

Unit 0.2 C++ Review

Numerical Analysis

EE/NTHU

Feb. 13, 2017

Mathematical Operations in C and C++

- In **C** or **C++** we have the following basic numerical types defined already.
 - Integer types: **int**, **short**, **long**, **unsigned**, **singed**, etc.
 - Floating-point types: **float**, **double**, **long double**.
- Using these basic types, mathematical expressions can be coded using simple operators.
 - For example, to evaluate

$$y = \frac{x^2 + 1}{x^3 + 2x^2 + 3x + 4}.$$

- One simply writes

```
y=(x*x+1)/(x*x*x+2*x*x+3*x+4);
```

- Thus, simple mathematical expressions can be coded with ease.
- This is because the operators associated with the basic types have been defined.

Needs of Vector and Matrix Operations

- In this course, we will need to solve linear and nonlinear systems.
- Often these systems are expressed by matrices and vectors.
- Solution methods are often expressed using matrices and vectors as well.
- The vectors and matrices are not basic types in **C** or **C++**.
- And no mathematical operations defined.
- For example, given an $n \times n$ matrix **A**, an n -vector **x** and a real number λ , if λ is an eigenvector of **A** and **x** is the associated eigenvector, then

$$\mathbf{Ax} - \lambda\mathbf{x} = \mathbf{0}. \quad (0.2.1)$$

- Vectors are usually defined as 1-dimensional array in **C** or **C++**; while matrices defined as 2-dimensional array. To check if Eq. (0.2.1) is satisfied, multiple loops are needed.
- Fortunately, using **class** in **C++** one can defined mathematical operators to match mathematical equations.
- Translating numerical theories to program becomes an easy job.

Classes in C++

- Class in **C++** is designed to provide programmers with a tool for creating new types that can be used as conveniently as the basic types.
- It needs to define two aspects: data stored and the operations associated with.
 - **Data members** to store necessary data for the new type.
 - **Function members** to define the operations.
- Data members are similar to **struct** in **C**.
 - Storage for any predefined types can be declared.
- The members of a class are further classified into two categories:
 - **public**: members can be accessed by any functions,
 - **private**: members can only be accessed by member functions.
- Using private data, data hiding is made possible.
 - Clear responsibility,
 - Easier debugging,
 - Enable better teamwork, etc.

Class Example

```
class Complex {
public:
    Complex(double r=0,double i=0);    // constructor
    Complex(const Complex &C);        // copy constructor
    double r() const;                  // get real part
    double i() const;                  // get imaginary part
    Complex& operator+= (Complex &C2); // C1 += C2;
    Complex& operator+= (double);      // C1 += dbl;
    Complex& operator-= (Complex &C2); // C1 -= C2;
    Complex& operator-= (double);      // C1 -= dbl;
    Complex& operator*= (Complex &C2); // C1 *= C2;
    Complex& operator*= (double);      // C1 *= dbl;
    Complex& operator/= (Complex &C2); // C1 /= C2;
    Complex& operator/= (double);      // C1 /= dbl;
private:
    double x,y;
};
```

Classes in C++, II

- Data members can be divided into **public** and **private**.
 - Data accessible by all functions should be declared **public**,
 - Other data declared **private**.
- Member functions can have public and private as well.
 - Most member functions are public to define operations on the class.
 - Private functions can be used by member functions only.
 - Utility functions.
- The **struct** in **C** is a special case of class in **C++**.
 - It has only public members.
 - And no member functions.
- In declaring a class, the member function should be declared.
- All members (data and function) are accessed through the member operator `.` or `->`.
 - Accessing data: `z1.x`, `z1.y`,
 - Accessing function: `z1.r()`, `z1.i()`,
 - If `zp` is a pointer to `z`, then
 - Accessing data: `zp->x`, `zp->y`,
 - Accessing function: `zp->r()`, `zp->i()`.

Member Function Accessing

- In **C** or **C++**, a function, f , is invoked by
 - $f(x)$, calling f with one argument.
 - $f(x1, x2)$, calling f with two arguments.
 - $f()$, calling f with no argument.
 - The parentheses `()` are needed to signify a function call.
- When calling a member function, (assuming z is an **Complex** object)
 - `z.r()` or `z.i()`.
 - Parentheses are still needed to signify a function call.
 - There is an implicit argument supplied, `this`.
 - `this` is a pointer pointing to the object.
 - In `z.r()`, `this = &z`.
 - In the function definition of `r()`, data member can be accessed by the name of the data or with `this->` preceding the name of the data.
 - For example, the member function `r()` is defined as

```
double Complex::r()  
{  
    return x;  
}
```

```
double Complex::r()  
{  
    return this->x;  
}
```

Member Function Definition

- To define a member function, the name should be preceded with `class name::`, as the examples shown above.
 - The `return type` needs to precede the `class name`.
 - Note that in **C++**, the return type should always be defined.
- Two special functions need no return type.
 - **constructor** and **destructor** functions.
- In **C** or **C++**, a variable is created by a declaration statement.
 - For example, two variables are declared

```
double x1, x2;
```

- In **C++**, an object is instantiated by declaration as

```
Complex z1;
```

When this declaration statement is executed, the constructor function is actually called such that the data member can be initialized (or not) properly.

Constructors

- Three types of constructors are possible (using Complex class as an example).
 - Initialization constructor:
`Complex z1(1.0,1.0); // initializing z1 to (1.0,1.0)`
 - Uninitialization constructor, two usages:
`Complex z2; // uninit constructor`
`Complex z2(); // uninit constructor`
 - Copy constructor, two usages:
`Complex z3=z1; // copy constructor`
`Complex z3(z1); // copy constructor`
- Each constructors need to declared and defined separately.

```
Complex::Complex(double r,double i) // init constructor
{
    x=r; y=i;
}
Complex::Complex() // uninit constructor, no actions required
{ }
Complex::Complex(const Complex &z) // copy constructor
{
    x=z.x; y=z.y;
}
```

Constructors, II

- Note that the name of the constructor functions are simply the name of the class.
 - In defining the member function, class name and scope operator `::` need to precede function name.
- Constructor functions need no return type.
- Three functions defined with the same name – **function overload**.
 - The number of arguments or the argument types need to be different.
- In **C++**, function arguments can have default values.

```
Complex::Complex(double r=0,double i=0) // init constructor
{
    x=r; y=i;
}
```

- In this case, when parameters are not supplied in function calls, the default values are taken.

```
Complex z1(1.0,1.0); // initialize both real and imaginary
Complex z2(1.0); // z2=(1.0,0)
Complex z3(); // z3=(0,0)
```

- In this case, the uninit constructor should not be declared or defined.

Destructor

- **Destructor** function is called when a variable is no longer needed.
 - Local variables (automatic storage duration) going out of scope.
 - Temporary variables no longer needed.

```
z4=z1+z2*z3;  
z5=sqrt(z6);
```

A temporary variable may be created in forming the product $z2*z3$, and returned by a function before assignment.

- Compiler generates calls to destructor functions.
- In most cases, destructor needs no actions.
 - But, if dynamic memory is allocated by constructor then destructor should release the memory space acquired.
- The name of destructor function is `~` preceding the class name.
- Destructor has no return type.

```
Complex::~Complex()  
{ } // no action needed
```

References

- In **C**, function parameters are passed by value.
 - A copy of the variable is created and passed to the function.
 - The return value of a function is also a copy that is destroyed after its use.
 - Thus, the value of the parameter cannot be modified by a function.
 - Pointer should be passed, if its value is to be modified.
- In **C++**, the variable itself can be passed and returned by a function – [pass-by-reference](#).
- To do so, the parameter needs to be declared with a reference symbol `&` after the type name.
- Example member function declaration:

```
Complex& operator+= (Complex &C2); // C1 += C2;
```

- Example member function definition:

```
Complex & Complex::operator+=(Complex &z)  
{  
    x+=z.x;  
    y+=z.y;  
    return *this;  
}
```

References, II

- Using a referenced parameter or return, the variable itself is passed between functions.
 - No copies are created – improved efficiency.
 - Variable can be modified.
- Reference can be used to create an alias for a variable.

```
Complex z1;  
Complex &za=z1;
```

The variable za is an alias of z1. Modifying za changes the value of z1, vice versa.

- Note that reference can only be created for variables that are existing physically. Temporary variables created during expression evaluation may not have references.

```
z4=z1+z2*z3;
```

- The temporary variable created for $z2*z3$ may not be physically present and, thus, no reference possible.

const

- Using reference can increase program efficiency by eliminating the need of copying function arguments repeatedly.
- The function is also able to access the variables with the capability of modifying their values.
- To clearly state that a function argument will not be modified, the `const` key word should be used.

```
Complex& operator+= (const Complex &C2); // C1 += C2;
```

- With `const` inserted, the variable `C2` cannot be modified.
- Any attempts of modifying `C2` will result in a compilation error.
- Note that the copy constructor should have the argument declared as a `const reference` of the class type.

```
class Complex {  
    // ...  
    Complex(const Complex &C);    // copy constructor  
    // ...  
}
```

- Note that `const` can be used for any argument, not necessary of reference type, with the same effect.
- A member function of a class that will not modify any of its data members can be declared to be a `const` member function.

```
class Complex {
    // ...
    double r() const;           // return real part
    double i() const;           // return imaginary part
    // ...
}
double Complex:r() const
{
    return x;
}
```

- Note that in function definition, the `const` key word needs to appear after the parentheses as well.

Defining Operators for Classes

- In `C++`, most operators can be defined for classes

`+` `-` `*` `/` `%` `^` `&` `|` `~` `!` `=` `<` `>` `+=` `-=` `*=` `/=` `%=`
`^=` `&=` `|=` `<<` `>>` `>>=` `<<=` `==` `!=` `<=` `>=` `&&` `||` `++` `--`
`->*` `,` `->` `[]` `()` `new` `new[]` `delete` `delete[]` .

- The following operators **cannot** be redefined.
 - `::` scope resolution,
 - `.` member selection,
 - `.*` member selection through pointer to member,
 - `? :` ternary condition expression,
 - `sizeof`,
 - `typeid`.
- The first 3 operators take a name, rather than value, as the second operand.

Defining Operators for Classes, II

- In **C++**, the operators are functions.
 - They have the name `operator` preceding the `operator symbol`.
 - For example, `operator+`.
- If `z` is an object of class `Complex` and `operator+` function is defined as a member function of class `Complex`, then it is possible to use both shorthand or explicit operator function call.
- If `operator+` is defined as a **member** function of `z1`.

```
z3=z1+z2;  
z3=z1.operator+(z2);
```

- If `operator-` is defined as a **nonmember** function, then the following are equivalent.

```
z3=z1-z2;  
z3=operator-(z1,z2);
```

- Shorthand operator symbols are usually preferred.

Defining Operators for Classes, III

- In defining an operator function as a **member** function, it should be **declared** in the class declaration as well, and then **defined** with the class name preceding the function name.

```
class Complex {                                // class declaration  
public:  
    // ...  
    Complex operator+(Complex C);             // + function declaration  
    // ...  
};  
Complex Complex::operator+(Complex C)         // + function definition  
{ ... }
```

- In defining operator function as **nonmember** function, it follows regular function declaration and definition.

```
Complex operator-(Complex C1,Complex C2);     // - function declaration  
  
Complex operator-(Complex C1,Complex C2)      // - function definition  
{ ... }
```

Defining Operators for Classes, IV

- Note in the preceding function declaration and definition, the addition of `z1+z1` must have both operands of type `Complex`.
- If `z1` or `z2` is not `Complex`, then the addition is not defined.
- To handle the case that `z2` is a `double`, the function needs to be overloaded by declare and define the following:

```
class Complex {                                // class declaration
public:
    // ...
    Complex operator+(double dbl);            // + can handle Complex + dbl
    // ...
};
Complex Complex::operator+(double dbl) // + function definition
{ ... }
```

- But the case of `double z1` can only be handled by declaring a `nonmember` function.

```
Complex operator+(double dbl, Complex C);
```

- This is due to that the first operand of the operator defined as member function must be an implicit argument, which has the same type as the class.

friend of a Class

- In `C++`, private members cannot be accessed by nonmember functions.
- Exceptions can be made by declaring a function as a friend.
 - `friend` functions can access private members (data and functions).
 - `friend` functions must be declared in class declaration.

```
class Complex {                                // class declaration
public:
    // ...
    friend Complex sqrt();                    // + function declaration
    // ...
};
```

- Note that `sqrt()` is a nonmember function but it can access class `Complex`'s private data.
- Properties of friendship.
 - Friendship is granted not taken.
 - Friendship is not symmetric.
 - Friendship is not transitive.

Header Files and Source Files

- A well defined class can be reused for many applications.
- It is a good practice to put the declarations associated with a class into a header file that ends with `.h` file extension.
- The function definitions, member and nonmember, are put into a separated source file that ends with `.cpp` file extension.
- The source file can then be compiled by

```
$ g++ -c Complex.cpp
```

This produces a `Complex.o` object file that can be linked with application programs.

```
$ g++ prog.cpp Complex.o
```

- This practice saves compilation time and the header file serves as a well defined interface for the users.
- As long as the function declarations are not changed, modifying the the function definitions in `Complex.cpp` file does not affect the application source file.
 - But the program needs to be re-linked.

Complex.h (1/2)

```
// complex number class
#ifndef COMPLEX_H
#define COMPLEX_H
#include <stdio.h>
#include <stdlib.h>
class Complex {
public:
    Complex(double r=0,double i=0);           // constructor
    Complex(const Complex &C);                 // copy constructor
    double r() const;                          // get real part
    double i() const;                          // get imaginary part
    Complex& operator+= (Complex&);            // C1 += C2;
    Complex& operator+= (double);              // C1 += dbl;
    Complex& operator*= (Complex&);            // C1 *= C2;
    Complex& operator*= (double);              // C1 *= dbl;
    Complex& operator/= (Complex&);            // C1 /= C2;
    Complex& operator/= (double);              // C1 /= dbl;
private:
    double x,y;
};
```

Complex.h (2/2)

```
Complex operator+(Complex);           // unary plus
Complex operator+(Complex,Complex);   // C1 + C2
Complex operator+(Complex,double);    // C1 + dbl
Complex operator+(double,Complex);    // dbl + C1
Complex operator-(Complex);           // unary minus
Complex operator-(Complex,Complex);   // C1 - C2
Complex operator-(Complex,double);    // C1 - dbl
Complex operator-(double,Complex);    // dbl - C1
Complex operator*(Complex,Complex);   // C1 * C2
Complex operator*(Complex,double);    // C1 * dbl
Complex operator*(double,Complex);    // dbl * C1
Complex operator/(Complex,Complex);   // C1 / C2
Complex operator/(Complex,double);    // C1 / dbl
Complex operator/(double,Complex);    // dbl / C1
double fabs(Complex z);               // |C1|
Complex& operator++(Complex&);        // prefix increment
Complex operator++(Complex&,int);     // postfix increment
int operator==(Complex,Complex);      // testing for equality
int operator!=(Complex,Complex);
#endif
```

Complex.cpp (1/4)

```
// Function definitions
#include <math.h>
#include "Complex.h"
Complex::Complex(double r=0,double i=0) // init constructor
{
    x=r; y=i;
}
Complex::Complex(const Complex &z)      // copy constructor
{
    x=z.x; y=z.y;
}
double Complex::r() const              // get real part
{
    return x;
}
double Complex::i() const              // get imaginary part
{
    return y;
}
```


Complex.cpp (2/4)

```
Complex & Complex::operator+=(Complex &z)    // C += Z;
{
    x+=z.x;
    y+=z.y;
    return *this;
}
Complex & Complex::operator+=(double d1)    // C += double;
{
    x+=d1;
    return *this;
}
Complex operator+(Complex z)                // unary plus
{
    Complex z1(z);
    return z1;
}
Complex operator+(Complex z1,Complex z2)    // z1 + z2
{
    Complex z(z1);
    return z+=z2;
}
```

Complex.cpp (3/4)

```
Complex operator+(Complex z1,double d)    // z1 + double
{
    Complex z(z1);
    return z+=d;
}
Complex operator+(double d,Complex z1)    // double + z1
{
    Complex z(z1);
    return z+=d;
}
double fabs(Complex z)                    // |z|
{
    return sqrt(z.r()*z.r()+z.i()*z.i());
}
Complex &operator++(Complex &z1)          // prefix increment
{
    return z1+=1.0;
}
```

```
Complex operator++(Complex &z1,int)           // postfix increment
{
    Complex z(z1);
    ++z1;
    return z;
}
int operator==(Complex z1,Complex z2)        // test for equality
{
    if (z1.r()==z2.r() && z1.i()==z2.i()) return 1;
    return 0;
}
```

- With these example functions, all functions defined in [Complex.h](#) can be easily implemented.
- And coding for complex number expressions are very straightforward.
- Try to code the following expressions.

$$f(z) = \frac{3z^3 + 2z^2 + z + 1}{z^4 - z^3 + z^2 - z + 1},$$
$$g(z) = (z + 1)(z + 2)(z + 3).$$

Summary

- Needs of classes
- Class in C++
- Member functions
- Constructor
- Destructor
- References
- Const
- Defining operator functions
- Friends
- Header and source files
- Example: Complex class

