Digital Design & Computer Architecture

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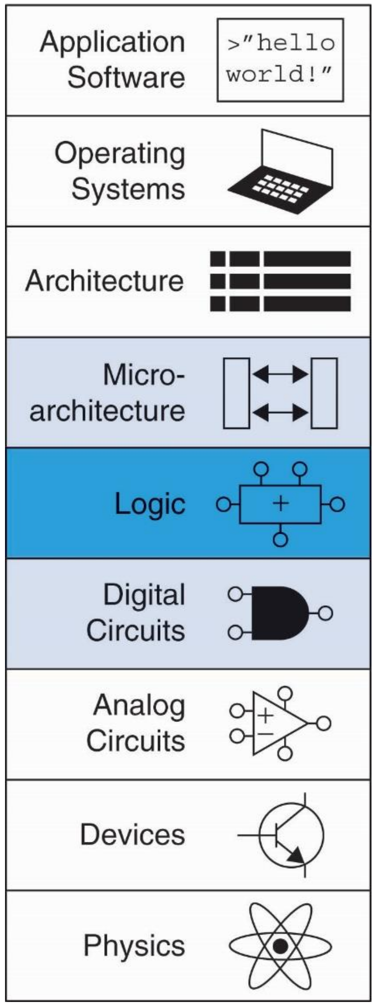
Chapter 5: Digital Building Blocks

Chapter 5 :: Topics

- **Introduction**
- **Arithmetic Circuits**
- **Number Systems**
- **Sequential Building Blocks**
- **Memory Arrays**
- **Logic Arrays**



Introduction

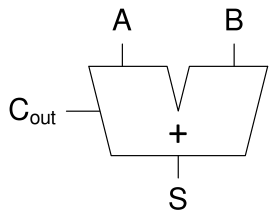
- **Digital building blocks:**
  - Gates, multiplexers, decoders, registers, arithmetic circuits, counters, memory arrays, logic arrays
- **Building blocks demonstrate hierarchy, modularity, and regularity:**
  - Hierarchy of simpler components
  - Well-defined interfaces and functions
  - Regular structure easily extends to different sizes
- **We'll use these building blocks in Chapter 7 to build a microprocessor**

Chapter 5: Digital Building Blocks

Adders

1-Bit Adders

Half Adder

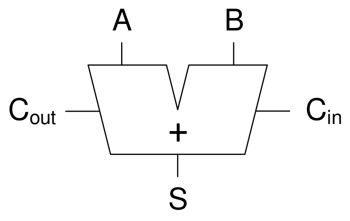


A	B	C_{out}	S
0	0		
0	1		
1	0		
1	1		

S =

C_{out} =

Full Adder



C_{in}	A	B	C_{out}	S
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

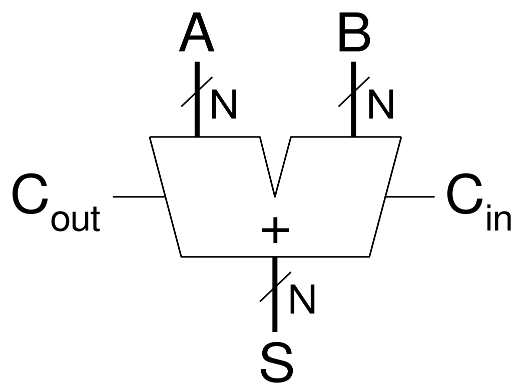
S =

C_{out} =

Multibit Adders: CPAs

- Types of carry propagate adders (CPAs):
  - **Ripple-carry** (slow)
  - **Carry-lookahead** (fast)
  - **Prefix** (faster)
- Carry-lookahead and prefix adders faster for large adders but require more hardware

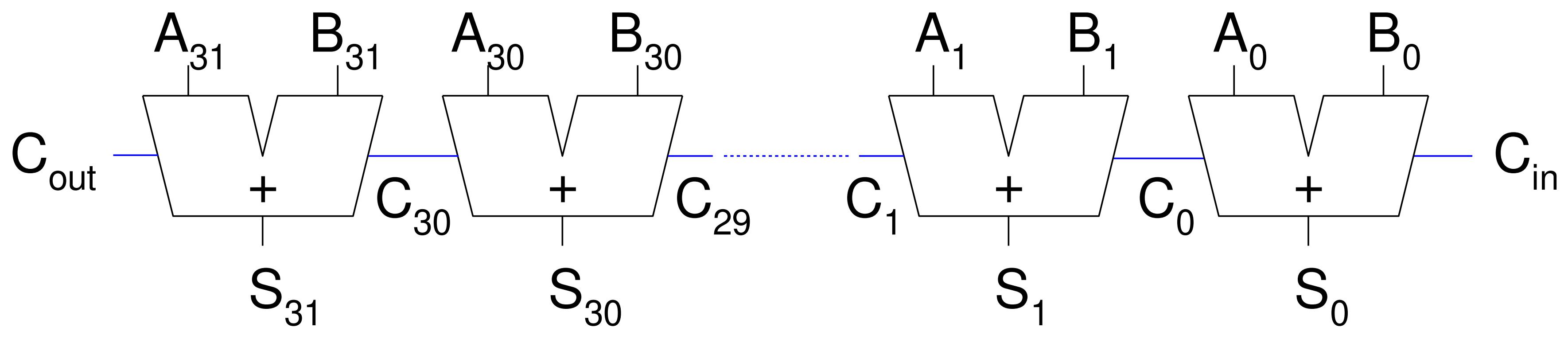
Symbol



Ripple Carry Addition

Ripple-Carry Adder

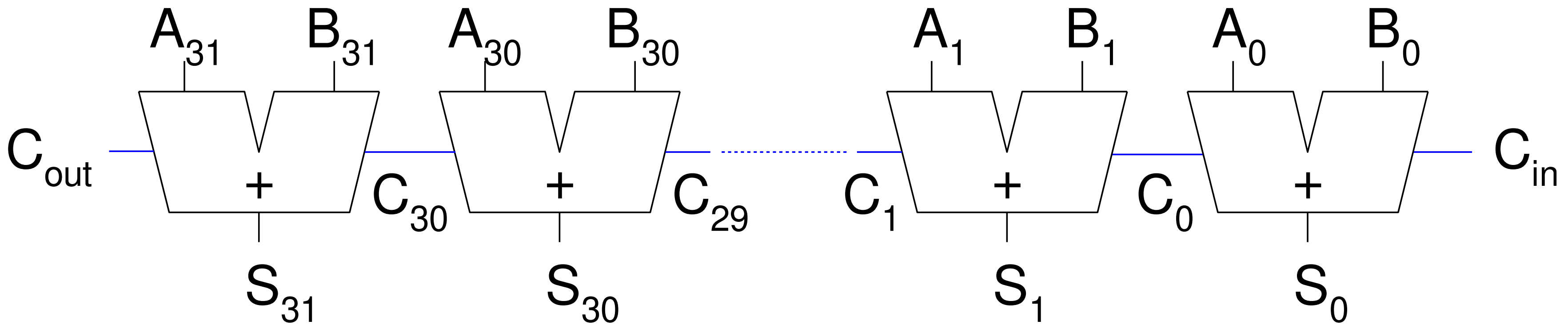
- Chain 1-bit adders together
- Carry ripples through entire chain
- Disadvantage: **slow**



Ripple-Carry Adder Delay

$$t_{\text{ripple}} =$$

where t_{FA} is the delay of a 1-bit full adder



Chapter 5: Digital Building Blocks

Carry Lookahead Addition

Carry-Lookahead Adder

Compute C_{out} for *k*-bit blocks using *generate* and *propagate* signals

Some definitions:

- Column *i* produces a carry out by either ***generating*** a carry out or ***propagating*** a carry in to the carry out
- Calculate generate (G_i) and propagate (P_i) signals for each column:
  - **Generate:** Column *i* will generate a carry out if A_i **and** B_i are both 1.

$$G_i = A_i B_i$$

  - **Propagate:** Column *i* will propagate a carry in to the carry out if A_i **or** B_i is 1.

$$P_i = A_i + B_i$$

  - **Carry out:** The carry out of column *i* (C_i) is:

$$C_i = A_i B_i + (A_i + B_i) C_{i-1} = G_i + P_i C_{i-1}$$

Propagate and Generate Signals

Examples: Column propagate and generate signals:

Column propagate: $P_i = A_i + B_i$

Column generate: $G_i = A_i B_i$

```
   1011      A3:0
 + 0110      B3:0
 ------
```

```
   1011      A3:0
 + 1001      B3:0
 ------
```

Block Propagate and Generate

Now use column Propagate and Generate signals to compute **Block Propagate** and **Block Generate** signals for k-bit blocks, i.e.:

- Compute if a **k-bit group** will **propagate** a carry in (of the block) to the carry out (of the block)
- Compute if a **k-bit group** will **generate** a carry out (of the block)

Block Propagate and Generate

- **Example:** 4-bit blocks
  - **Block propagate signal:** $P_{3:0}$ (single-bit signal)
    - A carry-in would propagate through all 4 bits of the block:

      $$P_{3:0} = P_3 P_2 P_1 P_0$$

    - **Examples:**

```
  1011
+ 0100
------
```

```
  1011
+ 0001
------
```

Block Propagate and Generate

- **Example:** 4-bit blocks
  - **Block propagate signal:** $P_{3:0}$ (single-bit signal)
    - A carry-in would propagate through all 4 bits of the block:

$$P_{3:0} = P_3 P_2 P_1 P_0$$

  - **Block generate signal:** $G_{3:0}$ (single-bit signal)
    - A carry is generated:
      - in column 3, **or**
      - in column 2 and propagated through column 3, **or**
      - in column 1 and propagated through columns 2 and 3, **or**
      - in column 0 and propagated through columns 1-3

$$G_{3:0} = G_3 + G_2 P_3 + G_1 P_2 P_3 + G_0 P_1 P_2 P_3$$

$$G_{3:0} = G_3 + P_3\,[G_2 + P_2\,(G_1 + P_1 G_0\,)]$$

Block Propagate and Generate

- **Example:** 4-bit blocks
  - **Block generate signal:** $G_{3:0}$ (single-bit signal)
    - A carry is: generated in column 3, **or** generated in column 2 and propagated through column 3, **or** ...

$$G_{3:0} = G_3 + G_2P_3 + G_1P_2P_3 + G_0P_1P_2P_3$$

$$\begin{array}{r} 1001 \\ +\ 1100 \\ \hline \end{array} \qquad \begin{array}{r} 1110 \\ +\ 0100 \\ \hline \end{array} \qquad \begin{array}{r} 0110 \\ +\ 0010 \\ \hline \end{array}$$

Block Propagate and Generate

- **Example:** 4-bit blocks
  - **Block propagate signal:** $\boldsymbol{P_{3:0}}$ (single-bit signal)
    - A carry-in would propagate through all 4 bits of the block:

$$P_{3:0} = P_3P_2P_1P_0$$

  - **Block generate signal:** $\boldsymbol{G_{3\text{-}0}}$ (single-bit signal)
    - A carry is generated:
      - in column 3, **or**
      - in column 2 and propagated through column 3, **or**
      - in column 1 and propagated through columns 2 and 3, **or**
      - in column 0 and propagated through columns 1-3

$$G_{3:0} = G_3 + G_2P_3 + G_1P_2P_3 + G_0P_1P_2P_3$$

$$G_{3:0} = G_3 + P_3\,[G_2 + P_2\,(G_1 + P_1G_0\,)]$$

$$\boxed{C_3 = G_{3:0} + P_{3:0}\,C_{-1}}$$

Block Propagate and Generate

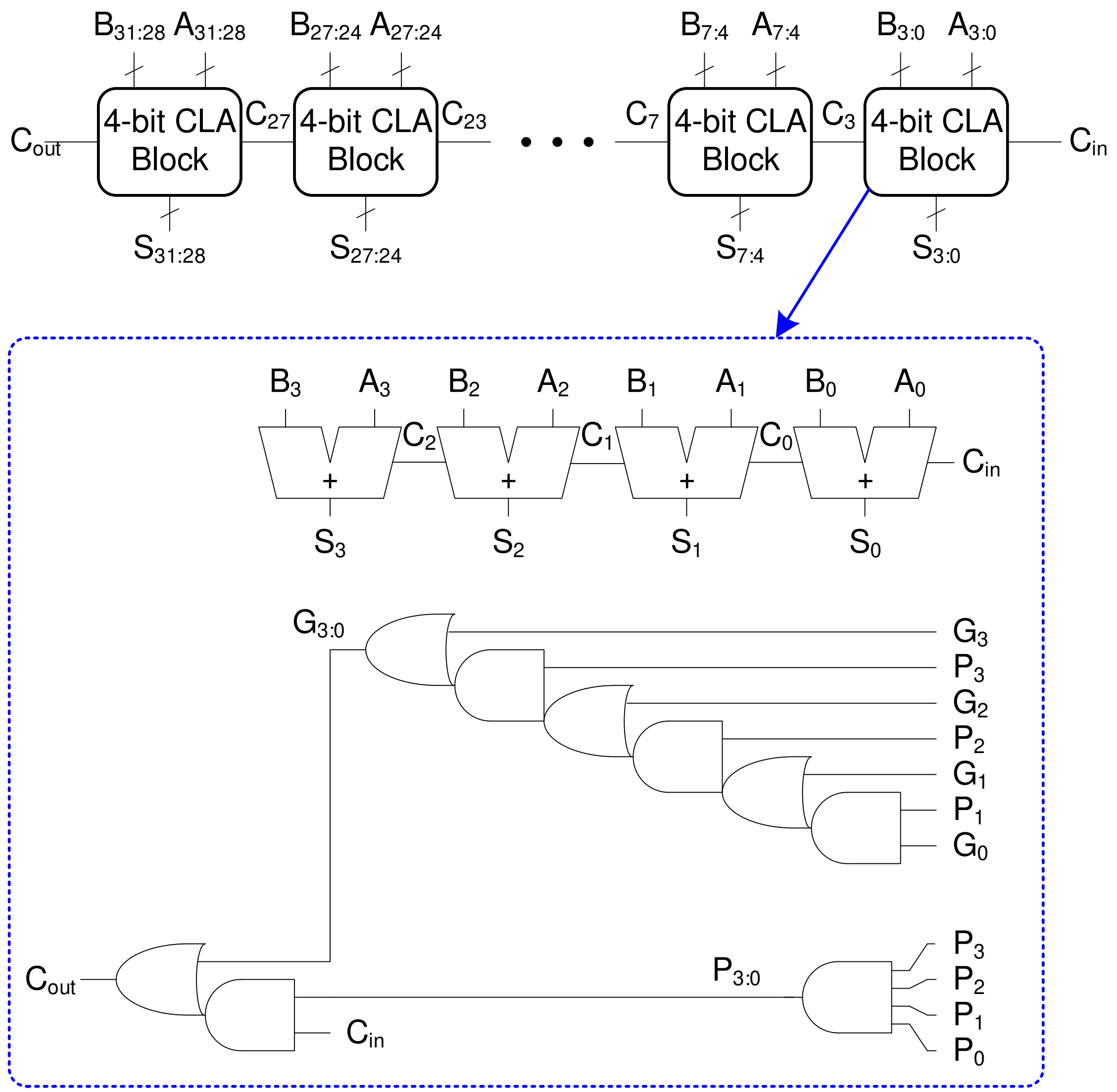
- **Example:** Block propagate and generate signals for 4-bit blocks ($P_{3:0}$ and $G_{3:0}$):

$$P_{3:0} = P_3 P_2 P_1 P_0$$

$$G_{3:0} = G_3 + P_3 (G_2 + P_2 (G_1 + P_1 G_0)$$

$$C_3 = G_{3:0} + P_{3:0} C_{-1}$$

32-bit CLA with 4-bit Blocks



Carry-Lookahead Addition

- **Step 1:** Compute G_i and P_i for all columns
- **Step 2:** Compute G and P for k-bit blocks
- **Step 3:** C_{in} propagates through each k-bit propagate/generate logic (meanwhile computing sums)
- **Step 4:** Compute sum for most significant k-bit block

Carry-Lookahead Addition

- **Step 1:** Compute G_i and P_i for all columns

$$G_i = A_i B_i$$
$$P_i = A_i + B_i$$

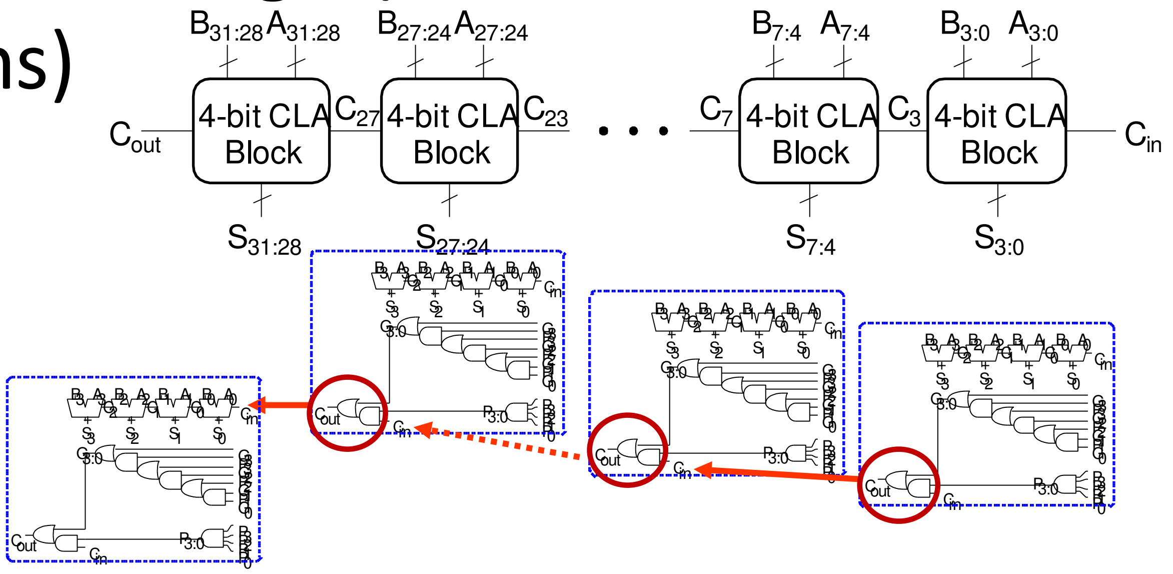
Carry-Lookahead Addition

- **Step 1:** Compute G_i and P_i for all columns
- **Step 2:** Compute G and P for k-bit blocks

$$P_{3:0} = P_3 P_2 P_1 P_0$$
$$G_{3:0} = G_3 + P_3 \left(G_2 + P_2 \left(G_1 + P_1 G_0\right)\right)$$

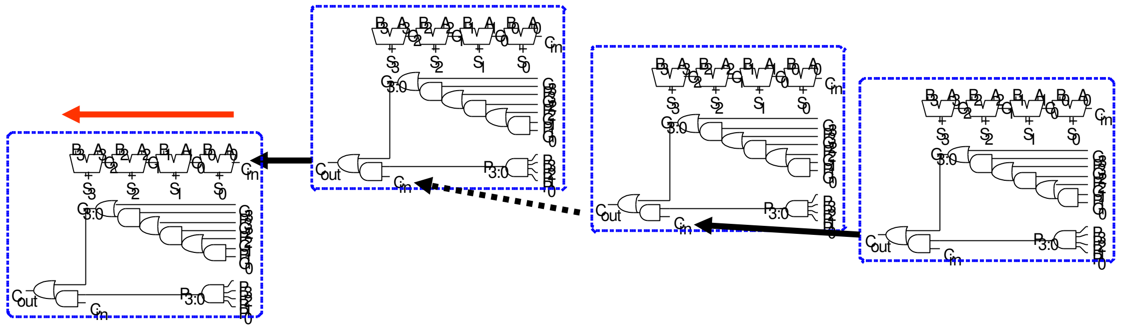
Carry-Lookahead Addition

- **Step 1:** Compute G_i and P_i for all columns
- **Step 2:** Compute G and P for k-bit blocks
- **Step 3:** C_{in} propagates through each k-bit propagate/generate logic (meanwhile computing sums)

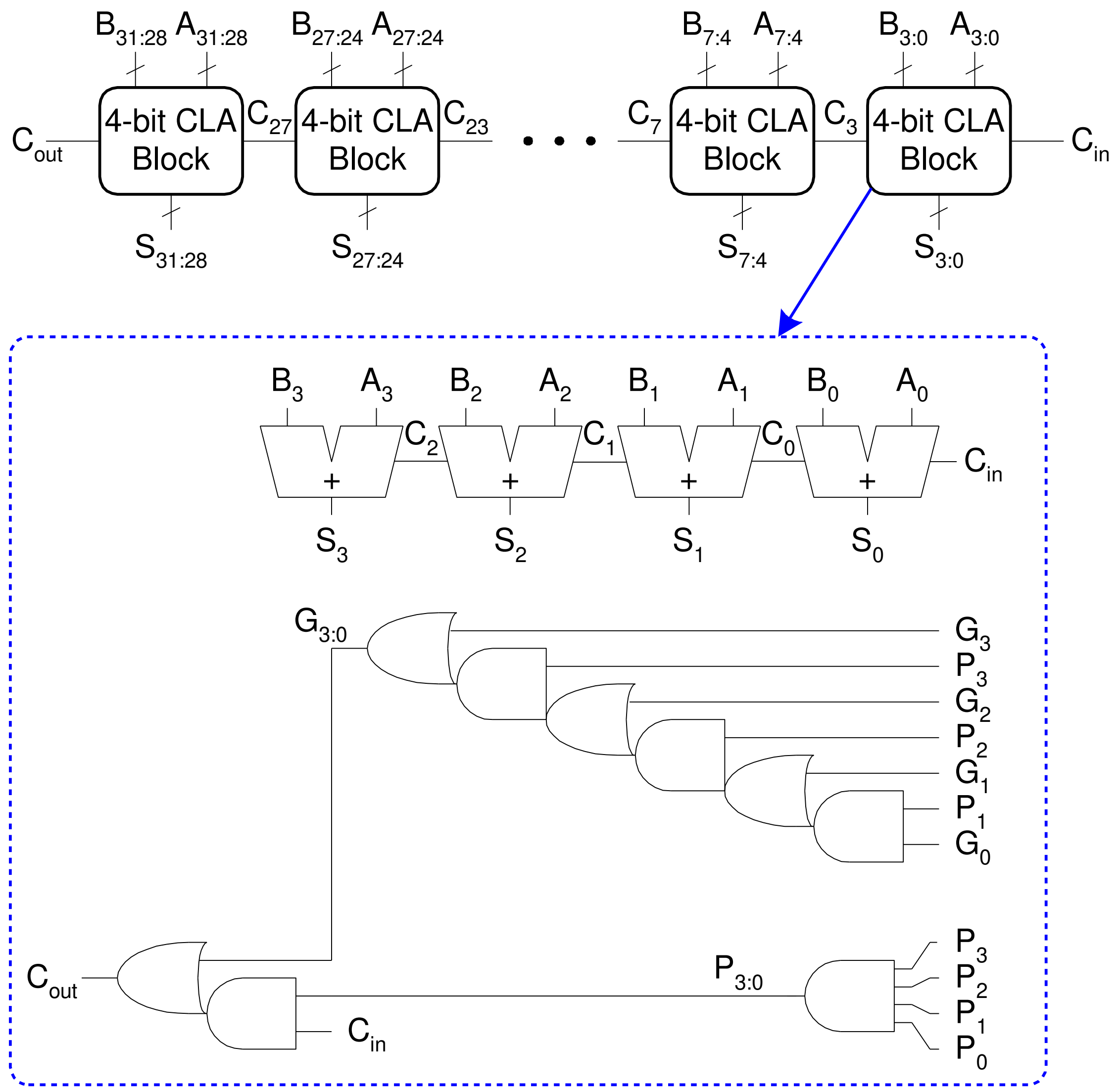


Carry-Lookahead Addition

- **Step 1:** Compute G_i and P_i for all columns
- **Step 2:** Compute *G* and *P* for *k*-bit blocks
- **Step 3:** C_{in} propagates through each *k*-bit propagate/generate logic (meanwhile computing sums)
- **Step 4:** Compute sum for most significant k-bit block



32-bit CLA with 4-bit Blocks



Carry-Lookahead Adder Delay

For *N*-bit CLA with *k*-bit blocks:

$$t_{CLA} = t_{pg} + t_{pg\_block} + (N/k - 1)t_{\text{AND\_OR}} + kt_{FA}$$

- t_{pg} : delay to generate all P_i, G_i
- t_{pg_block} : delay to generate all $P_{i:j}$, $G_{i:j}$
- $t_{\text{AND\_OR}}$: delay from C_{in} to C_{out} of final AND/OR gate in *k*-bit CLA block

An *N*-bit carry-lookahead adder is generally much faster than a ripple-carry adder for *N* > 16

Prefix Addition

Prefix Adder

- Computes carry in (C_{i-1}) for each column, then computes sum:

$$S_i = (A_i \oplus B_i) \oplus C_{i-1}$$

- It computes C_{i-1} by:
  - Computing G and P for 1-, 2-, 4-, 8-bit blocks, etc. until all G_i (carry in) known
  - $G_i = C_i$
- $\log_2 N$ stages

Prefix Adder

- Carry out either *generated* in a column or *propagated* from a previous column.
- Column -1 holds C_{in}, so

  $$G_{-1} = C_{in}, P_{-1} = \text{X (not used)}$$

- Carry in to column *i* = carry out of column *i-1*:

  $$C_{i-1} = G_{i-1:-1}$$

  $G_{i-1:-1}$: generate signal spanning columns *i*-1 to -1

- Sum equation:

  $$S_i = (A_i \oplus B_i) \oplus G_{i-1:-1}$$

- **Goal:** Quickly compute $G_{0:-1}, G_{1:-1}, G_{2:-1}, G_{3:-1}, G_{4:-1}, G_{5:-1}, \ldots$ (called ***prefixes***) (= $C_0, C_1, C_2, C_3, C_4, C_5, \ldots$)

Prefix Adder

- Generate and propagate signals for a block spanning bits $i{:}j$

  $$G_{i:j} = G_{i:k} + P_{i:k}\, G_{k-1:j}$$

  $$P_{i:j} = P_{i:k} P_{k-1:j}$$

- In words:
  - **Generate:** block $i{:}j$ will generate a carry if:
    - **upper part** $(i{:}k)$ generates a carry $(G_{i:k})$ **or**
    - upper part $(i{:}k)$ propagates a carry $(P_{i:k})$ generated in lower part $(k\text{-}1{:}j)$ $(G_{k-1:j})$
  - **Propagate:** block $i{:}j$ will propagate a carry if *both* the upper and lower parts propagate the carry $(P_{i:k}\, AND\ P_{k-1:j})$

Prefix Adder Example

Step 1. Calculate P's and G's for **1-bit block**

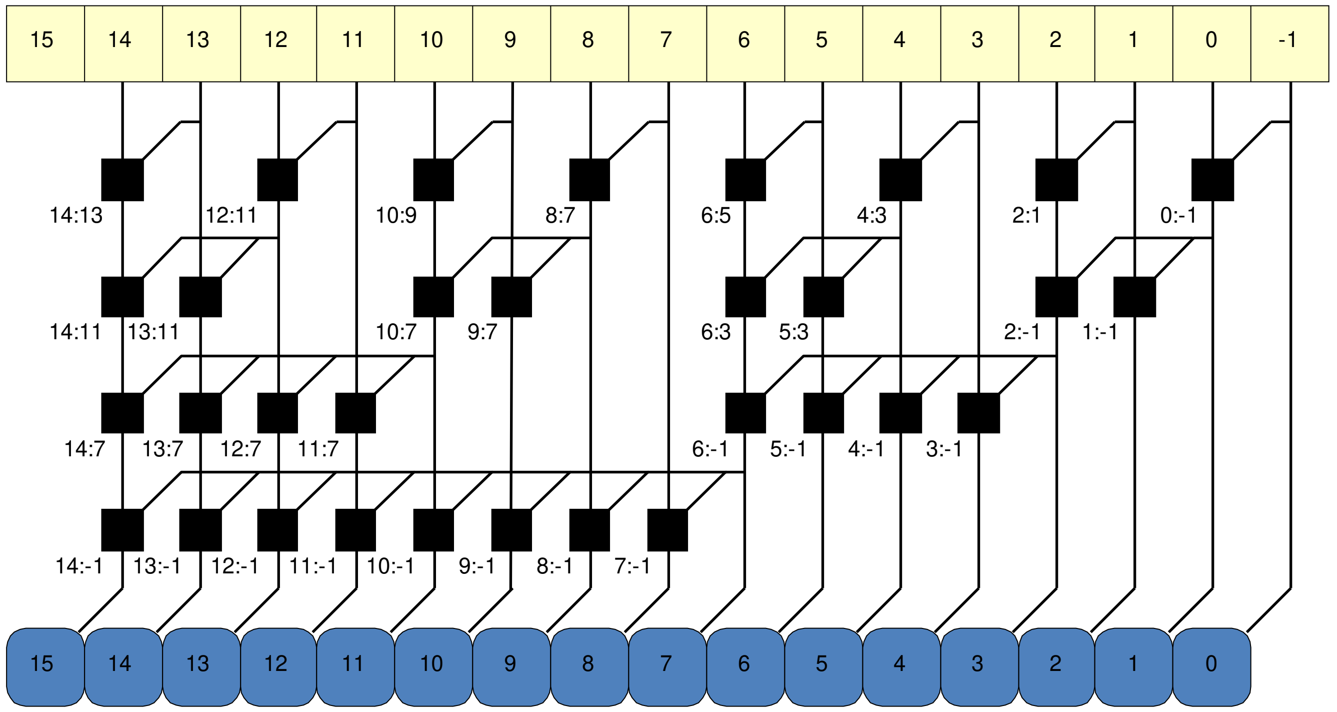
Step 2. Calculate P's and G's for **2-bit blocks**

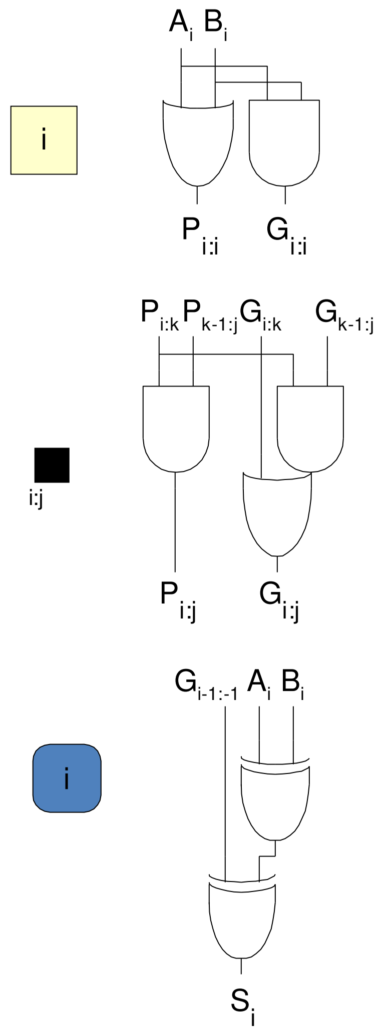
Step 3. Calculate P's and G's for **4-bit blocks**

Step 4. Continue to calculate P's and G's for larger blocks (8-bit, 16-bit, etc.)

Step 5. Use prefixes to calculate sums

Prefix Adder Schematic





Prefix Adder Delay

$$t_{PA} = t_{pg} + \log_2 N(t_{pg\_prefix}) + t_{XOR}$$

t_{pg}: delay to produce P_i, G_i (AND or OR gate)

t_{pg_prefix}: delay of black prefix cell (AND-OR gate)

Adder Delay Comparisons

Compare the delay of: 32-bit ripple-carry, CLA, and prefix adders

- CLA has 4-bit blocks
- 2-input gate delay = 100 ps; full adder delay = 300 ps

t_{ripple}

t_{CLA}

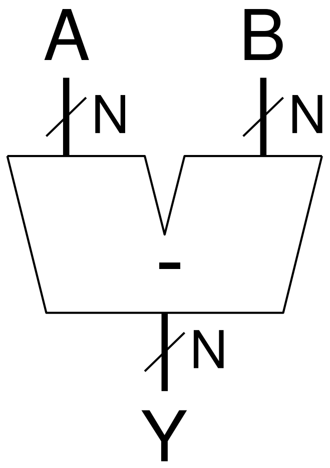
t_{PA}

Subtracters & Comparators

Subtracter

$$A - B = A + \overline{B} + 1$$

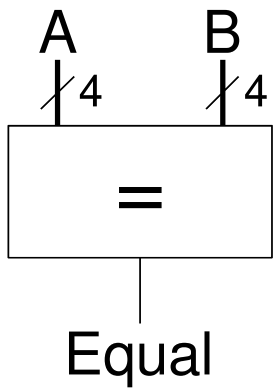
Symbol **Implementation**



Comparator: Equality

Symbol

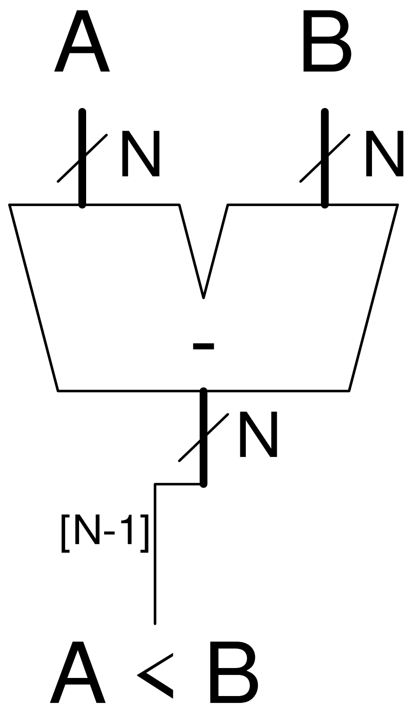
Implementation



Comparator: Signed Less Than

A < B if A-B is negative

Beware of overflow



Chapter 5: Digital Building Blocks

ALU:

Arithmetic Logic Unit

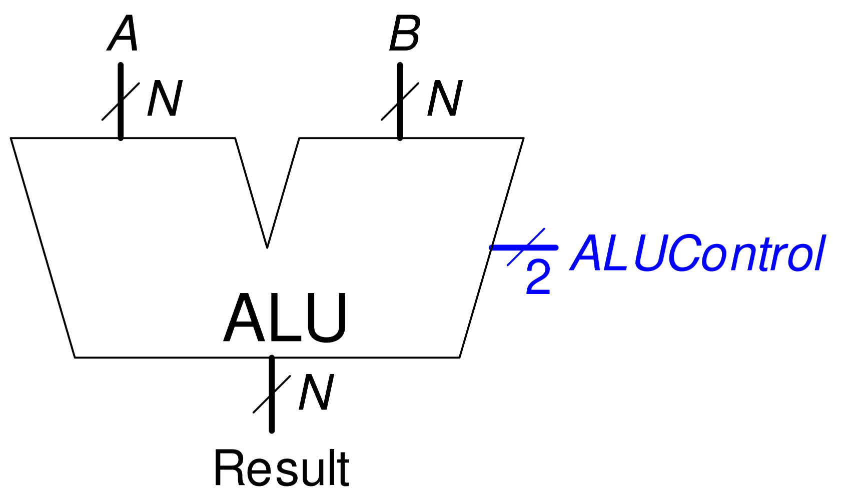
ALU: Arithmetic Logic Unit

ALU should perform:

- Addition
- Subtraction
- AND
- OR

ALU: Arithmetic Logic Unit

$ALUControl_{1:0}$	Function
00	Add
01	Subtract
10	AND
11	OR



Example: Perform *A* OR *B*

$ALUControl_{1:0} = 11$

Result* = *A* OR *B

ALU: Arithmetic Logic Unit

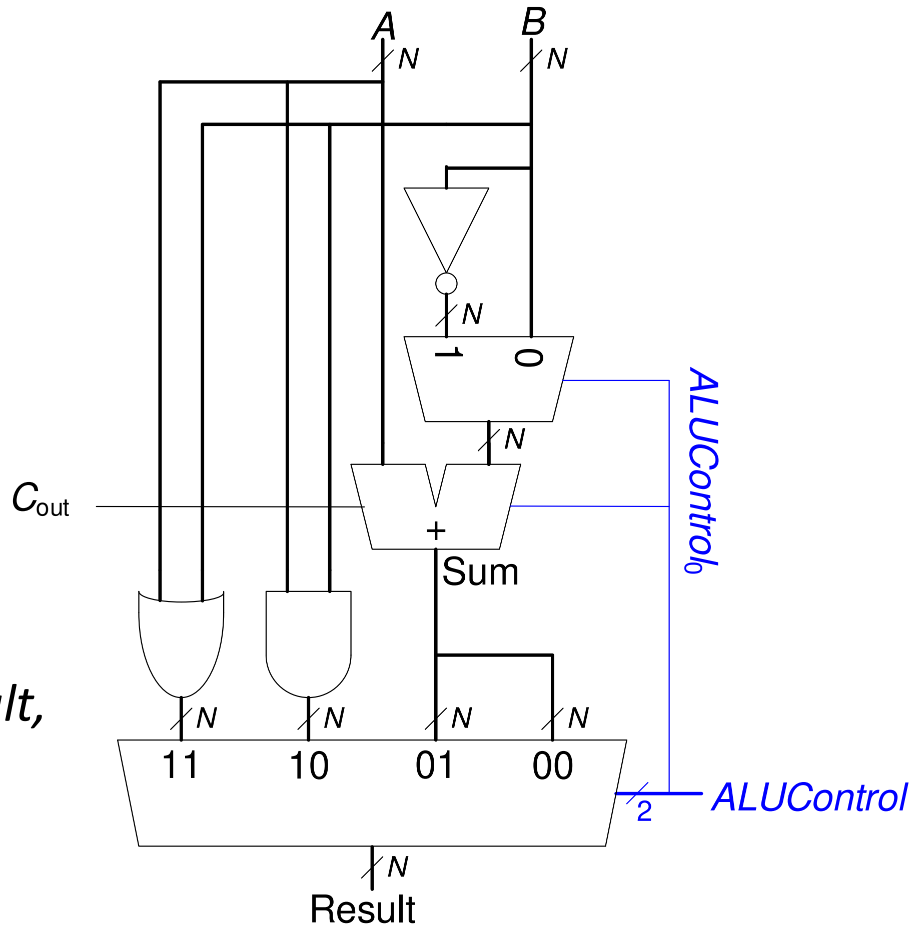
$ALUControl_{1:0}$	Function
00	Add
01	Subtract
10	AND
11	OR

Example: Perform *A* OR *B*

$ALUControl_{1:0}$ = 11

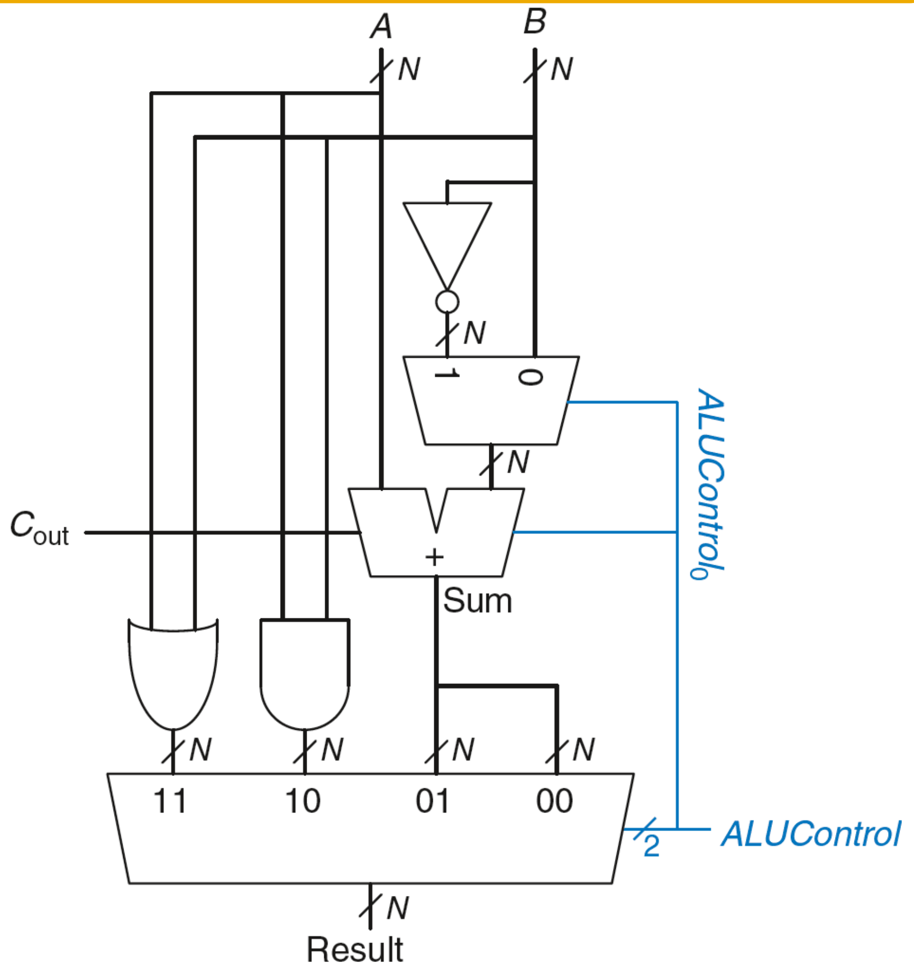
Mux selects output of OR gate as *Result*, so:

Result* = *A* OR *B



ALU: Arithmetic Logic Unit

$ALUControl_{1:0}$	Function
00	Add
01	Subtract
10	AND
11	OR



Example: Perform *A* + *B*

ALUControl$_{1:0}$ = 00

ALUControl$_0$ = 0, so:

  C_{in} to adder = 0

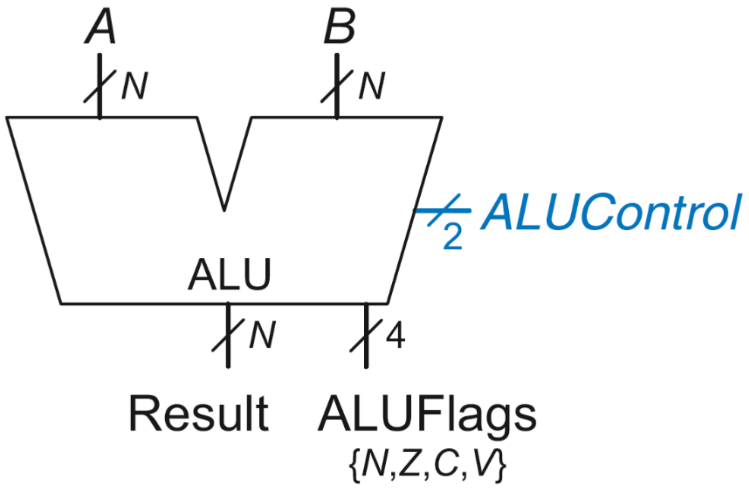
  2nd input to adder is *B*

Mux selects *Sum* as *Result,* so

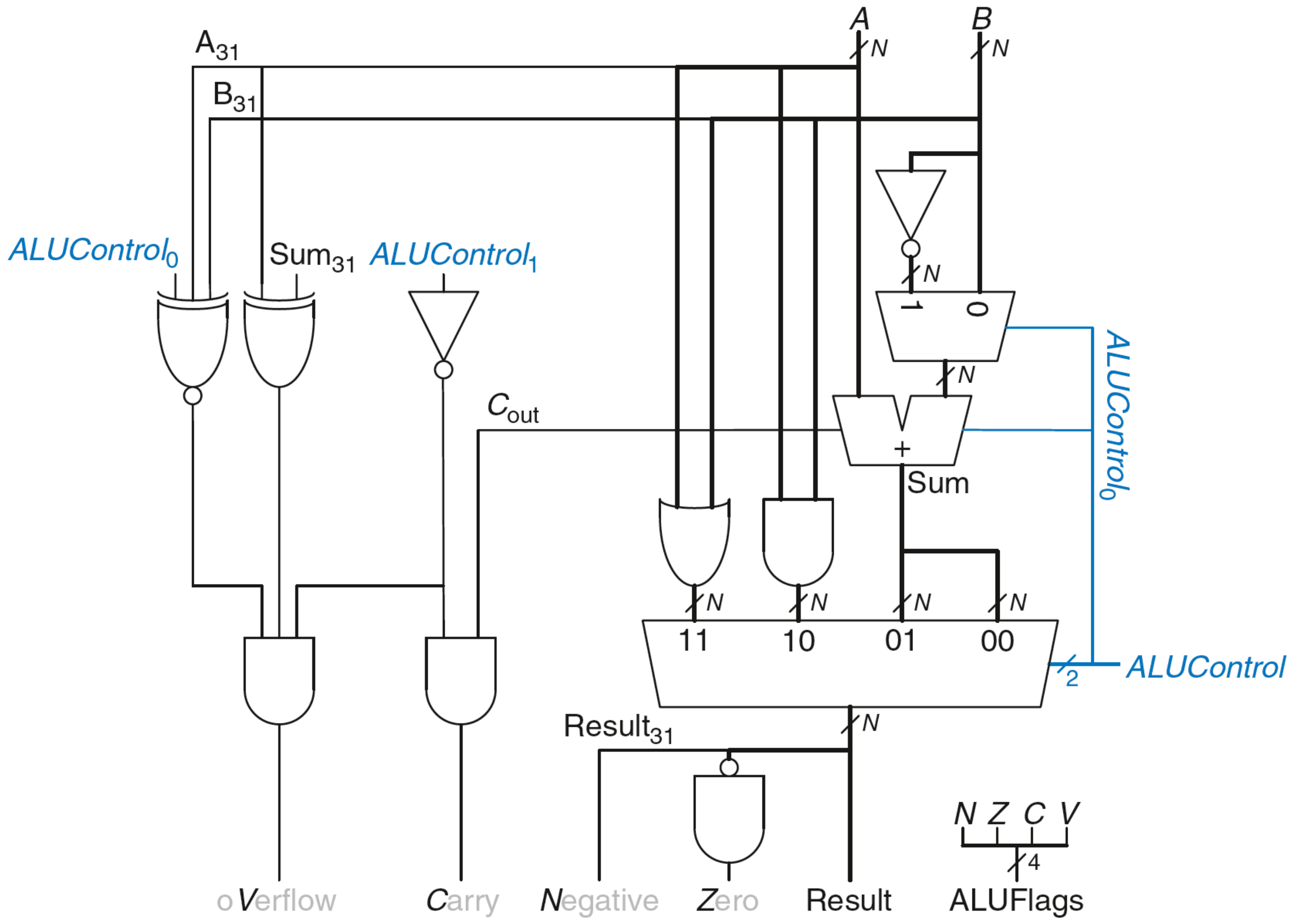
  Result* = *A* + *B

ALU with Status Flags

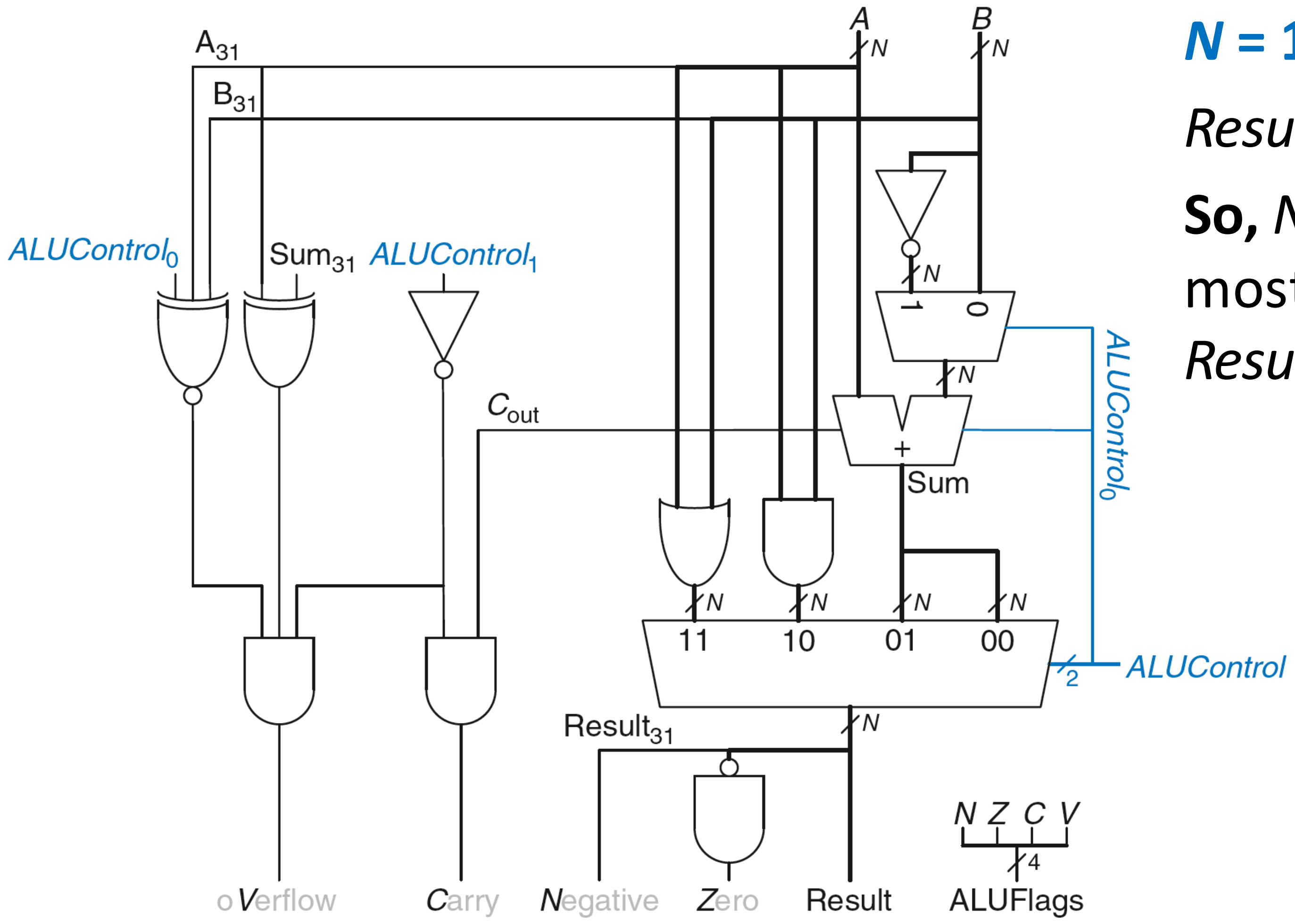
Flag	**Description**
N	*Result* is **N**egative
Z	*Result* is **Z**ero
C	Adder produces **C**arry out
V	Adder o**V**erflowed



ALU with Status Flags



ALU with Status Flags: **N**egative

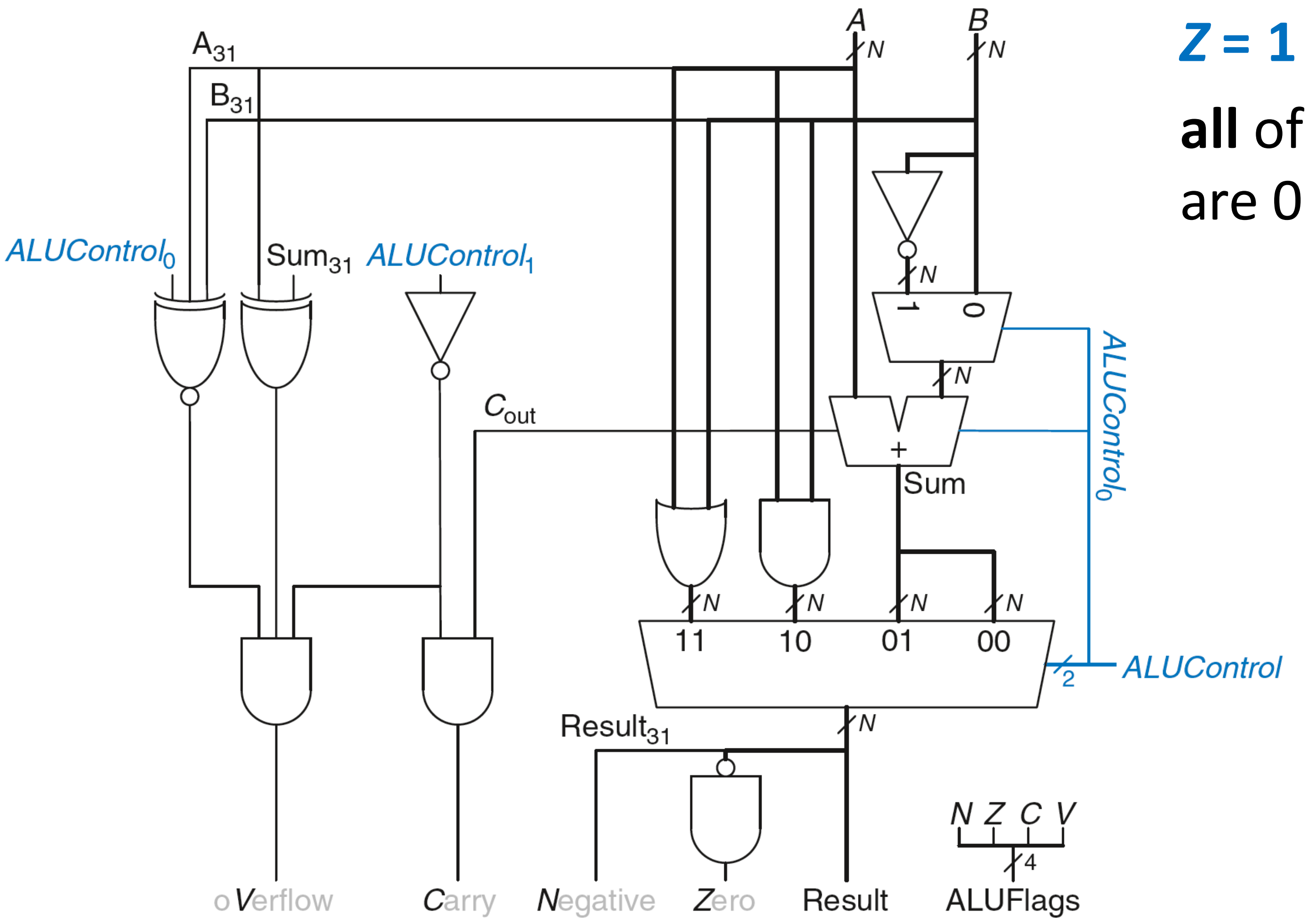


***N* = 1** if:

Result is **negative**

So, *N* is connected to most significant bit of *Result.*

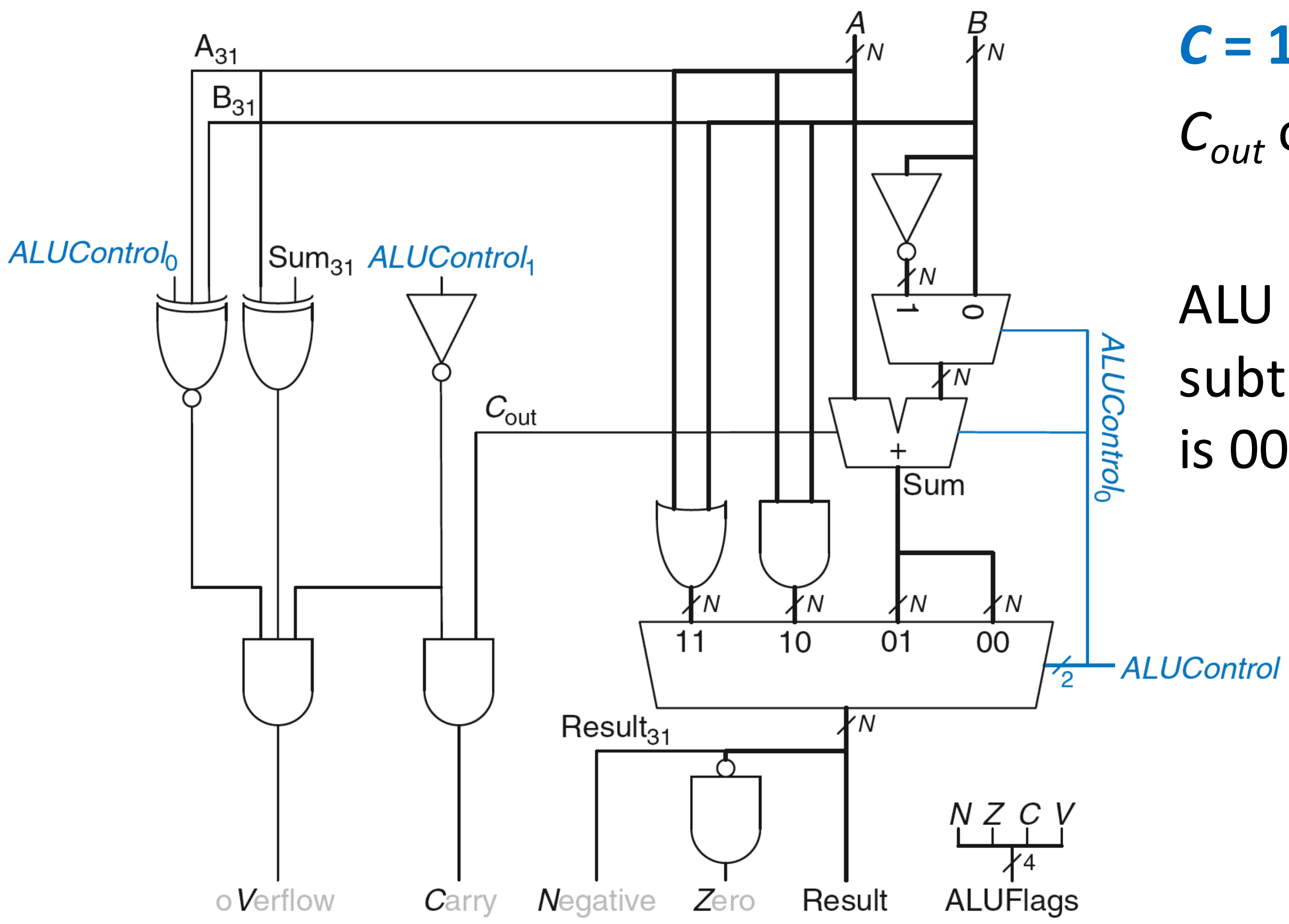
ALU with Status Flags: Zero



***Z* = 1** if:

all of the bits of *Result* are 0

ALU with Status Flags: Carry



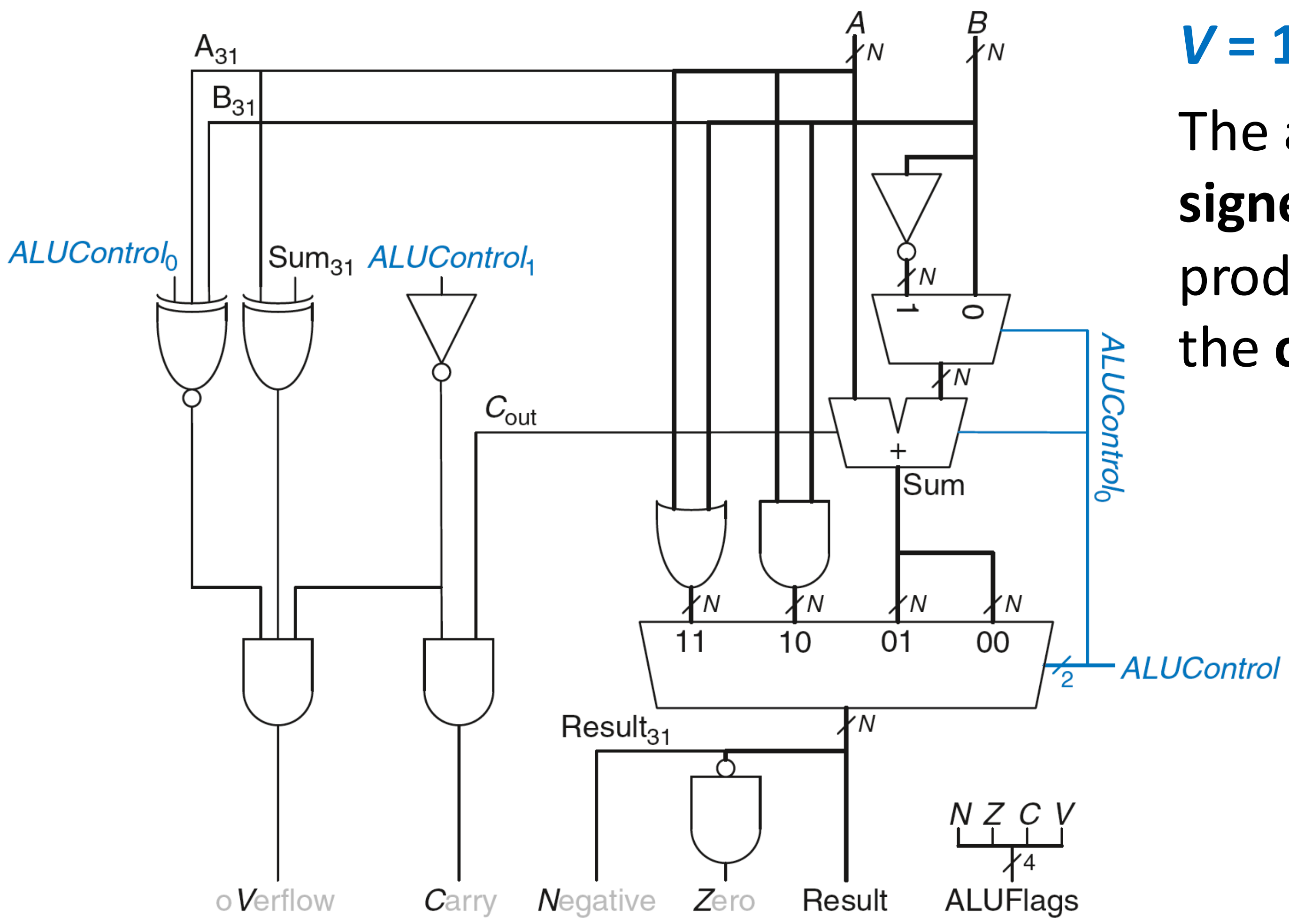
***C* = 1** if:

C_{out} of Adder is 1

AND

ALU is adding or subtracting (ALUControl is 00 or 01)

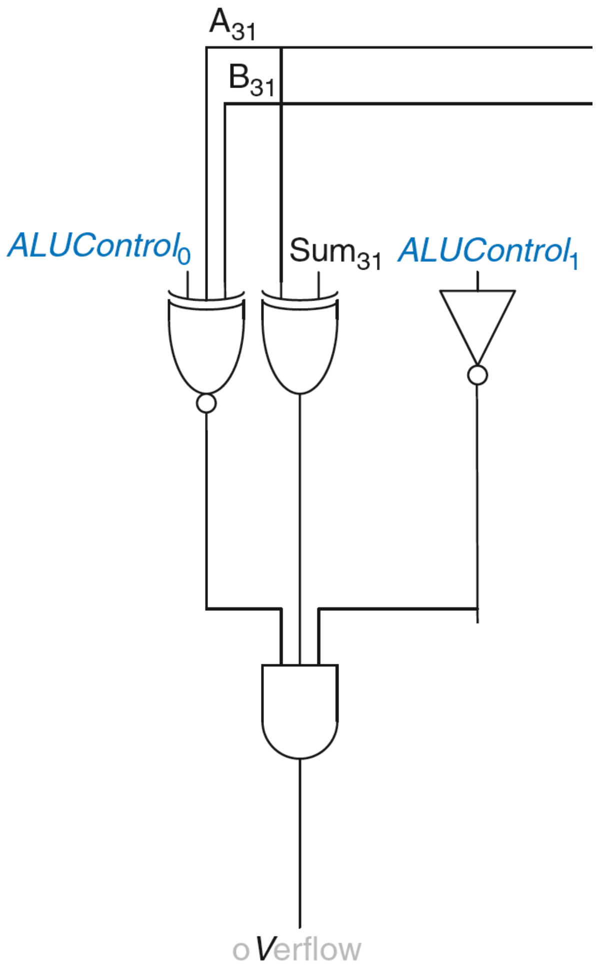
ALU with Status Flags: oVerflow



***V* = 1** if:

The addition of 2 **same-signed numbers** produces a result with the **opposite sign**

ALU with Status Flags: oVerflow



***V* = 1** if:

ALU is performing addition or subtraction

($ALUControl_1$ = 0)

AND

A and Sum have opposite signs

AND

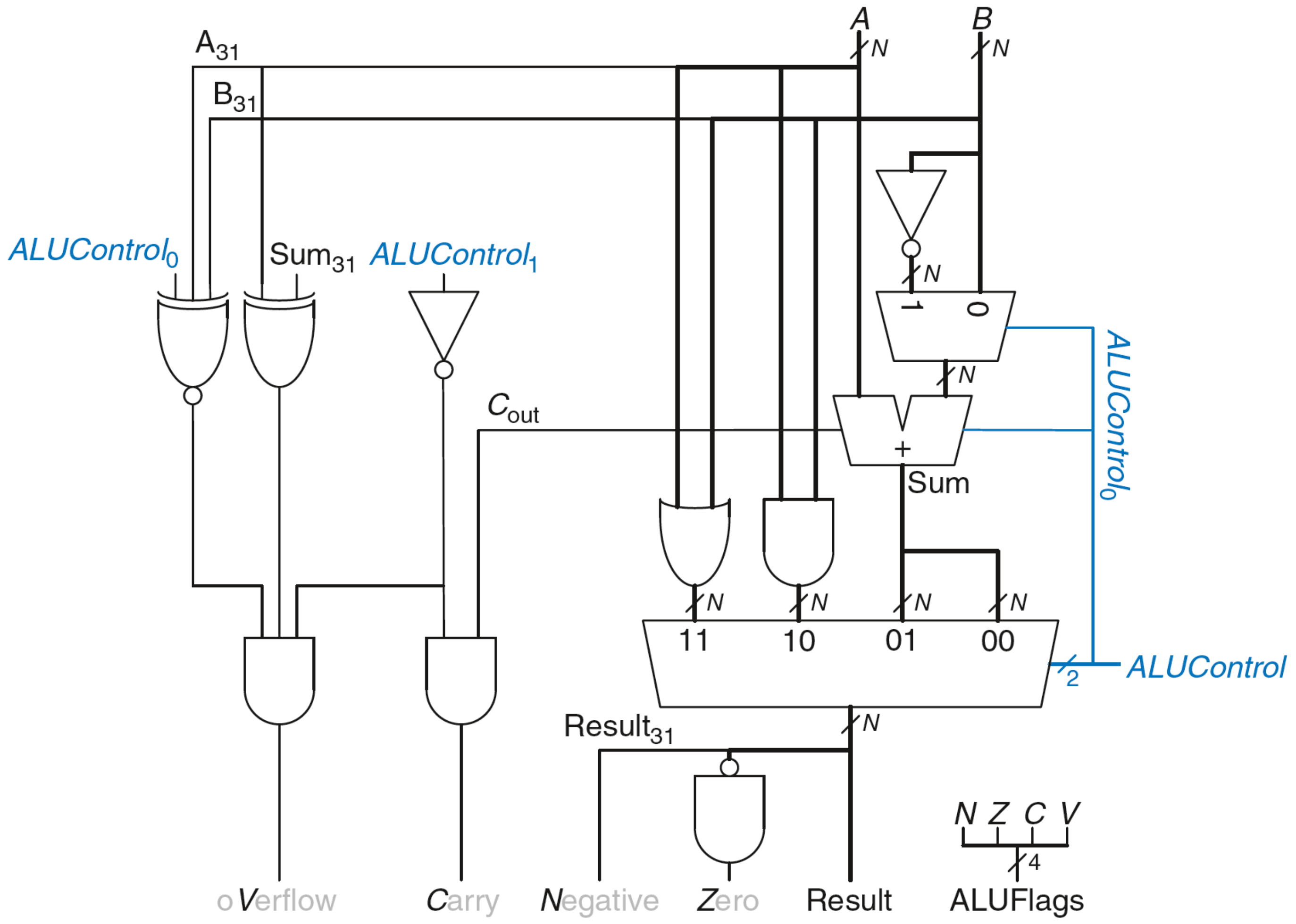
A and B have same signs for addition

($ALUControl_0$ = 0) **OR**

A and B have different signs for subtraction

($ALUControl_0$ = 1)

ALU with Status Flags



Comparison based on Flags

Compare by subtracting and checking flags

Different for signed and unsigned

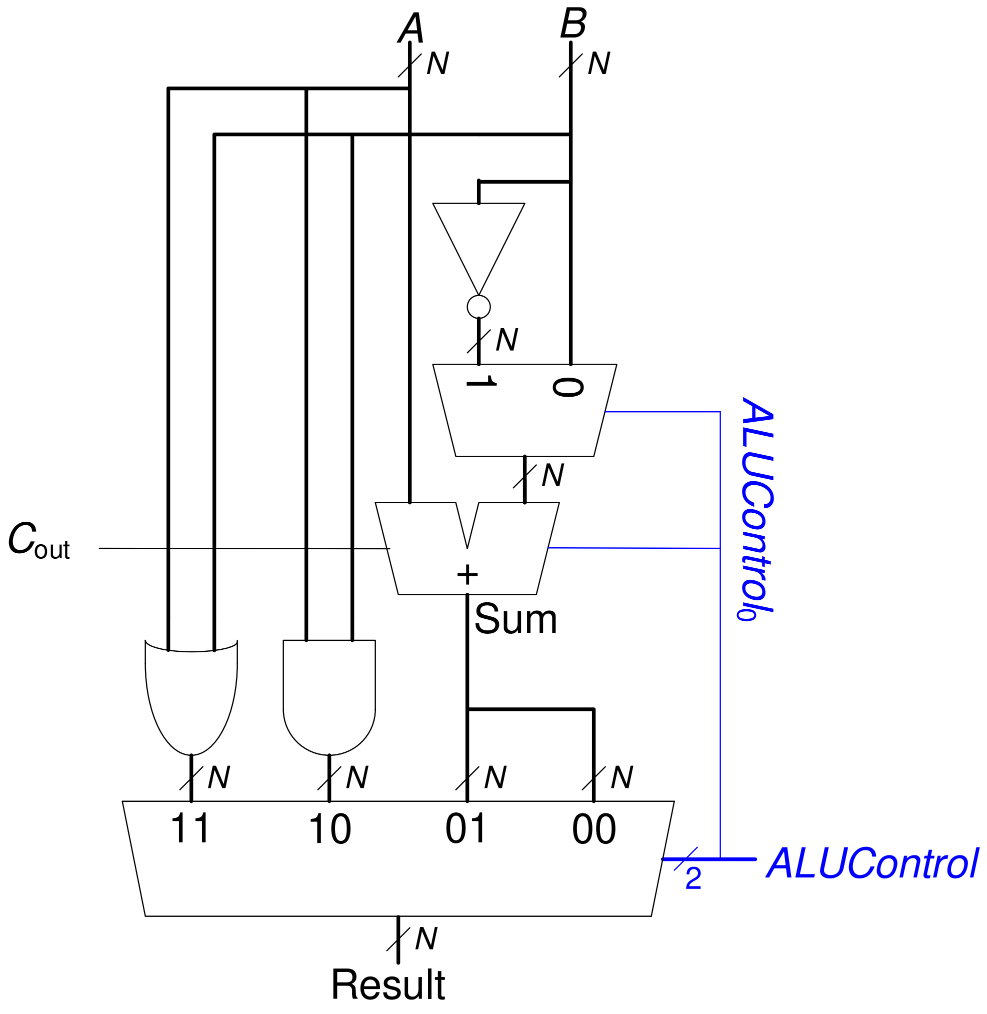
Comparison	Signed	Unsigned
==	Z	Z
!=	~Z	~Z
<	N ^ V	~C
<=	Z | (N ^ V)	Z | ~C
>	~Z & ~(N ^ V)	~Z & C
>=	~(N ^ V)	C

Other ALU Operations

- **Set Less Than** (also called Set if Less Than)
  - **Sets lsb** of result **if A < B**
    - Result = 0000…001 if A < B
    - Result = 0000…000 otherwise
  - Comes in signed and unsigned flavors
- **XOR**
  - Result = A XOR B

Extending ALU: SLT

$ALUControl_{2:0}$	Function
00	add
01	subtract
10	and
11	or
	SLT



Shifters, Multipliers, & Dividers

Shifters

Logical shifter: shifts value to left or right and fills empty spaces with 0's

- Ex: 11001 >> 2 =
- Ex: 11001 << 2 =

Arithmetic shifter: same as logical shifter, but on right shift, fills empty spaces with the old most significant bit (msb)

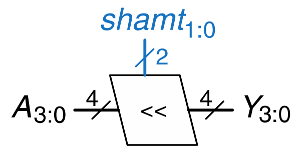
- Ex: 11001 >>> 2 =
- Ex: 11001 <<< 2 =

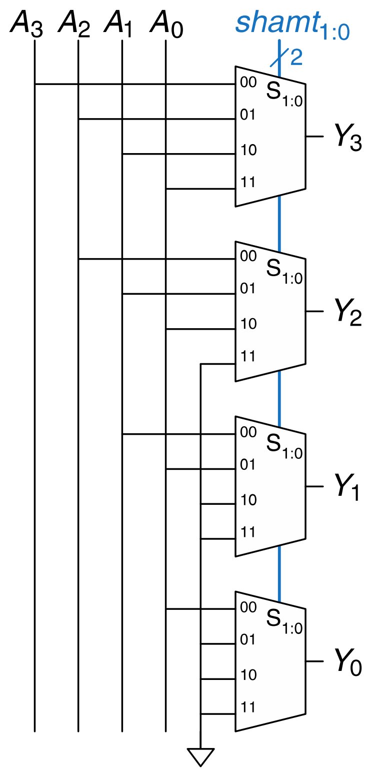
Rotator: rotates bits in a circle, such that bits shifted off one end are shifted into the other end

- Ex: 11001 ROR 2 =
- Ex: 11001 ROL 2 =

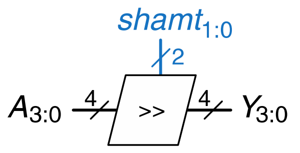
Shifter Design

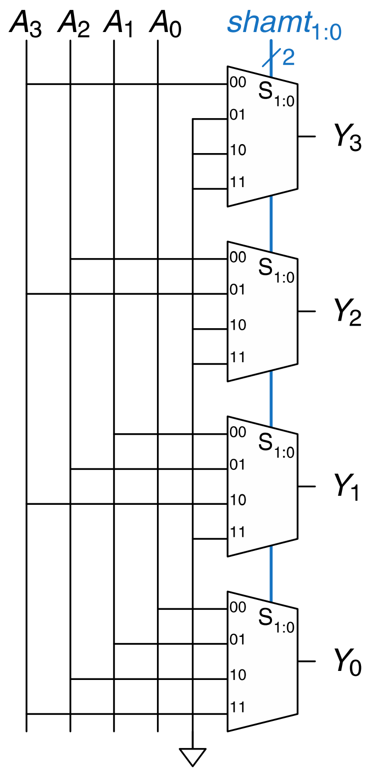
Shift Left



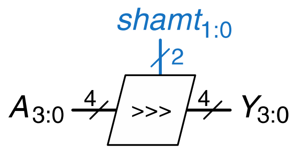


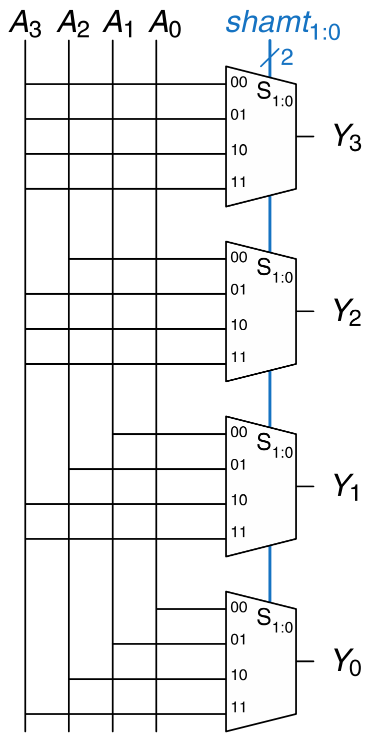
Logical Shift Right





Arithmetic Shift Right





Shifters as Multipliers and Dividers

- $A << N = A \times 2^N$

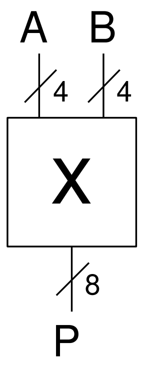
- $A >>> N = A \div 2^N$

Multipliers

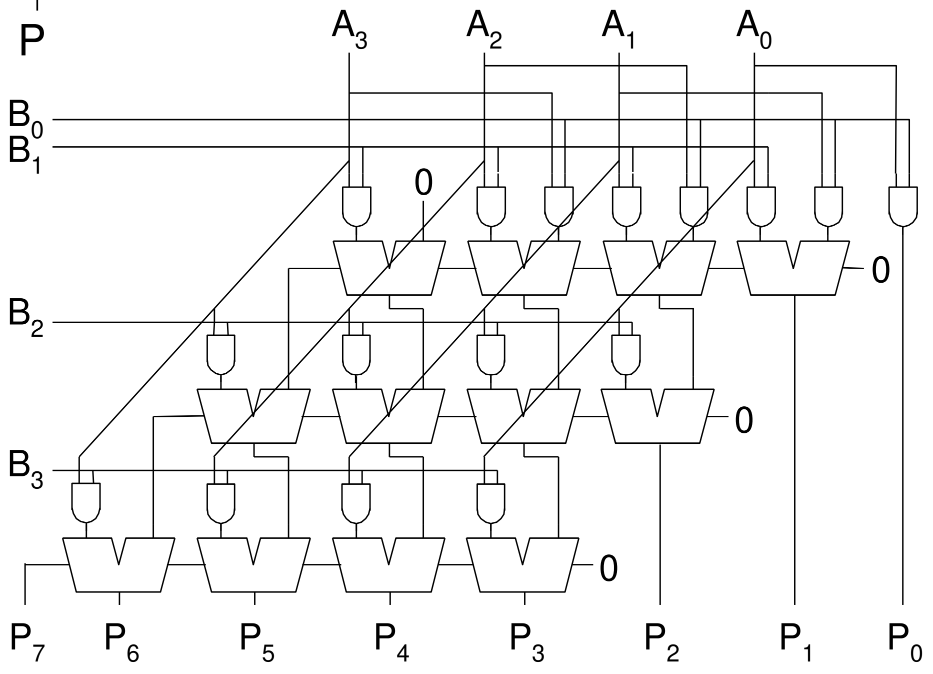
- **Partial products** formed by multiplying a single digit of the multiplier with multiplicand
- **Shifted** partial products **summed** to form result

```
     Decimal                               Binary

        230      multiplicand                0101
   x     42      multiplier                x 0111
   --------                            ----------
        460      partial                     0101
     + 920       products                   0101
   --------                                0101
       9660                             + 0000
                                       ----------
                 result                   0100011

 230 x 42 = 9660                       5 x 7 = 35
```

4 x 4 Multiplier



$$
\begin{array}{ccccccccc}
 & & & & & A_3 & A_2 & A_1 & A_0 \\
 & & & & \times & B_3 & B_2 & B_1 & B_0 \\
\hline
 & & & & & A_3B_0 & A_2B_0 & A_1B_0 & A_0B_0 \\
 & & & & A_3B_1 & A_2B_1 & A_1B_1 & A_0B_1 & \\
 & & & A_3B_2 & A_2B_2 & A_1B_2 & A_0B_2 & & \\
+ & & A_3B_3 & A_2B_3 & A_1B_3 & A_0B_3 & & & \\
\hline
 & P_7 & P_6 & P_5 & P_4 & P_3 & P_2 & P_1 & P_0
\end{array}
$$



Dividers

A/B = Q + R/B

Decimal Example: 2584/15 = 172 R4

Long-Hand:

```
      172  R4
   ┌──────
15 │2584
   -15
   ───
    108
   -105
   ────
     34
    -30
    ───
      4
```

Long-Hand Revisited:

```
 0002
-  15
─────
  -13
```

$\underset{3}{\underline{0}}\ \underset{2}{\underline{\quad}}\ \underset{1}{\underline{\quad}}\ \underset{0}{\underline{\quad}}$

```
 0025
-  15
─────
   10
```

$\underset{3}{\underline{0}}\ \underset{2}{\underline{1}}\ \underset{1}{\underline{\quad}}\ \underset{0}{\underline{\quad}}$

```
 0108
- 105
─────
    3
```

$\underset{3}{\underline{0}}\ \underset{2}{\underline{1}}\ \underset{1}{\underline{7}}\ \underset{0}{\underline{\quad}}$

```
 0034
-  30
─────
    4
```

$\underset{3}{\underline{0}}\ \underset{2}{\underline{1}}\ \underset{1}{\underline{7}}\ \underset{0}{\underline{2}}$

Dividers

A/B = Q + R/B

Decimal: 2584/15 = 172 R4 **Binary: 1101/0010 = 0110 R1**

$$\begin{array}{r} 0002 \\ -\quad 15 \\ \hline -13 \end{array} \qquad \underset{3}{\underline{0}}\ \underset{2}{\underline{\ \ }}\ \underset{1}{\underline{\ \ }}\ \underset{0}{\underline{\ \ }} \qquad\qquad \begin{array}{r} 0001 \\ -\ 0010 \\ \hline 1111 \end{array} \qquad \underset{3}{\underline{0}}\ \underset{2}{\underline{\ \ }}\ \underset{1}{\underline{\ \ }}\ \underset{0}{\underline{\ \ }}$$

$$\begin{array}{r} 0025 \\ -\quad 15 \\ \hline 10 \end{array} \qquad \underset{3}{\underline{0}}\ \underset{2}{\underline{1}}\ \underset{1}{\underline{\ \ }}\ \underset{0}{\underline{\ \ }} \qquad\qquad \begin{array}{r} 0011 \\ -\ 0010 \\ \hline 0001 \end{array} \qquad \underset{3}{\underline{0}}\ \underset{2}{\underline{1}}\ \underset{1}{\underline{\ \ }}\ \underset{0}{\underline{\ \ }}$$

$$\begin{array}{r} 0108 \\ -\ 105 \\ \hline 3 \end{array} \qquad \underset{3}{\underline{0}}\ \underset{2}{\underline{1}}\ \underset{1}{\underline{7}}\ \underset{0}{\underline{\ \ }} \qquad\qquad \begin{array}{r} 0010 \\ -\ 0010 \\ \hline 0000 \end{array} \qquad \underset{3}{\underline{0}}\ \underset{2}{\underline{1}}\ \underset{1}{\underline{1}}\ \underset{0}{\underline{\ \ }}$$

$$\begin{array}{r} 0034 \\ -\quad 30 \\ \hline 4 \end{array} \qquad \underset{3}{\underline{0}}\ \underset{2}{\underline{1}}\ \underset{1}{\underline{7}}\ \underset{0}{\underline{2}} \qquad\qquad \begin{array}{r} 0001 \\ -\ 0010 \\ \hline 1111 \end{array} \qquad \underset{3}{\underline{0}}\ \underset{2}{\underline{1}}\ \underset{1}{\underline{1}}\ \underset{0}{\underline{0}}\ \ \text{R1}$$

Dividers

A/B = Q + R/B

```
$R' = 0$
for $i$ = N-1 to 0
    $R = \{R' << 1, A_i\}$
    $D = R - B$
    if $D < 0$, $Q_i = 0$;  $R' = R$
    else        $Q_i = 1$;  $R' = D$
$R = R'$
```

Binary: **1101/10 = 0110 R1**

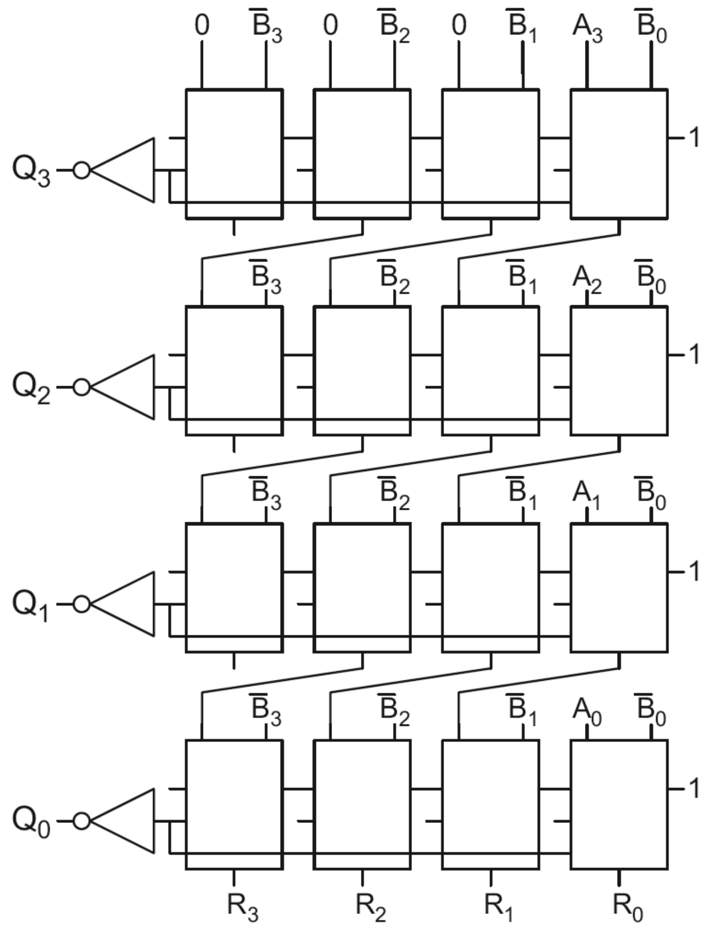
$$\begin{array}{r} 0001 \\ -\,0010 \\ \hline 1111 \end{array} \qquad \underset{3}{\underline{0}}\ \underset{2}{\underline{\ \ }}\ \underset{1}{\underline{\ \ }}\ \underset{0}{\underline{\ \ }}$$

$$\begin{array}{r} 0011 \\ -\,0010 \\ \hline 0001 \end{array} \qquad \underset{3}{\underline{0}}\ \underset{2}{\underline{1}}\ \underset{1}{\underline{\ \ }}\ \underset{0}{\underline{\ \ }}$$

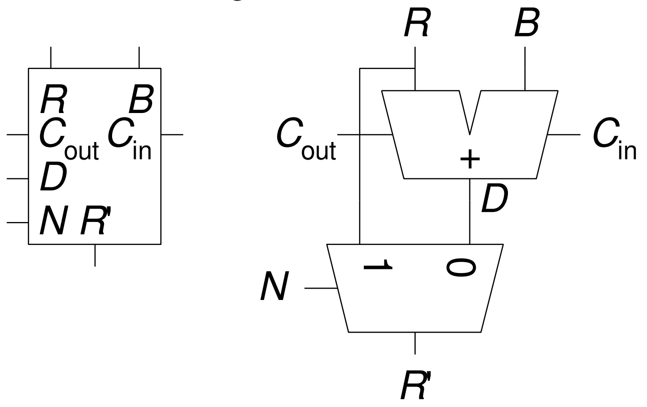
$$\begin{array}{r} 0010 \\ -\,0010 \\ \hline 0000 \end{array} \qquad \underset{3}{\underline{0}}\ \underset{2}{\underline{1}}\ \underset{1}{\underline{1}}\ \underset{0}{\underline{\ \ }}$$

$$\begin{array}{r} 0001 \\ -\,0010 \\ \hline 1111 \end{array} \qquad \underset{3}{\underline{0}}\ \underset{2}{\underline{1}}\ \underset{1}{\underline{1}}\ \underset{0}{\underline{0}}\ \ \text{R1}$$

4 x 4 Divider



Legend

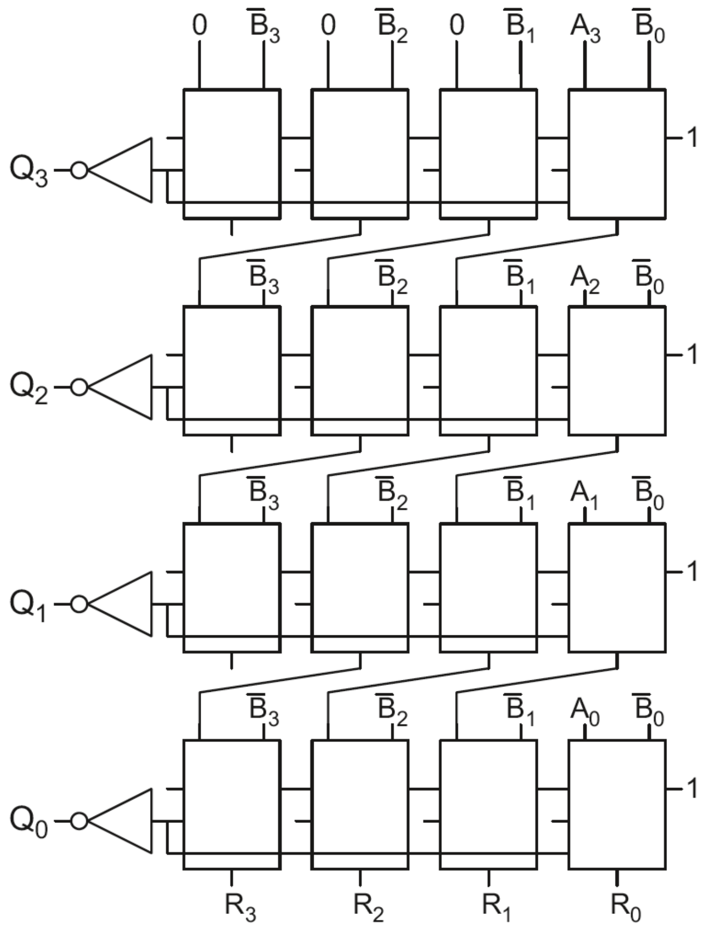


Division: A/B = Q + R/B

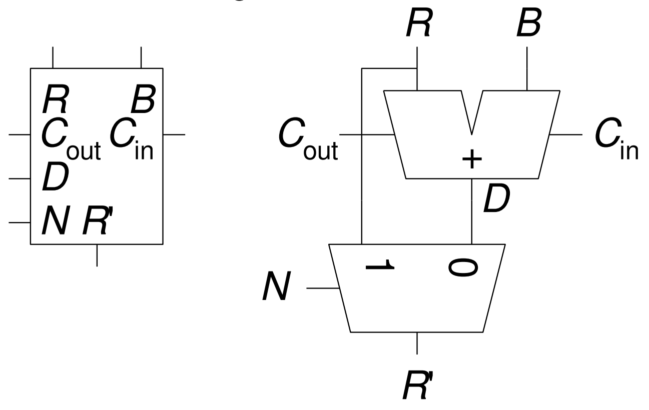
```
R′ = 0
for i = N-1 to 0
  R = {R′ << 1, A_i}
  D = R - B
  if D < 0, Q_i=0, R′=R
  else      Q_i=1, R′=D
R=R′
```

Each row computes one iteration of the division algorithm.

4 x 4 Divider



Legend



```
 0001
-0010
 1111      0 _ _ _
           3 2 1 0

 0011
-0010
 0001      0 1 _ _
           3 2 1 0

 0010
-0010
 0000      0 1 1 _
           3 2 1 0

 0001
-0010      0 1 1 0   R1
 1111      3 2 1 0
```

Each row computes one iteration of the division algorithm.

Fixed-Point Numbers

Number Systems

Numbers we can represent using binary representations

- **Positive numbers**
  - Unsigned binary
- **Negative numbers**
  - Two’s complement
  - Sign/magnitude numbers

What about **fractions**?

Numbers with Fractions

Two common notations:

- **Fixed-point:** binary point fixed
- **Floating-point:** binary point floats to the right of the most significant 1

Fixed-Point Numbers

- 6.75 using 4 integer bits and 4 fraction bits:

  01101100

  0110.1100

  $2^2 + 2^1 + 2^{-1} + 2^{-2} = 6.75$

  - Binary point is implied
  - The number of integer and fraction bits must be agreed upon beforehand

Unsigned Fixed Point Formats

- **Ua.b:** unsigned number with
  - **a** integer bits
  - **b** fractional bits.
- **Example: 6.75** is
  - **U4.4:** 01101100
  - **U3.5:** 11011000
  - **U6.2:** 00011011
- 8, 16, and 32-bit fixed point numbers are common
  - **U8.8** often represents sensor data, audio, pixels
  - **U16.16** used for higher precision signal processing

Signed Fixed Point Formats

- **Qa.b:** signed 2’s complement number with
  - **a** integer bits (including the sign bit)
  - **b** fractional bits
- **To negate a Q fixed point number:**
  - Invert the bits
  - Add one to the LSB
- **Example:** write **-6.75** in Q4.4
  - 6.75 =   01101100
  - Invert: 10010011
  - Add 1 LSB: 10010100
- **Q1.15** (aka Q15) is common for signal processing (1, -1]

Saturating Arithmetic

- Fixed point **overflow** is usually bad
  - Produces undesired artifacts:
    - Video: dark pixel in middle of bright pixels
    - Audio: clicking sounds
- **Saturating arithmetic**
  - Instead of overflowing, use largest value
  - In U4.4: 11000000 + 01111000 = 11111111

    12 + 7.5 = 15.9375

Floating-Point Numbers

Floating-Point Numbers

- Binary point floats to the right of the most significant 1
- Similar to decimal scientific notation

- For example, write 273_{10} in scientific notation:

$$\mathbf{273 = 2.73 \times 10^2}$$

- In general, a number is written in scientific notation as:

$$\pm \mathbf{M} \times \mathbf{B}^{\mathbf{E}}$$

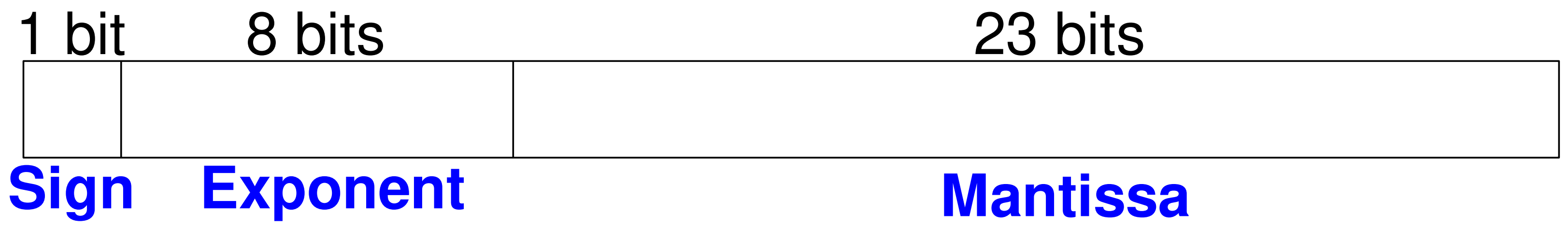
  - **M** = mantissa
  - **B** = base
  - **E** = exponent
  - In the example, M = 2.73, B = 10, and E = 2

Floating vs. Fixed Point Numbers

- Floating point numbers are like **scientific notation**
  - Allow a **greater dynamic range** of smallest to largest
  - **Arithmetic is harder**
    - Mantissa must be aligned before adding
    - This costs performance and power
- Fixed point numbers are **harder for the programmer**
  - **Smaller dynamic range**
  - Take care of **overflow**

- **Floating Point** is preferred for general-purpose computing where programming time is most important
- **Fixed Point** is preferred for signal processing performance, power, and hardware cost matter most
  - Machine learning, video

Floating-Point Numbers



- **Example:** represent the value 228_{10} using a 32-bit floating point representation

We show three versions – **the final version** is called the:

IEEE 754 floating-point standard

Floating-Point Representation 1

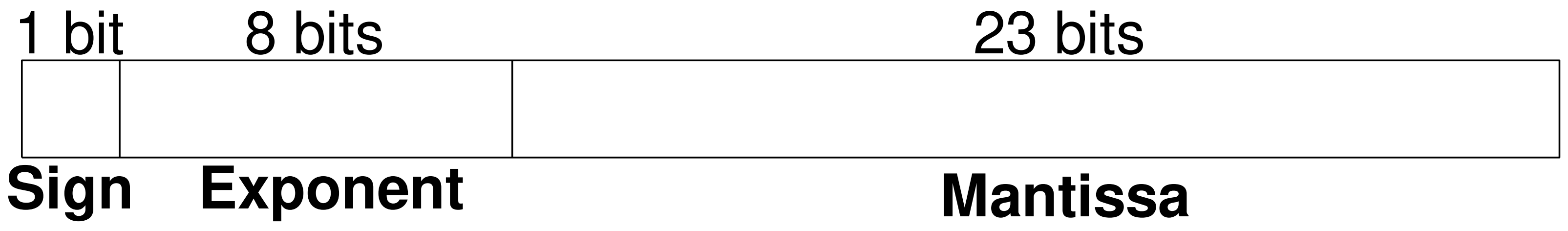
1. Convert decimal to binary:

   $228_{10} = 11100100_2$

2. Write the number in “binary scientific notation”:

   $11100100_2 = 1.11001_2 \times 2^7$

3. Fill in each field of the 32-bit floating point number:
   - The sign bit is positive (0)
   - The 8 exponent bits represent the value 7
   - The remaining 23 bits are the mantissa



Floating-Point Representation 2

- First bit of the mantissa is always 1:
  - $228_{10} = 11100100_2 = \mathbf{1.11001} \times 2^7$
- So, no need to store it: *implicit leading 1*
- Store just **fraction bits** in 23-bit field

1 bit	8 bits	23 bits
0	00000111	110 0100 0000 0000 0000 0000
Sign	**Exponent**	**Fraction**

Floating-Point Representation 3

- ***Biased exponent***: bias = 127 (01111111_2)
  - Biased exponent = bias + exponent
  - Exponent of 7 is stored as:

    127 + 7 = 134 = $0x10000110_2$

- The **IEEE 754 32-bit floating-point representation** of 228_{10}

1 bit	8 bits	23 bits
0	10000110	110 0100 0000 0000 0000 0000
Sign	**Biased Exponent**	**Fraction**

in hexadecimal: **0x43640000**

Floating-Point Example

Write **-58.25_{10}** in floating point (IEEE 754)

1. Convert magnitude of decimal to binary:

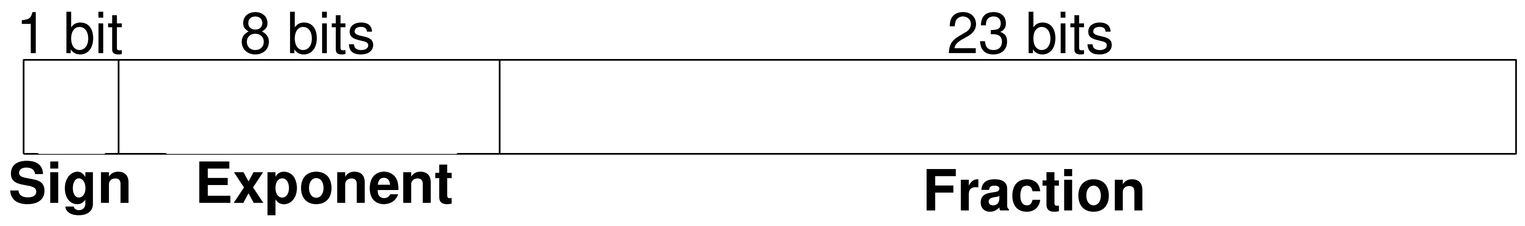
   58.25_{10} = 111010.01_2

2. Write in binary scientific notation:

   1.1101001×2^5

3. Fill in fields:

   Sign bit: **1** (negative)
   8 exponent bits: (127 + 5) = 132 = **10000100_2**
   23 fraction bits: **110 1001 0000 0000 0000 0000**



in hexadecimal: **0xC2690000**

Floating-Point Special Cases

Number	Sign	Exponent	Fraction
0	X	00000000	00000000000000000000000
∞	0	11111111	00000000000000000000000
- ∞	1	11111111	00000000000000000000000
NaN	X	11111111	non-zero

Floating-Point Precision

- **Single-Precision:**
  - 32-bit
  - 1 sign bit, 8 exponent bits, 23 fraction bits
  - bias = 127

- **Double-Precision:**
  - 64-bit
  - 1 sign bit, 11 exponent bits, 52 fraction bits
  - bias = 1023

Floating-Point Rounding & Overflow

- **Overflow:** number too large to be represented
- **Underflow:** number too small to be represented
- **Rounding modes:**
  - Down
  - Up
  - Toward zero
  - To nearest
- **Example:** round 1.100101 (1.578125) to only 3 fraction bits

– **Down:**	1.100
– **Up:**	1.101
– **Toward zero:**	1.100
– **To nearest:**	1.101 (1.625 is closer to 1.578125 than 1.5 is)

Chapter 5: Digital Building Blocks

Floating-Point Addition

Floating-Point Addition

1. **Extract** exponent and fraction bits
2. **Prepend** leading 1 to form mantissa
3. **Compare** exponents
4. **Shift** smaller mantissa if necessary
5. **Add** mantissas
6. **Normalize** mantissa and adjust exponent if necessary
7. **Round** result
8. **Assemble** exponent and fraction back into floating-point format

Floating-Point Addition Example

Add the following floating-point numbers:

0x3FC00000

0x40500000

Floating-Point Addition Example

1. Extract exponent and fraction bits

	1 bit	8 bits	23 bits
0x3FC00000	0	01111111	100 0000 0000 0000 0000 0000
	Sign	**Exponent**	**Fraction**

	1 bit	8 bits	23 bits
0x40500000	0	10000000	101 0000 0000 0000 0000 0000
	Sign	**Exponent**	**Fraction**

For first number (N1):	S = 0, E = 127, F = .1
For second number (N2):	S = 0, E = 128, F = .101

2. Prepend leading 1 to form mantissa

N1:	1.1
N2:	1.101

Floating-Point Addition Example

3. **Compare exponents**

   127 – 128 = -1, so shift N1 right by 1 bit

4. **Shift smaller mantissa if necessary**

   shift N1's mantissa: 1.1 >> 1 = 0.11 $(\times 2^1)$

5. **Add mantissas**

$$\begin{array}{rl} & 0.11 \times 2^1 \\ + & 1.101 \times 2^1 \\ \hline & 10.011 \times 2^1 \end{array}$$

Floating-Point Addition Example

6. **Normalize mantissa and adjust exponent if necessary**

   $10.011 \times 2^1 = 1.0011 \times 2^2$

7. **Round result**

   No need (fits in 23 bits)

8. **Assemble exponent and fraction back into floating-point format**

   $S = 0, E = 2 + 127 = 129 = 10000001_2, F = 001100...$

1 bit	8 bits	23 bits
0	10000001	001 1000 0000 0000 0000 0000
Sign	**Exponent**	**Fraction**

in hexadecimal: **0x40980000**

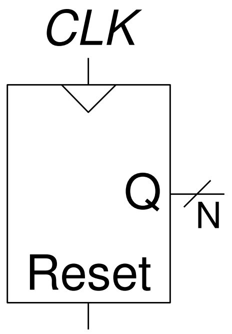
Chapter 5: Digital Building Blocks

Counters & Shift Registers

Counters

- Increments on each clock edge
- Used to cycle through numbers. For example,
  - 000, 001, 010, 011, 100, 101, 110, 111, 000, 001…
- **Example uses:**
  - Digital clock displays
  - Program counter: keeps track of current instruction executing

Symbol **Implementation**



Counter SystemVerilog Idiom

```
module counter(input  logic       clk, reset,
               output logic [7:0] q);

  always_ff @(posedge clk)
    if (reset) q <= 0;    // synchronous reset
    else       q <= q+1; //
end

// alternative more verbose
module counter(input  logic       clk, reset,
               output logic [7:0] q);

  logic [7:0] nextq;

  assign nextq = q + 1; // adder
  always_ff @(posedge clk) // state register with synchronous reset
    if (reset) q <= 0;
    else       q <= nextq;
end
```

Divide-by-2^N Counter

- Most significant bit of an N-bit counter toggles every 2^N cycles.
- Useful for slowing a clock. Ex: blink an LED
- Example: 50 MHz clock, 24-bit counter
  - 2.98 Hz

Digitally Controlled Oscillator

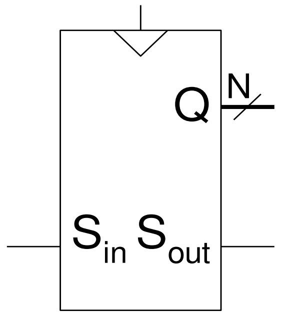
- N-bit counter
- Add p on each cycle, instead of 1
- Most significant bit toggles at $f_{out} = f_{clk} * p / 2^N$

- Example: f_{clk} = 50 MHz clock
  - How to generate a f_{out} = 200 Hz signal?
  - $p/2^N$ = 200 / 50 MHz
- Try N = 24, p = 67 ➔ f_{out} = 199.676 Hz
- Or N = 32, p = 17179 ➔ f_{out} = 199.990 Hz

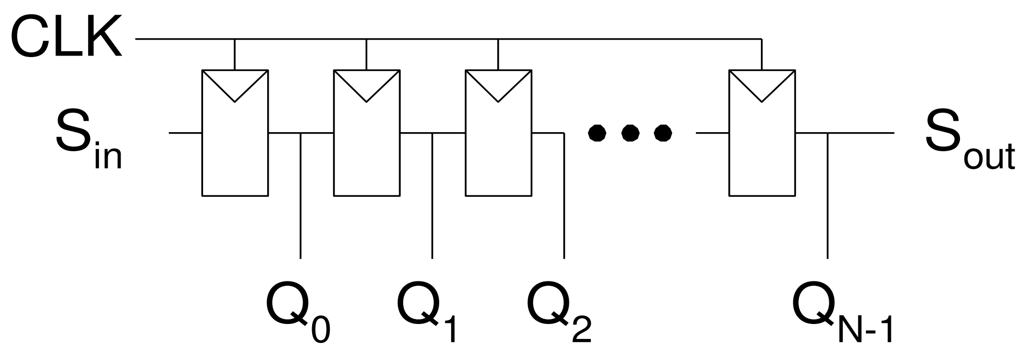
Shift Registers

- Shift a new bit in on each clock edge
- Shift a bit out on each clock edge
- *Serial-to-parallel converter*: converts serial input (S_{in}) to parallel output ($Q_{0:N-1}$)

Symbol:

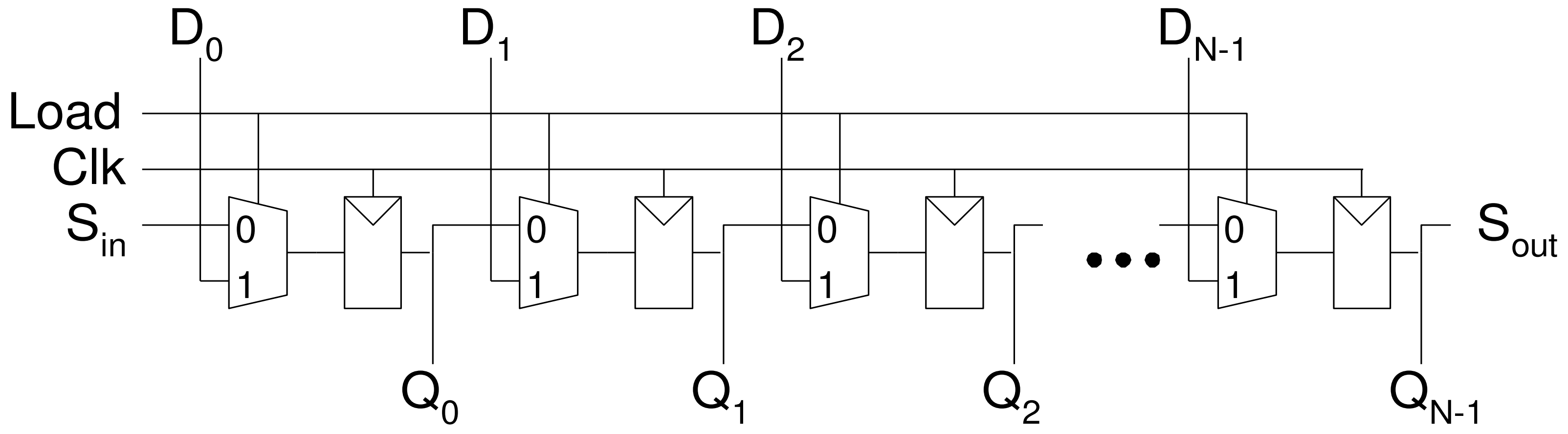


Implementation:



Shift Register with Parallel Load

- When *Load* = 1, acts as a normal *N*-bit register
- When *Load* = 0, acts as a shift register
- Now can act as a *serial-to-parallel converter* (S_{in} to $Q_{0:N-1}$) or a *parallel-to-serial converter* ($D_{0:N-1}$ to S_{out})



Shift Register SystemVerilog Idiom

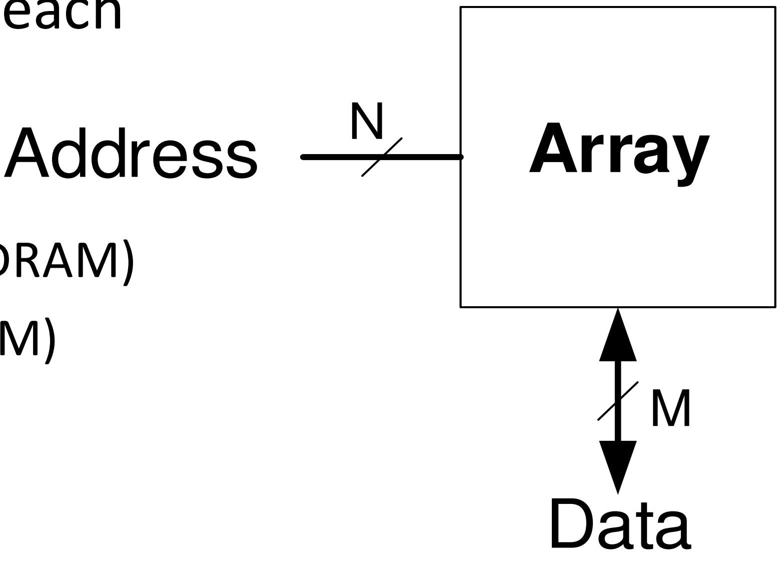
```
module shiftreg #(parameter N=8)
                 (input  logic         clk,
                  input  logic         reset, load,
                  input  logic         sin,
                  input  logic [N-1:0] d,
                  output logic [N-1:0] q,
                  output logic         sout);

  always_ff @(posedge clk, posedge reset)
    if (reset)       q <= 0;
    else if (load) q <= d;
    else             q <= {q[N-2:0], sin};

  assign sout = q[N-1];
end
```

Chapter 5: Digital Building Blocks

Memory

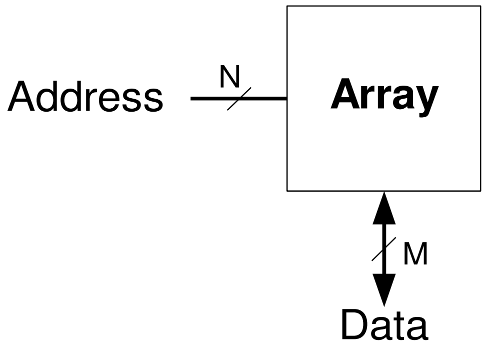
Memory Arrays

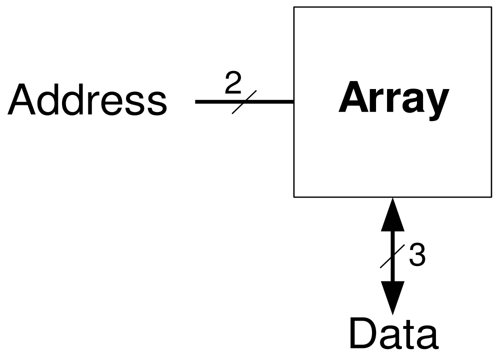
- Efficiently store large amounts of data
- *M*-bit data value read/written at each unique *N*-bit address
- 3 common types:
  - Dynamic random access memory (DRAM)
  - Static random access memory (SRAM)
  - Read only memory (ROM)

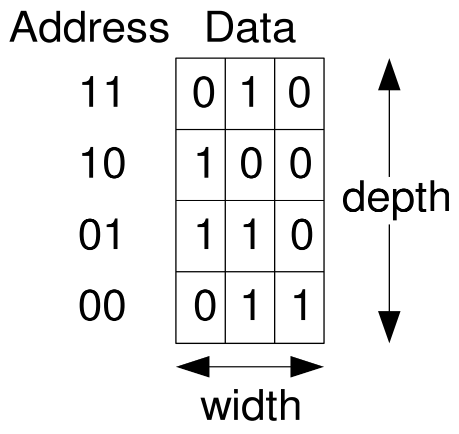


Memory Arrays

- 2-dimensional array of bit cells
- Each bit cell stores one bit
- *N* address bits and *M* data bits:
  - 2^N rows and *M* columns
  - **Depth:** number of rows (number of words)
  - **Width:** number of columns (size of word)
  - **Array size:** depth × width = $2^N \times M$



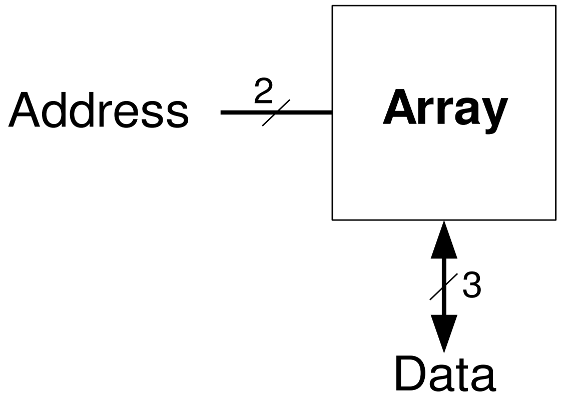


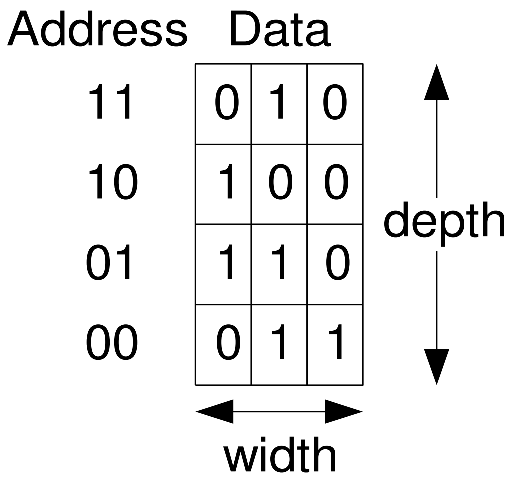


Address	Data		
11	0	1	0
10	1	0	0
01	1	1	0
00	0	1	1

Memory Array Example

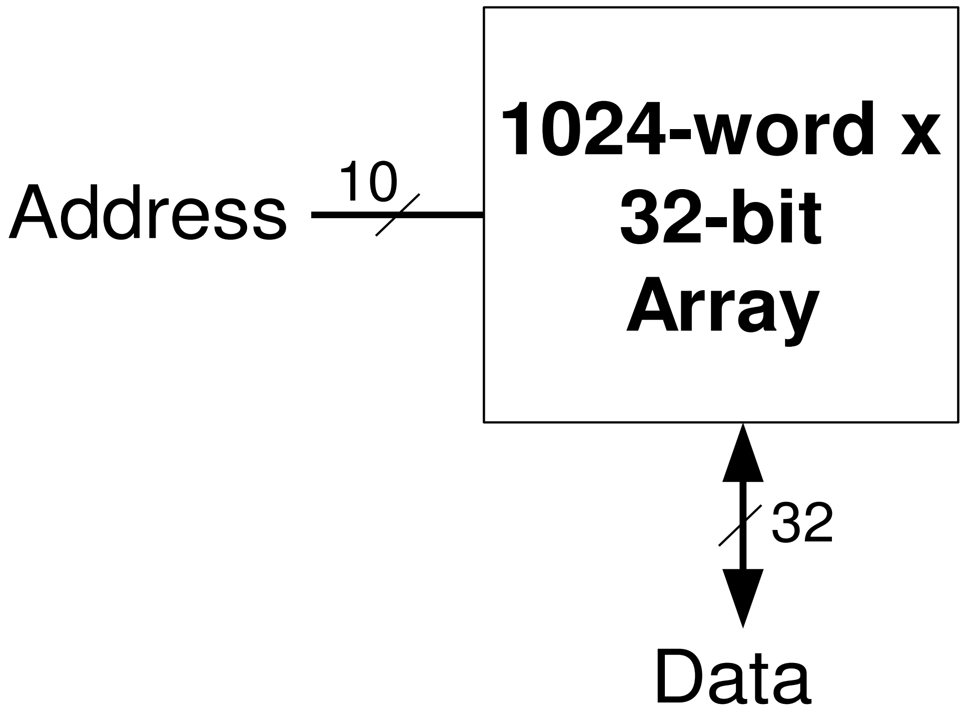
- **$2^2 \times$ 3-bit** array
- **Number of words:** 4
- **Word size:** 3-bits
- For example, the 3-bit word stored at address 10 is 100



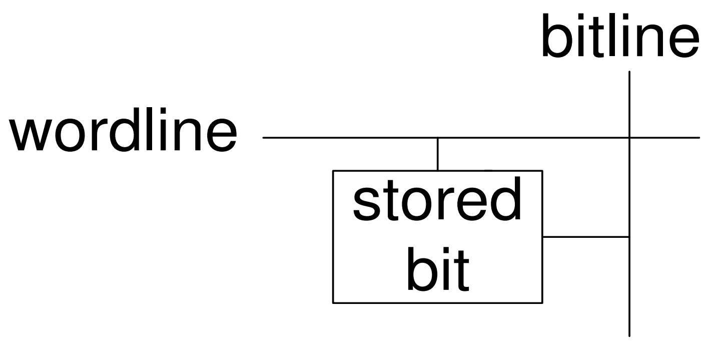


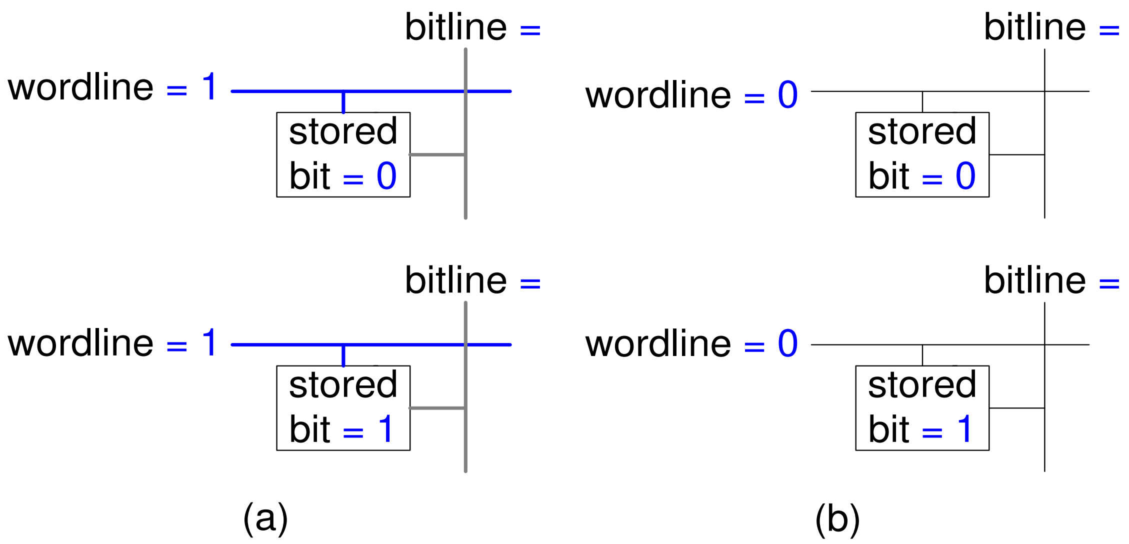
Address	Data
11	0 1 0
10	1 0 0
01	1 1 0
00	0 1 1

Memory Arrays



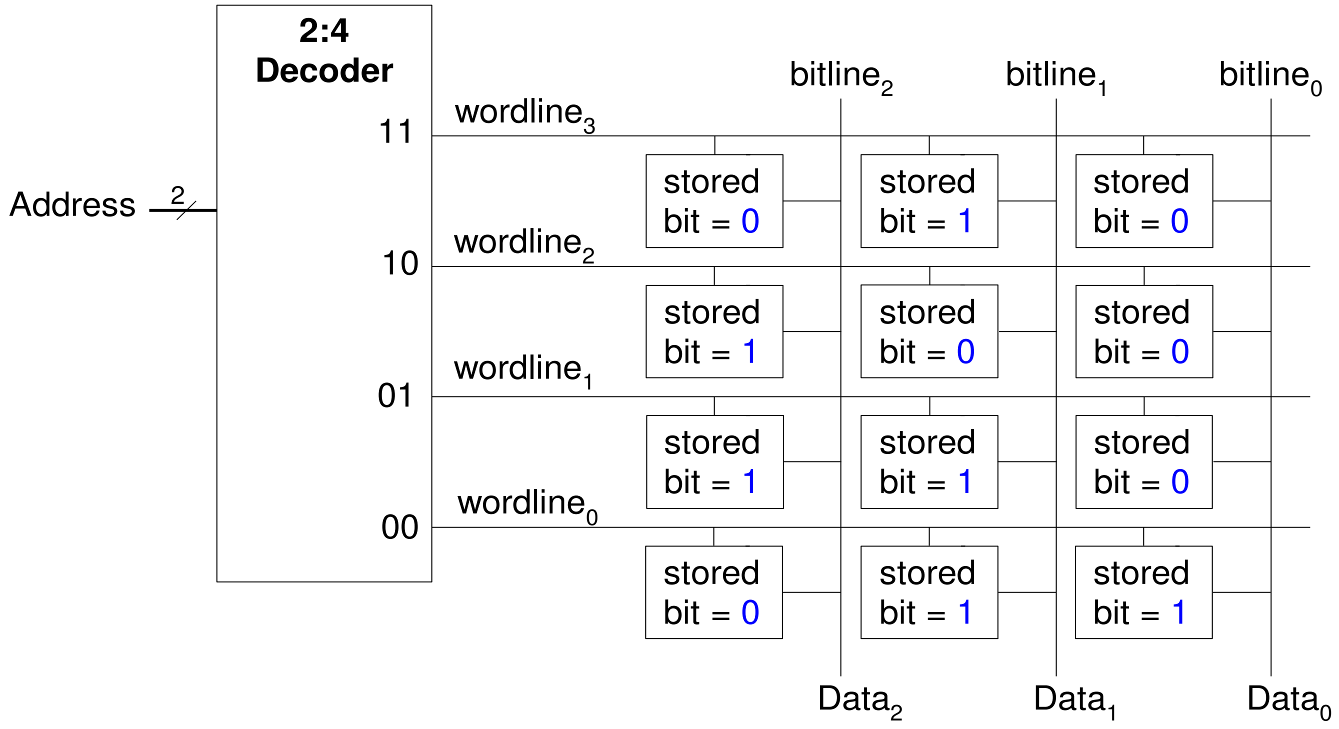
Memory Array Bit Cells





Memory Array

- **Wordline:**
  - like an enable
  - single row in memory array read/written
  - corresponds to unique address
  - only one wordline **HIGH** at once



Types of Memory

- Random access memory (RAM): **volatile**
- Read only memory (ROM): **nonvolatile**

RAM: Random Access Memory

- **Volatile:** loses its data when power off
- Read and written quickly
- Main memory in your computer is RAM (DRAM)

Historically called ***random access*** memory because any data word accessed as easily as any other (in contrast to sequential access memories such as a tape recorder)

ROM: Read Only Memory

- **Nonvolatile:** retains data when power off
- Read quickly, but writing is impossible or slow
- Flash memory in cameras, thumb drives, and digital cameras are all ROMs

Historically called ***read only*** memory because ROMs were written at time of fabrication or by burning fuses. Once a ROM was configured, it could not be written again. This is no longer the case for Flash memory and other types of ROMs.

Chapter 5: Digital Building Blocks

RAM

Types of RAM

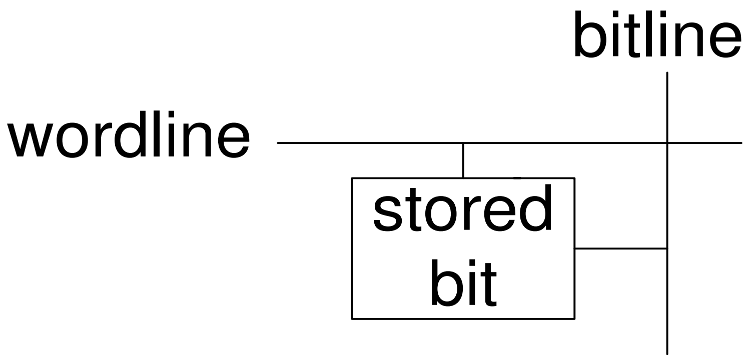
- **DRAM** (Dynamic random access memory)
- **SRAM** (Static random access memory)
- Differ in how they store data:
  - DRAM uses a capacitor
  - SRAM uses cross-coupled inverters

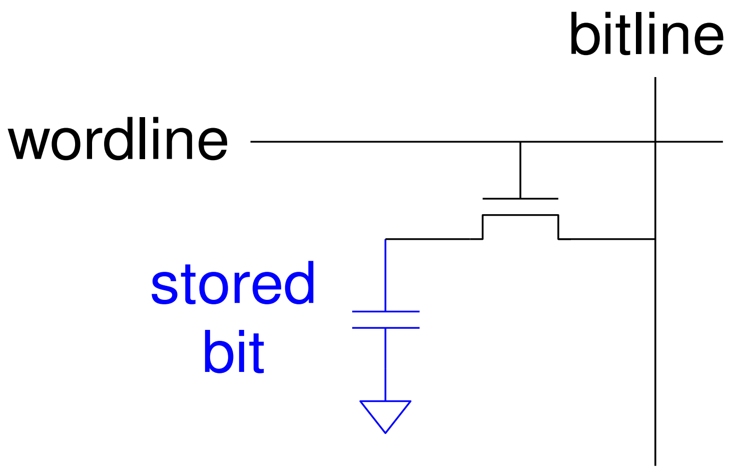
Robert Dennard, 1932 -

- Invented DRAM in 1966 at IBM
- Others were skeptical that the idea would work
- By the mid-1970's DRAM in virtually all computers

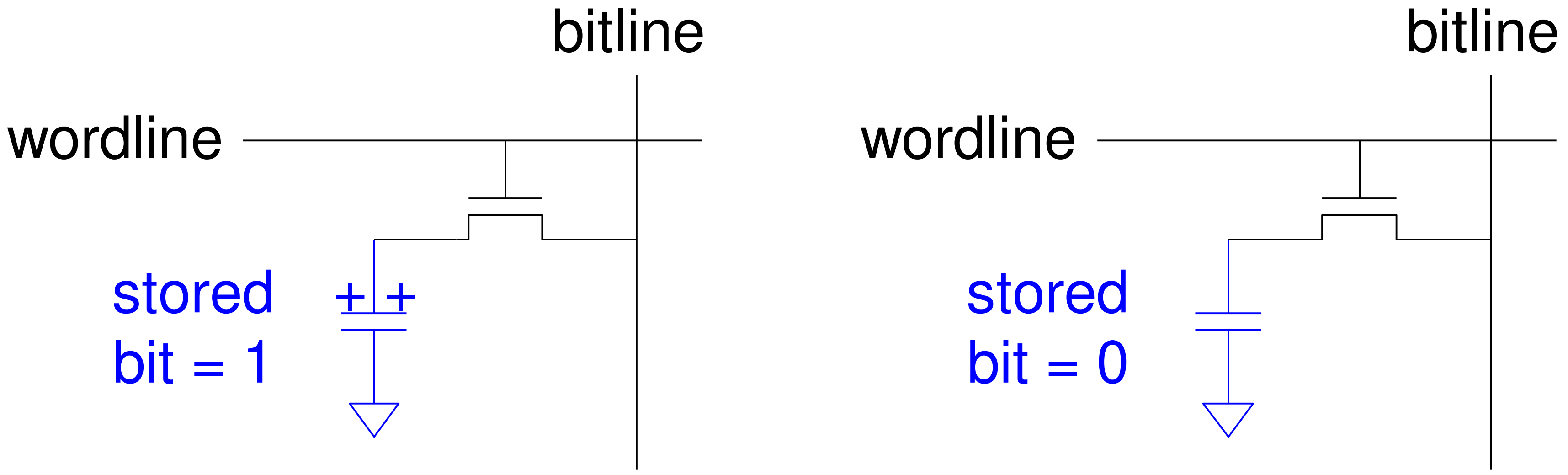
DRAM

- Data bits stored on **capacitor**
- *Dynamic* because the value needs to be **refreshed** (rewritten) periodically and after read:
  - Charge leakage from the capacitor degrades the value
  - Reading destroys the stored value

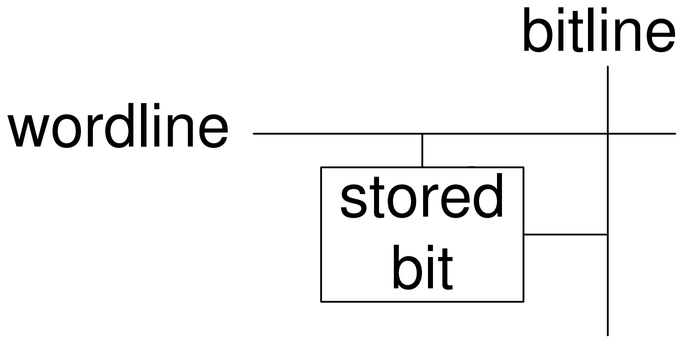


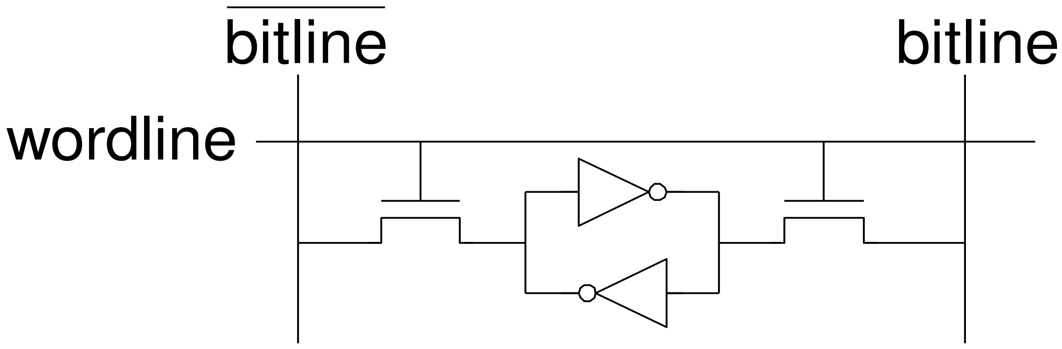


DRAM

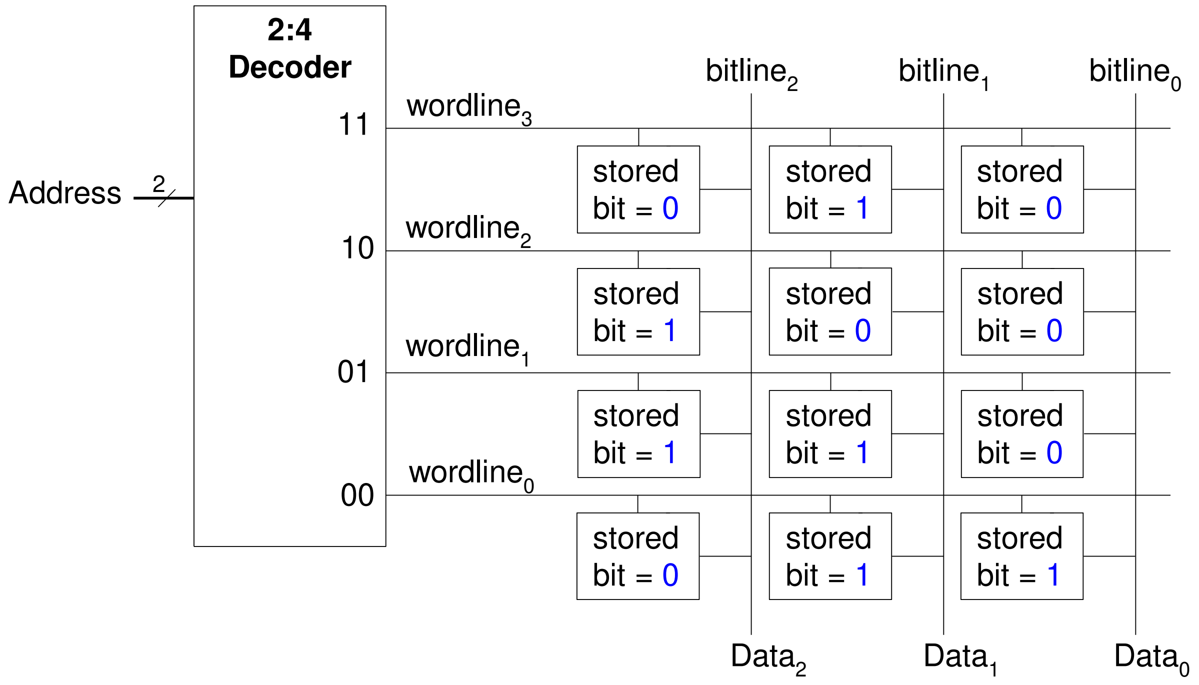

bitline
wordline
stored bit = 1
+ +
bitline
wordline
stored bit = 0

SRAM

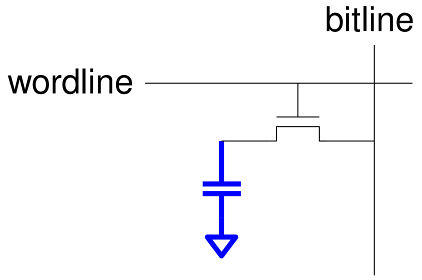

bitline
wordline
stored bit


$\overline{\text{bitline}}$
bitline
wordline

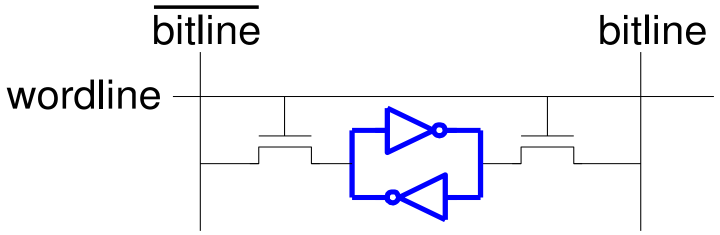
Memory Arrays Review



DRAM bit cell:

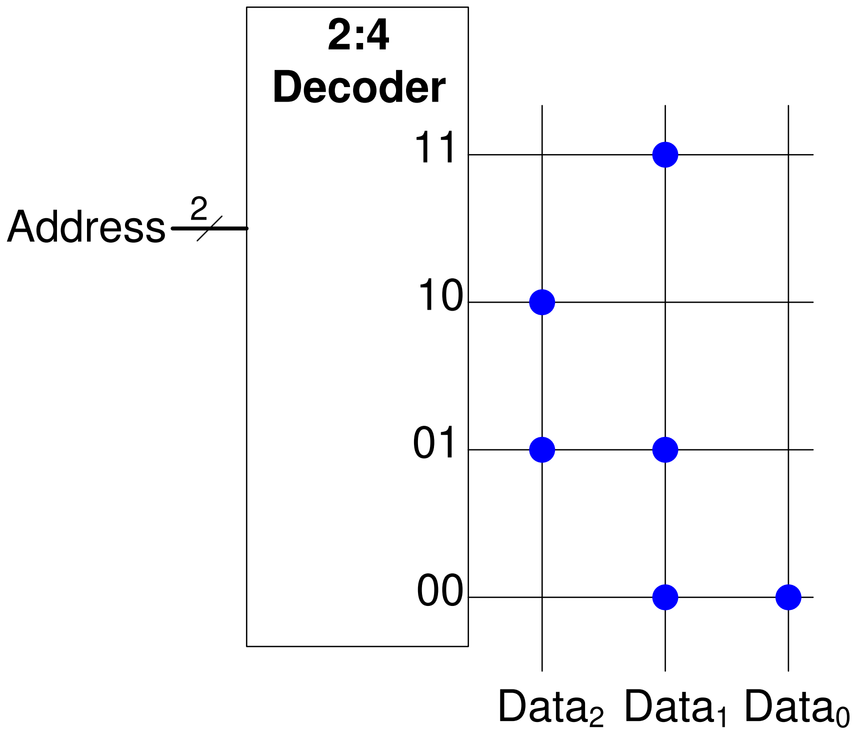


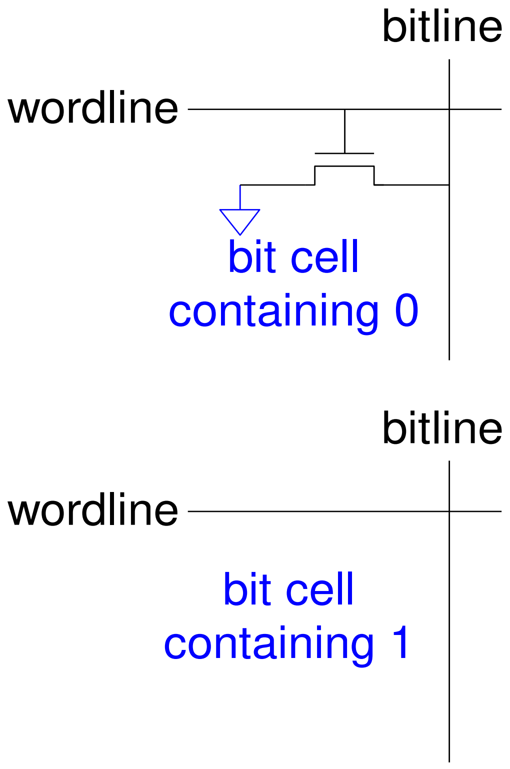
SRAM bit cell:



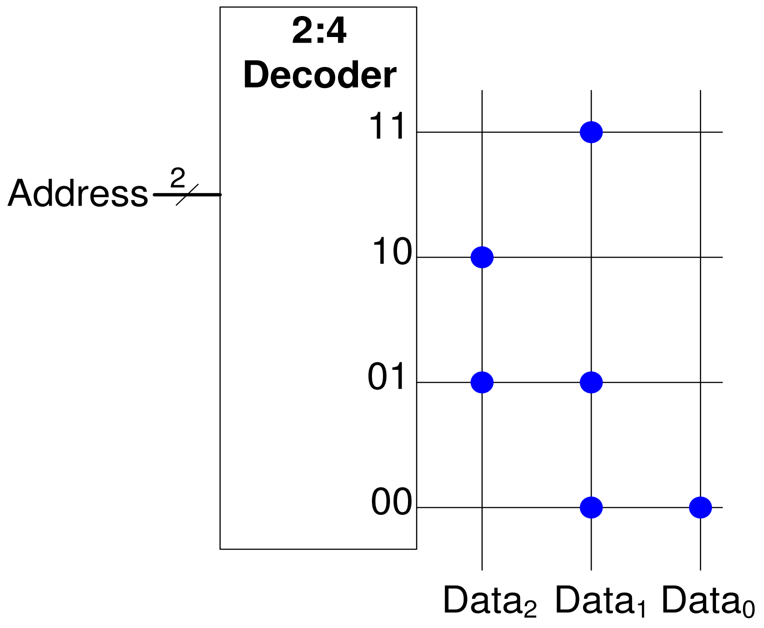
ROM

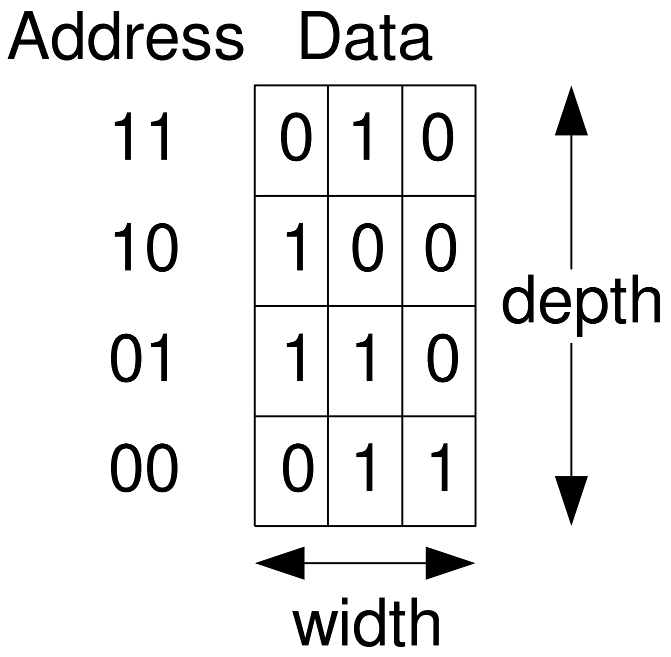
ROM: Dot Notation





ROM Storage



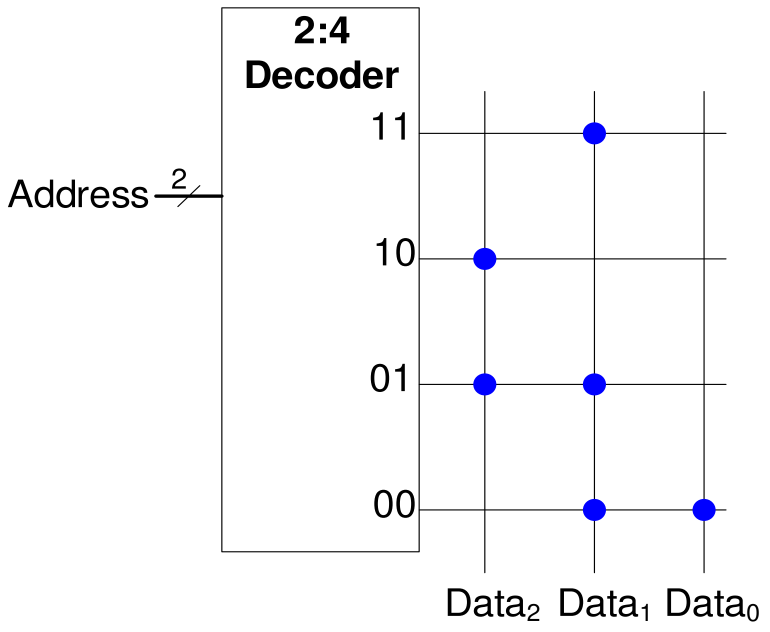


Address	Data
11	0 1 0
10	1 0 0
01	1 1 0
00	0 1 1

Fujio Masuoka, 1944 -

- Developed memories and high speed circuits at Toshiba, 1971-1994
- Invented Flash memory as an unauthorized project pursued during nights and weekends in the late 1970's
- The process of erasing the memory reminded him of the flash of a camera
- Toshiba slow to commercialize the idea; Intel was first to market in 1988
- Flash has grown into a $25 billion per year market

ROM Logic



$$Data_2 = A_1 \oplus A_0$$

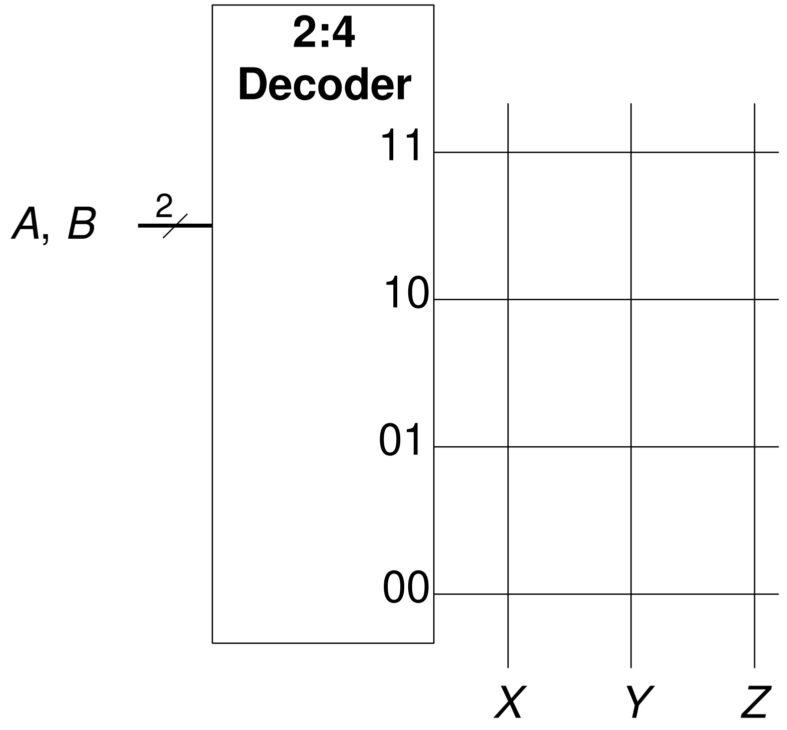
$$Data_1 = \overline{A_1} + A_0$$

$$Data_0 = \overline{A_1}\,\overline{A_0}$$

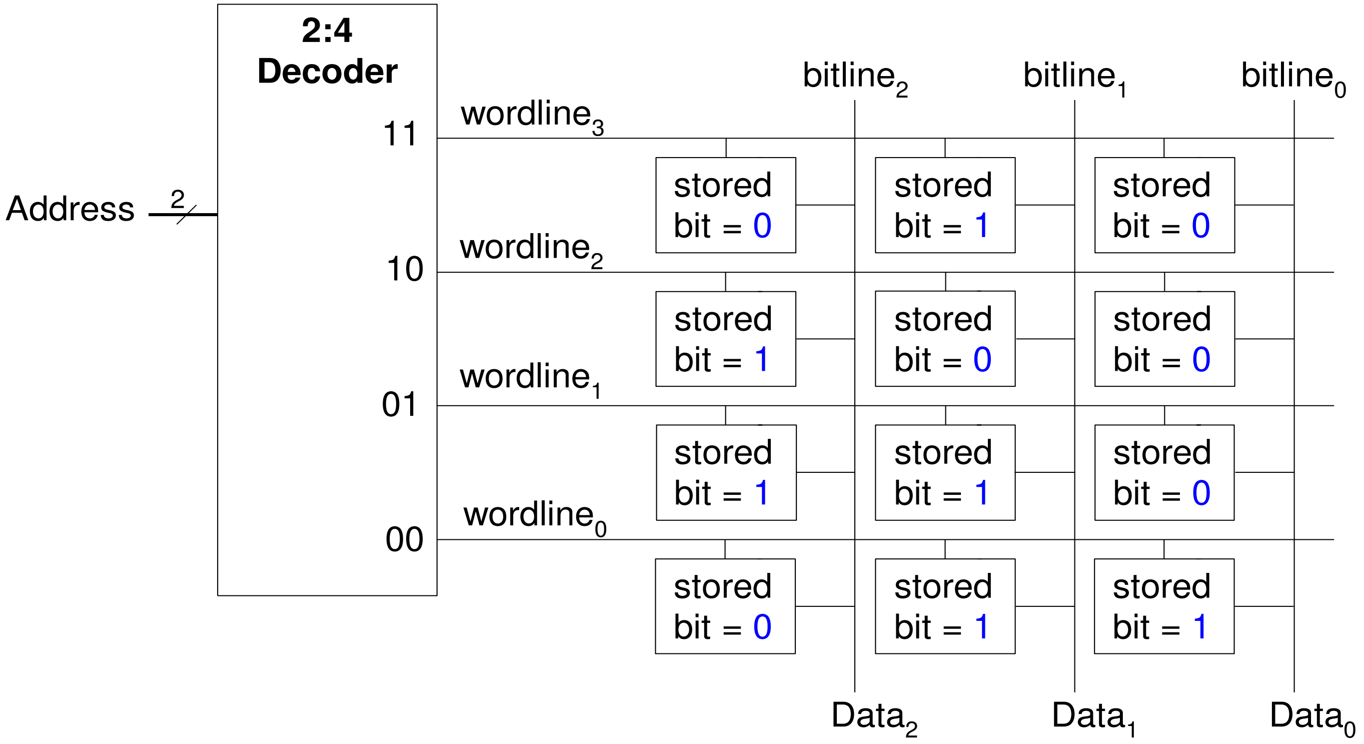
Example: Logic with ROMs

Implement the following logic functions using a $2^2 \times 3$-bit ROM:

- $X = AB$
- $Y = A + B$
- $Z = A\overline{B}$



Logic with Any Memory Array



$Data_2 =$

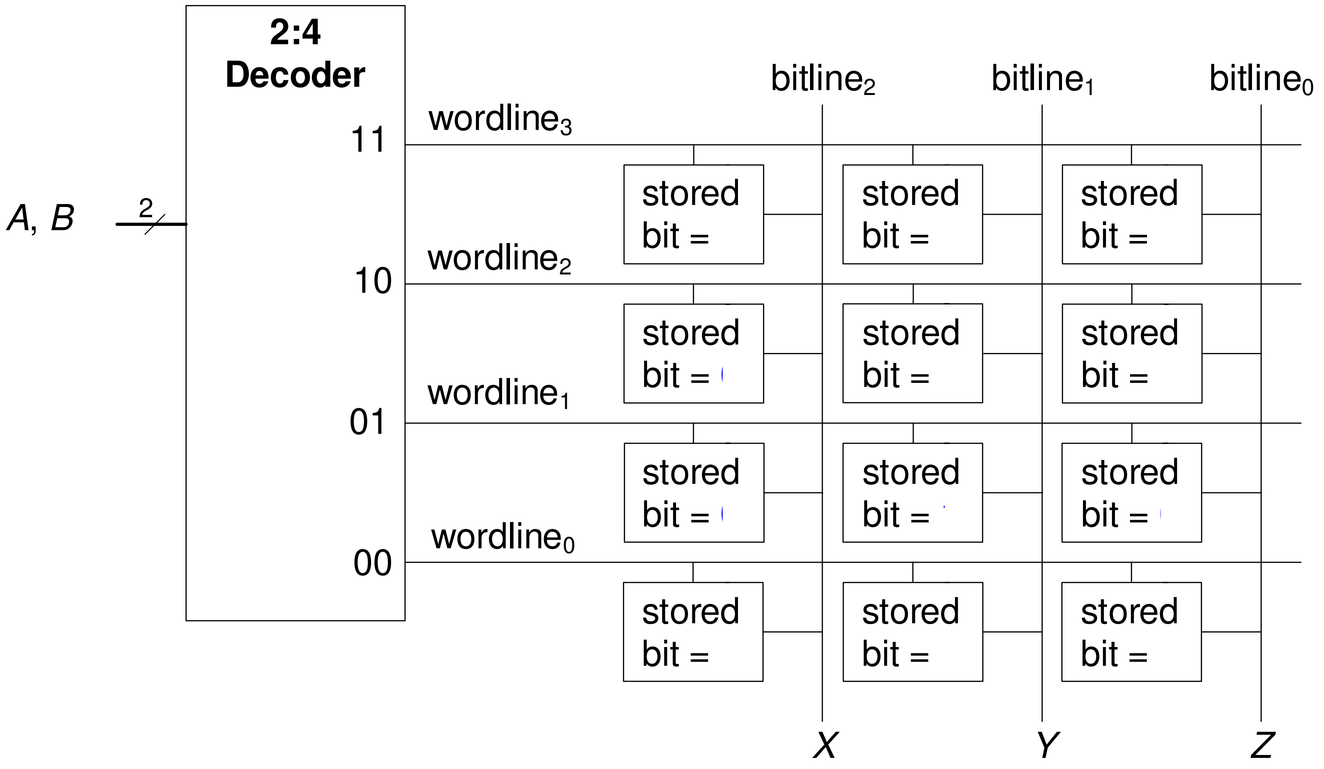
$Data_1 =$

$Data_0 =$

Logic with Memory Arrays

Implement the following logic functions using a $2^2 \times$ 3-bit memory array:

- $X = AB$
- $Y = A + B$
- $Z = A\overline{B}$

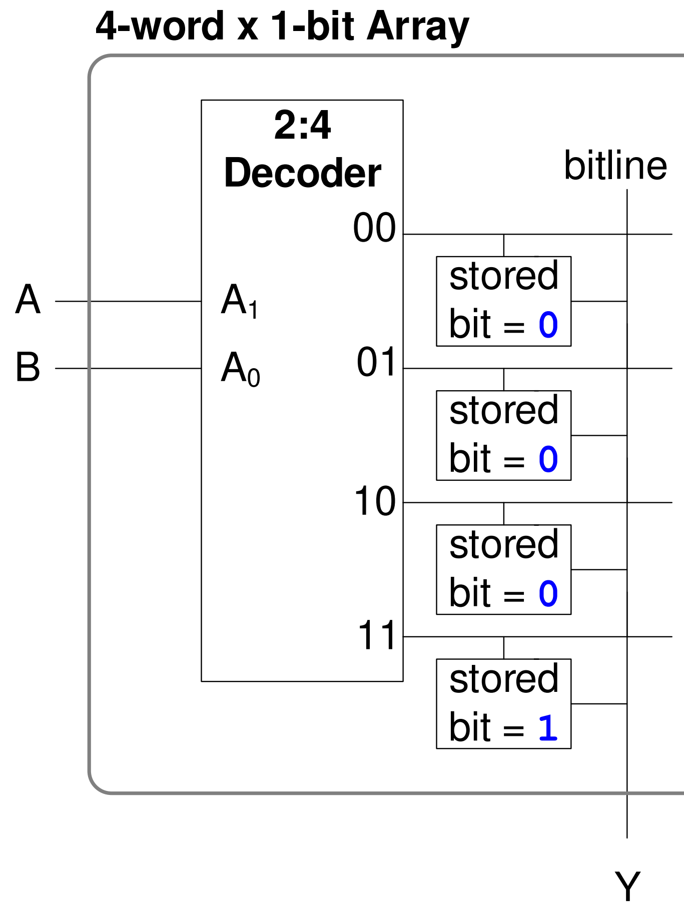


Logic with Memory Arrays

Called ***lookup tables*** (**LUTs**): look up output at each input combination (address)

Truth Table

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1



SystemVerilog & Multiported Memories

SystemVerilog RAM

```
// 256 x 3 RAM with one read/write port
module ram(input  logic       clk, we,
           input  logic [7:0] a,
           input  logic [2:0] wd,
           output logic [2:0] rd);

  logic  [2:0] RAM[255:0];

  assign rd = RAM[a];

  always @(posedge clk)
    if  (we)
      RAM[a] <= wd;
endmodule
```

SystemVerilog ROM

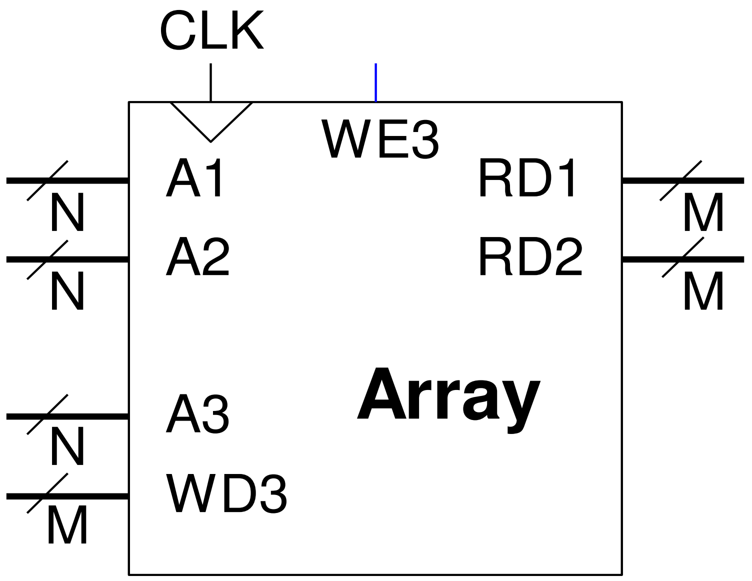
```
// 128 x 32 ROM with one read port
// Contents initialized from file
module rom(input  logic [6:0]  a,
           output logic [31:0] rd);

  logic  [31:0] ROM[127:0];

  // initialize contents from file
  initial
    $readmemh("memfile.dat", ROM);

  // read port
  assign rd = ROM[a];
endmodule
```

SystemVerilog ROM memfile

```
// memfile.dat
// Contains up to 128 lines of 32-bit hex numbers
// defining the contents of the ROM
01234567
89ABCDEF
FFFFFFFF
A5A5A5A5
…
```

Multi-ported Memories

- **Port:** address/data pair
- 3-ported memory
  - 2 read ports (A1/RD1, A2/RD2)
  - 1 write port (A3/WD3, WE3 enables writing)
- **Register file:** small multi-ported memory



SystemVerilog Memory Arrays

```
// 32 x 32 register file with 2 read, 1 write port
// register 0 hardwired to read as 0
module regfile(input  logic        clk,
               input  logic        we3,
               input  logic [4:0]  ra1, ra2, wa3,
               input  logic [31:0] wd3,
               output logic [31:0] rd1, rd2);

  logic [31:0] rf[31:0];

  always_ff @(posedge clk)
    if (we3) rf[wa3] <= wd3;

  assign rd1 = (ra1 == 5'b00000) ? 32'b0 : rf[ra1];
  assign rd2 = (ra2 == 5'b00000) ? 32'b0 : rf[ra2];
endmodule
```

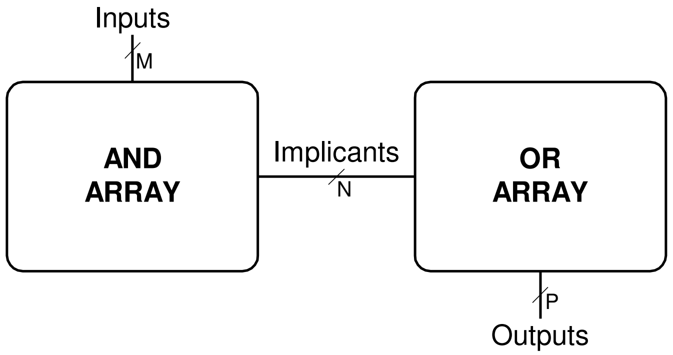
Logic Arrays: PLAs & FPGAs

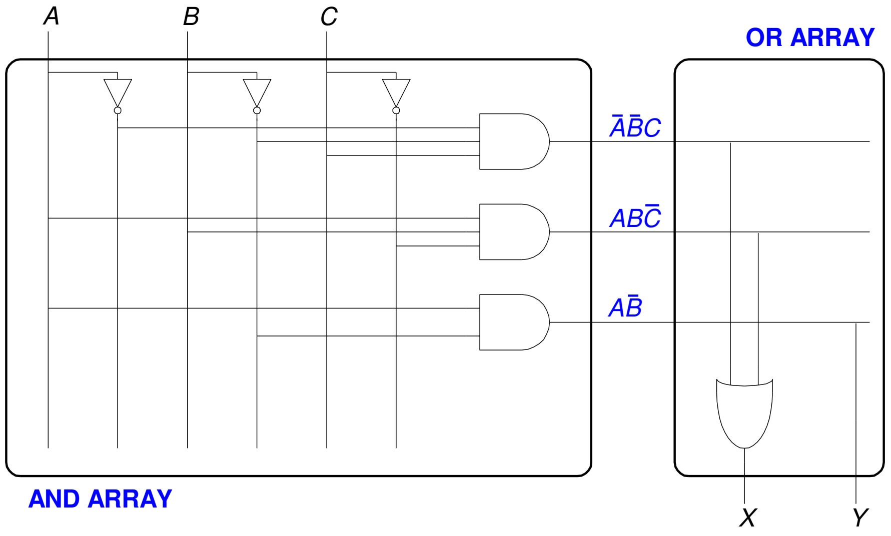
Logic Arrays

- **PLAs** (Programmable logic arrays)
  - AND array followed by OR array
  - Combinational logic only
  - Fixed internal connections
- **FPGAs** (Field programmable gate arrays)
  - Array of Logic Elements (LEs)
  - Combinational and sequential logic
  - Programmable internal connections

PLAs: Programmable Logic Arrays

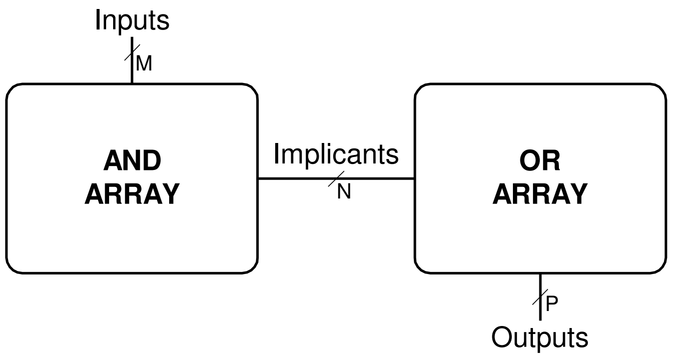
- $X = \overline{A}\overline{B}C + AB\overline{C}$
- $Y = A\overline{B}$

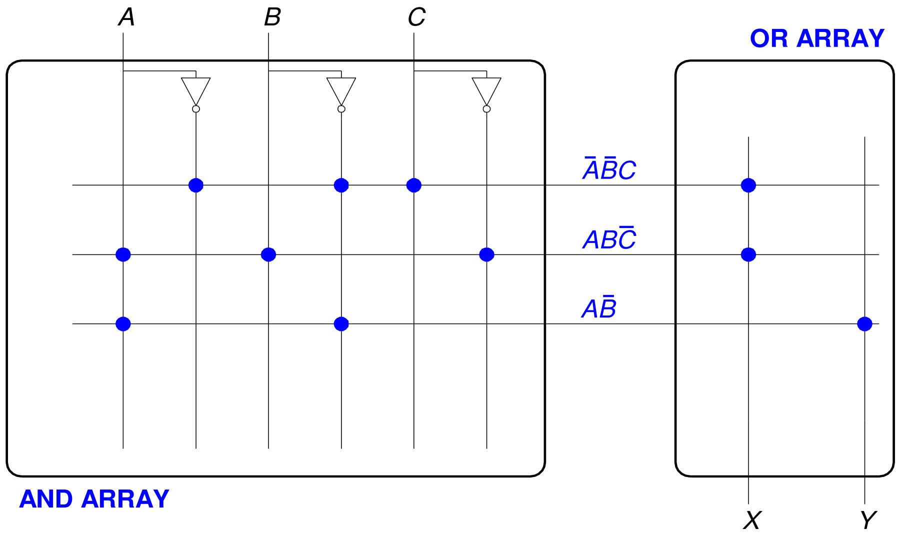




PLAs: Dot Notation

- X = ABC + ABC
- Y = AB

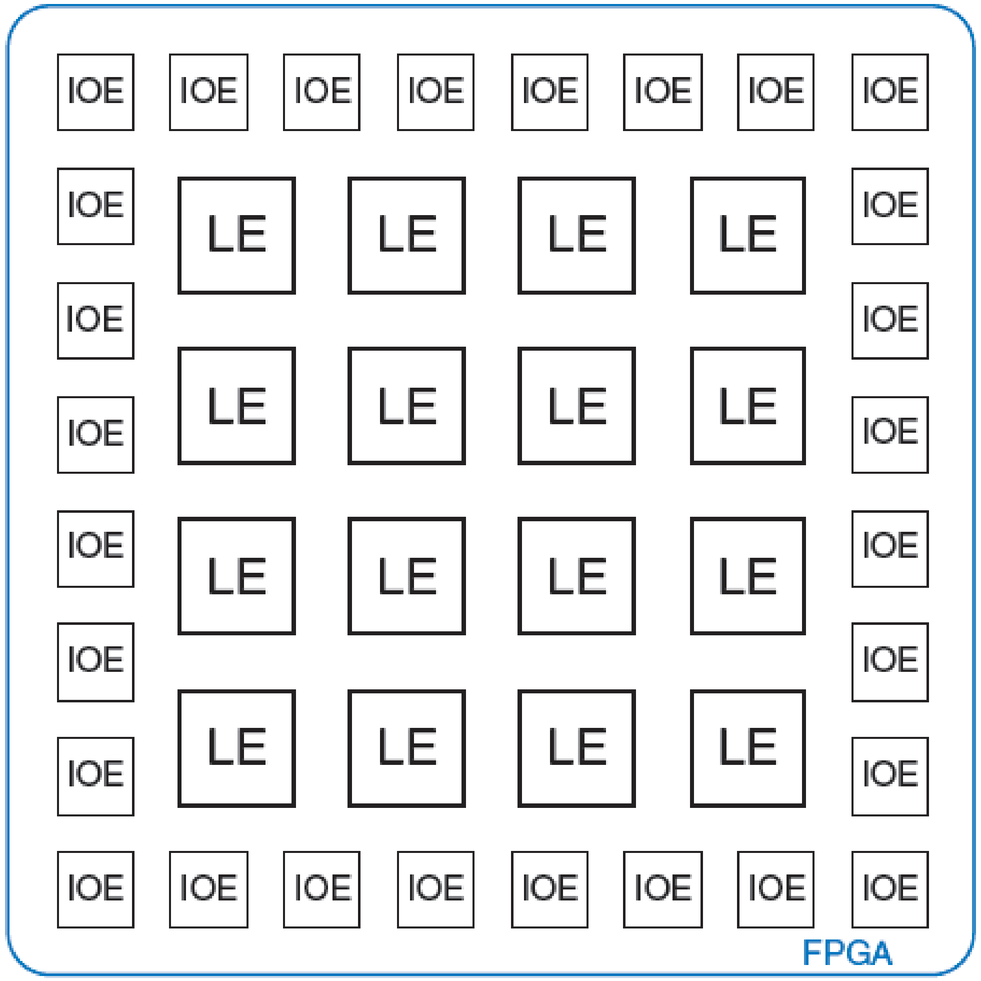




FPGAs: Field Programmable Gate Arrays

- Composed of:
  - **LEs** (Logic elements): perform logic
  - **IOEs** (Input/output elements): interface with outside world
  - **Programmable interconnection:** connect LEs and IOEs
  - Some FPGAs include other building blocks such as multipliers and RAMs

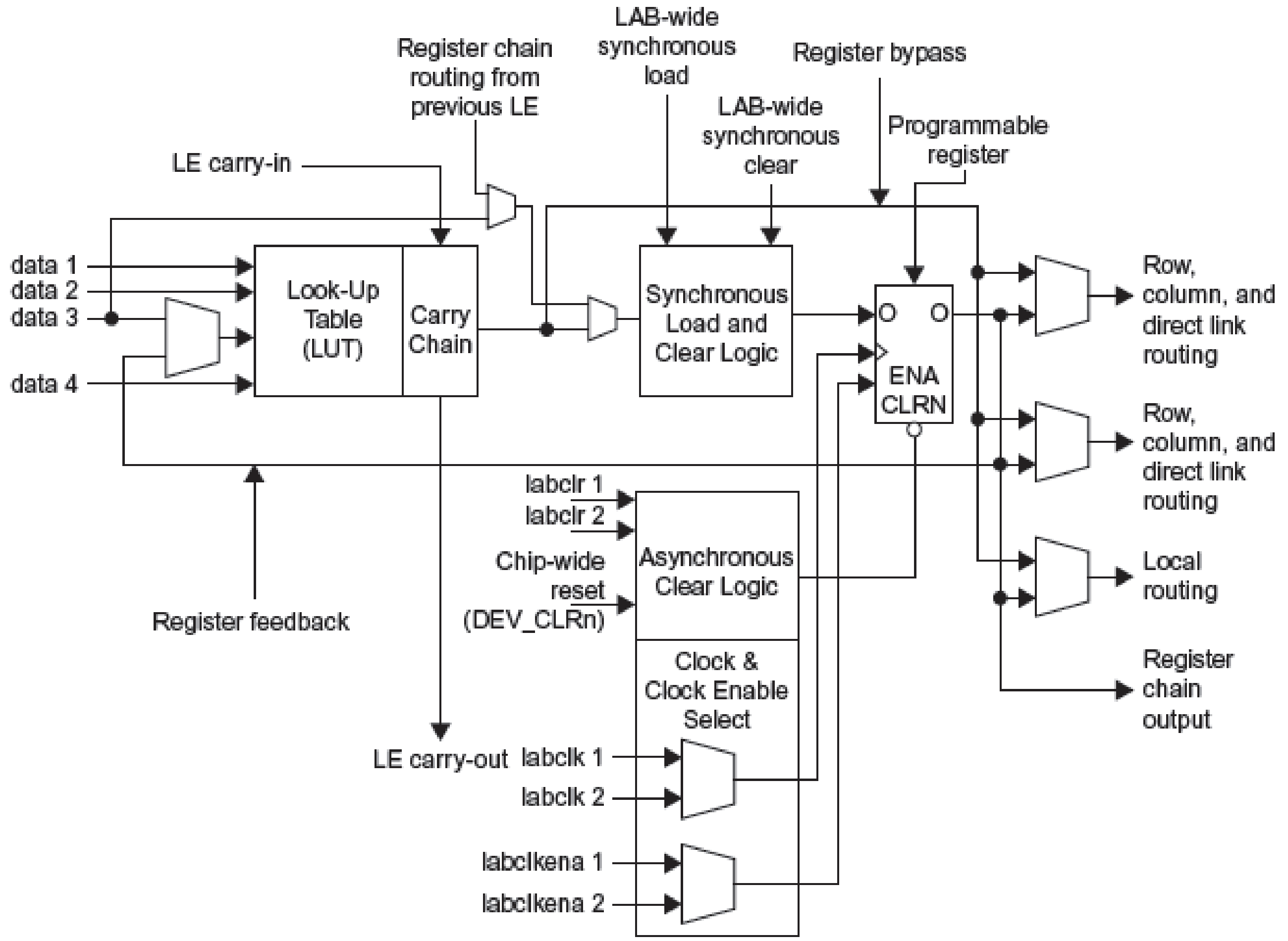
General FPGA Layout



LE: Logic Element

- Composed of:
  - **LUTs** (lookup tables): perform combinational logic
  - **Flip-flops:** perform sequential logic
  - **Multiplexers:** connect LUTs and flip-flops

Altera Cyclone IV LE



From Cyclone IV datasheet

Altera Cyclone IV LE

- **The Altera Cyclone IV LE has:**
  - 1 four-input **LUT**
  - 1 **registered output**
  - 1 **combinational output**

LE Configuration Example

Show how to configure a Cyclone IV LE to perform the following functions:

- $X = \overline{A}\,\overline{B}C + AB\overline{C}$
- $Y = A\overline{B}$

Logic Elements Example 1

How many Cyclone IV LEs are required to build

$$Y = A1 \oplus A2 \oplus A3 \oplus A4 \oplus A5 \oplus A6$$

Solution:

Logic Elements Example 2

How many Cyclone IV LEs are required to build

32-bit 2:1 multiplexer

Solution:

Logic Elements Example 3

How many Cyclone IV LEs are required to build

Arbitrary FSM with 2 bits of state, 2 inputs, 3 outputs

Solution:

FPGA Design Flow

Using a CAD tool (such as Altera’s Quartus II)

- **Enter the design** using schematic entry or an HDL
- **Simulate** the design
- **Synthesize** design and map it onto FPGA
- **Download the configuration** onto the FPGA
- **Test** the design

About these Notes