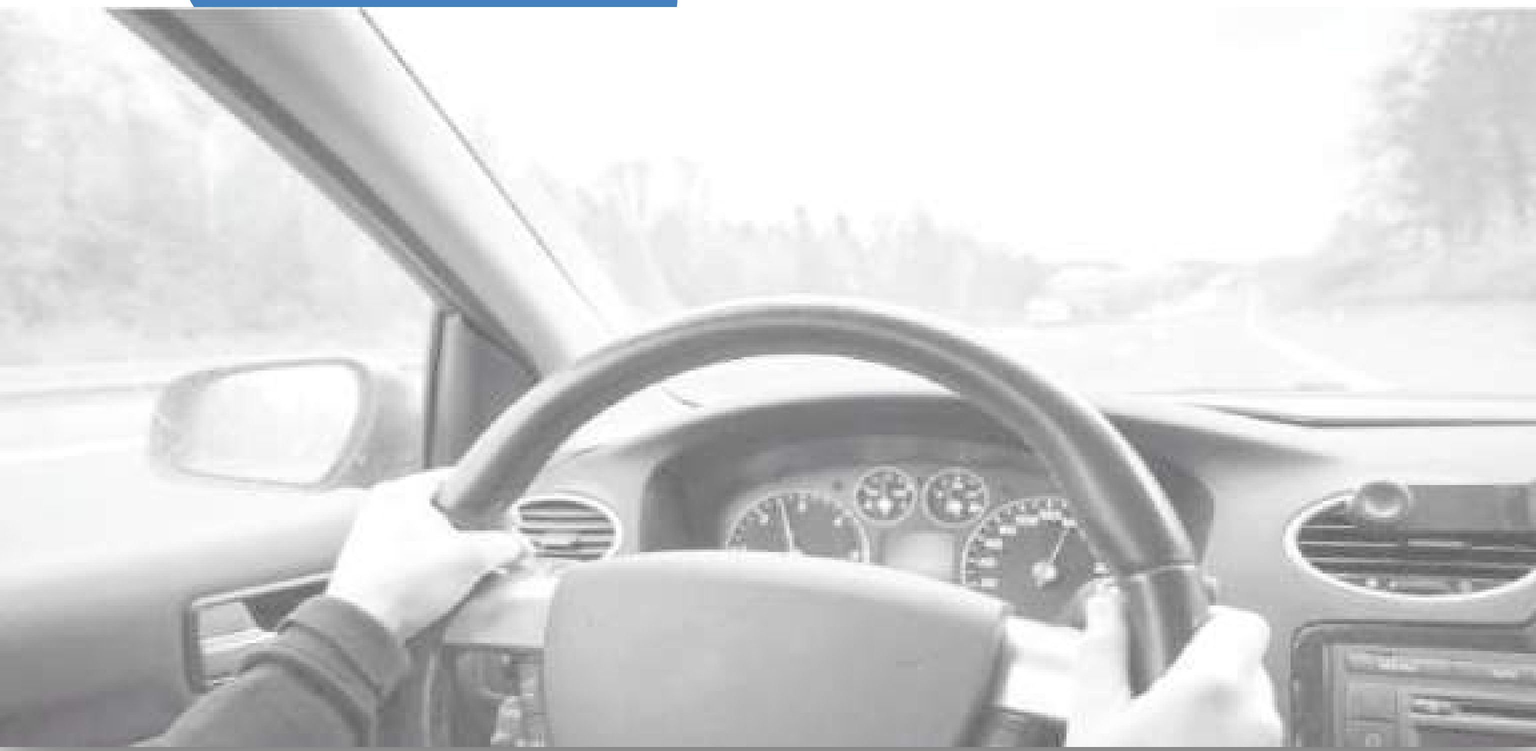
Number Systems 1

NCERT SOLUTIONS



What's inside

– *Textbook Exercise Q's (solved)*

EduCart

Exercise – 1.1

1. **Is zero a rational number ? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?**

Sol. When zero is a rational number then it can be written in the form $\frac{p}{q}$ as:

$0 = \frac{0}{1}$, where $p = 0$ and $q = 1$

According to the question, the value of q in $\frac{p}{q}$ can be any number, for example: $\frac{0}{10}, \frac{0}{20}, \frac{0}{30}$ etc.

2. Find six rational numbers between 3 and 4. [

Sol. The rational number between two rational numbers

$$= \frac{\text{First number + Second number}}{2}$$

The rational number between 3 and 4 $= \frac{3+4}{2} = \frac{7}{2}$

$\therefore 3 < \frac{7}{2} < 4$

Now, rational number between 3 and $\frac{7}{2} = \frac{1}{2}\left(3+\frac{7}{2}\right)$

$$= \frac{1}{2}\left(\frac{6+7}{2}\right)$$

$$= \frac{13}{4}$$

$\therefore 3 < \frac{13}{4} < \frac{7}{2} < 4$

Now, rational number between $\frac{7}{2}$ and 4 $= \frac{1}{2}\left(\frac{7}{2}+4\right)$

$$= \frac{1}{2}\left(\frac{7+8}{2}\right)$$

$$= \frac{15}{4}$$

$\therefore 3 < \frac{13}{4} < \frac{7}{2} < \frac{15}{4} < 4$

Now, rational number between 3 and $\frac{13}{4} = \frac{1}{2}\left(3+\frac{13}{4}\right)$

$$= \frac{1}{2}\left(\frac{12+13}{4}\right)$$

Click here for all Solved PYQs & New pattern Q's Educart book

$$= \frac{25}{8}$$

$$\therefore 3 < \frac{25}{8} < \frac{13}{4} < \frac{7}{2} < \frac{15}{4} < 4$$

Now, rational number between $\frac{15}{4}$ and $4 = \frac{1}{2}\left(\frac{15}{4} + 4\right)$

$$= \frac{1}{2}\left(\frac{15+16}{4}\right)$$

$$= \frac{31}{8}$$

$$\therefore 3 < \frac{25}{8} < \frac{13}{4} < \frac{7}{2} < \frac{15}{4} < \frac{31}{8} < 4$$

Now, rational number between $\frac{31}{8}$ and $4 = \frac{1}{2}\left(\frac{31}{8} + 4\right)$

$$= \frac{1}{2}\left(\frac{31+32}{8}\right)$$

$$= \frac{63}{16}$$

Hence, six rational numbers between 3 and 4 are:
$\frac{25}{8}, \frac{13}{4}, \frac{7}{2}, \frac{15}{4}, \frac{31}{8}$ and $\frac{63}{16}$.

3. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Sol. Rational number between $\frac{3}{5}$ and $\frac{4}{5} = \frac{1}{2}\left(\frac{3}{5} + \frac{4}{5}\right)$

$$= \frac{1}{2}\left(\frac{3+4}{5}\right)$$

$$= \frac{7}{10}$$

Rational number between $\frac{3}{5}$ and $\frac{7}{10} = \frac{1}{2}\left(\frac{3}{5} + \frac{7}{10}\right)$

$$= \frac{1}{2}\left(\frac{6+7}{10}\right)$$

$$= \frac{13}{20}$$

Rational number between $\frac{7}{10}$ and $\frac{4}{5} = \frac{1}{2}\left(\frac{7}{10} + \frac{4}{5}\right)$

$$= \frac{1}{2}\left(\frac{7+8}{10}\right)$$

$$= \frac{15}{20}$$

Click here for all Solved PYQs & New pattern Q's Educart book

Rational number between $\frac{13}{20}$ and $\frac{7}{10} = \frac{1}{2}\left(\frac{13}{20}+\frac{7}{10}\right)$

$$= \frac{1}{2}\left(\frac{13+14}{20}\right)$$

$$= \frac{27}{40}$$

Rational number between $\frac{15}{20}$ and $\frac{4}{5} = \frac{1}{2}\left(\frac{15}{20}+\frac{4}{5}\right)$

$$= \frac{1}{2}\left(\frac{15+16}{20}\right)$$

$$= \frac{31}{40}$$

Hence, five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are

$\frac{13}{20}, \frac{27}{40}, \frac{7}{10}, \frac{15}{20}$ and $\frac{31}{40}$.

4.State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Sol. (i) True, every natural number is a whole number.

(ii) False, because negative numbers are included in integers, but whole numbers are 0, 1, 2, 3, ...

(iii) False, for example $\frac{1}{2}$ is a rational number but not a whole number.

Exercise – 1.2

1.State whether the following statements are true or false. Justify your answer.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form, where *m* is a natural number.

(iii) Every real number is an irrational number.

Sol. (i) This statement is true because irrational and rational numbers together form real numbers. Therefore, every irrational number is a real number.

(ii) This statement is false, because all real numbers can be represented on the number line. Here m is a natural number which show that only the points $\sqrt{1}, \sqrt{2}, \sqrt{3}$, ... lie on the number line. Whereas the reality is that there is a wide gap between the points representing any two consecutive numbers; For example $\sqrt{2}$ = 1.414 and $\sqrt{3}$ = 1.732, then the next number between 1.414 and 1.732 also get a place on the number line. Apart from this, negative number lie on the number line. No negative number can be the square roof of any natural number. Hence, every point on the number line cannot be represented by $\sqrt{m}$, where m is a natural number.

Click here for all Solved PYQs & New pattern Q's Educart book

(iii) This statement is false, because the set of real numbers is formed by the collection of rational and irrational numbers. So, every irrational number can be a real number but every real number need not be irrational.

2.Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Sol. No, the square roots of all positive integers are not irrational. Such as 4, 9, 16, 25, 36, ... etc. are positive integers, but their square roots are rational numbers, not an irrational number, such as:

$\sqrt{4}$ = 2 a rational number

$\sqrt{9}$ = 3 a rational number etc.

3.Show how $\sqrt{5}$ can be represented on the number line.

Sol. Here we write by $5 = (2)^2 + 1$.

First draw a number line. Construct a right angled ΔOAB on the number line by taking OA = 2 units and AB = 1 unit.

In ΔAOB,

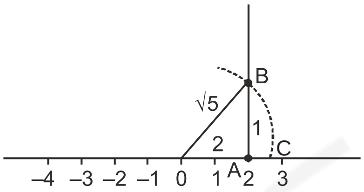
$OB = \sqrt{OA^2 + AB^2}$ [From Pythagoras Theorem]

$= \sqrt{(2)^2 + (1)^2} = \sqrt{5}$

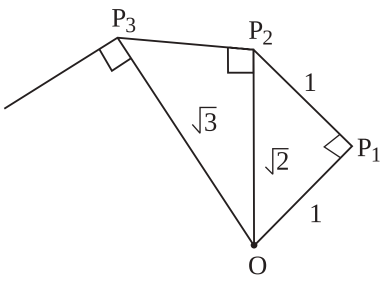
Taking O as the centre and OB as the radius mark an arc with the help of compass which intersects the number line at point C.

$OC = OB = \sqrt{5}$

Hence, point C is represented on the number line.



4.Classroom activity (Constructing the 'square root spiral') : Take a large sheet of paper and construct the 'square root spiral' in the following fashion. Start with a point O and draw a line segment OP_1 of unit length. Draw a line segment $P_1 P_2$ perpendicular to OP_1 of unit length (see Fig.). Now draw a line segment P_2P_3 perpendicular to OP_2. Then draw a line segment P_3P_4 perpendicular to OP_3. Continuing in this manner, you can get the line segment $P_{n-1} P_n$ by drawing a line segment of unit length perpendicular to OP_{n-1}. In this manner, you will have created the points P_2, P_3 ...., P_n, ... , and joined them to create a beautiful spiral depicting $\sqrt{2}, \sqrt{3}, \sqrt{4}, ...$



Click here for all Solved PYQs & New pattern Q's Educart book

Constructing square root spiral

Sol. Take a large sheet of paper. First take a point O on it and draw a line segment OP_1 of unit length. Draw a line segment P_1P_2 perpendicular to point P_1 of unit length, now join OP_2. Line segment OP_2 represents $\sqrt{2}$. Similarly, draw a line segment P_2P_3 of unit length on point P_2, then join OP_3 which represents $\sqrt{3}$. Similarly draw a perpendicular line segment P_3P_4 on P_3 of unit length and join OP_4 which represents $\sqrt{4}$. By doing a similar activity we get a square roof spiral showing $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, ...

Exercise – 1.3

1. Write the following in decimal form and say what kind of decimal expansion each has :

(i) $\frac{36}{100}$ **(ii)** $\frac{1}{11}$

(iii) $4\frac{1}{8}$ **(iv)** $\frac{3}{13}$

(v) $\frac{2}{11}$ **(vi)** $\frac{329}{400}$

Sol. (i) $\frac{36}{100} = 0.36$

So, $\frac{36}{100}$ has terminating decimal expansion.

(ii) $\frac{1}{11}$

By division method :

$$\begin{array}{r|l} & 0.09090909 \\ \hline 11 & 100 \\ & \ \ 99 \\ \hline & \ \ \ 100 \\ & \ \ \ \ 99 \\ \hline & \ \ \ \ \ 100 \\ & \ \ \ \ \ \ 99 \\ \hline & \ \ \ \ \ \ \ 100 \\ & \ \ \ \ \ \ \ \ 99 \\ \hline & \ \ \ \ \ \ \ \ \ 1 \end{array}$$

$\frac{1}{11} = 0.09090909 \ldots = 0.\overline{09}$

So, $\frac{1}{11}$ has a non-terminating repeating decimal expansion.

(iii) $4\frac{1}{8}$

Click here for all Solved PYQs & New pattern Q's Educart book

By division method:

	4·125
8	33
	32
	10
	8
	20
	16
	40
	40
	0

$4\frac{1}{8} = 41.25$

So, $4\frac{1}{8}$ has a terminating decimal expansion.

(iv) $\frac{3}{13}$

	0·230769230769
13	30
	26
	40
	39
	100
	91
	90
	78
	120
	117
	30
	26
	40
	39
	100
	91
	90
	78
	120
	117
	3

$\frac{3}{13} = 0.230769230769 \ldots = 0.\overline{230769}$

EduCart

Click here for all Solved PYQs & New pattern Q's Educart book

So, $\frac{3}{13}$ has a non-terminating repeating decimal expansion.

(v) $\frac{2}{11}$

	0·1818
11	20
	11
	90
	88
	20
	11
	90
	88
	2

$\frac{2}{11}$ = 0.1818 ... = $0.\overline{18}$

So, $\frac{2}{11}$ has a non-terminating repeating decimal expansion.

(vi) $\frac{329}{400}$

	0·8225
400	3290
	3200
	900
	800
	1000
	800
	2000
	2000
	0

$\frac{329}{400}$ = 0.8225

So, $\frac{329}{400}$ has a terminating decimal expansion.

2. You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division ? If yes, how ?

Sol. We have, $\frac{1}{7} = 0.\overline{142857}$

Click here for all Solved PYQs & New pattern Q's Educart book

$$\therefore \quad \frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$$

Similarly,

$$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times 0.\overline{142857} = 0.\overline{428571}$$

$$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times 0.\overline{142857} = 0.\overline{571428}$$

$$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times 0.\overline{142857} = 0.\overline{714285}$$

$$\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times 0.\overline{142857} = 0.\overline{857142}$$

3. .Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$:

(i) $0.\overline{6}$ **(ii)** $0.\overline{47}$ **(iii)** $0.\overline{001}$

Sol. (i) Let $x = 0.\overline{6}$

$x = 0.6666...$ ...(i)

Multiplying by 10 both sides

$10x = 6.6666...$ ...(ii)

Subtracting eq. (i) from (ii), we get

$9x = 6$

$x = \frac{6}{9} = \frac{2}{3}$

Hence, $0.\overline{6} = \frac{2}{3}$

(ii) Let $x = 0.47$

$x = 0.47777...$ ...(i)

Multiplying eq. (i) by 10

$10x = 4.7777...$ ...(ii)

Multiplying eq. (i) by 100

$100x = 47.7777...$ ...(iii)

Subtracting eq. (ii) from (iii), we get

$90x = 43$

Click here for all Solved PYQs & New pattern Q's Educart book

$$x = \frac{43}{90}$$

Hence, $0.47 = \frac{43}{90}$

(iii) Let $x = 0.\overline{001}$

$x = 0.001001001...$ ...(i)

Multiplying eq. (i) by 1000, we get

$1000x = 1.001001001...$ ...(ii)

Subtracting eq. (i) from (ii), we get

$999x = 1$

$$x = \frac{1}{999}$$

Hence, $0.\overline{001} = \frac{1}{999}$

4. Express 0.999999.... in the form $\frac{p}{q}$. Are you surprised by your answer ? With your teacher and classmates discuss why the answer makes sense ?

Sol. Let $x = 0.9999...$ ...(i)

Multiplying eq. (i) by 10

$10x = 9.9999...$ ...(ii)

Subtracting eq. (i) from (ii), we get

$9x = 9$

$$x = \frac{9}{9} = 1$$

Hence, $0.9999... = 1$

Yes, we are surprised to get an integer 1, because after decimal, 9 is repeatedly infinitely and this will tend to an integer.

5. .What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Click here for all Solved PYQs & New pattern Q's Educart book

Sol.

	0·0588235294117647…
17	1·0000000000000000
	−85
	150
	−136
	140
	−136
	40
	−34
	60
	−51
	90
	−85
	50
	−34
	160
	−153
	70
	−68
	20
	−17
	30
	−17
	130
	−119
	110
	−102
	80
	−68
	120
	−119
	1

$\therefore \ \frac{1}{17} = 0.588235294117647$

Thus, $\frac{1}{17} = 0.\overline{0588235294117647}$, a block of 16 digits is repeated.

6. .Look at several examples of rational numbers in the form $\frac{p}{q}$ ($q \neq 0$), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy ?

Sol. Rational numbers of the form $\frac{p}{q}$, ($q \neq 0$) can be $\frac{1}{4}, \frac{7}{8}, \frac{37}{25}, \frac{639}{250}, \frac{7}{16}, \frac{11}{25}$, ... etc., which have terminating decimal representations. According to the definition of terminating decimal, when the denominator of rational number is in the power of 2 or 5 or both, then such rational

Click here for all Solved PYQs & New pattern Q's Educart book

numbers give a terminating decimal. In other words, it can also be said that to represent the rational number $\frac{p}{q}$. ($q \neq 0$) in terminating decimal form, it is necessary that every q be taken as the prime factorization of q only to the power of 2 or power of 5 or both.

7. .Write three numbers whose decimal expansions are non-terminating non-recurring.

Sol. We know that all the rational numbers are non-terminating non-recurring.

So, three numbers whose decimal expansions are non-terminating non-securring are:

0.04004000400004..., 0.505005000500005... and 0.007000700007...

8. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Sol. Decimal representation of rational number $\frac{5}{7}$

$$
\begin{array}{r|l}
 & 0{\cdot}714285\ldots \\
\hline
7 & 5{\cdot}0 \\
 & -49 \\
\hline
 & 10 \\
 & -7 \\
\hline
 & 30 \\
 & -28 \\
\hline
 & 20 \\
 & -14 \\
\hline
 & 60 \\
 & -56 \\
\hline
 & 40 \\
 & -35 \\
\hline
 & 5
\end{array}
$$

So, $\frac{5}{7} = 0.\overline{714285}$

Now, decimal representation of rational number $\frac{9}{11}$

$$
\begin{array}{r|l}
 & 0{\cdot}81 \\
\hline
11 & 9{\cdot}0 \\
 & -88 \\
\hline
 & 20 \\
 & -11 \\
\hline
 & 9
\end{array}
$$

So, $\frac{9}{11} = 0.\overline{81}$

The required irrational numbers will lie between $0.\overline{714285}$ and $0.\overline{81}$.

Also, the irrational numbers have non-terminating non-recurring.

$\therefore$ Required three irrational numbers are:

Click here for all Solved PYQs & New pattern Q's Educart book

0.75075007500075000075..., 0.767607600760076...... and 0.808008000800000...

9. Classify the following numbers as rational or irrational :

(i) $\sqrt{23}$ **(ii)** $\sqrt{225}$

(iii) 0.3796 **(iv) 7.478478......**

(v) 1.101001000100001.....

Sol. (i) $\sqrt{23}$ is irrational number because 23 is not a perfect square.

(ii) $\sqrt{225} = \sqrt{3\times3\times5\times5}$

$= 3 \times 5 = 15$

Hence, $\sqrt{225}$ is a rational number.

(iii) 0.3796 is terminating, so 0.3796 is a rational number.

(iv) 7.478478... is non terminating repeating decimal. So it is a rational number.

(v) 1.101001000100001... is non-terminating non-recurring decimal. So, it is an irrational number.

Exercise – 1.4

1. Classify the following numbers as rational or irrational:

(i) $2-\sqrt{5}$ **(ii)** $\left(3+\sqrt{23}\right)-\sqrt{23}$ **(iii)** $\frac{2\sqrt{7}}{7\sqrt{7}}$

(iv) $\frac{1}{\sqrt{2}}$ **(v)** 2π

Sol. (i) $2-\sqrt{5}$

In the given number 2 is rational number and $\sqrt{5}$ is irrational number. So, $\sqrt{5}$ is subtracted from 2. Now subtracting we will get an irrational number.

Hence, $2-\sqrt{5}$ is an irrational number.

(ii) $(3+\sqrt{23})-\sqrt{23} = 3+\sqrt{23}-\sqrt{23}$

= 3 which is a rational number

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}} = \frac{2}{7}$ which is a rational number

(iv) $\frac{1}{\sqrt{2}}$ is irrational number.

Because the quotient of a non-zero rational number with an irrational number is irrational.

(v) 2π

2 is rational number and π is an irrational number. So, we know that the product of a non-zero rational number with an irrational number is irrational.

Hence, 2π is irrational number.

2. Simplify each of the following expressions :

(i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$ **(ii)** $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

(iii) $\left(\sqrt{5}+\sqrt{2}\right)^2$ **(iv)** $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$

Click here for all Solved PYQs & New pattern Q's Educart book

Sol. (i) $(3+\sqrt{3})(2+\sqrt{2}) = 3\times 2+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

$= 6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $(3+\sqrt{3})(3-\sqrt{3}) = (3)^2-(\sqrt{3})^2$

$[(a+b)(a-b) = a^2 - b^2]$

$= 9 - 3$

$= 6$

(iii) $(\sqrt{5}+\sqrt{2})^2 = (\sqrt{5})^2+(\sqrt{2})^2+(2)(\sqrt{5})(\sqrt{2})$

$= 5+2+2\sqrt{10}$

$= 7+2\sqrt{10}$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = (\sqrt{5})^2-(\sqrt{2})^2$

$= 5 - 2 = 3$

3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction ?

Sol. We often take $\frac{22}{7}$ as an approximate value for π. So, π is an irrational number.

$\therefore\ \pi = \frac{22}{7}$ (approx), which is irrational.

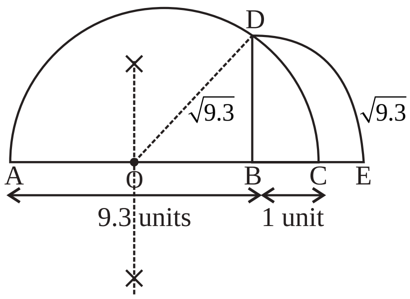
$\frac{c}{d} = \frac{22}{7}$ (approx), which is irrational.

$\left[\because \pi = \frac{c}{d}\right]$

4. Represent $\sqrt{9.3}$ on the number line.

Sol. To represent $\sqrt{9.3}$ on the number line first draw a line and take a point A on it and take another point B on it which is at a distance of 9.3 units from point A *i.e.*, now AB = 9.3 units. Another point C is marked at a distance of 1 unit from point B. Now bisect the line AC and find the point O. Now taking O as centre and OC as radius draw a semicircle. Now draw a perpendicular to AC which passes through B and intersects the semicircle at D. Thus, obtained is BD = $\sqrt{9.3}$. Draw an arc with centre B and radius BD, which intersects the number line at point E, then the point E represents $\sqrt{9.3}$.

Click here for all Solved PYQs & New pattern Q's Educart book



5. Rationalise the denominators each of the following :

1. $\frac{1}{\sqrt{7}}$

Sol. $\frac{1}{\sqrt{7}}$

Multiplying numerator and denominator by $\sqrt{7}$

$$\therefore \frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$$

Exercise – 1.5

1. Find : (i) $64^{\frac{1}{2}}$ **(ii)** $32^{\frac{1}{5}}$ **(iii)** $125^{\frac{1}{3}}$

Sol. (i) $(64)^{\frac{1}{2}} = (8 \times 8)^{\frac{1}{2}} = (8^2)^{\frac{1}{2}} = 8$

(ii) $(32)^{\frac{1}{5}} = (2 \times 2 \times 2 \times 2 \times 2)^{\frac{1}{5}}$

$$= (2^5)^{\frac{1}{5}} = 2$$

(iii) $(125)^{\frac{1}{3}} = (5 \times 5 \times 5)^{\frac{1}{3}}$

$$= (5^3)^{\frac{1}{3}} = 5$$

2. Find : (i) $9^{\frac{3}{2}}$ **(ii)** $32^{\frac{2}{5}}$ **(iii)** $16^{\frac{3}{4}}$ **(iv)** $125^{\frac{-1}{3}}$

Sol. (i) $(9)^{\frac{3}{2}} = (3 \times 3)^{\frac{3}{2}}$

$$= (3^2)^{\frac{3}{2}} = (3)^3 = 27$$

(ii) $(32)^{\frac{2}{5}} = (2 \times 2 \times 2 \times 2 \times 2)^{\frac{2}{5}}$

$$= (2^5)^{\frac{2}{5}} = (2)^2 = 4$$

Click here for all Solved PYQs & New pattern Q's Educart book

(iii) $(16)^{\frac{3}{4}} = (2\times2\times2\times2)^{\frac{3}{4}}$

$$= (2^4)^{\frac{3}{4}} = (2)^3 = 8$$

(iv) $(125)^{-\frac{1}{3}} = (5\times5\times5)^{-\frac{1}{3}}$

$$= (5^3)^{-\frac{1}{3}} = (5)^{-1} = \frac{1}{5}$$

3. Simplify : (i) $2^{\frac{2}{3}}.2^{\frac{1}{5}}$ **(ii)** $\left(\frac{1}{3^3}\right)^7$ **(iii)** $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ **(iv)** $7^{\frac{1}{2}}.8^{\frac{1}{2}}$

Sol. (i) $2^{\frac{2}{3}}.2^{\frac{1}{5}} = 2^{\frac{2}{3}+\frac{1}{5}}$ $[\because a^m.a^n = a^{m+n}]$

$$2^{\frac{10+3}{15}} = 2^{\frac{13}{15}}$$

(ii) $\left(\frac{1}{3^3}\right)^7 = (3^{-3})^7$ $\left[\because \frac{1}{a^m} = a^{-m}\right]$

$$= (3)^{-3\times7} = 3^{-21}$$

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}}$ $\left[\because \frac{a^m}{a^n} = a^{m-n}\right]$

$$= 11^{\frac{1}{4}}$$

(iv) $7^{\frac{1}{2}}\times8^{\frac{1}{2}} = (7\times8)^{\frac{1}{2}}$ $[\because a^m.b^m = (ab)^m]$

$$= (56)^{\frac{1}{2}}$$

Click here for all Solved PYQs & New pattern Q's Educart book

Polynomials 2

NCERT SOLUTIONS

What's inside

– *Textbook Exercise Q's (solved)*

EduCart

Exercise – 2.1

1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$ **(ii)** $y^2 + \sqrt{2}$

(iii) $3\sqrt{t} + t\sqrt{2}$ **(iv)** $y + \frac{2}{y}$

(v) $x^{10} + y^3 + t^{50}$

Sol. (i) $4x^2 - 3x + 7 = 4x^2 - 3x + 7x^\circ$

it is a polynomial in one variable *i.e.*, x because each exponent of x is a whole.

(ii) We have $y^2 + \sqrt{2} = y^2 + \sqrt{2}y^\circ$

it is a polynomial in one variable *i.e.*, y because each exponent of y is a whole number.

(iii) We have $3\sqrt{t} + t\sqrt{2} = 3t^{1/2} + \sqrt{2}t$

it is not a polynomial, because one of the exponents of t is 1/2, which is not a whole number.

(iv) We have $y + \frac{2}{y} = y + 2y^{-1}$

it is not a polynomial, because one of the exponent of y is –1 which is not a whole number.

(v) $x^{10} + y^3 + t^{50}$ is a polynomial in x, y and t *i.e.*, three variable, so it is not a polynomial in one variable.

2. Write the coefficients of x^2 in each of the following :

(i) $2 + x^2 + x$ **(ii)** $2 - x^2 + x^3$

(iii) $\frac{\pi}{2}x^2 + x$ **(iv)** $\sqrt{2}x - 1$

Sol. (i) The given polynomial is $2 + x^2 + x$, the coefficient of x^2 is 1.

(ii) The given polynomial is $2 - x^2 + x$, the coefficient of x^2 is –1.

(iii) The given polynomial is $\frac{\pi}{2}x^2 + x$, thee coefficient of x^2 is $\frac{\pi}{2}$.

(iv) The given polynomial is $\sqrt{2}x - 1$. The coefficient of x^2 is 0.

3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Sol. (i) A binomial of degree 35 can be $2x^{35} + 87$

(ii) A monomial of degree 100 can be $\frac{9}{2}x^{100}$.

4. Write the degree of each of the following polynomials :

(i) $5x^3 + 4x^2 + 7x$ **(ii)** $4 - y^2$

(iii) $5t - \sqrt{7}$ **(iv)** 3

Click here for all Solved PYQs & New pattern Q's Educart book

Sol. (i) The given polynomial is $5x^3 + 4x^2 + 7x$. The highest power of the variable x is 3. So the degree of the polynomial is 3.

(ii) The given polynomial is $4 - y^2$. The highest power of the variable y is 2. So the degree of the polynomial is 2.

(iii) The given polynomial is $5t - \sqrt{7}$. The highest power of variable t is 1. So the degree of the polynomial is 1.

(iv) Since, $3 = 3x^\circ$ [$\because x^\circ = 1$]

So the degree of the polynomial is 0.

5. Classify the following as linear, quadratic and cubic polynomials :

(i) $x^2 + x$ **(ii)** $x - x^3$

(iii) $y + y^2 + 4$ **(iv)** $1 + x$

(v) $3t$ **(vi)** r^2

(vii) $7x^3$

Sol. (i) The degree of $x^2 + x$ is 2. So it is a quadratic polynomial.

(ii) The degree of $x - x^3$ is 3. So it is a cubic polynomial.

(iii) The degree of $y + y^2 + 4$ is 2. So it is a quadratic polynomial.

(iv) The degree of $1 + x$ is 1. So it is a linear polynomial.

(v) The degree $3t$ is 1. So it is a linear polynomial.

(vi) The degree of r^2 is 2. So it is a quadratic polynomial.

(vii) The degree of $7x^3$ is 3. So it is a cubic polynomial.

Exercise – 2.2

1. Find the value of the polynomial $5x - 4x^2 + 3$ at

(i) $x = 0$

(ii) $x = -1$

(iii) $x = 2$

Sol. Let $p(x) = 5x - 4x^2 + 3$

(i) $P(0) = 5(0) - 4(0)^2 + 3$

$= 0 - 0 + 3 = 3$

Thus the value of $5x - 4x^2 + 3$ at $x = 0$ is 3.

(ii) $P(-1) = 5(-1) - 4(-1)^2 + 3$

$= -5 - 4 + 3$

$= -9 + 3$

$= -6$

Thus, the value of $5x - 4x^2 + 3$ at $x = 1$ is -6.

(iii $P(2) = 5(2) - 4(2)^2 + 3$

$= 10 - 4(4) + 3$

Click here for all Solved PYQs & New pattern Q's Educart book

$= 10 - 16 + 3 = -3$

Thus the value of $5x - 4x^2 + 3$ at $x = 2$ is -3.

2. Find P(0), P(1) and P(2) for each of the following polynomials :

(i) $P(y) = y^2 - y + 1$

(ii) $P(t) = 2 + t + 2t^2 - t^3$

(iii) $P(x) = x^3$

(iv) $P(x) = (x - 1)(x + 1)$

Sol. (i) Given, $P(y) = y^2 - y + 1$

$\therefore$ $P(0) = (0)^2 = 0 + 1$

$= 0 - 0 + 1 = 1$

$P(1) = (1)^2 - 1 + 1$

$= 1 - 1 + 1 = 1$

$P(2) = (2)^2 - 2 + 1$

$= 4 - 2 + 1 = 3$

(ii) Given, $P(t) = 2 + t + 2t^2 - t^3$

$\therefore$ $P(0) = 2 + (0) + 2(0)2 - (0)^3$

$= 2 + 0 + 2(0) - 0$

$= 2 + 0 + 0 - 0 = 2$

$P(1) = 2 + (1) + 2(1)^2 - (1)^3$

$= 2 + 1 + 2 \times 1 - 1$

$= 2 + 1 + 2 - 1 = 4$

$P(2) = 2 + (2)^2 + 2(2)^2 - (2)^2$

$= 2 + 2 + 2(4) - 8$

$= 2 + 2 + 8 - 8$

$= 4$

(iii) Given $P(x) = x^3$

$\therefore$ $P(0) = (0)^3 = 0$

$P(1) = (1)^3 = 1$

$P(2) = (2)^3 = 8$

(iv) Given $P(x) = (x - 1)(x + 1)$

$\therefore$ $P(0) = (0 - 1)(0 + 1)$

$= (-1)(1) = -1$

$P(1) = (1 - 1)(1 + 1)$

$= (0)(2) = 0$

$P(2) = (2 - 1)(2 + 1)$

$= (1)(3) = 3$

Click here for all Solved PYQs & New pattern Q's Educart book

3. Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $P(x) = 3x + 1; x = -\frac{1}{3}$

(ii) $P(x) = 5x - \pi; x = \frac{4}{5}$

(iii) $P(x) = x^2 - 1, x = 1, -1$

(iv) $P(x) = (x + 1)(x - 2), x = -1, 2$

(v) $P(x) = x^2, x = 0$

(vi) $P(x) = lx + m, x = -\frac{m}{l}$

(vii) $P(x) = 3x^2 - 1, x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(viii) $P(x) = 2x + 1, x = \frac{1}{2}$

Sol. (i) Given, $P(x) = 3x + 1$

$\therefore \quad P\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right)+1 = -1 + 1 = 0$

Since, $P\left(-\frac{1}{3}\right) = 0$

So, $x = -\frac{1}{3}$ is a zero of $P(x)$.

(ii) Given, $P(x) = 5x - \pi$

$\therefore \quad P\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right)-\pi = 4 - \pi$

Since, $P\left(\frac{4}{5}\right) \neq 0$, So, $x = \frac{4}{5}$ is not a zero of $P(x)$.

(iii) Given, $P(x) = x^2 - 1$

$\therefore \quad P(1) = 1^2 - 1 = 1 - 1 = 0$

Since, $P(1) = 0$, So, $x = 1$ is a zero of $P(x)$.

Also $P(-1) = (-1)^2 - 1 = 1 - 1 = 0$

Since, $P(-1) = 0$, So, $x = -1$ is also a zero of $P(x)$.

(iv) Given, $P(x) = (x + 1)(x - 2)$

$\therefore \quad P(-1) = (-1 + 1)(-1 - 2)$

$= (0)(-3) = 0$

Since, $P(-1) = 0$, So, $x = -1$ is a zero of $P(x)$.

Also, $P(2) = (2 + 1)(2 - 2)$

$= (3)(0) = 0$

Since, $P(2) = 0$, So, $x = 2$ is also a zero of $P(x)$.

(v) Given, $P(x) = x^2$

Click here for all Solved PYQs & New pattern Q's Educart book

$\therefore \quad P(0) = 0^2 = 0$

Since, P(0) = 0, So, $x = 0$ is zero of P(x).

(vi) Given, $\quad P(x) = lx + m$

$$\therefore \quad P\left(-\frac{m}{l}\right) = l\left(-\frac{m}{l}\right)+m$$

$$= -m + m = 0$$

Since, $P\left(-\frac{m}{l}\right) = 0$, So, $x = -\frac{m}{l}$ is a zero of P(x).

(vii) Given, $\quad P(x) = 3x^2 - 1$

$$\therefore \quad P\left(-\frac{1}{\sqrt{3}}\right) = 3\left(-\frac{1}{\sqrt{3}}\right)^2 -1$$

$$= 3\left(\frac{1}{3}\right)-1 = 1 - 1 = 0$$

Since, $P\left(-\frac{1}{\sqrt{3}}\right) = 0$, So, $x = -\frac{1}{\sqrt{3}}$ is a zero of P(x).

Also, $P\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 -1$

$$= 3\left(\frac{4}{3}\right)-1 = 4 - 1 = 3$$

Since, $P\left(\frac{2}{\sqrt{3}}\right) \neq 0$, So, $x = \frac{2}{\sqrt{3}}$ is not a zero of P(x).

(viii) Given, $\quad P(x) = 2x + 1$

$$\therefore \quad P\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)+1 = 1 + 1 = 2$$

Since, $P\left(\frac{1}{2}\right) \neq 0$, So, $x = \frac{1}{2}$ is not a zero of P(x).

4. Find the zero of the polynomial in each of the following cases.

(i) $P(x) = x + 5$

(ii) $P(x) = x - 5$

(iii) $P(x) = 2x + 5$

(iv) $P(x) = 3x - 2$

(v) $P(x) = 3x$

(vi) $P(x) = ax;\ a \neq 0$

(vii) $P(x) = cx + d;\ c \neq 0$, c, d are real numbers.

Sol. (i) Given, $\quad P(x) = x + 5$

To find zero of P(x), put P(x) = 0

$\Rightarrow \quad x + 5 = 0$

$\Rightarrow \quad x = -5$

Click here for all Solved PYQs & New pattern Q's Educart book

Thus, zero of $P(x)$ is -5.

(ii) Given, $P(x) = x - 5$

To find zero of $P(x)$, put $P(x) = 0$

$\Rightarrow \quad x - 5 = 0$

$\Rightarrow \quad x = 5$

Thus, zero of $P(x)$ is 5.

(iii)Given, $P(x) = 2x + 5$

To find zero of $P(x)$, put $P(x) = 0$

$\Rightarrow \quad 2x + 5 = 0$

$\Rightarrow \quad 2x = -5$

$\Rightarrow \quad x = -\frac{5}{2}$

Thus, zero of $P(x)$ is $-\frac{5}{2}$

(iv)Given, $P(x) = 3x - 2$

To find zero of $P(x)$, put $P(x) = 0$

$\Rightarrow \quad 3x - 2 = 0$

$\Rightarrow \quad 3x = 2$

$\Rightarrow \quad x = \frac{2}{3}$

Thus, zero of $P(x)$ is $\frac{2}{3}$

(v) Given, $P(x) = 3x$

To find of $P(x)$, put $P(x) = 0$

$\Rightarrow \quad 3x = 0$

$\Rightarrow \quad x = 0$

Thus, zero of $P(x)$ is 0.

(vi)Given, $P(x) = ax,\ a \neq 0$

To find zero of $P(x)$, put $P(x) = 0$

$\Rightarrow \quad ax = 0$

$\Rightarrow \quad x = 0$

Thus, zero of $P(x)$ is 0.

(vii) Given, $P(x) = (x + 0)$

To find zero of polynomial put $P(x) = 0$

$\Rightarrow \quad (x + 0) = 0$

$\Rightarrow \quad cx = -d$

$\Rightarrow \quad x = -\frac{d}{c}$

Click here for all Solved PYQs & New pattern Q's Educart book

Thus, zero of P(x) is $-\frac{d}{c}$

Exercise – 2.3

1. **Determine which of the following polymials has (x + 1) a factor.**

(i) $x^3 + x^2 + x + 1$

(ii) $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

Sol. The zero of x + 1 is –1.

(i) Let $P(x) = x^3 + x^2 + x + 1$

$\therefore$ $P(-1) = (-1)^3 + (-1)^2 + (-1)^{+1}$

$= -1 + 1 - 1 + 1 = 0$

$\Rightarrow$ $P(-1) = 0$

So, x + 1 is a factor of $x^3 + x^2 + x + 1$

(ii) Let $P(x) = x^4 + x^3 + x^2 + x + 1$

$\therefore$ $P(-1) = (-1)4 + (-1)^3 + (-1)^3 + (-1)^2$

$\neq (-1) + 1$

$= 1 - 1 + 1 - 1 + 1 = 1$

$P(-) \neq 0$

So, (x + 1) is not a factor of $x^4 + x^3 + x^2 + x + 1$.

(iii)Let $P(x) = x^4 + 3x^3 + 3x^2 + x + 1$

$P(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1 = 1$

$\therefore$ $P(-1) \neq 0$

So, (x + 1) is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$

(iv)Let $P(x) = x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

$\therefore$ $P(-1) = (-1)^3 - (-1)^2 - (2+\sqrt{2})(-1) + \sqrt{2}$

$= -1 - 1 + (2+\sqrt{2}) + \sqrt{2}$

$= 2\sqrt{2}$

$P(-1) \neq 0$

So, (x + 1) is not a factor of $x^3 + x^2(2+\sqrt{2})x + \sqrt{2}$

2. Use the factor Theorem to determine whether $g(x)$ is a factor of P(x) in each off the following cases.

(i) $P(x) = 2x^3 + x^2 - 2x - 1,\ g(x) = x + 1$

(ii) $P(x) = x^3 + 3x^2 + 3x + 1,\ g(x) = x + 2$

Click here for all Solved PYQs & New pattern Q's Educart book

(iii) $P(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$

Sol. (i) Given, $P(x) = 2x^3 + x^2 - 2x - 1$

and $g(x) = x + 1$

The zeroes of $g(x)$ is -1.

$\therefore$ $P(-1) = 2(-1)3 + (-1)2 - 2(-1) - 1$

$= 2(-1) + 1 + 2 - 1$

$= -2 + 1 + 2 - 1 = 0$

$\Rightarrow$ $P(-1) = 0$

So $g(x)$ is a factor of $P(x)$.

(ii) Given, $P(x) = x^3 + 3x^2 + 3x + 1$

and $g(x) = x + 2$

The zero of $g(x)$ is -2.

$\therefore$ $P(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

$= -8 + 12 - 6 + 1$

$= -14 + 13$

$= -1$

$P(-2) \neq 0$

So, $g(x)$ is not factor of $P(x)$.

(iii) Given, $P(x) = x^3 - 4x^2 + x + 6$

and $g(x) = x - 3$

The zero of $g(x)$ is 3.

$\therefore$ $P(3) = 33 - 4(3)2 + 3 + 6$

$= 27 - 4(9) + 3 + 6$

$= 27 - 36 + 3 + 6 = 0$

$P(3) = 0$

So, $g(x)$ is a factor of $P(x)$.

3. Find the value of k, if $x - 1$ is a factor of $P(x)$ in each of the following cases :

(i) $P(x) = x^2 + x + k$

(ii) $P(x) = 2x^2 + kx + \sqrt{2}$

(iii) $P(x) = kx^2 - \sqrt{2}x + 1$

(iv) $P(x) = kx^2 - 3x + k$

Sol. (i) Here, $P(x) = x^2 + x + k$

Since, $P(1) = 12 + 1 + k$

$\Rightarrow$ $P(1) = k + 2 = 0$

$\Rightarrow$ $k = -2$

(ii) Here, $P(x) = 2x^2 + kx + \sqrt{2}$

Since, $P(1) = 2(1)^2 + k(1) + \sqrt{2}$

Click here for all Solved PYQs & New pattern Q's Educart book

$$= 2+k+\sqrt{2} = 0$$

$$k = -2-\sqrt{2}$$

$$k = -(2+\sqrt{2})$$

(iii)Here, $\quad P(x) = kx^2-\sqrt{2}x+1$

Since, $\quad P(1) = k(1)^2-\sqrt{2}(1)+1$

$$= k-\sqrt{2}+1 = 0$$

$$\Rightarrow \quad k = \sqrt{2}-1$$

(iv)Here, $\quad P(x) = kx^2 - 3x + k$

$$P(1) = k(1)^2 - 3(1) + k$$

$$= k - 3 + k = 0$$

$$= 2k - 3 = 0$$

$$\Rightarrow \quad k = \frac{3}{2}$$

4. Factorise :

(i) 12x 2 – 7x + 1

(ii) 2x 2 + 7x + 3

(iii) 6x 2+ 5x – 6 (iv) 3x 2 – x – 4

5. Factorise :

(i) x3– 2x 2– x + 2

(ii) x3– 3x 2– 9x – 5

(iii) x 3+ 13x2 + 32x + 20

(iv) 2y 3+ y2– 2y – 1

Exercise – 2.4

1. Use suitable identities to find the following products :

(i) $(x + 4)(x + 10)$ (ii) $(x + 8)(x - 10)$

(iii)$(3x + 4)(3x - 5)$ (iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

(v) $(3 - 2x)(3 + 2x)$

Sol. (i)We have, $(x + 4)(x + 10)$

Using identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

We have, $(x + 8)(x - 10) = x^2 + (8 - 10)x + 8 \times (-10)$

$$= x^2 - 2x - 80$$

(iii)We have, $(3x + 4)(3x - 5)$

Using identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

We have,

$(3x + 4)(3x - 5)= (3x)^2 + (4 - 5)(3x) + 4 \times (-5)$

Click here for all Solved PYQs & New pattern Q's Educart book

$= 9x^2 - 3x - 20$

(iv) We have, $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

Using the identity, $(a + b)(a - b) = a^2 - b^2$

We have, $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right) = (y^2)^2-\left(\frac{3}{2}\right)^2$

$= y^4-\frac{9}{4}$

(v) We have, $(3 - 2x)(3 + 2x)$

Using the identity, $(a + b)(a - b) = a^2 - b^2$

We have, $(3 - 2x)(3 + 2x) = 3^2 - (2x)^2$

$= 9 - 4x^2$

2. Evaluate the following products without multiplying directly :

(i) 103 × 107 (ii) 95 × 96

(iii) 104 × 96

Sol. (i) $103 \times 107 = (100 + 3)(100 + 7)$

$= 100 \times 100 + 100 \times 7 + 3 \times 100 + 3 \times 7$

$= 10000 + 700 + 300 + 21$

$= 11021$

(ii) $95 \times 96 = (100 - 5)(100 - 4)$

$= (100)^2 + (-5 - 4)100 + (-5) \times (-4)$

$= 10000 - 900 + 20 = 9120$

(iii) $104 \times 96 = (100 + 4)(100 - 4)$

$= (100)^2 - (4)^2$

$= 10000 - 16$

$= 9984$

3. Factorise the following using appropriate identities :

(i) $9x^2 + 6xy + y^2$ (ii) $4y^2 - 4y + 1$

(iii) $x^2 - \frac{y^2}{100}$

Sol. (i) $9x^2 + 6xy + y^2 = (3x)^2 + 2 \times (3x)(y) + (y)^2$

$= (3x + y)^2$

$= (3x + y)(3x + y)$

(ii) $4y^2 - 4y + 1 = (2y)^2 - 2(2y) \times 1 + (1)^2$

$= (2y - 1)^2$

$= (2y - 1)(2y - 1)$

(iii) $x^2-\frac{y^2}{100} = (x)^2-\left(\frac{y}{100}\right)^2$

Click here for all Solved PYQs & New pattern Q's Educart book

$$= \left(x+\frac{y}{10}\right)\left(x-\frac{y}{10}\right)$$

4. Expand each of the following using suitable identies.

(i) $(x + 2y + 4z)^2$

(ii) $(2x - y + z)^2$

(iii) $(-2x + 3y + 2z)^2$

(iv) $(3a - 7b - c)^2$

(v) $(-2x + 5y - 3z)^2$

(vi) $\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2$

Sol. We know that,

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$$

(i) $(x + 2y + 4z)^2$

$= x^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(4z)(x)$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx$

(ii) $(2x - y + z)^2$

$= (2x)^2 + (-y)^2 + z^2 + 2(2x)(-y) + 2(y)(z) + 2(z)(2x)$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx$

(iii) $(-2x + 3y + 2z)^2$

$= (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(2z)(-2x)$

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12y^2 - 8zx$

(iv) $(3a - 7b - c)^2 = (3a)^2 + (-7b)^2 + (-1)^2 + 2(3a)(-7b) + 2(-7b)(-1) + 2(-1)(3a)$

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac$

(v) $(-2x + 5y - 3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) + 2(5y)(-3z) + 2(-3z)(-2x)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx$

(vi) $\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2 = \left(\frac{1}{4}a\right)^2+\left(-\frac{1}{2}b\right)^2+(1)^2+2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right)+2\left(-\frac{1}{2}b\right)(1)+2(1)\left(\frac{1}{4}a\right)$

$$= \frac{1}{16}a^2+\frac{1}{4}b^2+1-\frac{1}{4}ab-b+\frac{1}{2}a$$

5. Factorise:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$.

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Sol. (i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$

$= (2x + 3y - 4z)^2$

$= (2x + 3y - 4z)(2x + 3y - 4z)$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Click here for all Solved PYQs & New pattern Q's Educart book

$$= (-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + 2(-\sqrt{2}x)(y)(2\sqrt{2}z) \quad + (2\sqrt{2}z)(-\sqrt{2}x)$$

$$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2$$

$$= (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)$$

6. Write the following cubes in expanded form :

(i) $(2x + 1)^3$ **(ii) $(2a - 3b)^3$**

(iii) $\left[\frac{3}{2}x + 1\right]^3$ **(iv) $\left[x - \frac{2}{3}y\right]^2$**

Sol. We have, $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$ ...(i)

and $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ ...(ii)

(i) $(2x + 1)^3 = (2x)^3 + (1)^3 + 3(2x)(1)(2x + 1)$ [By (i)]

$= 8x^3 + 1 + 6x(2x + 1)$

$= 8x^3 + 12x^2 + 6x + 1$

(ii) $(2a - 3b)^3 = (2a)^3 - (3b)^3 - 3(2a)(2b)(2a - 3b)$ [By (ii)]

$= 8a^3 - 27b^3 - 18ab(2a - 3b)$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

(iii) $$\left(\frac{3}{2}x+1\right)^3 = \left(\frac{3}{2}x\right)^3 + (1)^3 + 3\left(\frac{3}{2}x\right)(1)\left(\frac{3}{2}x+1\right)$$ [By (i)]

$$= \frac{27}{8}x^3 + 1 + \frac{9}{2}x\left[\frac{3}{2}x+1\right]$$

$$= \frac{27}{8}x^3 + 1 + \frac{27x^2}{4} + \frac{9}{2}x$$

$$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$$

(iv) $$\left(x - \frac{2}{3}y\right)^3 = x^3 - \left(\frac{2}{3}y\right)^3 - 3(x)\left(\frac{2}{3}y\right)\left(x - \frac{2}{3}y\right)$$ [By (ii)]

$$= x^3 - \frac{8}{27}y^3 - 2xy\left(x - \frac{2}{3}y\right)$$

$$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$$

7. Evaluate the following using suitable identities :

(i) $(99)^3$ **(ii) $(102)^3$**

(iii) $(998)^3$

Sol. (i) We have, $99 = (100 - 1)$

$\therefore (100 - 1)^3 = 99$

$= (100)^3 - (1)^3 - 3(100)(1)(100 - 1)$

[Using $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$]

$= 1000000 - 1 - 300(100 - 1)$

Click here for all Solved PYQs & New pattern Q's Educart book

$= 1000000 - 1 - 30000 + 300$

$= 1000300 - 30001$

$= 970299$

(ii) We have, $102 = 100 + 2$

$\therefore \quad (102)^3 = (100 + 2)^3$

$= (100)^3 + (2)^3 + 3(100)(2)(100 + 2)$

[Using $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$]

$= 1000000 + 8 + 600(100 + 2)$

$= 1000000 + 8 + 60000 + 1200$

$= 1061208$

(iii) We have, $998 = 1000 - 2$

$\therefore \quad (998)^3 = (1000 - 2)^3$

$= (1000)^3 - (2)^3 - 3(1000)(2)(1000 - 2)$

[Using $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$]

$= 1000000000 - 8 - 6000(1000 - 2)$

$= 1000000000 - 8 - 6000000 + 12000$

$= 1000012000 - 6000008$

$= 994011992$

8. Factorise each of the following :

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Sol. (i) $8a^3 + b^3 + 12a^2b + 6a^2b$

$= (2a)^3 + (b)^3 + 6ab(2a + b)$

$= (2a)^3 + (b)^3 + 3(2a)(b)(2a + b)$

$= (2a + b)^3$ [Using $a^3 + b^3 + 3ab(a + b) = (a + b)^3$]

$= (2a + b)(2a + b)(2a + b)$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

$= (2a)^3 - (b)^3 - 3(2a)(b)(2a - b)$

$= (2a - b)^3$ [Using $a^3 - b^3 - 3ab(a - b) = (a - b)^3$]

$= (2a - b)(2a - b)(2a - b)$

(iii) $27 - 125a^3 - 135a + 225a^2$

$= (3)^3 - (5a)^3 - 3(3)(5a)(3 - 5a)$

$= (3 - 5a)^3$ [Using $a^3 - b^3 - 3ab(a - b) = (a - b)^3$]

Click here for all Solved PYQs & New pattern Q's Educart book

$= (3 - 5a)\ (3 - 5a)\ (3 - 5a)$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

Using $a^3 - b^3 - 3ab(a - b) = (a - b)^3$

$\therefore$ Here, $a = 4a$ and $b = 3b$

$= (4a)^3 - (3b)^3 - 3(4a)(3b)(4a - 3b)$

$= (4a - 3b)^3$

$= (4a - 3b)\ (4a - 3b)\ (4a - 3b)$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)\left(\frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$

$= \left(3p - \frac{1}{6}\right)^3$ [Using $a^3 - b^3 - 3ab(a - b) = (a - b)^3$]

$= \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$

9. Verify :

(i) $x^3 + y^3 = (x + y)\ (x^2 - xy + y^2)$

(ii) $x^3 - y^3 = (x - y)\ (x^2 + xy + y^2)$

Sol. (i) $\because (x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$\Rightarrow (x + y)^3 - 3(x + y)(xy) = x^3 + y^3$

$\Rightarrow (x + y)[(x + y)^2 - 3xy] = x^3 + y^3$

$\Rightarrow (x + y)(x^2 + y^2 - xy) = x^3 + y^3$

Hence, Verified.

(ii) $\because (x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$\Rightarrow (x - y)^3 + 3xy(x - y) = x^3 - y^3$

$\Rightarrow (x - y)[(x - y)^2 + 3xy] = x^3 - y^3$

$\Rightarrow (x - y)(x^2 + y^2 + xy) = x^3 - y^3$

Hence, Verified.

10. Factorise each of the following :

(i) $27y^3 + 125z^3$ **(ii) $64m^3 - 343n^3$**

Sol. (i) We know that

$x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$

We have, $27y^3 + 125z^3 = (3y)^3 + (5z)^3$

$= (3y + 5z)\ [(3y)^2 + (5z)^2 - (3y)(5z)]$

$= (3y + 5z)\ (9y^2 + 25z^2 - 15yz)$

(ii) We know that

$x^3 - y^3 = (x + y)\ (x^2 + xy + y^2)$

Click here for all Solved PYQs & New pattern Q's Educart book

We have, $64m^3 - 343n^3 = (4m)^3 - (7n)^3$

$= (4m - 7n)[(4m)^2 + (4m)(7n) + (7n)^2]$

$= (4m - 7n)(16m^2 + 28mn + 49n^2)$

11. Factorise : $27x^2 + y^3 + z^3 - 9xyz$

Sol. We have,

$27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$

Using the identity,

$\therefore x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

We have, $(3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$

$= (3x + y + z)[(3x)^2 + y^2 + z^2 - (3x \times y) - (y \times z) - (z \times 3x)]$

$= (3x + y + z)(4x^2 + y^2 + z^2 - 3xy - zy - 3zx)$

12. Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

Sol. R.H.S.

$= \frac{1}{2}(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$

$= \frac{1}{2}(x+y+z)[(x^2+y^2-2xy)+(y^2+z^2-2yz)+(z^2+x^2-2zx)]$

$= \frac{1}{2}(x+y+z)(x^2+y^2+y^2+z^2+z^2+x^2-2xy-2yz-2zx)$

$= \frac{1}{2}(x+y+z)[2(x^2+y^2+z^2-xy-yz-zx)]$

$= 2\times\frac{1}{2}(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= x^3 + y^3 + z^3 - 3xyz =$ L.H.S.

Hence, Verified.

13. If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.

Sol. Since, $x + y + z = 0$

$\Rightarrow x + y = -z$

$\Rightarrow (x + y)^3 = (-z)^3$

$\Rightarrow x^3 + y^3 + 3xy(x + y) = (-z)^3 [\because x + y = -z]$

$\Rightarrow x^3 + y^3 + 3xy(-z) = -z^3$

$\Rightarrow x^3 + y^3 + z^3 = 3xyz$

Hence, if $x + y + z = 0$

then $x^3 + y^3 + z^3 = 3xyz$

14. Without actually calculating the cubes, find the value of each of the following :

(i) $(-12)^3 + (7)^3 + (5)^3$

Click here for all Solved PYQs & New pattern Q's Educart book

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Sol. (i) We have, $(-12)^3 + (7)^3 + (5)^3$

Let $x = -12$, $y = 7$ and $z = 5$

Then, $x + y + z = -12 + 7 + 5 = 0$

We know that if $x + y + z = 0$, then

$x^3 + y^3 + z^3 = 3xyz$

$\therefore (-12)^3 + (7)^3 + (5)^3 = 3[(-12)(7)(5)]$

$= 3[-420] = -1260$

(ii) We have, $(28)^3 + (-15)^3 + (-13)^3$

Let $x = 28$, $y = -15$ and $z = -13$

Then, $x + y + z = 28 - 15 - 13 = 0$

$x^3 + y^3 + z^3 = 3xyz$

$\therefore (28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$

$= 3(5460)$

$= 16380$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given :

(i) Area : $25a^2 - 35a + 12$

(ii) Area : $35y^2 + 13y - 12$

Sol. Area of a rectangle = (Length) × (Breadth)

(i) $25a^2 - 35a + 12 = 25a^2 - 20a - 15a + 12$

$= 5a(5a - 4) - 3(5a - 4)$

$= (5a - 4)(5a - 3)$

Thus, the possible length and breadth are $(5a - 3)$ and $(5a + 4)$.

(ii) $35y^2 + 13y - 12 = 35y^2 + 28y - 15y - 12$

$= 7y(5y + 4) - 3(5y + 4)$

$= (5y + 4)(7y - 3)$

Thus, the possible length and breadth are $(7y - 3)$ and $(5y + 4)$.

16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below ?

(i) Volume : $3x^2 - 12x$

(ii) Volume : $12ky^2 + 8ky - 20k$

Sol. Volume of a cuboid = (Length) × (Breadth) × (Height)

(i) We have,

$3x^2 - 12x = 3x(x - 4)$

$= 3 \times x \times (x - 4)$

$\therefore$ The possible dimensions of the cuboid are 3, x and $(x - 4)$.

Click here for all Solved PYQs & New pattern Q's Educart book

(ii) We have, $12ky^2 + 8ky - 20k$

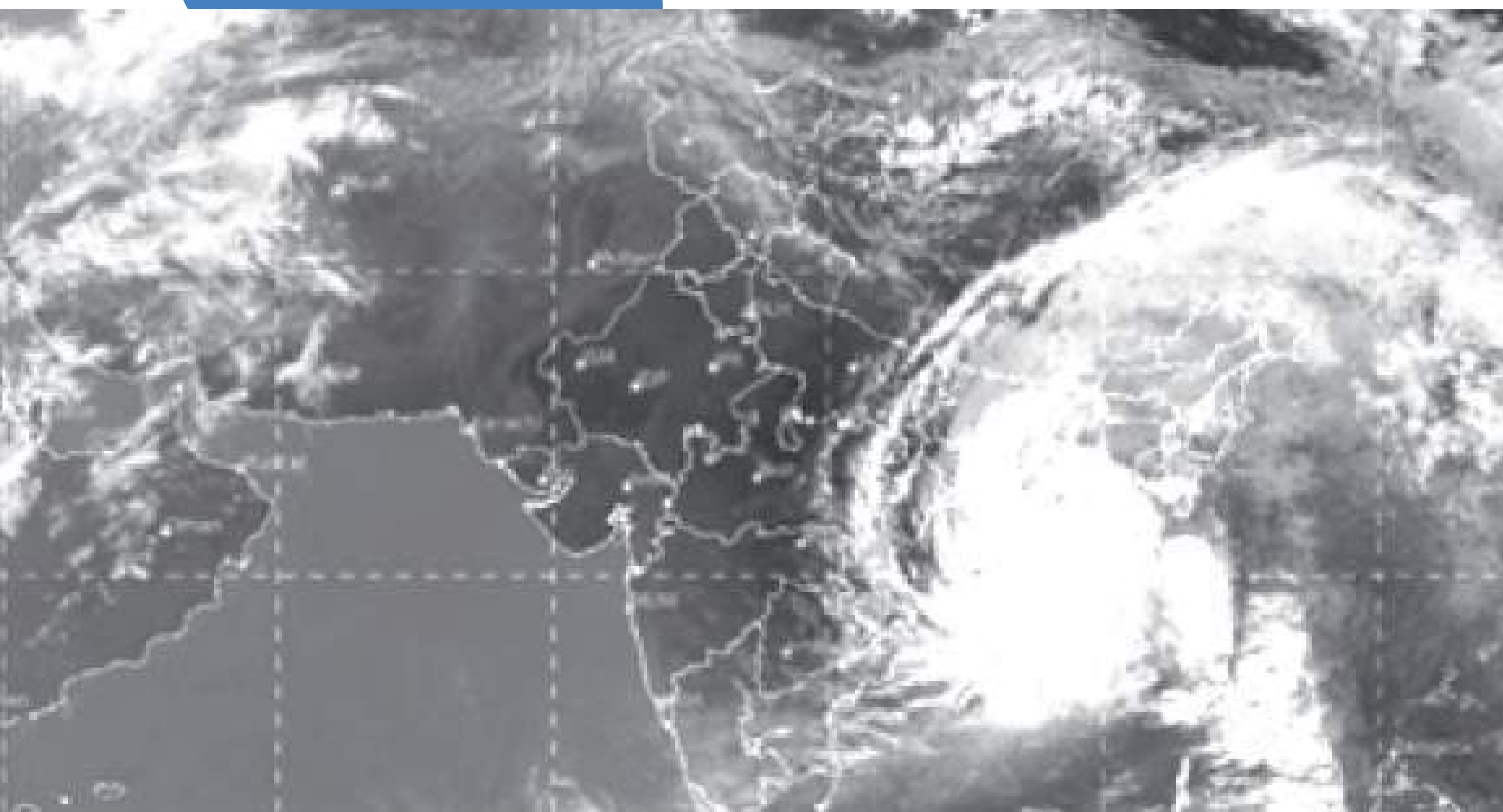
$$= 4(3ky^2 + 2ky - 5k)$$
$$= 4[k(3y^2 + 2y - 5)]$$
$$= 4 \times k(3y^2 + 2y - 5)$$
$$= 4k(3y^2 - 3y + 5y - 5)$$
$$= 4k[3y(y - 1) + 5(y - 1)]$$
$$= 4k[(y - 1)(3y + 5)]$$
$$= 4k \times (y - 1) \times (3y + 5)$$

Thus, the possible dimensions of the cuboid are $4k$, $(3y + 5)$ and $(y - 1)$.

Click here for all Solved PYQs & New pattern Q's Educart book

Coordinate Geometry 3

NCERT SOLUTIONS



What's inside

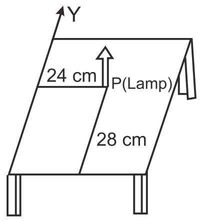
– *Textbook Exercise Q's (solved)*

EduCart

Exercise – 3.1

1. How will you describe the position of a table lamp on your study table to another person ?

Sol.



Consider the table lamp as a point and the table as a plane. Now take any two perpendicular edges on the table. The table has a shorter edge and a larger edge. Measure the distance of the lamp from the larger edge, let it be 24 cm. Again, measure the distance of the lamp from the shorter edge, let it be 28 cm.

Therefore, depending on the order of the axes, the position of the lamp can be written as (24, 28) or (28, 24).

2. (Street Plan) : A city has two main roads which cross each other at the centre of the city. These two roads are along the North-South direction and East-West direction.

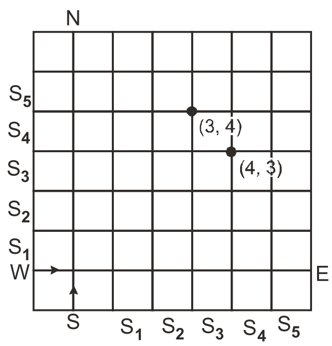
All the other streets of the city run parallel to these roads and are 200 m apart. There are 5 streets in each direction. Using 1 cm = 300 m, draw a model of the city on your notebook. Represent the roads/streets by single lines.

There are many cross-streets in your model. A particular cross-street is made by two streets, one running in the North-South direction and another in the East-West direction. Each cross street is referred to in the following manner : If the 2nd street running in the North-South direction and 5th in the East-West direction meet at some crossing, then we will call this corss-street (2, 5). Using this convention, find :

(i) how many cross-streets can be referred to as (4, 3)?

(ii) how many cross-streets can be referred to as (3, 4)?

Sol. The street plan is shown by the following figure:



(i) There is only one cross-street which can be referred as (4, 3).

(ii) There is only one cross-street which can be referred as (3, 4).

Click here for all Solved PYQs & New pattern Q's Educart book

Exercise – 3.2

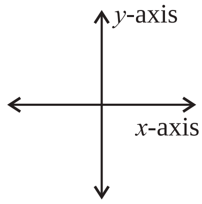
1. Write the answer of each of the following questions :

(i)What is the name of horizontal and the vertical lines drawn to determine the position of any point in the Cartesian plane ?

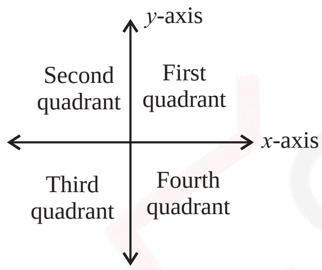
(ii)What is the name of each part of the plane formed by these two lines ?

(iii)Write the name of the point where these two lines intersect.

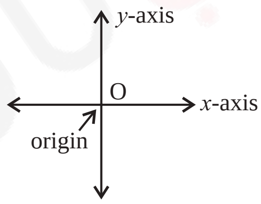
Sol. (i)To determine the position of any point in the Cartesian plane, the horizontal line is known as x-axis and vertical line is known as y-axis.



(ii)Each part of the plane formed by horizontal and vertical lines is known as quadrant.



(iii)The point where the horizontal and vertical lines intersect is known as origin.



2. See the figure given below and write the following :

(i)The coordinates of B.

(ii) The coordinates of C.

(iii) The point identified by the coordinates (–3, –5).

(iv) The point identified by the coordinates (2, – 4).

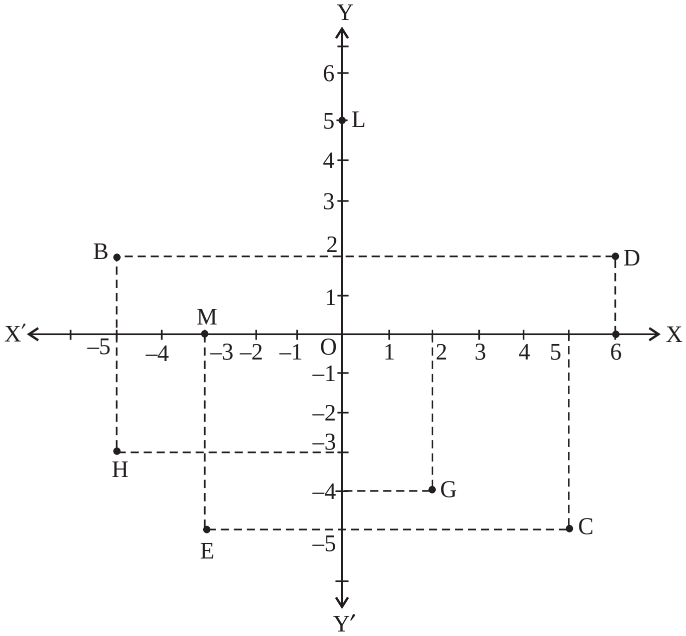
(v) The abscissa of the point D.

(vi) The ordinate of the point H.

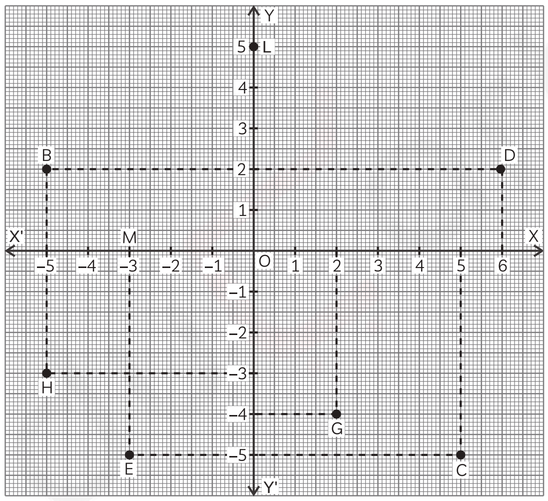
(vii) The coordinates of the point L.

(viii) The coordinates of the point M.

Click here for all Solved PYQs & New pattern Q's Educart book



Sol.



(i) The coordinates of B = (–5, 2)

(ii) The coordinates of C = (5, –5)

(iii) The point identified by the coordinates (–3, –5) is E.

(iv) The point identified by the coordinates (2, –4) is G.

(v) The abscissa of the point D = 6

(vi) The ordinate of the point H = –3

(vii) The coordinates of the point L = (0, 5)

(viii) The coordinates of the point M = (–3, 0)

Click here for all Solved PYQs & New pattern Q's Educart book

Linear Equations in Two Variables

4

NCERT SOLUTIONS

What's inside

– *Textbook Exercise Q's (solved)*

EduCart

Exercise – 4.1

1. The cost of a notebook is twice the cost of a pen. Write a linear equation in two variables to represent this statement.

(Take the cost of a notebook to be ₹ x and that of a pen to be ₹ y).

Sol. The cost of a pen $= ₹y$

and the cost of a notebook $= ₹x$

According to the question,

$$x = 2y$$

$$\Rightarrow \quad x - 2y = 0$$

The above equation is required linear equation in two variables.

2. Express the following linear equations in the form $ax + by + c = 0$ and indicate the values of a, b and c in each case :

(i) $2x + 3y = 9.3\overline{5}$ **(ii)** $x - \frac{y}{5} - 10 = 0$

(iii) $-2x + 3y = 6$ **(iv)** $x = 3y$

(v) $2x = -5y$ **(vi)** $3x + 2 = 0$

(vii) $y - 2 = 0$ **(viii)** $5 = 2x$

Sol. (i) $2x + 3y = 9.3\overline{5}$

$\Rightarrow 2x + 3y - 9.3\overline{5} = 0$

Comparing the equations with $ax + by + c = 0$,

we get, $a = 2$, $b = 3$, $c = -9.3\overline{5}$

(ii) $x - \frac{y}{5} - 10 = 0$

Comparing the equations with $ax + by + c = 0$,

we get, $a = 1$, $b = -\frac{1}{5}$, $c = -10$

(iii) $-2x + 3y = 6$

$\Rightarrow -2x + 3y - 6 = 0$

Comparing the equations with $ax + by + c = 0$,

we get, $a = -2$, $b = 3$, $c = -6$

(iv) $x = 3y$

$\Rightarrow x - 3y + 0 = 0$

Comparing the equations with $ax + by + c = 0$,

we get, $a = 1$, $b = -3$, $c = 0$

(v) $2x = -5y$

$\Rightarrow 2x + 5y + 0 = 0$

Comparing the equations with $ax + by + c = 0$,

we get, $a = 2$, $b = 5$, $c = 0$

(vi) $3x + 2 = 0$

$\Rightarrow 3x + 0.5y + 2 = 0$

Click here for all Solved PYQs & New pattern Q's Educart book

Comparing the equations with $ax + by + c = 0$,

we get, $a = 3, b = 0$ and $c = 2$

(vii) $y - 2 = 0$

$\Rightarrow 0.x + y - 2 = 0$

Comparing the equations with $ax + by + c = 0$,

we get, $a = 0, b = 1$ and $c = -2$

(vii) $5 = 2x$

$\Rightarrow -2x + 0.4 + 5 = 0$

Comparing the equations with $ax + by + c = 0$,

we get, $a = -2, b = 0$ and $c = 5$

Exercise – 4.2

1. Which one of the following options is true, and why ?

$y = 3x + 5$ has

(i) a unique solution,

(ii) only two solution,

(iii) infinitely many solutions

Sol. We know that a linear equation in two variables has infinitely many solutions.

Here, the given equation $y = 3x + 5$ is linear equation in two variables x and y. So, the equation has infinitely many solutions.

Hence, option (iii) is true.

2. Write four solutions for each of the following equations :

(i) $2x + y = 7$ (ii) $\pi x + y = 9$ (iii) $x = 4y$

Sol. (i) Given equation,

$$2x + y = 7$$

Put $x = 0$,

$$2 \times 0 + y = 7$$
$$0 + y = 7$$
$$y = 7$$

Therefore, (0, 7) is a solution of the equation.

Put $x = 1$, then

$$2 \times 1 + y = 7$$
$$y = 7 - 2 = 5$$

Therefore, (1, 5) is a solution of the given equation.

Put $x = 2$, then

$$2 \times 2 + y = 7$$
$$4 + y = 7$$
$$y = 3$$

Click here for all Solved PYQs & New pattern Q's Educart book

Therefore, (2, 3) is a solution of the equation.

Put $x = 4$ then

$$2 \times 4 + y = 7$$
$$8 + y = 7$$
$$y = 7 - 8 = -1$$

Therefore, $x = 4$ and $y = -1$ is a solution of the equation.

Hence, four solutions are:

(0, 7), (1, 5), (2, 3) and (4, –1)

(ii) Given equation,

$$\pi x + y = 9$$

Put $x = 1$

$$\pi \times 1 + y = 9$$
$$\pi + y = 9$$
$$y = 9 - \pi$$

Therefore, $(1, 9 - \pi)$ is a solution of the equation.

Put $x = 0$, then

$$\pi \times 0 + y = 9$$
$$0 + y = 9$$
$$y = 9$$

Therefore, (0, 9) is a solution of the equation.

Put $x = -1$, then

$$\pi \times (-1) + y = 9$$
$$-\pi + y = 9$$
$$y = 9 + \pi$$

Therefore, $(-1, 9 + \pi)$ is a solution of the given equation.

Put $x = \frac{9}{\pi}$, then

$$\pi \times \frac{9}{\pi} + y = 9$$
$$9 + y = 9$$
$$y = 0$$

Therefore, $\left(\frac{9}{\pi}, 0\right)$ is a solution of the given equation.

Hence, four solutions are:

$(1, 9 - \pi)$, (0, 9), $(-1, 9 + \pi)$ and $\left(\frac{9}{\pi}, 0\right)$

(iii) Given equation,

$$x = 4y$$

Put $x = 0$, then

$$0 = 4y$$

Click here for all Solved PYQs & New pattern Q's Educart book

$$y = 0$$

Therefore, (0, 0) is a solution of the equation.

Put $x = 2$, then

$$2 = 4y$$

$$y = \frac{1}{2}$$

Therefore, $\left(2, \frac{1}{2}\right)$ is a solution of the equation.

Put $x = 4$, then

$$4 = 4y$$

$$y = 1$$

Therefore, (4, 1) is a solution of the equation.

Put $x = -4$, then

$$-4 = 4y$$

$$y = -1$$

Therefore, (–4, –1) is a solution of the equation.

Hence, four solutions are:

(0, 0), $\left(2, \frac{1}{2}\right)$, (4, 1) and (–4, –1).

3. Check which of the following are solutions of the equation $x - 2y = 4$ and which are not ?

(i) (0, 2) **(ii) (2, 0)**

(iii)(4, 0) **(iv) $\left(\sqrt{2}, 4\sqrt{2}\right)$**

(v) (1, 1)

Sol. (i) To check (0, 2) is a solution of the euqation $x - 2y = 4$ or not, then put $x = 0$ and $y = 2$ in the given equation.

$$x - 2y = 4$$

$$0 - 2 \times 2 = 4$$

$$0 - 4 = 4$$

$$-4 = 4$$

$$\text{LHS} \neq \text{RHS}$$

Now, (0, 2) does not satisfy the equation.

Hence, (0, 2) is not the solution of the given equation.

(ii) To check (2, 0) is a solution of the equation $x - 2y = 4$ or not, then put $x = 2$ and $y = 0$ in the given equation.

$$x - 2y = 4$$

$$2 - 2 \times 0 = 4$$

$$2 - 0 = 4$$

$$2 = 4$$

$$\text{LHS} \neq \text{RHS}$$

Click here for all Solved PYQs & New pattern Q's Educart book

Now, (2, 0) does not satisfy the equation.

Hence, (2, 0) is not the solution of the given equation.

(iii) To check (4, 0) is a solution of the equation $x - 2y = 4$ or not, then put $x = 4$ and $y = 0$ in the given equation.

$$x - 2y = 4$$
$$4 - 2 \times 0 = 4$$
$$4 - 0 = 4$$
$$4 = 4$$
$$\text{LHS} \neq \text{RHS}$$

Now, (4, 0) satisfies the equation.

Hence, (4, 0) is the solution of the given equation.

(iv) To check $(\sqrt{4}, 4\sqrt{2})$ is a solution of the equation $x - 2y = 4$ or not, then put $x = \sqrt{2}$ and $y = 4\sqrt{2}$ in the given equation.

$$x - 2y = 4$$
$$\sqrt{2} - 2 \times 4\sqrt{2} = 4$$
$$\sqrt{2} - 8\sqrt{2} = 4$$
$$-7\sqrt{2} = 4$$
$$\text{LHS} \neq \text{RHS}$$

Now, $(\sqrt{2}, 4\sqrt{2})$ does not satisfy the equation.

Hence, $(\sqrt{2}, 4\sqrt{2})$ is not the solution of the given equation.

(v) To check (1, 1) is a solution of the equation $x - 2y = 4$ or not, then put $x = 1$ and $y = 1$ in the given equation.

$$x - 2y = 4$$
$$1 - 2 \times 1 = 4$$
$$1 - 2 = 4$$
$$-1 = 4$$
$$\text{LHS} \neq \text{RHS}$$

Now, (1, 1) does not satisfy the equation.

Hence, (1, 1) is not the solution of the given equation.

4. If $x = 2$, $y = 1$ is a solution of the equation $2x + 3y + k = 0$, then find the value of k.

Sol. Given equation,

$$2x + 3y + k = 0$$

$x = 2$ and $y = 1$ is a solution of the equation, then

$$2 \times 2 + 3 \times 1 + k = 0$$
$$\Rightarrow \quad 4 + 3 + k = 0$$
$$\Rightarrow \quad 7 + k = 0$$
$$\Rightarrow \quad k = -7$$

Click here for all Solved PYQs & New pattern Q's Educart book

Introduction to Euclid's Geometry

5

NCERT SOLUTIONS

What's inside

– *Textbook Exercise Q's (solved)*

EduCart

Exercise – 5.1

1. Which of the following statements are true and which are false ? Give reasons for your answers.

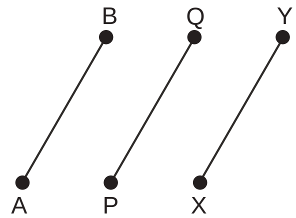
(i) Only one line can pass through a single point.

(ii) There are an infinite number of lines which pass through two distinct points.

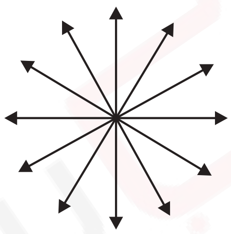
(iii) A terminated line can be produced indefinitely on both the sides.

(iv) If two circles are equal, then their radii are equal.

(v) In the given figure, if AB = PQ and PQ = XY, then AB = XY.



Sol. (i) False, because there can be infinite number of lines that can be drawn through a single point.



(ii) False, because two distinct points, there is a unique line *i.e.*, one line that passes through them.

In the following figure, it can be seen that there is only one single line that can pass through two distinct points A and B.

(iii) True, because Euclid's second postulate says that a terminated line can be produced infinitely on both sides.

(iv) True, because if two circles are equal, then their centre and circumference will coincide and hence, the radii will also be equal.

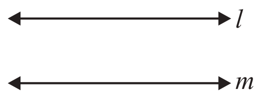
(v) True, because Euclid's first axiom says that things which are equal to the same thing equal to one another.

2. Give a definition for each of the following terms. Are there other terms that need to be defined first ? What are they, and how might you define them ?

Click here for all Solved PYQs & New pattern Q's Educart book

(i) **Parallel lines** (ii) **Perpendicular lines**
(iii) **Line segment** (iv) **Radius of a circle**
(v) **Square**

Sol. (i) Parallel lines: If the perpendicular distance between two lines is always constant, then these are called parallel lines. In other words, the lines which never intersect each other are called parallel lines.



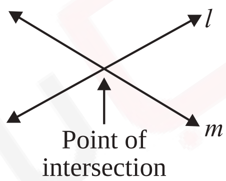
Here, l and m are parallel lines.

To define parallel lines, we must know about point, lines, and the point of intersection.

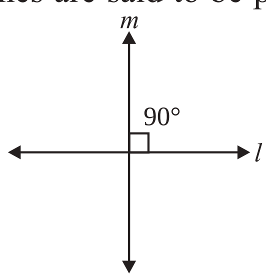
Point : A dimensionless dot which is drawn on a plane surface is known is point. A point is that which has no part.

Line : A collection of points that has only length and no breadth known as a line. And it can be extended on both directions. A line is breadthless length.

Point of intersection : A point where two or more lines intersect or meet is known as point of intersection.



(ii) Perpendicular lines : Perpendicular lines are these lines which intersect each other in a plane at right angles then the lines are said to be perpendicular to each other.



Here, l and m are perpendicular lines.

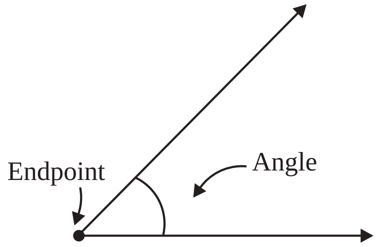
To define perpendicular lines, we must know about plane, line and angle.

Plane : Flat surface in which geometric figures can be drawn are known are plane. A plane surface is a surface which lies evenly with the straight lines on itself.

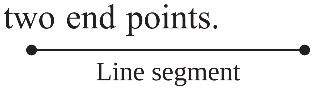
Line : A collection of points that has only length and no breadth is known as a line. And it can be extended on both direction. A line is breadthless length.

Angle : An angle is formed when two straight lines or rays meet at a common endpoint.

Click here for all Solved PYQs & New pattern Q's Educart book



(iii)Line segment : A straight line drawn from any point to any other point is called as line segment. A line segment has two end points.



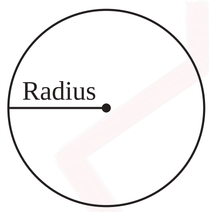
Line segment

To define line segment, we must know about point and straight line.

Point : A dimensionless dot which is drawn on a plane surface is known as point. A point is that which has no part.

Straight line : A straight line is a line which lies evenly with the points on itself.

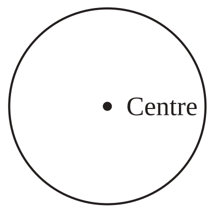
(iv)Radius of a circle : It is the distance between the centre of a circle to any point lying on the circle.



To define radius of a circle we must know about point and circle.

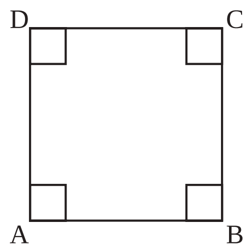
Point : A dimensionless dot which is drawn on a plane surface is known as point. A point is that which has no part.

Circle : A circle is the set of all those points in a plane whose distance from a fixed point remains constant. The fixed point is called the centre of the circle.



Circle

(v) Square : A square is a quadrilateral having all four line segments of equal length and each of its internal angle is right angles *i.e.*, 90°.



Click here for all Solved PYQs & New pattern Q's Educart book

Here, ABCD is a square; AB, BC, CD, DA are four line segments with equal length; and its all four angles *i.e.*, $\angle A$, $\angle B$, $\angle C$, $\angle D$ are right angles.

To define sequare, we must know about quadrilateral, line segment and angle.

Quadrilateral : A closed figure made of four line segments is called a quadrilateral.

Line segment : A line segment is a part of line and having a definite length. It has two end points.

Angle : An angle is formed when two straight lines or rays meet at a common end point.

3. Consider two 'postulates' given below :

(i) Given any two distinct points A and B, there exists a third point C which is in between A and B.

(ii) These exist at least three points that are not on the same line.

Do these postulates contain any undefined terms ? Are these postulates consistent ? Do they follow Euclid's postulates ? Explain.

Sol. These are various undefined terms is the given postulates. They are consistent because they deal with two different situations that is :

(i) If two points A and B are given, then there exists a third point C which is in between A and B.

(ii) If two points A and B are given, then we can take a point C which do not lie on the line passes through the point A and B.

These postulates do not follow from Euclid's postulates. They follow from the axiom, "Given two distinct points, there is a unique line that passes through them."

4. If a point C lies between two points A and B such that AC = BC, then prove that AC = $\frac{1}{2}$ AB. Explain by drawing the figure.

Sol.

Given, AC = BC

Adding AC both sides,

$$AC + AC = BC + AC$$

[If equals added to equals then whole are equal]

$$2AC = AB$$

$$AC = \frac{1}{2}AB$$

5. In question 15, point C is called a mid-point line segment AB. Prove that every line segment has one and only one mid-point.

Sol. Given,

C is the mid-point of line segment AB.

And, there is another point on the line segment AB.

Click here for all Solved PYQs & New pattern Q's Educart book

Let D another mid-point of line segment AB.

To prove : Every line segment has one and only one mid-point.

Proof : C is mid-point of line segment AB.

$\therefore \quad AC = BC$

$AC + AC = AC + BC$ [$\because$ Equals added to equals then whole are euqal]

$2AC = AB$

$AC = \frac{1}{2}AB$ ...(i)

Now, D is the mid-point of line segment AB.

$\therefore \quad AD = BD$

$AD + AD = AD + BD$ [$\because$ Equals added to equals then whole are equal]

$2AD = AB$

$AD = \frac{1}{2}AB$ ...(ii)

From equation (i) and (ii), we get

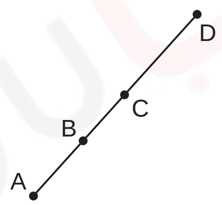
$AC = AD$

Which is only possible when C and D coincide.

So, point C line on D.

Hence, every line sgement has one and only one mid-point.

6. In the given figure, if AC = BD, then prove that AB = CD.



Sol. Given, $AC = BD$ ...(i)

Now, $AC = AB + BC$

and $BD = BC + CD$

From equation (i),

$AB + BC = BC + CD$

According to Euclid's axiom, if equals are substracted from equals, the remainders are euqal.

$\therefore \quad AB + BC - BC = BC + CD - BC$

$AB = CD$

7. Why is axiom 5, in the list of Euclid's axioms, considered a 'universal truth' ? (Note that the question is not about the fifth postulate)

Sol. Axiom 5 states that the whole is greater than the part. This axiom is known as a 'universal truth' because it holds true in any field, and not just in the field of methematics.

Click here for all Solved PYQs & New pattern Q's Educart book

Lines and Angles 6

NCERT SOLUTIONS

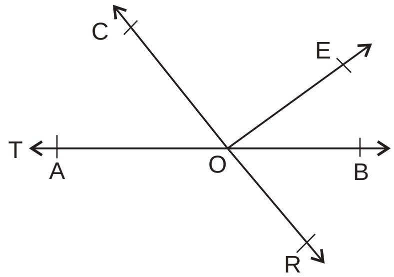
What's inside

– Textbook Exercise Q's (solved)

EduCart

Exercise – 6.1

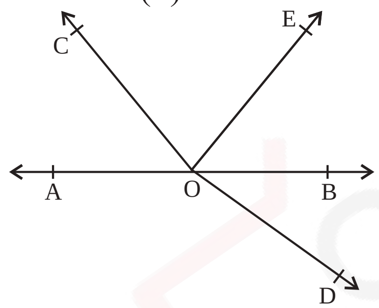
1. In fig., lines AB and CD intersect at O. If ∠AOC + ∠BOE = 70° and ∠BOD = 40°, find ∠BOE and reflex ∠COE :



Sol. It is clear from the figure that, ∠AOC, ∠BOE and ∠COE and also ∠COE, ∠BOD and ∠BOE forms a straight line.

∴ ∠AOC + ∠BOE + ∠COE = 180° ...(i)

and ∠COE + ∠BOD + ∠BOE = 180° ...(ii)



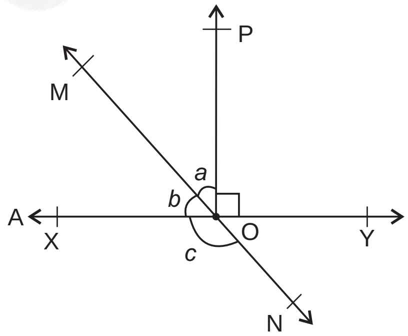
∵ It is given that, ∠AOC + ∠BOE = 70° and ∠BOD = 40°

So, from equation (i) and (ii) we get

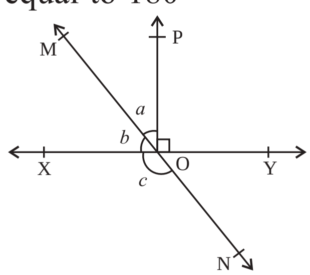
∠COE = 110° and ∠BOE = 30°

and reflex ∠COE = 360° – 110° = 250°

2. In fig, lines XY and MN intersect at O. If ∠POY = 90° and a : b = 2 : 3, find c :



Sol. ∵ The sum of linear pair always equal to 180°



EduCart

Click here for all Solved PYQs & New pattern Q's Educart book

So from the figure,

$\angle POY + a + b = 180°$

$\because \quad \angle POY = 90°$

So, $\quad a + b = 90° \quad ...(i)$

and also, $\quad a : b = 2 : 3$

Let $\quad a = 2x$ and $b = 3x$

so, from equation (i)

$$2x + 3x = 90° \Rightarrow 5x = 90° \Rightarrow x = 18°$$

$\therefore \quad a = 2 \times 18° = 36°$

and $\quad b = 3 \times 18° = 54°$

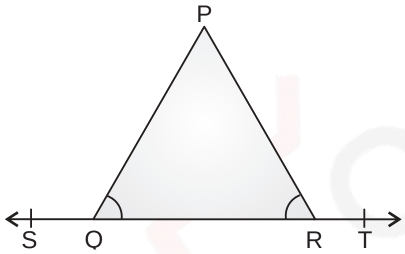
From the figure, b and c also forms a straight angle.

So, $\quad b + c = 180°$

$\Rightarrow \quad 54° + c = 180°$

$\Rightarrow \quad c = 126°$

3. In fig, ∠PQR = ∠PRQ, then prove that ∠PQS = ∠PRT :



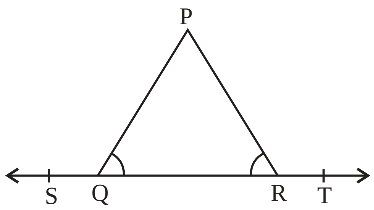
Sol. From the figure, it is clear that

ST is a straight line so,

$\angle PQS + \angle PQR = 180°$ (linear pair)

and $\angle PRT + \angle PRQ = 180°$ (linear pair)

Now, $\angle PQS + \angle PQR = \angle PRT + \angle PRQ$



$\because$ It is given that

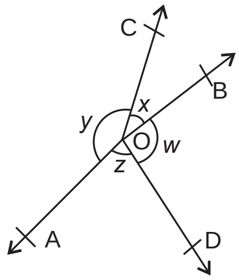
$\angle PQR = \angle PRQ$

So, $\angle PRQ + \angle PQS = \angle PRT + \angle PRQ$

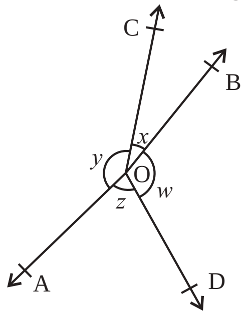
$\therefore \quad \angle PQS = \angle PRT$ **Hence Proved.**

4. In fig, if $x + y = w + z$, then prove that AOB is a line.

Click here for all Solved PYQs & New pattern Q's Educart book



Sol. $\because$ The angle around a point are 360° so, from the figure,



$x + y + w + z = 360°$

$\because$ It is given that, $x + y = w + z$

so, $x + y + x + y = 360°$

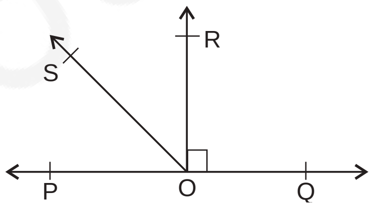
or $2(x + y) = 360°$

or $x + y = 180°$

$\therefore$ AOB is a line.

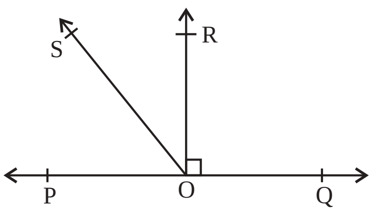
5. In fig, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR.

Prove that : $\frac{1}{2}$(∠QOS – ∠POS)



Sol. Given that, OR ⊥ PQ and ∠POQ = 180°

so, from the figure,



$\angle POS + \angle ROS + \angle ROQ = 180°$

$\therefore \quad \angle POS + \angle ROS = 180° - 90° \qquad (\because \angle POR = \angle ROQ = 90°)$

so $\quad \angle POS + \angle ROS = 90° \qquad$ ...(i)

and also, $\quad \angle QOS = \angle ROQ + \angle ROS$

It is given that $\quad \angle ROQ = 90°$

Click here for all Solved PYQs & New pattern Q's Educart book

$\therefore \quad \angle QOS = 90° + \angle ROS$

and $\quad \angle QOS = \angle ROS + 90°$...(ii)

From equation (i) and (ii) we get

$$\angle POS + \angle ROS = \angle QOS - \angle ROS$$

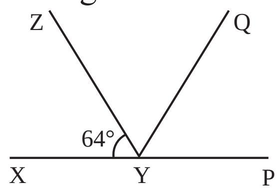
$\Rightarrow \quad 2\angle ROS = \angle QOS - \angle POS$

$\Rightarrow \quad \angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$

Hence Proved

6. It is given that $\angle XYZ = 64°$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.

Sol. It is clear from the figure that XP is a straight line.



$\therefore \quad \angle XYZ + \angle ZYP = 180°$ ...(i)

It is given that $\angle XYZ = 64°$

From equation (i)

$$64 + \angle ZYP = 180°$$

$\therefore \quad \angle ZYP = 180° - 64°$

$\angle ZYP = 116°$

and also, $\quad \angle ZYP = \angle ZYQ + \angle QYP$...(ii)

[From the figure]

$\because$ YQ is the bisecfor of $\angle ZYP$

so $\quad \angle ZYQ = \angle QYP$

$\therefore$ From the equation (ii)

$$\angle ZYP = 2\angle ZYQ$$

$\therefore \quad \angle ZYQ = \frac{1}{2}\angle ZYP = \frac{1}{2} \times 116°$

$\Rightarrow \quad \angle ZYQ = 58°$

Again $\quad \angle XYQ = \angle XYZ + \angle XYQ$

$\therefore \quad \angle XYQ = 64° + 58° = 122°$

Now reflex $\quad \angle QYP = 180° + \angle XYQ$

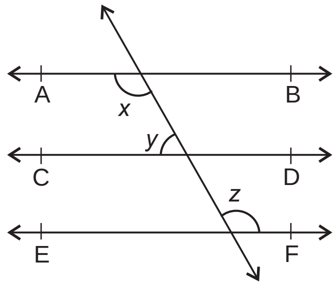
$= 180° + 122°$

$= 302°$

Exercise – 6.2

1. In Fig. if BA || CD, CD || EF and $y : z = 3 : 7$, find x.

Click here for all Solved PYQs & New pattern Q's Educart book



Sol. Given that, AB | | CD and CD | | EF

$\therefore \quad x + y = 180°$ ...(i)

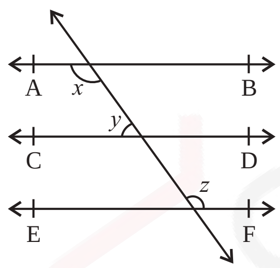
[The angles on the same side of a transversal line sums up to 180°.]

Also, $\angle O = z$ (corresponding angles)

and $y + \angle O = 180°$ (linear pair)

$\therefore \quad y + z = 180°$

It is also given that $y : z = 3 : 7$



Let $y = 3k, z = 7k$

$\therefore \quad 3k + 7k = 180°$

$\Rightarrow \quad 10k = 180°$

$\Rightarrow \quad k = 18°$

Now $y = 3 \times 18° = 54°$

$z = 7 \times 18° = 126°$

So from equation (i)

$x + 54° = 180°$

$x = 126°$

2. In Fig. 6.24, if AB || CD, EF ⊥ CD and ∠ GED = 126°, find ∠ AGE, ∠ GEF and ∠ FGE

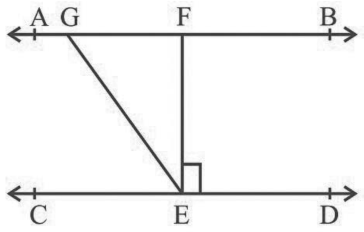
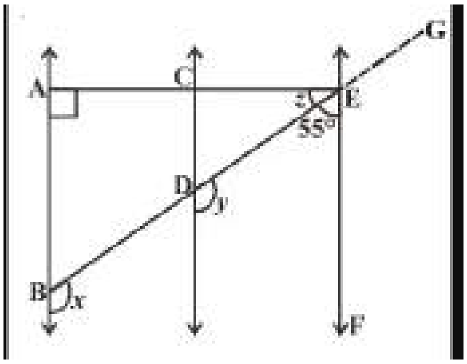


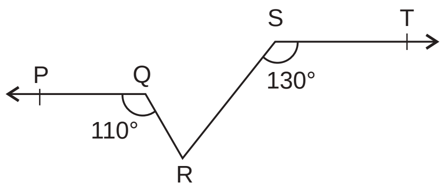
Fig. 6.24

Click here for all Solved PYQs & New pattern Q's Educart book



3. In Fig., if PQ || ST, ∠PQR = 110° and ∠RST = 130°, find ∠QRS.

[**Hint :** Draw a line parallel to ST through point R.]

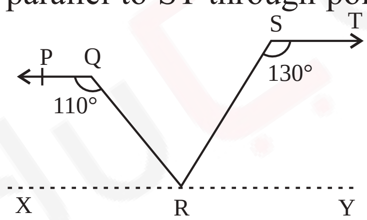


Sol. Given, PQ | | ST,

∠PQR = 110°

and ∠RST = 130°

Construction–Draw a line XY parallel to ST through point R.



So, ∠PQR + ∠QRX = 180°

[Angles same side of transversal]

∠QRX = 180° – 110°

[∵ ∠PQR = 110°]

∠QRX = 70°

Similarly, ∠RST + ∠SRY = 180°

or ∠SRY = 180° – 130°

[∵ ∠RST = 130°]

∴ ∠SRY = 50°

Now, ∠QRX + ∠QRS + ∠SRY = 180°

[Linear pairs on the line XY]

∴ ∠QRS = 180° – 70° – 50°

∴ ∠QRS = 60°

4. In Fig. 6.26, if AB || CD, ∠ APQ = 50° and ∠ PRD = 127°, find x and y

Click here for all Solved PYQs & New pattern Q's Educart book

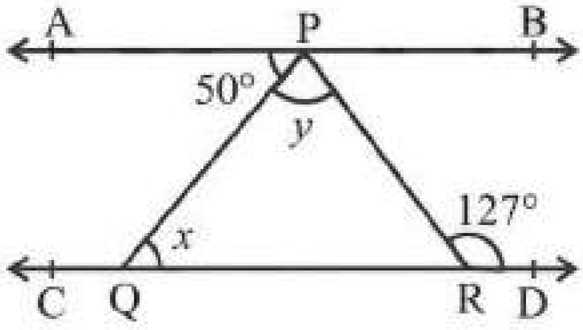


Fig. 6.26

5. In Fig. 6.27, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikesthe mirror RS at C and again reflects back along CD. Prove that AB || CD

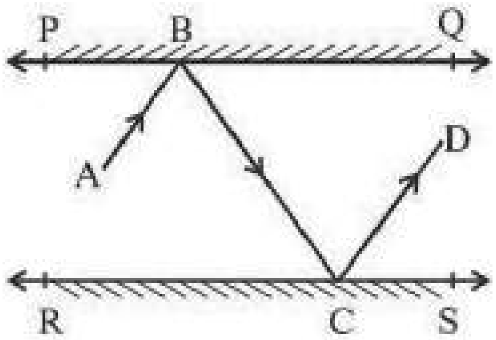


Fig. 6.27

Click here for all Solved PYQs & New pattern Q's Educart book

Triangles 7

NCERT SOLUTIONS

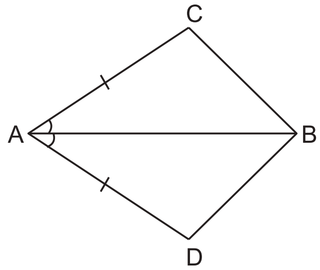
What's inside

– *Textbook Exercise Q's (solved)*

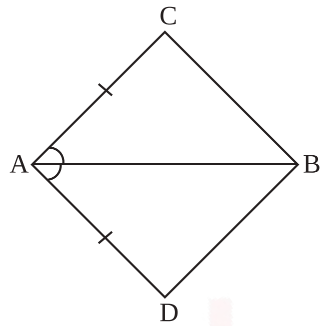
EduCart

Exercise – 7.1

1. In quadrilateral ACBD. AC = AD and AB bisect ∠A (see fig.). Show that $\Delta ABC \cong \Delta ABD$. What can you say about BC and BD?



Sol.



In ΔABC and ΔABD,

$AC = AD$ (given)

$\angle CAB = \angle DAB$ ($\because$ AB bisects $\angle A$)

$AB = AB$(Common side)

$\therefore\ \Delta ABC \cong \Delta ABD$ (by SAS congruency)

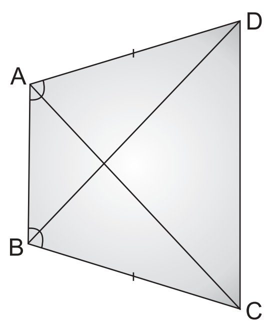
Now, $BC = BD$ [by CPCT]

2. ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA (see fig.) Prove that :

(i) $\Delta ABD \cong \Delta BAC$

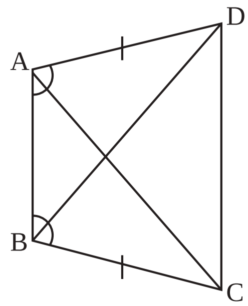
(ii) BD = AC

(iii) ∠ABD = ∠BAC



Click here for all Solved PYQs & New pattern Q's Educart book

Sol.



(i) In ΔABC and ΔABD,

$BC = AD$ (given)

$\angle CBA = \angle DAB$ (given)

$AB = AB$ (common side)

$\therefore$ $\Delta ABD \cong \Delta BAC$[by SAS congruency]

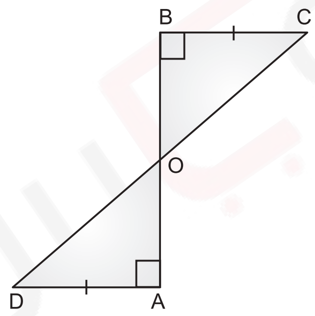
(ii) Now, $\Delta ABD \cong \Delta BAC$

$\therefore$ $BD = AC$ [by CPCT]

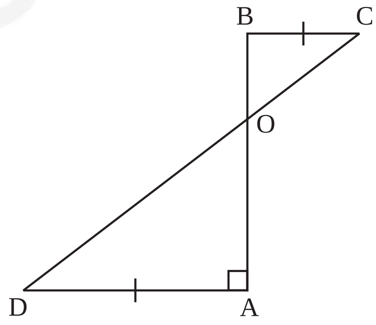
(iii) Now, $\Delta ABD \cong \Delta BAC$

$\therefore$ $\angle BAD = \angle BAC$ [by CPCT]

3. AD and BC are equal perpendiculars to a line segment AB (see fig.). Show that CD bisects AB.



Sol.



In ΔAOD and ΔBOC,

$AD = BC$ [given]

$\angle DAO = \angle CBO = 90°$

$\angle AOD = \angle COB$

[vertically opposite angles]

So, $\Delta AOD \cong \Delta BOC$[by ASA congruency]

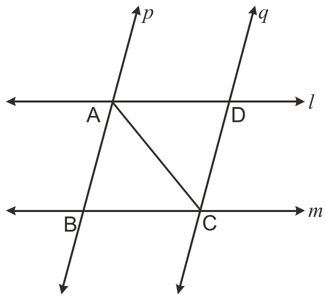
$\therefore$ $AO = BO$

Click here for all Solved PYQs & New pattern Q's Educart book

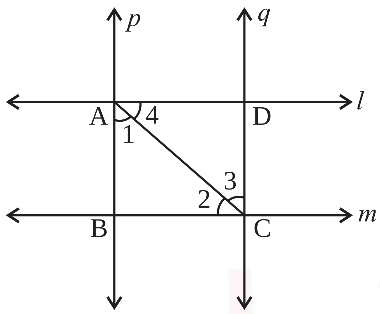
i.e., O is the mid-point of AB.

Hence, CD bisects AB.

4. *l* and *m* are two parallel lines intersected by another pair of parallel lines *p* and *q* (see fig.) Show that $\Delta ABC \cong \Delta CDA$.



Sol.



Given, $l \,||\, m$

$\therefore$ $\angle 2 = \angle 4$ [alternate interior angles]

$p \,||\, q$

$\angle 1 = \angle 3$ [alternate interior angles]

Now, in ΔABC and ΔACD,

$\angle 2 = \angle 4$

$\angle 1 = \angle 3$

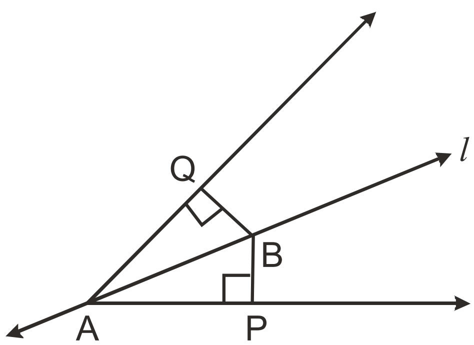
and AC = AC [common side]

$\therefore$ $\Delta ABC \cong \Delta CDA$[by ASA congruency]

5. Line λ is the bisector of an angle $\angle A$ and B is any point on *l*. BP and BQ are perpendiculars from B to the arms of $\angle A$ (see fig.). Show that :

(i) $\Delta APB \cong \Delta AQB$

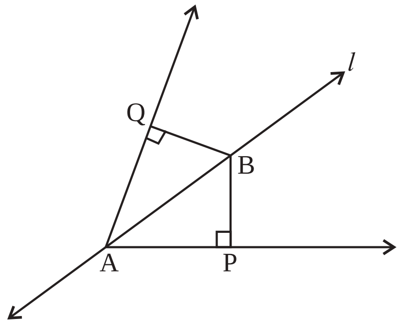
(ii) BP = BQ or B is equidistant from the arms of $\angle A$.



EduCart

Click here for all Solved PYQs & New pattern Q's Educart book

Sol.



(i) In ΔAPB and ΔAQB,

$\angle APB = \angle AQB = 90°$

$\angle PAB = \angle QAB$ [$\because$ l is the bisector of $\angle A$]

$AB = AB$ (common side)

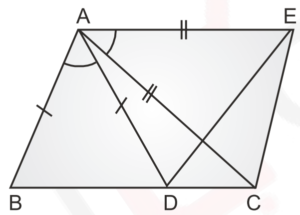
$\therefore$ $\Delta APB \cong \Delta AQB$[by ASA congruency]

(ii) Now, $\Delta APB \cong \Delta AQB$

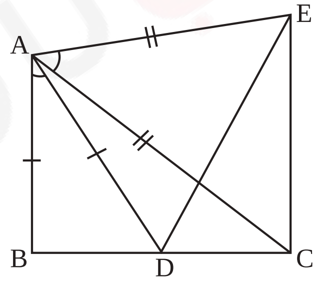
$\therefore$ $BP = BQ$ [by CPCT]

i.e., B is equidistant from the arms of $\angle A$.

6. In fig., AC = AE, AB = AD and $\angle$BAD = $\angle$EAC. Show that BC = DE.



Sol.



In ΔABC and ΔADE,

$AC = AE$ [given]

$AB = AD$ [given]

Now, $\angle BAD = \angle EAC$

Adding $\angle DAC$ to both sides,

$\angle BAD + \angle DAC = \angle EAC + \angle DAC$

$\angle BAC = \angle EAD$

$\therefore$ $\Delta ABC \cong \Delta ADE$[by SAS congruency]

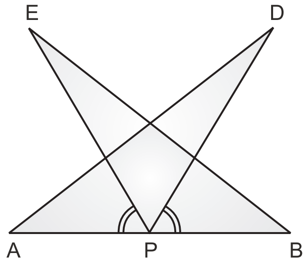
So, $BC = DE$ [by CPCT]

7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that $\angle$BAD = $\angle$ABE and $\angle$EPA = $\angle$DPB (see fig.). Show that :

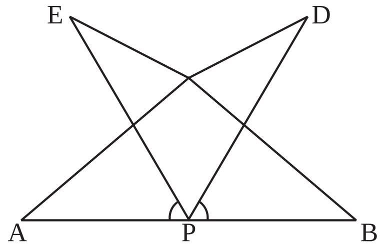
Click here for all Solved PYQs & New pattern Q's Educart book

(i) ΔDAP ≅ ΔEBP

(ii) AD = BE



Sol.



Given, that P is the mid-point of AB.

∴ AP = BP ...(i)

∠BAD = ∠ABE [given]

or ∠PAD = ∠PBE ...(ii)

and∠EPA= ∠DPB [given]

Adding ∠EPD to both sides,

∠EPA + ∠EPD = ∠DPB + ∠EPD

∠APD = ∠BPE ...(iii)

(i) In ΔDAP and ΔEBP, we have

AP = BP [From eq. (i)]

∠PAD = ∠PBE [From eq. (ii)]

∠APD = ∠BPE [From eq. (iii)]

∴ ΔDAP ≅ ΔEBP [by ASA congruency]

(ii) Now, ΔDAP ≅ ΔEBP

∴ AD = BE [by CPCT]

8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see fig.). Show that :

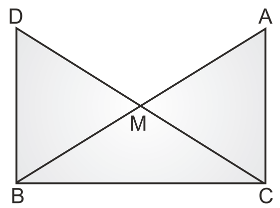
(i) ΔAMC ≅ ΔBMD

(ii) ∠DBC is a right angle.

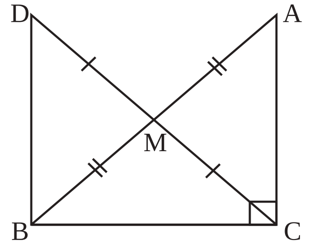
(iii) ΔDBC ≅ ΔACB

(iv) CM = $\frac{1}{2}$AB

Click here for all Solved PYQs & New pattern Q's Educart book



Sol.



(i) In ΔAMC and ΔBMD,

$AM = BM$ [$\because$ M is the mid-point of AB]

$CM = DM$ [Given]

$\angle AMC = \angle DMC$ [vertically opposite angles]

$\therefore \quad \Delta AMC \cong \Delta BMD$ [by SAS congruency]

(ii) Now, $\Delta AMC \cong \Delta BMD$

$AC = BD$ [by CPCT]

$\angle ACM = \angle BDM$ [by CPCT]

but these angles are alternate interior angles

$\therefore \quad AC \,||\, BD$

Now, AC || BD and BC is transversal.

$\therefore \angle ACB + \angle DBC = 180°$

[$\because$ Sum of interior angles on the same side of transversal is equal to 180°.]

$90° + \angle DBC = 180°$

$\angle DBC = 180° - 90° = 90°$

(iii) In ΔDBC and ΔACB,

$BC = BC$ [common side]

$BD = AC$

$\angle CBD = \angle BCA = 90°$

$\Delta DBC \cong \Delta ACB$ [by SAS congruency]

(iv) Now, $\Delta DBC \cong \Delta ACB$

So, $DC = AB$ [by CPCT]

$DM + CM = AB$

$CM + CM = AB$ [$\because$ DM = CM]

$2CM = AB$

$CM = \frac{1}{2}AB$

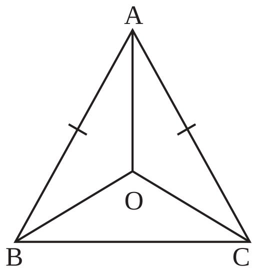
Click here for all Solved PYQs & New pattern Q's Educart book

Exercise – 7.2

1. In an isosceles triangle ABC, with AB = AC, the bisectors of ∠B and ∠C intersect each other at O. Join A to O. Show that :

(i) OB = OC (ii) AO bisects ∠A

Sol. (i)



Given, AB = AC

∴ ∠B = ∠C ...(i) [Angles opposite to equal sides of a triangle are equal.]

BO and CO are bisectors of ∠B and ∠C respectively.

So, $\angle ABO = \angle OBC = \frac{1}{2}\angle B$ ...(ii)

and $\angle ACO = \angle OCB = \frac{1}{2}\angle C$ ...(iii)

From equation (i),

∠B = ∠C

$\frac{1}{2}\angle B = \frac{1}{2}\angle C$ [Dividing by 2]

∠OBC = ∠OCB[From eq. (ii) and (iii)]

∴ OB = OC [∵ Sides opposite to equal angles of a triangle are equal.]

(ii) Given, AB = AC (Given)

OB = OC

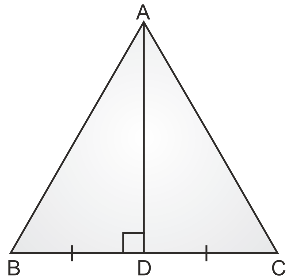
AO = AO (Common side)

∴ ΔABO ≅ ΔACO [by SSS congruency]

So, ∠BAO = ∠CAO [by CPCT]

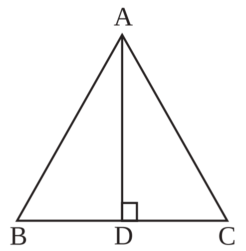
Hence, AO bisects ∠A.

2. In ΔABC, AD is the perpendicular bisector of BC (see fig.) Show that ΔABC is an isosceles triangle in which AB = AC.



Click here for all Solved PYQs & New pattern Q's Educart book

Sol.



In ΔABD and ΔACD,

$\angle$ADB = $\angle$ADC = 90° [$\because$ AD is perpendicular to BC]

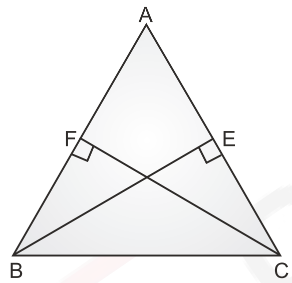
BD = DC [$\because$ AD is bisector of BC]

AD = AD [Common side]

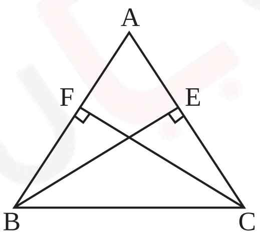
$\therefore$ ΔABD $\cong$ ΔACD [by SAS congruency]

So, AB = AC [by CPCT]

3. ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see fig.). Show that these altitudes are equal.



Sol.



Given, ABC is an isosceles triangle in which

AB = AC

In ΔABE and ΔACF,

$\angle$AEB = $\angle$AFC = 90°

[$\because$ BE $\perp$ AC and CF $\perp$ AB]

AB = AC [given]

$\angle$A = $\angle$A [Common angle]

$\therefore$ ΔABE $\cong$ ΔACF [by ASA congruency]

So, BE = CF [by CPCT]

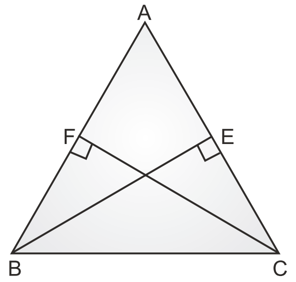
Hence, the altitudes BE and CF are equal.

4. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see fig.) Show that :

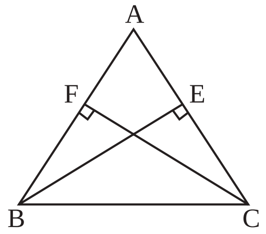
(i) ΔABE $\cong$ ΔACF

(ii) AB = AC, *i.e.*, ABC is an isosceles triangle.

Click here for all Solved PYQs & New pattern Q's Educart book



Sol.



(i) In ΔABE and ΔACF,

$BE = CF$ [given]

$\angle AEB = \angle AFC = 90°$

$\angle BAE = \angle CAF$ [Common angle]

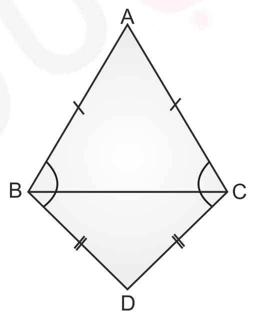
$\therefore$ $\Delta ABE \cong \Delta ACF$[by ASA congruency]

(ii) Now, $\Delta ABE \cong \Delta ACF$ (Given)

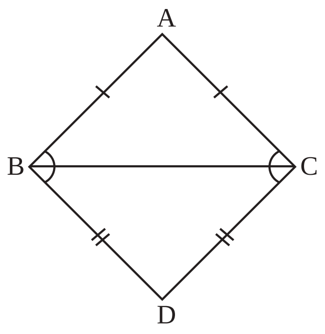
So, $AB = AC$ [by CPCT]

Hence, ABC is an isosceles triangle.

5. ABC and DBC are two isosceles triangles on the same base BC (see fig.). Show that $\angle ABD = \angle ACD$.



Sol.



Click here for all Solved PYQs & New pattern Q's Educart book

In ΔABC,

$$AB = AC$$

$\therefore \quad \angle ABC = \angle ACB \quad ...(i)$

[$\because$ Angles opposite to equal sides of a triangle are equal.]

In ΔBDC,

$$BD = CD$$

$\therefore \quad \angle DBC = \angle DCB \quad ...(ii)$

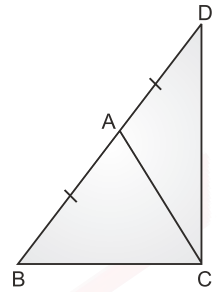
[$\because$ Angles opposite to equal sides of a triangle are equal.]

Adding equations (i) and (ii),

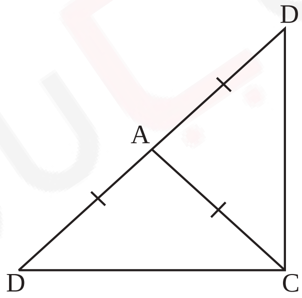
$$\angle ABC + \angle DBC = \angle ABC + \angle DCB$$

$$\angle ABC = \angle DCB$$

6. ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD =AB (see fig.). Show that ∠BCD is a right angle.



Sol.



In ΔABC,

$AB = AC \quad$ [given]

$\therefore \quad \angle ABC = \angle ACB \quad ...(i)$

[$\because$ Angles opposite to equal sides of a triangle are equal.]

Also given, $\quad AB = AD$

$\therefore \quad AC = AD \quad [\because AB = AC]$

Now, in ΔACD,

$$AC = AD$$

$\therefore \quad \angle ADC = \angle ACD \quad ...(ii)$

[$\because$ Angles opposite to equal sides of a triangle are equal.]

Adding equations (i) and (ii),

$$\angle ABC + \angle ADC = \angle ACB + \angle ACD$$

$\angle DBC + \angle BDC = \angle BCD \quad ...(iii)$

Click here for all Solved PYQs & New pattern Q's Educart book

[$\because$ $\angle ADC = \angle BDC$ and $\angle ABC = \angle DBC$]

Now, in ΔBCD, by angle sum property,

$\angle BCD + \angle BDC + \angle CBD = 180°$

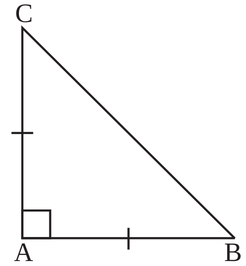
$\angle BCD + \angle BCD = 180°$ [From eq. (iii)]

$2\angle BCD = 180°$

$\angle BCD = 90°$

7. ABC is a right angled triangle in which $\angle A = 90°$ and AB = AC. Find $\angle B$ and $\angle C$.

Sol.



Given, $\angle A = 90°$

$AB = AC$

$\therefore$ $\angle B = \angle C$ [$\because$ Angles opposite to equal sides of a triangle are equal.]

Now, in ΔABC, by angle sum property,

$\angle A + \angle B + \angle C = 180°$

$90° + \angle B + \angle B = 180°$ [$\because$ $\angle B = \angle C$]

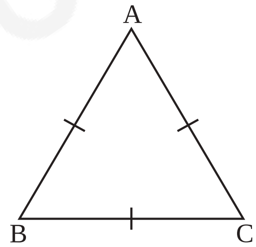
$2\angle B = 90°$

$\angle B = 45°$

and $\angle C = \angle B = 45°$

8. Show that the angles of an equilateral triangle are 60° each.

Sol.



Given, $AB = BC = CA$ [$\because$ ABC is an equilateral triangle]

Now, $AB = BC$

$\therefore$ $\angle A = \angle C$ ...(i)

[$\because$ Angles opposite to equal sides of a triangle are equal.]

Now, $BC = CA$

$\therefore$ $\angle A = \angle B$ ...(ii)

[$\because$ Angles opposite to equal sides of a triangle are equal.]

From equations (i) and (ii), we get

$\angle A = \angle B = \angle C$ ...(iii)

Now, in ΔABC, by angle sum property,

$\angle A + \angle B + \angle C = 180°$

Click here for all Solved PYQs & New pattern Q's Educart book

$$\angle A + \angle A + \angle A = 180°$$
$$3\angle A = 180°$$
$$\angle A = 60°$$

From equation (iii),

$$\angle A = \angle B = \angle C = 60°$$

Hence, the angles of an equilateral triangle are 60° each.

Exercise – 7.3

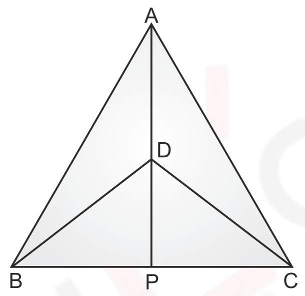
1. .ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see fig.) If AD is extended to intersect BC at P, show that :

(i) ΔABD ≅ ΔACD

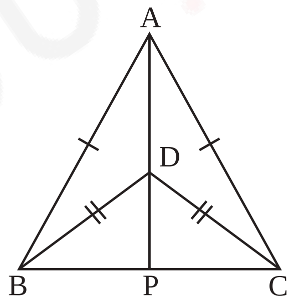
(ii) ΔABP ≅ ΔACP

(iii) AP bisects ∠A as well as ∠D

(iv) AP is the perpendicular bisector of BC.



Sol.



Given, ΔABC and ΔDBC are isosceles triangles.

So, AB = AC and BD = CD

(i) In ΔABD and ΔACD,

AB = AC [given]

BD = CD [given]

AD = AD [Common side]

∴ ΔABD ≅ ΔACD [by SSS congruency]

(ii) In ΔABC,

AB = AC [given]

Click here for all Solved PYQs & New pattern Q's Educart book

So, $\angle ABC = \angle ACB$ ...(i)

[$\because$ Angles opposite to equal sides of a triangle are equal.]

In ΔABP and ΔACP,

$AB = AC$ [given]

So, $\angle ABP = \angle ACP$ [From eq. (i)]

$AP = AP$ [Common side]

$\therefore$ $\Delta ABP \cong \Delta ACP$ [by SAS congruency]

(iii) We have proved that,

$\Delta ABP \cong \Delta ACP$

$\therefore$ $\angle BAP = \angle CAP$ [by CPCT]

Hence, AP bisects $\angle A$.

Also, we have proved that

$\Delta ABD \cong \Delta ACD$

$\therefore$ $\angle BDA = \angle CDA$ [by CPCT]

$180° - \angle BDA = 180° - \angle CDA$

$\angle BDP = \angle CDP$

Hence, AP bisects $\angle D$.

(iv) We have proved that

$\Delta ABP \cong \Delta ACP$

$\therefore$ $\angle APD = \angle APC$ [by CPCT]

and $BP = CP$ [by CPCT]

Now, $\angle APB + \angle APC = 180°$ [by linear pair axiom]

$\angle APB + \angle APB = 180°$[$\angle APB = \angle APC$]

$2\angle APB = 180°$

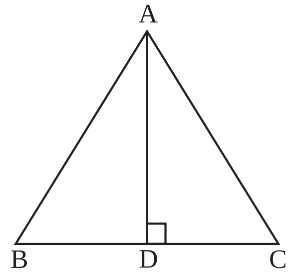
$\angle APB = 90°$

Hence, AP is the perpendicular bisector of BC.

2. AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that :

(i) AD bisects BC (ii) AD bisects $\angle A$

Sol.



Given, ΔABC is an isosceles triangle in which

$AB = AC$

(i) In ΔABD and ΔACD,

$AB = AC$ [given]

$\angle ADB = \angle ADC = 90°$[$\because$ $AD \perp BC$]

$AD = AD$ [Common side]

Click here for all Solved PYQs & New pattern Q's Educart book

$\therefore$ $\Delta ADB \cong \Delta ADC$[by RHS congruency]

So, $BD = CD$ [by CPCT]

Hence, AD bisects BC.

(ii) We have prove that,

$\Delta ADB \cong \Delta ADC$

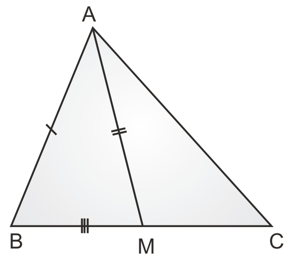
$\therefore$ $\angle BAD = \angle CAD$ [by CPCT]

Hence, AD bisects $\angle A$.

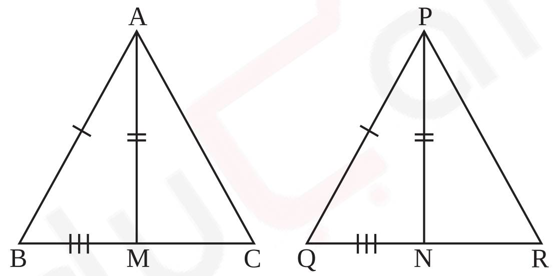
3. Two sides AB and BC median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR (see fig.). Show that :

(i) $\Delta ABM \cong \Delta PQN$

(ii) $\Delta ABC = \Delta PQR$



Sol.



AM is the median of ΔABC.

$\therefore$ $BM = CM = \frac{1}{2}BC$ ...(i)

and $QN = NR = \frac{1}{2}QR$ ...(ii)

Now, BC= QR [Given]

$2BM = 2QM$ [From eq. (i) and (ii)]

$BM = QN$ ...(iii)

(i) In ΔABM and ΔPQN,

$AB = PQ$ [given]

$AM = PN$ [given]

$BM = QN$ [from eq. (iii)]

$\therefore$ $\Delta ABM \cong \Delta PQN$ [by SSS congruency]

(ii) In ΔABC and ΔPQR,

$AB = PQ$ [given]

$\angle B = \angle Q$ [$\because$ $\Delta ABM \cong \Delta PQN$]

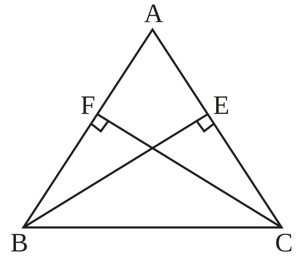
Click here for all Solved PYQs & New pattern Q's Educart book

$BC = QR$ [given]

$\therefore \quad \Delta ABC \cong \Delta PQR$ [by SSS congruency]

4. BE and CF are two equal altitudes of a triangle ABC, Using RHS congruence rule, prove that the triangle ABC is isosceles.

Sol.



In ΔBEC and ΔCFB,

$\angle BEC = \angle CFB = 90°$ [$\because$ $BE \perp AC$ and $CF \perp AB$]

$BC = BC$ [Common side]

$BE = CF$ [given]

$\therefore \quad \Delta BEC \cong \Delta CFB$ [by RHS congruency]

So, $EC = FB$ ...(i) [by CPCT]

In ΔAEB and ΔAFC,

$\angle A = \angle A$ [Common angle]

$\angle AEB = \angle AFC = 90°$ [$\because$ $BE \perp AC$ and $CF \perp AB$]

$BE = CF$ [given]

$\therefore \quad \Delta AEB \cong \Delta AFC$ [by ASA congruency]

So, $AE = AF$ ...(i) [by CPCT]

Adding equations (i) and (ii),

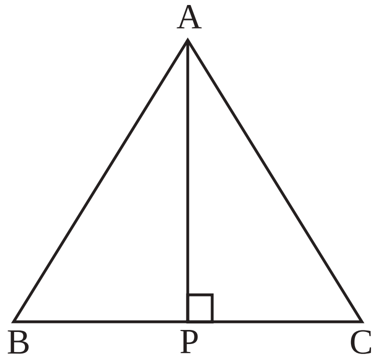
$EC + AE = FB + AF$

$AC = AB$

Hence, ΔABC is an isosceles triangle.

5. ABC is an isosceles triangle with AB = AC. Draw AP ⊥ BC to show that ∠B = ∠C.

Sol.



In ΔABP and ΔACP,

$AB = AC$ [given]

$\angle APB = \angle APC = 90°$ [$\because$ $AP \perp BC$]

Click here for all Solved PYQs & New pattern Q's Educart book

$$AP = AP \qquad \text{[Common side]}$$

$$\therefore \quad \Delta ABP \cong \Delta ACP \qquad \text{[by RHS congruency]}$$

$$\angle B = \angle C \qquad \text{[by CPCT]}$$

Click here for all Solved PYQs & New pattern Q's Educart book

Quadrilaterals 8

NCERT SOLUTIONS

What's inside

– *Textbook Exercise Q's (solved)*

EduCart

Exercise – 8.1

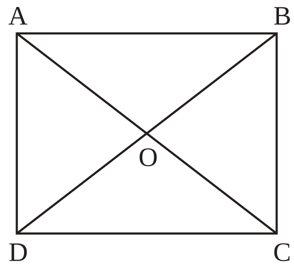
1. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Sol. Given that AB = BD,

To show that, ABCD is a rectangle if the diagonals of a parallelogram are equal.

To show ABCD is a rectangle we have to prove that one of its interior angles is right angles.

Proof: In ΔABC and ΔBAD,



$AB = BA$ (Common)

$BC = AD$ (Opposite side of ||gm)

$AC = BD$ (Given)

$\therefore \quad \Delta ABC \cong \Delta BAD$ [SSS congruency]

$\angle A = \angle B$ [By C.P.C.T.]

Also, $\angle A + \angle B = 180°$ [Sum of angles on the same side of the transversal]

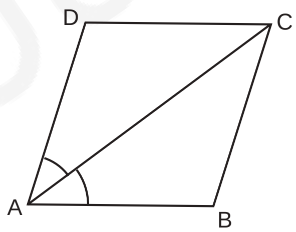
$\Rightarrow \quad 2\angle A = 180°$

$\Rightarrow \quad \angle A = 90° = \angle B$

Therefore, ABCD is a rectangle. **Hence Proved**

2. Show that the diagonals of a square are equal and bisect each other at right angles

3. Diagonal AC of a parallelogram ABCD bisects $\angle$A. Show that :



(i) it bisects $\angle$C also,

(ii) ABCD is a rhombus.

Sol. (i) In ΔADC and ΔCBA,

$AD = CB$ (Opposite sides of a parallelogram)

$DC = BA$ (Opposite sides of a parallelogram)

$AC = CA$ (Common side)

$\Delta ADC \cong \Delta CBA$ [SSS congruency]

Thus, $\angle ACD = \angle CAB$ [by CPCT]

and $\angle CAB = \angle CAD$ (Given)

$\Rightarrow \angle ACD = \angle BCA$

Thus, AC bisect $\angle$C also.

Click here for all Solved PYQs & New pattern Q's Educart book

(ii) $\angle ACD = \angle CAD$ (Proved above)

$\Rightarrow$ $AD = CD$ (Opposite sides of equal angles of a triangle are equal)

Also, $AB = BC = CD = DA$ (Opposite sides of a parallelogram)

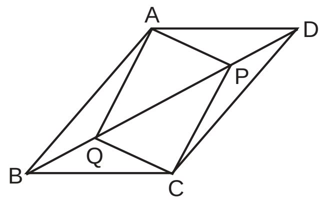
Thus, ABCD is a rhombus.

4. ABCD is a rectangle in which diagonal AC bisects $\angle$ A as well as $\angle$ C. Show that:

(i) ABCD is a square

(ii) diagonal BD bisects $\angle$ B as well as $\angle$ D.

5. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ. Show that :



(i) $\Delta APD \cong \Delta CQB$

(ii) AP = CQ

(iii) $\Delta AQB \cong \Delta CPD$

(iv) AQ = CP

(v) APCQ is a parallelogram

Sol. (i) In ΔAPD and ΔCQB

$DP = BQ$ (Given)

$\angle ADP = \angle CBQ$ [Alternate interior angles]

$AD = BC$

(Opposite sides of a parallelogram)

Thus, $\Delta APD \cong \Delta CQB$ [SAS congruency]

(ii) $\because$ $AP = CQ$ [by CPCT]

as $\Delta APD \cong \Delta CQB$.

(iii) In ΔAQB and ΔCPD,

$BQ = DP$ (Given)

$\angle ABQ = \angle CDP$ [Alternative interior angles]

$AB = CD$

(Opposite sides of a parallelogram)

Thus, $\Delta AQB \cong \Delta CPD$ [SAS congruency]

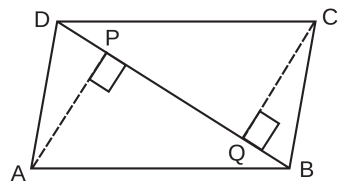
(iv) As $\Delta AQB \cong \Delta CPD$

$AQ = CP$ [By CPCT]

(v) From (ii) and (iv), it is clear that APCQ has equal opposite sides and also has equal and opposite angles, APCQ is a parallelogram.

6. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD. Show that :

Click here for all Solved PYQs & New pattern Q's Educart book



(i) $\Delta APB \cong \Delta CQD$

(ii) AP = CQ

Sol. (i) In ΔAPB and ΔCQD,

$\angle ABP = \angle CDQ$ [Alternate interior angles]

$\angle APB = \angle CQD$ (= 90° as AP and CQ are perpendiculars)

$AB = CD$ (ABCD is a parallelogram)

$\Delta APB \cong \Delta CQD$ [AAS congruency]

(ii) As $\Delta APB \cong \Delta CQD$

So, $AP = CQ$ [by CPCT

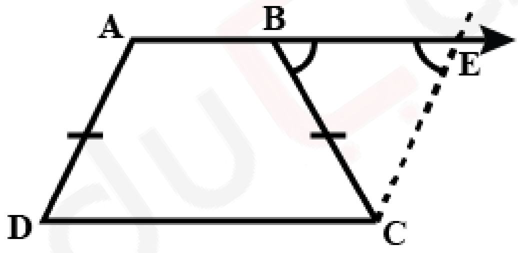
7. ABCD is a trapezium in which AB || CD and AD = BC (see Fig. 8.14). Show that

(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

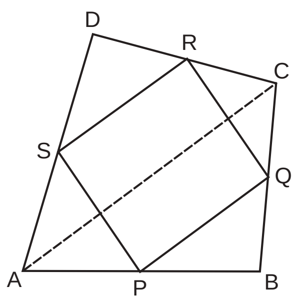
(iii) $\Delta ABC \cong \Delta BAD$

(iv) diagonal AC = diagonal BD [Hint : Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]



Exercise – 8.2

1. ABCD is a quadrilateral in which P, Q, R and S are points of the sides AB, BC, CD and DA. AC is a diagonal. Show that :



(i) SR || AC and SR = $\frac{1}{2}$ AC

(ii) PQ = SR

Click here for all Solved PYQs & New pattern Q's Educart book

(iii) PQRS is parallelogram

Sol. **(i)** R is the mid-point of DC and S is the mid-point of DA.

Thus, by mid-point theorem, SR || AC and SR = $\frac{1}{2}$AC

(ii) In ΔBAC,

P is the mid-point of AB and Q is the mid-point of BC.

Thus, by mid-point theorem, PQ || AC and PQ = $\frac{1}{2}$AC

also, SR = $\frac{1}{2}$AC

PQ = SR

(iii) SR|| AC .... from question (i)

and PQ || AC .... from question (ii)

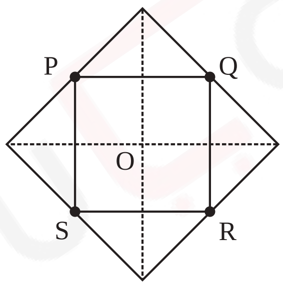
$\Rightarrow$SR || PQ [From (i) and (ii)]

also PQ = SR

$\therefore$ PQRS is a parallelogram.

2. ABCD is a rhombus and P, Q, R, and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Sol. Given in the question,



ABCD is a rhombus and P, Q, R and S are the mid-points of the sides.
AB, BC, CD and DA respectively.

To Prove: PQRS is a rectangle.

Construction: Join AC and BD.

Proof: In ΔDRS and ΔBPQ,

DS = BQ (Halves of the opposite side of the rhombus)

$\angle$SDR = $\angle$QBP (Opposite angles of the rhombus)

DR = BP (Halves of the opposite sides of the rhombus)

ΔDRS $\cong$ ΔBPQ [SAS Congruency]

RS = PQ [by CPCT]...(i)

In ΔQCR and ΔSAP,

RC = PA (Halves of the opposite side of the rhombus)

Click here for all Solved PYQs & New pattern Q's Educart book

$\angle RCQ = \angle PAS$ (Opposite angles of the rhombus)

$CQ = AS$ (Halves of the opposite sides of the rhombus)

$\Delta QCR \cong \Delta SAP$ [SAS congruency]

$RQ = SP$ [by CPCT] ...(ii)

Now, In ΔCDB,

R and Q are the mid-points of CD and BC respectively.

$\Rightarrow$ $QR \parallel BD$

also, P and S are the mid-points of AD and AB respectively.

$\Rightarrow$ $PS \parallel BD$ and $QR \parallel PS$

PQRS is a parallelogram.

also, $\angle PQR = 90°$

Now, In PQRS,

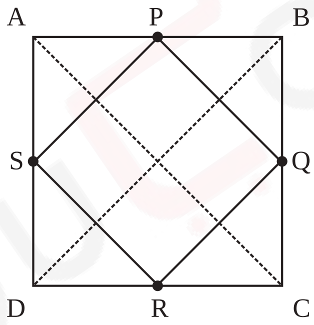
$RS = PQ$ and $RQ = SP$ from (i) and (ii)

$\angle Q = 90°$

PQRS is a rectangle.

3. ABCD is a rectangle and P, Q, R and S are mid-pints of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Sol. Given in the question,



ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively.

Construction: Join AC and BD.

To prove: PQRS is a rhombus.

Proof: In ΔABC,

P and Q are the mid-points of AB and BC respectively.

$PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ (mid-point theorem) ...(i)

In ΔADC

$SR \parallel AC$ and $SR = \frac{1}{2}AC$ (mid-point theorem) ...(ii)

So, $PQ \parallel SR$ and $PQ = SR$

As in quadrilateral PQRS one pair of opposite sides is equal and parallel to each other, so it is a parallelogram.

$PS \parallel QR$ and $PS = QR$ (Opposite sides of parallelogram) ...(iii)

Click here for all Solved PYQs & New pattern Q's Educart book

Now, In ΔBCD,

Q and R are mid-points of side BC and CD respectively.

QR || BD and QR = $\frac{1}{2}$BD (mid-point theorem) ...(iv)

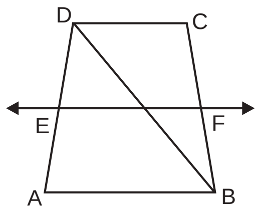
AC = BD (Diagonals of a rectangle are equal) ...(v)

From equations (i), (ii), (iii), (iv) and (v)

PQ = QR = SR = PS

So, PQRS is a rhombus. **Hence Proved**

4. ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F. Show that F is the mid-point of BC.



Sol. Given that,

ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD.

To Prove: F is the mid-point of BC.

Proof: BD, intersected EF at G.

In ΔBAD,

E is the mid-point of AD and also EG || AB

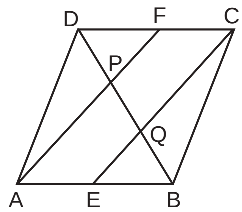
Thus, G is the mid-point of BD (Converse of mid-point theorem)

Now, In ΔBDC

G is the mid-point of BD and also GP || AB || DC.

Thus, F is the mid-point of BC (Converse of mid-point theorem).

5. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively. Show that the line segments AF and EC trisect the diagonal BD.



Sol. Given that,

ABCD is a parallelogram. E and F are the mid-points of sides AB and CD respectively.

To show: AF and EC trisect the diagonal BD.

Proof: ABCD is a parallelogram,

AB || CD

also, AE || FC

Now, AB = CD (Opposite sides of parallelogram ABCD)

Click here for all Solved PYQs & New pattern Q's Educart book

$\Rightarrow \quad \frac{1}{2}AB = \frac{1}{2}CD$

$\Rightarrow \quad AE = FC$ (E and F are mid-points of side AB and CD)

AECF is a parallelogram (AE and CF are parallel and equal to each other)

AP || EC (Opposite sides of a parallelogram)

Now, In ΔDQC

F is the mid-point of side DC and FP || QC

(as AF || EC)

P is the mid-point of DQ (Converse of mid-point theorem).

$\Rightarrow \quad DP = PQ$ ...(i)

Similarly, In ΔAPB,

E is mid-point of side AB and EQ || AP (as AF || EC)

Q is the mid-point of PB (Converse of mid-point theorem)

$\Rightarrow \quad PQ = QB$ ...(ii)

From equations (i) and (ii)

DP = PQ = BQ

Hence, the line segments. AF and EC trisect the diagonal BD. **Hence Proved**

6. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that :

(i) D is the mid-point of AC

(ii) MD $\perp$ AC

(iii) CM = MA = $\frac{1}{2}$ AB

Sol. (i) In ΔABC, M is the mid-point of side AB. According to question given that,

MD || BC

$\therefore \quad AD = DC$ (Converse of mid-point theorem)

Therefore, D is the mid-point of side AC.

(ii) Let, M is the line drawn parallel to side BC and AC is the transversal. (Given)

$\therefore \quad \angle 1 = \angle C$ (Corresponding Angles)

Click here for all Solved PYQs & New pattern Q's Educart book

or $\angle 1 = 90°$ [$\because$ $\angle C = 90°$] (Given)

Therefore, $MD \perp AC$.

(iii) In ΔAMD and ΔCMD.

$\angle 1 = \angle 2 = 90°$ (Proved above)

$AD = DC$ (Proved above)

and $MD = MD$ (Common sides)

Therefore, $\Delta AMD \cong \Delta CMD$ [by SAS Congruency]

$AM = CM$ [by CPCT] ...(i)

Given that, M is the mid-point of side AB.

$$AM = \frac{1}{2}AB \quad ...(ii)$$

From equation (i) and (ii)

$$CM = AM = \frac{1}{2}AB$$

Click here for all Solved PYQs & New pattern Q's Educart book

Circles 9

NCERT SOLUTIONS

What's inside

– *Textbook Exercise Q's (solved)*

EduCart

Exercise – 9.1

1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

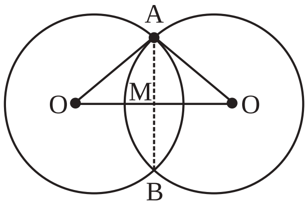
2. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal

Exercise – 9.2

1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

Sol. Let O and O' be the centres at the circles at radii 5 cm and 3 cm, respectively.

Let AB be their common chord.



Given, OA = 5 cm, O'A = 3 cm and OO' = 4 cm

$\therefore$ $AO'^2 + OO'^2 = 3^2 + 4^2 = 9 + 16 - 25 = OA^2$

$\therefore$ OO'A is a right angled triangle and right angled at O'

$$\text{Area of } \Delta OO'A = \frac{1}{2} \times O'A \times OO'$$

$$= \frac{1}{2} \times 3 \times 4$$

$$= 6 \text{ sq. units} \quad ...(i)$$

$$\text{Also, Area of } \Delta OO'A = \frac{1}{2} \times OO' \times AM$$

$$= \frac{1}{2} \times 4 \times AM = 2AM \quad ...(ii)$$

From eqs. (i) and (ii), we get

$$2AM = 6$$

$$\Rightarrow \quad AM = 3$$

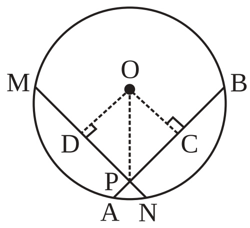
Since, when two circles intersect at two points, then their centre lie on the perpendicular bisector of the common chord.

$$\therefore \quad AB = 2 \times AM = 2 \times 3 = 6 \text{ cm}$$

2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Sol. Given: MN and AS are two chords at a circle with centre O, AS and MN intersect at P and MN = AB

Click here for all Solved PYQs & New pattern Q's Educart book



To prove: MP = PB and PN = AP

Construction: Draw OD ⊥ MN and OC ⊥ AB. Join OP

Proof: $\because$ $DM = DN = \frac{1}{2}MN$

(Perpendicular from centre bisects the chord)

MD = BC and DN = AC ...(i) ($\because$ MN = AS)

In ΔODP and ΔOPC

OD = OC (Equal chords at a circle are equidistant from the centre)

$\angle ODP = \angle OCP$

OP = OP (Common)

$\therefore$ RHS criterion of congruence,

$\Delta ODP \cong \Delta OCP$

$\therefore$ DP = PC (By CPCT) ...(ii)

On adding eq. (i) and (ii), we get

MD + DP = BG + PC

MP = PB

On subtraction eq. (ii) from eq. and (i), we get

DN – DP = AC – PC

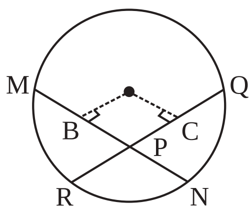
PN = AP

Hence, MP = PB and PN = AP are proved.

3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Sol. Given: RQ and MN are chords at a with centre O. MN and RQ intersect at P and MN = RQ

To prove: $\angle OPC = \angle OPB$



Construction: Draw OC ⊥ RQ and OB ⊥ MN.

Join OP.

Proof: In ΔOCP and ΔOBP, we get

$\angle OCP = \angle OBP$ (Each = 90°)

OP = OP (Common)

Click here for all Solved PYQs & New pattern Q's Educart book

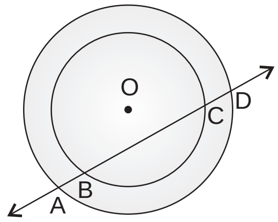
OC = OB (Equal chords at a circle are equidistant from the centre)

∴ By RHS criterion of congruence, we get

$\Delta OCP \cong \Delta OBP$

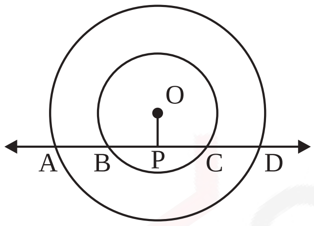
∴ $\angle OPC = \angle OPB$ (By CPCT)

4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD.



Sol. Let OP be the perpendicular from O on line 1.

Since, the perpendicular from the centre at a circle to a chord.



Now, BC is the chord of the smaller circle and OP ⊥ BC.

∴ BP = PC ...(i)

Since, AD is a chord at the larger circle and OP ⊥ AD.

∴ AP = PD ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

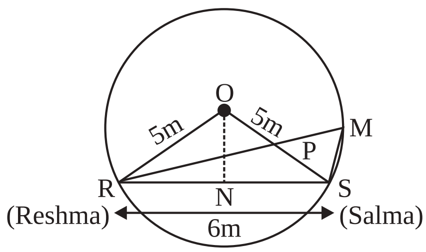
AP – BP = PD – PC

⇒ AB = CD **Hence Proved**

5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip ?

Sol. Let O be the centre at the circle and Reshma, Salma and Mandip are represented by the points P, S and M respectively.

Let RP = XM



Click here for all Solved PYQs & New pattern Q's Educart book

Area of $\Delta QRS = \frac{1}{2} \times x \times 5 = \frac{5x}{2}$ ...(i)

($\because$ In ORM, RM is a chord therefore OP $\perp$ RM)

Again, Area of $\Delta QRS = \frac{1}{2} \times RS \times ON$

$= \frac{1}{2} \times 6 \times 4 = 12$ ...(ii)

[$\because$ RS is a chord, therefore ON $\perp$ RS.

In right ΔRON, $QR^2 = RN^2 + NO^2 \Rightarrow 5^2 = 3^2 + NO^2$

$\Rightarrow NO^2 = 25 - 9 = 16 \Rightarrow NO = 4$ cm]

From eq. (i) and (ii), we get

$$\frac{5x}{2} = 12$$

$$\Rightarrow \qquad x = \frac{24}{5}$$

Since, P is the mid-point of RM

$$RM = 2RP = 2 \times \frac{24}{5}$$

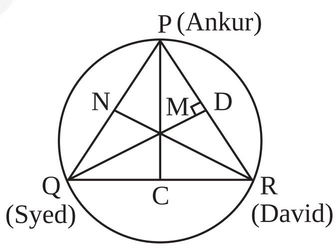
$$= \frac{48}{5} = 9.6 \text{ m}$$

Hence, the distance between Reshma and Mandip is 9.6 m.

6. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Sol. Let Ankur, Syed and David standing on the point P, Q and R.

Let $\qquad PQ = QR = PR = x$



Therefore, ΔPQR is an equilateral triangle. Drawn altitudes PC, QD and RN from vertices to the sides of a triangle and intersect these altitude at the centre at a circle.

As PQR is an equilateral, therefore these altitudes bisects their sides.

In ΔPQC, $\qquad PQ^2 = PC^2 + QC^2$

(By Pythagoras theorem)

$$x^2 = PC^2 + \left(\frac{x}{2}\right)^2$$

Click here for all Solved PYQs & New pattern Q's Educart book

$$PC^2 = x^2 - \frac{x^2}{4} = \frac{3x^2}{4} \quad \left(\because\ QC = \frac{1}{2}QR = \frac{x}{2}\right)$$

$$PC = \frac{\sqrt{3}x}{2}$$

$$MC = PC - PM = \frac{\sqrt{3}x}{2} - 20$$

($\because$ PM = radius = 20 cm)

In ΔQCM, $QM^2 = QC^2 + MC^2$

$$(20)^2 = \left(\frac{x}{2}\right)^2 + \left(\frac{\sqrt{3}x}{2} - 20\right)^2$$

$$400 = \frac{x^2}{4} + \frac{3x^2}{4} - 20\sqrt{3} + 400$$

$$0 = x^2 - 20\sqrt{3}x$$

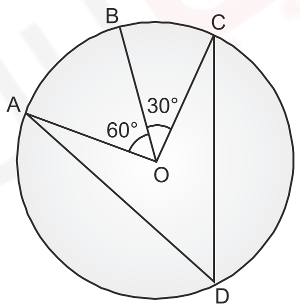
$$x^2 = 20\sqrt{3}x$$

$$x = 20\sqrt{3}$$

Hence, PQ = QR = PR = $20\sqrt{3}$ m

Exercise – 9.3

1. In figure below, A, B and C are three points on a circle with centre O such that ∠BOC = 30° and ∠AOB = 60°. If D is a point on the circle other than the arc ABC, find ∠ADC.



Sol. ∴ ∠AOC= ∠AOB + ∠BOC

= 60P + 30° = 90°

∴ Arc ABC makes 90° at the centre of the circle.

∴ ADC = $\frac{1}{2}$∠AOC

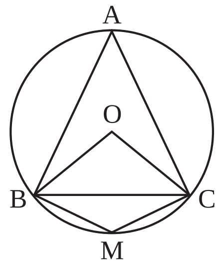
($\because$ The angle subtended by an arc at the centre is double the angle subtended by it any part of the circle.)

$$= \frac{1}{2} \times 90° = 45°$$

Click here for all Solved PYQs & New pattern Q's Educart book

2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Sol. Let BC be chord, which is equal to the radius. Join OB and OC.



Given, BC = OB = OC

$\therefore$ ΔOBC is an equilateral triangle $\angle$BOC = 60°

$\therefore$ $BAC = \frac{1}{2} \angle BOC$

$= \frac{1}{2} \times 60° = 30°$

($\because$ The angle subtend by an arc at the centre is double the angle subtended by it any part of the circle.)

Here, AMC is a cyclic quadrilateral.

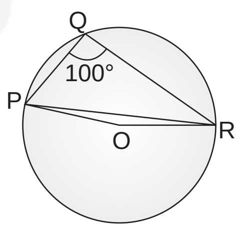
$\therefore$ $\angle BAC + \angle BMC = 180°$

($\because$ In a cyclic quadrilateral the sum of opposite angles is 180°)

$\Rightarrow$ $\angle BMC = 180° - 30°$

$= 150°$

3. In figure below, $\angle$PQR = 100°, where P, Q and R are points on a circle with centre O. Find $\angle$OPR.



Sol. $\therefore$ $\angle POR = 2\angle PQR = 2 \times 100° = 200°$

(Since, the angle subtended by the centre is double he angle subtended by circumference)

Since, In ΔOPR, $\angle POR = 360° - 200° = 160°$ ...(i)

Again, ΔOPR, OP= OR(Radii of the circle)

$\therefore$ $\angle OPR = \angle ORP$

(By property of isosceles triangle)

In ΔPOR, $\angle OPR + \angle ORP + \angle POR = 180°$ ...(ii)

From eq. (i) and (ii), we get

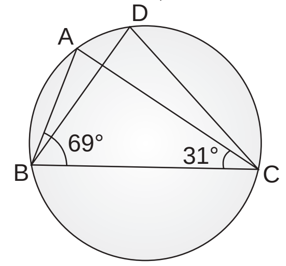
$\angle OPR + \angle OPR + 160° = 180°$

Click here for all Solved PYQs & New pattern Q's Educart book

$\therefore \quad 2\angle OPR = 180° - 160° = 20°$

$\therefore \quad \angle OPR = 120°^{circle/2} = 10°$

4. In figure below ∠ABC = 69°, ∠ACB = 31°, find ∠BDC.



Sol. $\because \quad \angle BDC = \angle BAC \quad ...(i)$ (Since, the angles in the same segment are equal)

Now, in ΔABC

$\therefore \quad \angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad \angle A + 69° + 31° = 180°$

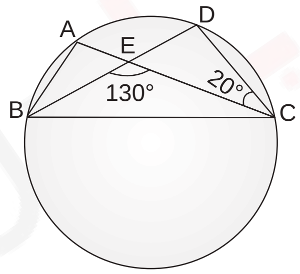
$\Rightarrow \quad \angle A + 100° = 180°$

$\therefore \quad \angle A = 180° - 100° = 80°$

$\Rightarrow \quad \angle BAC = 80°$

$\therefore$ From eq. (i) $\angle BDC = 80°$

5. In figure below, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠BEC = 130° and ∠ECD = 20°. Find ∠BAC.



Sol. $\therefore \quad \angle AEB = 180° - 130°$

$= 50°$ (Linear Pair) ...(i)

$\Rightarrow \quad \angle CED = \angle AEB = 50°$

(Vertically opposite)

Again $\quad \angle ABD = \angle ACD$ (Since, the angles in the same segment are equal)

$\angle ABE = \angle ECD$

$\Rightarrow \quad \angle ABE = 180° \quad ...(ii)$

$\therefore$ In ΔCDE

$\angle A + 20° + 50° = 180°$ [From eq. (i) and (ii)]

$\angle A + 70° = 180°$

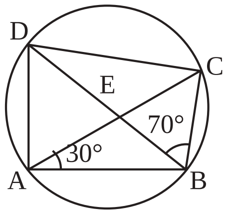
$\therefore \quad \angle A = 180° - 70° = 110°$

Hence $\quad \angle BAC = 110°$

Click here for all Solved PYQs & New pattern Q's Educart book

6. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70°$, $\angle BAC$ is 30°, find $\angle BCD$. Further, if AB = BC, find $\angle ECD$.

Sol. Angles in the same segment are equal.



$\therefore \quad \angle BDC = \angle BAC$

$\therefore \quad \angle BDC = 30°$

In ΔBCD, we have

$\therefore \angle BDC + \angle DBC + \angle BCD = 180°$ (Given, $\angle OBC = 70°$ and $\angle BDC = 30°$)

$\therefore 30° + 70° + \angle BCD = 180°$

$\therefore \quad \angle BCD = 180° - 30° - 70° = 80°$

If AB = BC, then $\angle BCA = \angle BAC = 80°$

(Angles opposite to equal sides in a triangle are equal)

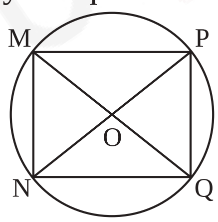
Now, $\quad \angle ECD = \angle BCD - \angle BCA$

$= 80° - 36P = 50°$ ($\because \angle BCD = 80°$ and $\angle BCA = 30°$)

Hence, $\quad \angle BCD = 80°$ and $\angle ECD = 50°$

7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Sol. Given: Diagonals NP and QM at a cyclic quadrilateral NQPM.



To prove: Quadrilateral NQM is a rectangle.

Proof: $\because$ ON = OP = QM = OM (Radii circle)

Now, $\quad ON = OP = \frac{1}{2} = <Q$

and $\quad OM \quad = \frac{1}{2}MQ$

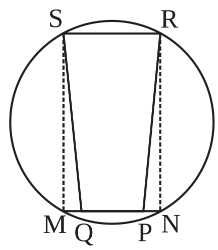
$\therefore \quad NP = MQ$

Hence, the diagonals of the quadrilateral MPQN are equal and bisect each other. So quadrilateral NQPM is a rectangle.

8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Sol. Given: Non-parallel sides PS and BR of a trapezium PQRS are equal.

Click here for all Solved PYQs & New pattern Q's Educart book



To prove: ABCD is a cyclic trapezium

Construction: Draw SM $\perp$ PQ and RN $\perp$ PQ.

Proof: In ΔSMP and ΔRNQ, we et SP = AQ (Given)

$\angle$SMP = $\angle$RNQ (Each = 90°) and SM = RN

($\therefore$ Distance between two parallel lines is always equal)

$\therefore$ By RHS criterion, we get $\Delta SMP \cong \Delta RNQ$

So, $\angle P = \angle Q$ (By CPCT)

and $\angle PSM = \angle QRN$

Now, $\angle PSM = \angle QRN$

$\therefore$ $90° + \angle PSM = 90° + \angle QRN$ (Adding both sides 90°)

So, $\angle PSR = \angle QRS$

i.e., $\angle S = \angle R$

Thus, $\angle P = \angle Q$ and $\angle R = \angle S$ ...(i)

$\therefore$ $\angle P + \angle Q + \angle R + \angle S = 360°$

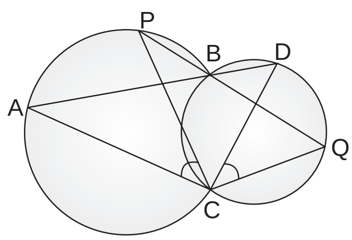
($\because$ Sum of the angles of a quadrilateral is 360°)

$\therefore$ $2\angle S + \angle Q = 360°$ [From eq. (i)]

$\angle S + \angle D = 180°$

Hence, PQRS is a cyclic trapezium.

9. Two circles intersect at two points B and C. Through B, two line segments ABCD and PBQ are drawn to intersect the circles at A, D and P, Q respectively. Prove that $\angle$ACP = $\angle$QCD.



Sol. Given: Two circles intersect at two points B and C. Through B two line segment ABD and PBQ are drawn to intersect the circles at A, D and P, Q, respectively.

To prove: $\angle ACP = \angle QCD$

Proof: In circle, $\angle ACP = \angle ABP$ ...(i) (Angles in the same segment)

In circle 11, $\angle QCD = \angle QBD$ ...(ii) (Angles in the same segment)

Click here for all Solved PYQs & New pattern Q's Educart book

$\angle ABP = \angle QBD$ (Vertically opposite angles)

From eq. (i) and (ii), we get $\angle ACP = \angle QCD$

10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Sol. Given: Two circles are drawn with sides AC and AB at AABC as diameters. Both circles intersect each other at O.

To prove: D lies on BC.

Construction: Join AD.

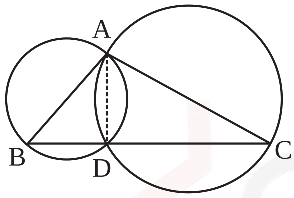
Proof: Since, AC and AB are the diameter of the two circles.

$\angle ADB = 90°$ ...(i)

($\therefore$ Angles in a semi-circle)

and $\angle ADC = 90°$ ...(ii)

(Angles in a semi-circle)



On adding eq. (i) and (ii), we get

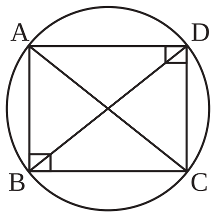
$\angle ADB + \angle ADC = 90° + 90° = 180°$

Hence, BCD is a straight line.

So, D lies on BC.

11. ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

Sol. Since, $\angle ADC$ and ΔABC are right angled triangles with answer common hypotenuse.



Draw a circle with AC as diameter passing through B and D. Join BD.

$\because$ Angles in the same segment are equal.

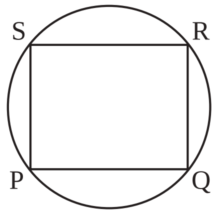
$\therefore$ $\angle CBD = \angle CAD$

12. Prove that a cyclic parallelogram is a rectangle.

Sol. Given: PQRS is a parallelogram inscribed in a circle.

To prove: PQRS is a rectangle.

Click here for all Solved PYQs & New pattern Q's Educart book



Proof: Since, PQRS is a cyclic quadrilateral.

$\therefore \quad \angle P + \angle R = 180°$ ...(i) $\quad$ ($\because$ Sum of opposite angles in a cyclic quadrilateral is 180°)

Put $\quad \angle P = \angle R$ ...(ii) $\quad$ ($\because$ In a ||gm opposite angles are equal)

From eq. (i) and (ii), we get

$$\angle P = \angle R = 90°$$

Similarly, $\quad \angle Q = \angle S = 90°$

$\therefore$ Each angle of PQRS is 90°.

Hence, PQRS is a rectangle.

Click here for all Solved PYQs & New pattern Q's Educart book

NCERT SOLUTIONS

Heron's Formula 10

What's inside

– *Textbook Exercise Q's (solved)*

EduCart

Exercise – 10.1

1. A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board ?

Sol. Let each side of equilateral triangle be a semi-perimeter

$$(s) = \frac{a+a+a}{2}$$

$$= \frac{3}{2}a$$

$$\text{Area of triangle} = \sqrt{\left(\frac{3a}{2}\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$$

$$= \sqrt{\frac{3a}{2}\times\frac{a}{2}\times\frac{a}{2}\times\frac{a}{2}}$$

$$= \frac{\sqrt{3}}{4}a^2 \text{ sq. unit}$$

To find area of triangle, when its perimeter

$$= 180 \text{ cm.}$$

$$\Rightarrow \quad a + a + a = 180 \text{ cm}$$

$$\Rightarrow \quad a = \frac{180}{3}$$

$$= 60 \text{ cm}$$

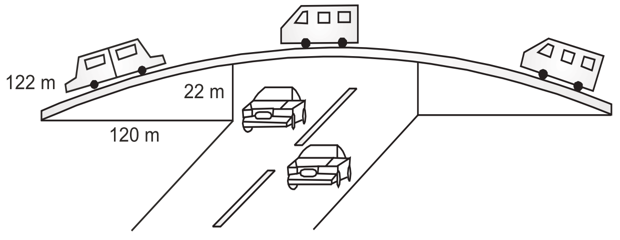
$$\text{Required area} = \frac{\sqrt{3}}{4}a^2$$

$$= \frac{\sqrt{3}}{4}\times 60^2 \text{ cm}^2$$

$$= \frac{\sqrt{3}}{4}\times 60\times 60 \text{ cm}^2$$

$$= 900\sqrt{3} \text{ cm}^2$$

2. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m. The advertisements yield an earning of ₹ 5000 per m² per year. A company hired one of its walls for 3 months. How much rent did it pay ?



Click here for all Solved PYQs & New pattern Q's Educart book

Sol. Let sides are,

$$a = 122 \text{ m},$$
$$b = 22 \text{ m},$$
$$c = 120 \text{ m}$$

then, $$s = \frac{a+b+c}{2}$$

$$= \frac{122+22+120}{2}$$

$$= 132 \text{ m}$$

Area of triangular wall $= \sqrt{s(s-a)(s-b)(s-c)}$

(by Heron's formula)

$$= \sqrt{132(132-122)(132-22)(132-120)} \text{ m}^2$$

$$= \sqrt{132 \times 10 \times 110 \times 12} \text{ m}^2$$

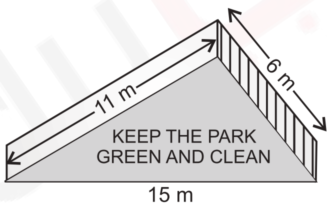
$$= 1320 \text{ m}^2$$

Rate of hiring = ₹5000 per m^2 per year

Total rent for 3 months $= ₹\left(5000 \times 13205 \times \frac{3}{12}\right)$

= ₹16,50,000

3. There is a slide in a park. One of its side walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN". If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour.



Sol. Let sides are :

$$a = 15 \text{ m},$$
$$b = 11 \text{ m},$$
$$c = 6 \text{ m}$$

then, $$s = \frac{a+b+c}{2}$$

$$= \frac{15+11+6}{2}$$

$$= 16 \text{ m}$$

The required area, using Heron's formula

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{16(16-15)(16-11)(16-6)}$$

$$= \sqrt{16 \times 1 \times 5 \times 10}$$

Click here for all Solved PYQs & New pattern Q's Educart book

$= 4 \times 5 \times \sqrt{2}\ m^2$

$= 20\sqrt{2}\ m^2$

4. Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42 cm.

Sol. Let a, b and c be the sides of triangle then,

$a = 18$ cm,

$b = 10$ cm

and $a + b + c = 42$ cm

$\therefore$ $c = 42 - a - b$

$\Rightarrow$ $c = (42 - 18 - 10)$ cm

$= 14$ cm

Now, $s = \frac{1}{2}(a + b + c)$

$= \frac{1}{2} \times 42$ cm

$= 21$ cm

$s - a = (21 - 18)$ cm

$= 3$ cm

$s - b = (21 - 10)$ cm

$= 11$ cm

and $s - c = (21 - 14)$ cm

$= 7$ cm

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21 \times 3 \times 11 \times 7}$

$= \sqrt{3 \times 7 \times 3 \times 11 \times 7}$

$= 3 \times 7\sqrt{11}\ cm^2$

$= 21\sqrt{11}\ cm^2$

5. Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540 cm. Find its area.

Sol. Perimeter of triangle = 540 cm

Let sides of triangles be

$a = 12x$,

$b = 17x$,

$c = 25x$

Now, $a + b + c = 540$

$12x + 17x + 25x = 540$

$54x = 540$

$x = 10$

$a = 12x$

Click here for all Solved PYQs & New pattern Q's Educart book

$= 12 \times 10$

$= 120$ cm

$b = 17x$

$= 17 \times 10$

$= 170$ cm

$c = 25x$

$= 25 \times 10$

$= 250$ cm

$$s = \frac{a+b+c}{2}$$

$$= \frac{540}{2}$$

$= 270$ cm

$\therefore$ Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{270(270-120)(270-170)(270-250)} \text{ cm}^2$$

$$= \sqrt{270 \times 150 \times 100 \times 20} \text{ cm}^2$$

$$= \sqrt{9 \times 3 \times 10 \times 5 \times 3 \times 10 \times 10 \times 10 \times 10 \times 2} \text{ cm}^2$$

$= 9 \times 10 \times 10 \times 10 \text{ cm}^2$

$= 9000 \text{ cm}^2$

6. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

Sol. According to question, Perimeter of triangle = 30 cm

i.e 12 cm + 12 cm + x = 30 cm

or 24 cm + x = 30 cm

or x = 30 cm – 24 cm

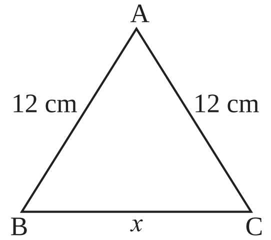
x = 6 cm

$\therefore$ Semi-perimeter of triangle

$$(s) = \frac{12+12+6}{2}$$

$$= \frac{30}{2}$$

= 15 cm.



According to Heron's Formula, area of triangle

Click here for all Solved PYQs & New pattern Q's Educart book

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{15(15-12)(15-12)(15-6)}$

$= \sqrt{15 \times 3 \times 3 \times 9}$ cm^2

$= \sqrt{15 \times 3 \times 3 \times 3 \times 3}$ cm^2

$= 3 \times 3\sqrt{15}$ cm^2

$= 9\sqrt{15}$ cm^2

Click here for all Solved PYQs & New pattern Q's Educart book

Surface Areas and Volumes 11

NCERT SOLUTIONS

What's inside

– *Textbook Exercise Q's (solved)*

EduCart

Exercise – 11.1

1. Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area.

Sol. Given, diameter of the base of a cone (d) = 10.5 cm

So, radius (r) = $\frac{10.5}{2}$ = 5.25 cm

and slant height (l) = 10 cm

Now, curved surface area of cone $= \pi rl$

$$= \frac{22}{7} \times 5.25 \times 10$$

$$= 22 \times 0.75 \times 10$$

$$= 165 \text{ cm}^2$$

2. Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.

Sol. Given, slant height of cone (l) = 21 m

and, diameter of the base of cone (d) = 24 m

and, radius (r) = $\frac{24}{2}$ = 12 m

Now, total surface area of cone $= \pi r(l + r)$

$$= \frac{22}{7} \times 12(21+12)$$

$$= \frac{22}{7} \times 12 \times 33$$

$$= 1244.57 \text{ m}^2$$

3. Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find (i) radius of the base and (ii) total surface area of the cone.

Sol. (i) Given, slant height of cone (l) = 14 cm

and curved surface area of cone = 308 cm^2

$\therefore$ $\pi rl = 308$

$$\frac{22}{7} \times r \times 14 = 308$$

$$r = \frac{308 \times 7}{22 \times 14} = 7 \text{ cm}$$

Hence, radius of the base of the cone is 7 cm.

(ii) Total surface area of the cone $= \pi r(l + r)$

$$= \frac{22}{7} \times 7(14+7)$$

$$= \frac{22}{7} \times 7 \times 21$$

$$= 462 \text{ cm}^2$$

Click here for all Solved PYQs & New pattern Q's Educart book

4. A conical tent is 10 m high and the radius of its base is 24 m. Find :

(i) slant height of the tent.

(ii) cost of the canvas required to make the tent, if the cost of 1 m² canvas is ₹ 70.

Sol. Given, height of conical tent (h) = 10 m

and radius of the base of the conical tent (r) = 24 m

(i) Now, slant height, $l = \sqrt{h^2 + r^2}$

$= \sqrt{(10)^2 + (24)^2}$

$= \sqrt{100 + 576}$

$= \sqrt{676}$

= 26 m

(ii) Curved surface area of the conical tent = πrl

$= \frac{22}{7} \times 24 \times 26$

$= \frac{13728}{7}$ m²

Therefore, $\frac{13728}{7}$ m² canvas is required to make the tent.

Now, required total cost = $70 \times \frac{13728}{7}$

= ₹137280

5. What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm (Use : π = 3.14).

Sol. Given, height of conical tent (h) = 8 m

and radius of the base of the conical tent (r) = 6 m

Now, slant height, $l = \sqrt{h^2 + r^2}$

$= \sqrt{8^2 + 6^2}$

$= \sqrt{64 + 36}$

$= \sqrt{100}$

= 10 m

Now, curved surface area of cone = πrl

= 3.14 × 6 × 10

= 188.4 m²

Therefore, required area of tarpaulin to make the conical tent is 188.4 m².

$\therefore$ Length of tarpaulin $= \frac{\text{Area}}{\text{width}}$

$= \frac{188.4}{3}$ = 62.8 m

Click here for all Solved PYQs & New pattern Q's Educart book

Now, extra length for stitching margins and wastage in cutting = 20 cm = 0.2 m

Hence, required total length of tarpaulin = 62.8 + 0.2

= 63 m

6. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of ₹ 210 per 100 m².

Sol. Given, slant height of conical tomb (l) = 25 m

and base diameter of conical tomb (d) = 14 m

So, base radius (r) = $\frac{14}{2}$ = 7 m

Now, curved surface area of the conical tomb

$$= \pi r l$$

$$= \frac{22}{7}\times 7\times 25$$

$$= 550 \text{ m}^2$$

Now, rate of white washing= ₹210 per 100 m²

∴ Required total cost for white washing

$$= \frac{210\times 550}{100}$$

= ₹1155

7. A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

Sol. Given, base radius of conical joker's cap (r) = 7 cm

and height of conical joker's cap (h) = 24 cm

Now, slant height $l = \sqrt{h^2 + r^2}$

$$= \sqrt{(24)^2 + (7)^2}$$

$$= \sqrt{576 + 49}$$

$$= \sqrt{625}$$

$$= 25 \text{ cm}$$

Now, curved surface area of cone = $\pi r l$

$$= \frac{22}{7}\times 7\times 25$$

$$= 550 \text{ cm}^2$$

Therefore, required area of sheet of make a cap

$$= 550 \text{ cm}^2$$

Hence, required total area of sheet to make 10 such caps

$$= 10 \times 550 = 5500 \text{ cm}^2$$

8. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the

Click here for all Solved PYQs & New pattern Q's Educart book

outer side of each of the cones is to be painted and the cost of painting is ₹ 12 per m², what will be the cost of painting all these cones ?
(Use : π = 3.14 and take $\sqrt{1.04}$ = 1.02)

Sol. Given, base diameter of a cone (d) = 40 cm

So, base radius of the cone (r) = $\frac{40}{2}$ = 20 cm

= 0.2 m

and height of the cone (h) = 1 m

Now, slant height

$$l = \sqrt{h^2 + r^2}$$

$$= \sqrt{(1)^2 + (0.2)^2}$$

$$= \sqrt{1 + 0.04}$$

$$= \sqrt{1.04}$$

= 1.02 m

Now, curved surface area of the cone = πrl

= 3.14 × 0.2 × 1.02

= 0.64056 m²

Rate of painting a cone = ₹12 per m²

∴ Cost of painting such 50 cones = 50 × 12 × 0.64056

= ₹384.34

Exercise – 11.2

1. Find the surface area of a sphere of radius :

(i) 10.5 cm (ii) 5.6 cm (iii) 14 cm

Sol. (i) Given, radius (r) = 10.5 cm

∴ Surface area of sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times (10.5)^2$$

= 4 × 22 × 1.5 × 10.5

= 1386 cm²

(ii) Given, radius (r) = 5.6 cm

∴ Surface area of sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times (5.6)^2$$

= 4 × 22 × 0.8 × 5.6

= 394.24 cm²

(iii) Given, radius (r) = 14 cm

∴ Surface area of sphese = $4\pi r^2$

Click here for all Solved PYQs & New pattern Q's Educart book

$$= 4\times\frac{22}{7}\times(14)^2$$

$= 4 \times 22 \times 2 \times 14$

$= 2464\ \text{cm}^2$

2. Find the surface area of a sphere of diameter :

(i) 14 cm (ii) 21 cm (iii) 3.5 cm

Sol. (i) Given, diameter = 14 cm

then radius (r) = $\frac{14}{2}$ = 7 cm

$\therefore$ Surface area of sphere $= 4\pi r^2$

$$= 4\times\frac{22}{7}\times(7)^2$$

$= 4 \times 22 \times 7$

$= 616\ \text{cm}^2$

(ii) Given, diameter = 21 cm

then radius (r) = $\frac{21}{2}$ = 10.5 cm

$\therefore$ Surface area of sphere $= 4\pi r^2$

$$= 4\times\frac{22}{7}\times(10.5)^2$$

$= 4 \times 22 \times 1.5 \times 10.5$

$= 1386\ \text{cm}^2$

(iii) Given, diameter = 3.5 cm

then radius (r) = $\frac{3.5}{2}$ cm

$\therefore$ Surface area of sphere $= 4\pi r^2$

$$= 4\times\frac{22}{7}\times\left(\frac{3.5}{2}\right)^2$$

$$= 4\times\frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2}$$

$= 38.5\ \text{cm}^2$

3. Find the total surface area of a hemisphere of radius 10 cm. (Use : π = 3.14)

Sol. Given, radius of hemisphere (r) = 10 cm

$\therefore$ Total surface area of hemisphere $= 3\pi r^2$

$= 3 \times 3.14 \times (10)^2$

$= 942\ \text{cm}^2$

4. The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Sol. **Case I :** radius of spherical balloon (r_1) = 7 cm

Click here for all Solved PYQs & New pattern Q's Educart book

$\therefore$ Surface area of the balloon (S_1) $= 4\pi r_1^2$

$$= 4\times\frac{22}{7}\times(7)^2$$

$= 616 \text{ cm}^2$

Case II : radius of spherical balloon (r_2) = 14 cm

$\therefore$ Surface area of the balloon (S_2) $= 4\pi r_2^2$

$$= 4\times\frac{22}{7}\times(14)^2$$

$= 2464 \text{ cm}^2$

Now, required ratio $= S_1 : S_2$

$= 616 : 2464$

$= 1 : 4$

5. A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin-plating it on the inside at the rate of ₹ 16 per 100 cm².

Sol. Given, inner diameter of hemispherical bowl (d)

= 10.5 cm

then, inner radius of hemispherical bowl (r)

$$= \frac{10.5}{2}$$

= 5.25 cm

Now, inner curved surface of the hemispherical bowl

$= 2\pi r^2$

$$= 2\times\frac{22}{7}\times(5.25)^2$$

$= 2 \times 22 \times 0.75 \times 5.25$

$= 173.25 \text{ cm}^2$

Rate of cost of tin-plating the bowl = ₹16 per 100 cm²

$\therefore$ Total cost of tin-plating the bowl $= \dfrac{16\times173.25}{100}$

= ₹27.72

6. Find the radius of a sphere whose surface area is 154 cm².

Sol. Given, Surface area of sphere = 154 cm²

$$\therefore \quad 4\pi r^2 = 154$$

$$4\times\frac{22}{7}\times r^2 = 154$$

$$r^2 = \frac{154\times7}{4\times22} = \frac{49}{4}$$

$$r = \sqrt{\frac{49}{4}}$$

$$r = \frac{7}{2} = 3.5 \text{ cm}$$

Click here for all Solved PYQs & New pattern Q's Educart book

Hence, radius of sphere is 3.5 cm.

7. The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.

Sol. Let, diameter of the earth is d unit.

then, radius of the earth $(r_1) = \frac{d}{2}$ unit

Now, diameter of the moon $= \frac{d}{4}$ unit

then, radius of the moon $(r_2) = \frac{d}{4\times 2} = \frac{d}{8}$ unit

Now, required ratio = Surface area of earth : Surface area of moon

$$= 4\pi r_1^2 : 4\pi r_2^2$$
$$= r_1^2 : r_2^2$$
$$= \left(\frac{d}{2}\right)^2 : \left(\frac{d}{8}\right)^2$$
$$= \frac{d^2}{4} : \frac{d^2}{64}$$
$$= \frac{1}{4} : \frac{1}{64}$$
$$= 1 : \frac{1}{16}$$
$$= 16 : 1$$

8. A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.

Sol. Given, inner radius of hemispherical bowl (r)

$= 5$ cm

and thickness $= 0.25$ cm

then outer radius of hemispherical bowl (R)

$= 5 + 0.25$

$= 5.25$ cm

$\therefore$ Outer curved surface of hemispherical bowl

$$= 2\pi R^2$$
$$= 2\times\frac{22}{7}\times(5.25)^2$$
$$= 173.25 \text{ cm}^2$$

9. A right circular cylinder just encloses a sphere of radius r (see Fig. 11.10). Find

(i) surface area of the sphere,

(ii) curved surface area of the cylinder,

(iii) ratio of the areas obtained in (i) and (ii).

Click here for all Solved PYQs & New pattern Q's Educart book

Exercise – 11.3

1. Find the volume of the right circular cone with :

(i) radius 6 cm, height 7 cm

(ii) radius 3.5 cm, height 12 cm

Sol. (i) Given, radius (r) = 6 cm

and height (h) = 7 cm

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\frac{22}{7}\times(6)^2\times7$

$= \frac{1}{3}\times22\times6\times6$

$= 264 \text{ cm}^2$

(ii) Given, radius (r) = 3.5 cm

and height (h) = 12 cm

$\therefore$ Volume of the cone$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\frac{22}{7}\times(3.5)^2\times12$

$= \frac{1}{3}\times22\times0.5\times3.5\times12$

$= 154 \text{ cm}^2$

2. Find the capacity in litres of a conical vessel with :

(i) radius 7 cm, slant height 25 cm

(ii) height 12 cm, slant height 13 cm

Sol. (i) Given, radius (r) = 7 cm

and slant height (l) = 25 cm

Now, $l^2 = h^2 + r^2$

$h^2 = l^2 - r^2$

$h = \sqrt{(25)^2-(7)^2}$

$= \sqrt{625-49}$

Click here for all Solved PYQs & New pattern Q's Educart book

$= \sqrt{576}$

$= 24$ cm

$\therefore$ Capacity of conical vessel $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\frac{22}{7}\times(7)^2\times24$

$= \frac{1}{3}\times22\times7\times24$

$= 1232$ cm^3

$= \frac{1232}{1000}$ litres [$\because$ 1000 cm^2 = 1L]

$= 1.232$ litres

(ii) Given, height (h) = 12 cm

and slant height (l) = 13 cm

Now, $l^2 = h^2 + r^2$

$r^2 = l^2 - h^2$

$r = \sqrt{(13)^2-(12)^2}$

$= \sqrt{169-144}$

$= \sqrt{25}$

$= 5$ cm

$\therefore$ Capacity of conical vessel$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\frac{22}{7}\times(5)^2\times12$

$= \frac{1}{3}\times\frac{22}{7}\times25\times12$

$= \frac{2200}{7}$ cm^3

$= \frac{2200}{7\times1000}$ [$\because$ 1000 cm^2 = 1 litre]

$= \frac{1.1}{35}$ litres

3. The height of a cone is 15 cm. If its volume is 1570 cm^3, find the radius of the base. (Use : π = 3.14)

Sol. Given, height of a cone (h) = 15 cm

and volume of the cone = 1570 cm^3

$\therefore$ $\frac{1}{3}\pi r^2 h = 1570$

$\frac{1}{3}\times3.14\times r^2\times15 = 1570$

Click here for all Solved PYQs & New pattern Q's Educart book

$$15.7 \times r^2 = 1570$$

$$r^2 = \frac{1570}{15.7} = 100$$

$$r = \sqrt{100} = 10 \text{ cm}$$

Hence, radius of the base of the cone is 10 cm.

4. If the volume of a right circular cone of height 9 cm is 48 π cm^3, find the diameter of its base.

Sol. Given, height of a cone (h) = 9 cm

and volume of the cone = 48π cm^3

$$\therefore \quad \frac{1}{3}\pi r^2 h = 48\pi$$

$$\frac{1}{3}\pi r^2 \times 9 = 48\pi$$

$$3\pi r^2 = 48\pi$$

$$r^2 = 16$$

$$r = \sqrt{16} = 4 \text{ cm}$$

Now, required diameter $= 2 \times r$

$= 2 \times 4 = 8$ cm

5. A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres ?

Sol. Given, diameter of conical pit (d) = 3.5 cm

So, radius of conical pit (r)= $\frac{3.5}{2}$ m

and height of conical pit (h) = 12 m

$\therefore$ Capacity of the conical pit = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{3.5}{2}\right)^2 \times 12$$

$= 38.5$ m^3

$= 38.5$ kilolitre [$\because$ 1 m^3 = 1 kilolitre]

6. The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find :

(i) height of the cone

(ii) slant height of the cone

(iii) curved surface area of the cone

Sol. Given, diameter of the base of a cone (d) = 28 cm

then, radius of the base of a cone (r) = $\frac{28}{2}$ = 14 cm

(i) Volume of cone = 9856 cm^3

$$\therefore \quad \frac{1}{3}\pi r^2 h = 9856$$

Click here for all Solved PYQs & New pattern Q's Educart book

$$\frac{1}{3}\times\frac{22}{7}\times(14)^2\times h = 9856$$

$$\frac{1}{3}\times 22\times 28\times h = 9856$$

$$h = \frac{9856\times 3}{22\times 28} = 48 \text{ cm}$$

Hence, height of the cone is 48 cm.

(ii) $$l = \sqrt{h^2+r^2}$$

$$= \sqrt{(48)^2+(14)^2}$$

$$= \sqrt{2304+196}$$

$$= \sqrt{2500}$$

$$= 50 \text{ cm}$$

Hence, slant height of the cone is 50 cm.

(iii) Curved surface area of the cone = πrl

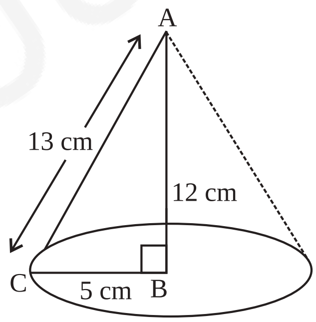
$$= \frac{22}{7}\times 14\times 50$$

$$= 22 \times 2 \times 50$$

$$= 2200 \text{ cm}^2$$

7. A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Sol. Given a right triangle ABC with sides 5 cm, 12 cm and 13 cm. It is revolved about the side 12 cm, then the solid is a right circular cone obtained with radius (r) = 5 cm and height (h) = 12 cm.



$\therefore$ Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi\times(5)^2\times 12$$

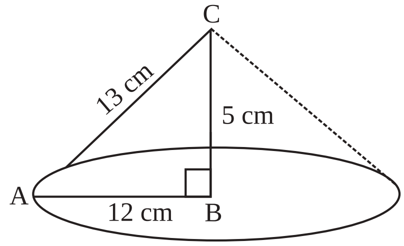
$$= 100\pi \text{ cm}^3$$

8. If the triangle ABC in the Question 15 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 15 and 16.

Sol. Given a right triangle ABC with sides 5 cm, 12 cm and 13 cm. It is revolved about the side 5

Click here for all Solved PYQs & New pattern Q's Educart book

cm, then the solid is a right circular cone obtained with radius (r) = 12 cm and height (h) = 5 cm.



$\therefore$ Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times (12)^2 \times 5$

$= 240\pi \text{ cm}^3$

Now, required ratio $= 100\pi : 240\pi = 5 : 12$

9. A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

Sol. Given, diameter of the heap of wheat (d) = 10.5 m

So, radius (r) $= \frac{10.5}{2} = 5.25$ m

and height of the heap of wheat (h) = 3 cm

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (5.25)^2 \times 3$

$= \frac{1}{3} \times 22 \times 0.75 \times 5.25 \times 3$

$= 86.625 \text{ m}^2$

Now, $l = \sqrt{h^2 + r^2}$

$= \sqrt{(3)^2 + (5.25)^2}$

$= \sqrt{9 + 27.5625}$

$= \sqrt{36.5625}$

$= 6.05$ m

Now, required area of the canvas to cover it

= curved surface area

$= \pi r l$

$= \frac{22}{7} \times 5.25 \times 6.05$

$= 99.825 \text{ m}^2$

Click here for all Solved PYQs & New pattern Q's Educart book

Exercise – 11.4

1. Find the volume of a sphere whose radius is :

(i) 7 cm (ii) 0.63 cm

Sol. (i) Given, radius (r) = 7 cm

$\therefore$ Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (7)^3$$

$$= \frac{4}{3} \times 22 \times 49$$

$$= \frac{4312}{3}$$

$$= 1437\frac{1}{3} \text{ cm}^3$$

(ii) Given, radius (r) = 0.63 m

$\therefore$ Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (0.63)^3$$

$$= \frac{4}{3} \times 22 \times 0.09 \times 0.63 \times 0.63$$

$$= 4 \times 22 \times 0.03 \times 0.3969$$

$$= 1.05 \text{ m}^3 \text{ (approx.)}$$

2. Find the amount of water displaced by a solid spherical ball of diameter :

(i) 28 cm (ii) 0.21 cm

Sol. (i) Given, diameter of spherical ball (d) = 28 cm

then radius of spherical ball (r) $= \frac{28}{2} = 14$ cm

Now, Volume of spherical ball $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (14)^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14$$

$$= \frac{34496}{3}$$

$$= 11498\frac{2}{3} \text{ cm}^3$$

Hence, $11498\frac{2}{3}$ cm^3 amount of water displaced by the ball.

(ii) Given, diameter of spherical ball (d) = 0.21 cm

Click here for all Solved PYQs & New pattern Q's Educart book

then radius of spherical ball (r) = $\frac{0.21}{2}$ m

Now, Volume of spherical ball = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3}\times\frac{22}{7}\times\left(\frac{0.21}{2}\right)^3$$

$$= \frac{4}{3}\times\frac{22}{7}\times\frac{0.21}{2}\times\frac{0.21}{2}\times\frac{0.21}{2}$$

$$= 0.004851\text{m}^3$$

Hence, 0.004851m^3 amount of water displaced by the ball.

3. The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm³ ?

Sol. Given, diameter of metallic ball = 4.2 cm

then, radius of metallic ball (r) = $\frac{4.2}{2}$ = 2.1 cm

$\therefore$ Volume of the ball = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3}\times\frac{22}{7}\times(2.1)^3$$

$$= 38.808 \text{ cm}^3$$

Now, density of the metal = 8.9 g per cm^3

Therefore, mass of the ball = 8.9 × 38.808

= 345.39 g

4. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon ?

Sol. Let, diameter of the earth is d unit.

then, radius of the earth (r_1)= $\frac{d}{2}$ unit

and, diameter of the moon = $\frac{d}{4}$ unit

then, radius of the moon (r_2)= $\frac{d}{4\times 2}$ = $\frac{d}{8}$ unit

$$\frac{\text{Volume of the earth}}{\text{Volume of the moon}} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3}$$

$$\frac{\text{Volume of the earth}}{\text{Volume of the moon}} = \left(\frac{r_1}{r_2}\right)^3$$

$$\frac{\text{Volume of the earth}}{\text{Volume of the moon}} = \left(\frac{\frac{d}{2}}{\frac{d}{8}}\right)^3 = 64$$

Click here for all Solved PYQs & New pattern Q's Educart book

Volume of the moon = $\frac{1}{64}$ Volume of the earth

Hence, Volume of the moon is $\frac{1}{64}$ of the Volume of the earth.

5. How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold ?

Sol. Given, diameter of hemispherical bowl = 10.5 cm

then, radius of hemispherical bowl = $\frac{10.5}{2}$ = 5.25 cm

$$\text{Volume of the bowl} = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3}\times\frac{22}{7}\times(5.25)^3$$

$$= 303.1875 \text{ cm}^3$$

$$= \frac{303.1875}{1000} \text{ litres} \qquad [\because 1000 \text{ cm}^3 = 1 \text{ litre}]$$

$$= 0.303 \text{ litres}$$

Hence, 0.303 litres of milk can be hold by the bowl.

6. A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m. then find the volume of the iron used to make the tank.

Sol. Given, inner radius of hemispherical tank (r) = 1 m

and thickness (t) = 1 cm = 1.01 m

then, outer radius of hemispherical tank (R) = $r + t$

= 1 + 0.01 = 0.01 m

Now, volume of iron used to make the tank

= Outer volume of the tank – Inner volume of the tank

$$= \frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3$$

$$= \frac{2}{3}\pi[R^3 - r^3]$$

$$= \frac{2}{3}\times\frac{22}{7}[(1.01)^3 - (1)^3]$$

$$= \frac{44}{21}\times 0.030301$$

$$= 0.6348 \text{ m}^3 \text{ (approx.)}$$

7. Find the volume of a sphere whose surface area is 154 cm^2.

Sol. Given, Surface area of sphere = 154 cm^2

$$\therefore \quad 4\pi r^2 = 154$$

$$4\times\frac{22}{7}\times r^2 = 154$$

$$r^2 = \frac{154\times 7}{4\times 22} = \frac{49}{4}$$

Click here for all Solved PYQs & New pattern Q's Educart book

$$r = \sqrt{\frac{49}{4}} = \frac{7}{2}$$

$$r = 3.5 \text{ cm}$$

Now, Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (3.5)^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5$$

$$= \frac{539}{3}$$

$$= 179\frac{2}{3} \text{ cm}^3$$

8. A dome of a building is in the form of a hemisphere. From inside, it was white washed at the cost of ₹ 4989.60. If the cost of the white washing is ₹ 20 per square metre, find the :

(i) inside surface area of the dome.

(ii) volume of the air inside the dome.

Sol. Given, total cost of white washing inside the hemispherical dome = ₹ 4989.60
and rate of cost of white washing inside the hemispherical dome = ₹ 20 per m^2

(i) $\therefore$ Inside surface area of the dome $= \frac{\text{Total cost}}{\text{Rate}}$

$$= \frac{4989.60}{20}$$

$$= 249.48 \text{ m}^2$$

(ii) Now, inside surface area of hemispherical dome

$$= 249.48 \text{ m}^2$$

$$\therefore \quad 2\pi r^2 = 249.48$$

$$2 \times \frac{22}{7} \times r^2 = 249.48$$

$$r^2 = \frac{249.48 \times 7}{2 \times 22} = 39.69$$

$$r = \sqrt{39.69} = 6.3 \text{ m}$$

$\therefore$ Volume of the air inside the dome

$$= \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times (6.3)^3$$

$$= \frac{44}{21} \times 6.3 \times 6.3 \times 6.3$$

Click here for all Solved PYQs & New pattern Q's Educart book

$= 523.9 \text{ m}^3$ (approx.)

9. Twenty seven solid iron spheres, each of radius *r* and surface area S are melted to form a sphere with surface area S′. Find the :

(i) radius *r*′ of the new sphere.

(ii) ratio of S and S′.

Sol. Given, radius of a solid iron sphere = r

and surface area of solid iron sphere = S

Now, volume of iron sphere = $\frac{4}{3}\pi r^3$

then, volume of 27 iron spheres = $27 \times \frac{4}{3}\pi r^3$

$= 36\pi r^3$

(i) 27 iron spheres are melted to form a new sphere whose radius r'.

$\therefore$ Volume of 27 sphere = Volume of new sphere

$$36\pi r^3 = \frac{4}{3}\pi (r')^3$$

$$(r')^3 = \frac{36\pi r^3 \times 3}{4\pi} = 27r^3$$

$$r' = \sqrt[3]{27r^3} = 3r$$

Hence, radius of the new sphere is $3r$.

(ii) Surface area of a iron sphere (S) = $4\pi r^2$

and Surface area of new sphere (S') = $4\pi(r')^2$

$= 4\pi(3r)^2$

$= 4\pi \times 9r^2$

$= 36\pi r^2$

$\therefore$ Required ratio = S : S'

$= 4\pi r^2 : 36\pi r^2$

$= 1 : 9$

10. A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule ?

Click here for all Solved PYQs & New pattern Q's Educart book

Sol. Given, diameter of a spherical capsule (d) = 3.5 mm

then, radius of the spherical capsule (r) $= \frac{3.5}{2}$ mm

$\therefore$ Volume of the capsule $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\times\frac{22}{7}\times\left(\frac{3.5}{2}\right)^3$$

$$= \frac{4}{3}\times\frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2}\times\frac{3.5}{2}$$

$= 22.46\ \text{mm}^3$ (approx.)

Hence, approximate $22.46\ \text{mm}^3$ medicine is needed to fill the capsule.

Click here for all Solved PYQs & New pattern Q's Educart book

Statistics 12

NCERT SOLUTIONS

What's inside

– Textbook Exercise Q's (solved)

EduCart

Exercise – 12.1

1. A survey conducted by an organisation for the cause of illness and death among the women between the ages 15 – 44 (in years) worldwide, found the following figures (in %) :

S. No.	Causes	Female Fatality Rate (%)
1.	Reproductive health conditions	31.8
2.	Neuropsychiatric conditions	25.4
3.	Injuries	12.4
4.	Cardiovascular conditions	4.3
5.	Respiratory conditions	4.1
6.	Other causes	22.0

(i) Represent the information given above graphically.

(ii) Which condition is the major cause of women's ill health and death worldwide?

(iii) Try to find out, with the help of your teacher, any two factors which play a major role in the cause in (ii) above being the major cause.

Sol. (i) We draw the bar graph of this data in the following steps. The unit in the second column in the table is percentage.

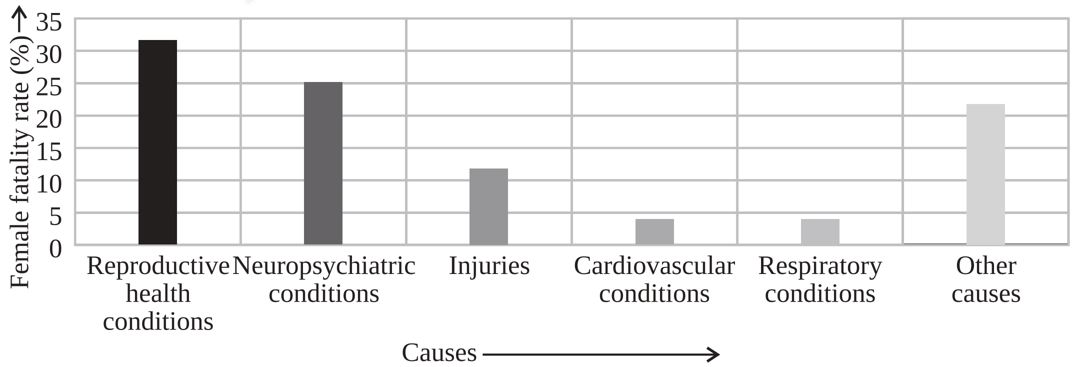
Step 1 : We represent the cause (variable) on the horizontal axis choosing any scale, since the width of the bar is not important. But for clarity, we take equal widths for all bars and maintain equal gaps in between. Let one cause be represented by one unit.

Step 2 : We represent the female fatality rate (value) on the vertical axis. Since, the maximum female fatality rate is 31.8, we can choose the scale as 1 unit = 5%.

Step 3 : To represent the first cause, *i.e.*, Reproductive health conditions, we draw a rectangular bar with width 1 unit and height as $31.8 \div 5 = 6.36$ units.

Step 4 : Similarly, other causes are represented leaving a gap of 1 unit in between two consecutive bars.

The required bar graph is drawn below :



(ii) From graph it is clear that the major cause of women's ill health and death worldwide is 'Reproductive health conditions'.

(iii) Two factors are uneducated and poor background.

Click here for all Solved PYQs & New pattern Q's Educart book

2. **The following data on the number of girls (to the nearest ten) per thousand boys in different sections of Indian society is given below :**

Section	Number of Girls Per Thousand Boys
Scheduled Caste (SC)	940
Scheduled Tribe (ST)	970
Non SC/ST	920
Backward districts	950
Non-backward districts	920
Rural	930
Urban	910

(i) Represent the information above by a bar graph.

(ii) In the classroom discuss what conclusions can be arrived at from the graph.

Sol. (i) We draw the bar graph of this data in the following steps. The unit in the second column in the table is number of girls per thousand boys.

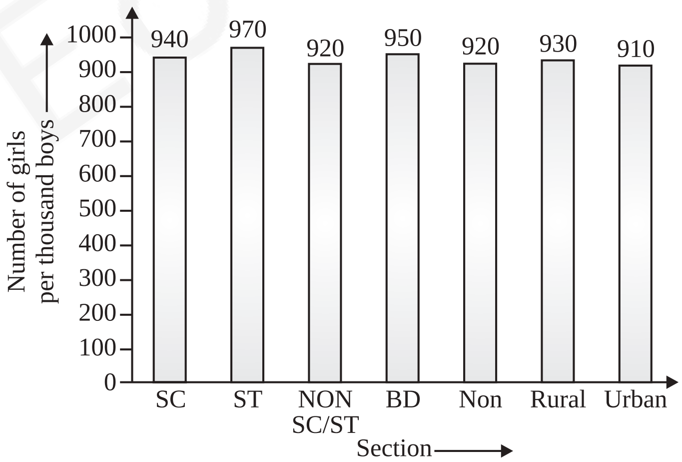
Step 1 : We represent the section (variable) on the horizontal axis choosing any scale, since the width of the bar is not important. But for clarity, we take equal widths for all bars and maintain equal gaps in between. Let one section be represented by one unit.

Step 2 : We represent the number of girls per thousand boys (value) on the vertical axis. Since, the maximum value is 940, we can choose the scale as 1 unit = 100.

Step 3 : To represent the first section, *i.e.*, Scheduled caste (SC), we draw a rectangular bar with width 1 unit and height as 940 ÷ 100 = 9.4 units.

Step 4 : Similarly, other sections are represented leaving a gap of 1 unit in between two consecutive bars.

The required bar graph is drawn below :



(ii) We conclude that number of girls per thousand boys are maximum in scheduled tribe (ST) section whereas minimum in urban section.

3. **Given below are the seats won by different political parties in the polling outcome of a state assembly elections :**

Click here for all Solved PYQs & New pattern Q's Educart book

Political Party	A	B	C	D	E	F
Seats Won	75	55	37	29	10	37

(i) Draw a bar graph to represent the polling results.

(ii) Which political party won the maximum number of seats ?

Sol. (i) We draw the bar graph of this data in the following steps. The unit in the second column in the table is seats won by the different political parties.

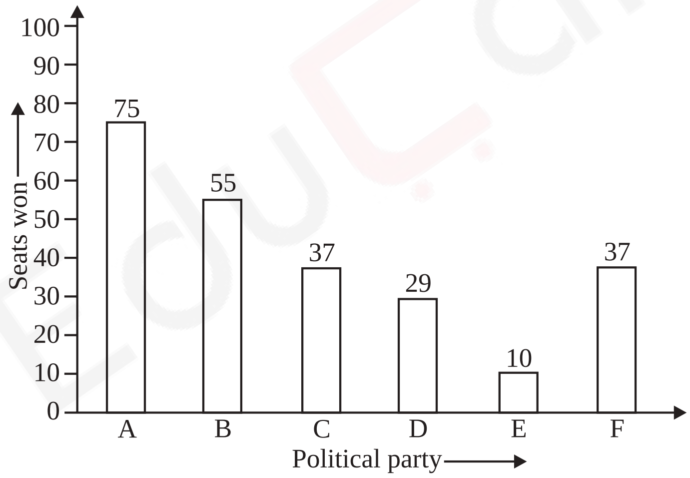
Step 1 : We represent the political party (variable) on the horizontal axis choosing any scale, since the width of the bar is not important. But for clarity, we take equal widths for all bars and maintain equal gaps in between. Let one political party be represented by one unit.

Step 2 : We represent seats won by the political parties (value) on the vertical axis. Since, the maximum value is 75, we can choose the scale as 1 unit = 10.

Step 3 : To represent the first party, *i.e.,* A, we draw a rectangular bar with width 1 unit and height as $75 \div 10 = 7.5$ units.

Step 4 : Similarly, other parties are represented leaving a gap of 1 unit in between two consecutive bars.

The required bar graph is drawn below :



(ii) From the graph, party A won the maximum number of seats.

4. The length of 40 leaves of a plant are measured correct to one millimetre, and the obtained data is represented in the following table :

Length (in mm)	Number of Leaves
118–126	3
127–135	5
136–144	9
145–153	12
154–162	5
163–171	4
172–180	2

Click here for all Solved PYQs & New pattern Q's Educart book

(i) Draw a histogram to represent the given data.

(ii) Is there any other suitable graphical representation for the same data ?

(iii) Is it correct to conclude that the maximum number of leaves are 153 mm long ? Why?

Sol. (i) The following frequency distribution table by converting the continuous class-intervals :

Length (in mm)	Number of Leaves
117.5 – 126.5	3
126.5 – 135.5	5
135.5 – 144.5	9
144.5 – 153.5	12
153.5 – 162.5	5
162.5 – 171.5	4
171.5 – 180.5	2

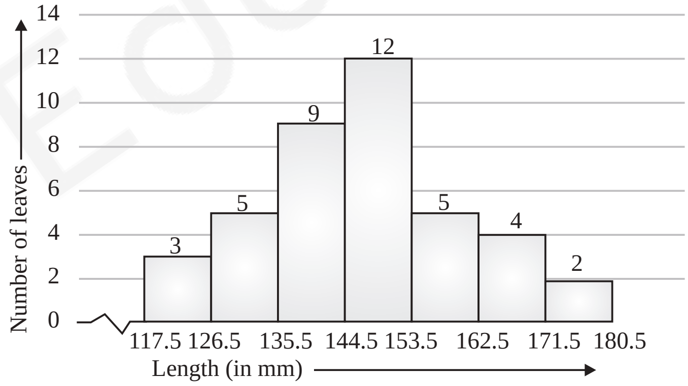
To represent the above data graphically by a histogram, we follow the steps :

Step 1 : We represent the length on the horizontal axis on a suitable scale. We can choose the scale as 1 unit = 9 mm. Also, since the first class-interval is starting from 117.5 and not zero, we show it on the graph by marking a kink or a break on the axis.

Step 2 : We represent the number of leaves (frequency) on the vertical axis on a suitable scale. Since the maximum frequency is 12, we need to choose the scale to accommodate this maximum frequency.

Step 3 : We now draw rectangles (or rectangular bars) of width equal to the class-size and lengths according to the frequencies of the corresponding class intervals.

In this way, we obtain the graph as shown below :



(ii) Yes, frequency polygon is another graphical representation for the same data.

(iii) No, because the maximum number of leaves have their lengths lying in the class interval 145-153.

5. The following table gives the life times of 400 neon lamps :

Life Time (in hours)	Number of Lamps
300-400	14
400-500	56

Click here for all Solved PYQs & New pattern Q's Educart book

500-600	60
600-700	86
700-800	74
800-900	62
900-1000	48

(i) Represent the given information with the help of a histogram.

(ii) How many lamps have a life time of more than 700 hours ?

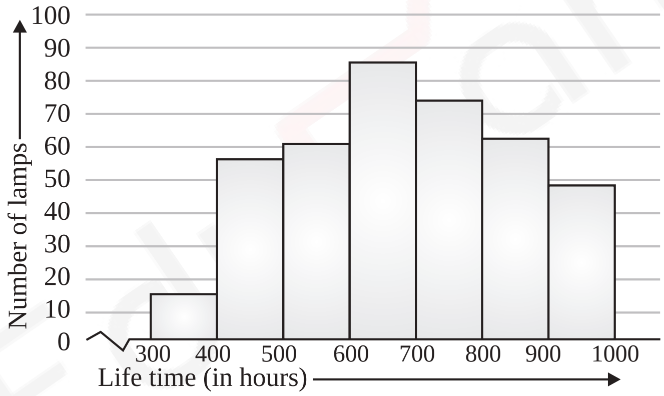
Sol. (i) To represent the data graphically by a histogram, we follow the steps :

Step 1 : We represent the life time on the horizontal axis on a suitable scale. We can choose the scale as 1 unit = 100 hours. Also, since the first class-interval is starting from 300 and not zero, we show it on the graph by marking a kink or a break on the axis.

Step 2 : We represent the number of neon lamps (frequency) on the vertical axis on a suitable scale. Since the maximum frequency is 86, we need to choose the scale to accommodate this maximum frequency.

Step 3 : We now draw rectangles (or rectangular bars) of width equal to the class-size and lengths according to the frequencies of the corresponding class intervals.

In this way, we obtain the graph as shown below :



(ii) The number of lamps having life time more than 700 hours = 74 + 62 + 48 = 184

6. The following table gives the distribution of students of two sections according to the marks obtained by them :

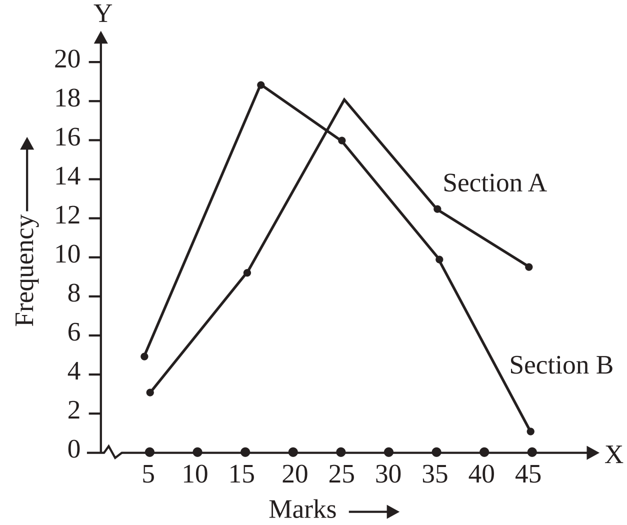
Section A		Section B	
Marks	**Frequency**	**Marks**	**Frequency**
0–10	3	0–10	5
10–20	9	10–20	19
20–30	17	20–30	15
30–40	12	30–40	10
40–50	9	40–50	1

Represent the marks of the students of both the sections on the same graph by two frequency polygons. From the two polygons compare the performance of the two sections.

Click here for all Solved PYQs & New pattern Q's Educart book

Sol. A frequency polygon is the polygon which is obtained by joining the mid-points of upper sides of the adjacent rectangles of the histogram.

The frequency polygon of the data is given below :



It is clear that from the polygon that the performance of section A is better than section B.

7. The runs scored by two teams A and B on the first 60 balls in a cricket match are given below :

Number of Balls	Team A	Team B
1–6	2	5
7–12	1	6
13–18	8	2
19–24	9	10
25–30	4	5
31–36	5	6
37–42	6	3
43–48	10	4
49–54	6	8
55–60	2	10

Represent the data of both the teams on the same graph by frequency polygons.

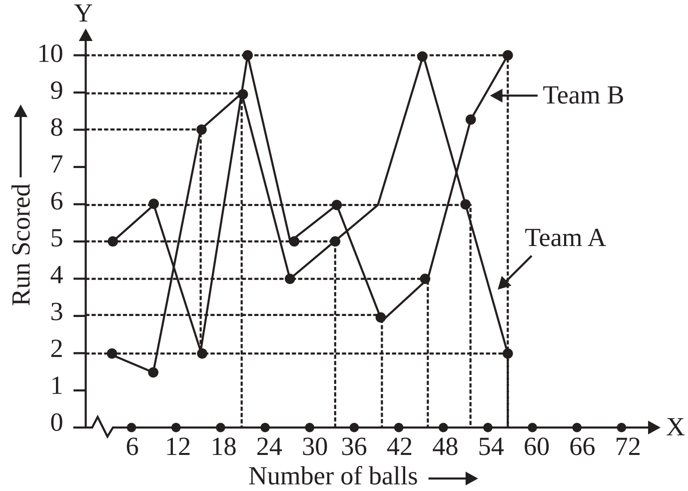
Sol. The following frequency distribution table by converting the continuous class-intervals :

Number of balls	Team A	Team B
0.5 – 6.5	2	5
6.5 – 12.5	1	6
12.5 – 18.5	8	2
18.5 – 24.5	9	10
24.5 – 30.5	4	5
30.5 – 36.5	5	6
36.5 – 42.5	6	3

Click here for all Solved PYQs & New pattern Q's Educart book

42.5 – 48.5	10	4
48.5 – 54.5	6	8
54.5 – 60.5	2	10

The frequency polygon of the data is given below :



8. A random survey of the number of children of various age groups playing in a park was found as follows :

Age (in years)	Number of Children
1–2	5
2–3	3
3–5	6
5–7	12
7–10	9
10–15	10
15–17	4

Draw a histogram to represent the data above.

Sol. Here, class widths are different. So, we need to make certain modifications in the lengths of the rectangles.

$\therefore$ Length of rectangle (Adjusted frequency)

$$= \left[\frac{\text{Minimum Class Size}}{\text{Class Size}}\right] \times \text{Frequency}$$

Here, minimum class size = 1

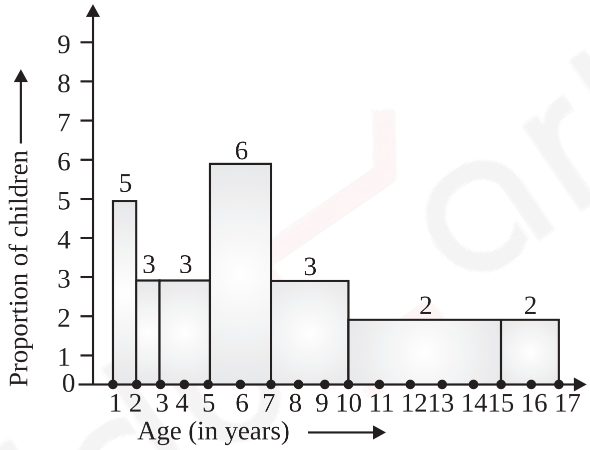
The following table for length of rectangles is given below :

Age (in years)	Number of children (frequency)	Width of the class	Length of the rectangle
1 – 2	5	1	$\frac{1}{1} \times 5 = 5$
2 – 3	3	1	$\frac{1}{1} \times 3 = 3$

Click here for all Solved PYQs & New pattern Q's Educart book

Age (in years)	Number of children (frequency)	Width of the class	Length of the rectangle
3 – 5	6	2	$\frac{1}{2}\times 6=3$
5 – 7	12	2	$\frac{1}{2}\times 12=6$
7 – 10	9	3	$\frac{1}{3}\times 93=$
10 – 15	10	5	$\frac{1}{5}\times 10=2$
15 – 17	4	2	$\frac{1}{2}\times 4=2$

The required the histogram is shown below :



9. 100 surnames were randomly picked up from a local telephone directory and a frequency distribution of the number of letters in the English alphabet in the surnames was found as follows :

Number of Letters	Number of Surnames
1–4	6
4–6	30
6–8	44
8–12	16
12–20	4

(i) Draw a histogram to depict the given information.

(ii) Write the class interval in which the maximum number of surnames lie.

Sol. (i) Here, class widths are different. So, we need to make certain modifications in the lengths of the rectangles.

$\therefore$ Length of rectangle (Adjusted frequency)

Click here for all Solved PYQs & New pattern Q's Educart book

$$= \left[\frac{\text{Minimum Class Size}}{\text{Class Size}}\right] \times \text{Frequency}$$

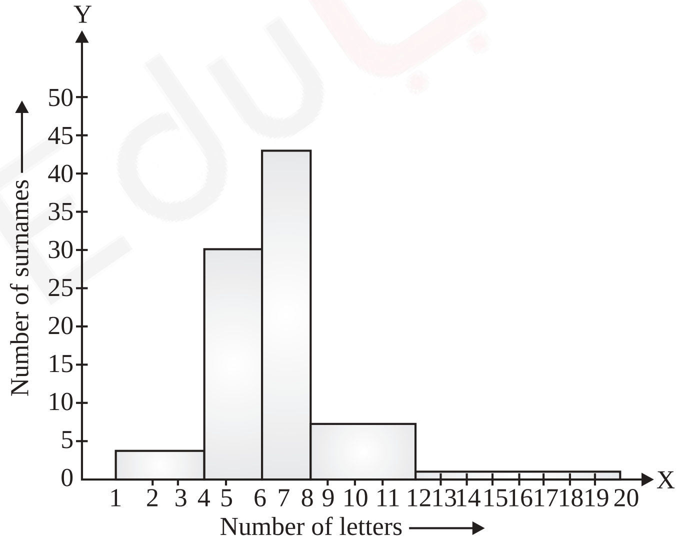
Here, minimum class size = 2

The following table for length of rectangles is given below :

Number of lettersNumber of surnames (frequency) Width of the class Length of the rectangle

Age (in years)	**Number of children (frequency)**	**Width of the class**	**Length of the rectangle**
1 – 4	6	3	$\frac{2}{3} \times 6 = 4$
4 – 6	30	2	$\frac{2}{2} \times 30 = 30$
6 – 8	44	2	$\frac{2}{2} \times 44 = 44$
8 – 12	16	4	$\frac{2}{4} \times 16 = 8$
12 – 20	4	8	$\frac{2}{8} \times 4 = 1$

The required histogram is shown below :



(ii) The maximum number of surnames lie in the class interval 6 – 8.

Click here for all Solved PYQs & New pattern Q's Educart book