# Numerical Analysis

# Homework 10. Numerical Integration

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## 1. Objective

In this homework, I will implement Newton-Cotes integration formulas to find the integral, I, of f(x) over a fixed range, $x \in [0, 2]$.

$$\mathrm{f(x)} = e^x$$
$$\mathrm{I} = \int_0^2 f(x)dx.$$

The closed form solution for I* is known and can be used to find the integration errors.

$$I^* = e^2 - e^0 \approx 6.389056.$$

Now I will divide the interval, $[0, 2]$, into 12, 24, 48, 96, 192, 384, 768, 1536 equal sized regions, and sum up the integration in each regions, which will be my result,

$$\mathrm{I}_{\mathrm{\#ofRegions}}^{(\mathrm{order})}.$$

Error is defined as

$$\mathrm{E}_{\mathrm{\#ofRegions}}^{(\mathrm{order})} = \left|\mathrm{I}^* - \mathrm{I}_{\mathrm{\#ofRegions}}^{(\mathrm{order})}\right|.$$

For example, 12 equal sized regions, the error of first order is

$$\mathrm{E}_{12}^{(1)} = \left|\mathrm{I}^* - \mathrm{I}_{12}^{(1)}\right|.$$

## 2. Approach

I implement Numerical Integration as the following function.

$$\boldsymbol{double\ \ integral}((\boldsymbol{double})(*\,\boldsymbol{f})(\boldsymbol{double}),\boldsymbol{double\ \ min},\boldsymbol{double\ \ max},\boldsymbol{int\ \ N},\boldsymbol{int\ \ order})$$

Note that **f** is the exponential function, **min** is the minimum of the integral range, **max** is the maximum of the integral range, **N** is the number of subinterval, and the **order** is the n-th order in Newton-Cotes method.

**Algorithm. integral**

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$\mathrm{w[\ ]:\ \ Newton-Cotes\ \ coefficients\ \ for\ \ the\ \ order.}$

$\mathrm{segment:}\ \ \frac{\mathrm{max-min}}{\mathrm{N}}.$

$\mathrm{h:}\ \ \frac{\mathrm{max-min}}{\mathrm{N*order}}.$

$for(\mathrm{i=0;\ \ i<N;\ \ i++})\{$

$\quad for(\mathrm{j=min+i*segment,k=0;\ \ k<=order;\ \ k++})\{$

$\quad\quad \mathrm{result+=f(j)*w[k]}$

$\quad\}$

$\}$

$\mathrm{result=result*h}$

$return\ \ \mathrm{result}$

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# 3. Results

| # of regions | order | $I_{12}^{(order)}$ | $E_{12}^{(order)}$ |
|---|---|---|---|
| 12 | 1 | 6.40384 | 0.014783 |
| 12 | 2 | 6.38906 | 1.71E-06 |
| 12 | 3 | 6.38906 | 7.60E-07 |
| 12 | 4 | 6.38906 | 7.07E-11 |
| 12 | 5 | 6.38906 | 3.98E-11 |
| 12 | 6 | 6.38906 | 0 |

| # of regions | order | $I_{24}^{(order)}$ | $E_{24}^{(order)}$ |
|---|---|---|---|
| 24 | 1 | 6.39275 | 0.003697 |
| 24 | 2 | 6.38906 | 1.07E-07 |
| 24 | 3 | 6.38906 | 4.75E-08 |
| 24 | 4 | 6.38906 | 1.10E-12 |
| 24 | 5 | 6.38906 | 6.18E-13 |
| 24 | 6 | 6.38906 | 1.78E-15 |

| # of regions | order | $I_{48}^{(order)}$ | $E_{48}^{(order)}$ |
|---|---|---|---|
| 48 | 1 | 6.38998 | 0.000924 |
| 48 | 2 | 6.38906 | 6.69E-09 |
| 48 | 3 | 6.38906 | 2.97E-09 |
| 48 | 4 | 6.38906 | 1.69E-14 |
| 48 | 5 | 6.38906 | 6.22E-15 |
| 48 | 6 | 6.38906 | 0 |

| # of regions | order | $I_{96}^{(order)}$ | $E_{96}^{(order)}$ |
|---|---|---|---|
| 96 | 1 | 6.38929 | 0.000231 |
| 96 | 2 | 6.38906 | 4.18E-10 |
| 96 | 3 | 6.38906 | 1.86E-10 |
| 96 | 4 | 6.38906 | 3.55E-15 |
| 96 | 5 | 6.38906 | 1.78E-15 |
| 96 | 6 | 6.38906 | 1.78E-15 |

| # of regions | order | $I_{192}^{(order)}$ | $E_{192}^{(order)}$ |
|---|---|---|---|
| 192 | 1 | 6.38911 | 5.78E-05 |
| 192 | 2 | 6.38906 | 2.61E-11 |
| 192 | 3 | 6.38906 | 1.16E-11 |
| 192 | 4 | 6.38906 | 0 |
| 192 | 5 | 6.38906 | 1.78E-15 |
| 192 | 6 | 6.38906 | 5.33E-15 |

| # of regions | order | $I_{384}^{(order)}$ | $E_{384}^{(order)}$ |
|---|---|---|---|
| 384 | 1 | 6.38907 | 1.44E-05 |
| 384 | 2 | 6.38906 | 1.64E-12 |
| 384 | 3 | 6.38906 | 7.27E-13 |
| 384 | 4 | 6.38906 | 1.78E-15 |
| 384 | 5 | 6.38906 | 3.55E-15 |
| 384 | 6 | 6.38906 | 1.78E-15 |

| # of regions | order | $I_{768}^{(order)}$ | $E_{768}^{(order)}$ |
|---|---|---|---|
| 768 | 1 | 6.38906 | 3.61E-06 |
| 768 | 2 | 6.38906 | 1.06E-13 |
| 768 | 3 | 6.38906 | 4.53E-14 |
| 768 | 4 | 6.38906 | 8.88E-16 |
| 768 | 5 | 6.38906 | 7.11E-15 |
| 768 | 6 | 6.38906 | 0 |

| # of regions | order | $I_{1536}^{(order)}$ | $E_{1536}^{(order)}$ |
|---|---|---|---|
| 1536 | 1 | 6.38906 | 9.03E-07 |
| 1536 | 2 | 6.38906 | 3.55E-15 |
| 1536 | 3 | 6.38906 | 0 |
| 1536 | 4 | 6.38906 | 8.88E-15 |
| 1536 | 5 | 6.38906 | 7.99E-15 |
| 1536 | 6 | 6.38906 | 2.93E-14 |

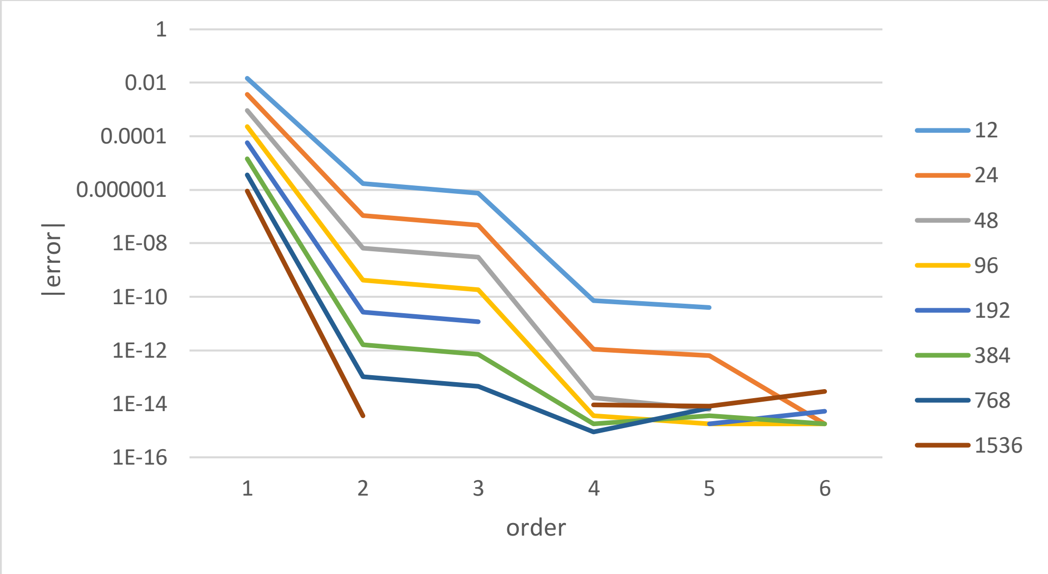


Figure 1. |error| vs. order, different number of regions

## 4. Observations

- **Same number of regions**
  When order increases, the absolute error will decrease.

- **Same order**
  When number of regions increases, the absolute error will decrease.

- **Between different order**
  When the order varies from even number to odd number, the absolute error just decreases little. However, when the order varies from odd number to even number, the absolute error decreases very much.