Network traffic

Traffic of requests to a web server can be categorized in three different level: HIGH, MEDIUM, and LOW. Transitions from one level to another occurs according to exponential distributions, characterized by the following rates:

- LOW to MEDIUM -> $\lambda = 0.33\ h^{-1}$
- MEDIUM to LOW -> $\lambda = 0.6\ h^{-1}$
- MEDIUM to HIGH -> $\lambda = 0.4\ h^{-1}$
- HIGH to MEDIUM -> $\lambda = 1\ h^{-1}$

Moreover, in each traffic state, the network might go to a DOWN state, according to an exponential distribution of rate $\lambda = 0.05\ h^{-1}$. The network remains in the DOWN state for an exponentially distributed amount of time, with rate $\lambda = 6\ h^{-1}$, then returns to a traffic state, with the following probabilities:

- DOWN to LOW -> $p = 0.6$
- DOWN to MEDIUM -> $p = 0.3$
- DOWN to HIGH -> $p = 0.1$

Analyze the considered network using a CTMC:

1. Draw the corresponding Markov Chain.
2. Compute the infinitesimal generator.
3. Show on a plot, the evolution of the states of the system starting from the MEDIUM traffic state, in time interval $T = [0, 8]$.
4. Show on a plot, the evolution of the states of the system starting from the DOWN state, in time interval $T = [0, 8]$.