Homework 2

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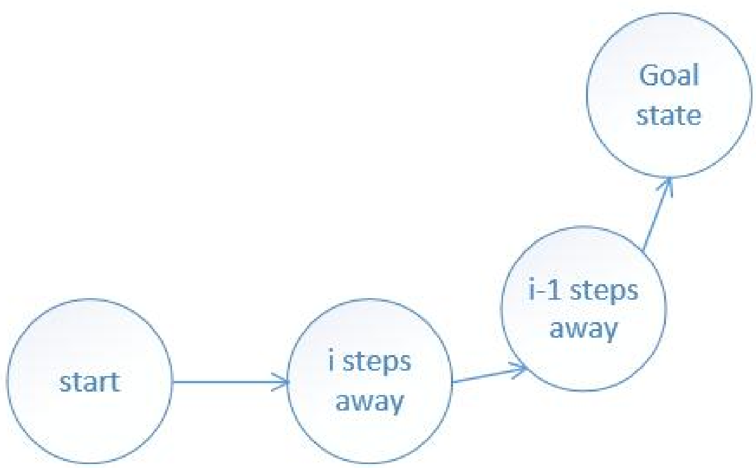
1 Prove that if a heuristic is consistent, it must be admissible.

Proof 1 *Prove by induction*
Let d(n) be the cost of the optimal path from n to the goal state.
BS: If node n is 0 step away from the goal state, we have $h(n) = 0 \le d(n)$
IS: If n is i steps away from the goal state, then some successor n' *of n generated by action a must exist and s.t.* n' *is on the optimal path from n to the goal state, which means that* n' *is i-1 steps away from the goal state. Therefore, we have*

$$h(n) \le c(n, a, n') + h(n') \le c(n, a, n') + d(n') = d(n)$$

Then the function h(n) is admissible 1



2 Give an example of heuristic, which is admissible but not consistent

Proof 2 *Consider a search problem, nodes are along the path P,*n_0 *is the start state,*n_m *is the goal state, each action from* n_i *to* n_{i+1} *costs 1, so we have the optimal path to the goal from the node* n_i $d(n_i)$*=m-i.*
Define function h as follows:

$$h(n_i) = m - 2\lceil i/2 \rceil$$

Then for node n_i, the actual distance from the goal state $h*(n_i) = m - i$, we can see that no matter i is odd or even, $h(n_i) < h*(n_i)$
Which means function h is admissible.
However, if i is even, $h(n_i) > 1 + h(n_{i+1})$, which means h is not consistent

3 Judge: A* of graph search is optimal with admissible heuristic

Proof 3 *3 Graph is described below, the number in the circle represents h(n). Obviously, the function is admissible.*
However, if we run A on this graph, we will get the wrong answer: the path above will be choosen.*
Which means the assumption is wrong