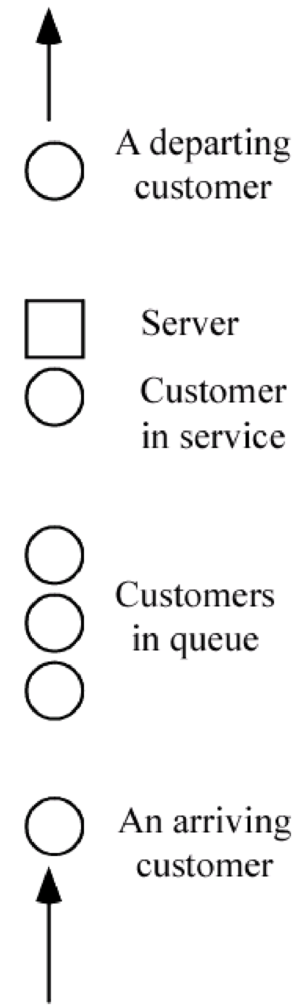
SIMULATION OF A SINGLE-SERVER QUEUEING SYSTEM

- Will show how to simulate a specific version of the single-server queuing system
- Though simple, it contains many features found in all simulation models

1- Problem Statement

- Recall single-server queuing model
- Assume interarrival times are independent and identically distributed (IID) random variables
- Assume service times are IID, and are independent of interarrival times
- Queue discipline is FIFO
- Start empty and idle at time 0
- First customer arrives after an interarrival time, not at time 0
- Stopping rule: When *n*th customer has completed delay in queue (i.e., *enters* service) … *n* will be specified as input



1- Problem Statement (cont'd.)

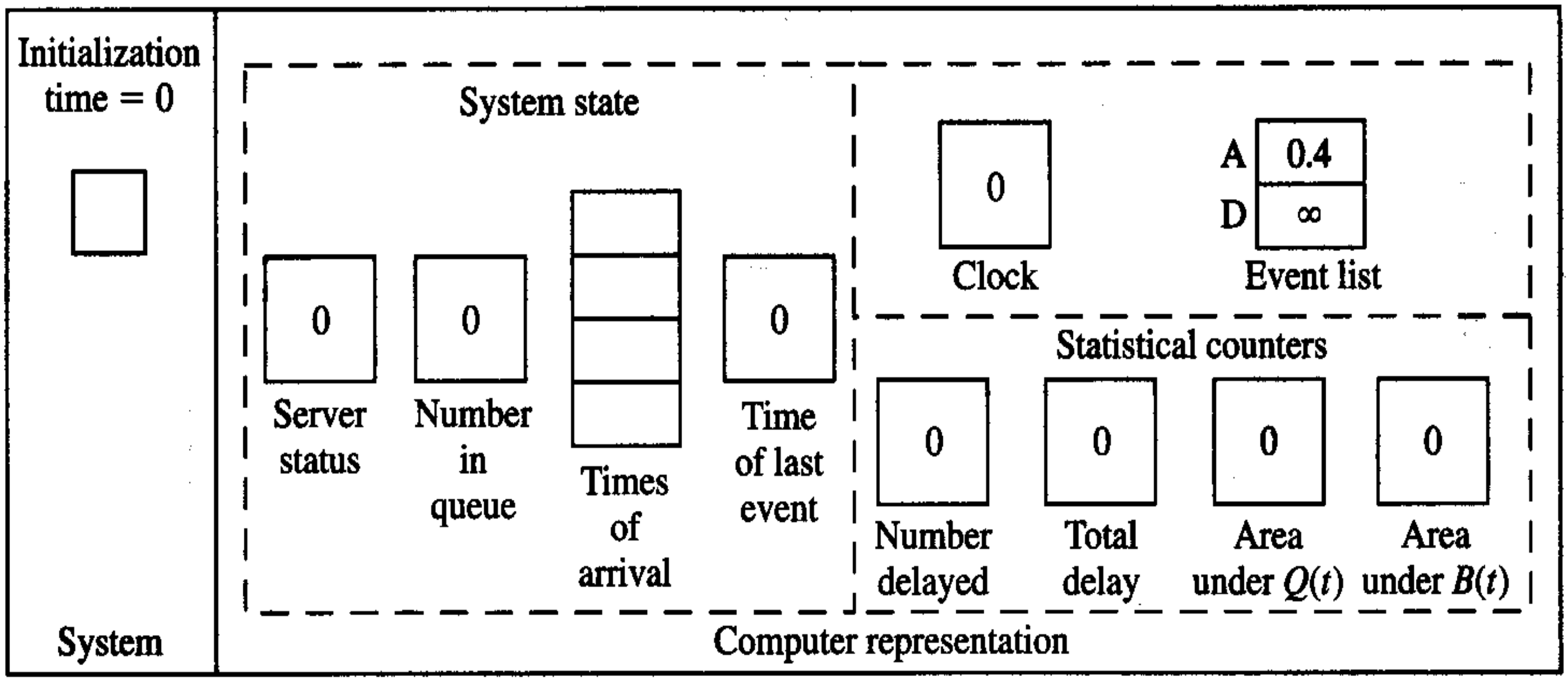
- Quantities to be estimated
  - *Expected average delay in queue* (excluding service time) of the n customers completing their delays
    - Why "expected?"
  - *Expected average number of customers in queue* (excluding any in service)
    - A *continuous-time average*
    - Area under $Q(t)$ = queue length at time t, divided by $T(n)$ = time simulation ends … see book for justification and details
  - *Expected utilization (proportion of time busy) of the server*
    - Another continuous-time average
    - Area under $B(t)$ = server-busy function (1 if busy, 0 if idle at time t), divided by $T(n)$ … justification and details in book
  - Many others are possible (maxima, minima, time or number in system, proportions, quantiles, variances …)

2- Intuitive Explanation

- Given (for now) interarrival times (all times are in minutes):
  0.4, 1.2, 0.5, 1.7, 0.2, 1.6, 0.2, 1.4, 1.9, …
- Given service times:
  2.0, 0.7, 0.2, 1.1, 3.7, 0.6, …
- n = 6 delays in queue desired
- "Hand" simulation:
  - Display system, state variables, clock, event list, statistical counters … all *after* execution of each event
  - Use above lists of interarrival, service times to "drive" simulation
  - Stop when number of delays hits n = 6, compute output performance measures

2- Intuitive Explanation (cont'd)

Status shown i *after* all change: have been made in each case …



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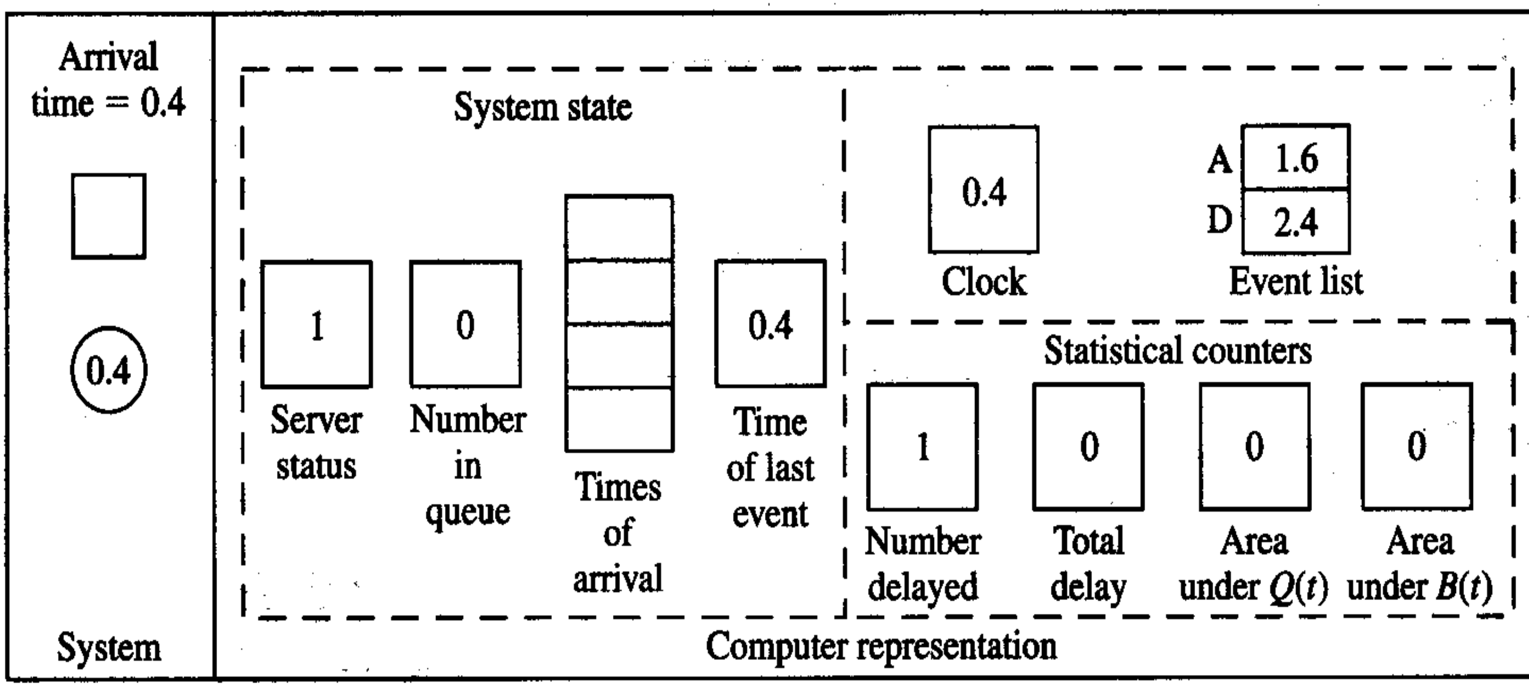

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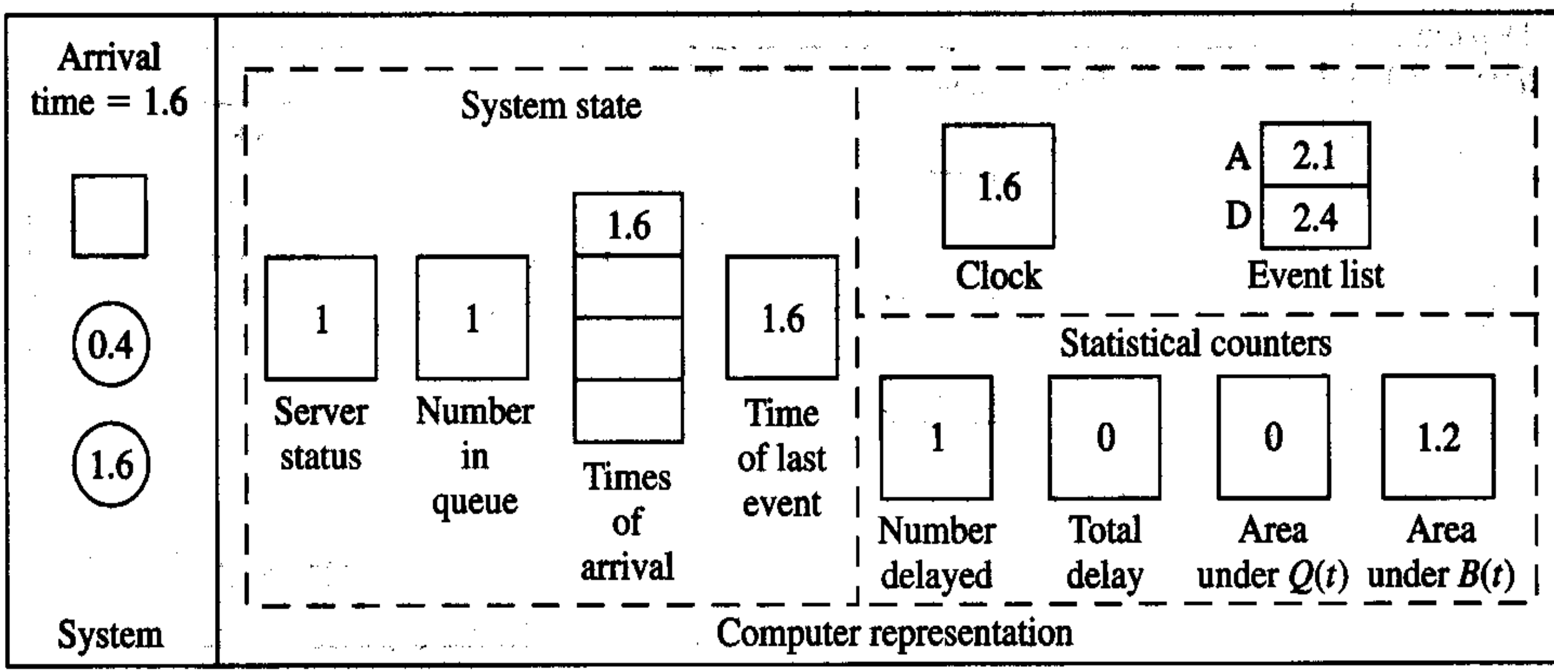

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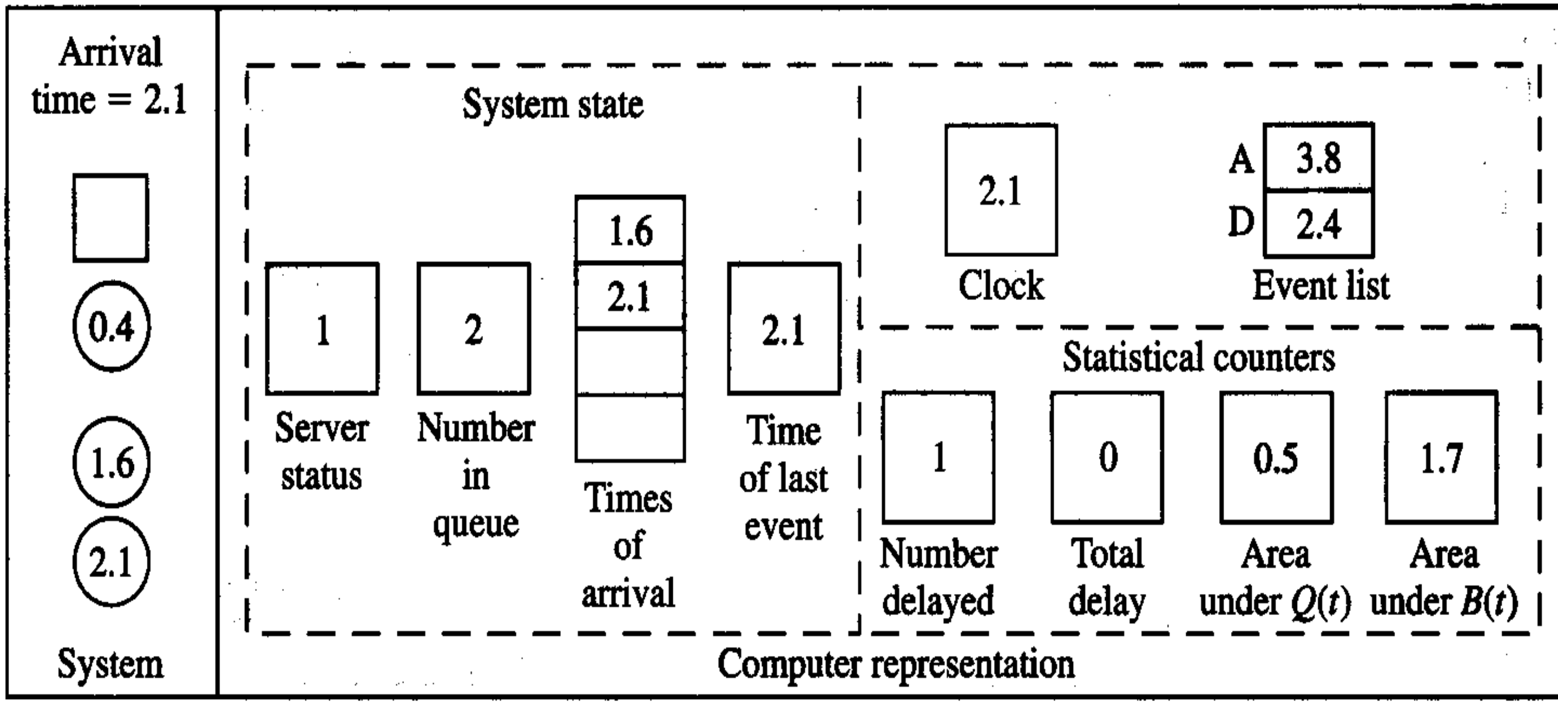

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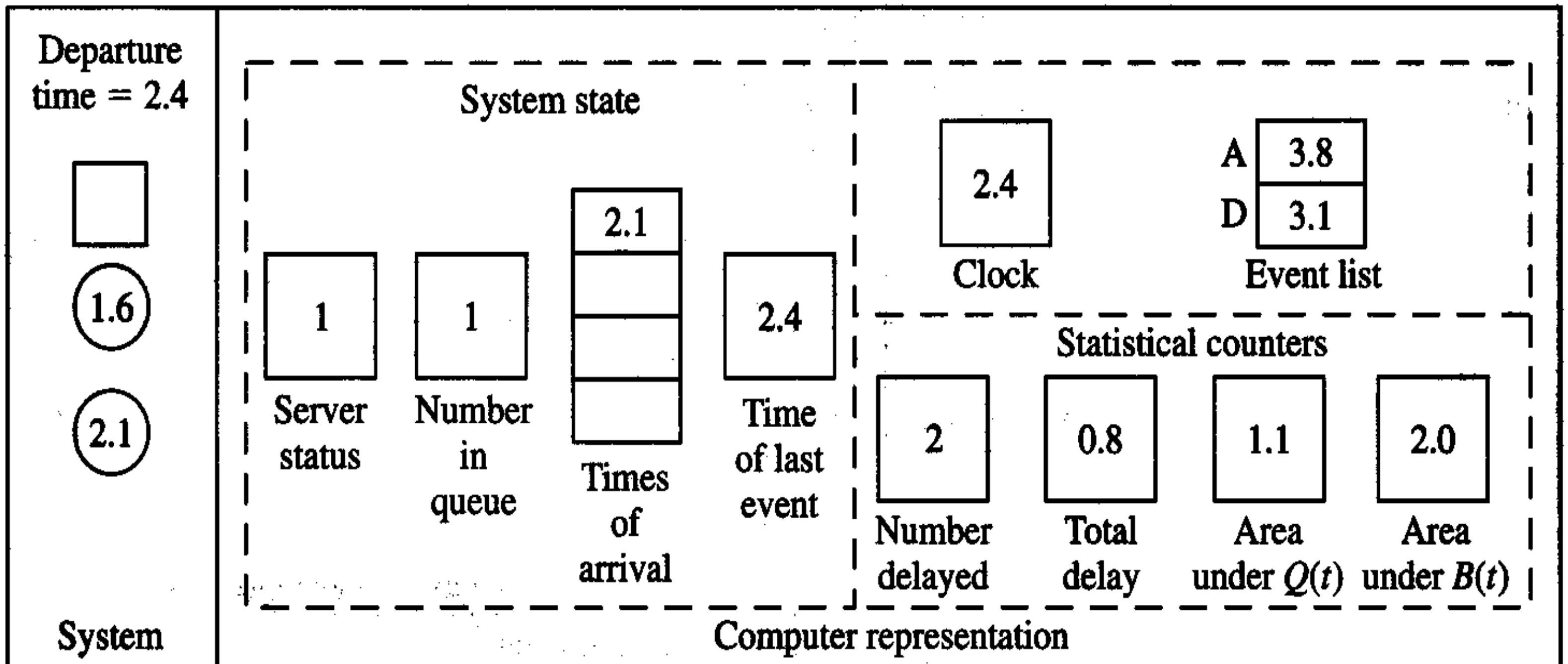

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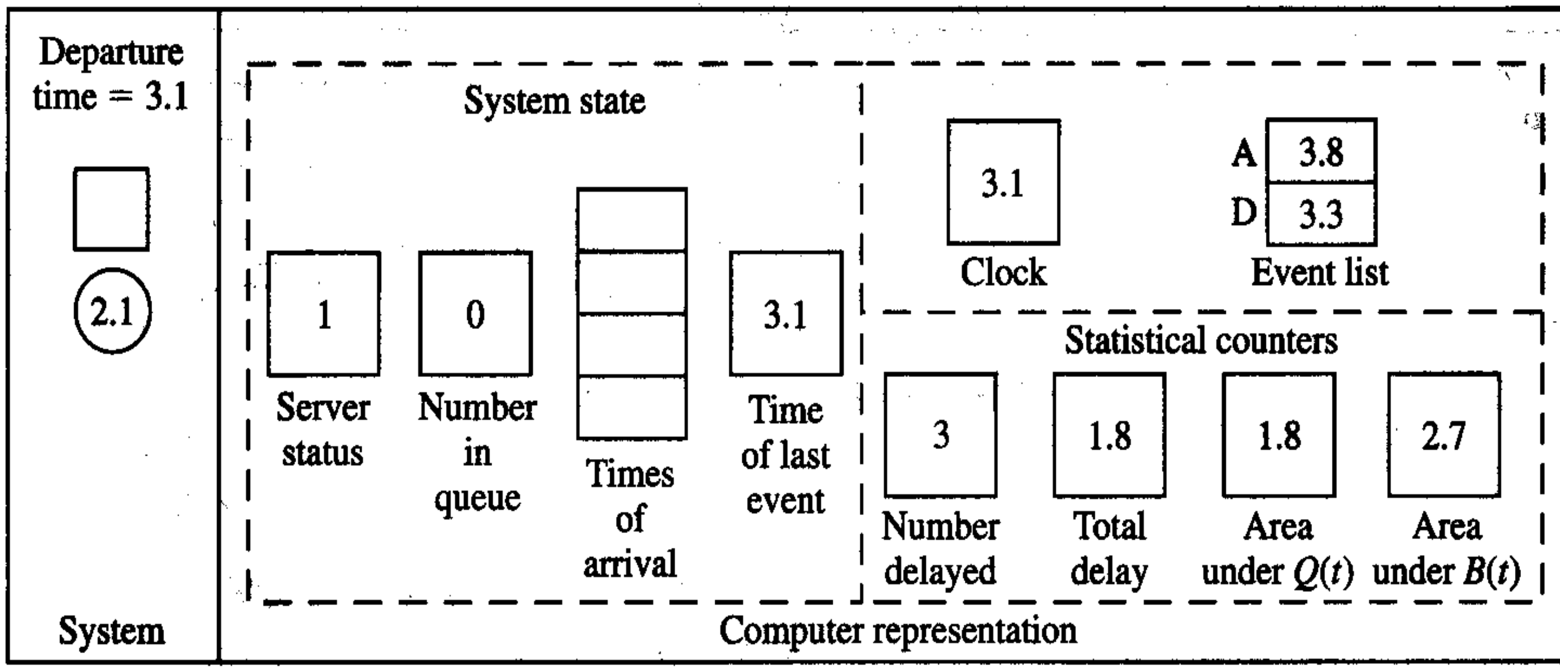

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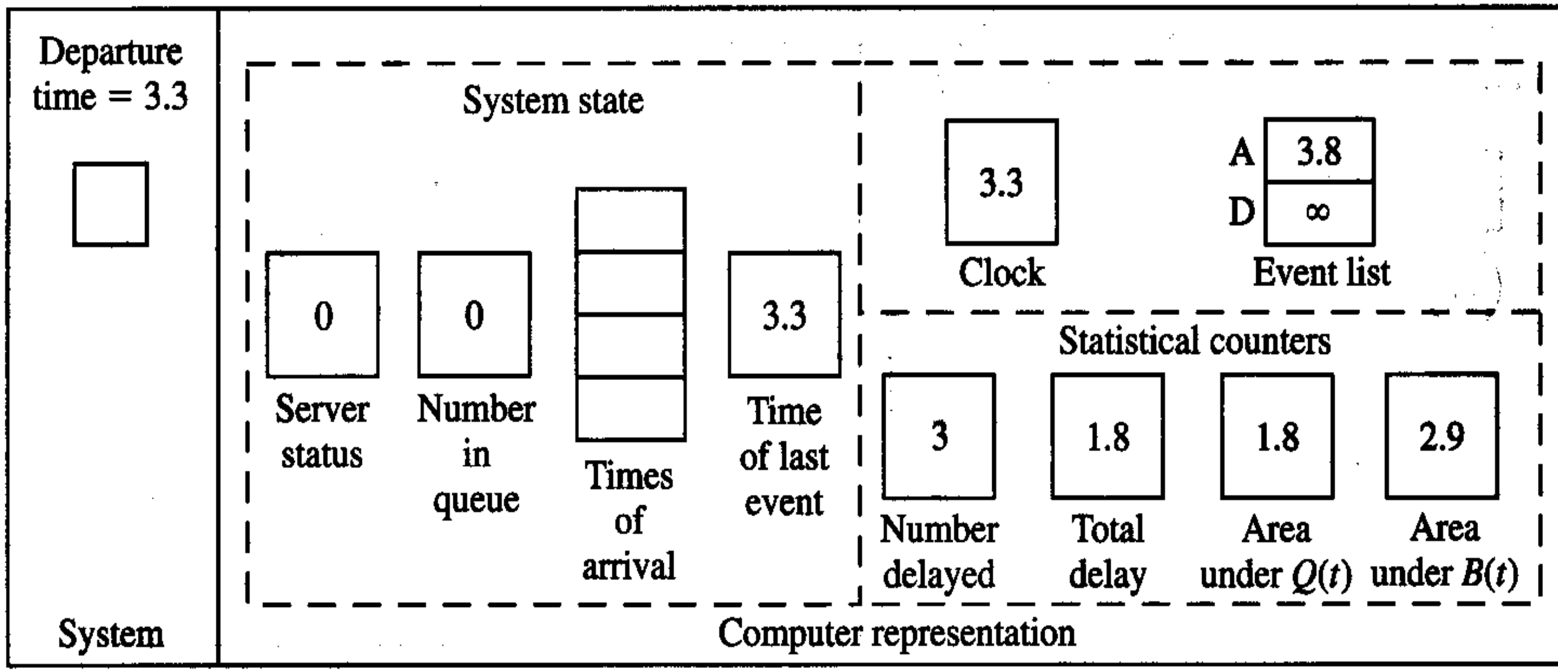

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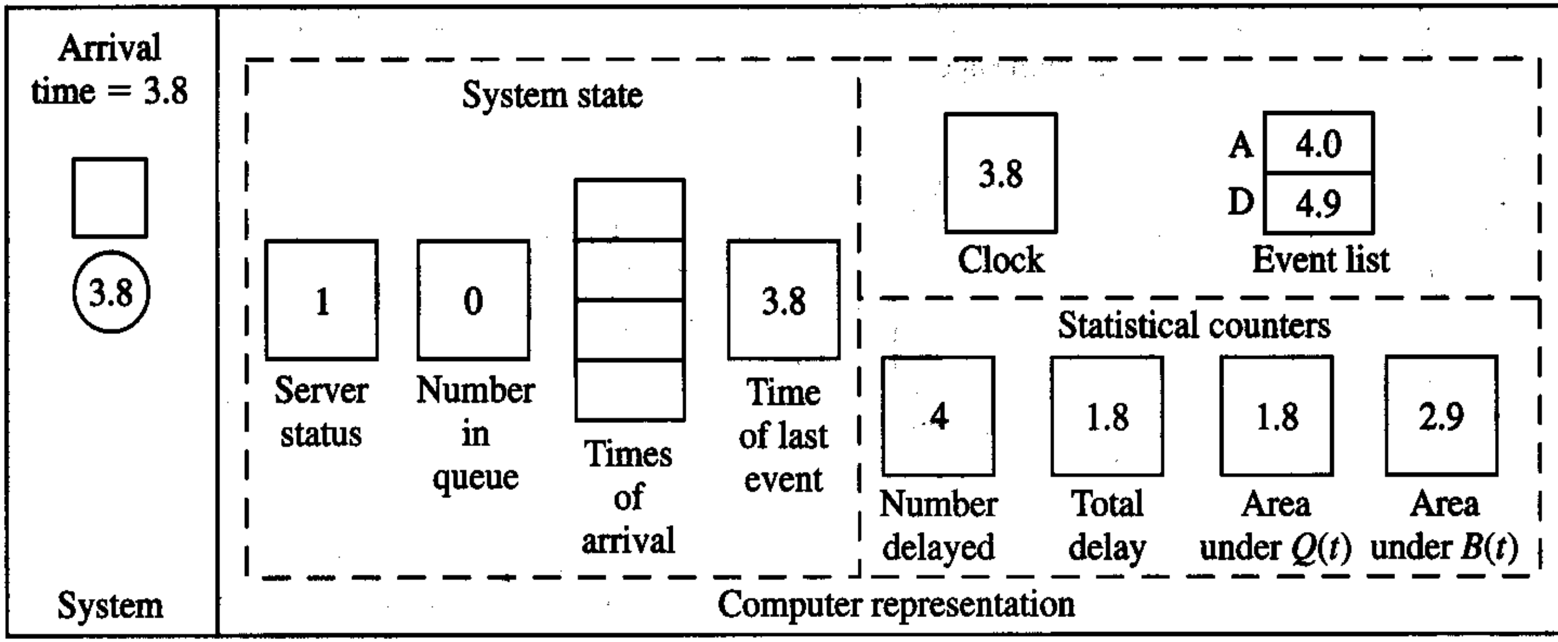

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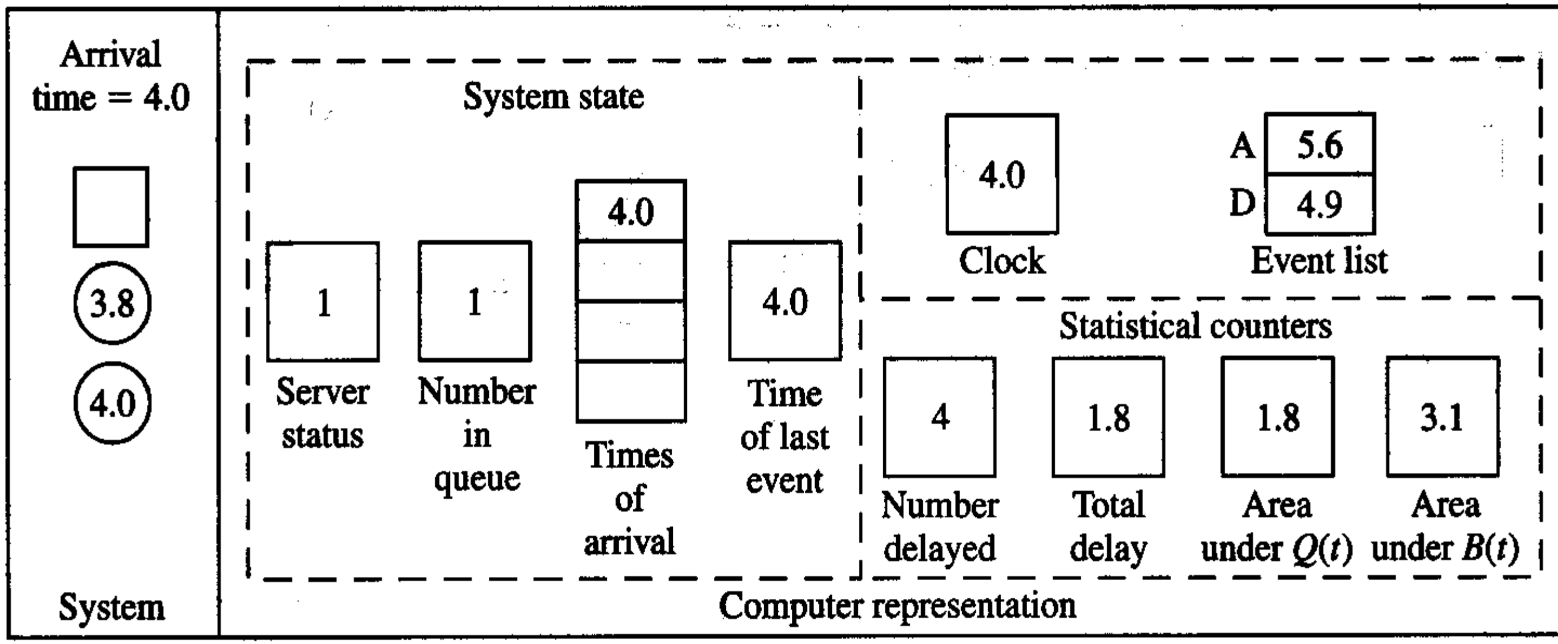

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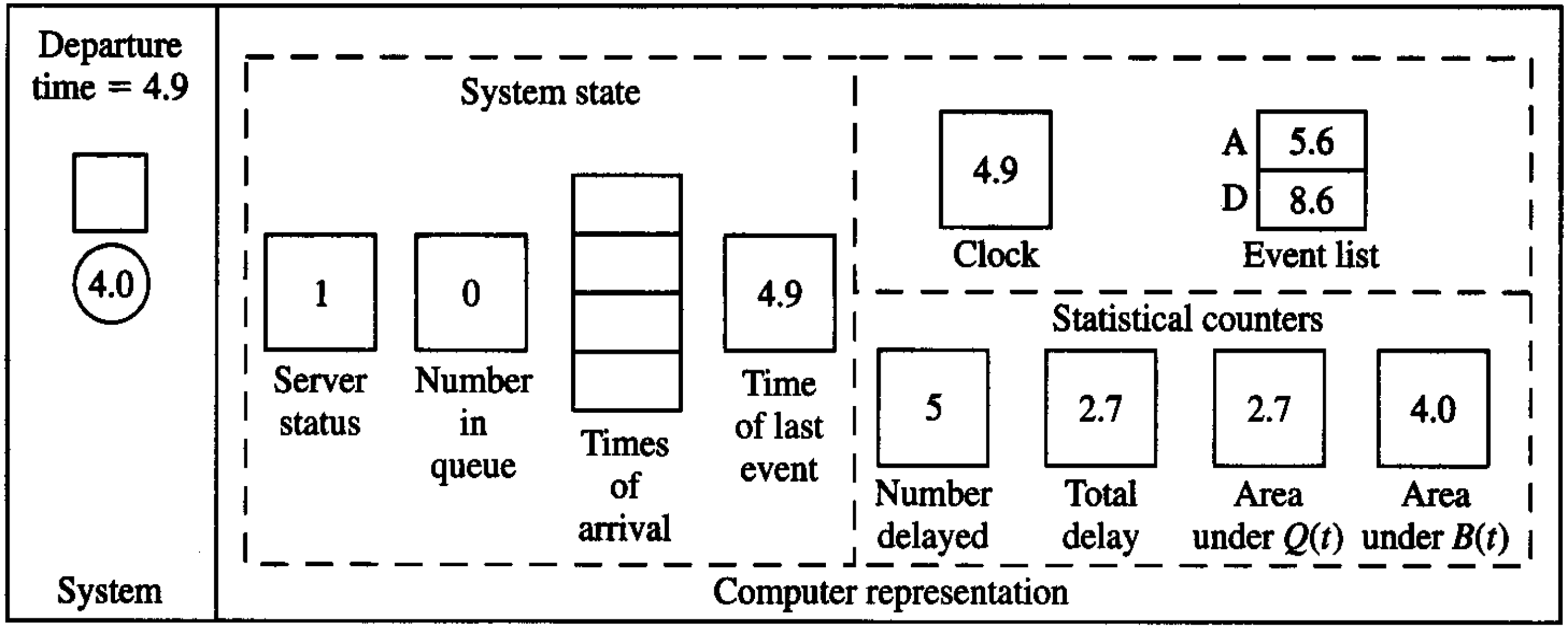

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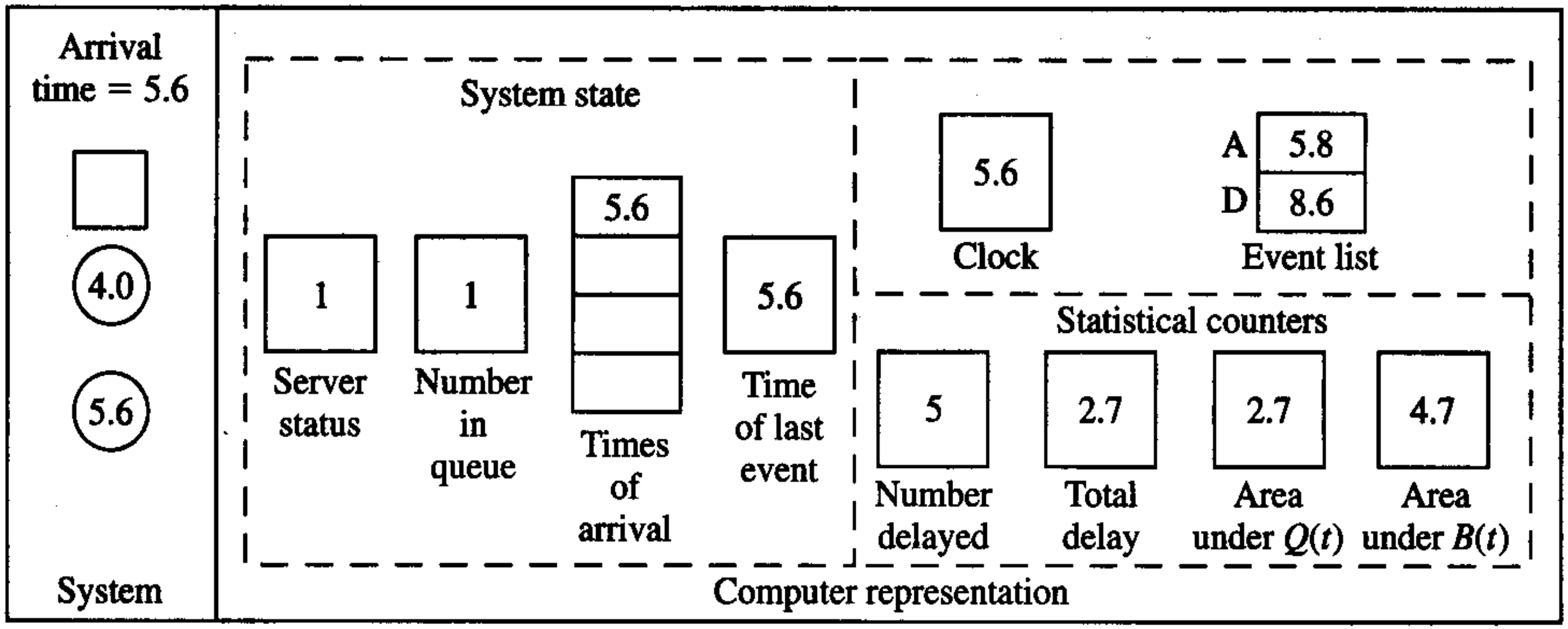

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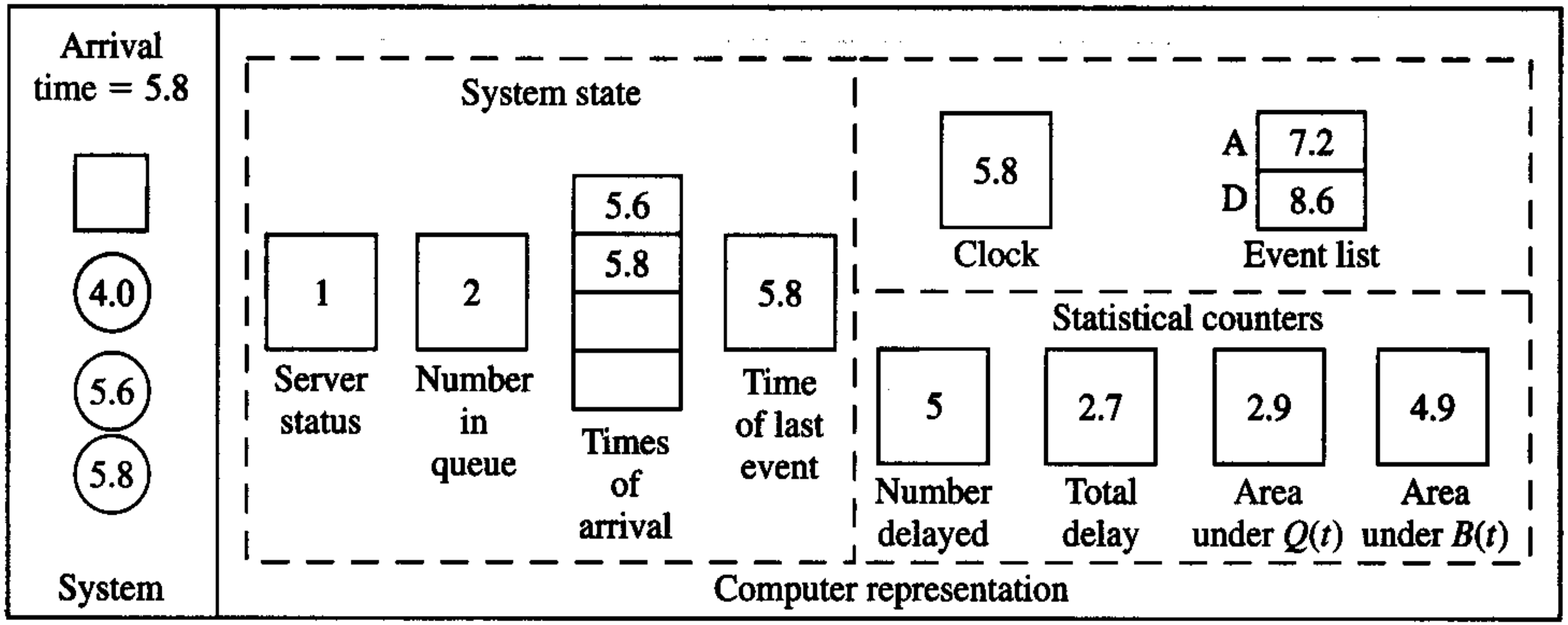

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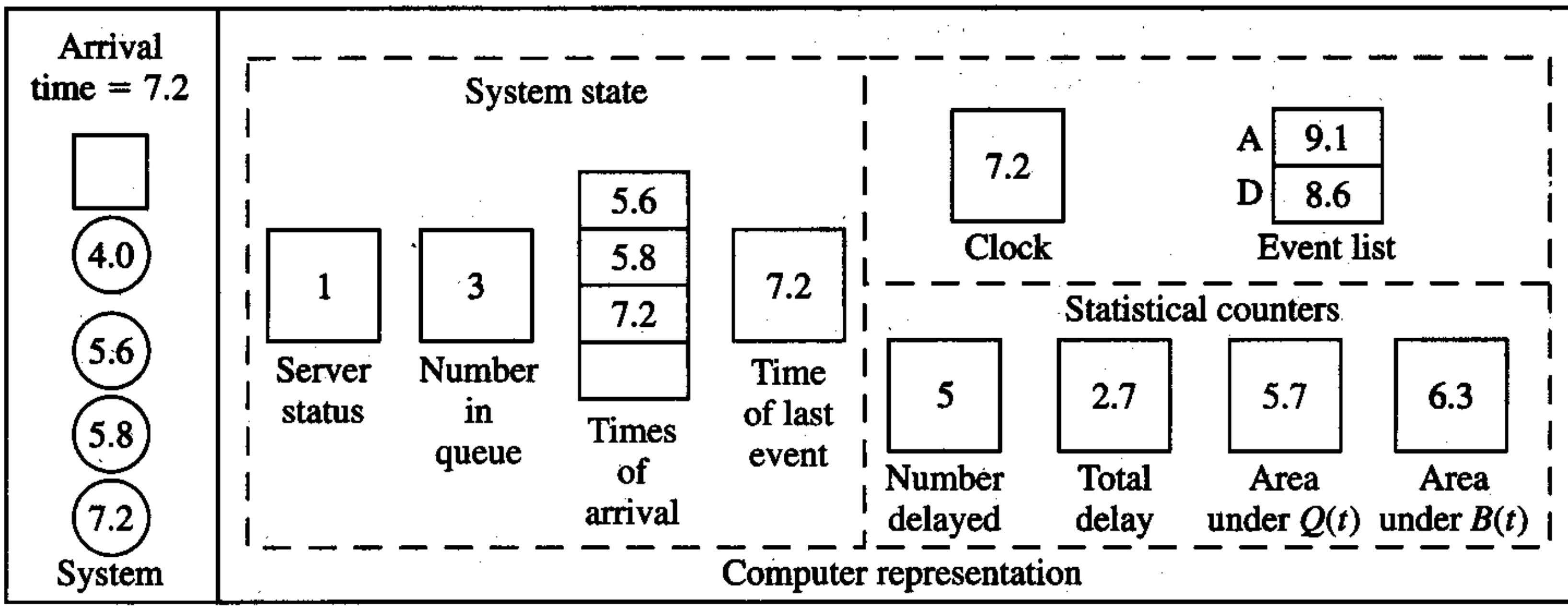

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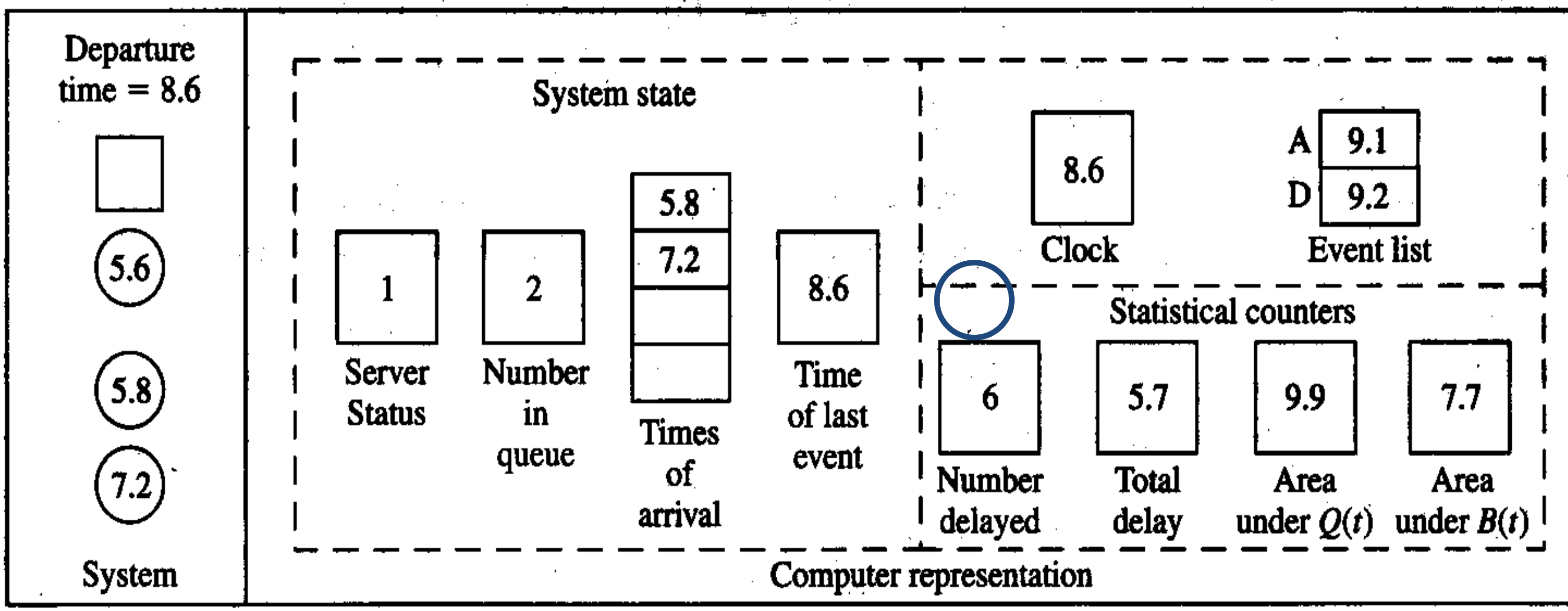

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Final output performance measures:

Average delay in queue = 5.7/6 = 0.95 min./cust.

Time-average number in queue = 9.9/8.6 = 1.15 custs.

Server utilization = 7.7/8.6 = 0.90 (dimensionless)

3- Program Organization and Logic

- C program to do this model (FORTRAN as well is in book)
  - Event types: 1 for arrival, 2 for departure
  - Modularize for initialization, timing, events, library, report, main
- Changes from hand simulation:
  - Stopping rule: *n* = 1000 (rather than 6)
  - Interarrival and service times "drawn" from an exponential distribution (mean β = 1 for interarrivals, 0.5 for service times)
    - Density function $f(x) = \begin{cases} \frac{1}{\beta} e^{-x/\beta} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$
    - Cumulative ($F(x) \equiv P(X \leq x) = \int_{-\infty}^{x} f(t)\,dt = \begin{cases} 1 - e^{-x/\beta} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

3- Program Organization and Logic

(cont'd.)

- How to "draw" (or generate) an observation (*variate*) from an exponential distribution?
- Proposal:
  - Assume a perfect *random-number generator* that generates IID variates from a continuous uniform distribution on [0, 1] …
  - Algorithm:
    1. Generate a random number U
    2. Return $X = -\beta \ln U$
  - Proof that algorithm is correct

$$
\begin{aligned}
P(\text{generated } X \le x) &= P(-\beta \ln U \le x) \\
&= P(\ln U \ge -x/\beta) \\
&= P(U \ge e^{-x/\beta}) \\
&= P(e^{-x/\beta} \le U \le 1) \\
&= 1 - e^{-x/\beta}
\end{aligned}
$$

ALTERNATIVE APPROACHES TO MODELING AND CODING SIMULATIONS

- Parallel and distributed simulation
  - Various kinds of parallel and distributed architectures
  - Break up a simulation model in some way, run the different parts simultaneously on different parallel processors
  - Different ways to break up model
    - By support functions – random-number generation, variate generation, event-list management, event routines, etc.
    - Decompose the model itself; assign different parts of model to different processors – message-passing to maintain synchronization, or forget synchronization and do “rollbacks” if necessary … “virtual time”
- Web-based simulation
  - Central simulation engine, submit “jobs” over the web
  - Wide-scope parallel/distributed simulation