국방 정보공학과 2학년 2020032706 송민정

공업수학 HW5

14-1

24. $\int_c \cos 2z\,dz$

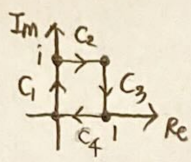
피적분 함수가 analytic 하므로

$\int_c \cos 2z\,dz = \int_{-\pi i}^{\pi i} \cos 2z\,dz$

$= \left[\frac{1}{2}\sin 2z\right]_{-\pi i}^{\pi i} = \frac{1}{2}\sin 2\pi j - \frac{1}{2}\sin(-2\pi j)$

$= \sin(2\pi j) = j\sinh 2\pi$

30. $\int_c \mathrm{Re}\, z^2 dz$



$\mathrm{Re}\, z^2 = x^2 - y^2$

$C_1 : z(t) = it \ (0 \le t \le 1)$

$C_2 : z(t) = t + i \ (0 \le t \le 1)$

$C_3 : z(t) = 1 - i - ti \ (0 \le t \le 1)$

$C_4 : z(t) = 1 - t \ (0 \le t \le 1)$

$\int_c \mathrm{Im}\, z^2 dz$

$= \int_{C_1} \mathrm{Im}\, z^2 dz + \int_{C_2} \mathrm{Im}\, z^2 dz + \int_{C_3} \mathrm{Im}\, z^2 dz$

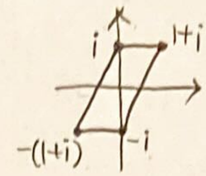
$+ \int_{C_4} \mathrm{Im}\, z^2 dz$

$= \int_0^1 -t^2 i\,dt + \int_0^1 (t^2-1)\,dt + \int_0^1 -[1-(1-t)^2]\,i\,dt$

$+ \int_0^1 -(1-t)^2 dt$

$= -1 - i$

14-2



20. $\int_c \mathrm{Ln}(1-z)\,dz$

피적분 함수의 singular point가 적분범위 (평행 사변형) 외부에 있으므로, 적분값은 0이다.

23. $\int_c \frac{2z-1}{z^2-z} dz$

$= \int_c \left(\frac{1}{z} + \frac{1}{z-1}\right) dz$

$= j2\pi + j2\pi = j4\pi$

24. 왼쪽, 오른쪽 방향이 다르므로, 각각 계산해 준다.

왼쪽 : $|z+1| = 1$

$\frac{1}{2}\oint_0^{2\pi} e^{-it} i e^{it} dt = \left[\frac{1}{2} it\right]_0^{2\pi} = \pi i$

오른쪽 : $|z-1| = 1$

$\frac{1}{2}\oint_0^{2\pi} e^{-it} i e^{it} dt = \left[\frac{1}{2} it\right]_0^{2\pi} = \pi i$

$\therefore \oint_c \frac{1}{z^2-1} dz = 2\pi i$