parallel Currents
Chap. 29-3 Ampere's Law
Chap. 29-4 Solenoids and Toroids
Chap. 29-5 A Current-Carrying Coil as a Magnetic Dipole

Chap. 29-1 Magnetic Field due to a Current

- 전류가 만드는 자기장
  - **Hans Christian Ørsted**, 1820 – 전류가 흐르는 도선 주변의 나침반 바늘의 방향이 달라지는 것 발견



전류가 흐르는 도선 주변에 자기장선이 원형으로 형성된다.

  - 전기와 자기가 관계되어 있다는 것을 처음으로 발견

André Marie Ampère

Jean-Baptiste Biot

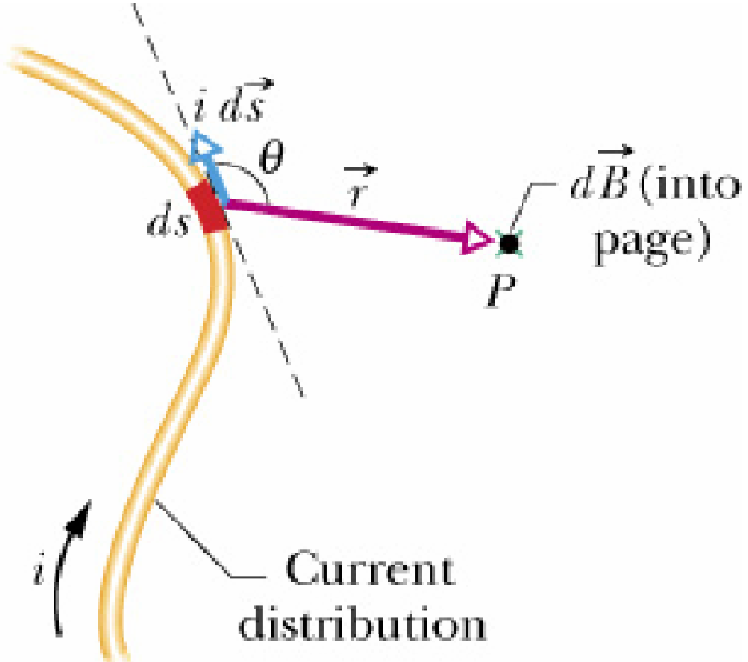
Félix Savart

Chap. 29-1 Magnetic Field due to a Current

비오-사바르 법칙 (Biot-Savart Law)

전류 *i* 가 흐르는 도선토막 *ds* 가 만드는 자기장 *dB* :

실험적 법칙

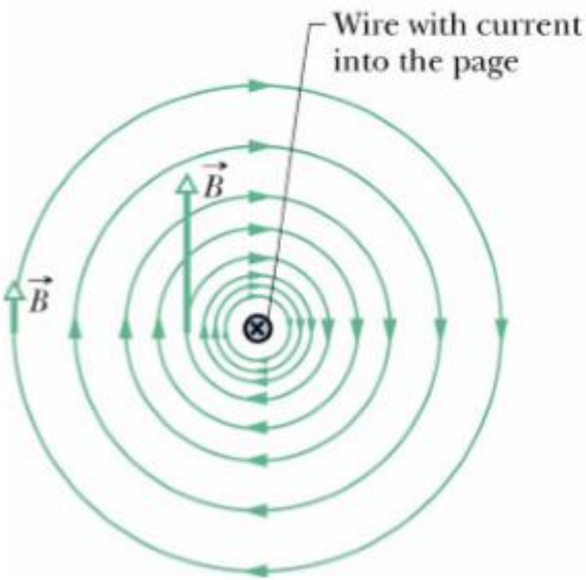


$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{s}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{i d\vec{s}\times\vec{r}}{r^3}$$

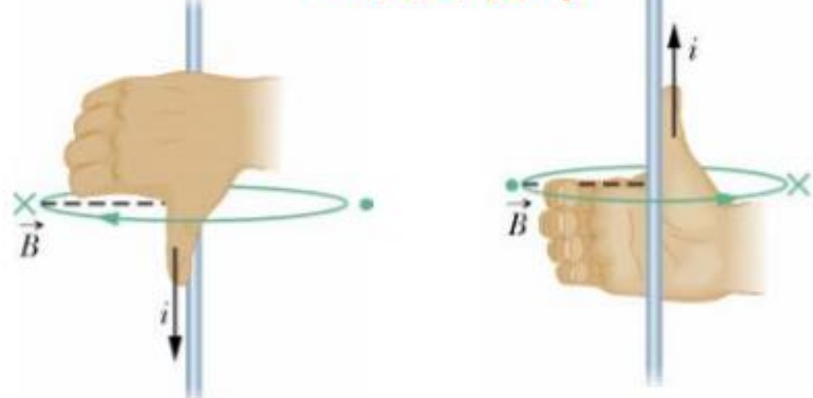
$$\mu_0 = 4\pi\times10^{-7}\,T\cdot m/A$$
$$\approx 1.26\times10^{-6}\,T\cdot m/A$$

: Permeability (투자율)

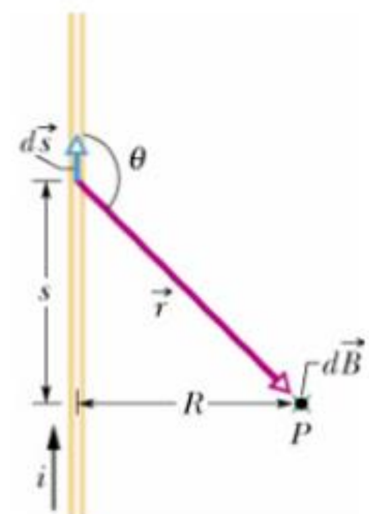
긴 직선도선 (Biot-Savart Law)



오른손 법칙



R 떨어진 위치에서의 B ?



$$B = \frac{\mu_0 i}{2\pi R}$$?

Chap. 29-1 Magnetic Field due to a Current

(증명) 무한히 긴 직선도선

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{s}\times\vec{r}}{r^3}$$ (Biot-Savart Law)

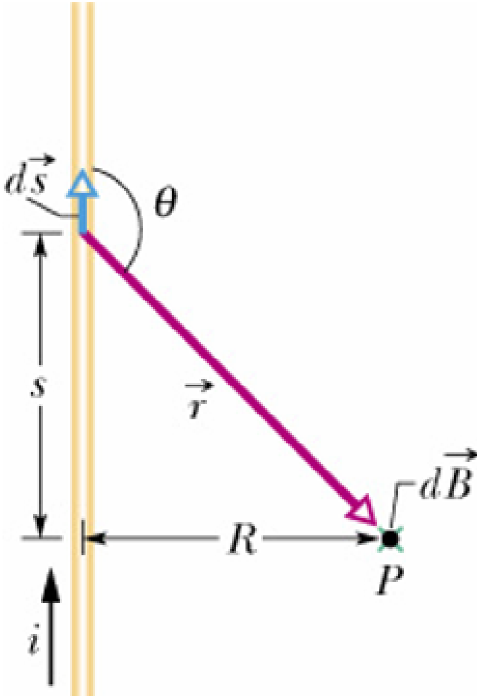
$$\left|d\vec{B}\right| = dB = \frac{\mu_0}{4\pi}\frac{ids\sin\theta}{r^2}$$

$$B = 2\int_0^\infty dB = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{\sin\theta}{r^2}ds$$

$$r = \sqrt{s^2 + R^2}$$

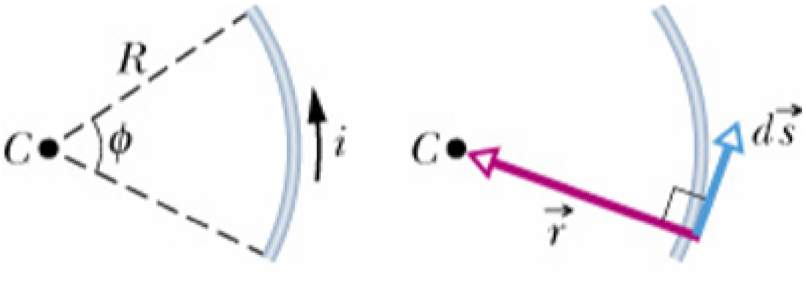
$$\sin\theta = \sin(\pi - \theta) = \frac{R}{r} = \frac{R}{\sqrt{s^2+R^2}}$$

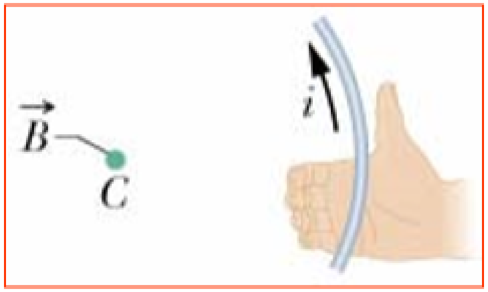
$$B = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{R}{\left(s^2+R^2\right)^{3/2}}ds = \frac{\mu_0 i}{2\pi}\left[\frac{s}{\left(s^2+R^2\right)^{1/2}}\right]_0^\infty \Rightarrow \boxed{B = \frac{\mu_0 i}{2\pi R}}$$



Chap. 29-1 Magnetic Field due to a Current

원형 모양의 도선





$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{s} \times \vec{r}}{r^3}$$ (Biot-Savart Law)

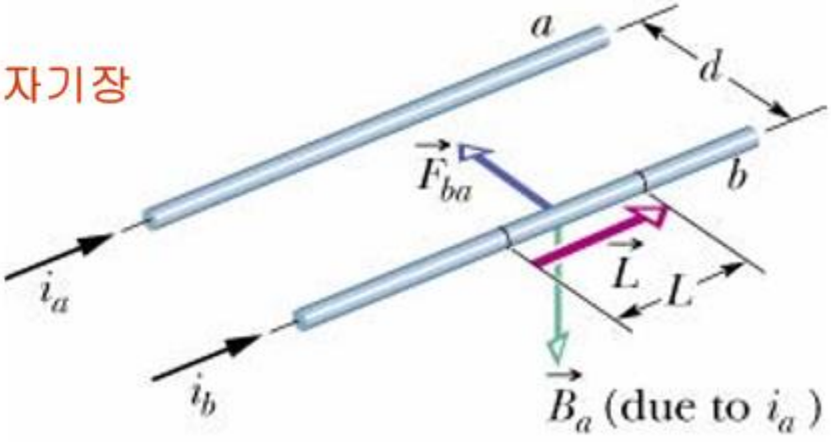
$$dB = \frac{\mu_0}{4\pi}\frac{ids\sin 90^\circ}{r^2} = \frac{\mu_0}{4\pi}\frac{ids}{r^2} \longleftarrow ds = Rd\phi$$

$$B = \frac{\mu_0}{4\pi}\int_0^{\phi}\frac{iR}{R^2}d\phi = \frac{\mu_0 i}{4\pi R}\phi$$: 원호 중심 (C) 에서의 **B** 크기

원형 고리도선인 경우 → $B = \frac{\mu_0 i}{4\pi R}(2\pi) = \frac{\mu_0 i}{2R}$

Chap. 29-2 Force Between Two Parallel Currents

$$B_a = \frac{\mu_0 i_a}{2\pi d}$$: 도선 a (i_a) 가 만드는 자기장



$$\vec{F}_{ba} = i_b \cdot \vec{L} \times \vec{B}_a$$

$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L}{2\pi d} i_a i_b$$

$$\vec{F}_{ba} = -\vec{F}_{ab}$$

(작용-반작용)

전류방향이 같으면 서로 당기고, 반대면 밀어낸다

- **암페어의 정의**

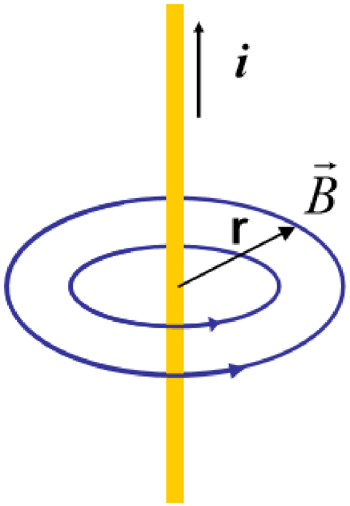
같은 크기의 전류가 흐르는 평행한 두 도선 사이에 작용하는 힘의 크기를 사용하여 정의한다.

1 A = 진공에서 1m 떨어진 평행한 두 도선에 1m당 $2 \cdot 10^{-7}$ N의 힘이 작용하는 전류의 크기

- 1 A의 정의는 μ_0의 값을 정해준다.

$$\mu_0 = \frac{(2\pi d) F_{1\to 2}}{i_1 i_2 L} = \frac{2\pi (1\,\mathrm{m})(2 \cdot 10^{-7}\,\mathrm{N})}{(1\,\mathrm{A})(1\,\mathrm{A})(1\,\mathrm{m})} = 4\pi \cdot 10^{-7}\,\frac{\mathrm{T\,m}}{\mathrm{A}}$$

Chap. 29-3 Ampere's Law



$$B(r) = \frac{\mu_0 i}{2\pi r}$$

$$2\pi r \cdot B(r) = \mu_0 i$$

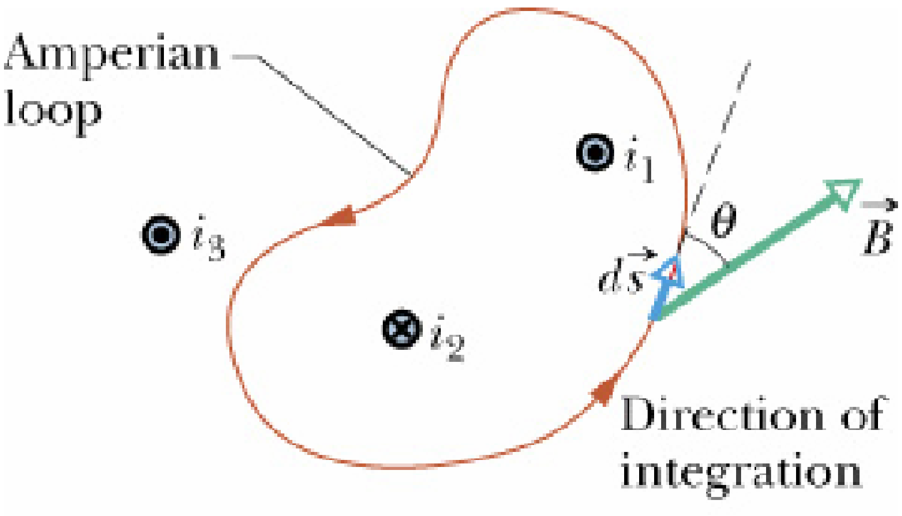
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \quad \longleftrightarrow \quad \oint \vec{E} \cdot d\vec{A} = \frac{q_{inc}}{\varepsilon_0}$$

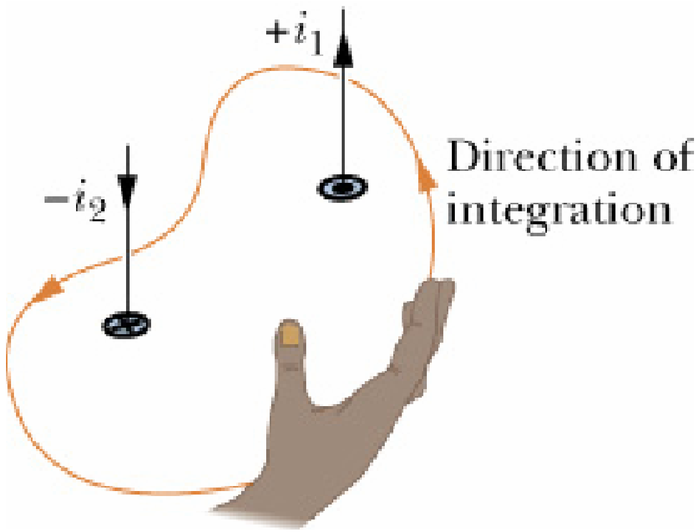
Ampere's Law **Gauss's Law**

폐곡선 (Ampere loop) 적분

Chap. 29-3 Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$





전류방향의 부호 정의

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta ds = \mu_0 i_{enc}$$

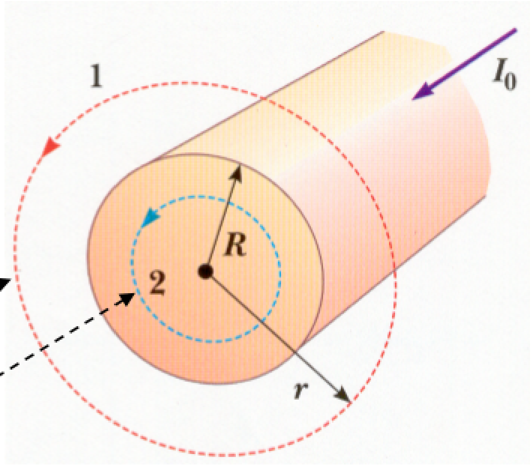
$$i_{enc} = i_1 - i_2$$

긴 직선 도선의 외부 및 내부 자기장

도선 단면(A)에 균일한 전류 밀도를 가정

$$j = \frac{i}{A} = \frac{i}{\pi R^2}$$ (전류 밀도 : current density)

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$ *(Ampere's Law)*

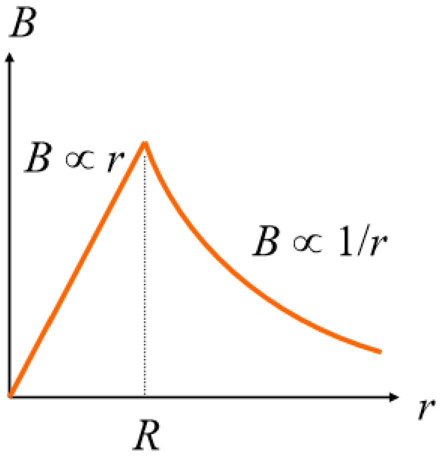


(1) r > R (도선 외부)

$$2\pi r \cdot B = \mu_0 i \quad \Rightarrow \quad B = \left(\frac{\mu_0 I}{2\pi}\right)\frac{1}{r}$$

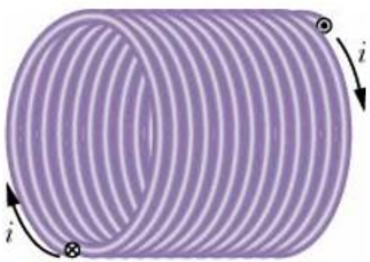
(2) r < R (도선 내부)

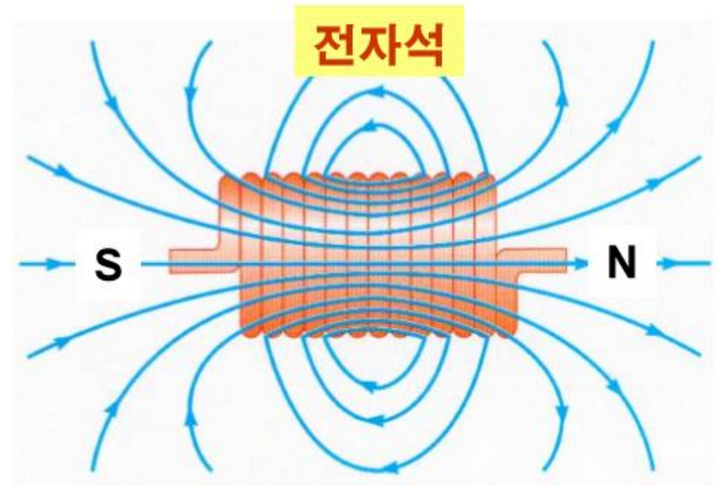
$$2\pi r \cdot B = \mu_0 \pi r^2 \cdot j = \mu_0 \frac{r^2}{R^2} i \quad \Rightarrow \quad B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r$$

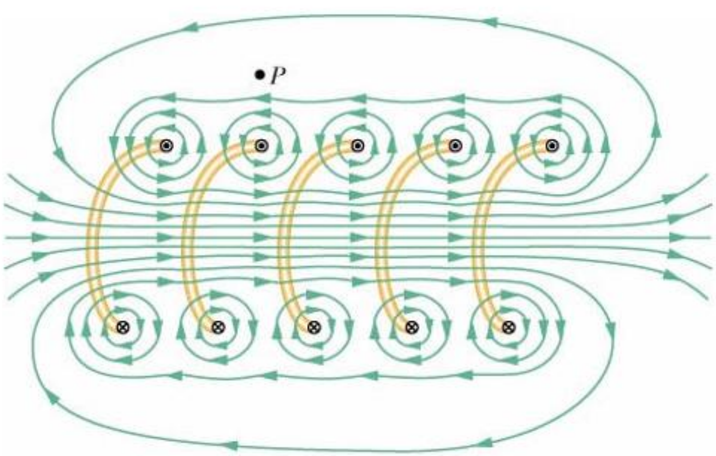


Chap. 29-4 Solenoids and Toroids

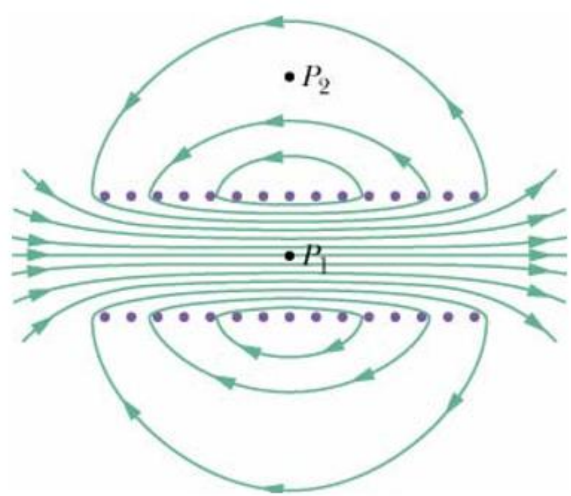
Solenoid (솔레노이드)







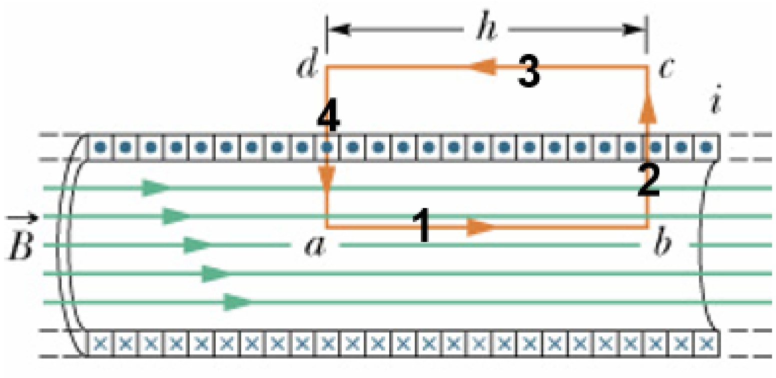
Solenoid coil이 느슨한 경우



Solenoid coil이 촘촘한 경우

Chap. 29-4 Solenoids and Toroids

매우 긴 이상적인 Solenoid



외부 자기장은 없음

내부 자기장은 균일

얼마일까?

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$$

$$\oint \vec{B}\cdot d\vec{s} = \vec{B}_1\cdot\vec{l}_1 + \vec{B}_2\cdot\vec{w}_2 + \vec{B}_3\cdot\vec{l}_3 + \vec{B}_4\cdot\vec{w}_4$$

$$B_3 = 0 \qquad B_2 = B_4 = 0 \quad (\text{or, } \mathrm{B} \perp w \text{ at 2 and 4})$$

$$\vec{B}_1\cdot\vec{l}_1 = B\cdot h = \mu_0 (nh) i$$

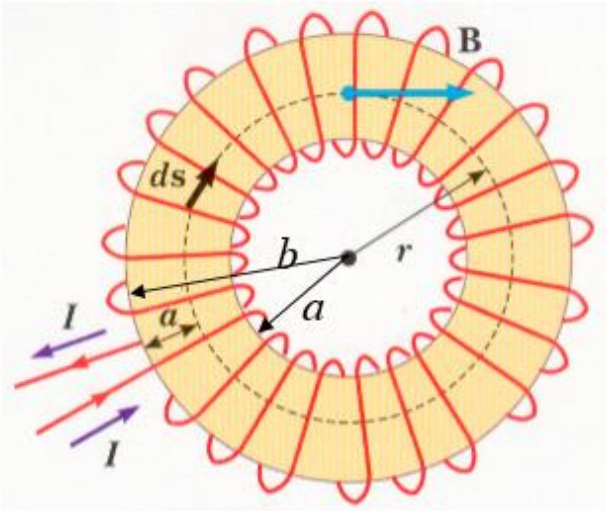
n **:** 단위 길이당 감은 횟수

$$\boxed{B = \mu_0 n i}$$

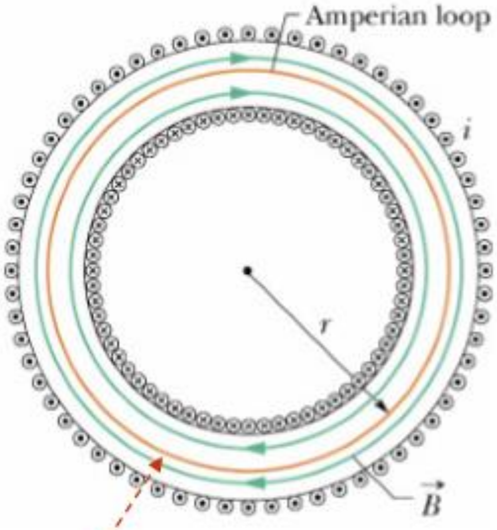
: Uniform field inside of Solenoid

Chap. 29-4 Solenoids and Toroids

Toroid (토로이드)



Total N turns



얼마일까?

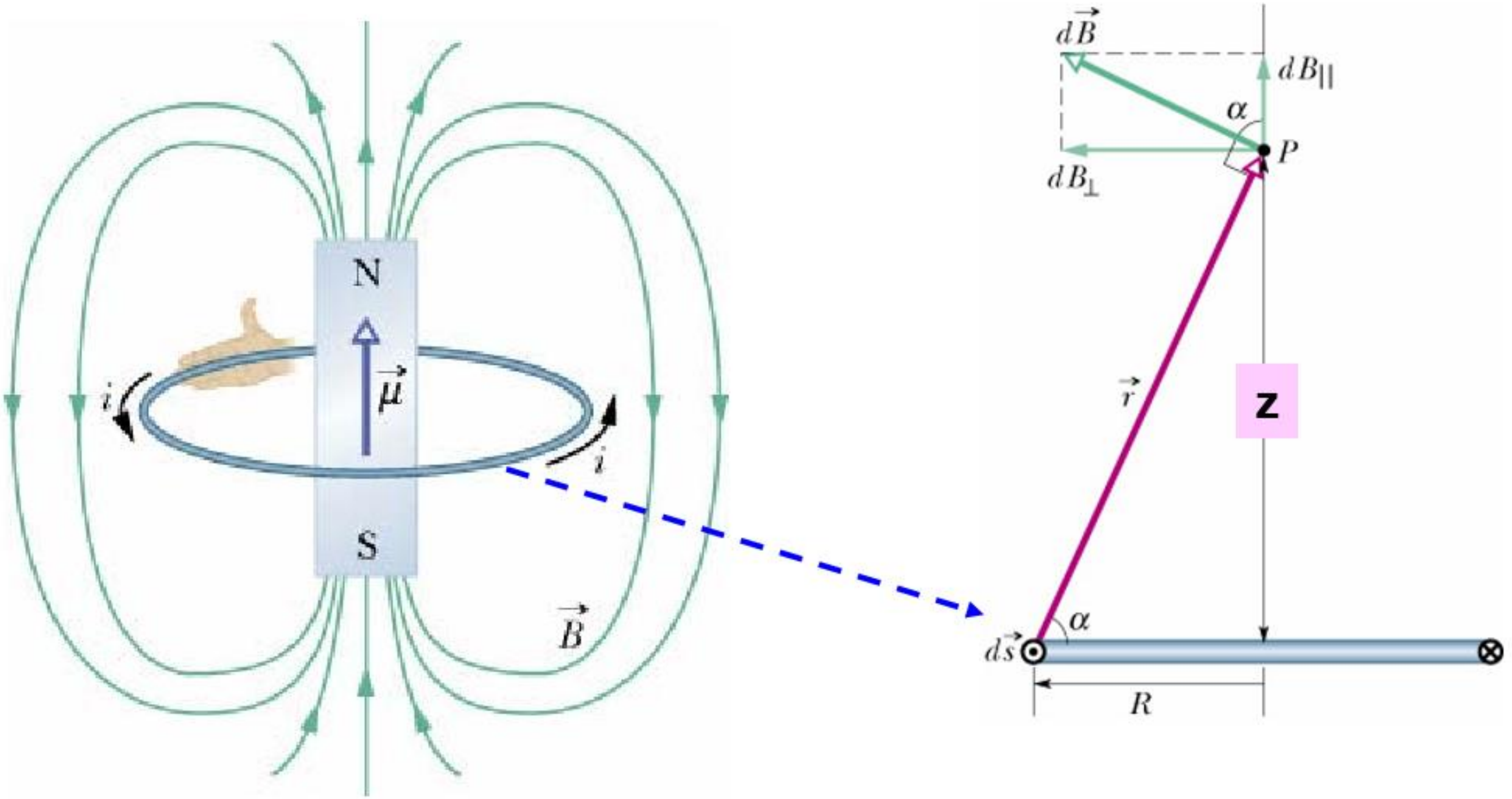
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

(i) $r < a$ ➔ $B = 0$

(ii) $r > b$ ➔ $B = 0$

(iii) $a < r < b$ ➔ $\oint \vec{B} \cdot d\vec{s} = 2\pi r \cdot B = \mu_0 N i$ ➔ $\boxed{B = \left(\frac{\mu_0 N i}{2\pi}\right)\frac{1}{r}}$

Chap. 29-5 A Current-Carrying Coil as a Magnetic Dipole



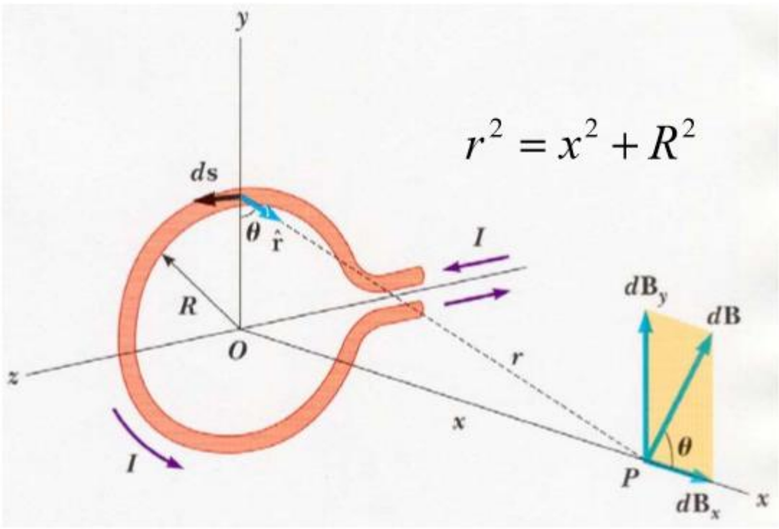
자기쌍극자 모멘트 **(magnetic dipole moment)**

$$\vec{\mu} = (Ni)\vec{A}$$ (N : 감은 횟수)

$$\vec{B}(z) = \left(\frac{\mu_0}{2\pi}\right)\frac{\vec{\mu}}{z^3}$$

Chap. 29-5 A Current-Carrying Coil as a Magnetic Dipole

전류고리가 만드는 자기장 증명



$$dB = \frac{\mu_0}{4\pi}\frac{I}{r^2}\cos\theta\,\hat{x} \qquad \cos\theta = \frac{R}{r}$$

$$dB = \frac{\mu_0 I}{4\pi}\frac{R}{(x^2+R^2)^{3/2}}Rd\theta$$

$$\vec{B} = \frac{\mu_0 I}{2}\frac{R^2}{(x^2+R^2)^{3/2}}\hat{x}$$

(i) $z = 0 \quad \Rightarrow \quad \vec{B} = \frac{\mu_0 I}{2R}\hat{x}$

(ii) $z >> R \quad \Rightarrow \quad \vec{B} \approx \frac{\mu_0 I}{2}\cdot\frac{R^2}{x^3}\hat{x}$

$$\mu \equiv IA = I\pi R^2$$

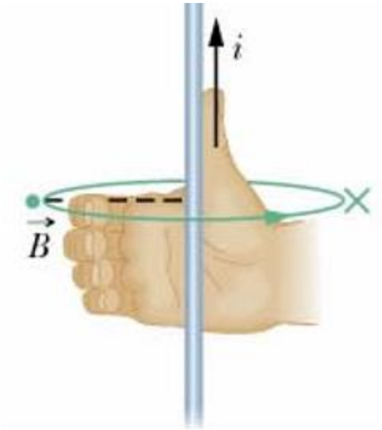
$$B \cong \frac{\mu_0}{2\pi}\frac{\mu}{x^3}$$

$$\vec{B} = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{x^3} \qquad (\vec{\mu} \equiv I\vec{A})$$

Summary

Biot-Savart Law

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{s} \times \vec{r}}{r^3}$$



Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

두 도선

$$F_{ba} = \frac{\mu_0 L}{2\pi d} i_a i_b$$

Solenoid

$$B = \mu_0 n i$$