# OC3140
# HW/Lab 6 Sampling Distribution

1. The heights of 1000 students are approximately normal distributed with a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters. If 200 random samples of size 25 are drawn from this population and the means recorded to the nearest tenth of a centimeter, determine
   (a) The mean and standard error of the sampling distribution of $\bar{X}$ ;
   (b) The number of sample means that fall between 172.5 and 175.8 centimeters inclusive;
   (c) The number of sample means falling below 172.0 centimeters.

## Solution:

(a) $\mu_{\bar{X}} = \mu = 174.5$ and $\sigma_{\bar{X}} = \sigma / \sqrt{n} = 6.9/5 = 1.38$

(b) $Z1 = \dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \dfrac{172.5-174.5}{1.38} = -1.45$, $Z2 = \dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \dfrac{175.8-174.5}{1.38} = 0.94$,

$P(-0.4 < z < 0.26) = P(0.94) - P(-1.45) = P(0.94) - 1 + P(1.45) = 0.82639 - 1 + 0.92647 = 0.7529$, N=0.7529*200=155.8=156;

(c) $Z = \dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \dfrac{172-174.5}{1.38} = -1.8116$,

$P(-1.8116) = 1 - P(1.8116) = 1 - 0.9649 = 0.0351$, $N = 0.0351*200 = 7$.

2. The daily high temperature of May in Monterey bay area is approximately normally distribution with mean equal to $67^o$ F and a standard deviation of $4^o$ F.
   (a). Find the probability daily high temperature of May hot than $69^o$ F.
   (b). Find the probability that the average of daily high temperature of the coming May hot than $69^o$ F.

## Solution:

(a). $Z = \dfrac{x-\mu}{\sigma} = 0.5$, From the standard normal distribution table, F(Z<1)=0.6915, so F(Z>1)=1-0.6915=0.3085. The probability daily high temperature of May hot than $69^o$ F is 30.85 %.

(b). That will be the mean of a random sample of size 31 taken from a population with mean $\mu = 67$ and finite variance $\sigma^2 = 4^2$. According Central Limit Theorem (CLT), the limiting probability distribution of $Z = \dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ follows the standard normal distribution.

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{69 - 67}{4 / \sqrt{31}} = 2.7839 \approx 2.8$$

From the standard normal distribution table, F(Z<2.8)=0.9974, so F(Z>2.8)=1-0.9974=0.0026.

The probability that the average of daily high temperature of the coming May hot than $69^o$ F will be 0.26 %.

3. A manufacturer of car batteries guarantees that his batteries will last, on the average, 3 years with a standard deviation of 1 year. If five of these batteries have lifetimes of 1.9, 2.4, 3.0, 3.5 and 4.2 years, is the manufacturer still convinced that his batteries have a standard deviation of 1 year? (using 5 %)

## Solution:

Try Chi square $\left(\chi^2\right)$ distribution (see Chapter 5, Page 8)

$x = 1.9, 2.4, 3.0, 3.5, 4.2$, n=5, $\sigma = 1$ (standard deviation of 1 year).

$\bar{x} = 3.0$, $s^2 = \frac{\sum\left(x_i - \bar{x}\right)^2}{n-1} = 0.815$, $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = 3.26$.

From $\chi^2$ distribution table (Chapter 5 Page 12) with

$df = n - 1 = 4$, $P(0.711) = 0.95$, $P(9.488) = 0.05$.

Since

$0.711 < \chi^2 (= 3.26) < 9.488$,

the standard deviation of 1 year is reasonable.

4. A normal population with unknown variance has a mean of 20. Is one likely to obtain a random sample of size 9 from this population with a mean of 24 and a standard deviation of 4.1?

## Solution:

$\mu = 20$, n=9, $\bar{x} = 24$, s=4.1, $t = \frac{\bar{x} - \mu}{s / \sqrt{n}} = \frac{24 - 20}{4.1/3} = 2.9268$, $\upsilon = n - 1 = 8$, and $t_{0.95} = 1.86$, as $t = 2.93 > t_{0.95}$, it unlikely obtained from this population and $\mu > 20$.

5. A manufacturer of light bulbs claims that his bulbs will burn on the average 500 hours. To maintain this average, he tests 25 bulbs each month. If the computed t-value falls between $-t_{0.05}$ and $t_{0.05}$, he is satisfied with his claim. What conclusion should he draw from a sample that has a mean ($\bar{x} = 518$ hours) and a standard deviation $s = 40$ hours?

## Solution:

Compute the *t*-value (see Chapter 5, Page 13)

$\mu = 500$, $n = 25$, $\bar{x} = 518$ and $s = 40$.

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}} = 2.25.$$

Use the t-distribution table (Chapter 5 Page 17),

$df = n - 1 = 24$, $t_{0.05} = 1.711$.

Since $t = 2.25 > t_{0.05}$, the average hours may be more than 500.

6. Two GPS receivers record the height at same location and same time period. The GPS receiver-A recorded 121 data with variance $s_A^2 = 16$, and the receiver-B got 61 data with variance $s_B^2 = 20$. Determine if these two GPS receivers get the data with equal variances. $(\alpha = 0.05)$

| GPS receiver | Sample size n | Sample Variance $s^2$ |
|---|---|---|
| A | 121 | 16 |
| B | 61 | 20 |

## Solution:

That is the problem about two samples with two different sample sizes. As the data from same location and same time period, so we can assume $\sigma_A^2 = \sigma_B^2$. The F- distribution can be used as:

$$F = \frac{s_B^2}{s_A^2} = \frac{20}{16} = 1.2$$

We have $\nu_1 = \nu_B = 61 - 1 = 60$ and $\nu_2 = \nu_A = 121 - 1 = 120$. From F-table, $f_{0.05}(60,120) = 1.43$. As $F = 1.2 < 1.43$, these two GPS receivers get the data with equal variances.