공업수학 I 과제 (4장)

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[4.n.10]

$$\begin{cases} y_1' = -4y_1 + 5y_2 \\ y_2' = -y_1 + 2y_2 \end{cases} \Rightarrow y' = Ay = \begin{pmatrix} -4 & 5 \\ -1 & 2 \end{pmatrix} y$$

$\det(A-\lambda I)=0$ 에서, $\det(A-\lambda I) = \begin{vmatrix} -4-\lambda & 5 \\ -1 & 2-\lambda \end{vmatrix} = \lambda^2+2\lambda-n=0$

$= (\lambda-1)(\lambda+n)=0$

$\therefore \lambda_1=-n, \quad \lambda_2=1$

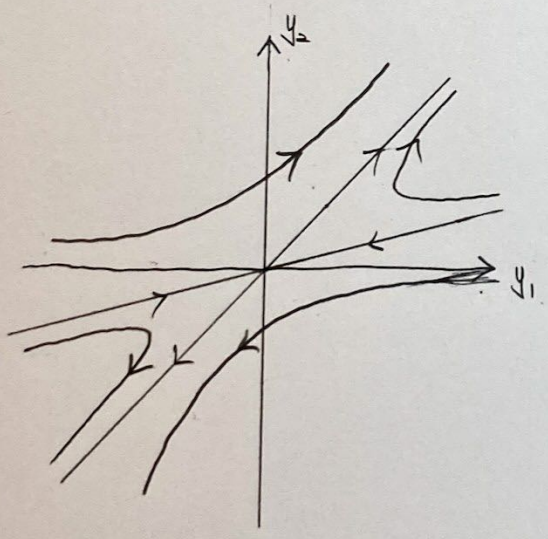
① $\lambda_1=-3$일 때, $-x_1+5x_2=0$, $x_{(1)}=\begin{bmatrix} 5 \\ 1 \end{bmatrix}$

② $\lambda_2=1$일 때, $-x_1+x_2=0$, $x_{(2)}=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\Rightarrow y = C_1\begin{bmatrix} 5 \\ 1 \end{bmatrix}e^{-3t} + C_2\begin{bmatrix} 1 \\ 1 \end{bmatrix}e^{t}$ (일반해)

초기 조건 $y_1(0)=0$, $y_2(0)=4$에 의해, $\begin{cases} 5C_1+C_2=0 \\ C_1+C_2=4 \end{cases}$ $C_1=-1, C_2=5$

$\therefore y = \begin{bmatrix} -5 \\ -1 \end{bmatrix}e^{-3t} + \begin{bmatrix} 5 \\ 5 \end{bmatrix}e^{t}$ (특수해)



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[4.3.12]

$$\begin{cases} y_1' = y_1 + 3y_2 \\ y_2' = \frac{1}{3}y_1 + y_2 \end{cases} \Rightarrow y' = Ay = \begin{pmatrix} 1 & 3 \\ \frac{1}{3} & 1 \end{pmatrix} y$$

$\det(A-\lambda I)=0$ 에서, $\det(A-\lambda I) = \begin{vmatrix} 1-\lambda & 3 \\ \frac{1}{3} & 1-\lambda \end{vmatrix} = \lambda^2 - 2\lambda + 1 - 1$

$= \lambda(\lambda-2) = 0$

$\therefore \lambda_1 = 0, \quad \lambda_2 = 2.$

① $\lambda_1 = 0$ 일 때, $x_1 + 3x_2 = 0$, $x_{(1)} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

② $\lambda_2 = 2$ 일 때, $-x_1 + 3x_2 = 0$, $x_{(2)} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$\Rightarrow y = C_1 \begin{bmatrix} -3 \\ 1 \end{bmatrix} + C_2 \begin{bmatrix} 3 \\ 1 \end{bmatrix} e^{2t}$ (일반해)

초기조건 $y_1(0) = 12$, $y_2(0) = 2$에 의해, $\begin{cases} -3C_1 + 3C_2 = 12 \\ C_1 + C_2 = 2 \end{cases}$ $C_1 = -1, \quad C_2 = 3$

$\therefore y = \begin{bmatrix} 3 \\ -1 \end{bmatrix} + \begin{bmatrix} 9 \\ 3 \end{bmatrix} e^{2t}$ (특수해)

[4.4.4]

$$\begin{cases} y_1' = 2y_1 + y_2 \\ y_2' = 5y_1 - 2y_2 \end{cases} \Rightarrow y' = Ay = \begin{pmatrix} 2 & 1 \\ 5 & -2 \end{pmatrix} y$$

$$\det(A - \lambda I) = \begin{vmatrix} 2-\lambda & 1 \\ 5 & -2-\lambda \end{vmatrix} = \lambda^2 - 4 - 5 = \lambda^2 - 9 = 0$$

$$\therefore \lambda_1 = -3,\ \lambda_2 = 3$$

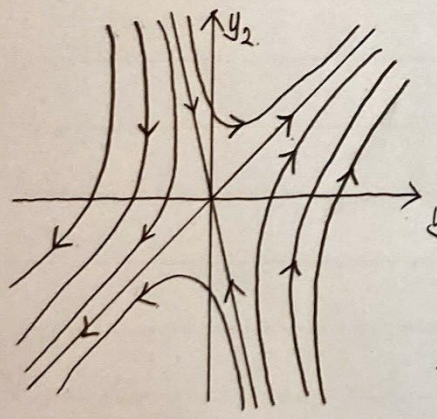
① $\lambda_1 = -3$ $\quad 5x_1 + x_2 = 0$, $x_{(1)} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

② $\lambda_2 = 3$ $\quad -x_1 + 3x_2 = 0$, $x_{(2)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\Rightarrow y = \begin{bmatrix} 5 \\ 1 \end{bmatrix} C_1 e^{-3t} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} C_2 e^{3t} \quad (\text{일반해})$$

$P = \lambda_1 + \lambda_2 = -3 + 3 = 0$, $q = \lambda_1 \lambda_2 = -9 < 0$, $\Delta = (\lambda_1 - \lambda_2)^2 = 36 > 0$

∴ 불안정한 안장점이다.



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