Numerical Analysis

Homework 12. RLC Circuit.

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1. Objective

The RLC circuit is shown below.

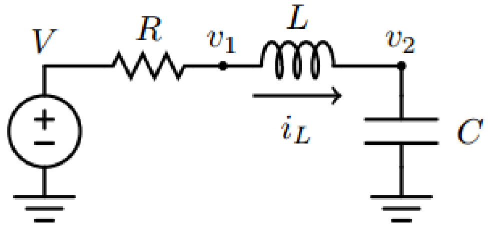


Figure 1. RLC circuit

Using these equations to solve v_1, v_2 and i_L:

$$\frac{v_1 - V}{R} + i_L = 0,$$

$$\frac{dv_2}{dt} = \frac{i_L}{C},$$

$$\frac{di_L}{dt} = \frac{v_1 - v_2}{L}.$$

And assuming

(1) $\mathrm{V(t) = 1 \ \ for \ \ all \ \ t,}$

(2) $\mathrm{at \ \ t = 0, v_1(0) = 1, v_2(0) = 0, and \ \ i_L(0) = 0,}$

(3) $\mathrm{R = 1 \ \ \Omega, L = 1 \ \ Henry, and \ \ C = 1 \ \ Farad.}$

2. Approach

2.1. Forward Euler Method

$$x(t+h) = x(t) + h \cdot f(t)$$

So the problem will become

$$i_L(t+h) = i_L(t) + h \cdot \frac{v_1(t) - v_2(t)}{L},$$

$$v_1(t+h) = -i_L(t+h) \cdot \mathrm{R} + V,$$

$$v_2(t+h) = v_2(t) + h \cdot \frac{i_L(t)}{C}.$$

2.2. Backward Euler Method

$$x(t+h) = x(t) + h \cdot f(t+h, x(t+h))$$

So the problem will become

$$i_L(t+h) = i_L(t) + h \cdot \frac{v_1(t+h) - v_2(t+h)}{L},$$

$$v_1(t+h) = -i_L(t+h) \cdot \mathrm{R} + V,$$

$$v_2(t+h) = v_2(t) + h \cdot \frac{i_L(t+h)}{C}.$$

2.3. Trapezoidal Method

$$x(t+h) = x(t) + h \cdot \frac{f\big(t+h, x(t+h)\big) + f(t, x(t))}{2}$$

So the problem will become

$$i_L(t+h) = i_L(t) + h \cdot \frac{[v_1(t+h) - v_2(t+h)] + [v_1(t) - v_2(t)]}{2L},$$

$$v_1(t+h) = -i_L(t+h) \cdot \mathrm{R} + V,$$

$$v_2(t+h) = v_2(t) + h \cdot \frac{i_L(t+h) + i_L(t)}{2C}.$$

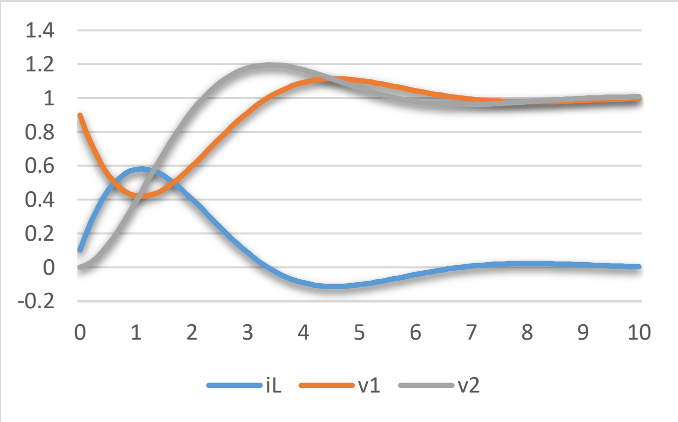
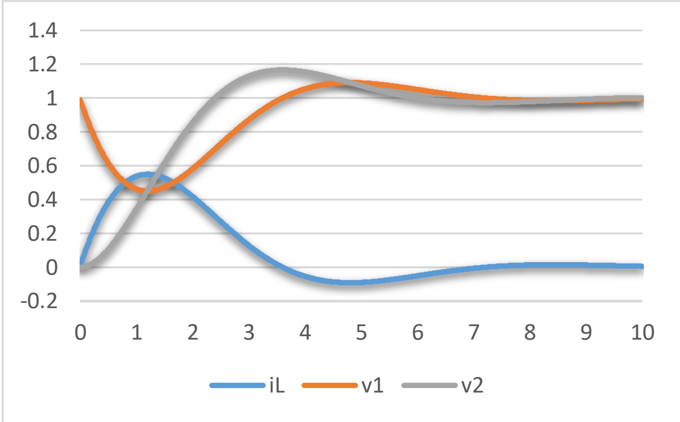
2.4. Algorithm

Algorithm. Methods for solving ODEs with a given initial value.

while (t < end_time) ***do***
    compute x(t + h)
    t = t + h
end while

3. Results

3.1. Plot

<table>
<tr><th>h=0.1</th><th>h=0.01</th></tr>
<tr><th colspan="2">Forward Euler</th></tr>
<tr><td>
</td><td>
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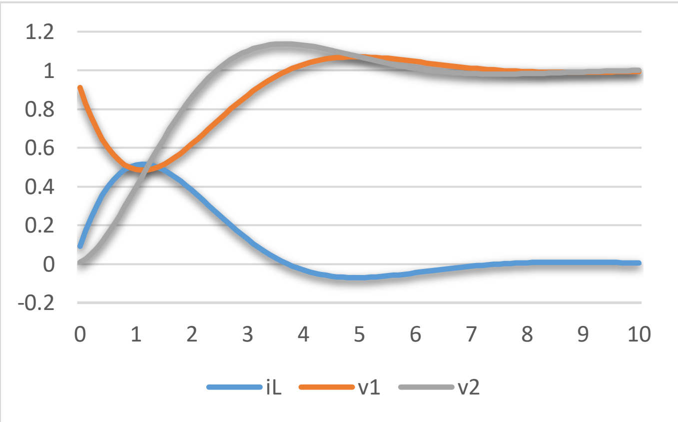
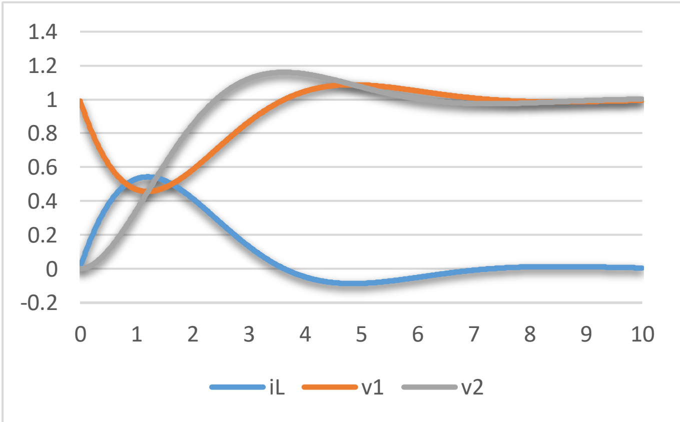
<tr><th colspan="2">Backward Euler</th></tr>
<tr><td>
</td><td>
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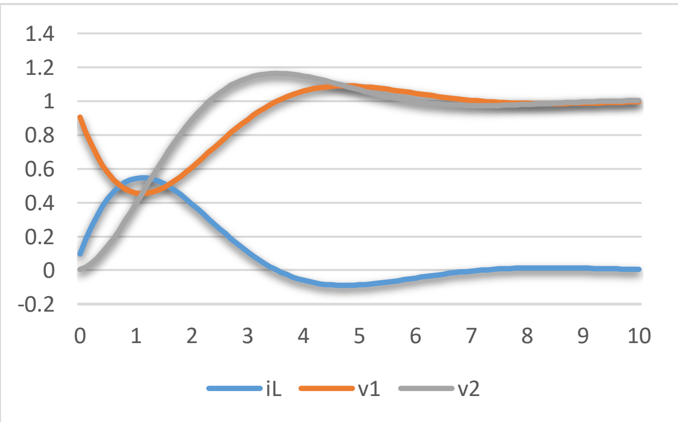
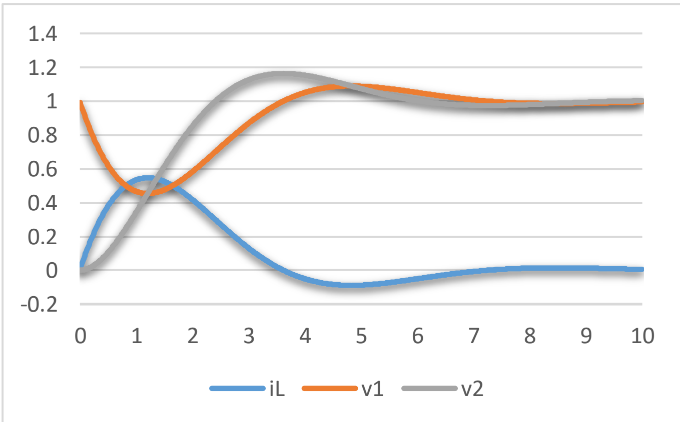
<tr><th colspan="2">Trapezoidal</th></tr>
<tr><td>
</td><td>
</td></tr>
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Table 1.

3.2. Maximum and minimum

	h=0.1			h=0.01		
Forward Euler						
	v_1	v_2	i_L	v_1	v_2	i_L
max	1.1139	1.19627	0.581654	1.09125	1.16602	0.549617
min	0.418346	0	-0.11391	0.450383	0	-0.09125
Backward Euler						
	v_1	v_2	i_L	v_1	v_2	i_L
max	1.07026	1.13651	0.515275	1.08694	1.16011	0.543012
min	0.484725	0.009009	-0.07026	0.456988	9.9e-05	-0.08694
Trapezoidal						
	v_1	v_2	i_L	v_1	v_2	i_L
max	1.08936	1.16346	0.546816	1.08907	1.16304	0.546298
min	0.453184	0.004751	-0.08936	0.453702	4.98e-05	-0.08907

Table 2.

4. Observations

4.1. Different in method (same in h, variable)

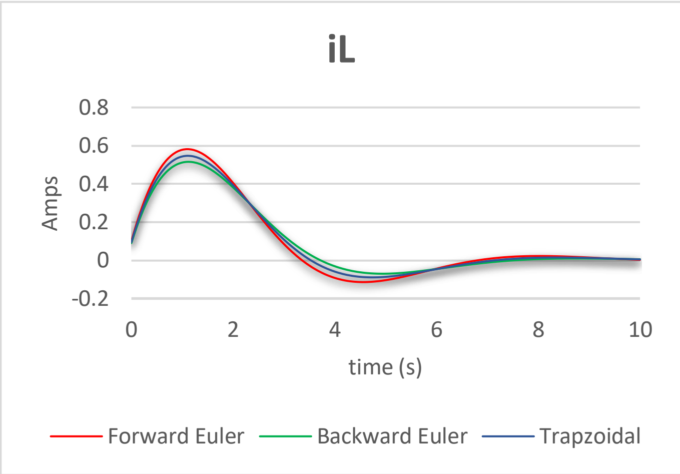
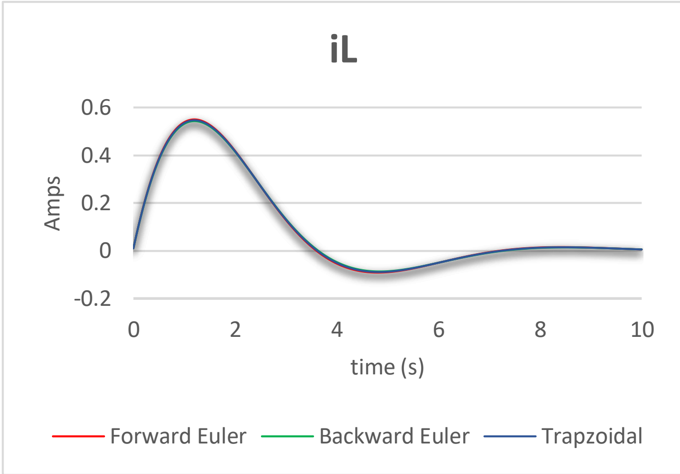
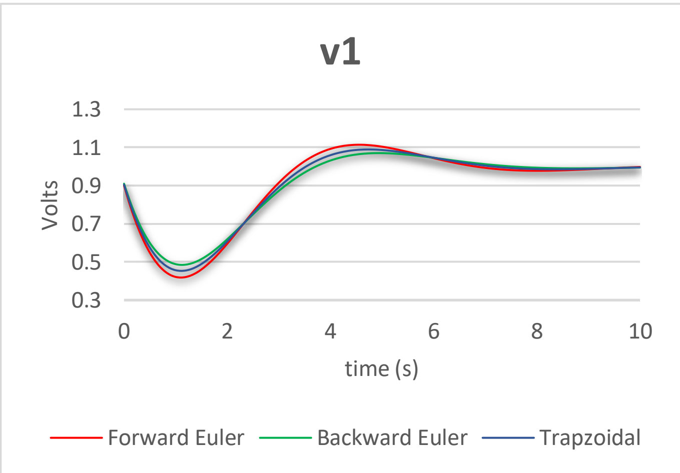
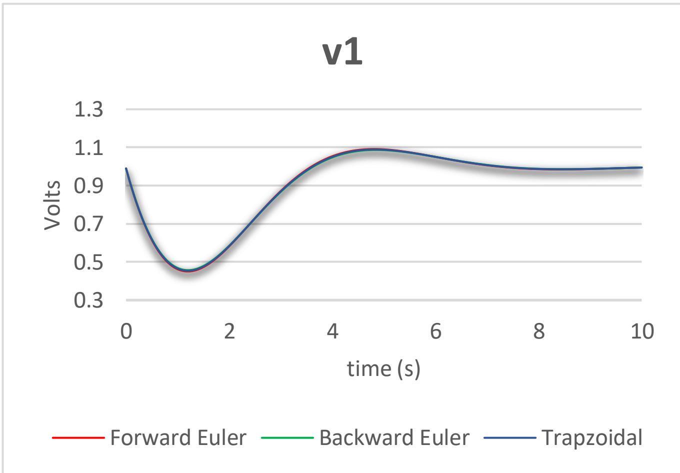
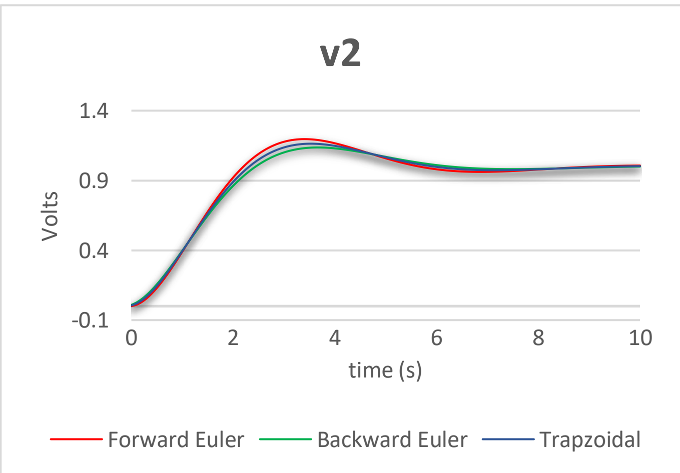
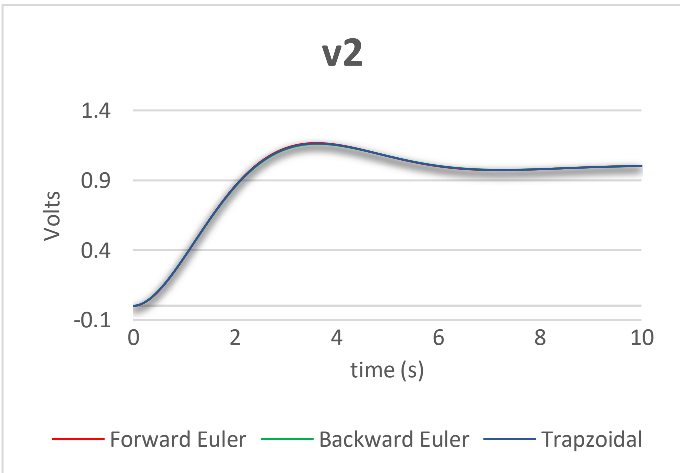
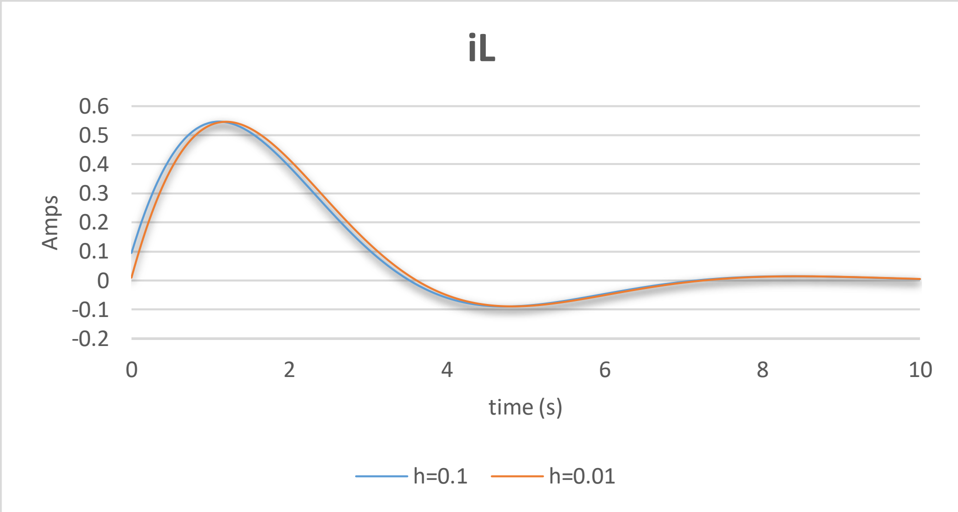
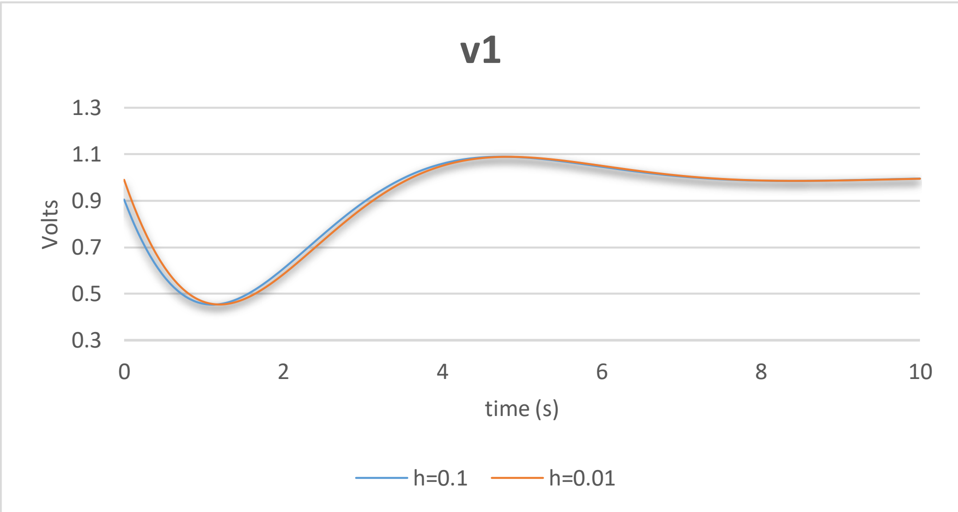
h=0.1	h=0.01
	
	
	

Table 3.

4.2. Different in h (same in method, variable)

- **Forward Euler**





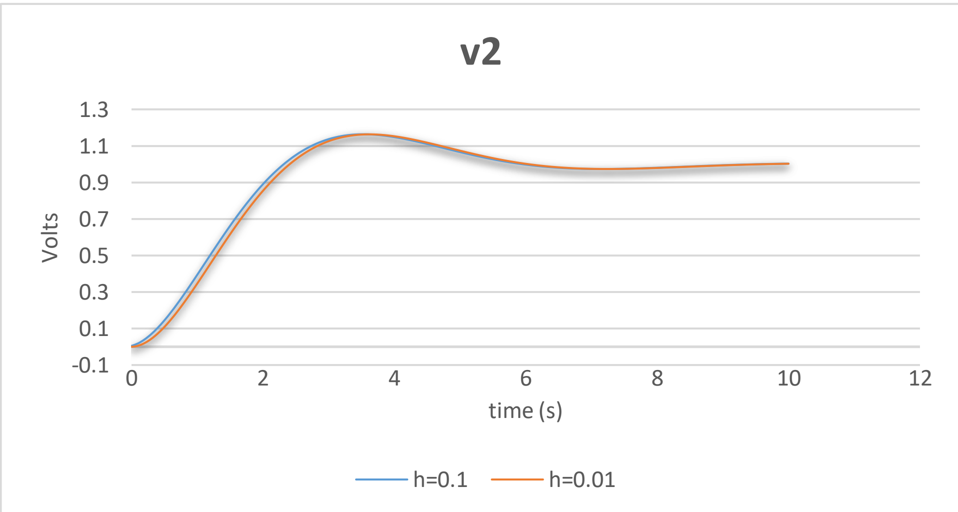
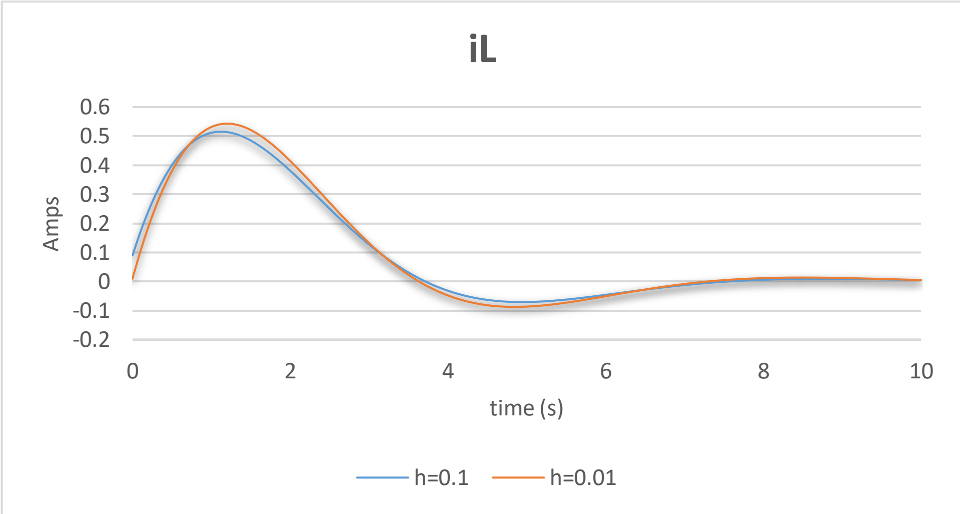
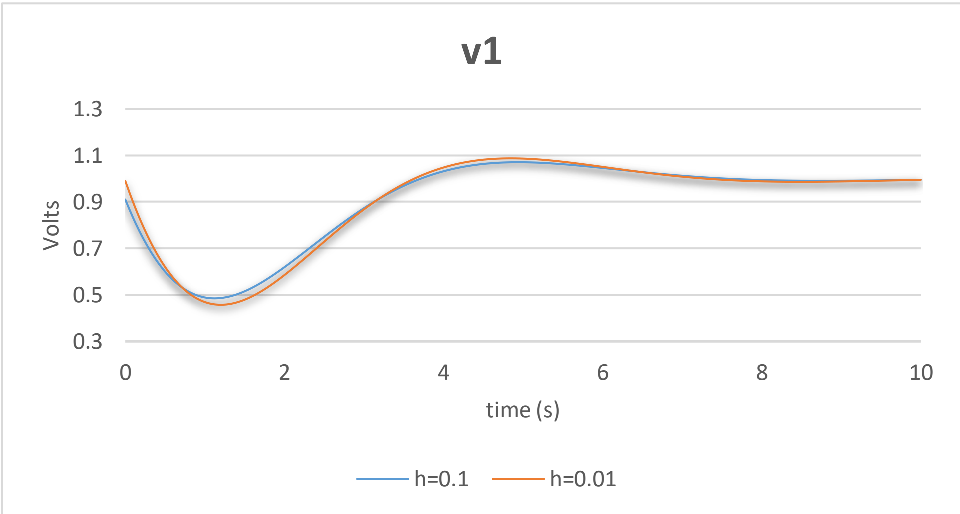


Table 4.

- **Backward Euler**





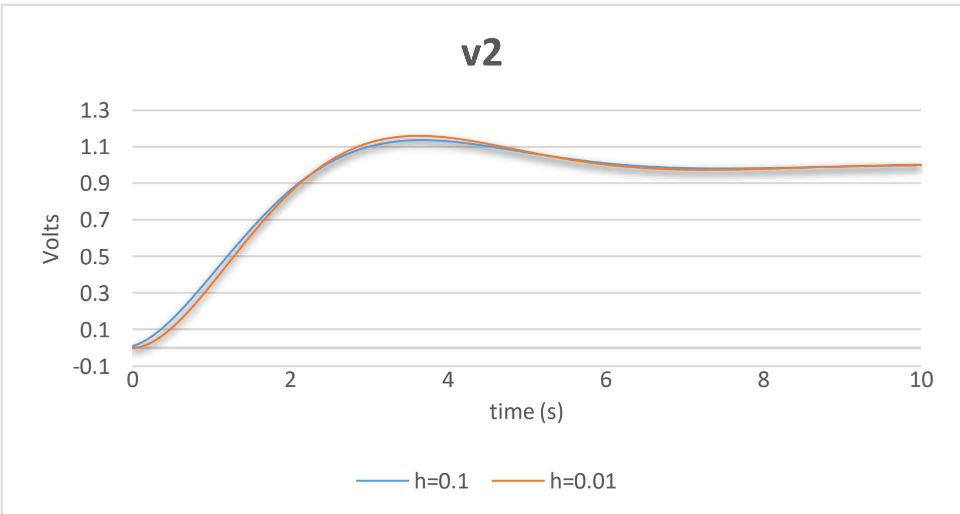
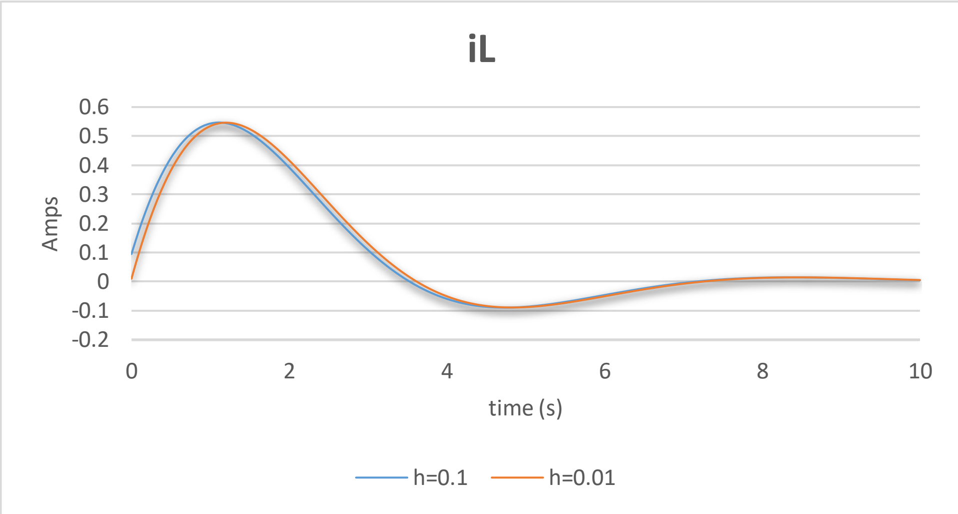
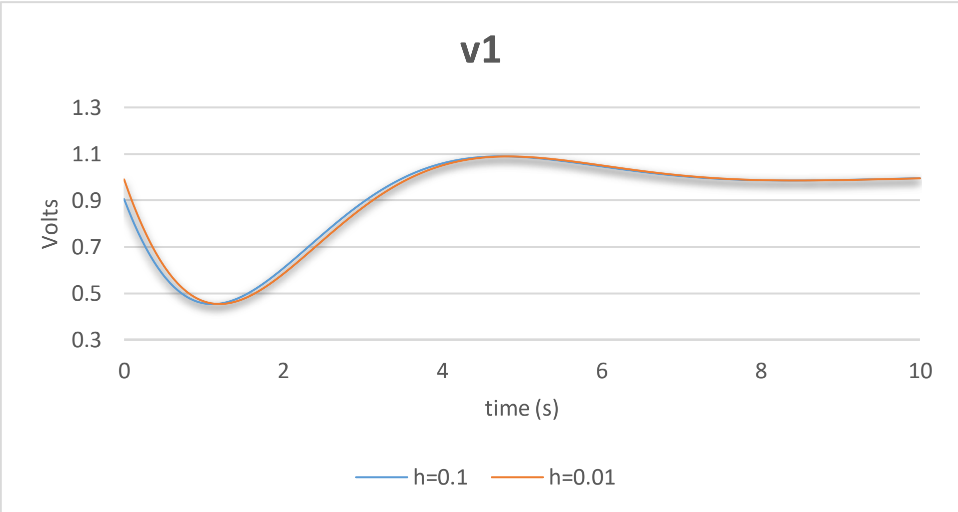


Table 5.

- **Trapezoidal**





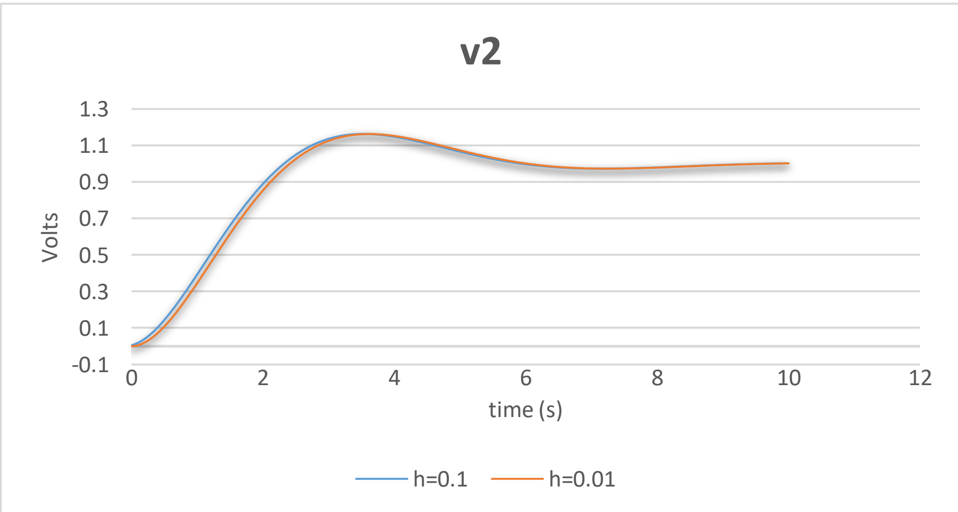


Table 6.

4.3. Conclusion

- Amplitude

  For h=0.1 and 0.01, from Table 2 and Table 3 we can obviously see the amplitude in damped oscillation region:

  Forward Euler > Trapezoidal > Backward Euler

  - Forward Euler

    From Table 2 and Table 4, the amplitude in damped oscillation region is smaller when h is smaller.

  - Backward Euler

    From Table 2 and Table 5, the amplitude in damped oscillation region is larger when h is smaller.

  - Trapezoidal

    From Table 2 and Table 6, the amplitude in damped oscillation region is smaller when h is smaller.