Homework 6

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1 Theorem proving for 2.6.4

Theorem 1 *Theorem 2.6.4*
$(i) A \to B, A \vdash B$
$(ii) A \vdash B \to A$
$(iii) A \to B, B \to A \vdash A \to C$
$(iv) A \to (B \to C), A \to B \vdash A \to C$

Proof 1 *Proof of theorem 2.6.4*
$(i) A \to B, A \vdash B$
Proof:
$(1) A \to B, A \vdash A \quad (\in)$
$(2) A \to B, A \vdash A \to B \quad (\in)$
$(3) A \to B, A \vdash B \quad (\to -)$

$(ii) A \vdash B \to A$
Proof:
$(1) A, B \vdash A \quad (\in)$
$(2) A \vdash B \to A \quad (\to +)$

$(iv) A \to (B \to C), A \to B \vdash A \to C$
Proof:
$(1) A \to (B \to C), A \to B, A \vdash A \to (B \to C) \quad (\in)$
$(2) A \to (B \to C), A \to B, A \vdash A \quad (\in)$
$(3) A \to (B \to C), A \to B, A \vdash (B \to C) \quad (\to -, (1), (2))$
$(4) A \to (B \to C), A \to B, A \vdash A \to B \quad (\in)$
$(5) A \to (B \to C), A \to B, A \vdash B \quad (\to -, (2), (4))$
$(6) A \to (B \to C), A \to B, A \vdash C \quad (\to -, (3), (5))$
$(7) A \to (B \to C), A \to B \vdash A \to C \quad (\to +, (6))$

2 Theorem proving for 2.6.9

Theorem 2 *Theorem 2.6.9*
$(i) A \vdash A \lor B, B \lor A$
$(ii) A \lor B \vdash\dashv B \lor A$
$(iii) (A \lor B) \lor C \vdash\dashv A \lor (B \lor C)$
$(iv) A \lor B \vdash\dashv \neg A \to B$
$(v) A \to B \vdash\dashv \neg A \lor B$
$(vi) \neg(A \lor B) \vdash\dashv \neg A \land \neg B$
$(vii) \neg(A \land B) \vdash\dashv \neg A \lor \neg B$
$(viii) \emptyset \vdash A \lor \neg A$

Proof 2 *Proof of theorem 2.6.9*
$(i) A \vdash A \lor B, B \lor A$
Proof:

$(1) A \vdash A \quad (\in)$
$(2) A \vdash A \lor B, B \lor A \quad (\lor+)$

$(ii) A \lor B \vdash\dashv B \lor A$
First we proof: $AlorB \vdash B \lor A$

$(1) A \lor B, A \vdash A \quad (\in)$
$(2) A \lor B, A \vdash B \lor A \quad (\lor+)$
Same reason we can proof that $(3) A \lor B, B \vdash B \lor A \quad (\lor+)$
$(4) A \lor B, A \lor B \vdash B \lor A \quad (\lor-)$

Then when we replace all A *in the proof above with* B *and all* B *in the proof above with* A
We will get the proof of $B \lor A \vdash A \lor B$
So, $A \lor B \vdash\dashv B \lor A$

$(iii) (A \lor B) \lor C \vdash\dashv A \lor (B \lor C)$
First we proof:$(A \lor B) \lor C \vdash A \lor (B \lor C)$

$(1) A \vdash A \lor (B \lor C) \quad (\lor+)$
$(2) B \vdash B \quad (\in)$
$(3) B \vdash B \lor C \quad (\lor+)$
$(4) B \vdash A \lor (B \lor C) \quad (\lor+)$

same reason we can get that $(5) C \vdash A \lor (B \lor C) \quad (\lor+)$

$(6) A \lor B \vdash A \lor (B \lor C) \quad (\lor-, (1), (4))$
$(7) (A \lor B) \lor C \vdash\dashv A \lor (B \lor C) \quad (\lor-, (5), (6))$

Then we proof:$A \lor (B \lor C) \vdash (A \lor B) \lor C$

$(1) C \vdash (A \lor B) \lor C \quad (\lor+)$
$(2) B \vdash B \quad (\in)$
$(3) B \vdash A \lor B \quad (\lor+)$
$(4) B \vdash (A \lor B) \lor C \quad (\lor+)$

same reason we can get that $(5) A \vdash (A \lor B) \lor C \quad (\lor+)$

$(6) B \lor C \vdash (A \lor B) \lor C \quad (\lor-, (1), (4))$
$(7) A \lor (B \lor C) \vdash\dashv (A \lor B) \lor C \quad (\lor-, (5), (6))$

Finally we get that $(A \lor B) \lor C \vdash\dashv A \lor (B \lor C)$

$(iv) A \lor B \vdash\dashv \neg A \to B$
First we proof: $A \lor B \vdash \neg A \to B$

$(1) A, \neg A, \neg B \vdash A \quad (\in)$
$(2) A, \neg A, \neg B \vdash \neg A \quad (\in)$
$(3) A, \neg A \vdash B \quad (\neg-)$
$(4) A \vdash \neg A \to B \quad (\to +)$
$(5) B \vdash \neg A \to B \quad (Theorem 2.6.4(ii))$
$(6) A \lor B \vdash \neg A \to B \quad (\lor-, (4), (5))$

Then we proof: $\neg A \to B \vdash A \lor B$

$(1) A \vdash A \lor B$ *(Theorem 2.6.9 (i))*
$(2) \neg(A \lor B) \vdash \neg A \quad (Theorem 2.6.6(v), (1))$
$(3) \neg A \to B, \neg(A \lor B) \vdash \neg A \quad (+, (2))$
$(4) \neg A \to B, \neg(A \lor B) \vdash \neg A \to B \quad (\in)$
$(5) \neg A \to B, \neg(A \lor B) \vdash B \quad (\to -, (3), (4))$
$(6) \neg A \to B, \neg(A \lor B) \vdash A \lor B \quad (\lor+)$

Finally we get that $A \lor B \vdash\dashv \neg A \to B$

$(v) A \to B \vdash\dashv \neg A \lor B$

Replace all A in the proof of (iv) with $\neg A$ *, then we get the proof of formula (v)*

$(vi)\neg(A \vee B) \vdash\dashv \neg A \wedge \neg B$
First we proof: $\neg(A \vee B) \vdash \neg A \wedge \neg B$
$(1) A \vdash A \vee B \quad (Theorem 2.6.9(i))$
$(2)\neg(A \vee B) \vdash \neg A \quad (Theorem 2.6.6(v),(1))$
$(3) B \vdash A \vee B \quad (Theorem 2.6.9(i))$
$(4)\neg(A \vee B) \vdash \neg B \quad (Theorem 2.6.6(v),(3))$
$(5)\neg(A \vee B) \vdash \neg A \wedge \neg B \quad (\wedge+)$
Then we proof: $\neg A \wedge \neg B \vdash \neg(A \vee B)$
$(1)\neg A \wedge \neg B, A \vee B \vdash \neg A \wedge \neg B \quad (\in)$
$(2)\neg A \wedge \neg B, A \vee B \vdash \neg A \quad (\wedge-)$
$(3)\neg A \wedge \neg B, A \vee B \vdash \neg B \quad (\wedge-)$
$(4) A \vee B \vdash\dashv \neg A \to B \quad (Theorem 2.6.9(iv)$
$(5)\neg A \wedge \neg B, A \vee B \vdash\dashv \neg A \to B \quad (+,(4))$
$(6))\neg A \wedge \neg B, A \vee B \vdash\dashv B \quad (\to -)$
$(7)\neg A \wedge \neg B \vdash \neg(A \vee B) \quad (\neg+)$
Finally we get that $\neg(A \vee B) \vdash\dashv \neg A \wedge \neg B$

$(vii)\neg(A \wedge B) \vdash\dashv \neg A \vee \neg B$
First we proof: $\neg(A \wedge B) \vdash \neg A \vee \neg B$
$(1)\neg(\neg A \vee \neg B) \vdash \neg\neg A \wedge \neg\neg B \quad (Theorem 2.6.9(vi))$
$(2)\neg(\neg A \vee \neg B) \vdash \neg\neg A \quad (\wedge-)$
$(3)\neg\neg A \vdash A \quad (Theorem 2.6.5(i))$
$(4)\emptyset \vdash \neg\neg A \to A \quad ((3), \to +)$
$(5)\neg(\neg A \vee \neg B) \vdash A \quad ((2),(4), \to -)$
$same reason we get that (6)\neg(\neg A \vee \neg B) \vdash B$
$(7)\neg(\neg A \vee \neg B) \vdash A \wedge B \quad ((5),(6), \wedge+)$
$(8)\neg(A \wedge B) \vdash \neg\neg(\neg A \vee \neg B) \quad (Theorem 2.6.5(i))$
$(9)\emptyset \vdash \neg(\neg A \vee \neg B) \to (\neg A \vee \neg B) \quad (Theorem 2.6.5(i), \to +)$
$(10)\neg(A \wedge B) \vdash \neg A \vee \neg B \quad ((8),(9), \to -)$
Then we proof: $\neg A \vee \neg B \vdash \neg(A \wedge B)$
$(1) A \wedge B \vdash A \quad (\wedge-)$
$(2) A \wedge B \vdash B \quad (\wedge-)$
$(3)\neg A \vdash \neg(A \wedge B) \quad (Theorem 2.6.6(v),(1))$
$(4)\neg B \vdash \neg(A \wedge B) \quad (Theorem 2.6.6(v),(2))$
$(5)\neg A \vee \neg B \vdash \neg(A \wedge B) \quad (\vee-,(3),(4))$
Finally we get that $\neg(A \wedge B) \vdash\dashv \neg A \vee \neg B$

$(viii)\emptyset \vdash A \vee \neg A$
Proof:
$(1)\neg(A \vee \neg A) \vdash \neg A \wedge \neg\neg A \quad (Teorem 2.6.9(vi))$
$(2)\neg(A \vee \neg A) \vdash \neg A \quad (\wedge-,(1))$
$(3)\neg(A \vee \neg A) \vdash \neg\neg A \quad (\wedge-,(1))$
$(4)\emptyset \vdash A \vee \neg A \quad (\neg-,(2),(3))$

3 Heuristic for A^*

(1) Formal definition
States: Different KBs in every step
The initial state : KB$\cup\neg\alpha$
Goal states: KBs which contains $\emptyset$
Action: In every step, we choose two clauses to apply the resolution inference rule

From the description above, we can easily know that the Heuristic which designed by ourselves is going to be used on a tree search. So we just need to find a admissible heuristic.

(2) A naive one

Simply define $h(n) = 1$

It is obviously that h is admissible. However, this solution is the same as brute force search.

(3) A better one

Notice that in many cases, the shortest clause in evey step has a length more than 1, so we define our $h(n) =$ length of the shortest clause

Proof:

From the resolution inference rule :

$$\frac{l_1 \vee l_2 \vee \ldots l_k, \qquad m_1 \vee m_2 \vee \ldots m_n}{l_1 \vee l_2 \vee \ldots l_{i-1} \vee l_{i+1} \ldots \vee l_k \vee m_1 \vee m_2 \vee \ldots m_{j-1} \vee m_{j+1} \ldots \vee m_n}$$

We know that the result clause is at most 1 literal shorter than the father clauses(if the length of the l or m is 1 Or we simplify the result with the rule : $A \cup A = A$). So if the shortest clause's length is L, it must take at least L steps to reach the goal. So the h is admissiable.