

- ***Build a Table for GF(2⁶)***

Table 1. GF(2⁶) with primitive polynomial $p(X) = 1 + X + X^6$

| GF | Binary Representation | | | | | | Decimal |
|---------------|-----------------------|---|---|---|---|---|---------|
| α^0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| α^1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| α^2 | 0 | 0 | 0 | 0 | 1 | 0 | 2 |
| α^3 | 0 | 0 | 0 | 1 | 0 | 0 | 4 |
| α^4 | 0 | 0 | 1 | 0 | 0 | 0 | 8 |
| α^5 | 0 | 1 | 0 | 0 | 0 | 0 | 16 |
| α^6 | 1 | 0 | 0 | 0 | 0 | 0 | 32 |
| α^7 | 0 | 0 | 0 | 0 | 1 | 1 | 3 |
| α^8 | 0 | 0 | 0 | 1 | 1 | 0 | 6 |
| α^9 | 0 | 0 | 1 | 1 | 0 | 0 | 12 |
| α^{10} | 0 | 1 | 1 | 0 | 0 | 0 | 24 |
| α^{11} | 1 | 1 | 0 | 0 | 0 | 0 | 48 |
| α^{12} | 1 | 0 | 0 | 0 | 1 | 1 | 35 |
| α^{13} | 0 | 0 | 0 | 1 | 0 | 1 | 5 |
| α^{14} | 0 | 0 | 1 | 0 | 1 | 0 | 10 |
| α^{15} | 0 | 1 | 0 | 1 | 0 | 0 | 20 |
| α^{16} | 1 | 0 | 1 | 0 | 0 | 0 | 40 |
| α^{17} | 0 | 1 | 0 | 0 | 1 | 1 | 19 |
| α^{18} | 1 | 0 | 0 | 1 | 1 | 0 | 38 |
| α^{19} | 0 | 0 | 1 | 1 | 1 | 1 | 15 |
| α^{20} | 0 | 1 | 1 | 1 | 1 | 0 | 30 |
| α^{21} | 1 | 1 | 1 | 1 | 0 | 0 | 60 |
| α^{22} | 1 | 1 | 1 | 0 | 1 | 1 | 59 |
| α^{23} | 1 | 1 | 0 | 1 | 0 | 1 | 53 |
| α^{24} | 1 | 0 | 1 | 0 | 0 | 1 | 41 |
| α^{25} | 0 | 1 | 0 | 0 | 0 | 1 | 17 |
| α^{26} | 1 | 0 | 0 | 0 | 1 | 0 | 34 |
| α^{27} | 0 | 0 | 0 | 1 | 1 | 1 | 7 |
| α^{28} | 0 | 0 | 1 | 1 | 1 | 0 | 14 |

| | | | | | | | |
|---------------|---|---|---|---|---|---|----|
| α^{29} | 0 | 1 | 1 | 1 | 0 | 0 | 28 |
| α^{30} | 1 | 1 | 1 | 0 | 0 | 0 | 56 |
| α^{31} | 1 | 1 | 0 | 0 | 1 | 1 | 51 |
| α^{32} | 1 | 0 | 0 | 1 | 0 | 1 | 37 |
| α^{33} | 0 | 0 | 1 | 0 | 0 | 1 | 9 |
| α^{34} | 0 | 1 | 0 | 0 | 1 | 0 | 18 |
| α^{35} | 1 | 0 | 0 | 1 | 0 | 0 | 36 |
| α^{36} | 0 | 0 | 1 | 0 | 1 | 1 | 11 |
| α^{37} | 0 | 1 | 0 | 1 | 1 | 0 | 22 |
| α^{38} | 1 | 0 | 1 | 1 | 0 | 0 | 44 |
| α^{39} | 0 | 1 | 1 | 0 | 1 | 1 | 27 |
| α^{40} | 1 | 1 | 0 | 1 | 1 | 0 | 54 |
| α^{41} | 1 | 0 | 1 | 1 | 1 | 1 | 47 |
| α^{42} | 0 | 1 | 1 | 1 | 0 | 1 | 29 |
| α^{43} | 1 | 1 | 1 | 0 | 1 | 0 | 58 |
| α^{44} | 1 | 1 | 0 | 1 | 1 | 1 | 55 |
| α^{45} | 1 | 0 | 1 | 1 | 0 | 1 | 45 |
| α^{46} | 0 | 1 | 1 | 0 | 0 | 1 | 25 |
| α^{47} | 1 | 1 | 0 | 0 | 1 | 0 | 50 |
| α^{48} | 1 | 0 | 0 | 1 | 1 | 1 | 39 |
| α^{49} | 0 | 0 | 1 | 1 | 0 | 1 | 13 |
| α^{50} | 0 | 1 | 1 | 0 | 1 | 0 | 26 |
| α^{51} | 1 | 1 | 0 | 1 | 0 | 0 | 52 |
| α^{52} | 1 | 0 | 1 | 0 | 1 | 1 | 43 |
| α^{53} | 0 | 1 | 0 | 1 | 0 | 1 | 21 |
| α^{54} | 1 | 0 | 1 | 0 | 1 | 0 | 42 |
| α^{55} | 0 | 1 | 0 | 1 | 1 | 1 | 23 |
| α^{56} | 1 | 0 | 1 | 1 | 1 | 0 | 46 |
| α^{57} | 0 | 1 | 1 | 1 | 1 | 1 | 31 |
| α^{58} | 1 | 1 | 1 | 1 | 1 | 0 | 62 |
| α^{59} | 1 | 1 | 1 | 1 | 1 | 1 | 63 |
| α^{60} | 1 | 1 | 1 | 1 | 0 | 1 | 61 |
| α^{61} | 1 | 1 | 1 | 0 | 0 | 1 | 57 |
| α^{62} | 1 | 1 | 0 | 0 | 0 | 1 | 49 |

| | | | | | | | |
|---------------|---|---|---|---|---|---|----|
| α^{63} | 1 | 0 | 0 | 0 | 0 | 1 | 33 |
|---------------|---|---|---|---|---|---|----|

● **Berlekamp – Massey Algorithm**

Table 2. Each steps for finding error-location polynomial

| r | B(X) | $\Lambda(X)$ | L |
|---------------|--|--|----------|
| α^{48} | α^{15} | $1 + \alpha^{48}X$ | 1 |
| α^8 | $\alpha^{15}X$ | $1 + \alpha^{34}X$ | 1 |
| α^{38} | $\alpha^{25} + \alpha^{59}X$ | $1 + \alpha^{34}X + \alpha^{53}X^2$ | 2 |
| α^{42} | $\alpha^{25}X + \alpha^{59}X^2$ | $1 + \alpha^{50}X + \alpha^{61}X^2$ | 2 |
| α^{55} | $\alpha^8 + \alpha^{58}X + \alpha^6X^3$ | $1 + \alpha^{50}X + \alpha^{54}X^2 + \alpha^{51}X^3$ | 3 |
| α^{10} | $\alpha^8X + \alpha^{58}X^2 + \alpha^6X^3$ | $1 + \alpha^{21}X + \alpha^{43}X^2 + \alpha^{19}X^3$ | 3 |
| α^{26} | $\alpha^{37} + \alpha^{58}X + \alpha^{17}X^2 + \alpha^3X^3$ | $1 + \alpha^{21}X + \alpha^{16}X^2 + \alpha^6X^3 + \alpha^{32}X^4$ | 4 |
| α^{50} | $\alpha^{37}X + \alpha^{58}X^2 + \alpha^{17}X^3 + \alpha^3X^4$ | $1 + \alpha^{53}X + \alpha^{13}X^2 + \alpha^{16}X^3 + \alpha^{11}X^4$ | 4 |
| α^{28} | $\alpha^{35} + \alpha^{25}X + \alpha^{48}X^2 + \alpha^{51}X^3 + \alpha^{46}X^4$ | $1 + \alpha^{53}X + \alpha^{27}X^2 + \alpha^{42}X^3 + \alpha^{42}X^4 + \alpha^{31}X^5$ | 5 |
| α^{30} | $\alpha^{35}X + \alpha^{25}X^2 + \alpha^{48}X^3 + \alpha^{51}X^4 + \alpha^{46}X^5$ | $1 + \alpha^{55}X + \alpha^5X^2 + \alpha^{33}X^3 + \alpha^{22}X^4 + \alpha^{40}X^5$ | 5 |

The **Error-Location Polynomial** is

$$\begin{aligned}\Lambda(X) &= 1 + \alpha^{55}X + \alpha^5X^2 + \alpha^{33}X^3 + \alpha^{22}X^4 + \alpha^{40}X^5 \\ &= (1 + \alpha^{51}X)(1 + \alpha^{35}X)(1 + \alpha^8X)(1 + \alpha^7X)(1 + \alpha^2X)\end{aligned}$$

The **Error Evaluator Polynomial** is

$$\Omega(X) = \alpha^{48} + \alpha^{61}X + \alpha^{29}X^2 + \alpha^{56}X^3 + \alpha^{34}X^4$$

So the **Error Pattern** is

$$e(X) = \alpha^6X^2 + \alpha^{20}X^7 + \alpha^3X^8 + \alpha^{62}X^{35} + \alpha^{15}X^{51}$$

Table 3. Error locations and error magnitudes

| Error Location | Error Magnitude |
|-----------------------|------------------------|
| Y_{51} | α^{15} |
| Y_{35} | α^{62} |
| Y_8 | α^3 |
| Y_7 | α^{20} |
| Y_2 | α^6 |

● ***Euclidean Algorithm***

Table 4. Each steps for finding error-location polynomial and error-value evaluator

| i | $Z_0^{(i)}(X)$ | $q_i(X)$ | $\sigma_i(X)$ |
|----|---|------------------------------|---|
| -1 | X^{10} | | 0 |
| 0 | $\alpha^{48} + \alpha^{19}X + \alpha^{61}X^2 + \alpha^{51}X^3 + \alpha^3X^4$ $+ \alpha^{42}X^5 + \alpha^{33}X^6 + \alpha^{29}X^7$ $+ \alpha^{38}X^8 + \alpha^{53}X^9$ | | 1 |
| 1 | $\alpha^{43} + \alpha^{51}X + \alpha^{47}X^2 + \alpha^{57}X^3 + \alpha^{17}X^5$ $+ \alpha^{32}X^6 + \alpha^{17}X^7 + \alpha^{34}X^8$ | $\alpha^{58} + \alpha^{10}X$ | $\alpha^{58} + \alpha^{10}X$ |
| 2 | $\alpha^{30} + \alpha^{21}X + \alpha^7X^2 + \alpha^{40}X^3 + \alpha X^4 + \alpha^{56}X^5$ $+ \alpha^{39}X^6 + \alpha^{39}X^7$ | $\alpha^{14} + \alpha^{19}X$ | $\alpha^{45} + \alpha^{12}X + \alpha^{29}X^2$ |
| 3 | $\alpha^{45} + \alpha^{21}X + \alpha^{59}X^2 + \alpha^{57}X^3 + \alpha^8X^4$ $+ \alpha^{14}X^5 + \alpha^5X^6$ | $\alpha^{25} + \alpha^{58}X$ | $\alpha^{60} + \alpha^{30}X + \alpha^{24}X^2 + \alpha^{24}X^3$ |
| 4 | $\alpha + \alpha^2X + \alpha^{44}X^2 + \alpha^{37}X^3 + \alpha^{18}X^4 + \alpha^{36}X^5$ | $\alpha^{16} + \alpha^{34}X$ | $\alpha^{16} + \alpha^{33}X + \alpha^{52}X^2 + \alpha^4X^3 + \alpha^{58}X^4$ |
| 5 | $\alpha^{35} + \alpha^{48}X + \alpha^{16}X^2 + \alpha^{43}X^3 + \alpha^{21}X^4$ | $\alpha^{32} + \alpha^{32}X$ | $\alpha^{50} + \alpha^{42}X + \alpha^{55}X^2 + \alpha^{20}X^3 + \alpha^9X^4 + \alpha^{27}X^5$ |

The error pattern is same as the results done by Berlekamp-Massey Algorithm.