

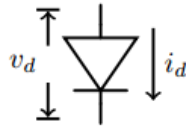
Numerical Analysis

Homework 11. Diode Networks

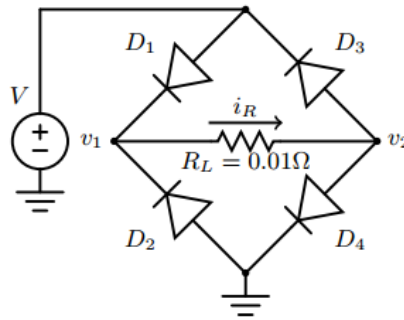
102061125 Kuan-Chun Chen

1. Objective

Diode is one of the most important elements in electronic circuits.



In this homework, I will solve the diode network below,



The equation describing the diode current as a function of diode voltage is:

$$i_d = I_s \left(e^{\frac{v_d}{\Phi}} - 1 \right),$$

Where I_s is a constant known as the saturation current and

$$\Phi = \frac{\phi_0 T}{300},$$

Is the built-in potential and is a function of temperature. For this homework, we set

$$I_s = 1 \text{ Amps},$$

$$\phi_0 = 0.026 \text{ Volts}.$$

2. Approach

2.1. Problem 1.

2.1.1. Model the system

Apply KCL at node v_1 and node v_2 ,

$$I_s \left(e^{\frac{v-v_1}{\Phi}} - 1 \right) + I_s \left(e^{\frac{-v_1}{\Phi}} - 1 \right) - \frac{v_1-v_2}{R_L} = 0,$$

$$I_s \left(e^{\frac{v_2-v}{\Phi}} - 1 \right) + I_s \left(e^{\frac{v_2}{\Phi}} - 1 \right) - \frac{v_1-v_2}{R_L} = 0.$$

Set $\mathbf{v} = [v_1, v_2]$ and then the problem becomes to find \mathbf{v}^* such that

$$F_1(\mathbf{v}) = I_s \left(e^{\frac{v-v_1}{\Phi}} - 1 \right) + I_s \left(e^{\frac{-v_1}{\Phi}} - 1 \right) - \frac{v_1-v_2}{R_L},$$

$$F_2(\mathbf{v}) = I_s \left(e^{\frac{v_2-v}{\Phi}} - 1 \right) + I_s \left(e^{\frac{v_2}{\Phi}} - 1 \right) - \frac{v_1-v_2}{R_L},$$

$$\mathbf{F}(\mathbf{v}^*) = \mathbf{0},$$

where $\mathbf{v} \in \mathbb{R}^2$, and is a 2-dimensional nonlinear system problem.

Define Jacobian matrix,

$$J_F(\mathbf{v}) = \begin{bmatrix} \frac{\partial F_i}{\partial v_i} \end{bmatrix}.$$

For the 2-dimensional problem above, we have

$$J_F(\mathbf{v}) = \begin{bmatrix} \frac{\partial F_1(v_1, v_2)}{\partial v_1} & \frac{\partial F_1(v_1, v_2)}{\partial v_2} \\ \frac{\partial F_2(v_1, v_2)}{\partial v_1} & \frac{\partial F_2(v_1, v_2)}{\partial v_2} \end{bmatrix}.$$

Newton's iteration is

$$\mathbf{v}^{(k+1)} = \mathbf{v}^{(k)} - J_F^{-1}(\mathbf{v}^{(k)}) \cdot \mathbf{F}(\mathbf{v}^{(k)}).$$

It can be written as

$$J_F(\mathbf{v}^{(k)}) \cdot \delta \mathbf{v}^{(k)} = -\mathbf{F}(\mathbf{v}^{(k)}),$$

$$\mathbf{v}^{(k+1)} = \mathbf{v}^{(k)} + \delta \mathbf{v}^{(k)}.$$

2.1.2. Algorithm

Algorithm. Cyclic Jacobian Updates

Given $\mathbf{v}^{(0)}$ and a small $\epsilon \geq 0$, let

```
k = 0, err(0) = 1 +  $\epsilon$ ,  
while (err(k) >  $\epsilon$ ) do  
    evaluate  $F(\mathbf{v}^{(k)})$ ,  
    if (k%p == 0) do  
        evaluate  $J_F(\mathbf{v}^{(k)})$ ,  
    end if  
    solve  $J_F(\mathbf{v}^{(k)}) \cdot \delta \mathbf{v} = -F(\mathbf{v}^{(k)})$ ,  
     $\mathbf{v}^{(k+1)} = \mathbf{v}^{(k)} + \delta \mathbf{v}$ ,  
    k = k + 1,  
    err(k) =  $\|F(\mathbf{v}^{(k)})\|$ ,  
end while
```

In case that explicit Jacobian matrix is difficult to evaluate, then the Jacobian can approximate numerically

$$(J_h^{(k)})_j = \frac{F(\mathbf{v}^{(k)} + h_j^{(k)} \mathbf{e}_j) - F(\mathbf{v}^{(k)})}{h_j^{(k)}}.$$

where \mathbf{e}_j is the j -th unit vector of the space \mathbb{R}^n and $h_j^{(k)} > 0$ is a small increment at iteration k .

2.2. Problem 2.

2.2.1. Model the system

Suppose the temperature for each diode is 300K when $V = 0$ Volts, and it will heat up when current flows through the diode with

$$T_d = 300 + \kappa \cdot i_d \cdot v_d,$$

where i_d is the current flows through the diode, v_d is the voltage across the diode, and $\kappa = 2$.

Apply KCL at node v_1 and node v_2 ,

$$I_s \left(e^{\frac{v-v_1}{\Phi}} - 1 \right) + I_s \left(e^{\frac{-v_1}{\Phi}} - 1 \right) - \frac{v_1-v_2}{R_L} = 0,$$

$$I_s \left(e^{\frac{v_2-V}{\Phi}} - 1 \right) + I_s \left(e^{\frac{v_2}{\Phi}} - 1 \right) - \frac{v_1-v_2}{R_L} = 0.$$

For each diode temperature,

$$T_1 = 300 + \kappa \cdot i_1 \cdot (V - v_1),$$

$$T_2 = 300 + \kappa \cdot i_2 \cdot (-v_1),$$

$$T_3 = 300 + \kappa \cdot i_3 \cdot (v_2 - V),$$

$$T_4 = 300 + \kappa \cdot i_4 \cdot v_2.$$

Set $\mathbf{v} = [v_1, v_2, T_1, T_2, T_3, T_4]$ and then the problem becomes to find \mathbf{v}^* such that

$$F_1(\mathbf{v}) = I_s \left(e^{\frac{v-v_1}{\Phi}} - 1 \right) + I_s \left(e^{\frac{-v_1}{\Phi}} - 1 \right) - \frac{v_1-v_2}{R_L},$$

$$F_2(\mathbf{v}) = I_s \left(e^{\frac{v_2-V}{\Phi}} - 1 \right) + I_s \left(e^{\frac{v_2}{\Phi}} - 1 \right) - \frac{v_1-v_2}{R_L},$$

$$F_3(\mathbf{v}) = T_1 - (300 + \kappa \cdot i_1 \cdot (V - v_1)),$$

$$F_4(\mathbf{v}) = T_2 - (300 + \kappa \cdot i_2 \cdot (-v_1)),$$

$$F_5(\mathbf{v}) = T_3 - (300 + \kappa \cdot i_3 \cdot (v_2 - V)),$$

$$F_6(\mathbf{v}) = T_4 - (300 + \kappa \cdot i_4 \cdot v_2),$$

$$F(\mathbf{v}^*) = 0,$$

Define Jacobian matrix,

$$J_F(\mathbf{v}) = \left[\frac{\partial F_i}{\partial v_i} \right].$$

Newton's iteration is

$$J_F(\mathbf{v}^{(k)}) \cdot \delta \mathbf{v}^{(k)} = -F(\mathbf{v}^{(k)}),$$

$$\mathbf{v}^{(k+1)} = \mathbf{v}^{(k)} + \delta \mathbf{v}^{(k)}.$$

2.2.2. Algorithm

Same as session 2.1.2.

3. Results

3.1. Voltages, currents and temperatures

3.1.1. Problem 1.

V	v_1	v_2	i_1	i_2	i_3	i_4	i_R
1	0.886308	0.113692	78.2617	-1	-1	78.2617	77.2617
0.98	0.866931	0.113069	76.3862	-1	-1	76.3862	75.3862
0.96	0.847568	0.112432	74.5136	-1	-1	74.5136	73.5136
0.94	0.82822	0.11178	72.6439	-1	-1	72.6439	71.6439
0.92	0.808887	0.111113	70.7774	-1	-1	70.7774	69.7774
0.9	0.789571	0.110429	68.9142	-1	-1	68.9142	67.9142
0.88	0.770272	0.109728	67.0544	-1	-1	67.0544	66.0544
0.86	0.750991	0.109009	65.1982	-1	-1	65.1982	64.1982
0.84	0.731729	0.108271	63.3458	-1	-1	63.3458	62.3458
0.82	0.712487	0.107513	61.4974	-1	-1	61.4974	60.4974
0.8	0.693266	0.106734	59.6531	-1	-1	59.6531	58.6531
0.78	0.674066	0.105934	57.8133	-1	-1	57.8133	56.8133
0.76	0.654891	0.105109	55.9781	-1	-1	55.9781	54.9781
0.74	0.63574	0.10426	54.1479	-1	-1	54.1479	53.1479
0.72	0.616614	0.103386	52.3229	-1	-1	52.3229	51.3229
0.7	0.597517	0.102483	50.5034	-1	-1	50.5034	49.5034
0.68	0.578449	0.101551	48.6898	-1	-1	48.6898	47.6898
0.66	0.559412	0.100588	46.8825	-1	-1	46.8825	45.8825
0.64	0.540409	0.099591	45.0818	-1	-1	45.0818	44.0818
0.62	0.521441	0.098559	43.2883	-1	-1	43.2883	42.2883
0.6	0.502511	0.097489	41.5023	-1	-1	41.5023	40.5023
0.58	0.483622	0.096378	39.7245	-1	-1	39.7245	38.7245
0.56	0.464777	0.095223	37.9554	-1	-1	37.9554	36.9554
0.54	0.445979	0.094021	36.1958	-1	-1	36.1958	35.1958
0.52	0.427232	0.092769	34.4463	-1	-1	34.4463	33.4463
0.5	0.408539	0.091461	32.7078	-1	-1	32.7078	31.7078
0.48	0.389906	0.090094	30.9812	-1	-1	30.9812	29.9812
0.46	0.371338	0.088662	29.2676	-1	-1	29.2676	28.2676
0.44	0.35284	0.08716	27.5681	-1	-1	27.5681	26.5681
0.42	0.33442	0.08558	25.884	-1	-1	25.884	24.884
0.4	0.316085	0.083915	24.2169	-1	-1	24.2169	23.2169
0.38	0.297842	0.082158	22.5685	-0.99999	-0.99999	22.5685	21.5685

0.36	0.279703	0.080297	20.9406	-0.99998	-0.99998	20.9406	19.9406
0.34	0.261678	0.078322	19.3356	-0.99996	-0.99996	19.3356	18.3357
0.32	0.243781	0.076219	17.756	-0.99992	-0.99992	17.756	16.7561
0.3	0.226025	0.073975	16.2048	-0.99983	-0.99983	16.2048	15.205
0.28	0.208429	0.071571	14.6855	-0.99967	-0.99967	14.6855	13.6858
0.26	0.191012	0.068988	13.2018	-0.99936	-0.99936	13.2018	12.2025
0.24	0.173799	0.066201	11.7585	-0.99875	-0.99875	11.7585	10.7598
0.22	0.156816	0.063184	10.3607	-0.9976	-0.9976	10.3607	9.36315
0.2	0.140095	0.059905	9.01447	-0.99543	-0.99543	9.01447	8.01904
0.18	0.123675	0.056325	7.72638	-0.99141	-0.99141	7.72638	6.73498
0.16	0.107599	0.052401	6.50387	-0.98405	-0.98405	6.50387	5.51982
0.14	0.09192	0.04808	5.35489	-0.97085	-0.97085	5.35489	4.38403
0.12	0.0767	0.0433	4.28768	-0.94766	-0.94766	4.28768	3.34002
0.1	0.062013	0.037987	3.31047	-0.90792	-0.90792	3.31047	2.40255
0.08	0.047946	0.032054	2.431	-0.84183	-0.84183	2.431	1.58917
0.06	0.034602	0.025398	1.65609	-0.73574	-0.73574	1.65609	0.920349
0.04	0.022093	0.017907	0.991156	-0.57248	-0.57248	0.991156	0.41868
0.02	0.010531	0.009469	0.439331	-0.33306	-0.33306	0.439331	0.106273
0	0	0	0	0	0	0	0
-0.02	-0.00947	-0.01053	-0.33306	0.439331	0.439331	-0.33306	0.106273
-0.04	-0.01791	-0.02209	-0.57248	0.991156	0.991156	-0.57248	0.41868
-0.06	-0.0254	-0.0346	-0.73574	1.65609	1.65609	-0.73574	0.920349
-0.08	-0.03205	-0.04795	-0.84183	2.431	2.431	-0.84183	1.58917
-0.1	-0.03799	-0.06201	-0.90792	3.31047	3.31047	-0.90792	2.40255
-0.12	-0.0433	-0.0767	-0.94766	4.28768	4.28768	-0.94766	3.34002
-0.14	-0.04808	-0.09192	-0.97085	5.35489	5.35489	-0.97085	4.38403
-0.16	-0.0524	-0.1076	-0.98405	6.50387	6.50387	-0.98405	5.51982
-0.18	-0.05633	-0.12368	-0.99141	7.72638	7.72638	-0.99141	6.73498
-0.2	-0.0599	-0.1401	-0.99543	9.01447	9.01447	-0.99543	8.01904
-0.22	-0.06318	-0.15682	-0.9976	10.3607	10.3607	-0.9976	9.36315
-0.24	-0.0662	-0.1738	-0.99875	11.7585	11.7585	-0.99875	10.7598
-0.26	-0.06899	-0.19101	-0.99936	13.2018	13.2018	-0.99936	12.2025
-0.28	-0.07157	-0.20843	-0.99967	14.6855	14.6855	-0.99967	13.6858
-0.3	-0.07398	-0.22603	-0.99983	16.2048	16.2048	-0.99983	15.205
-0.32	-0.07622	-0.24378	-0.99992	17.756	17.756	-0.99992	16.7561
-0.34	-0.07832	-0.26168	-0.99996	19.3356	19.3356	-0.99996	18.3357
-0.36	-0.0803	-0.2797	-0.99998	20.9406	20.9406	-0.99998	19.9406
-0.38	-0.08216	-0.29784	-0.99999	22.5685	22.5685	-0.99999	21.5685

-0.4	-0.08392	-0.31609	-1	24.2169	24.2169	-1	23.2169
-0.42	-0.08558	-0.33442	-1	25.884	25.884	-1	24.884
-0.44	-0.08716	-0.35284	-1	27.5681	27.5681	-1	26.5681
-0.46	-0.08866	-0.37134	-1	29.2676	29.2676	-1	28.2676
-0.48	-0.09009	-0.38991	-1	30.9812	30.9812	-1	29.9812
-0.5	-0.09146	-0.40854	-1	32.7078	32.7078	-1	31.7078
-0.52	-0.09277	-0.42723	-1	34.4463	34.4463	-1	33.4463
-0.54	-0.09402	-0.44598	-1	36.1958	36.1958	-1	35.1958
-0.56	-0.09522	-0.46478	-1	37.9554	37.9554	-1	36.9554
-0.58	-0.09638	-0.48362	-1	39.7245	39.7245	-1	38.7245
-0.6	-0.09749	-0.50251	-1	41.5023	41.5023	-1	40.5023
-0.62	-0.09856	-0.52144	-1	43.2883	43.2883	-1	42.2883
-0.64	-0.09959	-0.54041	-1	45.0818	45.0818	-1	44.0818
-0.66	-0.10059	-0.55941	-1	46.8825	46.8825	-1	45.8825
-0.68	-0.10155	-0.57845	-1	48.6898	48.6898	-1	47.6898
-0.7	-0.10248	-0.59752	-1	50.5034	50.5034	-1	49.5034
-0.72	-0.10339	-0.61661	-1	52.3229	52.3229	-1	51.3229
-0.74	-0.10426	-0.63574	-1	54.1479	54.1479	-1	53.1479
-0.76	-0.10511	-0.65489	-1	55.9781	55.9781	-1	54.9781
-0.78	-0.10593	-0.67407	-1	57.8133	57.8133	-1	56.8133
-0.8	-0.10673	-0.69327	-1	59.6531	59.6531	-1	58.6531
-0.82	-0.10751	-0.71249	-1	61.4974	61.4974	-1	60.4974
-0.84	-0.10827	-0.73173	-1	63.3458	63.3458	-1	62.3458
-0.86	-0.10901	-0.75099	-1	65.1982	65.1982	-1	64.1982
-0.88	-0.10973	-0.77027	-1	67.0544	67.0544	-1	66.0544
-0.9	-0.11043	-0.78957	-1	68.9142	68.9142	-1	67.9142
-0.92	-0.11111	-0.80889	-1	70.7774	70.7774	-1	69.7774
-0.94	-0.11178	-0.82822	-1	72.6439	72.6439	-1	71.6439
-0.96	-0.11243	-0.84757	-1	74.5136	74.5136	-1	73.5136
-0.98	-0.11307	-0.86693	-1	76.3862	76.3862	-1	75.3862
-1	-0.11369	-0.88631	-1	78.2617	78.2617	-1	77.2617

3.1.2. Problem 2.

v	v_1	v_2	i_1	i_2	i_3	i_4	i_R
1	0.879755	0.120245	76.9511	-1	-1	76.9511	75.9511
0.99	0.870188	0.119812	76.0377	-1	-1	76.0377	75.0377
0.98	0.860624	0.119376	75.1247	-1	-1	75.1247	74.1247
0.97	0.851062	0.118938	74.2124	-1	-1	74.2124	73.2124
0.96	0.841503	0.118497	73.3006	-1	-1	73.3006	72.3006
0.95	0.831947	0.118053	72.3893	-1	-1	72.3893	71.3893
0.94	0.822393	0.117607	71.4786	-1	-1	71.4786	70.4786
0.93	0.812843	0.117157	70.5686	-1	-1	70.5686	69.5686
0.92	0.803296	0.116704	69.6591	-1	-1	69.6591	68.6591
0.91	0.793752	0.116248	68.7503	-1	-1	68.7503	67.7503
0.9	0.784211	0.115789	67.8421	-1	-1	67.8421	66.8421
0.89	0.774673	0.115327	66.9347	-1	-1	66.9347	65.9347
0.88	0.765139	0.114861	66.0279	-1	-1	66.0279	65.0279
0.87	0.755609	0.114391	65.1218	-1	-1	65.1218	64.1218
0.86	0.746082	0.113918	64.2164	-1	-1	64.2164	63.2164
0.85	0.736559	0.113441	63.3118	-1	-1	63.3118	62.3118
0.84	0.72704	0.11296	62.408	-1	-1	62.408	61.408
0.83	0.717525	0.112475	61.505	-1	-1	61.505	60.505
0.82	0.708014	0.111986	60.6028	-1	-1	60.6028	59.6028
0.81	0.698508	0.111492	59.7015	-1	-1	59.7015	58.7015
0.8	0.689005	0.110995	58.8011	-1	-1	58.8011	57.8011
0.79	0.679508	0.110492	57.9015	-1	-1	57.9015	56.9015
0.78	0.670015	0.109985	57.0029	-1	-1	57.0029	56.0029
0.77	0.660526	0.109474	56.1053	-1	-1	56.1053	55.1053
0.76	0.651043	0.108957	55.2086	-1	-1	55.2086	54.2086
0.75	0.641565	0.108435	54.313	-1	-1	54.313	53.313
0.74	0.632092	0.107908	53.4185	-1	-1	53.4185	52.4185
0.73	0.622625	0.107375	52.525	-1	-1	52.525	51.525
0.72	0.613164	0.106836	51.6327	-1	-1	51.6327	50.6327
0.71	0.603708	0.106292	50.7416	-1	-1	50.7416	49.7416
0.7	0.594258	0.105742	49.8517	-1	-1	49.8517	48.8517
0.69	0.584815	0.105185	48.963	-1	-1	48.963	47.963
0.68	0.575378	0.104622	48.0756	-1	-1	48.0756	47.0756
0.67	0.565948	0.104052	47.1896	-1	-1	47.1896	46.1896
0.66	0.556525	0.103475	46.305	-1	-1	46.305	45.305
0.65	0.547109	0.102891	45.4218	-1	-1	45.4218	44.4218

0.64	0.537701	0.102299	44.5401	-1	-1	44.5401	43.5401
0.63	0.5283	0.1017	43.66	-1	-1	43.66	42.66
0.62	0.518907	0.101093	42.7815	-1	-1	42.7815	41.7815
0.61	0.509523	0.100477	41.9046	-1	-1	41.9046	40.9046
0.6	0.500148	0.099852	41.0295	-1	-1	41.0295	40.0295
0.59	0.490781	0.099219	40.1562	-1	-1	40.1562	39.1562
0.58	0.481424	0.098576	39.2848	-1	-1	39.2848	38.2848
0.57	0.472076	0.097924	38.4153	-1	-1	38.4153	37.4153
0.56	0.462739	0.097261	37.5478	-1	-1	37.5478	36.5478
0.55	0.453412	0.096588	36.6824	-1	-1	36.6824	35.6824
0.54	0.444096	0.095904	35.8193	-1	-1	35.8193	34.8193
0.53	0.434792	0.095208	34.9584	-1	-1	34.9584	33.9584
0.52	0.425499	0.094501	34.0999	-1	-1	34.0999	33.0999
0.51	0.416219	0.093781	33.2439	-1	-1	33.2439	32.2439
0.5	0.406952	0.093048	32.3905	-1	-1	32.3905	31.3905
0.49	0.397699	0.092301	31.5397	-1	-1	31.5397	30.5397
0.48	0.388459	0.091541	30.6919	-1	-1	30.6919	29.6919
0.47	0.379235	0.090765	29.847	-1	-1	29.847	28.847
0.46	0.370026	0.089974	29.0052	-1	-1	29.0052	28.0052
0.45	0.360833	0.089167	28.1666	-1	-1	28.1666	27.1666
0.44	0.351657	0.088343	27.3314	-1	-1	27.3314	26.3314
0.43	0.342499	0.087501	26.4999	-1	-1	26.4999	25.4999
0.42	0.33336	0.08664	25.672	-1	-1	25.672	24.672
0.41	0.324241	0.085759	24.8481	-1	-1	24.8481	23.8481
0.4	0.315142	0.084858	24.0283	-0.99999	-0.99999	24.0283	23.0283
0.39	0.306065	0.083935	23.2129	-0.99999	-0.99999	23.2129	22.2129
0.38	0.297011	0.08299	22.4021	-0.99999	-0.99999	22.4021	21.4021
0.37	0.287981	0.082019	21.5961	-0.99998	-0.99998	21.5961	20.5961
0.36	0.278976	0.081024	20.7952	-0.99998	-0.99998	20.7952	19.7952
0.35	0.269998	0.080002	19.9997	-0.99997	-0.99997	19.9997	18.9997
0.34	0.261049	0.078951	19.2098	-0.99996	-0.99996	19.2098	18.2099
0.33	0.25213	0.07787	18.426	-0.99994	-0.99994	18.426	17.4261
0.32	0.243243	0.076757	17.6485	-0.99991	-0.99991	17.6485	16.6486
0.31	0.23439	0.07561	16.8778	-0.99988	-0.99988	16.8778	15.8779
0.3	0.225572	0.074428	16.1142	-0.99983	-0.99983	16.1142	15.1144
0.29	0.216792	0.073208	15.3582	-0.99976	-0.99976	15.3582	14.3585
0.28	0.208053	0.071947	14.6103	-0.99966	-0.99966	14.6103	13.6106
0.27	0.199357	0.070643	13.8709	-0.99953	-0.99953	13.8709	12.8714

0.26	0.190707	0.069294	13.1406	-0.99934	-0.99934	13.1406	12.1413
0.25	0.182105	0.067895	12.4201	-0.99908	-0.99908	12.4201	11.421
0.24	0.173555	0.066445	11.7098	-0.99873	-0.99873	11.7098	10.7111
0.23	0.165062	0.064939	11.0105	-0.99824	-0.99824	11.0105	10.0123
0.22	0.156627	0.063373	10.323	-0.99757	-0.99757	10.323	9.32542
0.21	0.148256	0.061744	9.64791	-0.99664	-0.99664	9.64791	8.65126
0.2	0.139954	0.060046	8.98612	-0.99538	-0.99538	8.98612	7.99074
0.19	0.131724	0.058276	8.33848	-0.99367	-0.99367	8.33848	7.34482
0.18	0.123573	0.056427	7.70591	-0.99134	-0.99134	7.70591	6.71457
0.17	0.115506	0.054494	7.08935	-0.98819	-0.98819	7.08935	6.10115
0.16	0.107529	0.052471	6.48978	-0.98396	-0.98396	6.48978	5.50582
0.15	0.09965	0.05035	5.90824	-0.9783	-0.9783	5.90824	4.92995
0.14	0.091875	0.048125	5.34577	-0.97074	-0.97074	5.34577	4.37502
0.13	0.084213	0.045787	4.80341	-0.96073	-0.96073	4.80341	3.84268
0.12	0.076674	0.043327	4.28222	-0.94753	-0.94753	4.28222	3.33469
0.11	0.069265	0.040735	3.78326	-0.93025	-0.93025	3.78326	2.853
0.1	0.061999	0.038001	3.30753	-0.90779	-0.90779	3.30753	2.39974
0.09	0.054886	0.035114	2.85601	-0.8788	-0.8788	2.85601	1.97721
0.08	0.04794	0.03206	2.42963	-0.84171	-0.84171	2.42963	1.58792
0.07	0.041173	0.028827	2.02923	-0.79469	-0.79469	2.02923	1.23454
0.06	0.0346	0.0254	1.65558	-0.73566	-0.73566	1.65558	0.919921
0.05	0.028235	0.021765	1.30934	-0.66238	-0.66238	1.30934	0.646967
0.04	0.022093	0.017907	0.991028	-0.57244	-0.57244	0.991028	0.41859
0.03	0.016188	0.013812	0.700984	-0.46344	-0.46344	0.700984	0.237546
0.02	0.010531	0.009469	0.439318	-0.33305	-0.33305	0.439318	0.106267
0.01	0.005133	0.004867	0.205841	-0.17917	-0.17917	0.205841	0.026673
0	0	0	0	0	0	0	0
-0.01	-0.00487	-0.00513	-0.17917	0.205841	0.205841	-0.17917	0.026673
-0.02	-0.00947	-0.01053	-0.33305	0.439318	0.439318	-0.33305	0.106267
-0.03	-0.01381	-0.01619	-0.46344	0.700984	0.700984	-0.46344	0.237546
-0.04	-0.01791	-0.02209	-0.57244	0.991028	0.991028	-0.57244	0.41859
-0.05	-0.02177	-0.02823	-0.66238	1.30934	1.30934	-0.66238	0.646967
-0.06	-0.0254	-0.0346	-0.73566	1.65558	1.65558	-0.73566	0.919921
-0.07	-0.02883	-0.04117	-0.79469	2.02923	2.02923	-0.79469	1.23454
-0.08	-0.03206	-0.04794	-0.84171	2.42963	2.42963	-0.84171	1.58792
-0.09	-0.03511	-0.05489	-0.8788	2.85601	2.85601	-0.8788	1.97721
-0.1	-0.038	-0.062	-0.90779	3.30753	3.30753	-0.90779	2.39974
-0.11	-0.04074	-0.06927	-0.93025	3.78326	3.78326	-0.93025	2.853

-0.12	-0.04333	-0.07667	-0.94753	4.28222	4.28222	-0.94753	3.33469
-0.13	-0.04579	-0.08421	-0.96073	4.80341	4.80341	-0.96073	3.84268
-0.14	-0.04812	-0.09188	-0.97074	5.34577	5.34577	-0.97074	4.37502
-0.15	-0.05035	-0.09965	-0.9783	5.90824	5.90824	-0.9783	4.92995
-0.16	-0.05247	-0.10753	-0.98396	6.48978	6.48978	-0.98396	5.50582
-0.17	-0.05449	-0.11551	-0.98819	7.08935	7.08935	-0.98819	6.10115
-0.18	-0.05643	-0.12357	-0.99134	7.70591	7.70591	-0.99134	6.71457
-0.19	-0.05828	-0.13172	-0.99367	8.33848	8.33848	-0.99367	7.34482
-0.2	-0.06005	-0.13995	-0.99538	8.98612	8.98612	-0.99538	7.99074
-0.21	-0.06174	-0.14826	-0.99664	9.64791	9.64791	-0.99664	8.65126
-0.22	-0.06337	-0.15663	-0.99757	10.323	10.323	-0.99757	9.32542
-0.23	-0.06494	-0.16506	-0.99824	11.0105	11.0105	-0.99824	10.0123
-0.24	-0.06644	-0.17356	-0.99873	11.7098	11.7098	-0.99873	10.7111
-0.25	-0.0679	-0.18211	-0.99908	12.4201	12.4201	-0.99908	11.421
-0.26	-0.06929	-0.19071	-0.99934	13.1406	13.1406	-0.99934	12.1413
-0.27	-0.07064	-0.19936	-0.99953	13.8709	13.8709	-0.99953	12.8714
-0.28	-0.07195	-0.20805	-0.99966	14.6103	14.6103	-0.99966	13.6106
-0.29	-0.07321	-0.21679	-0.99976	15.3582	15.3582	-0.99976	14.3585
-0.3	-0.07443	-0.22557	-0.99983	16.1142	16.1142	-0.99983	15.1144
-0.31	-0.07561	-0.23439	-0.99988	16.8778	16.8778	-0.99988	15.8779
-0.32	-0.07676	-0.24324	-0.99991	17.6485	17.6485	-0.99991	16.6486
-0.33	-0.07787	-0.25213	-0.99994	18.426	18.426	-0.99994	17.4261
-0.34	-0.07895	-0.26105	-0.99996	19.2098	19.2098	-0.99996	18.2099
-0.35	-0.08	-0.27	-0.99997	19.9997	19.9997	-0.99997	18.9997
-0.36	-0.08102	-0.27898	-0.99998	20.7952	20.7952	-0.99998	19.7952
-0.37	-0.08202	-0.28798	-0.99998	21.5961	21.5961	-0.99998	20.5961
-0.38	-0.08299	-0.29701	-0.99999	22.4021	22.4021	-0.99999	21.4021
-0.39	-0.08394	-0.30607	-0.99999	23.2129	23.2129	-0.99999	22.2129
-0.4	-0.08486	-0.31514	-0.99999	24.0283	24.0283	-0.99999	23.0283
-0.41	-0.08576	-0.32424	-1	24.8481	24.8481	-1	23.8481
-0.42	-0.08664	-0.33336	-1	25.672	25.672	-1	24.672
-0.43	-0.0875	-0.3425	-1	26.4999	26.4999	-1	25.4999
-0.44	-0.08834	-0.35166	-1	27.3314	27.3314	-1	26.3314
-0.45	-0.08917	-0.36083	-1	28.1666	28.1666	-1	27.1666
-0.46	-0.08997	-0.37003	-1	29.0052	29.0052	-1	28.0052
-0.47	-0.09077	-0.37924	-1	29.847	29.847	-1	28.847
-0.48	-0.09154	-0.38846	-1	30.6919	30.6919	-1	29.6919
-0.49	-0.0923	-0.3977	-1	31.5397	31.5397	-1	30.5397

-0.5	-0.09305	-0.40695	-1	32.3905	32.3905	-1	31.3905
-0.51	-0.09378	-0.41622	-1	33.2439	33.2439	-1	32.2439
-0.52	-0.0945	-0.4255	-1	34.0999	34.0999	-1	33.0999
-0.53	-0.09521	-0.43479	-1	34.9584	34.9584	-1	33.9584
-0.54	-0.0959	-0.4441	-1	35.8193	35.8193	-1	34.8193
-0.55	-0.09659	-0.45341	-1	36.6824	36.6824	-1	35.6824
-0.56	-0.09726	-0.46274	-1	37.5478	37.5478	-1	36.5478
-0.57	-0.09792	-0.47208	-1	38.4153	38.4153	-1	37.4153
-0.58	-0.09858	-0.48142	-1	39.2848	39.2848	-1	38.2848
-0.59	-0.09922	-0.49078	-1	40.1562	40.1562	-1	39.1562
-0.6	-0.09985	-0.50015	-1	41.0295	41.0295	-1	40.0295
-0.61	-0.10048	-0.50952	-1	41.9046	41.9046	-1	40.9046
-0.62	-0.10109	-0.51891	-1	42.7815	42.7815	-1	41.7815
-0.63	-0.1017	-0.5283	-1	43.66	43.66	-1	42.66
-0.64	-0.1023	-0.5377	-1	44.5401	44.5401	-1	43.5401
-0.65	-0.10289	-0.54711	-1	45.4218	45.4218	-1	44.4218
-0.66	-0.10348	-0.55653	-1	46.305	46.305	-1	45.305
-0.67	-0.10405	-0.56595	-1	47.1896	47.1896	-1	46.1896
-0.68	-0.10462	-0.57538	-1	48.0756	48.0756	-1	47.0756
-0.69	-0.10519	-0.58482	-1	48.963	48.963	-1	47.963
-0.7	-0.10574	-0.59426	-1	49.8517	49.8517	-1	48.8517
-0.71	-0.10629	-0.60371	-1	50.7416	50.7416	-1	49.7416
-0.72	-0.10684	-0.61316	-1	51.6327	51.6327	-1	50.6327
-0.73	-0.10738	-0.62263	-1	52.525	52.525	-1	51.525
-0.74	-0.10791	-0.63209	-1	53.4185	53.4185	-1	52.4185
-0.75	-0.10844	-0.64157	-1	54.313	54.313	-1	53.313
-0.76	-0.10896	-0.65104	-1	55.2086	55.2086	-1	54.2086
-0.77	-0.10947	-0.66053	-1	56.1053	56.1053	-1	55.1053
-0.78	-0.10999	-0.67002	-1	57.0029	57.0029	-1	56.0029
-0.79	-0.11049	-0.67951	-1	57.9015	57.9015	-1	56.9015
-0.8	-0.111	-0.68901	-1	58.8011	58.8011	-1	57.8011
-0.81	-0.11149	-0.69851	-1	59.7015	59.7015	-1	58.7015
-0.82	-0.11199	-0.70801	-1	60.6028	60.6028	-1	59.6028
-0.83	-0.11248	-0.71753	-1	61.505	61.505	-1	60.505
-0.84	-0.11296	-0.72704	-1	62.408	62.408	-1	61.408
-0.85	-0.11344	-0.73656	-1	63.3118	63.3118	-1	62.3118
-0.86	-0.11392	-0.74608	-1	64.2164	64.2164	-1	63.2164
-0.87	-0.11439	-0.75561	-1	65.1218	65.1218	-1	64.1218

-0.88	-0.11486	-0.76514	-1	66.0279	66.0279	-1	65.0279
-0.89	-0.11533	-0.77467	-1	66.9347	66.9347	-1	65.9347
-0.9	-0.11579	-0.78421	-1	67.8421	67.8421	-1	66.8421
-0.91	-0.11625	-0.79375	-1	68.7503	68.7503	-1	67.7503
-0.92	-0.1167	-0.8033	-1	69.6591	69.6591	-1	68.6591
-0.93	-0.11716	-0.81284	-1	70.5686	70.5686	-1	69.5686
-0.94	-0.11761	-0.82239	-1	71.4786	71.4786	-1	70.4786
-0.95	-0.11805	-0.83195	-1	72.3893	72.3893	-1	71.3893
-0.96	-0.1185	-0.8415	-1	73.3006	73.3006	-1	72.3006
-0.97	-0.11894	-0.85106	-1	74.2124	74.2124	-1	73.2124
-0.98	-0.11938	-0.86062	-1	75.1247	75.1247	-1	74.1247
-0.99	-0.11981	-0.87019	-1	76.0377	76.0377	-1	75.0377
-1	-0.12025	-0.87976	-1	76.9511	76.9511	-1	75.9511

V	T ₁	T ₂	T ₃	T ₄
1	318.506	301.76	301.76	318.506
0.99	318.22	301.74	301.74	318.22
0.98	317.936	301.721	301.721	317.936
0.97	317.653	301.702	301.702	317.653
0.96	317.372	301.683	301.683	317.372
0.95	317.092	301.664	301.664	317.092
0.94	316.813	301.645	301.645	316.813
0.93	316.535	301.626	301.626	316.535
0.92	316.259	301.607	301.607	316.259
0.91	315.984	301.588	301.588	315.984
0.9	315.711	301.568	301.568	315.711
0.89	315.439	301.549	301.549	315.439
0.88	315.168	301.53	301.53	315.168
0.87	314.899	301.511	301.511	314.899
0.86	314.631	301.492	301.492	314.631
0.85	314.364	301.473	301.473	314.364
0.84	314.099	301.454	301.454	314.099
0.83	313.836	301.435	301.435	313.836
0.82	313.573	301.416	301.416	313.573
0.81	313.313	301.397	301.397	313.313
0.8	313.053	301.378	301.378	313.053
0.79	312.795	301.359	301.359	312.795
0.78	312.539	301.34	301.34	312.539

0.77	312.284	301.321	301.321	312.284
0.76	312.031	301.302	301.302	312.031
0.75	311.779	301.283	301.283	311.779
0.74	311.529	301.264	301.264	311.529
0.73	311.28	301.245	301.245	311.28
0.72	311.033	301.226	301.226	311.033
0.71	310.787	301.207	301.207	310.787
0.7	310.543	301.189	301.189	310.543
0.69	310.3	301.17	301.17	310.3
0.68	310.06	301.151	301.151	310.06
0.67	309.82	301.132	301.132	309.82
0.66	309.583	301.113	301.113	309.583
0.65	309.347	301.094	301.094	309.347
0.64	309.113	301.075	301.075	309.113
0.63	308.88	301.057	301.057	308.88
0.62	308.65	301.038	301.038	308.65
0.61	308.421	301.019	301.019	308.421
0.6	308.194	301	301	308.194
0.59	307.969	300.982	300.982	307.969
0.58	307.745	300.963	300.963	307.745
0.57	307.524	300.944	300.944	307.524
0.56	307.304	300.925	300.925	307.304
0.55	307.086	300.907	300.907	307.086
0.54	306.87	300.888	300.888	306.87
0.53	306.657	300.87	300.87	306.657
0.52	306.445	300.851	300.851	306.445
0.51	306.235	300.832	300.832	306.235
0.5	306.028	300.814	300.814	306.028
0.49	305.822	300.795	300.795	305.822
0.48	305.619	300.777	300.777	305.619
0.47	305.418	300.758	300.758	305.418
0.46	305.219	300.74	300.74	305.219
0.45	305.023	300.722	300.722	305.023
0.44	304.829	300.703	300.703	304.829
0.43	304.638	300.685	300.685	304.638
0.42	304.448	300.667	300.667	304.448
0.41	304.262	300.648	300.648	304.262
0.4	304.078	300.63	300.63	304.078

0.39	303.897	300.612	300.612	303.897
0.38	303.718	300.594	300.594	303.718
0.37	303.543	300.576	300.576	303.543
0.36	303.37	300.558	300.558	303.37
0.35	303.2	300.54	300.54	303.2
0.34	303.033	300.522	300.522	303.033
0.33	302.87	300.504	300.504	302.87
0.32	302.709	300.486	300.486	302.709
0.31	302.552	300.469	300.469	302.552
0.3	302.399	300.451	300.451	302.399
0.29	302.249	300.433	300.433	302.249
0.28	302.102	300.416	300.416	302.102
0.27	301.96	300.399	300.399	301.96
0.26	301.821	300.381	300.381	301.821
0.25	301.687	300.364	300.364	301.687
0.24	301.556	300.347	300.347	301.556
0.23	301.43	300.33	300.33	301.43
0.22	301.308	300.312	300.312	301.308
0.21	301.191	300.296	300.296	301.191
0.2	301.079	300.279	300.279	301.079
0.19	300.972	300.262	300.262	300.972
0.18	300.87	300.245	300.245	300.87
0.17	300.773	300.228	300.228	300.773
0.16	300.681	300.212	300.212	300.681
0.15	300.595	300.195	300.195	300.595
0.14	300.515	300.178	300.178	300.515
0.13	300.44	300.162	300.162	300.44
0.12	300.371	300.145	300.145	300.371
0.11	300.308	300.129	300.129	300.308
0.1	300.251	300.113	300.113	300.251
0.09	300.201	300.096	300.096	300.201
0.08	300.156	300.081	300.081	300.156
0.07	300.117	300.065	300.065	300.117
0.06	300.084	300.051	300.051	300.084
0.05	300.057	300.037	300.037	300.057
0.04	300.035	300.025	300.025	300.035
0.03	300.019	300.015	300.015	300.019
0.02	300.008	300.007	300.007	300.008

0.01	300.002	300.002	300.002	300.002
0	300	300	300	300
-0.01	300.002	300.002	300.002	300.002
-0.02	300.007	300.008	300.008	300.007
-0.03	300.015	300.019	300.019	300.015
-0.04	300.025	300.035	300.035	300.025
-0.05	300.037	300.057	300.057	300.037
-0.06	300.051	300.084	300.084	300.051
-0.07	300.065	300.117	300.117	300.065
-0.08	300.081	300.156	300.156	300.081
-0.09	300.096	300.201	300.201	300.096
-0.1	300.113	300.251	300.251	300.113
-0.11	300.129	300.308	300.308	300.129
-0.12	300.145	300.371	300.371	300.145
-0.13	300.162	300.44	300.44	300.162
-0.14	300.178	300.515	300.515	300.178
-0.15	300.195	300.595	300.595	300.195
-0.16	300.212	300.681	300.681	300.212
-0.17	300.228	300.773	300.773	300.228
-0.18	300.245	300.87	300.87	300.245
-0.19	300.262	300.972	300.972	300.262
-0.2	300.279	301.079	301.079	300.279
-0.21	300.296	301.191	301.191	300.296
-0.22	300.312	301.308	301.308	300.312
-0.23	300.33	301.43	301.43	300.33
-0.24	300.347	301.556	301.556	300.347
-0.25	300.364	301.687	301.687	300.364
-0.26	300.381	301.821	301.821	300.381
-0.27	300.399	301.96	301.96	300.399
-0.28	300.416	302.102	302.102	300.416
-0.29	300.433	302.249	302.249	300.433
-0.3	300.451	302.399	302.399	300.451
-0.31	300.469	302.552	302.552	300.469
-0.32	300.486	302.709	302.709	300.486
-0.33	300.504	302.87	302.87	300.504
-0.34	300.522	303.033	303.033	300.522
-0.35	300.54	303.2	303.2	300.54
-0.36	300.558	303.37	303.37	300.558

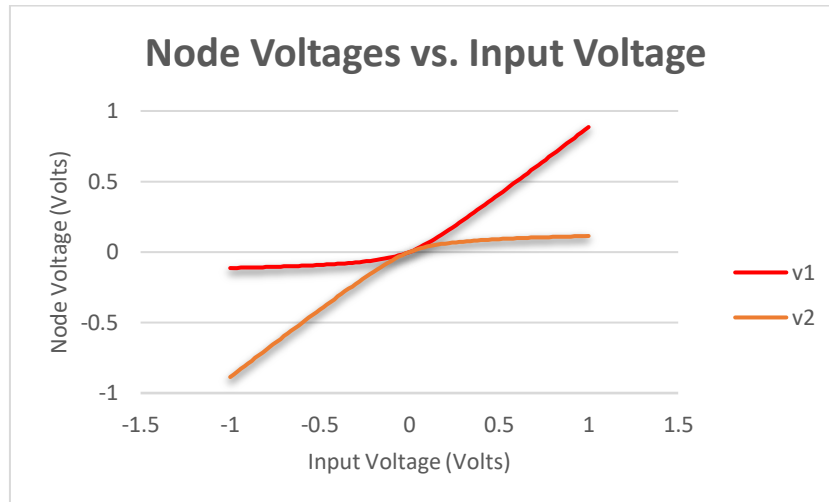
-0.37	300.576	303.543	303.543	300.576
-0.38	300.594	303.718	303.718	300.594
-0.39	300.612	303.897	303.897	300.612
-0.4	300.63	304.078	304.078	300.63
-0.41	300.648	304.262	304.262	300.648
-0.42	300.667	304.448	304.448	300.667
-0.43	300.685	304.638	304.638	300.685
-0.44	300.703	304.829	304.829	300.703
-0.45	300.722	305.023	305.023	300.722
-0.46	300.74	305.219	305.219	300.74
-0.47	300.758	305.418	305.418	300.758
-0.48	300.777	305.619	305.619	300.777
-0.49	300.795	305.822	305.822	300.795
-0.5	300.814	306.028	306.028	300.814
-0.51	300.832	306.235	306.235	300.832
-0.52	300.851	306.445	306.445	300.851
-0.53	300.87	306.657	306.657	300.87
-0.54	300.888	306.87	306.87	300.888
-0.55	300.907	307.086	307.086	300.907
-0.56	300.925	307.304	307.304	300.925
-0.57	300.944	307.524	307.524	300.944
-0.58	300.963	307.745	307.745	300.963
-0.59	300.982	307.969	307.969	300.982
-0.6	301	308.194	308.194	301
-0.61	301.019	308.421	308.421	301.019
-0.62	301.038	308.65	308.65	301.038
-0.63	301.057	308.88	308.88	301.057
-0.64	301.075	309.113	309.113	301.075
-0.65	301.094	309.347	309.347	301.094
-0.66	301.113	309.583	309.583	301.113
-0.67	301.132	309.82	309.82	301.132
-0.68	301.151	310.06	310.06	301.151
-0.69	301.17	310.3	310.3	301.17
-0.7	301.189	310.543	310.543	301.189
-0.71	301.207	310.787	310.787	301.207
-0.72	301.226	311.033	311.033	301.226
-0.73	301.245	311.28	311.28	301.245
-0.74	301.264	311.529	311.529	301.264

-0.75	301.283	311.779	311.779	301.283
-0.76	301.302	312.031	312.031	301.302
-0.77	301.321	312.284	312.284	301.321
-0.78	301.34	312.539	312.539	301.34
-0.79	301.359	312.795	312.795	301.359
-0.8	301.378	313.053	313.053	301.378
-0.81	301.397	313.313	313.313	301.397
-0.82	301.416	313.573	313.573	301.416
-0.83	301.435	313.836	313.836	301.435
-0.84	301.454	314.099	314.099	301.454
-0.85	301.473	314.364	314.364	301.473
-0.86	301.492	314.631	314.631	301.492
-0.87	301.511	314.899	314.899	301.511
-0.88	301.53	315.168	315.168	301.53
-0.89	301.549	315.439	315.439	301.549
-0.9	301.568	315.711	315.711	301.568
-0.91	301.588	315.984	315.984	301.588
-0.92	301.607	316.259	316.259	301.607
-0.93	301.626	316.535	316.535	301.626
-0.94	301.645	316.813	316.813	301.645
-0.95	301.664	317.092	317.092	301.664
-0.96	301.683	317.372	317.372	301.683
-0.97	301.702	317.653	317.653	301.702
-0.98	301.721	317.936	317.936	301.721
-0.99	301.74	318.22	318.22	301.74
-1	301.76	318.506	318.506	301.76

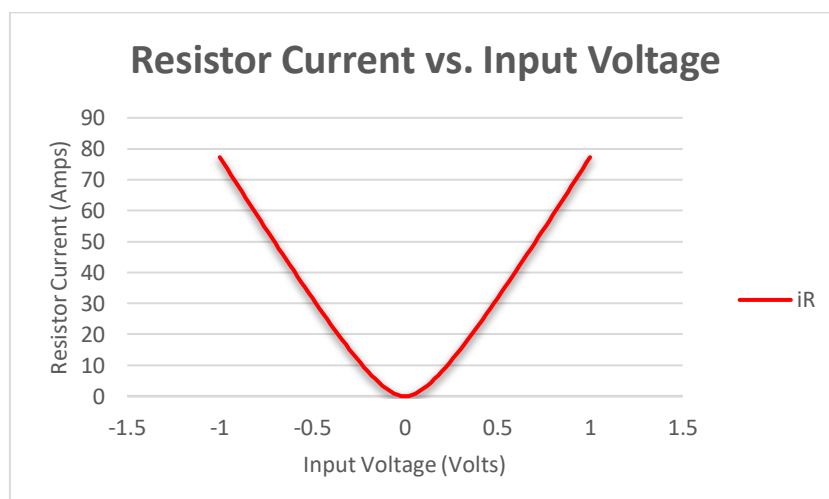
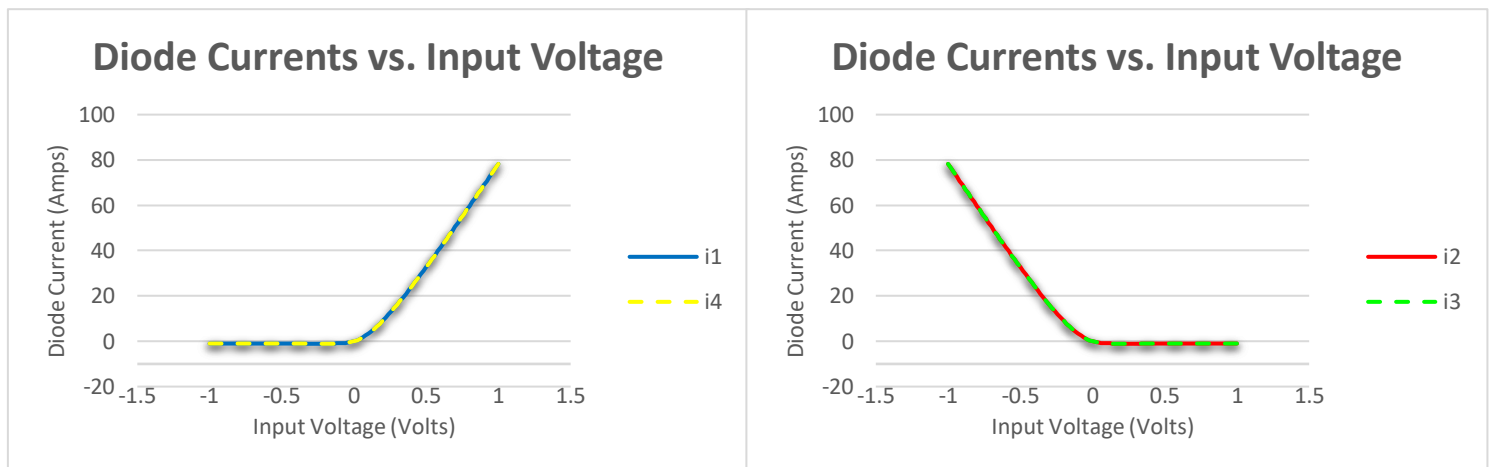
3.2. Plot

3.2.1. Problem 1.

- Voltages

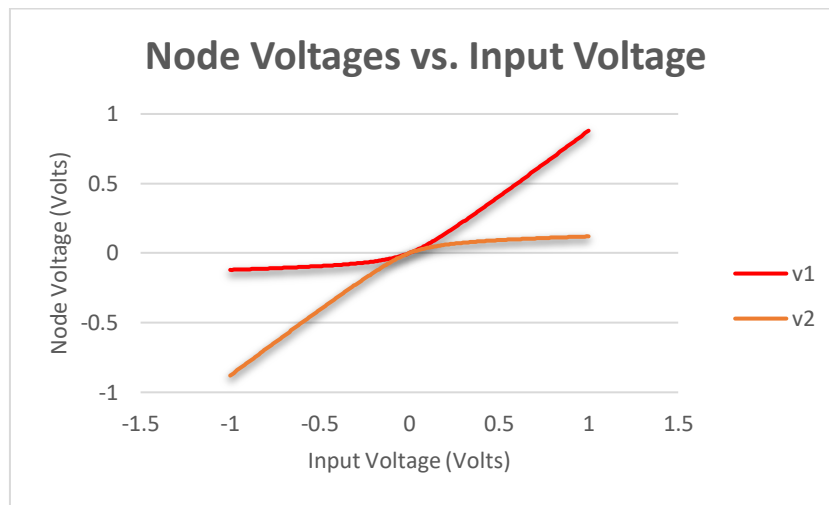


- Currents

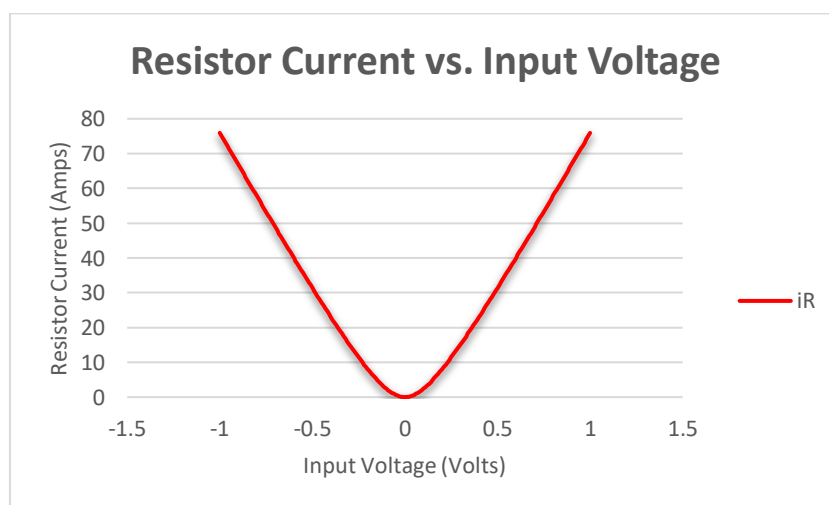
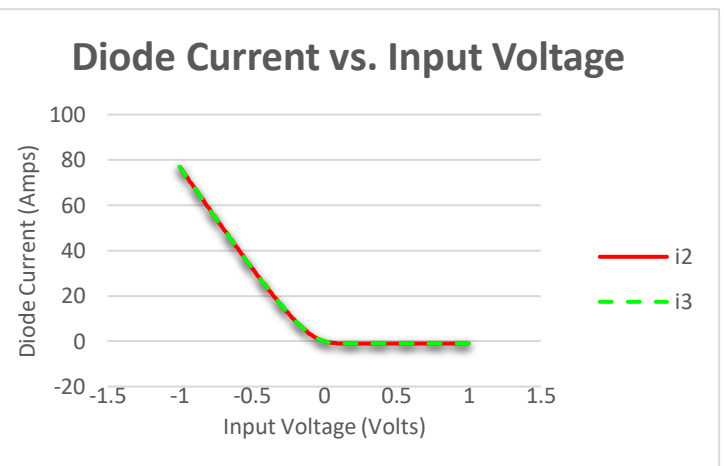
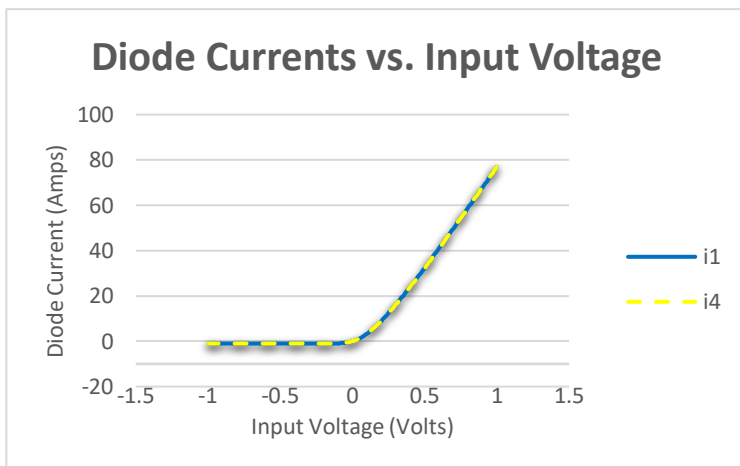


3.2.2. Problem 2.

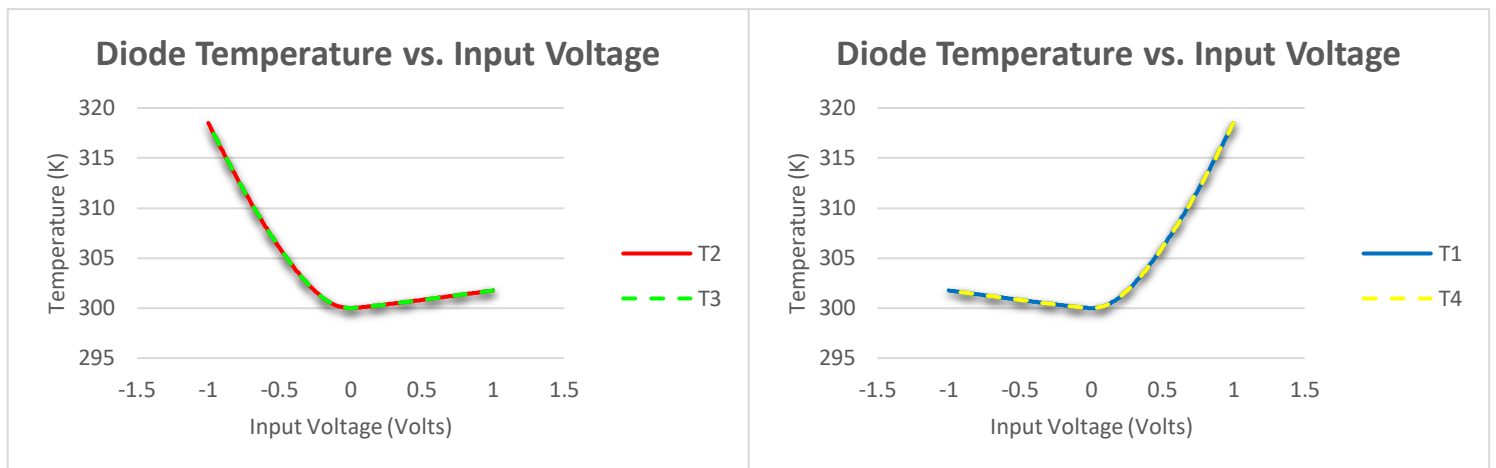
- Voltages



- Currents



- Temperature



4. Observations

- Both problem 1 and problem 2
 - The voltage of node v_1 and node v_2 have opposite trends.
 - The current and the temperature of diode 1 and diode 4 have same trends; the current and the temperature of diode 2 and diode 3 have same trends. The former and the latter have opposite trends.
 - The current trend of R_L is symmetric.