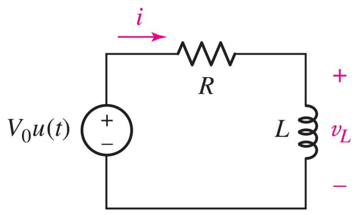
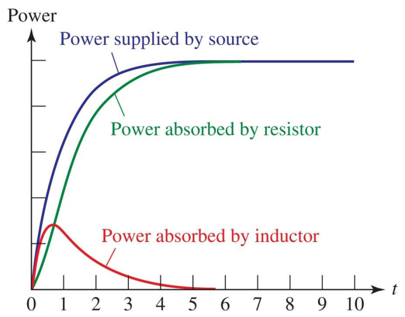
Chapter 11. AC Circuit: Power Analysis

1. Instantaneous Power
2. Average power and Effective Value
3. Apparent power, Power factor, and Complex power
4. Power factor correction

Instantaneous Power 순시 전력



- $p(t) = v(t)i(t)$
  - $p(t) = i^2(t)R = \frac{1}{R}v^2(t)$, in a resistor
  - $p(t) = Li(t)\frac{di(t)}{dt} = \frac{1}{L}v(t)\int^t v(\tau)d\tau$, in a conductor or
  - $p(t) = Cv(t)\frac{dv(t)}{dt} = \frac{1}{C}i(t)\int^t i(\tau)d\tau$, in a capacitor
- Given $i(t) = \frac{V_0}{R}\left(1 - e^{-\frac{Rt}{L}}\right)u(t)$
  - Power supplied: $p(t) = v_s(t)i(t) = \frac{V_0^2}{R}\left(1 - e^{-\frac{Rt}{L}}\right)u(t) = p_R(t) + p_L(t)$, where
    - $p_R(t) = i^2(t)R = \frac{V_0^2}{R}\left(1 - e^{-\frac{Rt}{L}}\right)^2 u(t)$
    - $p_L(t) = v_L(t)i(t) = V_0e^{-\frac{Rt}{L}} \cdot \frac{V_0}{R}\left(1 - e^{-\frac{Rt}{L}}\right)u(t)$

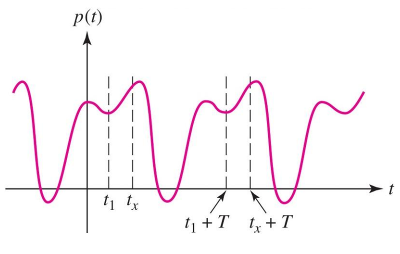


Power from Sinusoidal Source

- RL series circuit with $v_s(t) = V_m \cos \omega t$:
  - ∨ $i(t) = I_m \cos(\omega t + \phi)$

    where $I_m = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}$ and $\phi = -\tan^{-1}\frac{\omega L}{R}$
  - ∨ Instantaneous power,
    - † $p(t) = v_s(t)i(t) = V_m I_m \cos \omega t \cdot \cos(\omega t + \phi) = \frac{1}{2} V_m I_m (\cos\phi + \cos(2\omega t + \phi))$
    - † Constant term + double frequency term

Average Power 평균 전력

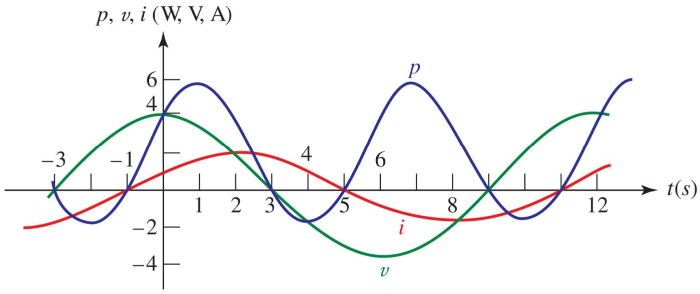


- Average power over an interval, $[t_1, t_2]$
  - ∨ $P_{avg} = \frac{1}{t_1 - t_2}\int_{t_1}^{t_2} p(t)dt$
- Periodic $p(t)$ with period T
  - ∨ $P_{avg} = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)dt$
- When $v(t) = V_m \cdot \cos(\omega t + \theta)$ and $i(t) = I_m \cdot \cos(\omega t + \phi)$ with $\omega = \pi/6$
  - ∨ $p(t) = v(t)i(t) = \frac{1}{2} V_m I_m (\cos(\theta - \phi) + \cos(2\omega t + \theta + \phi))$.
  - ∨ $P_{avg} = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{2} V_m I_m \cos(\theta - \phi)$
  - ∨ $P_{avg} = \frac{1}{2} Re\{\mathbf{VI}^*\}$

Average Power 평균 전력

- Example 11.2 Voltage $v(t) = 4\cos \pi t/6$ across an impedance $\mathbf{Z} = 2\angle 60°$
  - $\mathbf{V} = 4\angle 0°$
  - $\mathbf{I} = 2\angle -60°$
  - $P_{avg} = \frac{1}{2} 4 \cdot 2 \cdot \cos 60° = 2\ [W]$
  - $i(t) = 2\cos(\frac{\pi t}{6} - 60°)$
  - $p(t) = 8\cos\frac{\pi t}{6} \cdot 2\cos\left(\frac{\pi t}{6} - 60°\right)$

    $= 2 + 4\cos(\frac{\pi t}{3} - 60°)$



Average Power in Passive Elements

- Resistor, $\theta = \phi$ (in-phase)
  - $P_{avg}(t) = \frac{1}{2}\frac{V_m}{R}$
- Inductor and capacitor, $\theta - \phi = \pm 90°$
  - $P_{avg}(t) = 0$
    - † Average power absorbed by a purely reactive element(s) is zero, due to 90° phase difference between voltage and current.

Average Power: Example

- Example 11.3 Impedance $\mathbf{Z}_L = 8 - j11\ [\Omega]$ with current $\mathbf{I} = 5\angle 20°\ [A]$
  - $\mathbf{V} = \mathbf{Z}_L\mathbf{I} = (13.6015\angle -53.9726°) \cdot 5\angle 20° = 68.0074\angle -33.9726°\ [V]$
    - $P_{avg} = \frac{1}{2} 68.0074 \cdot 5 \cdot \cos(-33.9726° - 20°) = 100\ [W]$
  - $P_{avg} = \frac{1}{2} I_m^2 R = \frac{1}{2} \cdot 5^2 \cdot 8 = 100\ [W]$
    - The reactance part of impedance does not observe average power.
  - $P_{avg} = \frac{1}{2} Re\{\mathbf{V}\mathbf{I}^*\} = \frac{1}{2} Re\{(68.0074\angle -33.9726°) \cdot 5\angle 20°\}$

Average Power: Example

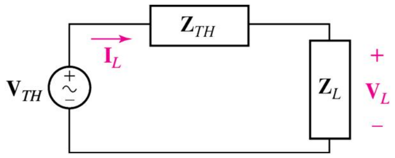
- Example 11.4
  - Note that average power absorbed by two reactive element is zero.
  - $j2\mathbf{I}_1 + 2(\mathbf{I}_1 - \mathbf{I}_2) = 20, \ -j2\mathbf{I}_2 + 10 = 2(\mathbf{I}_1 - \mathbf{I}_2)$
    - $\mathbf{I}_1 = 5 - j10 = 11.18\angle -63.43°$ and $\mathbf{I}_2 = 5 - j5 = 5\sqrt{2}\angle -45°$
    - $\mathbf{I}_1 - \mathbf{I}_2 = -j5 = 5\angle -90°$
  - Avg. power in resistor: $P_R = \frac{1}{2} I_m^2 R = \frac{1}{2} \cdot 5^2 \cdot 2 = 25\ [W]$
  - In the left voltage source,
    - $\mathbf{V}_L = 20\angle 0°$ and $\mathbf{I}_1 = 11.18\angle -63.43° \Rightarrow P_L = \frac{1}{2} 20 \cdot 11.18 \cdot \cos 63.43° = 50\ [W]$
  - In the right voltage source,
    - $\mathbf{V}_R = 10\angle 0°$ and $\mathbf{I}_2 = 5\sqrt{2}\angle -45° \Rightarrow P_R = \frac{1}{2} 10 \cdot 5\sqrt{2} \cdot \cos 45° = 25\ [W]$

Maximum Power Transfer

- A Thevenin equivalent circuit connected to load impedance:
  - $\mathbf{I}_L = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th} + \mathbf{Z}_L} = \frac{\mathbf{V}_{th}}{R_{th} + R_L + j(X_{th} + X_L)}$

    where $\mathbf{V}_{th} = R_{th} + jX_{th}$, $\mathbf{Z}_L = R_L + jX_L$

    $|\mathbf{I}_L| = \frac{|\mathbf{V}_{th}|}{\sqrt{(R_{th}+R_L)^2 + (X_{th}+X_L)^2}}$ and $\angle \mathbf{I}_L = \angle \mathbf{V}_{th} - \tan^{-1}\left(\frac{X_{th}+X_L}{R_{th}+R_L}\right)$



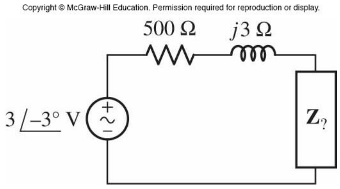
  - $\mathbf{V}_L = \frac{\mathbf{Z}_L}{\mathbf{Z}_{th} + \mathbf{Z}_L}\mathbf{V}_{th} = \frac{R_L + jX_L}{R_{th} + R_L + j(X_{th} + X_L)}\,\mathbf{V}_{th}$

    where $|\mathbf{V}_L| = \frac{|\mathbf{V}_{th}|\sqrt{R_L^2 + X_L^2}}{\sqrt{(R_{th}+R_L)^2 + (X_{th}+X_L)^2}}$ and $\angle \mathbf{V}_L = \angle \mathbf{V}_{th} + \tan^{-1}\frac{X_L}{R_L} - \tan^{-1}\left(\frac{X_{th}+X_L}{R_{th}+R_L}\right)$
  - $P_L = \frac{1}{2}|\mathbf{V}_{th}|^2 \frac{\sqrt{R_L^2 + X_L^2}}{(R_{th}+R_L)^2 + (X_{th}+X_L)^2}\cos\left(\tan^{-1}\frac{X_L}{R_L}\right) = \frac{1}{2}|\mathbf{V}_{th}|^2 \frac{R_L}{(R_{th}+R_L)^2 + (X_{th}+X_L)^2}$

    $\frac{\partial P}{\partial R_L} = 0$ and $\frac{\partial P}{\partial X_L} = 0 \Rightarrow R_L = R_{th}$ and $X_L = -X_{th}$ or $\mathbf{Z}_L = \mathbf{Z}_{th}^*$

Maximum Power Transfer

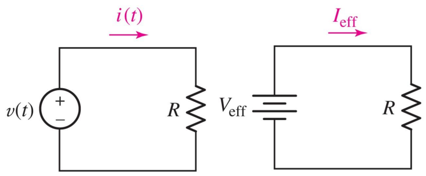
- Example 11.7 $v_s(t) = 3 \cdot \cos(100t - 3°)$
  - Series connection to an impedance, $\mathbf{Z}_L$
  - For maximum power transfer to load, $\mathbf{Z}_L = \mathbf{Z}_{th}^* = 500 - j3$



Effective Value 실효값

- Consider the voltage supplied to a household.
  - v $220[V]/60[Hz]$, compared 120/60 in America, 220/50 in Europe, 100/60 or 100/50 in Japan.
  - v $v(t) = 220\sqrt{2} \cdot \cos(2\pi 60t)$ with $V_m = 220\sqrt{2}\,[V]$
- Effective value of a periodic signal
  - v When $i(t)$ is periodic, its effective value is equal to the value of the direct current (dc) which delivers the same average power to the resistor as does the periodic current to a R $[\Omega]$ resistor.
  - v In DC, $P = I_{eff}^2 R$
  - v In periodic current,

    $P_{avg} = \frac{1}{T}\int_0^T i^2(t)Rdt = \frac{R}{T}\int_0^T i^2(t)dt$



Effective Value

- $I_{eff}^2 R = \frac{1}{2}\frac{R}{T}\int_0^T i^2(t)dt$
  - v $I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$ … rms(root mean squared) value
- Sinusoidal current
  - v $i(t) = I_m \cos(\omega t + \phi)$
  - v $I_{eff} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2(\omega t + \phi)\,dt} = \frac{1}{\sqrt{2}} I_m$
  - v $P_{avg} = \frac{1}{2} I_m^2 R = I_{eff}^2 R = \frac{V_{eff}^2}{R}$
  - v $P_{avg} = V_{eff} I_{eff} \cos(\theta - \phi)$

In practice, the effective value is usually used in the fields of power transmission or distribution and of rotating machinery; in the areas of electronics and communications, the amplitude is more often used.

Apparent Power 피상전력 & Power Factor 역률

- When $v(t) = V_m \cos(\omega t - \theta)$ and $i(t) = I_m \cos(\omega t - \phi)$ in a circuit element,
  - Voltage leads current by $\theta - \phi$.
  - Average power:
    - $P_{avg} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = V_{eff} I_{eff} \cos(\theta - \phi)$
  - Apparent power: $V_{eff} I_{eff}$ $[VA]$ (ignoring phase difference)
  - Power factor: ratio of average power to apparent power
    - $PF = \frac{P_{avg}}{V_{ff} I_{eff}} = \cos(\theta - \phi)$, where $\theta - \phi$ … PF angle
    - $0 \leq PF \leq 1$ ($PF = 1$ for resistive load, $PF = 0$ for purely reactive load)

Power Factor

- Resistor
  - Phase difference, $\theta - \phi = 0° \Rightarrow PF = 1$ (Average power = apparent power)
  - Even in a circuit with inductor and capacitor, it is possible to have equivalent unity power factor.
- Pure reactive component (capacitor or inductor)
  - Phase difference, $\theta - \phi = \pm 90°$ (+ for inductor, − for capacitor) $\Rightarrow PF = 0$
- PF is insensitive to sign of PF angle, $\theta - \phi$.
  - In inductive (유도성) load, current lags voltage ($\theta - \phi > 0$): Lagging PF 지상 역률
  - In capacitive (용량성) load, voltage leads current ($\theta - \phi < 0$): Leading PF 진상 역률

Complex Power 복소 전력

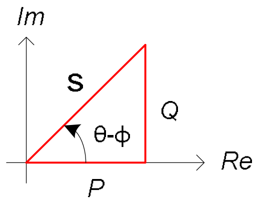
- Complex power, $\mathbf{S}$
  - $\mathbf{S} = \mathbf{V}_{eff}\mathbf{I}_{eff}^{*} = V_{eff}I_{eff}e^{j(\theta-\phi)} = P_{avg} + jQ$
    - $Re\{\mathbf{S}\} = P_{avg}\ [W]$ and $Im\{\mathbf{S}\} = Q\ [VAR]$ (reactive power, 무효 전력)
    - Reactive power represents the flow of energy back and forth to the load.
    - $P_{avg} = V_{eff}I_{eff}\cos(\theta-\phi)$
    - $Q = V_{eff}I_{eff}\sin(\theta-\phi)$
    - $|\mathbf{S}| = V_{eff}I_{eff}$

Various Powers

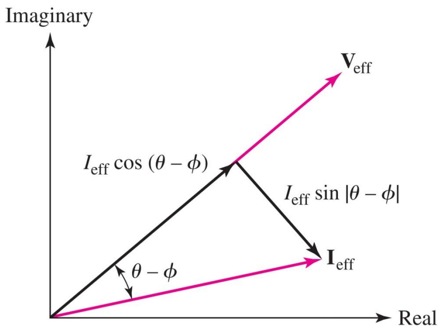
Quantity	Symbol	Formula	Units
Average power	P_{avg}	$V_{eff}I_{eff}\cos(\theta-\phi)$	Watt [W]
Reactive power	Q	$V_{eff}I_{eff}\sin(\theta-\phi)$	VAR, volt-ampere-reactive
Complex power	$\mathbf{S}$	$P_{avg} + jQ$ $V_{eff}I_{eff}\angle(\theta-\phi)$ $\mathbf{V}_{eff}\mathbf{I}_{eff}^{*}$	VA, volt-ampere
Apparent power	$\|\mathbf{S}\|$	$V_{eff}I_{eff}$	VA

Power Triangle

- Complex power, $\mathbf{S} = P_{avg} + jQ$



- Phasor diagram of $\mathbf{V}_{eff}$ and $\mathbf{I}_{eff}$

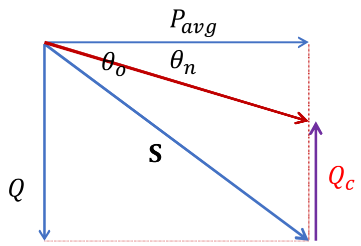


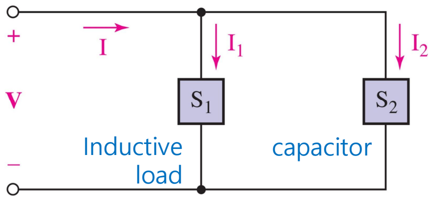
Phasor Factor Correction

- Consumer wants an electric power supply with constant average power and constant voltage.
  - $P_{avg} = V_{eff} I_{eff} \cos(\theta - \phi)$
  - Lower PF ⇒ larger current is required
    - † Large current requires large current-carrying capacity.
    - † Large current often causes increased loss in the electrical power transmission and distribution system.
    - † Thus, large current asks increased cost to power supplying company.
  - Almost all electric facilities are inductive and thus have lagging PF.
  - In order to improve the PF, we connect a capacitor in parallel to the load (역률 개선용 진상 condenser)

Phasor Factor Correction

1. Calculated the required reactive power
   - $\tan\theta_o = \frac{Q}{P_{avg}}$ and $\tan\theta_n = \frac{Q-Q_c}{P_{avg}}$
   - $Q_c = P_{avg}(\tan\theta_0 - \tan\theta_n)$
2. Connect a capacitor in parallel
   - $\mathbf{S} = \mathbf{V}\mathbf{I}^* = \mathbf{V}(\mathbf{I}_1 + \mathbf{I}_2)^* = \mathbf{S}_1 + \mathbf{S}_2$ (complex power after correction)
   - Choose $\mathbf{S}_2 = \mathbf{S} - \mathbf{S}_1 = -jQ_c$ (pure capacitive)
3. Find the capacitance
   - $\mathbf{S}_2 = \mathbf{V}\mathbf{I}_2^* = \mathbf{V}\cdot\left(\frac{\mathbf{V}}{\mathbf{Z}_2}\right)^* = \frac{V_{eff}^2}{\mathbf{Z}_2^*}$ (assume $\mathbf{V} = V_{eff}\angle 0°$)
   - $\mathbf{Z}_2 = \frac{V_{eff}^2}{\mathbf{S}_2^*}$ or $\frac{1}{j\omega C} = \frac{V_{eff}^2}{jQ_c}$ $\quad C = \frac{P_{avg}(\tan\theta_0 - \tan\theta_n)}{\omega V_{eff}^2}$





Phasor Factor Correction

- Example 11.10 Industrial customer is operating a 50 $[kW]$ induction motor at a lagging PF of 0.8 with $V_{eff} = 230\ [V\ rms]$.
   - Old PF, 0.8 $\Rightarrow \theta_1 = \cos^{-1}0.8 = 36.9°$ and new PF, 0.95 lagging $\Rightarrow \theta_n = \cos^{-1}0.95 = 18.195°$
   - Complex power to motor, $\mathbf{S}_1 = P_{avg} + jQ_1 = 50 + j\frac{P_{avg}}{0.8}\sin\theta_1 = 50 + j37.5$
   - Complex power to corrected load,
     - $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = P_{avg} + jQ_n = 50 + j\frac{P_{avg}}{0.95}\sin\theta_n = 50 + j16.43$
   - Complex power to correct, $\mathbf{S}_2 = \mathbf{S} - \mathbf{S}_1 = -j21.07\ [kVA]$
   - Capacitance, $\mathbf{Z}_2 = \frac{V_{eff}^2}{\mathbf{S}_2^*} = \frac{230^2}{j21.07} = -j2.51$