Digital Signal Processing

Homework 1. Digital Notch Filter

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1. Objective

I will implement a notch filter, and see its magnitude and phase response. Then give two different frequency signals and see the output responses after this filter.

2. Algorithm

To construct a system transfer function,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{r=0}^{M} b_r \cdot z^{-r}}{\sum_{k=0}^{N} a_r \cdot z^{-k}}$$

Let $\mathrm{a}_1 = 1.3711242$, $\mathrm{a}_2 = 0.93906244$.

$$H(z) = \frac{1}{2} \times \frac{(1 + a_2) - 2a_1 z^{-1} + (1 + a_2) z^{-2}}{1 - a_1 z^{-1} + a_2 z^{-2}}$$

The relationship among ω, f, and ν(normalized frequency) is

$$\omega T = 2\pi f \times \frac{1}{f_s} = \frac{2\pi f}{2 f_0} = \frac{\pi f}{f_0} = \pi \nu$$

$$\nu = \frac{f}{f_0} = \frac{2f}{f_s}$$

where f_0 is folding frequency, and f_s is sampling frequency. $f_s = 2f_0$

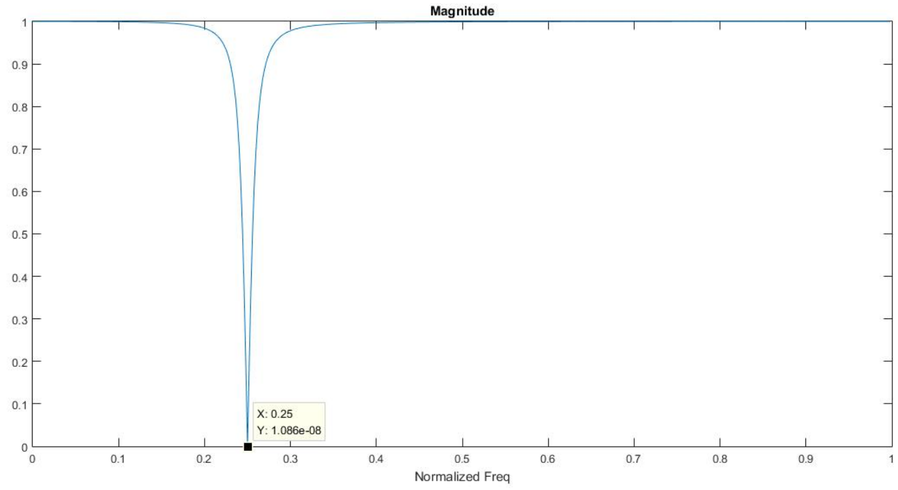
These equations are very important. When given a signal frequency, I have to figure out what ν it is. For example, in part 2, given $\mathrm{F} = 1250Hz$, and its normalized frequency is

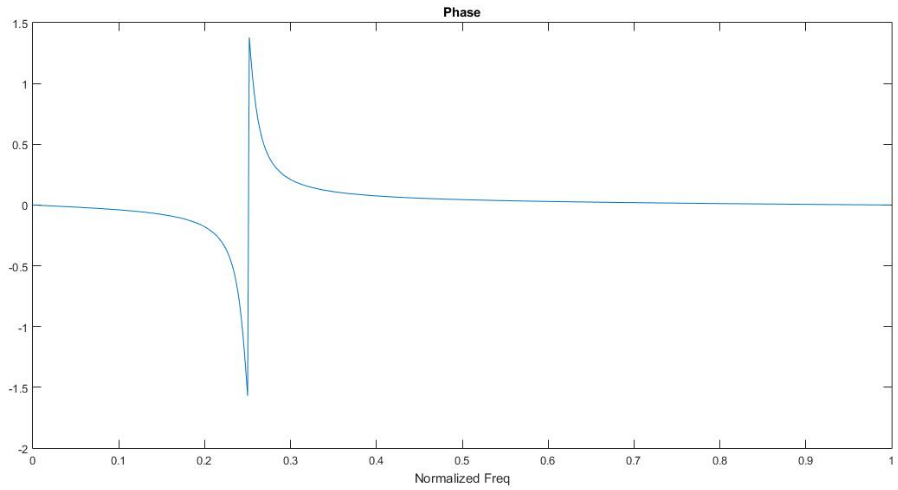
$$\nu = \frac{2\mathrm{F}}{f_s} = 2\mathrm{F} \cdot \mathrm{T}_{\text{sampling time}}$$

$$= 2 \times 1250 \times 0.0001 = 0.25$$

3. Results

3.1. Part 1



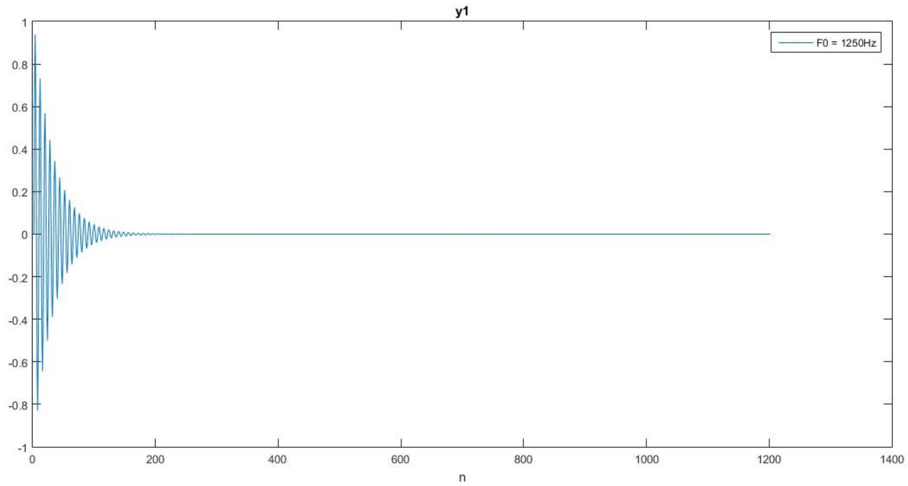


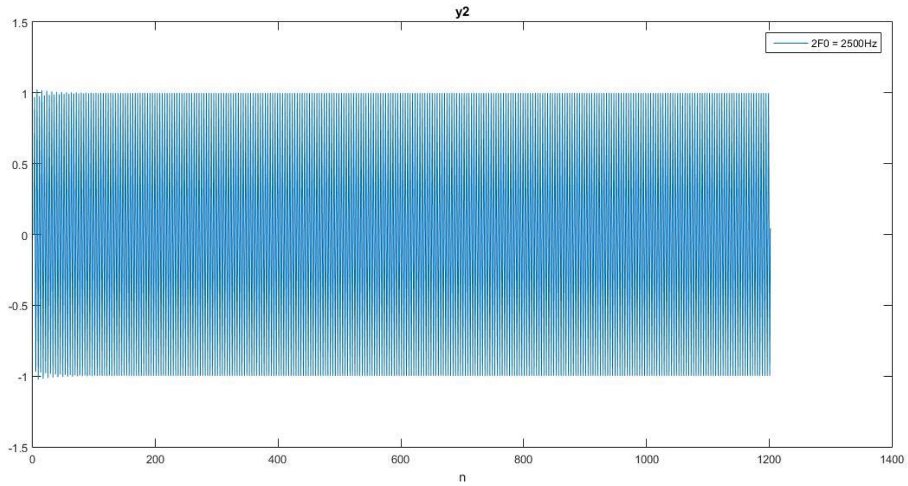
We can find that the stop-band normalized frequency is 0.25.

3.2. Part 2

$\mathrm{F}_0 = 1250\ Hz$,
$\mathrm{T} = 0.0001\ \mathrm{sec}$,
$\mathrm{I} = 1200\ \mathrm{points}$.

$$x_1 = sin(2 \cdot \pi \cdot F_0 \cdot t)$$
$$x_2 = sin(2 \cdot \pi \cdot 2F_0 \cdot t)$$





ν_1 for y_1 is

$$2 \times 1250 \times 0.0001 = 0.25$$

ν_2 for y_2 is

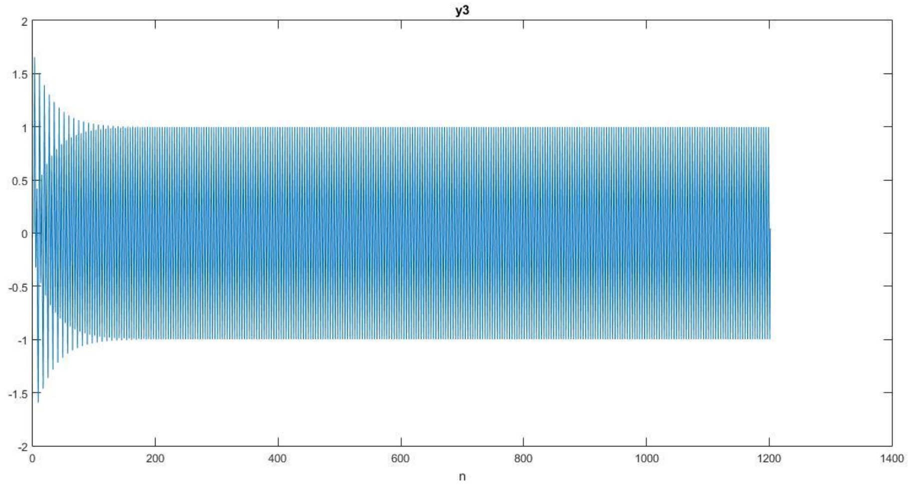
$$2 \times 2500 \times 0.0001 = 0.5$$

So, y_1 has an attenuation, but y_2 doesn't.

3.3. Part 3

Now let $x_3 = x_1 + x_2$,

$$x_3 = sin(2 \cdot \pi \cdot F_0 \cdot t) + sin(2 \cdot \pi \cdot 2F_0 \cdot t)$$



The output of the system y_3 will only remain y_2, which normalized frequency is 0.5.