Numerical Analysis
Homework 6. Matrix Condition Numbers

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1. Objective

There are two parts in this homework. The first part analyzes the termination conditions for the *power method* that finds the largest eigenvalue of an nxn matrix A. The second part applies *power method*, *inverse power method* and *inverse power method with shifting* to find out the matrix condition numbers of different size of resistor networks.

2. Computational Complexity

- Only one matrix-vector multiplication is needed for each iteration → $O(n^2)$
- Since the power method takes N_{iter} iterations, the overall complexity is $O(N_{iter} * n^2)$.

3. Results

- **Part 1**

  Four different termination conditions.

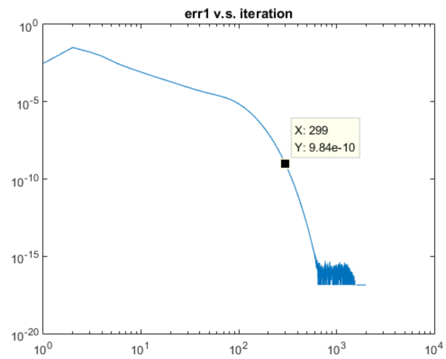
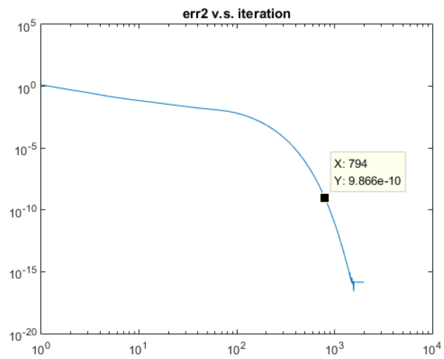
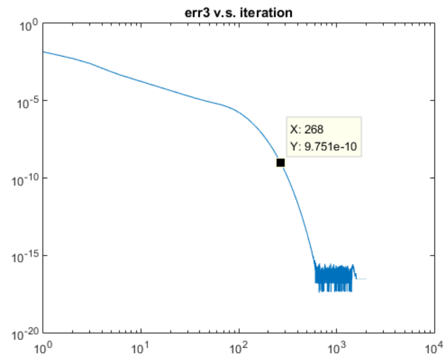
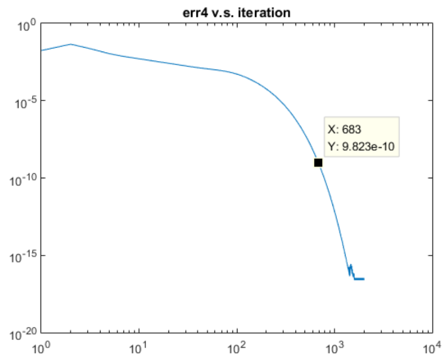
$$\epsilon_1 = \left|\lambda^{(k+1)} - \lambda^{(k)}\right|,$$

$$\epsilon_2 = \left\|q^{(k+1)} - q^{(k)}\right\|_2,$$

$$\epsilon_2 = \left\|r^{(k+1)}\right\|_2,$$

$$\epsilon_4 = \frac{\left\|r^{(k+1)}\right\|_2}{\left|(w^{(k)})^T q^{(k)}\right|}$$

Using power method, here are four different error vs. iterations.

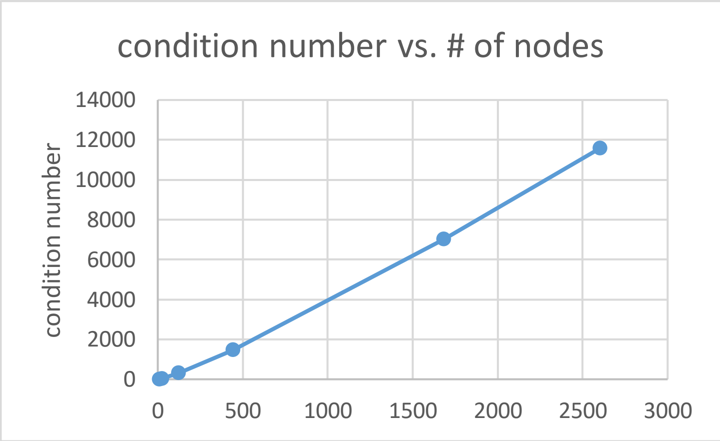
ϵ_1	ϵ_2
	
ϵ_3	**ϵ_4**
	

different error	iter when err smaller than 10^{-9}	time	time/iter
ϵ_1	299	0.00225	0.67275
ϵ_2	794	0.002297	1.823715
ϵ_3	268	0.002297	0.615596
ϵ_4	683	0.006875	4.695625

After analyzing each termination conditions, I prefer using ϵ_1 which is the difference of the calculated eigenvalues. The reason is that ϵ_1 has the least CPU time computing the eigenvalue of these resistor networks in this homework.

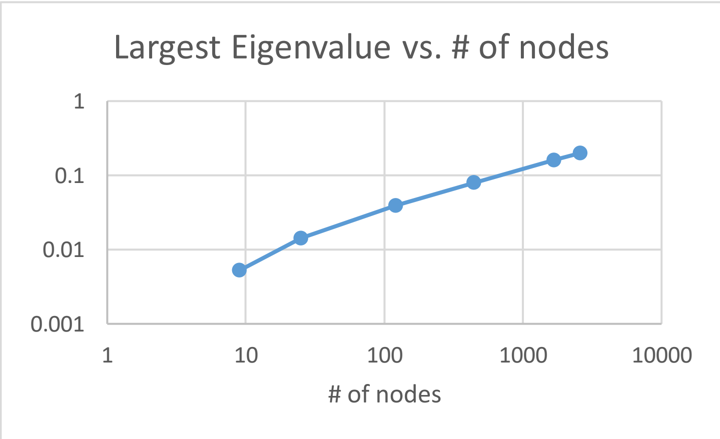
- **Part 2**
  - **Condition numbers**

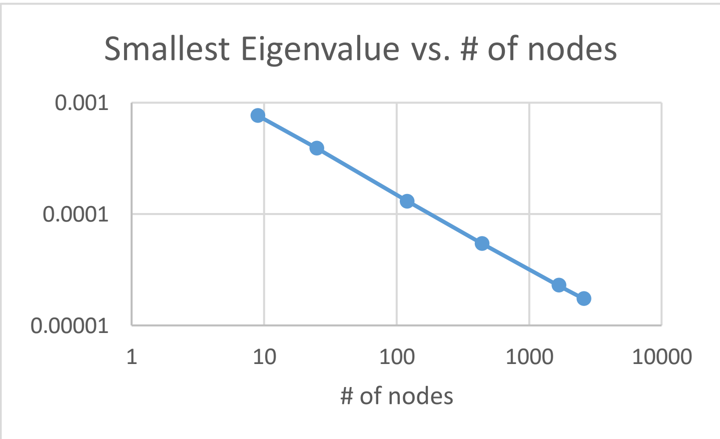
# of nodes	9	25	121	441	1681	2601
condition number	6.854104816	36.50760292	302.4332798	1467.204739	7016.86965	11574.26448



The condition number grows linearly as number of nodes increase.

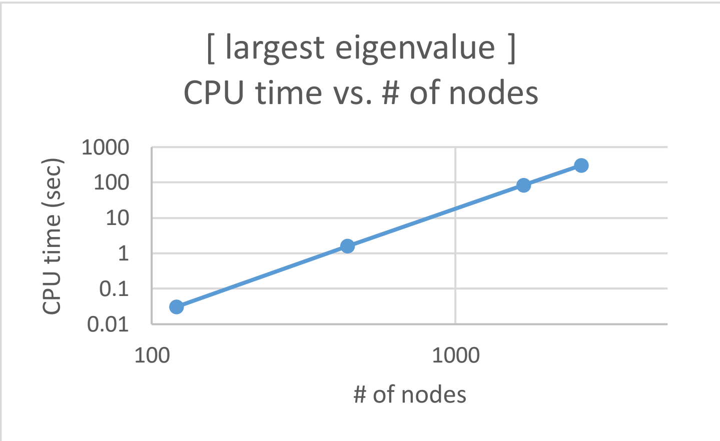
  - **Eigenvalues**

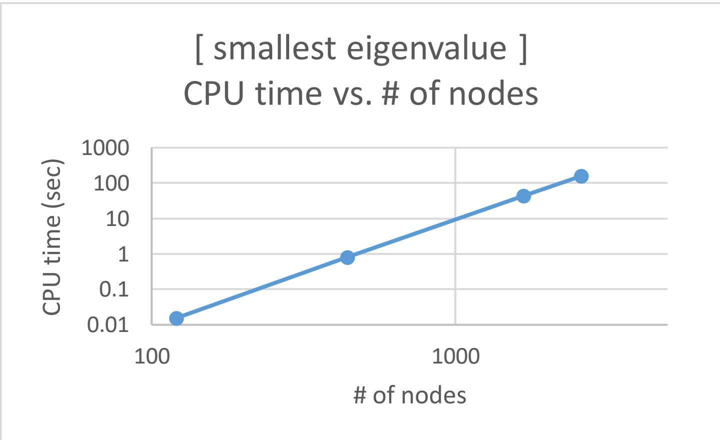




The largest eigenvalue increases exponentially and the smallest eigenvalue decreases exponentially.

■ **CPU time**





The CPU time to find either largest or smallest eigenvalue increases exponentially.

4. Conclusion

- CPU time matches the computational complexity.
- Using inverse power method with shifting, if the "shift" is properly positioned, the eigenvalue close to the "shift" will be found quickly.
- Initial eigenvector guess $q^{(0)}$ can affect the convergence rate, iterations to reach the termination condition, and the total time.