Numerical Analysis

Homework 5. Conjugate Gradient Methods

Due: April 4, 2017

Conjugate gradient method is very attractive from theoretical point of view. In this homework, you'll implement the conjugate gradient method to see if it is, indeed, a good iterative method. As in the last homework, you'll add the following nonmember function to your `MAT.h` and `MAT.cpp` files.

```
int cg(MAT &A,VEC b,VEC &x,int maxIter, double tol);
```

The five function arguments have the same meanings as before.

`A`: the matrix of the linear system,
`b`: the right hand side vector of the linear system,
`x`: an initial guess for the iterative solution method and the final solution when function returns,
`maxIter`: maximum number of iterations allowed,
`tol`: tolerance of the iterative method.

Again, the last argument `tol` is used for convergence check. In this homework, a different approach is taken. Since the residue $\mathbf{r} = \mathbf{b} - \mathbf{A}\mathbf{x}$ is calculated at each iteration, so is $\mathbf{r}^T\mathbf{r}$, we can use this scalar for convergence check. In particular, our iteration error is defined as

$$err = \sqrt{\frac{\mathbf{r}^T\mathbf{r}}{n}} \tag{5.1}$$

where n is the dimension of the linear system. When $err < tol$ then the conjugate iteration is terminated, or when the iteration number reaches `maxIter`.

1. Please use this conjugate gradient method to solve the resistor network problem in `HW04` with 20 resistors per side, and with the solution accuracy of 10^{-7} volts.
2. Please solve the same resistor network problem with 40 resistors per side.
3. Please solve the same resistor network problem with 60 resistors per side.
4. Please solve the same resistor network problem with 80 resistors per side.
5. Please solve the same resistor network problem with 100 resistors per side.
6. Compare to the solutions obtained using direct method whenever possible. And state your observations in the convergence behavior, CPU time, and the resistor network properties, etc.

Notes.

1. For this homework you need to turn in a set of C++ source codes. That includes `hw05.cpp`, which solves all four resistor networks, `MAT.h`, the new header file, `MAT.cpp`, which includes the conjugate gradient functions, and the `VEC.h` and `VEC.cpp` if these have also be modified.

2. A `pdf` file is also needed. Please name this file `hw05a.pdf`.

3. Submit your files on EE workstations. Please use the following command to submit your homework 5.

   ```
   $ ~ee407002/bin/submit hw05 hw05a.pdf hw05.cpp MAT.h MAT.cpp VEC.h VEC.cpp
   ```

   where `hw05` indicates homework 5.

4. Your report should be clearly written such that I can understand it. The writing, including English grammar, is part of the grading criteria.