Numerical Analysis

Homework 10. Numerical Integration

Due: May 9, 2017

In this home work, you will implement various Newton-Cotes integration formulas to find the integral, I, of $f(x)$ over a fixed range, $x \in [0, 2]$.

$$f(x) = e^x \tag{10.1}$$

$$I = \int_0^2 f(x)\,\mathrm{d}x. \tag{10.2}$$

Note that the closed form solution for I^* is known and can be used to find the integration errors.

1. Divide the interval, $[0, 2]$, into 12 equal sized regions, to generate an array $Y[k]$, $k = 0, 1, \cdots, 12$, with
$$Y[k] = e^{k \cdot h}, \tag{10.3}$$
where $h = 2/12$.

   1.1. Using the array $Y[\,]$ and the region size h to perform the first order Newton-Cotes integration to find the integral, $I_{12}^{(1)}$, and the error, $E_{12}^{(1)} = |I^* - I_{12}^{(1)}|$.

   1.2. Using the array $Y[\,]$ and the region size h to perform the second order Newton-Cotes integration to find the integral, $I_{12}^{(2)}$, and the error, $E_{12}^{(2)} = |I^* - I_{12}^{(2)}|$.

   1.3. Using the array $Y[\,]$ and the region size h to perform the third order Newton-Cotes integration to find the integral, $I_{12}^{(3)}$, and the error, $E_{12}^{(3)} = |I^* - I_{12}^{(3)}|$.

   1.4. Using the array $Y[\,]$ and the region size h to perform the fourth order Newton-Cotes integration to find the integral, $I_{12}^{(4)}$, and the error, $E_{12}^{(4)} = |I^* - I_{12}^{(4)}|$.

   1.5. Using the array $Y[\,]$ and the region size h to perform the sixth order Newton-Cotes integration to find the integral, $I_{12}^{(6)}$, and the error, $E_{12}^{(6)} = |I^* - I_{12}^{(6)}|$.

2. Divide the interval, $[0, 2]$, into 24 equal sized regions, to generate an array $Y[k]$, $k = 0, 1, \cdots, 24$, with
$$Y[k] = e^{k \cdot h},$$
where $h = 2/24$.

   2.1. Using the array $Y[\,]$ and the region size h to perform the first order Newton-Cotes integration to find the integral, $I_{24}^{(1)}$, and the error, $E_{24}^{(1)} = |I^* - I_{24}^{(1)}|$.

   2.2. Using the array $Y[\,]$ and the region size h to perform the second order Newton-Cotes integration to find the integral, $I_{24}^{(2)}$, and the error, $E_{24}^{(2)} = |I^* - I_{24}^{(2)}|$.

   2.3. Using the array $Y[\,]$ and the region size h to perform the third order Newton-Cotes integration to find the integral, $I_{24}^{(3)}$, and the error, $E_{24}^{(3)} = |I^* - I_{24}^{(3)}|$.

2.4. Using the array $Y[\,]$ and the region size h to perform the fourth order Newton-Cotes integration to find the integral, $I_{24}^{(4)}$, and the error, $E_{24}^{(4)} = |I^* - I_{24}^{(4)}|$.

2.5. Using the array $Y[\,]$ and the region size h to perform the sixth order Newton-Cotes integration to find the integral, $I_{24}^{(6)}$, and the error, $E_{24}^{(6)} = |I^* - I_{24}^{(6)}|$.

3. Repeat the same process and divide the interval into 48 regions to find $I_{48}^{(1)}$, $E_{48}^{(1)}$, $I_{48}^{(2)}$, $E_{48}^{(2)}$, $I_{48}^{(3)}$, $E_{48}^{(3)}$, $I_{48}^{(4)}$, $E_{48}^{(4)}$, $I_{48}^{(6)}$, and $E_{48}^{(6)}$.

4. Repeat the same process and divide the interval into 96 regions to find $I_{96}^{(1)}$, $E_{96}^{(1)}$, $I_{96}^{(2)}$, $E_{96}^{(2)}$, $I_{96}^{(3)}$, $E_{96}^{(3)}$, $I_{96}^{(4)}$, $E_{96}^{(4)}$, $I_{96}^{(6)}$, and $E_{96}^{(6)}$.

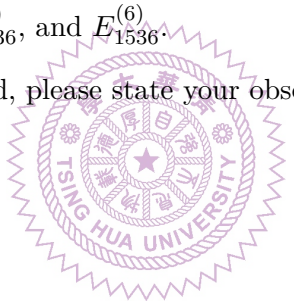
5. Repeat the same process and divide the interval into 192 regions to find $I_{192}^{(1)}$, $E_{192}^{(1)}$, $I_{192}^{(2)}$, $E_{192}^{(2)}$, $I_{192}^{(3)}$, $E_{192}^{(3)}$, $I_{192}^{(4)}$, $E_{192}^{(4)}$, $I_{192}^{(6)}$, and $E_{192}^{(6)}$.

6. Repeat the same process and divide the interval into 384 regions to find $I_{384}^{(1)}$, $E_{384}^{(1)}$, $I_{384}^{(2)}$, $E_{384}^{(2)}$, $I_{384}^{(3)}$, $E_{384}^{(3)}$, $I_{384}^{(4)}$, $E_{384}^{(4)}$, $I_{384}^{(6)}$, and $E_{384}^{(6)}$.

7. Repeat the same process and divide the interval into 768 regions to find $I_{768}^{(1)}$, $E_{768}^{(1)}$, $I_{768}^{(2)}$, $E_{768}^{(2)}$, $I_{768}^{(3)}$, $E_{768}^{(3)}$, $I_{768}^{(4)}$, $E_{768}^{(4)}$, $I_{768}^{(6)}$, and $E_{768}^{(6)}$.

8. Repeat the same process and divide the interval into 1536 regions to find $I_{1536}^{(1)}$, $E_{1536}^{(1)}$, $I_{1536}^{(2)}$, $E_{1536}^{(2)}$, $I_{1536}^{(3)}$, $E_{1536}^{(3)}$, $I_{1536}^{(4)}$, $E_{1536}^{(4)}$, $I_{1536}^{(6)}$, and $E_{1536}^{(6)}$.

9. Compare the solutions you obtained, please state your observations.



Notes.

1. For this homework you need to turn in a set of C++ source codes. That includes `hw09.cpp`, which solves question 5 above, `MAT.h`, the new header file, `MAT.cpp`, which includes the two functions above, `VEC.h` and `VEC.cpp` files.

2. A `pdf` file is also needed. Please name this file `hw09a.pdf`.

3. Submit your files on EE workstations. Please use the following command to submit your homework 9.

```
$ ~ee407002/bin/submit hw09 hw09a.pdf hw09.cpp MAT.h MAT.cpp VEC.h VEC.cpp
```

   where `hw09` indicates homework 9.

4. Your report should be clearly written such that I can understand it. The writing, including English grammar, is part of the grading criteria.