국방정보공학과 2학년 20200722706 송민영

공업수학2 _ HW3

1. $F(x,y,z) = (x^2+y^2,\ x^2-y^2)$, $R: 1 \le y \le 2-x^2$

"Green theorem"에 의해, $\iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dxdy = \oint_C (F_1 dx + F_2 dy)$ 이므로

$\oint_C F\cdot dr = \int_C [(x^2+y^2)dx + (x^2-y^2)dy]$

$= \iint_R (2x-2y)\,dxdy = \int_{-1}^{1}\int_{1}^{2-x^2} (2x-2y)\,dydx$

$= \int_{-1}^{1} [2xy - y^2]_1^{2-x^2} dx = \int_{-1}^{1} (2x(2-x^2) - (2-x^2)^2 - 2x + 1)\,dx$

$= \int_{-1}^{1} (4x - 2x^3 - 4 + 4x^2 - x^4 - 2x + 1)\,dx = \int_{-1}^{1} (-x^4 - 2x^3 + 4x^2 + 2x - 3)\,dx$

$= \left[-\frac{1}{5}x^5 - \frac{1}{2}x^4 + \frac{4}{3}x^3 + x^2 - 3x\right]_{-1}^{1} = \left(-\frac{1}{5} - \cancel{\frac{1}{2}} + \frac{4}{3} + \cancel{1} - 3 - \frac{1}{5} + \cancel{\frac{1}{2}} + \frac{4}{3} \cancel{-1} - 3\right)$

$= -\frac{2}{5} + \frac{8}{3} - 6 = -\frac{56}{15}$

$\therefore \oint_C F\cdot dr = -\frac{56}{15}$

2. $\iint_S (x+y+z)\,dA$, $S: z = x+2y$, $0 \le x < \pi$, $0 \le y \le x$

$S: r(u,v) = [u, v, u+2v]$, $0 \le u \le \pi$, $0 \le v \le u$

$r_u = (1,0,1)$, $r_v(0,1,2)$ 이므로

$N = \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{vmatrix} = (-1,-2,1)$, $|N| = \sqrt{1+4+1} = \sqrt{6}$

$\therefore \iint_S G(r)\,dA = \int_0^{\pi}\int_0^{u} \sqrt{6}(2u+3v)\,dv\,du$

$= \int_0^{\pi} \left[2\sqrt{6}uv + \frac{3\sqrt{6}}{2}v^2\right]_0^u du = \int_0^{\pi} \left(2\sqrt{6}u^2 + \frac{3\sqrt{6}}{2}u^2\right) du$

$= \sqrt{6}\int_0^{\pi} \frac{7}{2}u^2\,du = \sqrt{6} \times \left[\frac{7}{6}u^3\right]_0^{\pi}$

$= \frac{7\sqrt{6}}{6}\pi^3$

3. $F(x,y,z) = (xy^2,\ x^2y,\ 6\sin x)$, $S: z=\sqrt{x^2+y^2}$, $z=2$, $z=4$

"divergence theorem"에 의해, $\iint_S F\cdot n\,dA = \iiint_T \mathrm{div}(F)\,dV$ 이다.

$x = r\cos\theta$, $y = r\sin\theta$

$\mathrm{div}(F) = y^2 + x^2 + 0 = y^2 + x^2$

$\iint_S F\cdot n\,dA = \iiint_T (y^2+x^2)\,dV$

$$= \int_2^4\int_0^{2\pi}\int_0^z (r^2\sin^2\theta + r^2\cos^2\theta)\,r\,dr\,d\theta\,dz$$

$$= \int_2^4\int_0^{2\pi}\int_0^z r^3\,dr\,d\theta\,dz$$

$$= \int_2^4\int_0^{2\pi}\left[\frac{1}{4}r^4\right]_0^z d\theta\,dz$$

$$= \int_2^4\int_0^{2\pi}\frac{1}{4}z^4\,d\theta\,dz = \int_2^4\left[\frac{1}{4}z^4\theta\right]_0^{2\pi}dz$$

$$= \int_2^4 \frac{\pi}{2}z^4\,dz = \left[\frac{\pi}{10}z^5\right]_2^4$$

$$= \frac{\pi}{10}(4^5 - 2^5) = \frac{\pi}{10}\times 992 = \frac{992}{10}\pi.$$