Numerical Analysis

Homework 12. RLC Circuit.

Due: May 23, 2017

An RLC circuit is shown below.

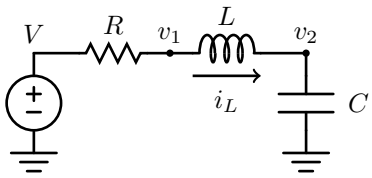

Figure 1. An RLC circuit.

This circuit has 3 variables, v_1, v_2 and i_L, where i_L is the inductor current. Using these 3 variables, the system of equation can be shown to be:

$$\begin{aligned} \frac{v_1 - V}{R} + i_L &= 0, \\ \frac{\mathrm{d}v_2}{\mathrm{d}t} &= \frac{i_L}{C}, \\ \frac{\mathrm{d}i_L}{\mathrm{d}t} &= \frac{v_1 - v_2}{L}. \end{aligned} \tag{12.1}$$

Assuming (1) $V(t) = 1$ for all t, (2) at $t = 0$, $v_1(0) = 1$, $v_2(0) = 0$, and $i_L(0) = 0$, (3) $R = 1$ Ω, $L = 1$ Henry, and $C = 1$ Farad.

1. Assuming the solution at time t is known, i.e., $v_1(t)$, $v_2(t)$, and $i_L(t)$ are given, please use Forward Euler method to derive the system solution to solve for $v_1(t + h)$, $v_2(t + h)$ and $i_L(t + j)$.
   - 1.1. Let $h = 0.1$ solve those three variables for $h \le t \le 10$.
     - 1.1.1. Plot $v_1(t)$, $v_2(t)$ and $i_L(t)$ for $0 \le t \le 10$.
     - 1.1.2. Find the minimums and maximums for those 3 variables for $0 \le t \le 10$.
   - 1.2. Let $h = 0.01$ solve those three variables for $h \le t \le 10$.
     - 1.2.1. Plot $v_1(t)$, $v_2(t)$ and $i_L(t)$ for $0 \le t \le 10$.
     - 1.2.2. Find the minimums and maximums for those 3 variables for $0 \le t \le 10$.
2. Assuming the solution at time t is known, i.e., $v_1(t)$, $v_2(t)$, and $i_L(t)$ are given, please use Backward Euler method to derive the system solution to solve for $v_1(t + h)$, $v_2(t + h)$ and $i_L(t + j)$.
   - 2.1. Let $h = 0.1$ solve those three variables for $h \le t \le 10$.
     - 2.1.1. Plot $v_1(t)$, $v_2(t)$ and $i_L(t)$ for $0 \le t \le 10$.

2.1.2. Find the minimums and maximums for those 3 variables for $0 \le t \le 10$.

2.2. Let $h = 0.01$ solve those three variables for $h \le t \le 10$.

2.2.1. Plot $v_1(t)$, $v_2(t)$ and $i_L(t)$ for $0 \le t \le 10$.

2.2.2. Find the minimums and maximums for those 3 variables for $0 \le t \le 10$.

3. Assuming the solution at time t is known, i.e., $v_1(t)$, $v_2(t)$, and $i_L(t)$ are given, please use Trapezoidal method to derive the system solution to solve for $v_1(t+h)$, $v_2(t+h)$ and $i_L(t+j)$.

3.1. Let $h = 0.1$ solve those three variables for $h \le t \le 10$.

3.1.1. Plot $v_1(t)$, $v_2(t)$ and $i_L(t)$ for $0 \le t \le 10$.

3.1.2. Find the minimums and maximums for those 3 variables for $0 \le t \le 10$.

3.2. Let $h = 0.01$ solve those three variables for $h \le t \le 10$.

3.2.1. Plot $v_1(t)$, $v_2(t)$ and $i_L(t)$ for $0 \le t \le 10$.

3.2.2. Find the minimums and maximums for those 3 variables for $0 \le t \le 10$.

4. Please state your observations.

Notes.

1. For this homework you need to turn in a set of C++ source codes. That includes `hw12.cpp`, which solves question 3 above, `MAT.h`, `MAT.cpp`, `VEC.h` and `VEC.cpp` files.
2. A `pdf` file is also needed. Please name this file `hw12a.pdf`.
3. Submit your files on EE workstations. Please use the following command to submit your homework 12.

```
$ ~ee407002/bin/submit hw12 hw12a.pdf hw12.cpp MAT.h MAT.cpp VEC.h VEC.cpp
```

   where `hw12` indicates homework 12.

4. Your report should be clearly written such that I can understand it. The writing, including English grammar, is part of the grading criteria.