

# Direct Investment and Intermediary Asset Pricing

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## Abstract

The wealth share of direct investors is priced in the cross-section of stocks and bonds. This is consistent with a theoretical model that expands the intermediary asset pricing model of He and Krishnamurthy (2013) to include households' direct investment. Using total assets of mutual funds as a proxy for direct investment, I show that a portfolio mimicking my proxy for the share of direct investment has a price of -0.9% to -2.0% annually. My factor has explanatory power even in a two-factor model that also includes the intermediary capital ratio factor of He, Kelly, and Manela (2017).

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## 1. Introduction

Financial intermediaries, such as banks and broker-dealers, play a central role in many markets. For example, more than 95% of the CDS market is underwritten by financial intermediaries (Aldasoro and Ehlers (2018)). This has prompted some researchers to develop theories of asset pricing based on the central role of intermediaries. Most notably, He and Krishnamurthy (2013) introduce a model in which capital constraints of financial intermediaries amplify the shocks to the risk premium during financial crises. On the empirical side, Adrian et al. (2014) and He et al. (2017) use proxies for tightness of financial constraints on intermediaries as a factor to explain the cross section of many asset classes. The success of these papers has given rise to a branch of literature dubbed intermediary asset pricing.

My paper shows that in an intermediary asset pricing model, households' direct investment is priced in the cross-section of stocks and bonds. Ignoring households' direct participation in the market has been a criticism of intermediary asset pricing (Cochrane (2017)). For example, in He and Krishnamurthy (2013), intermediaries are run by experts who have access to the market for the risky asset. On the other hand, households can decide to consume, lend to, or invest in the equity of the intermediaries but cannot directly invest in the risky asset. While financial intermediaries dominate many markets, in reality households can directly invest in the stock and bond markets through mutual funds and ETFs. Therefore models like He and Krishnamurthy (2013) do not apply to the stock and bond markets. However, inspired by models like He and Krishnamurthy (2013), empirical papers such as Adrian et al. (2014) and He et al. (2017) argue that intermediaries' leverage is a good proxy for

how constrained they are, and, consequently, their marginal utility of wealth. Hence, these papers use the leverage of broker-dealers and bank holding companies as an asset pricing factor in the cross-section of stocks and bonds.

In contrast, my model adds a direct investment path to the model of He and Krishnamurthy (2013), assuming that households can directly invest in the risky asset through an index fund. As a result, households can choose to lend to the bank, invest in its equity, consume, or invest in the index fund. Moreover, I remove the representative household assumption and replace it with two households with different relative risk aversion parameters. The difference in households' risk aversion parameters gives rise to time-varying risk aversion and risk premium, which are the driving forces of my model.

My model brings two implications. First, it predicts that bank leverage is negatively correlated with the return on the risky asset. The bank manager, who is as risk-averse as the least risk-averse household, can consume, borrow from households, or invest in the risky asset. In equilibrium, he chooses to borrow and hold a levered position in the risky asset. Consequently, the less risk-averse household invests in the bank's equity, whereas the more risk-averse household chooses to lend and invest in the index fund. When the risky asset appreciates, the less risk-averse household stands to gain more due to its levered position. Therefore, its wealth share increases, or equivalently, the wealth share of the more risk-averse household decreases. Hence, the effective risk aversion of the economy decreases, which, in turn, lowers the Sharpe ratio of the risky asset. A lower Sharpe ratio makes the risky asset less attractive to the less risk-averse household, reducing his optimal leverage. As a result, bank lever-

age must be negatively correlated with contemporaneous return and have a negative price of risk. This prediction is consistent with He and Krishnamurthy (2013) and He et al. (2017).

Second, my model predicts that the wealth share of direct investors is also negatively correlated with the return on the risky asset. When market appreciates, the wealth share of the more risk-averse household decreases. But the more risk-averse household is the only direct investor in equilibrium. Therefore, the wealth share of direct investors must also be negatively correlated with the contemporaneous return and have a negative price of risk. This implication is novel and cannot be derived from intermediary asset pricing models without a direct investment path for households.

Cross-sectional asset pricing tests confirm the predictions of my model. I use the intermediary capital risk factor of He et al. (2017) as a proxy for funding constraints of leveraged intermediaries. To construct a pricing factor for the wealth share of direct investors, I calculate the wealth share ratio as the ratio of the total AUM of all mutual funds plus the total bank deposits, to the sum of AUM of mutual funds, total deposits, and the total market equity of all financial companies. The wealth share factor is the percent changes in the wealth share ratio. I find that this wealth share factor is negatively priced in the cross-section of US equities and bonds. Moreover, this result is robust to including the intermediary capital factor or the market excess return in a two factor model, splitting the sample in half, and including only the stock or bond portfolios. I also construct a factor mimicking portfolio for the wealth share factor and find that its price of risk is between -0.88%

and -2.0% annually. I also find that my factor has an alpha of 40bps with respect to the Fama-French three-, five-, and six-factor models.

My paper belongs to the literature on intermediary asset pricing. The original purpose of intermediary asset pricing models was to understand the role of financial intermediaries and their implications for monetary policy. However, a few of these macro models, including He and Krishnamurthy (2012), He and Krishnamurthy (2013), and Brunnermeier and Pedersen (2009), have significant implications for asset pricing. For example, He and Krishnamurthy (2013) implies that funding constraints of financial intermediaries can significantly increase the risk premium. Since then, the literature has moved in various directions. Some researchers have developed specialized models for Federal Reserve primary dealers, short-term interest rates, the option market, OTC markets, and FX markets (Li (2019), Li and Wallen (2018), Fournier and Jacobs (2018), Randall (2015), Pham (2018)). Ayala (2016) incorporates a disaster model into an intermediary model to justify asset pricing puzzles. Ma (2018) and Kargar (2018) study heterogeneity among the intermediaries. My model contributes to this branch by adding a direct investment path for households.

On the empirical side, numerous papers show that different items on intermediaries' balance sheets are related to the prices in various markets including FX, OTC, CDS, commodity, European, treasury, REIT, and MBS (Ding and Ma (2013), Friewald and Nagler (Forthcoming), Siriwardane (Forthcoming), Etula (2013), Baltzer et al. (2019), Lou et al. (2013), Bond et al. (2018), Gabaix et al. (2007)). Furthermore, several other papers establish the causal effect of intermediaries on asset prices through natural experiments. These papers include evidence from the Ger-

man Black Friday of 1927, closed-end funds, introduction of CDS, and economic recessions (Gissler (2015), Tang (2014), Wang et al. (2016), Muir (2017)). My paper contributes to this branch by showing that a new factor corresponding to households' direct investment is also priced in the cross-section of stocks and bonds.

Another important question is which intermediaries should be used for pricing assets and what variables best measure their health. Researchers have experimented with broker-dealers, Federal Reserve primary dealers, mutual funds, pension funds, and insurance companies (Adrian et al. (2014), He et al. (2017), Kim (2018), Greenwood and Vissing-Jorgensen (2018)). My paper contributes to this literature by using the total AUM of mutual funds for pricing. To the best of my knowledge, Kim (2018) is the only other paper using mutual funds for pricing. However, Kim (2018) uses the cross-section of mutual funds to build a liquidity factor while my paper uses the aggregate wealth of mutual funds as a proxy for households' direct investment.

Haddad and Muir (2017) is the closest paper in intermediary asset pricing to mine in the sense that they also focus on direct investment. The authors show, through a static model, that financial intermediaries are "marginal investors" provided that the following two conditions hold: (1) households have a different risk aversion than intermediaries, and (2) households face a cost when directly investing in the market. On the other hand, my paper has a dynamic model and assumes costless direct investment. In other words, my results do not rely on direct investment being costly. On the empirical side, Haddad and Muir (2017) compare stock, bond, option, sovereign debt, commodity, foreign exchange, and CDS markets and show that the intermediary factors of Adrian et al. (2014) and He et al. (2017) have more

predictive power in more intermediated markets. In contrast, my paper introduces a new pricing factor representing direct investment and shows that it is priced in the cross-section of stocks and bonds.

## 2. Model

### 2.1. General Setup

There are three agents: a bank manager, who runs the bank, and two households. Each agent has a time-additive CRRA utility. I denote households' risk aversion by  $\gamma_i$ ,  $i = 1, 2$ , and bank manager's risk aversion by  $\gamma_b$  and assume  $\gamma_b = \gamma_2 < \gamma_1$ . There are three markets: one for a perishable consumption good, one for the risky asset, and a risk-free lending market where households can lend to the bank at a risk-free rate,  $r_t$ . Households can access the risky asset through investing in the index fund. Prices are normalized so that the consumption good is the numeraire at each moment. The risk-free bond is in zero net supply while the risky asset has a unit supply. The risky asset represents a claim to a stream of dividends following the process,

$$\frac{dD_t}{D_t} = \mu dt + \sigma dZ_t, \quad (1)$$

where  $\mu$  and  $\sigma$  are positive constants and  $Z_t$  is a standard Wiener process. I let  $P_t$  denote the price of the risky asset. As a result, its return can be written as

$$dR_t = \frac{D_t dt + dP_t}{P_t}. \quad (2)$$

Each agent is endowed with a fractional ownership in the risky asset. I denote households' endowments by  $e_i$ ,  $i = 1, 2$ , and bank manager's endowment by  $e_b$ . Obviously,  $e_1 + e_2 + e_b = 1$ . I do not assume any restrictions on the relative magnitudes

of  $e_1$ ,  $e_2$ , and  $e_b$ . Thus, all the results hold when  $e_b$  is much smaller than  $e_1$  and  $e_2$ , which is the realistic case.

### 2.2. Index Fund

Households invest in the risky asset through an index fund. Therefore, the return on investing in the index fund is equal to the return on the risky asset.

### 2.3. Bank

The bank is run by a manager with a wealth process,  $w_{bt}$ , and its capital consists of bank manager's wealth, households' investment in bank's equity, and the capital raised through borrowing in the bond market. The bank manager seeks to maximize his expected utility, which depends on his own consumption stream,  $c_{bt}$ . His decision, therefore, comprises choosing a consumption process and a borrowing process,  $b_t$ . As the bank can be levered, the return on its equity is not equal to the return on the risky asset. Let

$$\alpha_t = \frac{\text{Bank's total investment in the risky asset}}{\text{Bank's total equity}} = \frac{\text{Bank's total equity} + b_t}{\text{Bank's total equity}} \quad (3)$$

denote the bank's leverage. We can compute the return on the bank's equity as

$$d\tilde{R}_t = r_t dt + \alpha_t(dR_t - r_t dt). \quad (4)$$

Now we can write the bank manager's decision problem as

$$\begin{aligned} \max_{c_{bt}, b_t} \quad & E \int_0^\infty e^{-\delta t} u_b(c_{bt}) dt \\ \text{s.t.} \quad & dw_{bt} = -c_{bt} dt + w_{bt} d\tilde{R}_t. \end{aligned} \quad (5)$$

## 2.4. Households

There are two types of households with identical choice problems but different risk aversions. I denote them by subscripts  $i = 1, 2$  and assume household 1 is the more risk-averse household (MRA) while household 2 is the less risk-averse one (LRA). Let  $c_{it}$  and  $w_{it}$  denote the consumption and wealth processes of the households. Their decision would then consist of  $L_{it}$ , the amount of lending in the bond market,  $B_{it}$ , their position in bank's equity, and  $M_{it}$ , their investment in the index fund. We can write household  $i$ 's choice problem as

$$\begin{aligned} \max_{c_{it}, L_{it}, B_{it}, M_{it}} \quad & E \int_0^\infty e^{-\delta t} u_i(c_{it}) dt \\ \text{s.t.} \quad & dw_{it} = -c_{it} dt + L_{it} r_t dt + B_{it} d\tilde{R}_t + M_{it} dR_t. \end{aligned} \tag{6}$$

Using this notation, in equilibrium

$$\alpha_t = \frac{w_{bt} + B_{1t} + B_{2t} + b_t}{w_{bt} + B_{1t} + B_{2t}}. \tag{7}$$

## 2.5. Equilibrium

An equilibrium is a set of processes, consisting of: a price for the risky asset,  $P_t$ ; a risk-free rate,  $r_t$ ; and a set of decisions for each agent,  $\{c_{bt}, b_t, c_{1t}, L_{1t}, B_{1t}, M_{1t}, c_{2t}, L_{2t}, B_{2t}, M_{2t}\}$  such that each agent's decision solves his optimization problem and markets clear as follows

$$w_{1t} + w_{2t} + w_{bt} = P_t, \tag{8}$$

$$L_{1t} + L_{2t} = b_t, \tag{9}$$

$$c_{1t} + c_{2t} + c_{bt} = D_t. \tag{10}$$

## 2.6. Equilibrium Characterization

I solve the model for the specific case of  $\gamma_1 = 2\gamma_2$ . The following theorem characterizes the equilibrium.

**Lemma 1.** *Let  $f$  and  $g$  denote the following functions*

$$\begin{aligned} f(x) &= \frac{2}{\eta} E_0 \int_0^\infty e^{-\delta t} \left( \frac{\sqrt{1+\eta x} - 1}{\sqrt{1+\eta x Y_t} - 1} \right)^{\gamma_1} (\sqrt{1+\eta x Y_t} - 1) dt \\ g(x) &= E_0 \int_0^\infty e^{-\delta t} \left( \frac{\sqrt{1+\eta x} - 1}{\sqrt{1+\eta x Y_t} - 1} \right)^{\gamma_1} x Y_t dt, \end{aligned} \quad (11)$$

where  $Y_t$  is a geometric Brownian motion with the same dynamic as  $D_t$  but an initial value of one. The economy described in section 2.1 has a market equilibrium as follows.

$$\begin{aligned} r_t &= \delta + \frac{\gamma_1 \eta \mu D_t}{2\sqrt{1+\eta D_t}(\sqrt{1+\eta D_t} - 1)} - \frac{\gamma_1 \eta^2 \sigma^2 D_t^2 ((\gamma_1 + 2)\sqrt{1+\eta D_t} - 1)}{8(1+\eta D_t)^{\frac{3}{2}}(\sqrt{1+\eta D_t} - 1)^2} \\ P_t &= g(D_t) \\ c_{1t} &= \frac{2}{\eta} [\sqrt{1+\eta D_t} - 1] \\ c_{2t} &= \frac{1}{1+\zeta} (D_t - c_{1t}) \\ c_{bt} &= \frac{\zeta}{1+\zeta} (D_t - c_{1t}) \\ M_{1t} &= g(D_t) \frac{f'(D_t)}{g'(D_t)} \\ L_{1t} &= -b_t = f(D_t) - M_{1t} \\ B_{1t} = L_{2t} = M_{2t} &= 0 \\ B_{2t} &= \frac{1}{1+\zeta} (g(D_t) - M_{1t}), \end{aligned} \quad (12)$$

where  $\eta$  and  $\zeta$  are determined through

$$\begin{aligned} f(D_0) &= e_1 g(D_0) \\ \zeta &= \frac{e_b}{e_2}. \end{aligned} \quad (13)$$

*Proof.* The main idea behind the proof is to map the model into Wang (1996) and prove that there exists a one-to-one mapping between the Pareto-optimal allocation of Wang (1996) and the economy studied in this paper. Subsequently, I calculate the Pareto-optimal consumption processes  $c_{1t}$ ,  $c_{2t}$ , and  $c_{bt}$ . The stochastic discount factor will then be

$$m_t = e^{-\delta t} \left( \frac{2}{\eta} (\sqrt{1 + \eta D_t} - 1) \right)^{-\gamma_1}. \quad (14)$$

Hence, the price of a given consumption process  $\{c_s\}_{s=t}^{\infty}$  at time  $t$  will be

$$\text{Price of } \{c_s\}_{s=t}^{\infty} = E_t \int_t^{\infty} \frac{m_s}{m_t} c_s ds. \quad (15)$$

Using equation 15, I prove that the wealth of the MRA household is given by  $f(D_t)$ . Moreover, the price of the risky asset is given by  $g(D_t)$ . Given the interpretation of  $f$  and  $g$ , the rest of the formulas in lemma 1 can be easily calculated. The appendix contains a complete version of the proof.  $\square$

The following theorem describes the variables of interest.

**Lemma 2.** *The wealth share of the MRA household and the bank's leverage can be written as*

$$x_t = \frac{f(D_t)}{g(D_t)}, \quad (16)$$

$$\alpha_t = \frac{1 - \frac{f'(D_t)}{g'(D_t)}}{1 - \frac{f(D_t)}{g(D_t)}}. \quad (17)$$

Furthermore, they exhibit the following covariance processes with the SDF

$$E_t[dm_t d\alpha_t] = -\gamma_1 \left( \frac{2}{\eta} \right)^{-\gamma_1-1} \left( -\frac{f''g' - g''f'}{(g')^2(1 - \frac{f}{g})} + \frac{f'g - g'f}{g^2(1 - \frac{f}{g})^2} \left( 1 - \frac{f'}{g'} \right) \right) \frac{(\sqrt{1 + \eta D_t} - 1)^{-\gamma_1-1}}{\sqrt{1 + \eta D_t}} \sigma^2 D_t^2 dt, \quad (18)$$

$$E_t[dm_t dx_t] = -\gamma_1 \left( \frac{2}{\eta} \right)^{-\gamma_1-1} \frac{f'g - g'f}{g^2} \frac{(\sqrt{1 + \eta D_t} - 1)^{-\gamma_1-1}}{\sqrt{1 + \eta D_t}} \sigma^2 D_t^2 dt \quad (19)$$

*Proof.* Because  $f$  is the total wealth of the MRA household and  $g$  is the price of the risky asset, equation 16 holds. The rest of the results are derived from equilibrium

quantities in lemma 1 using straightforward algebra. A complete derivation is in the appendix.  $\square$

The discount factor is high when the marginal utility of wealth is low and vice versa. Therefore, it is negatively correlated with contemporaneous return. This means that the SDF has a negative price of risk and that risk premia are opposite in sign to covariances with the SDF. As a result, a variable that positively correlates with the discount factor must have a negative price of risk. The following theorem states that the price of risk for the bank leverage and the wealth share of the MRA household is negative.

**Theorem 1.** *Both the bank leverage and the wealth share of direct investors are positively correlated with the SDF and negatively correlated with the contemporaneous excess return.*

*Proof.* Intuitively, theorem 1 states that when the SDF is high (low), the bank's leverage and the wealth share of direct investors are both high (low). Section 2.7 explains the economic mechanism behind theorem 1 in details. A complete proof of this theorem is presented in the appendix.  $\square$

Because the bank's leverage and the wealth share of direct investors are positively correlated with the SDF, they must have negative prices of risk. In section 3, I test this implication in the equity and bond markets.

### 2.7. Interpretation

To illustrate the economic mechanism behind the model, I calculate the equilibrium numerically. To calibrate the model, I use  $\mu = 6.02\%$  and  $\sigma = 7.29\%$ , matching the mean and the standard deviation of the dividend growth rate for the S&P 500 index between 1990 and 2018. For risk aversions, I use  $\gamma_1 = 2.0$  and  $\gamma_2 = 1.0$ , as used by He and Krishnamurthy (2013). For the discount rate, I use  $\delta = 4.0\%$ .

When the market goes up more than the risk free rate, because the less risk-averse investor has a levered position in the risky asset, he stands to gain more relative to the more risk-averse investor, resulting in an increase in his wealth share. To see this, let  $\beta < 1$  denote the fraction of the more risk-averse household's wealth invested in the risky asset. One can write

$$\begin{aligned}\frac{dw_{1t}}{w_{1t}} &= \beta(dR_t - r_t dt) + r_t dt - \frac{c_{1t}dt}{w_{1t}} \\ \frac{dw_{2t}}{w_{2t}} &= \alpha(dR_t - r_t dt) + r_t dt - \frac{c_{2t}dt}{w_{2t}}.\end{aligned}$$

The more risk averse household is always the buyer in the consumption market. In other words, its consumption always exceeds its share of the dividends. Thus  $\frac{c_{1t}dt}{w_{1t}} > \frac{c_{2t}dt}{w_{2t}}$ . Hence, given that  $r_t dt < dR_t$  and  $\beta < 1 < \alpha$ , it is clear that  $\frac{dw_{1t}}{w_{1t}} < \frac{dw_{2t}}{w_{2t}}$ . As a result, the wealth share of the MRA household shrinks while its total wealth increases due to a positive return on both its lending and its investment in the risky asset.

A decrease in the wealth share of the MRA household lowers the effective risk-aversion in the economy. As a result, the Sharpe ratio of the risky asset, which is the price of one unit of risk, decreases. Consequently, the risky asset becomes less attractive to the less risk-averse investor, resulting in a lower level of optimal leverage for the bank manager. Figures 1 and 2 demonstrate these effects.

### 3. Empirical Tests

#### 3.1. Data

I use total financial assets of mutual funds and exchange-traded funds and total deposits of commercial banks from Flow of Funds, which are available as series

FL484090005 and B1058NCBDM on the website of the Federal Reserve. In addition, I use monthly stock data from CRSP, stock portfolios and factors from Kenneth French's website, 10 maturity-sorted bond portfolios from WRDS, and yield-sorted corporate bond portfolios from Asaf Manela's website, which he acquired from Nozawa (2017). Also, I use the intermediary capital risk factor of He et al. (2017) as a proxy for bank's leverage and refer to it as HKM capital factor. He et al. (2017) calculate their factor as follows. First, they identify the primary dealers of the New York Fed and their parent bank holding companies. Next, they calculate the capital ratio of the bank holding companies as the ratio of their aggregated market equity to the sum of their aggregated market equity and aggregated book value of debt. Finally, they remove the secular trend in the capital ratio factor using an AR(1) specification. For this paper, I use the factor as provided on Asaf Manela's website. To calculate my proxy for the wealth share of direct investors, I aggregate the market cap of financial companies defined as the securities whose SIC codes are between 6000 and 6799 in CRSP. In using total market cap of financial companies I follow Kargar (2018). Then I construct my proxy as

$$x_t = \frac{AUM \text{ of mutual funds} + \text{total deposits}}{\text{market cap of financial companies} + AUM \text{ of mutual funds} + \text{total deposits}}. \quad (20)$$

I use percent changes of this ratio as my pricing factor and refer to it as the wealth share factor. Figure 3 shows the time series of the wealth share ratio as well as the wealth share factor. Moreover, Table 1 shows summary statistics of the wealth share factor, wealth share ratio, the HKM factor, and the market excess return in the period 1973Q2-2012Q4. Over this period, the wealth share factor has a mean

of 0% and standard deviation of 1.2%. Moreover, its minimum and maximum are -3.0% and 4.0% respectively.

### 3.2. Main Results

Theorem 1 predicts that the wealth share factor must be negatively correlated with the market. To verify this result, I report the correlation matrix among the wealth share factor, the HKM capital factor, and the market excess return. The correlation between the wealth share factor and the market is -74%. In addition to the correct sign, its magnitude suggests that the wealth share factor is likely capturing some other risk than the market. The same is true for the correlation between the wealth share factor and HKM's capital ratio factor, which is -81%. Moreover, the theory predicts a negative correlation between the market and bank leverage. Because HKM's capital ratio factor is inversely related to changes in leverage, it must have a positive correlation with market. This is also reflected in Table 2 and Figure 4.

Next, I follow He et al. (2017) in conducting cross-sectional asset pricing tests of the form

$$R_{it} - R_f = \alpha + \beta_i \lambda_t + \epsilon_{it}. \quad (21)$$

In equation 21,  $R_{it}$ ,  $R_f$ ,  $\beta_i$ ,  $\lambda_t$ , and  $\epsilon_{it}$  are the return of the  $i$ -th portfolio in period  $t$ , the risk-free rate, the vector of risk exposures, the vector of risk prices, and the pricing errors, respectively. The test portfolios consist of 25 size- and book-to-market sorted equity portfolios from Kenneth French's website, 10 maturity-sorted treasury portfolios from WRDS, and 10 yield-sorted corporate bond portfolios from Nozawa (2017). All these portfolios are available on Asaf Manela's website and I used his

data for my tests. Table 3 shows the price of risk for each regression. All standard errors are calculated using the GMM method from Cochrane (2009), following He et al. (2017). In column 1, I run quarterly regressions using the wealth ratio factor. As predicted by the model, the price of risk is -0.31% and statistically significant at 5% level. The intercept is 0.43% and the Mean Absolute Price Error (MAPE) is 0.33%. Columns 2 and 3 show the results for stocks and bonds separately. Figure 5 visualizes the performance of the wealth share factor in the cross-section of stocks and bonds.

Column 4 shows the results for cross-sectional regressions with the wealth share factor and HKM's capital ratio factor. These results demonstrate that the wealth share factor captures a different aspect of risk than the leverage of bank holding companies. As predicted by the theory, the capital ratio factor has a positive price of risk while the wealth share factor has a negative price of risk. Column 5 shows that the negative price of risk for the wealth share factor is robust to including the market return as a second factor.

### *3.3. Robustness*

To make sure that my results are not driven by secular trends in the data, I divide my sample in half and repeat the cross-sectional regressions. The results are shown in Table 4. Even though significance levels are reduced because of the smaller sample size and thus reduced power, the point estimates are still negative.

To mitigate the concerns expressed in Lewellen et al. (2010), I repeat my cross-sectional tests adding 10 investment-, 10 momentum-, and 10 profitability-sorted portfolios from Kenneth French's website. Table 5 reports the results of these tests.

The magnitudes of the estimates remain roughly the same and the results are mostly significant.

### *3.4. Factor Mimicking Portfolio*

Because the wealth share factor is not a traded portfolio, I construct a portfolio of traded assets mimicking the wealth share factor, following the method described in Adrian et al. (2014). The portfolios I use to find the projection of the wealth share factor are 30 industry portfolios from Kenneth French's website. I run the following time-series regression

$$WSF_t = \alpha + \beta F_t + \epsilon_t, \quad (22)$$

where  $WSF$  is the wealth share factor and  $F$  is the vector of all spanning portfolios. Therefore, my mimicking portfolio returns are

$$WSMP_t = \hat{\beta} F_t. \quad (23)$$

I use this portfolio to run my cross-sectional asset pricing tests. The results are shown in Table 6. Columns 1-3 report the results for the original 45 test portfolios used in Table 3 while columns 4-6 also include the additional portfolios of Table 5. This table shows that the price of the wealth share factor mimicking portfolio is between -0.88% and -2.0% annually, depending on the test portfolios and other included factors. The MAPE and significance levels are similar to the main results in Table 3.

Finally, I test if conventional asset pricing factors can explain the wealth share factor. I run time-series regressions of the wealth share mimicking portfolio on the Fama-French 3-, 5-, and 6-factor models. Table 7 reports the results of these regres-

sions. The wealth share factor has an alpha of approximately 10 bps per quarter, or 40 bps annually. These results suggest that the wealth share factor is capturing a different aspect of risk than the Fama-French factors.

#### 4. Conclusion

Papers of intermediary asset pricing often consider only the leverage of banks or broker-dealers and ignore households' direct investment. But in the stock and bond markets, households can directly, and virtually costlessly, invest in the market. I develop a model incorporating a direct investment path for households into an intermediary asset pricing model. My model predicts that bank leverage and the wealth share of direct investors are both negatively correlated with the market and, therefore, have a negative price of risk. When the risky asset appreciates, leveraged investors stand to gain more. So the wealth share of direct investors, who are more risk averse and unlevered, decreases. Moreover, the effective risk aversion in the economy decreases, reducing the Sharpe ratio of the risky asset. A lower Sharpe ratio makes the risky asset less attractive to investors. I test these implications in the cross-section of stocks and bonds in the U.S. and find evidence supporting them. Most importantly, the price of risk for my wealth share factor is between -0.88% and -2.0% annually. Moreover, this result is robust to the inclusion of HKM's capital ratio factor or the market excess return in a two-factor pricing model. I also show that the wealth share factor has an alpha of 40 bps with respect to Fama-French 3-, 5-, and 6-factor models.

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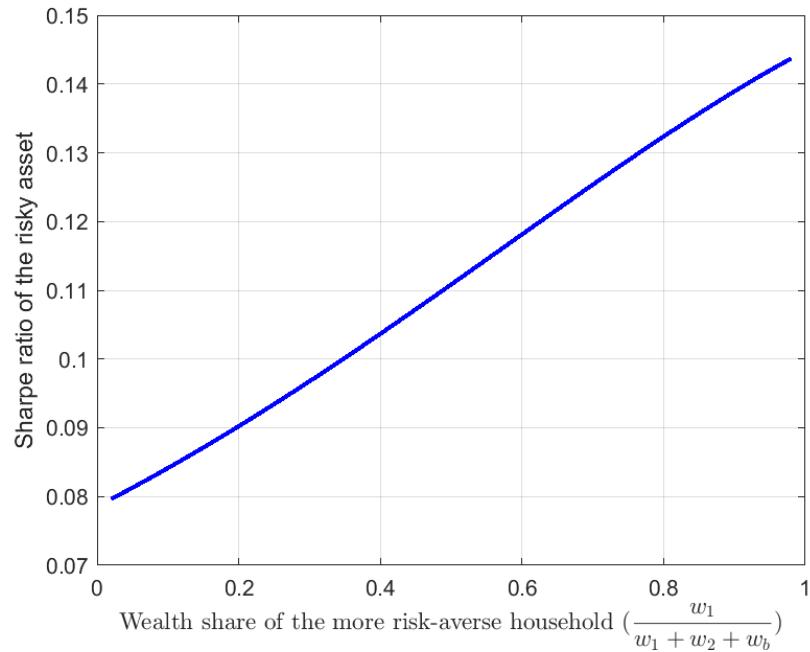


Figure 1: The Sharpe ratio of the risky asset as a function of the wealth share of the more risk-averse household. Values are numerically calculated using the equilibrium in lemma 1 and assuming  $\gamma_1 = 2\gamma_2 = 2$ ,  $\mu = 6.02\%$ ,  $\sigma = 7.29\%$ ,  $\delta = 4.0\%$ .

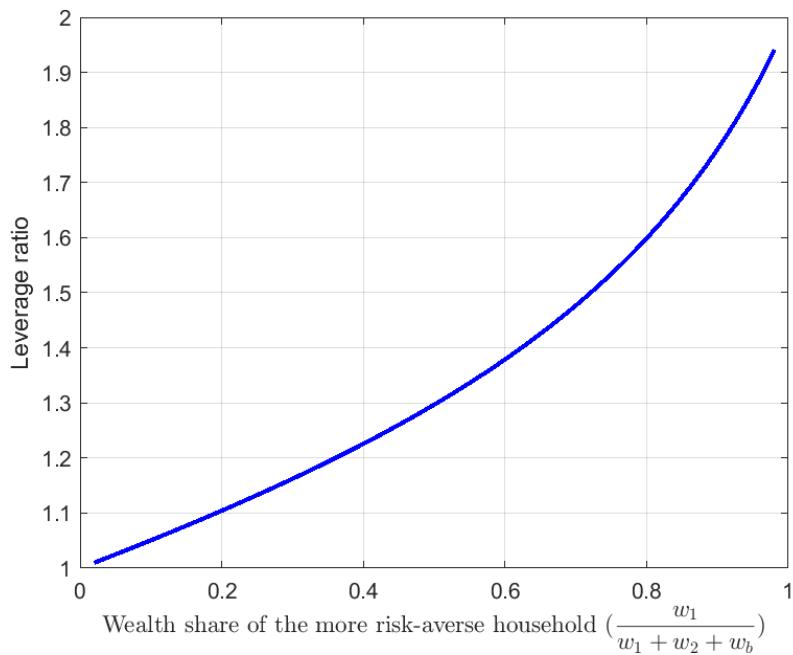


Figure 2: Bank leverage as a function of the wealth share of the more risk-averse household. Values are numerically calculated using the equilibrium in lemma 1 and assuming  $\gamma_1 = 2\gamma_2 = 2$ ,  $\mu = 6.02\%$ ,  $\sigma = 7.29\%$ ,  $\delta = 4.0\%$ .

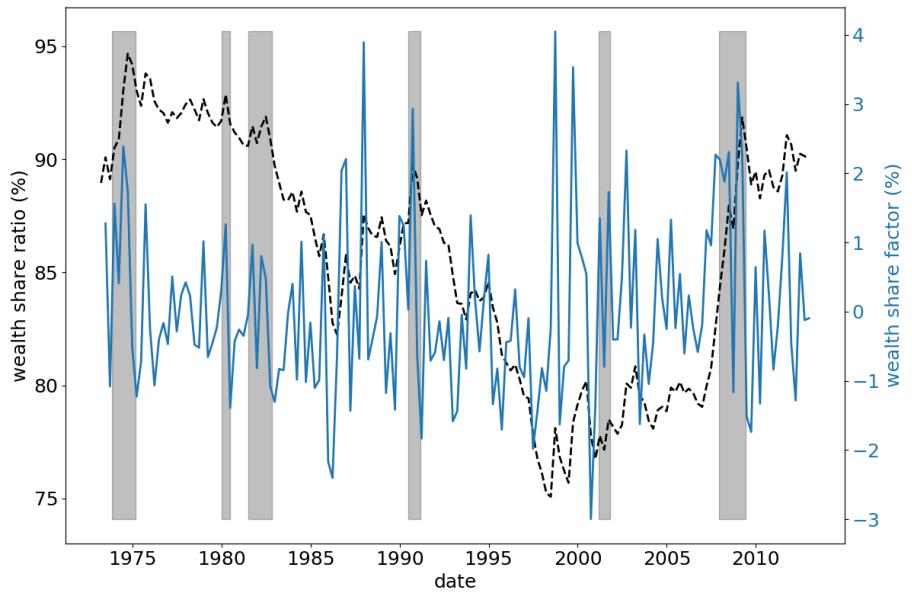


Figure 3: Quarterly time series of the wealth share ratio (black, dashed) and the wealth share factor (purple, solid), from 1973Q2 to 2012Q4. The wealth share ratio is the ratio of total AUM of mutual funds plus total deposits to the sum of total AUM of mutual funds, total deposits, and total market capitalization of financial companies. The wealth share factor is the percent changes in the wealth share ratio. The shaded areas indicate NBER recessions.

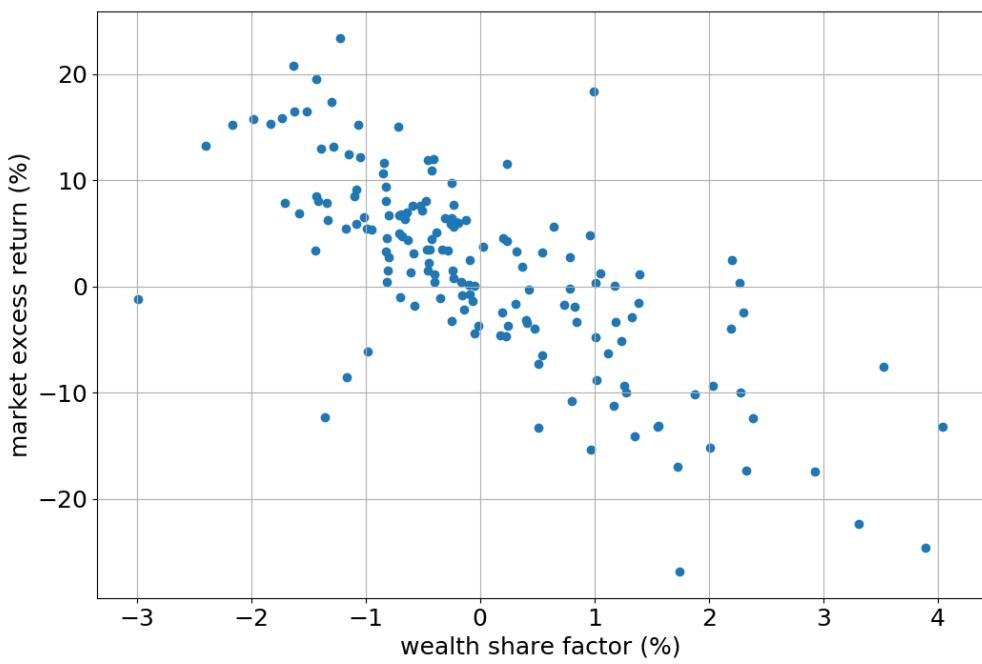


Figure 4: Scatter plot of market excess return versus the wealth share factor. The wealth share ratio is the ratio of total AUM of mutual funds plus total deposits to the sum of total AUM of mutual funds, total deposits, and total market capitalization of financial companies. The wealth share factor is the percent changes in the wealth share ratio. The data is from 1973Q2 to 2012Q4, in quarterly frequency.

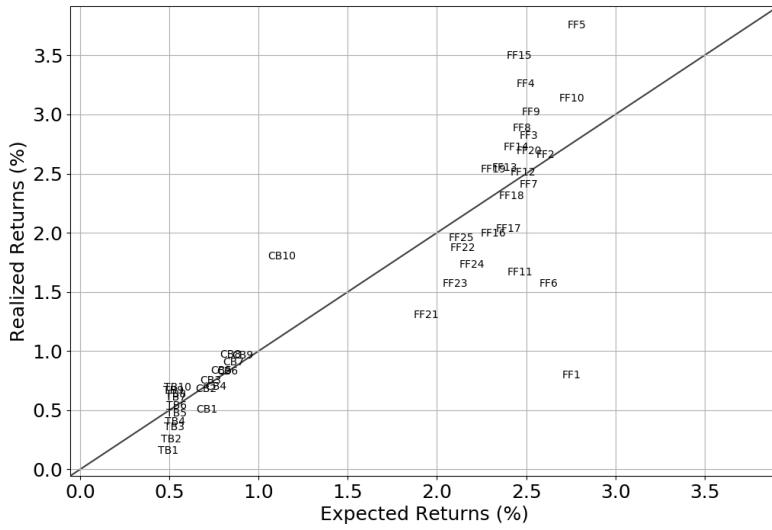


Figure 5: Realized returns versus expected returns in the cross-section of stocks and bonds using the wealth share factor. Test portfolios are 25 size- and book-to-market-sorted stock portfolios from Kenneth French's website, 10 maturity-sorted treasury portfolios from WRDS, and 10 yield-sorted portfolios from Nozawa (2017). On the horizontal axis, the expected returns are calculated using the price of risk in Table 3, column 1 and their corresponding betas. On the vertical axis is the mean realized return of each portfolio over the entire span of the data. The data is from 1973Q2 to 2012Q4, in quarterly frequency.

Table 1: Summary Statistics

*Wealth share ratio* is the ratio of AUM of all mutual funds plus total deposits to the sum of AUM of all mutual funds, total deposits, and market cap of all financial companies. *Wealth share factor* is the percent changes in the wealth share ratio. *HKM* is the intermediary capital ratio factor of He et al. (2017). *Market* is the excess return on the market portfolio. The data is in quarterly frequency and covers 1973Q2 to 2012Q4.

	count	mean	std	min	1%	10%	25%	50%	75%	90%	99%	max
Wealth share factor	159	0.0%	1.2%	-3.0%	-2.3%	-1.4%	-0.8%	-0.2%	0.8%	1.7%	3.7%	4.0%
Wealth share ratio	159	85.7%	5.3%	75.1%	75.5%	78.2%	80.2%	86.6%	90.3%	92.0%	94.0%	94.7%
HKM capital ratio factor	159	-0.1%	12.8%	-43.7%	-31.5%	-15.3%	-7.7%	1.4%	7.9%	13.7%	30.9%	44.5%
Market	159	1.5%	9.1%	-26.9%	-23.3%	-10.9%	-3.4%	2.5%	6.8%	12.5%	20.0%	23.4%

Table 2: Pairwise Correlations

*Wealth share factor* is the percent changes in the wealth share ratio, defined as the ratio of the AUM of all mutual funds plus total deposits to the sum of the AUM of all mutual funds, total deposits, and market cap of all financial companies. *HKM capital factor* is the intermediary capital ratio factor of He et al. (2017). *Market* is the excess return of the market portfolio. The data is in quarterly frequency and covers 1973Q2 to 2012Q4.

	Wealth share factor	HKM capital factor	Market
Wealth share factor	1.00		
HKM capital factor	-0.81	1.00	
Market	-0.73	0.80	1.00

Table 3: Cross-Sectional Asset Pricing Tests

In this test, I calculate the beta for each test portfolio from a time-series regression of portfolio returns on the factors. Next, I run a cross-sectional regression of the mean portfolio returns on the betas and report the price of risk for each factor together with its t-statistic calculated using the GMM method in Cochrane (2009) in parentheses. Test portfolios are: 25 size- and book-to-market-sorted stock portfolios from Kenneth French's website, 10 maturity-sorted treasury portfolios from WRDS, and 10 yield-sorted portfolios from Nozawa (2017). The data has quarterly frequency from 1973Q2 to 2012Q4. *MAPE* is the mean absolute pricing error for all portfolios. *Wealth share factor* is percent changes in the ratio of the AUM of all mutual funds plus total deposits to the sum of the AUM of all mutual funds, total deposits, and market cap of all financial companies. *HKM* is the intermediary capital factor of He et al. (2017). *Market* is the excess return of the market portfolio.

	1	2	3	4	5
Wealth share factor	-0.31 (-2.11)	-0.45 (-1.53)	-0.56 (-2.59)	-0.51 (-2.92)	-0.55 (-3.24)
HKM				1.07 (1.07)	
Market					1.37 (1.60)
Intercept	0.43 (1.83)	-0.48 (-0.34)	0.29 (1.40)	0.46 (2.03)	0.39 (1.60)
MAPE	0.33 148	0.44 159	0.13 148	0.46 148	0.25 148
N					
Portfolios	stocks and bonds	stocks only	bonds only	stocks and bonds	stocks and bonds

Table 4: Subsample Tests

In this test, I calculate the beta for each test portfolio from a time-series regression of portfolio returns on the factors. Next, I run a cross-sectional regression of the mean portfolio returns on the betas and report the price of risk for each factor together with its t-statistic calculated using the GMM method in Cochrane (2009) in parentheses. Test portfolios are: 25 size- and book-to-market-sorted stock portfolios from Kenneth French's website, 10 maturity-sorted treasury portfolios from WRDS, and 10 yield-sorted portfolios from Nozawa (2017). The data has quarterly frequency from 1973Q2 to 2012Q4. Columns 1-3 report the results for the first half of the sample. Columns 4-6 report the results for the second half of the sample. *MAPE* is the mean absolute pricing error for all portfolios. *Wealth share factor* is percent changes in the ratio of the AUM of all mutual funds plus total deposits to the sum of the AUM of all mutual funds, total deposits, and market cap of all financial companies. *HKM* is the intermediary capital factor of He et al. (2017). *Market* is the excess return of the market portfolio.

	First Half (1973Q2-1992Q4)			Second Half (1993Q1-2012Q4)		
Wealth share factor	-0.27 (-1.61)	0.00 (-0.01)	-0.83 (-2.37)	-0.31 (-1.34)	-0.38 (-1.79)	-0.40 (-1.84)
HKM		6.70 (2.51)			2.31 (1.06)	
Market			1.51 (1.03)			1.22 (1.06)
Intercept	0.27 (0.59)	-0.06 (-0.11)	0.02 (0.04)	0.67 (4.19)	0.67 (4.11)	0.67 (4.13)
MAPE	0.44	0.48	0.23	0.36	0.36	0.36
N	73	73	73	76	76	76

Table 5: More Test Assets

In this test, I calculate the beta for each test portfolio from a time-series regression of portfolio returns on the factors. Next, I run a cross-sectional regression of the mean portfolio returns on the betas and report the price of risk for each factor together with its t-statistic calculated using the GMM method in Cochrane (2009) in parentheses. Test portfolios are: 25 size- and book-to-market-sorted, 10 momentum-sorted, 10 investment-sorted, and 10 profitability-sorted stock portfolios from Kenneth French's website, 10 maturity-sorted treasury portfolios from WRDS, and 10 yield-sorted portfolios from Nozawa (2017). The data has quarterly frequency from 1973Q2 to 2012Q4. *MAPE* is the mean absolute pricing error for all portfolios. *Wealth share factor* is percent changes in the ratio of the AUM of all mutual funds plus total deposits to the sum of the AUM of all mutual funds, total deposits, and market cap of all financial companies. *HKM* is the intermediary capital factor of He et al. (2017). *Market* is the excess return of the market portfolio.

	1	2	3
Wealth share factor	-0.25 (-1.79)	-0.56 (-3.17)	-0.49 (-3.01)
HKM		0.36 (0.21)	
Market			1.19 (1.46)
Intercept	0.44 (1.87)	0.45 (2.01)	0.42 (1.76)
MAPE	0.44	0.39	0.39
N	148	148	148

Table 6: Cross-Sectional Tests with the Factor Mimicking Portfolio

In this test, I calculate the beta for each test portfolio from a time-series regression of portfolio returns on the factors. Next, I run a cross-sectional regression of the mean portfolio returns on the betas and report the price of risk for each factor together with its t-statistic calculated using the GMM method in Cochrane (2009) in parentheses. Test portfolios are: 25 size- and book-to-market-sorted stock portfolios from Kenneth French's website, 10 maturity-sorted treasury portfolios from WRDS, and 10 yield-sorted portfolios from Nozawa (2017). The data has quarterly frequency from 1973Q2 to 2012Q4. In columns 4-6, I add 10 momentum-sorted, 10 investment-sorted, and 10 profitability-sorted stock portfolios. *MAPE* is the mean absolute pricing error for all portfolios. *WSMP* is the mimicking portfolio for the wealth share factor. *HKM* is the intermediary capital ratio factor of He et al. (2017). *Market* is the excess return on the market portfolio.

	1	2	3	4	5	6
WSMP	-0.28 (-2.09)	-0.45 (-2.92)	-0.50 (-3.28)	-0.22 (-1.74)	-0.43 (-3.00)	-0.40 (-2.86)
HKM		0.90 (0.47)			0.41 (0.24)	
Market			1.51 (1.78)			1.28 (1.59)
Intercept	0.39 (1.56)	0.23 (0.96)	0.23 (0.92)	0.44 (1.79)	0.23 (0.98)	0.34 (1.38)
MAPE	0.33	0.28	0.25	0.37	0.39	0.40
N	148	148	148	148	148	148
Num. Port.	45	45	45	75	75	75

Table 7: Time-Series Regressions of the Wealth Share Mimicking Portfolio on Fama-French Factors

In this test, I run regressions of the form  $WSMP_t = \alpha + \beta F_t + \epsilon_t$ , where  $WSMP$  is the wealth share mimicking portfolio and  $F_t$  is a vector of asset pricing factors. The table reports  $\alpha$  and  $\beta$  for each regression alongside their t-statistics. The factors are Fama-French 3-, 5-, and 6- factor models. The data has quarterly frequency from 1973Q2 to 2012Q4.

	3-Factor	5-Factor	6-Factor
alpha (%)	0.09 (1.92)	0.11 (2.22)	0.11 (1.98)
Mkt	-0.12 (-20.0)	-0.12 (-19.5)	-0.12 (-19.2)
SMB	0.01 (0.63)	0.00 (0.43)	0.00 (0.52)
HML	-0.08 (-9.31)	-0.08 (-7.52)	-0.08 (-7.12)
RMW		-0.03 (-2.68)	-0.03 (-2.70)
CMA		0.02 (1.09)	0.02 (1.00)
MOM			0.00 (0.47)
R-sq (%)	75.5	76.8	76.8
N	148	148	148

## A. Proofs

*Lemma 1*

First, I prove that each equilibrium of this economy corresponds to an equilibrium in Wang's (1996) two-agent, pure-exchange economy. Then I use the results in Wang (1996) to characterize the equilibrium. Notice the market is complete. So the first welfare theorem holds and every competitive equilibrium gives a Pareto-optimal allocation. We can find all Pareto-optimal allocations through solving the social planner's problem. Let us assume that the social planner assigns weights  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_b$  to household 1, household 2, and the bank respectively. We can write social planner's problem as

$$\begin{aligned} \max_{c_{1t}, c_{2t}, c_{bt}} \quad & E \int_0^\infty e^{-\delta t} (\lambda_1 u_1(c_{1t}) + \lambda_2 u_2(c_{2t}) + \lambda_b u_b(c_{bt})) dt \\ \text{s.t.} \quad & c_{1t} + c_{2t} + c_{bt} = D_t. \end{aligned} \tag{A.1}$$

The Kuhn-Tucker conditions for this problem can be written as

$$\lambda_1 u'_1(c_{1t}) = \lambda_2 u'_2(c_{2t}) = \lambda_b u'_b(c_{bt}) \tag{A.2}$$

$$c_{1t} + c_{2t} + c_{bt} = D_t. \tag{A.3}$$

This is a system of three equations with three unknowns. Using the fact that  $\gamma_2 = \gamma_b$ , we can prove  $c_{2t} = \zeta c_{bt}$ , for  $\zeta = \exp(\frac{1}{\gamma_2} \ln(\frac{\lambda_2}{\lambda_b}))$ . I claim that  $\{c_{2t}(1 + \zeta), c_{1t}\}$  is a Pareto optimal allocation of Wang (1996). To find the weighting scheme that gives this allocation, notice the first order conditions for this allocation in Wang's (1996) model are

$$\lambda'_1 [c_{1t}(1 + \zeta)]^{-\gamma_1} = \lambda'_2 c_{2t}^{-\gamma_2} \tag{A.4}$$

$$c_{1t}(1 + \zeta) + c_{2t} = D_t, \quad (\text{A.5})$$

where  $\{\lambda'_1, \lambda'_2\}$  denote the weighting scheme of social planner's problem in Wang (1996). Notice that (A.5) is the same as (A.3). Equation (A.4) is also equivalent to (A.2) given that  $\lambda'_1$  and  $\lambda'_2$  satisfy the following conditions

$$\begin{aligned} \frac{(1 + \zeta)^{-\gamma_1}}{\lambda_1} \lambda'_1 - \frac{1}{\lambda_2} \lambda'_2 &= 0 \\ \lambda'_1 + \lambda'_2 &= 1 \end{aligned} \quad (\text{A.6})$$

The system in (A.6) has a unique solution, which gives a weighting scheme that maps the given Pareto-optimal allocation in my model to an equilibrium in Wang (1996). The formulas in equation (12) can now be derived from theorem 1 in Wang (1996) using straightforward algebra.

### *Lemma 2*

Because the total wealth of the economy is the price of the risky asset, we can say  $\frac{M_t}{P_t} = \frac{1}{g} \frac{gf'}{g'} = \frac{f'}{g'}$ . Moreover, since bank manager's leverage and household 2's leverage are equal and both equal to bank's leverage, we can write

$$\alpha_t = \frac{\text{Bank's AUM}}{\text{Bank's wealth}} = \frac{1 - \frac{f'}{g'}}{1 - \frac{f}{g}} \quad (\text{A.7})$$

To calculate the covariances, we need to calculate the derivative of SDF,  $\alpha_t$ , and  $\theta_{1t}$  with respect to  $D_t$ .

$$dm_t - E_t[dm_t] = -\gamma_1 \left( \frac{2}{\eta} \right)^{-\gamma_1-1} \frac{(\sqrt{1 + \eta D_t} - 1)^{-\gamma_1-1}}{\sqrt{1 + \eta D_t}} \sigma D_t dZ_t, \quad (\text{A.8})$$

$$d\alpha_t - E_t[d\alpha_t] = \left( -\frac{f''g' - g''f'}{(g')^2(1 - \frac{f}{g})} + \frac{f'g - g'f}{g^2(1 - \frac{f}{g})^2} \left( 1 - \frac{f'}{g'} \right) \right) \sigma D_t dZ_t, \quad (\text{A.9})$$

$$d\theta_{1t} - E_t[d\theta_{1t}] = \frac{f''g' - g''f'}{(g')^2} \sigma D_t dZ_t. \quad (\text{A.10})$$

Multiply A.8 by A.9 and A.10 to get equations 18 and 19, respectively.

*Theorem 1*

Section 2.7 proves that if the return on the risky asset exceeds the return on the risk-free asset, then the wealth share of the more risk-averse household decreases. This is equivalent to the wealth share of the more risk-averse household being negatively correlated with the contemporaneous return. To prove that bank leverage is negatively priced, notice that  $dm_t - E_t[dm_t] < 0$ . So it suffices to show that both  $d\theta_{1t} - E_t[d\theta_{1t}]$  and  $d\alpha_t - E_t[d\alpha_t]$  are negative. Also notice  $\frac{f}{g}$  is the wealth share of the more risk-averse household and  $\frac{f'}{g'}$  is the fraction of the risky asset owned by the more risk-averse household. As a result, both these quantities are between 0 and 1. To prove that  $d\alpha_t - E_t[d\alpha_t] \leq 0$ , notice that  $d\alpha_t - E_t[d\alpha_t] = \frac{d\alpha_t}{dD_t} \sigma D_t dZ_t$ . So we have to prove that  $\frac{d\alpha_t}{dD_t} < 0$ . Calculating  $\frac{d\alpha_t}{dD_t}$  gives

$$\frac{d\alpha_t}{dD_t} = \frac{\frac{d}{dD_t}(1 - \frac{f'}{g'})}{1 - \frac{f}{g}} - \frac{\frac{d}{dD_t}(1 - \frac{f}{g})(1 - \frac{f'}{g'})}{(1 - \frac{f}{g})^2}.$$

As a result, we have to show that

$$\frac{\frac{d}{dD_t}(1 - \frac{f'}{g'})}{1 - \frac{f}{g}} \leq \frac{\frac{d}{dD_t}(1 - \frac{f}{g})(1 - \frac{f'}{g'})}{(1 - \frac{f}{g})^2}.$$

Rearranging the terms, we get

$$\frac{\frac{d}{dD_t}(\frac{f'}{g'})}{\frac{d}{dD_t}(\frac{f}{g})} \leq \frac{1 - \frac{f'}{g'}}{1 - \frac{f}{g}}.$$

The left-hand side is equal to  $\frac{d\theta_{1t}}{dx_t}$ , which is less than one because the more risk-averse household is the marginal lender and does not invest all its wealth in the risky asset. The right-hand side is the bank leverage which is larger than one. So the inequality must hold.