**Numerical Analysis**

**Homework 6. Matrix Condition Numbers**

***102061125 陳冠鈞***

1. **Objective**

There are two parts in this homework. The first part analyzes the termination conditions for the *power method* that finds the largest eigenvalue of an nxn matrix A. The second part applies *power method*, *inverse power method* and *inverse power method with shifting* to find out the matrix condition numbers of different size of resistor networks.

1. **Computational Complexity**

* Only one matrix-vector multiplication is needed for each iteration 🡪
* Since the power method takes iterations, the overall complexity is .

1. **Results**
   * **Part 1**

Four different termination conditions.

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Using power method, here are four different error vs. iterations.

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| --- | --- | --- | --- |
| different error | iter when err smaller than | time | time/iter |
|  | 299 | 0.00225 | 0.67275 |
|  | 794 | 0.002297 | 1.823715 |
|  | 268 | 0.002297 | 0.615596 |
|  | 683 | 0.006875 | 4.695625 |

After analyzing each termination conditions, I prefer using which is the difference of the calculated eigenvalues. The reason is that has the least CPU time computing the eigenvalue of these resistor networks in this homework.

* **Part 2**
  + **Condition numbers**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| # of nodes | 9 | 25 | 121 | 441 | 1681 | 2601 |
| condition number | 6.854104816 | 36.50760292 | 302.4332798 | 1467.204739 | 7016.86965 | 11574.26448 |

The condition number grows linearly as number of nodes increase.

* + **Eigenvalues**

The largest eigenvalue increases exponentially and the smallest eigenvalue decreases exponentially.

* + **CPU time**

The CPU time to find either largest or smallest eigenvalue increases exponentially.

1. **Conclusion**

* CPU time matches the computational complexity.
* Using inverse power method with shifting, if the “shift” is properly positioned, the eigenvalue close to the “shift” will be found quickly.
* Initial eigenvector guess can affect the convergence rate, iterations to reach the termination condition, and the total time.