Homework 5

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October 10, 2017

Abstract

In this homework, 2 methods were used to proof these equivalences
(1) Using Contradiction .
(2) Using Truth Table .

1 Implication elimination : $(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta)$

Proof 1 *We have the definition that* $\alpha \equiv \beta$ *if and only if* $\alpha \vDash \beta$ *and* $\beta \vDash \alpha$
Prove by contradiction.
First, we proof $(\alpha \Rightarrow \beta) \vDash (\neg\alpha \vee \beta)$
Assume that $(\alpha \Rightarrow \beta) \nvDash (\neg\alpha \vee \beta)$
Then it means that there is a set of value v s.t.
(1) $(\alpha \Rightarrow \beta)^v = 1$
(2) $(\neg\alpha \vee \beta)^v = 0$
From (2) *we can know that* $\alpha^v = 1 \quad \beta^v = 0$
But from (1) *we can know that if* $(\alpha \Rightarrow \beta)^v = 1$ *and* $\alpha^v = 1$*, then we must have* $\beta^v = 1$
Contradiction!
Then we know that$(\alpha \Rightarrow \beta) \vDash (\neg\alpha \vee \beta)$ *is correct*

Second, we proof $(\neg\alpha \vee \beta) \vDash (\alpha \Rightarrow \beta)$
Assume that $(\neg\alpha \vee \beta) \nvDash (\alpha \Rightarrow \beta)$
Then it means that there is a set of value v s.t.
(1) $(\neg\alpha \vee \beta)^v = 1$
(2) $(\alpha \Rightarrow \beta)^v = 0$
From (2) *we can know that* $\alpha^v = 1 \quad \beta^v = 0$
But from (1) *we can know that if* $(\neg\alpha \vee \beta)^v = 1$ *and* $\alpha^v = 1$*, then we must have* $\beta^v = 1$
Contradiction!
Then we know that $(\neg\alpha \vee \beta) \vDash (\alpha \Rightarrow \beta)$*is correct*

Above all, we know that $(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta)$

2 Biconditional elimination : $(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$

Proof 2 *We have the definition that* $\alpha \equiv \beta$ *if and only if* $\alpha \vDash \beta$ *and* $\beta \vDash \alpha$
Prove by contradiction.
First, we proof$(\alpha \Leftrightarrow \beta) \vDash ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$
Assume that $(\alpha \Leftrightarrow \beta) \nvDash ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$
Then it means that there is a set of value v s.t.
(1) $(\alpha \Leftrightarrow \beta)^v = 1$
(2) $((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))^v = 0$
From (2) *we can know that* $\alpha^v = 1 \quad \beta^v = 0$ *or* $\alpha^v = 0 \quad \beta^v = 1$
But from (1) *we can know that if* $(\alpha \Leftrightarrow \beta)^v = 1$*, then we must have* $\alpha^v = \beta^v$

Contradiction!
Then we know that $(\alpha \Leftrightarrow \beta) \vDash ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$ *is correct*

Second, we proof that $((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \vDash (\alpha \Leftrightarrow \beta)$
Assume that $((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \nvDash (\alpha \Leftrightarrow \beta)$
Then it means that there is a set of value v s.t.
(1) $((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))^v = 1$
(2) $(\alpha \Leftrightarrow \beta)^v = 0$
From (2) *we can know that* $\alpha^v = 1 \quad \beta^v = 0 \quad or \quad \alpha^v = 0 \quad \beta^v = 1$
But from (1) *we can know that if* $((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))^v = 1$, *then we must have* $\alpha^v = \beta^v$
Contradiction!
Then we know that $((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \nvDash (\alpha \Leftrightarrow \beta)$ *is correct*

Above all, we know that $(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$

3 De Morgan : $\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$

Proof 3 *Using Truth table*

α	β	$\neg(\alpha \wedge \beta)$	$(\neg\alpha \vee \neg\beta)$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

Table 1: Truth table 1

4 De Morgan : $\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta)$

Proof 4 *Using Truth table*

α	β	$\neg(\alpha \vee \beta)$	$(\neg\alpha \wedge \neg\beta)$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Table 2: Truth table 2

5 Distributvity of $\wedge$ over $\vee$: $(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$

α	β	γ	$(\alpha \wedge (\beta \vee \gamma))$	$((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

Table 3: Truth table 3

6 Distributvity of $\vee$ over $\wedge$: $(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$

α	β	γ	$(\alpha \vee (\beta \wedge \gamma))$	$((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

Table 4: Truth table 4