2장 7절 연습문제 풀이

2006년 3월 27일

19.

$$y = \sinh^{-1} 3x \Longrightarrow y' = \frac{3}{\sqrt{1+9x^2}}.$$

20.

$$y = (\cosh^{-1} \frac{1}{2}x)^2 \Longrightarrow y' = 2(\cosh^{-1} \frac{1}{2}x)\frac{\frac{1}{2}}{\sqrt{\frac{1}{4}x^2 - 1}}.$$

21.

$$y = \cosh^{-1}(\sec x) \Longrightarrow y' = \frac{\sec x \tan x}{\sqrt{\sec^2 x - 1}} = \frac{\sec x \tan x}{\sqrt{\tan^2 x}} = \sec x$$

22.

$$\begin{aligned} y &= (\tanh^{-1}(1-x^2))^3 \\ \Longrightarrow y' &= 3(\tanh^{-1}(1-x^2))^2 \cdot (\tanh^{-1}(1-x^2))' \\ &= 3(\tanh^{-1}(1-x^2))^2 \left(\frac{-2x}{1-(1-x^2)^2}\right) \\ &= \frac{-6x}{x^2(2-x^2)}(\tanh^{-1}(1-x^2))^2 \\ &= \frac{6}{x(x^2-2)}(\tanh^{-1}(1-x^2))^2. \end{aligned}$$

23.

$$y = \cosh^{-1}(\tan x) \Longrightarrow y' = \frac{\sec^2 x}{\sqrt{\tan^2 x - 1}}$$

24.

$$y = \cosh(\sinh^{-1} x) \Longrightarrow y' = \sinh(\sinh^{-1} x) \cdot \frac{1}{\sqrt{1+x^2}} = \frac{x}{\sqrt{1+x^2}}.$$

25.

$$y = \tanh^{-1} e^x \Longrightarrow y' = \frac{e^x}{1-e^{2x}}.$$

26.

$$y = e^{\coth^{-1} x} \Longrightarrow y' = e^{\coth^{-1} x} \cdot \frac{1}{1-x^2} = \frac{e^{\coth^{-1} x}}{1-x^2}.$$

27.

$$\begin{aligned} y = \text{sech}^{-1}(\sin^{-1} x) & \\ \Longrightarrow y' &= \frac{(\sin^{-1} x)'}{\sin^{-1} x \sqrt{(\sin^{-1} x)^2 - 1}} \\ &= \frac{1}{\sin^{-1} x \sqrt{(\sin^{-1} x)^2 - 1}} \frac{1}{\sqrt{1-x^2}}. \end{aligned}$$

28.

$$y = \sinh^{-1}(\sec x) \Longrightarrow y' = \frac{\sec x \tan x}{\sqrt{\sec^2 x - 1}}.$$

29.

$$y = \tan(\tanh^{-1} x) \Longrightarrow y' = \sec^2(\tanh^{-1} x) \frac{1}{1-x^2} = \frac{\sec^2(\tanh^{-1} x)}{1-x^2}.$$

30.

$$y = \tanh^{-1}(\tan^{-1} x) \Longrightarrow y' = \frac{(\tan^{-1} x)'}{1-(\tan^{-1} x)^2} = \frac{1}{1-(\tan^{-1} x)^2} \cdot \frac{1}{1+x^2}.$$

31.

$$y = \sinh^{-1}(\tan x) \Longrightarrow y' = \frac{\sec^2 x}{\sqrt{1+\tan^2 x}} = \frac{\sec^2 x}{\sqrt{\sec^2 x}} = |\sec x|.$$

32.

$$y = \sinh^{-1}(\tan^{-1} e^x) \Longrightarrow y' = \frac{\frac{e^x}{1+e^{2x}}}{\sqrt{1+(\tan^{-1} e^x)^2}}.$$

35. $\sinh^{-1} x = \cosh^{-1} \sqrt{1+x^2}$ 의 증명.

증명. $\sinh^{-1} x = X$ 라하자. 그러면

$$\begin{aligned}
\sinh^{-1} x = X &\Longrightarrow \sinh X = x \\
&\Longrightarrow \cosh^2 X = 1 + \sinh^2 X \\
&\Longrightarrow \cosh X = \sqrt{1+\sinh^2 X} = \sqrt{1+x^2} \\
&\Longrightarrow X = \cosh^{-1} \sqrt{1+x^2}
\end{aligned}$$

그러므로, $\sinh^{-1} x = \cosh^{-1} \sqrt{1+x^2}$ ♣

37. $\tanh^{-1} x = \sinh^{-1} \frac{x}{\sqrt{1-x^2}}$ 의 증명.

증명. $\tanh^{-1} x = X$ 라하자. 그러면

$$\begin{aligned}
\tanh^{-1} x = X &\Longrightarrow \tanh X = x \\
&\Longrightarrow \operatorname{sech}^2 X = 1 - \tanh^2 X = 1 - x^2 \\
&\Longrightarrow \cosh^2 X = \frac{1}{1-x^2} \\
&\Longrightarrow \sinh^2 X = \cosh^2 X - 1 = \frac{1}{1-x^2} - 1 = \frac{x^2}{1-x^2} \\
&\Longrightarrow \sinh X = \sqrt{\frac{x^2}{1-x^2}} \\
&\Longrightarrow X = \sinh^{-1} \frac{x}{\sqrt{1-x^2}}.
\end{aligned}$$

그러므로, $\tanh^{-1} x = \sinh^{-1} \frac{x}{\sqrt{1-x^2}}$ ♣