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Methods for estimating adjusted risk ratios

Peter Cummings
Department of Epidemiology
School of Public Health and
Harborview Injury Prevention and Research Center
University of Washington
Seattle, WA
peterc@u.washington.edu

Abstract. The risk ratio can be a useful statistic for summarizing the results of cross-sectional, cohort, and randomized trial studies. I discuss several methods for estimating adjusted risk ratios and show how they can be executed in Stata, including 1) Mantel—Haenszel and inverse-variance stratified methods; 2) generalized linear regression with a log link and binomial distribution; 3) generalized linear regression with a log link, normal distribution, and robust variance estimator; 4) Poisson regression with a robust variance estimator; 5) Cox proportional hazards regression with a robust variance estimator; 6) standardized risk ratios from logistic, probit, complementary log-log, and log-log regression; and 7) a substitution method. Advantages and drawbacks are noted for some methods.

Keywords: st0162, risk ratio, odds ratio

1 Introduction

The case—control study design is typically (but not always) used when outcomes are rare in the population from which study subjects are sampled. In 1951, Cornfield noted that when outcomes are sufficiently rare, the odds ratio from a case—control study will approximate the population risk ratio for the association of an exposure with a disease outcome. It was later realized that if controls are sampled as each case arises in time, the odds ratio will estimate the incidence-rate ratio even when outcomes are common (Greenland and Thomas 1982; Rodrigues and Kirkwood 1990; Rothman, Greenland, and Lash 2008, 113—114). In Stata, case—control data can be analyzed using Mantel—Haenszel stratified methods (cc, tabodds, mhodds), logistic regression (logistic), or conditional logistic regression (clogit) to estimate adjusted odds ratios that usually can be interpreted either as risk ratios (when outcomes are rare) or incidence-rate ratios (when incidence density sampling is used).

Cross-sectional, cohort, and randomized controlled trial designs with binary outcomes can often be summarized by estimating odds ratios or risk ratios. If the study outcome is sufficiently rare among exposed and unexposed study subjects, the odds ratio for the exposure—outcome association will closely approximate the risk ratio. But if the outcome is common and the risk ratio is not close to 1, the odds ratio will be further from 1 compared with the risk ratio. Even if the outcome is rare in the entire sample, if an adjustment is made for other variables, then the adjusted odds ratio will

be further from 1 than the adjusted risk ratio if the outcome is common in adjustment variable subgroups that contribute a noteworthy portion of the outcomes (Greenland 1987).

When summary odds and risk ratios differ, there is debate regarding which is preferable. Some have argued that odds ratios are preferred because they are symmetric with regard to the outcome definition (Walter 1998; Olkin 1998; Senn 1999; Newman 2001, 35–40; Cook 2002). Furthermore, when outcomes are common, a constant (homogeneous) adjusted odds ratio for all subjects may be more plausible than a constant risk ratio (Levin 1991; Senn 1998; Cook 2002).

Some who favor risk ratios feel they are more easily understood by physicians (Sackett, Deeks, and Altman 1996). Others have noted that risk ratios have a desirable feature called collapsibility; in the absence of confounding, a weighted average of stratum-specific risk ratios will equal the ratio from one 2×2 table of the pooled (collapsed) counts from the stratum-specific tables (Miettinen and Cook 1981; Greenland 1987, 1991b; Greenland, Robins, and Pearl 1999; Newman 2001, 52–55; Rothman, Greenland, and Lash 2008, 62). This means that a crude (unadjusted) risk ratio will not change if we adjust for a variable that is not a confounder. In the absence of confounding, the risk ratio estimates the change in risk, on a ratio scale, for the entire exposed group due to exposure. Because of collapsibility, this risk ratio has a useful interpretation as the ratio change in the average risk in the exposed group due to exposure. It is not the average ratio change in risk (i.e., the average risk ratio) among exposed individuals, except in the unlikely event that the risk ratios for all individuals are the same (Greenland 1987).

Odds ratios lack the property of collapsibility and therefore the interpretation of an odds ratio is more limited; in the absence of confounding, it estimates the change in odds, on a ratio scale, in the exposed group due to exposure. But it does not estimate either the change in the average odds of the exposed due to exposure or the average change in odds (i.e., the average odds ratio) among exposed individuals, not even if all individuals had the same change in odds when exposed (Greenland 1987). The odds ratio will estimate the average change in odds for exposed individuals only if all individual odds ratios are the same and all individual risks without exposure are the same. Except in this unlikely situation, the crude odds ratio will be closer to 1 than the average of stratum-specific or individual odds ratios. Even in the absence of confounding, the adjusted (conditional) odds ratio will be further from 1 than the crude (unadjusted or marginal) odds ratio (Gail et al. 1984; Greenland 1987; Hauck et al. 1998; Steyerberg et al. 2000; Newman 2001, 52–55; Rothman, Greenland, and Lash 2008, 62; Cummings 2009).

For analysts who wish to estimate odds ratios for the association of exposure with disease in a cross-sectional study, cohort study, or randomized trial, the statistical methods in Stata's cc, tabodds, mhodds, and logistic commands can be used. If the goal is to estimate risk ratios, these same methods can be used if outcomes are sufficiently rare that odds ratios will closely approximate risk ratios. But if risk ratios are desired when outcomes are common, odds ratio estimates will not suffice. In this article, I describe methods for estimating adjusted risk ratios with confidence intervals (CIs) in Stata.

2 Data used to illustrate the methods

I will show how to reproduce the risk-ratio estimates and CIs that Greenland (2004a) gave in a review of risk-ratio estimation. The data (table 1) are from table 5.3 in Newman's (2001, 98 and 126) textbook. Newman described 192 women who were diagnosed with breast cancer in Canada and followed for 5 years; 28% (54/192) of the women died, so the outcome was not rare. Greenland estimated the risk ratio for death at 5 years among women with low estrogen-receptor levels in their breast cancer tissue compared with women who had high receptor levels; these risk ratios were adjusted for cancer stage (I, II, or III) so that women with the same cancer stage were compared.

Table 1. Deaths, total subjects, and risk of death for 192 women with breast cancer followed for 5 years, by stage at diagnosis (I, II, III) and estrogen-receptor-level category (low, high). Also, risk ratios within each cancer stage for death among women with low versus high receptor levels.

Receptor levels	Stage	Died	Total	Risk	Risk ratio for death comparing women with low versus high receptor levels
Low	Ι	2	12	0.17	1.8
High	I	5	55	0.09	1.0 (reference group)
Low	II	9	22	0.41	1.8
High	II	17	74	0.23	1.0 (reference group)
Low	III	12	14	0.86	1.4
High	III	9	15	0.60	1.0 (reference group)

3 Method 1: Mantel-Haenszel and inverse-variance stratified methods

Mantel-Haenszel methods for odds ratios were described in 1959 (Mantel and Haenszel 1959) and extended to risk ratios in 1981 (Nurminen 1981; Tarone 1981; Kleinbaum, Kupper, and Morgenstern 1982; Newman 2001, 148–149; Rothman, Greenland, and Lash 2008, 274–275). We can estimate the adjusted risk ratio for death associated with low estrogen-receptor levels (the low variable) compared with high estrogen-receptor levels by using the cs command:

(Continued on next page)

. use brcadat
(Breast cancer data)

. cs died low, by(stage) pool

Cancer stage	RR	[95% Conf	. Interva	l] M-H Weight
1	1.833333	.4024728	8.351	15 .8955224
2	1.780749	.9269437	3.4209	91 3.895833
3	1.428571	.8971062	2.2748	88 4.344828
Crude	2.225806	1.449035	3.4189	74
Pooled (direct)	1.554553	1.076372	2.2451	69
M-H combined	1.618421	1.093775	2.3947	19
Test of homogeneity Test of homogeneity		chi2(2) = chi2(2) =		r>chi2 = 0.8443 r>chi2 = 0.8251

I invoked the pool option so that the output shows both the Mantel-Haenszel combined risk ratio and the pooled risk ratio obtained using inverse-variance weights. These methods require that variables be treated as categorical, not continuous.

4 Method 2: Generalized linear regression with a log link and binomial distribution

Estimation of risk ratios using a generalized linear model with a log link and binomial distribution was proposed in 1986 (Wacholder 1986). This approach has been described in several articles (Robbins, Chao, and Fonseca 2002; McNutt et al. 2003; Barros and Hirakata 2003); it has been called log-binomial (Blizzard and Hosmer 2006) or binomial log-linear regression (Greenland 2004a). This approach can be implemented in Stata:

```
. glm died low stage2 stage3, family(binomial) link(log) eform difficult
Iteration 0:
               log\ likelihood = -154.81266
               log\ likelihood = -99.23223
Iteration 1:
Iteration 2:
               log\ likelihood = -94.764125
Iteration 3:
               log likelihood = -93.93508
               log\ likelihood = -93.050613
Iteration 4:
Iteration 5:
               log likelihood = -92.930394
Iteration 6:
               log\ likelihood = -92.927072
Iteration 7:
               log likelihood = -92.927069
Generalized linear models
                                                    No. of obs
                                                                             192
                 : ML
                                                    Residual df
                                                                             188
Optimization
                                                    Scale parameter =
                                                                               1
                 = 185.8541388
                                                     (1/df) Deviance =
                                                                        .9885858
Deviance
                 = 190.2212968
                                                     (1/df) Pearson = 1.011815
Pearson
Variance function: V(u) = u*(1-u)
                                                     [Bernoulli]
Link function
                 : g(u) = ln(u)
                                                     [Log]
                                                    AIC
                                                                        1.009657
Log likelihood
                 = -92.92706941
                                                    BIC
                                                                        -802.555
                                OIM
        died
               Risk Ratio
                             Std. Err.
                                            z
                                                 P>|z|
                                                            [95% Conf. Interval]
                 1.558321
                             .3148624
                                          2.20
                                                 0.028
                                                            1.048745
                                                                        2.315497
         low
                                                                        5.490288
      stage2
                 2.538159
                             .9991488
                                          2.37
                                                 0.018
                                                             1.17339
      stage3
                 5.868042
                             2.273768
                                          4.57
                                                 0.000
                                                            2.745787
                                                                        12.54064
```

Above Stata reported that for iteration 0, the likelihood region was not concave. When the command is run without the difficult option, Stata 10.0 will repeatedly report a not-concave region and fail to converge. The difficult option changed Stata's convergence algorithm and solved the problem in this example, but that option may not always work. The convergence problem arose because among women with Stage III cancer and low estrogen-receptor levels, the risk of death was close to 1: 12/14 = 0.86. When the risk is close to 1 in a stratum of the data, maximum-likelihood convergence may fail. This problem has been discussed in several articles (Carter, Lipsitz, and Tilley 2005; Blizzard and Hosmer 2006; Lumley, Kronmal, and Ma 2006; Localio, Margolis, and Berlin 2007).

Wacholder (1986; Lumley, Kronmal, and Ma 2006) described a method that modified the convergence by truncating estimated risks to values slightly greater than 0 and less than 1. This is implemented in Stata's binreg command:

(Continued on next page)

```
. xi: binreg died low i.stage, rr
                                       (naturally coded; _Istage_1 omitted)
i.stage
                  _Istage_1-3
Iteration 1:
               deviance = 309.6253
Iteration 2:
             deviance =
                            190.79
Iteration 3:
             deviance = 185.9416
              deviance = 185.8543
Iteration 4:
Iteration 5:
              deviance = 185.8541
Iteration 6:
               deviance = 185.8541
Generalized linear models
                                                    No. of obs
                                                                            192
Optimization
                : MQL Fisher scoring
                                                    Residual df
                                                                            188
                   (IRLS EIM)
                                                    Scale parameter =
                                                                              1
                                                                       .9885858
                 = 185.8541388
                                                    (1/df) Deviance =
Deviance
                 = 190.2164034
                                                    (1/df) Pearson = 1.011789
Pearson
Variance function: V(u) = u*(1-u)
                                                    [Bernoulli]
                 : g(u) = ln(u)
Link function
                                                    [Log]
                                                    BIC
                                                                       -802.555
                               EIM
        died
               Risk Ratio
                            Std. Err.
                                            z
                                                P>|z|
                                                           [95% Conf. Interval]
                            .3067419
                                                 0.024
         low
                 1.558326
                                          2.25
                                                           1.059515
                                                                       2.291972
                 2.538158
                            .9976133
                                          2.37
                                                 0.018
                                                           1.174781
                                                                       5.483782
   _Istage_2
   _Istage_3
                 5.868047
                            2.259727
                                          4.60
                                                 0.000
                                                           2.758699
                                                                       12.48196
```

The risk ratios and standard errors estimated by glm, family(binomial) link(log) and by binreg, rr are similar, but not identical, because the convergence methods differ. The default for glm is maximum likelihood and the default for binreg is iterated, reweighted least squares; by changing the default optimization, either command can produce the estimates obtained by the other, provided that both methods achieve convergence. Because binreg constrains risk estimates to be greater than 0 and less than 1, it may converge when maximum likelihood will not. Sometimes both methods will fail to converge.

Another approach to convergence difficulty is to make several copies of the original data and append them into one data file (Deddens, Petersen, and Lei 2003; Petersen and Deddens 2006; Deddens and Petersen 2008). Then make one more copy, but recode all the 0 outcomes to 1 and all the 1 outcomes to 0 in that copy, and append this recoded copy to all the other copies. Then analyze all these data together; including one set of data with reversed outcomes may help the maximum-likelihood algorithm converge. If the number of copies is sufficiently large, the risk-ratio estimates will approximate those from maximum-likelihood methods. Because a set of records larger than the original data is used, corrections must be made to the standard errors and CIs. This extra step of correcting the standard errors can be avoided by using just two copies of the data, one with recoded outcomes, with appropriate weights (Lumley, Kronmal, and Ma 2006). Below I used importance weights of 0.999 and 0.001, and produced the risk ratio I would get by analyzing 999 copies of the original data with just 1 copy of recoded data. The difficult option was still required.

```
. generate iweight = .999
. append using breadat
(label noves already defined)
. recode died 0=1 1=0 if iweight==.
(died: 192 changes made)
. replace iweight = .001 if iweight == .
(192 real changes made)
. glm died low stage2 stage3 [iweight=iweight], family(binomial) link(log)
> eform difficult
Iteration 0:
              log likelihood = -155.22946 (not concave)
Iteration 1: log likelihood = -99.401459
Iteration 2: log likelihood = -94.953215
Iteration 3: log likelihood = -94.13882
Iteration 4: log likelihood = -93.23838
Iteration 5: log likelihood = -93.113209
Iteration 6:
              log likelihood = -93.109614
Iteration 7:
              log likelihood = -93.109611
Generalized linear models
                                                   No. of obs
                                                                           384
                : ML
                                                   Residual df
                                                                           380
Optimization
                                                   Scale parameter =
                                                                             1
Deviance
                                                   (1/df) Deviance = .4900506
                 = 186.2192211
Pearson
                    190.253604
                                                   (1/df) Pearson = .5006674
Variance function: V(u) = u*(1-u)
                                                   [Bernoulli]
Link function : g(u) = ln(u)
                                                   [Log]
                                                   AIC
                                                                   = .5057792
                                                                   = -2075.025
Log likelihood = -93.10961053
                               OIM
                            Std. Err.
                                                          [95% Conf. Interval]
        died
               Risk Ratio
                                                P>|z|
                                         2.19
                                                0.028
                                                                      2.312583
         low
                 1.556724
                            .3143539
                                                          1.047915
      stage2
                 2.523499
                            .9897355
                                         2.36
                                                0.018
                                                          1.169918
                                                                      5.443157
      stage3
                 5.823722
                            2.248418
                                         4.56
                                                0.000
                                                          2.732558
                                                                      12.41172
```

5 Method 3: Generalized linear regression with a log link, normal distribution, and robust variance estimator

Convergence problems for generalized linear regression with a log link can also be resolved by using a Gaussian (normal) distribution (Lumley, Kronmal, and Ma 2006). The resulting standard errors may be too big or small, but a robust variance estimator will correct the standard errors. Below convergence was achieved without the difficult option:

(Continued on next page)

```
. use brcadat, clear
(Breast cancer data)
. glm died low stage2 stage3, family(gaussian) link(log) eform robust
Iteration 0:
               log pseudolikelihood = -169.42788
               log pseudolikelihood = -119.04978
Iteration 1:
Iteration 2:
               log pseudolikelihood = -98.365962
Iteration 3:
               log pseudolikelihood = -94.332246
Iteration 4:
               log pseudolikelihood = -94.242364
Iteration 5:
               log pseudolikelihood = -94.242364
Generalized linear models
                                                    No. of obs
                                                                            192
Optimization
                : ML
                                                    Residual df
                                                                   =
                                                                            188
                                                    Scale parameter =
                                                                       .1595924
Deviance
                                                    (1/df) Deviance = .1595924
                 = 30.00336652
Pearson
                 = 30.00336652
                                                    (1/df) Pearson = .1595924
Variance function: V(u) = 1
                                                    [Gaussian]
Link function
               : g(u) = ln(u)
                                                    [Log]
                                                    AIC
                                                                    = 1.023358
Log pseudolikelihood = -94.24236355
                                                    BIC
                                                                    = -958.4058
                             Robust
                                                           [95% Conf. Interval]
                   exp(b)
                                                P>|z|
        died
                            Std. Err.
                                           z
                 1.553274
                            .3193155
                                         2.14
                                                0.032
                                                           1.038154
                                                                       2.323992
         low
                 2.524622
                            1.005045
                                         2.33
                                                0.020
                                                           1.157006
                                                                       5.508803
      stage2
                  5.86819
                            2.312817
                                         4.49
                                                0.000
                                                           2.710329
                                                                       12.70534
      stage3
```

6 Method 4: Poisson regression with a robust variance estimator

Poisson regression is a generalized linear model with a log link and a Poisson distribution. When the outcome is binary, the exponentiated coefficients are risk ratios instead of incidence-rate ratios (Gourieroux, Monfort, and Trognon 1984a,b; Lloyd 1999, 85–86; Wooldridge 2002, 648–649; Greenland 2004a; Zou 2004; Carter, Lipsitz, and Tilley 2005). Methods that rely on the Poisson distribution assume that the mean count and its variance are equal. In the breast cancer data, the mean count of deaths per woman was 54/192 = 0.28125. If the variance of the mean count is also 0.28125, then the standard error of the mean count is the square root of the variance divided by the square root of the number of women (192) = 0.0382733. This is indeed the standard error that Stata reports using the ci, poisson command or using lincom after the poisson command:

. ci died, poisson

Variable	Exposure	Mean	Std. Err.	— Poisson [95% Conf.	
died	192	. 28125	.0382733	.2112837	.3669702

i, nolog						
ssion			Numbe	er of obs	=	192
			LR cl	ni2(0)	=	0.00
					=	
d = -122.4996	1 		Pseud	io R2	-	0.0000
Coef.	Std. Err.	z	P> z	[95% Co	nf.	Interval]
-1.268511	.1360828	-9.32	0.000	-1.53522	9	-1.001794
s, irr						
_cons = 0						
IRR	Std. Err.	z	P> z	[95% Co	nf.	Interval]
.28125	.0382733	-9.32	0.000	.215406	4	.3672201
	Coef1.268511 s, irr cons = 0 IRR	Coef. Std. Err1.268511 .1360828 s, irr cons = 0 IRR Std. Err.	Coef. Std. Err. z -1.268511 .1360828 -9.32 s, irr cons = 0 IRR Std. Err. z	Number Number	Number of obs LR chi2(0) Prob > chi2 d = -122.49961 Pseudo R2 Coef. Std. Err. z P> z [95% Cost -1.268511	Number of obs = LR chi2(0) = Prob > chi2 = Prob > chi2 = Pseudo R2 = Pseud

If the deaths were from a Poisson distribution, women would have nonnegative integer counts of 0, 1, 2, 3, ..., or more deaths. The data cannot be Poisson, because no woman dies more than once. The data are from a binomial distribution, and the binomial variance is assumed to be the proportion that died multiplied by 1 minus that proportion. The standard error of the mean proportion is the square root of the variance divided by the square root of the number of women, which is 0.0324477. This is the standard error reported by ci, binomial:

. ci died, binomial

Variable	Obs	Mean	Std. Err.	— Binomia [95% Conf.	
died	192	.28125	.0324477	.2188833	.3505085

If we use Poisson methods for these binomial data, the standard error for the outcome proportion (risk) is too large: 0.03827 instead of 0.03245. As the outcome becomes less common, the Poisson standard error will converge toward the binomial standard error (Armitage, Berry, and Matthews 2002, 71–76). But in the breast cancer data, use of Poisson regression to estimate risk ratios will produce standard errors, p-values, and CIs that are too large:

. poisson died	d low stage2 s	stage3, irr r	ıoLog				
Poisson regres	ssion			Number	of obs	; =	192
•				LR chi	.2(3)	=	26.71
				Prob >	chi2	=	0.0000
Log likelihoo	d = -109.1460	1		Pseudo	R2	=	0.1090
	r						
died	IRR	Std. Err.	z	P> z	[95%	Conf.	Interval]
low	1.630775	.4688634	1.70	0.089	.9282	513	2.864987
stage2	2.520742	1.074375	2.17	0.030	1.093	288	5.811955
stage3	5.913372	2.645148	3.97	0.000	2.460	814	14.20992

Above, the 95% CI for the low variable is wide, 0.93 to 2.86, compared with the CIs from other methods. We can obtain standard errors and CIs that are approximately correct by using a robust variance estimator, which can relax the assumption that the data are from a Poisson distribution (Wooldridge 2002, 650–651; Greenland 2004a; Zou 2004; Carter, Lipsitz, and Tilley 2005). The robust variance estimator is sometimes called the Huber, White, Huber–White, sandwich, or survey estimator, as well as other names (Hardin and Hilbe 2007, 35–36). In Stata, we can invoke this estimator with the vce(robust) option and the CI for the low variable becomes narrower, 1.07 to 2.48:

```
. poisson died low stage2 stage3, irr nolog vce(robust)
Poisson regression
                                                                               192
                                                     Wald chi2(3)
                                                                             53.61
                                                     Prob > chi2
                                                                            0.0000
Log pseudolikelihood = -109.14601
                                                     Pseudo R2
                                                                            0.1090
                              Robust
                       IRR
                             Std. Err.
                                                  P>|z|
                                                             [95% Conf. Interval]
        died
                                             z
                              .3480542
                  1.630775
                                           2.29
                                                  0.022
                                                             1.073305
                                                                          2.477792
         low
                  2.520742
                              .9937819
                                           2.35
                                                  0.019
                                                              1.16399
                                                                          5.458932
      stage2
                  5.913372
                              2.28568
                                           4.60
                                                  0.000
                                                             2.772187
                                                                          12.61386
      stage3
```

7 Method 5: Cox proportional hazards regression with a robust variance estimator

Cox proportional hazards regression can estimate risk ratios if we set the follow-up time to 1, or any quantity that is the same for all subjects, and use the Breslow method to break ties. The robust variance estimator should be used, because otherwise the standard errors will be too large:

. stcox low s	tage2 stage3,	hr breslow v	ce(robus	st) nolog			
Cox regression	n Breslow m	method for to	ies				
No. of subject	ts =	192		Numb	er of obs	=	192
No. of failure	es =	54					
Time at risk	=	192					
				Wald	chi2(3)	=	53.61
Log pseudolik	elihood = -:	270.55115		Prob	> chi2	=	0.0000
		Robust					
_t	Haz. Ratio	Std. Err.	z	P> z	[95% Cor	nf.	Interval]
low	1.630775	.3480542	2.29	0.022	1.073309	5	2.477792
stage2	2.520742	.9937819	2.35	0.019	1.16399	9	5.458932
stage3	5.913372	2.28568	4.60	0.000	2.772187	7	12.61386

The results above reproduce exactly the results from Poisson regression with the robust variance estimator; the Poisson and Cox methods are identical when implemented in this way. Options other than the Breslow method for dealing with tied survival times will produce risk-ratio estimates for exposure to a low estrogen-receptor-level tumor that are too large: 1) the efron option produces a risk ratio of 1.91, 2) the exactm option yields 2.04, and 3) the exactp option risk ratio is 2.49.

8 Method 6: Regression-based standardized risk ratios

Any regression model for binomial outcomes can estimate the probability (risk) of death for women with high and low estrogen-receptor—level tumors within each cancer stage. With this information, we can estimate the average risk of death that would be expected if all 192 women had low estrogen-receptor—level tumors and the distribution of cancer stages observed in the data. This estimate is said to be standardized to the distribution of the other variables, cancer stage in this example, in the regression model (Lane and Nelder 1982; Flanders and Rhodes 1987; Joffe and Greenland 1995; Greenland 1991a, 2004a; Localio, Margolis, and Berlin 2007; Rothman, Greenland, and Lash 2008, 442–446). The average risk can also be estimated assuming that all 192 women had a high estrogen-receptor—level tumor. The ratio of the low estrogen-receptor—level average risk divided by the high estrogen-receptor—level average risk can be calculated and the standard error for this risk ratio can be estimated using the delta method (Casella and Berger 2002, 240–245). The word "standardized" is used just as for standardized mortality rates or any statistic standardized to a given population distribution. We can estimate this standardized risk ratio by using logistic regression:

(Continued on next page)

```
. use brcadat, clear
(Breast cancer data)
. logistic died low stage2 stage3, nolog

Logistic regression

Number of obs = 192

LR chi2(3) = 42.27

Prob > chi2 = 0.0000

Log likelihood = -92.939847

Pseudo R2 = 0.1853
```

died	Odds Ratio	Std. Err.	z	P> z	[95% Conf.	Interval]
low	2.508065	.9916923	2.33	0.020	1.155507	5.443836
stage2	3.109772	1.44851	2.44	0.015	1.248087	7.748406
stage3	18.8389	11.03231	5.01	0.000	5.978344	59.36498

```
. #delimit ;
delimiter now ;
. predictn1 lnrr =
> ln(
> sum(1/
> (1+exp(-(_b[_cons]+_b[stage2]*stage2+_b[stage3]*stage3+_b[low]))))
> /
> sum(1/
> (1+exp(-(_b[_cons]+_b[stage2]*stage2+_b[stage3]*stage3)))))
> , se(lnrr_se);
. #delimit cr
delimiter now cr
. scalar rr = exp(lnrr[_N])
. scalar upper = exp(lnrr[_N] + invnormal(1-.05/2)*lnrr_se[_N])
. scalar lower = exp(lnrr[_N] - invnormal(1-.05/2)*lnrr_se[_N])
. display "Risk ratio = " rr " 95% CI = " lower ", " upper
Risk ratio = 1.6755988 95% CI = 1.0935713, 2.5673969
```

The adjusted odds ratio for death among women with a low estrogen-receptor level-tumor, compared with women with a high estrogen-receptor-level tumor, was 2.5. Because the outcome of death was common, this odds ratio does not closely approximate the risk ratio.

Above I used predictnl to estimate the ln of the risk ratio (lnrr variable); this command can estimate nonlinear comparisons from regression coefficients. The se option estimated the standard error for the ln risk ratio using the delta method. To make the output less cluttered, I used delimit to change how Stata recognizes the end of a command line. To estimate the risk or probability of death, I used the expression $1/\{1 + \exp(-\text{linear predictor})\}$. The first sum used by predictnl is for risk estimates if all women had a low estrogen-receptor-level tumor, because the ln odds term for low receptor status, b[low], is included in the sum, regardless of each woman's actual receptor status. In this first sum, the regression coefficients (which are ln odds estimates) for _Istage_2 and _Istage_3 were both multiplied by each woman's observed cancer stage, thereby standardizing the estimate to the observed distribution of cancer stage. Stata's sum() function is the running sum from the first record to the last, so the sum in the last record of the data is the sum of all the estimated risks if all 192 women had a low estrogen-receptor-level tumor but each had her observed cancer stage. The second sum

in the expression, after the line with only a division sign, is the sum of all estimated risks if all women had a high estrogen-receptor—level tumor, again standardized to the observed cancer-stage distribution. The first sum is divided by the second and the ln taken of this ratio so that the ln of the risk ratio is estimated. I then estimated the risk ratio, which is exp(lnrr), and the 95% upper and lower confidence limits for the risk ratio, and used the display command to show these results for the last record by using the subscript [_N]: risk ratio = 1.7, 95% CI is [1.1, 2.6].

We can use simpler commands to estimate the risk ratio, but they do not provide a CI. Still, these commands show how the risks and risk ratio may be estimated and are shown below to clarify how Stata is using the regression estimates. After fitting the logistic model, the commands are

```
. replace low=0
(48 real changes made)
. predict risk0
(option pr assumed; Pr(died))
. summ risk0, meanonly
. local avrisk0 = r(mean)
. replace low=1
(192 real changes made)
. predict risk1
(option pr assumed; Pr(died))
. summ risk1, meanonly
. local avrisk1 = r(mean)
. local rr = `avrisk1'/`avrisk0'
. display "Risk1 = " `avrisk1' " Risk0 = " `avrisk0' " Risk ratio = " `rr'
Risk1 = .40087948 Risk0 = .23924549 Risk ratio = 1.6755989
```

Risks for binomial outcomes can also be estimated after probit regression. In the probit model, the outcome risk estimate applies the cumulative standard normal distribution function (normal()) to the linear predictor, instead of the ln odds function used in logistic regression:

```
. use brcadat, clear
(Breast cancer data)
. probit died low stage2 stage3, nolog
Probit regression
                                                     Number of obs
                                                                               192
                                                     LR chi2(3)
                                                                             42.21
                                                     Prob > chi2
                                                                            0.0000
Log likelihood = -92.968357
                                                     Pseudo R2
                                                                            0.1850
                                                              [95% Conf. Interval]
        died
                             Std. Err.
                                                  P>|z|
                     Coef.
                                             z
                              .2343501
                                                   0.022
         low
                  .5386148
                                           2.30
                                                               .079297
                                                                          .9979327
      stage2
                  .6290485
                              .2503224
                                           2.51
                                                   0.012
                                                              .1384256
                                                                          1.119671
                                                   0.000
                                                                          2.386454
      stage3
                  1.739085
                              .3302966
                                           5.27
                                                             1.091715
                                                  0.000
       _cons
                 -1.376363
                             .2165793
                                          -6.36
                                                            -1.800851
                                                                         -.9518752
```

```
. #delimit ;
delimiter now ;
. predictnl lnrr = ln(
> sum(normal(_b[_cons]+_b[stage2]*stage2+_b[stage3]*stage3+_b[low]))
> /
> sum(normal(_b[_cons]+_b[stage2]*stage2+_b[stage3]*stage3)))
> , se(lnrr_se);
. #delimit cr
delimiter now cr
. scalar rr = exp(lnrr[_N])
. scalar upper = exp(lnrr[_N] + invnormal(1-.05/2)*lnrr_se[_N])
. scalar lower = exp(lnrr[_N] - invnormal(1-.05/2)*lnrr_se[_N])
. display "Risk ratio = " rr " 95% CI = " lower ", " upper
Risk ratio = 1.6751332 95% CI = 1.0913484, 2.5711965
```

Nelder (2001) has suggested that when a dichotomous outcome is common, the complementary log-log regression model may fit the data well:

```
. use brcadat, clear
(Breast cancer data)
. cloglog died low stage2 stage3, nolog
Complementary log-log regression
                                             Number of obs
                                                                       192
                                             Zero outcomes
                                                                       138
                                             Nonzero outcomes
                                                                       54
                                             LR chi2(3)
                                                                     42.60
Log likelihood = -92.771237
                                             Prob > chi2
                                                                    0.0000
       died
                  Coef. Std. Err.
                                             P>|z|
                                                      [95% Conf. Interval]
                                       Z
                .7138795 .2936374
                                      2.43 0.015
                                                      .1383607
                                                                 1.289398
        low
                1.022028
                                      2.39 0.017
                                                      .1855198
     stage2
                         .426798
                                                                1.858537
                2.302261 .4522776
                                      5.09 0.000
                                                      1.415813
                                                                 3.188709
     stage3
                                     -6.10 0.000 -3.130655 -1.608683
      _cons
               -2.369669 .3882655
```

```
. #delimit;
delimiter now;
. predictnl lnrr = ln(
> sum(1-exp(-exp(_b[_cons]+_b[stage2]*stage2+_b[stage3]*stage3+_b[low])))
> /
> sum(1-exp(-exp(_b[_cons]+_b[stage2]*stage2+_b[stage3]*stage3))))
> , se(lnrr_se);
. #delimit cr
delimiter now cr
. scalar rr = exp(lnrr[_N])
. scalar upper = exp(lnrr[_N] + invnormal(1-.05/2)*lnrr_se[_N])
. scalar lower = exp(lnrr[_N] - invnormal(1-.05/2)*lnrr_se[_N])
. display "Risk ratio = " rr " 95% CI = " lower ", " upper
Risk ratio = 1.6652749 95% CI = 1.1009763, 2.5188013
```

Hardin and Hilbe (2007, 147) note that if most subjects either have or do not have the outcome, the complementary log-log and log-log models may fit better than logistic or probit models. We can fit the log-log model using the glm command:

```
. use brcadat, clear
(Breast cancer data)
. glm died low stage2 stage3, nolog family(bin) link(loglog) eform
Generalized linear models
                                                    No. of obs
                                                                             192
Optimization
                 : ML
                                                    Residual df
                                                                             188
                                                    Scale parameter =
                                                                              1
Deviance
                 = 186.5949538
                                                    (1/df) Deviance = .9925263
Pearson
                 = 192.8460376
                                                    (1/df) Pearson = 1.025777
Variance function: V(u) = u*(1-u)
                                                    [Bernoulli]
Link function
               : g(u) = -\ln(-\ln(u))
                                                    [Log-log]
                                                    AIC
                                                                    = 1.013515
                                                                    = -801.8142
                 = -93.2974769
                                                    BTC
Log likelihood
                                MIO
        died
                   exp(b)
                            Std. Err.
                                                 P>|z|
                                                           [95% Conf. Interval]
                 1.642276
                             .3938706
                                          2.07
                                                 0.039
                                                           1.026362
                                                                        2.627796
                 1.695259
                             .3494023
                                          2.56
                                                 0.010
                                                                        2.539063
      stage2
                                                           1.131876
                                                           2.835022
      stage3
                 6.133979
                            2.415435
                                          4.61
                                                 0.000
                                                                       13.27174
. #delimit ;
delimiter now;
. predictnl lnrr = ln(
    sum(exp(-exp(-(_b[_cons]+_b[stage2]*stage2+_b[stage3]*stage3+_b[low]))))
    sum(exp(-exp(-(_b[_cons]+_b[stage2]*stage2+_b[stage3]*stage3)))))
    , se(lnrr_se);
. #delimit cr
delimiter now cr
. scalar rr = exp(lnrr[_N])
. scalar upper = exp(lnrr[_N] + invnormal(1-.05/2)*lnrr_se[_N])
. scalar lower = exp(lnrr[_N] - invnormal(1-.05/2)*lnrr_se[_N])
. display "Risk ratio = " rr " 95% CI = " lower ", " upper
```

9 Method 7: A substitution method

Risk ratio = 1.6312705 95% CI = 1.0545183, 2.5234682

A crude (unadjusted) odds ratio can be converted to a risk ratio: crude risk ratio = $(\text{crude odds ratio})/\{(1-\text{Po})+(\text{Po}\times\text{crude odds ratio})\}$, where Po is the proportion of all unexposed subjects who had the outcome in the data. Zhang and Yu (1998) suggested that, in a cohort study, one can use this same formula to convert an adjusted odds ratio to an adjusted risk ratio. This substitution method was described by Holland (1989), who used it to estimate an adjusted risk difference from a Mantel-Haenszel summary odds ratio. Greenland and Holland (1991) reported that this will produce ratio estimates biased away from 1 when outcomes are common and risk among those not exposed varies substantially. The bias occurs because the summary odds ratio is not a weighted average of stratum-specific odds ratios; odds ratios lack the property of collapsibility (Greenland 1987). The method can be implemented in Stata:

```
. use brcadat, clear
(Breast cancer data)
. summ died if low==0, meanonly
. local p0 = r(mean)
. display `p0´
.21527778
. logistic died low stage2 stage3, nolog
                                                                           192
Logistic regression
                                                  Number of obs =
                                                  LR chi2(3)
                                                                         42.27
                                                                        0.0000
                                                  Prob > chi2
                                                  Pseudo R2
Log likelihood = -92.939847
                                                                        0.1853
        died
               Odds Ratio
                           Std. Err.
                                           z
                                                P>|z|
                                                          [95% Conf. Interval]
         low
                 2.508065
                            .9916923
                                         2.33
                                                0.020
                                                          1.155507
                                                                      5.443836
                 3.109772
                            1.44851
                                         2.44
                                               0.015
                                                          1.248087
                                                                      7.748406
      stage2
      stage3
                  18.8389
                           11.03231
                                         5.01
                                                0.000
                                                          5.978344
                                                                      59.36498
. scalar rr = \exp(b[low])/[(1-p0)+(p0*exp(b[low]))]
 scalar lower = \exp(b[low]-invnormal(1-.05/2)*_se[low])/[(1-p0')+
> (`p0'*exp(_b[low]-invnormal(1-.05/2)*_se[low]))]
. scalar upper = \exp(b[low]+invnormal(1-.05/2)*_se[low])/[(1-p0')+
```

10 Bootstrap Cls

In some examples above, approximately correct CIs were obtained using robust or delta methods. Bootstrap methods can also be used for CIs. Here are commands to estimate bootstrap CIs for the risk ratio by using a logistic model:

```
. use brcadat, clear
(Breast cancer data)
. program stlogit, rclass
            version 10
 1.
 2.
            logistic died low stage2 stage3, nolog
            preserve
 4.
            replace low=0
            predict risk0
 5.
 6.
            summ risk0, meanonly
 7.
            scalar avrisk0 = r(mean)
 8.
            replace low=1
            predict risk1
 9.
10.
            summ risk1, meanonly
            scalar avrisk1 = r(mean)
11.
12
            return scalar lnrr = ln(avrisk1/avrisk0)
13
            restore
14. end
. set seed 93514
```

> (`p0'*exp(_b[low]+invnormal(1-.05/2)*_se[low]))]

. display "Risk ratio = " rr " 95% CI = " lower ", " upper Risk ratio = 1.8933751 95% CI = 1.1180767, 2.7822097

```
. bootstrap lnrr=r(lnrr), saving(bsanrr3a, replace) reps(400001) nowarn nodots:
```

> stlogit

(output omitted)

. estat bootstrap, all eform

Bootstrap results Number of obs = 192Replications = 400001

command: stlogit
lnrr: r(lnrr)

	Observed exp(b)	Bias	Bootstrap Std. Err.	[95% Conf.	Interval]	
lnrr	1.6755989	0049751	.37575458	1.079661	2.600476	(N)
				1.068839	2.587837	(P)
				1.071832	2.594932	(BC)

- (N) normal confidence interval
- (P) percentile confidence interval
- (BC) bias-corrected confidence interval

Stata's bootstrap command simplifies the task of estimating bootstrap CIs by using four methods: 1) normal, 2) percentile, 3) bias corrected, and 4) bias corrected and accelerated. Other methods are available (Carpenter and Bithell 2000). For the risk ratios estimated in this article, the choice among Stata's four methods makes little difference. But for some epidemiologic data, the normal and percentile methods should be used with caution because they may have substantial coverage error (Efron and Tibshirani 1993; Carpenter and Bithell 2000; Greenland 2004b).

11 Risk-ratio methods for matched data

Adjusted risk ratios for matched data can be estimated using conditional Poisson regression, which Stata implements in the **xtpoisson**, **fe** command. I have previously reviewed the analysis of matched cohort data in the *Stata Journal* (Cummings and McKnight 2004) and elsewhere (Cummings, McKnight, and Weiss 2003; Cummings, McKnight, and Greenland 2003).

12 Summary

When the risk-ratio estimates in this article are rounded to one decimal, nearly all the methods produced estimates of 1.6 or 1.7 (table 2). They also differed little with regard to estimated CIs: the 95% lower bound was 1.0 or 1.1 and the upper bound, 2.3 to 2.6.

The risk ratio that stands out as different came from the substitution method: risk ratio = 1.9 and 95% CI is [1.1, 3.1]. The substitution method has nothing to recommend it; it will usually produce estimates biased away from 1 when outcomes are common, and in Stata it offers little advantage in terms of simplicity. Stata users who wish to estimate an adjusted risk ratio have better methods that they can use, all of which are fairly easy to implement.

Table 2. Risk-ratio estimates for death within 5 years among 192 women with breast cancer, comparing women with low estrogen-receptor—level tumors with women with high estrogen-receptor—level tumors, adjusted for cancer stage at diagnosis. Results are shown using the methods described in this article.

Method	Risk ratio	95% CI	Bootstrap 95% CI†	Akaike information criteria‡
1. Stratified methods				
Mantel-Haenszel	1.62	1.09, 2.39	1.07, 2.48	
Inverse-variance weights	1.55	1.08, 2.25		
2. Generalized linear regression				
with log link and				
binomial distribution				
Maximum likelihood	1.56	1.05, 2.32		193.9
Wacholder's truncated				
method	1.56	1.06, 2.29	1.03, 2.44	
Copy method	1.56	1.05, 2.31		
3. Generalized linear regression				
with log link, Gaussian				
distribution, robust variance				
estimator	1.55	1.04, 2.32	1.04, 2.43	196.5
4. Poisson regression with				
robust variance estimator	1.63	1.07, 2.48	1.06, 2.55	226.3
5. Cox proportional hazards				
with robust variance estimator	1.63	1.07, 2.48	1.06, 2.55	547.1
6. Regression-based				
standardized risk ratios				
Logistic	1.68	1.09, 2.57	1.07, 2.59	193.9
Probit	1.68	1.09, 2.57	1.07, 2.59	193.9
Complementary log-log	1.67	1.10, 2.52	1.08, 2.56	193.5
Log-log	1.63	1.05, 2.52	1.03, 2.54	194.6
7. Substitution method	1.89	1.12, 2.78	1.08, 3.11	

[†] Bias-corrected bootstrap CIs based upon 400001 replications. Not estimated for the inverse-variance stratified method because the cs command does not return the pooled risk ratio from this method. Not estimated for the maximum-likelihood version of the generalized linear model with a log link and binomial distribution because convergence failed in many bootstrap samples. Convergence also failed in 100 bootstrap samples (0.025%) using Wacholder's truncated method (binreg), 10 samples (0.0025%) using the generalized linear model with a log link and Gaussian distribution, and 5 samples (0.00125%) using the complementary log-log method.

[‡] Akaike information criteria statistic for models fit using maximum likelihood. The statistic compares the fitted model with a model that has only the outcome variable. In Stata, smaller Akaike information criteria statistics indicate better fit.

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About the author

Peter Cummings is a professor in the Department of Epidemiology in the School of Public Health and a core faculty member at the Harborview Injury Research and Prevention Research Center, University of Washington, Seattle, WA.