1stNBwithBHP

June 3, 2024

[ ]:
```
import sys
import os

import matplotlib.pyplot as plt
%matplotlib inline
import numpy as np

# import joblib
# sys.modules['sklearn.externals.joblib'] = joblib
# from sklearn.preprocessing import Imputer
# sys.modules['sklearn.preprocessing.Imputer'] = Imputer
from few.trajectory.inspiral import EMRIInspiral
from few.amplitude.romannet import RomanAmplitude
from few.amplitude.interp2dcubicspline import Interp2DAmplitude
from few.waveform import FastSchwarzschildEccentricFlux,␣
↪SlowSchwarzschildEccentricFlux, GenerateEMRIWaveform
from few.utils.utility import (get_overlap,
                               get_mismatch,
                               get_fundamental_frequencies,
                               get_separatrix,
                               get_mu_at_t,
                               get_p_at_t,
                               get_kerr_geo_constants_of_motion,
                               xI_to_Y,
                               Y_to_xI)

from few.utils.ylm import GetYlms
from few.utils.modeselector import ModeSelector
from few.summation.interpolatedmodesum import CubicSplineInterpolant
from few.waveform import SchwarzschildEccentricWaveformBase
from few.summation.interpolatedmodesum import InterpolatedModeSum
from few.summation.directmodesum import DirectModeSum
from few.utils.constants import *
from few.summation.aakwave import AAKSummation
from few.waveform import Pn5AAKWaveform, AAKWaveformBase
```

[ ]:
```python
use_gpu = True

# keyword arguments for inspiral generator (RunSchwarzEccFluxInspiral)
inspiral_kwargs={
        "DENSE_STEPPING": 0,  # we want a sparsely sampled trajectory
        "max_init_len": int(1e3),  # all of the trajectories will be well under␣
 ↪len = 1000
    }

# keyword arguments for inspiral generator (RomanAmplitude)
amplitude_kwargs = {
    "max_init_len": int(1e3),  # all of the trajectories will be well under len␣
 ↪= 1000
    "use_gpu": use_gpu  # GPU is available in this class
}

# keyword arguments for Ylm generator (GetYlms)
Ylm_kwargs = {
    "assume_positive_m": False  # if we assume positive m, it will generate␣
 ↪negative m for all m>0
}

# keyword arguments for summation generator (InterpolatedModeSum)
sum_kwargs = {
    "use_gpu": use_gpu,  # GPU is availabel for this type of summation
    "pad_output": False,
}

from few.waveform import FastSchwarzschildEccentricFlux,␣
 ↪SlowSchwarzschildEccentricFlux, GenerateEMRIWaveform

few = FastSchwarzschildEccentricFlux(
    inspiral_kwargs=inspiral_kwargs,
    amplitude_kwargs=amplitude_kwargs,
    Ylm_kwargs=Ylm_kwargs,
    sum_kwargs=sum_kwargs,
    use_gpu=use_gpu,
)
```

LISA PSD function

[ ]:
```python
def power_spectral_density_RCLfit(freq):
    r"""
    Return the effective power spectral density (PSD) of the detector noise
    at a given frequency, according to the analytical fit by Robson, Cornish
    and Liu, :arxiv:`1803.01944`
```

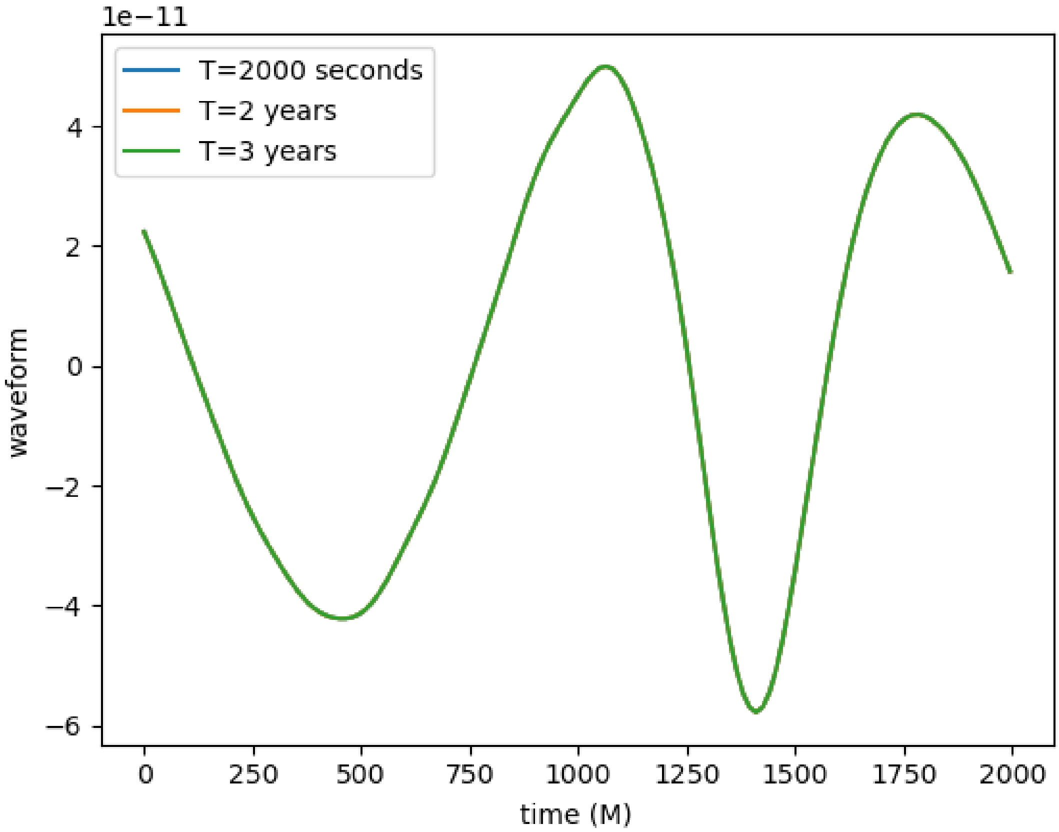
```
    INPUT:

    - ``freq`` -- frequency `f` (in `\mathrm{Hz}`)

    OUTPUT:

    - effective power spectral density `S(f)` (in `\mathrm{Hz}^{-1}`)

    EXAMPLES::

        sage: from kerrgeodesic_gw import lisa_detector
        sage: Sn = lisa_detector.power_spectral_density_RCLfit
        sage: Sn(1.e-1)  # tol 1.0e-13
        2.12858262120861e-39
        sage: Sn(1.e-2)  # tol 1.0e-13
        1.44307343517977e-40
        sage: Sn(1.e-3)  # tol 1.0e-13
        1.63410027259543e-38

    """
    p_oms = 2.25e-22 * (1 + (2.e-3/freq)**4)
    p_acc = 9.e-30 * (1 +(4.e-4/freq)**2) * (1 + (freq/8.e-3)**4)
    L = 2.5e9
    f_star = 1.909e-2
    p_n = (p_oms + 2*(1 + np.cos(freq/f_star)**2)*p_acc/(2*(np.pi)*freq)**4)/
↪L**2
    return 10./3.*p_n*(1 + 0.6*(freq/f_star)**2)
```

In This section I will explore the Black hole perturbation tool kit. Key idea is to reproduce the results of [3] where Sage Math has been used instade of BHP toolkit. Since we are interested in the $SrgA^*$,I will use the mass of the black hole to be $M = 4.15 \pm 0.27 \times 106 M_\odot$ form [4]. Lets Consider an object orbiting the Black hole with mass $\mu = 5 \times 10^{-2}$.

[ ]:
```
# This block of code is setting up the initial conditions for generating a␣
↪gravitational waveform.



M = 4.15e6 # mass of the black hole in solar masses
mu = 5e-2 # mass of the compact object in solar masses
dist1 = 0.000008277 # distance to the source in gigaparsecs
e0 = 0.0 # initial eccentricity
theta = np.pi / 2  # polar viewing angle
phi = 0  # azimuthal viewing angle
dt = 15 # time step in seconds
p0 = 6.15 # initial semilatus rectum in this case our orbit is circular. thus p␣
↪=r = 6.15.
t0 = 0.0 # initial time
```

```
T = T=2000/(365.25*24*3600) # total time in years to generate the waveform
wave = few(M, mu, p0, e0, theta, phi, dist=dist1, dt=dt, T=T) # generate the␣
 ↪waveform

wave2 = few(M, mu, p0, e0, theta, phi, dist=dist1, dt=dt, T=2) # generate the␣
 ↪waveform
wave3 = few(M, mu, p0, e0, theta, phi, dist=dist1, dt=dt, T=3) # generate the␣
 ↪waveform
```

[ ]:
```
# this block of code is generating the waveform and plotting it.
# dt =1
t= np.arange(len(wave)) * dt  # time array
# make a constant with p0/mu and multiply it with the wave.real
# to get the waveform in the right units
waveformReal = (M*p0)/mu * wave.real
waveformReal2 = (M*p0)/mu * wave2.real
waveformReal3 = (M*p0)/mu * wave3.real
# plot waveformreal vs t , waveformreal2 vs t, waveformreal3 vs t
plt.figure()
plt.plot(t,waveformReal[:134].get(), label='T=2000 seconds')
plt.plot(t, waveformReal2[:134].get(), label='T=2 years')
plt.plot(t, waveformReal3[:134].get(), label='T=3 years')
plt.xlabel('time (M)')
plt.ylabel('waveform')
plt.legend()
plt.show()
```



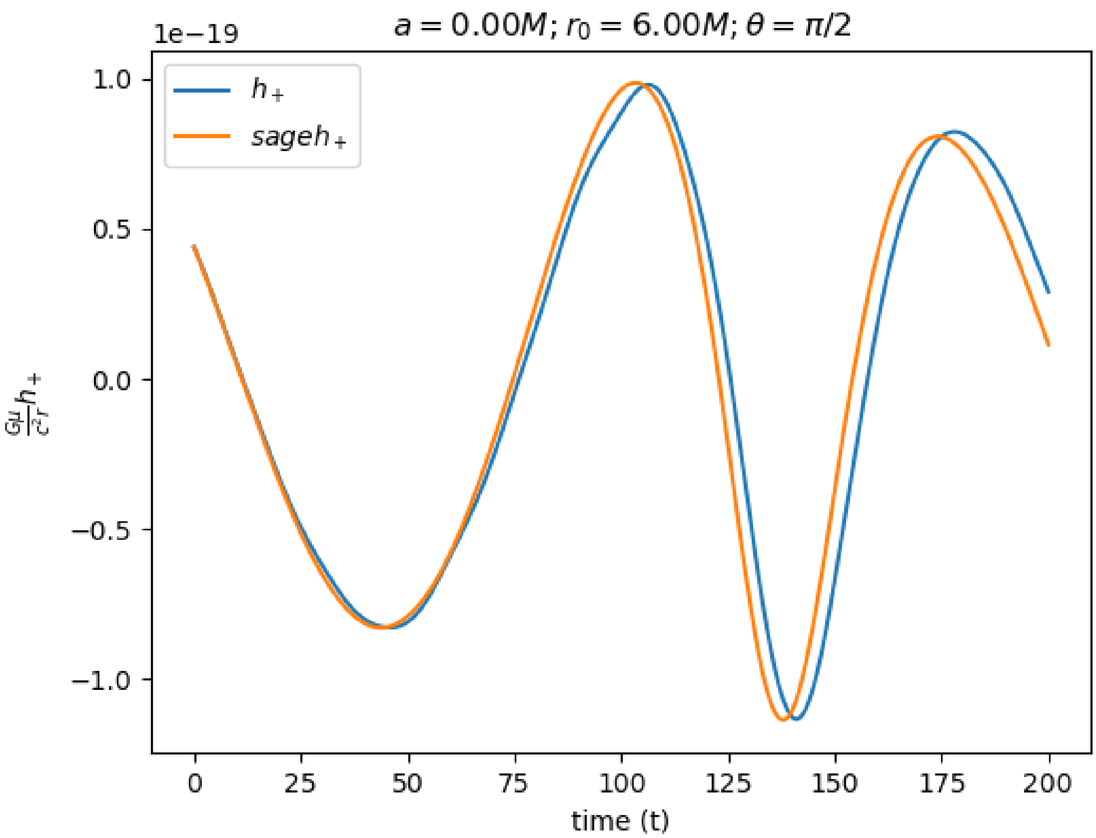
The plot above show that depending on the integration time the waveform does not changes.

```
[ ]: theta = np.pi/2  # polar viewing angle
     phi =0 # azimuthal viewing angle
     dt = 10
     dist1 = 0.000008277
     # wave = few(M, mu, p0, e0, theta, phi, dt=dt, T=2000/(365.25*24*3600)  )

     wave1 = few(M, mu, p0, e0, theta, phi, dist=dist1, dt=dt, T=2000/(365.
      ↪25*24*3600))
```

```
[ ]: # this data has been created by a sage file. a two column file named time_and_A.
      ↪txt is created with the time and amplitude values. read the file and store␣
      ↪the values in a list
     time = []
     amplitude = []
     with open('time_and_h.txt', 'r') as file:
         for line in file:
             t, a = line.split()
             time.append(float(t))
```

```
        amplitude.append(float(a))

Time= np.array(time)
Amplitude= np.array(amplitude)
```

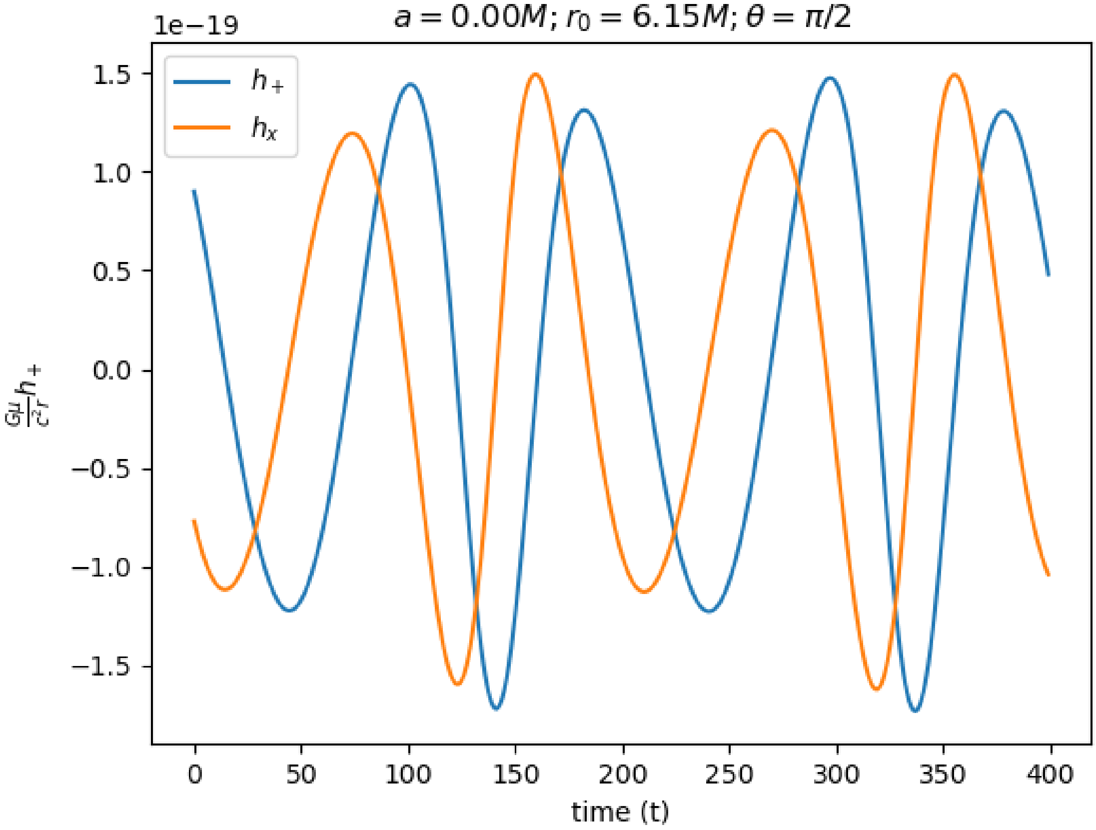
[ ]:
```
t = np.arange(len(wave1))

plt.plot(wave1.real.get(), label=r'$h_+$')
plt.plot(Amplitude, label=r'$sage h_+$')
# plt.plot(wave.imag[:220],label=r'$h_x$')
plt.xlabel(r'time (t)')
plt.ylabel(r'$\frac{G \mu}{c^2 r} h_+ $')
# title a = 0:00M; r0 = 6:000M; theta = pi ¼
plt.title(r'$a = 0.00M; r_0 = 6.00M; \theta = \pi/2$')
plt.legend()
plt.show()
```



[ ]:
```
wave2 = few(M, mu, p0, e0, np.pi/4, phi, dist=dist1, dt=dt, T=0.1)
t = np.arange(len(wave2))*dt
```

[ ]:
```python
# this data has been created by a sage file. a two column file named time_and_A.
↪txt is created with the time and amplitude values. read the file and store␣
↪the values in a list
time = []
amplitude = []
with open('time_and_h_4.txt', 'r') as file:
    for line in file:
        t, a = line.split()
        time.append(float(t))
        amplitude.append(float(a))

Time4= np.array(time)
Amplitude4= np.array(amplitude)
```

[ ]:
```python
t = np.arange(len(wave1))

plt.plot(wave2.real[:400].get(), label=r'$h_+$')
plt.plot(wave2.imag[:400].get(), label=r'$ h_x$')
# plt.plot(Amplitude, label=r'$sage h_+$')
# plt.plot(wave.imag[:220],label=r'$h_x$')
plt.xlabel(r'time (t)')
plt.ylabel(r'$\frac{G \mu}{c^2 r} h_+ $')
# title a = 0:00M; r0 = 6:000M; theta = pi ¼
plt.title(r'$a = 0.00M; r_0 = 6.15M; \theta = \pi/2$')
plt.legend()
plt.show()
```



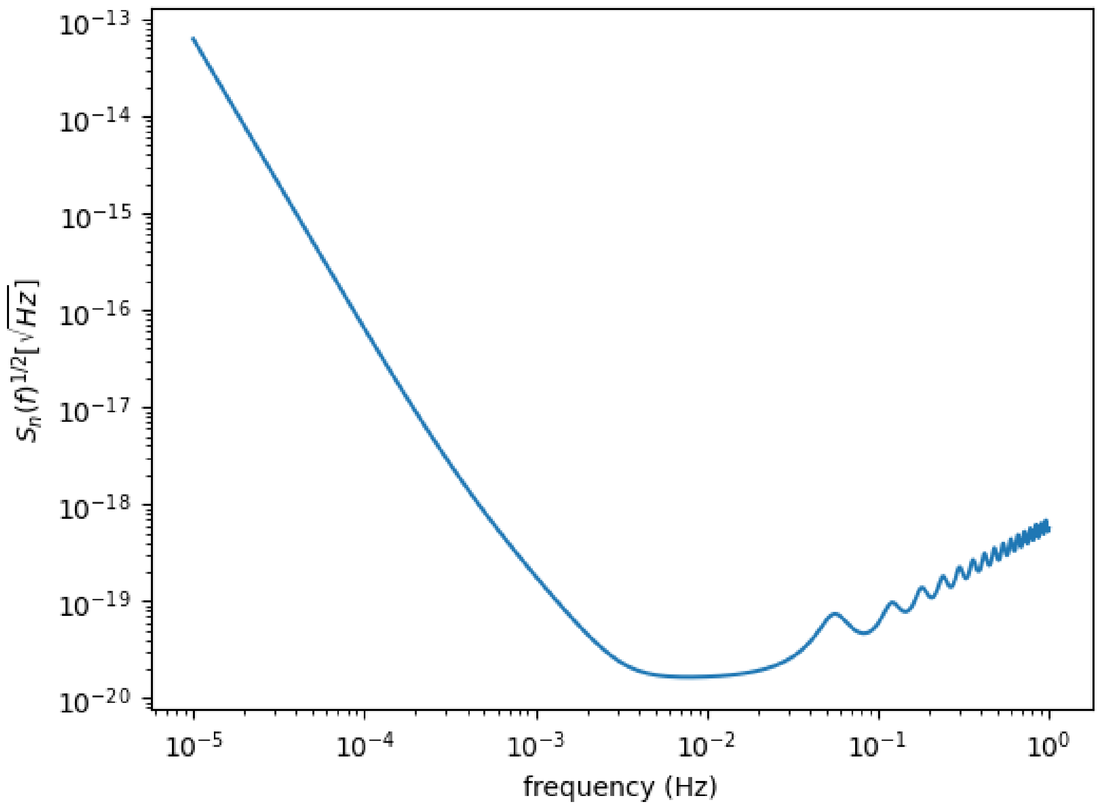
Results here matches with the paper. Now we focus into the S/R of the waveform and its detectibility by LISA. We will take $a=0, = /2 $ and $ 9 M< r_0 < 16.15M$ and vary the eccentricity of the orbit. The observation day would be one day or 1/365 year.

But before that I will bring the PSD from sage.

[ ]:
```
# read psd.txt and strote two colum in two different arrarys
freq = []
psd = []
with open('psd.txt', 'r') as file:
    for line in file:
        f, p = line.split()
        freq.append(float(f))
        psd.append(float(p))
sumpsd = np.sum(psd)
```

[ ]:
```
# log plot freq vs  sqrt of psd)
plt.figure()
plt.loglog(freq, np.sqrt(psd))

plt.ylabel(r"$S_n(f)^{ 1/2} [\sqrt {Hz} ]$")
```

```
plt.xlabel('frequency (Hz)')
plt.show()
```



[ ]:
```
wave_forSNR = few(M, mu, 9, e0, theta, phi, dt=dt, T=1/365) # generate the␣
 ↪waveform
# take fourier transform of the waveform
wave_forSNR = np.fft.fft(wave_forSNR.real)
# same for the imaginary part
wave_forSNR_imag = np.fft.fft(wave_forSNR.imag)
# get the power spectrum
realval = np.abs(wave_forSNR)**2
# same for the imaginary part
imagval = np.abs(wave_forSNR_imag)**2

# add = power + power_imag and take a square root of the suma
addval= realval + imagval
sumval = np.sqrt(np.sum(addval))
```

[ ]:
```
# adding all the values in the power spectrum and taking a square root of the␣
 ↪sum gives the SNR value.
```

```
print(sumval)
sumvalpsd = np.sqrt(sumpsd)
snr = sumval/sumvalpsd
print(snr)
```

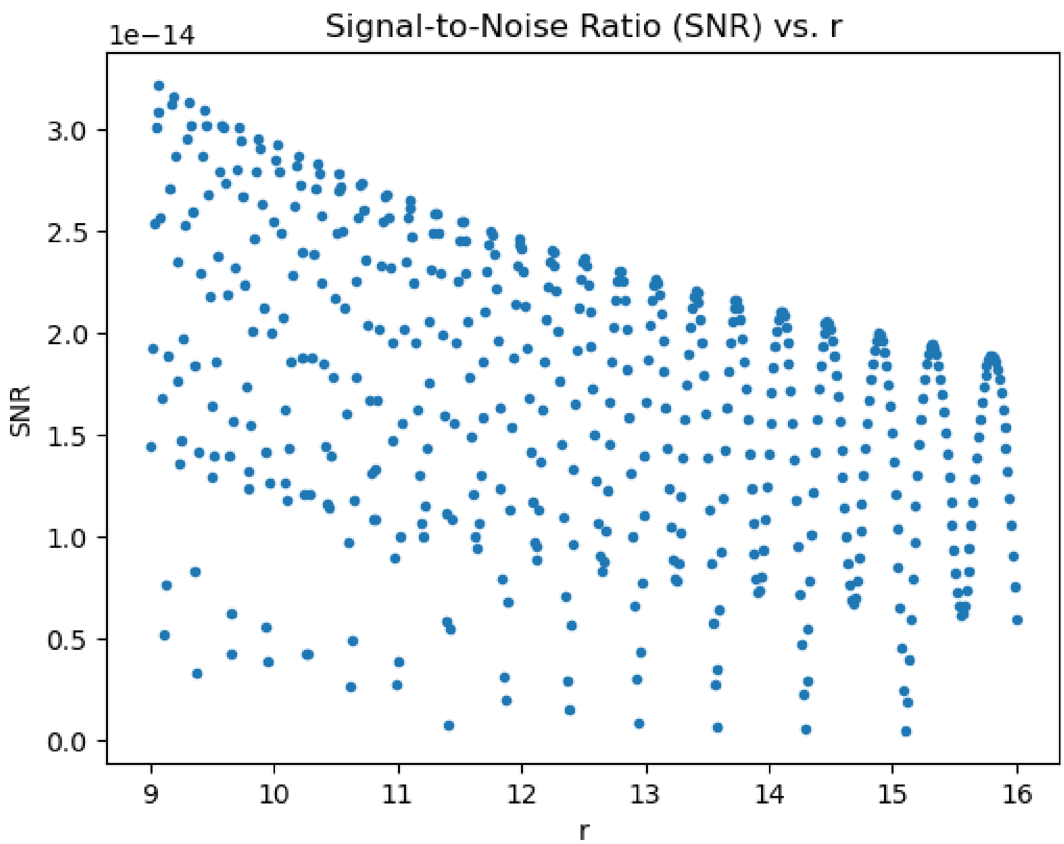
```
49896.04074754956
2.053001128145932e+17
```

[ ]:
```
# # plot
# plt.figure()
# plt.plot(cp.asnumpy(r_cp), cp.asnumpy(snr_cp), '.')
# plt.xlabel('r')
# plt.ylabel('SNR')
# plt.title('Signal-to-Noise Ratio (SNR) vs. Radial Coordinate (r)')
# plt.show()
# plt.show()
# plt.figure()
# plt.plot(np.array(cupy.array(r).get())  ,np.array(cupy.array(snr).get()), '.')
# plt.xlabel()
# plt.show()



    # plt.plot(cp.asnumpy(r.get()), cp.asnumpy(cp.stack(snrProfile).get()), '.')
```

[ ]:
```
import cupy as cp
```

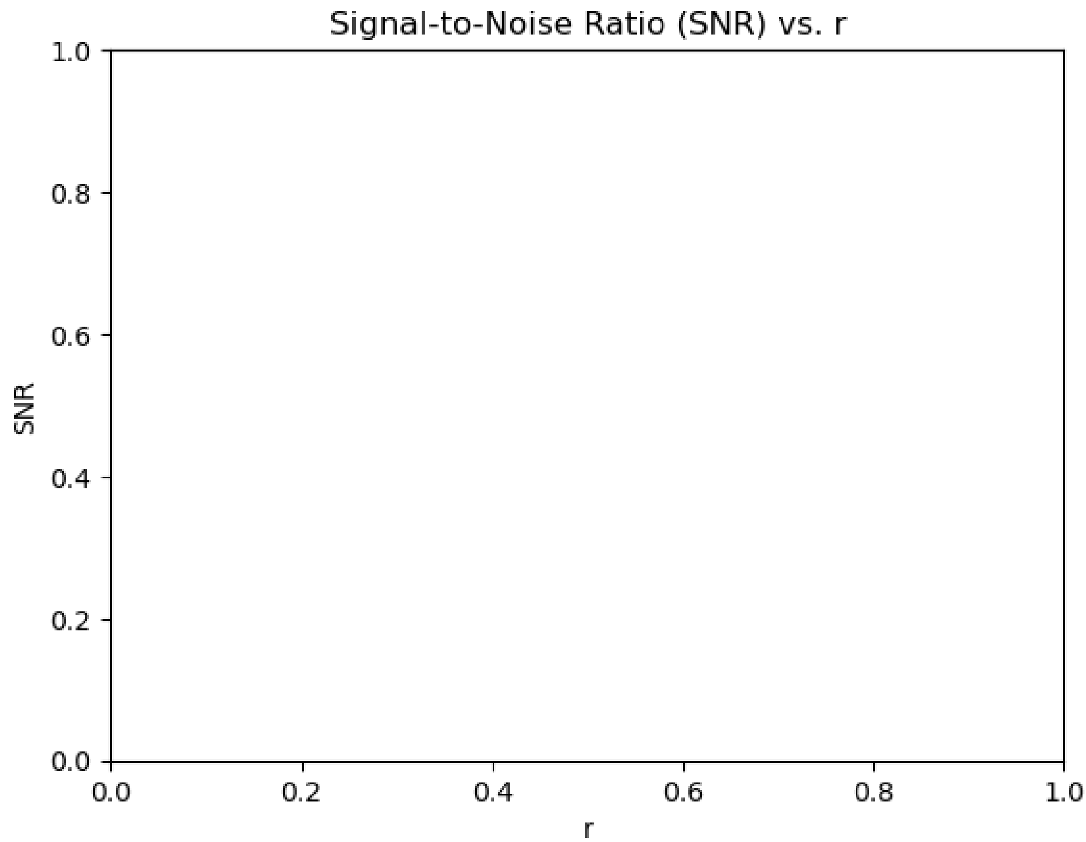
[ ]:
```
import numpy as np
import matplotlib.pyplot as plt


Msol=2e30
r = np.linspace(9, 16, 500)
sumvalr=[]
snr = []
for r_val in r:
    wave_forSNR = few(M, mu, r_val, e0, theta,phi,dist=dist1, dt=dt, T=1/365) #␣
 ↪generate the waveform
    wave_forSNR = np.fft.fft(wave_forSNR.real) # take fourier transform of the␣
 ↪waveform
    wave_forSNR_imag = np.fft.fft(wave_forSNR.imag) # same for the imaginary␣
 ↪part
    realval = np.abs(wave_forSNR)**2 # get the power spectrum
    imagval = np.abs(wave_forSNR_imag)**2 # same for the imaginary part
    addval = realval + imagval # add = power + power_imag
    sumval = np.sqrt(np.sum(addval)) # take a square root of the sum
```

```
    sumvalr.append(sumval)
    snr.append(sumval/sumvalpsd) # store the snr value
# now in another loop print r an arrow and snr value
# i want to multiply my snr with solar mass and divide it by mu to get the snr␣
↪in the right units.

plt.figure()
plt.plot( np.array(cp.array(r).get())  ,np.array(cp.array(sumvalr).get()), '.')
# plt.plot( np.array(cupy.array(r).get())  ,np.array(cupy.array(snr).get()), '.
↪')
plt.xlabel('r')
plt.ylabel('SNR')
plt.title('Signal-to-Noise Ratio (SNR) vs. r')
plt.show()
```



[ ]:
```
plt.figure()
# plt.plot( np.array(cupy.array(r).get())  ,np.array(cupy.array(sumvalr).
↪get()), '.')
# plt.plot( np.array(cupy.array(r).get())  ,np.array(cupy.array(snr).get()), '.
↪')
```

```
plt.xlabel('r')
plt.ylabel('SNR')
plt.title('Signal-to-Noise Ratio (SNR) vs. r')
plt.show()
```



[ ]:
```
def calculate_signalMod(r, L):
    wave = few(M, mu, r, e0, theta, phi, dist=dist1, dt=dt, T=L)
    fft_plus = cp.fft.fft(wave.real)
    fft_corss = cp.fft.fft(wave.imag)
    power_spectrum_real = cp.abs(fft_plus) ** 2
    power_spectrum_imag = cp.abs(fft_corss) ** 2
    power_spectrum = power_spectrum_real + power_spectrum_imag
    return power_spectrum

def calculate_snr(r_values, L):
    snr = []

    for r_val in r_values:
        signal_to = calculate_signalMod(r_val, 1/365)  # Assuming this is␣
 ↪defined elsewhere
```

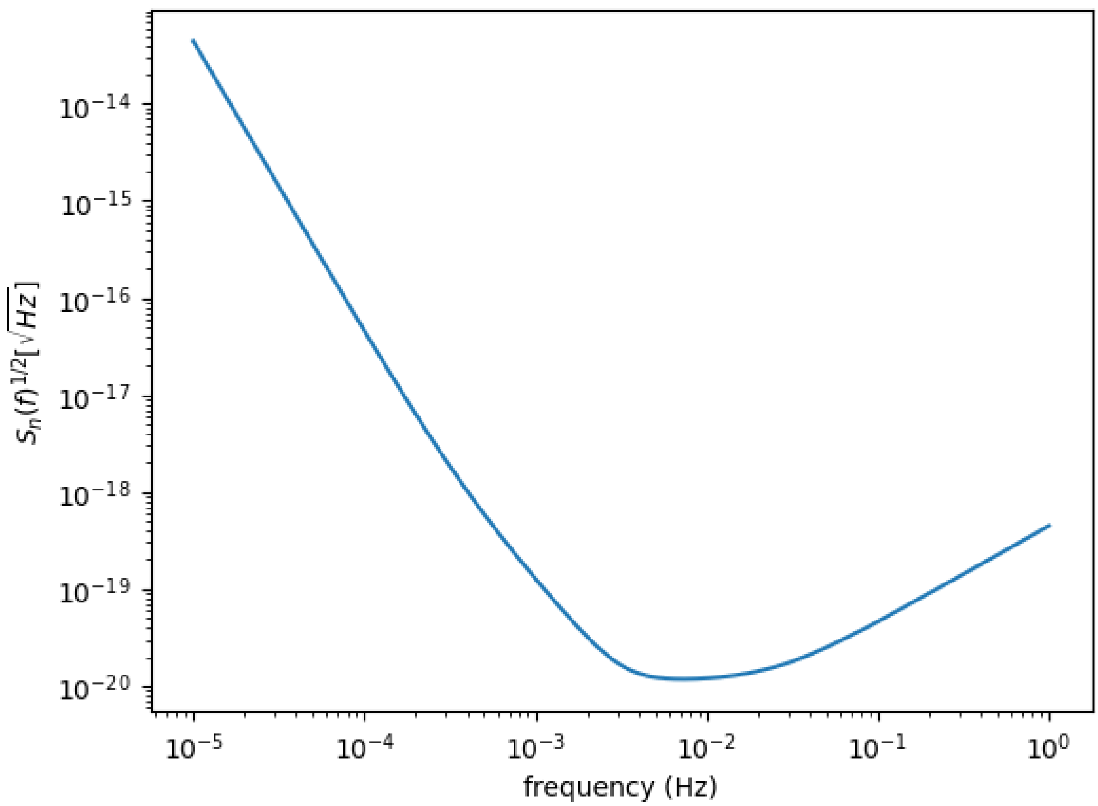
```
        freq = np.logspace(-5, 0, len(signal_to))
        psd = power_spectral_density_RCLfit(freq)  # Assuming this is defined␣
↪elsewhere
        templ = []
        for i in range(len(psd)):
            ratio = signal_to[i] / psd[i]
            templ.append(ratio)
        templ = cp.array(templ)
        sum_temp = cp.sum(templ)
        snr.append(cp.sqrt(sum_temp).get())  # Retrieve result from GPU and␣
↪append to list
    return snr
```

[ ]:
```
# # Iterate over different values of T
# # let l be an array of 9-16 with 20 values
# l= np.linspace(9, 16, 20)
# for L in [1 / 365]:
#     snrProfile = []
#     for p0 in r:
#         snrProfile.append(calculate_snr(l, L))
```

[ ]:
```
# rplt=np.array(cp.array(r).get())
# snrplt=np.array(cp.array(snrProfile).get())
# plt.figure()
# plt.plot( rplt  ,snrplt/mu, '.')
# plt.xlabel('r')
# plt.title(f'SNR Profile for T = {L} years')
# plt.show()
```

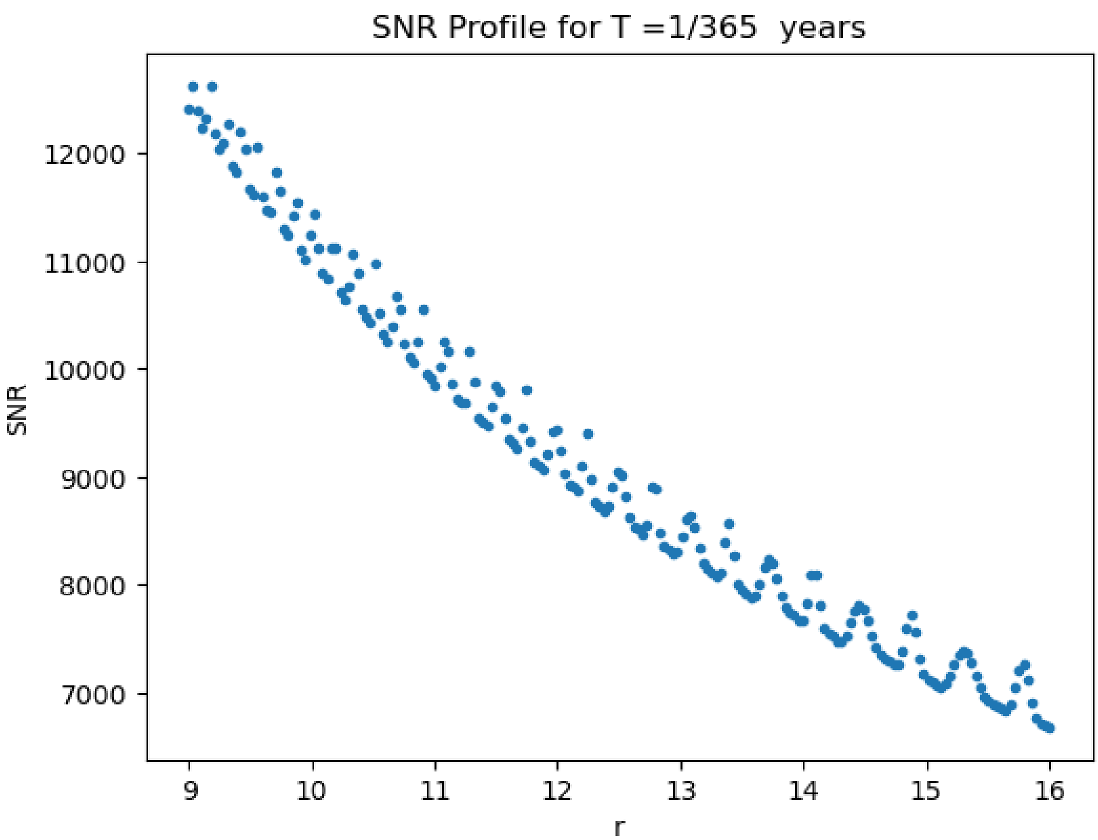
[ ]:
```

```

[ ]:
```
# a logsppace arrary frin -5 to 0 with 1000 points
freq = np.logspace(-5, 0, 1000)
power_spectral_density = power_spectral_density_RCLfit(freq)
plt.figure()
plt.loglog(freq, np.sqrt(power_spectral_density))
plt.xlabel('frequency (Hz)')
plt.ylabel(r'$S_n(f)^{1/2} [\sqrt{Hz}]$')
plt.show()
```



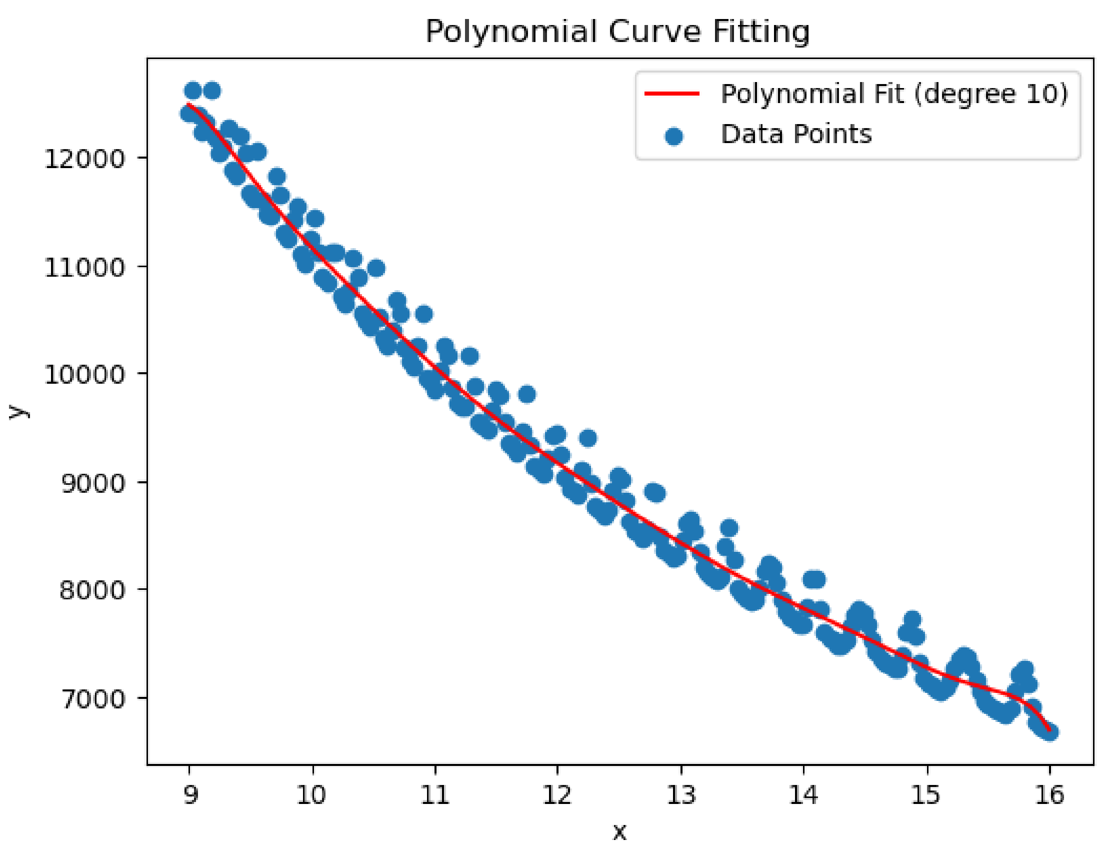

```
[ ]: # store snr in a list
snr = []
r = np.linspace(9, 16, 200)

#make a loop to on r values
for r_val in range(len(r)):
    signalto= calculate_signalMod(r[r_val], 1/365)
    freq=np.logspace(-5, 0, len(signalto))
    psd= power_spectral_density_RCLfit(freq)
    templ=[]
    sumtemp=0
    for i in range(len(psd)):
        ratio = signalto[i]/psd[i]
        templ.append(ratio)
    # convert the list to a cupy array
    templ = cp.array(templ)
    # sum the array

    sumtemp = np.sum(templ.get())

    snr.append(np.sqrt(sumtemp))
```

```
rplt=np.array(cp.array(r).get())
snrplt=np.array(cp.array(snr).get())
plt.figure()
plt.plot( rplt  ,snrplt/mu, '.')
# plot a best fit line for the snr values

plt.xlabel('r')
plt.ylabel('SNR')
plt.title(f'SNR Profile for T =1/365  years')
plt.show()
```



[ ]:
```
import numpy as np
x = rplt
y = snrplt/mu
```

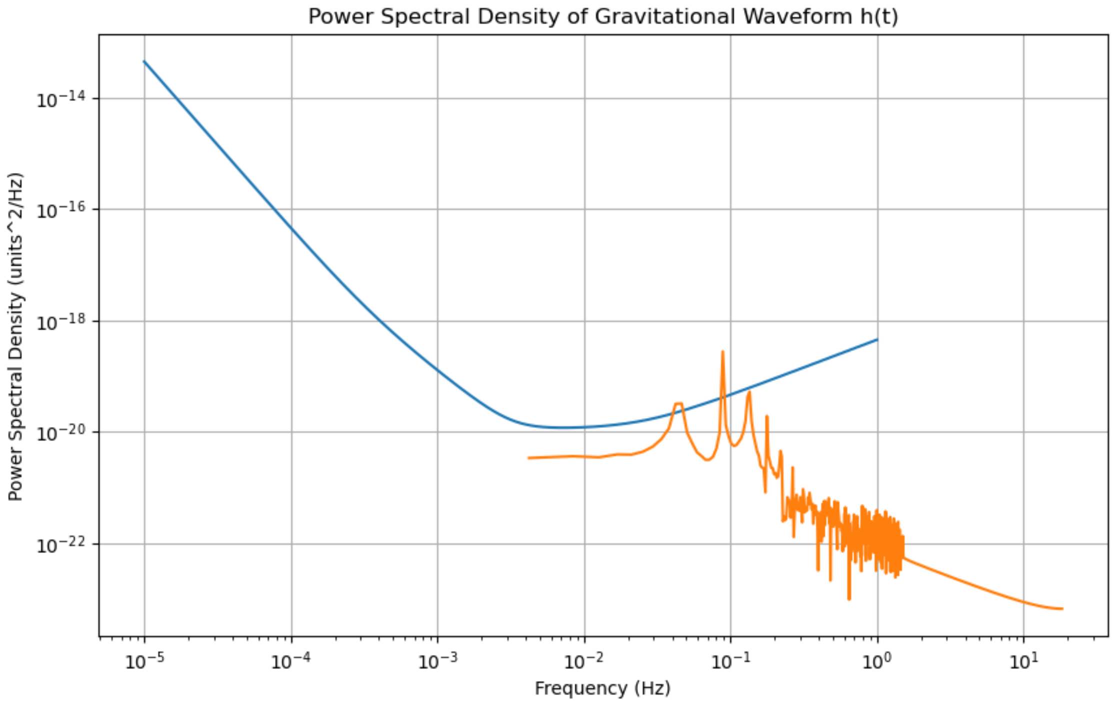
```
degree = 10
coefficients = np.polyfit(x, y, degree)
polynomial = np.poly1d(coefficients)

# Generate x values for plotting the fit
x_fit = np.linspace(min(x), max(x), 3000)
y_fit = polynomial(x_fit)

# Plot the original data points
plt.scatter(x, y, label='Data Points')

# Plot the fitted polynomial curve
plt.plot(x_fit, y_fit, label=f'Polynomial Fit (degree {degree})', color='red')

# Add labels and a legend
plt.xlabel('x')
plt.ylabel('y')
plt.title('Polynomial Curve Fitting')
plt.legend()

# Show the plot
plt.show()
```



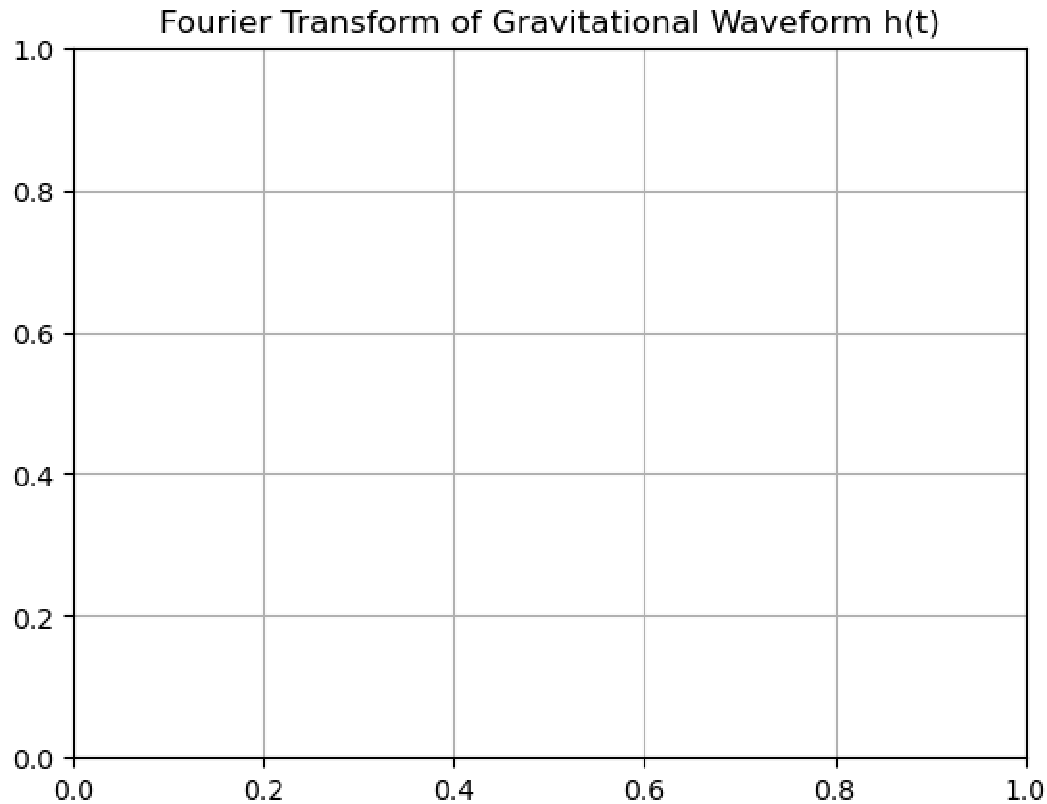
[ ]:
```
dt=10
wave5 = few(M, 10e-5, 16,0.0, theta, phi, dist=dist1, dt=dt, T=1/365)
```

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt

# Assuming `wave5` is generated using the `few` function and is a CuPy array

# Total duration in days and convert to seconds
T_days = 1.0 / 365  # Total duration in days
T = T_days * 24 * 3600  # Convert duration to seconds
N = len(wave5)  # Number of sample points

# Create a time array
t = np.linspace(0.0, T, N)

# Compute Fourier transform
H_f = np.fft.fft(wave5.get().real)  # Explicitly convert to NumPy array

# Compute frequency bins
frequencies = np.fft.fftfreq(N, d=t[1] - t[0])

# Truncate to positive frequencies
positive_frequencies = frequencies > 0
frequencies = frequencies[positive_frequencies]
H_f = H_f[positive_frequencies]

# # Limit to 0.1 MHz (100 kHz)
# limit_frequency = 1  # 0.1 MHz = 100,000 Hz
# limited_frequencies = frequencies < limit_frequency
# frequencies = frequencies[limited_frequencies]
# H_f = H_f[limited_frequencies]

# Compute the Power Spectral Density (PSD)
psd = np.sqrt((np.abs(H_f) ** 2))
 # PSD calculation
# lisa sensitivity curve
freq = np.logspace(-5, 0, 1000)
power_spectral_density = power_spectral_density_RCLfit(freq)


# Plot the Power Spectral Density
plt.figure(figsize=(10, 6))
plt.loglog(freq, np.sqrt(power_spectral_density), label='LISA Sensitivity␣
↪Curve')
```

```
plt.loglog(frequencies, psd)
plt.title('Power Spectral Density of Gravitational Waveform h(t)')
plt.xlabel('Frequency (Hz)')
plt.ylabel('Power Spectral Density (units^2/Hz)')
plt.grid(True)
plt.show()
```



[ ]:
```
# Limit to 0.1 MHz (100 kHz)
limit_frequency = 1  # 0.1 MHz = 100,000 Hzpower_spectral_density
plt.title('Fourier Transform of Gravitational Waveform h(t)')
plt.grid(True)
plt.show()
```



[ ]:
```
wave5 = few(M, mu, 16,0.75, theta, phi, dist=dist1, dt=dt, T=15/365)
```

[ ]:
```
fft_TD = np.fft.fftshift(np.fft.fft(wave5.real)) * dt
fft_TDX = np.fft.fftshift(np.fft.fft(wave5.imag)) * dt
freq = np.fft.fftshift(np.fft.fftfreq(len(wave5.real) , dt))

# define the positive frequencies
positive_frequency_mask = (freq>=0.0)
```

[ ]:

[ ]:
```
print(freq[positive_frequency_mask])
```

```
[0.00000000e+00 7.71057583e-07 1.54211517e-06 … 4.99976868e-02
 4.99984579e-02 4.99992289e-02]
```

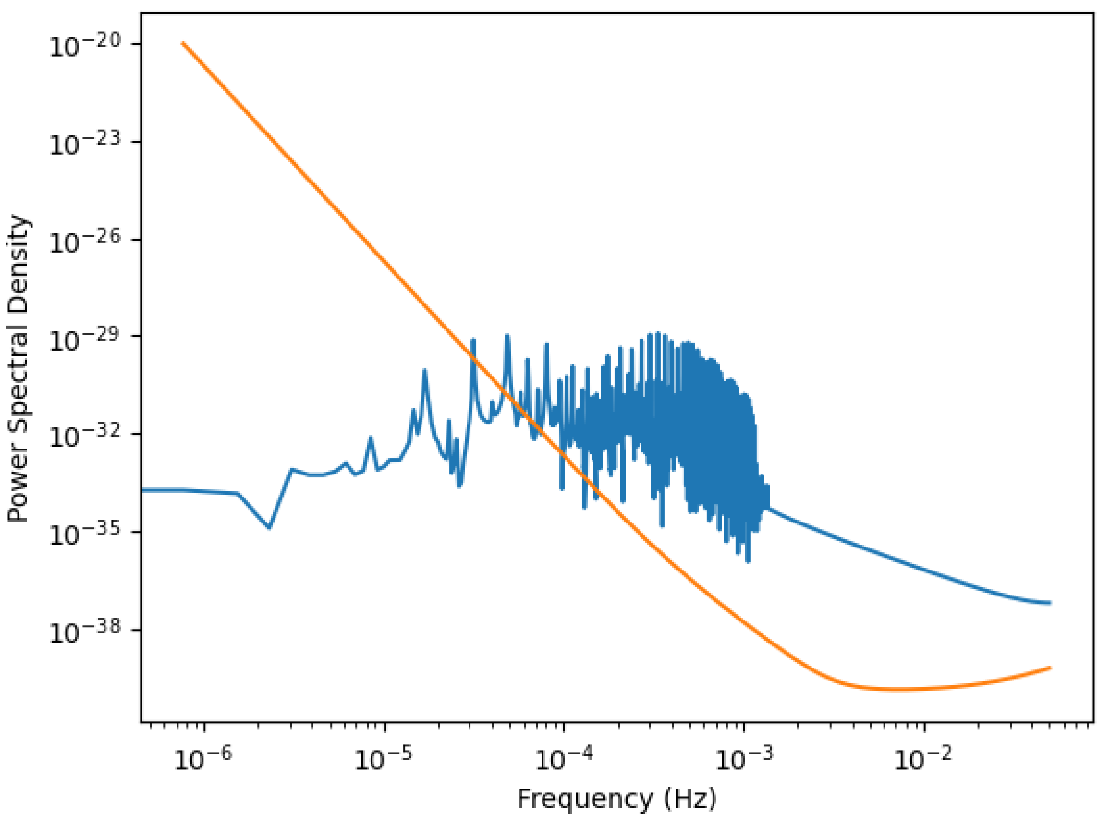
[ ]:
```
a1= freq[positive_frequency_mask]
b1=np.abs(fft_TD[positive_frequency_mask])**2
a=cp.array(a1)
b=cp.array(b1)
```

```
power_spectral_density =␣
 ↪power_spectral_density_RCLfit(freq[positive_frequency_mask])
```

```
/home/masrukuddin/anaconda3/envs/few_env/lib/python3.7/site-
packages/ipykernel_launcher.py:27: RuntimeWarning: divide by zero encountered in
true_divide
/home/masrukuddin/anaconda3/envs/few_env/lib/python3.7/site-
packages/ipykernel_launcher.py:28: RuntimeWarning: divide by zero encountered in
true_divide
```

[ ]:
```
plt.figure()
plt.loglog(a.get(), b.get())
plt.loglog(a1, power_spectral_density, label='LISA Sensitivity Curve')
plt.xlabel('Frequency (Hz)')
plt.ylabel('Power Spectral Density')
```

[ ]:
```
Text(0, 0.5, 'Power Spectral Density')
```



[ ]:
```
pluspart = np.abs(fft_TD)**2
crosspart = np.abs(fft_TDX)**2
```

[ ]:
```
df = freq[1] - freq[0]
# empty numpy array
integralp = np.zeros(len(pluspart))
numerator = pluspart + crosspart
denominator = power_spectral_density_RCLfit(freq)
for i in range(len(numerator)):
    integralp[i]=numerator[i]/denominator[i]
snrsQ = 4*np.sum(integralp)*df
print(np.sqrt(snrsQ))
```

```
/home/masrukuddin/anaconda3/envs/few_env/lib/python3.7/site-
packages/ipykernel_launcher.py:27: RuntimeWarning: divide by zero encountered in
true_divide
/home/masrukuddin/anaconda3/envs/few_env/lib/python3.7/site-
packages/ipykernel_launcher.py:28: RuntimeWarning: divide by zero encountered in
true_divide
```

```
70.42176080584017
```

[ ]:
```
64846*2
```

[ ]:
```
129692
```

[ ]:
```
def snrRev(r):
    wave5 = few(M, mu, r,0, theta, phi, dist=dist1, dt=dt, T=15/365)
    fft_TD = np.fft.fftshift(np.fft.fft(wave5.real)) * dt
    fft_TDX = np.fft.fftshift(np.fft.fft(wave5.imag)) * dt
    freq = np.fft.fftshift(np.fft.fftfreq(len(wave5.real) , dt))

    pluspart = np.abs(fft_TD)**2
    crosspart = np.abs(fft_TDX)**2
    df = freq[1] - freq[0]
    # empty numpy array
    integralp = np.zeros(len(pluspart))
    numerator = pluspart + crosspart
    denominator = power_spectral_density_RCLfit(freq)
    for i in range(len(numerator)):
        integralp[i]=numerator[i]/denominator[i]
    snr = np.sqrt((4*np.sum(integralp)*df))
    return float(snr)
```

[ ]:
```
snrRev(16)
```

```
/home/masrukuddin/anaconda3/envs/few_env/lib/python3.7/site-
packages/ipykernel_launcher.py:27: RuntimeWarning: divide by zero encountered in
true_divide
/home/masrukuddin/anaconda3/envs/few_env/lib/python3.7/site-
packages/ipykernel_launcher.py:28: RuntimeWarning: divide by zero encountered in
```

true_divide

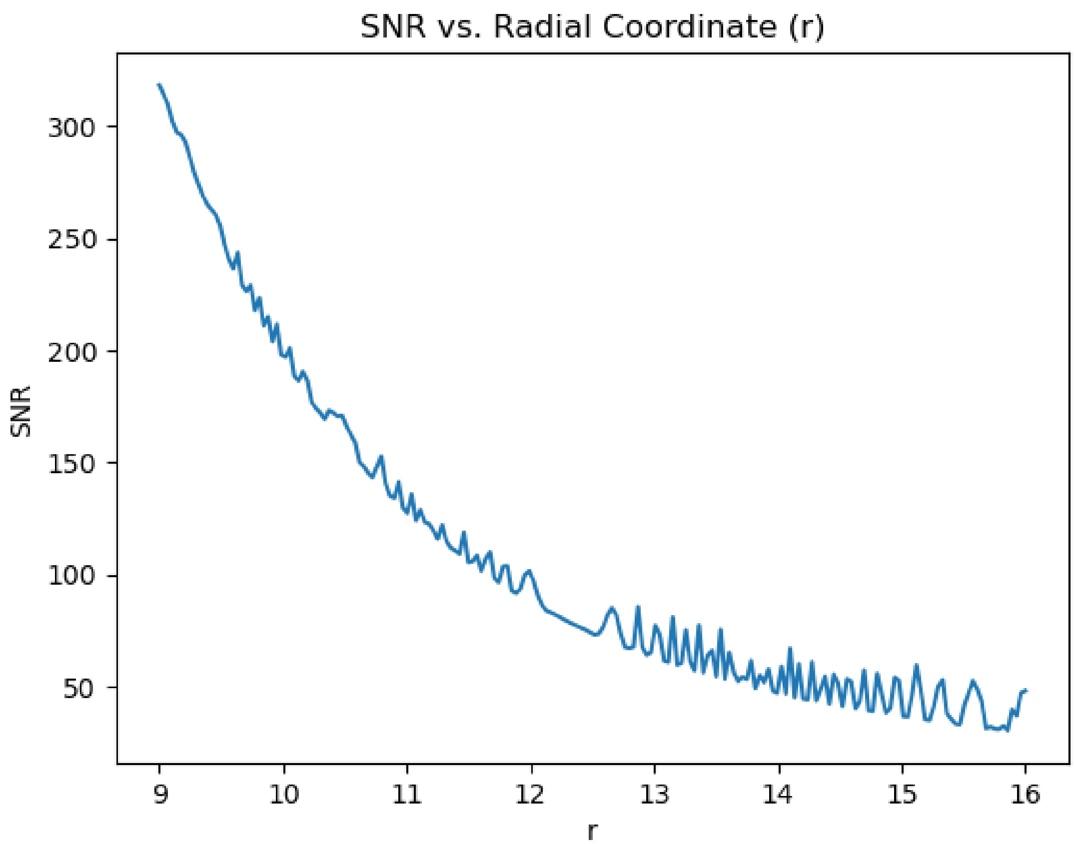
[ ]: 47.85176761910726

[ ]:
```
# r= np.linspace(9, 16, 200)
# snr = []
# for r_val in r:
#     snr.append(snrRev(r))
```

[ ]:
```
def snrRev(r):
    wave5 = few(M, mu, r, 0, theta, phi, dist=dist1, dt=dt, T=15/365)
    fft_TD = np.fft.fftshift(np.fft.fft(wave5.real)) * dt
    fft_TDX = np.fft.fftshift(np.fft.fft(wave5.imag)) * dt
    freq = np.fft.fftshift(np.fft.fftfreq(len(wave5.real), dt))

    pluspart = np.abs(fft_TD)**2
    crosspart = np.abs(fft_TDX)**2
    df = freq[1] - freq[0]
    integralp = np.zeros(len(pluspart))
    numerator = pluspart + crosspart
    denominator = power_spectral_density_RCLfit(freq)

    for i in range(len(numerator)):
        integralp[i] = numerator[i] / denominator[i]

    snr = np.sqrt((4 * np.sum(integralp) * df))
    return float(snr)

r = np.linspace(9, 16, 200)
snr = []

for r_val in r:
    snr.append(snrRev(r_val))
```

/home/masrukuddin/anaconda3/envs/few_env/lib/python3.7/site-packages/ipykernel_launcher.py:27: RuntimeWarning: divide by zero encountered in true_divide
/home/masrukuddin/anaconda3/envs/few_env/lib/python3.7/site-packages/ipykernel_launcher.py:28: RuntimeWarning: divide by zero encountered in true_divide

[ ]:
```
plt.figure()
plt.plot(r, snr)
plt.xlabel('r')
plt.ylabel('SNR')
plt.title('SNR vs. Radial Coordinate (r)')
plt.show()
```

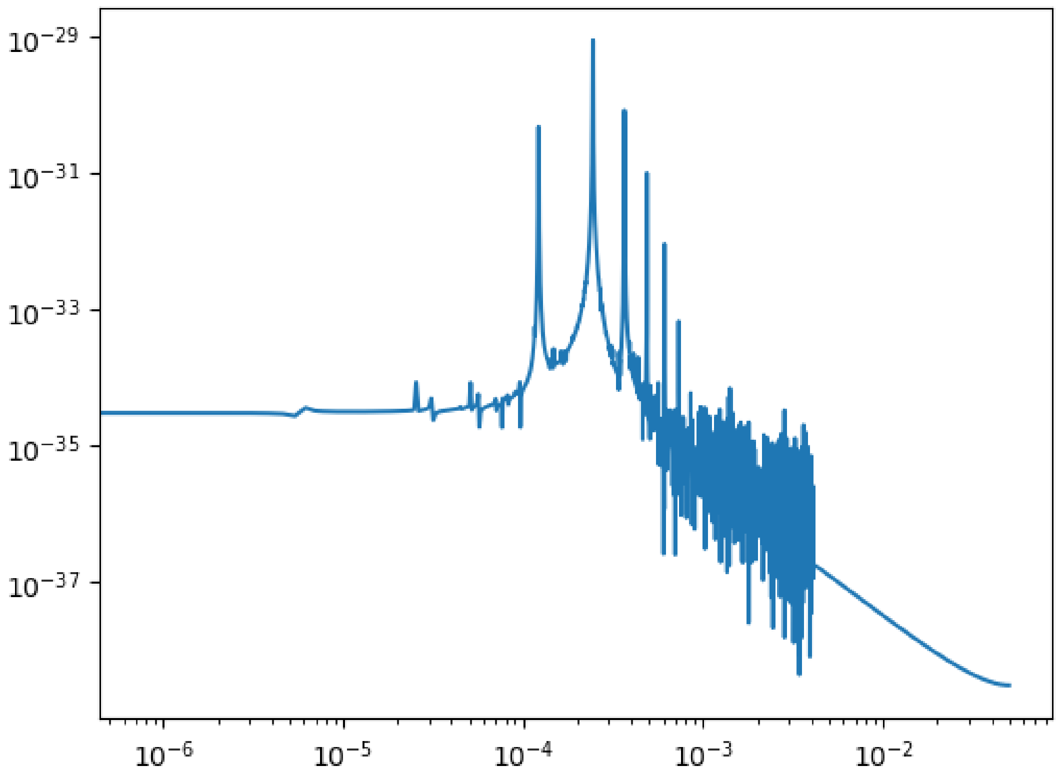


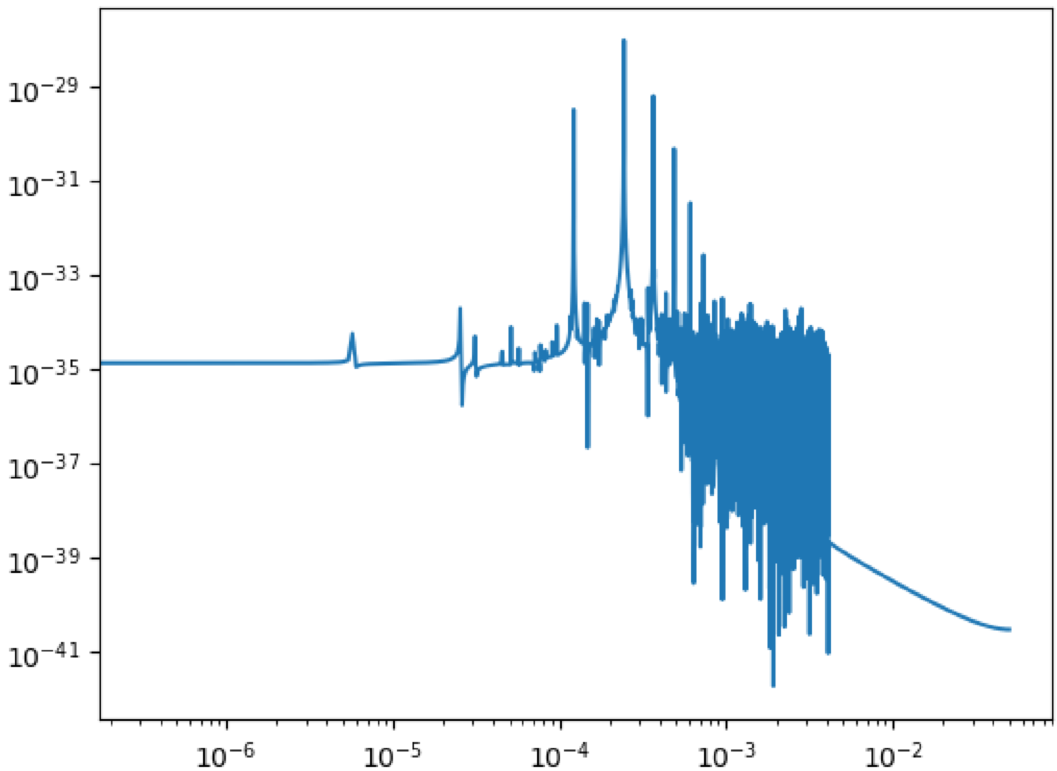
[ ]:
```
def sensitivitYimposed(x, p0,e,intg_time):
    wave5 = few(M, x, p0, e, theta, phi, dist=dist1, dt=dt, T=intg_time)
    fft_TD = np.fft.fftshift(np.fft.fft(wave5.real)) * dt
    fft_TDX = np.fft.fftshift(np.fft.fft(wave5.imag)) * dt
    freq = np.fft.fftshift(np.fft.fftfreq(len(wave5.real) , dt))
    positive_frequency_mask = (freq>=0.0)
    a1= freq[positive_frequency_mask]
    b1=np.abs(fft_TD[positive_frequency_mask])**2
    a=cp.array(a1)
    b=cp.array(b1)
    return a,b
```

[ ]:
```
y1, y2 = sensitivitYimposed(1e-2,16, 0.0, 15/365)
y3, y4 = sensitivitYimposed(1e-2,16, 0.0, 0.1)
y5, y6 = sensitivitYimposed(1e-2,16, 0.0, 1)
```

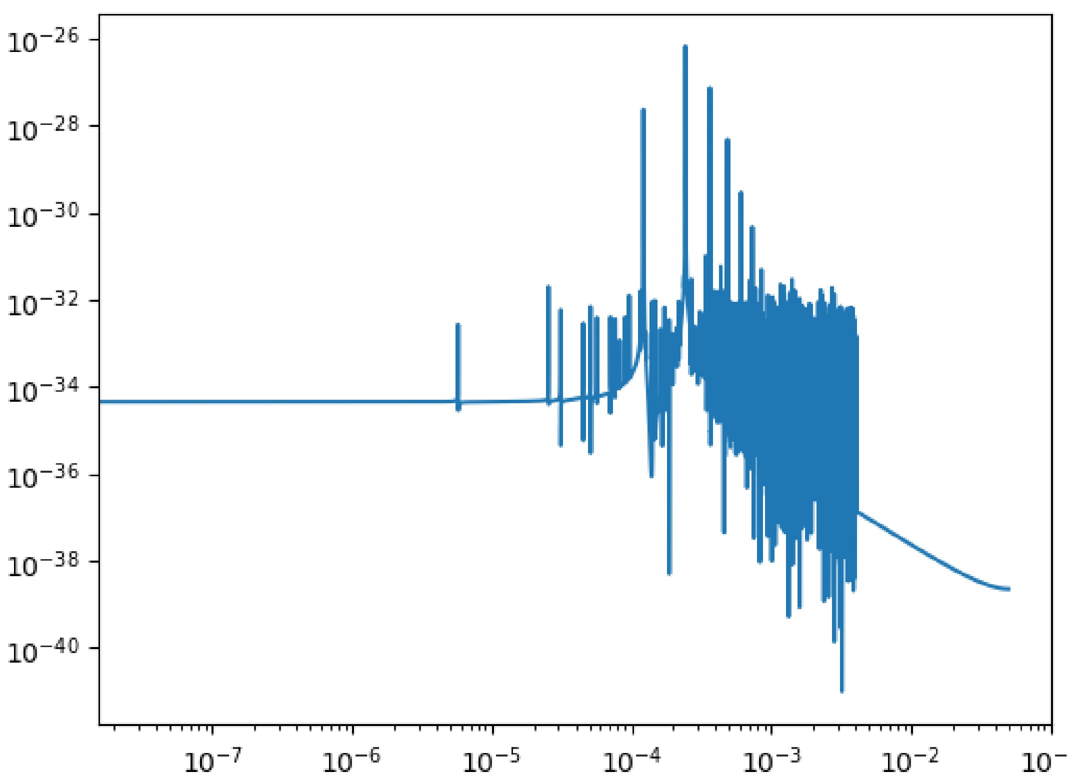
[ ]:
```
plt.figure()
plt.loglog(y1.get(), y2.get())
plt.show()
plt.figure()
```

```
plt.loglog(y3.get(), y4.get())
plt.show()
plt.figure()
plt.loglog(y5.get(), y6.get())
plt.show()
plt.Figure()
plt.plot(y1.get(), y2.get())
plt.show()
plt.Figure()
plt.plot(y3.get(), y4.get())
plt.show()
```

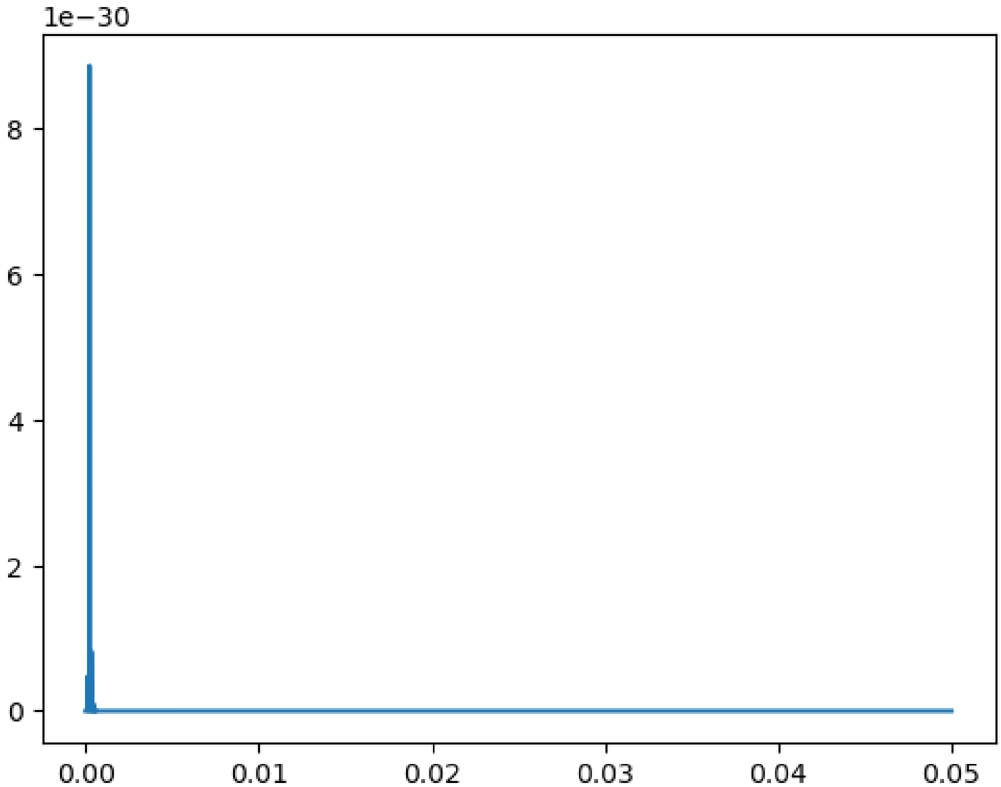


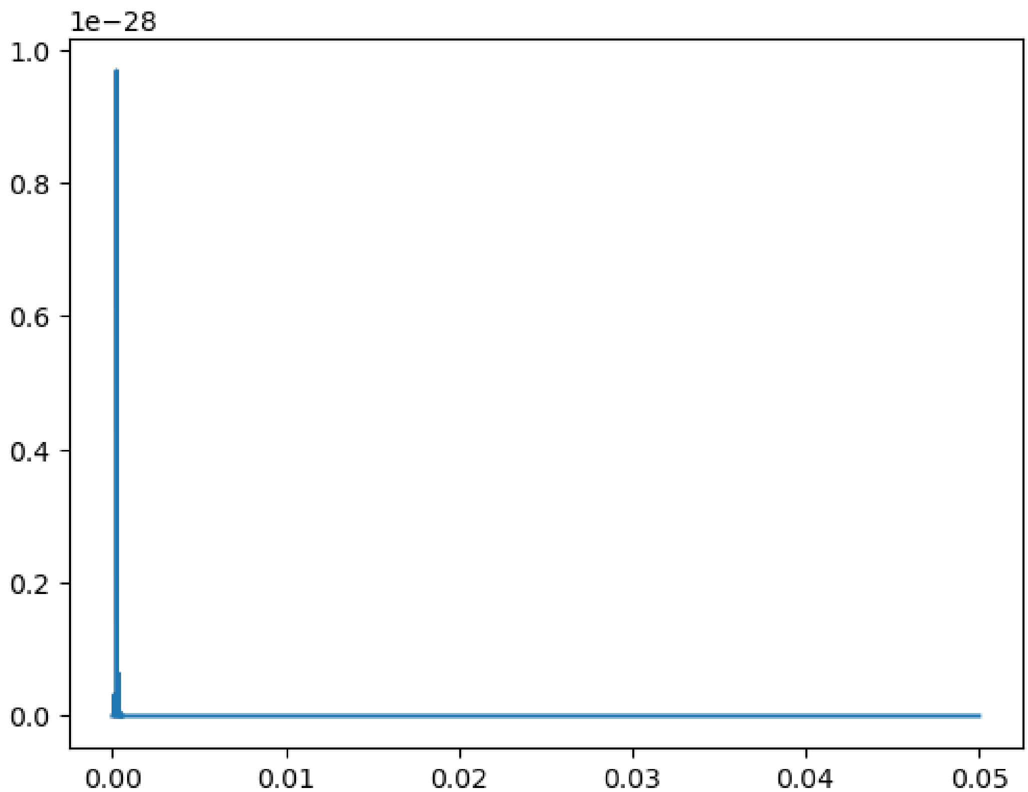


10^{-29}
10^{-31}
10^{-33}
10^{-35}
10^{-37}
10^{-39}
10^{-41}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}

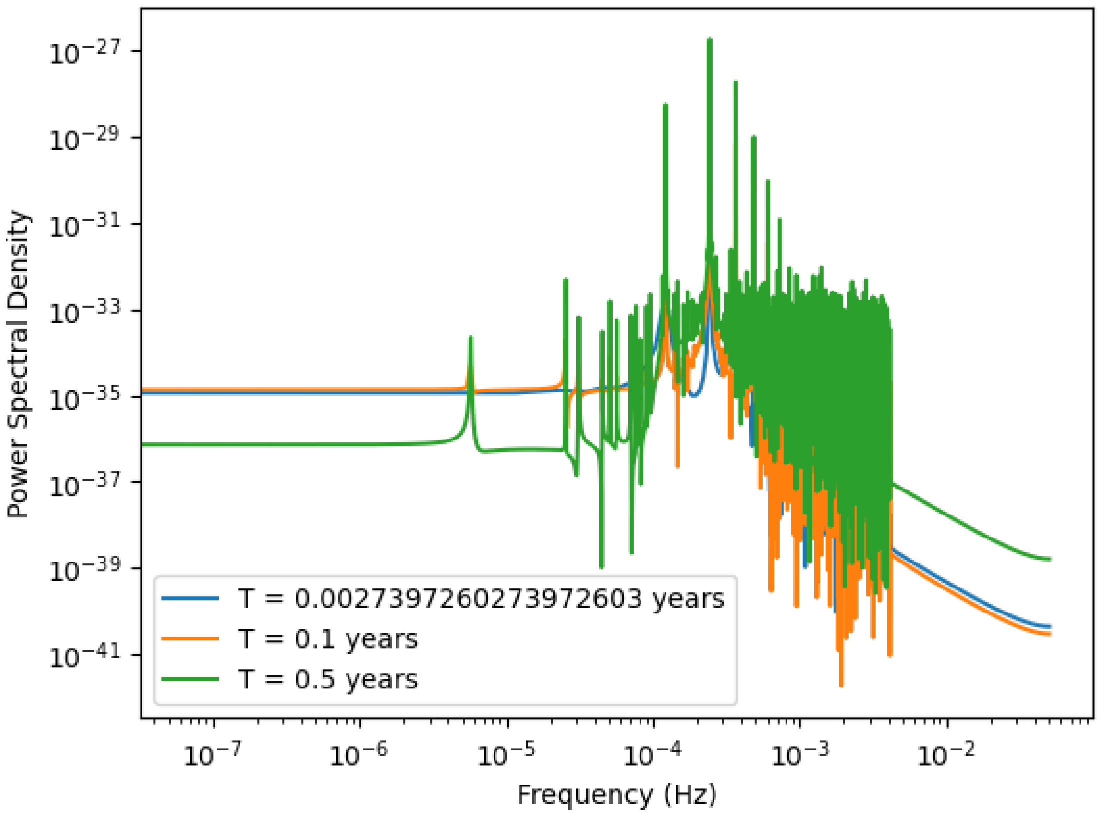


10^{-26}
10^{-28}
10^{-30}
10^{-32}
10^{-34}
10^{-36}
10^{-38}
10^{-40}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}


1e−30
8
6
4
2
0
0.00
0.01
0.02
0.03
0.04
0.05



[ ]:
```
# for T in range(1, 16):
# an tratty with T = 1/365, 0.1 ,0.5
plt.figure()

for T in [1/365, 0.1, 0.5]:
    sen =  sensitivitYimposed(1e-2, 16, 0.0, T)
    plt.loglog(sen[0].get(), sen[1].get(), label=f'T = {T} years')
plt.xlabel('Frequency (Hz)')
plt.ylabel('Power Spectral Density')
plt.legend()
plt.show()
# find the index for maxima of sen[1]
mval=max_index = np.argmax(sen[1].get())
print(mval)
print(sen[0].get()[mval])
# print(sen[0].get()[np.where(sen[1].get() == mval)])
# max(sen[1].get())
```

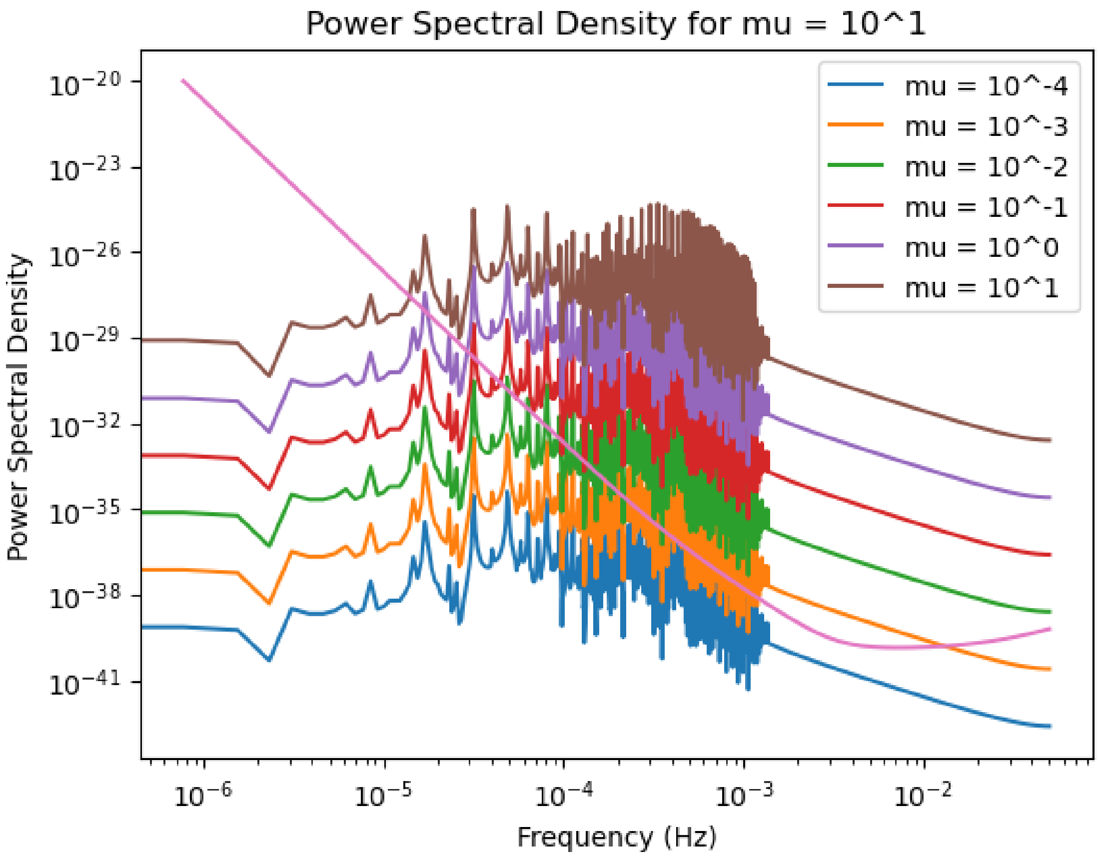



```
3839
0.00024329682085394078
```

```
[ ]: # mu to be from 10^-7 to 10^2
power_spectral_density =␣
 ↪power_spectral_density_RCLfit(freq[positive_frequency_mask])
plt.figure()

for mu in range(-4, 2):
    y=sensitivitYimposed(10**mu, 16, 0.75,15/365)
    plt.loglog(y[0].get(), y[1].get(), label=f'mu = 10^{mu}')
    plt.legend()
plt.loglog(freq[positive_frequency_mask], power_spectral_density, label='LISA␣
 ↪Sensitivity Curve')
plt.xlabel('Frequency (Hz)')
plt.ylabel('Power Spectral Density')
plt.title(f'Power Spectral Density for mu = 10^{mu}')
plt.show()
```

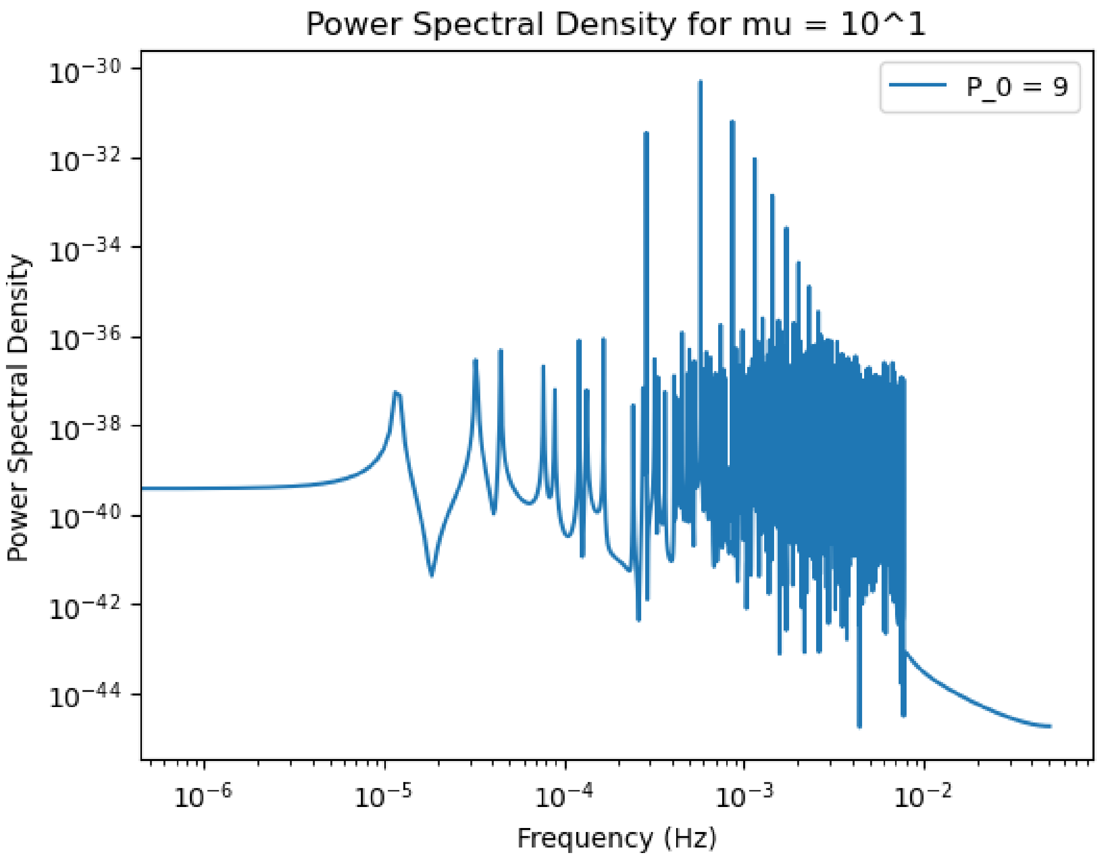
```
/home/masrukuddin/anaconda3/envs/few_env/lib/python3.7/site-
packages/ipykernel_launcher.py:27: RuntimeWarning: divide by zero encountered in
```

```
true_divide
/home/masrukuddin/anaconda3/envs/few_env/lib/python3.7/site-
packages/ipykernel_launcher.py:28: RuntimeWarning: divide by zero encountered in
true_divide
```



[ ]:
```
# mu to be from 10^-7 to 10^2
power_spectral_density =␣
↪power_spectral_density_RCLfit(freq[positive_frequency_mask])
plt.figure()

for R in range(9, 10, 1):
    y=sensitivitYimposed(10**-3, R, 0,15/365)
    plt.loglog(y[0].get(), y[1].get(), label=f'P_0 = {R}')
    plt.legend()
# plt.loglog(freq[positive_frequency_mask], power_spectral_density, label='LISA␣
↪Sensitivity Curve')
plt.xlabel('Frequency (Hz)')
plt.ylabel('Power Spectral Density')
plt.title(f'Power Spectral Density for mu = 10^{mu}')
plt.show()
```

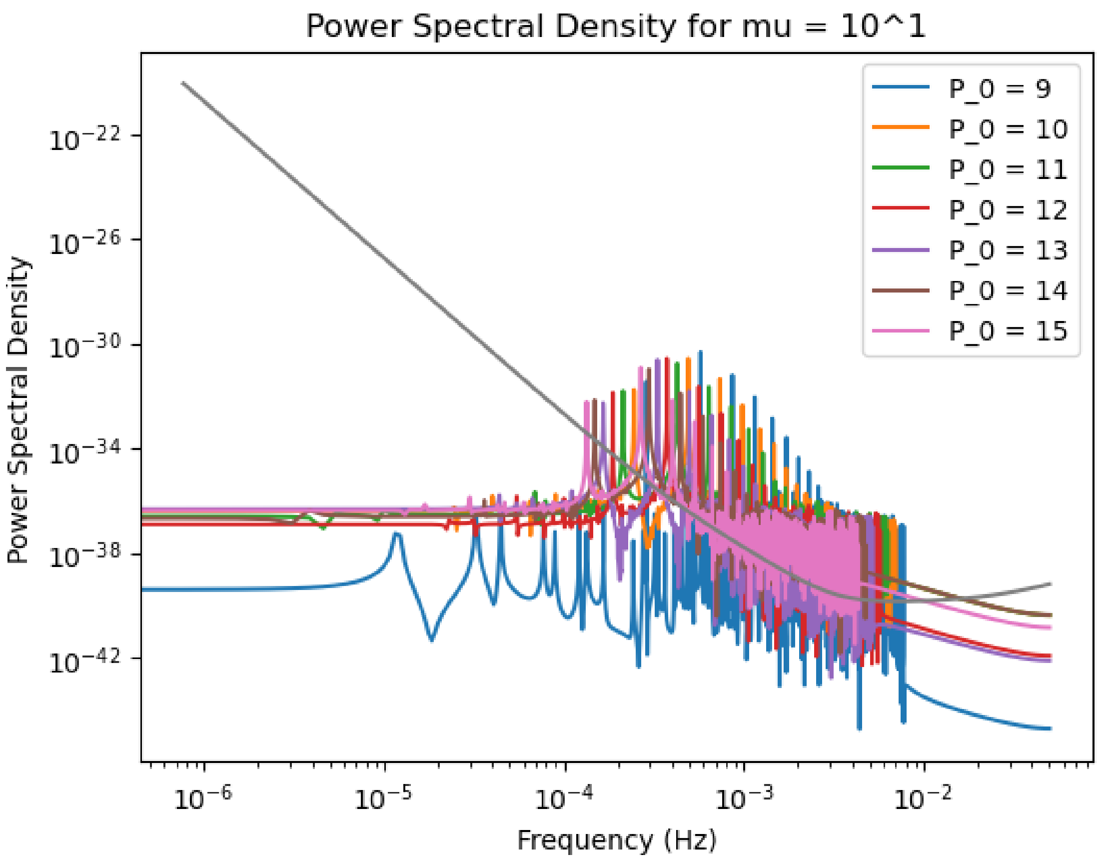
```
/home/masrukuddin/anaconda3/envs/few_env/lib/python3.7/site-
packages/ipykernel_launcher.py:27: RuntimeWarning: divide by zero encountered in
true_divide
/home/masrukuddin/anaconda3/envs/few_env/lib/python3.7/site-
packages/ipykernel_launcher.py:28: RuntimeWarning: divide by zero encountered in
true_divide
```



[ ]:
```
# mu to be from 10^-7 to 10^2
power_spectral_density =␣
 ↪power_spectral_density_RCLfit(freq[positive_frequency_mask])
plt.figure()

for R in range(9, 16, 1):
    y=sensitivitYimposed(10**-3, R, 0,15/365)
    plt.loglog(y[0].get(), y[1].get(), label=f'P_0 = {R}')
    plt.legend()
plt.loglog(freq[positive_frequency_mask], power_spectral_density, label='LISA␣
 ↪Sensitivity Curve')
plt.xlabel('Frequency (Hz)')
plt.ylabel('Power Spectral Density')
```

```
plt.title(f'Power Spectral Density for mu = 10^{mu}')
plt.show()
```

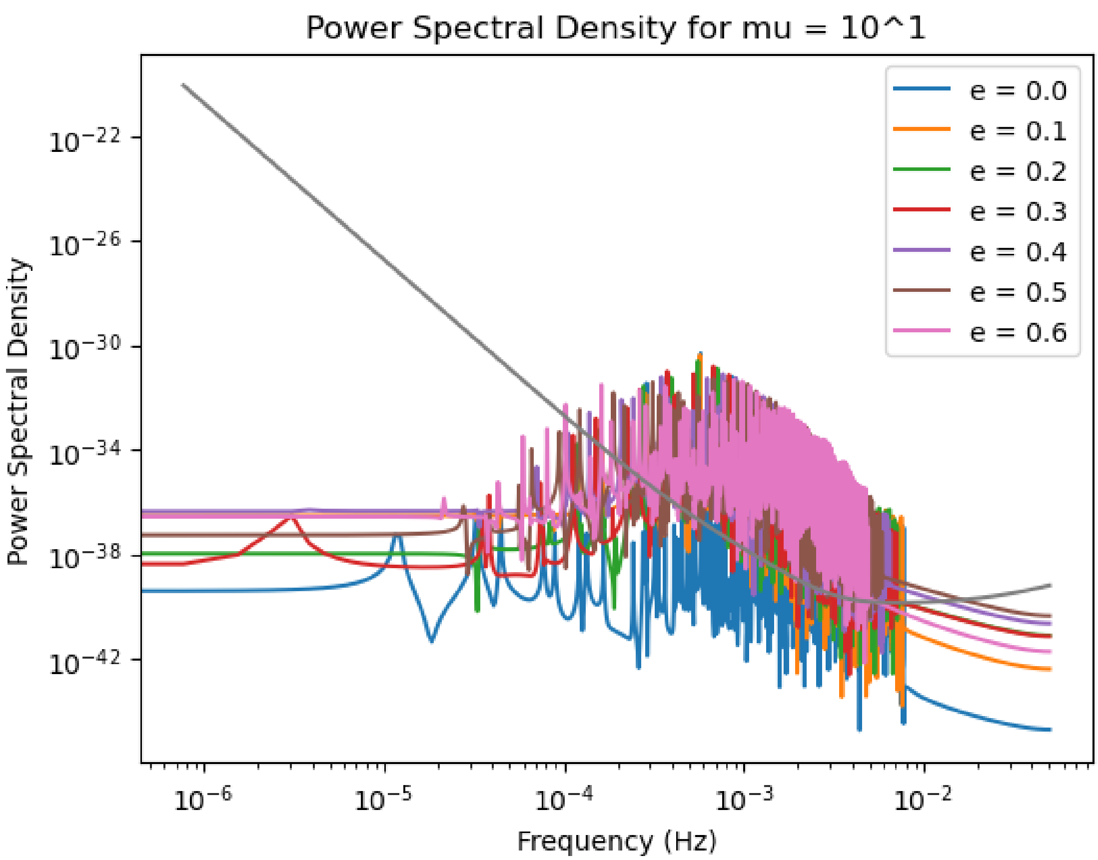
```
/home/masrukuddin/anaconda3/envs/few_env/lib/python3.7/site-
packages/ipykernel_launcher.py:27: RuntimeWarning: divide by zero encountered in
true_divide
/home/masrukuddin/anaconda3/envs/few_env/lib/python3.7/site-
packages/ipykernel_launcher.py:28: RuntimeWarning: divide by zero encountered in
true_divide
```



[ ]:
```
# mu to be from 10^-7 to 10^2
power_spectral_density =␣
↪power_spectral_density_RCLfit(freq[positive_frequency_mask])
plt.figure()

for E in range(0, 7, 1):
    y=sensitivitYimposed(10**-3, 9, E/10,15/365)
    plt.loglog(y[0].get(), y[1].get(), label=f'e = {E/10}')
    plt.legend()
```

```
plt.loglog(freq[positive_frequency_mask], power_spectral_density, label='LISA␣
  ↪Sensitivity Curve')
plt.xlabel('Frequency (Hz)')
plt.ylabel('Power Spectral Density')
plt.title(f'Power Spectral Density for mu = 10^{mu}')
plt.show()
```

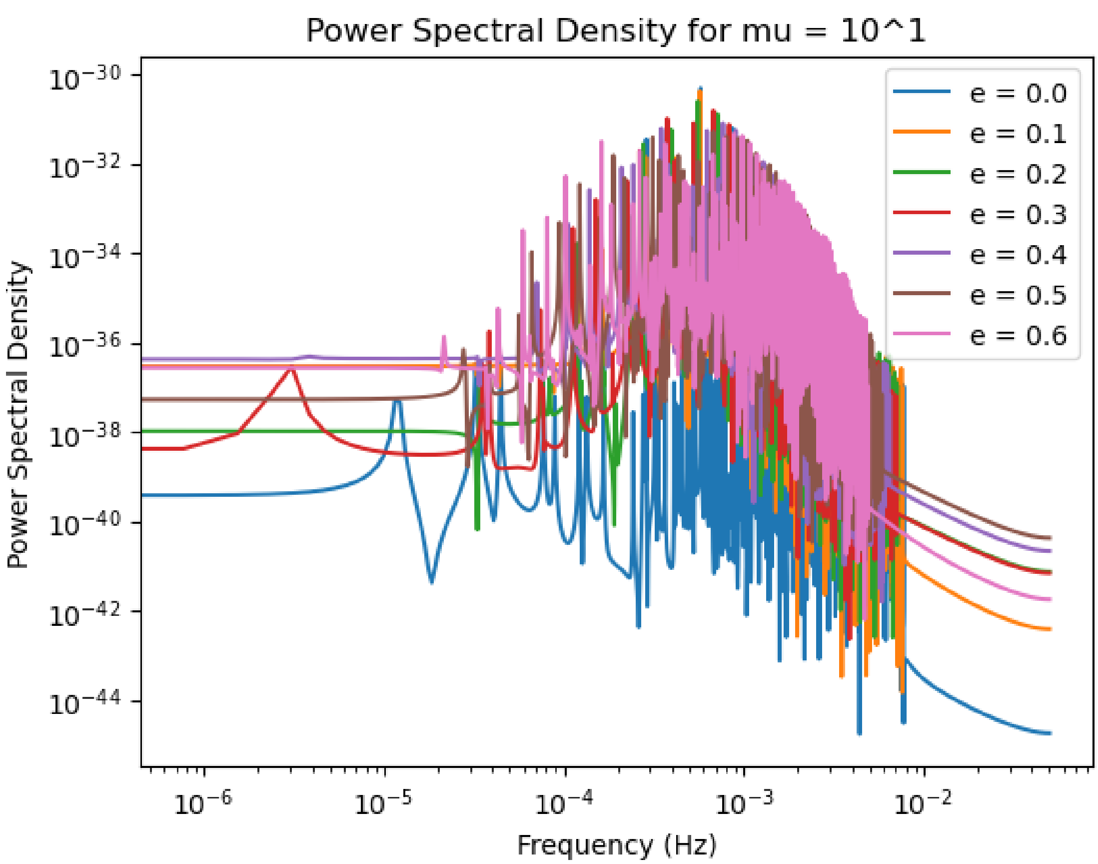
```
/home/masrukuddin/anaconda3/envs/few_env/lib/python3.7/site-
packages/ipykernel_launcher.py:27: RuntimeWarning: divide by zero encountered in
true_divide
/home/masrukuddin/anaconda3/envs/few_env/lib/python3.7/site-
packages/ipykernel_launcher.py:28: RuntimeWarning: divide by zero encountered in
true_divide
```



[ ]:
```
# mu to be from 10^-7 to 10^2
power_spectral_density =␣
  ↪power_spectral_density_RCLfit(freq[positive_frequency_mask])
plt.figure()

for E in range(0, 7, 1):
```

```
    y=sensitivitYimposed(10**-3, 9, E/10,15/365)
    plt.loglog(y[0].get(), y[1].get(), label=f'e = {E/10}')
    plt.legend()
# plt.loglog(freq[positive_frequency_mask], power_spectral_density, label='LISA␣
↪Sensitivity Curve')
plt.xlabel('Frequency (Hz)')
plt.ylabel('Power Spectral Density')
plt.title(f'Power Spectral Density for mu = 10^{mu}')
plt.show()
```

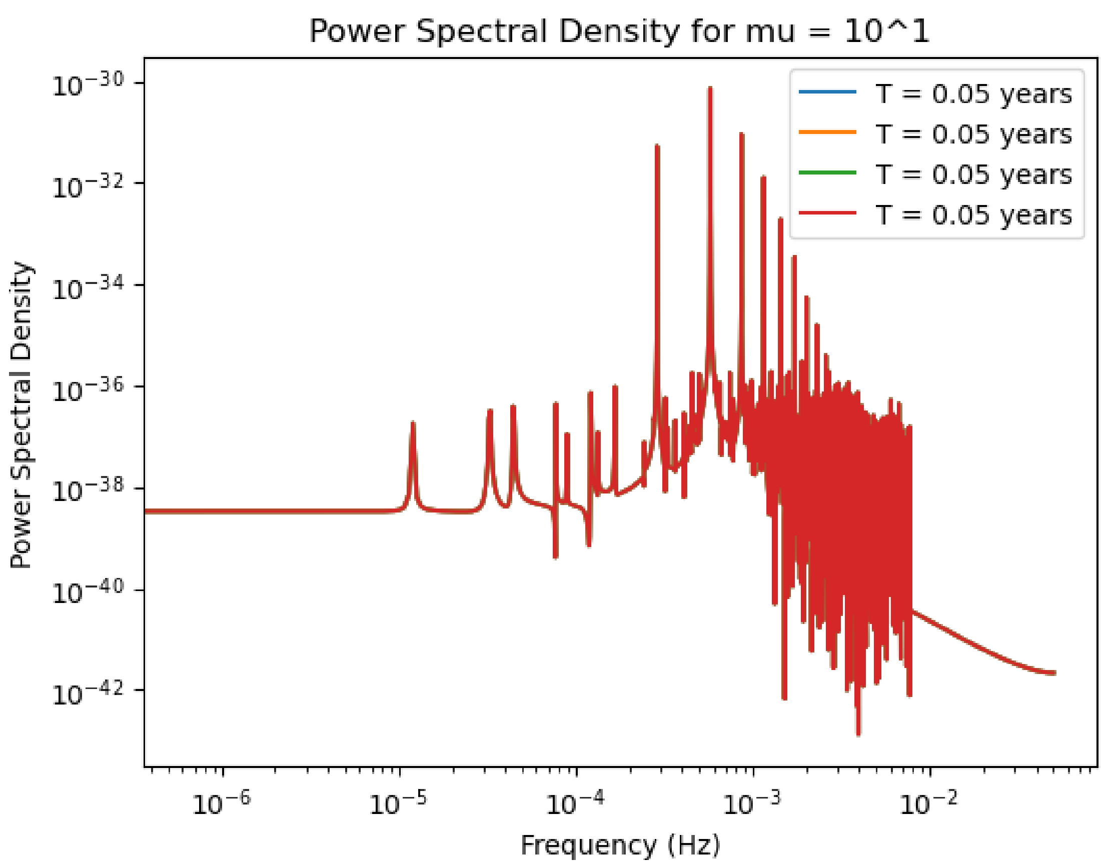
```
/home/masrukuddin/anaconda3/envs/few_env/lib/python3.7/site-
packages/ipykernel_launcher.py:27: RuntimeWarning: divide by zero encountered in
true_divide
/home/masrukuddin/anaconda3/envs/few_env/lib/python3.7/site-
packages/ipykernel_launcher.py:28: RuntimeWarning: divide by zero encountered in
true_divide
```



[ ]:
```
power_spectral_density =␣
↪power_spectral_density_RCLfit(freq[positive_frequency_mask])
plt.figure()
```

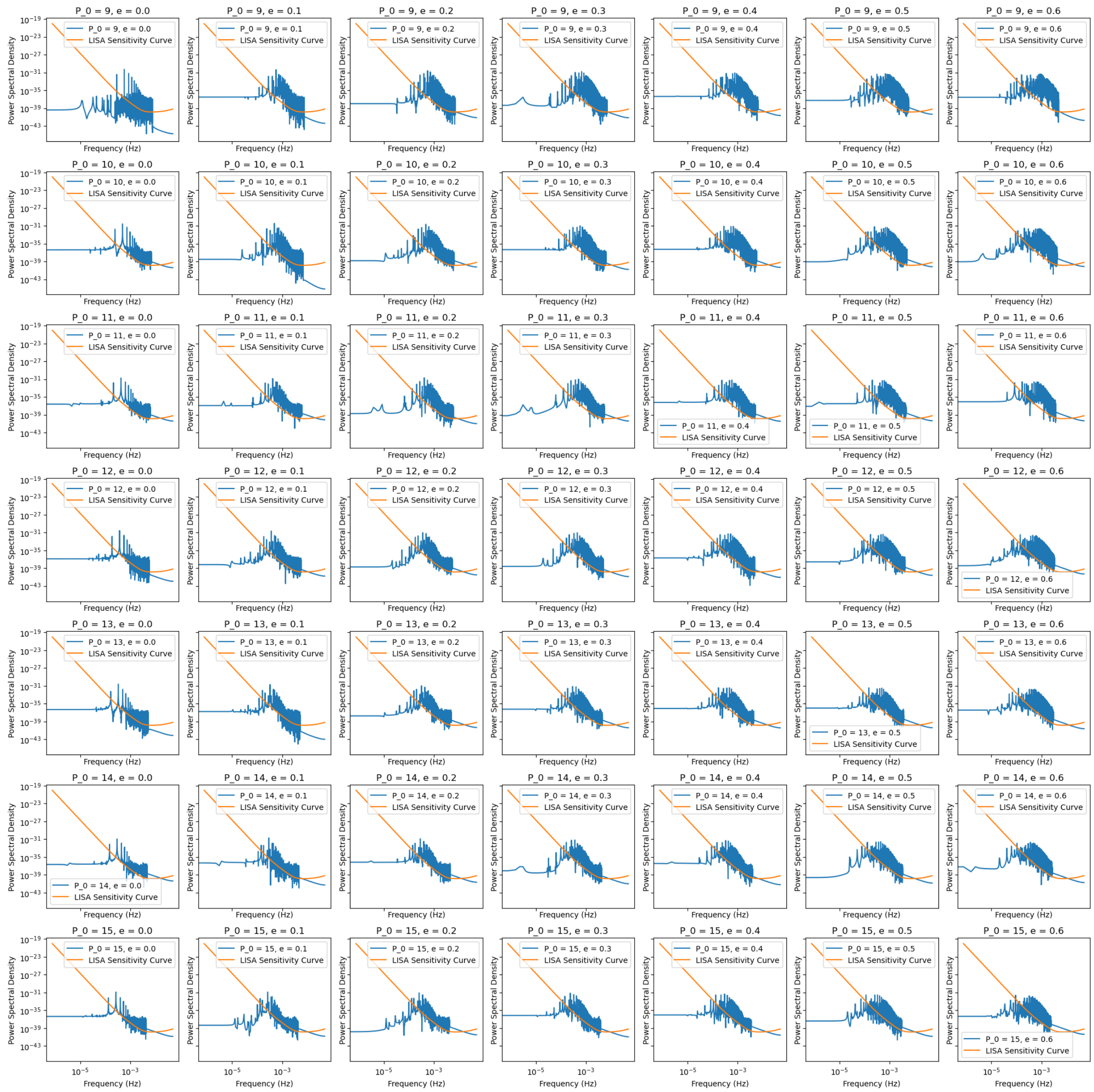
```
for E in range(2, 10, 2):
    y=sensitivitYimposed(10**-3, 9, 0,T/10)
    plt.loglog(y[0].get(), y[1].get(), label=f'T = {T/10} years')
    plt.legend()
# plt.loglog(freq[positive_frequency_mask], power_spectral_density, label='LISA␣
 ↪Sensitivity Curve')
plt.xlabel('Frequency (Hz)')
plt.ylabel('Power Spectral Density')
plt.title(f'Power Spectral Density for mu = 10^{mu}')
plt.show()
```

```
/home/masrukuddin/anaconda3/envs/few_env/lib/python3.7/site-
packages/ipykernel_launcher.py:27: RuntimeWarning: divide by zero encountered in
true_divide
/home/masrukuddin/anaconda3/envs/few_env/lib/python3.7/site-
packages/ipykernel_launcher.py:28: RuntimeWarning: divide by zero encountered in
true_divide
```



[ ]:

```
import numpy as np
import matplotlib.pyplot as plt




# freq = np.logspace(-4, 1, 100)
positive_frequency_mask = freq > 0
power_spectral_density =␣
 ↪power_spectral_density_RCLfit(freq[positive_frequency_mask])

P0_values = np.arange(9, 16, 1)
e_values = np.arange(0, 7, 1) / 10

# Create a grid of subplots
fig, axes = plt.subplots(len(P0_values), len(e_values), figsize=(20, 20),␣
 ↪sharex=True, sharey=True)

# Iterate over combinations of P0 and e
for i, P0 in enumerate(P0_values):
    for j, e in enumerate(e_values):
        ax = axes[i, j]
        y = sensitivitYimposed(10**-3, P0, e, 15/365)
        ax.loglog(y[0].get(), y[1].get(), label=f'P_0 = {P0}, e = {e}')
        ax.loglog(freq[positive_frequency_mask], power_spectral_density,␣
 ↪label='LISA Sensitivity Curve')
        ax.legend()
        ax.set_xlabel('Frequency (Hz)')

        ax.set_ylabel('Power Spectral Density')
        ax.set_title(f'P_0 = {P0}, e = {e}')

plt.tight_layout()
plt.show()
```



[ ]: