국방정보공학과 2학년 2020072706 송민섭

공업수학2 HW4

1. $x^2-4y^2+z^2=16$ at $P(2,1,4)$ tangent plane

$\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) = (2x, -8y, 2z)$

$\nabla f(2,1,4) = (4, -8, 8)$

즉, $P(2,1,4)$에서 접평면의 법선 벡터가 $(4,-8,8)$ 이다.

$\therefore\ \nabla f(2,1,4)\cdot(x-2, y-1, z-4) = 0$

$(4,-8,8)\cdot(x-2, y-1, z-4) = 0$

$4(x-2)-8(y-1)+8(z-4)=0$, $\boxed{x-2y+2z=8}$

2. $F(x,y,z) = (x, y, z^2+1)$, S: $x^2+y^2=a^2$, $z=0$, $z=c$

"divergence theorem"에 의해, $\iint_S F\cdot n\,dA = \iiint_T \mathrm{div}(F)\,dV$ 이다.

$\mathrm{div}(F) = 1+1+2z = 2+2z$

$\iint_S F\cdot n\,dA = \iiint_T (2+2z)\,dV = \int_0^c\int_0^{2\pi}\int_0^a (2+2z)\,r\,dr\,d\theta\,dz$

$= \int_0^c\int_0^{2\pi}\left[\frac{1}{2}(2+2z)r^2\right]_0^a d\theta\,dz = \int_0^c\int_0^{2\pi}(1+z)a^2\,d\theta\,dz$

$= \int_0^c\left[(1+z)a^2\theta\right]_0^{2\pi}dz = \int_0^c 2\pi a^2(1+z)\,dz = \int_0^c(2\pi a^2+2\pi a^2 z)\,dz$

$= \left[2\pi a^2 z+\pi a^2 z^2\right]_0^c = 2\pi a^2 c+\pi a^2 c^2 = \boxed{\pi a^2 c\,(2+c)}$

3. $\sqrt[4]{-1}$

$w = \sqrt[n]{z} = \sqrt[n]{r}\left(\cos\frac{\theta+2k\pi}{n} + j\sin\frac{\theta+2k\pi}{n}\right)$, $k=0,1,\cdots,n-1$이므로

$n=4$, $r=1$, $\theta=\pi$

$\therefore\ \sqrt[4]{-1} = \sqrt[4]{1}\left(\cos\frac{\pi+2k\pi}{4} + j\sin\frac{\pi+2k\pi}{4}\right)$, $k=0,1,2,3$

$\Rightarrow \boxed{\frac{1\pm i}{\sqrt{2}},\ \frac{-1\pm i}{\sqrt{2}}}$