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This is a system of linear equations with 4-unknowns and 4 equations.

$$\begin{aligned} x+2y-w&=3 \\ 2x-y+z+3w&=2 \\ x+7y-z-6w&=7 \\ x-3y+z+4w&=-1 \end{aligned} \quad \rightarrow \quad [Ax:b] = \begin{bmatrix} 1 & 2 & 0 & -1 & 3 \\ 2 & -1 & 1 & 3 & 2 \\ 1 & 7 & -1 & -6 & 7 \\ 1 & -3 & 1 & 4 & -1 \end{bmatrix}$$

<1> $(R_2-2R_1) \rightarrow R_2$

$$\begin{bmatrix} 1 & 2 & 0 & -1 & 3 \\ 0 & -5 & 1 & 5 & -4 \\ 1 & 7 & -1 & -6 & 7 \\ 1 & -3 & 1 & 4 & -1 \end{bmatrix} \rightarrow$$

<2> $(R_3-R_1) \rightarrow R_3$

$$\begin{bmatrix} 1 & 2 & 0 & -1 & 3 \\ 0 & -5 & 1 & 5 & -4 \\ 0 & 5 & -1 & -5 & 4 \\ 1 & -3 & 1 & 4 & -1 \end{bmatrix}$$

<3> $(R_4-R_1) \rightarrow R_4$

$$\begin{bmatrix} 1 & 2 & 0 & -1 & 3 \\ 0 & -5 & 1 & 5 & -4 \\ 0 & 5 & -1 & -5 & 4 \\ 0 & -5 & 1 & 5 & -4 \end{bmatrix} \rightarrow$$

<4> $(R_3+R_2) \rightarrow R_3$

$$\begin{bmatrix} 1 & 2 & 0 & -1 & 3 \\ 0 & -5 & 1 & 5 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -5 & 1 & 5 & -4 \end{bmatrix}$$

<5> $(R_4-R_2) \rightarrow R_4$

$$\begin{bmatrix} 1 & 2 & 0 & -1 & 3 \\ 0 & -5 & 1 & 5 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow$$

<6> $R_2 \times (-\frac{1}{5}) \Rightarrow$ REF

$$\begin{array}{c} \begin{bmatrix} 1 & 2 & 0 & -1 & 3 \\ 0 & 1 & -\frac{1}{5} & -1 & \frac{4}{5} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{ccccc} x & y & z & w & \end{array} \end{array}$$

In this case, the leading variables are x, y and free variables are z, w. So, to get the solution, we should assign parameters to free variables ; $z=r$, $w=s$

$\therefore$ $x+2y-s=3$

$y-\frac{1}{5}r-s=\frac{4}{5} \Rightarrow y=\dfrac{r+5s+4}{5}$, $x=-\dfrac{2r+5s-7}{5}$

Answer: $x=-\dfrac{2r+5s-7}{5}$, $y=\dfrac{r+5s+4}{5}$, $z=r$, $w=s$