Numerical Analysis

Homework 8. Polynomial Interpolations

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1. Objective

In this homework, I will find the functions that approximate the simulated waveform shown below.

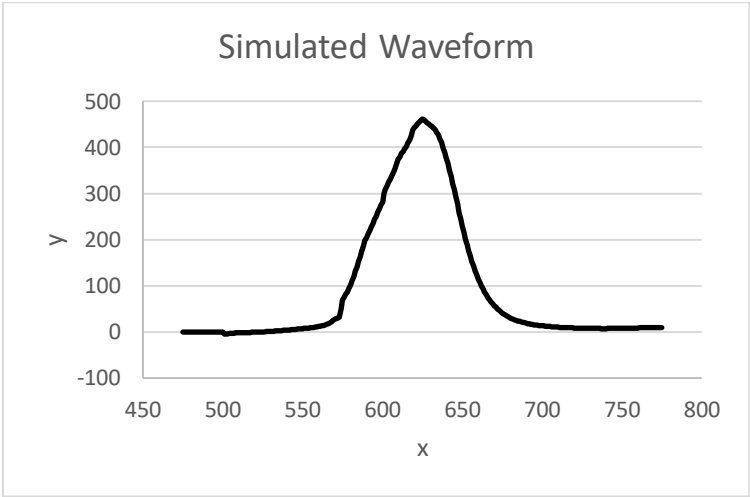


Figure 1. A simulated waveform

The data for this waveform are given in the file f301.dat. Other support points data for approximating the simulated waveform are given in f3.dat, f5.dat, f7.dat, f13.dat, and f21.dat.

2. Approach

I implement the following function for Lagrange Interpolation.

double Lagrange(double x, VEC &XDATA, VEC &YDATA);

This function interpolate the function of the given support points (**XDATA[i], YDATA[i]**) and find the value at **x**.

Algorithm. Lagrange

Input : x, the x-coordinate of the interpolated point;
XDATA, support points $\{x_i, 0 \leq \mathrm{i} \leq \mathrm{n}\}$ on the x-axis;
YDATA, support points $\{y_i, 0 \leq \mathrm{i} \leq \mathrm{n}\}$ on the y-axis;

Output : y, a interpolated point;

$$F(x) = \sum_{i=0}^{n} y_i \prod_{k=0, k\neq i}^{n} \frac{x-x_k}{x_i-x_k}.$$

Or let

$$L_i(x) = \prod_{k=0, k\neq i}^{n} \frac{x-x_k}{x_i-x_k}.$$

then

$$F(x) = \sum_{i=0}^{n} y_i \, L_i(x).$$

3. Results

3.1. Interpolated Values

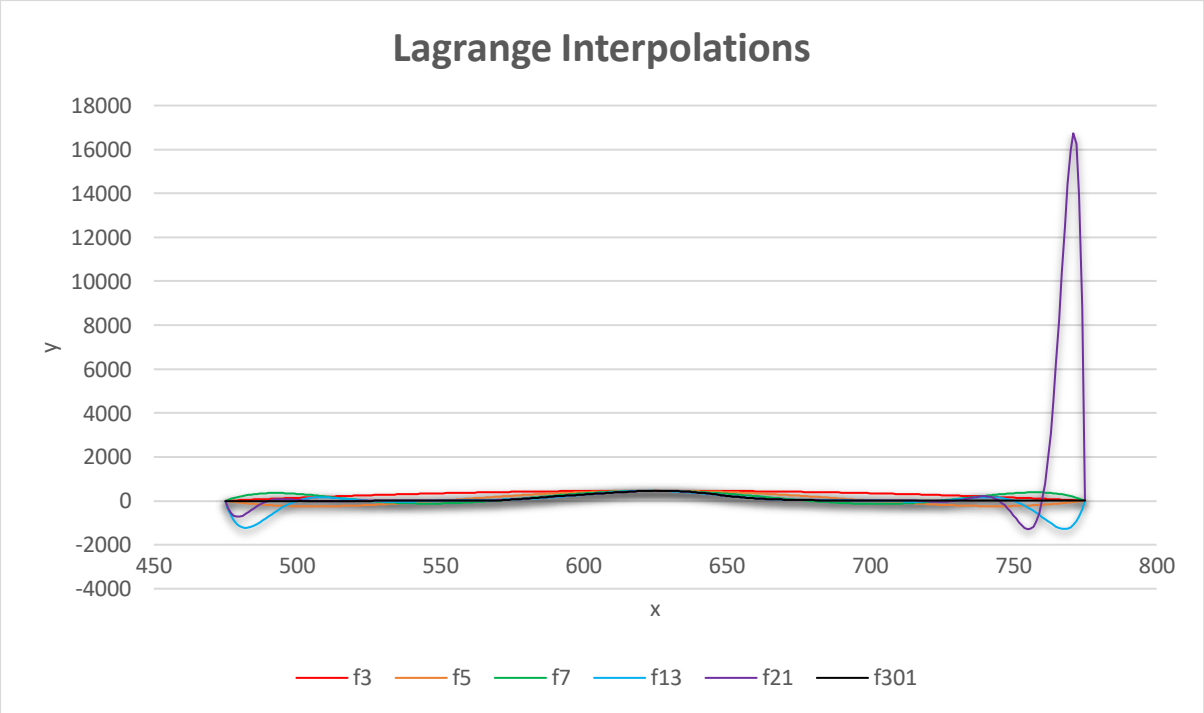


Figure 2. Lagrange Interpolations with different support points data

The interpolation with f21.dat has an extreme data, so it is hard to observe the behavior of other interpolations with other data in this figure. Let's zoom in the plot.

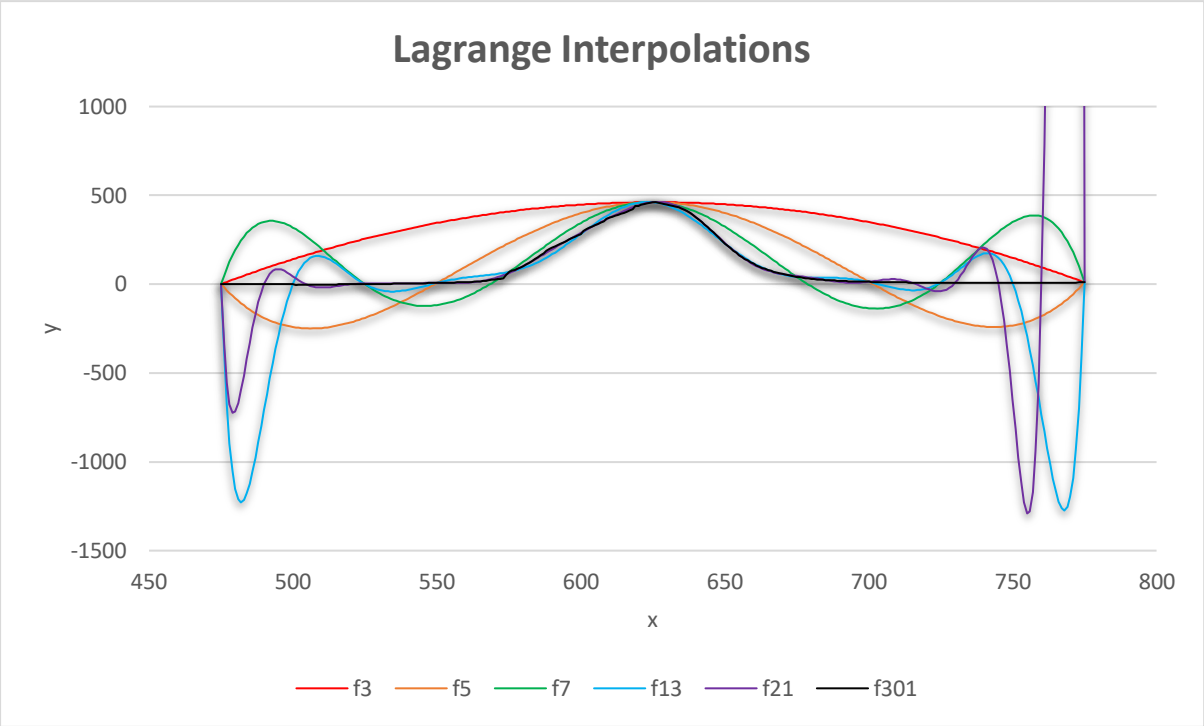


Figure 3. Lagrange Interpolations with different support points data (zoom in)

3.2. Error

	f3	f5	f7	f13	f21
max err (x: 475~775)	372.867	248.341	379.107	1283.451	16728.61
max err (x: 550~700)	372.867	233.364	148.891	39.619	17.804

Table 1. Maximum Error

4. Observations

f301, the black waveform, is the data of the simulated waveform, and now I am going to compare the other waveforms against it.

First, look at the behavior of the sidebands and the middleband.

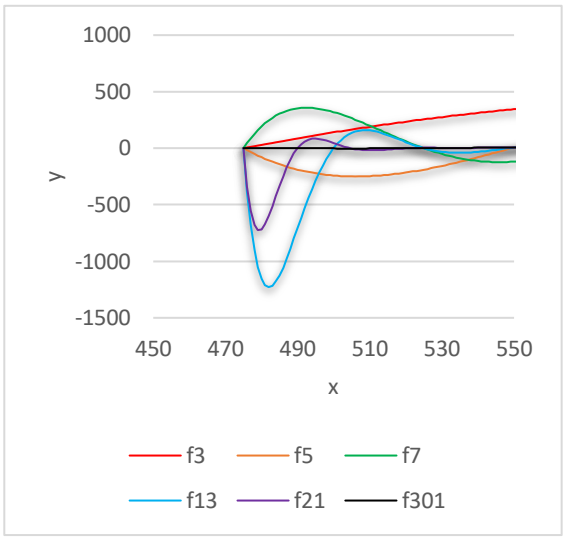
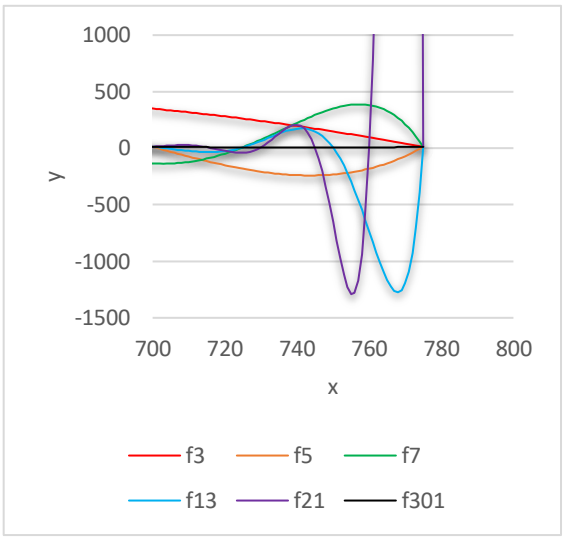
Lower-band	Higher-band
	

Figure 4. Behavior of the sidebands

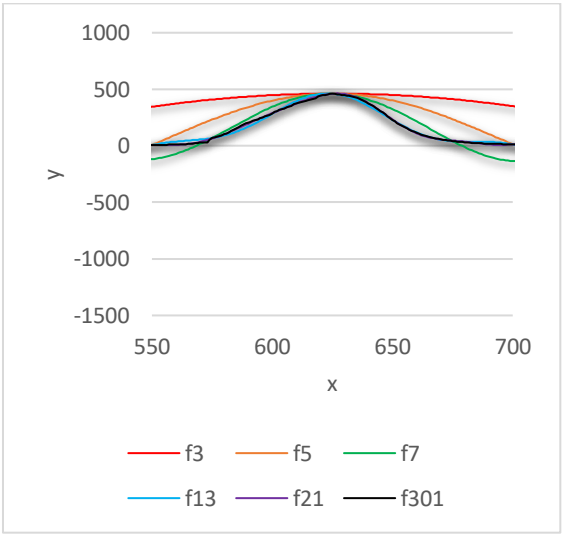


Figure 5. Behavior of the middleband

We can see that if we have more support points, the ripple effect will be more on the sidebands. On the contrary, if we have more support points, it will be more accurate on the middleband.

The following error plot can prove my observations.

Figure 6. Maximum Error

If we look at the orange curve, the error decreases as support points increase. Now we look at the blue curve. This curve is the max error in the range $475 \leq x \leq 775$, and it includes the sidebands. The max error will be dominated by the error of the sidebands, so it increases as support points increase. These two conclusions satisfy my observations above.