Numerical Analysis

Project. Polynomial Roots Finder.

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1. Objective

Given a polynomial of degree n as the following

with all coefficients real, .

In this project, I want to find a method that can find all roots given a polynomial. There are three different methods will be discussed in this project.

1. Newton-Horner Method
2. Lin’s Quadratic Method
3. Bairstow’s Method

However, each of these three methods has their constraints, so I will go through some polynomials and see the results to compare them.

1. Approach
   1. Newton-Horner Method

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And

|  |  |
| --- | --- |
| ,  ,  ,  …  ,  . | ,  ,  ,  …  ,  . |

This process is called deflation, and the coefficients can be rewrite as recursive form

,

,

Now if we define

And when is a root of then .

Next, we need the derivative, , when using Newton’s method to find a root.

Then

Now we do deflation again, and we can get ,  
where is the value of the derivative.

And the solution can be solved in iterative form

|  |
| --- |
| **Algorithm.** Newton-Horner Method |
| Given an *n*-degree polynomial with coefficients , and initial guess , a small number *ε* and an integer *maxiter*, |

* 1. Lin’s Quadratic Method

Given the polynomial as before

We assume

In case that is a factor of , then

Comparing the coefficients, we have

|  |  |
| --- | --- |
| … | … |

Or in recursive form can be found by

,

,

.

If is a factor of

, .

, .

, .

Once the quadratic factor, , is found then the real roots or the complex conjugates can be calculated quickly.

can e deflated again to the polynomial. The same process can be carried out for the next factors.

* 1. Bairstow’s Method

Given the polynomial as before

We assume

…

To apply Newton’s method, we need to find , to form the iterations.

Thus we have

|  |  |
| --- | --- |
| … | … |

Let and , then we have

|  |  |
| --- | --- |
| … | … |

Thus, to find a quadratic factor of an n degree polynomial, we have the following algorithm.

|  |
| --- |
| **Algorithm.** Bairstow’s Method |
| Given , and integer maxiter and a small number ε,  Let |

1. Results
   1. Order of Convergence

|  |  |  |
| --- | --- | --- |
|  | Simple roots: 1, 2, 3. | |
| Plot | Order of convergence |
| Newton-Horner Method  initial guess: |  | 2 |
| Lin’s Quadratic Method  initial guess:  , |  | 1 |
| Bairstow’s Method  initial guess:  , |  | 2 |

Table 1.

|  |  |  |
| --- | --- | --- |
|  | Double roots: 1, 1. Simple root: 2. | |
| Plot | Order of convergence |
| Newton-Horner Method  initial guess: |  | 1 |
| Lin’s Quadratic Method  initial guess:  , |  | 1 |
| Bairstow’s Method  initial guess:  , |  | 2 |

Table 2.

|  |  |  |
| --- | --- | --- |
| Order of convergence |  |  |
| Simple roots: 1, 2, 3. | Double roots: 1, 1. Simple root: 2. |
| Newton-Horner Method | 2 | 1 |
| Lin’s Quadratic Method | 1 | 1 |
| Bairstow’s Method | 2 | 2 |

Table 3.

* 1. Roots and Error

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Simple roots: 1, 2, 3. | | Double roots: 1, 1. Simple root: 2. | |
| roots | error | roots | error |
| Newton-Horner Method  initial guess: | 1-1.45948e-23i | 2.22E-16 | 1+2.48856e-06i | 7.44E-12 |
| 2+3.04679e-12i | 7.08E-12 | 1-4.97708e-06i | 2.98E-11 |
| 3-1.52339e-12i | 7.07E-12 | 2-2.78168e-11i | 3.72E-11 |
| Lin’s Quadratic Method  initial guess:  , | 1 | 2.22E-16 | 1 | 1.82E-11 |
| 2 | 9.41E-10 | 0.999996 | 1.82E-11 |
| 3 | 1.88E-09 | 2 | 6.08E-10 |
| Bairstow’s Method  initial guess:  , | 1 | 0 | 1 | 0 |
| 2 | 0 | 1 | 0 |
| 3 | 1.47E-12 | 2 | 1.49E-13 |

Table 4.

1. Conclusion

These three methods, Newton-Horner Method, Lin’s Quadratic Method and Bairstow’s Method, all need initial guess, but guess for different thing.

* Newton-Horner Method needs initial guess for .
* Lin’s Quadratic Method needs initial guess for .
* Bairstow’s Method needs initial guess for .

With good initial guess, each method can find all roots of polynomials. However, when we go through the algorithm, there may have some problem.

For example, in Lin’s Quadratic Method algorithm

, ,

when becomes 0, and will go to infinity (or negative infinity).

Anothoer example, in Bairstow’s Method algorithm

,

if the determinant of is 0, and will go to infinity (or negative infinity).

As the result, when we find that the roots do not make sense, maybe we can fine-tune the initial guess to solve this problem.

Let’s look at table 2, table 3 and table 4:

* The **accuracy** of these three methods:

Bairstow’s Method > Newton-Horner Method > Lin’s Quadratic Method

* The **order of convergence** when polynomial contains only simple roots:

Bairstow’s Method = Newton-Horner Method > Lin’s Quadratic Method

* The **order of convergence** when polynomial contains double roots:

Bairstow’s Method > Newton-Horner Method = Lin’s Quadratic Method

By the results above, I think Bairstow’s Method is the better method to find all roots of polynomials.