Homework 1

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1 Graph separation property

Theorem 1 *There is a graph $G = (V_0, E)$, we define set V(visited), set F(fringe), set R(remains). At the beginning, a starting node S and an ending node E are given and $X \in V$, nodes which are adjacent to S are included in set F, $R = V_0/(V \bigcup F)$*
In every step, a node in F is chosen to be included in V (for example X), nodes from R which are adjacent to X are added in F, set R changes according to the description above. The program stop when one of situations below occurs
1.node E is included in set V
2.set F is empty
In the case which is described above, the Graph separation property which means that the fringe(set F)separates the visited(set V) and the unexplored nodes(set R) exists. To be more precise, we say that every path which connect a node from V and a node from R include a node from F.

Proof 1 *Prove by contradiction.*
Assume that there is a path which connect set V and set R doesn't contain any nodes in F. We have already know that nodes in set V are adjacent to nodes either in set V or set F. So if such path do exist, we can certainly find a pair of nodes that one is in set V and the other is in set R, which contradicts to the definition of set F.

2 Uniform-cost search

Theorem 2 *Uniform-cost search gives us a optimal solution in which the sum of all the actions' costs is minimal. In other words, in a state graph G, Uniform-cost search gives us a path which connect the beginning state and the ending state with the least cost.*

Proof 2 *Prove by induction*
Use the definition of graph and sets in Therome 1.
1.Basic step
The initial state is added in set V first, because obviously it costs zero. Then set F and R are constructed.
2.Induction
Assume that we have a set V in which nodes' optimal solutions are found. It's suggested that we also find the optimal solution to the lowest-cost node in set F.Because if this solution isn't the optimal solution, then it represents that its optimal solution contains a node which is now in the set F, so it contradicts to the assumption that this is the lowest-cost node in F.

So, by induction, we can say that once a node was added in the set V, its optimal solution must have been found.